

Computer Algebra Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.1-Inverse-sine/266-5.1.4

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3.172	$\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$	1585
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3.198	$\int \frac{x^4 (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$	1877
3.199	$\int \frac{x^3 (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$	1890
3.200	$\int \frac{x^2 (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$	1899
3.201	$\int \frac{x (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$	1910
3.202	$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$	1918
3.203	$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx$	1929

3.204	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^3} dx$	1941
3.205	$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^3} dx$	1958
3.206	$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^3} dx$	1974
3.207	$\int x^3 \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 dx$	1993
3.208	$\int x^2 \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 dx$	2004
3.209	$\int x \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 dx$	2014
3.210	$\int \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 dx$	2022
3.211	$\int \frac{\sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2}{x} dx$	2029
3.212	$\int \frac{\sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2}{x^2} dx$	2038
3.213	$\int \frac{\sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2}{x^3} dx$	2047
3.214	$\int \frac{\sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2}{x^4} dx$	2058
3.215	$\int x^3 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 dx$	2067
3.216	$\int x^2 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 dx$	2082
3.217	$\int x (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 dx$	2096
3.218	$\int (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 dx$	2104
3.219	$\int \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{x} dx$	2113
3.220	$\int \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{x^2} dx$	2125
3.221	$\int \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{x^3} dx$	2138
3.222	$\int \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{x^4} dx$	2151
3.223	$\int x^3 (d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2164
3.224	$\int x^2 (d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2181
3.225	$\int x (d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2199
3.226	$\int (d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2207
3.227	$\int \frac{(d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2}{x} dx$	2218
3.228	$\int \frac{(d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2}{x^2} dx$	2232
3.229	$\int \frac{(d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2}{x^3} dx$	2249
3.230	$\int \frac{(d-c^2dx^2)^{5/2} (a+b \arcsin(cx))^2}{x^4} dx$	2266
3.231	$\int \frac{x^5 (a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2279
3.232	$\int \frac{x^4 (a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2290
3.233	$\int \frac{x^3 (a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2300
3.234	$\int \frac{x^2 (a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2309
3.235	$\int \frac{x (a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2317
3.236	$\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2323

3.237	$\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2 dx^2}} dx$	2328
3.238	$\int \frac{(a+b \arcsin(cx))^2}{x^2\sqrt{d-c^2 dx^2}} dx$	2335
3.239	$\int \frac{(a+b \arcsin(cx))^2}{x^3\sqrt{d-c^2 dx^2}} dx$	2343
3.240	$\int \frac{(a+b \arcsin(cx))^2}{x^4\sqrt{d-c^2 dx^2}} dx$	2353
3.241	$\int \frac{x^5(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2363
3.242	$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2379
3.243	$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2391
3.244	$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2401
3.245	$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2409
3.246	$\int \frac{(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2416
3.247	$\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2 dx^2)^{3/2}} dx$	2423
3.248	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2 dx^2)^{3/2}} dx$	2433
3.249	$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2 dx^2)^{3/2}} dx$	2444
3.250	$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2 dx^2)^{3/2}} dx$	2459
3.251	$\int \frac{x^5(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2471
3.252	$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2485
3.253	$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2496
3.254	$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2506
3.255	$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2515
3.256	$\int \frac{(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2523
3.257	$\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2 dx^2)^{5/2}} dx$	2533
3.258	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2 dx^2)^{5/2}} dx$	2548
3.259	$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2 dx^2)^{5/2}} dx$	2562
3.260	$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2 dx^2)^{5/2}} dx$	2577
3.261	$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2 x^2}} dx$	2593
3.262	$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2 x^2}} dx$	2602
3.263	$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2 x^2}} dx$	2609
3.264	$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2 x^2}} dx$	2615
3.265	$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2 x^2}} dx$	2620
3.266	$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2 x^2}} dx$	2625

3.267	$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx$	2631
3.268	$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx$	2638
3.269	$\int x^m(d-c^2dx^2)^3(a+b\arcsin(cx))^2 dx$	2646
3.270	$\int x^m(d-c^2dx^2)^2(a+b\arcsin(cx))^2 dx$	2662
3.271	$\int x^m(d-c^2dx^2)(a+b\arcsin(cx))^2 dx$	2674
3.272	$\int \frac{x^m(a+b\arcsin(cx))^2}{d-c^2dx^2} dx$	2682
3.273	$\int \frac{x^m(a+b\arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	2687
3.274	$\int \frac{x^m(a+b\arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	2693
3.275	$\int x^m(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 dx$	2700
3.276	$\int x^m(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 dx$	2710
3.277	$\int x^m\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 dx$	2717
3.278	$\int \frac{x^m(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2723
3.279	$\int \frac{x^m(a+b\arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$	2728
3.280	$\int \frac{x^m(a+b\arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2733
3.281	$\int \frac{x^m\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2738
3.282	$\int \frac{x^4\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2743
3.283	$\int \frac{x^3\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2752
3.284	$\int \frac{x^2\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2760
3.285	$\int \frac{x\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2766
3.286	$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2772
3.287	$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx$	2777
3.288	$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx$	2784
3.289	$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx$	2791
3.290	$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$	2801
3.291	$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$	2807
3.292	$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$	2813
3.293	$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$	2819
3.294	$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$	2825
3.295	$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx$	2831
3.296	$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))} dx$	2836
3.297	$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))} dx$	2841
3.298	$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx$	2846

3.299	$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2853
3.300	$\int \frac{x(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2859
3.301	$\int \frac{(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2865
3.302	$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))} dx$	2871
3.303	$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))} dx$	2876
3.304	$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))} dx$	2881
3.305	$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2886
3.306	$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2893
3.307	$\int \frac{x(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2900
3.308	$\int \frac{(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2907
3.309	$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))} dx$	2913
3.310	$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b \arcsin(cx))} dx$	2918
3.311	$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))} dx$	2923
3.312	$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2928
3.313	$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2933
3.314	$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2938
3.315	$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2943
3.316	$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2948
3.317	$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2953
3.318	$\int \frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2957
3.319	$\int \frac{1}{x^2\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2962
3.320	$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2967
3.321	$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2973
3.322	$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2979
3.323	$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2985
3.324	$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2991
3.325	$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2997
3.326	$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	3002
3.327	$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	3007
3.328	$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3012

3.329	$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$	3017
3.330	$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$	3022
3.331	$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$	3027
3.332	$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$	3032
3.333	$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$	3037
3.334	$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$	3042
3.335	$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$	3047
3.336	$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$	3052
3.337	$\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$	3057
3.338	$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arcsin(cx)} dx$	3062
3.339	$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx$	3067
3.340	$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$	3072
3.341	$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$	3077
3.342	$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$	3082
3.343	$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$	3087
3.344	$\int \frac{x^m}{\sqrt{1-a^2x^2}\arcsin(ax)} dx$	3092
3.345	$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx$	3097
3.346	$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx$	3102
3.347	$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$	3107
3.348	$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$	3112
3.349	$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$	3117
3.350	$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))^2} dx$	3122
3.351	$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$	3127
3.352	$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$	3134
3.353	$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$	3144
3.354	$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$	3153
3.355	$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx$	3161
3.356	$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx$	3167
3.357	$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx$	3172
3.358	$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx$	3177
3.359	$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx$	3185

3.360	$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3193
3.361	$\int \frac{(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3202
3.362	$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2} dx$	3209
3.363	$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2} dx$	3215
3.364	$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2} dx$	3220
3.365	$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3225
3.366	$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3233
3.367	$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3241
3.368	$\int \frac{(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3250
3.369	$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2} dx$	3258
3.370	$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b \arcsin(cx))^2} dx$	3264
3.371	$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2} dx$	3269
3.372	$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3274
3.373	$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3281
3.374	$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3288
3.375	$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3294
3.376	$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3302
3.377	$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3309
3.378	$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3314
3.379	$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3319
3.380	$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3324
3.381	$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3329
3.382	$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3334
3.383	$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3339
3.384	$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3344
3.385	$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3349
3.386	$\int \left(-\frac{1}{(1-x^2) \arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2} \arcsin(x)} \right) dx$	3354
3.387	$\int \frac{x^3(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$	3359
3.388	$\int \frac{x^2(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$	3367
3.389	$\int \frac{x(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$	3376

3.390	$\int \frac{d-c^2 dx^2}{(a+b \arcsin(cx))^{3/2}} dx$	3384
3.391	$\int \frac{d-c^2 dx^2}{x(a+b \arcsin(cx))^{3/2}} dx$	3392
3.392	$\int \frac{d-c^2 dx^2}{x^2(a+b \arcsin(cx))^{3/2}} dx$	3398
3.393	$\int \frac{x^3(d-c^2 dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3404
3.394	$\int \frac{x^2(d-c^2 dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3414
3.395	$\int \frac{x(d-c^2 dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3425
3.396	$\int \frac{(d-c^2 dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3434
3.397	$\int \frac{(d-c^2 dx^2)^2}{x(a+b \arcsin(cx))^{3/2}} dx$	3442
3.398	$\int \frac{(d-c^2 dx^2)^2}{x^2(a+b \arcsin(cx))^{3/2}} dx$	3448
3.399	$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx$	3454
3.400	$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx$	3460
3.401	$\int x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^n dx$	3466
3.402	$\int x \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^n dx$	3472
3.403	$\int \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^n dx$	3478
3.404	$\int \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^n}{x} dx$	3484
3.405	$\int \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^n}{x^2} dx$	3489
3.406	$\int x^2 (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^n dx$	3494
3.407	$\int x (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^n dx$	3501
3.408	$\int (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^n dx$	3508
3.409	$\int \frac{(d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^n}{x} dx$	3515
3.410	$\int \frac{(d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^n}{x^2} dx$	3520
3.411	$\int x^2 (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^n dx$	3525
3.412	$\int x (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^n dx$	3533
3.413	$\int (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^n dx$	3541
3.414	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^n}{x} dx$	3548
3.415	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^n}{x^2} dx$	3553
3.416	$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2 x^2}} dx$	3558
3.417	$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2 x^2}} dx$	3563
3.418	$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2 x^2}} dx$	3569
3.419	$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2 x^2}} dx$	3574
3.420	$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2 x^2}} dx$	3579
3.421	$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2 x^2}} dx$	3584

3.422	$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx$	3589
3.423	$\int x^4(d+ex^2)(a+b\arcsin(cx)) dx$	3594
3.424	$\int x^3(d+ex^2)(a+b\arcsin(cx)) dx$	3602
3.425	$\int x^2(d+ex^2)(a+b\arcsin(cx)) dx$	3610
3.426	$\int x(d+ex^2)(a+b\arcsin(cx)) dx$	3618
3.427	$\int (d+ex^2)(a+b\arcsin(cx)) dx$	3625
3.428	$\int \frac{(d+ex^2)(a+b\arcsin(cx))}{x} dx$	3632
3.429	$\int \frac{(d+ex^2)(a+b\arcsin(cx))}{x^2} dx$	3638
3.430	$\int \frac{(d+ex^2)(a+b\arcsin(cx))}{x^3} dx$	3645
3.431	$\int \frac{(d+ex^2)(a+b\arcsin(cx))}{x^4} dx$	3651
3.432	$\int x^4(d+ex^2)^2(a+b\arcsin(cx)) dx$	3661
3.433	$\int x^3(d+ex^2)^2(a+b\arcsin(cx)) dx$	3670
3.434	$\int x^2(d+ex^2)^2(a+b\arcsin(cx)) dx$	3682
3.435	$\int x(d+ex^2)^2(a+b\arcsin(cx)) dx$	3691
3.436	$\int (d+ex^2)^2(a+b\arcsin(cx)) dx$	3701
3.437	$\int \frac{(d+ex^2)^2(a+b\arcsin(cx))}{x} dx$	3709
3.438	$\int \frac{(d+ex^2)^2(a+b\arcsin(cx))}{x^2} dx$	3716
3.439	$\int \frac{(d+ex^2)^2(a+b\arcsin(cx))}{x^3} dx$	3725
3.440	$\int \frac{(d+ex^2)^2(a+b\arcsin(cx))}{x^4} dx$	3732
3.441	$\int x^4(d+ex^2)^3(a+b\arcsin(cx)) dx$	3742
3.442	$\int x^3(d+ex^2)^3(a+b\arcsin(cx)) dx$	3752
3.443	$\int x^2(d+ex^2)^3(a+b\arcsin(cx)) dx$	3764
3.444	$\int x(d+ex^2)^3(a+b\arcsin(cx)) dx$	3773
3.445	$\int (d+ex^2)^3(a+b\arcsin(cx)) dx$	3783
3.446	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x} dx$	3793
3.447	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x^2} dx$	3800
3.448	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x^3} dx$	3809
3.449	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x^4} dx$	3816
3.450	$\int \frac{x^4(a+b\arcsin(cx))}{d+ex^2} dx$	3827
3.451	$\int \frac{x^3(a+b\arcsin(cx))}{d+ex^2} dx$	3836
3.452	$\int \frac{x^2(a+b\arcsin(cx))}{d+ex^2} dx$	3844
3.453	$\int \frac{x(a+b\arcsin(cx))}{d+ex^2} dx$	3851
3.454	$\int \frac{a+b\arcsin(cx)}{d+ex^2} dx$	3858
3.455	$\int \frac{a+b\arcsin(cx)}{x(d+ex^2)} dx$	3865
3.456	$\int \frac{a+b\arcsin(cx)}{x^2(d+ex^2)} dx$	3872

3.457	$\int \frac{a+b \arcsin(cx)}{x^3(d+ex^2)} dx$	3880
3.458	$\int \frac{a+b \arcsin(cx)}{x^4(d+ex^2)} dx$	3889
3.459	$\int \frac{x^3(a+b \arcsin(cx))}{(d+ex^2)^2} dx$	3898
3.460	$\int \frac{x(a+b \arcsin(cx))}{(d+ex^2)^2} dx$	3907
3.461	$\int \frac{a+b \arcsin(cx)}{x(d+ex^2)^2} dx$	3914
3.462	$\int \frac{a+b \arcsin(cx)}{x^3(d+ex^2)^2} dx$	3923
3.463	$\int \frac{x^4(a+b \arcsin(cx))}{(d+ex^2)^2} dx$	3932
3.464	$\int \frac{x^2(a+b \arcsin(cx))}{(d+ex^2)^2} dx$	3941
3.465	$\int \frac{a+b \arcsin(cx)}{(d+ex^2)^2} dx$	3950
3.466	$\int \frac{a+b \arcsin(cx)}{x^2(d+ex^2)^2} dx$	3959
3.467	$\int \frac{x^5(a+b \arcsin(cx))}{(d+ex^2)^3} dx$	3968
3.468	$\int \frac{x^3(a+b \arcsin(cx))}{(d+ex^2)^3} dx$	3977
3.469	$\int \frac{x(a+b \arcsin(cx))}{(d+ex^2)^3} dx$	3985
3.470	$\int \frac{a+b \arcsin(cx)}{x(d+ex^2)^3} dx$	3993
3.471	$\int \frac{a+b \arcsin(cx)}{x^3(d+ex^2)^3} dx$	4002
3.472	$\int \frac{x^4(a+b \arcsin(cx))}{(d+ex^2)^3} dx$	4012
3.473	$\int \frac{x^2(a+b \arcsin(cx))}{(d+ex^2)^3} dx$	4021
3.474	$\int \frac{a+b \arcsin(cx)}{(d+ex^2)^3} dx$	4030
3.475	$\int \frac{x \arcsin(2x)}{1-4x^2} dx$	4039
3.476	$\int \frac{x \arcsin(2x)}{1+4x^2} dx$	4046
3.477	$\int (fx)^m (d+ex^2)^3 (a+b \arcsin(cx)) dx$	4052
3.478	$\int (fx)^m (d+ex^2)^2 (a+b \arcsin(cx)) dx$	4062
3.479	$\int (fx)^m (d+ex^2) (a+b \arcsin(cx)) dx$	4070
3.480	$\int \frac{(fx)^m (a+b \arcsin(cx))}{d+ex^2} dx$	4076
3.481	$\int \frac{(fx)^m (a+b \arcsin(cx))}{(d+ex^2)^2} dx$	4081
3.482	$\int x^2 \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$	4086
3.483	$\int x \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$	4096
3.484	$\int \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$	4104
3.485	$\int \frac{\sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2}{x} dx$	4112
3.486	$\int \frac{\sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2}{x^2} dx$	4122
3.487	$\int x^2 (d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2 dx$	4131
3.488	$\int x (d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2 dx$	4142
3.489	$\int (d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2 dx$	4150

3.490	$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2}{x} dx$	4159
3.491	$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2}{x^2} dx$	4171
3.492	$\int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx$	4183
3.493	$\int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx$	4191
3.494	$\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx$	4197
3.495	$\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}} dx$	4203
3.496	$\int \frac{(a+b \arcsin(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}} dx$	4211
3.497	$\int \frac{x^2(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx$	4219
3.498	$\int \frac{x(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx$	4228
3.499	$\int \frac{(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx$	4235
3.500	$\int \frac{(a+b \arcsin(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}} dx$	4243
3.501	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}} dx$	4253
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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [501]. This is test number [266].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.80 (500)	0.20 (1)
Rubi	99.40 (498)	0.60 (3)
Maple	93.81 (470)	6.19 (31)
Fricas	41.32 (207)	58.68 (294)
Maxima	38.12 (191)	61.88 (310)
Giac	32.14 (161)	67.86 (340)
Sympy	29.34 (147)	70.66 (354)
Reduce	27.94 (140)	72.06 (361)
Mupad	17.96 (90)	82.04 (411)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

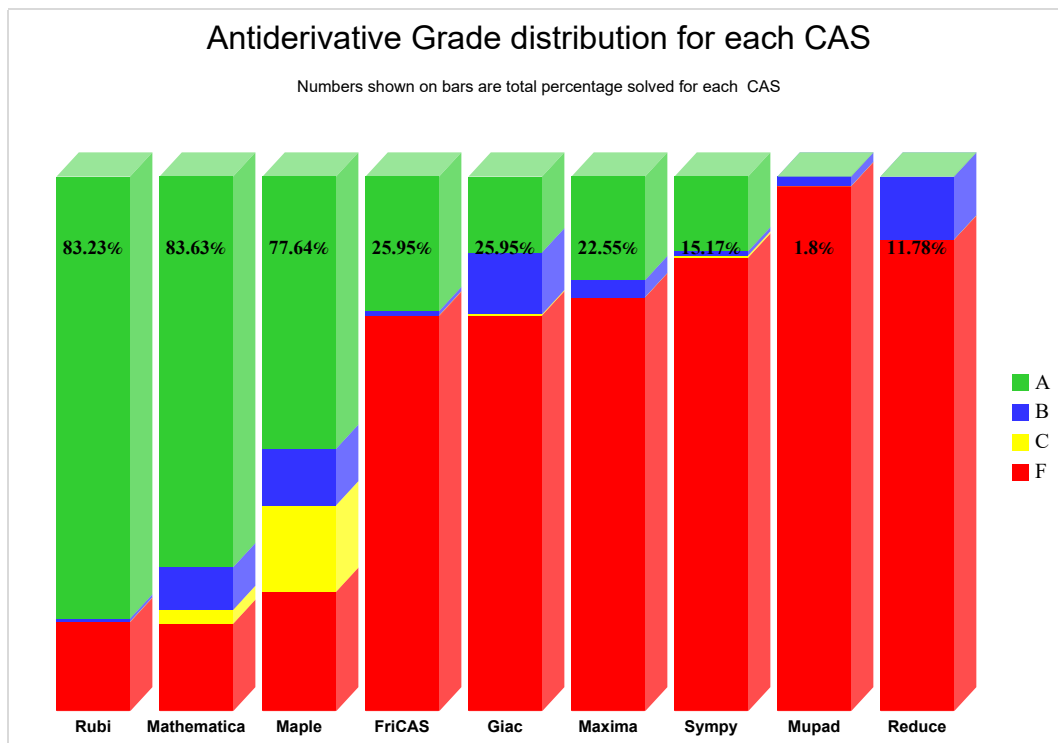
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

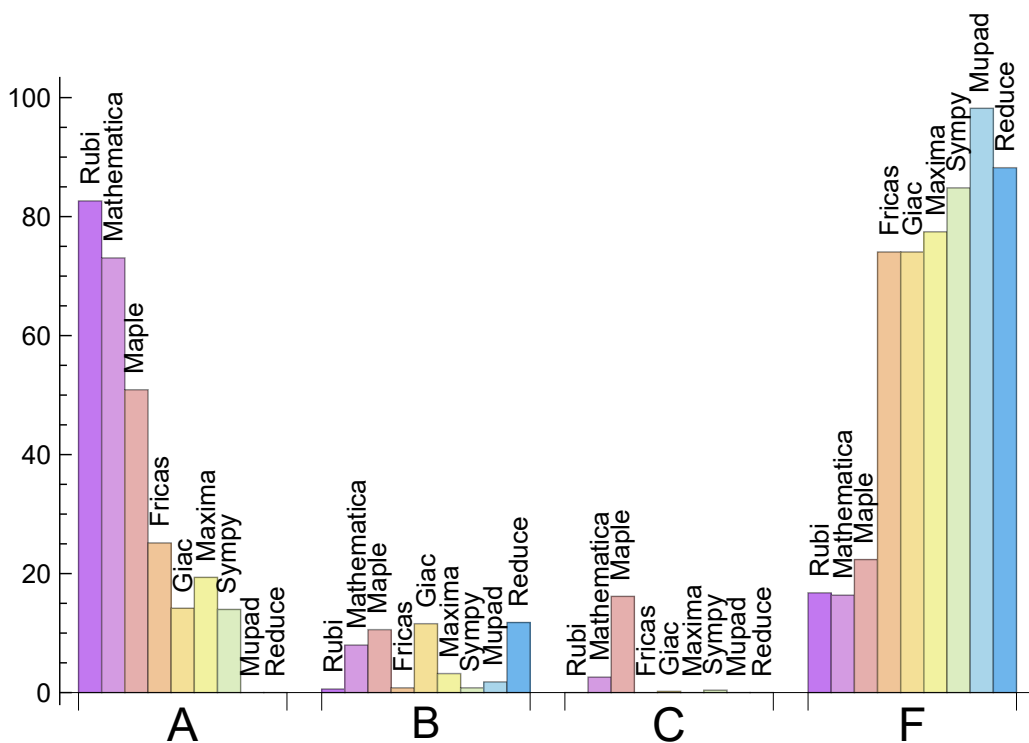
System	% A grade	% B grade	% C grade	% F grade
Rubi	82.635	0.599	0.000	16.766
Mathematica	73.054	7.984	2.595	16.367
Maple	50.898	10.579	16.168	22.355
Fricas	25.150	0.798	0.000	74.052
Maxima	19.361	3.194	0.000	77.445
Giac	14.172	11.577	0.200	74.052
Sympy	13.972	0.798	0.399	84.830
Mupad	0.000	1.796	0.000	98.204
Reduce	0.000	11.776	0.000	88.224

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Mathematica	1	100.00	0.00	0.00
Rubi	3	100.00	0.00	0.00
Maple	31	100.00	0.00	0.00
Fricas	294	95.24	0.00	4.76
Giac	340	20.88	3.53	75.59
Maxima	310	87.42	0.00	12.58
Sympy	354	82.77	16.67	0.56
Reduce	361	100.00	0.00	0.00
Mupad	411	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.12
Mupad	0.22
Reduce	0.22
Maxima	0.32
Rubi	1.05
Mathematica	1.50
Giac	1.97
Maple	4.88
Sympy	5.05

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	27.57	1.01	28.00	1.00
Reduce	133.71	2.79	102.00	1.50
Sympy	145.50	1.38	54.00	1.08
Maxima	156.58	1.75	130.00	1.04
Fricas	166.08	1.63	104.00	1.18
Rubi	218.73	0.98	170.50	1.00
Mathematica	251.57	1.13	180.50	1.01
Maple	485.22	1.95	259.50	1.34
Giac	528.40	3.31	202.00	1.35

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

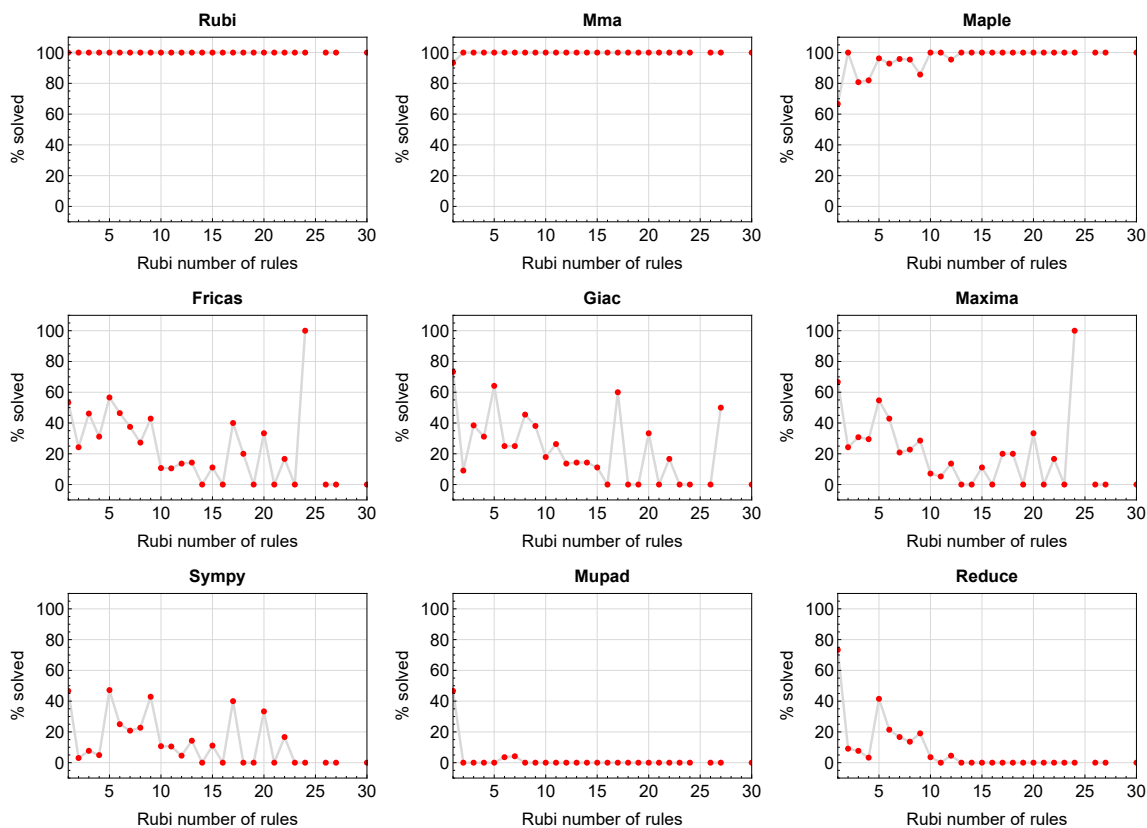


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

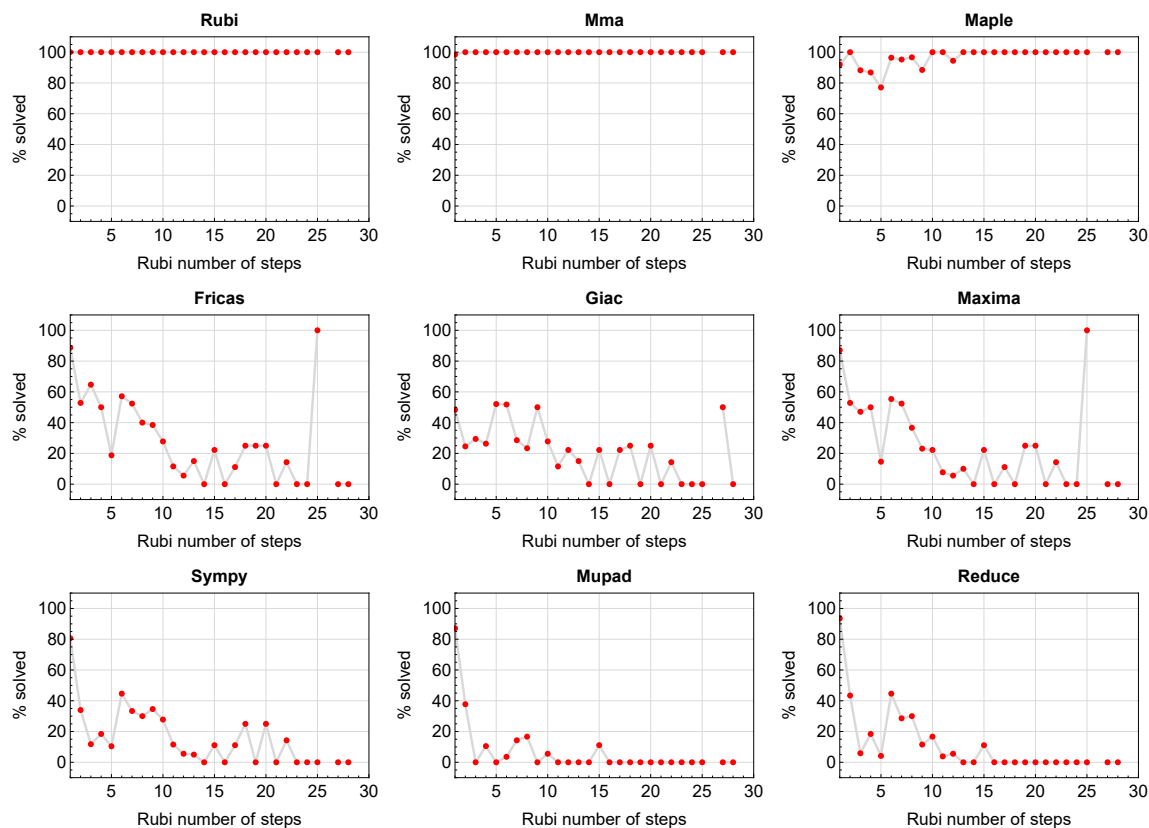


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram shows that the percentage of solved integrals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

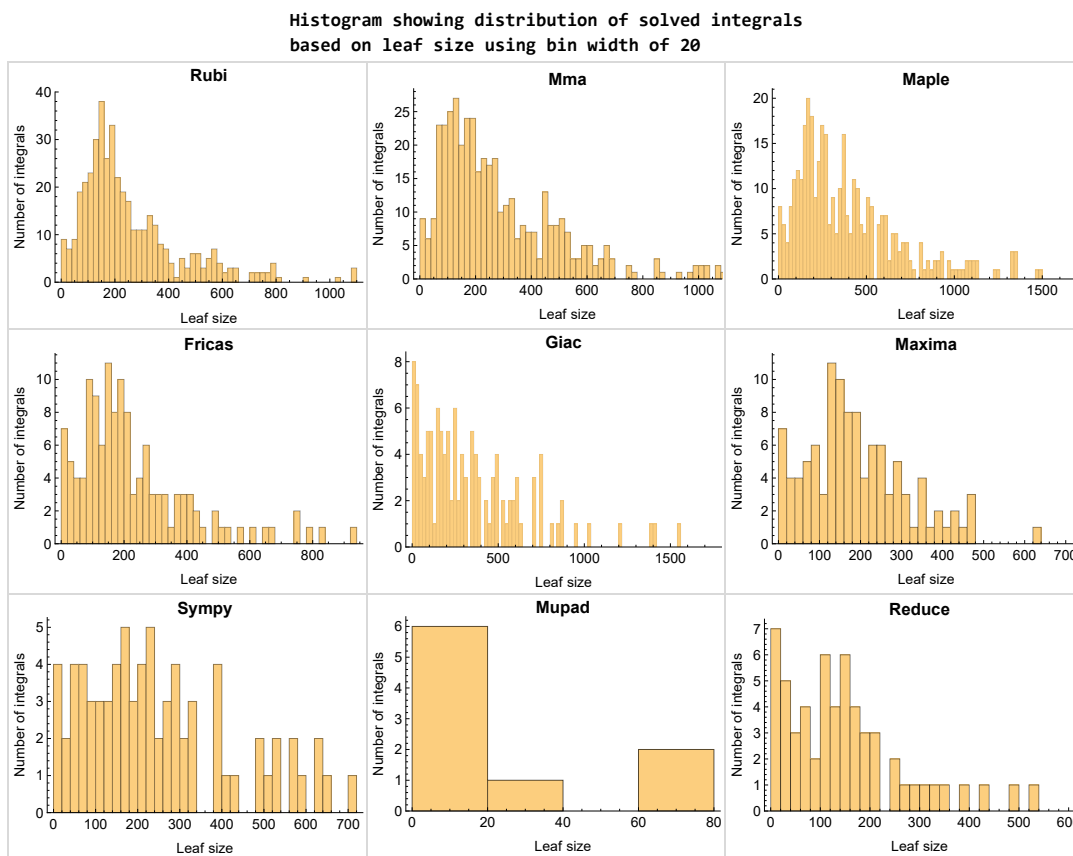


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

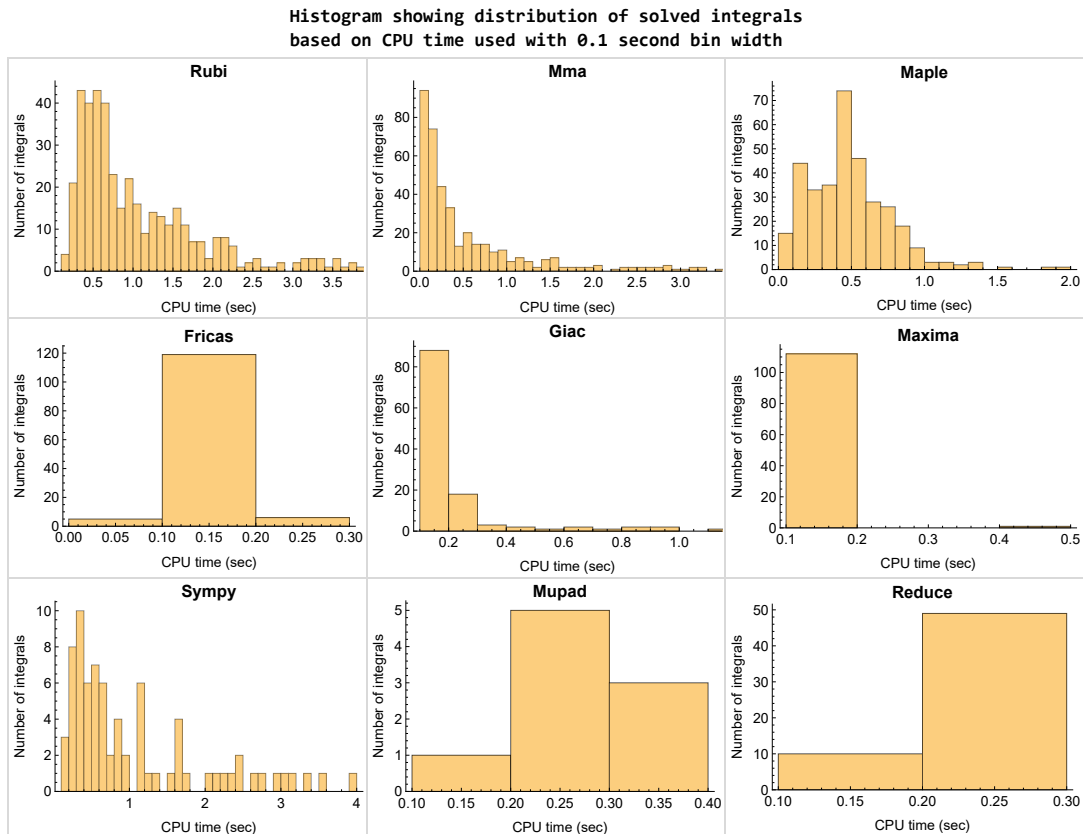


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

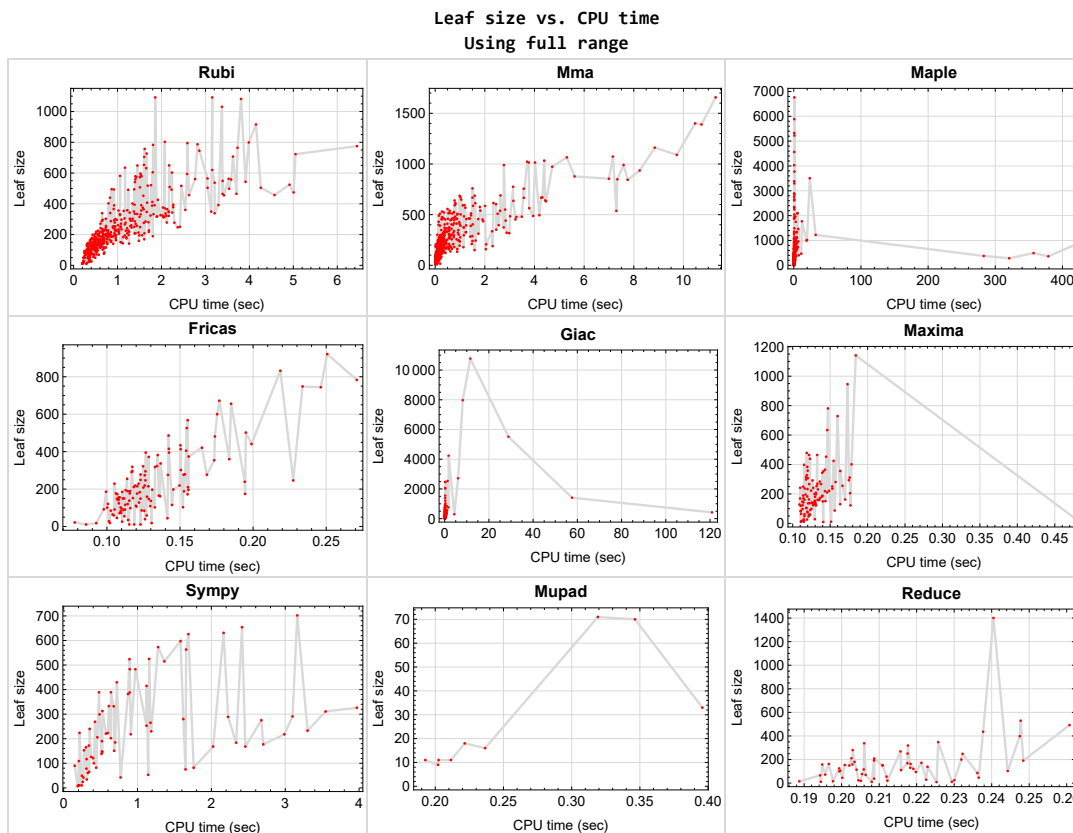


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{143, 144, 145, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 295, 296, 297, 302, 303, 304, 309, 310, 311, 318, 319, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 355, 356, 357, 362, 363, 364, 369, 370, 371, 378, 379, 380, 381, 382, 383, 384, 385, 391, 392, 397, 398, 404, 405, 409, 410, 414, 415, 416, 421, 422, 480, 481}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {16, 18, 27, 229, 438, 440, 449}

Mathematica {195, 197, 204, 206, 249, 257, 269, 270, 271, 467, 470, 471, 472, 473, 474, 501}

Maple {259, 451, 453, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 473, 474, 500}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'beselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
if x is None:
    return 1
else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.15 Current tree layout of integration tests

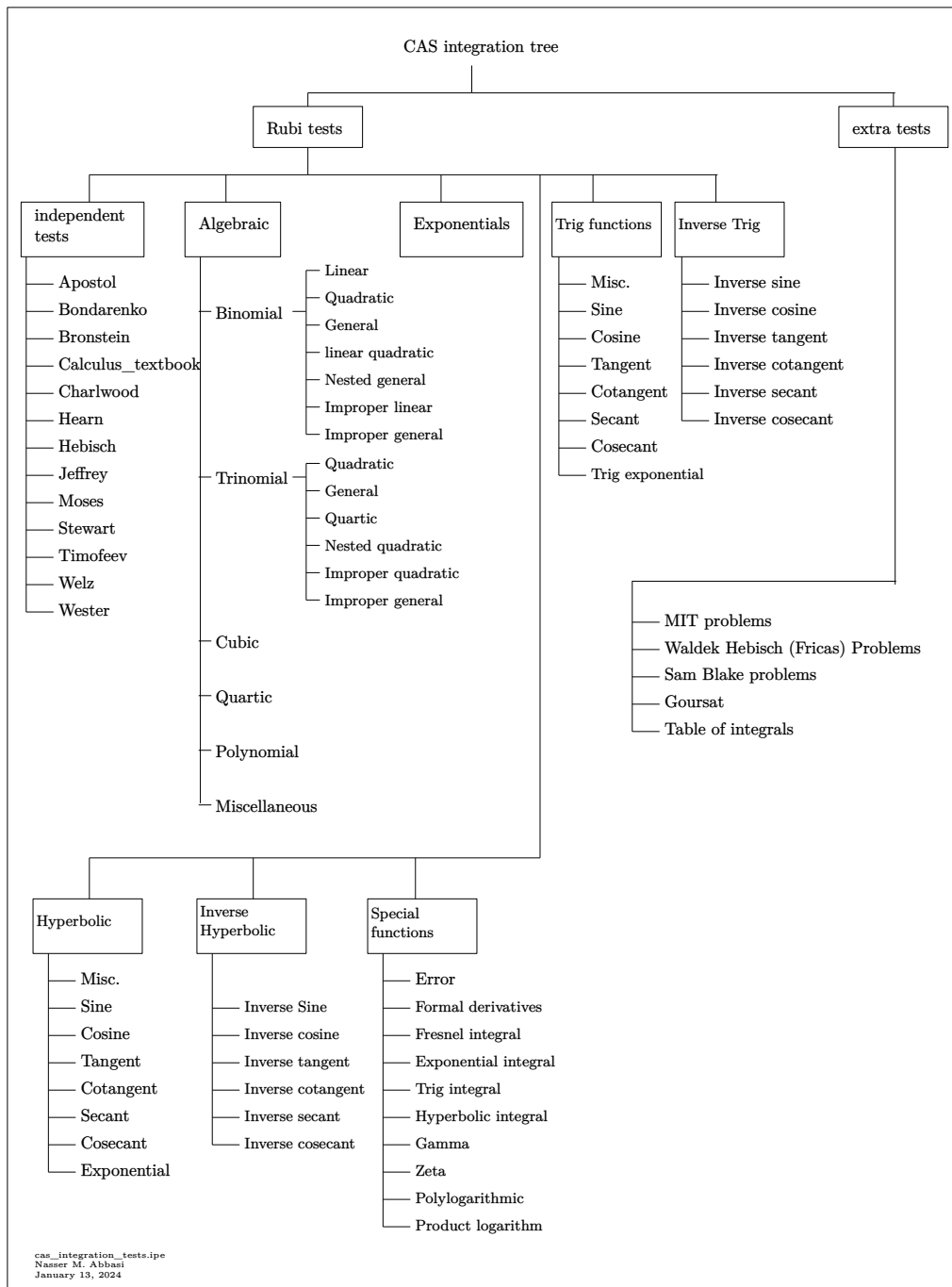
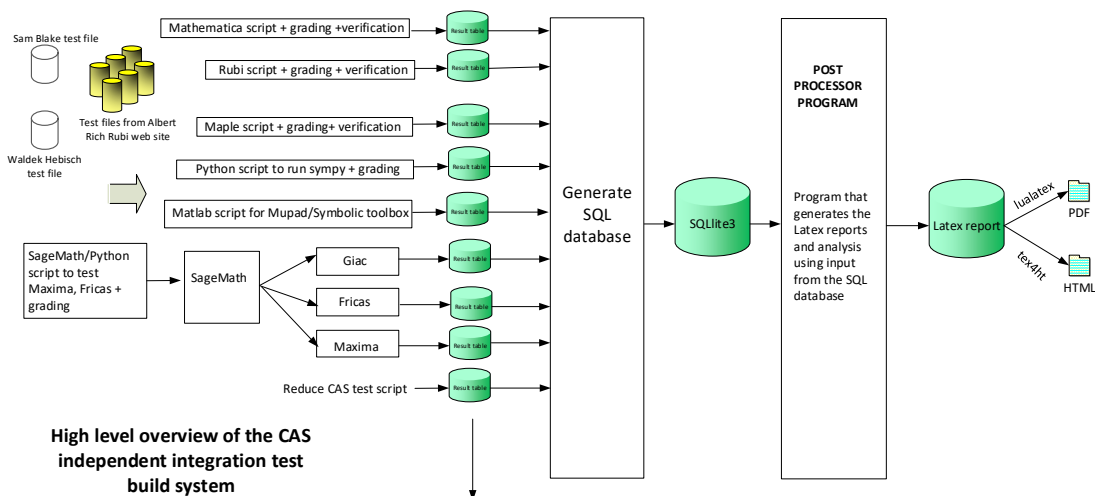


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "E"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in Rubi Table file

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi
January 13, 2024
Design note

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi	39
Mma	40
Maple	41
Fricas	41
Maxima	42
Giac	43
Mupad	44
Sympy	45
Reduce	46

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 317, 320, 321, 322, 323, 324, 325, 351, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 386, 388, 389, 390, 393, 394, 395, 396, 399, 400, 401, 402, 403, 406, 407, 408, 411, 412, 413, 417, 418, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501 }

B grade { 172, 352, 387 }

C grade { }

F normal fail { 178, 230, 259 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 37, 40, 43, 45, 47, 49, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 120, 121, 122, 123, 124, 125, 126, 127, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 184, 191, 192, 193, 198, 199, 200, 201, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 317, 320, 321, 322, 323, 324, 325, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 386, 401, 402, 403, 406, 407, 408, 411, 412, 413, 417, 418, 419, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 496, 498, 500, 501 }

B grade { 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 44, 46, 48, 50, 51, 52, 53, 54, 181, 183, 185, 186, 187, 188, 189, 190, 194, 195, 196, 197, 202, 203, 204, 205, 206, 475, 494, 497, 499 }

C grade { 117, 119, 128, 130, 387, 388, 389, 390, 393, 394, 395, 396, 400 }

F normal fail { 399 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 62, 63, 64, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 109, 112, 113, 115, 123, 125, 134, 136, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 186, 188, 189, 190, 191, 193, 195, 197, 198, 200, 201, 202, 204, 206, 207, 211, 213, 215, 220, 221, 223, 225, 228, 229, 231, 237, 239, 245, 246, 247, 248, 249, 251, 253, 255, 257, 259, 261, 262, 263, 264, 265, 266, 267, 268, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 317, 320, 321, 322, 323, 324, 325, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 387, 388, 389, 390, 393, 394, 395, 396, 400, 420, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 475, 476, 491, 498, 500, 501 }

B grade { 33, 49, 102, 108, 110, 111, 160, 169, 182, 185, 187, 192, 194, 196, 203, 205, 209, 212, 214, 217, 219, 222, 227, 230, 232, 233, 234, 236, 238, 240, 241, 242, 243, 244, 250, 252, 254, 256, 258, 260, 460, 468, 469, 483, 485, 486, 488, 490, 494, 495, 496, 497, 499 }

C grade { 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 76, 84, 85, 86, 87, 88, 89, 90, 91, 92, 114, 116, 117, 118, 119, 120, 121, 122, 124, 126, 127, 128, 129, 130, 131, 132, 133, 135, 137, 199, 208, 210, 216, 218, 224, 226, 235, 399, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 473, 474, 482, 484, 487, 489, 492, 493 }

F normal fail { 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 269, 270, 271, 386, 401, 402, 403, 406, 407, 408, 411, 412, 413, 417, 418, 419, 477, 478, 479 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 60, 61, 62, 63, 64, 73, 74, 75, 76, 77, 78, 79, 80, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 105, 107, 109, 111, 114, 116, 117, 119, 121, 128, 130, 132, 153, 154, 155, 156, 157, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 192, 199, 201, 207, 209, 215, 217, 223, 225, 231, 233, 235, 261, 262, 263, 264, 265, 282, 283, 284, 285, 286, 317, 325, 377, 386, 420, 423, 424, 425, 426, 427, 429, 431, 432, 433, 434, 435, 436, 438, 440, 441, 442, 443, 444, 445, 447, 449, 483, 488, 493 }

B grade { 59, 460, 468, 469 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 104, 106, 108, 110, 112, 113, 115, 118, 120, 122, 123, 124, 125, 126, 127, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 167, 168, 169, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 208, 210, 211, 212, 213, 214, 216, 218, 219, 220, 221, 222, 224, 226, 227, 228, 229, 230, 232, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 266, 267, 268, 269, 270, 271, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 401, 402, 403, 406, 407, 408, 411, 412, 413, 417, 418, 419, 428, 430, 437, 439, 446, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 484, 485, 486, 487, 489, 490, 491, 492, 494, 495, 496, 497, 498, 499, 500, 501 }

F(-1) timedout fail { }

F(-2) exception fail { 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 9, 11, 12, 14, 16, 18, 23, 25, 27, 59, 60, 61, 62, 63, 64, 73, 74, 75, 76, 77, 78, 79, 80, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 105, 107, 109, 111, 112, 114, 116, 119, 122, 124, 130, 131, 133, 137, 153, 155, 207, 209, 215, 217, 223, 225, 231, 233, 235, 236, 262, 264, 265, 283, 285, 286, 317, 325, 377, 420, 423, 424, 425, 426, 427, 429, 431, 432, 433, 434, 435, 436, 438, 440, 441, 442, 443, 444, 445, 447, 449 }

B grade { 10, 13, 19, 20, 21, 22, 40, 157, 162, 164, 166, 171, 173, 175, 192, 386 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 104, 106, 108, 110, 113, 115, 117, 118, 120, 121, 123, 125, 126, 127, 128, 129, 132, 134, 135, 136, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 154, 156, 158, 159, 160, 161, 163, 165, 167, 168, 169, 170, 172, 174, 176, 177, 178, 179,

180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 210, 211, 212, 213, 214, 216, 218, 219, 220, 221, 222, 224, 226, 227, 228, 229, 230, 232, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 266, 267, 268, 269, 270, 271, 282, 284, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 387, 388, 389, 390, 393, 394, 395, 396, 401, 402, 403, 406, 407, 408, 411, 412, 413, 428, 430, 437, 439, 446, 448, 451, 453, 455, 457, 459, 461, 462, 467, 468, 469, 470, 471, 475, 476, 477, 478, 479, 498, 499 }

F(-1) timedout fail { }

F(-2) exception fail { 399, 400, 416, 417, 418, 419, 421, 422, 450, 452, 454, 456, 458, 460, 463, 464, 465, 466, 472, 473, 474, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501 }

Giac

A grade { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 40, 47, 55, 56, 68, 69, 84, 85, 99, 101, 102, 103, 108, 110, 112, 153, 155, 156, 157, 163, 165, 166, 172, 208, 216, 224, 232, 234, 236, 261, 263, 264, 265, 282, 284, 285, 286, 293, 294, 301, 312, 314, 315, 316, 317, 321, 323, 324, 325, 377, 399, 420, 424, 426, 427, 436 }

B grade { 7, 9, 16, 18, 25, 49, 105, 154, 162, 164, 171, 173, 174, 175, 192, 199, 201, 290, 292, 298, 299, 300, 305, 306, 307, 308, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 373, 375, 376, 386, 423, 425, 429, 431, 432, 433, 434, 435, 438, 440, 441, 442, 443, 444, 445, 447, 449 }

C grade { 400 }

F normal fail { 28, 30, 37, 39, 46, 48, 104, 106, 138, 139, 140, 141, 142, 152, 180, 182, 189, 191, 198, 200, 266, 267, 268, 269, 270, 271, 287, 288, 289, 387, 388, 389, 390, 393, 394, 395, 396, 401, 406, 411, 418, 419, 463, 464, 472, 473, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501 }

F(-1) timedout fail { 27, 170, 177, 179, 197, 206, 416, 461, 462, 466, 470, 471 }

F(-2) exception fail { 6, 8, 15, 17, 24, 26, 29, 31, 32, 33, 34, 35, 36, 38, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 107, 109, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 151, 158, 159, 160, 161, 167,

168, 169, 176, 178, 181, 183, 184, 185, 186, 187, 188, 190, 193, 194, 195, 196, 202, 203, 204, 205, 207, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 233, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 291, 295, 297, 302, 304, 309, 311, 313, 320, 322, 326, 329, 331, 334, 336, 338, 339, 340, 345, 346, 347, 351, 355, 357, 362, 364, 369, 371, 372, 374, 378, 380, 382, 384, 391, 397, 402, 403, 404, 405, 407, 408, 409, 410, 412, 413, 414, 415, 417, 428, 430, 437, 439, 446, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 465, 467, 468, 469, 474, 480, 481
}

Mupad

A grade { }

B grade { 7, 103, 265, 286, 317, 325, 377, 420, 429 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 386, 387, 388, 389, 390, 393, 394, 395, 396, 399, 400, 401, 402, 403, 406, 407, 408, 411, 412, 413, 417, 418, 419, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 99, 100, 101, 102, 103, 153, 154, 155, 156, 157, 162, 163, 164, 166, 171, 172, 173, 175, 261, 262, 263, 264, 265, 282, 283, 284, 285, 286, 317, 423, 424, 425, 426, 427, 429, 431, 432, 433, 434, 435, 436, 438, 440, 441, 442, 443, 445, 447, 449 }

B grade { 165, 174, 420, 444 }

C grade { 325, 377 }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 79, 80, 81, 82, 83, 89, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 140, 141, 142, 148, 149, 150, 152, 158, 159, 160, 161, 167, 168, 169, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 266, 267, 268, 269, 270, 271, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 386, 387, 388, 389, 390, 393, 394, 395, 396, 399, 400, 401, 402, 403, 417, 418, 419, 428, 430, 437, 439, 446, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 478, 479, 482, 483, 484, 485, 486, 493, 494, 495, 496, 497, 498, 499, 500, 501 }

F(-1) timedout fail { 50, 51, 68, 69, 74, 75, 76, 77, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 137, 139, 145, 146, 147, 151, 202, 216, 223, 224, 225, 226, 227, 275, 276, 280, 338, 345, 406, 407, 408, 410, 411, 412, 413, 414, 415, 471, 477, 481, 487, 488, 489, 490, 491, 492 }

F(-2) exception fail { 111, 235 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 102, 103, 111, 112, 235, 236, 264, 265, 285, 286, 317, 325, 377, 386, 399, 420, 423, 424, 425, 426, 427, 429, 431, 432, 433, 434, 435, 436, 438, 440, 441, 442, 443, 444, 445, 447, 449, 460 }

C grade { }

F normal fail { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 282, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 300, 301, 305, 306, 307, 308, 312, 313, 314, 315, 316, 320, 321, 322, 323, 324, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 367, 368, 372, 373, 374, 375, 376, 387, 388, 389, 390, 393, 394, 395, 396, 400, 401, 402, 403, 406, 407, 408, 411, 412, 413, 417, 418, 419, 428, 430, 437, 439, 446, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501 }

F(-1) timeout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	126	87	126	189	101	151	195	125	0
N.S.	1	0.98	0.68	0.98	1.48	0.79	1.18	1.52	0.98	0.00
time (sec)	N/A	0.306	0.087	0.186	0.114	0.100	0.684	0.129	0.199	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	135	89	114	169	96	138	144	114	0
N.S.	1	1.10	0.72	0.93	1.37	0.78	1.12	1.17	0.93	0.00
time (sec)	N/A	0.300	0.058	0.137	0.110	0.110	0.513	0.125	0.204	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	85	106	148	91	126	142	105	0
N.S.	1	1.00	0.81	1.01	1.41	0.87	1.20	1.35	1.00	0.00
time (sec)	N/A	0.316	0.062	0.153	0.113	0.105	0.372	0.124	0.200	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	93	77	92	128	86	117	100	94	0
N.S.	1	1.03	0.86	1.02	1.42	0.96	1.30	1.11	1.04	0.00
time (sec)	N/A	0.247	0.056	0.242	0.109	0.124	0.310	0.123	0.220	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	88	80	97	71	90	80	77	0
N.S.	1	1.03	1.14	1.04	1.26	0.92	1.17	1.04	1.00	0.00
time (sec)	N/A	0.265	0.036	0.059	0.113	0.109	0.150	0.123	0.205	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	135	115	157	0	0	0	0	66	0
N.S.	1	1.12	0.95	1.30	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.562	0.090	0.375	0.000	0.000	0.000	0.000	0.188	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	78	64	82	98	82	856	66	71
N.S.	1	1.03	1.13	0.93	1.19	1.42	1.19	12.41	0.96	1.03
time (sec)	N/A	0.286	0.022	0.078	0.111	0.116	1.760	0.938	0.194	0.319

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	138	112	172	0	0	0	0	63	0
N.S.	1	0.99	0.81	1.24	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.590	0.075	0.264	0.000	0.000	0.000	0.000	0.203	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	93	87	123	109	177	296	72	0
N.S.	1	0.99	1.15	1.07	1.52	1.35	2.19	3.65	0.89	0.00
time (sec)	N/A	0.285	0.028	0.098	0.113	0.130	2.702	4.372	0.196	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	174	119	168	328	153	230	284	169	0
N.S.	1	0.94	0.64	0.90	1.76	0.82	1.24	1.53	0.91	0.00
time (sec)	N/A	0.450	0.071	0.174	0.119	0.108	1.185	0.133	0.218	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	191	115	156	298	149	218	205	158	0
N.S.	1	1.04	0.62	0.85	1.62	0.81	1.18	1.11	0.86	0.00
time (sec)	N/A	0.472	0.063	0.152	0.118	0.123	0.909	0.136	0.203	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	153	111	148	267	141	202	227	149	0
N.S.	1	0.95	0.69	0.92	1.66	0.88	1.25	1.41	0.93	0.00
time (sec)	N/A	0.474	0.060	0.159	0.122	0.108	0.639	0.140	0.202	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	121	94	127	237	137	190	157	138	0
N.S.	1	0.98	0.76	1.02	1.91	1.10	1.53	1.27	1.11	0.00
time (sec)	N/A	0.325	0.042	0.146	0.116	0.130	0.518	0.135	0.223	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	127	95	119	196	121	165	158	121	0
N.S.	1	0.97	0.73	0.91	1.50	0.92	1.26	1.21	0.92	0.00
time (sec)	N/A	0.390	0.030	0.096	0.117	0.101	0.302	0.129	0.219	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	226	166	185	0	0	0	0	111	0
N.S.	1	1.23	0.90	1.01	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.980	0.305	0.344	0.000	0.000	0.000	0.000	0.219	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	116	126	115	160	152	184	2717	111	0
N.S.	1	0.94	1.02	0.93	1.30	1.24	1.50	22.09	0.90	0.00
time (sec)	N/A	0.422	0.058	0.115	0.116	0.118	2.335	6.086	0.216	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	234	185	258	0	0	0	0	119	0
N.S.	1	1.16	0.92	1.28	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.950	0.096	0.424	0.000	0.000	0.000	0.000	0.211	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	137	136	113	170	162	233	1409	116	0
N.S.	1	1.07	1.06	0.88	1.33	1.27	1.82	11.01	0.91	0.00
time (sec)	N/A	0.441	0.059	0.118	0.114	0.136	3.303	57.637	0.206	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	216	143	210	479	189	289	353	211	0
N.S.	1	0.93	0.62	0.91	2.06	0.81	1.25	1.52	0.91	0.00
time (sec)	N/A	0.643	0.116	0.187	0.119	0.114	2.226	0.145	0.203	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	199	139	198	439	185	280	250	200	0
N.S.	1	0.97	0.67	0.96	2.13	0.90	1.36	1.21	0.97	0.00
time (sec)	N/A	0.433	0.117	0.164	0.123	0.110	1.620	0.137	0.232	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	195	135	190	398	177	265	296	191	0
N.S.	1	0.94	0.65	0.92	1.92	0.86	1.28	1.43	0.92	0.00
time (sec)	N/A	0.679	0.104	0.168	0.115	0.106	1.176	0.141	0.248	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	110	160	358	173	253	202	180	0
N.S.	1	0.97	0.73	1.07	2.39	1.15	1.69	1.35	1.20	0.00
time (sec)	N/A	0.327	0.049	0.147	0.120	0.107	1.122	0.138	0.203	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	167	119	162	307	157	221	224	163	0
N.S.	1	0.95	0.68	0.93	1.75	0.90	1.26	1.28	0.93	0.00
time (sec)	N/A	0.549	0.061	0.091	0.120	0.109	0.572	0.135	0.197	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	335	207	221	0	0	0	0	153	0
N.S.	1	1.43	0.88	0.94	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	1.393	0.454	0.375	0.000	0.000	0.000	0.000	0.197	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	123	152	250	188	291	5513	154	0
N.S.	1	0.91	0.75	0.93	1.52	1.15	1.77	33.62	0.94	0.00
time (sec)	N/A	0.640	0.141	0.111	0.114	0.153	3.099	28.835	0.201	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	343	226	293	0	0	0	0	162	0
N.S.	1	1.30	0.86	1.11	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.276	0.297	0.492	0.000	0.000	0.000	0.000	0.197	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	160	162	159	242	196	326	0	158	0
N.S.	1	0.90	0.91	0.89	1.36	1.10	1.83	0.00	0.89	0.00
time (sec)	N/A	0.608	0.121	0.115	0.110	0.156	3.970	0.000	0.195	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	186	286	229	0	0	0	0	74	0
N.S.	1	1.08	1.66	1.33	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.853	0.587	0.465	0.000	0.000	0.000	0.000	0.192	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	145	312	160	0	0	0	0	102	0
N.S.	1	1.01	2.17	1.11	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.647	0.337	0.378	0.000	0.000	0.000	0.000	0.190	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	114	238	191	0	0	0	0	82	0
N.S.	1	0.92	1.92	1.54	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.538	0.374	0.345	0.000	0.000	0.000	0.000	0.219	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	78	244	95	0	0	0	0	58	0
N.S.	1	0.95	2.98	1.16	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.415	0.272	0.208	0.000	0.000	0.000	0.000	0.204	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	73	207	140	0	0	0	0	54	0
N.S.	1	0.87	2.46	1.67	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.404	0.027	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	67	274	193	0	0	0	0	58	0
N.S.	1	0.94	3.86	2.72	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.467	0.152	0.320	0.000	0.000	0.000	0.000	0.202	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	110	259	204	0	0	0	0	65	0
N.S.	1	0.95	2.23	1.76	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.685	0.334	0.423	0.000	0.000	0.000	0.000	0.200	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	115	392	244	0	0	0	0	87	0
N.S.	1	0.93	3.16	1.97	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.717	0.198	0.408	0.000	0.000	0.000	0.000	0.226	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	176	350	250	0	0	0	0	85	0
N.S.	1	1.02	2.02	1.45	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.959	0.395	0.497	0.000	0.000	0.000	0.000	0.233	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	199	332	259	0	0	0	0	167	0
N.S.	1	1.06	1.78	1.39	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.938	0.453	0.469	0.000	0.000	0.000	0.000	0.207	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	153	334	161	0	0	0	0	160	0
N.S.	1	0.99	2.15	1.04	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.692	0.501	0.288	0.000	0.000	0.000	0.000	0.201	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	133	463	201	0	0	0	0	156	0
N.S.	1	0.92	3.22	1.40	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.567	0.347	0.350	0.000	0.000	0.000	0.000	0.213	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	98	136	55	0	89	89	0
N.S.	1	1.00	0.88	1.72	2.39	0.96	0.00	1.56	1.56	0.00
time (sec)	N/A	0.242	0.029	0.162	0.120	0.112	0.000	0.137	0.199	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	130	334	201	0	0	0	0	148	0
N.S.	1	0.92	2.37	1.43	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.523	0.364	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	119	364	251	0	0	0	0	164	0
N.S.	1	0.98	2.98	2.06	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.615	0.467	0.405	0.000	0.000	0.000	0.000	0.197	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	201	348	268	0	0	0	0	165	0
N.S.	1	1.16	2.00	1.54	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.841	0.936	0.521	0.000	0.000	0.000	0.000	0.223	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	198	461	278	0	0	0	0	195	0
N.S.	1	1.32	3.07	1.85	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.965	0.688	0.483	0.000	0.000	0.000	0.000	0.217	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	303	426	312	0	0	0	0	184	0
N.S.	1	1.30	1.83	1.34	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	1.279	0.973	0.601	0.000	0.000	0.000	0.000	0.245	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	220	445	262	0	0	0	0	273	0
N.S.	1	1.08	2.18	1.28	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	1.034	0.619	0.443	0.000	0.000	0.000	0.000	0.213	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	99	79	176	0	91	0	124	162	0
N.S.	1	0.99	0.79	1.76	0.00	0.91	0.00	1.24	1.62	0.00
time (sec)	N/A	0.321	0.134	0.143	0.000	0.112	0.000	0.143	0.236	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	188	445	260	0	0	0	0	271	0
N.S.	1	0.93	2.20	1.29	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.845	0.771	0.435	0.000	0.000	0.000	0.000	0.211	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	62	151	0	88	0	172	158	0
N.S.	1	0.98	0.75	1.82	0.00	1.06	0.00	2.07	1.90	0.00
time (sec)	N/A	0.285	0.062	0.133	0.000	0.111	0.000	0.137	0.200	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	182	501	262	0	0	0	0	262	0
N.S.	1	0.93	2.56	1.34	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.786	0.882	0.239	0.000	0.000	0.000	0.000	0.192	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	190	524	325	0	0	0	0	289	0
N.S.	1	1.10	3.03	1.88	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.906	0.759	0.492	0.000	0.000	0.000	0.000	0.209	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	271	512	327	0	0	0	0	285	0
N.S.	1	1.18	2.23	1.42	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	1.080	1.445	0.562	0.000	0.000	0.000	0.000	0.204	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	291	568	385	0	0	0	0	328	0
N.S.	1	1.26	2.46	1.67	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	1.331	1.215	0.588	0.000	0.000	0.000	0.000	0.204	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	391	587	372	0	0	0	0	304	0
N.S.	1	1.34	2.02	1.28	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	1.540	1.523	0.681	0.000	0.000	0.000	0.000	0.230	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	215	169	673	0	0	0	243	98	0
N.S.	1	0.82	0.65	2.57	0.00	0.00	0.00	0.93	0.37	0.00
time (sec)	N/A	0.847	0.076	0.413	0.000	0.000	0.000	0.348	0.217	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	164	140	367	0	0	0	173	77	0
N.S.	1	0.87	0.74	1.94	0.00	0.00	0.00	0.92	0.41	0.00
time (sec)	N/A	0.645	0.058	0.260	0.000	0.000	0.000	0.340	0.222	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	280	0	0	0	0	51	0
N.S.	1	1.00	0.96	2.41	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.397	0.027	0.240	0.000	0.000	0.000	0.000	0.206	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	142	298	0	0	0	0	69	0
N.S.	1	1.00	1.29	2.71	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.438	0.265	0.426	0.000	0.000	0.000	0.000	0.216	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	84	122	158	137	415	0	0	68	0
N.S.	1	0.76	1.10	1.42	1.23	3.74	0.00	0.00	0.61	0.00
time (sec)	N/A	0.358	0.237	0.490	0.122	0.142	0.000	0.000	0.233	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	130	172	1903	140	502	0	0	88	0
N.S.	1	0.70	0.92	10.18	0.75	2.68	0.00	0.00	0.47	0.00
time (sec)	N/A	0.451	0.190	0.530	0.130	0.195	0.000	0.000	0.217	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	174	213	2751	199	568	0	0	109	0
N.S.	1	0.66	0.81	10.46	0.76	2.16	0.00	0.00	0.41	0.00
time (sec)	N/A	0.516	0.220	0.575	0.129	0.155	0.000	0.000	0.214	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	171	157	195	197	177	0	0	109	0
N.S.	1	0.67	0.61	0.76	0.77	0.69	0.00	0.00	0.43	0.00
time (sec)	N/A	0.498	0.100	1.235	0.131	0.116	0.000	0.000	0.212	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	128	134	179	138	150	0	0	89	0
N.S.	1	0.70	0.73	0.98	0.75	0.82	0.00	0.00	0.49	0.00
time (sec)	N/A	0.413	0.058	0.674	0.126	0.111	0.000	0.000	0.206	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	83	70	153	75	116	0	0	66	0
N.S.	1	0.75	0.64	1.39	0.68	1.05	0.00	0.00	0.60	0.00
time (sec)	N/A	0.303	0.054	0.398	0.120	0.113	0.000	0.000	0.218	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	147	187	314	0	0	0	0	53	0
N.S.	1	0.72	0.92	1.55	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.661	0.374	0.431	0.000	0.000	0.000	0.000	0.228	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	163	239	289	0	0	0	0	66	0
N.S.	1	0.72	1.06	1.28	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.692	1.502	0.496	0.000	0.000	0.000	0.000	0.209	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	209	321	349	0	0	0	0	85	0
N.S.	1	0.69	1.07	1.16	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.930	2.880	0.549	0.000	0.000	0.000	0.000	0.215	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	305	193	770	0	0	0	353	146	0
N.S.	1	0.90	0.57	2.26	0.00	0.00	0.00	1.04	0.43	0.00
time (sec)	N/A	1.162	0.115	0.339	0.000	0.000	0.000	0.647	0.205	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	254	170	682	0	0	0	276	126	0
N.S.	1	0.96	0.64	2.57	0.00	0.00	0.00	1.04	0.48	0.00
time (sec)	N/A	0.922	0.098	0.363	0.000	0.000	0.000	0.575	0.207	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	204	210	480	0	0	0	0	101	0
N.S.	1	1.10	1.14	2.59	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.552	0.366	0.302	0.000	0.000	0.000	0.000	0.216	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	197	222	248	0	0	0	0	115	0
N.S.	1	1.06	1.20	1.34	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.705	0.430	0.474	0.000	0.000	0.000	0.000	0.210	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	198	211	276	0	0	0	0	130	0
N.S.	1	1.04	1.10	1.45	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.791	0.560	0.534	0.000	0.000	0.000	0.000	0.218	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	94	156	2350	172	526	0	0	120	0
N.S.	1	0.61	1.01	15.26	1.12	3.42	0.00	0.00	0.78	0.00
time (sec)	N/A	0.396	0.208	0.579	0.137	0.155	0.000	0.000	0.227	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	139	198	3384	151	600	0	0	140	0
N.S.	1	0.60	0.86	14.65	0.65	2.60	0.00	0.00	0.61	0.00
time (sec)	N/A	0.463	0.244	0.631	0.144	0.175	0.000	0.000	0.276	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	187	238	4563	210	672	0	0	160	0
N.S.	1	0.61	0.77	14.81	0.68	2.18	0.00	0.00	0.52	0.00
time (sec)	N/A	0.612	0.273	0.711	0.146	0.177	0.000	0.000	0.222	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	227	278	5886	269	744	0	0	180	0
N.S.	1	0.59	0.72	15.29	0.70	1.93	0.00	0.00	0.47	0.00
time (sec)	N/A	0.790	0.309	0.807	0.133	0.246	0.000	0.000	0.219	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	225	174	255	267	249	0	0	177	0
N.S.	1	0.60	0.46	0.68	0.71	0.66	0.00	0.00	0.47	0.00
time (sec)	N/A	0.661	0.117	1.133	0.138	0.123	0.000	0.000	0.237	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	182	150	239	208	219	0	0	157	0
N.S.	1	0.60	0.50	0.79	0.69	0.73	0.00	0.00	0.52	0.00
time (sec)	N/A	0.504	0.101	0.625	0.131	0.126	0.000	0.000	0.228	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	139	126	223	149	189	0	0	137	0
N.S.	1	0.61	0.56	0.98	0.66	0.83	0.00	0.00	0.60	0.00
time (sec)	N/A	0.421	0.084	0.493	0.140	0.116	0.000	0.000	0.232	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	94	84	198	87	159	0	0	115	0
N.S.	1	0.61	0.55	1.29	0.57	1.04	0.00	0.00	0.75	0.00
time (sec)	N/A	0.305	0.039	0.259	0.156	0.127	0.000	0.000	0.217	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	225	278	525	0	0	0	0	103	0
N.S.	1	0.80	0.99	1.88	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.947	0.747	0.487	0.000	0.000	0.000	0.000	0.226	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	234	389	437	0	0	0	0	126	0
N.S.	1	0.79	1.31	1.47	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.023	1.436	0.494	0.000	0.000	0.000	0.000	0.240	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	253	494	371	0	0	0	0	117	0
N.S.	1	0.82	1.61	1.21	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.068	4.196	0.562	0.000	0.000	0.000	0.000	0.216	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	407	220	1106	0	0	0	440	195	0
N.S.	1	0.95	0.51	2.57	0.00	0.00	0.00	1.02	0.45	0.00
time (sec)	N/A	1.674	0.147	0.518	0.000	0.000	0.000	0.946	0.233	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	356	196	907	0	0	0	361	175	0
N.S.	1	1.01	0.56	2.58	0.00	0.00	0.00	1.03	0.50	0.00
time (sec)	N/A	1.540	0.131	0.422	0.000	0.000	0.000	0.835	0.232	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	278	266	691	0	0	0	0	150	0
N.S.	1	1.06	1.02	2.64	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.936	0.343	0.367	0.000	0.000	0.000	0.000	0.219	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	300	257	306	0	0	0	0	165	0
N.S.	1	0.98	0.84	1.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.948	0.672	0.596	0.000	0.000	0.000	0.000	0.226	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	296	243	343	0	0	0	0	179	0
N.S.	1	1.07	0.88	1.24	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.982	0.966	0.572	0.000	0.000	0.000	0.000	0.241	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	297	234	2615	0	0	0	0	182	0
N.S.	1	1.07	0.84	9.44	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	1.089	0.997	0.605	0.000	0.000	0.000	0.000	0.216	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	107	190	4031	205	656	0	0	171	0
N.S.	1	0.53	0.94	19.86	1.01	3.23	0.00	0.00	0.84	0.00
time (sec)	N/A	0.378	0.277	0.674	0.177	0.185	0.000	0.000	0.234	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	168	232	5324	162	748	0	0	191	0
N.S.	1	0.60	0.82	18.88	0.57	2.65	0.00	0.00	0.68	0.00
time (sec)	N/A	0.446	0.283	0.737	0.170	0.234	0.000	0.000	0.217	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	199	272	6761	221	832	0	0	212	0
N.S.	1	0.55	0.75	18.73	0.61	2.30	0.00	0.00	0.59	0.00
time (sec)	N/A	0.546	0.311	0.863	0.149	0.219	0.000	0.000	0.217	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	194	160	255	219	291	0	0	206	0
N.S.	1	0.55	0.45	0.72	0.62	0.82	0.00	0.00	0.58	0.00
time (sec)	N/A	0.560	0.125	0.745	0.143	0.117	0.000	0.000	0.248	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	149	137	239	160	255	0	0	186	0
N.S.	1	0.54	0.49	0.86	0.58	0.92	0.00	0.00	0.67	0.00
time (sec)	N/A	0.469	0.108	0.354	0.133	0.114	0.000	0.000	0.245	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	104	93	214	98	215	0	0	163	0
N.S.	1	0.51	0.46	1.06	0.49	1.06	0.00	0.00	0.81	0.00
time (sec)	N/A	0.329	0.048	0.483	0.125	0.128	0.000	0.000	0.246	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	315	394	652	0	0	0	0	152	0
N.S.	1	0.86	1.08	1.79	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.501	1.248	0.547	0.000	0.000	0.000	0.000	0.212	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	324	484	659	0	0	0	0	176	0
N.S.	1	0.84	1.25	1.71	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.751	3.020	0.533	0.000	0.000	0.000	0.000	0.229	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	332	640	529	0	0	0	0	178	0
N.S.	1	0.85	1.65	1.36	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.819	4.430	0.561	0.000	0.000	0.000	0.000	0.225	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	64	76	85	60	82	91	23	0
N.S.	1	1.09	0.73	0.86	0.97	0.68	0.93	1.03	0.26	0.00
time (sec)	N/A	0.481	0.021	0.385	0.111	0.105	0.440	0.130	0.206	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	75	49	95	61	44	65	0	23	0
N.S.	1	1.04	0.68	1.32	0.85	0.61	0.90	0.00	0.32	0.00
time (sec)	N/A	0.362	0.017	0.224	0.122	0.141	0.344	0.000	0.194	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	40	56	39	42	53	23	0
N.S.	1	1.00	0.86	0.80	1.12	0.78	0.84	1.06	0.46	0.00
time (sec)	N/A	0.318	0.009	0.209	0.122	0.126	0.268	0.139	0.207	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	62	27	26	24	27	25	0
N.S.	1	1.00	1.00	2.14	0.93	0.90	0.83	0.93	0.86	0.00
time (sec)	N/A	0.223	0.007	0.189	0.115	0.110	0.258	0.121	0.230	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.85
time (sec)	N/A	0.197	0.004	0.081	0.111	0.115	0.213	0.111	0.208	0.211

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	71	103	0	0	0	0	23	0
N.S.	1	1.00	1.37	1.98	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.372	0.085	0.240	0.000	0.000	0.000	0.000	0.198	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	32	26	28	0	67	23	0
N.S.	1	1.00	1.00	1.14	0.93	1.00	0.00	2.39	0.82	0.00
time (sec)	N/A	0.237	0.016	0.268	0.119	0.101	0.000	0.127	0.193	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	93	137	171	0	0	0	0	23	0
N.S.	1	0.95	1.40	1.74	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.522	0.613	0.740	0.000	0.000	0.000	0.000	0.214	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	235	119	177	180	150	0	0	93	0
N.S.	1	1.05	0.53	0.79	0.80	0.67	0.00	0.00	0.42	0.00
time (sec)	N/A	0.658	0.043	0.786	0.130	0.112	0.000	0.000	0.216	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	208	161	377	0	0	0	141	82	0
N.S.	1	1.04	0.80	1.88	0.00	0.00	0.00	0.70	0.41	0.00
time (sec)	N/A	0.662	0.891	0.323	0.000	0.000	0.000	0.441	0.213	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	151	92	160	121	120	0	0	73	0
N.S.	1	1.02	0.62	1.08	0.82	0.81	0.00	0.00	0.49	0.00
time (sec)	N/A	0.452	0.034	0.473	0.131	0.114	0.000	0.000	0.219	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	134	268	0	0	0	88	61	0
N.S.	1	1.00	1.08	2.16	0.00	0.00	0.00	0.71	0.49	0.00
time (sec)	N/A	0.401	0.887	0.268	0.000	0.000	0.000	0.406	0.215	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-2)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	132	58	92	0	0	46	0
N.S.	1	1.00	0.96	1.97	0.87	1.37	0.00	0.00	0.69	0.00
time (sec)	N/A	0.263	0.020	0.199	0.123	0.098	0.000	0.000	0.237	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	86	28	0	0	24	24	0
N.S.	1	1.00	1.02	1.76	0.57	0.00	0.00	0.49	0.49	0.00
time (sec)	N/A	0.222	0.012	0.127	0.129	0.000	0.000	0.208	0.204	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	88	146	180	0	0	0	0	41	0
N.S.	1	0.61	1.01	1.24	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.510	0.249	0.419	0.000	0.000	0.000	0.000	0.201	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	81	204	104	219	0	0	49	0
N.S.	1	1.00	1.23	3.09	1.58	3.32	0.00	0.00	0.74	0.00
time (sec)	N/A	0.363	0.162	0.445	0.118	0.150	0.000	0.000	0.231	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	166	244	277	0	0	0	0	69	0
N.S.	1	0.72	1.07	1.21	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.819	1.632	0.490	0.000	0.000	0.000	0.000	0.200	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	145	135	850	124	434	0	0	73	0
N.S.	1	0.99	0.92	5.78	0.84	2.95	0.00	0.00	0.50	0.00
time (sec)	N/A	0.553	0.180	0.492	0.127	0.150	0.000	0.000	0.209	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	158	166	425	0	441	0	0	108	0
N.S.	1	0.71	0.75	1.92	0.00	2.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.568	0.205	0.562	0.000	0.199	0.000	0.000	0.232	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	231	173	432	0	0	0	0	189	0
N.S.	1	1.08	0.81	2.02	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.937	0.445	0.480	0.000	0.000	0.000	0.000	0.213	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	109	136	307	142	382	0	0	98	0
N.S.	1	0.77	0.96	2.16	1.00	2.69	0.00	0.00	0.69	0.00
time (sec)	N/A	0.436	0.156	0.457	0.125	0.135	0.000	0.000	0.223	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	160	274	0	0	0	0	125	0
N.S.	1	1.00	1.19	2.03	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.437	0.183	0.409	0.000	0.000	0.000	0.000	0.217	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	194	0	279	0	0	85	0
N.S.	1	1.00	0.70	2.66	0.00	3.82	0.00	0.00	1.16	0.00
time (sec)	N/A	0.289	0.015	0.378	0.000	0.153	0.000	0.000	0.217	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	177	62	0	0	0	80	0
N.S.	1	1.00	0.96	2.21	0.78	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.250	0.052	0.334	0.116	0.000	0.000	0.000	0.204	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	158	300	224	0	0	0	0	114	0
N.S.	1	0.71	1.34	1.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.698	0.666	0.520	0.000	0.000	0.000	0.000	0.213	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	118	171	229	129	0	0	0	96	0
N.S.	1	0.80	1.16	1.55	0.87	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.403	0.377	0.474	0.130	0.000	0.000	0.000	0.214	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	249	404	425	0	0	0	0	147	0
N.S.	1	0.79	1.28	1.34	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.068	1.516	0.594	0.000	0.000	0.000	0.000	0.226	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	173	224	1048	0	0	0	0	108	0
N.S.	1	0.74	0.96	4.48	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.518	0.443	0.568	0.000	0.000	0.000	0.000	0.243	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	358	253	426	0	0	0	0	261	0
N.S.	1	1.22	0.86	1.45	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	1.223	0.485	0.677	0.000	0.000	0.000	0.000	0.208	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	171	169	414	0	481	0	0	217	0
N.S.	1	0.78	0.77	1.89	0.00	2.20	0.00	0.00	0.99	0.00
time (sec)	N/A	0.477	0.221	0.554	0.000	0.174	0.000	0.000	0.225	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	252	213	359	0	0	0	0	252	0
N.S.	1	1.19	1.00	1.69	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.892	0.345	0.546	0.000	0.000	0.000	0.000	0.210	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	134	143	252	160	421	0	0	208	0
N.S.	1	0.89	0.95	1.68	1.07	2.81	0.00	0.00	1.39	0.00
time (sec)	N/A	0.465	0.166	0.499	0.135	0.165	0.000	0.000	0.215	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	104	103	1242	153	0	0	0	196	0
N.S.	1	0.83	0.82	9.94	1.22	0.00	0.00	0.00	1.57	0.00
time (sec)	N/A	0.431	0.125	0.521	0.130	0.000	0.000	0.000	0.211	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	103	85	223	0	374	0	0	195	0
N.S.	1	0.87	0.71	1.87	0.00	3.14	0.00	0.00	1.64	0.00
time (sec)	N/A	0.354	0.031	0.460	0.000	0.156	0.000	0.000	0.222	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	113	1072	141	0	0	0	197	0
N.S.	1	1.03	0.73	6.96	0.92	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.491	0.074	0.403	0.128	0.000	0.000	0.000	0.235	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	260	456	512	0	0	0	0	284	0
N.S.	1	0.88	1.55	1.74	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	1.313	1.256	0.632	0.000	0.000	0.000	0.000	0.215	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	169	238	1346	0	0	0	0	217	0
N.S.	1	0.75	1.06	6.01	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.569	0.435	0.570	0.000	0.000	0.000	0.000	0.212	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	375	537	605	0	0	0	0	315	0
N.S.	1	0.96	1.37	1.54	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	1.574	7.299	0.684	0.000	0.000	0.000	0.000	0.213	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	222	277	1877	255	0	0	0	226	0
N.S.	1	0.72	0.90	6.09	0.83	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.684	0.321	0.583	0.166	0.000	0.000	0.000	0.229	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	0	0	0	0	0	120	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	0.290	0.031	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	97	0	0	0	0	0	125	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.365	0.029	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	324	256	0	0	0	0	0	613	0
N.S.	1	1.03	0.81	0.00	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	2.019	0.292	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	187	0	0	0	0	0	346	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.595	0.012	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	127	118	0	0	0	0	0	155	0
N.S.	1	0.98	0.91	0.00	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.370	0.033	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	30	29	37	0	49	27
N.S.	1	1.00	1.08	1.00	1.20	1.16	1.48	0.00	1.96	1.08
time (sec)	N/A	0.295	3.159	1.085	0.425	0.111	2.828	0.000	0.209	0.377

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	28	41	54	0	63	27
N.S.	1	1.00	1.08	1.00	1.12	1.64	2.16	0.00	2.52	1.08
time (sec)	N/A	0.513	4.744	1.194	0.472	0.122	16.829	0.000	0.200	0.392

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	30	55	0	0	81	27
N.S.	1	1.00	1.08	1.00	1.20	2.20	0.00	0.00	3.24	1.08
time (sec)	N/A	0.758	5.439	0.864	0.433	0.116	0.000	0.000	0.205	0.397

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	452	338	0	0	0	0	0	159	0
N.S.	1	0.71	0.53	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.587	0.760	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	399	327	237	0	0	0	0	0	103	0
N.S.	1	0.82	0.59	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.060	0.335	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	218	181	0	0	0	0	0	46	0
N.S.	1	0.89	0.74	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.513	0.048	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	129	0	0	0	0	0	52	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.374	0.041	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	271	207	0	0	0	0	0	99	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.613	0.153	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	408	390	279	0	0	0	0	0	131	0
N.S.	1	0.96	0.68	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.923	0.229	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.293	0.023	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	331	203	275	453	229	388	495	175	0
N.S.	1	1.14	0.70	0.95	1.56	0.79	1.34	1.71	0.60	0.00
time (sec)	N/A	1.525	0.160	0.344	0.145	0.105	0.893	0.154	0.232	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	334	192	330	0	211	332	377	164	0
N.S.	1	1.65	0.95	1.63	0.00	1.04	1.64	1.87	0.81	0.00
time (sec)	N/A	1.445	0.096	0.441	0.000	0.118	0.677	0.146	0.236	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	241	179	279	354	194	313	356	154	0
N.S.	1	1.14	0.85	1.32	1.68	0.92	1.48	1.69	0.73	0.00
time (sec)	N/A	1.217	0.135	0.276	0.141	0.113	0.525	0.156	0.217	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	153	165	192	0	176	269	248	184	0
N.S.	1	1.04	1.12	1.31	0.00	1.20	1.83	1.69	1.25	0.00
time (sec)	N/A	0.756	0.149	0.264	0.000	0.121	0.425	0.147	0.221	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	140	137	173	233	146	224	196	145	0
N.S.	1	1.09	1.07	1.35	1.82	1.14	1.75	1.53	1.13	0.00
time (sec)	N/A	0.672	0.084	0.000	0.120	0.107	0.214	0.143	0.226	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	195	256	368	0	0	0	0	151	0
N.S.	1	1.10	1.44	2.07	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	1.495	0.372	0.496	0.000	0.000	0.000	0.000	0.211	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	175	203	250	0	0	0	0	142	0
N.S.	1	1.17	1.36	1.68	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	1.202	0.321	0.430	0.000	0.000	0.000	0.000	0.205	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	195	236	458	0	0	0	0	113	0
N.S.	1	1.01	1.22	2.37	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	1.558	0.302	0.455	0.000	0.000	0.000	0.000	0.217	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	226	266	261	0	0	0	0	123	0
N.S.	1	1.28	1.51	1.48	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	1.242	0.565	0.422	0.000	0.000	0.000	0.000	0.226	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	517	253	530	781	337	563	702	243	0
N.S.	1	1.31	0.64	1.34	1.98	0.85	1.43	1.78	0.62	0.00
time (sec)	N/A	2.451	0.136	0.546	0.147	0.137	1.658	0.162	0.212	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	594	239	423	0	319	515	522	232	0
N.S.	1	1.97	0.79	1.40	0.00	1.06	1.71	1.73	0.77	0.00
time (sec)	N/A	2.582	0.141	0.473	0.000	0.117	1.363	0.168	0.255	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	388	229	399	634	296	483	553	222	0
N.S.	1	1.25	0.74	1.29	2.05	0.95	1.56	1.78	0.72	0.00
time (sec)	N/A	1.788	0.121	0.293	0.146	0.117	0.971	0.163	0.236	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	209	218	270	0	278	430	383	252	0
N.S.	1	0.96	1.00	1.24	0.00	1.28	1.97	1.76	1.16	0.00
time (sec)	N/A	0.792	0.232	0.283	0.000	0.123	0.720	0.154	0.240	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	244	193	275	465	247	389	374	213	0
N.S.	1	1.11	0.88	1.26	2.12	1.13	1.78	1.71	0.97	0.00
time (sec)	N/A	0.897	0.078	0.000	0.133	0.126	0.478	0.146	0.207	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	346	371	428	0	0	0	0	221	0
N.S.	1	1.25	1.34	1.55	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	2.164	0.379	0.505	0.000	0.000	0.000	0.000	0.216	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	313	322	372	0	0	0	0	213	0
N.S.	1	1.26	1.29	1.49	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	2.047	0.733	0.537	0.000	0.000	0.000	0.000	0.211	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	343	361	640	0	0	0	0	246	0
N.S.	1	1.20	1.26	2.23	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	2.244	0.659	0.665	0.000	0.000	0.000	0.000	0.217	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	365	374	378	0	0	0	0	225	0
N.S.	1	1.36	1.40	1.41	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	2.128	0.568	0.563	0.000	0.000	0.000	0.000	0.207	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	707	301	618	1141	413	702	865	309	0
N.S.	1	1.49	0.63	1.30	2.40	0.87	1.47	1.82	0.65	0.00
time (sec)	N/A	3.627	0.242	0.570	0.184	0.150	3.163	0.179	0.234	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	916	287	518	0	395	654	631	298	0
N.S.	1	2.39	0.75	1.35	0.00	1.03	1.70	1.64	0.78	0.00
time (sec)	N/A	4.151	0.248	0.498	0.000	0.127	2.415	0.180	0.273	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	537	277	524	946	372	626	716	288	0
N.S.	1	1.37	0.71	1.34	2.42	0.95	1.60	1.83	0.74	0.00
time (sec)	N/A	3.209	0.212	0.327	0.173	0.129	1.689	0.169	0.221	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	261	257	336	0	354	573	492	318	0
N.S.	1	0.94	0.93	1.21	0.00	1.28	2.07	1.78	1.15	0.00
time (sec)	N/A	1.013	0.276	0.301	0.000	0.173	1.279	0.168	0.235	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	350	241	384	729	323	524	528	279	0
N.S.	1	1.17	0.81	1.29	2.45	1.08	1.76	1.77	0.94	0.00
time (sec)	N/A	1.372	0.147	0.209	0.160	0.134	0.891	0.165	0.211	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	543	466	498	0	0	0	0	287	0
N.S.	1	1.51	1.29	1.38	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	3.913	0.735	0.588	0.000	0.000	0.000	0.000	0.226	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	465	483	416	0	0	0	0	280	0
N.S.	1	1.41	1.47	1.26	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	3.705	1.162	0.577	0.000	0.000	0.000	0.000	0.224	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	0	556	708	0	0	0	0	315	0
N.S.	1	0.00	1.40	1.79	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.000	0.499	0.887	0.000	0.000	0.000	0.000	0.216	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	549	480	488	0	0	0	0	294	0
N.S.	1	1.58	1.38	1.40	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	3.392	0.801	0.786	0.000	0.000	0.000	0.000	0.222	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	316	508	570	0	0	0	0	114	0
N.S.	1	1.06	1.71	1.92	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.890	0.902	1.197	0.000	0.000	0.000	0.000	0.207	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	208	459	371	0	0	0	0	206	0
N.S.	1	0.99	2.19	1.77	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	1.611	0.517	0.566	0.000	0.000	0.000	0.000	0.246	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	198	412	507	0	0	0	0	157	0
N.S.	1	0.91	1.89	2.33	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.255	0.863	0.795	0.000	0.000	0.000	0.000	0.219	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	114	342	206	0	0	0	0	92	0
N.S.	1	0.97	2.92	1.76	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.605	0.569	0.340	0.000	0.000	0.000	0.000	0.196	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	135	302	375	0	0	0	0	85	0
N.S.	1	0.87	1.94	2.40	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.561	0.088	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	126	453	452	0	0	0	0	92	0
N.S.	1	0.96	3.46	3.45	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.636	0.403	0.464	0.000	0.000	0.000	0.000	0.206	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	217	525	474	0	0	0	0	102	0
N.S.	1	0.91	2.21	1.99	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.711	0.750	0.673	0.000	0.000	0.000	0.000	0.193	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	189	614	610	0	0	0	0	128	0
N.S.	1	0.90	2.92	2.90	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.802	0.814	0.635	0.000	0.000	0.000	0.000	0.204	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	349	849	561	0	0	0	0	126	0
N.S.	1	1.05	2.55	1.68	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	3.132	7.337	0.810	0.000	0.000	0.000	0.000	0.200	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	313	614	618	0	0	0	0	262	0
N.S.	1	1.04	2.05	2.06	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	2.051	2.347	0.683	0.000	0.000	0.000	0.000	0.207	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	502	364	0	0	0	0	253	0
N.S.	1	1.00	2.21	1.60	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	1.716	1.023	0.451	0.000	0.000	0.000	0.000	0.220	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	218	457	446	0	0	0	0	249	0
N.S.	1	0.94	1.96	1.91	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	1.265	1.735	0.544	0.000	0.000	0.000	0.000	0.215	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	75	172	293	102	0	204	167	0
N.S.	1	0.97	0.84	1.93	3.29	1.15	0.00	2.29	1.88	0.00
time (sec)	N/A	0.397	0.114	0.289	0.175	0.133	0.000	0.161	0.221	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	213	458	446	0	0	0	0	233	0
N.S.	1	0.93	1.99	1.94	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	1.476	0.884	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	207	612	604	0	0	0	0	252	0
N.S.	1	0.98	2.90	2.86	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	1.390	1.615	0.583	0.000	0.000	0.000	0.000	0.202	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	338	1161	605	0	0	0	0	256	0
N.S.	1	1.04	3.58	1.87	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	3.210	8.844	0.745	0.000	0.000	0.000	0.000	0.218	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	323	759	661	0	0	0	0	292	0
N.S.	1	1.24	2.91	2.53	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	2.110	1.503	0.740	0.000	0.000	0.000	0.000	0.224	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	524	1390	707	0	0	0	0	279	0
N.S.	1	1.19	3.17	1.61	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	4.912	10.725	0.886	0.000	0.000	0.000	0.000	0.216	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	360	667	570	0	0	0	0	439	0
N.S.	1	1.05	1.94	1.66	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	2.542	4.269	0.680	0.000	0.000	0.000	0.000	0.212	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	187	192	386	0	198	0	318	311	0
N.S.	1	1.09	1.12	2.24	0.00	1.15	0.00	1.85	1.81	0.00
time (sec)	N/A	0.880	0.344	0.517	0.000	0.145	0.000	0.200	0.214	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	326	670	568	0	0	0	0	437	0
N.S.	1	0.96	1.96	1.67	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	1.978	4.320	0.660	0.000	0.000	0.000	0.000	0.211	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	140	162	270	0	165	0	395	304	0
N.S.	1	0.93	1.08	1.80	0.00	1.10	0.00	2.63	2.03	0.00
time (sec)	N/A	0.507	0.675	0.308	0.000	0.135	0.000	0.190	0.231	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	318	666	570	0	0	0	0	417	0
N.S.	1	0.96	2.01	1.72	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	1.903	3.572	0.644	0.000	0.000	0.000	0.000	0.221	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	337	756	759	0	0	0	0	455	0
N.S.	1	1.14	2.55	2.56	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	2.091	3.567	0.740	0.000	0.000	0.000	0.000	0.218	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	504	1400	730	0	0	0	0	454	0
N.S.	1	1.17	3.26	1.70	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	4.259	10.468	0.856	0.000	0.000	0.000	0.000	0.219	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	498	989	921	0	0	0	0	505	0
N.S.	1	1.24	2.45	2.29	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	3.538	7.587	0.839	0.000	0.000	0.000	0.000	0.227	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	775	1657	821	0	0	0	0	477	0
N.S.	1	1.42	3.05	1.51	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	6.449	11.295	0.979	0.000	0.000	0.000	0.000	0.235	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	364	242	503	311	277	0	0	127	0
N.S.	1	1.13	0.75	1.57	0.97	0.86	0.00	0.00	0.40	0.00
time (sec)	N/A	1.594	0.180	2.222	0.176	0.152	0.000	0.000	0.259	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	246	678	0	0	0	385	115	0
N.S.	1	1.00	0.81	2.24	0.00	0.00	0.00	1.27	0.38	0.00
time (sec)	N/A	1.345	0.205	0.391	0.000	0.000	0.000	0.640	0.223	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	149	120	439	188	208	0	0	100	0
N.S.	1	0.82	0.66	2.43	1.04	1.15	0.00	0.00	0.55	0.00
time (sec)	N/A	0.477	0.162	0.506	0.140	0.125	0.000	0.000	0.198	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	169	128	531	0	0	0	0	82	0
N.S.	1	0.88	0.67	2.77	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.579	0.130	0.385	0.000	0.000	0.000	0.000	0.207	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	234	391	659	0	0	0	0	88	0
N.S.	1	0.62	1.03	1.74	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.090	1.020	0.685	0.000	0.000	0.000	0.000	0.202	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	182	257	565	0	0	0	0	119	0
N.S.	1	0.80	1.13	2.49	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.201	0.861	0.648	0.000	0.000	0.000	0.000	0.203	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	248	480	589	0	0	0	0	102	0
N.S.	1	0.62	1.21	1.48	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.513	3.491	0.734	0.000	0.000	0.000	0.000	0.206	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	206	248	2041	0	0	0	0	104	0
N.S.	1	0.66	0.79	6.50	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.033	0.926	0.760	0.000	0.000	0.000	0.000	0.209	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	561	244	528	356	360	0	0	209	0
N.S.	1	1.29	0.56	1.22	0.82	0.83	0.00	0.00	0.48	0.00
time (sec)	N/A	2.765	0.186	0.839	0.164	0.184	0.000	0.000	0.253	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	509	297	1320	0	0	0	566	198	0
N.S.	1	1.21	0.71	3.14	0.00	0.00	0.00	1.34	0.47	0.00
time (sec)	N/A	2.186	0.199	0.584	0.000	0.000	0.000	1.123	0.224	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	189	159	465	236	295	0	0	183	0
N.S.	1	0.73	0.62	1.80	0.91	1.14	0.00	0.00	0.71	0.00
time (sec)	N/A	0.568	0.108	0.767	0.153	0.126	0.000	0.000	0.229	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	332	247	929	0	0	0	0	166	0
N.S.	1	1.12	0.83	3.14	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	1.052	0.107	0.510	0.000	0.000	0.000	0.000	0.233	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	378	576	1072	0	0	0	0	170	0
N.S.	1	0.71	1.08	2.01	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	2.266	1.801	0.766	0.000	0.000	0.000	0.000	0.234	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	370	396	709	0	0	0	0	197	0
N.S.	1	0.87	0.93	1.67	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	2.079	2.558	0.790	0.000	0.000	0.000	0.000	0.233	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	376	854	884	0	0	0	0	201	0
N.S.	1	0.64	1.45	1.50	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.034	6.999	0.826	0.000	0.000	0.000	0.000	0.204	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	392	493	2250	0	0	0	0	222	0
N.S.	1	0.98	1.23	5.62	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	3.296	1.445	0.834	0.000	0.000	0.000	0.000	0.216	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	799	270	594	401	486	0	0	292	0
N.S.	1	1.43	0.48	1.06	0.72	0.87	0.00	0.00	0.52	0.00
time (sec)	N/A	3.986	0.235	0.844	0.179	0.142	0.000	0.000	0.279	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	764	348	1939	0	0	0	751	281	0
N.S.	1	1.37	0.63	3.49	0.00	0.00	0.00	1.35	0.51	0.00
time (sec)	N/A	3.730	0.246	0.770	0.000	0.000	0.000	1.696	0.235	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	228	176	531	281	405	0	0	265	0
N.S.	1	0.66	0.51	1.55	0.82	1.18	0.00	0.00	0.77	0.00
time (sec)	N/A	0.751	0.178	0.873	0.157	0.154	0.000	0.000	0.245	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	521	299	1349	0	0	0	0	249	0
N.S.	1	1.27	0.73	3.28	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.762	0.659	0.675	0.000	0.000	0.000	0.000	0.237	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	563	775	1490	0	0	0	0	253	0
N.S.	1	0.85	1.17	2.26	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	3.529	3.141	0.921	0.000	0.000	0.000	0.000	0.231	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	620	586	972	0	0	0	0	282	0
N.S.	1	1.13	1.07	1.77	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	3.156	2.008	0.917	0.000	0.000	0.000	0.000	0.232	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	728	558	1073	1326	0	0	0	0	286	0
N.S.	1	0.77	1.47	1.82	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	3.585	7.158	0.910	0.000	0.000	0.000	0.000	0.220	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	0	690	2650	0	0	0	0	306	0
N.S.	1	0.00	1.19	4.58	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.000	2.631	0.925	0.000	0.000	0.000	0.000	0.239	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	412	230	507	365	276	0	0	133	0
N.S.	1	1.22	0.68	1.50	1.08	0.82	0.00	0.00	0.39	0.00
time (sec)	N/A	1.793	0.105	1.335	0.143	0.142	0.000	0.000	0.244	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	347	283	722	0	0	0	307	122	0
N.S.	1	1.07	0.88	2.24	0.00	0.00	0.00	0.95	0.38	0.00
time (sec)	N/A	1.663	1.414	0.539	0.000	0.000	0.000	0.824	0.209	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	245	176	479	251	210	0	0	111	0
N.S.	1	1.09	0.79	2.14	1.12	0.94	0.00	0.00	0.50	0.00
time (sec)	N/A	1.322	0.074	0.752	0.153	0.155	0.000	0.000	0.222	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	176	210	517	0	0	0	185	99	0
N.S.	1	0.85	1.02	2.51	0.00	0.00	0.00	0.90	0.48	0.00
time (sec)	N/A	0.700	1.472	0.451	0.000	0.000	0.000	0.706	0.197	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	99	86	316	130	147	0	0	100	0
N.S.	1	0.95	0.83	3.04	1.25	1.41	0.00	0.00	0.96	0.00
time (sec)	N/A	0.365	0.055	0.316	0.163	0.129	0.000	0.000	0.199	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	64	143	47	0	0	38	38	0
N.S.	1	1.00	1.31	2.92	0.96	0.00	0.00	0.78	0.78	0.00
time (sec)	N/A	0.259	0.016	0.217	0.121	0.000	0.000	0.268	0.209	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	144	301	387	0	0	0	0	74	0
N.S.	1	0.56	1.17	1.51	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.688	0.567	0.634	0.000	0.000	0.000	0.000	0.209	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	138	159	427	0	0	0	0	83	0
N.S.	1	0.75	0.87	2.33	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.767	0.542	0.638	0.000	0.000	0.000	0.000	0.202	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	251	487	585	0	0	0	0	107	0
N.S.	1	0.62	1.21	1.46	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.384	3.950	0.780	0.000	0.000	0.000	0.000	0.228	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	255	269	2319	0	0	0	0	111	0
N.S.	1	0.82	0.86	7.43	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.292	0.607	0.776	0.000	0.000	0.000	0.000	0.245	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	504	453	1087	0	0	0	0	180	0
N.S.	1	1.15	1.03	2.47	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	3.048	0.656	0.855	0.000	0.000	0.000	0.000	0.237	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	394	312	807	0	0	0	0	376	0
N.S.	1	0.95	0.75	1.95	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	2.267	1.742	0.743	0.000	0.000	0.000	0.000	0.223	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	286	369	668	0	0	0	0	168	0
N.S.	1	0.90	1.16	2.11	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.393	0.549	0.671	0.000	0.000	0.000	0.000	0.210	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	195	295	504	0	0	0	0	213	0
N.S.	1	0.78	1.18	2.02	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.968	0.761	0.645	0.000	0.000	0.000	0.000	0.220	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	137	276	401	0	0	0	0	151	0
N.S.	1	0.66	1.33	1.93	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.598	1.010	0.586	0.000	0.000	0.000	0.000	0.210	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	140	165	354	0	0	0	0	142	0
N.S.	1	0.72	0.85	1.82	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.637	0.186	0.520	0.000	0.000	0.000	0.000	0.208	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	278	667	591	0	0	0	0	181	0
N.S.	1	0.60	1.43	1.27	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	2.121	1.502	0.858	0.000	0.000	0.000	0.000	0.230	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	277	322	513	0	0	0	0	164	0
N.S.	1	0.83	0.97	1.54	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	2.294	0.917	0.731	0.000	0.000	0.000	0.000	0.238	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	457	844	869	0	0	0	0	221	0
N.S.	1	0.72	1.33	1.37	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	4.572	7.755	0.958	0.000	0.000	0.000	0.000	0.209	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	464	462	2844	0	0	0	0	180	0
N.S.	1	0.98	0.98	6.01	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	3.390	0.841	0.887	0.000	0.000	0.000	0.000	0.206	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	565	594	815	0	0	0	0	392	0
N.S.	1	1.26	1.33	1.82	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	3.047	1.182	0.883	0.000	0.000	0.000	0.000	0.213	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	426	374	801	0	0	0	0	429	0
N.S.	1	1.01	0.89	1.90	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	1.917	1.149	0.843	0.000	0.000	0.000	0.000	0.210	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	350	511	505	0	0	0	0	381	0
N.S.	1	1.05	1.54	1.52	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	2.166	0.697	0.753	0.000	0.000	0.000	0.000	0.206	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	218	303	3298	0	0	0	0	364	0
N.S.	1	0.66	0.91	9.93	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	1.021	0.949	0.790	0.000	0.000	0.000	0.000	0.209	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	197	461	468	0	0	0	0	362	0
N.S.	1	0.67	1.57	1.59	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.810	1.396	0.683	0.000	0.000	0.000	0.000	0.204	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	270	320	2895	0	0	0	0	361	0
N.S.	1	0.87	1.03	9.31	0.00	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	1.412	0.394	0.635	0.000	0.000	0.000	0.000	0.219	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	474	935	924	0	0	0	0	460	0
N.S.	1	0.82	1.62	1.60	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	5.006	8.236	1.017	0.000	0.000	0.000	0.000	0.236	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	455	352	3773	0	0	0	0	392	0
N.S.	1	1.01	0.78	8.35	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	3.423	2.460	0.929	0.000	0.000	0.000	0.000	0.228	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	752	0	1090	1121	0	0	0	0	500	0
N.S.	1	0.00	1.45	1.49	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.000	9.732	1.085	0.000	0.000	0.000	0.000	0.216	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	723	441	5225	0	0	0	0	405	0
N.S.	1	1.34	0.82	9.71	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	5.050	2.838	0.944	0.000	0.000	0.000	0.000	0.218	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	225	100	129	0	84	146	143	25	0
N.S.	1	1.43	0.64	0.82	0.00	0.54	0.93	0.91	0.16	0.00
time (sec)	N/A	1.189	0.033	0.483	0.000	0.126	0.523	0.151	0.219	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	159	81	127	105	64	121	0	25	0
N.S.	1	1.26	0.64	1.01	0.83	0.51	0.96	0.00	0.20	0.00
time (sec)	N/A	0.856	0.031	0.362	0.118	0.104	0.398	0.000	0.224	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	73	71	0	59	78	81	25	0
N.S.	1	1.09	0.82	0.80	0.00	0.66	0.88	0.91	0.28	0.00
time (sec)	N/A	0.546	0.018	0.456	0.000	0.127	0.308	0.165	0.204	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	56	51	80	49	35	49	49	45	0
N.S.	1	1.02	0.93	1.45	0.89	0.64	0.89	0.89	0.82	0.00
time (sec)	N/A	0.331	0.010	0.304	0.116	0.115	0.247	0.143	0.200	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.85
time (sec)	N/A	0.198	0.004	0.150	0.115	0.123	0.199	0.115	0.194	0.202

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	98	116	161	0	0	0	0	25	0
N.S.	1	1.07	1.26	1.75	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.510	0.099	0.393	0.000	0.000	0.000	0.000	0.196	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	72	141	0	0	0	0	25	0
N.S.	1	1.16	0.95	1.86	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.527	0.212	0.410	0.000	0.000	0.000	0.000	0.197	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	194	254	0	0	0	0	25	0
N.S.	1	1.00	1.19	1.56	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.966	0.983	0.476	0.000	0.000	0.000	0.000	0.197	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1312	1031	539	0	0	0	0	0	1135	0
N.S.	1	0.79	0.41	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	3.377	0.584	0.000	0.000	0.000	0.000	0.000	0.292	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	756	650	403	0	0	0	0	0	649	0
N.S.	1	0.86	0.53	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	2.174	0.380	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	354	267	0	0	0	0	0	297	0
N.S.	1	0.95	0.72	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	1.142	0.218	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	43	66	0	80	29
N.S.	1	1.00	1.07	1.00	1.19	1.59	2.44	0.00	2.96	1.07
time (sec)	N/A	0.296	4.957	0.898	0.592	0.111	3.874	0.000	0.222	0.349

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	30	55	92	0	102	29
N.S.	1	1.00	1.07	1.00	1.11	2.04	3.41	0.00	3.78	1.07
time (sec)	N/A	1.239	6.719	0.968	0.702	0.111	14.950	0.000	0.222	0.221

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	69	119	0	128	29
N.S.	1	1.00	1.07	1.00	1.19	2.56	4.41	0.00	4.74	1.07
time (sec)	N/A	1.987	7.903	0.930	0.743	0.122	100.058	0.000	0.218	0.225

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	134	0	0	264	29
N.S.	1	1.00	1.07	0.93	1.00	4.62	0.00	0.00	9.10	1.00
time (sec)	N/A	3.190	7.919	9.844	0.756	0.131	0.000	0.000	0.312	0.301

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	85	0	0	171	29
N.S.	1	1.00	1.07	0.93	1.00	2.93	0.00	0.00	5.90	1.00
time (sec)	N/A	1.888	0.493	4.091	0.591	0.116	0.000	0.000	0.271	0.316

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	41	31	0	77	29
N.S.	1	1.00	1.07	0.93	1.00	1.41	1.07	0.00	2.66	1.00
time (sec)	N/A	0.981	0.331	1.698	0.398	0.114	18.048	0.000	0.229	0.413

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	56	31	0	85	29
N.S.	1	1.00	1.07	0.93	1.00	1.93	1.07	0.00	2.93	1.00
time (sec)	N/A	0.353	3.291	0.890	0.546	0.113	7.080	0.000	0.198	0.397

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	68	31	0	153	29
N.S.	1	1.00	1.07	0.93	1.00	2.34	1.07	0.00	5.28	1.00
time (sec)	N/A	0.363	3.693	1.167	0.508	0.120	10.781	0.000	0.207	0.308

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	82	0	0	202	29
N.S.	1	1.00	1.07	0.93	1.00	2.83	0.00	0.00	6.97	1.00
time (sec)	N/A	0.385	3.852	1.258	0.542	0.136	0.000	0.000	0.218	0.300

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	36	26	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.50	1.08	1.00	1.04	1.00
time (sec)	N/A	0.272	0.756	0.799	0.334	0.105	2.607	0.464	0.191	0.193

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	276	125	160	0	111	185	192	25	0
N.S.	1	1.45	0.65	0.84	0.00	0.58	0.97	1.01	0.13	0.00
time (sec)	N/A	1.598	0.040	0.437	0.000	0.120	0.697	0.160	0.209	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	208	100	180	131	85	148	0	25	0
N.S.	1	1.32	0.64	1.15	0.83	0.54	0.94	0.00	0.16	0.00
time (sec)	N/A	1.220	0.035	0.415	0.130	0.103	0.519	0.000	0.206	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	116	85	85	0	73	100	108	25	0
N.S.	1	1.08	0.79	0.79	0.00	0.68	0.93	1.01	0.23	0.00
time (sec)	N/A	0.734	0.021	0.561	0.000	0.110	0.409	0.181	0.226	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	61	107	64	46	61	62	55	0
N.S.	1	1.07	0.91	1.60	0.96	0.69	0.91	0.93	0.82	0.00
time (sec)	N/A	0.389	0.012	0.380	0.111	0.107	0.327	0.156	0.212	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.85
time (sec)	N/A	0.200	0.005	0.209	0.151	0.119	0.245	0.113	0.229	0.193

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	150	180	225	0	0	0	0	25	0
N.S.	1	1.09	1.30	1.63	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.646	0.127	0.468	0.000	0.000	0.000	0.000	0.230	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	117	108	205	0	0	0	0	25	0
N.S.	1	1.18	1.09	2.07	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.679	0.182	0.498	0.000	0.000	0.000	0.000	0.215	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	257	317	399	0	0	0	0	25	0
N.S.	1	0.97	1.20	1.51	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.440	2.984	0.554	0.000	0.000	0.000	0.000	0.226	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	171	152	157	0	0	0	472	27	0
N.S.	1	0.83	0.74	0.76	0.00	0.00	0.00	2.29	0.13	0.00
time (sec)	N/A	0.760	0.286	0.217	0.000	0.000	0.000	0.151	0.206	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	154	135	138	0	0	0	0	27	0
N.S.	1	0.84	0.74	0.75	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.666	0.217	0.176	0.000	0.000	0.000	0.000	0.215	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	71	66	68	0	0	0	169	27	0
N.S.	1	0.87	0.80	0.83	0.00	0.00	0.00	2.06	0.33	0.00
time (sec)	N/A	0.523	0.157	0.308	0.000	0.000	0.000	0.141	0.200	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	104	91	92	0	0	0	172	25	0
N.S.	1	0.86	0.75	0.76	0.00	0.00	0.00	1.42	0.21	0.00
time (sec)	N/A	0.536	0.164	0.166	0.000	0.000	0.000	0.154	0.217	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	71	62	63	0	0	0	102	24	0
N.S.	1	0.87	0.76	0.77	0.00	0.00	0.00	1.24	0.29	0.00
time (sec)	N/A	0.465	0.192	0.204	0.000	0.000	0.000	0.147	0.214	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	27	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	0.96	1.00
time (sec)	N/A	0.600	2.135	0.661	0.237	0.110	0.725	0.000	0.201	0.212

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	60	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	2.14	1.00
time (sec)	N/A	0.522	1.413	0.836	0.219	0.102	0.633	0.328	0.213	0.210

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	0	31	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	0.00	1.11	1.00
time (sec)	N/A	0.315	3.853	1.696	0.268	0.110	0.642	0.000	0.201	0.212

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	204	179	184	0	0	0	614	60	0
N.S.	1	0.83	0.73	0.75	0.00	0.00	0.00	2.51	0.24	0.00
time (sec)	N/A	0.736	0.505	0.191	0.000	0.000	0.000	0.178	0.211	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	171	165	157	0	0	0	473	60	0
N.S.	1	0.83	0.80	0.76	0.00	0.00	0.00	2.30	0.29	0.00
time (sec)	N/A	0.587	0.431	0.167	0.000	0.000	0.000	0.163	0.196	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	154	136	139	0	0	0	360	58	0
N.S.	1	0.84	0.74	0.76	0.00	0.00	0.00	1.97	0.32	0.00
time (sec)	N/A	0.582	0.361	0.159	0.000	0.000	0.000	0.160	0.317	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	121	121	111	0	0	0	252	57	0
N.S.	1	0.84	0.84	0.77	0.00	0.00	0.00	1.75	0.40	0.00
time (sec)	N/A	0.457	0.305	0.198	0.000	0.000	0.000	0.164	0.276	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	58	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	2.07	1.00
time (sec)	N/A	0.928	2.124	0.357	0.328	0.095	2.044	0.000	0.259	0.202

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	90	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	3.21	1.00
time (sec)	N/A	0.802	1.919	0.414	0.266	0.104	1.623	0.355	0.284	0.196

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	0	64	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	0.00	2.29	1.00
time (sec)	N/A	0.332	3.852	1.196	0.348	0.088	2.085	0.000	0.309	0.207

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	204	180	185	0	0	0	746	91	0
N.S.	1	0.83	0.73	0.76	0.00	0.00	0.00	3.04	0.37	0.00
time (sec)	N/A	0.762	0.773	0.191	0.000	0.000	0.000	0.170	0.306	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	221	209	203	0	0	0	757	91	0
N.S.	1	0.82	0.78	0.76	0.00	0.00	0.00	2.82	0.34	0.00
time (sec)	N/A	0.771	0.726	0.181	0.000	0.000	0.000	0.165	0.231	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	204	180	185	0	0	0	614	89	0
N.S.	1	0.83	0.73	0.76	0.00	0.00	0.00	2.51	0.36	0.00
time (sec)	N/A	0.580	0.642	0.177	0.000	0.000	0.000	0.171	0.213	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	171	165	157	0	0	0	472	88	0
N.S.	1	0.83	0.80	0.76	0.00	0.00	0.00	2.29	0.43	0.00
time (sec)	N/A	0.488	0.549	0.220	0.000	0.000	0.000	0.161	0.218	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	45	26	0	89	28
N.S.	1	1.00	1.07	0.93	1.00	1.61	0.93	0.00	3.18	1.00
time (sec)	N/A	1.183	2.124	0.367	0.333	0.104	4.971	0.000	0.196	0.204

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	28	122	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	1.00	4.36	1.00
time (sec)	N/A	1.068	1.908	0.365	0.339	0.088	3.926	0.376	0.244	0.198

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	0	93	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	0.00	3.32	1.00
time (sec)	N/A	0.301	3.834	0.831	0.322	0.086	4.076	0.000	0.229	0.211

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	33	30	0	0	0	35	25	0
N.S.	1	0.88	0.80	0.73	0.00	0.00	0.00	0.85	0.61	0.00
time (sec)	N/A	0.373	0.103	0.418	0.000	0.000	0.000	0.145	0.198	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	22	23	0	0	0	0	25	0
N.S.	1	0.93	0.81	0.85	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.407	0.091	0.343	0.000	0.000	0.000	0.000	0.208	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	22	21	0	0	0	23	25	0
N.S.	1	0.93	0.81	0.78	0.00	0.00	0.00	0.85	0.93	0.00
time (sec)	N/A	0.352	0.078	0.209	0.000	0.000	0.000	0.132	0.231	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	22	21	0	0	0	23	25	0
N.S.	1	0.93	0.81	0.78	0.00	0.00	0.00	0.85	0.93	0.00
time (sec)	N/A	0.370	0.010	0.000	0.000	0.000	0.000	0.129	0.223	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	9	23	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	1.00	2.56	0.00
time (sec)	N/A	0.291	0.066	0.191	0.000	0.000	0.000	0.136	0.207	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	10	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.78	1.11	1.00	1.00
time (sec)	N/A	0.199	0.014	0.078	0.141	0.086	0.190	0.121	0.225	0.202

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	24	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	1.00	1.00	1.04	1.00
time (sec)	N/A	0.259	1.334	0.525	0.222	0.091	0.442	0.135	0.214	0.170

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	26	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	1.08	1.00	1.04	1.00
time (sec)	N/A	0.276	0.654	0.311	0.214	0.100	0.455	0.132	0.197	0.174

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	154	136	139	0	0	0	0	39	0
N.S.	1	0.84	0.74	0.76	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.597	0.288	0.224	0.000	0.000	0.000	0.000	0.210	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	121	108	111	0	0	0	254	39	0
N.S.	1	0.84	0.75	0.77	0.00	0.00	0.00	1.76	0.27	0.00
time (sec)	N/A	0.511	0.244	0.189	0.000	0.000	0.000	0.155	0.223	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	104	92	93	0	0	0	0	39	0
N.S.	1	0.86	0.76	0.77	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.502	0.209	0.184	0.000	0.000	0.000	0.000	0.218	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	71	64	65	0	0	0	104	39	0
N.S.	1	0.87	0.78	0.79	0.00	0.00	0.00	1.27	0.48	0.00
time (sec)	N/A	0.451	0.195	0.182	0.000	0.000	0.000	0.152	0.203	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	45	46	0	0	0	50	37	0
N.S.	1	0.91	0.83	0.85	0.00	0.00	0.00	0.93	0.69	0.00
time (sec)	N/A	0.492	0.146	0.157	0.000	0.000	0.000	0.148	0.213	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	19	42	17	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.19	2.62	1.06	1.00	1.00
time (sec)	N/A	0.226	0.051	0.078	0.112	0.102	0.771	0.112	0.198	0.237

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	0	37	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	0.00	1.32	1.00
time (sec)	N/A	0.304	3.634	0.281	0.238	0.095	1.180	0.000	0.201	0.198

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	53	29	28	41	28
N.S.	1	1.00	1.07	0.93	1.00	1.89	1.04	1.00	1.46	1.00
time (sec)	N/A	0.336	0.044	0.436	0.231	0.095	1.000	0.324	0.205	0.205

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	63	27	28	102	28
N.S.	1	1.00	1.07	0.93	1.00	2.25	0.96	1.00	3.64	1.00
time (sec)	N/A	0.388	5.250	0.589	0.269	0.102	1.856	0.769	0.214	0.181

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	61	26	0	83	26
N.S.	1	1.00	1.08	0.92	1.00	2.35	1.00	0.00	3.19	1.00
time (sec)	N/A	0.298	0.061	0.702	0.249	0.110	1.820	0.000	0.236	0.178

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	60	26	25	81	25
N.S.	1	1.00	1.08	0.92	1.00	2.40	1.04	1.00	3.24	1.00
time (sec)	N/A	0.249	0.095	0.214	0.269	0.106	2.128	0.381	0.225	0.182

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	64	27	0	83	28
N.S.	1	1.00	1.07	0.93	1.00	2.29	0.96	0.00	2.96	1.00
time (sec)	N/A	0.342	6.906	2.043	0.263	0.093	3.795	0.000	0.206	0.203

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	68	29	28	87	28
N.S.	1	1.00	1.07	0.93	1.00	2.43	1.04	1.00	3.11	1.00
time (sec)	N/A	0.341	6.054	1.072	0.257	0.106	2.626	3.574	0.235	0.228

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	86	27	28	125	28
N.S.	1	1.00	1.07	0.93	1.00	3.07	0.96	1.00	4.46	1.00
time (sec)	N/A	0.337	4.812	1.441	0.327	0.091	1.914	2.747	0.203	0.165

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	84	26	0	123	26
N.S.	1	1.00	1.08	0.92	1.00	3.23	1.00	0.00	4.73	1.00
time (sec)	N/A	0.303	0.067	1.224	0.272	0.107	1.999	0.000	0.206	0.168

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	83	26	25	121	25
N.S.	1	1.00	1.08	0.92	1.00	3.32	1.04	1.00	4.84	1.00
time (sec)	N/A	0.252	0.100	0.986	0.260	0.097	2.184	0.917	0.201	0.166

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	85	27	0	123	28
N.S.	1	1.00	1.07	0.93	1.00	3.04	0.96	0.00	4.39	1.00
time (sec)	N/A	0.353	5.094	3.375	0.273	0.112	4.594	0.000	0.215	0.194

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	89	29	28	127	28
N.S.	1	1.00	1.07	0.93	1.00	3.18	1.04	1.00	4.54	1.00
time (sec)	N/A	0.348	9.434	2.740	0.288	0.128	3.054	7.203	0.224	0.206

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	45	0	0	97	28
N.S.	1	1.00	1.07	0.93	1.00	1.61	0.00	0.00	3.46	1.00
time (sec)	N/A	0.362	0.928	1.641	0.366	0.114	0.000	0.000	0.226	0.182

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	38	27	0	63	28
N.S.	1	1.00	1.07	0.93	1.00	1.36	0.96	0.00	2.25	1.00
time (sec)	N/A	0.346	0.516	1.539	0.296	0.098	13.205	0.000	0.214	0.189

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	27	0	27	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.96	0.00	0.96	1.00
time (sec)	N/A	0.298	0.101	1.542	0.231	0.111	0.704	0.000	0.194	0.176

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	50	27	28	39	28
N.S.	1	1.00	1.07	0.93	1.00	1.79	0.96	1.00	1.39	1.00
time (sec)	N/A	0.299	0.806	0.977	0.220	0.106	0.881	0.304	0.199	0.184

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	63	27	28	85	28
N.S.	1	1.00	1.07	0.93	1.00	2.25	0.96	1.00	3.04	1.00
time (sec)	N/A	0.311	1.173	1.197	0.280	0.108	9.720	0.528	0.202	0.173

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	86	27	28	125	28
N.S.	1	1.00	1.07	0.93	1.00	3.07	0.96	1.00	4.46	1.00
time (sec)	N/A	0.325	1.739	1.278	0.273	0.100	16.330	1.081	0.212	0.170

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	36	24	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.50	1.00	1.00	1.04	1.00
time (sec)	N/A	0.257	0.355	0.464	0.231	0.102	0.646	0.274	0.214	0.166

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	186	59	0	0	139	28
N.S.	1	1.00	1.07	0.93	6.64	2.11	0.00	0.00	4.96	1.00
time (sec)	N/A	0.309	0.994	2.085	2.029	0.122	0.000	0.000	0.242	0.220

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	161	52	29	0	91	28
N.S.	1	1.00	1.07	0.93	5.75	1.86	1.04	0.00	3.25	1.00
time (sec)	N/A	0.310	0.546	2.011	1.576	0.122	31.864	0.000	0.232	0.212

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	138	42	29	0	41	28
N.S.	1	1.00	1.07	0.93	4.93	1.50	1.04	0.00	1.46	1.00
time (sec)	N/A	0.297	0.105	1.804	1.250	0.106	1.504	0.000	0.242	0.205

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	112	80	29	28	71	28
N.S.	1	1.00	1.07	0.93	4.00	2.86	1.04	1.00	2.54	1.00
time (sec)	N/A	0.405	0.820	0.885	0.665	0.094	2.740	0.422	0.204	0.205

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	218	106	29	28	141	28
N.S.	1	1.00	1.07	0.93	7.79	3.79	1.04	1.00	5.04	1.00
time (sec)	N/A	0.308	1.189	1.181	1.120	0.108	73.211	0.931	0.208	0.198

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	278	144	29	28	212	28
N.S.	1	1.00	1.07	0.93	9.93	5.14	1.04	1.00	7.57	1.00
time (sec)	N/A	0.318	1.761	1.296	1.292	0.103	74.329	2.240	0.196	0.209

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	292	175	340	0	0	0	0	41	0
N.S.	1	1.36	0.82	1.59	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.924	0.362	0.224	0.000	0.000	0.000	0.000	0.201	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	194	82	136	0	0	0	563	41	0
N.S.	1	2.06	0.87	1.45	0.00	0.00	0.00	5.99	0.44	0.00
time (sec)	N/A	1.165	0.248	0.355	0.000	0.000	0.000	0.221	0.207	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	183	125	223	0	0	0	608	39	0
N.S.	1	1.22	0.83	1.49	0.00	0.00	0.00	4.05	0.26	0.00
time (sec)	N/A	1.130	0.224	0.185	0.000	0.000	0.000	0.221	0.210	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	83	72	134	0	0	0	290	38	0
N.S.	1	0.97	0.84	1.56	0.00	0.00	0.00	3.37	0.44	0.00
time (sec)	N/A	0.698	0.145	0.221	0.000	0.000	0.000	0.211	0.224	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	128	43	27	0	42	28
N.S.	1	1.00	1.07	0.93	4.57	1.54	0.96	0.00	1.50	1.00
time (sec)	N/A	1.076	8.288	1.251	0.655	0.132	1.018	0.000	0.239	0.210

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	126	49	29	28	171	28
N.S.	1	1.00	1.07	0.93	4.50	1.75	1.04	1.00	6.11	1.00
time (sec)	N/A	0.445	1.682	0.847	0.538	0.109	0.977	0.599	0.231	0.194

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	135	49	29	0	48	28
N.S.	1	1.00	1.07	0.93	4.82	1.75	1.04	0.00	1.71	1.00
time (sec)	N/A	0.342	10.740	5.530	0.715	0.107	1.094	0.000	0.217	0.199

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	394	399	455	0	0	0	2065	88	0
N.S.	1	1.42	1.44	1.64	0.00	0.00	0.00	7.43	0.32	0.00
time (sec)	N/A	1.387	0.722	0.397	0.000	0.000	0.000	0.255	0.223	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	252	306	364	0	0	0	1553	88	0
N.S.	1	1.15	1.39	1.65	0.00	0.00	0.00	7.06	0.40	0.00
time (sec)	N/A	0.948	0.554	0.405	0.000	0.000	0.000	0.244	0.211	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	291	295	341	0	0	0	1215	86	0
N.S.	1	1.36	1.38	1.59	0.00	0.00	0.00	5.68	0.40	0.00
time (sec)	N/A	1.298	0.380	0.401	0.000	0.000	0.000	0.238	0.234	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	140	122	250	0	0	0	747	85	0
N.S.	1	0.93	0.81	1.67	0.00	0.00	0.00	4.98	0.57	0.00
time (sec)	N/A	0.563	0.414	0.444	0.000	0.000	0.000	0.227	0.205	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	146	43	27	0	87	28
N.S.	1	1.00	1.07	0.93	5.21	1.54	0.96	0.00	3.11	1.00
time (sec)	N/A	1.013	7.035	0.906	0.857	0.086	2.919	0.000	0.214	0.210

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	146	49	29	28	266	28
N.S.	1	1.00	1.07	0.93	5.21	1.75	1.04	1.00	9.50	1.00
time (sec)	N/A	0.539	2.836	0.918	0.830	0.102	2.897	0.612	0.225	0.202

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	153	49	29	0	96	28
N.S.	1	1.00	1.07	0.93	5.46	1.75	1.04	0.00	3.43	1.00
time (sec)	N/A	0.307	10.897	4.954	0.853	0.084	3.715	0.000	0.253	0.205

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	494	408	455	0	0	0	2479	133	0
N.S.	1	1.78	1.47	1.64	0.00	0.00	0.00	8.92	0.48	0.00
time (sec)	N/A	1.380	0.983	0.407	0.000	0.000	0.000	0.261	0.249	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	402	414	479	0	0	0	2461	133	0
N.S.	1	1.43	1.47	1.70	0.00	0.00	0.00	8.73	0.47	0.00
time (sec)	N/A	1.339	0.759	0.403	0.000	0.000	0.000	0.251	0.225	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	391	404	455	0	0	0	2026	131	0
N.S.	1	1.42	1.46	1.65	0.00	0.00	0.00	7.34	0.47	0.00
time (sec)	N/A	1.843	0.659	0.407	0.000	0.000	0.000	0.257	0.234	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	190	311	364	0	0	0	1394	130	0
N.S.	1	0.88	1.43	1.68	0.00	0.00	0.00	6.42	0.60	0.00
time (sec)	N/A	0.742	0.563	0.449	0.000	0.000	0.000	0.232	0.223	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	161	60	27	0	132	28
N.S.	1	1.00	1.07	0.93	5.75	2.14	0.96	0.00	4.71	1.00
time (sec)	N/A	1.180	8.616	0.934	0.934	0.090	7.363	0.000	0.228	0.199

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	162	66	29	28	365	28
N.S.	1	1.00	1.07	0.93	5.79	2.36	1.04	1.00	13.04	1.00
time (sec)	N/A	0.556	2.717	1.045	1.064	0.085	6.891	0.672	0.240	0.197

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	169	66	29	0	139	28
N.S.	1	1.00	1.07	0.93	6.04	2.36	1.04	0.00	4.96	1.00
time (sec)	N/A	0.295	10.893	3.721	1.021	0.091	7.167	0.000	0.270	0.195

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	177	157	340	0	0	0	0	64	0
N.S.	1	0.87	0.77	1.67	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.652	0.233	0.480	0.000	0.000	0.000	0.000	0.221	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	131	117	250	0	0	0	876	64	0
N.S.	1	0.93	0.83	1.77	0.00	0.00	0.00	6.21	0.45	0.00
time (sec)	N/A	0.605	0.207	0.470	0.000	0.000	0.000	0.231	0.208	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	127	113	227	0	0	0	0	64	0
N.S.	1	0.89	0.80	1.60	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.594	0.182	0.474	0.000	0.000	0.000	0.000	0.205	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	75	70	136	0	0	0	346	64	0
N.S.	1	0.95	0.89	1.72	0.00	0.00	0.00	4.38	0.81	0.00
time (sec)	N/A	0.691	0.144	0.467	0.000	0.000	0.000	0.230	0.210	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	59	108	0	0	0	200	62	0
N.S.	1	0.94	0.82	1.50	0.00	0.00	0.00	2.78	0.86	0.00
time (sec)	N/A	0.571	0.104	0.437	0.000	0.000	0.000	0.221	0.242	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	18	53	18	21	18
N.S.	1	1.00	1.00	1.06	1.00	1.00	2.94	1.00	1.17	1.00
time (sec)	N/A	0.217	0.007	0.107	0.114	0.093	1.145	0.122	0.212	0.222

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	111	80	29	0	63	28
N.S.	1	1.00	1.07	0.93	3.96	2.86	1.04	0.00	2.25	1.00
time (sec)	N/A	0.442	4.899	0.569	0.690	0.098	1.744	0.000	0.216	0.200

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	120	86	31	28	69	28
N.S.	1	1.00	1.07	0.93	4.29	3.07	1.11	1.00	2.46	1.00
time (sec)	N/A	0.433	1.041	0.371	0.625	0.086	1.534	0.640	0.218	0.206

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	205	106	29	0	141	28
N.S.	1	1.00	1.07	0.93	7.32	3.79	1.04	0.00	5.04	1.00
time (sec)	N/A	0.373	41.347	1.442	0.838	0.088	3.656	0.000	0.217	0.198

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	193	106	29	28	309	28
N.S.	1	1.00	1.07	0.93	6.89	3.79	1.04	1.00	11.04	1.00
time (sec)	N/A	0.507	4.712	1.056	0.617	0.109	3.454	1.720	0.225	0.206

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	198	104	27	0	139	26
N.S.	1	1.00	1.08	0.92	7.62	4.00	1.04	0.00	5.35	1.00
time (sec)	N/A	0.312	36.722	0.400	0.712	0.090	3.610	0.000	0.232	0.197

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	192	103	27	25	137	25
N.S.	1	1.00	1.08	0.92	7.68	4.12	1.08	1.00	5.48	1.00
time (sec)	N/A	0.402	1.872	0.398	0.621	0.135	4.039	0.792	0.206	0.221

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	209	108	29	0	140	28
N.S.	1	1.00	1.07	0.93	7.46	3.86	1.04	0.00	5.00	1.00
time (sec)	N/A	0.346	34.177	5.811	0.924	0.138	6.368	0.000	0.211	0.213

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	218	114	31	28	146	28
N.S.	1	1.00	1.07	0.93	7.79	4.07	1.11	1.00	5.21	1.00
time (sec)	N/A	0.353	22.551	1.496	0.859	0.087	5.609	17.741	0.226	0.203

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	38	22	0	73	15	0
N.S.	1	1.00	1.00	0.00	2.38	1.38	0.00	4.56	0.94	0.00
time (sec)	N/A	0.311	0.199	0.000	0.476	0.078	0.000	0.164	0.189	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	507	287	304	0	0	0	0	86	0
N.S.	1	2.02	1.14	1.21	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.510	0.354	0.788	0.000	0.000	0.000	0.000	0.261	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	585	514	447	0	0	0	0	86	0
N.S.	1	0.99	0.87	0.76	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.533	0.537	0.947	0.000	0.000	0.000	0.000	0.245	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	285	276	303	0	0	0	0	84	0
N.S.	1	1.18	1.15	1.26	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.638	0.210	0.792	0.000	0.000	0.000	0.000	0.249	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	256	348	304	0	0	0	0	1522	0
N.S.	1	1.01	1.38	1.20	0.00	0.00	0.00	0.00	6.02	0.00
time (sec)	N/A	0.738	0.566	0.273	0.000	0.000	0.000	0.000	0.491	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	30	0	87	0	85	27
N.S.	1	1.00	1.07	0.93	1.11	0.00	3.22	0.00	3.15	1.00
time (sec)	N/A	1.641	0.391	0.767	0.776	0.000	4.028	0.000	0.221	0.244

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	30	0	94	29	250	27
N.S.	1	1.00	1.07	0.93	1.11	0.00	3.48	1.07	9.26	1.00
time (sec)	N/A	1.585	3.537	0.804	0.792	0.000	5.286	0.808	0.304	0.261

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	605	540	591	0	0	0	0	131	0
N.S.	1	1.25	1.11	1.22	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.801	1.822	1.344	0.000	0.000	0.000	0.000	0.333	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	803	686	594	0	0	0	0	131	0
N.S.	1	1.57	1.34	1.16	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	2.076	0.987	1.318	0.000	0.000	0.000	0.000	0.287	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	603	410	449	0	0	0	0	129	0
N.S.	1	1.62	1.10	1.20	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	2.239	0.493	0.885	0.000	0.000	0.000	0.000	0.285	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	366	522	451	0	0	0	0	0	0
N.S.	1	0.94	1.34	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.912	0.353	0.362	0.000	0.000	0.000	0.000	0.471	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	30	0	133	0	130	29
N.S.	1	1.00	1.07	0.93	1.03	0.00	4.59	0.00	4.48	1.00
time (sec)	N/A	2.279	0.743	1.456	1.087	0.000	5.317	0.000	0.270	0.209

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	30	0	143	30	347	29
N.S.	1	1.00	1.07	0.93	1.03	0.00	4.93	1.03	11.97	1.00
time (sec)	N/A	2.472	10.221	0.632	1.071	0.000	5.129	1.204	0.342	0.218

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F(-2)	F(-2)	F	A	B	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	47	0	0	0	46	29	0
N.S.	1	1.00	0.00	1.12	0.00	0.00	0.00	1.10	0.69	0.00
time (sec)	N/A	0.342	0.000	1.568	0.000	0.000	0.000	0.281	0.223	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	53	20	0	0	0	37	30	0
N.S.	1	1.00	2.12	0.80	0.00	0.00	0.00	1.48	1.20	0.00
time (sec)	N/A	0.350	0.053	0.669	0.000	0.000	0.000	0.145	0.228	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	198	191	0	0	0	0	0	30	0
N.S.	1	0.76	0.74	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.676	0.600	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	303	272	0	0	0	0	0	28	0
N.S.	1	0.77	0.70	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.733	0.560	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	198	182	0	0	0	0	0	27	0
N.S.	1	0.76	0.70	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.570	0.527	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	29	0	30	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.00	0.00	1.03	1.00
time (sec)	N/A	0.831	0.233	1.207	0.561	0.131	2.331	0.000	0.239	0.196

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	31	0	107	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.07	0.00	3.69	1.00
time (sec)	N/A	0.567	0.285	0.832	0.574	0.118	3.217	0.000	0.234	0.201

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	684	493	509	0	0	0	0	0	64	0
N.S.	1	0.72	0.74	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.918	2.517	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	440	464	0	0	0	0	0	62	0
N.S.	1	0.74	0.78	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.829	1.354	0.000	0.000	0.000	0.000	0.000	0.283	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	466	339	327	0	0	0	0	0	61	0
N.S.	1	0.73	0.70	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.649	1.240	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	29	0	62	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.00	0.00	2.14	1.00
time (sec)	N/A	1.384	0.247	0.825	0.663	0.127	104.273	0.000	0.213	0.190

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	0	0	169	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.00	0.00	5.83	1.00
time (sec)	N/A	1.039	0.912	0.777	0.677	0.126	0.000	0.000	0.243	0.186

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	906	635	989	0	0	0	0	0	97	0
N.S.	1	0.70	1.09	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.169	2.768	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	815	582	603	0	0	0	0	0	95	0
N.S.	1	0.71	0.74	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.058	2.453	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	698	495	477	0	0	0	0	0	94	0
N.S.	1	0.71	0.68	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.867	3.274	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	0	0	95	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.00	0.00	3.28	1.00
time (sec)	N/A	2.322	0.265	0.632	0.795	0.136	0.000	0.000	0.221	0.191

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	0	0	236	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.00	0.00	8.14	1.00
time (sec)	N/A	1.605	0.614	0.832	0.790	0.137	0.000	0.000	0.602	0.199

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	36	26	0	25	24
N.S.	1	1.00	1.08	0.92	0.00	1.50	1.08	0.00	1.04	1.00
time (sec)	N/A	0.279	0.480	0.586	0.000	0.121	3.600	0.000	0.199	0.176

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	155	153	0	0	0	0	0	25	0
N.S.	1	0.95	0.94	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.513	0.208	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	104	109	0	0	0	0	0	25	0
N.S.	1	0.95	1.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.470	0.175	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	73	70	0	0	0	0	0	23	0
N.S.	1	0.97	0.93	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.365	0.050	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	18	34	17	19	33
N.S.	1	1.00	1.00	1.06	1.00	1.06	2.00	1.00	1.12	1.94
time (sec)	N/A	0.213	0.007	0.122	0.115	0.131	0.312	0.120	0.205	0.396

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	35	24	24	25	24
N.S.	1	1.00	1.08	0.92	0.00	1.46	1.00	1.00	1.04	1.00
time (sec)	N/A	0.264	2.331	0.684	0.000	0.130	0.633	0.176	0.202	0.178

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	26	24	25	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	1.08	1.00	1.04	1.00
time (sec)	N/A	0.278	0.746	0.727	0.000	0.132	0.978	0.181	0.197	0.189

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	153	115	194	183	128	223	316	192	0
N.S.	1	1.01	0.76	1.28	1.20	0.84	1.47	2.08	1.26	0.00
time (sec)	N/A	0.396	0.069	0.446	0.119	0.122	0.598	0.125	0.209	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	145	116	170	163	124	206	254	172	0
N.S.	1	0.97	0.78	1.14	1.09	0.83	1.38	1.70	1.15	0.00
time (sec)	N/A	0.342	0.056	0.390	0.122	0.108	0.456	0.130	0.221	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	96	154	142	107	172	210	150	0
N.S.	1	1.02	0.80	1.28	1.18	0.89	1.43	1.75	1.25	0.00
time (sec)	N/A	0.346	0.059	0.414	0.114	0.120	0.341	0.126	0.211	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	136	95	161	122	102	153	168	130	0
N.S.	1	1.13	0.79	1.34	1.02	0.85	1.28	1.40	1.08	0.00
time (sec)	N/A	0.315	0.041	0.568	0.177	0.109	0.274	0.124	0.218	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	84	71	100	91	82	109	109	103	0
N.S.	1	1.04	0.88	1.23	1.12	1.01	1.35	1.35	1.27	0.00
time (sec)	N/A	0.294	0.036	0.162	0.124	0.109	0.205	0.119	0.244	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	142	114	161	0	0	0	0	80	0
N.S.	1	1.08	0.86	1.22	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.587	0.125	1.077	0.000	0.000	0.000	0.000	0.208	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	71	79	79	103	75	1032	70	70
N.S.	1	1.08	1.08	1.20	1.20	1.56	1.14	15.64	1.06	1.06
time (sec)	N/A	0.314	0.034	0.180	0.119	0.152	1.650	0.370	0.206	0.346

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	127	101	181	0	0	0	0	61	0
N.S.	1	1.07	0.85	1.52	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.630	0.098	1.189	0.000	0.000	0.000	0.000	0.224	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	84	109	113	119	115	168	424	86	0
N.S.	1	0.99	1.28	1.33	1.40	1.35	1.98	4.99	1.01	0.00
time (sec)	N/A	0.334	0.030	0.194	0.111	0.145	2.460	120.872	0.236	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	187	329	314	219	415	598	348	0
N.S.	1	1.00	0.78	1.37	1.30	0.91	1.72	2.48	1.44	0.00
time (sec)	N/A	0.617	0.126	0.449	0.118	0.119	1.122	0.137	0.226	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	225	190	293	284	215	382	498	319	0
N.S.	1	0.93	0.79	1.22	1.18	0.89	1.59	2.07	1.32	0.00
time (sec)	N/A	0.553	0.105	0.415	0.136	0.121	0.871	0.135	0.218	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	199	158	269	253	186	333	429	280	0
N.S.	1	1.01	0.80	1.36	1.28	0.94	1.68	2.17	1.41	0.00
time (sec)	N/A	0.564	0.108	0.443	0.132	0.099	0.612	0.128	0.203	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	205	159	253	223	183	299	350	251	0
N.S.	1	1.16	0.90	1.43	1.26	1.03	1.69	1.98	1.42	0.00
time (sec)	N/A	0.496	0.093	0.604	0.125	0.122	0.485	0.129	0.218	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	152	125	194	182	151	240	265	208	0
N.S.	1	1.01	0.83	1.29	1.21	1.01	1.60	1.77	1.39	0.00
time (sec)	N/A	0.390	0.063	0.197	0.139	0.108	0.352	0.128	0.209	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	232	218	240	0	0	0	0	174	0
N.S.	1	1.01	0.95	1.05	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.716	0.207	1.228	0.000	0.000	0.000	0.000	0.239	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	114	129	157	151	172	168	4243	163	0
N.S.	1	0.90	1.02	1.25	1.20	1.37	1.33	33.67	1.29	0.00
time (sec)	N/A	0.485	0.101	0.218	0.130	0.155	2.023	1.792	0.218	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	194	184	246	0	0	0	0	147	0
N.S.	1	1.05	0.99	1.33	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.745	0.250	1.930	0.000	0.000	0.000	0.000	0.214	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	142	140	156	159	174	218	2534	152	0
N.S.	1	1.13	1.11	1.24	1.26	1.38	1.73	20.11	1.21	0.00
time (sec)	N/A	0.448	0.104	0.224	0.135	0.195	2.988	1.336	0.211	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	340	271	484	465	322	631	943	530	0
N.S.	1	1.00	0.79	1.42	1.36	0.94	1.85	2.77	1.55	0.00
time (sec)	N/A	0.834	0.175	0.467	0.122	0.125	2.166	0.141	0.248	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	403	276	436	425	318	597	807	492	0
N.S.	1	1.20	0.82	1.30	1.27	0.95	1.78	2.41	1.47	0.00
time (sec)	N/A	0.794	0.159	0.436	0.155	0.133	1.582	0.141	0.261	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	231	404	384	277	525	711	436	0
N.S.	1	1.00	0.80	1.41	1.34	0.97	1.83	2.48	1.52	0.00
time (sec)	N/A	0.848	0.145	0.445	0.119	0.168	1.157	0.141	0.238	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	288	232	365	344	274	483	597	398	0
N.S.	1	1.15	0.92	1.45	1.37	1.09	1.92	2.38	1.59	0.00
time (sec)	N/A	0.642	0.125	0.596	0.139	0.126	0.895	0.137	0.248	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	226	187	305	292	229	389	480	338	0
N.S.	1	1.00	0.83	1.36	1.30	1.02	1.73	2.13	1.50	0.00
time (sec)	N/A	0.681	0.092	0.191	0.126	0.122	0.640	0.142	0.206	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	356	322	341	0	0	0	0	295	0
N.S.	1	1.00	0.90	0.96	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.022	0.326	2.129	0.000	0.000	0.000	0.000	0.214	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	183	250	241	239	275	10769	269	0
N.S.	1	1.00	0.96	1.32	1.27	1.26	1.45	56.68	1.42	0.00
time (sec)	N/A	0.659	0.125	0.226	0.127	0.194	2.677	11.623	0.216	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	266	262	357	0	0	0	0	246	0
N.S.	1	1.02	1.00	1.36	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	1.167	0.277	2.960	0.000	0.000	0.000	0.000	0.232	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	172	194	235	231	246	311	7971	248	0
N.S.	1	0.92	1.04	1.26	1.24	1.32	1.67	42.85	1.33	0.00
time (sec)	N/A	0.691	0.160	0.237	0.134	0.227	3.547	8.180	0.232	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	515	376	0	0	0	0	65	0
N.S.	1	1.00	0.79	0.58	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.602	0.595	282.731	0.000	0.000	0.000	0.000	0.211	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	475	2088	0	0	0	0	97	0
N.S.	1	1.00	0.85	3.74	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.406	0.278	5.133	0.000	0.000	0.000	0.000	0.196	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	456	285	0	0	0	0	78	0
N.S.	1	1.00	0.79	0.49	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.405	0.275	320.761	0.000	0.000	0.000	0.000	0.230	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	399	1965	0	0	0	0	37	0
N.S.	1	1.00	0.81	4.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.243	0.136	2.432	0.000	0.000	0.000	0.000	0.224	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	490	236	0	0	0	0	46	0
N.S.	1	1.00	0.91	0.44	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.396	0.148	1.888	0.000	0.000	0.000	0.000	0.201	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	430	344	0	0	0	0	44	0
N.S.	1	1.00	0.83	0.66	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.686	0.141	2.334	0.000	0.000	0.000	0.000	0.202	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	579	579	455	363	0	0	0	0	58	0
N.S.	1	1.00	0.79	0.63	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.547	0.257	379.011	0.000	0.000	0.000	0.000	0.226	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	509	421	0	0	0	0	65	0
N.S.	1	1.00	0.89	0.73	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.510	0.190	2.316	0.000	0.000	0.000	0.000	0.229	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	531	491	0	0	0	0	75	0
N.S.	1	1.00	0.82	0.76	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.452	0.276	356.983	0.000	0.000	0.000	0.000	0.258	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	593	2101	0	0	0	0	123	0
N.S.	1	1.00	1.03	3.66	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.428	0.788	5.778	0.000	0.000	0.000	0.000	0.238	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	83	87	404	0	395	0	0	1401	0
N.S.	1	0.97	1.01	4.70	0.00	4.59	0.00	0.00	16.29	0.00
time (sec)	N/A	0.251	0.097	7.368	0.000	0.142	0.000	0.000	0.240	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	640	468	0	0	0	0	138	0
N.S.	1	1.00	1.07	0.78	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.522	0.783	11.188	0.000	0.000	0.000	0.000	0.212	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	687	589	0	0	0	0	168	0
N.S.	1	1.00	1.09	0.93	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.653	1.592	2.926	0.000	0.000	0.000	0.000	0.262	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	649	914	0	0	0	0	149	0
N.S.	1	1.00	0.82	1.16	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.820	1.046	5.409	0.000	0.000	0.000	0.000	0.230	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	603	811	0	0	0	0	144	0
N.S.	1	1.00	0.81	1.09	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.861	0.812	417.554	0.000	0.000	0.000	0.000	0.237	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	757	757	591	843	0	0	0	0	137	0
N.S.	1	1.00	0.78	1.11	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.618	1.112	3.808	0.000	0.000	0.000	0.000	0.197	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	672	965	0	0	0	0	155	0
N.S.	1	1.00	0.85	1.21	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.589	1.021	6.720	0.000	0.000	0.000	0.000	0.254	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	705	705	973	3508	0	0	0	0	238	0
N.S.	1	1.00	1.38	4.98	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.608	4.716	23.833	0.000	0.000	0.000	0.000	0.230	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	178	152	1020	0	921	0	0	180	0
N.S.	1	1.16	0.99	6.67	0.00	6.02	0.00	0.00	1.18	0.00
time (sec)	N/A	0.419	0.320	19.654	0.000	0.251	0.000	0.000	0.218	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	137	141	998	0	783	0	0	172	0
N.S.	1	1.03	1.06	7.50	0.00	5.89	0.00	0.00	1.29	0.00
time (sec)	N/A	0.313	0.329	18.997	0.000	0.271	0.000	0.000	0.235	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	727	727	1022	1130	0	0	0	0	265	0
N.S.	1	1.00	1.41	1.55	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.662	3.704	4.478	0.000	0.000	0.000	0.000	0.259	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	783	783	1065	1344	0	0	0	0	299	0
N.S.	1	1.00	1.36	1.72	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.809	5.303	4.612	0.000	0.000	0.000	0.000	0.296	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1082	1082	1014	1752	0	0	0	0	272	0
N.S.	1	1.00	0.94	1.62	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	3.812	3.768	2.804	0.000	0.000	0.000	0.000	0.236	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1014	1224	0	0	0	0	271	0
N.S.	1	1.00	0.93	1.12	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	3.155	4.021	32.563	0.000	0.000	0.000	0.000	0.241	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1033	1772	0	0	0	0	263	0
N.S.	1	1.00	0.95	1.62	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.858	4.394	12.170	0.000	0.000	0.000	0.000	0.233	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	63	213	59	0	0	0	0	19	0
N.S.	1	1.17	3.94	1.09	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.373	0.228	0.577	0.000	0.000	0.000	0.000	0.245	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	213	169	0	0	0	0	17	0
N.S.	1	1.00	0.87	0.69	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.732	0.076	0.784	0.000	0.000	0.000	0.000	0.263	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	492	310	0	0	0	0	0	656	0
N.S.	1	1.02	0.64	0.00	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	2.230	0.308	0.000	0.000	0.000	0.000	0.000	0.304	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	296	224	0	0	0	0	0	360	0
N.S.	1	1.01	0.76	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.661	0.159	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	170	122	0	0	0	0	0	149	0
N.S.	1	1.06	0.76	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.393	0.107	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	0	43	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	0.00	1.87	1.09
time (sec)	N/A	0.254	1.673	1.475	0.372	0.114	9.706	0.000	0.263	0.257

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	0	65	25
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	0.00	2.83	1.09
time (sec)	N/A	0.242	2.951	0.583	0.383	0.111	0.000	0.000	0.308	0.272

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	280	297	759	0	0	0	0	135	0
N.S.	1	0.80	0.85	2.16	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.704	1.156	4.335	0.000	0.000	0.000	0.000	0.421	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	152	178	649	0	197	0	0	110	0
N.S.	1	0.68	0.79	2.88	0.00	0.88	0.00	0.00	0.49	0.00
time (sec)	N/A	0.848	1.139	4.129	0.000	0.131	0.000	0.000	0.358	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	146	288	634	0	0	0	0	100	0
N.S.	1	0.66	1.30	2.86	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.767	1.498	3.415	0.000	0.000	0.000	0.000	0.304	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	209	434	895	0	0	0	0	185	0
N.S.	1	0.48	1.00	2.07	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.409	1.935	3.601	0.000	0.000	0.000	0.000	0.307	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	159	374	687	0	0	0	0	105	0
N.S.	1	0.62	1.46	2.67	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.485	1.155	3.858	0.000	0.000	0.000	0.000	0.300	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	456	452	1464	0	0	0	0	224	0
N.S.	1	0.90	0.89	2.88	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	2.626	1.907	6.729	0.000	0.000	0.000	0.000	0.493	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	193	207	679	0	302	0	0	200	0
N.S.	1	0.57	0.61	2.01	0.00	0.89	0.00	0.00	0.59	0.00
time (sec)	N/A	0.941	2.056	4.113	0.000	0.151	0.000	0.000	0.475	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	277	373	1101	0	0	0	0	190	0
N.S.	1	0.77	1.03	3.04	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.183	2.761	4.031	0.000	0.000	0.000	0.000	0.411	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	321	632	1326	0	0	0	0	275	0
N.S.	1	0.50	0.98	2.05	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	2.166	4.465	4.454	0.000	0.000	0.000	0.000	0.377	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	317	538	629	0	0	0	0	192	0
N.S.	1	0.63	1.07	1.25	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	2.078	2.613	4.516	0.000	0.000	0.000	0.000	0.420	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	150	326	1006	0	0	0	0	123	0
N.S.	1	0.60	1.30	4.02	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.084	1.235	3.071	0.000	0.000	0.000	0.000	0.326	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	102	150	363	0	138	0	0	97	0
N.S.	1	0.75	1.10	2.67	0.00	1.01	0.00	0.00	0.71	0.00
time (sec)	N/A	0.660	1.587	2.269	0.000	0.115	0.000	0.000	0.315	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	159	197	0	0	0	0	91	0
N.S.	1	1.00	2.89	3.58	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.452	2.042	2.136	0.000	0.000	0.000	0.000	0.272	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	150	336	601	0	0	0	0	180	0
N.S.	1	0.52	1.17	2.09	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.069	2.293	3.081	0.000	0.000	0.000	0.000	0.269	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	141	189	539	0	0	0	0	97	0
N.S.	1	0.66	0.88	2.52	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	1.262	2.315	2.432	0.000	0.000	0.000	0.000	0.273	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	169	636	725	0	0	0	0	212	0
N.S.	1	0.57	2.16	2.46	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.608	3.149	6.327	0.000	0.000	0.000	0.000	0.358	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	143	453	383	0	0	0	0	174	0
N.S.	1	0.59	1.86	1.57	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	1.119	3.232	3.443	0.000	0.000	0.000	0.000	0.299	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	140	550	705	0	0	0	0	165	0
N.S.	1	0.61	2.38	3.05	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.901	2.818	3.253	0.000	0.000	0.000	0.000	0.287	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	548	250	877	1083	0	0	0	0	323	0
N.S.	1	0.46	1.60	1.98	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	2.421	5.614	3.995	0.000	0.000	0.000	0.000	0.333	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	247	564	723	0	0	0	0	187	0
N.S.	1	0.62	1.42	1.83	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	2.369	3.788	4.258	0.000	0.000	0.000	0.000	0.309	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [176] had the largest ratio of [1.1111000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.98	23	0.217
2	A	6	6	1.10	23	0.261
3	A	6	5	1.00	23	0.217
4	A	4	4	1.03	21	0.190
5	A	6	5	1.03	20	0.250
6	A	12	11	1.12	23	0.478
7	A	8	7	1.03	23	0.304
8	A	12	11	0.99	23	0.478
9	A	7	6	0.99	23	0.261
10	A	6	5	0.94	25	0.200
11	A	9	9	1.04	25	0.360
12	A	6	5	0.95	25	0.200
13	A	5	5	0.98	23	0.217
14	A	6	5	0.97	22	0.227
15	A	17	16	1.23	25	0.640
16	A	8	7	0.94	25	0.280
17	A	17	16	1.16	25	0.640
18	A	11	10	1.07	25	0.400
19	A	6	5	0.93	25	0.200
20	A	8	8	0.97	25	0.320
21	A	6	5	0.94	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	6	0.97	23	0.261
23	A	6	5	0.95	22	0.227
24	A	22	21	1.43	25	0.840
25	A	6	5	0.91	25	0.200
26	A	22	21	1.30	25	0.840
27	A	10	9	0.90	25	0.360
28	A	13	12	1.08	25	0.480
29	A	11	10	1.01	25	0.400
30	A	9	8	0.92	25	0.320
31	A	7	6	0.95	23	0.261
32	A	6	5	0.87	22	0.227
33	A	7	6	0.94	25	0.240
34	A	11	10	0.95	25	0.400
35	A	10	9	0.93	25	0.360
36	A	16	15	1.02	25	0.600
37	A	13	12	1.06	25	0.480
38	A	11	10	0.99	25	0.400
39	A	9	8	0.92	25	0.320
40	A	2	2	1.00	23	0.087
41	A	9	8	0.92	22	0.364
42	A	10	9	0.98	25	0.360
43	A	14	13	1.16	25	0.520
44	A	13	12	1.32	25	0.480
45	A	20	19	1.30	25	0.760
46	A	13	12	1.08	25	0.480
47	A	4	4	0.99	25	0.160
48	A	11	10	0.93	25	0.400
49	A	3	3	0.98	23	0.130
50	A	11	10	0.93	22	0.455
51	A	13	12	1.10	25	0.480
52	A	17	16	1.18	25	0.640
53	A	17	16	1.26	25	0.640

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	24	23	1.34	25	0.920
55	A	7	7	0.82	27	0.259
56	A	5	5	0.87	27	0.185
57	A	3	3	1.00	24	0.125
58	A	3	3	1.00	27	0.111
59	A	3	3	0.76	27	0.111
60	A	4	4	0.70	27	0.148
61	A	4	4	0.66	27	0.148
62	A	3	3	0.67	27	0.111
63	A	3	3	0.70	27	0.111
64	A	2	2	0.75	25	0.080
65	A	8	7	0.72	27	0.259
66	A	8	7	0.72	27	0.259
67	A	10	9	0.69	27	0.333
68	A	10	10	0.90	27	0.370
69	A	8	8	0.96	27	0.296
70	A	6	6	1.10	24	0.250
71	A	6	6	1.06	27	0.222
72	A	6	6	1.04	27	0.222
73	A	5	4	0.61	27	0.148
74	A	6	5	0.60	27	0.185
75	A	6	5	0.61	27	0.185
76	A	6	5	0.59	27	0.185
77	A	4	4	0.60	27	0.148
78	A	4	4	0.60	27	0.148
79	A	4	4	0.61	27	0.148
80	A	3	3	0.61	25	0.120
81	A	10	9	0.80	27	0.333
82	A	11	10	0.79	27	0.370
83	A	11	10	0.82	27	0.370
84	A	15	14	0.95	27	0.519
85	A	13	12	1.01	27	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	8	8	1.06	24	0.333
87	A	11	10	0.98	27	0.370
88	A	11	10	1.07	27	0.370
89	A	11	10	1.07	27	0.370
90	A	5	4	0.53	27	0.148
91	A	7	6	0.60	27	0.222
92	A	6	5	0.55	27	0.185
93	A	4	4	0.55	27	0.148
94	A	4	4	0.54	27	0.148
95	A	3	3	0.51	25	0.120
96	A	13	12	0.86	27	0.444
97	A	13	12	0.84	27	0.444
98	A	14	13	0.85	27	0.481
99	A	5	5	1.09	22	0.227
100	A	4	4	1.04	22	0.182
101	A	3	3	1.00	22	0.136
102	A	2	2	1.00	20	0.100
103	A	1	1	1.00	19	0.053
104	A	6	5	1.00	22	0.227
105	A	2	2	1.00	22	0.091
106	A	8	7	0.95	22	0.318
107	A	6	6	1.05	27	0.222
108	A	5	5	1.04	27	0.185
109	A	4	4	1.02	27	0.148
110	A	3	3	1.00	27	0.111
111	A	2	2	1.00	25	0.080
112	A	1	1	1.00	24	0.042
113	A	6	5	0.61	27	0.185
114	A	2	2	1.00	27	0.074
115	A	8	7	0.72	27	0.259
116	A	4	4	0.99	27	0.148
117	A	4	4	0.71	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	7	1.08	27	0.259
119	A	4	4	0.77	27	0.148
120	A	3	3	1.00	27	0.111
121	A	2	2	1.00	25	0.080
122	A	2	2	1.00	24	0.083
123	A	8	7	0.71	27	0.259
124	A	7	6	0.80	27	0.222
125	A	11	10	0.79	27	0.370
126	A	6	5	0.74	27	0.185
127	A	12	11	1.22	27	0.407
128	A	6	6	0.78	27	0.222
129	A	8	7	1.19	27	0.259
130	A	4	4	0.89	27	0.148
131	A	5	4	0.83	27	0.148
132	A	3	3	0.87	25	0.120
133	A	4	4	1.03	24	0.167
134	A	11	10	0.88	27	0.370
135	A	6	5	0.75	27	0.185
136	A	15	14	0.96	27	0.519
137	A	6	5	0.72	27	0.185
138	A	1	1	1.00	30	0.033
139	A	1	1	1.00	31	0.032
140	A	9	9	1.03	25	0.360
141	A	7	7	1.00	25	0.280
142	A	4	4	0.98	23	0.174
143	N/A	1	0	1.00	25	0.000
144	N/A	4	0	1.00	25	0.000
145	N/A	6	0	1.00	25	0.000
146	A	9	9	0.71	27	0.333
147	A	6	6	0.82	27	0.222
148	A	3	3	0.89	27	0.111
149	A	1	1	1.00	27	0.037

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	3	3	1.00	27	0.111
151	A	5	5	0.96	27	0.185
152	A	1	1	1.00	22	0.045
153	A	11	11	1.14	25	0.440
154	A	9	9	1.65	25	0.360
155	A	9	9	1.14	25	0.360
156	A	7	7	1.04	23	0.304
157	A	5	5	1.09	22	0.227
158	A	14	13	1.10	25	0.520
159	A	12	11	1.17	25	0.440
160	A	14	13	1.01	25	0.520
161	A	10	9	1.28	25	0.360
162	A	17	17	1.31	27	0.630
163	A	13	13	1.97	27	0.481
164	A	15	15	1.25	27	0.556
165	A	9	9	0.96	25	0.360
166	A	8	8	1.11	24	0.333
167	A	22	21	1.25	27	0.778
168	A	17	16	1.26	27	0.593
169	A	22	21	1.20	27	0.778
170	A	16	15	1.36	27	0.556
171	A	22	22	1.49	27	0.815
172	B	18	17	2.39	27	0.630
173	A	20	20	1.37	27	0.741
174	A	11	11	0.94	25	0.440
175	A	9	9	1.17	24	0.375
176	A	31	30	1.51	27	1.111
177	A	22	21	1.41	27	0.778
178	F	0	0	N/A	0.000	N/A
179	A	20	19	1.58	27	0.704
180	A	13	12	1.06	27	0.444
181	A	13	12	0.99	27	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	11	10	0.91	27	0.370
183	A	8	7	0.97	25	0.280
184	A	7	6	0.87	24	0.250
185	A	8	7	0.96	27	0.259
186	A	14	13	0.91	27	0.481
187	A	12	11	0.90	27	0.407
188	A	16	15	1.05	27	0.556
189	A	16	15	1.04	27	0.556
190	A	13	12	1.00	27	0.444
191	A	11	10	0.94	27	0.370
192	A	3	3	0.97	25	0.120
193	A	11	10	0.93	24	0.417
194	A	12	11	0.98	27	0.407
195	A	19	18	1.04	27	0.667
196	A	18	17	1.24	27	0.630
197	A	22	21	1.19	27	0.778
198	A	16	15	1.05	27	0.556
199	A	9	8	1.09	27	0.296
200	A	13	12	0.96	27	0.444
201	A	5	5	0.93	25	0.200
202	A	13	12	0.96	24	0.500
203	A	17	16	1.14	27	0.593
204	A	24	23	1.17	27	0.852
205	A	23	22	1.24	27	0.815
206	A	28	27	1.42	27	1.000
207	A	13	12	1.13	29	0.414
208	A	10	10	1.00	29	0.345
209	A	7	6	0.82	27	0.222
210	A	5	5	0.88	26	0.192
211	A	9	8	0.62	29	0.276
212	A	11	10	0.80	29	0.345
213	A	12	11	0.62	29	0.379

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	13	12	0.66	29	0.414
215	A	19	18	1.29	29	0.621
216	A	17	17	1.21	29	0.586
217	A	7	6	0.73	27	0.222
218	A	10	10	1.12	26	0.385
219	A	15	14	0.71	29	0.483
220	A	18	17	0.87	29	0.586
221	A	16	15	0.64	29	0.517
222	A	23	22	0.98	29	0.759
223	A	25	24	1.43	29	0.828
224	A	27	27	1.37	29	0.931
225	A	7	6	0.66	27	0.222
226	A	12	12	1.27	26	0.462
227	A	21	20	0.85	29	0.690
228	A	27	26	1.13	29	0.897
229	A	23	22	0.77	29	0.759
230	F	0	0	N/A	0.000	N/A
231	A	13	12	1.22	29	0.414
232	A	10	10	1.07	29	0.345
233	A	8	7	1.09	29	0.241
234	A	5	5	0.85	29	0.172
235	A	2	2	0.95	27	0.074
236	A	1	1	1.00	26	0.038
237	A	7	6	0.56	29	0.207
238	A	10	9	0.75	29	0.310
239	A	12	11	0.62	29	0.379
240	A	13	12	0.82	29	0.414
241	A	19	18	1.15	29	0.621
242	A	15	14	0.95	29	0.483
243	A	11	10	0.90	29	0.345
244	A	9	8	0.78	29	0.276
245	A	7	6	0.66	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	7	0.72	26	0.269
247	A	13	12	0.60	29	0.414
248	A	15	14	0.83	29	0.483
249	A	23	22	0.72	29	0.759
250	A	17	16	0.98	29	0.552
251	A	15	14	1.26	29	0.483
252	A	12	11	1.01	29	0.379
253	A	15	14	1.05	29	0.483
254	A	11	10	0.66	29	0.345
255	A	9	8	0.67	27	0.296
256	A	11	10	0.87	26	0.385
257	A	21	20	0.82	29	0.690
258	A	20	19	1.01	29	0.655
259	F	0	0	N/A	0.000	N/A
260	A	23	22	1.34	29	0.759
261	A	10	10	1.43	24	0.417
262	A	9	8	1.26	24	0.333
263	A	5	5	1.09	24	0.208
264	A	3	3	1.02	22	0.136
265	A	1	1	1.00	21	0.048
266	A	7	6	1.07	24	0.250
267	A	10	9	1.16	24	0.375
268	A	12	11	1.00	24	0.458
269	A	12	12	0.79	27	0.444
270	A	9	9	0.86	27	0.333
271	A	5	5	0.95	25	0.200
272	N/A	1	0	1.00	27	0.000
273	N/A	6	0	1.00	27	0.000
274	N/A	8	0	1.00	27	0.000
275	N/A	15	0	1.00	29	0.000
276	N/A	8	0	1.00	29	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
277	N/A	4	0	1.00	29	0.000
278	N/A	1	0	1.00	29	0.000
279	N/A	1	0	1.00	29	0.000
280	N/A	1	0	1.00	29	0.000
281	N/A	1	0	1.00	24	0.000
282	A	9	9	1.45	24	0.375
283	A	10	10	1.32	24	0.417
284	A	6	6	1.08	24	0.250
285	A	4	4	1.07	22	0.182
286	A	1	1	1.00	21	0.048
287	A	8	7	1.09	24	0.292
288	A	11	10	1.18	24	0.417
289	A	12	11	0.97	24	0.458
290	A	4	3	0.83	28	0.107
291	A	5	4	0.84	28	0.143
292	A	4	3	0.87	28	0.107
293	A	5	4	0.86	26	0.154
294	A	5	4	0.87	25	0.160
295	N/A	2	0	1.00	28	0.000
296	N/A	2	0	1.00	28	0.000
297	N/A	1	0	1.00	28	0.000
298	A	5	4	0.83	28	0.143
299	A	4	3	0.83	28	0.107
300	A	5	4	0.84	26	0.154
301	A	5	4	0.84	25	0.160
302	N/A	2	0	1.00	28	0.000
303	N/A	2	0	1.00	28	0.000
304	N/A	1	0	1.00	28	0.000
305	A	5	4	0.83	28	0.143
306	A	4	3	0.82	28	0.107
307	A	5	4	0.83	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	5	4	0.83	25	0.160
309	N/A	2	0	1.00	28	0.000
310	N/A	2	0	1.00	28	0.000
311	N/A	1	0	1.00	28	0.000
312	A	5	4	0.88	24	0.167
313	A	5	4	0.93	24	0.167
314	A	5	4	0.93	24	0.167
315	A	5	4	0.93	24	0.167
316	A	4	3	1.00	22	0.136
317	A	1	1	1.00	21	0.048
318	N/A	1	0	1.00	24	0.000
319	N/A	1	0	1.00	24	0.000
320	A	6	5	0.84	28	0.179
321	A	5	4	0.84	28	0.143
322	A	6	5	0.86	28	0.179
323	A	5	4	0.87	28	0.143
324	A	9	8	0.91	26	0.308
325	A	1	1	1.00	25	0.040
326	N/A	1	0	1.00	28	0.000
327	N/A	1	0	1.00	28	0.000
328	N/A	1	0	1.00	28	0.000
329	N/A	1	0	1.00	26	0.000
330	N/A	1	0	1.00	25	0.000
331	N/A	1	0	1.00	28	0.000
332	N/A	1	0	1.00	28	0.000
333	N/A	1	0	1.00	28	0.000
334	N/A	1	0	1.00	26	0.000
335	N/A	1	0	1.00	25	0.000
336	N/A	1	0	1.00	28	0.000
337	N/A	1	0	1.00	28	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	N/A	1	0	1.00	28	0.000
339	N/A	1	0	1.00	28	0.000
340	N/A	1	0	1.00	28	0.000
341	N/A	1	0	1.00	28	0.000
342	N/A	1	0	1.00	28	0.000
343	N/A	1	0	1.00	28	0.000
344	N/A	1	0	1.00	24	0.000
345	N/A	1	0	1.00	28	0.000
346	N/A	1	0	1.00	28	0.000
347	N/A	1	0	1.00	28	0.000
348	N/A	2	0	1.00	28	0.000
349	N/A	1	0	1.00	28	0.000
350	N/A	1	0	1.00	28	0.000
351	A	5	4	1.36	28	0.143
352	B	13	12	2.06	28	0.429
353	A	12	11	1.22	26	0.423
354	A	12	11	0.97	25	0.440
355	N/A	10	0	1.00	28	0.000
356	N/A	2	0	1.00	28	0.000
357	N/A	1	0	1.00	28	0.000
358	A	5	4	1.42	28	0.143
359	A	6	5	1.15	28	0.179
360	A	9	8	1.36	26	0.308
361	A	6	5	0.93	25	0.200
362	N/A	7	0	1.00	28	0.000
363	N/A	2	0	1.00	28	0.000
364	N/A	1	0	1.00	28	0.000
365	A	5	4	1.78	28	0.143
366	A	6	5	1.43	28	0.179
367	A	9	8	1.42	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	6	5	0.88	25	0.200
369	N/A	7	0	1.00	28	0.000
370	N/A	2	0	1.00	28	0.000
371	N/A	1	0	1.00	28	0.000
372	A	5	4	0.87	28	0.143
373	A	6	5	0.93	28	0.179
374	A	5	4	0.89	28	0.143
375	A	12	11	0.95	28	0.393
376	A	9	8	0.94	26	0.308
377	A	1	1	1.00	25	0.040
378	N/A	2	0	1.00	28	0.000
379	N/A	2	0	1.00	28	0.000
380	N/A	1	0	1.00	28	0.000
381	N/A	2	0	1.00	28	0.000
382	N/A	1	0	1.00	26	0.000
383	N/A	2	0	1.00	25	0.000
384	N/A	1	0	1.00	28	0.000
385	N/A	1	0	1.00	28	0.000
386	A	1	1	1.00	33	0.030
387	B	5	4	2.02	27	0.148
388	A	6	5	0.99	27	0.185
389	A	9	8	1.18	25	0.320
390	A	6	5	1.01	24	0.208
391	N/A	8	0	1.00	27	0.000
392	N/A	4	0	1.00	27	0.000
393	A	5	4	1.25	29	0.138
394	A	6	5	1.57	29	0.172
395	A	9	8	1.62	27	0.296
396	A	6	5	0.94	26	0.192
397	N/A	8	0	1.00	29	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
398	N/A	4	0	1.00	29	0.000
399	A	1	1	1.00	38	0.026
400	A	5	4	1.00	19	0.211
401	A	4	3	0.76	29	0.103
402	A	5	4	0.77	27	0.148
403	A	5	4	0.76	26	0.154
404	N/A	2	0	1.00	29	0.000
405	N/A	2	0	1.00	29	0.000
406	A	4	3	0.72	29	0.103
407	A	5	4	0.74	27	0.148
408	A	5	4	0.73	26	0.154
409	N/A	2	0	1.00	29	0.000
410	N/A	2	0	1.00	29	0.000
411	A	4	3	0.70	29	0.103
412	A	5	4	0.71	27	0.148
413	A	5	4	0.71	26	0.154
414	N/A	2	0	1.00	29	0.000
415	N/A	2	0	1.00	29	0.000
416	N/A	1	0	1.00	24	0.000
417	A	5	4	0.95	24	0.167
418	A	5	4	0.95	24	0.167
419	A	5	4	0.97	22	0.182
420	A	1	1	1.00	21	0.048
421	N/A	1	0	1.00	24	0.000
422	N/A	1	0	1.00	24	0.000
423	A	6	5	1.01	19	0.263
424	A	6	6	0.97	19	0.316
425	A	6	5	1.02	19	0.263
426	A	5	5	1.13	17	0.294
427	A	6	5	1.04	16	0.312
428	A	4	4	1.08	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
429	A	7	6	1.08	19	0.316
430	A	4	4	1.07	19	0.211
431	A	7	6	0.99	19	0.316
432	A	6	5	1.00	21	0.238
433	A	8	8	0.93	21	0.381
434	A	6	5	1.01	21	0.238
435	A	7	7	1.16	19	0.368
436	A	6	5	1.01	18	0.278
437	A	4	4	1.01	21	0.190
438	A	8	7	0.90	21	0.333
439	A	4	4	1.05	21	0.190
440	A	9	8	1.13	21	0.381
441	A	6	5	1.00	21	0.238
442	A	12	12	1.20	21	0.571
443	A	6	5	1.00	21	0.238
444	A	9	9	1.15	19	0.474
445	A	6	5	1.00	18	0.278
446	A	4	4	1.00	21	0.190
447	A	6	5	1.00	21	0.238
448	A	4	4	1.02	21	0.190
449	A	10	9	0.92	21	0.429
450	A	2	2	1.00	21	0.095
451	A	2	2	1.00	21	0.095
452	A	2	2	1.00	21	0.095
453	A	2	2	1.00	19	0.105
454	A	2	2	1.00	18	0.111
455	A	2	2	1.00	21	0.095
456	A	2	2	1.00	21	0.095
457	A	2	2	1.00	21	0.095
458	A	2	2	1.00	21	0.095
459	A	2	2	1.00	21	0.095
460	A	4	3	0.97	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	A	2	2	1.00	21	0.095
462	A	2	2	1.00	21	0.095
463	A	2	2	1.00	21	0.095
464	A	2	2	1.00	21	0.095
465	A	2	2	1.00	18	0.111
466	A	2	2	1.00	21	0.095
467	A	2	2	1.00	21	0.095
468	A	8	7	1.16	21	0.333
469	A	5	4	1.03	19	0.211
470	A	2	2	1.00	21	0.095
471	A	2	2	1.00	21	0.095
472	A	2	2	1.00	21	0.095
473	A	2	2	1.00	21	0.095
474	A	2	2	1.00	18	0.111
475	A	7	6	1.17	15	0.400
476	A	2	2	1.00	15	0.133
477	A	8	8	1.02	23	0.348
478	A	6	6	1.01	23	0.261
479	A	4	4	1.06	21	0.190
480	N/A	1	0	1.00	23	0.000
481	N/A	1	0	1.00	23	0.000
482	A	11	11	0.80	35	0.314
483	A	8	7	0.68	33	0.212
484	A	6	6	0.66	32	0.188
485	A	10	9	0.48	35	0.257
486	A	12	11	0.62	35	0.314
487	A	18	18	0.90	35	0.514
488	A	8	7	0.57	33	0.212
489	A	11	11	0.77	32	0.344
490	A	16	15	0.50	35	0.429
491	A	19	18	0.63	35	0.514
492	A	6	6	0.60	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
493	A	3	3	0.75	33	0.091
494	A	2	2	1.00	32	0.062
495	A	8	7	0.52	35	0.200
496	A	11	10	0.66	35	0.286
497	A	10	9	0.57	35	0.257
498	A	8	7	0.59	33	0.212
499	A	9	8	0.61	32	0.250
500	A	14	13	0.46	35	0.371
501	A	16	15	0.62	35	0.429

CHAPTER 3

LISTING OF INTEGRALS

3.1	$\int x^4(d - c^2 dx^2)(a + b \arcsin(cx)) dx$	206
3.2	$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx$	213
3.3	$\int x^2(d - c^2 dx^2)(a + b \arcsin(cx)) dx$	220
3.4	$\int x(d - c^2 dx^2)(a + b \arcsin(cx)) dx$	227
3.5	$\int (d - c^2 dx^2)(a + b \arcsin(cx)) dx$	233
3.6	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx$	240
3.7	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx$	248
3.8	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx$	256
3.9	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$	264
3.10	$\int x^4(d - c^2 dx^2)^2(a + b \arcsin(cx)) dx$	272
3.11	$\int x^3(d - c^2 dx^2)^2(a + b \arcsin(cx)) dx$	280
3.12	$\int x^2(d - c^2 dx^2)^2(a + b \arcsin(cx)) dx$	289
3.13	$\int x(d - c^2 dx^2)^2(a + b \arcsin(cx)) dx$	297
3.14	$\int (d - c^2 dx^2)^2(a + b \arcsin(cx)) dx$	304
3.15	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))}{x} dx$	311
3.16	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))}{x^2} dx$	321
3.17	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))}{x^3} dx$	330
3.18	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))}{x^4} dx$	341
3.19	$\int x^4(d - c^2 dx^2)^3(a + b \arcsin(cx)) dx$	351
3.20	$\int x^3(d - c^2 dx^2)^3(a + b \arcsin(cx)) dx$	360
3.21	$\int x^2(d - c^2 dx^2)^3(a + b \arcsin(cx)) dx$	369
3.22	$\int x(d - c^2 dx^2)^3(a + b \arcsin(cx)) dx$	377
3.23	$\int (d - c^2 dx^2)^3(a + b \arcsin(cx)) dx$	385
3.24	$\int \frac{(d - c^2 dx^2)^3(a + b \arcsin(cx))}{x} dx$	392
3.25	$\int \frac{(d - c^2 dx^2)^3(a + b \arcsin(cx))}{x^2} dx$	403

3.26	$\int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))}{x^3} dx$	412
3.27	$\int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))}{x^4} dx$	424
3.28	$\int \frac{x^4 (a+b \arcsin(cx))}{d-c^2 dx^2} dx$	433
3.29	$\int \frac{x^3 (a+b \arcsin(cx))}{d-c^2 dx^2} dx$	442
3.30	$\int \frac{x^2 (a+b \arcsin(cx))}{d-c^2 dx^2} dx$	450
3.31	$\int \frac{x (a+b \arcsin(cx))}{d-c^2 dx^2} dx$	457
3.32	$\int \frac{a+b \arcsin(cx)}{d-c^2 dx^2} dx$	463
3.33	$\int \frac{a+b \arcsin(cx)}{x (d-c^2 dx^2)} dx$	469
3.34	$\int \frac{a+b \arcsin(cx)}{x^2 (d-c^2 dx^2)} dx$	476
3.35	$\int \frac{a+b \arcsin(cx)}{x^3 (d-c^2 dx^2)} dx$	484
3.36	$\int \frac{a+b \arcsin(cx)}{x^4 (d-c^2 dx^2)} dx$	492
3.37	$\int \frac{x^4 (a+b \arcsin(cx))}{(d-c^2 dx^2)^2} dx$	501
3.38	$\int \frac{x^3 (a+b \arcsin(cx))}{(d-c^2 dx^2)^2} dx$	511
3.39	$\int \frac{x^2 (a+b \arcsin(cx))}{(d-c^2 dx^2)^2} dx$	520
3.40	$\int \frac{x (a+b \arcsin(cx))}{(d-c^2 dx^2)^2} dx$	528
3.41	$\int \frac{a+b \arcsin(cx)}{(d-c^2 dx^2)^2} dx$	534
3.42	$\int \frac{a+b \arcsin(cx)}{x (d-c^2 dx^2)^2} dx$	542
3.43	$\int \frac{a+b \arcsin(cx)}{x^2 (d-c^2 dx^2)^2} dx$	550
3.44	$\int \frac{a+b \arcsin(cx)}{x^3 (d-c^2 dx^2)^2} dx$	560
3.45	$\int \frac{a+b \arcsin(cx)}{x^4 (d-c^2 dx^2)^2} dx$	569
3.46	$\int \frac{x^4 (a+b \arcsin(cx))}{(d-c^2 dx^2)^3} dx$	580
3.47	$\int \frac{x^3 (a+b \arcsin(cx))}{(d-c^2 dx^2)^3} dx$	589
3.48	$\int \frac{x^2 (a+b \arcsin(cx))}{(d-c^2 dx^2)^3} dx$	595
3.49	$\int \frac{x (a+b \arcsin(cx))}{(d-c^2 dx^2)^3} dx$	604
3.50	$\int \frac{a+b \arcsin(cx)}{(d-c^2 dx^2)^3} dx$	610
3.51	$\int \frac{a+b \arcsin(cx)}{x (d-c^2 dx^2)^3} dx$	619
3.52	$\int \frac{a+b \arcsin(cx)}{x^2 (d-c^2 dx^2)^3} dx$	628
3.53	$\int \frac{a+b \arcsin(cx)}{x^3 (d-c^2 dx^2)^3} dx$	639
3.54	$\int \frac{a+b \arcsin(cx)}{x^4 (d-c^2 dx^2)^3} dx$	650
3.55	$\int x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx$	663
3.56	$\int x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx$	671
3.57	$\int \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx$	678

3.58	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^2} dx$	684
3.59	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^4} dx$	690
3.60	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^6} dx$	696
3.61	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^8} dx$	704
3.62	$\int x^5 \sqrt{d-c^2dx^2}(a+b \arcsin(cx)) dx$	712
3.63	$\int x^3 \sqrt{d-c^2dx^2}(a+b \arcsin(cx)) dx$	719
3.64	$\int x \sqrt{d-c^2dx^2}(a+b \arcsin(cx)) dx$	726
3.65	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x} dx$	732
3.66	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^3} dx$	739
3.67	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^5} dx$	747
3.68	$\int x^4 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	755
3.69	$\int x^2 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	765
3.70	$\int (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	773
3.71	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^2} dx$	781
3.72	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^4} dx$	789
3.73	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^6} dx$	797
3.74	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^8} dx$	804
3.75	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^{10}} dx$	812
3.76	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^{12}} dx$	820
3.77	$\int x^7 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	828
3.78	$\int x^5 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	835
3.79	$\int x^3 (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	842
3.80	$\int x (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) dx$	849
3.81	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x} dx$	855
3.82	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^3} dx$	863
3.83	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^5} dx$	872
3.84	$\int x^4 (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) dx$	881
3.85	$\int x^2 (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) dx$	892
3.86	$\int (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) dx$	902
3.87	$\int \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{x^2} dx$	910
3.88	$\int \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{x^4} dx$	919
3.89	$\int \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{x^6} dx$	928
3.90	$\int \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{x^8} dx$	937

3.91	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{x^{10}} dx$	944
3.92	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{x^{12}} dx$	952
3.93	$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$	960
3.94	$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$	967
3.95	$\int x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$	974
3.96	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{x} dx$	980
3.97	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{x^3} dx$	990
3.98	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{x^5} dx$	1000
3.99	$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2 x^2}} dx$	1011
3.100	$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2 x^2}} dx$	1017
3.101	$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2 x^2}} dx$	1023
3.102	$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2 x^2}} dx$	1028
3.103	$\int \frac{\arcsin(ax)}{\sqrt{1-a^2 x^2}} dx$	1033
3.104	$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2 x^2}} dx$	1038
3.105	$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2 x^2}} dx$	1043
3.106	$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2 x^2}} dx$	1048
3.107	$\int \frac{x^5 (a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1054
3.108	$\int \frac{x^4 (a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1061
3.109	$\int \frac{x^3 (a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1068
3.110	$\int \frac{x^2 (a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1075
3.111	$\int \frac{x (a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1081
3.112	$\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx$	1086
3.113	$\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2 dx^2}} dx$	1091
3.114	$\int \frac{a+b \arcsin(cx)}{x^2\sqrt{d-c^2 dx^2}} dx$	1097
3.115	$\int \frac{a+b \arcsin(cx)}{x^3\sqrt{d-c^2 dx^2}} dx$	1102
3.116	$\int \frac{a+b \arcsin(cx)}{x^4\sqrt{d-c^2 dx^2}} dx$	1109
3.117	$\int \frac{x^5 (a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx$	1116
3.118	$\int \frac{x^4 (a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx$	1123
3.119	$\int \frac{x^3 (a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx$	1131
3.120	$\int \frac{x^2 (a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx$	1138
3.121	$\int \frac{x (a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx$	1144
3.122	$\int \frac{a+b \arcsin(cx)}{(d-c^2 dx^2)^{3/2}} dx$	1149

3.123	$\int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{3/2}} dx$	1154
3.124	$\int \frac{a+b \arcsin(cx)}{x^2(d-c^2 dx^2)^{3/2}} dx$	1161
3.125	$\int \frac{a+b \arcsin(cx)}{x^3(d-c^2 dx^2)^{3/2}} dx$	1167
3.126	$\int \frac{a+b \arcsin(cx)}{x^4(d-c^2 dx^2)^{3/2}} dx$	1176
3.127	$\int \frac{x^6(a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1183
3.128	$\int \frac{x^5(a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1192
3.129	$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1200
3.130	$\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1207
3.131	$\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1214
3.132	$\int \frac{x(a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1221
3.133	$\int \frac{a+b \arcsin(cx)}{(d-c^2 dx^2)^{5/2}} dx$	1227
3.134	$\int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{5/2}} dx$	1234
3.135	$\int \frac{a+b \arcsin(cx)}{x^2(d-c^2 dx^2)^{5/2}} dx$	1243
3.136	$\int \frac{a+b \arcsin(cx)}{x^3(d-c^2 dx^2)^{5/2}} dx$	1250
3.137	$\int \frac{a+b \arcsin(cx)}{x^4(d-c^2 dx^2)^{5/2}} dx$	1262
3.138	$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx$	1270
3.139	$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1275
3.140	$\int x^m (d-c^2 dx^2)^3 (a+b \arcsin(cx)) dx$	1280
3.141	$\int x^m (d-c^2 dx^2)^2 (a+b \arcsin(cx)) dx$	1289
3.142	$\int x^m (d-c^2 dx^2) (a+b \arcsin(cx)) dx$	1297
3.143	$\int \frac{x^m (a+b \arcsin(cx))}{d-c^2 dx^2} dx$	1303
3.144	$\int \frac{x^m (a+b \arcsin(cx))}{(d-c^2 dx^2)^2} dx$	1308
3.145	$\int \frac{x^m (a+b \arcsin(cx))}{(d-c^2 dx^2)^3} dx$	1313
3.146	$\int x^m (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx)) dx$	1319
3.147	$\int x^m (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx)) dx$	1328
3.148	$\int x^m \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx$	1335
3.149	$\int \frac{x^m (a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx$	1341
3.150	$\int \frac{x^m (a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx$	1346
3.151	$\int \frac{x^m (a+b \arcsin(cx))}{(d-c^2 dx^2)^{5/2}} dx$	1352
3.152	$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2 x^2}} dx$	1359
3.153	$\int x^4 (d-c^2 dx^2) (a+b \arcsin(cx))^2 dx$	1364
3.154	$\int x^3 (d-c^2 dx^2) (a+b \arcsin(cx))^2 dx$	1377

3.155	$\int x^2(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx$	1387
3.156	$\int x(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx$	1398
3.157	$\int (d - c^2 dx^2)(a + b \arcsin(cx))^2 dx$	1407
3.158	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx$	1415
3.159	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx$	1425
3.160	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx$	1435
3.161	$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx$	1445
3.162	$\int x^4(d - c^2 dx^2)^2(a + b \arcsin(cx))^2 dx$	1454
3.163	$\int x^3(d - c^2 dx^2)^2(a + b \arcsin(cx))^2 dx$	1468
3.164	$\int x^2(d - c^2 dx^2)^2(a + b \arcsin(cx))^2 dx$	1482
3.165	$\int x(d - c^2 dx^2)^2(a + b \arcsin(cx))^2 dx$	1496
3.166	$\int (d - c^2 dx^2)^2(a + b \arcsin(cx))^2 dx$	1506
3.167	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))^2}{x} dx$	1517
3.168	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))^2}{x^2} dx$	1531
3.169	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))^2}{x^3} dx$	1543
3.170	$\int \frac{(d - c^2 dx^2)^2(a + b \arcsin(cx))^2}{x^4} dx$	1557
3.171	$\int x^4(d - c^2 dx^2)^3(a + b \arcsin(cx))^2 dx$	1569
3.172	$\int x^3(d - c^2 dx^2)^3(a + b \arcsin(cx))^2 dx$	1585
3.173	$\int x^2(d - c^2 dx^2)^3(a + b \arcsin(cx))^2 dx$	1599
3.174	$\int x(d - c^2 dx^2)^3(a + b \arcsin(cx))^2 dx$	1613
3.175	$\int (d - c^2 dx^2)^3(a + b \arcsin(cx))^2 dx$	1625
3.176	$\int \frac{(d - c^2 dx^2)^3(a + b \arcsin(cx))^2}{x} dx$	1637
3.177	$\int \frac{(d - c^2 dx^2)^3(a + b \arcsin(cx))^2}{x^2} dx$	1653
3.178	$\int \frac{(d - c^2 dx^2)^3(a + b \arcsin(cx))^2}{x^3} dx$	1668
3.179	$\int \frac{(d - c^2 dx^2)^3(a + b \arcsin(cx))^2}{x^4} dx$	1681
3.180	$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$	1695
3.181	$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$	1705
3.182	$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$	1715
3.183	$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$	1724
3.184	$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$	1732
3.185	$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx$	1739
3.186	$\int \frac{(a + b \arcsin(cx))^2}{x^2(d - c^2 dx^2)} dx$	1747
3.187	$\int \frac{(a + b \arcsin(cx))^2}{x^3(d - c^2 dx^2)} dx$	1757
3.188	$\int \frac{(a + b \arcsin(cx))^2}{x^4(d - c^2 dx^2)} dx$	1767

3.189	$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	1779
3.190	$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	1791
3.191	$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	1801
3.192	$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	1810
3.193	$\int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	1816
3.194	$\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^2} dx$	1825
3.195	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^2} dx$	1835
3.196	$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^2} dx$	1849
3.197	$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^2} dx$	1862
3.198	$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	1877
3.199	$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	1890
3.200	$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	1899
3.201	$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	1910
3.202	$\int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	1918
3.203	$\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^3} dx$	1929
3.204	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^3} dx$	1941
3.205	$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^3} dx$	1958
3.206	$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^3} dx$	1974
3.207	$\int x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 dx$	1993
3.208	$\int x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 dx$	2004
3.209	$\int x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 dx$	2014
3.210	$\int \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 dx$	2022
3.211	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x} dx$	2029
3.212	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx$	2038
3.213	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^3} dx$	2047
3.214	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^4} dx$	2058
3.215	$\int x^3(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 dx$	2067
3.216	$\int x^2(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 dx$	2082
3.217	$\int x(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 dx$	2096
3.218	$\int (d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 dx$	2104
3.219	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2}{x} dx$	2113
3.220	$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2}{x^2} dx$	2125

3.221	$\int \frac{(d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^2}{x^3} dx$	2138
3.222	$\int \frac{(d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^2}{x^4} dx$	2151
3.223	$\int x^3 (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2164
3.224	$\int x^2 (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2181
3.225	$\int x (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2199
3.226	$\int (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2 dx$	2207
3.227	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2}{x} dx$	2218
3.228	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2}{x^2} dx$	2232
3.229	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2}{x^3} dx$	2249
3.230	$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2}{x^4} dx$	2266
3.231	$\int \frac{x^5 (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx$	2279
3.232	$\int \frac{x^4 (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx$	2290
3.233	$\int \frac{x^3 (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx$	2300
3.234	$\int \frac{x^2 (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx$	2309
3.235	$\int \frac{x (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx$	2317
3.236	$\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx$	2323
3.237	$\int \frac{(a+b \arcsin(cx))^2}{x \sqrt{d-c^2 dx^2}} dx$	2328
3.238	$\int \frac{(a+b \arcsin(cx))^2}{x^2 \sqrt{d-c^2 dx^2}} dx$	2335
3.239	$\int \frac{(a+b \arcsin(cx))^2}{x^3 \sqrt{d-c^2 dx^2}} dx$	2343
3.240	$\int \frac{(a+b \arcsin(cx))^2}{x^4 \sqrt{d-c^2 dx^2}} dx$	2353
3.241	$\int \frac{x^5 (a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2363
3.242	$\int \frac{x^4 (a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2379
3.243	$\int \frac{x^3 (a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2391
3.244	$\int \frac{x^2 (a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2401
3.245	$\int \frac{x (a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2409
3.246	$\int \frac{(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{3/2}} dx$	2416
3.247	$\int \frac{(a+b \arcsin(cx))^2}{x (d-c^2 dx^2)^{3/2}} dx$	2423
3.248	$\int \frac{(a+b \arcsin(cx))^2}{x^2 (d-c^2 dx^2)^{3/2}} dx$	2433
3.249	$\int \frac{(a+b \arcsin(cx))^2}{x^3 (d-c^2 dx^2)^{3/2}} dx$	2444
3.250	$\int \frac{(a+b \arcsin(cx))^2}{x^4 (d-c^2 dx^2)^{3/2}} dx$	2459
3.251	$\int \frac{x^5 (a+b \arcsin(cx))^2}{(d-c^2 dx^2)^{5/2}} dx$	2471

3.252	$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2485
3.253	$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2496
3.254	$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2506
3.255	$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2515
3.256	$\int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2523
3.257	$\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx$	2533
3.258	$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^{5/2}} dx$	2548
3.259	$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^{5/2}} dx$	2562
3.260	$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^{5/2}} dx$	2577
3.261	$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$	2593
3.262	$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$	2602
3.263	$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$	2609
3.264	$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$	2615
3.265	$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$	2620
3.266	$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx$	2625
3.267	$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx$	2631
3.268	$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx$	2638
3.269	$\int x^m(d-c^2dx^2)^3(a+b \arcsin(cx))^2 dx$	2646
3.270	$\int x^m(d-c^2dx^2)^2(a+b \arcsin(cx))^2 dx$	2662
3.271	$\int x^m(d-c^2dx^2)(a+b \arcsin(cx))^2 dx$	2674
3.272	$\int \frac{x^m(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$	2682
3.273	$\int \frac{x^m(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$	2687
3.274	$\int \frac{x^m(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$	2693
3.275	$\int x^m(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))^2 dx$	2700
3.276	$\int x^m(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 dx$	2710
3.277	$\int x^m\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 dx$	2717
3.278	$\int \frac{x^m(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$	2723
3.279	$\int \frac{x^m(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$	2728
3.280	$\int \frac{x^m(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$	2733
3.281	$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2738
3.282	$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2743
3.283	$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2752

3.284	$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2760
3.285	$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2766
3.286	$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$	2772
3.287	$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx$	2777
3.288	$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx$	2784
3.289	$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx$	2791
3.290	$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$	2801
3.291	$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$	2807
3.292	$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$	2813
3.293	$\int \frac{x\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$	2819
3.294	$\int \frac{\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$	2825
3.295	$\int \frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))} dx$	2831
3.296	$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))} dx$	2836
3.297	$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))} dx$	2841
3.298	$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2846
3.299	$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2853
3.300	$\int \frac{x(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2859
3.301	$\int \frac{(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	2865
3.302	$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))} dx$	2871
3.303	$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))} dx$	2876
3.304	$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))} dx$	2881
3.305	$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2886
3.306	$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2893
3.307	$\int \frac{x(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2900
3.308	$\int \frac{(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	2907
3.309	$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))} dx$	2913
3.310	$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b \arcsin(cx))} dx$	2918
3.311	$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))} dx$	2923
3.312	$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2928
3.313	$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2933

3.314	$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2938
3.315	$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2943
3.316	$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2948
3.317	$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2953
3.318	$\int \frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2957
3.319	$\int \frac{1}{x^2\sqrt{1-a^2x^2} \arcsin(ax)} dx$	2962
3.320	$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2967
3.321	$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2973
3.322	$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2979
3.323	$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2985
3.324	$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2991
3.325	$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	2997
3.326	$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	3002
3.327	$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	3007
3.328	$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3012
3.329	$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3017
3.330	$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3022
3.331	$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3027
3.332	$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3032
3.333	$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$	3037
3.334	$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$	3042
3.335	$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$	3047
3.336	$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$	3052
3.337	$\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$	3057
3.338	$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$	3062
3.339	$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$	3067
3.340	$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$	3072
3.341	$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$	3077
3.342	$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$	3082
3.343	$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$	3087
3.344	$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$	3092
3.345	$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3097

3.346	$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3102
3.347	$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$	3107
3.348	$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3112
3.349	$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3117
3.350	$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))^2} dx$	3122
3.351	$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$	3127
3.352	$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$	3134
3.353	$\int \frac{x\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$	3144
3.354	$\int \frac{\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$	3153
3.355	$\int \frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))^2} dx$	3161
3.356	$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))^2} dx$	3167
3.357	$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))^2} dx$	3172
3.358	$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3177
3.359	$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3185
3.360	$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3193
3.361	$\int \frac{(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$	3202
3.362	$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2} dx$	3209
3.363	$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2} dx$	3215
3.364	$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2} dx$	3220
3.365	$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3225
3.366	$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3233
3.367	$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3241
3.368	$\int \frac{(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$	3250
3.369	$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2} dx$	3258
3.370	$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b \arcsin(cx))^2} dx$	3264
3.371	$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2} dx$	3269
3.372	$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3274
3.373	$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3281
3.374	$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3288
3.375	$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3294

3.376	$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3302
3.377	$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3309
3.378	$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3314
3.379	$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$	3319
3.380	$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3324
3.381	$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3329
3.382	$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3334
3.383	$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3339
3.384	$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3344
3.385	$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$	3349
3.386	$\int \left(-\frac{1}{(1-x^2) \arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2} \arcsin(x)} \right) dx$	3354
3.387	$\int \frac{x^3(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$	3359
3.388	$\int \frac{x^2(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$	3367
3.389	$\int \frac{x(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$	3376
3.390	$\int \frac{d-c^2dx^2}{(a+b \arcsin(cx))^{3/2}} dx$	3384
3.391	$\int \frac{d-c^2dx^2}{x(a+b \arcsin(cx))^{3/2}} dx$	3392
3.392	$\int \frac{d-c^2dx^2}{x^2(a+b \arcsin(cx))^{3/2}} dx$	3398
3.393	$\int \frac{x^3(d-c^2dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3404
3.394	$\int \frac{x^2(d-c^2dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3414
3.395	$\int \frac{x(d-c^2dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3425
3.396	$\int \frac{(d-c^2dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$	3434
3.397	$\int \frac{(d-c^2dx^2)^2}{x(a+b \arcsin(cx))^{3/2}} dx$	3442
3.398	$\int \frac{(d-c^2dx^2)^2}{x^2(a+b \arcsin(cx))^{3/2}} dx$	3448
3.399	$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx$	3454
3.400	$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx$	3460
3.401	$\int x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n dx$	3466
3.402	$\int x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n dx$	3472
3.403	$\int \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n dx$	3478
3.404	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x} dx$	3484
3.405	$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x^2} dx$	3489
3.406	$\int x^2(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^n dx$	3494
3.407	$\int x(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^n dx$	3501

3.408	$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx$	3508
3.409	$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx$	3515
3.410	$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx$	3520
3.411	$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$	3525
3.412	$\int x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$	3533
3.413	$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$	3541
3.414	$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx$	3548
3.415	$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx$	3553
3.416	$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1 - a^2 x^2}} dx$	3558
3.417	$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1 - a^2 x^2}} dx$	3563
3.418	$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1 - a^2 x^2}} dx$	3569
3.419	$\int \frac{x \arcsin(ax)^n}{\sqrt{1 - a^2 x^2}} dx$	3574
3.420	$\int \frac{\arcsin(ax)^n}{\sqrt{1 - a^2 x^2}} dx$	3579
3.421	$\int \frac{\arcsin(ax)^n}{x \sqrt{1 - a^2 x^2}} dx$	3584
3.422	$\int \frac{\arcsin(ax)^n}{x^2 \sqrt{1 - a^2 x^2}} dx$	3589
3.423	$\int x^4 (d + ex^2) (a + b \arcsin(cx)) dx$	3594
3.424	$\int x^3 (d + ex^2) (a + b \arcsin(cx)) dx$	3602
3.425	$\int x^2 (d + ex^2) (a + b \arcsin(cx)) dx$	3610
3.426	$\int x (d + ex^2) (a + b \arcsin(cx)) dx$	3618
3.427	$\int (d + ex^2) (a + b \arcsin(cx)) dx$	3625
3.428	$\int \frac{(d + ex^2) (a + b \arcsin(cx))}{x} dx$	3632
3.429	$\int \frac{(d + ex^2) (a + b \arcsin(cx))}{x^2} dx$	3638
3.430	$\int \frac{(d + ex^2) (a + b \arcsin(cx))}{x^3} dx$	3645
3.431	$\int \frac{(d + ex^2) (a + b \arcsin(cx))}{x^4} dx$	3651
3.432	$\int x^4 (d + ex^2)^2 (a + b \arcsin(cx)) dx$	3661
3.433	$\int x^3 (d + ex^2)^2 (a + b \arcsin(cx)) dx$	3670
3.434	$\int x^2 (d + ex^2)^2 (a + b \arcsin(cx)) dx$	3682
3.435	$\int x (d + ex^2)^2 (a + b \arcsin(cx)) dx$	3691
3.436	$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$	3701
3.437	$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx$	3709
3.438	$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx$	3716
3.439	$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx$	3725
3.440	$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx$	3732
3.441	$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx$	3742
3.442	$\int x^3 (d + ex^2)^3 (a + b \arcsin(cx)) dx$	3752

3.443	$\int x^2(d+ex^2)^3(a+b\arcsin(cx))dx$	3764
3.444	$\int x(d+ex^2)^3(a+b\arcsin(cx))dx$	3773
3.445	$\int (d+ex^2)^3(a+b\arcsin(cx))dx$	3783
3.446	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x}dx$	3793
3.447	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x^2}dx$	3800
3.448	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x^3}dx$	3809
3.449	$\int \frac{(d+ex^2)^3(a+b\arcsin(cx))}{x^4}dx$	3816
3.450	$\int \frac{x^4(a+b\arcsin(cx))}{d+ex^2}dx$	3827
3.451	$\int \frac{x^3(a+b\arcsin(cx))}{d+ex^2}dx$	3836
3.452	$\int \frac{x^2(a+b\arcsin(cx))}{d+ex^2}dx$	3844
3.453	$\int \frac{x(a+b\arcsin(cx))}{d+ex^2}dx$	3851
3.454	$\int \frac{a+b\arcsin(cx)}{d+ex^2}dx$	3858
3.455	$\int \frac{a+b\arcsin(cx)}{x(d+ex^2)}dx$	3865
3.456	$\int \frac{a+b\arcsin(cx)}{x^2(d+ex^2)}dx$	3872
3.457	$\int \frac{a+b\arcsin(cx)}{x^3(d+ex^2)}dx$	3880
3.458	$\int \frac{a+b\arcsin(cx)}{x^4(d+ex^2)}dx$	3889
3.459	$\int \frac{x^3(a+b\arcsin(cx))}{(d+ex^2)^2}dx$	3898
3.460	$\int \frac{x(a+b\arcsin(cx))}{(d+ex^2)^2}dx$	3907
3.461	$\int \frac{a+b\arcsin(cx)}{x(d+ex^2)^2}dx$	3914
3.462	$\int \frac{a+b\arcsin(cx)}{x^3(d+ex^2)^2}dx$	3923
3.463	$\int \frac{x^4(a+b\arcsin(cx))}{(d+ex^2)^2}dx$	3932
3.464	$\int \frac{x^2(a+b\arcsin(cx))}{(d+ex^2)^2}dx$	3941
3.465	$\int \frac{a+b\arcsin(cx)}{(d+ex^2)^2}dx$	3950
3.466	$\int \frac{a+b\arcsin(cx)}{x^2(d+ex^2)^2}dx$	3959
3.467	$\int \frac{x^5(a+b\arcsin(cx))}{(d+ex^2)^3}dx$	3968
3.468	$\int \frac{x^3(a+b\arcsin(cx))}{(d+ex^2)^3}dx$	3977
3.469	$\int \frac{x(a+b\arcsin(cx))}{(d+ex^2)^3}dx$	3985
3.470	$\int \frac{a+b\arcsin(cx)}{x(d+ex^2)^3}dx$	3993
3.471	$\int \frac{a+b\arcsin(cx)}{x^3(d+ex^2)^3}dx$	4002
3.472	$\int \frac{x^4(a+b\arcsin(cx))}{(d+ex^2)^3}dx$	4012
3.473	$\int \frac{x^2(a+b\arcsin(cx))}{(d+ex^2)^3}dx$	4021
3.474	$\int \frac{a+b\arcsin(cx)}{(d+ex^2)^3}dx$	4030

3.475	$\int \frac{x \arcsin(2x)}{1-4x^2} dx$	4039
3.476	$\int \frac{x \arcsin(2x)}{1+4x^2} dx$	4046
3.477	$\int (fx)^m (d+ex^2)^3 (a+b \arcsin(cx)) dx$	4052
3.478	$\int (fx)^m (d+ex^2)^2 (a+b \arcsin(cx)) dx$	4062
3.479	$\int (fx)^m (d+ex^2) (a+b \arcsin(cx)) dx$	4070
3.480	$\int \frac{(fx)^m (a+b \arcsin(cx))}{d+ex^2} dx$	4076
3.481	$\int \frac{(fx)^m (a+b \arcsin(cx))}{(d+ex^2)^2} dx$	4081
3.482	$\int x^2 \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$	4086
3.483	$\int x \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$	4096
3.484	$\int \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$	4104
3.485	$\int \frac{\sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2}{x} dx$	4112
3.486	$\int \frac{\sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2}{x^2} dx$	4122
3.487	$\int x^2 (d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2 dx$	4131
3.488	$\int x (d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2 dx$	4142
3.489	$\int (d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2 dx$	4150
3.490	$\int \frac{(d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2}{x} dx$	4159
3.491	$\int \frac{(d+cdx)^{3/2} (e-cex)^{3/2} (a+b \arcsin(cx))^2}{x^2} dx$	4171
3.492	$\int \frac{x^2 (a+b \arcsin(cx))^2}{\sqrt{d+cdx} \sqrt{e-cex}} dx$	4183
3.493	$\int \frac{x (a+b \arcsin(cx))^2}{\sqrt{d+cdx} \sqrt{e-cex}} dx$	4191
3.494	$\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d+cdx} \sqrt{e-cex}} dx$	4197
3.495	$\int \frac{(a+b \arcsin(cx))^2}{x \sqrt{d+cdx} \sqrt{e-cex}} dx$	4203
3.496	$\int \frac{(a+b \arcsin(cx))^2}{x^2 \sqrt{d+cdx} \sqrt{e-cex}} dx$	4211
3.497	$\int \frac{x^2 (a+b \arcsin(cx))^2}{(d+cdx)^{3/2} (e-cex)^{3/2}} dx$	4219
3.498	$\int \frac{x (a+b \arcsin(cx))^2}{(d+cdx)^{3/2} (e-cex)^{3/2}} dx$	4228
3.499	$\int \frac{(a+b \arcsin(cx))^2}{(d+cdx)^{3/2} (e-cex)^{3/2}} dx$	4235
3.500	$\int \frac{(a+b \arcsin(cx))^2}{x (d+cdx)^{3/2} (e-cex)^{3/2}} dx$	4243
3.501	$\int \frac{(a+b \arcsin(cx))^2}{x^2 (d+cdx)^{3/2} (e-cex)^{3/2}} dx$	4253

3.1 $\int x^4(d - c^2 dx^2) (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 23, antiderivative size = 128

$$\int x^4(d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{2bd\sqrt{1 - c^2x^2}}{35c^5} + \frac{bd(1 - c^2x^2)^{3/2}}{105c^5} - \frac{8bd(1 - c^2x^2)^{5/2}}{175c^5} + \frac{bd(1 - c^2x^2)^{7/2}}{49c^5}$$

$$+ \frac{1}{5}dx^5(a + b \arcsin(cx)) - \frac{1}{7}c^2dx^7(a + b \arcsin(cx))$$

output $2/35*b*d*(-c^2*x^2+1)^(1/2)/c^5+1/105*b*d*(-c^2*x^2+1)^(3/2)/c^5-8/175*b*d*(-c^2*x^2+1)^(5/2)/c^5+1/49*b*d*(-c^2*x^2+1)^(7/2)/c^5+1/5*d*x^5*(a+b*\arcsin(c*x))-1/7*c^2*d*x^7*(a+b*\arcsin(c*x))$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.68

$$\int x^4(d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{d(-105ax^5(-7 + 5c^2x^2) + \frac{b\sqrt{1-c^2x^2}(152+76c^2x^2+57c^4x^4-75c^6x^6)}{c^5} - 105bx^5(-7 + 5c^2x^2) \arcsin(cx))}{3675}$$

input `Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]`

output $(d*(-105*a*x^5*(-7 + 5*c^2*x^2) + (b*\text{Sqrt}[1 - c^2*x^2]*(152 + 76*c^2*x^2 + 57*c^4*x^4 - 75*c^6*x^6)))/c^5 - 105*b*x^5*(-7 + 5*c^2*x^2)*\text{ArcSin}[c*x])/3675$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5192, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d - c^2 dx^2)(a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{dx^5(7 - 5c^2 x^2)}{35\sqrt{1 - c^2 x^2}} dx - \frac{1}{7}c^2 dx^7(a + b \arcsin(cx)) + \frac{1}{5}dx^5(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{35}bcd \int \frac{x^5(7 - 5c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{7}c^2 dx^7(a + b \arcsin(cx)) + \frac{1}{5}dx^5(a + b \arcsin(cx))$$

$$\downarrow 354$$

$$-\frac{1}{70}bcd \int \frac{x^4(7 - 5c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx^2 - \frac{1}{7}c^2 dx^7(a + b \arcsin(cx)) + \frac{1}{5}dx^5(a + b \arcsin(cx))$$

$$\downarrow 86$$

$$-\frac{1}{70}bcd \int \left(\frac{5(1 - c^2 x^2)^{5/2}}{c^4} - \frac{8(1 - c^2 x^2)^{3/2}}{c^4} + \frac{\sqrt{1 - c^2 x^2}}{c^4} + \frac{2}{c^4 \sqrt{1 - c^2 x^2}} \right) dx^2 - \frac{1}{7}c^2 dx^7(a + b \arcsin(cx)) + \frac{1}{5}dx^5(a + b \arcsin(cx))$$

$$\downarrow 2009$$

$$-\frac{1}{7}c^2 dx^7(a + b \arcsin(cx)) + \frac{1}{5}dx^5(a + b \arcsin(cx)) - \frac{1}{70}bcd \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{3c^6} - \frac{4\sqrt{1-c^2x^2}}{c^6} \right)$$

input `Int[x^4*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]`

output `-1/70*(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^6 - (2*(1 - c^2*x^2)^(3/2))/(3*c^6) + (16*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1 - c^2*x^2)^(7/2))/(7*c^6))) + (d*x^5*(a + b*ArcSin[c*x]))/5 - (c^2*d*x^7*(a + b*ArcSin[c*x]))/7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 86 `Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

method	result
parts	$-ad\left(\frac{1}{7}c^2x^7 - \frac{1}{5}x^5\right) - \frac{db\left(\frac{\arcsin(cx)c^7x^7}{7} - \frac{c^5x^5\arcsin(cx)}{5} - \frac{19c^4x^4\sqrt{-c^2x^2+1}}{1225} - \frac{76c^2x^2\sqrt{-c^2x^2+1}}{3675} - \frac{152\sqrt{-c^2x^2+1}}{3675} + \frac{c^6}{3675}\right)}{c^5}$
derivativedivides	$-ad\left(\frac{1}{7}c^7x^7 - \frac{1}{5}c^5x^5\right) - db\left(\frac{\arcsin(cx)c^7x^7}{7} - \frac{c^5x^5\arcsin(cx)}{5} - \frac{19c^4x^4\sqrt{-c^2x^2+1}}{1225} - \frac{76c^2x^2\sqrt{-c^2x^2+1}}{3675} - \frac{152\sqrt{-c^2x^2+1}}{3675} + \frac{c^6}{3675}\right)$
default	$-ad\left(\frac{1}{7}c^7x^7 - \frac{1}{5}c^5x^5\right) - db\left(\frac{\arcsin(cx)c^7x^7}{7} - \frac{c^5x^5\arcsin(cx)}{5} - \frac{19c^4x^4\sqrt{-c^2x^2+1}}{1225} - \frac{76c^2x^2\sqrt{-c^2x^2+1}}{3675} - \frac{152\sqrt{-c^2x^2+1}}{3675} + \frac{c^6}{3675}\right)$
orering	$\frac{(975c^8x^8 - 1377c^6x^6 - 228c^4x^4 - 608c^2x^2 + 608)(-c^2dx^2 + d)(a + b\arcsin(cx))}{3675c^6x(c^2x^2 - 1)} - \frac{(75c^6x^6 - 57c^4x^4 - 76c^2x^2 - 152)}{3675c^5}$

input

```
int(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-a*d*(1/7*c^2*x^7-1/5*x^5)-d*b/c^5*(1/7*arcsin(c*x)*c^7*x^7-1/5*c^5*x^5*ar
csin(c*x)-19/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)-76/3675*c^2*x^2*(-c^2*x^2+1)^(
1/2)-152/3675*(-c^2*x^2+1)^(1/2)+1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int x^4(d - c^2dx^2)(a + b\arcsin(cx)) dx = \frac{525ac^7dx^7 - 735ac^5dx^5 + 105(5bc^7dx^7 - 7bc^5dx^5)\arcsin(cx) + (75bc^6dx^6 - 57bc^4dx^4 - 76bc^2dx^2 - 152)}{3675c^5}$$

input `integrate(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output
$$-1/3675*(525*a*c^7*d*x^7 - 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 - 7*b*c^5*d*x^5)*arcsin(c*x) + (75*b*c^6*d*x^6 - 57*b*c^4*d*x^4 - 76*b*c^2*d*x^2 - 152*b*d)*sqrt(-c^2*x^2 + 1))/c^5$$

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \begin{cases} -\frac{ac^2 dx^7}{7} + \frac{adx^5}{5} - \frac{bc^2 dx^7 \arcsin(cx)}{7} - \frac{bcdx^6 \sqrt{-c^2 x^2 + 1}}{49} + \frac{bdx^5 \arcsin(cx)}{5} + \frac{19bdx^4 \sqrt{-c^2 x^2 + 1}}{1225c} + \frac{76bdx^2 \sqrt{-c^2 x^2 + 1}}{3675c^3} + \frac{152bd\sqrt{-c^2 x^2 + 1}}{3675c^5} \\ \frac{adx^5}{5} \end{cases}$$

input `integrate(x**4*(-c**2*d*x**2+d)*(a+b*asin(c*x)),x)`

output `Piecewise((-a*c**2*d*x**7/7 + a*d*x**5/5 - b*c**2*d*x**7*asin(c*x)/7 - b*c*d*x**6*sqrt(-c**2*x**2 + 1)/49 + b*d*x**5*asin(c*x)/5 + 19*b*d*x**4*sqrt(-c**2*x**2 + 1)/(1225*c) + 76*b*d*x**2*sqrt(-c**2*x**2 + 1)/(3675*c**3) + 152*b*d*sqrt(-c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (a*d*x**5/5, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.48

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{7} ac^2 dx^7 + \frac{1}{5} adx^5$$

$$- \frac{1}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right)$$

$$+ \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bd$$

input `integrate(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.52

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{7} ac^2 dx^7 + \frac{1}{5} adx^5 - \frac{(c^2 x^2 - 1)^3 b dx \arcsin(cx)}{7 c^4} - \frac{8 (c^2 x^2 - 1)^2 b dx \arcsin(cx)}{35 c^4} - \frac{(c^2 x^2 - 1) b dx \arcsin(cx)}{35 c^4} - \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd}{49 c^5} + \frac{2 b dx \arcsin(cx)}{35 c^4} - \frac{8 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd}{175 c^5} + \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} bd}{105 c^5} + \frac{2 \sqrt{-c^2 x^2 + 1} bd}{35 c^5}$$

input `integrate(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/7*(c^2*x^2 - 1)^3*b*d*x*arcsin(c*x)/c^4 - 8/35*(c^2*x^2 - 1)^2*b*d*x*arcsin(c*x)/c^4 - 1/35*(c^2*x^2 - 1)*b*d*x*arcsin(c*x)/c^4 - 1/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d/c^5 + 2/35*b*d*x*arcsin(c*x)/c^4 - 8/175*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d/c^5 + 1/105*(-c^2*x^2 + 1)^(3/2)*b*d/c^5 + 2/35*sqrt(-c^2*x^2 + 1)*b*d/c^5`

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2) dx$$

input `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2),x)`

output `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2), x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{d(-525 \operatorname{asin}(cx) b c^7 x^7 + 735 \operatorname{asin}(cx) b c^5 x^5 - 75 \sqrt{-c^2 x^2 + 1} b c^6 x^6 + 57 \sqrt{-c^2 x^2 + 1} b c^4 x^4 + 76 \sqrt{-c^2 x^2 + 1} b c^2 x^2) + 525 a c^7 x^7 + 735 a c^5 x^5 - 75 \sqrt{-c^2 x^2 + 1} b c^6 x^6 + 57 \sqrt{-c^2 x^2 + 1} b c^4 x^4 + 76 \sqrt{-c^2 x^2 + 1} b c^2 x^2}{3675 c^5}$$

input `int(x^4*(-c^2*d*x^2+d)*(a+b*asin(c*x)),x)`

output `(d*(-525*asin(c*x)*b*c**7*x**7 + 735*asin(c*x)*b*c**5*x**5 - 75*sqrt(-c**2*x**2 + 1)*b*c**6*x**6 + 57*sqrt(-c**2*x**2 + 1)*b*c**4*x**4 + 76*sqrt(-c**2*x**2 + 1)*b*c**2*x**2 + 152*sqrt(-c**2*x**2 + 1)*b - 525*a*c**7*x**7 + 735*a*c**5*x**5))/(3675*c**5)`

3.2 $\int x^3(d - c^2 dx^2) (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 23, antiderivative size = 123

$$\int x^3(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{bdx\sqrt{1 - c^2x^2}}{24c^3} + \frac{bdx^3\sqrt{1 - c^2x^2}}{36c} - \frac{1}{36}bcdx^5\sqrt{1 - c^2x^2} - \frac{bd \arcsin(cx)}{24c^4} + \frac{1}{4}dx^4(a + b \arcsin(cx)) - \frac{1}{6}c^2dx^6(a + b \arcsin(cx))$$

output

```
1/24*b*d*x*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*d*x^3*(-c^2*x^2+1)^(1/2)/c-1/36*b*c*d*x^5*(-c^2*x^2+1)^(1/2)-1/24*b*d*arcsin(c*x)/c^4+1/4*d*x^4*(a+b*arcsin(c*x))-1/6*c^2*d*x^6*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx$$

$$= \frac{d(-6ac^4x^4(-3 + 2c^2x^2) + bcx\sqrt{1 - c^2x^2}(3 + 2c^2x^2 - 2c^4x^4) - 3b(1 - 6c^4x^4 + 4c^6x^6) \arcsin(cx))}{72c^4}$$

input

```
Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]
```

output

```
(d*(-6*a*c^4*x^4*(-3 + 2*c^2*x^2) + b*c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2 - 2*c^4*x^4) - 3*b*(1 - 6*c^4*x^4 + 4*c^6*x^6)*ArcSin[c*x]))/(72*c^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5192, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{dx^4(3 - 2c^2x^2)}{12\sqrt{1 - c^2x^2}} dx - \frac{1}{6}c^2 dx^6(a + b \arcsin(cx)) + \frac{1}{4}dx^4(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{12}bcd \int \frac{x^4(3 - 2c^2x^2)}{\sqrt{1 - c^2x^2}} dx - \frac{1}{6}c^2 dx^6(a + b \arcsin(cx)) + \frac{1}{4}dx^4(a + b \arcsin(cx))$$

$$\downarrow 363$$

$$-\frac{1}{12}bcd \left(\frac{4}{3} \int \frac{x^4}{\sqrt{1 - c^2x^2}} dx + \frac{1}{3}x^5\sqrt{1 - c^2x^2} \right) - \frac{1}{6}c^2 dx^6(a + b \arcsin(cx)) + \frac{1}{4}dx^4(a + b \arcsin(cx))$$

$$\begin{aligned}
& \downarrow 262 \\
& -\frac{1}{12}bcd \left(\frac{4}{3} \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{1}{3}x^5\sqrt{1-c^2x^2} \right) - \frac{1}{6}c^2dx^6(a + \\
& \quad b \arcsin(cx)) + \frac{1}{4}dx^4(a + b \arcsin(cx)) \\
& \downarrow 262 \\
& -\frac{1}{12}bcd \left(\frac{4}{3} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{1}{3}x^5\sqrt{1-c^2x^2} \right) - \frac{1}{6}c^2dx^6(a + \\
& \quad b \arcsin(cx)) + \frac{1}{4}dx^4(a + b \arcsin(cx)) \\
& \downarrow 223 \\
& -\frac{1}{6}c^2dx^6(a + b \arcsin(cx)) + \frac{1}{4}dx^4(a + b \arcsin(cx)) - \\
& \frac{1}{12}bcd \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{1}{3}x^5\sqrt{1-c^2x^2} \right)
\end{aligned}$$

input `Int[x^3*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]`

output `(d*x^4*(a + b*ArcSin[c*x]))/4 - (c^2*d*x^6*(a + b*ArcSin[c*x]))/6 - (b*c*d*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2))/3))/12`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

```
rule 262 Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1))) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

```
rule 363 Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

```
rule 5192 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

method	result
parts	$-ad\left(\frac{1}{6}c^2x^6 - \frac{1}{4}x^4\right) - \frac{db\left(\frac{\arcsin(cx)c^6x^6}{6} - \frac{c^4x^4\arcsin(cx)}{4} - \frac{c^3x^3\sqrt{-c^2x^2+1}}{36} - \frac{cx\sqrt{-c^2x^2+1}}{24} + \frac{\arcsin(cx)}{24} + \frac{c^5x^5\sqrt{-c^2x^2+1}}{36}\right)}{c^4}$
derivativedivides	$-ad\left(\frac{1}{6}c^6x^6 - \frac{1}{4}c^4x^4\right) - db\left(\frac{\arcsin(cx)c^6x^6}{6} - \frac{c^4x^4\arcsin(cx)}{4} - \frac{c^3x^3\sqrt{-c^2x^2+1}}{36} - \frac{cx\sqrt{-c^2x^2+1}}{24} + \frac{\arcsin(cx)}{24} + \frac{c^5x^5\sqrt{-c^2x^2+1}}{36}\right)$
default	$-ad\left(\frac{1}{6}c^6x^6 - \frac{1}{4}c^4x^4\right) - db\left(\frac{\arcsin(cx)c^6x^6}{6} - \frac{c^4x^4\arcsin(cx)}{4} - \frac{c^3x^3\sqrt{-c^2x^2+1}}{36} - \frac{cx\sqrt{-c^2x^2+1}}{24} + \frac{\arcsin(cx)}{24} + \frac{c^5x^5\sqrt{-c^2x^2+1}}{36}\right)$
oring	$\frac{(22c^6x^6 - 34c^4x^4 - 9c^2x^2 + 12)(-c^2dx^2 + d)(a + b\arcsin(cx))}{72c^4(c^2x^2 - 1)} - \frac{(2c^4x^4 - 2c^2x^2 - 3)\left(3x^2(-c^2dx^2 + d)(a + b\arcsin(cx))\right)}{72x^2c^4}$

```
input int(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-a*d*(1/6*c^2*x^6-1/4*x^4)-d*b/c^4*(1/6*arcsin(c*x)*c^6*x^6-1/4*c^4*x^4*arcsin(c*x)-1/36*c^3*x^3*(-c^2*x^2+1)^(1/2)-1/24*c*x*(-c^2*x^2+1)^(1/2)+1/24*arcsin(c*x)+1/36*c^5*x^5*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx = \frac{12 ac^6 dx^6 - 18 ac^4 dx^4 + 3(4 bc^6 dx^6 - 6 bc^4 dx^4 + bd) \arcsin(cx) + (2 bc^5 dx^5 - 2 bc^3 dx^3 - 3 bcdx) \sqrt{-c^2 x^2 + 1}}{72 c^4}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
-1/72*(12*a*c^6*d*x^6 - 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 - 6*b*c^4*d*x^4 + b*d)*arcsin(c*x) + (2*b*c^5*d*x^5 - 2*b*c^3*d*x^3 - 3*b*c*d*x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx = \begin{cases} -\frac{ac^2 dx^6}{6} + \frac{adx^4}{4} - \frac{bc^2 dx^6 \arcsin(cx)}{6} - \frac{bcdx^5 \sqrt{-c^2 x^2 + 1}}{36} + \frac{bdx^4 \arcsin(cx)}{4} + \frac{bdx^3 \sqrt{-c^2 x^2 + 1}}{36c} + \frac{bdx \sqrt{-c^2 x^2 + 1}}{24c^3} - \frac{bd \arcsin(cx)}{24c^4} \\ \frac{adx^4}{4} \end{cases}$$

input

```
integrate(x**3*(-c**2*d*x**2+d)*(a+b*asin(c*x)),x)
```

output

```
Piecewise((-a*c**2*d*x**6/6 + a*d*x**4/4 - b*c**2*d*x**6*asin(c*x)/6 - b*c*d*x**5*sqrt(-c**2*x**2 + 1)/36 + b*d*x**4*asin(c*x)/4 + b*d*x**3*sqrt(-c**2*x**2 + 1)/(36*c) + b*d*x*sqrt(-c**2*x**2 + 1)/(24*c**3) - b*d*asin(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.37

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx = -\frac{1}{6} ac^2 dx^6 + \frac{1}{4} adx^4$$

$$- \frac{1}{288} \left(48 x^6 \arcsin(cx) + \left(\frac{8 \sqrt{-c^2 x^2 + 1} x^5}{c^2} + \frac{10 \sqrt{-c^2 x^2 + 1} x^3}{c^4} + \frac{15 \sqrt{-c^2 x^2 + 1} x}{c^6} - \frac{15 \arcsin(cx)}{c^7} \right) \right. \\ \left. + \frac{1}{32} \left(8 x^4 \arcsin(cx) + \left(\frac{2 \sqrt{-c^2 x^2 + 1} x^3}{c^2} + \frac{3 \sqrt{-c^2 x^2 + 1} x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) c \right) \right) bd$$

input `integrate(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx)) dx = -\frac{1}{6} ac^2 dx^6 + \frac{1}{4} adx^4$$

$$- \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b dx}{36 c^3}$$

$$- \frac{(c^2 x^2 - 1)^3 b d \arcsin(cx)}{6 c^4}$$

$$+ \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} b dx}{36 c^3} - \frac{(c^2 x^2 - 1)^2 b d \arcsin(cx)}{4 c^4}$$

$$+ \frac{\sqrt{-c^2 x^2 + 1} b dx}{24 c^3} + \frac{b d \arcsin(cx)}{24 c^4}$$

input `integrate(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```
-1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/36*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b
*d*x/c^3 - 1/6*(c^2*x^2 - 1)^3*b*d*arcsin(c*x)/c^4 + 1/36*(-c^2*x^2 + 1)^(
3/2)*b*d*x/c^3 - 1/4*(c^2*x^2 - 1)^2*b*d*arcsin(c*x)/c^4 + 1/24*sqrt(-c^2*
x^2 + 1)*b*d*x/c^3 + 1/24*b*d*arcsin(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int x^3 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2) dx$$

input

```
int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2),x)
```

output

```
int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int x^3 (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{d(-12 \operatorname{asin}(cx) b c^6 x^6 + 18 \operatorname{asin}(cx) b c^4 x^4 - 3 \operatorname{asin}(cx) b - 2\sqrt{-c^2 x^2 + 1} b c^5 x^5 + 2\sqrt{-c^2 x^2 + 1} b c^3 x^3 + 3)}{72c^4}$$

input

```
int(x^3*(-c^2*d*x^2+d)*(a+b*asin(c*x)),x)
```

output

```
(d*( - 12*asin(c*x)*b*c**6*x**6 + 18*asin(c*x)*b*c**4*x**4 - 3*asin(c*x)*b
- 2*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 + 2*sqrt( - c**2*x**2 + 1)*b*c**3*
x**3 + 3*sqrt( - c**2*x**2 + 1)*b*c*x - 12*a*c**6*x**6 + 18*a*c**4*x**4))/
(72*c**4)
```


3.3 $\int x^2(d - c^2 dx^2) (a + b \arcsin(cx)) dx$

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Maxima [A] (verification not implemented)	224
Giac [A] (verification not implemented)	225
Mupad [F(-1)]	225
Reduce [B] (verification not implemented)	226

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int x^2(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{2bd\sqrt{1 - c^2 x^2}}{15c^3} + \frac{bd(1 - c^2 x^2)^{3/2}}{45c^3} - \frac{bd(1 - c^2 x^2)^{5/2}}{25c^3} + \frac{1}{3} dx^3(a + b \arcsin(cx)) - \frac{1}{5} c^2 dx^5(a + b \arcsin(cx))$$

output

```
2/15*b*d*(-c^2*x^2+1)^(1/2)/c^3+1/45*b*d*(-c^2*x^2+1)^(3/2)/c^3-1/25*b*d*(-c^2*x^2+1)^(5/2)/c^3+1/3*d*x^3*(a+b*arcsin(c*x))-1/5*c^2*d*x^5*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int x^2(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{d(b\sqrt{1 - c^2 x^2}(26 + 13c^2 x^2 - 9c^4 x^4) + a(75c^3 x^3 - 45c^5 x^5) + 15bc^3 x^3(5 - 3c^2 x^2) \arcsin(cx))}{225c^3}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]
```

output

```
(d*(b*Sqrt[1 - c^2*x^2]*(26 + 13*c^2*x^2 - 9*c^4*x^4) + a*(75*c^3*x^3 - 45*c^5*x^5) + 15*b*c^3*x^3*(5 - 3*c^2*x^2)*ArcSin[c*x]))/(225*c^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5192, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 (d - c^2 dx^2) (a + b \arcsin(cx)) dx \\
 & \quad \downarrow \text{5192} \\
 & -bc \int \frac{dx^3 (5 - 3c^2 x^2)}{15\sqrt{1 - c^2 x^2}} dx - \frac{1}{5} c^2 dx^5 (a + b \arcsin(cx)) + \frac{1}{3} dx^3 (a + b \arcsin(cx)) \\
 & \quad \downarrow \text{27} \\
 & -\frac{1}{15} bcd \int \frac{x^3 (5 - 3c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{5} c^2 dx^5 (a + b \arcsin(cx)) + \frac{1}{3} dx^3 (a + b \arcsin(cx)) \\
 & \quad \downarrow \text{354} \\
 & -\frac{1}{30} bcd \int \frac{x^2 (5 - 3c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx^2 - \frac{1}{5} c^2 dx^5 (a + b \arcsin(cx)) + \frac{1}{3} dx^3 (a + b \arcsin(cx)) \\
 & \quad \downarrow \text{86} \\
 & -\frac{1}{30} bcd \int \left(-\frac{3(1 - c^2 x^2)^{3/2}}{c^2} + \frac{\sqrt{1 - c^2 x^2}}{c^2} + \frac{2}{c^2 \sqrt{1 - c^2 x^2}} \right) dx^2 - \frac{1}{5} c^2 dx^5 (a + b \arcsin(cx)) + \\
 & \quad \frac{1}{3} dx^3 (a + b \arcsin(cx)) \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$-\frac{1}{5}c^2 dx^5(a + b \arcsin(cx)) + \frac{1}{3}dx^3(a + b \arcsin(cx)) - \frac{1}{30}bcd \left(\frac{6(1 - c^2x^2)^{5/2}}{5c^4} - \frac{2(1 - c^2x^2)^{3/2}}{3c^4} - \frac{4\sqrt{1 - c^2x^2}}{c^4} \right)$$

input `Int[x^2*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]`

output `-1/30*(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^4 - (2*(1 - c^2*x^2)^(3/2))/(3*c^4) + (6*(1 - c^2*x^2)^(5/2))/(5*c^4))) + (d*x^3*(a + b*ArcSin[c*x]))/3 - (c^2*d*x^5*(a + b*ArcSin[c*x]))/5`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 86 `Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

method	result
parts	$-ad\left(\frac{1}{5}c^2x^5 - \frac{1}{3}x^3\right) - \frac{db\left(\frac{c^5x^5 \arcsin(cx)}{5} - \frac{c^3x^3 \arcsin(cx)}{3} - \frac{13c^2x^2\sqrt{-c^2x^2+1}}{225} - \frac{26\sqrt{-c^2x^2+1}}{225} + \frac{c^4x^4\sqrt{-c^2x^2+1}}{25}\right)}{c^3}$
derivativedivides	$\frac{-ad\left(\frac{1}{5}c^5x^5 - \frac{1}{3}c^3x^3\right) - db\left(\frac{c^5x^5 \arcsin(cx)}{5} - \frac{c^3x^3 \arcsin(cx)}{3} - \frac{13c^2x^2\sqrt{-c^2x^2+1}}{225} - \frac{26\sqrt{-c^2x^2+1}}{225} + \frac{c^4x^4\sqrt{-c^2x^2+1}}{25}\right)}{c^3}$
default	$\frac{-ad\left(\frac{1}{5}c^5x^5 - \frac{1}{3}c^3x^3\right) - db\left(\frac{c^5x^5 \arcsin(cx)}{5} - \frac{c^3x^3 \arcsin(cx)}{3} - \frac{13c^2x^2\sqrt{-c^2x^2+1}}{225} - \frac{26\sqrt{-c^2x^2+1}}{225} + \frac{c^4x^4\sqrt{-c^2x^2+1}}{25}\right)}{c^3}$
orering	$\frac{(81c^6x^6 - 145c^4x^4 - 78c^2x^2 + 52)(-c^2dx^2 + d)(a + b \arcsin(cx))}{225c^4x(c^2x^2 - 1)} - \frac{(9c^4x^4 - 13c^2x^2 - 26)\left(2x(-c^2dx^2 + d)(a + b \arcsin(cx))\right)}{225}$

```
input int(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output -a*d*(1/5*c^2*x^5-1/3*x^3)-d*b/c^3*(1/5*c^5*x^5*arcsin(c*x)-1/3*c^3*x^3*ar
csin(c*x)-13/225*c^2*x^2*(-c^2*x^2+1)^(1/2)-26/225*(-c^2*x^2+1)^(1/2)+1/25
*c^4*x^4*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int x^2(d - c^2dx^2)(a + b \arcsin(cx)) dx = \frac{45ac^5dx^5 - 75ac^3dx^3 + 15(3bc^5dx^5 - 5bc^3dx^3) \arcsin(cx) + (9bc^4dx^4 - 13bc^2dx^2 - 26bd)\sqrt{-c^2x^2}}{225c^3}$$

input `integrate(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output
$$-1/225*(45*a*c^5*d*x^5 - 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 - 5*b*c^3*d*x^3)*arcsin(c*x) + (9*b*c^4*d*x^4 - 13*b*c^2*d*x^2 - 26*b*d)*sqrt(-c^2*x^2 + 1))/c^3$$

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \begin{cases} -\frac{ac^2 dx^5}{5} + \frac{adx^3}{3} - \frac{bc^2 dx^5 \arcsin(cx)}{5} - \frac{bcdx^4 \sqrt{-c^2 x^2 + 1}}{25} + \frac{bdx^3 \arcsin(cx)}{3} + \frac{13bdx^2 \sqrt{-c^2 x^2 + 1}}{225c} + \frac{26bd \sqrt{-c^2 x^2 + 1}}{225c^3} & \text{for } c \neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(x**2*(-c**2*d*x**2+d)*(a+b*asin(c*x)),x)`

output `Piecewise((-a*c**2*d*x**5/5 + a*d*x**3/3 - b*c**2*d*x**5*asin(c*x)/5 - b*c*d*x**4*sqrt(-c**2*x**2 + 1)/25 + b*d*x**3*asin(c*x)/3 + 13*b*d*x**2*sqrt(-c**2*x**2 + 1)/(225*c) + 26*b*d*sqrt(-c**2*x**2 + 1)/(225*c**3), Ne(c, 0)), (a*d*x**3/3, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{5} ac^2 dx^5$$

$$- \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bc^2 d$$

$$+ \frac{1}{3} adx^3 + \frac{1}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bd$$

input `integrate(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/5*a*c^2*d*x^5 - 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^2*d + 1/3*a*d*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.35

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{5} ac^2 dx^5 + \frac{1}{3} adx^3 - \frac{(c^2 x^2 - 1)^2 b dx \arcsin(cx)}{5 c^2} - \frac{(c^2 x^2 - 1) b dx \arcsin(cx)}{15 c^2} + \frac{2 b dx \arcsin(cx)}{15 c^2} - \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b d}{25 c^3} + \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} b d}{45 c^3} + \frac{2 \sqrt{-c^2 x^2 + 1} b d}{15 c^3}$$

input `integrate(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-1/5*a*c^2*d*x^5 + 1/3*a*d*x^3 - 1/5*(c^2*x^2 - 1)^2*b*d*x*arcsin(c*x)/c^2 - 1/15*(c^2*x^2 - 1)*b*d*x*arcsin(c*x)/c^2 + 2/15*b*d*x*arcsin(c*x)/c^2 - 1/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d/c^3 + 1/45*(-c^2*x^2 + 1)^(3/2)*b*d/c^3 + 2/15*sqrt(-c^2*x^2 + 1)*b*d/c^3`

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int x^2 (a + b \arcsin(cx)) (d - c^2 dx^2) dx$$

input `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2),x)`

output `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2), x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int x^2(d - c^2 dx^2)(a + b \arcsin(cx)) dx$$

$$= \frac{d(-45a \sin(cx) b c^5 x^5 + 75a \sin(cx) b c^3 x^3 - 9\sqrt{-c^2 x^2 + 1} b c^4 x^4 + 13\sqrt{-c^2 x^2 + 1} b c^2 x^2 + 26\sqrt{-c^2 x^2 + 1} b - 45a^2 c^5 x^5 + 75a^2 c^3 x^3)}{225c^3}$$

input `int(x^2*(-c^2*d*x^2+d)*(a+b*asin(c*x)),x)`

output `(d*(- 45*asin(c*x)*b*c**5*x**5 + 75*asin(c*x)*b*c**3*x**3 - 9*sqrt(- c**2*x**2 + 1)*b*c**4*x**4 + 13*sqrt(- c**2*x**2 + 1)*b*c**2*x**2 + 26*sqrt(- c**2*x**2 + 1)*b - 45*a*c**5*x**5 + 75*a*c**3*x**3))/(225*c**3)`

3.4 $\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 21, antiderivative size = 90

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{3bdx\sqrt{1 - c^2x^2}}{32c} + \frac{bdx(1 - c^2x^2)^{3/2}}{16c} + \frac{3bd \arcsin(cx)}{32c^2} - \frac{d(1 - c^2x^2)^2 (a + b \arcsin(cx))}{4c^2}$$

output

```
3/32*b*d*x*(-c^2*x^2+1)^(1/2)/c+1/16*b*d*x*(-c^2*x^2+1)^(3/2)/c+3/32*b*d*a
rcsin(c*x)/c^2-1/4*d*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{d(cx(8acx(-2 + c^2x^2) + b\sqrt{1 - c^2x^2}(-5 + 2c^2x^2)) + b(5 - 16c^2x^2 + 8c^4x^4) \arcsin(cx))}{32c^2}$$

input

```
Integrate[x*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]
```


output

$$-1/32*(d*(c*x*(8*a*c*x*(-2 + c^2*x^2) + b*sqrt[1 - c^2*x^2]*(-5 + 2*c^2*x^2)) + b*(5 - 16*c^2*x^2 + 8*c^4*x^4)*ArcSin[c*x]))/c^2$$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5182, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$\downarrow 5182$$

$$\frac{bd \int (1 - c^2 x^2)^{3/2} dx}{4c} - \frac{d(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{4c^2}$$

$$\downarrow 211$$

$$\frac{bd \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)}{4c} - \frac{d(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{4c^2}$$

$$\downarrow 211$$

$$\frac{bd \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)}{4c} - \frac{d(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{4c^2}$$

$$\downarrow 223$$

$$\frac{bd \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)}{4c} - \frac{d(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{4c^2}$$

input

$$\text{Int}[x*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]$$

output

$$-1/4*(d*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/c^2 + (b*d*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)$$

Definitions of rubi rules used

rule 211 $\text{Int}[(a_) + (b_)*(x_)^2]^{(p_)} , x_Symbol] \rightarrow \text{Simp}[x*((a + b*x^2)^p/(2*p + 1)), x] + \text{Simp}[2*a*(p/(2*p + 1)) \text{Int}[(a + b*x^2)^{(p - 1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[p, 0] \&\& (\text{IntegerQ}[4*p] \parallel \text{IntegerQ}[6*p])$

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

rule 5182 $\text{Int}[(a_) + \text{ArcSin}[c_)*(x_)]*(b_)^{(n_)}*(x_)*((d_) + (e_)*(x_)^2)^{(p_)} , x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[p, -1]$

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{ad(c^2x^2-1)^2}{4} - db\left(\frac{c^4x^4 \arcsin(cx)}{4} - \frac{c^2x^2 \arcsin(cx)}{2} + \frac{5 \arcsin(cx)}{32} + \frac{c^3x^3 \sqrt{-c^2x^2+1}}{16} - \frac{5cx \sqrt{-c^2x^2+1}}{32}\right)}{c^2}$
default	$\frac{-\frac{ad(c^2x^2-1)^2}{4} - db\left(\frac{c^4x^4 \arcsin(cx)}{4} - \frac{c^2x^2 \arcsin(cx)}{2} + \frac{5 \arcsin(cx)}{32} + \frac{c^3x^3 \sqrt{-c^2x^2+1}}{16} - \frac{5cx \sqrt{-c^2x^2+1}}{32}\right)}{c^2}$
parts	$-\frac{ad(c^2x^2-1)^2}{4c^2} - \frac{db\left(\frac{c^4x^4 \arcsin(cx)}{4} - \frac{c^2x^2 \arcsin(cx)}{2} + \frac{5 \arcsin(cx)}{32} + \frac{c^3x^3 \sqrt{-c^2x^2+1}}{16} - \frac{5cx \sqrt{-c^2x^2+1}}{32}\right)}{c^2}$
orering	$\frac{(14c^4x^4 - 33c^2x^2 + 10)(-c^2dx^2 + d)(a + b \arcsin(cx))}{32c^2(c^2x^2 - 1)} - \frac{(2c^2x^2 - 5)\left((-c^2dx^2 + d)(a + b \arcsin(cx)) - 2x^2c^2d(a + b \arcsin(cx))\right)}{32c^2}$

input $\text{int}(x*(-c^2*d*x^2+d)*(a+b*\arcsin(c*x)),x,\text{method}=_RETURNVERBOSE)$

output $1/c^2*(-1/4*a*d*(c^2*x^2-1)^2-d*b*(1/4*c^4*x^4*\arcsin(c*x)-1/2*c^2*x^2*\arcsin(c*x)+5/32*\arcsin(c*x)+1/16*c^3*x^3*(-c^2*x^2+1)^{(1/2)}-5/32*c*x*(-c^2*x^2+1)^{(1/2))}$

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{8ac^4 dx^4 - 16ac^2 dx^2 + (8bc^4 dx^4 - 16bc^2 dx^2 + 5bd) \arcsin(cx) + (2bc^3 dx^3 - 5bcdx) \sqrt{-c^2 x^2 + 1}}{32c^2}$$

input `integrate(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`output `-1/32*(8*a*c^4*d*x^4 - 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 - 16*b*c^2*d*x^2 + 5*b*d)*arcsin(c*x) + (2*b*c^3*d*x^3 - 5*b*c*d*x)*sqrt(-c^2*x^2 + 1))/c^2`**Sympy [A] (verification not implemented)**

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \begin{cases} -\frac{ac^2 dx^4}{4} + \frac{adx^2}{2} - \frac{bc^2 dx^4 \arcsin(cx)}{4} - \frac{bcdx^3 \sqrt{-c^2 x^2 + 1}}{16} + \frac{bdx^2 \arcsin(cx)}{2} + \frac{5bdx \sqrt{-c^2 x^2 + 1}}{32c} - \frac{5bd \arcsin(cx)}{32c^2} & \text{for } c \neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(x*(-c**2*d*x**2+d)*(a+b*asin(c*x)),x)`output `Piecewise((-a*c**2*d*x**4/4 + a*d*x**2/2 - b*c**2*d*x**4*asin(c*x)/4 - b*c*d*x**3*sqrt(-c**2*x**2 + 1)/16 + b*d*x**2*asin(c*x)/2 + 5*b*d*x*sqrt(-c**2*x**2 + 1)/(32*c) - 5*b*d*asin(c*x)/(32*c**2), Ne(c, 0)), (a*d*x**2/2, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{4} ac^2 dx^4 - \frac{1}{32} \left(8x^4 \arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5} \right) c \right) bc^2d + \frac{1}{2} adx^2 + \frac{1}{4} \left(2x^2 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3} \right) \right) bd$$

input `integrate(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/4*a*c^2*d*x^4 - 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{4} ac^2 dx^4 + \frac{(-c^2x^2 + 1)^{\frac{3}{2}} b dx}{16c} - \frac{(c^2x^2 - 1)^2 bd \arcsin(cx)}{4c^2} + \frac{3\sqrt{-c^2x^2+1} b dx}{32c} + \frac{(c^2x^2 - 1) ad}{2c^2} + \frac{3bd \arcsin(cx)}{32c^2}$$

input `integrate(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-1/4*a*c^2*d*x^4 + 1/16*(-c^2*x^2 + 1)^(3/2)*b*d*x/c - 1/4*(c^2*x^2 - 1)^2*b*d*arcsin(c*x)/c^2 + 3/32*sqrt(-c^2*x^2 + 1)*b*d*x/c + 1/2*(c^2*x^2 - 1)*a*d/c^2 + 3/32*b*d*arcsin(c*x)/c^2`

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int x(a + b \arcsin(cx)) (d - c^2 dx^2) dx$$

input `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2),x)`output `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2), x)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{d(-8a \arcsin(cx) b c^4 x^4 + 16a \arcsin(cx) b c^2 x^2 - 5a \arcsin(cx) b - 2\sqrt{-c^2 x^2 + 1} b c^3 x^3 + 5\sqrt{-c^2 x^2 + 1} b c x - 8a c^2 x^2) + 8a^2 \arcsin(cx) b c^2 x^2 - 8a^2 \arcsin(cx) b c^2 x^2}{32c^2}$$

input `int(x*(-c^2*d*x^2+d)*(a+b*asin(c*x)),x)`output `(d*(- 8*asin(c*x)*b*c**4*x**4 + 16*asin(c*x)*b*c**2*x**2 - 5*asin(c*x)*b - 2*sqrt(- c**2*x**2 + 1)*b*c**3*x**3 + 5*sqrt(- c**2*x**2 + 1)*b*c*x - 8*a*c**4*x**4 + 16*a*c**2*x**2))/(32*c**2)`

3.5 $\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \frac{2bd\sqrt{1 - c^2 x^2}}{3c} + \frac{bd(1 - c^2 x^2)^{3/2}}{9c} + dx(a + b \arcsin(cx)) - \frac{1}{3}c^2 dx^3(a + b \arcsin(cx))$$

output

```
2/3*b*d*(-c^2*x^2+1)^(1/2)/c+1/9*b*d*(-c^2*x^2+1)^(3/2)/c+d*x*(a+b*arcsin(c*x))-1/3*c^2*d*x^3*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx = adx - \frac{1}{3}ac^2 dx^3 + \frac{7bd\sqrt{1 - c^2 x^2}}{9c} - \frac{1}{9}bcdx^2\sqrt{1 - c^2 x^2} + bdx \arcsin(cx) - \frac{1}{3}bc^2 dx^3 \arcsin(cx)$$

input

```
Integrate[(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]
```

output

```
a*d*x - (a*c^2*d*x^3)/3 + (7*b*d*Sqrt[1 - c^2*x^2])/(9*c) - (b*c*d*x^2*Sqr
t[1 - c^2*x^2])/9 + b*d*x*ArcSin[c*x] - (b*c^2*d*x^3*ArcSin[c*x])/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5154, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5154}$$

$$-bc \int \frac{dx(3 - c^2 x^2)}{3\sqrt{1 - c^2 x^2}} dx - \frac{1}{3} c^2 dx^3 (a + b \arcsin(cx)) + dx(a + b \arcsin(cx))$$

$$\downarrow \text{27}$$

$$-\frac{1}{3} bcd \int \frac{x(3 - c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{3} c^2 dx^3 (a + b \arcsin(cx)) + dx(a + b \arcsin(cx))$$

$$\downarrow \text{353}$$

$$-\frac{1}{6} bcd \int \frac{3 - c^2 x^2}{\sqrt{1 - c^2 x^2}} dx^2 - \frac{1}{3} c^2 dx^3 (a + b \arcsin(cx)) + dx(a + b \arcsin(cx))$$

$$\downarrow \text{53}$$

$$-\frac{1}{6} bcd \int \left(\sqrt{1 - c^2 x^2} + \frac{2}{\sqrt{1 - c^2 x^2}} \right) dx^2 - \frac{1}{3} c^2 dx^3 (a + b \arcsin(cx)) + dx(a + b \arcsin(cx))$$

$$\downarrow \text{2009}$$

$$-\frac{1}{3} c^2 dx^3 (a + b \arcsin(cx)) + dx(a + b \arcsin(cx)) - \frac{1}{6} bcd \left(-\frac{2(1 - c^2 x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1 - c^2 x^2}}{c^2} \right)$$

input

```
Int[(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]
```

output

```
-1/6*(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2))
) + d*x*(a + b*ArcSin[c*x]) - (c^2*d*x^3*(a + b*ArcSin[c*x]))/3
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
:= Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5154

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x
] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```


Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result	size
parts	$-ad\left(\frac{1}{3}c^2x^3 - x\right) - \frac{db\left(\frac{c^3x^3 \arcsin(cx)}{3} - cx \arcsin(cx) + \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} - \frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	80
derivativedivides	$\frac{-ad\left(\frac{1}{3}c^3x^3 - cx\right) - bd\left(\frac{c^3x^3 \arcsin(cx)}{3} - cx \arcsin(cx) + \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} - \frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	82
default	$\frac{-ad\left(\frac{1}{3}c^3x^3 - cx\right) - bd\left(\frac{c^3x^3 \arcsin(cx)}{3} - cx \arcsin(cx) + \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} - \frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	82
oring	$\frac{x(5c^2x^2-23)(-c^2dx^2+d)(a+b \arcsin(cx))}{9c^2x^2-9} - \frac{(c^2x^2-7)\left(-2c^2dx(a+b \arcsin(cx)) + \frac{(-c^2dx^2+d)bc}{\sqrt{-c^2x^2+1}}\right)}{9c^2}$	101

input `int((-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `-a*d*(1/3*c^2*x^3-x)-d*b/c*(1/3*c^3*x^3*arcsin(c*x)-c*x*arcsin(c*x)+1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-7/9*(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= -\frac{3ac^3dx^3 - 9acdx + 3(bc^3dx^3 - 3bcdx) \arcsin(cx) + (bc^2dx^2 - 7bd)\sqrt{-c^2x^2 + 1}}{9c}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `-1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*arcsin(c*x) + (b*c^2*d*x^2 - 7*b*d)*sqrt(-c^2*x^2 + 1))/c`

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \begin{cases} -\frac{ac^2 dx^3}{3} + adx - \frac{bc^2 dx^3 \arcsin(cx)}{3} - \frac{bcdx^2 \sqrt{-c^2 x^2 + 1}}{9} + bdx \arcsin(cx) + \frac{7bd\sqrt{-c^2 x^2 + 1}}{9c} & \text{for } c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x)),x)`output `Piecewise((-a*c**2*d*x**3/3 + a*d*x - b*c**2*d*x**3*asin(c*x)/3 - b*c*d*x**2*sqrt(-c**2*x**2 + 1)/9 + b*d*x*asin(c*x) + 7*b*d*sqrt(-c**2*x**2 + 1)/(9*c), Ne(c, 0)), (a*d*x, True))`**Maxima [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= -\frac{1}{3} ac^2 dx^3 - \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bc^2 d$$

$$+ adx + \frac{(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1})bd}{c}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`output `-1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d/c`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx = -\frac{1}{3} ac^2 dx^3 - \frac{1}{3} (c^2 x^2 - 1) b dx \arcsin(cx) + \frac{2}{3} b dx \arcsin(cx) + a dx + \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} bd}{9c} + \frac{2\sqrt{-c^2 x^2 + 1} bd}{3c}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`output `-1/3*a*c^2*d*x^3 - 1/3*(c^2*x^2 - 1)*b*d*x*arcsin(c*x) + 2/3*b*d*x*arcsin(c*x) + a*d*x + 1/9*(-c^2*x^2 + 1)^(3/2)*b*d/c + 2/3*sqrt(-c^2*x^2 + 1)*b*d/c`**Mupad [F(-1)]**

Timed out.

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \begin{cases} \frac{bd(\sqrt{1-c^2x^2} + cx \arcsin(cx))}{c} - bc^2 d \left(\frac{\sqrt{\frac{1}{c^2} - x^2} \left(\frac{2}{c^2} + x^2 \right)}{9} + \frac{x^3 \arcsin(cx)}{3} \right) - \frac{a dx (c^2 x^2 - 3)}{3} & \text{if } 0 < c \\ \int (a + b \arcsin(cx)) (d - c^2 dx^2) dx & \text{if } -0 < c \end{cases}$$

input `int((a + b*asin(c*x))*(d - c^2*d*x^2),x)`output `piecewise(0 < c, - b*c^2*d*((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 + (x^3*a*asin(c*x))/3) + (b*d*((-c^2*x^2 + 1)^(1/2) + c*x*asin(c*x)))/c - (a*d*x*(c^2*x^2 - 3))/3, -0 < c, int((a + b*asin(c*x))*(d - c^2*d*x^2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{d(-3a \sin(cx) b c^3 x^3 + 9a \sin(cx) b c x - \sqrt{-c^2 x^2 + 1} b c^2 x^2 + 7\sqrt{-c^2 x^2 + 1} b - 3a c^3 x^3 + 9a c x)}{9c}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x)),x)`

output `(d*(-3*asin(c*x)*b*c**3*x**3 + 9*asin(c*x)*b*c*x - sqrt(-c**2*x**2 + 1)*b*c**2*x**2 + 7*sqrt(-c**2*x**2 + 1)*b - 3*a*c**3*x**3 + 9*a*c*x))/(9*c)`

3.6 $\int \frac{(d-c^2 dx^2)(a+b \arcsin(cx))}{x} dx$

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Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = -\frac{1}{4}bcdx\sqrt{1 - c^2x^2} - \frac{1}{4}bd \arcsin(cx) + \frac{1}{2}d(1 - c^2x^2)(a + b \arcsin(cx)) - \frac{id(a + b \arcsin(cx))^2}{2b} + d(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) - \frac{1}{2}ibd \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-1/4*b*c*d*x*(-c^2*x^2+1)^(1/2)-1/4*b*d*arcsin(c*x)+1/2*d*(-c^2*x^2+1)*(a+b*arcsin(c*x))-1/2*I*d*(a+b*arcsin(c*x))^2/b+d*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = -\frac{1}{2}ac^2 dx^2 - \frac{1}{4}bcdx\sqrt{1 - c^2x^2} \\ + \frac{1}{4}bd \arcsin(cx) - \frac{1}{2}bc^2 dx^2 \arcsin(cx) \\ + bd \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) + ad \log(x) \\ - \frac{1}{2}ibd(\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)}))$$

input

```
Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x,x]
```

output

```
-1/2*(a*c^2*d*x^2) - (b*c*d*x*Sqrt[1 - c^2*x^2])/4 + (b*d*ArcSin[c*x])/4 -
(b*c^2*d*x^2*ArcSin[c*x])/2 + b*d*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x
])] + a*d*Log[x] - (I/2)*b*d*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c
*x])])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx \\ \downarrow \text{5188} \\ d \int \frac{a + b \arcsin(cx)}{x} dx - \frac{1}{2}bcd \int \sqrt{1 - c^2x^2} dx + \frac{1}{2}d(1 - c^2x^2)(a + b \arcsin(cx)) \\ \downarrow \text{211} \\ d \int \frac{a + b \arcsin(cx)}{x} dx - \frac{1}{2}bcd \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2x^2}} dx + \frac{1}{2}x\sqrt{1 - c^2x^2} \right) + \frac{1}{2}d(1 - c^2x^2)(a + b \arcsin(cx))$$

↓ 223

$$d \int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 5136

$$d \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{cx} d \arcsin(cx) + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 3042

$$d \int - \left((a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) \right) d \arcsin(cx) + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 25

$$-d \int (a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 4200

$$d \left(2i \int - \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 25

$$d \left(-2i \int \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 2620

$$d \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{2} i b \int \log \left(1 - e^{2i \arcsin(cx)} \right) d \arcsin(cx) \right) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) + \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bcd \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right)$$

↓ 2715

$$\begin{aligned}
& d\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx))-\frac{1}{4}b\int e^{-2i\arcsin(cx)}\log\left(1-e^{2i\arcsin(cx)}\right)de^{2i\arcsin(cx)}\right)-\frac{i}{2}d(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right) \\
& \quad \downarrow \text{2838} \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))+ \\
& d\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)\right)-\frac{i(a+b\arcsin(cx))^2}{2b}\right)- \\
& \quad \frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)
\end{aligned}$$

input `Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x,x]`

output `(d*(1 - c^2*x^2)*(a + b*ArcSin[c*x]))/2 - (b*c*d*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + d*(((-1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4))`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]`

rule 211 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 2620

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5136

```
Int[(((a_) + ArcSin[(c_)*(x_)])*(b_))^(n_)/(x_), x_Symbol] := Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

rule 5188

```
Int[(((a_) + ArcSin[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_)^2)^(p_)/(x_),
x_Symbol] := Simp[(d + e*x^2)^p*((a + b*ArcSin[c*x])/(2*p)), x] + (Simp[d
Int[(d + e*x^2)^(p - 1)*((a + b*ArcSin[c*x])/x), x], x] - Simp[b*c*(d^p/(2
*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.30

method	result
parts	$-ad\left(\frac{c^2x^2}{2} - \ln(x)\right) - db\left(\frac{i\arcsin(cx)^2}{2} - \arcsin(cx)\ln(1 - icx - \sqrt{-c^2x^2 + 1}) + i\text{poly}\right)$
derivativedivides	$-ad\left(\frac{c^2x^2}{2} - \ln(cx)\right) - db\left(\frac{i\arcsin(cx)^2}{2} - \arcsin(cx)\ln(1 - icx - \sqrt{-c^2x^2 + 1}) + i\text{poly}\right)$
default	$-ad\left(\frac{c^2x^2}{2} - \ln(cx)\right) - db\left(\frac{i\arcsin(cx)^2}{2} - \arcsin(cx)\ln(1 - icx - \sqrt{-c^2x^2 + 1}) + i\text{poly}\right)$

input `int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output `-a*d*(1/2*c^2*x^2-ln(x))-d*b*(1/2*I*arcsin(c*x)^2-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-1/4*arcsin(c*x)*cos(2*arcsin(c*x))+1/8*sin(2*arcsin(c*x)))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x,x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))/x, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = -d \left(\int \left(-\frac{a}{x} \right) dx + \int ac^2 x dx \right. \\ \left. + \int \left(-\frac{b \arcsin(cx)}{x} \right) dx + \int bc^2 x \arcsin(cx) dx \right)$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))/x,x)`

output `-d*(Integral(-a/x, x) + Integral(a*c**2*x, x) + Integral(-b*asin(c*x)/x, x) + Integral(b*c**2*x*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `-1/2*a*c^2*d*x^2 + a*d*log(x) - integrate((b*c^2*d*x^2 - b*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx))(d - c^2 dx^2)}{x} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x} dx$$

$$= \frac{d(-2a \arcsin(cx) b c^2 x^2 + \arcsin(cx) b - \sqrt{-c^2 x^2 + 1} b c x + 4 \left(\int \frac{a \arcsin(cx)}{x} dx \right) b + 4 \log(x) a - 2a c^2 x^2)}{4}$$

input

```
int((-c^2*d*x^2+d)*(a+b*asin(c*x))/x,x)
```

output

```
(d*(- 2*asin(c*x)*b*c**2*x**2 + asin(c*x)*b - sqrt(- c**2*x**2 + 1)*b*c*
x + 4*int(asin(c*x)/x,x)*b + 4*log(x)*a - 2*a*c**2*x**2))/4
```

3.7 $\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx$

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Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx = -bcd\sqrt{1 - c^2x^2} - \frac{d(a + b \arcsin(cx))}{x} - c^2 dx(a + b \arcsin(cx)) - bcd \operatorname{arctanh}(\sqrt{1 - c^2x^2})$$

```
output -b*c*d*(-c^2*x^2+1)^(1/2)-d*(a+b*arcsin(c*x))/x-c^2*d*x*(a+b*arcsin(c*x))-b*c*d*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx = -\frac{ad}{x} - ac^2 dx - bcd\sqrt{1 - c^2x^2} - \frac{bd \arcsin(cx)}{x} - bc^2 dx \arcsin(cx) - bcd \operatorname{arctanh}(\sqrt{1 - c^2x^2})$$

```
input Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```

-((a*d)/x) - a*c^2*d*x - b*c*d*Sqrt[1 - c^2*x^2] - (b*d*ArcSin[c*x])/x - b
*c^2*d*x*ArcSin[c*x] - b*c*d*ArcTanh[Sqrt[1 - c^2*x^2]]

```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5192, 25, 27, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx \\
 & \quad \downarrow \text{5192} \\
 & -bc \int -\frac{d(c^2 x^2 + 1)}{x\sqrt{1 - c^2 x^2}} dx + c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{25} \\
 & bc \int \frac{d(c^2 x^2 + 1)}{x\sqrt{1 - c^2 x^2}} dx + c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{27} \\
 & bcd \int \frac{c^2 x^2 + 1}{x\sqrt{1 - c^2 x^2}} dx + c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{354} \\
 & \frac{1}{2}bcd \int \frac{c^2 x^2 + 1}{x^2\sqrt{1 - c^2 x^2}} dx^2 + c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{90} \\
 & \frac{1}{2}bcd \left(\int \frac{1}{x^2\sqrt{1 - c^2 x^2}} dx^2 - 2\sqrt{1 - c^2 x^2} \right) + c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{73}
 \end{aligned}$$

$$\frac{1}{2}bcd \left(-\frac{2 \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1-c^2x^2}}{c^2} - 2\sqrt{1-c^2x^2} \right) + c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x}$$

↓ 221

$$c^2(-d)x(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))}{x} + \frac{1}{2}bcd \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right)$$

input `Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x^2,x]`

output `-((d*(a + b*ArcSin[c*x]))/x) - c^2*d*x*(a + b*ArcSin[c*x]) + (b*c*d*(-2*Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] :> Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 90 `Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)) Int[(c + d*x)^(n)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 354 `Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 5192 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result
parts	$-ad\left(c^2x + \frac{1}{x}\right) - dbc\left(cx \arcsin(cx) + \frac{\arcsin(cx)}{cx} + \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) + \sqrt{-c^2x^2+1}\right)$
derivativedivides	$c\left(-ad\left(cx + \frac{1}{cx}\right) - db\left(cx \arcsin(cx) + \frac{\arcsin(cx)}{cx} + \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) + \sqrt{-c^2x^2+1}\right)\right)$
default	$c\left(-ad\left(cx + \frac{1}{cx}\right) - db\left(cx \arcsin(cx) + \frac{\arcsin(cx)}{cx} + \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) + \sqrt{-c^2x^2+1}\right)\right)$

input `int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)`

output `-a*d*(c^2*x+1/x)-d*b*c*(c*x*arcsin(c*x)+arcsin(c*x)/c/x+arctanh(1/(-c^2*x^2+1)^(1/2))+(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx = \frac{2ac^2 dx^2 + bcdx \log(\sqrt{-c^2 x^2 + 1} + 1) - bcdx \log(\sqrt{-c^2 x^2 + 1} - 1) + 2\sqrt{-c^2 x^2 + 1}bcdx + 2ad + 2}{2x}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")`

output `-1/2*(2*a*c^2*d*x^2 + b*c*d*x*log(sqrt(-c^2*x^2 + 1) + 1) - b*c*d*x*log(sqrt(-c^2*x^2 + 1) - 1) + 2*sqrt(-c^2*x^2 + 1)*b*c*d*x + 2*a*d + 2*(b*c^2*d*x^2 + b*d)*arcsin(c*x))/x`

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx = -ac^2 dx - \frac{ad}{x} - bc^2 d \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2 x^2 + 1}}{c} & \text{otherwise} \end{cases} \right) + bcd \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2 x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{bd \operatorname{asin}(cx)}{x}$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))/x**2,x)`

output `-a*c**2*d*x - a*d/x - b*c**2*d*Piecewise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True)) + b*c*d*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*d*asin(c*x)/x`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx$$

$$= -ac^2 dx - \left(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1} \right) bcd$$

$$- \left(c \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd - \frac{ad}{x}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `-a*c^2*d*x - (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*c*d - (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d - a*d/x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(65) = 130.

Time = 0.94 (sec) , antiderivative size = 856, normalized size of antiderivative = 12.41

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx = \text{Too large to display}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output

```

-1/2*b*c^5*d*x^4*arcsin(c*x)/((c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(s
qrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^4) - 1/2*a*c^5*d*x^4/((c^
3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^
2*x^2 + 1) + 1)^4) + b*c^4*d*x^3*log(abs(c)*abs(x))/((c^3*x^3/(sqrt(-c^2*x
^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^3)
- b*c^4*d*x^3*log(sqrt(-c^2*x^2 + 1) + 1)/((c^3*x^3/(sqrt(-c^2*x^2 + 1) +
1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^3) + b*c^4*
d*x^3/((c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))
*(sqrt(-c^2*x^2 + 1) + 1)^3) - 3*b*c^3*d*x^2*arcsin(c*x)/((c^3*x^3/(sqrt(-
c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) +
1)^2) - 3*a*c^3*d*x^2/((c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^
2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^2) + b*c^2*d*x*log(abs(c)*abs(x)
)/((c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sq
rt(-c^2*x^2 + 1) + 1)) - b*c^2*d*x*log(sqrt(-c^2*x^2 + 1) + 1)/((c^3*x^3/(
sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 +
1) + 1)) - b*c^2*d*x/((c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^
2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)) - 1/2*b*c*d*arcsin(c*x)/(c^3*x^
3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1)) - 1/2*a*c*d/(
c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))

```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx = -\frac{ad(c^2 x^2 + 1)}{x} - bcd \left(\sqrt{1 - c^2 x^2} + cx \arcsin(cx) \right) - \frac{bd \arcsin(cx)}{x} - bcd \operatorname{atanh} \left(\frac{1}{\sqrt{1 - c^2 x^2}} \right)$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x^2,x)
```

output

```

-(a*d*(c^2*x^2 + 1))/x - b*c*d*((1 - c^2*x^2)^(1/2) + c*x*asin(c*x)) - (b
*d*asin(c*x))/x - b*c*d*atanh(1/(1 - c^2*x^2)^(1/2))

```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{d(-\operatorname{asin}(cx) b c^2 x^2 - \operatorname{asin}(cx) b - \sqrt{-c^2 x^2 + 1} b c x + \log\left(\tan\left(\frac{\operatorname{asin}(cx)}{2}\right)\right) b c x - a c^2 x^2 - a)}{x}$$

input

```
int((-c^2*d*x^2+d)*(a+b*asin(c*x))/x^2,x)
```

output

```
(d*( - asin(c*x)*b*c**2*x**2 - asin(c*x)*b - sqrt( - c**2*x**2 + 1)*b*c*x
+ log(tan(asin(c*x)/2))*b*c*x - a*c**2*x**2 - a))/x
```

3.8 $\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx$

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Optimal result

Integrand size = 23, antiderivative size = 139

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = -\frac{bcd\sqrt{1 - c^2x^2}}{2x} - \frac{1}{2}bc^2d \arcsin(cx) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{ic^2d(a + b \arcsin(cx))^2}{2b} - c^2d(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) + \frac{1}{2}ibc^2d \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-1/2*b*c*d*(-c^2*x^2+1)^(1/2)/x-1/2*b*c^2*d*arcsin(c*x)-1/2*d*(-c^2*x^2+1)
*(a+b*arcsin(c*x))/x^2+1/2*I*c^2*d*(a+b*arcsin(c*x))^2/b-c^2*d*(a+b*arcsin
(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*I*b*c^2*d*polylog(2,(I*c*x+
-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = -\frac{ad}{2x^2} - \frac{bcd\sqrt{1 - c^2x^2}}{2x} - \frac{bd \arcsin(cx)}{2x^2} - bc^2 d \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) - ac^2 d \log(x) + \frac{1}{2} ibc^2 d (\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)}))$$

input

```
Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/2*(a*d)/x^2 - (b*c*d*Sqrt[1 - c^2*x^2])/(2*x) - (b*d*ArcSin[c*x])/(2*x^2) - b*c^2*d*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - a*c^2*d*Log[x] + (I/2)*b*c^2*d*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5190, 247, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx$$

$$\downarrow \text{5190}$$

$$c^2(-d) \int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{2} bcd \int \frac{\sqrt{1 - c^2x^2}}{x^2} dx - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2}$$

$$\downarrow \text{247}$$

$$c^2(-d) \int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{2} bcd \left(c^2 \left(- \int \frac{1}{\sqrt{1 - c^2x^2}} dx \right) - \frac{\sqrt{1 - c^2x^2}}{x} \right) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2}$$

↓ 223

$$c^2(-d) \int \frac{a + b \arcsin(cx)}{x} dx - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right)$$

↓ 5136

$$c^2(-d) \int \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{cx} d \arcsin(cx) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right)$$

↓ 3042

$$c^2(-d) \int - \left((a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) \right) d \arcsin(cx) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right)$$

↓ 25

$$c^2d \int (a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right)$$

↓ 4200

$$c^2(-d) \left(2i \int - \frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right)$$

↓ 25

$$c^2(-d) \left(-2i \int \frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right)$$

↓ 2620

$$c^2(-d) \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{2} ib \int \log \left(1 - e^{2i \arcsin(cx)} \right) d \arcsin(cx) \right) - \frac{i(a + b \arcsin(cx))^2}{2b} \right. \\ \left. \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right) \right)$$

↓ 2715

$$c^2(-d) \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log \left(1 - e^{2i \arcsin(cx)} \right) de^{2i \arcsin(cx)} \right) - \frac{i(a + b \arcsin(cx))^2}{2b} \right. \\ \left. \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right) \right)$$

↓ 2838

$$c^2(-d) \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) + \frac{1}{4} b \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) \right) - \frac{i(a + b \arcsin(cx))^2}{2b} \right. \\ \left. \frac{d(1 - c^2x^2)(a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd \left(-c \arcsin(cx) - \frac{\sqrt{1 - c^2x^2}}{x} \right) \right)$$

input

```
Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/2*(d*(1 - c^2*x^2)*(a + b*ArcSin[c*x]))/x^2 + (b*c*d*(-(Sqrt[1 - c^2*x^2]/x) - c*ArcSin[c*x])/2 - c^2*d*((-1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```


rule 247 $\text{Int}[\left((c_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \left((a_{.}) + (b_{.}) \cdot (x_{.})^2\right)^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[(c \cdot x)^{(m+1)} \cdot (a + b \cdot x^2)^p / (c \cdot (m+1)), x] - \text{Simp}[2 \cdot b \cdot (p / (c^2 \cdot (m+1))) \text{Int}[(c \cdot x)^{(m+2)} \cdot (a + b \cdot x^2)^{(p-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{!LtQ}[(m + 2 \cdot p + 3) / 2, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 2620 $\text{Int}[\left((F_{.})^{\left((g_{.}) \cdot (e_{.}) + (f_{.}) \cdot (x_{.})\right)}\right)^{(n_{.})} \cdot \left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} / \left((a_{.}) + (b_{.}) \cdot (F_{.})^{\left((g_{.}) \cdot (e_{.}) + (f_{.}) \cdot (x_{.})\right)}\right)^{(n_{.})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\left((c + d \cdot x)^m / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F])\right) \cdot \text{Log}[1 + b \cdot (F^{(g \cdot (e + f \cdot x))})^n / a], x] - \text{Simp}[d \cdot (m / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F])) \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 + b \cdot (F^{(g \cdot (e + f \cdot x))})^n / a], x], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_{.}) + (b_{.}) \cdot (F_{.})^{\left((e_{.}) \cdot (c_{.}) + (d_{.}) \cdot (x_{.})\right)}], x_{\text{Symbol}}] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_{.}) \cdot \left((d_{.}) + (e_{.}) \cdot (x_{.})^{(n_{.})}\right)] / (x_{.}), x_{\text{Symbol}}] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n / n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

rule 3042 $\text{Int}[u_{.}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4200 $\text{Int}[\left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \tan\left[(e_{.}) + \text{Pi} \cdot (k_{.}) + (f_{.}) \cdot (x_{.})\right], x_{\text{Symbol}}] \rightarrow \text{Simp}[I \cdot (c + d \cdot x)^{(m+1)} / (d \cdot (m+1)), x] - \text{Simp}[2 \cdot I \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot (E^{(2 \cdot I \cdot (e + f \cdot x))} / (1 + E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}))], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[4 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5136 $\text{Int}[\left((a_{.}) + \text{ArcSin}[(c_{.}) \cdot (x_{.})] \cdot (b_{.})\right)^{(n_{.})} / (x_{.}), x_{\text{Symbol}}] \rightarrow \text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Cot}[x], x], x, \text{ArcSin}[c \cdot x]] /;$ $\text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{IGtQ}[n, 0]$

rule 5190

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x
])/ (f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1))) Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

method	result
parts	$-ad c^2 \ln(x) - \frac{ad}{2x^2} - db c^2 \left(-\frac{i \arcsin(cx)^2}{2} + \frac{-ic^2x^2 + cx\sqrt{-c^2x^2+1} + \arcsin(cx)}{2c^2x^2} + \arcsin(cx) \ln \right)$
derivativedivides	$c^2 \left(-ad \left(\ln(cx) + \frac{1}{2c^2x^2} \right) - db \left(-\frac{i \arcsin(cx)^2}{2} + \frac{-ic^2x^2 + cx\sqrt{-c^2x^2+1} + \arcsin(cx)}{2c^2x^2} + \arcsin(cx) \right) \right)$
default	$c^2 \left(-ad \left(\ln(cx) + \frac{1}{2c^2x^2} \right) - db \left(-\frac{i \arcsin(cx)^2}{2} + \frac{-ic^2x^2 + cx\sqrt{-c^2x^2+1} + \arcsin(cx)}{2c^2x^2} + \arcsin(cx) \right) \right)$

input

```
int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output

```
-a*d*c^2*ln(x)-1/2*a*d/x^2-d*b*c^2*(-1/2*I*arcsin(c*x)^2+1/2*(-I*c^2*x^2+c
*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+arcsin(c*x)*ln(1-I*c*x-(-c^2*x^
2+1)^(1/2))+arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,I*c*x+(
-c^2*x^2+1)^(1/2))-I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")
```

output

```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = -d \left(\int \left(-\frac{a}{x^3} \right) dx + \int \frac{ac^2}{x} dx \right. \\ \left. + \int \left(-\frac{b \arcsin(cx)}{x^3} \right) dx + \int \frac{bc^2 \arcsin(cx)}{x} dx \right)$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))/x**3,x)`

output `-d*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*asin(c*x)/x**3, x) + Integral(b*c**2*asin(c*x)/x, x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `-b*c^2*d*integrate(arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x) - a*c^2*d*log(x) - 1/2*b*d*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) - 1/2*a*d/x^2`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx))(d - c^2 dx^2)}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x^3,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{d(-\arcsin(cx)b - \sqrt{-c^2 x^2 + 1}bcx - 2\left(\int \frac{\arcsin(cx)}{x} dx\right)bc^2 x^2 - 2\log(x)ac^2 x^2 - a)}{2x^2}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x))/x^3,x)`

output `(d*(-asin(c*x)*b - sqrt(-c**2*x**2 + 1)*b*c*x - 2*int(asin(c*x)/x,x)*b
*c**2*x**2 - 2*log(x)*a*c**2*x**2 - a))/(2*x**2)`

3.9 $\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$

Optimal result	264
Mathematica [A] (verified)	264
Rubi [A] (verified)	265
Maple [A] (verified)	267
Fricas [A] (verification not implemented)	267
Sympy [A] (verification not implemented)	268
Maxima [A] (verification not implemented)	269
Giac [B] (verification not implemented)	269
Mupad [F(-1)]	271
Reduce [B] (verification not implemented)	271

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx = -\frac{bcd\sqrt{1 - c^2x^2}}{6x^2} - \frac{d(a + b \arcsin(cx))}{3x^3} + \frac{c^2d(a + b \arcsin(cx))}{x} + \frac{5}{6}bc^3 \operatorname{darctanh}(\sqrt{1 - c^2x^2})$$

output

```
-1/6*b*c*d*(-c^2*x^2+1)^(1/2)/x^2-1/3*d*(a+b*arcsin(c*x))/x^3+c^2*d*(a+b*arcsin(c*x))/x+5/6*b*c^3*d*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx = -\frac{ad}{3x^3} + \frac{ac^2d}{x} - \frac{bcd\sqrt{1 - c^2x^2}}{6x^2} - \frac{bd \arcsin(cx)}{3x^3} + \frac{bc^2d \arcsin(cx)}{x} + \frac{5}{6}bc^3 \operatorname{darctanh}(\sqrt{1 - c^2x^2})$$

input

```
Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x^4,x]
```

output

$$-1/3*(a*d)/x^3 + (a*c^2*d)/x - (b*c*d*\text{Sqrt}[1 - c^2*x^2])/(6*x^2) - (b*d*ArcSin[c*x])/(3*x^3) + (b*c^2*d*ArcSin[c*x])/x + (5*b*c^3*d*ArcTanh[\text{Sqrt}[1 - c^2*x^2]])/6$$
Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5192, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$$

$$\downarrow 5192$$

$$-bc \int -\frac{d(1 - 3c^2 x^2)}{3x^3 \sqrt{1 - c^2 x^2}} dx + \frac{c^2 d(a + b \arcsin(cx))}{x} - \frac{d(a + b \arcsin(cx))}{3x^3}$$

$$\downarrow 27$$

$$\frac{1}{3}bcd \int \frac{1 - 3c^2 x^2}{x^3 \sqrt{1 - c^2 x^2}} dx + \frac{c^2 d(a + b \arcsin(cx))}{x} - \frac{d(a + b \arcsin(cx))}{3x^3}$$

$$\downarrow 354$$

$$\frac{1}{6}bcd \int \frac{1 - 3c^2 x^2}{x^4 \sqrt{1 - c^2 x^2}} dx^2 + \frac{c^2 d(a + b \arcsin(cx))}{x} - \frac{d(a + b \arcsin(cx))}{3x^3}$$

$$\downarrow 87$$

$$\frac{1}{6}bcd \left(-\frac{5}{2}c^2 \int \frac{1}{x^2 \sqrt{1 - c^2 x^2}} dx^2 - \frac{\sqrt{1 - c^2 x^2}}{x^2} \right) + \frac{c^2 d(a + b \arcsin(cx))}{x} - \frac{d(a + b \arcsin(cx))}{3x^3}$$

$$\downarrow 73$$

$$\frac{1}{6}bcd \left(5 \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1 - c^2 x^2} - \frac{\sqrt{1 - c^2 x^2}}{x^2} \right) + \frac{c^2 d(a + b \arcsin(cx))}{x} - \frac{d(a + b \arcsin(cx))}{3x^3}$$

$$\downarrow 221$$

$$\frac{c^2 d(a + b \arcsin(cx))}{x} - \frac{d(a + b \arcsin(cx))}{3x^3} + \frac{1}{6}bcd \left(5c^2 \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) - \frac{\sqrt{1 - c^2 x^2}}{x^2} \right)$$

input `Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(d*(a + b*ArcSin[c*x]))/x^3 + (c^2*d*(a + b*ArcSin[c*x]))/x + (b*c*d*(-(Sqrt[1 - c^2*x^2]/x^2) + 5*c^2*ArcTanh[Sqrt[1 - c^2*x^2]]))/6`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 73 `Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntegerQ[a, b, c, d, m, n, x]`

rule 87 `Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_] := Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)) Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

method	result
parts	$-ad\left(\frac{1}{3x^3} - \frac{c^2}{x}\right) - db c^3 \left(\frac{\arcsin(cx)}{3c^3x^3} - \frac{\arcsin(cx)}{cx} + \frac{\sqrt{-c^2x^2+1}}{6c^2x^2} - \frac{5 \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6} \right)$
derivativedivides	$c^3 \left(-ad\left(\frac{1}{3c^3x^3} - \frac{1}{cx}\right) - db \left(\frac{\arcsin(cx)}{3c^3x^3} - \frac{\arcsin(cx)}{cx} + \frac{\sqrt{-c^2x^2+1}}{6c^2x^2} - \frac{5 \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6} \right) \right)$
default	$c^3 \left(-ad\left(\frac{1}{3c^3x^3} - \frac{1}{cx}\right) - db \left(\frac{\arcsin(cx)}{3c^3x^3} - \frac{\arcsin(cx)}{cx} + \frac{\sqrt{-c^2x^2+1}}{6c^2x^2} - \frac{5 \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6} \right) \right)$

input

```
int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
-a*d*(1/3/x^3-c^2/x)-d*b*c^3*(1/3*arcsin(c*x)/c^3/x^3-arcsin(c*x)/c/x+1/6/
c^2/x^2*(-c^2*x^2+1)^(1/2)-5/6*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{5bc^3 dx^3 \log(\sqrt{-c^2x^2+1}+1) - 5bc^3 dx^3 \log(\sqrt{-c^2x^2+1}-1) + 12ac^2 dx^2 - 2\sqrt{-c^2x^2+1}bcdx - 4a}{12x^3}$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")
```


output

```
1/12*(5*b*c^3*d*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(sqrt(-
c^2*x^2 + 1) - 1) + 12*a*c^2*d*x^2 - 2*sqrt(-c^2*x^2 + 1)*b*c*d*x - 4*a*d
+ 4*(3*b*c^2*d*x^2 - b*d)*arcsin(c*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{ac^2d}{x} - \frac{ad}{3x^3} - bc^3d \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) + \frac{bc^2d \operatorname{asin}(cx)}{x}$$

$$+ \frac{bcd \left(\begin{cases} -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1 + \frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1 + \frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1 - \frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases} \right)}{3} - \frac{bd \operatorname{asin}(cx)}{3x^3}$$

input

```
integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))/x**4,x)
```

output

```
a*c**2*d/x - a*d/(3*x**3) - b*c**3*d*Piecewise((-acosh(1/(c*x)), 1/Abs(c**
2*x**2) > 1), (I*asin(1/(c*x)), True)) + b*c**2*d*asin(c*x)/x + b*c*d*Piec
ewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2)))) - 1/(2*c*
x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x
))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 - b*d*asin(c*x)/(3*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.52

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= \left(c \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bc^2 d$$

$$- \frac{1}{6} \left(\left(c^2 \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2 x^2 + 1}}{x^2} \right) c + \frac{2 \arcsin(cx)}{x^3} \right) bd$$

$$+ \frac{ac^2 d}{x} - \frac{ad}{3x^3}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output `(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*c^2*d - 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*b*d + a*c^2*d/x - 1/3*a*d/x^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(71) = 142.

Time = 4.37 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.65

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx = -\frac{bc^6 dx^3 \arcsin(cx)}{24(\sqrt{-c^2 x^2 + 1} + 1)^3} - \frac{ac^6 dx^3}{24(\sqrt{-c^2 x^2 + 1} + 1)^3}$$

$$+ \frac{bc^5 dx^2}{24(\sqrt{-c^2 x^2 + 1} + 1)^2} + \frac{3bc^4 dx \arcsin(cx)}{8(\sqrt{-c^2 x^2 + 1} + 1)}$$

$$+ \frac{3ac^4 dx}{8(\sqrt{-c^2 x^2 + 1} + 1)} - \frac{5}{6} bc^3 d \log(|c||x|)$$

$$+ \frac{5}{6} bc^3 d \log(\sqrt{-c^2 x^2 + 1} + 1)$$

$$+ \frac{3bc^2 d(\sqrt{-c^2 x^2 + 1} + 1) \arcsin(cx)}{8x}$$

$$+ \frac{3ac^2 d(\sqrt{-c^2 x^2 + 1} + 1)}{8x}$$

$$- \frac{bcd(\sqrt{-c^2 x^2 + 1} + 1)^2}{24x^2}$$

$$- \frac{bd(\sqrt{-c^2 x^2 + 1} + 1)^3 \arcsin(cx)}{24x^3}$$

$$- \frac{ad(\sqrt{-c^2 x^2 + 1} + 1)^3}{24x^3}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output `-1/24*b*c^6*d*x^3*arcsin(c*x)/(sqrt(-c^2*x^2 + 1) + 1)^3 - 1/24*a*c^6*d*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + 1/24*b*c^5*d*x^2/(sqrt(-c^2*x^2 + 1) + 1)^2 + 3/8*b*c^4*d*x*arcsin(c*x)/(sqrt(-c^2*x^2 + 1) + 1) + 3/8*a*c^4*d*x/(sqrt(-c^2*x^2 + 1) + 1) - 5/6*b*c^3*d*log(abs(c)*abs(x)) + 5/6*b*c^3*d*log(sqrt(-c^2*x^2 + 1) + 1) + 3/8*b*c^2*d*(sqrt(-c^2*x^2 + 1) + 1)*arcsin(c*x)/x + 3/8*a*c^2*d*(sqrt(-c^2*x^2 + 1) + 1)/x - 1/24*b*c*d*(sqrt(-c^2*x^2 + 1) + 1)^2/x^2 - 1/24*b*d*(sqrt(-c^2*x^2 + 1) + 1)^3*arcsin(c*x)/x^3 - 1/24*a*d*(sqrt(-c^2*x^2 + 1) + 1)^3/x^3`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx))(d - c^2 dx^2)}{x^4} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x^4, x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2))/x^4, x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{d \left(6 \arcsin(cx) b c^2 x^2 - 2 \arcsin(cx) b - \sqrt{-c^2 x^2 + 1} b c x - 5 \log \left(\tan \left(\frac{\arcsin(cx)}{2} \right) \right) b c^3 x^3 + 6 a c^2 x^2 - 2 a \right)}{6 x^3}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x))/x^4, x)`

output `(d*(6*asin(c*x)*b*c**2*x**2 - 2*asin(c*x)*b - sqrt(-c**2*x**2 + 1)*b*c*x - 5*log(tan(asin(c*x)/2))*b*c**3*x**3 + 6*a*c**2*x**2 - 2*a))/(6*x**3)`

3.10 $\int x^4(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	272
Mathematica [A] (verified)	273
Rubi [A] (verified)	273
Maple [A] (verified)	275
Fricas [A] (verification not implemented)	276
Sympy [A] (verification not implemented)	276
Maxima [B] (verification not implemented)	277
Giac [A] (verification not implemented)	278
Mupad [F(-1)]	278
Reduce [B] (verification not implemented)	279

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int x^4(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{8bd^2\sqrt{1 - c^2x^2}}{315c^5} + \frac{4bd^2(1 - c^2x^2)^{3/2}}{945c^5} + \frac{bd^2(1 - c^2x^2)^{5/2}}{525c^5}$$

$$- \frac{10bd^2(1 - c^2x^2)^{7/2}}{441c^5} + \frac{bd^2(1 - c^2x^2)^{9/2}}{81c^5}$$

$$+ \frac{1}{5}d^2x^5(a + b \arcsin(cx)) - \frac{2}{7}c^2d^2x^7(a + b \arcsin(cx)) + \frac{1}{9}c^4d^2x^9(a + b \arcsin(cx))$$

output

```
8/315*b*d^2*(-c^2*x^2+1)^(1/2)/c^5+4/945*b*d^2*(-c^2*x^2+1)^(3/2)/c^5+1/525*b*d^2*(-c^2*x^2+1)^(5/2)/c^5-10/441*b*d^2*(-c^2*x^2+1)^(7/2)/c^5+1/81*b*d^2*(-c^2*x^2+1)^(9/2)/c^5+1/5*d^2*x^5*(a+b*arcsin(c*x))-2/7*c^2*d^2*x^7*(a+b*arcsin(c*x))+1/9*c^4*d^2*x^9*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.64

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2 (315ac^5 x^5 (63 - 90c^2 x^2 + 35c^4 x^4) + b\sqrt{1 - c^2 x^2} (2104 + 1052c^2 x^2 + 789c^4 x^4 - 2650c^6 x^6 + 1225c^8 x^8))}{99225c^5}$$

input

```
Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(d^2*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcSin[c*x]))/(99225*c^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5192, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{d^2 x^5 (35c^4 x^4 - 90c^2 x^2 + 63)}{315\sqrt{1 - c^2 x^2}} dx + \frac{1}{9} c^4 d^2 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 d^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} d^2 x^5 (a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{315} bcd^2 \int \frac{x^5 (35c^4 x^4 - 90c^2 x^2 + 63)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{9} c^4 d^2 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 d^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} d^2 x^5 (a + b \arcsin(cx))$$

$$\downarrow 1578$$

$$-\frac{1}{630}bcd^2 \int \frac{x^4(35c^4x^4 - 90c^2x^2 + 63)}{\sqrt{1-c^2x^2}} dx^2 + \frac{1}{9}c^4d^2x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2d^2x^7(a + b \arcsin(cx)) + \frac{1}{5}d^2x^5(a + b \arcsin(cx))$$

$$\downarrow 1195$$

$$-\frac{1}{630}bcd^2 \int \left(\frac{35(1-c^2x^2)^{7/2}}{c^4} - \frac{50(1-c^2x^2)^{5/2}}{c^4} + \frac{3(1-c^2x^2)^{3/2}}{c^4} + \frac{4\sqrt{1-c^2x^2}}{c^4} + \frac{8}{c^4\sqrt{1-c^2x^2}} \right) dx^2 + \frac{1}{9}c^4d^2x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2d^2x^7(a + b \arcsin(cx)) + \frac{1}{5}d^2x^5(a + b \arcsin(cx))$$

$$\downarrow 2009$$

$$\frac{1}{9}c^4d^2x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2d^2x^7(a + b \arcsin(cx)) + \frac{1}{5}d^2x^5(a + b \arcsin(cx)) - \frac{1}{630}bcd^2 \left(-\frac{70(1-c^2x^2)^{9/2}}{9c^6} + \frac{100(1-c^2x^2)^{7/2}}{7c^6} - \frac{6(1-c^2x^2)^{5/2}}{5c^6} - \frac{8(1-c^2x^2)^{3/2}}{3c^6} - \frac{16\sqrt{1-c^2x^2}}{c^6} \right)$$

input `Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/630*(b*c*d^2*((-16*sqrt[1 - c^2*x^2])/c^6 - (8*(1 - c^2*x^2)^(3/2))/(3*c^6) - (6*(1 - c^2*x^2)^(5/2))/(5*c^6) + (100*(1 - c^2*x^2)^(7/2))/(7*c^6) - (70*(1 - c^2*x^2)^(9/2))/(9*c^6))) + (d^2*x^5*(a + b*ArcSin[c*x]))/5 - (2*c^2*d^2*x^7*(a + b*ArcSin[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcSin[c*x]))/9`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1195 `Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 1578

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5192

```
Int[((a_) + ArcSin[(c_)*(x)])*(b_)*((f_)*(x))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.90

method	result
parts	$a d^2 \left(\frac{1}{9} c^4 x^9 - \frac{2}{7} c^2 x^7 + \frac{1}{5} x^5 \right) + \frac{d^2 b \left(\frac{\arcsin(cx) c^9 x^9}{9} - \frac{2 \arcsin(cx) c^7 x^7}{7} + \frac{c^5 x^5 \arcsin(cx)}{5} + \frac{263 c^4 x^4 \sqrt{-c^2 x^2 + 1}}{33075} + \frac{1052 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{99225} \right)}{c^5}$
derivativedivides	$a d^2 \left(\frac{1}{9} c^9 x^9 - \frac{2}{7} c^7 x^7 + \frac{1}{5} c^5 x^5 \right) + d^2 b \left(\frac{\arcsin(cx) c^9 x^9}{9} - \frac{2 \arcsin(cx) c^7 x^7}{7} + \frac{c^5 x^5 \arcsin(cx)}{5} + \frac{263 c^4 x^4 \sqrt{-c^2 x^2 + 1}}{33075} + \frac{1052 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{99225} \right)$
default	$a d^2 \left(\frac{1}{9} c^9 x^9 - \frac{2}{7} c^7 x^7 + \frac{1}{5} c^5 x^5 \right) + d^2 b \left(\frac{\arcsin(cx) c^9 x^9}{9} - \frac{2 \arcsin(cx) c^7 x^7}{7} + \frac{c^5 x^5 \arcsin(cx)}{5} + \frac{263 c^4 x^4 \sqrt{-c^2 x^2 + 1}}{33075} + \frac{1052 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{99225} \right)$
oring	$\frac{(20825 c^{10} x^{10} - 54450 c^8 x^8 + 36757 c^6 x^6 + 5260 c^4 x^4 + 12624 c^2 x^2 - 8416) (-c^2 d x^2 + d)^2 (a + b \arcsin(cx))}{99225 c^6 (cx - 1) x (cx + 1) (c^2 x^2 - 1)} - \frac{(1225 c^8 x^8 - \dots)}{\dots}$

input

```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
a*d^2*(1/9*c^4*x^9-2/7*c^2*x^7+1/5*x^5)+d^2*b/c^5*(1/9*arcsin(c*x)*c^9*x^9-2/7*arcsin(c*x)*c^7*x^7+1/5*c^5*x^5*arcsin(c*x)+263/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)+1052/99225*c^2*x^2*(-c^2*x^2+1)^(1/2)+2104/99225*(-c^2*x^2+1)^(1/2)-106/3969*c^6*x^6*(-c^2*x^2+1)^(1/2)+1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```


Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{11025 ac^9 d^2 x^9 - 28350 ac^7 d^2 x^7 + 19845 ac^5 d^2 x^5 + 315 (35 bc^9 d^2 x^9 - 90 bc^7 d^2 x^7 + 63 bc^5 d^2 x^5) \arcsin(cx)}{99225 c^5}$$

input `integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")`output `1/99225*(11025*a*c^9*d^2*x^9 - 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 + 315*(35*b*c^9*d^2*x^9 - 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*arcsin(c*x) + (1225*b*c^8*d^2*x^8 - 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x^4 + 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(-c^2*x^2 + 1))/c^5`**Sympy [A] (verification not implemented)**

Time = 1.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ac^4 d^2 x^9}{9} - \frac{2ac^2 d^2 x^7}{7} + \frac{ad^2 x^5}{5} + \frac{bc^4 d^2 x^9 \arcsin(cx)}{9} + \frac{bc^3 d^2 x^8 \sqrt{-c^2 x^2 + 1}}{81} - \frac{2bc^2 d^2 x^7 \arcsin(cx)}{7} - \frac{106bcd^2 x^6 \sqrt{-c^2 x^2 + 1}}{3969} + \frac{bd^2 x^5}{5} \\ \frac{ad^2 x^5}{5} \end{cases}$$

input `integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*asin(c*x)),x)`output `Piecewise((a*c**4*d**2*x**9/9 - 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c**4*d**2*x**9*asin(c*x)/9 + b*c**3*d**2*x**8*sqrt(-c**2*x**2 + 1)/81 - 2*b*c**2*d**2*x**7*asin(c*x)/7 - 106*b*c*d**2*x**6*sqrt(-c**2*x**2 + 1)/3969 + b*d**2*x**5*asin(c*x)/5 + 263*b*d**2*x**4*sqrt(-c**2*x**2 + 1)/(33075*c) + 1052*b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(99225*c**3) + 2104*b*d**2*sqrt(-c**2*x**2 + 1)/(99225*c**5), Ne(c, 0)), (a*d**2*x**5/5, True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(160) = 320$.

Time = 0.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.76

$$\int x^4(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{9} ac^4 d^2 x^9 - \frac{2}{7} ac^2 d^2 x^7$$

$$+ \frac{1}{2835} \left(315 x^9 \arcsin(cx) + \left(\frac{35 \sqrt{-c^2 x^2 + 1} x^8}{c^2} + \frac{40 \sqrt{-c^2 x^2 + 1} x^6}{c^4} + \frac{48 \sqrt{-c^2 x^2 + 1} x^4}{c^6} + \frac{64 \sqrt{-c^2 x^2 + 1} x^2}{c^8} \right) \right.$$

$$+ \frac{1}{5} ad^2 x^5$$

$$- \frac{2}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right.$$

$$\left. + \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) \right) bd^2$$

input `integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*c^4*d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^2`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.53

$$\begin{aligned}
& \int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx \\
&= \frac{1}{9} ac^4 d^2 x^9 - \frac{2}{7} ac^2 d^2 x^7 + \frac{1}{5} ad^2 x^5 + \frac{(c^2 x^2 - 1)^4 bd^2 x \arcsin(cx)}{9 c^4} \\
&+ \frac{10 (c^2 x^2 - 1)^3 bd^2 x \arcsin(cx)}{63 c^4} + \frac{(c^2 x^2 - 1)^2 bd^2 x \arcsin(cx)}{105 c^4} \\
&+ \frac{(c^2 x^2 - 1)^4 \sqrt{-c^2 x^2 + 1} bd^2}{81 c^5} - \frac{4 (c^2 x^2 - 1) bd^2 x \arcsin(cx)}{315 c^4} \\
&+ \frac{10 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd^2}{441 c^5} + \frac{8 bd^2 x \arcsin(cx)}{315 c^4} \\
&+ \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^2}{525 c^5} + \frac{4 (-c^2 x^2 + 1)^{\frac{3}{2}} bd^2}{945 c^5} + \frac{8 \sqrt{-c^2 x^2 + 1} bd^2}{315 c^5}
\end{aligned}$$

input `integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/5*a*d^2*x^5 + 1/9*(c^2*x^2 - 1)^4*b*d^2*x*arcsin(c*x)/c^4 + 10/63*(c^2*x^2 - 1)^3*b*d^2*x*arcsin(c*x)/c^4 + 1/105*(c^2*x^2 - 1)^2*b*d^2*x*arcsin(c*x)/c^4 + 1/81*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*d^2/c^5 - 4/315*(c^2*x^2 - 1)*b*d^2*x*arcsin(c*x)/c^4 + 10/441*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^2/c^5 + 8/315*b*d^2*x*arcsin(c*x)/c^4 + 1/525*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^2/c^5 + 4/945*(-c^2*x^2 + 1)^(3/2)*b*d^2/c^5 + 8/315*sqrt(-c^2*x^2 + 1)*b*d^2/c^5`

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^2 dx$$

input `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^2,x)`

output `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2 (11025 \operatorname{asin}(cx) b c^9 x^9 - 28350 \operatorname{asin}(cx) b c^7 x^7 + 19845 \operatorname{asin}(cx) b c^5 x^5 + 1225 \sqrt{-c^2 x^2 + 1} b c^8 x^8 - 2650 \sqrt{-c^2 x^2 + 1} b c^6 x^6 + 789 \sqrt{-c^2 x^2 + 1} b c^4 x^4 + 1052 \sqrt{-c^2 x^2 + 1} b c^2 x^2 + 2104 \sqrt{-c^2 x^2 + 1} b + 11025 a c^9 x^9 - 28350 a c^7 x^7 + 19845 a c^5 x^5)}{(99225 c^5)}$$

input

```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*asin(c*x)),x)
```

output

```
(d**2*(11025*asin(c*x)*b*c**9*x**9 - 28350*asin(c*x)*b*c**7*x**7 + 19845*asin(c*x)*b*c**5*x**5 + 1225*sqrt(-c**2*x**2 + 1)*b*c**8*x**8 - 2650*sqrt(-c**2*x**2 + 1)*b*c**6*x**6 + 789*sqrt(-c**2*x**2 + 1)*b*c**4*x**4 + 1052*sqrt(-c**2*x**2 + 1)*b*c**2*x**2 + 2104*sqrt(-c**2*x**2 + 1)*b + 11025*a*c**9*x**9 - 28350*a*c**7*x**7 + 19845*a*c**5*x**5))/(99225*c**5)
```

3.11 $\int x^3(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 25, antiderivative size = 184

$$\int x^3(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{73bd^2 x \sqrt{1 - c^2 x^2}}{3072c^3} + \frac{73bd^2 x^3 \sqrt{1 - c^2 x^2}}{4608c} - \frac{43bcd^2 x^5 \sqrt{1 - c^2 x^2}}{1152} + \frac{1}{64} bc^3 d^2 x^7 \sqrt{1 - c^2 x^2} - \frac{73bd^2 \arcsin(cx)}{3072c^4} + \frac{1}{4} d^2 x^4 (a + b \arcsin(cx)) - \frac{1}{3} c^2 d^2 x^6 (a + b \arcsin(cx)) + \frac{1}{8} c^4 d^2 x^8 (a + b \arcsin(cx))$$

output

```
73/3072*b*d^2*x*(-c^2*x^2+1)^(1/2)/c^3+73/4608*b*d^2*x^3*(-c^2*x^2+1)^(1/2)/c-43/1152*b*c*d^2*x^5*(-c^2*x^2+1)^(1/2)+1/64*b*c^3*d^2*x^7*(-c^2*x^2+1)^(1/2)-73/3072*b*d^2*arcsin(c*x)/c^4+1/4*d^2*x^4*(a+b*arcsin(c*x))-1/3*c^2*d^2*x^6*(a+b*arcsin(c*x))+1/8*c^4*d^2*x^8*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.62

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2 (384ac^4 x^4 (6 - 8c^2 x^2 + 3c^4 x^4) + bcx \sqrt{1 - c^2 x^2} (219 + 146c^2 x^2 - 344c^4 x^4 + 144c^6 x^6) + 3b(-73 + 768c^4 x^4 - 1024c^6 x^6 + 384c^8 x^8) \operatorname{ArcSin}[cx])}{9216c^4}$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(d^2*(384*a*c^4*x^4*(6 - 8*c^2*x^2 + 3*c^4*x^4) + b*c*x*Sqrt[1 - c^2*x^2]*
(219 + 146*c^2*x^2 - 344*c^4*x^4 + 144*c^6*x^6) + 3*b*(-73 + 768*c^4*x^4 -
1024*c^6*x^6 + 384*c^8*x^8)*ArcSin[c*x]))/(9216*c^4)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5192, 27, 1590, 25, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{d^2 x^4 (3c^4 x^4 - 8c^2 x^2 + 6)}{24\sqrt{1 - c^2 x^2}} dx + \frac{1}{8} c^4 d^2 x^8 (a + b \arcsin(cx)) - \frac{1}{3} c^2 d^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4} d^2 x^4 (a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{24} bcd^2 \int \frac{x^4 (3c^4 x^4 - 8c^2 x^2 + 6)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{8} c^4 d^2 x^8 (a + b \arcsin(cx)) - \frac{1}{3} c^2 d^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4} d^2 x^4 (a + b \arcsin(cx))$$

$$\begin{aligned}
& \downarrow 1590 \\
& -\frac{1}{24}bcd^2 \left(-\frac{\int -\frac{c^2x^4(48-43c^2x^2)}{\sqrt{1-c^2x^2}} dx}{8c^2} - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right) + \frac{1}{8}c^4d^2x^8(a+b\arcsin(cx)) - \\
& \quad \frac{1}{3}c^2d^2x^6(a+b\arcsin(cx)) + \frac{1}{4}d^2x^4(a+b\arcsin(cx)) \\
& \downarrow 25 \\
& -\frac{1}{24}bcd^2 \left(\frac{\int \frac{c^2x^4(48-43c^2x^2)}{\sqrt{1-c^2x^2}} dx}{8c^2} - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right) + \frac{1}{8}c^4d^2x^8(a+b\arcsin(cx)) - \\
& \quad \frac{1}{3}c^2d^2x^6(a+b\arcsin(cx)) + \frac{1}{4}d^2x^4(a+b\arcsin(cx)) \\
& \downarrow 27 \\
& -\frac{1}{24}bcd^2 \left(\frac{1}{8} \int \frac{x^4(48-43c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right) + \frac{1}{8}c^4d^2x^8(a+b\arcsin(cx)) - \\
& \quad \frac{1}{3}c^2d^2x^6(a+b\arcsin(cx)) + \frac{1}{4}d^2x^4(a+b\arcsin(cx)) \\
& \downarrow 363 \\
& -\frac{1}{24}bcd^2 \left(\frac{1}{8} \left(\frac{73}{6} \int \frac{x^4}{\sqrt{1-c^2x^2}} dx + \frac{43}{6}x^5\sqrt{1-c^2x^2} \right) - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right) + \frac{1}{8}c^4d^2x^8(a + \\
& \quad b\arcsin(cx)) - \frac{1}{3}c^2d^2x^6(a+b\arcsin(cx)) + \frac{1}{4}d^2x^4(a+b\arcsin(cx)) \\
& \downarrow 262 \\
& -\frac{1}{24}bcd^2 \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{43}{6}x^5\sqrt{1-c^2x^2} \right) - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right) + \\
& \quad \frac{1}{8}c^4d^2x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2d^2x^6(a+b\arcsin(cx)) + \frac{1}{4}d^2x^4(a+b\arcsin(cx)) \\
& \downarrow 262 \\
& -\frac{1}{24}bcd^2 \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{43}{6}x^5\sqrt{1-c^2x^2} \right) - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right) + \\
& \quad \frac{1}{8}c^4d^2x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2d^2x^6(a+b\arcsin(cx)) + \frac{1}{4}d^2x^4(a+b\arcsin(cx)) \\
& \downarrow 223
\end{aligned}$$

$$\frac{1}{8}c^4d^2x^8(a + b \arcsin(cx)) - \frac{1}{3}c^2d^2x^6(a + b \arcsin(cx)) + \frac{1}{4}d^2x^4(a + b \arcsin(cx)) - \frac{1}{24}bcd^2 \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{43}{6}x^5\sqrt{1-c^2x^2} \right) - \frac{3}{8}c^2x^7\sqrt{1-c^2x^2} \right)$$

input `Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `(d^2*x^4*(a + b*ArcSin[c*x]))/4 - (c^2*d^2*x^6*(a + b*ArcSin[c*x]))/3 + (c^4*d^2*x^8*(a + b*ArcSin[c*x]))/8 - (b*c*d^2*((-3*c^2*x^7*sqrt[1 - c^2*x^2])/8 + ((43*x^5*sqrt[1 - c^2*x^2])/6 + (73*(-1/4*(x^3*sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/6)/8)/24`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 262 `Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

output

```
a*d^2*(1/8*c^4*x^8-1/3*c^2*x^6+1/4*x^4)+d^2*b/c^4*(1/8*arcsin(c*x)*c^8*x^8
-1/3*arcsin(c*x)*c^6*x^6+1/4*c^4*x^4*arcsin(c*x)+73/4608*c^3*x^3*(-c^2*x^2
+1)^(1/2)+73/3072*c*x*(-c^2*x^2+1)^(1/2)-73/3072*arcsin(c*x)-43/1152*c^5*x
^5*(-c^2*x^2+1)^(1/2)+1/64*c^7*x^7*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{1152 ac^8 d^2 x^8 - 3072 ac^6 d^2 x^6 + 2304 ac^4 d^2 x^4 + 3(384 bc^8 d^2 x^8 - 1024 bc^6 d^2 x^6 + 768 bc^4 d^2 x^4 - 73 bd^2) \arcsin(cx)}{9216 c^4}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
1/9216*(1152*a*c^8*d^2*x^8 - 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(
384*b*c^8*d^2*x^8 - 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*arc
sin(c*x) + (144*b*c^7*d^2*x^7 - 344*b*c^5*d^2*x^5 + 146*b*c^3*d^2*x^3 + 21
9*b*c*d^2*x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ac^4 d^2 x^8}{8} - \frac{ac^2 d^2 x^6}{3} + \frac{ad^2 x^4}{4} + \frac{bc^4 d^2 x^8 \arcsin(cx)}{8} + \frac{bc^3 d^2 x^7 \sqrt{-c^2 x^2 + 1}}{64} - \frac{bc^2 d^2 x^6 \arcsin(cx)}{3} - \frac{43bcd^2 x^5 \sqrt{-c^2 x^2 + 1}}{1152} + \frac{bd^2 x^4 \arcsin(cx)}{4} \\ \frac{ad^2 x^4}{4} \end{cases}$$

input

```
integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*c**4*d**2*x**8/8 - a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**
4*d**2*x**8*asin(c*x)/8 + b*c**3*d**2*x**7*sqrt(-c**2*x**2 + 1)/64 - b*c**
2*d**2*x**6*asin(c*x)/3 - 43*b*c*d**2*x**5*sqrt(-c**2*x**2 + 1)/1152 + b*d
**2*x**4*asin(c*x)/4 + 73*b*d**2*x**3*sqrt(-c**2*x**2 + 1)/(4608*c) + 73*b
*d**2*x*sqrt(-c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*asin(c*x)/(3072*c**4)
, Ne(c, 0)), (a*d**2*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.62

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{8} ac^4 d^2 x^8 - \frac{1}{3} ac^2 d^2 x^6$$

$$+ \frac{1}{3072} \left(384 x^8 \arcsin(cx) + \left(\frac{48 \sqrt{-c^2 x^2 + 1} x^7}{c^2} + \frac{56 \sqrt{-c^2 x^2 + 1} x^5}{c^4} + \frac{70 \sqrt{-c^2 x^2 + 1} x^3}{c^6} + \frac{105 \sqrt{-c^2 x^2 + 1}}{c^8} \right. \right.$$

$$\left. \left. + \frac{1}{4} ad^2 x^4 \right. \right.$$

$$\left. - \frac{1}{144} \left(48 x^6 \arcsin(cx) + \left(\frac{8 \sqrt{-c^2 x^2 + 1} x^5}{c^2} + \frac{10 \sqrt{-c^2 x^2 + 1} x^3}{c^4} + \frac{15 \sqrt{-c^2 x^2 + 1} x}{c^6} - \frac{15 \arcsin(cx)}{c^7} \right) \right. \right.$$

$$\left. \left. + \frac{1}{32} \left(8 x^4 \arcsin(cx) + \left(\frac{2 \sqrt{-c^2 x^2 + 1} x^3}{c^2} + \frac{3 \sqrt{-c^2 x^2 + 1} x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) \right) c \right) bd^2$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```
1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arcsin(c*x) + (48*
sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*
x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*
b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 +
1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6
- 15*arcsin(c*x)/c^7)*c)*b*c^2*d^2 + 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^
2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*
d^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\begin{aligned}
& \int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx \\
&= \frac{1}{8} ac^4 d^2 x^8 - \frac{1}{3} ac^2 d^2 x^6 + \frac{1}{4} ad^2 x^4 + \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd^2 x}{64 c^3} \\
&\quad + \frac{(c^2 x^2 - 1)^4 bd^2 \arcsin(cx)}{8 c^4} + \frac{11 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^2 x}{1152 c^3} \\
&\quad + \frac{(c^2 x^2 - 1)^3 bd^2 \arcsin(cx)}{6 c^4} + \frac{55 (-c^2 x^2 + 1)^{\frac{3}{2}} bd^2 x}{4608 c^3} \\
&\quad + \frac{55 \sqrt{-c^2 x^2 + 1} bd^2 x}{3072 c^3} + \frac{55 bd^2 \arcsin(cx)}{3072 c^4}
\end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/4*a*d^2*x^4 + 1/64*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^2*x/c^3 + 1/8*(c^2*x^2 - 1)^4*b*d^2*arcsin(c*x)/c^4 + 11/1152*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^2*x/c^3 + 1/6*(c^2*x^2 - 1)^3*b*d^2*arcsin(c*x)/c^4 + 55/4608*(-c^2*x^2 + 1)^(3/2)*b*d^2*x/c^3 + 55/3072*sqrt(-c^2*x^2 + 1)*b*d^2*x/c^3 + 55/3072*b*d^2*arcsin(c*x)/c^4`

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int x^3 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^2 dx$$

input `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^2,x)`

output `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.86

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2 (1152 \operatorname{asin}(cx) b c^8 x^8 - 3072 \operatorname{asin}(cx) b c^6 x^6 + 2304 \operatorname{asin}(cx) b c^4 x^4 - 219 \operatorname{asin}(cx) b + 144 \sqrt{-c^2 x^2 + 1} b}{9216 c^4}$$

input

```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*asin(c*x)),x)
```

output

```
(d**2*(1152*asin(c*x)*b*c**8*x**8 - 3072*asin(c*x)*b*c**6*x**6 + 2304*asin(c*x)*b*c**4*x**4 - 219*asin(c*x)*b + 144*sqrt(-c**2*x**2 + 1)*b*c**7*x**7 - 344*sqrt(-c**2*x**2 + 1)*b*c**5*x**5 + 146*sqrt(-c**2*x**2 + 1)*b*c**3*x**3 + 219*sqrt(-c**2*x**2 + 1)*b*c*x + 1152*a*c**8*x**8 - 3072*a*c**6*x**6 + 2304*a*c**4*x**4))/(9216*c**4)
```

3.12 $\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 25, antiderivative size = 161

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{8bd^2\sqrt{1 - c^2x^2}}{105c^3} + \frac{4bd^2(1 - c^2x^2)^{3/2}}{315c^3} + \frac{bd^2(1 - c^2x^2)^{5/2}}{175c^3} - \frac{bd^2(1 - c^2x^2)^{7/2}}{49c^3}$$

$$+ \frac{1}{3}d^2x^3(a + b \arcsin(cx)) - \frac{2}{5}c^2d^2x^5(a + b \arcsin(cx)) + \frac{1}{7}c^4d^2x^7(a + b \arcsin(cx))$$

output

```
8/105*b*d^2*(-c^2*x^2+1)^(1/2)/c^3+4/315*b*d^2*(-c^2*x^2+1)^(3/2)/c^3+1/17
5*b*d^2*(-c^2*x^2+1)^(5/2)/c^3-1/49*b*d^2*(-c^2*x^2+1)^(7/2)/c^3+1/3*d^2*x
^3*(a+b*arcsin(c*x))-2/5*c^2*d^2*x^5*(a+b*arcsin(c*x))+1/7*c^4*d^2*x^7*(a+
b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.69

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2(105ac^3x^3(35 - 42c^2x^2 + 15c^4x^4) + b\sqrt{1 - c^2x^2}(818 + 409c^2x^2 - 612c^4x^4 + 225c^6x^6) + 105bc^3x^3(35 - 42c^2x^2 + 15c^4x^4))}{11025c^3}$$

input `Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output $(d^2*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) + b*\text{Sqrt}[1 - c^2*x^2]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4)*\text{ArcSin}[c*x]))/(11025*c^3)$

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5192, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{d^2 x^3 (15c^4 x^4 - 42c^2 x^2 + 35)}{105\sqrt{1 - c^2 x^2}} dx + \frac{1}{7} c^4 d^2 x^7 (a + b \arcsin(cx)) - \frac{2}{5} c^2 d^2 x^5 (a + b \arcsin(cx)) + \frac{1}{3} d^2 x^3 (a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{105} bcd^2 \int \frac{x^3 (15c^4 x^4 - 42c^2 x^2 + 35)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{7} c^4 d^2 x^7 (a + b \arcsin(cx)) - \frac{2}{5} c^2 d^2 x^5 (a + b \arcsin(cx)) + \frac{1}{3} d^2 x^3 (a + b \arcsin(cx))$$

$$\downarrow 1578$$

$$-\frac{1}{210} bcd^2 \int \frac{x^2 (15c^4 x^4 - 42c^2 x^2 + 35)}{\sqrt{1 - c^2 x^2}} dx^2 + \frac{1}{7} c^4 d^2 x^7 (a + b \arcsin(cx)) - \frac{2}{5} c^2 d^2 x^5 (a + b \arcsin(cx)) + \frac{1}{3} d^2 x^3 (a + b \arcsin(cx))$$

$$\downarrow 1195$$

$$-\frac{1}{210}bcd^2 \int \left(-\frac{15(1-c^2x^2)^{5/2}}{c^2} + \frac{3(1-c^2x^2)^{3/2}}{c^2} + \frac{4\sqrt{1-c^2x^2}}{c^2} + \frac{8}{c^2\sqrt{1-c^2x^2}} \right) dx^2 + \frac{1}{7}c^4d^2x^7(a+b\arcsin(cx)) - \frac{2}{5}c^2d^2x^5(a+b\arcsin(cx)) + \frac{1}{3}d^2x^3(a+b\arcsin(cx))$$

↓ 2009

$$\frac{1}{7}c^4d^2x^7(a+b\arcsin(cx)) - \frac{2}{5}c^2d^2x^5(a+b\arcsin(cx)) + \frac{1}{3}d^2x^3(a+b\arcsin(cx)) - \frac{1}{210}bcd^2 \left(\frac{30(1-c^2x^2)^{7/2}}{7c^4} - \frac{6(1-c^2x^2)^{5/2}}{5c^4} - \frac{8(1-c^2x^2)^{3/2}}{3c^4} - \frac{16\sqrt{1-c^2x^2}}{c^4} \right)$$

input `Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/210*(b*c*d^2*((-16*sqrt[1 - c^2*x^2])/c^4 - (8*(1 - c^2*x^2)^(3/2))/(3*c^4) - (6*(1 - c^2*x^2)^(5/2))/(5*c^4) + (30*(1 - c^2*x^2)^(7/2))/(7*c^4)) + (d^2*x^3*(a + b*ArcSin[c*x]))/3 - (2*c^2*d^2*x^5*(a + b*ArcSin[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcSin[c*x]))/7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1195 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 1578 `Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

method	result
parts	$a d^2 \left(\frac{1}{7} c^4 x^7 - \frac{2}{5} c^2 x^5 + \frac{1}{3} x^3 \right) + \frac{d^2 b \left(\frac{\arcsin(cx) c^7 x^7}{7} - \frac{2 c^5 x^5 \arcsin(cx)}{5} + \frac{c^3 x^3 \arcsin(cx)}{3} + \frac{409 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{11025} + \frac{818 \sqrt{-c^2 x^2}}{11025} \right)}{c^3}$
derivativedivides	$\frac{a d^2 \left(\frac{1}{7} c^7 x^7 - \frac{2}{5} c^5 x^5 + \frac{1}{3} c^3 x^3 \right) + d^2 b \left(\frac{\arcsin(cx) c^7 x^7}{7} - \frac{2 c^5 x^5 \arcsin(cx)}{5} + \frac{c^3 x^3 \arcsin(cx)}{3} + \frac{409 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{11025} + \frac{818 \sqrt{-c^2 x^2}}{11025} \right)}{c^3}$
default	$\frac{a d^2 \left(\frac{1}{7} c^7 x^7 - \frac{2}{5} c^5 x^5 + \frac{1}{3} c^3 x^3 \right) + d^2 b \left(\frac{\arcsin(cx) c^7 x^7}{7} - \frac{2 c^5 x^5 \arcsin(cx)}{5} + \frac{c^3 x^3 \arcsin(cx)}{3} + \frac{409 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{11025} + \frac{818 \sqrt{-c^2 x^2}}{11025} \right)}{c^3}$
orering	$\frac{(2925 c^8 x^8 - 8532 c^6 x^6 + 7353 c^4 x^4 + 4090 c^2 x^2 - 1636) (-c^2 d x^2 + d)^2 (a + b \arcsin(cx))}{11025 c^4 (cx-1)x(cx+1)(c^2 x^2-1)} - \frac{(225 c^6 x^6 - 612 c^4 x^4 + 409 c^2 x^2)}{11025 c^3}$

input

```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
a*d^2*(1/7*c^4*x^7-2/5*c^2*x^5+1/3*x^3)+d^2*b/c^3*(1/7*arcsin(c*x)*c^7*x^7
-2/5*c^5*x^5*arcsin(c*x)+1/3*c^3*x^3*arcsin(c*x)+409/11025*c^2*x^2*(-c^2*x
^2+1)^(1/2)+818/11025*(-c^2*x^2+1)^(1/2)-68/1225*c^4*x^4*(-c^2*x^2+1)^(1/2
)+1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{1575 ac^7 d^2 x^7 - 4410 ac^5 d^2 x^5 + 3675 ac^3 d^2 x^3 + 105 (15 bc^7 d^2 x^7 - 42 bc^5 d^2 x^5 + 35 bc^3 d^2 x^3) \arcsin(cx) + (225 c^6 x^6 - 612 c^4 x^4 + 409 c^2 x^2)}{11025 c^3}$$

input `integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `1/11025*(1575*a*c^7*d^2*x^7 - 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105*(15*b*c^7*d^2*x^7 - 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*arcsin(c*x) + (225*b*c^6*d^2*x^6 - 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 + 818*b*d^2)*sqrt(-c^2*x^2 + 1))/c^3`

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ac^4 d^2 x^7}{7} - \frac{2ac^2 d^2 x^5}{5} + \frac{ad^2 x^3}{3} + \frac{bc^4 d^2 x^7 \arcsin(cx)}{7} + \frac{bc^3 d^2 x^6 \sqrt{-c^2 x^2 + 1}}{49} - \frac{2bc^2 d^2 x^5 \arcsin(cx)}{5} - \frac{68bcd^2 x^4 \sqrt{-c^2 x^2 + 1}}{1225} + \frac{bd^2 x^3 \arcsin(cx)}{3} \\ \frac{ad^2 x^3}{3} \end{cases}$$

input `integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*asin(c*x)),x)`

output `Piecewise((a*c**4*d**2*x**7/7 - 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c**4*d**2*x**7*asin(c*x)/7 + b*c**3*d**2*x**6*sqrt(-c**2*x**2 + 1)/49 - 2*b*c**2*d**2*x**5*asin(c*x)/5 - 68*b*c*d**2*x**4*sqrt(-c**2*x**2 + 1)/1225 + b*d**2*x**3*asin(c*x)/3 + 409*b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(11025*c) + 818*b*d**2*sqrt(-c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (a*d**2*x**3/3, True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.66

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{7} ac^4 d^2 x^7 - \frac{2}{5} ac^2 d^2 x^5$$

$$+ \frac{1}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right)$$

$$- \frac{2}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bc^2 d^2$$

$$+ \frac{1}{3} ad^2 x^3 + \frac{1}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bd^2$$

input `integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*b*c^4*d^2 - 2/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{1}{7} ac^4 d^2 x^7 - \frac{2}{5} ac^2 d^2 x^5 + \frac{1}{3} ad^2 x^3 + \frac{(c^2 x^2 - 1)^3 bd^2 x \arcsin(cx)}{7 c^2}$$

$$+ \frac{(c^2 x^2 - 1)^2 bd^2 x \arcsin(cx)}{35 c^2} - \frac{4(c^2 x^2 - 1) bd^2 x \arcsin(cx)}{105 c^2}$$

$$+ \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd^2}{49 c^3} + \frac{8 bd^2 x \arcsin(cx)}{105 c^2}$$

$$+ \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^2}{175 c^3} + \frac{4(-c^2 x^2 + 1)^{\frac{3}{2}} bd^2}{315 c^3} + \frac{8 \sqrt{-c^2 x^2 + 1} bd^2}{105 c^3}$$

input `integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output
$$\frac{1}{7}ac^4d^2x^7 - \frac{2}{5}a^2c^2d^2x^5 + \frac{1}{3}ad^2x^3 + \frac{1}{7}(c^2x^2 - 1)^3 b^2d^2x \arcsin(cx) / c^2 + \frac{1}{35}(c^2x^2 - 1)^2 b^2d^2x \arcsin(cx) / c^2 - \frac{4}{105}(c^2x^2 - 1) b^2d^2x \arcsin(cx) / c^2 + \frac{1}{49}(c^2x^2 - 1)^3 \sqrt{-c^2x^2 + 1} b^2d^2 / c^3 + \frac{8}{105} b^2d^2x \arcsin(cx) / c^2 + \frac{1}{175}(c^2x^2 - 1)^2 \sqrt{-c^2x^2 + 1} b^2d^2 / c^3 + \frac{4}{315}(c^2x^2 - 1)^{3/2} b^2d^2 / c^3 + \frac{8}{105} \sqrt{-c^2x^2 + 1} b^2d^2 / c^3$$

Mupad [F(-1)]

Timed out.

$$\int x^2(d - c^2dx^2)^2(a + b \arcsin(cx)) dx = \int x^2(a + b \arcsin(cx))(d - c^2dx^2)^2 dx$$

input `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^2,x)`

output `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int x^2(d - c^2dx^2)^2(a + b \arcsin(cx)) dx = \frac{d^2(1575a \sin(cx) b c^7 x^7 - 4410a \sin(cx) b c^5 x^5 + 3675a \sin(cx) b c^3 x^3 + 225\sqrt{-c^2x^2 + 1} b c^6 x^6 - 612\sqrt{-c^2x^2 + 1} b c^4 x^4 + 11025 a^2 \arcsin(cx) b c^2 x^2 - 11025 a^2 \arcsin(cx) b c^2 x^2)}{11025}$$

input `int(x^2*(-c^2*d*x^2+d)^2*(a+b*asin(c*x)),x)`

output

```
(d**2*(1575*asin(c*x)*b*c**7*x**7 - 4410*asin(c*x)*b*c**5*x**5 + 3675*asin
(c*x)*b*c**3*x**3 + 225*sqrt(-c**2*x**2 + 1)*b*c**6*x**6 - 612*sqrt(-c
**2*x**2 + 1)*b*c**4*x**4 + 409*sqrt(-c**2*x**2 + 1)*b*c**2*x**2 + 818*s
qrt(-c**2*x**2 + 1)*b + 1575*a*c**7*x**7 - 4410*a*c**5*x**5 + 3675*a*c**
3*x**3))/(11025*c**3)
```

3.13 $\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	297
Mathematica [A] (verified)	297
Rubi [A] (verified)	298
Maple [A] (verified)	300
Fricas [A] (verification not implemented)	300
Sympy [A] (verification not implemented)	301
Maxima [B] (verification not implemented)	301
Giac [A] (verification not implemented)	302
Mupad [F(-1)]	303
Reduce [B] (verification not implemented)	303

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{5bd^2 x \sqrt{1 - c^2 x^2}}{96c} + \frac{5bd^2 x(1 - c^2 x^2)^{3/2}}{144c} + \frac{bd^2 x(1 - c^2 x^2)^{5/2}}{36c} + \frac{5bd^2 \arcsin(cx)}{96c^2} - \frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))}{6c^2}$$

output

```
5/96*b*d^2*x*(-c^2*x^2+1)^(1/2)/c+5/144*b*d^2*x*(-c^2*x^2+1)^(3/2)/c+1/36*
b*d^2*x*(-c^2*x^2+1)^(5/2)/c+5/96*b*d^2*arcsin(c*x)/c^2-1/6*d^2*(-c^2*x^2+
1)^3*(a+b*arcsin(c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{d^2 \left(48a(-1 + c^2 x^2)^3 + bcx \sqrt{1 - c^2 x^2} (33 - 26c^2 x^2 + 8c^4 x^4) + 3b(-11 + 48c^2 x^2 - 48c^4 x^4 + 16c^6 x^6) \arcsin(cx) \right)}{288c^2}$$

input `Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output $(d^2*(48*a*(-1 + c^2*x^2)^3 + b*c*x*\text{Sqrt}[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4) + 3*b*(-11 + 48*c^2*x^2 - 48*c^4*x^4 + 16*c^6*x^6)*\text{ArcSin}[c*x]))/(288*c^2)$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5182, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx \\
 & \quad \downarrow 5182 \\
 & \frac{bd^2 \int (1 - c^2 x^2)^{5/2} dx}{6c} - \frac{d^2 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{6c^2} \\
 & \quad \downarrow 211 \\
 & \frac{bd^2 \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} dx + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right)}{6c} - \frac{d^2 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{6c^2} \\
 & \quad \downarrow 211 \\
 & \frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right)}{6c} - \frac{d^2 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{6c^2} \\
 & \quad \downarrow 211 \\
 & \frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right)}{6c} - \frac{d^2 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{6c^2}
 \end{aligned}$$

$$\frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)}{d^2(1-c^2x^2)^3 \frac{6c}{6c^2} (a + b \arcsin(cx))}$$

input `Int[x*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/6*(d^2*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x]))/c^2 + (b*d^2*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c))/4))/6))/(6*c)`

Defintions of rubi rules used

rule 211 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a d^2 (c^2 x^2 - 1)^3}{6} + d^2 b \left(\frac{\arcsin(cx) c^6 x^6}{6} - \frac{c^4 x^4 \arcsin(cx)}{2} + \frac{c^2 x^2 \arcsin(cx)}{2} - \frac{11 \arcsin(cx)}{96} + \frac{c^5 x^5 \sqrt{-c^2 x^2 + 1}}{36} - \frac{13 c^3 x^3 \sqrt{-c^2 x^2 + 1}}{144} \right) \frac{1}{c^2}$
default	$\frac{a d^2 (c^2 x^2 - 1)^3}{6} + d^2 b \left(\frac{\arcsin(cx) c^6 x^6}{6} - \frac{c^4 x^4 \arcsin(cx)}{2} + \frac{c^2 x^2 \arcsin(cx)}{2} - \frac{11 \arcsin(cx)}{96} + \frac{c^5 x^5 \sqrt{-c^2 x^2 + 1}}{36} - \frac{13 c^3 x^3 \sqrt{-c^2 x^2 + 1}}{144} \right) \frac{1}{c^2}$
parts	$\frac{a d^2 (c^2 x^2 - 1)^3}{6 c^2} + \frac{d^2 b \left(\frac{\arcsin(cx) c^6 x^6}{6} - \frac{c^4 x^4 \arcsin(cx)}{2} + \frac{c^2 x^2 \arcsin(cx)}{2} - \frac{11 \arcsin(cx)}{96} + \frac{c^5 x^5 \sqrt{-c^2 x^2 + 1}}{36} - \frac{13 c^3 x^3 \sqrt{-c^2 x^2 + 1}}{144} \right)}{c^2}$
orering	$\frac{(88 c^6 x^6 - 282 c^4 x^4 + 335 c^2 x^2 - 66) (-c^2 d x^2 + d)^2 (a + b \arcsin(cx))}{288 c^2 (cx - 1)(cx + 1)(c^2 x^2 - 1)} - \frac{(8 c^4 x^4 - 26 c^2 x^2 + 33) \left((-c^2 d x^2 + d)^2 (a + b \arcsin(cx)) \right)}{288 c^2}$

```
input int(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output 1/c^2*(1/6*a*d^2*(c^2*x^2-1)^3+d^2*b*(1/6*arcsin(c*x)*c^6*x^6-1/2*c^4*x^4*
arcsin(c*x)+1/2*c^2*x^2*arcsin(c*x)-11/96*arcsin(c*x)+1/36*c^5*x^5*(-c^2*x
^2+1)^(1/2)-13/144*c^3*x^3*(-c^2*x^2+1)^(1/2)+11/96*c*x*(-c^2*x^2+1)^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{48 ac^6 d^2 x^6 - 144 ac^4 d^2 x^4 + 144 ac^2 d^2 x^2 + 3(16 bc^6 d^2 x^6 - 48 bc^4 d^2 x^4 + 48 bc^2 d^2 x^2 - 11 bd^2) \arcsin(cx)}{288 c^2}$$

```
input integrate(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

$$\frac{1}{288}(48ac^6d^2x^6 - 144a^2c^4d^2x^4 + 144a^2c^2d^2x^2 + 3(16b^2c^6d^2x^6 - 48b^2c^4d^2x^4 + 48b^2c^2d^2x^2 - 11b^2d^2) \arcsin(cx) + (8b^2c^5d^2x^5 - 26b^2c^3d^2x^3 + 33b^2cd^2x) \sqrt{-c^2x^2 + 1}) / c^2$$

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.53

$$\int x(d - c^2dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ac^4d^2x^6}{6} - \frac{ac^2d^2x^4}{2} + \frac{ad^2x^2}{2} + \frac{bc^4d^2x^6 \arcsin(cx)}{6} + \frac{bc^3d^2x^5 \sqrt{-c^2x^2+1}}{36} - \frac{bc^2d^2x^4 \arcsin(cx)}{2} - \frac{13bcd^2x^3 \sqrt{-c^2x^2+1}}{144} + \frac{bd^2x^2 \arcsin(cx)}{2} \\ \frac{ad^2x^2}{2} \end{cases}$$

input

```
integrate(x*(-c**2*d*x**2+d)**2*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*c**4*d**2*x**6/6 - a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4*d**2*x**6*asin(c*x)/6 + b*c**3*d**2*x**5*sqrt(-c**2*x**2 + 1)/36 - b*c**2*d**2*x**4*asin(c*x)/2 - 13*b*c*d**2*x**3*sqrt(-c**2*x**2 + 1)/144 + b*d**2*x**2*asin(c*x)/2 + 11*b*d**2*x*sqrt(-c**2*x**2 + 1)/(96*c) - 11*b*d**2*asin(c*x)/(96*c**2), Ne(c, 0)), (a*d**2*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(107) = 214.

Time = 0.12 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.91

$$\int x(d - c^2dx^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{6} ac^4d^2x^6 - \frac{1}{2} ac^2d^2x^4$$

$$+ \frac{1}{288} \left(48x^6 \arcsin(cx) + \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15 \arcsin(cx)}{c^7} \right. \right.$$

$$\left. \left. - \frac{1}{16} \left(8x^4 \arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) c \right) bc^2d^2 \right.$$

$$\left. \left. + \frac{1}{2} ad^2x^2 + \frac{1}{4} \left(2x^2 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3} \right) \right) \right) bd^2$$

input `integrate(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output
$$\begin{aligned} & 1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^4*d^2 - 1/16*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d^2 \end{aligned}$$

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.27

$$\begin{aligned} \int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx &= \frac{1}{6} ac^4 d^2 x^6 - \frac{1}{2} ac^2 d^2 x^4 \\ &+ \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b d^2 x}{36 c} \\ &+ \frac{(c^2 x^2 - 1)^3 b d^2 \arcsin(cx)}{6 c^2} \\ &+ \frac{5(-c^2 x^2 + 1)^{\frac{3}{2}} b d^2 x}{144 c} + \frac{5 \sqrt{-c^2 x^2 + 1} b d^2 x}{96 c} \\ &+ \frac{(c^2 x^2 - 1) a d^2}{2 c^2} + \frac{5 b d^2 \arcsin(cx)}{96 c^2} \end{aligned}$$

input `integrate(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output
$$\begin{aligned} & 1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/36*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^2*x/c + 1/6*(c^2*x^2 - 1)^3*b*d^2*arcsin(c*x)/c^2 + 5/144*(-c^2*x^2 + 1)^{(3/2)}*b*d^2*x/c + 5/96*sqrt(-c^2*x^2 + 1)*b*d^2*x/c + 1/2*(c^2*x^2 - 1)*a*d^2/c^2 + 5/96*b*d^2*arcsin(c*x)/c^2 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int x(a + b \arcsin(cx)) (d - c^2 dx^2)^2 dx$$

input `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^2,x)`output `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^2, x)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2(48 \arcsin(cx) b c^6 x^6 - 144 \arcsin(cx) b c^4 x^4 + 144 \arcsin(cx) b c^2 x^2 - 33 \arcsin(cx) b + 8\sqrt{-c^2 x^2 + 1} b c^5 x^5 - 26 \sqrt{-c^2 x^2 + 1} b c^3 x^3 + 33 \sqrt{-c^2 x^2 + 1} b c x + 48 a c^6 x^6 - 144 a c^4 x^4 + 144 a c^2 x^2)}{288 c^2}$$

input `int(x*(-c^2*d*x^2+d)^2*(a+b*asin(c*x)),x)`output `(d**2*(48*asin(c*x)*b*c**6*x**6 - 144*asin(c*x)*b*c**4*x**4 + 144*asin(c*x)*b*c**2*x**2 - 33*asin(c*x)*b + 8*sqrt(-c**2*x**2 + 1)*b*c**5*x**5 - 26*sqrt(-c**2*x**2 + 1)*b*c**3*x**3 + 33*sqrt(-c**2*x**2 + 1)*b*c*x + 48*a*c**6*x**6 - 144*a*c**4*x**4 + 144*a*c**2*x**2))/(288*c**2)`

3.14 $\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 22, antiderivative size = 131

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{8bd^2\sqrt{1 - c^2x^2}}{15c} + \frac{4bd^2(1 - c^2x^2)^{3/2}}{45c} + \frac{bd^2(1 - c^2x^2)^{5/2}}{25c} + d^2x(a + b \arcsin(cx)) - \frac{2}{3}c^2d^2x^3(a + b \arcsin(cx)) + \frac{1}{5}c^4d^2x^5(a + b \arcsin(cx))$$

```
output 8/15*b*d^2*(-c^2*x^2+1)^(1/2)/c+4/45*b*d^2*(-c^2*x^2+1)^(3/2)/c+1/25*b*d^2
*(-c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*arcsin(c*x))-2/3*c^2*d^2*x^3*(a+b*arcsin(
c*x))+1/5*c^4*d^2*x^5*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2(15acx(15 - 10c^2x^2 + 3c^4x^4) + b\sqrt{1 - c^2x^2}(149 - 38c^2x^2 + 9c^4x^4) + 15bcx(15 - 10c^2x^2 + 3c^4x^4) \arcsin(cx))}{225c}$$

input

```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(d^2*(15*a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*(149 - 38*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)*ArcSin[c*x]))/(225*c)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5154, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5154$$

$$-bc \int \frac{d^2 x (3c^4 x^4 - 10c^2 x^2 + 15)}{15\sqrt{1 - c^2 x^2}} dx + \frac{1}{5} c^4 d^2 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 d^2 x^3 (a + b \arcsin(cx)) + d^2 x (a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{15} bcd^2 \int \frac{x(3c^4 x^4 - 10c^2 x^2 + 15)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{5} c^4 d^2 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 d^2 x^3 (a + b \arcsin(cx)) + d^2 x (a + b \arcsin(cx))$$

$$\downarrow 1576$$

$$\begin{aligned}
& -\frac{1}{30}bcd^2 \int \frac{3c^4x^4 - 10c^2x^2 + 15}{\sqrt{1-c^2x^2}} dx^2 + \frac{1}{5}c^4d^2x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2d^2x^3(a + \\
& \quad b \arcsin(cx)) + d^2x(a + b \arcsin(cx)) \\
& \quad \downarrow \text{1140} \\
& -\frac{1}{30}bcd^2 \int \left(3(1-c^2x^2)^{3/2} + 4\sqrt{1-c^2x^2} + \frac{8}{\sqrt{1-c^2x^2}} \right) dx^2 + \frac{1}{5}c^4d^2x^5(a + b \arcsin(cx)) - \\
& \quad \frac{2}{3}c^2d^2x^3(a + b \arcsin(cx)) + d^2x(a + b \arcsin(cx)) \\
& \quad \downarrow \text{2009} \\
& \frac{1}{5}c^4d^2x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2d^2x^3(a + b \arcsin(cx)) + d^2x(a + b \arcsin(cx)) - \\
& \quad \frac{1}{30}bcd^2 \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} - \frac{8(1-c^2x^2)^{3/2}}{3c^2} - \frac{16\sqrt{1-c^2x^2}}{c^2} \right)
\end{aligned}$$

input `Int[(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/30*(b*c*d^2*((-16*sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2))) + d^2*x*(a + b*ArcSin[c*x]) - (2*c^2*d^2*x^3*(a + b*ArcSin[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcSin[c*x]))/5`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1140 `Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]`

rule 1576 `Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]`

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5154 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

method	result
parts	$a d^2 \left(\frac{1}{5} c^4 x^5 - \frac{2}{3} c^2 x^3 + x \right) + \frac{b d^2 \left(\frac{c^5 x^5 \arcsin(cx)}{5} - \frac{2 c^3 x^3 \arcsin(cx)}{3} + c x \arcsin(cx) + \frac{149 \sqrt{-c^2 x^2 + 1}}{225} - \frac{38 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{225} \right)}{c}$
derivativedivides	$\frac{d^2 a \left(\frac{1}{5} c^5 x^5 - \frac{2}{3} c^3 x^3 + c x \right) + d^2 b \left(\frac{c^5 x^5 \arcsin(cx)}{5} - \frac{2 c^3 x^3 \arcsin(cx)}{3} + c x \arcsin(cx) + \frac{149 \sqrt{-c^2 x^2 + 1}}{225} - \frac{38 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{225} \right) + c^4}{c}$
default	$\frac{d^2 a \left(\frac{1}{5} c^5 x^5 - \frac{2}{3} c^3 x^3 + c x \right) + d^2 b \left(\frac{c^5 x^5 \arcsin(cx)}{5} - \frac{2 c^3 x^3 \arcsin(cx)}{3} + c x \arcsin(cx) + \frac{149 \sqrt{-c^2 x^2 + 1}}{225} - \frac{38 c^2 x^2 \sqrt{-c^2 x^2 + 1}}{225} \right) + c^4}{c}$
oring	$\frac{x(81c^4x^4 - 302c^2x^2 + 821)(-c^2dx^2 + d)^2(a + b \arcsin(cx))}{225(cx - 1)(cx + 1)(c^2x^2 - 1)} - \frac{(9c^4x^4 - 38c^2x^2 + 149) \left(-4(-c^2dx^2 + d)(a + b \arcsin(cx)) \right)}{225c^2(cx - 1)(cx + 1)}$

```
input int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*d^2*(1/5*c^4*x^5-2/3*c^2*x^3+x)+b*d^2/c*(1/5*c^5*x^5*arcsin(c*x)-2/3*c^3*x^3*arcsin(c*x)+c*x*arcsin(c*x)+149/225*(-c^2*x^2+1)^(1/2)-38/225*c^2*x^2*(-c^2*x^2+1)^(1/2)+1/25*c^4*x^4*(-c^2*x^2+1)^(1/2))
```


Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{45 ac^5 d^2 x^5 - 150 ac^3 d^2 x^3 + 225 acd^2 x + 15 (3 bc^5 d^2 x^5 - 10 bc^3 d^2 x^3 + 15 bcd^2 x) \arcsin(cx) + (9 bc^4 d^2 x^4 - 38 bc^2 d^2 x^2 + 149 b^2 d^2) \sqrt{-c^2 x^2 + 1}}{225 c}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*arcsin(c*x) + (9*b*c^4*d^2*x^4 - 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(-c^2*x^2 + 1))/c`

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.26

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ac^4 d^2 x^5}{5} - \frac{2ac^2 d^2 x^3}{3} + ad^2 x + \frac{bc^4 d^2 x^5 \arcsin(cx)}{5} + \frac{bc^3 d^2 x^4 \sqrt{-c^2 x^2 + 1}}{25} - \frac{2bc^2 d^2 x^3 \arcsin(cx)}{3} - \frac{38bcd^2 x^2 \sqrt{-c^2 x^2 + 1}}{225} + bd^2 x a \\ ad^2 x \end{cases}$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x)),x)`

output `Piecewise((a*c**4*d**2*x**5/5 - 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d**2*x**5*asin(c*x)/5 + b*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)/25 - 2*b*c**2*d**2*x**3*asin(c*x)/3 - 38*b*c*d**2*x**2*sqrt(-c**2*x**2 + 1)/225 + b*d**2*x*asin(c*x) + 149*b*d**2*sqrt(-c**2*x**2 + 1)/(225*c), Ne(c, 0)), (a*d**2*x, True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.50

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{5} ac^4 d^2 x^5 + \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bc^4 d^2 - \frac{2}{3} ac^2 d^2 x^3 - \frac{2}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bc^2 d^2 + ad^2 x + \frac{(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1}) bd^2}{c}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2 - 2/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d^2/c`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{5} ac^4 d^2 x^5 - \frac{2}{3} ac^2 d^2 x^3 + \frac{1}{5} (c^2 x^2 - 1)^2 bd^2 x \arcsin(cx) - \frac{4}{15} (c^2 x^2 - 1) bd^2 x \arcsin(cx) + \frac{8}{15} bd^2 x \arcsin(cx) + \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^2}{25 c} + ad^2 x + \frac{4(-c^2 x^2 + 1)^{\frac{3}{2}} bd^2}{45 c} + \frac{8 \sqrt{-c^2 x^2 + 1} bd^2}{15 c}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `1/5*a*c^4*d^2*x^5 - 2/3*a*c^2*d^2*x^3 + 1/5*(c^2*x^2 - 1)^2*b*d^2*x*arcsin(c*x) - 4/15*(c^2*x^2 - 1)*b*d^2*x*arcsin(c*x) + 8/15*b*d^2*x*arcsin(c*x) + 1/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^2/c + a*d^2*x + 4/45*(-c^2*x^2 + 1)^(3/2)*b*d^2/c + 8/15*sqrt(-c^2*x^2 + 1)*b*d^2/c`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (d - c^2 dx^2)^2 dx$$

input `int((a + b*asin(c*x))*(d - c^2*d*x^2)^2,x)`

output `int((a + b*asin(c*x))*(d - c^2*d*x^2)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \frac{d^2(45 \arcsin(cx) b c^5 x^5 - 150 \arcsin(cx) b c^3 x^3 + 225 \arcsin(cx) b c x + 9\sqrt{-c^2 x^2 + 1} b c^4 x^4 - 38\sqrt{-c^2 x^2 + 1} b c^2 x^2 + 225 a c^5 x^5 - 150 a c^3 x^3 + 225 a c x)}{225c}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x)),x)`

output `(d**2*(45*asin(c*x)*b*c**5*x**5 - 150*asin(c*x)*b*c**3*x**3 + 225*asin(c*x)*b*c*x + 9*sqrt(-c**2*x**2 + 1)*b*c**4*x**4 - 38*sqrt(-c**2*x**2 + 1)*b*c**2*x**2 + 149*sqrt(-c**2*x**2 + 1)*b + 45*a*c**5*x**5 - 150*a*c**3*x**3 + 225*a*c*x))/(225*c)`

3.15 $\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx$

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Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = -\frac{11}{32}bcd^2x\sqrt{1 - c^2x^2} - \frac{1}{16}bcd^2x(1 - c^2x^2)^{3/2} - \frac{11}{32}bd^2 \arcsin(cx) + \frac{1}{2}d^2(1 - c^2x^2)(a + b \arcsin(cx)) + \frac{1}{4}d^2(1 - c^2x^2)^2(a + b \arcsin(cx)) - \frac{id^2(a + b \arcsin(cx))^2}{2b} + d^2(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) - \frac{1}{2}ibd^2 \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-11/32*b*c*d^2*x*(-c^2*x^2+1)^(1/2)-1/16*b*c*d^2*x*(-c^2*x^2+1)^(3/2)-11/32*b*d^2*arcsin(c*x)+1/2*d^2*(-c^2*x^2+1)*(a+b*arcsin(c*x))+1/4*d^2*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))-1/2*I*d^2*(a+b*arcsin(c*x))^2/b+d^2*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d^2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.90

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = \frac{1}{32} d^2 \left(-32ac^2 x^2 + 8ac^4 x^4 - 13bcx\sqrt{1 - c^2 x^2} + 2bc^3 x^3 \sqrt{1 - c^2 x^2} - 16ib \arcsin(cx)^2 + 26b \arctan\left(\frac{cx}{-1 + \sqrt{1 - c^2 x^2}}\right) + 8b \arcsin(cx) (-4c^2 x^2 + c^4 x^4) + 4 \log(1 - e^{2i \arcsin(cx)}) + 32a \log(x) - 16ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right)$$

input

```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x,x]
```

output

```
(d^2*(-32*a*c^2*x^2 + 8*a*c^4*x^4 - 13*b*c*x*Sqrt[1 - c^2*x^2] + 2*b*c^3*x^3*Sqrt[1 - c^2*x^2] - (16*I)*b*ArcSin[c*x]^2 + 26*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) + 8*b*ArcSin[c*x]*(-4*c^2*x^2 + c^4*x^4 + 4*Log[1 - E^((2*I)*ArcSin[c*x])]) + 32*a*Log[x] - (16*I)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/32
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5188, 27, 211, 211, 223, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx$$

↓ 5188

$$d \int \frac{d(1-c^2x^2)(a+b \arcsin(cx))}{x} dx - \frac{1}{4}bcd^2 \int (1-c^2x^2)^{3/2} dx + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx))$$

↓ 27

$$d^2 \int \frac{(1-c^2x^2)(a+b \arcsin(cx))}{x} dx - \frac{1}{4}bcd^2 \int (1-c^2x^2)^{3/2} dx + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx))$$

↓ 211

$$d^2 \int \frac{(1-c^2x^2)(a+b \arcsin(cx))}{x} dx - \frac{1}{4}bcd^2 \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx))$$

↓ 211

$$d^2 \int \frac{(1-c^2x^2)(a+b \arcsin(cx))}{x} dx - \frac{1}{4}bcd^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx))$$

↓ 223

$$d^2 \int \frac{(1-c^2x^2)(a+b \arcsin(cx))}{x} dx + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx)) - \frac{1}{4}bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)$$

↓ 5188

$$d^2 \left(\int \frac{a+b \arcsin(cx)}{x} dx - \frac{1}{2}bc \int \sqrt{1-c^2x^2} dx + \frac{1}{2}(1-c^2x^2)(a+b \arcsin(cx)) \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx)) - \frac{1}{4}bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)$$

↓ 211

$$d^2 \left(\int \frac{a+b \arcsin(cx)}{x} dx - \frac{1}{2}bc \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{2}(1-c^2x^2)(a+b \arcsin(cx)) \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b \arcsin(cx)) - \frac{1}{4}bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)$$

↓ 223

$$d^2 \left(\int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) \right) +$$

$$\frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) -$$

$$\frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)$$

↓ 5136

$$d^2 \left(\int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{cx} d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) \right) +$$

$$\frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) -$$

$$\frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)$$

↓ 3042

$$d^2 \left(\int - \left((a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) \right) d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) \right) +$$

$$\frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) -$$

$$\frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)$$

↓ 25

$$d^2 \left(- \int (a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) \right) +$$

$$\frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) -$$

$$\frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)$$

↓ 4200

$$d^2 \left(2i \int - \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{i(a + b \arcsin(cx))^2}{2b} - \frac{1}{2} \right) +$$

$$\frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) -$$

$$\frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right)$$

↓ 25

$$d^2 \left(-2i \int \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{i(a + b \arcsin(cx))^2}{2b} - \frac{1}{2} \right. \\ \left. \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) - \frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) \right)$$

↓ 2620

$$d^2 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2} i b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) + \frac{1}{2} (1 - c^2 x^2) \right. \\ \left. \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) - \frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) \right)$$

↓ 2715

$$d^2 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d e^{2i \arcsin(cx)} \right) + \right. \\ \left. \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) - \frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) \right)$$

↓ 2838

$$d^2 \left(\frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - 2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{4} b \text{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right. \\ \left. \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) - \frac{1}{4} bcd^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) \right)$$

input

```
Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x,x]
```


output

```
(d^2*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/4 - (b*c*d^2*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4 + d^2*((1 - c^2*x^2)*(a + b*ArcSin[c*x]))/2 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/4)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838 `Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*(c + d*x)^(m + 1)/(d*(m + 1)), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi) * (E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n * Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5188 `Int((((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_), x_Symbol] := Simp[(d + e*x^2)^p * ((a + b * ArcSin[c*x]) / (2*p)), x] + (Simp[d Int[(d + e*x^2)^(p - 1) * ((a + b * ArcSin[c*x]) / x), x], x] - Simp[b*c*(d^p / (2*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.01

method	result
parts	$a d^2 \left(\frac{c^4 x^4}{4} - c^2 x^2 + \ln(x) \right) + d^2 b \left(-\frac{i \arcsin(cx)^2}{2} + \arcsin(cx) \ln(1 - icx - \sqrt{-c^2 x^2 + 1}) \right)$
derivativedivides	$a d^2 \left(\frac{c^4 x^4}{4} - c^2 x^2 + \ln(cx) \right) + d^2 b \left(-\frac{i \arcsin(cx)^2}{2} + \arcsin(cx) \ln(1 - icx - \sqrt{-c^2 x^2 + 1}) \right)$
default	$a d^2 \left(\frac{c^4 x^4}{4} - c^2 x^2 + \ln(cx) \right) + d^2 b \left(-\frac{i \arcsin(cx)^2}{2} + \arcsin(cx) \ln(1 - icx - \sqrt{-c^2 x^2 + 1}) \right)$

input `int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output

```
a*d^2*(1/4*c^4*x^4-c^2*x^2+ln(x))+d^2*b*(-1/2*I*arcsin(c*x)^2+arcsin(c*x)*
ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+arcsi
n(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2
))+1/32*arcsin(c*x)*cos(4*arcsin(c*x))-1/128*sin(4*arcsin(c*x))+3/8*arcsin
(c*x)*cos(2*arcsin(c*x))-3/16*sin(2*arcsin(c*x)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x,x, algorithm="fricas")
```

output

```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arcsin(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = d^2 \left(\int \frac{a}{x} dx + \int (-2ac^2 x) dx + \int ac^4 x^3 dx \right. \\ \left. + \int \frac{b \arcsin(cx)}{x} dx + \int (-2bc^2 x \arcsin(cx)) dx \right. \\ \left. + \int bc^4 x^3 \arcsin(cx) dx \right)$$

input

```
integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))/x,x)
```

output

```
d**2*(Integral(a/x, x) + Integral(-2*a*c**2*x, x) + Integral(a*c**4*x**3,
x) + Integral(b*asin(c*x)/x, x) + Integral(-2*b*c**2*x*asin(c*x), x) + Int
egral(b*c**4*x**3*asin(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `1/4*a*c^4*d^2*x^4 - a*c^2*d^2*x^2 + a*d^2*log(x) + integrate((b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^2}{x} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x} dx$$

$$= \frac{d^2 \left(8 \operatorname{asin}(cx) b c^4 x^4 - 32 \operatorname{asin}(cx) b c^2 x^2 + 13 \operatorname{asin}(cx) b + 2 \sqrt{-c^2 x^2 + 1} b c^3 x^3 - 13 \sqrt{-c^2 x^2 + 1} b c x + 32 \right)}{32}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))/x,x)`

output `(d**2*(8*asin(c*x)*b*c**4*x**4 - 32*asin(c*x)*b*c**2*x**2 + 13*asin(c*x)*b + 2*sqrt(-c**2*x**2 + 1)*b*c**3*x**3 - 13*sqrt(-c**2*x**2 + 1)*b*c*x + 32*int(asin(c*x)/x,x)*b + 32*log(x)*a + 8*a*c**4*x**4 - 32*a*c**2*x**2)) /32`

3.16 $\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$

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Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx = -\frac{5}{3}bcd^2\sqrt{1 - c^2x^2} - \frac{1}{9}bcd^2(1 - c^2x^2)^{3/2} - \frac{d^2(a + b \arcsin(cx))}{x} - 2c^2d^2x(a + b \arcsin(cx)) + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - bcd^2\operatorname{arctanh}\left(\sqrt{1 - c^2x^2}\right)$$

output

```
-5/3*b*c*d^2*(-c^2*x^2+1)^(1/2)-1/9*b*c*d^2*(-c^2*x^2+1)^(3/2)-d^2*(a+b*arcsin(c*x))/x-2*c^2*d^2*x*(a+b*arcsin(c*x))+1/3*c^4*d^2*x^3*(a+b*arcsin(c*x))-b*c*d^2*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{d^2(-9a - 18ac^2x^2 + 3ac^4x^4 - 16bcx\sqrt{1 - c^2x^2} + bc^3x^3\sqrt{1 - c^2x^2} + 3b(-3 - 6c^2x^2 + c^4x^4) \arcsin(cx) + b^2c^2x^2 \arcsin^2(cx) + 9b^2c^2x^2 \arcsin(cx) \log|x| - 9b^2c^2x^2 \arcsin^2(cx))}{9x}$$

input

```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```
(d^2*(-9*a - 18*a*c^2*x^2 + 3*a*c^4*x^4 - 16*b*c*x*Sqrt[1 - c^2*x^2] + b*c^3*x^3*Sqrt[1 - c^2*x^2] + 3*b*(-3 - 6*c^2*x^2 + c^4*x^4)*ArcSin[c*x] + 9*b*c*x*Log[x] - 9*b*c*x*Log[1 + Sqrt[1 - c^2*x^2]]))/(9*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5192, 27, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$\downarrow 5192$$

$$-bc \int -\frac{d^2(-c^4x^4 + 6c^2x^2 + 3)}{3x\sqrt{1 - c^2x^2}} dx + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + b \arcsin(cx)) - \frac{d^2(a + b \arcsin(cx))}{x}$$

$$\downarrow 27$$

$$\frac{1}{3}bcd^2 \int \frac{-c^4x^4 + 6c^2x^2 + 3}{x\sqrt{1 - c^2x^2}} dx + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + b \arcsin(cx)) - \frac{d^2(a + b \arcsin(cx))}{x}$$

$$\begin{aligned}
& \downarrow 1578 \\
& \frac{1}{6}bcd^2 \int \frac{-c^4x^4 + 6c^2x^2 + 3}{x^2\sqrt{1-c^2x^2}} dx^2 + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + b \arcsin(cx)) - \\
& \qquad \qquad \qquad \frac{d^2(a + b \arcsin(cx))}{x} \\
& \downarrow 1192 \\
& \frac{bd^2 \int \frac{-c^4x^8 - 4c^4x^4 + 8c^4}{1-x^4} d\sqrt{1-c^2x^2}}{3c^3} + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + b \arcsin(cx)) - \\
& \qquad \qquad \qquad \frac{d^2(a + b \arcsin(cx))}{x} \\
& \downarrow 25 \\
& -\frac{bd^2 \int \frac{-c^4x^8 - 4c^4x^4 + 8c^4}{1-x^4} d\sqrt{1-c^2x^2}}{3c^3} + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + b \arcsin(cx)) - \\
& \qquad \qquad \qquad \frac{d^2(a + b \arcsin(cx))}{x} \\
& \downarrow 1467 \\
& -\frac{bd^2 \int \left(x^4c^4 + \frac{3c^4}{1-x^4} + 5c^4\right) d\sqrt{1-c^2x^2}}{3c^3} + \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + \\
& \qquad \qquad \qquad b \arcsin(cx)) - \frac{d^2(a + b \arcsin(cx))}{x} \\
& \downarrow 2009 \\
& \frac{1}{3}c^4d^2x^3(a + b \arcsin(cx)) - 2c^2d^2x(a + b \arcsin(cx)) - \frac{d^2(a + b \arcsin(cx))}{x} + \\
& \qquad \qquad \qquad \frac{bd^2 \left(-3c^4 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{1}{3}c^4x^6 - 5c^4\sqrt{1-c^2x^2}\right)}{3c^3}
\end{aligned}$$

input

$$\text{Int}[\left(\left(d - c^2*d*x^2\right)^2*(a + b*\text{ArcSin}[c*x])\right)/x^2,x]$$

output

$$\begin{aligned}
& -\left(\left(d^2*(a + b*\text{ArcSin}[c*x])\right)/x\right) - 2*c^2*d^2*x*(a + b*\text{ArcSin}[c*x]) + (c^4*d^2*x^3*(a + b*\text{ArcSin}[c*x]))/3 + (b*d^2*(-1/3*(c^4*x^6) - 5*c^4*\text{Sqrt}[1 - c^2*x^2] - 3*c^4*\text{ArcTanh}[\text{Sqrt}[1 - c^2*x^2]]))/(3*c^3)
\end{aligned}$$

Definitions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 1192 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[2/e^(n + 2*p + 1) Subst[Int[x^(2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]`
- rule 1467 `Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]`
- rule 1578 `Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 5192 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

method	result
parts	$a d^2 \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) + d^2 b c \left(\frac{c^3 x^3 \arcsin(cx)}{3} - 2cx \arcsin(cx) - \frac{\arcsin(cx)}{cx} + \frac{c^2 x^2 \sqrt{-c^2 x^2 + 1}}{9} \right)$
derivativedivides	$c \left(a d^2 \left(\frac{c^3 x^3}{3} - 2cx - \frac{1}{cx} \right) + d^2 b \left(\frac{c^3 x^3 \arcsin(cx)}{3} - 2cx \arcsin(cx) - \frac{\arcsin(cx)}{cx} + \frac{c^2 x^2 \sqrt{-c^2 x^2 + 1}}{9} \right) \right)$
default	$c \left(a d^2 \left(\frac{c^3 x^3}{3} - 2cx - \frac{1}{cx} \right) + d^2 b \left(\frac{c^3 x^3 \arcsin(cx)}{3} - 2cx \arcsin(cx) - \frac{\arcsin(cx)}{cx} + \frac{c^2 x^2 \sqrt{-c^2 x^2 + 1}}{9} \right) \right)$

input `int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)`

output `a*d^2*(1/3*c^4*x^3-2*c^2*x-1/x)+d^2*b*c*(1/3*c^3*x^3*arcsin(c*x)-2*c*x*arcsin(c*x)-arcsin(c*x)/c/x+1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/9*(-c^2*x^2+1)^(1/2)-arctanh(1/(-c^2*x^2+1)^(1/2)))`

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{6ac^4d^2x^4 - 36ac^2d^2x^2 - 9bcd^2x \log(\sqrt{-c^2x^2 + 1} + 1) + 9bcd^2x \log(\sqrt{-c^2x^2 + 1} - 1) - 18ad^2 + 6(bcd^2x^2 \arcsin(cx) + 2(bcd^2x^2 - 16bcd^2x) \sqrt{-c^2x^2 + 1})}{18x}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")`

output `1/18*(6*a*c^4*d^2*x^4 - 36*a*c^2*d^2*x^2 - 9*b*c*d^2*x*log(sqrt(-c^2*x^2 + 1) + 1) + 9*b*c*d^2*x*log(sqrt(-c^2*x^2 + 1) - 1) - 18*a*d^2 + 6*(b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - 3*b*d^2)*arcsin(c*x) + 2*(b*c^3*d^2*x^3 - 16*b*c*d^2*x)*sqrt(-c^2*x^2 + 1))/x`

Sympy [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{ac^4 d^2 x^3}{3} - 2ac^2 d^2 x - \frac{ad^2}{x} - \frac{bc^5 d^2 \left(\begin{cases} -\frac{x^2 \sqrt{-c^2 x^2 + 1}}{3c^2} - \frac{2\sqrt{-c^2 x^2 + 1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right)}{3}$$

$$+ \frac{bc^4 d^2 x^3 \operatorname{asin}(cx)}{3} - 2bc^2 d^2 \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2 x^2 + 1}}{c} & \text{otherwise} \end{cases} \right)$$

$$+ bcd^2 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2 x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{bd^2 \operatorname{asin}(cx)}{x}$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))/x**2,x)`output `a*c**4*d**2*x**3/3 - 2*a*c**2*d**2*x - a*d**2/x - b*c**5*d**2*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True))/3 + b*c**4*d**2*x**3*asin(c*x)/3 - 2*b*c**2*d**2*Piecewise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True)) + b*c*d**2*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x))), True)) - b*d**2*asin(c*x)/x`**Maxima [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{1}{3} ac^4 d^2 x^3 + \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bc^4 d^2$$

$$- 2ac^2 d^2 x - 2 \left(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1} \right) bcd^2$$

$$- \left(c \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd^2 - \frac{ad^2}{x}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^4*d^2 - 2*a*c^2*d^2*x - 2*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*c*d^2 - (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d^2 - a*d^2/x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2717 vs. $2(111) = 222$.

Time = 6.09 (sec) , antiderivative size = 2717, normalized size of antiderivative = 22.09

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx = \text{Too large to display}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output

```
-1/2*b*c^9*d^2*x^8*arcsin(c*x)/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^8 - 1/2*a*c^9*d^2*x^8/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^8)*(sqrt(-c^2*x^2 + 1) + 1)^8) + b*c^8*d^2*x^7*log(abs(c)*abs(x))/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^8*(sqrt(-c^2*x^2 + 1) + 1)^7) - b*c^8*d^2*x^7*log(sqrt(-c^2*x^2 + 1) + 1)/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^8*(sqrt(-c^2*x^2 + 1) + 1)^7) + 16/9*b*c^8*d^2*x^7/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^8*(sqrt(-c^2*x^2 + 1) + 1)^7) - 6*b*c^7*d^2*x^6*arcsin(c*x)/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^6) - 6*a*c^7*d^2*x^6/((c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))^6*(sqrt(-c^2*x^2 + 1) + 1)^6) + 3*b*c^6*d^2*x^5*log(abs(c)*abs(x))/((c^7*x^7/(sqrt(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \left\{ \begin{array}{l} b c^4 d^2 \left(\frac{\sqrt{\frac{1}{c^2} - x^2} \left(\frac{2}{c^2} + x^2 \right)}{9} + \frac{x^3 \arcsin(cx)}{3} \right) - \frac{a d^2 (-c^4 x^4 + 6 c^2 x^2 + 3)}{3x} - 2 b c d^2 (\sqrt{1 - c^2 x^2} + c x \arcsin(cx)) - b c \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^2}{x^2} dx \end{array} \right.$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x^2,x)
```

output

```

piecewise(0 < c, - b*c*d^2*atanh(1/(- c^2*x^2 + 1)^(1/2)) - (a*d^2*(6*c^2*
x^2 - c^4*x^4 + 3))/(3*x) - 2*b*c*d^2*((- c^2*x^2 + 1)^(1/2) + c*x*asin(c*
x)) + b*c^4*d^2*((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 + (x^3*asin(c*x))/3
) - (b*d^2*asin(c*x))/x, ~0 < c, int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2
/x^2, x))

```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{d^2 \left(3 \arcsin(cx) b c^4 x^4 - 18 \arcsin(cx) b c^2 x^2 - 9 \arcsin(cx) b + \sqrt{-c^2 x^2 + 1} b c^3 x^3 - 16 \sqrt{-c^2 x^2 + 1} b c x + 9 \log \right)}{9x}$$

input

```

int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))/x^2,x)

```

output

```

(d**2*(3*asin(c*x)*b*c**4*x**4 - 18*asin(c*x)*b*c**2*x**2 - 9*asin(c*x)*b
+ sqrt(- c**2*x**2 + 1)*b*c**3*x**3 - 16*sqrt(- c**2*x**2 + 1)*b*c*x + 9
*log(tan(asin(c*x)/2))*b*c*x + 3*a*c**4*x**4 - 18*a*c**2*x**2 - 9*a))/(9*x
)

```

3.17
$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx$$

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Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx = -\frac{1}{4}bc^3d^2x\sqrt{1 - c^2x^2} - \frac{bcd^2(1 - c^2x^2)^{3/2}}{2x} - \frac{1}{4}bc^2d^2 \arcsin(cx) - c^2d^2(1 - c^2x^2)(a + b \arcsin(cx)) - \frac{d^2(1 - c^2x^2)^2(a + b \arcsin(cx))}{2x^2} + \frac{ic^2d^2(a + b \arcsin(cx))^2}{b} - 2c^2d^2(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) + ibc^2d^2 \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-1/4*b*c^3*d^2*x*(-c^2*x^2+1)^(1/2)-1/2*b*c*d^2*(-c^2*x^2+1)^(3/2)/x-1/4*b*c^2*d^2*arcsin(c*x)-c^2*d^2*(-c^2*x^2+1)*(a+b*arcsin(c*x))-1/2*d^2*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))/x^2+I*c^2*d^2*(a+b*arcsin(c*x))^2/b-2*c^2*d^2*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+I*b*c^2*d^2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{d^2 \left(-2a + 2ac^4 x^4 - 2bcx\sqrt{1 - c^2 x^2} + bc^3 x^3 \sqrt{1 - c^2 x^2} + 4ibc^2 x^2 \arcsin(cx)^2 - 2bc^2 x^2 \arctan\left(\frac{cx}{-1 + \sqrt{1 - c^2 x^2}}\right) \right)}{x^3}$$

input

```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
(d^2*(-2*a + 2*a*c^4*x^4 - 2*b*c*x*Sqrt[1 - c^2*x^2] + b*c^3*x^3*Sqrt[1 - c^2*x^2] + (4*I)*b*c^2*x^2*ArcSin[c*x]^2 - 2*b*c^2*x^2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) + 2*b*ArcSin[c*x]*(-1 + c^4*x^4 - 4*c^2*x^2*Log[1 - E^((2*I)*ArcSin[c*x])]) - 8*a*c^2*x^2*Log[x] + (4*I)*b*c^2*x^2*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5190, 27, 247, 211, 223, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx$$

$$\downarrow 5190$$

$$-2c^2 d \int \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))}{x} dx + \frac{1}{2} b c d^2 \int \frac{(1 - c^2 x^2)^{3/2}}{x^2} dx -$$

$$\frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2}$$

$$\downarrow 27$$

$$\begin{aligned}
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx + \frac{1}{2}bcd^2 \int \frac{(1-c^2x^2)^{3/2}}{x^2} dx - \\
& \quad \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{247} \\
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx + \\
& \frac{1}{2}bcd^2 \left(-3c^2 \int \sqrt{1-c^2x^2} dx - \frac{(1-c^2x^2)^{3/2}}{x} \right) - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{211} \\
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx + \\
& \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) - \frac{(1-c^2x^2)^{3/2}}{x} \right) - \\
& \quad \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{223} \\
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) - \frac{(1-c^2x^2)^{3/2}}{x} \right) \\
& \quad \downarrow \text{5188} \\
& -2c^2 d^2 \left(\int \frac{a+b\arcsin(cx)}{x} dx - \frac{1}{2}bc \int \sqrt{1-c^2x^2} dx + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right) - \\
& \quad \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) - \frac{(1-c^2x^2)^{3/2}}{x} \right) \\
& \quad \downarrow \text{211} \\
& -2c^2 d^2 \left(\int \frac{a+b\arcsin(cx)}{x} dx - \frac{1}{2}bc \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right) - \\
& \quad \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) - \frac{(1-c^2x^2)^{3/2}}{x} \right)
\end{aligned}$$

↓ 223

$$-2c^2 d^2 \left(\int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) \right) - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right)$$

↓ 5136

$$-2c^2 d^2 \left(\int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{cx} d \arcsin(cx) + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) \right) - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right)$$

↓ 3042

$$-2c^2 d^2 \left(\int - \left((a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) \right) d \arcsin(cx) + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) \right) - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right)$$

↓ 25

$$-2c^2 d^2 \left(- \int (a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) \right) - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right)$$

↓ 4200

$$\begin{aligned}
& -2c^2 d^2 \left(2i \int -\frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2}(1 - c^2 x^2)(a + b \arcsin(cx)) - \frac{i(a + b \arcsin(cx))}{2b} \right. \\
& \quad \left. \frac{d^2(1 - c^2 x^2)^2(a + b \arcsin(cx))}{2x^2} + \right. \\
& \quad \left. \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{25}
\end{aligned}$$

$$\begin{aligned}
& -2c^2 d^2 \left(-2i \int \frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2}(1 - c^2 x^2)(a + b \arcsin(cx)) - \frac{i(a + b \arcsin(cx))}{2b} \right) \\
& \quad \frac{d^2(1 - c^2 x^2)^2(a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right) \\
& \qquad \qquad \qquad \downarrow \text{2620}
\end{aligned}$$

$$\begin{aligned}
& -2c^2 d^2 \left(-2i \left(\frac{1}{2}i \log(1 - e^{2i \arcsin(cx)}) \right) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) + \frac{1}{2}(1 - c^2 x^2) \\
& \quad \frac{d^2(1 - c^2 x^2)^2(a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right) \\
& \qquad \qquad \qquad \downarrow \text{2715}
\end{aligned}$$

$$\begin{aligned}
& -2c^2 d^2 \left(-2i \left(\frac{1}{2}i \log(1 - e^{2i \arcsin(cx)}) \right) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right) \\
& \quad \frac{d^2(1 - c^2 x^2)^2(a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2}bcd^2 \left(-3c^2 \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right) \\
& \qquad \qquad \qquad \downarrow \text{2838}
\end{aligned}$$

$$-2c^2d^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx)) + \frac{1}{4}b\text{PolyLog}\left(2, e^{2i\arcsin(cx)}\right)\right) + \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2}\right) - \frac{(1-c^2x^2)^{3/2}}{x}\right)$$

input `Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x^3,x]`

output `-1/2*(d^2*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/x^2 + (b*c*d^2*(-((1 - c^2*x^2)^(3/2)/x) - 3*c^2*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c))))/2 - 2*c^2*d^2(((1 - c^2*x^2)*(a + b*ArcSin[c*x]))/2 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c))))/2 - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4))`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 211 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 247 $\text{Int}[\left((c_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \left((a_{.}) + (b_{.}) \cdot (x_{.})^2\right)^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[(c \cdot x)^{(m+1)} \cdot (a + b \cdot x^2)^p / (c \cdot (m+1)), x] - \text{Simp}[2 \cdot b \cdot (p / (c^2 \cdot (m+1))) \text{Int}[(c \cdot x)^{(m+2)} \cdot (a + b \cdot x^2)^{(p-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{!LtQ}[(m + 2 \cdot p + 3) / 2, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 2620 $\text{Int}[\left((F_{.})^{\left((g_{.}) \cdot (e_{.}) + (f_{.}) \cdot (x_{.})\right)}\right)^{(n_{.})} \cdot \left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} / \left((a_{.}) + (b_{.}) \cdot (F_{.})^{\left((g_{.}) \cdot (e_{.}) + (f_{.}) \cdot (x_{.})\right)}\right)^{(n_{.})}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\left((c + d \cdot x)^m / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F])\right) \cdot \text{Log}[1 + b \cdot (F^{(g \cdot (e + f \cdot x))})^n / a], x] - \text{Simp}[d \cdot (m / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F])) \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 + b \cdot (F^{(g \cdot (e + f \cdot x))})^n / a], x], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_{.}) + (b_{.}) \cdot (F_{.})^{\left((e_{.}) \cdot (c_{.}) + (d_{.}) \cdot (x_{.})\right)}], x_{\text{Symbol}}] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_{.}) \cdot \left((d_{.}) + (e_{.}) \cdot (x_{.})^{(n_{.})}\right)] / (x_{.}), x_{\text{Symbol}}] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n / n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

rule 3042 $\text{Int}[u_{.}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4200 $\text{Int}[\left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \tan\left[\left(e_{.}\right) + \text{Pi} \cdot (k_{.}) + (f_{.}) \cdot (x_{.})\right], x_{\text{Symbol}}] \rightarrow \text{Simp}[I \cdot (c + d \cdot x)^{(m+1)} / (d \cdot (m+1)), x] - \text{Simp}[2 \cdot I \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot (E^{(2 \cdot I \cdot (e + f \cdot x))} / (1 + E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}))], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[4 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5136 $\text{Int}[\left((a_{.}) + \text{ArcSin}[(c_{.}) \cdot (x_{.})] \cdot (b_{.})\right)^{(n_{.})} / (x_{.}), x_{\text{Symbol}}] \rightarrow \text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Cot}[x], x], x, \text{ArcSin}[c \cdot x]] /;$ $\text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{IGtQ}[n, 0]$

rule 5188

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.)/(x_),
x_Symbol] := Simp[(d + e*x^2)^p*((a + b*ArcSin[c*x])/(2*p)), x] + (Simp[d
Int[(d + e*x^2)^(p - 1)*((a + b*ArcSin[c*x])/x), x], x] - Simp[b*c*(d^p/(2
*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5190

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x
])/((f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1))) Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2
)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.28

method	result
derivativedivides	$c^2 \left(a d^2 \left(\frac{c^2 x^2}{2} - 2 \ln(cx) - \frac{1}{2c^2 x^2} \right) + i d^2 b \arcsin(cx)^2 + \frac{bc d^2 x \sqrt{-c^2 x^2 + 1}}{4} + \frac{d^2 b \arcsin(cx) c^2 x^2}{2} \right)$
default	$c^2 \left(a d^2 \left(\frac{c^2 x^2}{2} - 2 \ln(cx) - \frac{1}{2c^2 x^2} \right) + i d^2 b \arcsin(cx)^2 + \frac{bc d^2 x \sqrt{-c^2 x^2 + 1}}{4} + \frac{d^2 b \arcsin(cx) c^2 x^2}{2} \right)$
parts	$a d^2 \left(\frac{c^4 x^2}{2} - 2c^2 \ln(x) - \frac{1}{2x^2} \right) + i d^2 b c^2 \arcsin(cx)^2 + \frac{b c^3 d^2 x \sqrt{-c^2 x^2 + 1}}{4} + \frac{d^2 b c^4 \arcsin(cx) x^2}{2}$

input

```
int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output

```
c^2*(a*d^2*(1/2*c^2*x^2-2*ln(c*x)-1/2/c^2/x^2)+I*d^2*b*arcsin(c*x)^2+1/4*b
*c*d^2*x*(-c^2*x^2+1)^(1/2)+1/2*d^2*b*arcsin(c*x)*c^2*x^2-1/4*b*d^2*arcsin
(c*x)+1/2*I*d^2*b-1/2*d^2*b/c/x*(-c^2*x^2+1)^(1/2)-1/2*d^2*b*arcsin(c*x)/c
^2/x^2-2*d^2*b*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*d^2*b*arcsin(c
*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*d^2*b*polylog(2,I*c*x+(-c^2*x^2+1)
^(1/2))+2*I*d^2*b*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")`

output `integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))/x^3, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx = d^2 \left(\int \frac{a}{x^3} dx + \int \left(-\frac{2ac^2}{x} \right) dx + \int ac^4 x dx \right. \\ \left. + \int \frac{b \arcsin(cx)}{x^3} dx + \int \left(-\frac{2bc^2 \arcsin(cx)}{x} \right) dx \right. \\ \left. + \int bc^4 x \arcsin(cx) dx \right)$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))/x**3,x)`

output `d**2*(Integral(a/x**3, x) + Integral(-2*a*c**2/x, x) + Integral(a*c**4*x, x) + Integral(b*asin(c*x)/x**3, x) + Integral(-2*b*c**2*asin(c*x)/x, x) + Integral(b*c**4*x*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `1/2*a*c^4*d^2*x^2 - 2*a*c^2*d^2*log(x) - 1/2*b*d^2*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate((b*c^4*d^2*x^2 - 2*b*c^2*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^2}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x^3,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{d^2 \left(2 \arcsin(cx) b c^4 x^4 - \arcsin(cx) b c^2 x^2 - 2 \arcsin(cx) b + \sqrt{-c^2 x^2 + 1} b c^3 x^3 - 2 \sqrt{-c^2 x^2 + 1} b c x - 8 \left(\int \frac{\arcsin}{x} \right) \right)}{4x^2}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))/x^3,x)`

output `(d**2*(2*asin(c*x)*b*c**4*x**4 - asin(c*x)*b*c**2*x**2 - 2*asin(c*x)*b + sqrt(-c**2*x**2 + 1)*b*c**3*x**3 - 2*sqrt(-c**2*x**2 + 1)*b*c*x - 8*int(asin(c*x)/x,x)*b*c**2*x**2 - 8*log(x)*a*c**2*x**2 + 2*a*c**4*x**4 - 2*a))/(4*x**2)`

3.18 $\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$

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Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx = bc^3 d^2 \sqrt{1 - c^2 x^2} - \frac{bcd^2 \sqrt{1 - c^2 x^2}}{6x^2} - \frac{d^2 (a + b \arcsin(cx))}{3x^3} + \frac{2c^2 d^2 (a + b \arcsin(cx))}{x} + c^4 d^2 x (a + b \arcsin(cx)) + \frac{11}{6} bc^3 d^2 \operatorname{arctanh}(\sqrt{1 - c^2 x^2})$$

output

```
b*c^3*d^2*(-c^2*x^2+1)^(1/2)-1/6*b*c*d^2*(-c^2*x^2+1)^(1/2)/x^2-1/3*d^2*(a+b*arcsin(c*x))/x^3+2*c^2*d^2*(a+b*arcsin(c*x))/x+c^4*d^2*x*(a+b*arcsin(c*x))+11/6*b*c^3*d^2*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{d^2(-2a + 12ac^2x^2 + 6ac^4x^4 - bcx\sqrt{1 - c^2x^2} + 6bc^3x^3\sqrt{1 - c^2x^2} + 2b(-1 + 6c^2x^2 + 3c^4x^4) \arcsin(cx) - 11bc^3x^3 \log|x| + 11b^2c^3x^3 \log|1 + \sqrt{1 - c^2x^2}|)}{6x^3}$$

input

```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x^4,x]
```

output

```
(d^2*(-2*a + 12*a*c^2*x^2 + 6*a*c^4*x^4 - b*c*x*Sqrt[1 - c^2*x^2] + 6*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*(-1 + 6*c^2*x^2 + 3*c^4*x^4)*ArcSin[c*x] - 11*b*c^3*x^3*Log[x] + 11*b*c^3*x^3*Log[1 + Sqrt[1 - c^2*x^2]]))/(6*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5192, 27, 1578, 1192, 25, 1471, 25, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$\downarrow \text{5192}$$

$$-bc \int -\frac{d^2(-3c^4x^4 - 6c^2x^2 + 1)}{3x^3\sqrt{1 - c^2x^2}} dx + c^4 d^2 x(a + b \arcsin(cx)) + \frac{2c^2 d^2 (a + b \arcsin(cx))}{x} - \frac{d^2(a + b \arcsin(cx))}{3x^3}$$

$$\downarrow \text{27}$$

$$\frac{1}{3}bcd^2 \int \frac{-3c^4x^4 - 6c^2x^2 + 1}{x^3\sqrt{1 - c^2x^2}} dx + c^4 d^2 x(a + b \arcsin(cx)) + \frac{2c^2 d^2 (a + b \arcsin(cx))}{x} - \frac{d^2(a + b \arcsin(cx))}{3x^3}$$

$$\begin{aligned}
& \downarrow 1578 \\
& \frac{1}{6}bcd^2 \int \frac{-3c^4x^4 - 6c^2x^2 + 1}{x^4\sqrt{1-c^2x^2}} dx^2 + c^4d^2x(a + b \arcsin(cx)) + \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \\
& \qquad \qquad \qquad \frac{d^2(a + b \arcsin(cx))}{3x^3} \\
& \downarrow 1192 \\
& -\frac{bd^2 \int -\frac{3c^4x^8 - 12c^4x^4 + 8c^4}{(1-x^4)^2} d\sqrt{1-c^2x^2}}{3c} + c^4d^2x(a + b \arcsin(cx)) + \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \\
& \qquad \qquad \qquad \frac{d^2(a + b \arcsin(cx))}{3x^3} \\
& \downarrow 25 \\
& \frac{bd^2 \int \frac{3c^4x^8 - 12c^4x^4 + 8c^4}{(1-x^4)^2} d\sqrt{1-c^2x^2}}{3c} + c^4d^2x(a + b \arcsin(cx)) + \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \\
& \qquad \qquad \qquad \frac{d^2(a + b \arcsin(cx))}{3x^3} \\
& \downarrow 1471 \\
& -\frac{bd^2 \left(\frac{1}{2} \int -\frac{c^4(17-6x^4)}{1-x^4} d\sqrt{1-c^2x^2} + \frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)} \right)}{3c} + c^4d^2x(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \frac{d^2(a + b \arcsin(cx))}{3x^3} \\
& \downarrow 25 \\
& -\frac{bd^2 \left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)} - \frac{1}{2} \int \frac{c^4(17-6x^4)}{1-x^4} d\sqrt{1-c^2x^2} \right)}{3c} + c^4d^2x(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \frac{d^2(a + b \arcsin(cx))}{3x^3} \\
& \downarrow 27 \\
& -\frac{bd^2 \left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)} - \frac{1}{2}c^4 \int \frac{17-6x^4}{1-x^4} d\sqrt{1-c^2x^2} \right)}{3c} + c^4d^2x(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \frac{d^2(a + b \arcsin(cx))}{3x^3} \\
& \downarrow 299 \\
& -\frac{bd^2 \left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)} - \frac{1}{2}c^4 \left(11 \int \frac{1}{1-x^4} d\sqrt{1-c^2x^2} + 6\sqrt{1-c^2x^2} \right) \right)}{3c} + c^4d^2x(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{2c^2d^2(a + b \arcsin(cx))}{x} - \frac{d^2(a + b \arcsin(cx))}{3x^3}
\end{aligned}$$

$$\begin{array}{c} \downarrow 219 \\ \frac{c^4 d^2 x(a + b \arcsin(cx)) + \frac{2c^2 d^2 (a + b \arcsin(cx))}{x} - \frac{d^2 (a + b \arcsin(cx))}{3x^3} -}{3c} \\ \frac{bd^2 \left(\frac{c^4 \sqrt{1-c^2 x^2}}{2(1-x^4)} - \frac{1}{2} c^4 \left(11 \operatorname{arctanh}(\sqrt{1-c^2 x^2}) + 6\sqrt{1-c^2 x^2} \right) \right)}{3c} \end{array}$$

input `Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(d^2*(a + b*ArcSin[c*x]))/x^3 + (2*c^2*d^2*(a + b*ArcSin[c*x]))/x + c^4*d^2*x*(a + b*ArcSin[c*x]) - (b*d^2*((c^4*Sqrt[1 - c^2*x^2])/(2*(1 - x^4))) - (c^4*(6*Sqrt[1 - c^2*x^2] + 11*ArcTanh[Sqrt[1 - c^2*x^2]]))/2)/(3*c)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 299 `Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*x*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2*p + 3)) Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ[2*p + 3, 0]`

rule 1192

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[2/e^(n + 2*p + 1) Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*(d + e*x^2)^(q + 1)/(2*d*(q + 1)), x] + Simp[1/(2*d*(q
+ 1)) Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

method	result
parts	$a d^2 \left(c^4 x - \frac{1}{3x^3} + \frac{2c^2}{x} \right) + d^2 b c^3 \left(cx \arcsin(cx) - \frac{\arcsin(cx)}{3c^3 x^3} + \frac{2 \arcsin(cx)}{cx} - \frac{\sqrt{-c^2 x^2 + 1}}{6c^2 x^2} + \frac{11}{6c^2 x^3} \right)$
derivativedivides	$c^3 \left(a d^2 \left(cx - \frac{1}{3c^3 x^3} + \frac{2}{cx} \right) + d^2 b \left(cx \arcsin(cx) - \frac{\arcsin(cx)}{3c^3 x^3} + \frac{2 \arcsin(cx)}{cx} - \frac{\sqrt{-c^2 x^2 + 1}}{6c^2 x^2} + \frac{11}{6c^2 x^3} \right) \right)$
default	$c^3 \left(a d^2 \left(cx - \frac{1}{3c^3 x^3} + \frac{2}{cx} \right) + d^2 b \left(cx \arcsin(cx) - \frac{\arcsin(cx)}{3c^3 x^3} + \frac{2 \arcsin(cx)}{cx} - \frac{\sqrt{-c^2 x^2 + 1}}{6c^2 x^2} + \frac{11}{6c^2 x^3} \right) \right)$

input `int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)`

output `a*d^2*(c^4*x-1/3/x^3+2*c^2/x)+d^2*b*c^3*(c*x*arcsin(c*x)-1/3*arcsin(c*x)/c^3/x^3+2*arcsin(c*x)/c/x-1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)+11/6*arctanh(1/(-c^2*x^2+1)^(1/2)))+(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.27

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{12 ac^4 d^2 x^4 + 11 bc^3 d^2 x^3 \log(\sqrt{-c^2 x^2 + 1} + 1) - 11 bc^3 d^2 x^3 \log(\sqrt{-c^2 x^2 + 1} - 1) + 24 ac^2 d^2 x^2 - 4 ad^2}{12 x^3}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `1/12*(12*a*c^4*d^2*x^4 + 11*b*c^3*d^2*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - 11*b*c^3*d^2*x^3*log(sqrt(-c^2*x^2 + 1) - 1) + 24*a*c^2*d^2*x^2 - 4*a*d^2 + 4*(3*b*c^4*d^2*x^4 + 6*b*c^2*d^2*x^2 - b*d^2)*arcsin(c*x) + 2*(6*b*c^3*d^2*x^3 - b*c*d^2*x)*sqrt(-c^2*x^2 + 1))/x^3`

Sympy [A] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.82

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= ac^4 d^2 x + \frac{2ac^2 d^2}{x} - \frac{ad^2}{3x^3} + bc^4 d^2 \left(\begin{cases} 0 & \text{for } c = 0 \\ x \arcsin(cx) + \frac{\sqrt{-c^2 x^2 + 1}}{c} & \text{otherwise} \end{cases} \right)$$

$$- 2bc^3 d^2 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2 x^2|} > 1 \\ i \arcsin\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) + \frac{2bc^2 d^2 \arcsin(cx)}{x}$$

$$+ \frac{bcd^2 \left(\begin{cases} -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1 + \frac{1}{c^2 x^2}}} - \frac{1}{2cx^3\sqrt{-1 + \frac{1}{c^2 x^2}}} & \text{for } \frac{1}{|c^2 x^2|} > 1 \\ \frac{ic^2 \arcsin\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1 - \frac{1}{c^2 x^2}}}{2x} & \text{otherwise} \end{cases} \right)}{3}$$

$$- \frac{bd^2 \arcsin(cx)}{3x^3}$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))/x**4,x)`output `a*c**4*d**2*x + 2*a*c**2*d**2/x - a*d**2/(3*x**3) + b*c**4*d**2*Piecewise(
(0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True)) - 2*b*c**3*d**
2*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), Tr
ue)) + 2*b*c**2*d**2*asin(c*x)/x + b*c*d**2*Piecewise((-c**2*acosh(1/(c*x)
)/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**
2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**
2*x**2))/(2*x), True))/3 - b*d**2*asin(c*x)/(3*x**3)`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.33

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= ac^4 d^2 x + \left(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1} \right) bc^3 d^2$$

$$+ 2 \left(c \log \left(\frac{2 \sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bc^2 d^2$$

$$- \frac{1}{6} \left(\left(c^2 \log \left(\frac{2 \sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2 x^2 + 1}}{x^2} \right) c + \frac{2 \arcsin(cx)}{x^3} \right) bd^2$$

$$+ \frac{2ac^2 d^2}{x} - \frac{ad^2}{3x^3}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output `a*c^4*d^2*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*c^3*d^2 + 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*c^2*d^2 - 1/6*(c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*b*d^2 + 2*a*c^2*d^2/x - 1/3*a*d^2/x^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(116) = 232.

Time = 57.64 (sec) , antiderivative size = 1409, normalized size of antiderivative = 11.01

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx = \text{Too large to display}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output

```

-1/24*b*c^11*d^2*x^8*arcsin(c*x)/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^
3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^8) - 1/24*a*c^1
1*d^2*x^8/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sqrt(-c^2*x^2 +
1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^8) + 1/24*b*c^10*d^2*x^7/((c^5*x^5/(sq
rt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x
^2 + 1) + 1)^7) + 5/6*b*c^9*d^2*x^6*arcsin(c*x)/((c^5*x^5/(sqrt(-c^2*x^2 +
1) + 1)^5 + c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^
6) + 5/6*a*c^9*d^2*x^6/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sq
rt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^6) - 11/6*b*c^8*d^2*x^5*log
(abs(c)*abs(x))/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sqrt(-c^
2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^5) + 11/6*b*c^8*d^2*x^5*log(sq
rt(-c^2*x^2 + 1) + 1)/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sq
rt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^5) - 23/24*b*c^8*d^2*x^5/(
(c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*
(sqrt(-c^2*x^2 + 1) + 1)^5) + 15/4*b*c^7*d^2*x^4*arcsin(c*x)/((c^5*x^5/(sq
rt(-c^2*x^2 + 1) + 1)^5 + c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x
^2 + 1) + 1)^4) + 15/4*a*c^7*d^2*x^4/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5
+ c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^4) - 11/6*b
*c^6*d^2*x^3*log(abs(c)*abs(x))/((c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^3
*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^3) + 11/6*b*c...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^2}{x^4} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x^4,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{d^2 \left(6a \sin(cx) b c^4 x^4 + 12a \sin(cx) b c^2 x^2 - 2a \sin(cx) b + 6\sqrt{-c^2 x^2 + 1} b c^3 x^3 - \sqrt{-c^2 x^2 + 1} b c x - 11 \log \right)}{6x^3}$$

input

```
int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))/x^4,x)
```

output

```
(d**2*(6*asin(c*x)*b*c**4*x**4 + 12*asin(c*x)*b*c**2*x**2 - 2*asin(c*x)*b
+ 6*sqrt(-c**2*x**2 + 1)*b*c**3*x**3 - sqrt(-c**2*x**2 + 1)*b*c*x - 11
*log(tan(asin(c*x)/2))*b*c**3*x**3 + 6*a*c**4*x**4 + 12*a*c**2*x**2 - 2*a)
)/(6*x**3)
```

3.19 $\int x^4(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 25, antiderivative size = 232

$$\int x^4(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{16bd^3\sqrt{1 - c^2x^2}}{1155c^5} + \frac{8bd^3(1 - c^2x^2)^{3/2}}{3465c^5} + \frac{2bd^3(1 - c^2x^2)^{5/2}}{1925c^5} + \frac{bd^3(1 - c^2x^2)^{7/2}}{1617c^5} - \frac{4bd^3(1 - c^2x^2)^{9/2}}{297c^5} + \frac{bd^3(1 - c^2x^2)^{11/2}}{121c^5} + \frac{1}{5}d^3x^5(a + b \arcsin(cx)) - \frac{3}{7}c^2d^3x^7(a + b \arcsin(cx)) + \frac{1}{3}c^4d^3x^9(a + b \arcsin(cx)) - \frac{1}{11}c^6d^3x^{11}(a + b \arcsin(cx))$$

output

```
16/1155*b*d^3*(-c^2*x^2+1)^(1/2)/c^5+8/3465*b*d^3*(-c^2*x^2+1)^(3/2)/c^5+2
/1925*b*d^3*(-c^2*x^2+1)^(5/2)/c^5+1/1617*b*d^3*(-c^2*x^2+1)^(7/2)/c^5-4/2
97*b*d^3*(-c^2*x^2+1)^(9/2)/c^5+1/121*b*d^3*(-c^2*x^2+1)^(11/2)/c^5+1/5*d^
3*x^5*(a+b*arcsin(c*x))-3/7*c^2*d^3*x^7*(a+b*arcsin(c*x))+1/3*c^4*d^3*x^9*
(a+b*arcsin(c*x))-1/11*c^6*d^3*x^11*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.62

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3 (-3465ac^5x^5(-231 + 495c^2x^2 - 385c^4x^4 + 105c^6x^6) + b\sqrt{1 - c^2x^2}(50488 + 25244c^2x^2 + 18933c^4x^4 - 117625c^6x^6 + 111475c^8x^8 - 33075c^{10}x^{10}) - 3465b*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6)*\text{ArcSin}[c*x])}{4002075*c^5}$$

input

```
Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(d^3*(-3465*a*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6) + b*
*sqrt[1 - c^2*x^2]*(50488 + 25244*c^2*x^2 + 18933*c^4*x^4 - 117625*c^6*x^6
+ 111475*c^8*x^8 - 33075*c^10*x^10) - 3465*b*c^5*x^5*(-231 + 495*c^2*x^2
- 385*c^4*x^4 + 105*c^6*x^6)*ArcSin[c*x]))/(4002075*c^5)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5192, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{d^3 x^5 (-105c^6 x^6 + 385c^4 x^4 - 495c^2 x^2 + 231)}{1155\sqrt{1 - c^2 x^2}} dx - \frac{1}{11} c^6 d^3 x^{11} (a + b \arcsin(cx)) + \frac{1}{3} c^4 d^3 x^9 (a + b \arcsin(cx)) - \frac{3}{7} c^2 d^3 x^7 (a + b \arcsin(cx)) + \frac{1}{5} d^3 x^5 (a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{bcd^3 \int \frac{x^5 (-105c^6 x^6 + 385c^4 x^4 - 495c^2 x^2 + 231)}{\sqrt{1 - c^2 x^2}} dx}{1155} - \frac{1}{11} c^6 d^3 x^{11} (a + b \arcsin(cx)) + \frac{1}{3} c^4 d^3 x^9 (a + b \arcsin(cx)) - \frac{3}{7} c^2 d^3 x^7 (a + b \arcsin(cx)) + \frac{1}{5} d^3 x^5 (a + b \arcsin(cx))$$

$$\begin{aligned}
& \downarrow \text{2331} \\
& \frac{bcd^3 \int \frac{x^4(-105c^6x^6+385c^4x^4-495c^2x^2+231)}{\sqrt{1-c^2x^2}} dx^2}{2310} - \frac{1}{11}c^6d^3x^{11}(a+b\arcsin(cx)) + \frac{1}{3}c^4d^3x^9(a+b\arcsin(cx)) \\
& \quad - \frac{3}{7}c^2d^3x^7(a+b\arcsin(cx)) + \frac{1}{5}d^3x^5(a+b\arcsin(cx)) \\
& \downarrow \text{2123} \\
& \frac{bcd^3 \int \left(\frac{105(1-c^2x^2)^{9/2}}{c^4} - \frac{140(1-c^2x^2)^{7/2}}{c^4} + \frac{5(1-c^2x^2)^{5/2}}{c^4} + \frac{6(1-c^2x^2)^{3/2}}{c^4} + \frac{8\sqrt{1-c^2x^2}}{c^4} + \frac{16}{c^4\sqrt{1-c^2x^2}} \right) dx^2}{2310} \\
& \quad - \frac{1}{11}c^6d^3x^{11}(a+b\arcsin(cx)) + \frac{1}{3}c^4d^3x^9(a+b\arcsin(cx)) - \frac{3}{7}c^2d^3x^7(a+b\arcsin(cx)) + \\
& \quad \quad \quad \frac{1}{5}d^3x^5(a+b\arcsin(cx)) \\
& \downarrow \text{2009} \\
& \quad - \frac{1}{11}c^6d^3x^{11}(a+b\arcsin(cx)) + \frac{1}{3}c^4d^3x^9(a+b\arcsin(cx)) - \frac{3}{7}c^2d^3x^7(a+b\arcsin(cx)) + \frac{1}{5}d^3x^5(a+b\arcsin(cx)) - \\
& \quad bcd^3 \left(-\frac{210(1-c^2x^2)^{11/2}}{11c^6} + \frac{280(1-c^2x^2)^{9/2}}{9c^6} - \frac{10(1-c^2x^2)^{7/2}}{7c^6} - \frac{12(1-c^2x^2)^{5/2}}{5c^6} - \frac{16(1-c^2x^2)^{3/2}}{3c^6} - \frac{32\sqrt{1-c^2x^2}}{c^6} \right) \\
& \quad \quad \quad \frac{2310}
\end{aligned}$$

input `Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/2310*(b*c*d^3*((-32*sqrt[1 - c^2*x^2])/c^6 - (16*(1 - c^2*x^2)^(3/2))/(3*c^6) - (12*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1 - c^2*x^2)^(7/2))/(7*c^6) + (280*(1 - c^2*x^2)^(9/2))/(9*c^6) - (210*(1 - c^2*x^2)^(11/2))/(11*c^6)) + (d^3*x^5*(a + b*ArcSin[c*x]))/5 - (3*c^2*d^3*x^7*(a + b*ArcSin[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcSin[c*x]))/3 - (c^6*d^3*x^11*(a + b*ArcSin[c*x]))/11`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2123 `Int[(Px_)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])`

rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`

rule 5192 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

method	result
parts	$-d^3 a \left(\frac{1}{11} c^6 x^{11} - \frac{1}{3} c^4 x^9 + \frac{3}{7} c^2 x^7 - \frac{1}{5} x^5 \right) - \frac{d^3 b \left(\frac{\arcsin(cx) c^{11} x^{11}}{11} - \frac{\arcsin(cx) c^9 x^9}{3} + \frac{3 \arcsin(cx) c^7 x^7}{7} - \frac{c^5 x^5}{5} \arcsin(cx) \right)}{c^2 d + e}$
derivativedivides	$-d^3 a \left(\frac{1}{11} c^{11} x^{11} - \frac{1}{3} c^9 x^9 + \frac{3}{7} c^7 x^7 - \frac{1}{5} c^5 x^5 \right) - d^3 b \left(\frac{\arcsin(cx) c^{11} x^{11}}{11} - \frac{\arcsin(cx) c^9 x^9}{3} + \frac{3 \arcsin(cx) c^7 x^7}{7} - \frac{c^5 x^5}{5} \arcsin(cx) \right)$
default	$-d^3 a \left(\frac{1}{11} c^{11} x^{11} - \frac{1}{3} c^9 x^9 + \frac{3}{7} c^7 x^7 - \frac{1}{5} c^5 x^5 \right) - d^3 b \left(\frac{\arcsin(cx) c^{11} x^{11}}{11} - \frac{\arcsin(cx) c^9 x^9}{3} + \frac{3 \arcsin(cx) c^7 x^7}{7} - \frac{c^5 x^5}{5} \arcsin(cx) \right)$
orering	$\frac{(694575c^{12}x^{12} - 2581075c^{10}x^{10} + 3337325c^8x^8 - 1460245c^6x^6 - 176708c^4x^4 - 403904c^2x^2 + 201952)(-c^2dx^2 + d)^3(a + b \arcsin(cx))}{4002075c^6(cx-1)^2x(cx+1)^2(c^2x^2-1)}$

input `int(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$-d^3*a*(1/11*c^6*x^11-1/3*c^4*x^9+3/7*c^2*x^7-1/5*x^5)-d^3*b/c^5*(1/11*\arcsin(c*x)*c^11*x^11-1/3*\arcsin(c*x)*c^9*x^9+3/7*\arcsin(c*x)*c^7*x^7-1/5*c^5*x^5*\arcsin(c*x)-6311/1334025*c^4*x^4*(-c^2*x^2+1)^{(1/2)}-25244/4002075*c^2*x^2*(-c^2*x^2+1)^{(1/2)}-50488/4002075*(-c^2*x^2+1)^{(1/2)}+4705/160083*c^6*x^6*(-c^2*x^2+1)^{(1/2)}-91/3267*c^8*x^8*(-c^2*x^2+1)^{(1/2)}+1/121*c^10*x^10*(-c^2*x^2+1)^{(1/2)})$$

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.81

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{363825 ac^{11} d^3 x^{11} - 1334025 ac^9 d^3 x^9 + 1715175 ac^7 d^3 x^7 - 800415 ac^5 d^3 x^5 + 3465 (105 bc^{11} d^3 x^{11} - 385 bc^9 d^3 x^9 + 495 bc^7 d^3 x^7 - 231 bc^5 d^3 x^5) \arcsin(cx) + (33075 bc^{10} d^3 x^{10} - 111475 bc^8 d^3 x^8 + 117625 bc^6 d^3 x^6 - 18933 bc^4 d^3 x^4 - 25244 bc^2 d^3 x^2 - 50488 b d^3) \sqrt{-c^2 x^2 + 1}}{c^5}$$

input `integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output
$$-1/4002075*(363825*a*c^11*d^3*x^11 - 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*d^3*x^7 - 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 - 385*b*c^9*d^3*x^9 + 495*b*c^7*d^3*x^7 - 231*b*c^5*d^3*x^5)*\arcsin(c*x) + (33075*b*c^10*d^3*x^10 - 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 - 18933*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 - 50488*b*d^3)*\sqrt{-c^2*x^2 + 1})/c^5$$

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.25

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \begin{cases} -\frac{ac^6 d^3 x^{11}}{11} + \frac{ac^4 d^3 x^9}{3} - \frac{3ac^2 d^3 x^7}{7} + \frac{ad^3 x^5}{5} - \frac{bc^6 d^3 x^{11} \arcsin(cx)}{11} - \frac{bc^5 d^3 x^{10} \sqrt{-c^2 x^2 + 1}}{121} + \frac{bc^4 d^3 x^9 \arcsin(cx)}{3} + \frac{91bc^3 d^3 x^8 \sqrt{-c^2 x^2 + 1}}{3267} \\ \frac{ad^3 x^5}{5} \end{cases}$$

input `integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*asin(c*x)),x)`

output `Piecewise((-a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 - 3*a*c**2*d**3*x**7/7 + a*d**3*x**5/5 - b*c**6*d**3*x**11*asin(c*x)/11 - b*c**5*d**3*x**10*sqrt(-c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*asin(c*x)/3 + 91*b*c**3*d**3*x**8*sqrt(-c**2*x**2 + 1)/3267 - 3*b*c**2*d**3*x**7*asin(c*x)/7 - 4705*b*c*d**3*x**6*sqrt(-c**2*x**2 + 1)/160083 + b*d**3*x**5*asin(c*x)/5 + 6311*b*d**3*x**4*sqrt(-c**2*x**2 + 1)/(1334025*c) + 25244*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(4002075*c**3) + 50488*b*d**3*sqrt(-c**2*x**2 + 1)/(4002075*c**5), Ne(c, 0)), (a*d**3*x**5/5, True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(200) = 400$.

Time = 0.12 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.06

$$\int x^4(d - c^2 dx^2)^3(a + b \arcsin(cx)) dx = -\frac{1}{11} ac^6 d^3 x^{11} + \frac{1}{3} ac^4 d^3 x^9 - \frac{3}{7} ac^2 d^3 x^7 - \frac{1}{7623} \left(693 x^{11} \arcsin(cx) + \left(\frac{63 \sqrt{-c^2 x^2 + 1} x^{10}}{c^2} + \frac{70 \sqrt{-c^2 x^2 + 1} x^8}{c^4} + \frac{80 \sqrt{-c^2 x^2 + 1} x^6}{c^6} + \frac{96 \sqrt{-c^2 x^2 + 1} x^4}{c^8} \right) \right) + \frac{1}{945} \left(315 x^9 \arcsin(cx) + \left(\frac{35 \sqrt{-c^2 x^2 + 1} x^8}{c^2} + \frac{40 \sqrt{-c^2 x^2 + 1} x^6}{c^4} + \frac{48 \sqrt{-c^2 x^2 + 1} x^4}{c^6} + \frac{64 \sqrt{-c^2 x^2 + 1} x^2}{c^8} \right) \right) + \frac{1}{5} ad^3 x^5 - \frac{3}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right) + \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) \right) c \right) bd^3$$

input `integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```

-1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693
*x^11*arcsin(c*x) + (63*sqrt(-c^2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1
)*x^8/c^4 + 80*sqrt(-c^2*x^2 + 1)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8
+ 128*sqrt(-c^2*x^2 + 1)*x^2/c^10 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c)*b*c^6*
d^3 + 1/945*(315*x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sq
rt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2
+ 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5
- 3/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*
x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^
8)*c)*b*c^2*d^3 + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2
+ 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^3

```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.52

$$\begin{aligned}
& \int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx \\
&= -\frac{1}{11} ac^6 d^3 x^{11} + \frac{1}{3} ac^4 d^3 x^9 - \frac{3}{7} ac^2 d^3 x^7 + \frac{1}{5} ad^3 x^5 - \frac{(c^2 x^2 - 1)^5 bd^3 x \arcsin(cx)}{11 c^4} \\
&\quad - \frac{4(c^2 x^2 - 1)^4 bd^3 x \arcsin(cx)}{33 c^4} - \frac{(c^2 x^2 - 1)^3 bd^3 x \arcsin(cx)}{231 c^4} \\
&\quad - \frac{(c^2 x^2 - 1)^5 \sqrt{-c^2 x^2 + 1} bd^3}{121 c^5} + \frac{2(c^2 x^2 - 1)^2 bd^3 x \arcsin(cx)}{385 c^4} \\
&\quad - \frac{4(c^2 x^2 - 1)^4 \sqrt{-c^2 x^2 + 1} bd^3}{297 c^5} - \frac{8(c^2 x^2 - 1) bd^3 x \arcsin(cx)}{1155 c^4} \\
&\quad - \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd^3}{1617 c^5} + \frac{16 bd^3 x \arcsin(cx)}{1155 c^4} \\
&\quad + \frac{2(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^3}{1925 c^5} + \frac{8(-c^2 x^2 + 1)^{\frac{3}{2}} bd^3}{3465 c^5} + \frac{16 \sqrt{-c^2 x^2 + 1} bd^3}{1155 c^5}
\end{aligned}$$

input

```

integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")

```

output

```
-1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 + 1/5*a*d^3*x^5 - 1/11*(c^2*x^2 - 1)^5*b*d^3*x*arcsin(c*x)/c^4 - 4/33*(c^2*x^2 - 1)^4*b*d^3*x*arcsin(c*x)/c^4 - 1/231*(c^2*x^2 - 1)^3*b*d^3*x*arcsin(c*x)/c^4 - 1/121*(c^2*x^2 - 1)^5*sqrt(-c^2*x^2 + 1)*b*d^3/c^5 + 2/385*(c^2*x^2 - 1)^2*b*d^3*x*arcsin(c*x)/c^4 - 4/297*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*d^3/c^5 - 8/1155*(c^2*x^2 - 1)*b*d^3*x*arcsin(c*x)/c^4 - 1/1617*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^3/c^5 + 16/1155*b*d^3*x*arcsin(c*x)/c^4 + 2/1925*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^3/c^5 + 8/3465*(-c^2*x^2 + 1)^(3/2)*b*d^3/c^5 + 16/1155*sqrt(-c^2*x^2 + 1)*b*d^3/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3 dx$$

input

```
int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^3,x)
```

output

```
int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.91

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3 (-363825 \operatorname{asin}(cx) b c^{11} x^{11} + 1334025 \operatorname{asin}(cx) b c^9 x^9 - 1715175 \operatorname{asin}(cx) b c^7 x^7 + 800415 \operatorname{asin}(cx) b c^5 x^5 - 161700 \operatorname{asin}(cx) b c^3 x^3 + 161700 \operatorname{asin}(cx) b c x - 161700 \operatorname{asin}(cx) b c^3 x^3 + 161700 \operatorname{asin}(cx) b c^5 x^5 - 161700 \operatorname{asin}(cx) b c^7 x^7 + 161700 \operatorname{asin}(cx) b c^9 x^9 - 161700 \operatorname{asin}(cx) b c^{11} x^{11})}{c^5}$$

input

```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*asin(c*x)),x)
```

output

```
(d**3*( - 363825*asin(c*x)*b*c**11*x**11 + 1334025*asin(c*x)*b*c**9*x**9 -
1715175*asin(c*x)*b*c**7*x**7 + 800415*asin(c*x)*b*c**5*x**5 - 33075*sqrt
( - c**2*x**2 + 1)*b*c**10*x**10 + 111475*sqrt( - c**2*x**2 + 1)*b*c**8*x*
*8 - 117625*sqrt( - c**2*x**2 + 1)*b*c**6*x**6 + 18933*sqrt( - c**2*x**2 +
1)*b*c**4*x**4 + 25244*sqrt( - c**2*x**2 + 1)*b*c**2*x**2 + 50488*sqrt( -
c**2*x**2 + 1)*b - 363825*a*c**11*x**11 + 1334025*a*c**9*x**9 - 1715175*a
*c**7*x**7 + 800415*a*c**5*x**5))/(4002075*c**5)
```

3.20 $\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	360
Mathematica [A] (verified)	361
Rubi [A] (verified)	361
Maple [A] (verified)	364
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Giac [A] (verification not implemented)	367
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Reduce [B] (verification not implemented)	368

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{49bd^3x\sqrt{1 - c^2x^2}}{5120c^3} + \frac{49bd^3x(1 - c^2x^2)^{3/2}}{7680c^3} + \frac{49bd^3x(1 - c^2x^2)^{5/2}}{9600c^3} + \frac{7bd^3x(1 - c^2x^2)^{7/2}}{1600c^3} - \frac{bd^3x(1 - c^2x^2)^{9/2}}{100c^3} + \frac{49bd^3 \arcsin(cx)}{5120c^4} - \frac{d^3(1 - c^2x^2)^4 (a + b \arcsin(cx))}{8c^4} + \frac{d^3(1 - c^2x^2)^5 (a + b \arcsin(cx))}{10c^4}$$

output

```
49/5120*b*d^3*x*(-c^2*x^2+1)^(1/2)/c^3+49/7680*b*d^3*x*(-c^2*x^2+1)^(3/2)/c^3+49/9600*b*d^3*x*(-c^2*x^2+1)^(5/2)/c^3+7/1600*b*d^3*x*(-c^2*x^2+1)^(7/2)/c^3-1/100*b*d^3*x*(-c^2*x^2+1)^(9/2)/c^3+49/5120*b*d^3*arcsin(c*x)/c^4-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arcsin(c*x))/c^4+1/10*d^3*(-c^2*x^2+1)^5*(a+b*arcsin(c*x))/c^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.67

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3 (-1920ac^4 x^4 (-10 + 20c^2 x^2 - 15c^4 x^4 + 4c^6 x^6) + bcx \sqrt{1 - c^2 x^2} (1185 + 790c^2 x^2 - 3208c^4 x^4 + 2736c^6 x^6 - 768c^8 x^8) - 15b(79 - 1280c^4 x^4 + 2560c^6 x^6 - 1920c^8 x^8 + 512c^{10} x^{10}) \arcsin(cx))}{76800c^4}$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(d^3*(-1920*a*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6) + b*c*x*
Sqrt[1 - c^2*x^2]*(1185 + 790*c^2*x^2 - 3208*c^4*x^4 + 2736*c^6*x^6 - 768*
c^8*x^8) - 15*b*(79 - 1280*c^4*x^4 + 2560*c^6*x^6 - 1920*c^8*x^8 + 512*c^1
0*x^10)*ArcSin[c*x]))/(76800*c^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5192, 27, 299, 211, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int -\frac{d^3(1 - c^2 x^2)^{7/2} (4c^2 x^2 + 1)}{40c^4} dx + \frac{d^3(1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} -$$

$$\frac{d^3(1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bd^3 \int (1 - c^2 x^2)^{7/2} (4c^2 x^2 + 1) dx}{40c^3} + \frac{d^3 (1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} - \\
& \quad \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4} \\
& \quad \downarrow \text{299} \\
& \frac{bd^3 \left(\frac{7}{5} \int (1 - c^2 x^2)^{7/2} dx - \frac{2}{5} x (1 - c^2 x^2)^{9/2} \right)}{40c^3} + \frac{d^3 (1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} - \\
& \quad \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4} \\
& \quad \downarrow \text{211} \\
& \frac{bd^3 \left(\frac{7}{5} \left(\frac{7}{8} \int (1 - c^2 x^2)^{5/2} dx + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right) - \frac{2}{5} x (1 - c^2 x^2)^{9/2} \right)}{40c^3} + \\
& \quad \frac{d^3 (1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4} \\
& \quad \downarrow \text{211} \\
& \frac{bd^3 \left(\frac{7}{5} \left(\frac{7}{8} \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} dx + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right) - \frac{2}{5} x (1 - c^2 x^2)^{9/2} \right)}{40c^3} + \\
& \quad \frac{d^3 (1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4} \\
& \quad \downarrow \text{211} \\
& \frac{bd^3 \left(\frac{7}{5} \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right) - \frac{2}{5} x (1 - c^2 x^2)^{9/2} \right)}{40c^3} + \\
& \quad \frac{d^3 (1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4} \\
& \quad \downarrow \text{211} \\
& \frac{bd^3 \left(\frac{7}{5} \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right) - \frac{2}{5} x (1 - c^2 x^2)^{9/2} \right)}{40c^3} + \\
& \quad \frac{d^3 (1 - c^2 x^2)^5 (a + b \arcsin(cx))}{10c^4} - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^4} \\
& \quad \downarrow \text{223}
\end{aligned}$$

$$\frac{d^3(1-c^2x^2)^5(a+b\arcsin(cx))}{10c^4} - \frac{d^3(1-c^2x^2)^4(a+b\arcsin(cx))}{8c^4} + \frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2}\right) + \frac{1}{4}x(1-c^2x^2)^{3/2}\right) + \frac{1}{6}x(1-c^2x^2)^{5/2}\right) + \frac{1}{8}x(1-c^2x^2)^{7/2}\right) - \frac{2}{5}x(1-c^2x^2)^{9/2}\right)}{40c^3}$$

input `Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcSin[c*x]))/c^4 + (d^3*(1 - c^2*x^2)^5*(a + b*ArcSin[c*x]))/(10*c^4) + (b*d^3*((-2*x*(1 - c^2*x^2)^(9/2))/5 + (7*((x*(1 - c^2*x^2)^(7/2))/8 + (7*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c))))/4))/8))/5))/(40*c^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 211 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 299 `Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*x*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2*p + 3)) Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ[2*p + 3, 0]`

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.96

method	result
parts	$-d^3 a \left(\frac{1}{10} c^6 x^{10} - \frac{3}{8} c^4 x^8 + \frac{1}{2} c^2 x^6 - \frac{1}{4} x^4 \right) - \frac{d^3 b \left(\frac{\arcsin(cx) c^{10} x^{10}}{10} - \frac{3 \arcsin(cx) c^8 x^8}{8} + \frac{\arcsin(cx) c^6 x^6}{2} - \frac{c^4 x^4}{4} \arcsin(cx) - \frac{7}{7680} c^3 x^3 (-c^2 x^2 + 1)^{1/2} - \frac{79}{5120} c x (-c^2 x^2 + 1)^{1/2} + \frac{79}{5120} \arcsin(cx) + \frac{401}{9600} c^5 x^5 (-c^2 x^2 + 1)^{1/2} - \frac{57}{1600} c^7 x^7 (-c^2 x^2 + 1)^{1/2} + \frac{1}{100} c^9 x^9 (-c^2 x^2 + 1)^{1/2} \right)}{c^4}$
derivativedivides	$\frac{-d^3 a \left(\frac{1}{10} c^{10} x^{10} - \frac{3}{8} c^8 x^8 + \frac{1}{2} c^6 x^6 - \frac{1}{4} c^4 x^4 \right) - d^3 b \left(\frac{\arcsin(cx) c^{10} x^{10}}{10} - \frac{3 \arcsin(cx) c^8 x^8}{8} + \frac{\arcsin(cx) c^6 x^6}{2} - \frac{c^4 x^4}{4} \arcsin(cx) - \frac{7}{7680} c^3 x^3 (-c^2 x^2 + 1)^{1/2} - \frac{79}{5120} c x (-c^2 x^2 + 1)^{1/2} + \frac{79}{5120} \arcsin(cx) + \frac{401}{9600} c^5 x^5 (-c^2 x^2 + 1)^{1/2} - \frac{57}{1600} c^7 x^7 (-c^2 x^2 + 1)^{1/2} + \frac{1}{100} c^9 x^9 (-c^2 x^2 + 1)^{1/2} \right)}{c^4}$
default	$\frac{-d^3 a \left(\frac{1}{10} c^{10} x^{10} - \frac{3}{8} c^8 x^8 + \frac{1}{2} c^6 x^6 - \frac{1}{4} c^4 x^4 \right) - d^3 b \left(\frac{\arcsin(cx) c^{10} x^{10}}{10} - \frac{3 \arcsin(cx) c^8 x^8}{8} + \frac{\arcsin(cx) c^6 x^6}{2} - \frac{c^4 x^4}{4} \arcsin(cx) - \frac{7}{7680} c^3 x^3 (-c^2 x^2 + 1)^{1/2} - \frac{79}{5120} c x (-c^2 x^2 + 1)^{1/2} + \frac{79}{5120} \arcsin(cx) + \frac{401}{9600} c^5 x^5 (-c^2 x^2 + 1)^{1/2} - \frac{57}{1600} c^7 x^7 (-c^2 x^2 + 1)^{1/2} + \frac{1}{100} c^9 x^9 (-c^2 x^2 + 1)^{1/2} \right)}{c^4}$
orering	$\frac{(4864c^{10}x^{10} - 18576c^8x^8 + 25160c^6x^6 - 11978c^4x^4 - 2765c^2x^2 + 1580)(-c^2dx^2 + d)^3(a + b \arcsin(cx))}{25600c^4(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(768c^8x^8 - 2765c^2x^2 + 1580)(-c^2dx^2 + d)^3(a + b \arcsin(cx))}{25600c^4(cx-1)^2(cx+1)^2(c^2x^2-1)}$

input

```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-d^3*a*(1/10*c^6*x^10-3/8*c^4*x^8+1/2*c^2*x^6-1/4*x^4)-d^3*b/c^4*(1/10*arc
sin(c*x)*c^10*x^10-3/8*arcsin(c*x)*c^8*x^8+1/2*arcsin(c*x)*c^6*x^6-1/4*c^4
*x^4*arcsin(c*x)-79/7680*c^3*x^3*(-c^2*x^2+1)^(1/2)-79/5120*c*x*(-c^2*x^2+
1)^(1/2)+79/5120*arcsin(c*x)+401/9600*c^5*x^5*(-c^2*x^2+1)^(1/2)-57/1600*c
^7*x^7*(-c^2*x^2+1)^(1/2)+1/100*c^9*x^9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.90

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{7680 ac^{10} d^3 x^{10} - 28800 ac^8 d^3 x^8 + 38400 ac^6 d^3 x^6 - 19200 ac^4 d^3 x^4 + 15 (512 bc^{10} d^3 x^{10} - 1920 bc^8 d^3 x^8$$

input `integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `-1/76800*(7680*a*c^10*d^3*x^10 - 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6 - 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 - 1920*b*c^8*d^3*x^8 + 2560*b*c^6*d^3*x^6 - 1280*b*c^4*d^3*x^4 + 79*b*d^3)*arcsin(c*x) + (768*b*c^9*d^3*x^9 - 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 - 790*b*c^3*d^3*x^3 - 185*b*c*d^3*x)*sqrt(-c^2*x^2 + 1))/c^4`

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.36

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \begin{cases} -\frac{ac^6 d^3 x^{10}}{10} + \frac{3ac^4 d^3 x^8}{8} - \frac{ac^2 d^3 x^6}{2} + \frac{ad^3 x^4}{4} - \frac{bc^6 d^3 x^{10} \operatorname{asin}(cx)}{10} - \frac{bc^5 d^3 x^9 \sqrt{-c^2 x^2 + 1}}{100} + \frac{3bc^4 d^3 x^8 \operatorname{asin}(cx)}{8} + \frac{57bc^3 d^3 x^7 \sqrt{-c^2 x^2 + 1}}{1600} \\ \frac{ad^3 x^4}{4} \end{cases}$$

input `integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*asin(c*x)),x)`

output `Piecewise((-a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 - a*c**2*d**3*x**6/2 + a*d**3*x**4/4 - b*c**6*d**3*x**10*asin(c*x)/10 - b*c**5*d**3*x**9*sqrt(-c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*asin(c*x)/8 + 57*b*c**3*d**3*x**7*sqrt(-c**2*x**2 + 1)/1600 - b*c**2*d**3*x**6*asin(c*x)/2 - 401*b*c*d**3*x**5*sqrt(-c**2*x**2 + 1)/9600 + b*d**3*x**4*asin(c*x)/4 + 79*b*d**3*x**3*sqrt(-c**2*x**2 + 1)/(7680*c) + 79*b*d**3*x*sqrt(-c**2*x**2 + 1)/(5120*c**3) - 79*b*d**3*asin(c*x)/(5120*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(178) = 356$.

Time = 0.12 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.13

$$\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = -\frac{1}{10} ac^6 d^3 x^{10} + \frac{3}{8} ac^4 d^3 x^8 - \frac{1}{2} ac^2 d^3 x^6 - \frac{1}{12800} \left(1280 x^{10} \arcsin(cx) + \left(\frac{128 \sqrt{-c^2 x^2 + 1} x^9}{c^2} + \frac{144 \sqrt{-c^2 x^2 + 1} x^7}{c^4} + \frac{168 \sqrt{-c^2 x^2 + 1} x^5}{c^6} + \frac{210 \sqrt{-c^2 x^2 + 1} x^3}{c^8} + \frac{315 \sqrt{-c^2 x^2 + 1} x}{c^{10}} - \frac{315 \arcsin(cx)}{c^{11}} \right) c \right) b c^6 d^3 + \frac{1}{1024} \left(384 x^8 \arcsin(cx) + \left(\frac{48 \sqrt{-c^2 x^2 + 1} x^7}{c^2} + \frac{56 \sqrt{-c^2 x^2 + 1} x^5}{c^4} + \frac{70 \sqrt{-c^2 x^2 + 1} x^3}{c^6} + \frac{105 \sqrt{-c^2 x^2 + 1} x}{c^8} - \frac{105 \arcsin(cx)}{c^9} \right) c \right) b c^4 d^3 + \frac{1}{4} a d^3 x^4 - \frac{1}{96} \left(48 x^6 \arcsin(cx) + \left(\frac{8 \sqrt{-c^2 x^2 + 1} x^5}{c^2} + \frac{10 \sqrt{-c^2 x^2 + 1} x^3}{c^4} + \frac{15 \sqrt{-c^2 x^2 + 1} x}{c^6} - \frac{15 \arcsin(cx)}{c^7} \right) c \right) b c^2 d^3 + \frac{1}{32} \left(8 x^4 \arcsin(cx) + \left(\frac{2 \sqrt{-c^2 x^2 + 1} x^3}{c^2} + \frac{3 \sqrt{-c^2 x^2 + 1} x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) c \right) b d^3$$

input `integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
-1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(1280*x^10*arcsin(c*x) + (128*sqrt(-c^2*x^2 + 1)*x^9/c^2 + 144*sqrt(-c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(-c^2*x^2 + 1)*x^5/c^6 + 210*sqrt(-c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(-c^2*x^2 + 1)*x/c^10 - 315*arcsin(c*x)/c^11)*c)*b*c^6*d^3 + 1/1024*(384*x^8*arcsin(c*x) + (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.21

$$\begin{aligned}
\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = & -\frac{1}{10} ac^6 d^3 x^{10} + \frac{3}{8} ac^4 d^3 x^8 - \frac{1}{2} ac^2 d^3 x^6 \\
& + \frac{1}{4} ad^3 x^4 - \frac{(c^2 x^2 - 1)^4 \sqrt{-c^2 x^2 + 1} bd^3 x}{100 c^3} \\
& - \frac{(c^2 x^2 - 1)^5 bd^3 \arcsin(cx)}{10 c^4} \\
& - \frac{7(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd^3 x}{1600 c^3} \\
& - \frac{(c^2 x^2 - 1)^4 bd^3 \arcsin(cx)}{8 c^4} \\
& + \frac{49(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^3 x}{9600 c^3} \\
& + \frac{49(-c^2 x^2 + 1)^{\frac{3}{2}} bd^3 x}{7680 c^3} \\
& + \frac{49 \sqrt{-c^2 x^2 + 1} bd^3 x}{5120 c^3} + \frac{49 bd^3 \arcsin(cx)}{5120 c^4}
\end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 + 1/4*a*d^3*x^4 - 1/100*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 - 1/10*(c^2*x^2 - 1)^5*b*d^3*arcsin(c*x)/c^4 - 7/1600*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 - 1/8*(c^2*x^2 - 1)^4*b*d^3*arcsin(c*x)/c^4 + 49/9600*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 + 49/7680*(-c^2*x^2 + 1)^(3/2)*b*d^3*x/c^3 + 49/5120*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 + 49/5120*b*d^3*arcsin(c*x)/c^4`

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int x^3 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3 dx$$

input `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^3,x)`

output `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^3, x)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3 (-7680 \operatorname{asin}(cx) b c^{10} x^{10} + 28800 \operatorname{asin}(cx) b c^8 x^8 - 38400 \operatorname{asin}(cx) b c^6 x^6 + 19200 \operatorname{asin}(cx) b c^4 x^4 - 118500 \operatorname{asin}(cx) b c^2 x^2 + 1185 \operatorname{asin}(cx) b c x - 7680 a c^{10} x^{10} + 28800 a c^8 x^8 - 38400 a c^6 x^6 + 19200 a c^4 x^4)}{(76800 c^4)}$$

input `int(x^3*(-c^2*d*x^2+d)^3*(a+b*asin(c*x)),x)`

output `(d**3*(- 7680*asin(c*x)*b*c**10*x**10 + 28800*asin(c*x)*b*c**8*x**8 - 38400*asin(c*x)*b*c**6*x**6 + 19200*asin(c*x)*b*c**4*x**4 - 1185*asin(c*x)*b - 768*sqrt(- c**2*x**2 + 1)*b*c**9*x**9 + 2736*sqrt(- c**2*x**2 + 1)*b*c**7*x**7 - 3208*sqrt(- c**2*x**2 + 1)*b*c**5*x**5 + 790*sqrt(- c**2*x**2 + 1)*b*c**3*x**3 + 1185*sqrt(- c**2*x**2 + 1)*b*c*x - 7680*a*c**10*x**10 + 28800*a*c**8*x**8 - 38400*a*c**6*x**6 + 19200*a*c**4*x**4))/(76800*c**4)`

3.21 $\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 25, antiderivative size = 207

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{16bd^3\sqrt{1 - c^2x^2}}{315c^3} + \frac{8bd^3(1 - c^2x^2)^{3/2}}{945c^3} + \frac{2bd^3(1 - c^2x^2)^{5/2}}{525c^3} + \frac{bd^3(1 - c^2x^2)^{7/2}}{441c^3} - \frac{bd^3(1 - c^2x^2)^{9/2}}{81c^3} + \frac{1}{3}d^3x^3(a + b \arcsin(cx)) - \frac{3}{5}c^2d^3x^5(a + b \arcsin(cx)) + \frac{3}{7}c^4d^3x^7(a + b \arcsin(cx)) - \frac{1}{9}c^6d^3x^9(a + b \arcsin(cx))$$

output

```
16/315*b*d^3*(-c^2*x^2+1)^(1/2)/c^3+8/945*b*d^3*(-c^2*x^2+1)^(3/2)/c^3+2/5
25*b*d^3*(-c^2*x^2+1)^(5/2)/c^3+1/441*b*d^3*(-c^2*x^2+1)^(7/2)/c^3-1/81*b*
d^3*(-c^2*x^2+1)^(9/2)/c^3+1/3*d^3*x^3*(a+b*arcsin(c*x))-3/5*c^2*d^3*x^5*(
a+b*arcsin(c*x))+3/7*c^4*d^3*x^7*(a+b*arcsin(c*x))-1/9*c^6*d^3*x^9*(a+b*ar
csin(c*x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3 (-315ac^3x^3(-105 + 189c^2x^2 - 135c^4x^4 + 35c^6x^6) + b\sqrt{1 - c^2x^2}(5258 + 2629c^2x^2 - 6297c^4x^4 + 4675c^6x^6 - 1225c^8x^8) - 315b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6)*\text{ArcSin}[c*x])}{99225c^3}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(d^3*(-315*a*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(5258 + 2629*c^2*x^2 - 6297*c^4*x^4 + 4675*c^6*x^6 - 1225*c^8*x^8) - 315*b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6)*ArcSin[c*x])/(99225*c^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5192, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5192}$$

$$-bc \int \frac{d^3 x^3 (-35c^6 x^6 + 135c^4 x^4 - 189c^2 x^2 + 105)}{315\sqrt{1 - c^2 x^2}} dx - \frac{1}{9} c^6 d^3 x^9 (a + b \arcsin(cx)) + \frac{3}{7} c^4 d^3 x^7 (a + b \arcsin(cx)) - \frac{3}{5} c^2 d^3 x^5 (a + b \arcsin(cx)) + \frac{1}{3} d^3 x^3 (a + b \arcsin(cx))$$

$$\downarrow \text{27}$$

$$-\frac{1}{315} bcd^3 \int \frac{x^3 (-35c^6 x^6 + 135c^4 x^4 - 189c^2 x^2 + 105)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{9} c^6 d^3 x^9 (a + b \arcsin(cx)) + \frac{3}{7} c^4 d^3 x^7 (a + b \arcsin(cx)) - \frac{3}{5} c^2 d^3 x^5 (a + b \arcsin(cx)) + \frac{1}{3} d^3 x^3 (a + b \arcsin(cx))$$

$$\begin{aligned} & \downarrow 2331 \\ & -\frac{1}{630}bcd^3 \int \frac{x^2(-35c^6x^6 + 135c^4x^4 - 189c^2x^2 + 105)}{\sqrt{1-c^2x^2}} dx^2 - \frac{1}{9}c^6d^3x^9(a + b \arcsin(cx)) + \\ & \quad \frac{3}{7}c^4d^3x^7(a + b \arcsin(cx)) - \frac{3}{5}c^2d^3x^5(a + b \arcsin(cx)) + \frac{1}{3}d^3x^3(a + b \arcsin(cx)) \end{aligned}$$

$$\begin{aligned} & \downarrow 2123 \\ & -\frac{1}{630}bcd^3 \int \left(-\frac{35(1-c^2x^2)^{7/2}}{c^2} + \frac{5(1-c^2x^2)^{5/2}}{c^2} + \frac{6(1-c^2x^2)^{3/2}}{c^2} + \frac{8\sqrt{1-c^2x^2}}{c^2} + \frac{16}{c^2\sqrt{1-c^2x^2}} \right) dx^2 - \\ & \quad \frac{1}{9}c^6d^3x^9(a + b \arcsin(cx)) + \frac{3}{7}c^4d^3x^7(a + b \arcsin(cx)) - \frac{3}{5}c^2d^3x^5(a + b \arcsin(cx)) + \\ & \quad \quad \quad \frac{1}{3}d^3x^3(a + b \arcsin(cx)) \end{aligned}$$

$$\begin{aligned} & \downarrow 2009 \\ & -\frac{1}{9}c^6d^3x^9(a + \\ & b \arcsin(cx)) + \frac{3}{7}c^4d^3x^7(a + b \arcsin(cx)) - \frac{3}{5}c^2d^3x^5(a + b \arcsin(cx)) + \frac{1}{3}d^3x^3(a + b \arcsin(cx)) - \\ & \frac{1}{630}bcd^3 \left(\frac{70(1-c^2x^2)^{9/2}}{9c^4} - \frac{10(1-c^2x^2)^{7/2}}{7c^4} - \frac{12(1-c^2x^2)^{5/2}}{5c^4} - \frac{16(1-c^2x^2)^{3/2}}{3c^4} - \frac{32\sqrt{1-c^2x^2}}{c^4} \right) \end{aligned}$$

input `Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/630*(b*c*d^3*((-32*sqrt[1 - c^2*x^2])/c^4 - (16*(1 - c^2*x^2)^(3/2))/(3*c^4) - (12*(1 - c^2*x^2)^(5/2))/(5*c^4) - (10*(1 - c^2*x^2)^(7/2))/(7*c^4) + (70*(1 - c^2*x^2)^(9/2))/(9*c^4))) + (d^3*x^3*(a + b*ArcSin[c*x]))/3 - (3*c^2*d^3*x^5*(a + b*ArcSin[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcSin[c*x]))/7 - (c^6*d^3*x^9*(a + b*ArcSin[c*x]))/9`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`


```
rule 2123 Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c,
d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

```
rule 2331 Int[(Pq_)*(x_)^m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2 S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

```
rule 5192 Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)
^2)^(p_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

method	result
parts	$-d^3 a \left(\frac{1}{9} c^6 x^9 - \frac{3}{7} c^4 x^7 + \frac{3}{5} c^2 x^5 - \frac{1}{3} x^3 \right) - \frac{d^3 b \left(\frac{\arcsin(cx) c^9 x^9}{9} - \frac{3 \arcsin(cx) c^7 x^7}{7} + \frac{3 c^5 x^5 \arcsin(cx)}{5} - \frac{c^3 x^3 \arcsin(cx)}{3} \right)}{c^3}$
derivativedivides	$\frac{-d^3 a \left(\frac{1}{9} c^9 x^9 - \frac{3}{7} c^7 x^7 + \frac{3}{5} c^5 x^5 - \frac{1}{3} c^3 x^3 \right) - d^3 b \left(\frac{\arcsin(cx) c^9 x^9}{9} - \frac{3 \arcsin(cx) c^7 x^7}{7} + \frac{3 c^5 x^5 \arcsin(cx)}{5} - \frac{c^3 x^3 \arcsin(cx)}{3} - \frac{2629 c^3}{3} \right)}{c^3}$
default	$\frac{-d^3 a \left(\frac{1}{9} c^9 x^9 - \frac{3}{7} c^7 x^7 + \frac{3}{5} c^5 x^5 - \frac{1}{3} c^3 x^3 \right) - d^3 b \left(\frac{\arcsin(cx) c^9 x^9}{9} - \frac{3 \arcsin(cx) c^7 x^7}{7} + \frac{3 c^5 x^5 \arcsin(cx)}{5} - \frac{c^3 x^3 \arcsin(cx)}{3} - \frac{2629 c^3}{3} \right)}{c^3}$
oring	$\frac{(20825 c^{10} x^{10} - 82375 c^8 x^8 + 119261 c^6 x^6 - 66701 c^4 x^4 - 36806 c^2 x^2 + 10516) (-c^2 d x^2 + d)^3 (a + b \arcsin(cx))}{99225 c^4 (cx - 1)^2 x (cx + 1)^2 (c^2 x^2 - 1)} - \frac{(1225 c^8)}{c^3}$

```
input int(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-d^3*a*(1/9*c^6*x^9-3/7*c^4*x^7+3/5*c^2*x^5-1/3*x^3)-d^3*b/c^3*(1/9*arcsin
(c*x)*c^9*x^9-3/7*arcsin(c*x)*c^7*x^7+3/5*c^5*x^5*arcsin(c*x)-1/3*c^3*x^3*
arcsin(c*x)-2629/99225*c^2*x^2*(-c^2*x^2+1)^(1/2)-5258/99225*(-c^2*x^2+1)^(
1/2)+2099/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(-c^2*x^2+1)^(
1/2)+1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx =$$

$$\frac{11025 ac^9 d^3 x^9 - 42525 ac^7 d^3 x^7 + 59535 ac^5 d^3 x^5 - 33075 ac^3 d^3 x^3 + 315 (35 bc^9 d^3 x^9 - 135 bc^7 d^3 x^7 + 189 bc^5 d^3 x^5 - 105 bc^3 d^3 x^3) \arcsin(cx) + (1225 b^2 c^8 d^3 x^8 - 4675 b^2 c^6 d^3 x^6 + 6297 b^2 c^4 d^3 x^4 - 2629 b^2 c^2 d^3 x^2 - 5258 b^2 d^3) \sqrt{-c^2 x^2 + 1}}{c^3}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
-1/99225*(11025*a*c^9*d^3*x^9 - 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5
- 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 - 135*b*c^7*d^3*x^7 + 189*b*
c^5*d^3*x^5 - 105*b*c^3*d^3*x^3)*arcsin(c*x) + (1225*b*c^8*d^3*x^8 - 4675*
b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 - 2629*b*c^2*d^3*x^2 - 5258*b*d^3)*sqrt
(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.28

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} -\frac{ac^6 d^3 x^9}{9} + \frac{3ac^4 d^3 x^7}{7} - \frac{3ac^2 d^3 x^5}{5} + \frac{ad^3 x^3}{3} - \frac{bc^6 d^3 x^9 \arcsin(cx)}{9} - \frac{bc^5 d^3 x^8 \sqrt{-c^2 x^2 + 1}}{81} + \frac{3bc^4 d^3 x^7 \arcsin(cx)}{7} + \frac{187bc^3 d^3 x^6 \sqrt{-c^2 x^2 + 1}}{3969} \\ \frac{ad^3 x^3}{3} \end{cases}$$

input

```
integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*asin(c*x)),x)
```

output

```
Piecewise((-a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 - 3*a*c**2*d**3*x**5/5 + a*d**3*x**3/3 - b*c**6*d**3*x**9*asin(c*x)/9 - b*c**5*d**3*x**8*sqrt(-c**2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*asin(c*x)/7 + 187*b*c**3*d**3*x**6*sqrt(-c**2*x**2 + 1)/3969 - 3*b*c**2*d**3*x**5*asin(c*x)/5 - 2099*b*c*d**3*x**4*sqrt(-c**2*x**2 + 1)/33075 + b*d**3*x**3*asin(c*x)/3 + 2629*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(99225*c) + 5258*b*d**3*sqrt(-c**2*x**2 + 1)/(99225*c**3), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(179) = 358$.

Time = 0.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.92

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = -\frac{1}{9} ac^6 d^3 x^9 + \frac{3}{7} ac^4 d^3 x^7 - \frac{1}{2835} \left(315 x^9 \arcsin(cx) + \left(\frac{35 \sqrt{-c^2 x^2 + 1} x^8}{c^2} + \frac{40 \sqrt{-c^2 x^2 + 1} x^6}{c^4} + \frac{48 \sqrt{-c^2 x^2 + 1} x^4}{c^6} + \frac{64 \sqrt{-c^2 x^2 + 1}}{c^8} - \frac{3}{5} ac^2 d^3 x^5 + \frac{3}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} - \frac{1}{25} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bc^2 d^3 + \frac{1}{3} ad^3 x^3 + \frac{1}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bd^3 \right)$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```
-1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*arcsin(c*x) + (35
*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2
*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)
/c^10)*c)*b*c^6*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arcsin(c*x) + (5*s
qrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2
+ 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 - 1/25*(15*x^5*arc
sin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsin
(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.43

$$\int x^2(d - c^2dx^2)^3(a + b \arcsin(cx)) dx$$

$$= -\frac{1}{9}ac^6d^3x^9 + \frac{3}{7}ac^4d^3x^7 - \frac{3}{5}ac^2d^3x^5 - \frac{(c^2x^2 - 1)^4bd^3x \arcsin(cx)}{9c^2}$$

$$+ \frac{1}{3}ad^3x^3 - \frac{(c^2x^2 - 1)^3bd^3x \arcsin(cx)}{63c^2} + \frac{2(c^2x^2 - 1)^2bd^3x \arcsin(cx)}{105c^2}$$

$$- \frac{(c^2x^2 - 1)^4\sqrt{-c^2x^2 + 1}bd^3}{81c^3} - \frac{8(c^2x^2 - 1)bd^3x \arcsin(cx)}{315c^2}$$

$$- \frac{(c^2x^2 - 1)^3\sqrt{-c^2x^2 + 1}bd^3}{441c^3} + \frac{16bd^3x \arcsin(cx)}{315c^2}$$

$$+ \frac{2(c^2x^2 - 1)^2\sqrt{-c^2x^2 + 1}bd^3}{525c^3} + \frac{8(-c^2x^2 + 1)^{\frac{3}{2}}bd^3}{945c^3} + \frac{16\sqrt{-c^2x^2 + 1}bd^3}{315c^3}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
-1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 3/5*a*c^2*d^3*x^5 - 1/9*(c^2*x^2
- 1)^4*b*d^3*x*arcsin(c*x)/c^2 + 1/3*a*d^3*x^3 - 1/63*(c^2*x^2 - 1)^3*b*d^
3*x*arcsin(c*x)/c^2 + 2/105*(c^2*x^2 - 1)^2*b*d^3*x*arcsin(c*x)/c^2 - 1/81
*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*d^3/c^3 - 8/315*(c^2*x^2 - 1)*b*d^3*
x*arcsin(c*x)/c^2 - 1/441*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^3/c^3 + 1
6/315*b*d^3*x*arcsin(c*x)/c^2 + 2/525*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b
*d^3/c^3 + 8/945*(-c^2*x^2 + 1)^(3/2)*b*d^3/c^3 + 16/315*sqrt(-c^2*x^2 + 1
)*b*d^3/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int x^2 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3 dx$$

input `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^3,x)`output `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^3, x)`**Reduce [B] (verification not implemented)**

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.92

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3 (-11025 \operatorname{asin}(cx) b c^9 x^9 + 42525 \operatorname{asin}(cx) b c^7 x^7 - 59535 \operatorname{asin}(cx) b c^5 x^5 + 33075 \operatorname{asin}(cx) b c^3 x^3 - 1225$$

input `int(x^2*(-c^2*d*x^2+d)^3*(a+b*asin(c*x)),x)`output `(d**3*(- 11025*asin(c*x)*b*c**9*x**9 + 42525*asin(c*x)*b*c**7*x**7 - 59535*asin(c*x)*b*c**5*x**5 + 33075*asin(c*x)*b*c**3*x**3 - 1225*sqrt(- c**2*x**2 + 1)*b*c**8*x**8 + 4675*sqrt(- c**2*x**2 + 1)*b*c**6*x**6 - 6297*sqrt(- c**2*x**2 + 1)*b*c**4*x**4 + 2629*sqrt(- c**2*x**2 + 1)*b*c**2*x**2 + 5258*sqrt(- c**2*x**2 + 1)*b - 11025*a*c**9*x**9 + 42525*a*c**7*x**7 - 59535*a*c**5*x**5 + 33075*a*c**3*x**3))/(99225*c**3)`

3.22 $\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 23, antiderivative size = 150

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{35bd^3 x \sqrt{1 - c^2 x^2}}{1024c} + \frac{35bd^3 x(1 - c^2 x^2)^{3/2}}{1536c} + \frac{7bd^3 x(1 - c^2 x^2)^{5/2}}{384c} + \frac{bd^3 x(1 - c^2 x^2)^{7/2}}{64c} + \frac{35bd^3 \arcsin(cx)}{1024c^2} - \frac{d^3(1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^2}$$

output

```
35/1024*b*d^3*x*(-c^2*x^2+1)^(1/2)/c+35/1536*b*d^3*x*(-c^2*x^2+1)^(3/2)/c+
7/384*b*d^3*x*(-c^2*x^2+1)^(5/2)/c+1/64*b*d^3*x*(-c^2*x^2+1)^(7/2)/c+35/10
24*b*d^3*arcsin(c*x)/c^2-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arcsin(c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{d^3 \left(384a(-1 + c^2 x^2)^4 + bcx\sqrt{1 - c^2 x^2}(-279 + 326c^2 x^2 - 200c^4 x^4 + 48c^6 x^6) + 3b(93 - 512c^2 x^2 + 768c^4 x^4 - 512c^6 x^6 + 128c^8 x^8) \right) \arcsin(cx)}{3072c^2}$$

input

```
Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
-1/3072*(d^3*(384*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[1 - c^2*x^2]*(-279 + 326*c^2*x^2 - 200*c^4*x^4 + 48*c^6*x^6) + 3*b*(93 - 512*c^2*x^2 + 768*c^4*x^4 - 512*c^6*x^6 + 128*c^8*x^8)*ArcSin[c*x]))/c^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5182, 211, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx \\ & \quad \downarrow \text{5182} \\ & \frac{bd^3 \int (1 - c^2 x^2)^{7/2} dx}{8c} - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^2} \\ & \quad \downarrow \text{211} \\ & \frac{bd^3 \left(\frac{7}{8} \int (1 - c^2 x^2)^{5/2} dx + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right)}{8c} - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))}{8c^2} \\ & \quad \downarrow \text{211} \end{aligned}$$

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} dx + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right)}{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))} \frac{8c}{8c^2}$$

↓ 211

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right)}{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))} \frac{8c}{8c^2}$$

↓ 211

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right)}{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))} \frac{8c}{8c^2}$$

↓ 223

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) + \frac{1}{8} x (1 - c^2 x^2)^{7/2} \right)}{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))} \frac{8c}{8c^2}$$

input `Int[x*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcSin[c*x]))/c^2 + (b*d^3*((x*(1 - c^2*x^2)^(7/2))/8 + (7*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/6))/8)/(8*c)`

Defintions of rubi rules used

```
rule 211 Int[((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

```
rule 223 Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

```
rule 5182 Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{d^3 a (c^2 x^2 - 1)^4}{8} - d^3 b \left(\frac{\arcsin(cx) c^8 x^8}{8} - \frac{\arcsin(cx) c^6 x^6}{2} + \frac{3c^4 x^4 \arcsin(cx)}{4} - \frac{c^2 x^2 \arcsin(cx)}{2} + \frac{93 \arcsin(cx)}{1024} + \frac{c^7 x^7 \sqrt{-c^2 x^2 + 1}}{64} \right)}{c^2}$
default	$\frac{-\frac{d^3 a (c^2 x^2 - 1)^4}{8} - d^3 b \left(\frac{\arcsin(cx) c^8 x^8}{8} - \frac{\arcsin(cx) c^6 x^6}{2} + \frac{3c^4 x^4 \arcsin(cx)}{4} - \frac{c^2 x^2 \arcsin(cx)}{2} + \frac{93 \arcsin(cx)}{1024} + \frac{c^7 x^7 \sqrt{-c^2 x^2 + 1}}{64} \right)}{c^2}$
parts	$-\frac{d^3 a (c^2 x^2 - 1)^4}{8c^2} - \frac{d^3 b \left(\frac{\arcsin(cx) c^8 x^8}{8} - \frac{\arcsin(cx) c^6 x^6}{2} + \frac{3c^4 x^4 \arcsin(cx)}{4} - \frac{c^2 x^2 \arcsin(cx)}{2} + \frac{93 \arcsin(cx)}{1024} + \frac{c^7 x^7 \sqrt{-c^2 x^2 + 1}}{64} \right)}{c^2}$
oring	$\frac{(720c^8 x^8 - 2984c^6 x^6 + 4786c^4 x^4 - 3815c^2 x^2 + 558)(-c^2 d x^2 + d)^3 (a + b \arcsin(cx))}{3072c^2 (cx - 1)^2 (cx + 1)^2 (c^2 x^2 - 1)} - \frac{(48c^6 x^6 - 200c^4 x^4 + 326c^2 x^2 - 27)}{c^2}$

```
input int(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/c^2*(-1/8*d^3*a*(c^2*x^2-1)^4-d^3*b*(1/8*arcsin(c*x)*c^8*x^8-1/2*arcsin(c*x)*c^6*x^6+3/4*c^4*x^4*arcsin(c*x)-1/2*c^2*x^2*arcsin(c*x)+93/1024*arcsin(c*x)+1/64*c^7*x^7*(-c^2*x^2+1)^(1/2)-25/384*c^5*x^5*(-c^2*x^2+1)^(1/2)+163/1536*c^3*x^3*(-c^2*x^2+1)^(1/2)-93/1024*c*x*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.15

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx =$$

$$\frac{-384 ac^8 d^3 x^8 - 1536 ac^6 d^3 x^6 + 2304 ac^4 d^3 x^4 - 1536 ac^2 d^3 x^2 + 3(128 bc^8 d^3 x^8 - 512 bc^6 d^3 x^6 + 768 bc^4 d^3 x^4 - 512 bc^2 d^3 x^2 + 93 b d^3) \arcsin(cx) + (48 b c^7 d^3 x^7 - 200 b c^5 d^3 x^5 + 326 b c^3 d^3 x^3 - 279 b c d^3 x) \sqrt{-c^2 x^2 + 1}}{c^2}$$

input

```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
-1/3072*(384*a*c^8*d^3*x^8 - 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 - 1536*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 - 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x^4 - 512*b*c^2*d^3*x^2 + 93*b*d^3)*arcsin(c*x) + (48*b*c^7*d^3*x^7 - 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 - 279*b*c*d^3*x)*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.69

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} -\frac{ac^6 d^3 x^8}{8} + \frac{ac^4 d^3 x^6}{2} - \frac{3ac^2 d^3 x^4}{4} + \frac{ad^3 x^2}{2} - \frac{bc^6 d^3 x^8 \arcsin(cx)}{8} - \frac{bc^5 d^3 x^7 \sqrt{-c^2 x^2 + 1}}{64} + \frac{bc^4 d^3 x^6 \arcsin(cx)}{2} + \frac{25bc^3 d^3 x^5 \sqrt{-c^2 x^2 + 1}}{384} \\ \frac{ad^3 x^2}{2} \end{cases}$$

input

```
integrate(x*(-c**2*d*x**2+d)**3*(a+b*asin(c*x)),x)
```

output

```
Piecewise((-a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 - 3*a*c**2*d**3*x**4/4
+ a*d**3*x**2/2 - b*c**6*d**3*x**8*asin(c*x)/8 - b*c**5*d**3*x**7*sqrt(-c
**2*x**2 + 1)/64 + b*c**4*d**3*x**6*asin(c*x)/2 + 25*b*c**3*d**3*x**5*sqrt
(-c**2*x**2 + 1)/384 - 3*b*c**2*d**3*x**4*asin(c*x)/4 - 163*b*c*d**3*x**3*
sqrt(-c**2*x**2 + 1)/1536 + b*d**3*x**2*asin(c*x)/2 + 93*b*d**3*x*sqrt(-c*
*2*x**2 + 1)/(1024*c) - 93*b*d**3*asin(c*x)/(1024*c**2), Ne(c, 0)), (a*d**
3*x**2/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(129) = 258$.

Time = 0.12 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.39

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = -\frac{1}{8} ac^6 d^3 x^8 + \frac{1}{2} ac^4 d^3 x^6$$

$$- \frac{1}{3072} \left(384 x^8 \arcsin(cx) + \left(\frac{48 \sqrt{-c^2 x^2 + 1} x^7}{c^2} + \frac{56 \sqrt{-c^2 x^2 + 1} x^5}{c^4} + \frac{70 \sqrt{-c^2 x^2 + 1} x^3}{c^6} + \frac{105 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right.$$

$$\left. - \frac{3}{4} ac^2 d^3 x^4 \right.$$

$$\left. + \frac{1}{96} \left(48 x^6 \arcsin(cx) + \left(\frac{8 \sqrt{-c^2 x^2 + 1} x^5}{c^2} + \frac{10 \sqrt{-c^2 x^2 + 1} x^3}{c^4} + \frac{15 \sqrt{-c^2 x^2 + 1} x}{c^6} - \frac{15 \arcsin(cx)}{c^7} \right) \right)$$

$$- \frac{3}{32} \left(8 x^4 \arcsin(cx) + \left(\frac{2 \sqrt{-c^2 x^2 + 1} x^3}{c^2} + \frac{3 \sqrt{-c^2 x^2 + 1} x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) c \right) bc^2 d^3$$

$$+ \frac{1}{2} ad^3 x^2 + \frac{1}{4} \left(2 x^2 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x}{c^2} - \frac{\arcsin(cx)}{c^3} \right) \right) bd^3$$

input

```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```
-1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*arcsin(c*x) + (48
*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2
*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)
*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x
^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/
c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^4*d^3 - 3/32*(8*x^4*arcsin(c*x) + (2*sqrt
(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)
)*b*c^2*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 +
1)*x/c^2 - arcsin(c*x)/c^3))*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.35

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = -\frac{1}{8} ac^6 d^3 x^8 + \frac{1}{2} ac^4 d^3 x^6 - \frac{3}{4} ac^2 d^3 x^4$$

$$- \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b d^3 x}{64 c}$$

$$- \frac{(c^2 x^2 - 1)^4 b d^3 \arcsin(cx)}{8 c^2}$$

$$+ \frac{7(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b d^3 x}{384 c}$$

$$+ \frac{35(-c^2 x^2 + 1)^{\frac{3}{2}} b d^3 x}{1536 c} + \frac{35 \sqrt{-c^2 x^2 + 1} b d^3 x}{1024 c}$$

$$+ \frac{(c^2 x^2 - 1) a d^3}{2 c^2} + \frac{35 b d^3 \arcsin(cx)}{1024 c^2}$$

input

```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
-1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 3/4*a*c^2*d^3*x^4 - 1/64*(c^2*x^2
- 1)^3*sqrt(-c^2*x^2 + 1)*b*d^3*x/c - 1/8*(c^2*x^2 - 1)^4*b*d^3*arcsin(c*
x)/c^2 + 7/384*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^3*x/c + 35/1536*(-c^
2*x^2 + 1)^(3/2)*b*d^3*x/c + 35/1024*sqrt(-c^2*x^2 + 1)*b*d^3*x/c + 1/2*(c
^2*x^2 - 1)*a*d^3/c^2 + 35/1024*b*d^3*arcsin(c*x)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int x(a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3 dx$$

input `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^3,x)`output `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^3, x)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.20

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3(-384 \operatorname{asin}(cx) b c^8 x^8 + 1536 \operatorname{asin}(cx) b c^6 x^6 - 2304 \operatorname{asin}(cx) b c^4 x^4 + 1536 \operatorname{asin}(cx) b c^2 x^2 - 279 \operatorname{asin}(cx) b c^2 x^2 - 279 \operatorname{asin}(cx) b c^2 x^2 - 279 \operatorname{asin}(cx) b c^2 x^2)}{(3072 c^2)}$$

input `int(x*(-c^2*d*x^2+d)^3*(a+b*asin(c*x)),x)`output `(d**3*(- 384*asin(c*x)*b*c**8*x**8 + 1536*asin(c*x)*b*c**6*x**6 - 2304*asin(c*x)*b*c**4*x**4 + 1536*asin(c*x)*b*c**2*x**2 - 279*asin(c*x)*b - 48*sqrt(- c**2*x**2 + 1)*b*c**7*x**7 + 200*sqrt(- c**2*x**2 + 1)*b*c**5*x**5 - 326*sqrt(- c**2*x**2 + 1)*b*c**3*x**3 + 279*sqrt(- c**2*x**2 + 1)*b*c*x - 384*a*c**8*x**8 + 1536*a*c**6*x**6 - 2304*a*c**4*x**4 + 1536*a*c**2*x**2))/(3072*c**2)`

3.23 $\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 22, antiderivative size = 175

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{16bd^3\sqrt{1 - c^2x^2}}{35c} + \frac{8bd^3(1 - c^2x^2)^{3/2}}{105c} + \frac{6bd^3(1 - c^2x^2)^{5/2}}{175c} + \frac{bd^3(1 - c^2x^2)^{7/2}}{49c}$$

$$+ d^3x(a + b \arcsin(cx)) - c^2d^3x^3(a + b \arcsin(cx)) + \frac{3}{5}c^4d^3x^5(a + b \arcsin(cx)) - \frac{1}{7}c^6d^3x^7(a + b \arcsin(cx))$$

```
output 16/35*b*d^3*(-c^2*x^2+1)^(1/2)/c+8/105*b*d^3*(-c^2*x^2+1)^(3/2)/c+6/175*b*d^3*(-c^2*x^2+1)^(5/2)/c+1/49*b*d^3*(-c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcsin(c*x))-c^2*d^3*x^3*(a+b*arcsin(c*x))+3/5*c^4*d^3*x^5*(a+b*arcsin(c*x))-1/7*c^6*d^3*x^7*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx =$$

$$\frac{d^3(105acx(-35 + 35c^2x^2 - 21c^4x^4 + 5c^6x^6) + b\sqrt{1 - c^2x^2}(-2161 + 757c^2x^2 - 351c^4x^4 + 75c^6x^6) + 3675c}{3675c}$$

input `Integrate[(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/3675*(d^3*(105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcSin[c*x]))/c`

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5154, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx \\
 & \quad \downarrow 5154 \\
 & -bc \int \frac{d^3 x (-5c^6 x^6 + 21c^4 x^4 - 35c^2 x^2 + 35)}{35\sqrt{1 - c^2 x^2}} dx - \frac{1}{7} c^6 d^3 x^7 (a + b \arcsin(cx)) + \frac{3}{5} c^4 d^3 x^5 (a + b \arcsin(cx)) - c^2 d^3 x^3 (a + b \arcsin(cx)) + d^3 x (a + b \arcsin(cx)) \\
 & \quad \downarrow 27 \\
 & -\frac{1}{35} bcd^3 \int \frac{x (-5c^6 x^6 + 21c^4 x^4 - 35c^2 x^2 + 35)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{7} c^6 d^3 x^7 (a + b \arcsin(cx)) + \frac{3}{5} c^4 d^3 x^5 (a + b \arcsin(cx)) - c^2 d^3 x^3 (a + b \arcsin(cx)) + d^3 x (a + b \arcsin(cx)) \\
 & \quad \downarrow 2331 \\
 & -\frac{1}{70} bcd^3 \int \frac{-5c^6 x^6 + 21c^4 x^4 - 35c^2 x^2 + 35}{\sqrt{1 - c^2 x^2}} dx^2 - \frac{1}{7} c^6 d^3 x^7 (a + b \arcsin(cx)) + \frac{3}{5} c^4 d^3 x^5 (a + b \arcsin(cx)) - c^2 d^3 x^3 (a + b \arcsin(cx)) + d^3 x (a + b \arcsin(cx)) \\
 & \quad \downarrow 2389 \\
 & -\frac{1}{70} bcd^3 \int \left(5(1 - c^2 x^2)^{5/2} + 6(1 - c^2 x^2)^{3/2} + 8\sqrt{1 - c^2 x^2} + \frac{16}{\sqrt{1 - c^2 x^2}} \right) dx^2 - \frac{1}{7} c^6 d^3 x^7 (a + b \arcsin(cx)) + \frac{3}{5} c^4 d^3 x^5 (a + b \arcsin(cx)) - c^2 d^3 x^3 (a + b \arcsin(cx)) + d^3 x (a + b \arcsin(cx))
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 2009 \\
 & -\frac{1}{7}c^6d^3x^7(a + b \arcsin(cx)) + \frac{3}{5}c^4d^3x^5(a + b \arcsin(cx)) - c^2d^3x^3(a + b \arcsin(cx)) + d^3x(a + \\
 & \quad b \arcsin(cx)) - \\
 & \quad \frac{1}{70}bcd^3 \left(-\frac{10(1 - c^2x^2)^{7/2}}{7c^2} - \frac{12(1 - c^2x^2)^{5/2}}{5c^2} - \frac{16(1 - c^2x^2)^{3/2}}{3c^2} - \frac{32\sqrt{1 - c^2x^2}}{c^2} \right)
 \end{aligned}$$

input `Int[(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/70*(b*c*d^3*((-32*sqrt[1 - c^2*x^2])/c^2 - (16*(1 - c^2*x^2)^(3/2))/(3*c^2) - (12*(1 - c^2*x^2)^(5/2))/(5*c^2) - (10*(1 - c^2*x^2)^(7/2))/(7*c^2)) + d^3*x*(a + b*ArcSin[c*x]) - c^2*d^3*x^3*(a + b*ArcSin[c*x]) + (3*c^4*d^3*x^5*(a + b*ArcSin[c*x]))/5 - (c^6*d^3*x^7*(a + b*ArcSin[c*x]))/7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2331 `Int[(P_q)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/2 Subst[Int[x^(m - 1)/2*SubstFor[x^2, P_q, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[P_q, x^2] && IntegerQ[(m - 1)/2]`

rule 2389 `Int[(P_q)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Int[ExpandIntegrand[P_q*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[P_q, x] && (IGtQ[p, 0] || EqQ[n, 1])`

rule 5154 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

method	result
parts	$-a d^3 \left(\frac{1}{7} c^6 x^7 - \frac{3}{5} c^4 x^5 + c^2 x^3 - x \right) - \frac{b d^3 \left(\frac{\arcsin(cx) c^7 x^7}{7} - \frac{3 c^5 x^5 \arcsin(cx)}{5} + c^3 x^3 \arcsin(cx) - c x \arcsin(cx) \right)}{c}$
derivativedivides	$\frac{-d^3 a \left(\frac{1}{7} c^7 x^7 - \frac{3}{5} c^5 x^5 + c^3 x^3 - c x \right) - d^3 b \left(\frac{\arcsin(cx) c^7 x^7}{7} - \frac{3 c^5 x^5 \arcsin(cx)}{5} + c^3 x^3 \arcsin(cx) - c x \arcsin(cx) \right) - \frac{2161 \sqrt{-c^2 x^2 + 1}}{3675}}{c}$
default	$\frac{-d^3 a \left(\frac{1}{7} c^7 x^7 - \frac{3}{5} c^5 x^5 + c^3 x^3 - c x \right) - d^3 b \left(\frac{\arcsin(cx) c^7 x^7}{7} - \frac{3 c^5 x^5 \arcsin(cx)}{5} + c^3 x^3 \arcsin(cx) - c x \arcsin(cx) \right) - \frac{2161 \sqrt{-c^2 x^2 + 1}}{3675}}{c}$
orering	$\frac{x(325c^6x^6 - 1437c^4x^4 + 2739c^2x^2 - 5547)(-c^2dx^2 + d)^3(a + b \arcsin(cx))}{1225(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(75c^6x^6 - 351c^4x^4 + 757c^2x^2 - 2161) \left(-\frac{2161 \sqrt{-c^2x^2 + 1}}{3675} \right)}{3675}$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `-a*d^3*(1/7*c^6*x^7-3/5*c^4*x^5+c^2*x^3-x)-b*d^3/c*(1/7*arcsin(c*x)*c^7*x^7-3/5*c^5*x^5*arcsin(c*x)+c^3*x^3*arcsin(c*x)-c*x*arcsin(c*x)-2161/3675*(-c^2*x^2+1)^(1/2)+757/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)+1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.90

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \frac{525 ac^7 d^3 x^7 - 2205 ac^5 d^3 x^5 + 3675 ac^3 d^3 x^3 - 3675 acd^3 x + 105 (5 bc^7 d^3 x^7 - 21 bc^5 d^3 x^5 + 35 bc^3 d^3 x^3 - 35 bc d^3 x) \arcsin(cx) + (75 bc^6 d^3 x^6 - 351 bc^4 d^3 x^4 + 757 bc^2 d^3 x^2 - 2161 b d^3) \sqrt{-c^2 x^2 + 1}}{3675 c}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `-1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 3675*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 - 35*b*c*d^3*x)*arcsin(c*x) + (75*b*c^6*d^3*x^6 - 351*b*c^4*d^3*x^4 + 757*b*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(-c^2*x^2 + 1))/c`

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.26

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} -\frac{ac^6 d^3 x^7}{7} + \frac{3ac^4 d^3 x^5}{5} - ac^2 d^3 x^3 + ad^3 x - \frac{bc^6 d^3 x^7 \arcsin(cx)}{7} - \frac{bc^5 d^3 x^6 \sqrt{-c^2 x^2 + 1}}{49} + \frac{3bc^4 d^3 x^5 \arcsin(cx)}{5} + \frac{117bc^3 d^3 x^4 \sqrt{-c^2 x^2 + 1}}{1225} \\ ad^3 x \end{cases}$$

input `integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x)),x)`

output `Piecewise((-a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 - a*c**2*d**3*x**3 + a*d**3*x - b*c**6*d**3*x**7*asin(c*x)/7 - b*c**5*d**3*x**6*sqrt(-c**2*x**2 + 1)/49 + 3*b*c**4*d**3*x**5*asin(c*x)/5 + 117*b*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)/1225 - b*c**2*d**3*x**3*asin(c*x) - 757*b*c*d**3*x**2*sqrt(-c**2*x**2 + 1)/3675 + b*d**3*x*asin(c*x) + 2161*b*d**3*sqrt(-c**2*x**2 + 1)/(3675*c), Ne(c, 0)), (a*d**3*x, True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.75

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = -\frac{1}{7} ac^6 d^3 x^7 + \frac{3}{5} ac^4 d^3 x^5$$

$$- \frac{1}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) bc^4 d^3 \right.$$

$$+ \frac{1}{25} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bc^4 d^3$$

$$- ac^2 d^3 x^3 - \frac{1}{3} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bc^2 d^3$$

$$+ ad^3 x + \frac{(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1}) bd^3}{c}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
-1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d^3 + a*d^3*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.28

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = -\frac{1}{7} ac^6 d^3 x^7 + \frac{3}{5} ac^4 d^3 x^5 - ac^2 d^3 x^3 - \frac{1}{7} (c^2 x^2 - 1)^3 b d^3 x \arcsin(cx) + \frac{6}{35} (c^2 x^2 - 1)^2 b d^3 x \arcsin(cx) - \frac{8}{35} (c^2 x^2 - 1) b d^3 x \arcsin(cx) - \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b d^3}{49 c} + \frac{16}{35} b d^3 x \arcsin(cx) + \frac{6 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b d^3}{175 c} + a d^3 x + \frac{8 (-c^2 x^2 + 1)^{\frac{3}{2}} b d^3}{105 c} + \frac{16 \sqrt{-c^2 x^2 + 1} b d^3}{35 c}$$

input

```
integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
-1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - a*c^2*d^3*x^3 - 1/7*(c^2*x^2 - 1)^3*b*d^3*x*arcsin(c*x) + 6/35*(c^2*x^2 - 1)^2*b*d^3*x*arcsin(c*x) - 8/35*(c^2*x^2 - 1)*b*d^3*x*arcsin(c*x) - 1/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^3/c + 16/35*b*d^3*x*arcsin(c*x) + 6/175*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^3/c + a*d^3*x + 8/105*(-c^2*x^2 + 1)^(3/2)*b*d^3/c + 16/35*sqrt(-c^2*x^2 + 1)*b*d^3/c
```

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3 dx$$

input `int((a + b*asin(c*x))*(d - c^2*d*x^2)^3,x)`

output `int((a + b*asin(c*x))*(d - c^2*d*x^2)^3, x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{d^3(-525 \operatorname{asin}(cx) b c^7 x^7 + 2205 \operatorname{asin}(cx) b c^5 x^5 - 3675 \operatorname{asin}(cx) b c^3 x^3 + 3675 \operatorname{asin}(cx) b c x - 75 \sqrt{-c^2 x^2 + d})}{(3675 c)}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x)),x)`

output `(d**3*(- 525*asin(c*x)*b*c**7*x**7 + 2205*asin(c*x)*b*c**5*x**5 - 3675*asin(c*x)*b*c**3*x**3 + 3675*asin(c*x)*b*c*x - 75*sqrt(- c**2*x**2 + 1)*b*c**6*x**6 + 351*sqrt(- c**2*x**2 + 1)*b*c**4*x**4 - 757*sqrt(- c**2*x**2 + 1)*b*c**2*x**2 + 2161*sqrt(- c**2*x**2 + 1)*b - 525*a*c**7*x**7 + 2205*a*c**5*x**5 - 3675*a*c**3*x**3 + 3675*a*c*x)/(3675*c)`

3.24 $\int \frac{(d-c^2dx^2)^3(a+b \arcsin(cx))}{x} dx$

Optimal result	392
Mathematica [A] (verified)	393
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Fricas [F]	400
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Maxima [F]	401
Giac [F(-2)]	401
Mupad [F(-1)]	402
Reduce [F]	402

Optimal result

Integrand size = 25, antiderivative size = 235

$$\int \frac{(d - c^2dx^2)^3 (a + b \arcsin(cx))}{x} dx = -\frac{19}{48}bcd^3x\sqrt{1 - c^2x^2} - \frac{7}{72}bcd^3x(1 - c^2x^2)^{3/2} - \frac{1}{36}bcd^3x(1 - c^2x^2)^{5/2} - \frac{19}{48}bd^3 \arcsin(cx) + \frac{1}{2}d^3(1 - c^2x^2)(a + b \arcsin(cx)) + \frac{1}{4}d^3(1 - c^2x^2)^2(a + b \arcsin(cx)) + \frac{1}{6}d^3(1 - c^2x^2)^3(a + b \arcsin(cx)) - \frac{id^3(a + b \arcsin(cx))^2}{2b} + d^3(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) - \frac{1}{2}ibd^3 \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-19/48*b*c*d^3*x*(-c^2*x^2+1)^(1/2)-7/72*b*c*d^3*x*(-c^2*x^2+1)^(3/2)-1/36
*b*c*d^3*x*(-c^2*x^2+1)^(5/2)-19/48*b*d^3*arcsin(c*x)+1/2*d^3*(-c^2*x^2+1)
*(a+b*arcsin(c*x))+1/4*d^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))+1/6*d^3*(-c^2*
x^2+1)^3*(a+b*arcsin(c*x))-1/2*I*d^3*(a+b*arcsin(c*x))^2/b+d^3*(a+b*arcsin
(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d^3*polylog(2,(I*c*x+(-c
^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx = -\frac{1}{144} d^3 \left(216ac^2 x^2 - 108ac^4 x^4 + 24ac^6 x^6 \right. \\ \left. + 75bcx\sqrt{1 - c^2 x^2} - 22bc^3 x^3 \sqrt{1 - c^2 x^2} \right. \\ \left. + 4bc^5 x^5 \sqrt{1 - c^2 x^2} + 72ib \arcsin(cx)^2 \right. \\ \left. - 150b \arctan\left(\frac{cx}{-1 + \sqrt{1 - c^2 x^2}}\right) \right. \\ \left. + 12b \arcsin(cx) (18c^2 x^2 - 9c^4 x^4 + 2c^6 x^6 \right. \\ \left. - 12 \log(1 - e^{2i \arcsin(cx)}) \right) - 144a \log(x) \\ \left. + 72ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right)$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x,x]
```

output

```
-1/144*(d^3*(216*a*c^2*x^2 - 108*a*c^4*x^4 + 24*a*c^6*x^6 + 75*b*c*x*Sqrt[
1 - c^2*x^2] - 22*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 4*b*c^5*x^5*Sqrt[1 - c^2*x
^2] + (72*I)*b*ArcSin[c*x]^2 - 150*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])
] + 12*b*ArcSin[c*x]*(18*c^2*x^2 - 9*c^4*x^4 + 2*c^6*x^6 - 12*Log[1 - E^((
2*I)*ArcSin[c*x])]) - 144*a*Log[x] + (72*I)*b*PolyLog[2, E^((2*I)*ArcSin[c
*x])])))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.43, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {5188, 27, 211, 211, 211, 223, 5188, 211, 211, 223, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx$$

$$\begin{aligned}
& \downarrow 5188 \\
& d \int \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx - \frac{1}{6}bcd^3 \int (1-c^2x^2)^{5/2} dx + \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) \\
& \downarrow 27 \\
& d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx - \frac{1}{6}bcd^3 \int (1-c^2x^2)^{5/2} dx + \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) \\
& \downarrow 211 \\
& d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx - \frac{1}{6}bcd^3 \left(\frac{5}{6} \int (1-c^2x^2)^{3/2} dx + \frac{1}{6}x(1-c^2x^2)^{5/2} \right) + \\
& \quad \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) \\
& \downarrow 211 \\
& d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx - \\
& \frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right) + \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) \\
& \downarrow 211 \\
& d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx - \\
& \frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right) + \\
& \quad \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) \\
& \downarrow 223 \\
& d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx + \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) - \\
& \frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right) \\
& \downarrow 5188
\end{aligned}$$

$$d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{1}{4}bc \int (1-c^2x^2)^{3/2} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) \right) +$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) -$$

$$\frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)$$

↓ 211

$$d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{1}{4}bc \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) \right) +$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) -$$

$$\frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)$$

↓ 211

$$d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{1}{4}bc \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) \right) +$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) -$$

$$\frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)$$

↓ 223

$$d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) \right) +$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) -$$

$$\frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)$$

↓ 5188

$$d^3 \left(\int \frac{a+b\arcsin(cx)}{x} dx - \frac{1}{2}bc \int \sqrt{1-c^2x^2} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right) +$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx)) -$$

$$\frac{1}{6}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)$$

↓ 211

$$d^3 \left(\int \frac{a + b \arcsin(cx)}{x} dx - \frac{1}{2} bc \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1-c^2x^2} \right) + \frac{1}{4} (1-c^2x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1-c^2x^2) (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1-c^2x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1-c^2x^2} \right) + \frac{1}{4} x (1-c^2x^2)^{3/2} \right) + \frac{1}{6} x (1-c^2x^2)^{5/2} \right) \right)$$

↓ 223

$$d^3 \left(\int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{4} (1-c^2x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1-c^2x^2) (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1-c^2x^2} \right) - \frac{1}{6} d^3 (1-c^2x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1-c^2x^2} \right) + \frac{1}{4} x (1-c^2x^2)^{3/2} \right) + \frac{1}{6} x (1-c^2x^2)^{5/2} \right) \right)$$

↓ 5136

$$d^3 \left(\int \frac{\sqrt{1-c^2x^2} (a + b \arcsin(cx))}{cx} d \arcsin(cx) + \frac{1}{4} (1-c^2x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1-c^2x^2) (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1-c^2x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1-c^2x^2} \right) + \frac{1}{4} x (1-c^2x^2)^{3/2} \right) + \frac{1}{6} x (1-c^2x^2)^{5/2} \right) \right)$$

↓ 3042

$$d^3 \left(\int - \left((a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) \right) d \arcsin(cx) + \frac{1}{4} (1-c^2x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1-c^2x^2) (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1-c^2x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1-c^2x^2} \right) + \frac{1}{4} x (1-c^2x^2)^{3/2} \right) + \frac{1}{6} x (1-c^2x^2)^{5/2} \right) \right)$$

↓ 25

$$d^3 \left(- \int (a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{4} (1-c^2x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1-c^2x^2) (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1-c^2x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1-c^2x^2} \right) + \frac{1}{4} x (1-c^2x^2)^{3/2} \right) + \frac{1}{6} x (1-c^2x^2)^{5/2} \right) \right)$$

↓ 4200

$$d^3 \left(2i \int -\frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{4}(1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) \right)$$

↓ 25

$$d^3 \left(-2i \int \frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{4}(1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) \right)$$

↓ 2620

$$d^3 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2} i b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) + \frac{1}{4}(1 - c^2 x^2)^2 (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) \right)$$

↓ 2715

$$d^3 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d e^{2i \arcsin(cx)} \right) + \frac{1}{4}(1 - c^2 x^2)^2 (a + b \arcsin(cx)) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) \right)$$

↓ 2838

$$d^3 \left(\frac{1}{4}(1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2}(1 - c^2 x^2) (a + b \arcsin(cx)) - 2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d e^{2i \arcsin(cx)} \right) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx)) - \frac{1}{6} bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) + \frac{1}{6} x (1 - c^2 x^2)^{5/2} \right) \right)$$

input `Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x,x]`

output `(d^3*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x]))/6 - (b*c*d^3*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/6 + d^3*(((1 - c^2*x^2)*(a + b*ArcSin[c*x]))/2 + ((1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/4 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 - (b*c*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4 - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/4)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 211 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 2620 `Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_)]], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4200 `Int[((c_) + (d_)*(x_)^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5188 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^2)^(p_)/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcSin[c*x])/(2*p)), x] + (Simp[d
Int[(d + e*x^2)^(p - 1)*((a + b*ArcSin[c*x])/x), x], x] - Simp[b*c*(d^p/(2
*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.94

method	result
parts	$-d^3a\left(\frac{c^6x^6}{6} - \frac{3c^4x^4}{4} + \frac{3c^2x^2}{2} - \ln(x)\right) - d^3b\left(\frac{i\arcsin(cx)^2}{2} - \arcsin(cx)\ln(1 - icx - \sqrt{-c^2x^2 - 1})\right)$
derivativedivides	$-d^3a\left(\frac{c^6x^6}{6} - \frac{3c^4x^4}{4} + \frac{3c^2x^2}{2} - \ln(cx)\right) - d^3b\left(\frac{i\arcsin(cx)^2}{2} - \arcsin(cx)\ln(1 - icx - \sqrt{-c^2x^2 - 1})\right)$
default	$-d^3a\left(\frac{c^6x^6}{6} - \frac{3c^4x^4}{4} + \frac{3c^2x^2}{2} - \ln(cx)\right) - d^3b\left(\frac{i\arcsin(cx)^2}{2} - \arcsin(cx)\ln(1 - icx - \sqrt{-c^2x^2 - 1})\right)$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output `-d^3*a*(1/6*c^6*x^6-3/4*c^4*x^4+3/2*c^2*x^2-ln(x))-d^3*b*(1/2*I*arcsin(c*x)^2-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-1/192*arcsin(c*x)*cos(6*arcsin(c*x))+1/1152*sin(6*arcsin(c*x))-1/16*arcsin(c*x)*cos(4*arcsin(c*x))+1/64*sin(4*arcsin(c*x))-29/64*arcsin(c*x)*cos(2*arcsin(c*x))+29/128*sin(2*arcsin(c*x)))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x,x, algorithm="fricas")`

output `integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arcsin(c*x))/x, x)`

Sympy [F]

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx = & -d^3 \left(\int \left(-\frac{a}{x} \right) dx + \int 3ac^2 x dx \right. \\ & + \int (-3ac^4 x^3) dx + \int ac^6 x^5 dx \\ & + \int \left(-\frac{b \operatorname{asin}(cx)}{x} \right) dx + \int 3bc^2 x \operatorname{asin}(cx) dx \\ & + \int (-3bc^4 x^3 \operatorname{asin}(cx)) dx \\ & \left. + \int bc^6 x^5 \operatorname{asin}(cx) dx \right) \end{aligned}$$

input `integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))/x,x)`

output `-d**3*(Integral(-a/x, x) + Integral(3*a*c**2*x, x) + Integral(-3*a*c**4*x**3, x) + Integral(a*c**6*x**5, x) + Integral(-b*asin(c*x)/x, x) + Integral(3*b*c**2*x*asin(c*x), x) + Integral(-3*b*c**4*x**3*asin(c*x), x) + Integral(b*c**6*x**5*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `-1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 - 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) - integrate((b*c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^3}{x} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x,x)`output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x, x)`**Reduce [F]**

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x} dx$$

$$= \frac{d^3 \left(-24 \arcsin(cx) b c^6 x^6 + 108 \arcsin(cx) b c^4 x^4 - 216 \arcsin(cx) b c^2 x^2 + 75 \arcsin(cx) b - 4 \sqrt{-c^2 x^2 + 1} b c^5 x^5 - \dots \right)}{144}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))/x,x)`output `(d**3*(- 24*asin(c*x)*b*c**6*x**6 + 108*asin(c*x)*b*c**4*x**4 - 216*asin(c*x)*b*c**2*x**2 + 75*asin(c*x)*b - 4*sqrt(- c**2*x**2 + 1)*b*c**5*x**5 + 22*sqrt(- c**2*x**2 + 1)*b*c**3*x**3 - 75*sqrt(- c**2*x**2 + 1)*b*c*x + 144*int(asin(c*x)/x,x)*b + 144*log(x)*a - 24*a*c**6*x**6 + 108*a*c**4*x**4 - 216*a*c**2*x**2))/144`

3.25
$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

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Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx = -\frac{11}{5}bcd^3\sqrt{1 - c^2x^2} - \frac{1}{5}bcd^3(1 - c^2x^2)^{3/2} - \frac{1}{25}bcd^3(1 - c^2x^2)^{5/2} - \frac{d^3(a + b \arcsin(cx))}{x} - 3c^2d^3x(a + b \arcsin(cx)) + c^4d^3x^3(a + b \arcsin(cx)) - \frac{1}{5}c^6d^3x^5(a + b \arcsin(cx)) - bcd^3\operatorname{arctanh}(\sqrt{1 - c^2x^2})$$

output

```
-11/5*b*c*d^3*(-c^2*x^2+1)^(1/2)-1/5*b*c*d^3*(-c^2*x^2+1)^(3/2)-1/25*b*c*d^3*(-c^2*x^2+1)^(5/2)-d^3*(a+b*arcsin(c*x))/x-3*c^2*d^3*x*(a+b*arcsin(c*x))+c^4*d^3*x^3*(a+b*arcsin(c*x))-1/5*c^6*d^3*x^5*(a+b*arcsin(c*x))-b*c*d^3*arctanh((-c^2*x^2+1)^(1/2))
```


Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx = \frac{1}{10} d^3 \left(-\frac{10(a + b \arcsin(cx))}{x} - 30c^2 x (a + b \arcsin(cx)) + 10c^4 x^3 (a + b \arcsin(cx)) - 2c^6 x^5 (a + b \arcsin(cx)) - \frac{2}{5} bc \left(\sqrt{1 - c^2 x^2} (61 - 7c^2 x^2 + c^4 x^4) + 25 \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) \right) \right)$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```
(d^3*((-10*(a + b*ArcSin[c*x]))/x - 30*c^2*x*(a + b*ArcSin[c*x]) + 10*c^4*x^3*(a + b*ArcSin[c*x]) - 2*c^6*x^5*(a + b*ArcSin[c*x]) - (2*b*c*(Sqrt[1 - c^2*x^2]*(61 - 7*c^2*x^2 + c^4*x^4) + 25*ArcTanh[Sqrt[1 - c^2*x^2]]))/5))/10
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5192, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

↓ 5192

$$-bc \int -\frac{d^3 (c^6 x^6 - 5c^4 x^4 + 15c^2 x^2 + 5)}{5x\sqrt{1 - c^2 x^2}} dx - \frac{1}{5} c^6 d^3 x^5 (a + b \arcsin(cx)) + c^4 d^3 x^3 (a + b \arcsin(cx)) - 3c^2 d^3 x (a + b \arcsin(cx)) - \frac{d^3 (a + b \arcsin(cx))}{x}$$

$$\begin{aligned}
& \downarrow 27 \\
& \frac{1}{5}bcd^3 \int \frac{c^6x^6 - 5c^4x^4 + 15c^2x^2 + 5}{x\sqrt{1-c^2x^2}} dx - \frac{1}{5}c^6d^3x^5(a + b \arcsin(cx)) + c^4d^3x^3(a + \\
& \quad b \arcsin(cx)) - 3c^2d^3x(a + b \arcsin(cx)) - \frac{d^3(a + b \arcsin(cx))}{x} \\
& \downarrow 2331 \\
& \frac{1}{10}bcd^3 \int \frac{c^6x^6 - 5c^4x^4 + 15c^2x^2 + 5}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{1}{5}c^6d^3x^5(a + b \arcsin(cx)) + c^4d^3x^3(a + \\
& \quad b \arcsin(cx)) - 3c^2d^3x(a + b \arcsin(cx)) - \frac{d^3(a + b \arcsin(cx))}{x} \\
& \downarrow 2123 \\
& \frac{1}{10}bcd^3 \int \left((1-c^2x^2)^{3/2}c^2 + 3\sqrt{1-c^2x^2}c^2 + \frac{11c^2}{\sqrt{1-c^2x^2}} + \frac{5}{x^2\sqrt{1-c^2x^2}} \right) dx^2 - \frac{1}{5}c^6d^3x^5(a + \\
& \quad b \arcsin(cx)) + c^4d^3x^3(a + b \arcsin(cx)) - 3c^2d^3x(a + b \arcsin(cx)) - \frac{d^3(a + b \arcsin(cx))}{x} \\
& \downarrow 2009 \\
& -\frac{1}{5}c^6d^3x^5(a + b \arcsin(cx)) + c^4d^3x^3(a + b \arcsin(cx)) - 3c^2d^3x(a + b \arcsin(cx)) - \\
& \quad \frac{d^3(a + b \arcsin(cx))}{x} + \\
& \frac{1}{10}bcd^3 \left(-10\operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{2}{5}(1-c^2x^2)^{5/2} - 2(1-c^2x^2)^{3/2} - 22\sqrt{1-c^2x^2} \right)
\end{aligned}$$

input

```
Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```
-((d^3*(a + b*ArcSin[c*x]))/x) - 3*c^2*d^3*x*(a + b*ArcSin[c*x]) + c^4*d^3*x^3*(a + b*ArcSin[c*x]) - (c^6*d^3*x^5*(a + b*ArcSin[c*x]))/5 + (b*c*d^3*(-22*sqrt[1 - c^2*x^2] - 2*(1 - c^2*x^2)^(3/2) - (2*(1 - c^2*x^2)^(5/2))/5 - 10*ArcTanh[Sqrt[1 - c^2*x^2]]))/10
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2123 `Int[(Px_)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])`

rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`

rule 5192 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

method	result
parts	$-d^3a\left(\frac{c^6x^5}{5} - c^4x^3 + 3c^2x + \frac{1}{x}\right) - d^3bc\left(\frac{c^5x^5 \arcsin(cx)}{5} - c^3x^3 \arcsin(cx) + 3cx \arcsin(cx)\right)$
derivativedivides	$c\left(-d^3a\left(\frac{c^5x^5}{5} - c^3x^3 + 3cx + \frac{1}{cx}\right) - d^3b\left(\frac{c^5x^5 \arcsin(cx)}{5} - c^3x^3 \arcsin(cx) + 3cx \arcsin(cx)\right)\right)$
default	$c\left(-d^3a\left(\frac{c^5x^5}{5} - c^3x^3 + 3cx + \frac{1}{cx}\right) - d^3b\left(\frac{c^5x^5 \arcsin(cx)}{5} - c^3x^3 \arcsin(cx) + 3cx \arcsin(cx)\right)\right)$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)`

output

```
-d^3*a*(1/5*c^6*x^5-c^4*x^3+3*c^2*x+1/x)-d^3*b*c*(1/5*c^5*x^5*arcsin(c*x)-
c^3*x^3*arcsin(c*x)+3*c*x*arcsin(c*x)+arcsin(c*x)/c/x+1/25*c^4*x^4*(-c^2*x
^2+1)^(1/2)-7/25*c^2*x^2*(-c^2*x^2+1)^(1/2)+61/25*(-c^2*x^2+1)^(1/2)+arcta
nh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.15

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx =$$

$$\frac{10 ac^6 d^3 x^6 - 50 ac^4 d^3 x^4 + 150 ac^2 d^3 x^2 + 25 bcd^3 x \log(\sqrt{-c^2 x^2 + 1} + 1) - 25 bcd^3 x \log(\sqrt{-c^2 x^2 + 1} - 1)}{x^2}$$

input

```
integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")
```

output

```
-1/50*(10*a*c^6*d^3*x^6 - 50*a*c^4*d^3*x^4 + 150*a*c^2*d^3*x^2 + 25*b*c*d^
3*x*log(sqrt(-c^2*x^2 + 1) + 1) - 25*b*c*d^3*x*log(sqrt(-c^2*x^2 + 1) - 1)
+ 50*a*d^3 + 10*(b*c^6*d^3*x^6 - 5*b*c^4*d^3*x^4 + 15*b*c^2*d^3*x^2 + 5*b
*d^3)*arcsin(c*x) + 2*(b*c^5*d^3*x^5 - 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sq
rt(-c^2*x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.77

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx \\
&= -\frac{ac^6 d^3 x^5}{5} + ac^4 d^3 x^3 - 3ac^2 d^3 x - \frac{ad^3}{x} \\
&\quad + \frac{bc^7 d^3 \left(\begin{cases} -\frac{x^4 \sqrt{-c^2 x^2 + 1}}{5c^2} - \frac{4x^2 \sqrt{-c^2 x^2 + 1}}{15c^4} - \frac{8\sqrt{-c^2 x^2 + 1}}{15c^6} & \text{for } c^2 \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases} \right)}{5} \\
&\quad - \frac{bc^6 d^3 x^5 \arcsin(cx)}{5} - bc^5 d^3 \left(\begin{cases} -\frac{x^2 \sqrt{-c^2 x^2 + 1}}{3c^2} - \frac{2\sqrt{-c^2 x^2 + 1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right) \\
&\quad + bc^4 d^3 x^3 \arcsin(cx) - 3bc^2 d^3 \left(\begin{cases} 0 & \text{for } c = 0 \\ x \arcsin(cx) + \frac{\sqrt{-c^2 x^2 + 1}}{c} & \text{otherwise} \end{cases} \right) \\
&\quad + bcd^3 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2 x^2}\right| > 1 \\ i \arcsin\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{bd^3 \arcsin(cx)}{x}
\end{aligned}$$

input

```
integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))/x**2,x)
```

output

```
-a*c**6*d**3*x**5/5 + a*c**4*d**3*x**3 - 3*a*c**2*d**3*x - a*d**3/x + b*c*
*7*d**3*Piecewise((-x**4*sqrt(-c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(-c**2
*x**2 + 1)/(15*c**4) - 8*sqrt(-c**2*x**2 + 1)/(15*c**6), Ne(c**2, 0)), (x*
*6/6, True))/5 - b*c**6*d**3*x**5*asin(c*x)/5 - b*c**5*d**3*Piecewise((-x*
*2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c**
2, 0)), (x**4/4, True)) + b*c**4*d**3*x**3*asin(c*x) - 3*b*c**2*d**3*Piece
wise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True)) + b*c*d*
*3*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), Tr
ue)) - b*d**3*asin(c*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.52

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx = -\frac{1}{5} ac^6 d^3 x^5 - \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bc^6 d^3 + ac^4 d^3 x^3 + \frac{1}{3} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bc^4 d^3 - 3 ac^2 d^3 x - 3 \left(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1} \right) bcd^3 - \left(c \log \left(\frac{2 \sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd^3 - \frac{ad^3}{x}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `-1/5*a*c^6*d^3*x^5 - 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^6*d^3 + a*c^4*d^3*x^3 + 1/3*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^4*d^3 - 3*a*c^2*d^3*x - 3*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*c*d^3 - (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d^3 - a*d^3/x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5513 vs. 2(148) = 296.

Time = 28.83 (sec) , antiderivative size = 5513, normalized size of antiderivative = 33.62

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx = \text{Too large to display}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output

```
-1/2*b*c^13*d^3*x^12*arcsin(c*x)/((c^11*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 +
5*c^9*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)
)^7 + 10*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^3*x^3/(sqrt(-c^2*x^2 + 1)
+ 1)^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^12) - 1/
2*a*c^13*d^3*x^12/((c^11*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 + 5*c^9*x^9/(sqr
t(-c^2*x^2 + 1) + 1)^9 + 10*c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 10*c^5*x^
5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/
(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^12) + b*c^12*d^3*x^11*1
og(abs(c)*abs(x))/((c^11*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 + 5*c^9*x^9/(sqr
t(-c^2*x^2 + 1) + 1)^9 + 10*c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 10*c^5*x^
5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/
(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^11) - b*c^12*d^3*x^11*1
og(sqrt(-c^2*x^2 + 1) + 1)/((c^11*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 + 5*c^9
*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 +
10*c^5*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)
^3 + c*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^11) + 61/25*b*
c^12*d^3*x^11/((c^11*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 + 5*c^9*x^9/(sqrt(-c
^2*x^2 + 1) + 1)^9 + 10*c^7*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 10*c^5*x^5/(s
qrt(-c^2*x^2 + 1) + 1)^5 + 5*c^3*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c*x/(sqr
t(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^11) - 9*b*c^11*d^3*x^10*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx = \int \frac{(a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3}{x^2} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x^2,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{d^3 \left(-5 \operatorname{asin}(cx) b c^6 x^6 + 25 \operatorname{asin}(cx) b c^4 x^4 - 75 \operatorname{asin}(cx) b c^2 x^2 - 25 \operatorname{asin}(cx) b - \sqrt{-c^2 x^2 + 1} b c^5 x^5 + 7 \sqrt{-c^2 x^2 + 1} b c^3 x^3 - 61 \sqrt{-c^2 x^2 + 1} b c x + 25 \log(\tan(\operatorname{asin}(cx)/2)) b c x - 5 a c^6 x^6 + 25 a c^4 x^4 - 75 a c^2 x^2 - 25 a \right)}{25 x}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))/x^2,x)`

output

```
(d**3*( - 5*asin(c*x)*b*c**6*x**6 + 25*asin(c*x)*b*c**4*x**4 - 75*asin(c*x)
)*b*c**2*x**2 - 25*asin(c*x)*b - sqrt( - c**2*x**2 + 1)*b*c**5*x**5 + 7*sq
rt( - c**2*x**2 + 1)*b*c**3*x**3 - 61*sqrt( - c**2*x**2 + 1)*b*c*x + 25*lo
g(tan(asin(c*x)/2))*b*c*x - 5*a*c**6*x**6 + 25*a*c**4*x**4 - 75*a*c**2*x**
2 - 25*a))/(25*x)
```


$$3.26 \quad \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx$$

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Optimal result

Integrand size = 25, antiderivative size = 263

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx = & \frac{3}{32} bc^3 d^3 x \sqrt{1 - c^2 x^2} - \frac{7}{16} bc^3 d^3 x (1 - c^2 x^2)^{3/2} \\ & - \frac{bcd^3 (1 - c^2 x^2)^{5/2}}{2x} + \frac{3}{32} bc^2 d^3 \arcsin(cx) \\ & - \frac{3}{2} c^2 d^3 (1 - c^2 x^2) (a + b \arcsin(cx)) \\ & - \frac{3}{4} c^2 d^3 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) \\ & - \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} \\ & + \frac{3ic^2 d^3 (a + b \arcsin(cx))^2}{2b} \\ & - 3c^2 d^3 (a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) \\ & + \frac{3}{2} ibc^2 d^3 \text{PolyLog}(2, e^{2i \arcsin(cx)}) \end{aligned}$$

output

```
3/32*b*c^3*d^3*x*(-c^2*x^2+1)^(1/2)-7/16*b*c^3*d^3*x*(-c^2*x^2+1)^(3/2)-1/
2*b*c*d^3*(-c^2*x^2+1)^(5/2)/x+3/32*b*c^2*d^3*arcsin(c*x)-3/2*c^2*d^3*(-c^
2*x^2+1)*(a+b*arcsin(c*x))-3/4*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))-1/
2*d^3*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))/x^2+3/2*I*c^2*d^3*(a+b*arcsin(c*x))
^2/b-3*c^2*d^3*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+3/2*I*
b*c^2*d^3*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.86

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx =$$

$$\frac{d^3 \left(16a - 48ac^4 x^4 + 8ac^6 x^6 + 16bcx\sqrt{1 - c^2 x^2} - 21bc^3 x^3 \sqrt{1 - c^2 x^2} + 2bc^5 x^5 \sqrt{1 - c^2 x^2} - 48ibc^2 x^2 \arcsin(cx) \right)}{x^3}$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/32*(d^3*(16*a - 48*a*c^4*x^4 + 8*a*c^6*x^6 + 16*b*c*x*Sqrt[1 - c^2*x^2]
- 21*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*c^5*x^5*Sqrt[1 - c^2*x^2] - (48*I)
*b*c^2*x^2*ArcSin[c*x]^2 + 42*b*c^2*x^2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^
2])]) + 8*b*ArcSin[c*x]*(2 - 6*c^4*x^4 + c^6*x^6 + 12*c^2*x^2*Log[1 - E^((2
*I)*ArcSin[c*x])]) + 96*a*c^2*x^2*Log[x] - (48*I)*b*c^2*x^2*PolyLog[2, E^((
2*I)*ArcSin[c*x])]))/x^2
```

Rubi [A] (verified)Time = 1.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.30, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {5190, 27, 247, 211, 211, 223, 5188, 211, 211, 223, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx \\
& \quad \downarrow \text{5190} \\
& -3c^2 d \int \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx + \frac{1}{2} bcd^3 \int \frac{(1 - c^2 x^2)^{5/2}}{x^2} dx - \\
& \quad \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{27} \\
& -3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx + \frac{1}{2} bcd^3 \int \frac{(1 - c^2 x^2)^{5/2}}{x^2} dx - \\
& \quad \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{247} \\
& -3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx + \\
& \frac{1}{2} bcd^3 \left(-5c^2 \int (1 - c^2 x^2)^{3/2} dx - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) - \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{211} \\
& -3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx + \\
& \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} dx + \frac{1}{4} x(1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) - \\
& \quad \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{211} \\
& -3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx + \\
& \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x(1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) - \\
& \quad \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{223}
\end{aligned}$$

$$-3c^2d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) - \frac{(1-c^2x^2)^{5/2}}{x} \right)$$

↓ 5188

$$-3c^2d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{1}{4}bc \int (1-c^2x^2)^{3/2} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) - \frac{(1-c^2x^2)^{5/2}}{x} \right)$$

↓ 211

$$-3c^2d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{1}{4}bc \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) - \frac{(1-c^2x^2)^{5/2}}{x} \right)$$

↓ 211

$$-3c^2d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx - \frac{1}{4}bc \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) - \frac{(1-c^2x^2)^{5/2}}{x} \right)$$

↓ 223

$$-3c^2d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) \right) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) - \frac{(1-c^2x^2)^{5/2}}{x} \right)$$

↓ 5188

$$-3c^2 d^3 \left(\int \frac{a + b \arcsin(cx)}{x} dx - \frac{1}{2} bc \int \sqrt{1 - c^2 x^2} dx + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\ \left. + \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) \right)$$

↓ 211

$$-3c^2 d^3 \left(\int \frac{a + b \arcsin(cx)}{x} dx - \frac{1}{2} bc \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\ \left. + \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) \right)$$

↓ 223

$$-3c^2 d^3 \left(\int \frac{a + b \arcsin(cx)}{x} dx + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) \right. \\ \left. + \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) \right)$$

↓ 5136

$$-3c^2 d^3 \left(\int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{cx} d \arcsin(cx) + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\ \left. + \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right) \right)$$

↓ 3042

$$\begin{aligned}
& -3c^2 d^3 \left(\int - \left((a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) \right) d \arcsin(cx) + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) \right) \\
& \quad \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right)
\end{aligned}$$

↓ 25

$$\begin{aligned}
& -3c^2 d^3 \left(- \int (a + b \arcsin(cx)) \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) \right) \\
& \quad \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right)
\end{aligned}$$

↓ 4200

$$\begin{aligned}
& -3c^2 d^3 \left(2i \int - \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right) \\
& \quad \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right)
\end{aligned}$$

↓ 25

$$\begin{aligned}
& -3c^2 d^3 \left(-2i \int \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right) \\
& \quad \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))}{2x^2} + \\
& \quad \frac{1}{2} bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) + \frac{1}{4} x (1 - c^2 x^2)^{3/2} \right) - \frac{(1 - c^2 x^2)^{5/2}}{x} \right)
\end{aligned}$$

↓ 2620

$$-3c^2d^3\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arcsin(cx)}\right)d\arcsin(cx)\right)+\frac{1}{4}(1-c^2x^2)^{5/2}/x\right) + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^{5/2}}{x}\right)$$

↓ 2715

$$-3c^2d^3\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx))-\frac{1}{4}b\int e^{-2i\arcsin(cx)}\log\left(1-e^{2i\arcsin(cx)}\right)de^{2i\arcsin(cx)}\right)+\frac{1}{4}(1-c^2x^2)^{5/2}/x\right) + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^{5/2}}{x}\right)$$

↓ 2838

$$-3c^2d^3\left(\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx))\right)\right) + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^{5/2}}{x}\right)$$

input

```
Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/2*(d^3*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x]))/x^2 + (b*c*d^3*(-((1 - c^2*x^2)^(5/2)/x) - 5*c^2*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4)))/2 - 3*c^2*d^3(((1 - c^2*x^2)*(a + b*ArcSin[c*x]))/2 + ((1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/4 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 - (b*c*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4 - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])]) + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4)
```

Defintions of rubi rules used

- rule 25 $\text{Int}[-(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[\text{Fx}, \text{x}], \text{x}]$
- rule 27 $\text{Int}[(\text{a}_)*(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{a} \quad \text{Int}[\text{Fx}, \text{x}], \text{x}] \text{ ; FreeQ}[\text{a}, \text{x}] \ \&\& \ \text{!MatchQ}[\text{Fx}, (\text{b}_)*(\text{Gx}_)] \text{ ; FreeQ}[\text{b}, \text{x}]$
- rule 211 $\text{Int}[(\text{a}_) + (\text{b}_)*(x_)^2)^{(\text{p}_)}, \text{x_Symbol}] \rightarrow \text{Simp}[\text{x}*((\text{a} + \text{b}*x^2)^{\text{p}}/(2*\text{p} + 1)), \text{x}] + \text{Simp}[2*\text{a}*(\text{p}/(2*\text{p} + 1)) \quad \text{Int}[(\text{a} + \text{b}*x^2)^{(\text{p} - 1)}, \text{x}], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{p}, 0] \ \&\& \ (\text{IntegerQ}[4*\text{p}] \ || \ \text{IntegerQ}[6*\text{p}])$
- rule 223 $\text{Int}[1/\text{Sqrt}[(\text{a}_) + (\text{b}_)*(x_)^2], \text{x_Symbol}] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-\text{b}, 2]*(\text{x}/\text{Sqrt}[\text{a}])]/\text{Rt}[-\text{b}, 2], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{a}, 0] \ \&\& \ \text{NegQ}[\text{b}]$
- rule 247 $\text{Int}[(\text{c}_)*(x_)^{(\text{m}_)}*((\text{a}_) + (\text{b}_)*(x_)^2)^{(\text{p}_)}, \text{x_Symbol}] \rightarrow \text{Simp}[(\text{c}*x)^{(\text{m} + 1)}*((\text{a} + \text{b}*x^2)^{\text{p}}/(\text{c}*(\text{m} + 1))), \text{x}] - \text{Simp}[2*\text{b}*(\text{p}/(\text{c}^2*(\text{m} + 1))) \quad \text{Int}[(\text{c}*x)^{(\text{m} + 2)}*(\text{a} + \text{b}*x^2)^{(\text{p} - 1)}, \text{x}], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{p}, 0] \ \&\& \ \text{LtQ}[\text{m}, -1] \ \&\& \ \text{!IntegerQ}[(\text{m} + 2*\text{p} + 3)/2, 0] \ \&\& \ \text{IntBinomialQ}[\text{a}, \text{b}, \text{c}, 2, \text{m}, \text{p}, \text{x}]$
- rule 2620 $\text{Int}[(\text{F}_)^{(\text{g}_)*(\text{e}_) + (\text{f}_)*(x_))^{(\text{n}_)}*((\text{c}_) + (\text{d}_)*(x_))^{(\text{m}_)}]/((\text{a}_) + (\text{b}_)*(\text{F}_)^{(\text{g}_)*(\text{e}_) + (\text{f}_)*(x_))^{(\text{n}_)}), \text{x_Symbol}] \rightarrow \text{Simp}[(\text{c} + \text{d}*x)^{\text{m}}/(\text{b}*f*g*n*\text{Log}[\text{F}]))*\text{Log}[1 + \text{b}*((\text{F}^{\text{g}}(\text{e} + \text{f}*x))^{\text{n}}/\text{a})], \text{x}] - \text{Simp}[\text{d}*(\text{m}/(\text{b}*f*g*n*\text{Log}[\text{F}])) \quad \text{Int}[(\text{c} + \text{d}*x)^{(\text{m} - 1)}*\text{Log}[1 + \text{b}*((\text{F}^{\text{g}}(\text{e} + \text{f}*x))^{\text{n}}/\text{a})], \text{x}], \text{x}] \text{ ; FreeQ}[\{\text{F}, \text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{g}, \text{n}\}, \text{x}] \ \&\& \ \text{IGtQ}[\text{m}, 0]$
- rule 2715 $\text{Int}[\text{Log}[(\text{a}_) + (\text{b}_)*(\text{F}_)^{(\text{e}_)*(\text{c}_) + (\text{d}_)*(x_))^{(\text{n}_)}], \text{x_Symbol}] \rightarrow \text{Simp}[1/(\text{d}*e*n*\text{Log}[\text{F}]) \quad \text{Subst}[\text{Int}[\text{Log}[\text{a} + \text{b}*x]/x, \text{x}], \text{x}, (\text{F}^{\text{e}}(\text{c} + \text{d}*x))^{\text{n}}], \text{x}] \text{ ; FreeQ}[\{\text{F}, \text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{n}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{a}, 0]$
- rule 2838 $\text{Int}[\text{Log}[(\text{c}_)*(\text{d}_) + (\text{e}_)*(x_)^{(\text{n}_)}]/(x_), \text{x_Symbol}] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-\text{c})*e*x^{\text{n}}/\text{n}, \text{x}] \text{ ; FreeQ}[\{\text{c}, \text{d}, \text{e}, \text{n}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c*d}, 1]$

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))]], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n * Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5188 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.)/(x_), x_Symbol] := Simp[(d + e*x^2)^p*((a + b*ArcSin[c*x])/(2*p)), x] + (Simp[d Int[(d + e*x^2)^(p - 1)*((a + b*ArcSin[c*x])/x), x], x] - Simp[b*c*(d^p/(2*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

rule 5190 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1))) Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]`

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.11

method	result
parts	$-d^3 a \left(\frac{c^6 x^4}{4} - \frac{3c^4 x^2}{2} + 3c^2 \ln(x) + \frac{1}{2x^2} \right) - d^3 b c^2 \left(-\frac{3i \arcsin(cx)^2}{2} - \frac{5(i+2 \arcsin(cx))(-2i\sqrt{-c^2 x^2 + 1})}{32} \right)$
derivativedivides	$c^2 \left(-d^3 a \left(\frac{c^4 x^4}{4} - \frac{3c^2 x^2}{2} + 3 \ln(cx) + \frac{1}{2c^2 x^2} \right) + \frac{3id^3 b \arcsin(cx)^2}{2} + \frac{5bc d^3 x \sqrt{-c^2 x^2 + 1}}{8} + \frac{5d^3 b \arcsin(cx)}{4} \right)$
default	$c^2 \left(-d^3 a \left(\frac{c^4 x^4}{4} - \frac{3c^2 x^2}{2} + 3 \ln(cx) + \frac{1}{2c^2 x^2} \right) + \frac{3id^3 b \arcsin(cx)^2}{2} + \frac{5bc d^3 x \sqrt{-c^2 x^2 + 1}}{8} + \frac{5d^3 b \arcsin(cx)}{4} \right)$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)`

output `-d^3*a*(1/4*c^6*x^4-3/2*c^4*x^2+3*c^2*ln(x)+1/2/x^2)-d^3*b*c^2*(-3/2*I*arcsin(c*x)^2-5/32*(I+2*arcsin(c*x))*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)-5/32*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(-I+2*arcsin(c*x))+1/2*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+3*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+3*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-3*I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-3*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+1/32*arcsin(c*x)*cos(4*arcsin(c*x))-1/128*sin(4*arcsin(c*x))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")`

output `integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arcsin(c*x))/x^3, x)`

Sympy [F]

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx = & -d^3 \left(\int \left(-\frac{a}{x^3} \right) dx + \int \frac{3ac^2}{x} dx \right. \\ & + \int (-3ac^4 x) dx + \int ac^6 x^3 dx \\ & + \int \left(-\frac{b \arcsin(cx)}{x^3} \right) dx + \int \frac{3bc^2 \arcsin(cx)}{x} dx \\ & + \int (-3bc^4 x \arcsin(cx)) dx \\ & \left. + \int bc^6 x^3 \arcsin(cx) dx \right) \end{aligned}$$

input `integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))/x**3,x)`

output `-d**3*(Integral(-a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(-3*a*c**4*x, x) + Integral(a*c**6*x**3, x) + Integral(-b*asin(c*x)/x**3, x) + Integral(3*b*c**2*asin(c*x)/x, x) + Integral(-3*b*c**4*x*asin(c*x), x) + Integral(b*c**6*x**3*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `-1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 - 3*a*c^2*d^3*log(x) - 1/2*b*d^3*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) - 1/2*a*d^3/x^2 - integrate((b*c^6*d^3*x^4 - 3*b*c^4*d^3*x^2 + 3*b*c^2*d^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^3}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x^3,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{d^3 \left(-8 \arcsin(cx) b c^6 x^6 + 48 \arcsin(cx) b c^4 x^4 - 21 \arcsin(cx) b c^2 x^2 - 16 \arcsin(cx) b - 2 \sqrt{-c^2 x^2 + 1} b c^5 x^5 + 21 \sqrt{-c^2 x^2 + 1} b c^3 x^3 - 16 \sqrt{-c^2 x^2 + 1} b c x - 96 \int \arcsin(cx)/x, x \right) b c^2 x^2 - 96 \log(x) a c^2 x^2 - 8 a c^6 x^6 + 48 a c^4 x^4 - 16 a}{(32 x^2)}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))/x^3,x)`

output `(d**3*(- 8*asin(c*x)*b*c**6*x**6 + 48*asin(c*x)*b*c**4*x**4 - 21*asin(c*x)*b*c**2*x**2 - 16*asin(c*x)*b - 2*sqrt(-c**2*x**2 + 1)*b*c**5*x**5 + 21*sqrt(-c**2*x**2 + 1)*b*c**3*x**3 - 16*sqrt(-c**2*x**2 + 1)*b*c*x - 96*int(asin(c*x)/x,x)*b*c**2*x**2 - 96*log(x)*a*c**2*x**2 - 8*a*c**6*x**6 + 48*a*c**4*x**4 - 16*a))/(32*x**2)`

$$3.27 \quad \int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))}{x^4} dx$$

Optimal result	424
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Giac [F(-1)]	431
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Reduce [B] (verification not implemented)	432

Optimal result

Integrand size = 25, antiderivative size = 178

$$\begin{aligned} \int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))}{x^4} dx = & \frac{8}{3} bc^3 d^3 \sqrt{1-c^2 x^2} - \frac{bcd^3 \sqrt{1-c^2 x^2}}{6x^2} \\ & + \frac{1}{9} bc^3 d^3 (1-c^2 x^2)^{3/2} - \frac{d^3 (a+b \arcsin(cx))}{3x^3} \\ & + \frac{3c^2 d^3 (a+b \arcsin(cx))}{x} \\ & + 3c^4 d^3 x (a+b \arcsin(cx)) \\ & - \frac{1}{3} c^6 d^3 x^3 (a+b \arcsin(cx)) \\ & + \frac{17}{6} bc^3 d^3 \operatorname{arctanh}(\sqrt{1-c^2 x^2}) \end{aligned}$$

output

```
8/3*b*c^3*d^3*(-c^2*x^2+1)^(1/2)-1/6*b*c*d^3*(-c^2*x^2+1)^(1/2)/x^2+1/9*b*
c^3*d^3*(-c^2*x^2+1)^(3/2)-1/3*d^3*(a+b*arcsin(c*x))/x^3+3*c^2*d^3*(a+b*ar
csin(c*x))/x+3*c^4*d^3*x*(a+b*arcsin(c*x))-1/3*c^6*d^3*x^3*(a+b*arcsin(c*x
))+17/6*b*c^3*d^3*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.91

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{d^3(-6a + 54ac^2x^2 + 54ac^4x^4 - 6ac^6x^6 - 3bcx\sqrt{1 - c^2x^2} + 50bc^3x^3\sqrt{1 - c^2x^2} - 2bc^5x^5\sqrt{1 - c^2x^2} - 6b($$

$$18x^3$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x^4,x]
```

output

```
(d^3*(-6*a + 54*a*c^2*x^2 + 54*a*c^4*x^4 - 6*a*c^6*x^6 - 3*b*c*x*Sqrt[1 -
c^2*x^2] + 50*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 2*b*c^5*x^5*Sqrt[1 - c^2*x^2]
- 6*b*(1 - 9*c^2*x^2 - 9*c^4*x^4 + c^6*x^6)*ArcSin[c*x] + 51*b*c^3*x^3*Arc
Tanh[Sqrt[1 - c^2*x^2]]))/(18*x^3)
```

Rubi [A] (warning: unable to verify)Time = 0.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5192, 27, 2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$\downarrow 5192$$

$$-bc \int -\frac{d^3(c^6 x^6 - 9c^4 x^4 - 9c^2 x^2 + 1)}{3x^3 \sqrt{1 - c^2 x^2}} dx - \frac{1}{3}c^6 d^3 x^3 (a + b \arcsin(cx)) + 3c^4 d^3 x (a +$$

$$b \arcsin(cx)) + \frac{3c^2 d^3 (a + b \arcsin(cx))}{x} - \frac{d^3 (a + b \arcsin(cx))}{3x^3}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{1}{3}bcd^3 \int \frac{c^6x^6 - 9c^4x^4 - 9c^2x^2 + 1}{x^3\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + 3c^4d^3x(a + b \arcsin(cx)) + \\
& \quad \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{2331} \\
& \frac{1}{6}bcd^3 \int \frac{c^6x^6 - 9c^4x^4 - 9c^2x^2 + 1}{x^4\sqrt{1-c^2x^2}} dx^2 - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + 3c^4d^3x(a + \\
& \quad b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{2124} \\
& \frac{1}{6}bcd^3 \left(- \int \frac{-2x^4c^6 + 18x^2c^4 + 17c^2}{2x^2\sqrt{1-c^2x^2}} dx^2 - \frac{\sqrt{1-c^2x^2}}{x^2} \right) - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + \\
& \quad 3c^4d^3x(a + b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{27} \\
& \frac{1}{6}bcd^3 \left(- \frac{1}{2} \int \frac{-2x^4c^6 + 18x^2c^4 + 17c^2}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{\sqrt{1-c^2x^2}}{x^2} \right) - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + \\
& \quad 3c^4d^3x(a + b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{1192} \\
& \frac{1}{6}bcd^3 \left(- \frac{\int -\frac{2c^6x^8 - 14c^6x^4 + 33c^6}{1-x^4} d\sqrt{1-c^2x^2}}{c^4} - \frac{\sqrt{1-c^2x^2}}{x^2} \right) - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + \\
& \quad 3c^4d^3x(a + b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{25} \\
& \frac{1}{6}bcd^3 \left(\frac{\int \frac{-2c^6x^8 - 14c^6x^4 + 33c^6}{1-x^4} d\sqrt{1-c^2x^2}}{c^4} - \frac{\sqrt{1-c^2x^2}}{x^2} \right) - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + \\
& \quad 3c^4d^3x(a + b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{1467} \\
& \frac{1}{6}bcd^3 \left(\frac{\int \left(2x^4c^6 + \frac{17c^6}{1-x^4} + 16c^6 \right) d\sqrt{1-c^2x^2}}{c^4} - \frac{\sqrt{1-c^2x^2}}{x^2} \right) - \frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + \\
& \quad 3c^4d^3x(a + b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{2009}
\end{aligned}$$

$$-\frac{1}{3}c^6d^3x^3(a + b \arcsin(cx)) + 3c^4d^3x(a + b \arcsin(cx)) + \frac{3c^2d^3(a + b \arcsin(cx))}{x} - \frac{d^3(a + b \arcsin(cx))}{3x^3} + \frac{1}{6}bcd^3 \left(-\frac{-17c^6 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{2}{3}c^6x^6 - 16c^6\sqrt{1-c^2x^2}}{c^4} - \frac{\sqrt{1-c^2x^2}}{x^2} \right)$$

input `Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(d^3*(a + b*ArcSin[c*x]))/x^3 + (3*c^2*d^3*(a + b*ArcSin[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcSin[c*x]) - (c^6*d^3*x^3*(a + b*ArcSin[c*x]))/3 + (b*c*d^3*(-(Sqrt[1 - c^2*x^2]/x^2) - ((-2*c^6*x^6)/3 - 16*c^6*Sqrt[1 - c^2*x^2] - 17*c^6*ArcTanh[Sqrt[1 - c^2*x^2]])/c^4))/6`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 1192 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[2/e^(n + 2*p + 1) Subst[Int[x^(2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]`

rule 1467 `Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2124 `Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c - a*d))), x] + Simp[1/((m + 1)*(b*c - a*d)) Int[(a + b*x)^(m + 1)*(c + d*x)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] || ! ILtQ[n, -1])`

rule 2331 `Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`

rule 5192 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

method	result
parts	$-d^3a\left(\frac{c^6x^3}{3} - 3c^4x + \frac{1}{3x^3} - \frac{3c^2}{x}\right) - d^3bc^3\left(\frac{c^3x^3 \arcsin(cx)}{3} - 3cx \arcsin(cx) + \frac{\arcsin(cx)}{3c^3x^3} - \dots\right)$
derivativedivides	$c^3\left(-d^3a\left(\frac{c^3x^3}{3} - 3cx + \frac{1}{3c^3x^3} - \frac{3}{cx}\right) - d^3b\left(\frac{c^3x^3 \arcsin(cx)}{3} - 3cx \arcsin(cx) + \frac{\arcsin(cx)}{3c^3x^3} - \dots\right)\right)$
default	$c^3\left(-d^3a\left(\frac{c^3x^3}{3} - 3cx + \frac{1}{3c^3x^3} - \frac{3}{cx}\right) - d^3b\left(\frac{c^3x^3 \arcsin(cx)}{3} - 3cx \arcsin(cx) + \frac{\arcsin(cx)}{3c^3x^3} - \dots\right)\right)$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)`

output

```
-d^3*a*(1/3*c^6*x^3-3*c^4*x+1/3/x^3-3*c^2/x)-d^3*b*c^3*(1/3*c^3*x^3*arcsin
(c*x)-3*c*x*arcsin(c*x)+1/3*arcsin(c*x)/c^3/x^3-3*arcsin(c*x)/c/x+1/6/c^2/
x^2*(-c^2*x^2+1)^(1/2)-17/6*arctanh(1/(-c^2*x^2+1)^(1/2))+1/9*c^2*x^2*(-c^
2*x^2+1)^(1/2)-25/9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.10

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx =$$

$$\frac{12 ac^6 d^3 x^6 - 108 ac^4 d^3 x^4 - 51 bc^3 d^3 x^3 \log(\sqrt{-c^2 x^2 + 1} + 1) + 51 bc^3 d^3 x^3 \log(\sqrt{-c^2 x^2 + 1} - 1) - 108 a^2 d^3 x^2 + 12 a d^3 + 12 (b^2 c^6 d^3 x^6 - 9 b^2 c^4 d^3 x^4 - 9 b^2 c^2 d^3 x^2 + b^2 d^3) \arcsin(cx) + 2 (2 b^2 c^5 d^3 x^5 - 50 b^2 c^3 d^3 x^3 + 3 b^2 c d^3 x) \sqrt{-c^2 x^2 + 1}}{x^3}$$

input

```
integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")
```

output

```
-1/36*(12*a*c^6*d^3*x^6 - 108*a*c^4*d^3*x^4 - 51*b*c^3*d^3*x^3*log(sqrt(-c
^2*x^2 + 1) + 1) + 51*b*c^3*d^3*x^3*log(sqrt(-c^2*x^2 + 1) - 1) - 108*a*c^
2*d^3*x^2 + 12*a*d^3 + 12*(b*c^6*d^3*x^6 - 9*b*c^4*d^3*x^4 - 9*b*c^2*d^3*x
^2 + b*d^3)*arcsin(c*x) + 2*(2*b*c^5*d^3*x^5 - 50*b*c^3*d^3*x^3 + 3*b*c*d^
3*x)*sqrt(-c^2*x^2 + 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.83

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx \\
&= -\frac{ac^6 d^3 x^3}{3} + 3ac^4 d^3 x + \frac{3ac^2 d^3}{x} - \frac{ad^3}{3x^3} \\
&\quad + \frac{bc^7 d^3 \left(\begin{cases} -\frac{x^2 \sqrt{-c^2 x^2 + 1}}{3c^2} - \frac{2\sqrt{-c^2 x^2 + 1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right)}{3} \\
&\quad - \frac{bc^6 d^3 x^3 \operatorname{asin}(cx)}{3} + 3bc^4 d^3 \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2 x^2 + 1}}{c} & \text{otherwise} \end{cases} \right) \\
&\quad - 3bc^3 d^3 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2 x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) + \frac{3bc^2 d^3 \operatorname{asin}(cx)}{x} \\
&\quad + \frac{bcd^3 \left(\begin{cases} -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1 + \frac{1}{c^2 x^2}}} - \frac{1}{2cx^3\sqrt{-1 + \frac{1}{c^2 x^2}}} & \text{for } \left|\frac{1}{c^2 x^2}\right| > 1 \\ \frac{ic^2 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1 - \frac{1}{c^2 x^2}}}{2x} & \text{otherwise} \end{cases} \right)}{3} \\
&\quad - \frac{bd^3 \operatorname{asin}(cx)}{3x^3}
\end{aligned}$$

input `integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))/x**4,x)`

output `-a*c**6*d**3*x**3/3 + 3*a*c**4*d**3*x + 3*a*c**2*d**3/x - a*d**3/(3*x**3) + b*c**7*d**3*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True))/3 - b*c**6*d**3*x**3*asin(c*x)/3 + 3*b*c**4*d**3*Piecewise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True)) - 3*b*c**3*d**3*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) + 3*b*c**2*d**3*asin(c*x)/x + b*c*d**3*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 - b*d**3*asin(c*x)/(3*x**3)`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.36

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx \\
&= -\frac{1}{3} ac^6 d^3 x^3 - \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bc^6 d^3 \\
&\quad + 3ac^4 d^3 x + 3 \left(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1} \right) bc^3 d^3 \\
&\quad + 3 \left(c \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bc^2 d^3 \\
&\quad - \frac{1}{6} \left(\left(c^2 \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2 x^2 + 1}}{x^2} \right) c + \frac{2 \arcsin(cx)}{x^3} \right) bd^3 \\
&\quad + \frac{3ac^2 d^3}{x} - \frac{ad^3}{3x^3}
\end{aligned}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output `-1/3*a*c^6*d^3*x^3 - 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*c^3*d^3 + 3*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*c^2*d^3 - 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 1/3*a*d^3/x^3`

Giac [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx = \text{Timed out}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output `Timed out`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^3}{x^4} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x^4,x)`output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^3)/x^4, x)`**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{d^3 \left(-6 \arcsin(cx) b c^6 x^6 + 54 \arcsin(cx) b c^4 x^4 + 54 \arcsin(cx) b c^2 x^2 - 6 \arcsin(cx) b - 2 \sqrt{-c^2 x^2 + 1} b c^5 x^5 + 50 \sqrt{-c^2 x^2 + 1} b c^3 x^3 - 3 \sqrt{-c^2 x^2 + 1} b c x - 51 \log(\tan(\arcsin(cx)/2)) b c^3 x^3 - 6 a c^6 x^6 + 54 a c^4 x^4 + 54 a c^2 x^2 - 6 a \right)}{(18 x^3)}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))/x^4,x)`output `(d**3*(- 6*asin(c*x)*b*c**6*x**6 + 54*asin(c*x)*b*c**4*x**4 + 54*asin(c*x)*b*c**2*x**2 - 6*asin(c*x)*b - 2*sqrt(- c**2*x**2 + 1)*b*c**5*x**5 + 50*sqrt(- c**2*x**2 + 1)*b*c**3*x**3 - 3*sqrt(- c**2*x**2 + 1)*b*c*x - 51*log(tan(asin(c*x)/2))*b*c**3*x**3 - 6*a*c**6*x**6 + 54*a*c**4*x**4 + 54*a*c**2*x**2 - 6*a))/(18*x**3)`

3.28 $\int \frac{x^4(a+b \arcsin(cx))}{d-c^2dx^2} dx$

Optimal result	433
Mathematica [A] (verified)	434
Rubi [A] (verified)	434
Maple [A] (verified)	438
Fricas [F]	439
Sympy [F]	439
Maxima [F]	440
Giac [F]	440
Mupad [F(-1)]	440
Reduce [F]	441

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{x^4(a+b \arcsin(cx))}{d-c^2dx^2} dx = -\frac{4b\sqrt{1-c^2x^2}}{3c^5d} + \frac{b(1-c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b \arcsin(cx))}{c^4d} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} - \frac{2i(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^5d} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^5d} - \frac{ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^5d}$$

output

```
-4/3*b*(-c^2*x^2+1)^(1/2)/c^5/d+1/9*b*(-c^2*x^2+1)^(3/2)/c^5/d-x*(a+b*arcsin(c*x))/c^4/d-1/3*x^3*(a+b*arcsin(c*x))/c^2/d-2*I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^5/d+I*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d-I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.66

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \frac{18acx + 6ac^3x^3 + 22b\sqrt{1 - c^2x^2} + 2bc^2x^2\sqrt{1 - c^2x^2} + 9ib\pi \arcsin(cx) + 18bcx \arcsin(cx) + 6bc^3x^3 \arcsin(cx)}{c^5d}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2),x]
```

output

```
-1/18*(18*a*c*x + 6*a*c^3*x^3 + 22*b*Sqrt[1 - c^2*x^2] + 2*b*c^2*x^2*Sqrt[1 - c^2*x^2] + (9*I)*b*Pi*ArcSin[c*x] + 18*b*c*x*ArcSin[c*x] + 6*b*c^3*x^3*ArcSin[c*x] - 9*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 18*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 9*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 18*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 9*a*Log[1 - c*x] - 9*a*Log[1 + c*x] + 9*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 9*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (18*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (18*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^5*d)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5210, 27, 243, 53, 2009, 5210, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

$$\downarrow 5210$$

$$\frac{\int \frac{x^2(a + b \arcsin(cx))}{d(1 - c^2 x^2)} dx}{c^2} + \frac{b \int \frac{x^3}{\sqrt{1 - c^2 x^2}} dx}{3cd} - \frac{x^3(a + b \arcsin(cx))}{3c^2 d}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} + \frac{b \int \frac{x^3}{\sqrt{1-c^2x^2}} dx}{3cd} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} \\
& \quad \downarrow \text{243} \\
& \frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} + \frac{b \int \frac{x^2}{\sqrt{1-c^2x^2}} dx^2}{6cd} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} \\
& \quad \downarrow \text{53} \\
& \frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} + \frac{b \int \left(\frac{1}{c^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{c^2} \right) dx^2}{6cd} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} \\
& \quad \downarrow \text{2009} \\
& \frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6cd} \\
& \quad \downarrow \text{5210} \\
& \frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \\
& \quad \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6cd} \\
& \quad \downarrow \text{241} \\
& \frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \\
& \quad \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6cd} \\
& \quad \downarrow \text{5164} \\
& \frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \\
& \quad \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6cd} \\
& \quad \downarrow \text{3042}
\end{aligned}$$

$$\frac{\frac{f(a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} - \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \frac{c^2d}{6cd} b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{4669}$$

$$\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} + \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \frac{c^2d}{6cd} b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{2715}$$

$$\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{c^3} + \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \frac{c^2d}{6cd} b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{2838}$$

$$\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} + \frac{x^3(a+b \arcsin(cx))}{3c^2d} + \frac{c^2d}{6cd} b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)$$

input `Int[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2), x]`

output `(b*((-2*sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/(6*c*d) - (x^3*(a + b*ArcSin[c*x]))/(3*c^2*d) + (-((b*sqrt[1 - c^2*x^2])/c^3) - (x*(a + b*ArcSin[c*x]))/c^2 + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/c^3)/(c^2*d)`

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 53 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$
- rule 241 $\text{Int}[(x_)*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p + 1)} / (2*b*(p + 1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 243 $\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m - 1)/2} * (a + b*x)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_.)*((F_)^{(e_.)*((c_.) + (d_.)(x_))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n)], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2838 $\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
-> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/((d_.) + (e_.)*(x_)^2), x_Symbol]
-> Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol]
-> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{a\left(\frac{c^3x^3}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{5b\sqrt{-c^2x^2+1}}{4d}-\frac{5b\arcsin(cx)cx}{4d}-\frac{b\arcsin(cx)\ln\left(1+i\left(icx+\sqrt{-c^2x^2+1}\right)\right)}{d}+\frac{b\arcsin(cx)}{d}$
default	$-\frac{a\left(\frac{c^3x^3}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{5b\sqrt{-c^2x^2+1}}{4d}-\frac{5b\arcsin(cx)cx}{4d}-\frac{b\arcsin(cx)\ln\left(1+i\left(icx+\sqrt{-c^2x^2+1}\right)\right)}{d}+\frac{b\arcsin(cx)}{d}$
parts	$-\frac{a\left(\frac{1}{3}c^2\frac{x^3}{c^4}+x+\frac{\ln(cx-1)}{2c^5}-\frac{\ln(cx+1)}{2c^5}\right)}{d}-\frac{5b\sqrt{-c^2x^2+1}}{4c^5d}-\frac{5b\arcsin(cx)x}{4dc^4}-\frac{b\arcsin(cx)\ln\left(1+i\left(icx+\sqrt{-c^2x^2+1}\right)\right)}{dc^5}$

input

```
int(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
1/c^5*(-a/d*(1/3*c^3*x^3+c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))-5/4*b/d*(-c^2*x^2+1)^(1/2)-5/4*b/d*arcsin(c*x)*c*x-b/d*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+b/d*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*b/d*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*b/d*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))+1/36*b/d*cos(3*arcsin(c*x))+1/12*b/d*arcsin(c*x)*sin(3*arcsin(c*x)))
```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^4}{c^2 dx^2 - d} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output

```
integral(-(b*x^4*arcsin(c*x) + a*x^4)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx = -\int \frac{ax^4}{c^2 x^2 - 1} dx + \int \frac{bx^4 \arcsin(cx)}{c^2 x^2 - 1} dx$$

input

```
integrate(x**4*(a+b*asin(c*x))/(-c**2*d*x**2+d),x)
```

output

```
-(Integral(a*x**4/(c**2*x**2 - 1), x) + Integral(b*x**4*asin(c*x)/(c**2*x**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^4}{c^2 dx^2 - d} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/6*a*(2*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x - 1)/(c^5*d)) + 1/6*(6*c^5*d*integrate(-1/6*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d*x^2 - c^4*d), x) - 2*(c^3*x^3 + 3*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*b/(c^5*d)`

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^4}{c^2 dx^2 - d} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")`

output `integrate(-(b*arcsin(c*x) + a)*x^4/(c^2*d*x^2 - d), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{d - c^2 dx^2} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2),x)`

output `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2), x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

$$= \frac{-6 \left(\int \frac{a \sin(cx) x^4}{c^2 x^2 - 1} dx \right) b c^5 - 3 \log(c^2 x - c) a + 3 \log(c^2 x + c) a - 2a c^3 x^3 - 6acx}{6c^5 d}$$

input `int(x^4*(a+b*asin(c*x))/(-c^2*d*x^2+d),x)`

output `(- 6*int((asin(c*x)*x**4)/(c**2*x**2 - 1),x)*b*c**5 - 3*log(c**2*x - c)*a + 3*log(c**2*x + c)*a - 2*a*c**3*x**3 - 6*a*c*x)/(6*c**5*d)`

3.29 $\int \frac{x^3(a+b \arcsin(cx))}{d-c^2dx^2} dx$

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Mupad [F(-1)]	449
Reduce [F]	449

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{x^3(a+b \arcsin(cx))}{d-c^2dx^2} dx = -\frac{bx\sqrt{1-c^2x^2}}{4c^3d} + \frac{b \arcsin(cx)}{4c^4d} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} + \frac{i(a+b \arcsin(cx))^2}{2bc^4d} - \frac{(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{c^4d} + \frac{ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2c^4d}$$

output

```
-1/4*b*x*(-c^2*x^2+1)^(1/2)/c^3/d+1/4*b*arcsin(c*x)/c^4/d-1/2*x^2*(a+b*arcsin(c*x))/c^2/d+1/2*I*(a+b*arcsin(c*x))^2/b/c^4/d-(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d+1/2*I*b*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 312 vs. $2(144) = 288$.

Time = 0.34 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.17

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \frac{2ac^2x^2 + bcx\sqrt{1 - c^2x^2} + 4ib\pi \arcsin(cx) + 2bc^2x^2 \arcsin(cx) - 2ib \arcsin(cx)^2 - 2b \arctan\left(\frac{cx}{-1 + \sqrt{1 - c^2x^2}}\right)}{d - c^2x^2}$$

input `Integrate[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2),x]`

output `-1/4*(2*a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2] + (4*I)*b*Pi*ArcSin[c*x] + 2*b*c^2*x^2*ArcSin[c*x] - (2*I)*b*ArcSin[c*x]^2 - 2*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) + 8*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] + 2*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 4*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 2*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 4*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*a*Log[1 - c^2*x^2] - 8*b*Pi*Log[Cos[ArcSin[c*x]/2]] + 2*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - 2*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (4*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (4*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^4*d)`

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5210, 27, 262, 223, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

↓ 5210

$$\begin{aligned}
& \frac{\int \frac{x(a+b \arcsin(cx))}{d(1-c^2x^2)} dx}{c^2} + \frac{b \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{2cd} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} \\
& \quad \downarrow 27 \\
& \frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} + \frac{b \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{2cd} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} \\
& \quad \downarrow 262 \\
& \frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} + \frac{b \left(\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2cd} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} \\
& \quad \downarrow 223 \\
& \frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2cd} \\
& \quad \downarrow 5180 \\
& \frac{\int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^4d} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2cd} \\
& \quad \downarrow 3042 \\
& \frac{\int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^4d} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} + \\
& \quad \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2cd} \\
& \quad \downarrow 4202 \\
& \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)} (a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^4d} - \frac{x^2(a+b \arcsin(cx))}{2c^2d} + \\
& \quad \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2cd} \\
& \quad \downarrow 2620 \\
& \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right)}{c^4d} - \\
& \quad \frac{x^2(a+b \arcsin(cx))}{2c^2d} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2cd} \\
& \quad \downarrow 2715
\end{aligned}$$

$$\frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(\frac{1}{4}b \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) dx\right) - \frac{1}{2}i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx))}{\frac{x^2(a + b \arcsin(cx))}{2c^2d} + \frac{b\left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}}$$

↓ 2838

$$\frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})\right)}{\frac{x^2(a + b \arcsin(cx))}{2c^2d} + \frac{b\left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}}$$

input `Int[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2), x]`

output `-1/2*(x^2*(a + b*ArcSin[c*x]))/(c^2*d) + (b*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(2*c*d) + (((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4))/(c^4*d)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 262 `Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(b*(m+2*p+1))), x] - Simp[a*c^2*((m-1)/(b*(m+2*p+1)) Int[(c*x)^(m-2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2-1] && NeQ[m+2*p+1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 2620 $\text{Int}[\left(\frac{(F_{-})^{((g_{-}) * (e_{-}) + (f_{-}) * (x_{-}))}}{(a_{-}) + (b_{-}) * (F_{-})^{((g_{-}) * (e_{-}) + (f_{-}) * (x_{-}))}}\right)^{(n_{-})} * ((c_{-}) + (d_{-}) * (x_{-}))^{(m_{-})} / ((a_{-}) + (b_{-}) * (F_{-})^{((g_{-}) * (e_{-}) + (f_{-}) * (x_{-}))})^{(n_{-})}, x_Symbol] \rightarrow \text{Simp}[\left(\frac{(c + d*x)^m}{(b*f*g*n*\text{Log}[F])}\right) * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_{-}) + (b_{-}) * (F_{-})^{((e_{-}) * (c_{-}) + (d_{-}) * (x_{-}))}]^{(n_{-})}, x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)))^n}], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_{-}) * ((d_{-}) + (e_{-}) * (x_{-})^{(n_{-})})] / (x_{-}), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_{-}, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[\left(\frac{(c_{-}) + (d_{-}) * (x_{-})^{(m_{-})}}{(c + d*x)^{m+1}}\right) * \tan[(e_{-}) + (f_{-}) * (x_{-})], x_Symbol] \rightarrow \text{Simp}[I * ((c + d*x)^{m+1} / (d*(m+1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * (E^{(2*I*(e + f*x))} / (1 + E^{(2*I*(e + f*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 5180 $\text{Int}[\left(\frac{(a_{-}) + \text{ArcSin}[(c_{-}) * (x_{-})] * (b_{-})}{(d_{-}) + (e_{-}) * (x_{-})^2}\right)^{(n_{-})} * (x_{-}), x_Symbol] \rightarrow \text{Simp}[-e^{(-1)} \text{Subst}[\text{Int}[(a + b*x)^n * \text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5210 $\text{Int}[\left(\frac{(a_{-}) + \text{ArcSin}[(c_{-}) * (x_{-})] * (b_{-})}{(d_{-}) + (e_{-}) * (x_{-})^2}\right)^{(n_{-})} * ((f_{-}) * (x_{-}))^{(m_{-})} * ((d_{-}) + (e_{-}) * (x_{-})^2)^{(p_{-})}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)} * (d + e*x^2)^{(p+1)} * ((a + b*\text{ArcSin}[c*x])^n / (e*(m + 2*p + 1))), x] + (\text{Simp}[f^2 * ((m-1) / (c^2*(m + 2*p + 1))) \text{Int}[(f*x)^{(m-2)} * (d + e*x^2)^p * (a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m + 2*p + 1))) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m-1)} * (1 - c^2*x^2)^{(p+1/2)} * (a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{IGtQ}[m, 1] \&\& \text{NeQ}[m + 2*p + 1, 0]$

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{a\left(\frac{c^2x^2}{2} + \frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2}\right)}{d} + \frac{ib \arcsin(cx)^2}{2d} - \frac{b\sqrt{-c^2x^2+1}cx}{4d} - \frac{b \arcsin(cx)c^2x^2}{2d} + \frac{b \arcsin(cx)}{4d} - \frac{b \arcsin(cx) \ln\left(1 + \frac{icx+1}{d}\right)}{d}$
default	$-\frac{a\left(\frac{c^2x^2}{2} + \frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2}\right)}{d} + \frac{ib \arcsin(cx)^2}{2d} - \frac{b\sqrt{-c^2x^2+1}cx}{4d} - \frac{b \arcsin(cx)c^2x^2}{2d} + \frac{b \arcsin(cx)}{4d} - \frac{b \arcsin(cx) \ln\left(1 + \frac{icx+1}{d}\right)}{d}$
parts	$-\frac{ax^2}{2dc^2} - \frac{a \ln(c^2x^2-1)}{2dc^4} + \frac{ib \arcsin(cx)^2}{2dc^4} - \frac{bx\sqrt{-c^2x^2+1}}{4c^3d} - \frac{b \arcsin(cx)x^2}{2dc^2} + \frac{b \arcsin(cx)}{4c^4d} - \frac{b \arcsin(cx) \ln\left(1 + \frac{icx+1}{d}\right)}{d}$

input `int(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{c^4} \left(-\frac{a}{d} \left(\frac{1}{2} c^2 x^2 + \frac{1}{2} \ln(cx-1) + \frac{1}{2} \ln(cx+1) \right) + \frac{1}{2} I \frac{b}{d} \arcsin(cx)^2 - \frac{1}{4} \frac{b}{d} (-c^2 x^2 + 1)^{(1/2)} cx - \frac{1}{2} \frac{b}{d} \arcsin(cx) c^2 x^2 + \frac{1}{4} \frac{b \arcsin(cx)}{d} - \frac{b}{d} \arcsin(cx) \ln\left(1 + \frac{Icx + (-c^2 x^2 + 1)^{(1/2)}}{d}\right) + \frac{1}{2} I \frac{b}{d} \operatorname{polylog}\left(2, -\frac{Icx + (-c^2 x^2 + 1)^{(1/2)}}{d}\right) \right)$$

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^3}{c^2 dx^2 - d} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*x^3*arcsin(c*x) + a*x^3)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx = -\frac{\int \frac{ax^3}{c^2 x^2 - 1} dx + \int \frac{bx^3 \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x**3*(a+b*asin(c*x))/(-c**2*d*x**2+d),x)`

output `-(Integral(a*x**3/(c**2*x**2 - 1), x) + Integral(b*x**3*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^3}{c^2 dx^2 - d} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*a*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) - 1/2*(2*c^4*d*integrate(1/2*(c^2*x^2*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) + e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) + e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(-c*x + 1))/(c^7*d*x^4 - c^5*d*x^2 + (c^5*d*x^2 - c^3*d)*e^(log(c*x + 1) + log(-c*x + 1))), x) + c^2*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + 1) + arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)*log(-c*x + 1))*b/(c^4*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{d - c^2 dx^2} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2),x)`

output `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2), x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

$$= \frac{-2 \operatorname{asin}(cx) b c^2 x^2 + \operatorname{asin}(cx) b - \sqrt{-c^2 x^2 + 1} b c x - 4 \left(\int \frac{\operatorname{asin}(cx) x}{c^2 x^2 - 1} dx \right) b c^2 - 2 \log(c^2 x - c) a - 2 \log(c^2 x + c) a}{4 c^4 d}$$

input `int(x^3*(a+b*asin(c*x))/(-c^2*d*x^2+d),x)`

output `(- 2*asin(c*x)*b*c**2*x**2 + asin(c*x)*b - sqrt(- c**2*x**2 + 1)*b*c*x -
4*int((asin(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - 2*log(c**2*x - c)*a - 2*log(c**2*x + c)*a - 2*a*c**2*x**2)/(4*c**4*d)`

3.30 $\int \frac{x^2(a+b \arcsin(cx))}{d-c^2dx^2} dx$

Optimal result	450
Mathematica [A] (verified)	450
Rubi [A] (verified)	451
Maple [A] (verified)	454
Fricas [F]	454
Sympy [F]	455
Maxima [F]	455
Giac [F]	455
Mupad [F(-1)]	456
Reduce [F]	456

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{x^2(a+b \arcsin(cx))}{d-c^2dx^2} dx = -\frac{b\sqrt{1-c^2x^2}}{c^3d} - \frac{x(a+b \arcsin(cx))}{c^2d} - \frac{2i(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^3d} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^3d} - \frac{ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^3d}$$

output

```
-b*(-c^2*x^2+1)^(1/2)/c^3/d-x*(a+b*arcsin(c*x))/c^2/d-2*I*(a+b*arcsin(c*x))
)*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^3/d+I*b*polylog(2,-I*(I*c*x+(-c^2*x^2
+1)^(1/2)))/c^3/d-I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.92

$$\int \frac{x^2(a+b \arcsin(cx))}{d-c^2dx^2} dx = \frac{2acx + 2b\sqrt{1-c^2x^2} + ib\pi \arcsin(cx) + 2bcx \arcsin(cx) - b\pi \log(1 - ie^{i \arcsin(cx)}) - 2b \arcsin(cx) \log(1 - ie^{i \arcsin(cx)})}{c^3d} - \frac{2acx + 2b\sqrt{1-c^2x^2} + ib\pi \arcsin(cx) + 2bcx \arcsin(cx) - b\pi \log(1 + ie^{i \arcsin(cx)}) - 2b \arcsin(cx) \log(1 + ie^{i \arcsin(cx)})}{c^3d}$$

input `Integrate[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2),x]`

output `-1/2*(2*a*c*x + 2*b*Sqrt[1 - c^2*x^2] + I*b*Pi*ArcSin[c*x] + 2*b*c*x*ArcSin[c*x] - b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 2*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + a*Log[1 - c*x] - a*Log[1 + c*x] + b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^3*d)`

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5210, 27, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx \\
 & \quad \downarrow \text{5210} \\
 & \frac{\int \frac{a+b \arcsin(cx)}{d(1-c^2x^2)} dx}{c^2} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x(a + b \arcsin(cx))}{c^2 d} \\
 & \quad \downarrow \text{27} \\
 & \frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2 d} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x(a + b \arcsin(cx))}{c^2 d} \\
 & \quad \downarrow \text{241} \\
 & \frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2 d} - \frac{x(a + b \arcsin(cx))}{c^2 d} - \frac{b\sqrt{1-c^2x^2}}{c^3 d} \\
 & \quad \downarrow \text{5164} \\
 & \frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3 d} - \frac{x(a + b \arcsin(cx))}{c^2 d} - \frac{b\sqrt{1-c^2x^2}}{c^3 d}
 \end{aligned}$$

$$\begin{aligned}
& \downarrow 3042 \\
& \frac{\int (a + b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^3 d} - \frac{x(a + b \arcsin(cx))}{c^2 d} - \frac{b\sqrt{1 - c^2 x^2}}{c^3 d} \\
& \downarrow 4669 \\
& \frac{-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{c^3 d} \\
& \quad \frac{x(a + b \arcsin(cx))}{c^2 d} - \frac{b\sqrt{1 - c^2 x^2}}{c^3 d} \\
& \downarrow 2715 \\
& \frac{ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{c^3 d} \\
& \quad \frac{x(a + b \arcsin(cx))}{c^2 d} - \frac{b\sqrt{1 - c^2 x^2}}{c^3 d} \\
& \downarrow 2838 \\
& \frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^3 d} \\
& \quad \frac{x(a + b \arcsin(cx))}{c^2 d} - \frac{b\sqrt{1 - c^2 x^2}}{c^3 d}
\end{aligned}$$

input `Int[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2),x]`

output `-((b*Sqrt[1 - c^2*x^2])/(c^3*d)) - (x*(a + b*ArcSin[c*x]))/(c^2*d) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^3*d)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)
Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f)
Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol]
:> Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5210 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*((d_) + (e_
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]`

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{a\left(\frac{cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}}{d}\right)-b\sqrt{-c^2x^2+1}}{d}+\frac{b\arcsin(cx)\ln\left(1-i\left(\frac{icx+\sqrt{-c^2x^2+1}}{d}\right)\right)}{d}-\frac{b\arcsin(cx)\ln\left(1+i\left(\frac{icx+\sqrt{-c^2x^2+1}}{d}\right)\right)}{c^3}$
default	$-\frac{a\left(\frac{cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}}{d}\right)-b\sqrt{-c^2x^2+1}}{d}+\frac{b\arcsin(cx)\ln\left(1-i\left(\frac{icx+\sqrt{-c^2x^2+1}}{d}\right)\right)}{d}-\frac{b\arcsin(cx)\ln\left(1+i\left(\frac{icx+\sqrt{-c^2x^2+1}}{d}\right)\right)}{c^3}$
parts	$-\frac{a\left(\frac{x}{c^2}+\frac{\ln(cx-1)}{2c^3}-\frac{\ln(cx+1)}{2c^3}\right)}{d}-\frac{b\sqrt{-c^2x^2+1}}{c^3d}-\frac{b\arcsin(cx)x}{dc^2}+\frac{ib\operatorname{dilog}\left(1+i\left(\frac{icx+\sqrt{-c^2x^2+1}}{d}\right)\right)}{dc^3}-\frac{ib\operatorname{dilog}\left(1-i\left(\frac{icx+\sqrt{-c^2x^2+1}}{d}\right)\right)}{dc^3}$

input `int(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output `1/c^3*(-a/d*(c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))-b/d*(-c^2*x^2+1)^(1/2)+b/d*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-b/d*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-b/d*arcsin(c*x)*c*x-I*b/d*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*b/d*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^2}{c^2 dx^2 - d} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*x^2*arcsin(c*x) + a*x^2)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx = -\frac{\int \frac{ax^2}{c^2 x^2 - 1} dx + \int \frac{bx^2 \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x**2*(a+b*asin(c*x))/(-c**2*d*x**2+d),x)`

output `-(Integral(a*x**2/(c**2*x**2 - 1), x) + Integral(b*x**2*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^2}{c^2 dx^2 - d} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*a*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d)) + 1/2*(2*c^3*d*integrate(-1/2*(2*c*x - log(c*x + 1) + log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d*x^2 - c^2*d), x) - 2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + 1) - arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*b/(c^3*d)`

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^2}{c^2 dx^2 - d} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")`

output `integrate(-(b*arcsin(c*x) + a)*x^2/(c^2*d*x^2 - d), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{d - c^2 dx^2} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2), x)`output `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2), x)`**Reduce [F]**

$$\int \frac{x^2(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

$$= \frac{-2 \operatorname{asin}(cx) b c x - 2 \sqrt{-c^2 x^2 + 1} b - 2 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^2 - 1} dx \right) b c - \log(c^2 x - c) a + \log(c^2 x + c) a - 2 a c x}{2 c^3 d}$$

input `int(x^2*(a+b*asin(c*x))/(-c^2*d*x^2+d), x)`output `(- 2*asin(c*x)*b*c*x - 2*sqrt(- c**2*x**2 + 1)*b - 2*int(asin(c*x)/(c**2*x**2 - 1), x)*b*c - log(c**2*x - c)*a + log(c**2*x + c)*a - 2*a*c*x)/(2*c**3*d)`

3.31 $\int \frac{x(a+b \arcsin(cx))}{d-c^2 dx^2} dx$

Optimal result	457
Mathematica [B] (verified)	457
Rubi [A] (verified)	458
Maple [A] (verified)	460
Fricas [F]	461
Sympy [F]	461
Maxima [F]	461
Giac [F(-2)]	462
Mupad [F(-1)]	462
Reduce [F]	462

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \frac{i(a + b \arcsin(cx))^2}{2bc^2d} - \frac{(a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)})}{c^2d} + \frac{ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2c^2d}$$

output

$$\frac{1}{2}i*(a+b*\arcsin(c*x))^2/b/c^2/d-(a+b*\arcsin(c*x))*\ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^2/d+1/2*I*b*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^2/d$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 244 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.98

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \frac{2ib\pi \arcsin(cx) - ib \arcsin(cx)^2 + 4b\pi \log(1 + e^{-i \arcsin(cx)}) + b\pi \log(1 - ie^{i \arcsin(cx)}) + 2b \arcsin(cx)}{d - c^2 dx^2}$$

input `Integrate[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2),x]`

output `-1/2*((2*I)*b*Pi*ArcSin[c*x] - I*b*ArcSin[c*x]^2 + 4*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] + b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + a*Log[1 - c^2*x^2] - 4*b*Pi*Log[Cos[ArcSin[c*x]/2]] + b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (2*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^2*d)`

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx \\
 & \quad \downarrow 5180 \\
 & \frac{\int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^2 d} \\
 & \quad \downarrow 3042 \\
 & \frac{\int (a + b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^2 d} \\
 & \quad \downarrow 4202 \\
 & \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^2 d} \\
 & \quad \downarrow 2620 \\
 & \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i(\frac{1}{2}ib \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2}i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)))}{c^2 d}
 \end{aligned}$$

↓ 2715

$$\frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(\frac{1}{4}b \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) dx e^{2i \arcsin(cx)} - \frac{1}{2}i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx))\right)}{c^2 d}$$

↓ 2838

$$\frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})\right)}{c^2 d}$$

input `Int[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2), x]`

output `((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)/(c^2*d)`

Defintions of rubi rules used

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[
I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x))))], x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5180

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)/((d_.) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

method	result
parts	$-\frac{a \ln(c^2 x^2 - 1)}{2d c^2} - \frac{b \left(-\frac{i \arcsin(cx)^2}{2} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{-c^2 x^2 + 1})^2 \right) - \frac{i \operatorname{polylog} \left(2, -\frac{icx + \sqrt{-c^2 x^2 + 1}}{2} \right)^2}{2} \right)}{d c^2}$
derivativedivides	$\frac{a \left(\frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} \right)}{d} - \frac{b \left(-\frac{i \arcsin(cx)^2}{2} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{-c^2 x^2 + 1})^2 \right) - \frac{i \operatorname{polylog} \left(2, -\frac{icx + \sqrt{-c^2 x^2 + 1}}{2} \right)^2}{2} \right)}{d c^2}$
default	$\frac{a \left(\frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} \right)}{d} - \frac{b \left(-\frac{i \arcsin(cx)^2}{2} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{-c^2 x^2 + 1})^2 \right) - \frac{i \operatorname{polylog} \left(2, -\frac{icx + \sqrt{-c^2 x^2 + 1}}{2} \right)^2}{2} \right)}{d c^2}$

input

```
int(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
-1/2*a/d/c^2*ln(c^2*x^2-1)-b/d/c^2*(-1/2*I*arcsin(c*x)^2+arcsin(c*x)*ln(1+
(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^
2))
```

Fricas [F]

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x}{c^2 dx^2 - d} dx$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*x*arcsin(c*x) + a*x)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = -\frac{\int \frac{ax}{c^2 x^2 - 1} dx + \int \frac{bx \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x*(a+b*asin(c*x))/(-c**2*d*x**2+d),x)`

output `-(Integral(a*x/(c**2*x**2 - 1), x) + Integral(b*x*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x}{c^2 dx^2 - d} dx$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*(2*c^2*d*integrate(1/2*(e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) + e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(-c*x + 1))/(c^5*d*x^4 - c^3*d*x^2 + (c^3*d*x^2 - c*d)*e^(log(c*x + 1) + log(-c*x + 1))), x) + arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) + arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1)*b/(c^2*d) - 1/2*a*log(c^2*d*x^2 - d)/(c^2*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{d - c^2 dx^2} dx$$

input `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2),x)`

output `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \frac{-2 \left(\int \frac{\operatorname{asin}(cx)x}{c^2 x^2 - 1} dx \right) b c^2 - \log(c^2 x - c) a - \log(c^2 x + c) a}{2c^2 d}$$

input `int(x*(a+b*asin(c*x))/(-c^2*d*x^2+d),x)`

output `(- 2*int((asin(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - log(c**2*x - c)*a - lo g(c**2*x + c)*a)/(2*c**2*d)`

3.32 $\int \frac{a+b \arcsin(cx)}{d-c^2dx^2} dx$

Optimal result	463
Mathematica [B] (verified)	463
Rubi [A] (verified)	464
Maple [A] (verified)	466
Fricas [F]	466
Sympy [F]	467
Maxima [F]	467
Giac [F(-2)]	467
Mupad [F(-1)]	468
Reduce [F]	468

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{a + b \arcsin(cx)}{d - c^2dx^2} dx = -\frac{2i(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{cd} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{cd} - \frac{ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{cd}$$

output

```
-2*I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c/d+I*b*polylog(2,
-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d-I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/
2)))/c/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 207 vs. 2(84) = 168.

Time = 0.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.46

$$\int \frac{a + b \arcsin(cx)}{d - c^2dx^2} dx = \frac{-ib\pi \arcsin(cx) + b\pi \log(1 - ie^{i \arcsin(cx)}) + 2b \arcsin(cx) \log(1 - ie^{i \arcsin(cx)}) + b\pi \log(1 + ie^{i \arcsin(cx)})}{d}$$

input `Integrate[(a + b*ArcSin[c*x])/(d - c^2*d*x^2),x]`

output `((-I)*b*Pi*ArcSin[c*x] + b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 2*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - a*Log[1 - c*x] + a*Log[1 + c*x] - b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (2*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (2*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c*d)`

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx \\
 & \quad \downarrow \text{5164} \\
 & \frac{\int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} d \arcsin(cx)}{cd} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\int (a + b \arcsin(cx)) \csc\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx)}{cd} \\
 & \quad \downarrow \text{4669} \\
 & \frac{-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{cd} \\
 & \quad \downarrow \text{2715} \\
 & \frac{ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{cd}
 \end{aligned}$$

↓ 2838

$$\frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{cd}$$

input `Int[(a + b*ArcSin[c*x])/(d - c^2*d*x^2),x]`

output `((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c*d)`

Defintions of rubi rules used

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{\frac{a \operatorname{arctanh}(cx)}{d} - \frac{b \left(-\operatorname{arctanh}(cx) \arcsin(cx) + i \operatorname{arctanh}(cx) \left(\ln \left(1 - \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) - \ln \left(1 + \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) \right) - i \operatorname{dilog} \left(1 + \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) \right)}{c}}{c}$
default	$\frac{\frac{a \operatorname{arctanh}(cx)}{d} - \frac{b \left(-\operatorname{arctanh}(cx) \arcsin(cx) + i \operatorname{arctanh}(cx) \left(\ln \left(1 - \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) - \ln \left(1 + \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) \right) - i \operatorname{dilog} \left(1 + \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) \right)}{c}}{c}$
parts	$-\frac{a \ln(cx-1)}{2dc} + \frac{a \ln(cx+1)}{2dc} - \frac{b \left(-\operatorname{arctanh}(cx) \arcsin(cx) + i \operatorname{arctanh}(cx) \left(\ln \left(1 - \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) - \ln \left(1 + \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) \right) - i \operatorname{dilog} \left(1 + \frac{i(cx+1)}{\sqrt{-c^2x^2+1}} \right) \right)}{dc}$

input `int((a+b*arcsin(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output `1/c*(a/d*arctanh(c*x)-b/d*(-arctanh(c*x)*arcsin(c*x)+I*arctanh(c*x)*(ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+I*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx = \int -\frac{b \arcsin(cx) + a}{c^2 dx^2 - d} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx = -\int \frac{a}{c^2 x^2 - 1} dx + \int \frac{b \arcsin(cx)}{c^2 x^2 - 1} dx$$

input `integrate((a+b*asin(c*x))/(-c**2*d*x**2+d),x)`

output `-(Integral(a/(c**2*x**2 - 1), x) + Integral(b*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx = \int -\frac{b \arcsin(cx) + a}{c^2 dx^2 - d} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) + 1/2*(2*c*d*integrate(1/2*sqrt(c*x + 1)*sqrt(-c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/(c^2*d*x^2 - d), x) + arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*b/(c*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{d - c^2 dx^2} dx$$

input

```
int((a + b*asin(c*x))/(d - c^2*d*x^2),x)
```

output

```
int((a + b*asin(c*x))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{d - c^2 dx^2} dx = \frac{-2 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^2 - 1} dx \right) bc - \log(c^2 x - c) a + \log(c^2 x + c) a}{2cd}$$

input

```
int((a+b*asin(c*x))/(-c^2*d*x^2+d),x)
```

output

```
( - 2*int(asin(c*x)/(c**2*x**2 - 1),x)*b*c - log(c**2*x - c)*a + log(c**2*
x + c)*a)/(2*c*d)
```

3.33 $\int \frac{a+b \arcsin(cx)}{x(d-c^2dx^2)} dx$

Optimal result	469
Mathematica [B] (verified)	469
Rubi [A] (verified)	470
Maple [B] (verified)	472
Fricas [F]	473
Sympy [F]	473
Maxima [F]	473
Giac [F(-2)]	474
Mupad [F(-1)]	474
Reduce [F]	474

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2dx^2)} dx = -\frac{2(a + b \arcsin(cx)) \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d} + \frac{ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2d} - \frac{ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{2d}$$

output

```
-2*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+1/2*I*b*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d-1/2*I*b*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 274 vs. 2(71) = 142.

Time = 0.15 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.86

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2dx^2)} dx = \frac{2ib\pi \arcsin(cx) + 4b\pi \log(1 + e^{-i \arcsin(cx)}) + b\pi \log(1 - ie^{i \arcsin(cx)}) + 2b \arcsin(cx) \log(1 - ie^{i \arcsin(cx)})}{d}$$

input `Integrate[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)),x]`

output `-1/2*((2*I)*b*Pi*ArcSin[c*x] + 4*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] + b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - 2*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - 2*a*Log[x] + a*Log[1 - c^2*x^2] - 4*b*Pi*Log[Cos[ArcSin[c*x]/2]] + b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (2*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])] + I*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/d`

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx \\
 & \quad \downarrow \text{5184} \\
 & \frac{\int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2x^2}} d \arcsin(cx)}{d} \\
 & \quad \downarrow \text{4919} \\
 & \frac{2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d} \\
 & \quad \downarrow \text{3042} \\
 & \frac{2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d} \\
 & \quad \downarrow \text{4671} \\
 & \frac{2\left(-\frac{1}{2}b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})(a + \right.}{d}
 \end{aligned}$$

↓ 2715

$$\frac{2\left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \right)}{d}$$

↓ 2838

$$\frac{2\left(-\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})\right)}{d}$$

input

```
Int[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)),x]
```

output

```
(2*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/d
```

Defintions of rubi rules used

rule 2715

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 4671

```
Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

```
rule 4919 Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

```
rule 5184 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(95) = 190.

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.72

method	result
parts	$-\frac{a\left(\frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} - \ln(x)\right)}{d} - \frac{b\left(\arcsin(cx) \ln\left(1 + \left(icx + \sqrt{-c^2x^2+1}\right)^2\right) - \frac{i \operatorname{polylog}\left(2, -\left(icx + \sqrt{-c^2x^2+1}\right)^2\right)}{2}\right)}{d}$
derivativedivides	$-\frac{a\left(\frac{\ln(cx+1)}{2} - \ln(cx) + \frac{\ln(cx-1)}{2}\right)}{d} - \frac{b\left(\arcsin(cx) \ln\left(1 + \left(icx + \sqrt{-c^2x^2+1}\right)^2\right) - \frac{i \operatorname{polylog}\left(2, -\left(icx + \sqrt{-c^2x^2+1}\right)^2\right)}{2}\right)}{d}$
default	$-\frac{a\left(\frac{\ln(cx+1)}{2} - \ln(cx) + \frac{\ln(cx-1)}{2}\right)}{d} - \frac{b\left(\arcsin(cx) \ln\left(1 + \left(icx + \sqrt{-c^2x^2+1}\right)^2\right) - \frac{i \operatorname{polylog}\left(2, -\left(icx + \sqrt{-c^2x^2+1}\right)^2\right)}{2}\right)}{d}$

```
input int((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d), x, method=_RETURNVERBOSE)
```

```
output -a/d*(1/2*ln(c*x-1)+1/2*ln(c*x+1)-ln(x))-b/d*(arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)/(c^2*d*x^3 - d*x), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx = -\frac{\int \frac{a}{c^2 x^3 - x} dx + \int \frac{b \arcsin(cx)}{c^2 x^3 - x} dx}{d}$$

input `integrate((a+b*asin(c*x))/x/(-c**2*d*x**2+d),x)`

output `-(Integral(a/(c**2*x**3 - x), x) + Integral(b*asin(c*x)/(c**2*x**3 - x), x))/d`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*a*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - b*integrate(arctan(2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/(c^2*d*x^3 - d*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx = \int \frac{a + b \operatorname{asin}(cx)}{x(d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)} dx \\ &= \frac{-2 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^3 - x} dx \right) b - \log(c^2 x - c) a - \log(c^2 x + c) a + 2 \log(x) a}{2d} \end{aligned}$$

input `int((a+b*asin(c*x))/x/(-c^2*d*x^2+d),x)`

output `(- 2*int(asin(c*x)/(c**2*x**3 - x),x)*b - log(c**2*x - c)*a - log(c**2*x + c)*a + 2*log(x)*a)/(2*d)`

3.34 $\int \frac{a+b \arcsin(cx)}{x^2(d-c^2dx^2)} dx$

Optimal result	476
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Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = -\frac{a + b \arcsin(cx)}{dx} - \frac{2ic(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{d} - \frac{bc \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{d} + \frac{ibc \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d} - \frac{ibc \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d}$$

output

$$-(a+b*\arcsin(c*x))/d/x-2*I*c*(a+b*\arcsin(c*x))*\arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d-b*c*\operatorname{arctanh}((-c^2*x^2+1)^(1/2))/d+I*b*c*\operatorname{polylog}(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d-I*b*c*\operatorname{polylog}(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 259 vs. 2(116) = 232.

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = \frac{2a + 2b \arcsin(cx) + ibc\pi x \arcsin(cx) + 2bcx \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) - bc\pi x \log(1 - ie^{i \arcsin(cx)}) - 2bcx \log(1 + ie^{i \arcsin(cx)})}{d}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)),x]`

output `-1/2*(2*a + 2*b*ArcSin[c*x] + I*b*c*Pi*x*ArcSin[c*x] + 2*b*c*x*ArcTanh[Sqrt[1 - c^2*x^2]] - b*c*Pi*x*Log[1 - I*E^(I*ArcSin[c*x])] - 2*b*c*x*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - b*c*Pi*x*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b*c*x*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + a*c*x*Log[1 - c*x] - a*c*x*Log[1 + c*x] + b*c*Pi*x*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + b*c*Pi*x*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*b*c*x*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2*I)*b*c*x*PolyLog[2, I*E^(I*ArcSin[c*x])])/(d*x)`

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5204, 27, 243, 73, 221, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx \\
 & \quad \downarrow \text{5204} \\
 & c^2 \int \frac{a + b \arcsin(cx)}{d(1 - c^2 x^2)} dx + \frac{bc \int \frac{1}{x\sqrt{1-c^2 x^2}} dx}{d} - \frac{a + b \arcsin(cx)}{dx} \\
 & \quad \downarrow \text{27} \\
 & \frac{c^2 \int \frac{a+b \arcsin(cx)}{1-c^2 x^2} dx}{d} + \frac{bc \int \frac{1}{x\sqrt{1-c^2 x^2}} dx}{d} - \frac{a + b \arcsin(cx)}{dx} \\
 & \quad \downarrow \text{243} \\
 & \frac{c^2 \int \frac{a+b \arcsin(cx)}{1-c^2 x^2} dx}{d} + \frac{bc \int \frac{1}{x^2 \sqrt{1-c^2 x^2}} dx^2}{2d} - \frac{a + b \arcsin(cx)}{dx} \\
 & \quad \downarrow \text{73} \\
 & \frac{c^2 \int \frac{a+b \arcsin(cx)}{1-c^2 x^2} dx}{d} - \frac{b \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1 - c^2 x^2}}{cd} - \frac{a + b \arcsin(cx)}{dx}
 \end{aligned}$$

$$\begin{aligned} & \downarrow 221 \\ & \frac{c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{d} - \frac{a+b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d} \\ & \downarrow 5164 \\ & \frac{c \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{d} - \frac{a+b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d} \\ & \downarrow 3042 \\ & \frac{c \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{d} - \frac{a+b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d} \\ & \downarrow 4669 \end{aligned}$$

$$\begin{aligned} & \frac{c(-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a + b \arcsin(cx))}{d} \\ & \frac{a+b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d} \\ & \downarrow 2715 \end{aligned}$$

$$\begin{aligned} & \frac{c(ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})(a + b \arcsin(cx))}{d} \\ & \frac{a+b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d} \\ & \downarrow 2838 \end{aligned}$$

$$\begin{aligned} & \frac{c(-2i \arctan(e^{i \arcsin(cx)})(a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d} \\ & \frac{a+b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d} \end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)), x]`

output

```

-((a + b*ArcSin[c*x])/(d*x)) - (b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/d + (c*((-
2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E
^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/d

```

Defintions of rubi rules used

rule 27

```

Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]

```

rule 73

```

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]

```

rule 221

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

rule 243

```

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int
[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]

```

rule 2715

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

rule 2838

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

rule 3042

```

Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]

```

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
  := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[
  d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
  x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))],
  x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /;
  FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)
*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.76

method	result
parts	$-\frac{a\left(\frac{c \ln(cx-1)}{2} - \frac{c \ln(cx+1)}{2} + \frac{1}{x}\right)}{d} - \frac{bc\left(\frac{\arcsin(cx)}{cx} + \arcsin(cx) \ln\left(1+i\left(icx+\sqrt{-c^2x^2+1}\right)\right) - \arcsin(cx) \ln\left(1-i\left(icx+\sqrt{-c^2x^2+1}\right)\right)\right)}{d}$
derivativedivides	$c\left(-\frac{a\left(-\frac{\ln(cx+1)}{2} + \frac{\ln(cx-1)}{2} + \frac{1}{cx}\right)}{d} - \frac{b\left(\frac{\arcsin(cx)}{cx} + \arcsin(cx) \ln\left(1+i\left(icx+\sqrt{-c^2x^2+1}\right)\right) - \arcsin(cx) \ln\left(1-i\left(icx+\sqrt{-c^2x^2+1}\right)\right)\right)}{d}\right)$
default	$c\left(-\frac{a\left(-\frac{\ln(cx+1)}{2} + \frac{\ln(cx-1)}{2} + \frac{1}{cx}\right)}{d} - \frac{b\left(\frac{\arcsin(cx)}{cx} + \arcsin(cx) \ln\left(1+i\left(icx+\sqrt{-c^2x^2+1}\right)\right) - \arcsin(cx) \ln\left(1-i\left(icx+\sqrt{-c^2x^2+1}\right)\right)\right)}{d}\right)$

input

```
int((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
-a/d*(1/2*c*ln(c*x-1)-1/2*c*ln(c*x+1)+1/x)-b/d*c*(arcsin(c*x)/c/x+arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-ln(I*c*x+(-c^2*x^2+1)^(1/2))-1)+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x^2} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output

```
integral(-(b*arcsin(c*x) + a)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = -\frac{\int \frac{a}{c^2 x^4 - x^2} dx + \int \frac{b \arcsin(cx)}{c^2 x^4 - x^2} dx}{d}$$

input

```
integrate((a+b*asin(c*x))/x**2/(-c**2*d*x**2+d),x)
```

output

```
-(Integral(a/(c**2*x**4 - x**2), x) + Integral(b*asin(c*x)/(c**2*x**4 - x**2), x))/d
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `1/2*a*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x)) + 1/2*(c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) + 2*d*x*integrate(1/2*(c^2*x*log(c*x + 1) - c^2*x*log(-c*x + 1) - 2*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^3 - d*x), x) - 2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*b/(d*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = \int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)} dx = \frac{-2 \left(\int \frac{a \sin(cx)}{c^2 x^4 - x^2} dx \right) bx - \log(c^2 x - c) acx + \log(c^2 x + c) acx - 2a}{2dx}$$

input `int((a+b*asin(c*x))/x^2/(-c^2*d*x^2+d), x)`

output `(- 2*int(asin(c*x)/(c**2*x**4 - x**2), x)*b*x - log(c**2*x - c)*a*c*x + lo
g(c**2*x + c)*a*c*x - 2*a)/(2*d*x)`

3.35 $\int \frac{a+b \arcsin(cx)}{x^3(d-c^2dx^2)} dx$

Optimal result	484
Mathematica [B] (verified)	485
Rubi [A] (verified)	485
Maple [A] (verified)	488
Fricas [F]	489
Sympy [F]	489
Maxima [F]	490
Giac [F(-2)]	490
Mupad [F(-1)]	490
Reduce [F]	491

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{a + b \arcsin(cx)}{x^3(d - c^2dx^2)} dx = -\frac{bc\sqrt{1 - c^2x^2}}{2dx} - \frac{a + b \arcsin(cx)}{2dx^2} - \frac{2c^2(a + b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{d} + \frac{ibc^2 \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2d} - \frac{ibc^2 \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{2d}$$

output

```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d/x-1/2*(a+b*arcsin(c*x))/d/x^2-2*c^2*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+1/2*I*b*c^2*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d-1/2*I*b*c^2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 392 vs. $2(124) = 248$.

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.16

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx =$$

$$a + bcx\sqrt{1 - c^2x^2} + b \arcsin(cx) + 2ibc^2\pi x^2 \arcsin(cx) + 4bc^2\pi x^2 \log(1 + e^{-i \arcsin(cx)}) + bc^2\pi x^2 \log($$

input `Integrate[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)),x]`

output

```
-1/2*(a + b*c*x*Sqrt[1 - c^2*x^2] + b*ArcSin[c*x] + (2*I)*b*c^2*Pi*x^2*Arc
Sin[c*x] + 4*b*c^2*Pi*x^2*Log[1 + E^((-I)*ArcSin[c*x])] + b*c^2*Pi*x^2*Log
[1 - I*E^(I*ArcSin[c*x])] + 2*b*c^2*x^2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[
c*x])] - b*c^2*Pi*x^2*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b*c^2*x^2*ArcSin[c*
x]*Log[1 + I*E^(I*ArcSin[c*x])] - 2*b*c^2*x^2*ArcSin[c*x]*Log[1 - E^((2*I)
*ArcSin[c*x])] - 2*a*c^2*x^2*Log[x] + a*c^2*x^2*Log[1 - c^2*x^2] - 4*b*c^2
*Pi*x^2*Log[Cos[ArcSin[c*x]/2]] + b*c^2*Pi*x^2*Log[-Cos[(Pi + 2*ArcSin[c*x]
)/4]] - b*c^2*Pi*x^2*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*b*c^2*x^2*P
olyLog[2, (-I)*E^(I*ArcSin[c*x])] - (2*I)*b*c^2*x^2*PolyLog[2, I*E^(I*ArcS
in[c*x])] + I*b*c^2*x^2*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(d*x^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5204, 27, 242, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx$$

↓ 5204

$$\begin{aligned}
 & c^2 \int \frac{a + b \arcsin(cx)}{dx(1 - c^2x^2)} dx + \frac{bc \int \frac{1}{x^2\sqrt{1-c^2x^2}} dx}{2d} - \frac{a + b \arcsin(cx)}{2dx^2} \\
 & \quad \downarrow 27 \\
 & \frac{c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d} + \frac{bc \int \frac{1}{x^2\sqrt{1-c^2x^2}} dx}{2d} - \frac{a + b \arcsin(cx)}{2dx^2} \\
 & \quad \downarrow 242 \\
 & \frac{c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d} - \frac{a + b \arcsin(cx)}{2dx^2} - \frac{bc\sqrt{1 - c^2x^2}}{2dx} \\
 & \quad \downarrow 5184 \\
 & \frac{c^2 \int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2x^2}} d \arcsin(cx)}{d} - \frac{a + b \arcsin(cx)}{2dx^2} - \frac{bc\sqrt{1 - c^2x^2}}{2dx} \\
 & \quad \downarrow 4919 \\
 & \frac{2c^2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d} - \frac{a + b \arcsin(cx)}{2dx^2} - \frac{bc\sqrt{1 - c^2x^2}}{2dx} \\
 & \quad \downarrow 3042 \\
 & \frac{2c^2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d} - \frac{a + b \arcsin(cx)}{2dx^2} - \frac{bc\sqrt{1 - c^2x^2}}{2dx} \\
 & \quad \downarrow 4671 \\
 & \frac{2c^2 \left(-\frac{1}{2}b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})) (a + b \arcsin(cx)) \right)}{d} \\
 & \quad \downarrow 2715 \\
 & \frac{2c^2 \left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right)}{d} \\
 & \quad \downarrow 2838 \\
 & \frac{2c^2 \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)})) (a + b \arcsin(cx)) \right) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d} \\
 & \quad \downarrow \\
 & \frac{a + b \arcsin(cx)}{2dx^2} - \frac{bc\sqrt{1 - c^2x^2}}{2dx}
 \end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)),x]`

output `-1/2*(b*c*Sqrt[1 - c^2*x^2])/(d*x) - (a + b*ArcSin[c*x])/(2*d*x^2) + (2*c^2*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])))/d`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 242 `Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)(x_)]^{(n_.)}((c_.) + (d_.)(x_))^{(m_.)}\text{Sec}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m \text{Csc}[2*a + 2*b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)} / ((x_)((d_.) + (e_.)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + b*x)^n / (\text{Cos}[x]\text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}((f_.)(x_))^{(m_.)}((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}(d + e*x^2)^{(p+1)}((a + b*\text{ArcSin}[c*x])^n / (d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1)) \text{Int}[(f*x)^{(m+2)}(d + e*x^2)^p(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}(1 - c^2*x^2)^{(p+1/2)}(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /;$ FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.97

method	result
derivativedivides	$c^2 \left(-\frac{a \left(\frac{\ln(cx+1)}{2} + \frac{1}{2c^2x^2} - \ln(cx) + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b \left(\frac{-ic^2x^2 + cx\sqrt{-c^2x^2+1} + \arcsin(cx)}{2c^2x^2} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{-c^2x^2+1}) \right) \right)}{d} \right)$
default	$c^2 \left(-\frac{a \left(\frac{\ln(cx+1)}{2} + \frac{1}{2c^2x^2} - \ln(cx) + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b \left(\frac{-ic^2x^2 + cx\sqrt{-c^2x^2+1} + \arcsin(cx)}{2c^2x^2} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{-c^2x^2+1}) \right) \right)}{d} \right)$
parts	$-\frac{a \left(\frac{c^2 \ln(cx-1)}{2} + \frac{c^2 \ln(cx+1)}{2} + \frac{1}{2x^2} - c^2 \ln(x) \right)}{d} - \frac{b c^2 \left(\frac{-ic^2x^2 + cx\sqrt{-c^2x^2+1} + \arcsin(cx)}{2c^2x^2} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{-c^2x^2+1}) \right) \right)}{d}$

input $\text{int}((a+b*\arcsin(c*x))/x^3/(-c^2*d*x^2+d), x, \text{method}=_RETURNVERBOSE)$

output

```
c^2*(-a/d*(1/2*ln(c*x+1)+1/2/c^2/x^2-ln(c*x)+1/2*ln(c*x-1))-b/d*(1/2*(-I*c
^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+arcsin(c*x)*ln(1+(I*c*x
+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arc
sin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1
/2))-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^
2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x^3} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output

```
integral(-(b*arcsin(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx = -\frac{\int \frac{a}{c^2 x^5 - x^3} dx + \int \frac{b \arcsin(cx)}{c^2 x^5 - x^3} dx}{d}$$

input

```
integrate((a+b*asin(c*x))/x**3/(-c**2*d*x**2+d),x)
```

output

```
-(Integral(a/(c**2*x**5 - x**3), x) + Integral(b*asin(c*x)/(c**2*x**5 - x*
*3), x))/d
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2)
)*a - b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*d*x^5 -
d*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)} dx$$

$$= \frac{-2 \left(\int \frac{a \arcsin(cx)}{c^2 x^5 - x^3} dx \right) b x^2 - \log(c^2 x - c) a c^2 x^2 - \log(c^2 x + c) a c^2 x^2 + 2 \log(x) a c^2 x^2 - a}{2 d x^2}$$

input `int((a+b*asin(c*x))/x^3/(-c^2*d*x^2+d),x)`

output `(- 2*int(asin(c*x)/(c**2*x**5 - x**3),x)*b*x**2 - log(c**2*x - c)*a*c**2*x**2 - log(c**2*x + c)*a*c**2*x**2 + 2*log(x)*a*c**2*x**2 - a)/(2*d*x**2)`

3.36 $\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)} dx$

Optimal result	492
Mathematica [B] (verified)	493
Rubi [A] (verified)	493
Maple [A] (verified)	498
Fricas [F]	498
Sympy [F]	499
Maxima [F]	499
Giac [F(-2)]	499
Mupad [F(-1)]	500
Reduce [F]	500

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{a + b \arcsin(cx)}{x^4(d - c^2dx^2)} dx = -\frac{bc\sqrt{1 - c^2x^2}}{6dx^2} - \frac{a + b \arcsin(cx)}{3dx^3} - \frac{c^2(a + b \arcsin(cx))}{dx}$$

$$- \frac{2ic^3(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{d}$$

$$- \frac{7bc^3 \operatorname{arctanh}(\sqrt{1 - c^2x^2})}{6d} + \frac{ibc^3 \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d}$$

$$- \frac{ibc^3 \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d}$$

output

```
-1/6*b*c*(-c^2*x^2+1)^(1/2)/d/x^2-1/3*(a+b*arcsin(c*x))/d/x^3-c^2*(a+b*arcsin(c*x))/d/x-2*I*c^3*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d-7/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))/d+I*b*c^3*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d-I*b*c^3*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 350 vs. $2(173) = 346$.

Time = 0.40 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.02

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx = \frac{2a + 6ac^2x^2 + bcx\sqrt{1 - c^2x^2} + 2b \arcsin(cx) + 6bc^2x^2 \arcsin(cx) + 3ibc^3\pi x^3 \arcsin(cx) + 7bc^3x^3 \arctan\left(\frac{cx}{\sqrt{1 - c^2x^2}}\right)}{d^2}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)),x]`

output

```
-1/6*(2*a + 6*a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2] + 2*b*ArcSin[c*x] + 6*b*c^2*x^2*ArcSin[c*x] + (3*I)*b*c^3*Pi*x^3*ArcSin[c*x] + 7*b*c^3*x^3*ArcTanh[Sqrt[1 - c^2*x^2]] - 3*b*c^3*Pi*x^3*Log[1 - I*E^(I*ArcSin[c*x])] - 6*b*c^3*x^3*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 3*b*c^3*Pi*x^3*Log[1 + I*E^(I*ArcSin[c*x])] + 6*b*c^3*x^3*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 3*a*c^3*x^3*Log[1 - c*x] - 3*a*c^3*x^3*Log[1 + c*x] + 3*b*c^3*Pi*x^3*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 3*b*c^3*Pi*x^3*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (6*I)*b*c^3*x^3*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (6*I)*b*c^3*x^3*PolyLog[2, I*E^(I*ArcSin[c*x])])/(d*x^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5204, 27, 243, 52, 73, 221, 5204, 243, 73, 221, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx$$

↓ 5204

$$\begin{aligned}
& c^2 \int \frac{a + b \arcsin(cx)}{dx^2(1 - c^2x^2)} dx + \frac{bc \int \frac{1}{x^3\sqrt{1-c^2x^2}} dx}{3d} - \frac{a + b \arcsin(cx)}{3dx^3} \\
& \quad \downarrow 27 \\
& \frac{c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)} dx}{d} + \frac{bc \int \frac{1}{x^3\sqrt{1-c^2x^2}} dx}{3d} - \frac{a + b \arcsin(cx)}{3dx^3} \\
& \quad \downarrow 243 \\
& \frac{c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)} dx}{d} + \frac{bc \int \frac{1}{x^4\sqrt{1-c^2x^2}} dx^2}{6d} - \frac{a + b \arcsin(cx)}{3dx^3} \\
& \quad \downarrow 52 \\
& \frac{c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)} dx}{d} + \frac{bc \left(\frac{1}{2} c^2 \int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{\sqrt{1-c^2x^2}}{x^2} \right)}{6d} - \frac{a + b \arcsin(cx)}{3dx^3} \\
& \quad \downarrow 73 \\
& \frac{c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)} dx}{d} + \frac{bc \left(- \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1 - c^2x^2} - \frac{\sqrt{1-c^2x^2}}{x^2} \right)}{6d} - \frac{a + b \arcsin(cx)}{3dx^3} \\
& \quad \downarrow 221 \\
& \frac{c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)} dx}{d} - \frac{a + b \arcsin(cx)}{3dx^3} + \frac{bc \left(c^2 \left(-\operatorname{arctanh} \left(\sqrt{1 - c^2x^2} \right) \right) - \frac{\sqrt{1-c^2x^2}}{x^2} \right)}{6d} \\
& \quad \downarrow 5204 \\
& \frac{c^2 \left(c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + bc \int \frac{1}{x\sqrt{1-c^2x^2}} dx - \frac{a+b \arcsin(cx)}{x} \right)}{d} - \frac{a + b \arcsin(cx)}{3dx^3} + \\
& \quad \frac{bc \left(c^2 \left(-\operatorname{arctanh} \left(\sqrt{1 - c^2x^2} \right) \right) - \frac{\sqrt{1-c^2x^2}}{x^2} \right)}{6d} \\
& \quad \downarrow 243 \\
& \frac{c^2 \left(c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{1}{2} bc \int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{a+b \arcsin(cx)}{x} \right)}{d} - \frac{a + b \arcsin(cx)}{3dx^3} + \\
& \quad \frac{bc \left(c^2 \left(-\operatorname{arctanh} \left(\sqrt{1 - c^2x^2} \right) \right) - \frac{\sqrt{1-c^2x^2}}{x^2} \right)}{6d} \\
& \quad \downarrow 73
\end{aligned}$$

↓ 2838

$$\frac{c^2 \left(c(-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) \right) - c^2 \left(\frac{a + b \arcsin(cx)}{3dx^3} + \frac{bc \left(c^2 \left(-\operatorname{arctanh} \left(\sqrt{1 - c^2 x^2} \right) \right) - \frac{\sqrt{1 - c^2 x^2}}{x^2} \right)}{6d} \right)}{d}$$

input `Int[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)),x]`

output `-1/3*(a + b*ArcSin[c*x])/(d*x^3) + (b*c*(-(Sqrt[1 - c^2*x^2]/x^2) - c^2*ArcTanH[Sqrt[1 - c^2*x^2]]))/(6*d) + (c^2*(-((a + b*ArcSin[c*x])/x) - b*c*ArcTanH[Sqrt[1 - c^2*x^2]] + c*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])))/d`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 243 $\text{Int}[(x_)^{(m_.)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m-1)/2)*(a+b*x)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c+d*x)})^n), x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c+d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e+f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c+d*x)^{(m-1)}*\text{Log}[1 - E^{(I*k*Pi)*E^{(I*(e+f*x))}], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c+d*x)^{(m-1)}*\text{Log}[1 + E^{(I*k*Pi)*E^{(I*(e+f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_.)}/((d_) + (e_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \text{ Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5204 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_.)}*((f_)*(x_))^{(m_.)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1))) \text{ Int}[(f*x)^{(m+2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.45

method	result
derivativedivides	$c^3 \left(-\frac{a \left(-\frac{\ln(cx+1)}{2} + \frac{1}{3c^3x^3} + \frac{1}{cx} + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b \left(\frac{6c^2x^2 \arcsin(cx) + cx\sqrt{-c^2x^2+1} + 2\arcsin(cx)}{6c^3x^3} - \frac{7 \ln \left(icx + \sqrt{-c^2x^2+1} \right)}{6} \right)}{d} \right)$
default	$c^3 \left(-\frac{a \left(-\frac{\ln(cx+1)}{2} + \frac{1}{3c^3x^3} + \frac{1}{cx} + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b \left(\frac{6c^2x^2 \arcsin(cx) + cx\sqrt{-c^2x^2+1} + 2\arcsin(cx)}{6c^3x^3} - \frac{7 \ln \left(icx + \sqrt{-c^2x^2+1} \right)}{6} \right)}{d} \right)$
parts	$-\frac{a \left(\frac{c^3 \ln(cx-1)}{2} - \frac{c^3 \ln(cx+1)}{2} + \frac{1}{3x^3} + \frac{c^2}{x} \right)}{d} - \frac{b c^3 \left(\frac{6c^2x^2 \arcsin(cx) + cx\sqrt{-c^2x^2+1} + 2\arcsin(cx)}{6c^3x^3} - \frac{7 \ln \left(icx + \sqrt{-c^2x^2+1} \right)}{6} \right)}{d}$

input `int((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output `c^3*(-a/d*(-1/2*ln(c*x+1)+1/3/c^3/x^3+1/c/x+1/2*ln(c*x-1))-b/d*(1/6*(6*c^2*x^2*arcsin(c*x)+c*x*(-c^2*x^2+1)^(1/2)+2*arcsin(c*x))/c^3/x^3-7/6*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)+7/6*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)/(c^2*d*x^6 - d*x^4), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx = -\frac{\int \frac{a}{c^2 x^6 - x^4} dx + \int \frac{b \arcsin(cx)}{c^2 x^6 - x^4} dx}{d}$$

input `integrate((a+b*asin(c*x))/x**4/(-c**2*d*x**2+d),x)`

output `-(Integral(a/(c**2*x**6 - x**4), x) + Integral(b*asin(c*x)/(c**2*x**6 - x**4), x))/d`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^3))*a + 1/6*(3*c^3*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(c*x + 1) - 3*c^3*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(-c*x + 1) + 6*d*x^3*integrate(1/6*(3*c^4*x^3*log(c*x + 1) - 3*c^4*x^3*log(-c*x + 1) - 6*c^3*x^2 - 2*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^5 - d*x^3), x) - 2*(3*c^2*x^2 + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*b/(d*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)} dx$$

$$= \frac{-6 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^6 - x^4} dx \right) b x^3 - 3 \log(c^2 x - c) a c^3 x^3 + 3 \log(c^2 x + c) a c^3 x^3 - 6 a c^2 x^2 - 2 a}{6 d x^3}$$

input `int((a+b*asin(c*x))/x^4/(-c^2*d*x^2+d),x)`

output `(- 6*int(asin(c*x)/(c**2*x**6 - x**4),x)*b*x**3 - 3*log(c**2*x - c)*a*c**
3*x**3 + 3*log(c**2*x + c)*a*c**3*x**3 - 6*a*c**2*x**2 - 2*a)/(6*d*x**3)`

3.37 $\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx$

Optimal result	501
Mathematica [A] (verified)	502
Rubi [A] (verified)	502
Maple [A] (verified)	507
Fricas [F]	507
Sympy [F]	508
Maxima [F]	508
Giac [F]	509
Mupad [F(-1)]	509
Reduce [F]	509

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx = -\frac{b}{2c^5d^2\sqrt{1-c^2x^2}} + \frac{b\sqrt{1-c^2x^2}}{c^5d^2} + \frac{3x(a+b \arcsin(cx))}{2c^4d^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} + \frac{3i(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^5d^2} - \frac{3ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{2c^5d^2} + \frac{3ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c^5d^2}$$

output

```
-1/2*b/c^5/d^2/(-c^2*x^2+1)^(1/2)+b*(-c^2*x^2+1)^(1/2)/c^5/d^2+3/2*x*(a+b*arcsin(c*x))/c^4/d^2+1/2*x^3*(a+b*arcsin(c*x))/c^2/d^2/(-c^2*x^2+1)+3*I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^5/d^2-3/2*I*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^2+3/2*I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^2
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.78

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

$$= \frac{4acx + 4b\sqrt{1 - c^2 x^2} + \frac{b\sqrt{1 - c^2 x^2}}{-1 + cx} - \frac{b\sqrt{1 - c^2 x^2}}{1 + cx} - \frac{2acx}{-1 + c^2 x^2} + 3ib\pi \arcsin(cx) + 4bcx \arcsin(cx) + \frac{b \arcsin(cx)}{1 - cx} - \dots}{1 - cx}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]
```

output

```
(4*a*c*x + 4*b*Sqrt[1 - c^2*x^2] + (b*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (b*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a*c*x)/(-1 + c^2*x^2) + (3*I)*b*Pi*ArcSin[c*x] + 4*b*c*x*ArcSin[c*x] + (b*ArcSin[c*x])/(1 - c*x) - (b*ArcSin[c*x])/(1 + c*x) - 3*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 6*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 3*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 6*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 3*a*Log[1 - c*x] - 3*a*Log[1 + c*x] + 3*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 3*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (6*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (6*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(4*c^5*d^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5206, 27, 243, 53, 2009, 5210, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

$$\downarrow \text{5206}$$

$$= \frac{3 \int \frac{x^2(a + b \arcsin(cx))}{d(1 - c^2 x^2)} dx}{2c^2 d} - \frac{b \int \frac{x^3}{(1 - c^2 x^2)^{3/2}} dx}{2cd^2} + \frac{x^3(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)}$$

$$\begin{aligned}
& \downarrow 27 \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \int \frac{x^3}{(1-c^2x^2)^{3/2}} dx}{2cd^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 243 \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \int \frac{x^2}{(1-c^2x^2)^{3/2}} dx^2}{4cd^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 53 \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \int \left(\frac{1}{c^2(1-c^2x^2)^{3/2}} - \frac{1}{c^2\sqrt{1-c^2x^2}} \right) dx^2}{4cd^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 2009 \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2d^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2} \\
& \downarrow 5210 \\
& -\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{2c^2d^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \\
& \quad \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2} \\
& \downarrow 241 \\
& -\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{2c^2d^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \\
& \quad \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2} \\
& \downarrow 5164 \\
& -\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{2c^2d^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \\
& \quad \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2} \\
& \downarrow 3042
\end{aligned}$$

$$3 \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right) + \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2}$$

4669

$$3 \left(\frac{-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} \right) - \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2}$$

2715

$$3 \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} \right) - \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2}$$

2838

$$3 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right) - \frac{x^3(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4cd^2}$$

input

Int[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]

output

-1/4*(b*(2/(c^4*sqrt[1 - c^2*x^2]) + (2*sqrt[1 - c^2*x^2])/c^4))/(c*d^2) + (x^3*(a + b*ArcSin[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (3*(-((b*sqrt[1 - c^2*x^2])/c^3) - (x*(a + b*ArcSin[c*x]))/c^2 + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/c^3))/(2*c^2*d^2)

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 53 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$
- rule 241 $\text{Int}[(x_)*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p + 1)} / (2*b*(p + 1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 243 $\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m - 1)/2} * (a + b*x)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_.)*((F_)^{(e_.)*((c_.) + (d_.)(x_))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n)], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2838 $\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
  => Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x]
  && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)^2), x_Symbol]
  => Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /;
  FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
  => Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
  Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))
  Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
  FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
  => Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1)))
  Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))
  Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
  FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a\left(cx - \frac{1}{4(cx+1)} - \frac{3\ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{3\ln(cx-1)}{4}\right)}{d^2} + \frac{b\sqrt{-c^2x^2+1}}{d^2} + \frac{b\arcsin(cx)cx}{d^2} - \frac{b\arcsin(cx)cx}{2d^2(c^2x^2-1)} + \frac{b\sqrt{-c^2x^2+1}}{2d^2(c^2x^2-1)} + \frac{3b\arcsin(cx)}{2d^2(c^2x^2-1)}$
default	$\frac{a\left(cx - \frac{1}{4(cx+1)} - \frac{3\ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{3\ln(cx-1)}{4}\right)}{d^2} + \frac{b\sqrt{-c^2x^2+1}}{d^2} + \frac{b\arcsin(cx)cx}{d^2} - \frac{b\arcsin(cx)cx}{2d^2(c^2x^2-1)} + \frac{b\sqrt{-c^2x^2+1}}{2d^2(c^2x^2-1)} + \frac{3b\arcsin(cx)}{2d^2(c^2x^2-1)}$
parts	$\frac{a\left(\frac{x}{c^4} - \frac{1}{4c^5(cx-1)} + \frac{3\ln(cx-1)}{4c^5} - \frac{1}{4c^5(cx+1)} - \frac{3\ln(cx+1)}{4c^5}\right)}{d^2} + \frac{b\sqrt{-c^2x^2+1}}{c^5d^2} + \frac{b\arcsin(cx)x}{d^2c^4} - \frac{b\arcsin(cx)x}{2d^2c^4(c^2x^2-1)} + \frac{b\sqrt{-c^2x^2+1}}{2d^2c^4(c^2x^2-1)}$

input `int(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output
$$\frac{1}{c^5} \left(\frac{a}{d^2} \left(cx - \frac{1}{4(cx+1)} - \frac{3}{4} \ln(cx+1) - \frac{1}{4(cx-1)} + \frac{3}{4} \ln(cx-1) \right) + \frac{b}{d^2} \sqrt{-c^2x^2+1} + \frac{b}{d^2} \arcsin(cx) cx - \frac{b}{2d^2} \frac{\arcsin(cx) cx}{c^2x^2-1} + \frac{b}{2d^2} \frac{\sqrt{-c^2x^2+1}}{c^2x^2-1} + \frac{3b}{2d^2} \frac{\arcsin(cx)}{c^2x^2-1} \right)$$

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(c^2dx^2 - d)^2} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*x^4*arcsin(c*x) + a*x^4)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{\frac{ax^4}{c^4 x^4 - 2c^2 x^2 + 1}}{d^2} dx + \int \frac{\frac{bx^4 \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1}}{d^2} dx$$

input `integrate(x**4*(a+b*asin(c*x))/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**4*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/4*a*(2*x/(c^6*d^2*x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^5*d^2) - 3*log(c*x - 1)/(c^5*d^2)) - 1/4*(3*(c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 3*(c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(2*c^3*x^3 - 3*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 4*(c^7*d^2*x^2 - c^5*d^2)*integrate(-1/4*(4*c^3*x^3 - 6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))*b/(c^7*d^2*x^2 - c^5*d^2)`

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)*x^4/(c^2*d*x^2 - d)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^2} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2,x)`

output `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

$$= \frac{4 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^7 x^2 - 4 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^5 + 3 \log(c^2 x - c) a c^2 x^2 - 3 \log(c^2 x - c) a - 3 \log(c^2 x - c) a}{4c^5 d^2 (c^2 x^2 - 1)}$$

input `int(x^4*(a+b*asin(c*x))/(-c^2*d*x^2+d)^2,x)`

output

```
(4*int((asin(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**7*x**2 - 4*int((asin(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**5 + 3*log(c**2*x - c)*a*c**2*x**2 - 3*log(c**2*x - c)*a - 3*log(c**2*x + c)*a*c**2*x**2 + 3*log(c**2*x + c)*a + 4*a*c**3*x**3 - 6*a*c*x)/(4*c**5*d**2*(c**2*x**2 - 1))
```

3.38
$$\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx$$

Optimal result	511
Mathematica [B] (verified)	512
Rubi [A] (verified)	512
Maple [A] (verified)	516
Fricas [F]	516
Sympy [F]	517
Maxima [F]	517
Giac [F(-2)]	518
Mupad [F(-1)]	518
Reduce [F]	518

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx = -\frac{bx}{2c^3d^2\sqrt{1-c^2x^2}} + \frac{b \arcsin(cx)}{2c^4d^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{i(a+b \arcsin(cx))^2}{2bc^4d^2} + \frac{(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{c^4d^2} - \frac{ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2c^4d^2}$$

output

```
-1/2*b*x/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*b*arcsin(c*x)/c^4/d^2+1/2*x^2*(a+b
*arcsin(c*x))/c^2/d^2/(-c^2*x^2+1)-1/2*I*(a+b*arcsin(c*x))^2/b/c^4/d^2+(a
b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*I*b*polylog(
2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 334 vs. $2(155) = 310$.

Time = 0.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.15

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

$$= \frac{b\sqrt{1-c^2x^2}}{-1+cx} + \frac{b\sqrt{1-c^2x^2}}{1+cx} - \frac{2a}{-1+c^2x^2} + 4ib\pi \arcsin(cx) + \frac{b \arcsin(cx)}{1-cx} + \frac{b \arcsin(cx)}{1+cx} - 2ib \arcsin(cx)^2 + 8b\pi \log(1 +$$

input `Integrate[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]`

output

```
((b*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (b*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a
)/(-1 + c^2*x^2) + (4*I)*b*Pi*ArcSin[c*x] + (b*ArcSin[c*x])/(1 - c*x) + (b
*ArcSin[c*x])/(1 + c*x) - (2*I)*b*ArcSin[c*x]^2 + 8*b*Pi*Log[1 + E^((-I)*A
rcSin[c*x])] + 2*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 4*b*ArcSin[c*x]*Log[1
- I*E^(I*ArcSin[c*x])] - 2*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 4*b*ArcSin
[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*a*Log[1 - c^2*x^2] - 8*b*Pi*Log[Cos
[ArcSin[c*x]/2]] + 2*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - 2*b*Pi*Log[S
in[(Pi + 2*ArcSin[c*x])/4]] - (4*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] -
(4*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(4*c^4*d^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5206, 27, 252, 223, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

↓ 5206

$$\begin{aligned}
& -\frac{\int \frac{x(a+b \arcsin(cx))}{d(1-c^2x^2)} dx}{c^2d} - \frac{b \int \frac{x^2}{(1-c^2x^2)^{3/2}} dx}{2cd^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow 27 \\
& -\frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d^2} - \frac{b \int \frac{x^2}{(1-c^2x^2)^{3/2}} dx}{2cd^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow 252 \\
& -\frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d^2} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{c^2} \right)}{2cd^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow 223 \\
& -\frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2d^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2cd^2} \\
& \quad \downarrow 5180 \\
& -\frac{\int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^4d^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2cd^2} \\
& \quad \downarrow 3042 \\
& -\frac{\int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^4d^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \\
& \quad \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2cd^2} \\
& \quad \downarrow 4202 \\
& -\frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^4d^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \\
& \quad \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2cd^2} \\
& \quad \downarrow 2620 \\
& -\frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right)}{c^4d^2} + \\
& \quad \frac{x^2(a+b \arcsin(cx))}{2c^2d^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2cd^2}
\end{aligned}$$

↓ 2715

$$\frac{\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(\frac{1}{4}b \int e^{-2i\arcsin(cx)} \log(1 + e^{2i\arcsin(cx)}) de^{2i\arcsin(cx)} - \frac{1}{2}i \log(1 + e^{2i\arcsin(cx)}) (a + b\arcsin(cx))\right)}{2c^2d^2(1 - c^2x^2)} - \frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

↓ 2838

$$\frac{\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1 + e^{2i\arcsin(cx)}) (a + b\arcsin(cx)) - \frac{1}{4}b \operatorname{PolyLog}(2, -e^{2i\arcsin(cx)})\right)}{2c^2d^2(1 - c^2x^2)} + \frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

input

```
Int[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]
```

output

```
(x^2*(a + b*ArcSin[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (b*(x/(c^2*sqrt[1 - c^2*x^2]) - ArcSin[c*x]/c^3))/(2*c*d^2) - (((1/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)/(c^4*d^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(2*b*(p+1))), x] - Simp[c^2*(m-1)/(2*b*(p+1)) Int[(c*x)^(m-2)*(a + b*x^2)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && LtQ[p, -1] && GtQ[m, 1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2620

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x))))], x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5180

```
Int[(((a_) + ArcSin[(c_)*(x_)])*(b_))^(n_)*(x_)/((d_) + (e_)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5206

```
Int[(((a_) + ArcSin[(c_)*(x_)])*(b_))^(n_)*((f_)*(x_))^(m_)*((d_) + (e_
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```


Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a \left(\frac{1}{4cx+4} + \frac{\ln(cx+1)}{2} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{2} \right)}{d^2} + \frac{b \left(-\frac{i \arcsin(cx)^2}{2} - \frac{ic^2x^2 - cx\sqrt{-c^2x^2+1} + \arcsin(cx) - i}{2(c^2x^2-1)} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{c^2x^2+1}) \right) \right)}{c^4 d^2}$
default	$\frac{a \left(\frac{1}{4cx+4} + \frac{\ln(cx+1)}{2} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{2} \right)}{d^2} + \frac{b \left(-\frac{i \arcsin(cx)^2}{2} - \frac{ic^2x^2 - cx\sqrt{-c^2x^2+1} + \arcsin(cx) - i}{2(c^2x^2-1)} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{c^2x^2+1}) \right) \right)}{c^4 d^2}$
parts	$\frac{a \left(-\frac{1}{4c^4(cx-1)} + \frac{\ln(cx-1)}{2c^4} + \frac{1}{4c^4(cx+1)} + \frac{\ln(cx+1)}{2c^4} \right)}{d^2} + \frac{b \left(-\frac{i \arcsin(cx)^2}{2} - \frac{ic^2x^2 - cx\sqrt{-c^2x^2+1} + \arcsin(cx) - i}{2(c^2x^2-1)} + \arcsin(cx) \ln \left(1 + (icx + \sqrt{c^2x^2+1}) \right) \right)}{d^2}$

input

```
int(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c^4*(a/d^2*(1/4/(c*x+1)+1/2*ln(c*x+1)-1/4/(c*x-1)+1/2*ln(c*x-1))+b/d^2*(-1/2*I*arcsin(c*x)^2-1/2*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x)-I)/(c^2*x^2-1)+arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^3}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*x^3*arcsin(c*x) + a*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{ax^3}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{bx^3 \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input `integrate(x**3*(a+b*asin(c*x))/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**3*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^3}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/2*a*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2)) + 1/2*((c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) + (c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) + 2*(c^6*d^2*x^2 - c^4*d^2)*integrate(1/2*((c^2*x^2 - 1)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) + (c^2*x^2 - 1)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(-c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x) - arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*b/(c^6*d^2*x^2 - c^4*d^2)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^2} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2,x)`

output `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

$$= \frac{2 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^6 x^2 - 2 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^4 + \log(c^2 x - c) a c^2 x^2 - \log(c^2 x - c) a + \log(c^2 x + c) a}{2c^4 d^2 (c^2 x^2 - 1)}$$

input `int(x^3*(a+b*asin(c*x))/(-c^2*d*x^2+d)^2,x)`

output

```
(2*int((asin(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**6*x**2 - 2*int((asin(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**4 + log(c**2*x - c)*a*c**2*x**2 - log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a - a*c**2*x**2)/(2*c**4*d**2*(c**2*x**2 - 1))
```

3.39 $\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx$

Optimal result	520
Mathematica [B] (verified)	521
Rubi [A] (verified)	521
Maple [A] (verified)	524
Fricas [F]	525
Sympy [F]	525
Maxima [F]	526
Giac [F]	526
Mupad [F(-1)]	526
Reduce [F]	527

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2dx^2)^2} dx = -\frac{b}{2c^3d^2\sqrt{1 - c^2x^2}} + \frac{x(a + b \arcsin(cx))}{2c^2d^2(1 - c^2x^2)} + \frac{i(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^3d^2} - \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{2c^3d^2} + \frac{ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c^3d^2}$$

output

```
-1/2*b/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsin(c*x))/c^2/d^2/(-c^2*x^2+1)+I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^3/d^2-1/2*I*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^2+1/2*I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 463 vs. $2(144) = 288$.

Time = 0.35 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.22

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = -\frac{ax}{2c^2 d^2 (-1 + c^2 x^2)} + \frac{a \log(1 - cx)}{4c^3 d^2} - \frac{a \log(1 + cx)}{4c^3 d^2} + b \left(\frac{\sqrt{1-c^2 x^2} - \arcsin(cx)}{4c^3 (-1+cx)} - \frac{\sqrt{1-c^2 x^2} + \arcsin(cx)}{4c^2 (c+c^2 x)} + \frac{3i\pi \arcsin(cx)}{2c} - \frac{i \arcsin(cx)^2}{2c} + \frac{2\pi \log(1+e^{-i \arcsin(cx)})}{c} - \frac{\pi \log(1+ie^{i \arcsin(cx)})}{c} + \frac{2 \arcsin(cx)}{c} \right)$$

input `Integrate[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]`

output

```
-1/2*(a*x)/(c^2*d^2*(-1 + c^2*x^2)) + (a*Log[1 - c*x])/(4*c^3*d^2) - (a*Log[1 + c*x])/(4*c^3*d^2) + (b*((Sqrt[1 - c^2*x^2] - ArcSin[c*x])/(4*c^3*(-1 + c*x)) - (Sqrt[1 - c^2*x^2] + ArcSin[c*x])/(4*c^2*(c + c^2*x)) + (((3*I)/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c - (Pi*Log[1 + I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c + (Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/c)/(4*c^2) - (((I/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c + (Pi*Log[1 - I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c - (Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, I*E^(I*ArcSin[c*x])])/c)/(4*c^2))/d^2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5206, 27, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx \\
& \quad \downarrow \text{5206} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{d(1-c^2 x^2)} dx}{2c^2 d} - \frac{b \int \frac{x}{(1-c^2 x^2)^{3/2}} dx}{2cd^2} + \frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} \\
& \quad \downarrow \text{27} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{1-c^2 x^2} dx}{2c^2 d^2} - \frac{b \int \frac{x}{(1-c^2 x^2)^{3/2}} dx}{2cd^2} + \frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} \\
& \quad \downarrow \text{241} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{1-c^2 x^2} dx}{2c^2 d^2} + \frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b}{2c^3 d^2 \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5164} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} d \arcsin(cx)}{2c^3 d^2} + \frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b}{2c^3 d^2 \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{3042} \\
& -\frac{\int (a + b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c^3 d^2} + \frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b}{2c^3 d^2 \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{4669} \\
& -\frac{-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{2c^3 d^2} \\
& \quad \downarrow \text{2715} \\
& -\frac{ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{2c^3 d^2} \\
& \quad \downarrow \text{2838} \\
& -\frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b}{2c^3 d^2 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

$$-\frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c^3 d^2} + \frac{x(a + b \arcsin(cx))}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b}{2c^3 d^2 \sqrt{1 - c^2 x^2}}$$

input `Int[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]`

output `-1/2*b/(c^3*d^2*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c^3*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x]
  && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /;
  FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)
  *(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
  b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
  Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
  [b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m -
  1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
  a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
  tQ[m, 1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a \left(-\frac{1}{4(cx+1)} - \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{4} \right) + b \left(-\frac{cx \arcsin(cx) - \sqrt{-c^2x^2+1}}{2(c^2x^2-1)} + \frac{\arcsin(cx) \ln(1+i \frac{icx + \sqrt{-c^2x^2+1}}{2})}{2} - \frac{\arcsin(cx) \ln(1-i \frac{icx + \sqrt{-c^2x^2+1}}{2})}{2} \right)}{d^2}$
default	$\frac{a \left(-\frac{1}{4(cx+1)} - \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{4} \right) + b \left(-\frac{cx \arcsin(cx) - \sqrt{-c^2x^2+1}}{2(c^2x^2-1)} + \frac{\arcsin(cx) \ln(1+i \frac{icx + \sqrt{-c^2x^2+1}}{2})}{2} - \frac{\arcsin(cx) \ln(1-i \frac{icx + \sqrt{-c^2x^2+1}}{2})}{2} \right)}{d^2}$
parts	$\frac{a \left(-\frac{1}{4c^3(cx-1)} + \frac{\ln(cx-1)}{4c^3} - \frac{1}{4c^3(cx+1)} - \frac{\ln(cx+1)}{4c^3} \right) + b \left(-\frac{cx \arcsin(cx) - \sqrt{-c^2x^2+1}}{2(c^2x^2-1)} + \frac{\arcsin(cx) \ln(1+i \frac{icx + \sqrt{-c^2x^2+1}}{2})}{2} - \frac{\arcsin(cx) \ln(1-i \frac{icx + \sqrt{-c^2x^2+1}}{2})}{2} \right)}{d^2}$

input

```
int(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c^3*(a/d^2*(-1/4/(c*x+1)-1/4*ln(c*x+1)-1/4/(c*x-1)+1/4*ln(c*x-1))+b/d^2*
(-1/2*(c*x*arcsin(c*x)-(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)+1/2*arcsin(c*x)*ln(
1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)
^(1/2)))-1/2*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/2*I*dilog(1-I*(I*c*
x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*x^2*arcsin(c*x) + a*x^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{\frac{ax^2}{c^4 x^4 - 2c^2 x^2 + 1} dx}{d^2} + \int \frac{bx^2 \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input

```
integrate(x**2*(a+b*asin(c*x))/(-c**2*d*x**2+d)**2,x)
```

output

```
(Integral(a*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**2*asin(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/4*a*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2)) - 1/4*(2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - (c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) + 4*(c^5*d^2*x^2 - c^3*d^2)*integrate(1/4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x))*b/(c^5*d^2*x^2 - c^3*d^2)`

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)*x^2/(c^2*d*x^2 - d)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^2} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2,x)`

output `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^5 x^2 - 4 \left(\int \frac{\arcsin(cx)x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^3 + \log(c^2 x - c) a c^2 x^2 - \log(c^2 x - c) a - \log(c^2 x + c) a c^2 x^2 + \log(c^2 x + c) a}{4c^3 d^2 (c^2 x^2 - 1)}$$

input `int(x^2*(a+b*asin(c*x))/(-c^2*d*x^2+d)^2,x)`

output `(4*int((asin(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**5*x**2 - 4*int((asin(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3 + log(c**2*x - c)*a*c**2*x**2 - log(c**2*x - c)*a - log(c**2*x + c)*a*c**2*x**2 + log(c**2*x + c)*a - 2*a*c*x)/(4*c**3*d**2*(c**2*x**2 - 1))`

3.40 $\int \frac{x(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx$

Optimal result	528
Mathematica [A] (verified)	528
Rubi [A] (verified)	529
Maple [A] (verified)	530
Fricas [A] (verification not implemented)	530
Sympy [F]	531
Maxima [B] (verification not implemented)	531
Giac [A] (verification not implemented)	532
Mupad [F(-1)]	532
Reduce [F]	532

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^2} dx = -\frac{bx}{2cd^2\sqrt{1 - c^2x^2}} + \frac{a + b \arcsin(cx)}{2c^2d^2(1 - c^2x^2)}$$

output $-1/2*b*x/c/d^2/(-c^2*x^2+1)^{(1/2)}+1/2*(a+b*\arcsin(c*x))/c^2/d^2/(-c^2*x^2+1)$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^2} dx = \frac{a - bcx\sqrt{1 - c^2x^2} + b \arcsin(cx)}{2c^2d^2 - 2c^4d^2x^2}$$

input `Integrate[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]`

output $(a - b*c*x*\text{Sqrt}[1 - c^2*x^2] + b*\text{ArcSin}[c*x])/(2*c^2*d^2 - 2*c^4*d^2*x^2)$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5182, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx$$

↓ 5182

$$\frac{a + b \arcsin(cx)}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b \int \frac{1}{(1 - c^2 x^2)^{3/2}} dx}{2cd^2}$$

↓ 208

$$\frac{a + b \arcsin(cx)}{2c^2 d^2 (1 - c^2 x^2)} - \frac{bx}{2cd^2 \sqrt{1 - c^2 x^2}}$$

input `Int[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]`

output `-1/2*(b*x)/(c*d^2*Sqrt[1 - c^2*x^2]) + (a + b*ArcSin[c*x])/(2*c^2*d^2*(1 - c^2*x^2))`

Defintions of rubi rules used

rule 208 `Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[x/(a*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b}, x]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{-\frac{a}{2d^2(c^2x^2-1)} + \frac{b\left(-\frac{\arcsin(cx)}{2(c^2x^2-1)} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{4cx-4} + \frac{\sqrt{-(cx+1)^2+2cx+2}}{4cx+4}\right)}{d^2}}{c^2}$
default	$\frac{-\frac{a}{2d^2(c^2x^2-1)} + \frac{b\left(-\frac{\arcsin(cx)}{2(c^2x^2-1)} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{4cx-4} + \frac{\sqrt{-(cx+1)^2+2cx+2}}{4cx+4}\right)}{d^2}}{c^2}$
parts	$-\frac{a}{2d^2c^2(c^2x^2-1)} + \frac{b\left(-\frac{\arcsin(cx)}{2(c^2x^2-1)} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{4cx-4} + \frac{\sqrt{-(cx+1)^2+2cx+2}}{4cx+4}\right)}{d^2c^2}$
oring	$-\frac{(cx-1)(cx+1)(3c^2x^2+2)(a+b\arcsin(cx))}{2c^2(-c^2dx^2+d)^2} - \frac{(cx-1)^2(cx+1)^2\left(\frac{a+b\arcsin(cx)}{(-c^2dx^2+d)^2} + \frac{xbc}{\sqrt{-c^2x^2+1}(-c^2dx^2+d)^2} + \frac{4x^2(a-b)}{(-c^2dx^2+d)^2}\right)}{2c^2}$

input `int(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output
$$\frac{1}{c^2} \left(-\frac{1}{2} \frac{a}{d^2} \frac{1}{(c^2x^2-1)} + \frac{b}{d^2} \left(-\frac{1}{2} \frac{\arcsin(cx)}{(c^2x^2-1)} + \frac{1}{4} \frac{(cx-1)\sqrt{-(cx-1)^2-2cx+2}}{(cx+1)\sqrt{-(cx+1)^2+2cx+2}} \right) \right)$$

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^2} dx = -\frac{ac^2x^2 - \sqrt{-c^2x^2 + 1}bcx + b \arcsin(cx)}{2(c^4d^2x^2 - c^2d^2)}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output
$$-\frac{1}{2} \frac{(ac^2x^2 - \sqrt{-c^2x^2 + 1}bcx + b\arcsin(cx))}{(c^4d^2x^2 - c^2d^2)}$$

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{\frac{ax}{c^4 x^4 - 2c^2 x^2 + 1}}{d^2} dx + \int \frac{\frac{bx \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1}}{d^2} dx$$

input `integrate(x*(a+b*asin(c*x))/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(50) = 100$.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\begin{aligned} & \int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx \\ &= \frac{1}{4} \left(\left(\frac{\sqrt{-c^2 x^2 + 1} c^2 d^2}{c^7 d^4 x + c^6 d^4} + \frac{\sqrt{-c^2 x^2 + 1} c^2 d^2}{c^7 d^4 x - c^6 d^4} \right) c^2 - \frac{2 \arcsin(cx)}{c^4 d^2 x^2 - c^2 d^2} \right) b \\ & \quad - \frac{a}{2(c^4 d^2 x^2 - c^2 d^2)} \end{aligned}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `1/4*((sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 - 2*arcsin(c*x)/(c^4*d^2*x^2 - c^2*d^2))*b - 1/2*a/(c^4*d^2*x^2 - c^2*d^2)`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.56

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = -\frac{bx^2 \arcsin(cx)}{2(c^2 x^2 - 1)d^2} - \frac{ax^2}{2(c^2 x^2 - 1)d^2} - \frac{bx}{2\sqrt{-c^2 x^2 + 1}cd^2} + \frac{b \arcsin(cx)}{2c^2 d^2} + \frac{a}{2c^2 d^2}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `-1/2*b*x^2*arcsin(c*x)/((c^2*x^2 - 1)*d^2) - 1/2*a*x^2/((c^2*x^2 - 1)*d^2) - 1/2*b*x/(sqrt(-c^2*x^2 + 1)*c*d^2) + 1/2*b*arcsin(c*x)/(c^2*d^2) + 1/2*a/(c^2*d^2)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^2} dx$$

input `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2,x)`

output `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \frac{2\left(\int \frac{\operatorname{asin}(cx)x}{c^4 x^4 - 2c^2 x^2 + 1} dx\right) b c^2 x^2 - 2\left(\int \frac{\operatorname{asin}(cx)x}{c^4 x^4 - 2c^2 x^2 + 1} dx\right) b - a x^2}{2d^2 (c^2 x^2 - 1)}$$

input `int(x*(a+b*asin(c*x))/(-c^2*d*x^2+d)^2,x)`

output

```
(2*int((asin(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**2*x**2 - 2*int(
(asin(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b - a*x**2)/(2*d**2*(c**2*x
**2 - 1))
```

3.41 $\int \frac{a+b \arcsin(cx)}{(d-c^2dx^2)^2} dx$

Optimal result	534
Mathematica [B] (verified)	535
Rubi [A] (verified)	535
Maple [A] (verified)	538
Fricas [F]	539
Sympy [F]	539
Maxima [F]	539
Giac [F(-2)]	540
Mupad [F(-1)]	540
Reduce [F]	541

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{a + b \arcsin(cx)}{(d - c^2dx^2)^2} dx = -\frac{b}{2cd^2\sqrt{1 - c^2x^2}} + \frac{x(a + b \arcsin(cx))}{2d^2(1 - c^2x^2)} - \frac{i(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{cd^2} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{2cd^2} - \frac{ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2cd^2}$$

output

```
-1/2*b/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)-I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c/d^2+1/2*I*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^2-1/2*I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 334 vs. $2(141) = 282$.

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.37

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx =$$

$$\frac{b\sqrt{1-c^2x^2}}{c-c^2x} + \frac{b\sqrt{1-c^2x^2}}{c+c^2x} + \frac{2ax}{-1+c^2x^2} + \frac{ib\pi \arcsin(cx)}{c} + \frac{b \arcsin(cx)}{c(-1+cx)} + \frac{b \arcsin(cx)}{c+c^2x} - \frac{b\pi \log(1-ie^{i \arcsin(cx)})}{c} - \frac{2b \arcsin(cx) \log(1-ie^{i \arcsin(cx)})}{c}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^2,x]
```

output

```
-1/4*((b*Sqrt[1 - c^2*x^2])/(c - c^2*x) + (b*Sqrt[1 - c^2*x^2])/(c + c^2*x)
) + (2*a*x)/(-1 + c^2*x^2) + (I*b*Pi*ArcSin[c*x])/c + (b*ArcSin[c*x])/(c*(
-1 + c*x)) + (b*ArcSin[c*x])/(c + c^2*x) - (b*Pi*Log[1 - I*E^(I*ArcSin[c*x]
)])/c - (2*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])])/c - (b*Pi*Log[1 +
I*E^(I*ArcSin[c*x])])/c + (2*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c
+ (a*Log[1 - c*x])/c - (a*Log[1 + c*x])/c + (b*Pi*Log[-Cos[(Pi + 2*ArcSin
[c*x])/4]])/c + (b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*b*PolyL
og[2, (-I)*E^(I*ArcSin[c*x])])/c + ((2*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x]
)]/c)/d^2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5162, 27, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx$$

↓ 5162

$$\begin{aligned}
& \frac{\int \frac{a+b \arcsin(cx)}{d(1-c^2x^2)} dx}{2d} - \frac{bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx}{2d^2} + \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 27 \\
& \frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{2d^2} - \frac{bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx}{2d^2} + \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 241 \\
& \frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{2d^2} + \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} - \frac{b}{2cd^2\sqrt{1-c^2x^2}} \\
& \quad \downarrow 5164 \\
& \frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2cd^2} + \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} - \frac{b}{2cd^2\sqrt{1-c^2x^2}} \\
& \quad \downarrow 3042 \\
& \frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2cd^2} + \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} - \frac{b}{2cd^2\sqrt{1-c^2x^2}} \\
& \quad \downarrow 4669 \\
& \frac{-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx))}{2cd^2} \\
& \quad \downarrow 2715 \\
& \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} - \frac{2cd^2}{b \sqrt{1-c^2x^2}} \\
& \quad \downarrow 2838 \\
& \frac{ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx))}{2cd^2} \\
& \quad \downarrow 2838 \\
& \frac{-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2cd^2} + \\
& \quad \frac{x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} - \frac{b}{2cd^2\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^2,x]`

output `-1/2*b/(c*d^2*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*d^2*(1 - c^2*x^2)) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:= Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x]
+ (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x]
+ Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x]
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:= Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{a \left(-\frac{1}{4(cx+1)} + \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{4} \right) + b \left(-\frac{cx \arcsin(cx) - \sqrt{-c^2x^2+1}}{2(c^2x^2-1)} - \frac{\arcsin(cx) \ln \left(1+i \frac{icx + \sqrt{-c^2x^2+1}}{2} \right)}{2} + \frac{\arcsin(cx) \ln \left(1-i \frac{icx + \sqrt{-c^2x^2+1}}{2} \right)}{2} \right)}{d^2}$
default	$\frac{a \left(-\frac{1}{4(cx+1)} + \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{4} \right) + b \left(-\frac{cx \arcsin(cx) - \sqrt{-c^2x^2+1}}{2(c^2x^2-1)} - \frac{\arcsin(cx) \ln \left(1+i \frac{icx + \sqrt{-c^2x^2+1}}{2} \right)}{2} + \frac{\arcsin(cx) \ln \left(1-i \frac{icx + \sqrt{-c^2x^2+1}}{2} \right)}{2} \right)}{d^2}$
parts	$\frac{a \left(-\frac{1}{4c(cx-1)} - \frac{\ln(cx-1)}{4c} - \frac{1}{4c(cx+1)} + \frac{\ln(cx+1)}{4c} \right) + b \left(-\frac{cx \arcsin(cx) - \sqrt{-c^2x^2+1}}{2(c^2x^2-1)} - \frac{\arcsin(cx) \ln \left(1+i \frac{icx + \sqrt{-c^2x^2+1}}{2} \right)}{2} + \frac{\arcsin(cx) \ln \left(1-i \frac{icx + \sqrt{-c^2x^2+1}}{2} \right)}{2} \right)}{d^2}$

input

```
int((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c*(a/d^2*(-1/4/(c*x+1)+1/4*ln(c*x+1)-1/4/(c*x-1)-1/4*ln(c*x-1))+b/d^2*(-1/2*(c*x*arcsin(c*x)-(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)-1/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/2*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/2*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx = \frac{\int \frac{a}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx}{d^2}$$

input `integrate((a+b*asin(c*x))/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output

```
-1/4*a*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2)) - 1/4*(2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) + (c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 4*(c^3*d^2*x^2 - c*d^2)*integrate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x))*b/(c^3*d^2*x^2 - c*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{(d - c^2 dx^2)^2} dx$$

input

```
int((a + b*asin(c*x))/(d - c^2*d*x^2)^2,x)
```

output

```
int((a + b*asin(c*x))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^2} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c^3 x^2 - 4 \left(\int \frac{\arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b c - \log(c^2 x - c) a c^2 x^2 + \log(c^2 x - c) a + \log(c^2 x + c) a c^2 x^2 - \log(c^2 x + c) a}{4c d^2 (c^2 x^2 - 1)}$$

input `int((a+b*asin(c*x))/(-c^2*d*x^2+d)^2,x)`

output `(4*int(asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3*x**2 - 4*int(asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c - log(c**2*x - c)*a*c**2*x**2 + log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a - 2*a*c*x)/(4*c*d**2*(c**2*x**2 - 1))`

3.42 $\int \frac{a+b \arcsin(cx)}{x(d-c^2dx^2)^2} dx$

Optimal result	542
Mathematica [B] (verified)	543
Rubi [A] (verified)	543
Maple [A] (verified)	546
Fricas [F]	547
Sympy [F]	547
Maxima [F]	548
Giac [F(-2)]	548
Mupad [F(-1)]	548
Reduce [F]	549

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2dx^2)^2} dx = -\frac{bcx}{2d^2\sqrt{1 - c^2x^2}} + \frac{a + b \arcsin(cx)}{2d^2(1 - c^2x^2)} - \frac{2(a + b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^2} + \frac{ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2d^2} - \frac{ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{2d^2}$$

output

```
-1/2*b*c*x/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)-2
*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2+1/2*I*b*polyl
og(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-1/2*I*b*polylog(2,(I*c*x+(-c^2*x^2
+1)^(1/2))^2)/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 364 vs. $2(122) = 244$.

Time = 0.47 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.98

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx$$

$$= \frac{b\sqrt{1-c^2x^2}}{-1+cx} + \frac{b\sqrt{1-c^2x^2}}{1+cx} - \frac{2a}{-1+c^2x^2} - 4ib\pi \arcsin(cx) + \frac{b \arcsin(cx)}{1-cx} + \frac{b \arcsin(cx)}{1+cx} - 8b\pi \log(1 + e^{-i \arcsin(cx)}) - 2b\pi \log(1 - e^{-i \arcsin(cx)})$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

output

```
((b*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (b*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a)/(-1 + c^2*x^2) - (4*I)*b*Pi*ArcSin[c*x] + (b*ArcSin[c*x])/(1 - c*x) + (b*ArcSin[c*x])/(1 + c*x) - 8*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 2*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 4*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 4*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 4*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 4*a*Log[x] - 2*a*Log[1 - c^2*x^2] + 8*b*Pi*Log[Cos[ArcSin[c*x]/2]] - 2*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (4*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])] - (2*I)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5208, 27, 208, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx$$

↓ 5208

$$\frac{\int \frac{a+b \arcsin(cx)}{dx(1-c^2x^2)} dx}{d} - \frac{bc \int \frac{1}{(1-c^2x^2)^{3/2}} dx}{2d^2} + \frac{a+b \arcsin(cx)}{2d^2(1-c^2x^2)}$$

↓ 27

$$\frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d^2} - \frac{bc \int \frac{1}{(1-c^2x^2)^{3/2}} dx}{2d^2} + \frac{a+b \arcsin(cx)}{2d^2(1-c^2x^2)}$$

↓ 208

$$\frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d^2} + \frac{a+b \arcsin(cx)}{2d^2(1-c^2x^2)} - \frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

↓ 5184

$$\frac{\int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2x^2}} d \arcsin(cx)}{d^2} + \frac{a+b \arcsin(cx)}{2d^2(1-c^2x^2)} - \frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

↓ 4919

$$\frac{2 \int (a+b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d^2} + \frac{a+b \arcsin(cx)}{2d^2(1-c^2x^2)} - \frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

↓ 3042

$$\frac{2 \int (a+b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d^2} + \frac{a+b \arcsin(cx)}{2d^2(1-c^2x^2)} - \frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

↓ 4671

$$\frac{2(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})(a+b \arcsin(cx)))}{d^2} - \frac{bcx}{2d^2\sqrt{1-c^2x^2}}}$$

↓ 2715

$$\frac{2(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1-e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - (\frac{1}{4}ib \operatorname{arctanh}(e^{2i \arcsin(cx)})(a+b \arcsin(cx))))}{d^2} - \frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

↓ 2838

$$\frac{2(-(\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}))}{\frac{a + b \arcsin(cx)}{2d^2(1 - c^2x^2)} - \frac{d^2}{2d^2\sqrt{1 - c^2x^2}}}$$

input `Int[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^2),x]`

output `-1/2*(b*c*x)/(d^2*sqrt[1 - c^2*x^2]) + (a + b*ArcSin[c*x])/(2*d^2*(1 - c^2*x^2)) + (2*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/d^2`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 208 `Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[x/(a*sqrt[a + b*x^2]), x] /; FreeQ[{a, b}, x]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

```
rule 4671 Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

```
rule 4919 Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

```
rule 5184 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_.) + (e_.)*(x_)^2)),
x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSi
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

```
rule 5208 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.06

method	result
parts	$\frac{a\left(-\frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{2} + \frac{1}{4cx+4} - \frac{\ln(cx+1)}{2} + \ln(x)\right)}{d^2} + \frac{b\left(-\frac{ic^2x^2 - cx\sqrt{-c^2x^2+1} + \arcsin(cx) - i}{2(c^2x^2-1)} - \arcsin(cx)\ln\left(1 + (icx + \dots)\right)\right)}{d^2}$
derivativedivides	$\frac{a\left(\frac{1}{4cx+4} - \frac{\ln(cx+1)}{2} + \ln(cx) - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{2}\right)}{d^2} + \frac{b\left(-\frac{ic^2x^2 - cx\sqrt{-c^2x^2+1} + \arcsin(cx) - i}{2(c^2x^2-1)} - \arcsin(cx)\ln\left(1 + (icx + \dots)\right)\right)}{d^2}$
default	$\frac{a\left(\frac{1}{4cx+4} - \frac{\ln(cx+1)}{2} + \ln(cx) - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{2}\right)}{d^2} + \frac{b\left(-\frac{ic^2x^2 - cx\sqrt{-c^2x^2+1} + \arcsin(cx) - i}{2(c^2x^2-1)} - \arcsin(cx)\ln\left(1 + (icx + \dots)\right)\right)}{d^2}$

input `int((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output `a/d^2*(-1/4/(c*x-1)-1/2*ln(c*x-1)+1/4/(c*x+1)-1/2*ln(c*x+1)+ln(x))+b/d^2*(-1/2*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x)-I)/(c^2*x^2-1)-arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2)+arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx = \int \frac{a}{c^4 x^5 - 2c^2 x^3 + x} dx + \int \frac{b \arcsin(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx$$

input `integrate((a+b*asin(c*x))/x/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b*asin(c*x)/(c**4*x**5 - 2*c**2*x**3 + x), x))/d**2`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/2*a*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*log(x)/d^2) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x(d - c^2 dx^2)^2} dx$$

input `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^2),x)`

output `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^2} dx$$

$$= \frac{2 \left(\int \frac{\arcsin(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx \right) b c^2 x^2 - 2 \left(\int \frac{\arcsin(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx \right) b - \log(c^2 x - c) a c^2 x^2 + \log(c^2 x - c) a - \log(c^2 x + c) a c^2 x^2 + \log(c^2 x + c) a}{2d^2 (c^2 x^2 - 1)}$$

input `int((a+b*asin(c*x))/x/(-c^2*d*x^2+d)^2,x)`

output `(2*int(asin(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*b*c**2*x**2 - 2*int(asin(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*b - log(c**2*x - c)*a*c**2*x**2 + log(c**2*x - c)*a - log(c**2*x + c)*a*c**2*x**2 + log(c**2*x + c)*a + 2*log(x)*a*c**2*x**2 - 2*log(x)*a - a*c**2*x**2)/(2*d**2*(c**2*x**2 - 1))`

3.43 $\int \frac{a+b \arcsin(cx)}{x^2(d-c^2dx^2)^2} dx$

Optimal result	550
Mathematica [A] (verified)	551
Rubi [A] (verified)	551
Maple [A] (verified)	556
Fricas [F]	557
Sympy [F]	557
Maxima [F]	557
Giac [F(-2)]	558
Mupad [F(-1)]	558
Reduce [F]	559

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = -\frac{bc}{2d^2 \sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{d^2 x} + \frac{c^2 x (a + b \arcsin(cx))}{2d^2 (1 - c^2 x^2)} - \frac{3ic(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{d^2} - \frac{bc \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{d^2} + \frac{3ibc \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{2d^2} - \frac{3ibc \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2d^2}$$

output

```
-1/2*b*c/d^2/(-c^2*x^2+1)^(1/2)-(a+b*arcsin(c*x))/d^2/x+1/2*c^2*x*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)-3*I*c*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^2-b*c*arctanh((-c^2*x^2+1)^(1/2))/d^2+3/2*I*b*c*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2-3/2*I*b*c*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.00

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = \frac{4a}{x} + \frac{bc\sqrt{1-c^2x^2}}{1-cx} + \frac{bc\sqrt{1-c^2x^2}}{1+cx} + \frac{2ac^2x}{-1+c^2x^2} + 3ibc\pi \arcsin(cx) + \frac{4b \arcsin(cx)}{x} + \frac{bc \arcsin(cx)}{-1+cx} + \frac{bc \arcsin(cx)}{1+cx} + 4bca$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

output

```
-1/4*((4*a)/x + (b*c*Sqrt[1 - c^2*x^2])/(1 - c*x) + (b*c*Sqrt[1 - c^2*x^2])/(1 + c*x) + (2*a*c^2*x)/(-1 + c^2*x^2) + (3*I)*b*c*Pi*ArcSin[c*x] + (4*b*ArcSin[c*x])/x + (b*c*ArcSin[c*x])/(-1 + c*x) + (b*c*ArcSin[c*x])/(1 + c*x) + 4*b*c*ArcTanh[Sqrt[1 - c^2*x^2]] - 3*b*c*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 6*b*c*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 3*b*c*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 6*b*c*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 3*a*c*Log[1 - c*x] - 3*a*c*Log[1 + c*x] + 3*b*c*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 3*b*c*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (6*I)*b*c*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (6*I)*b*c*PolyLog[2, I*E^(I*ArcSin[c*x])])/d^2
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5204, 27, 243, 61, 73, 221, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx$$

↓ 5204

$$3c^2 \int \frac{a + b \arcsin(cx)}{d^2 (1 - c^2 x^2)^2} dx + \frac{bc \int \frac{1}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{a + b \arcsin(cx)}{d^2 x (1 - c^2 x^2)}$$

$$\begin{aligned}
& \downarrow 27 \\
& \frac{3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{d^2} + \frac{bc \int \frac{1}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} \\
& \downarrow 243 \\
& \frac{3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{d^2} + \frac{bc \int \frac{1}{x^2(1-c^2x^2)^{3/2}} dx^2}{2d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} \\
& \downarrow 61 \\
& \frac{3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{d^2} + \frac{bc \left(\int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 + \frac{2}{\sqrt{1-c^2x^2}} \right)}{2d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} \\
& \downarrow 73 \\
& \frac{3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{d^2} + \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - \frac{2 \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1-c^2x^2}}{c^2} \right)}{2d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} \\
& \downarrow 221 \\
& \frac{3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \downarrow 5162 \\
& \frac{3c^2 \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2} bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \\
& \quad \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \downarrow 241 \\
& \frac{3c^2 \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \\
& \quad \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \downarrow 5164
\end{aligned}$$

$$\begin{aligned}
& \frac{3c^2 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{d^2} - \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \\
& \quad \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \quad \downarrow \text{3042} \\
& \frac{3c^2 \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{d^2} - \\
& \quad \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \quad \downarrow \text{4669} \\
& \frac{3c^2 \left(\frac{-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right)}{d^2} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \\
& \quad \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \quad \downarrow \text{2715} \\
& \frac{3c^2 \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right)}{d^2} \\
& \quad \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2} \\
& \quad \downarrow \text{2838} \\
& \frac{3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right)}{d^2} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \\
& \quad \frac{a+b \arcsin(cx)}{d^2x(1-c^2x^2)} + \frac{bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{2d^2}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^2), x]`

output

$$-\left(\frac{a + b \operatorname{ArcSin}[c x]}{d^2 x (1 - c^2 x^2)}\right) + \left(\frac{b c (2/\sqrt{1 - c^2 x^2} - 2 \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}])}{2 d^2} + \frac{3 c^2 (-1/2 b / (c \sqrt{1 - c^2 x^2}))}{2} + \frac{x (a + b \operatorname{ArcSin}[c x])}{2 (1 - c^2 x^2)} + \frac{(-2 I) (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[E^{(I \operatorname{ArcSin}[c x])}]}{2} + I b \operatorname{PolyLog}[2, (-I) E^{(I \operatorname{ArcSin}[c x])}] - I b \operatorname{PolyLog}[2, I E^{(I \operatorname{ArcSin}[c x])}]\right) / (2 c) / d^2$$

Defintions of rubi rules used

rule 27

$$\operatorname{Int}[(a_*)(F x_), x_Symbol] \rightarrow \operatorname{Simp}[a \operatorname{Int}[F x, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[F x, (b_*)(G x_) /; \operatorname{FreeQ}[b, x]]$$

rule 61

$$\operatorname{Int}[(a_.) + (b_.) (x_)^{(m_)} ((c_.) + (d_.) (x_))^{(n_)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b x)^{(m + 1)} ((c + d x)^{(n + 1)} / ((b c - a d)^{(m + 1)})), x] - \operatorname{Simp}[d ((m + n + 2) / ((b c - a d)^{(m + 1)})) \operatorname{Int}[(a + b x)^{(m + 1)} (c + d x)^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{!(LtQ}[n, -1] \&\& (\operatorname{EqQ}[a, 0] \operatorname{||} (\operatorname{NeQ}[c, 0] \&\& \operatorname{LtQ}[m - n, 0] \&\& \operatorname{IntegerQ}[n]))) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\operatorname{Int}[(a_.) + (b_.) (x_)^{(m_)} ((c_.) + (d_.) (x_))^{(n_)}, x_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Simp}[p/b \operatorname{Subst}[\operatorname{Int}[x^{(p(m + 1) - 1)} (c - a(d/b) + d(x^p/b))^{(n)}], x], x, (a + b x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221

$$\operatorname{Int}[(a_.) + (b_.) (x_)^2)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-a/b, 2]/a) \operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$$

rule 241

$$\operatorname{Int}[(x_.) ((a_.) + (b_.) (x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b x^2)^{(p + 1)} / (2 b (p + 1)), x] /; \operatorname{FreeQ}[\{a, b, p\}, x] \&\& \operatorname{NeQ}[p, -1]$$

rule 243

$$\operatorname{Int}[(x_)^{(m_)} ((a_.) + (b_.) (x_)^2)^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[1/2 \operatorname{Subst}[\operatorname{Int}[x^{((m - 1)/2)} (a + b x)^p, x], x, x^2], x] /; \operatorname{FreeQ}[\{a, b, m, p\}, x] \&\& \operatorname{IntegerQ}[(m - 1)/2]$$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^((n_))], x_Symbol]$
 $\text{:> Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] \text{:> Simp}[-\text{PolyLog}[2,$
 $(-c)*e*x^n/n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \text{:> Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinear}$
 $\text{Q}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol]$
 $\text{:> Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-\text{Simp}[d*(m/f)$
 $\text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],$
 $x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^(m - 1)*\text{Log}[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5162 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol]$
 $\text{:> Simp}[(-x)*(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(2*d*(p + 1))), x] + (\text{Simp}[(2*p + 3)/(2*d*(p + 1)) \text{ Int}[(d + e*x^2)^(p + 1)*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x]) /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p,$
 $-1] \ \&\& \ \text{NeQ}[p, -3/2]$

rule 5164 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)/((d_) + (e_)*(x_)^2), x_Symbol]$
 $\text{:> Simp}[1/(c*d) \text{ Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.54

method	result
derivativedivides	$c \left(\frac{a \left(-\frac{1}{4(cx+1)} + \frac{3 \ln(cx+1)}{4} - \frac{1}{cx} - \frac{1}{4(cx-1)} - \frac{3 \ln(cx-1)}{4} \right)}{d^2} + \frac{b \left(-\frac{3c^2 x^2 \arcsin(cx) - cx \sqrt{-c^2 x^2 + 1} - 2 \arcsin(cx)}{2cx(c^2 x^2 - 1)} + \ln(icx + \sqrt{-c^2 x^2 + 1}) \right)}{2cx(c^2 x^2 - 1)} \right)$
default	$c \left(\frac{a \left(-\frac{1}{4(cx+1)} + \frac{3 \ln(cx+1)}{4} - \frac{1}{cx} - \frac{1}{4(cx-1)} - \frac{3 \ln(cx-1)}{4} \right)}{d^2} + \frac{b \left(-\frac{3c^2 x^2 \arcsin(cx) - cx \sqrt{-c^2 x^2 + 1} - 2 \arcsin(cx)}{2cx(c^2 x^2 - 1)} + \ln(icx + \sqrt{-c^2 x^2 + 1}) \right)}{2cx(c^2 x^2 - 1)} \right)$
parts	$\frac{a \left(-\frac{c}{4(cx-1)} - \frac{3c \ln(cx-1)}{4} - \frac{c}{4(cx+1)} + \frac{3c \ln(cx+1)}{4} - \frac{1}{x} \right)}{d^2} + \frac{bc \left(-\frac{3c^2 x^2 \arcsin(cx) - cx \sqrt{-c^2 x^2 + 1} - 2 \arcsin(cx)}{2cx(c^2 x^2 - 1)} + \ln(icx + \sqrt{-c^2 x^2 + 1}) \right)}{2cx(c^2 x^2 - 1)}$

input

```
int((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
c*(a/d^2*(-1/4/(c*x+1)+3/4*ln(c*x+1)-1/c/x-1/4/(c*x-1)-3/4*ln(c*x-1))+b/d^
2*(-1/2*(3*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-2*arcsin(c*x))/c/x/(
c^2*x^2-1)+ln(I*c*x+(-c^2*x^2+1)^(1/2))-1)-ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-3
/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/2*arcsin(c*x)*ln(1-I*(
I*c*x+(-c^2*x^2+1)^(1/2)))+3/2*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/2
*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{\frac{a}{c^4 x^6 - 2c^2 x^4 + x^2} dx}{d^2} + \int \frac{\frac{b \arcsin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx}{d^2}$$

input `integrate((a+b*asin(c*x))/x**2/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b*asin(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output

```
-1/4*a*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x + 1)/d^2 + 3
*c*log(c*x - 1)/d^2) + 1/4*(3*(c^3*x^3 - c*x)*arctan2(c*x, sqrt(c*x + 1)*s
qrt(-c*x + 1))*log(c*x + 1) - 3*(c^3*x^3 - c*x)*arctan2(c*x, sqrt(c*x + 1)
*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(3*c^2*x^2 - 2)*arctan2(c*x, sqrt(c*x +
1)*sqrt(-c*x + 1)) + 4*(c^2*d^2*x^3 - d^2*x)*integrate(-1/4*(6*c^3*x^2 -
3*(c^4*x^3 - c^2*x)*log(c*x + 1) + 3*(c^4*x^3 - c^2*x)*log(-c*x + 1) - 4*c
)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)*
b/(c^2*d^2*x^3 - d^2*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx$$

input

```
int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^2), x)
```

output

```
int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^2} dx$$

$$= \frac{4 \left(\int \frac{a \sin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx \right) b c^2 x^3 - 4 \left(\int \frac{a \sin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx \right) b x - 3 \log(c^2 x - c) a c^3 x^3 + 3 \log(c^2 x - c) a c x + 3 \log(c^2 x + c) a c^3 x^3 - 3 \log(c^2 x + c) a c x - 6 a c^2 x^2 + 4 a}{4 d^2 x (c^2 x^2 - 1)}$$

input

```
int((a+b*asin(c*x))/x^2/(-c^2*d*x^2+d)^2,x)
```

output

```
(4*int(asin(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b*c**2*x**3 - 4*int(a
sin(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b*x - 3*log(c**2*x - c)*a*c**
3*x**3 + 3*log(c**2*x - c)*a*c*x + 3*log(c**2*x + c)*a*c**3*x**3 - 3*log(c
**2*x + c)*a*c*x - 6*a*c**2*x**2 + 4*a)/(4*d**2*x*(c**2*x**2 - 1))
```

3.44 $\int \frac{a+b \arcsin(cx)}{x^3(d-c^2dx^2)^2} dx$

Optimal result	560
Mathematica [B] (verified)	561
Rubi [A] (verified)	561
Maple [A] (verified)	566
Fricas [F]	566
Sympy [F]	567
Maxima [F]	567
Giac [F(-2)]	567
Mupad [F(-1)]	568
Reduce [F]	568

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{a + b \arcsin(cx)}{x^3(d - c^2dx^2)^2} dx = -\frac{bc}{2d^2x\sqrt{1 - c^2x^2}} - \frac{a + b \arcsin(cx)}{2d^2x^2} + \frac{c^2(a + b \arcsin(cx))}{2d^2(1 - c^2x^2)} - \frac{4c^2(a + b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^2} + \frac{ibc^2 \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d^2} - \frac{ibc^2 \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d^2}$$

output

```
-1/2*b*c/d^2/x/(-c^2*x^2+1)^(1/2)-1/2*(a+b*arcsin(c*x))/d^2/x^2+1/2*c^2*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)-4*c^2*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2+I*b*c^2*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-I*b*c^2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 461 vs. $2(150) = 300$.

Time = 0.69 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.07

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx$$

$$= \frac{-\frac{2a}{x^2} - \frac{2bc\sqrt{1-c^2x^2}}{x} + \frac{bc^2\sqrt{1-c^2x^2}}{-1+cx} + \frac{bc^2\sqrt{1-c^2x^2}}{1+cx} - \frac{2ac^2}{-1+c^2x^2} - 8ibc^2\pi \arcsin(cx) - \frac{2b \arcsin(cx)}{x^2} + \frac{bc^2 \arcsin(cx)}{1-cx} + bc^2 \arcsin(cx)}{d^2}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^2), x]`

output

```
((-2*a)/x^2 - (2*b*c*Sqrt[1 - c^2*x^2])/x + (b*c^2*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (b*c^2*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a*c^2)/(-1 + c^2*x^2) - (8*I)*b*c^2*Pi*ArcSin[c*x] - (2*b*ArcSin[c*x])/x^2 + (b*c^2*ArcSin[c*x])/(1 - c*x) + (b*c^2*ArcSin[c*x])/(1 + c*x) - 16*b*c^2*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 4*b*c^2*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 8*b*c^2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 4*b*c^2*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 8*b*c^2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 8*b*c^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 8*a*c^2*Log[x] - 4*a*c^2*Log[1 - c^2*x^2] + 16*b*c^2*Pi*Log[Cos[ArcSin[c*x]/2]] - 4*b*c^2*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 4*b*c^2*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (8*I)*b*c^2*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (8*I)*b*c^2*PolyLog[2, I*E^(I*ArcSin[c*x])] - (4*I)*b*c^2*PolyLog[2, E^((2*I)*ArcSin[c*x])])]/(4*d^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.32, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5204, 27, 245, 208, 5208, 208, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx$$

$$\begin{aligned}
& \downarrow 5204 \\
2c^2 \int \frac{a + b \arcsin(cx)}{d^2 x (1 - c^2 x^2)^2} dx + \frac{bc \int \frac{1}{x^2 (1 - c^2 x^2)^{3/2}} dx}{2d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} \\
& \downarrow 27 \\
\frac{2c^2 \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2} + \frac{bc \int \frac{1}{x^2 (1 - c^2 x^2)^{3/2}} dx}{2d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} \\
& \downarrow 245 \\
\frac{2c^2 \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2} + \frac{bc \left(2c^2 \int \frac{1}{(1 - c^2 x^2)^{3/2}} dx - \frac{1}{x\sqrt{1 - c^2 x^2}} \right)}{2d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} \\
& \downarrow 208 \\
\frac{2c^2 \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} + \frac{bc \left(\frac{2c^2 x}{\sqrt{1 - c^2 x^2}} - \frac{1}{x\sqrt{1 - c^2 x^2}} \right)}{2d^2} \\
& \downarrow 5208 \\
\frac{2c^2 \left(\int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)} dx - \frac{1}{2} bc \int \frac{1}{(1 - c^2 x^2)^{3/2}} dx + \frac{a + b \arcsin(cx)}{2(1 - c^2 x^2)} \right)}{d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} + \\
\frac{bc \left(\frac{2c^2 x}{\sqrt{1 - c^2 x^2}} - \frac{1}{x\sqrt{1 - c^2 x^2}} \right)}{2d^2} \\
& \downarrow 208 \\
\frac{2c^2 \left(\int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)} dx + \frac{a + b \arcsin(cx)}{2(1 - c^2 x^2)} - \frac{bcx}{2\sqrt{1 - c^2 x^2}} \right)}{d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} + \\
\frac{bc \left(\frac{2c^2 x}{\sqrt{1 - c^2 x^2}} - \frac{1}{x\sqrt{1 - c^2 x^2}} \right)}{2d^2} \\
& \downarrow 5184 \\
\frac{2c^2 \left(\int \frac{a + b \arcsin(cx)}{cx\sqrt{1 - c^2 x^2}} d \arcsin(cx) + \frac{a + b \arcsin(cx)}{2(1 - c^2 x^2)} - \frac{bcx}{2\sqrt{1 - c^2 x^2}} \right)}{d^2} - \frac{a + b \arcsin(cx)}{2d^2 x^2 (1 - c^2 x^2)} + \\
\frac{bc \left(\frac{2c^2 x}{\sqrt{1 - c^2 x^2}} - \frac{1}{x\sqrt{1 - c^2 x^2}} \right)}{2d^2} \\
& \downarrow 4919
\end{aligned}$$

$$\frac{2c^2 \left(2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2} - \frac{\frac{a + b \arcsin(cx)}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right)}{2d^2}}{d^2}$$

↓ 3042

$$\frac{2c^2 \left(2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2} - \frac{\frac{a + b \arcsin(cx)}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right)}{2d^2}}{d^2}$$

↓ 4671

$$\frac{2c^2 \left(2 \left(-\frac{1}{2} b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2} b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})) \right) \right)}{d^2}$$

$$\frac{\frac{a + b \arcsin(cx)}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right)}{2d^2}}{d^2}$$

↓ 2715

$$\frac{2c^2 \left(2 \left(\frac{1}{4} ib \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4} ib \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right) \right)}{d^2}$$

$$\frac{\frac{a + b \arcsin(cx)}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right)}{2d^2}}{d^2}$$

↓ 2838

$$\frac{2c^2 \left(2 \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)})) (a + b \arcsin(cx)) \right) + \frac{1}{4} ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4} ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right)}{d^2}$$

$$\frac{\frac{a + b \arcsin(cx)}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right)}{2d^2}}{d^2}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```


output

$$\frac{(b*c*(-1/(x*\sqrt{1-c^2*x^2})) + (2*c^2*x)/\sqrt{1-c^2*x^2}))/ (2*d^2) - (a + b*\text{ArcSin}[c*x])/(2*d^2*x^2*(1-c^2*x^2)) + (2*c^2*(-1/2*(b*c*x)/\sqrt{1-c^2*x^2} + (a + b*\text{ArcSin}[c*x])/(2*(1-c^2*x^2)) + 2*(-((a + b*\text{ArcSin}[c*x])*\text{ArcTanh}[E^{((2*I)*\text{ArcSin}[c*x])}] + (I/4)*b*\text{PolyLog}[2, -E^{((2*I)*\text{ArcSin}[c*x])}]) - (I/4)*b*\text{PolyLog}[2, E^{((2*I)*\text{ArcSin}[c*x])}])))/d^2$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_) /; \text{FreeQ}[b, x]]$$

rule 208

$$\text{Int}[(a_) + (b_)*(x_)^2)^{-3/2}, x_Symbol] \rightarrow \text{Simp}[x/(a*\sqrt{a + b*x^2}), x] /; \text{FreeQ}\{a, b\}, x]$$

rule 245

$$\text{Int}[(x_)^m * ((a_) + (b_)*(x_)^2)^p], x_Symbol] \rightarrow \text{Simp}[x^{m+1} * ((a + b*x^2)^{p+1}/(a*(m+1))), x] - \text{Simp}[b*(m+2*(p+1)+1)/(a*(m+1)) \text{Int}[x^{m+2}*(a + b*x^2)^p, x], x] /; \text{FreeQ}\{a, b, m, p\}, x] \ \&\& \ \text{ILtQ}[\text{Simplify}[(m+1)/2 + p + 1], 0] \ \&\& \ \text{NeQ}[m, -1]$$

rule 2715

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{(e_)*((c_) + (d_)*(x_))})^{(n_)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)(x_)]*((c_.) + (d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)(x_)]^{(n_.)}*((c_.) + (d_.)(x_))^{(m_.)}*\text{Sec}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IntegerQ}[n] \&\& \text{RationalQ}[m]$

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}/((x_)*((d_.) + (e_.)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + b*x)^n/(\text{Cos}[x]*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1))) \text{Int}[(f*x)^{(m+2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{ILtQ}[m, -1]$

rule 5208 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(-(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*d*f*(p+1))), x] + (\text{Simp}[(m+2*p+3)/(2*d*(p+1)) \text{Int}[(f*x)^m*(d + e*x^2)^{(p+1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*f*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& !\text{GtQ}[m, 1] \&\& (\text{IntegerQ}[m] || \text{IntegerQ}[p] || \text{EqQ}[n, 1])$

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.85

method	result
derivativedivides	$c^2 \left(\frac{a \left(\frac{1}{4cx+4} - \ln(cx+1) - \frac{1}{2c^2x^2} + 2 \ln(cx) - \frac{1}{4(cx-1)} - \ln(cx-1) \right)}{d^2} + \frac{b \left(-\frac{2c^2x^2 \arcsin(cx) - cx\sqrt{-c^2x^2+1} - \arcsin(cx)}{2c^2x^2(c^2x^2-1)} \right)}{d^2} \right)$
default	$c^2 \left(\frac{a \left(\frac{1}{4cx+4} - \ln(cx+1) - \frac{1}{2c^2x^2} + 2 \ln(cx) - \frac{1}{4(cx-1)} - \ln(cx-1) \right)}{d^2} + \frac{b \left(-\frac{2c^2x^2 \arcsin(cx) - cx\sqrt{-c^2x^2+1} - \arcsin(cx)}{2c^2x^2(c^2x^2-1)} \right)}{d^2} \right)$
parts	$\frac{a \left(-\frac{c^2}{4(cx-1)} - c^2 \ln(cx-1) + \frac{c^2}{4cx+4} - c^2 \ln(cx+1) - \frac{1}{2x^2} + 2c^2 \ln(x) \right)}{d^2} + \frac{b c^2 \left(-\frac{2c^2x^2 \arcsin(cx) - cx\sqrt{-c^2x^2+1} - \arcsin(cx)}{2c^2x^2(c^2x^2-1)} \right)}{d^2}$

input `int((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output `c^2*(a/d^2*(1/4/(c*x+1)-ln(c*x+1)-1/2/c^2/x^2+2*ln(c*x)-1/4/(c*x-1)-ln(c*x-1))+b/d^2*(-1/2*(2*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))/c^2/x^2/(c^2*x^2-1)-2*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*arcsin(c*x)*ln(1-I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-2*I*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))-2*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx = \frac{\int \frac{a}{c^4 x^7 - 2c^2 x^5 + x^3} dx + \int \frac{b \arcsin(cx)}{c^4 x^7 - 2c^2 x^5 + x^3} dx}{d^2}$$

input `integrate((a+b*asin(c*x))/x**3/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b*asin(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/2*a*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2 + (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vcteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d - c^2 dx^2)^2} dx$$

input `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^2),x)`

output `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^2} dx$$

$$= \frac{2 \left(\int \frac{\operatorname{asin}(cx)}{c^4 x^7 - 2c^2 x^5 + x^3} dx \right) b c^2 x^4 - 2 \left(\int \frac{\operatorname{asin}(cx)}{c^4 x^7 - 2c^2 x^5 + x^3} dx \right) b x^2 - 2 \log(c^2 x - c) a c^4 x^4 + 2 \log(c^2 x - c) a c^2 x^2 - 2 d^2 x^2 (c^2 x^2 - c)}{2 d^2 x^2 (c^2 x^2 - c)}$$

input `int((a+b*asin(c*x))/x^3/(-c^2*d*x^2+d)^2,x)`

output `(2*int(asin(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b*c**2*x**4 - 2*int(asin(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b*x**2 - 2*log(c**2*x - c)*a*c**4*x**4 + 2*log(c**2*x - c)*a*c**2*x**2 - 2*log(c**2*x + c)*a*c**4*x**4 + 2*log(c**2*x + c)*a*c**2*x**2 + 4*log(x)*a*c**4*x**4 - 4*log(x)*a*c**2*x**2 - 2*a*c**4*x**4 + a)/(2*d**2*x**2*(c**2*x**2 - 1))`

3.45 $\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)^2} dx$

Optimal result	569
Mathematica [A] (verified)	570
Rubi [A] (verified)	570
Maple [A] (verified)	576
Fricas [F]	577
Sympy [F]	577
Maxima [F]	578
Giac [F(-2)]	578
Mupad [F(-1)]	579
Reduce [F]	579

Optimal result

Integrand size = 25, antiderivative size = 233

$$\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)^2} dx = -\frac{bc^3}{2d^2\sqrt{1-c^2x^2}} - \frac{bc\sqrt{1-c^2x^2}}{6d^2x^2} - \frac{a+b \arcsin(cx)}{3d^2x^3} - \frac{2c^2(a+b \arcsin(cx))}{d^2x} + \frac{c^4x(a+b \arcsin(cx))}{2d^2(1-c^2x^2)} - \frac{5ic^3(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{d^2} - \frac{13bc^3 \operatorname{arctanh}(\sqrt{1-c^2x^2})}{6d^2} + \frac{5ibc^3 \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{2d^2} - \frac{5ibc^3 \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2d^2}$$

output

```
-1/2*b*c^3/d^2/(-c^2*x^2+1)^(1/2)-1/6*b*c*(-c^2*x^2+1)^(1/2)/d^2/x^2-1/3*(a+b*arcsin(c*x))/d^2/x^3-2*c^2*(a+b*arcsin(c*x))/d^2/x+1/2*c^4*x*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)-5*I*c^3*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^2-13/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))/d^2+5/2*I*b*c^3*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2-5/2*I*b*c^3*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.83

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx =$$

$$\frac{4a}{x^3} + \frac{24ac^2}{x} + \frac{2bc\sqrt{1-c^2x^2}}{x^2} - \frac{3bc^3\sqrt{1-c^2x^2}}{-1+cx} + \frac{3bc^3\sqrt{1-c^2x^2}}{1+cx} + \frac{6ac^4x}{-1+c^2x^2} + 15ibc^3\pi \arcsin(cx) + \frac{4b \arcsin(cx)}{x^3} + \frac{24bc^2}{x^3}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

output

```
-1/12*((4*a)/x^3 + (24*a*c^2)/x + (2*b*c*Sqrt[1 - c^2*x^2])/x^2 - (3*b*c^3*
*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (3*b*c^3*Sqrt[1 - c^2*x^2])/(1 + c*x) + (
6*a*c^4*x)/(-1 + c^2*x^2) + (15*I)*b*c^3*Pi*ArcSin[c*x] + (4*b*ArcSin[c*x]
)/x^3 + (24*b*c^2*ArcSin[c*x])/x + (3*b*c^3*ArcSin[c*x])/(-1 + c*x) + (3*b
*c^3*ArcSin[c*x])/(1 + c*x) + 26*b*c^3*ArcTanh[Sqrt[1 - c^2*x^2]] - 15*b*c
^3*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 30*b*c^3*ArcSin[c*x]*Log[1 - I*E^(I*A
rcSin[c*x])] - 15*b*c^3*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 30*b*c^3*ArcSin[
c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 15*a*c^3*Log[1 - c*x] - 15*a*c^3*Log[1
+ c*x] + 15*b*c^3*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 15*b*c^3*Pi*Log[
Sin[(Pi + 2*ArcSin[c*x])/4]] - (30*I)*b*c^3*PolyLog[2, (-I)*E^(I*ArcSin[c*
x])] + (30*I)*b*c^3*PolyLog[2, I*E^(I*ArcSin[c*x])])/d^2
```

Rubi [A] (verified)Time = 1.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.30, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {5204, 27, 243, 52, 61, 73, 221, 5204, 243, 61, 73, 221, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx$$

↓ 5204

$$\begin{aligned}
& \frac{5}{3}c^2 \int \frac{a + b \arcsin(cx)}{d^2 x^2 (1 - c^2 x^2)^2} dx + \frac{bc \int \frac{1}{x^3(1-c^2x^2)^{3/2}} dx}{3d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} \\
& \quad \downarrow 27 \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^2} dx}{3d^2} + \frac{bc \int \frac{1}{x^3(1-c^2x^2)^{3/2}} dx}{3d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} \\
& \quad \downarrow 243 \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^2} dx}{3d^2} + \frac{bc \int \frac{1}{x^4(1-c^2x^2)^{3/2}} dx^2}{6d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} \\
& \quad \downarrow 52 \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^2} dx}{3d^2} + \frac{bc \left(\frac{3}{2}c^2 \int \frac{1}{x^2(1-c^2x^2)^{3/2}} dx^2 - \frac{1}{x^2 \sqrt{1-c^2x^2}} \right)}{6d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} \\
& \quad \downarrow 61 \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^2} dx}{3d^2} + \frac{bc \left(\frac{3}{2}c^2 \left(\int \frac{1}{x^2 \sqrt{1-c^2x^2}} dx^2 + \frac{2}{\sqrt{1-c^2x^2}} \right) - \frac{1}{x^2 \sqrt{1-c^2x^2}} \right)}{6d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} \\
& \quad \downarrow 73 \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^2} dx}{3d^2} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - \frac{2 \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1-c^2x^2}}{c^2} \right) - \frac{1}{x^2 \sqrt{1-c^2x^2}} \right)}{6d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} \\
& \quad \downarrow 221 \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^2} dx}{3d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} + \\
& \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) \right) - \frac{1}{x^2 \sqrt{1-c^2x^2}} \right)}{6d^2} \\
& \quad \downarrow 5204 \\
& \frac{5c^2 \left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + bc \int \frac{1}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} \right)}{3d^2} - \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} + \\
& \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) \right) - \frac{1}{x^2 \sqrt{1-c^2x^2}} \right)}{6d^2}
\end{aligned}$$

$$\frac{5c^2 \left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{1}{2}bc \int \frac{1}{x^2(1-c^2x^2)^{3/2}} dx^2 - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} \right)}{3d^2} - \frac{a+b \arcsin(cx)}{3d^2x^3(1-c^2x^2)} +$$

$$\frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}} \right)}{6d^2}$$

↓ 243

$$\frac{5c^2 \left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{1}{2}bc \left(\int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 + \frac{2}{\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} \right)}{3d^2} -$$

$$\frac{a+b \arcsin(cx)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}} \right)}{6d^2}$$

↓ 61

$$\frac{5c^2 \left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - \frac{2 \int \frac{1}{c^2 - \frac{x^4}{c^2}} d\sqrt{1-c^2x^2}}{c^2} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} \right)}{3d^2} -$$

$$\frac{a+b \arcsin(cx)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}} \right)}{6d^2}$$

↓ 73

$$\frac{5c^2 \left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2} -$$

$$\frac{a+b \arcsin(cx)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}} \right)}{6d^2}$$

↓ 221

$$\frac{5c^2 \left(3c^2 \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2} -$$

$$\frac{a+b \arcsin(cx)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}} \right)}{6d^2}$$

↓ 5162

↓ 241

$$\frac{5c^2 \left(3c^2 \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}}}{6d^2}$$

↓ 5164

$$\frac{5c^2 \left(3c^2 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}}}{6d^2}$$

↓ 3042

$$\frac{5c^2 \left(3c^2 \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}}}{6d^2}$$

↓ 4669

$$\frac{5c^2 \left(3c^2 \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}}}{6d^2}$$

↓ 2715

$$\frac{5c^2 \left(3c^2 \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^2x^3(1-c^2x^2)} + \frac{bc \left(\frac{3}{2}c^2 \left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right) - \frac{1}{x^2\sqrt{1-c^2x^2}}}{6d^2}$$

↓ 2838

$$\frac{5c^2 \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} - \frac{1}{2c\sqrt{1 - c^2 x^2}} \right)}{3d^2} + \frac{a + b \arcsin(cx)}{3d^2 x^3 (1 - c^2 x^2)} + \frac{bc \left(\frac{3}{2} c^2 \left(\frac{2}{\sqrt{1 - c^2 x^2}} - 2 \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) \right) - \frac{1}{x^2 \sqrt{1 - c^2 x^2}} \right)}{6d^2}$$

input `Int[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^2), x]`

output `-1/3*(a + b*ArcSin[c*x])/(d^2*x^3*(1 - c^2*x^2)) + (b*c*(-(1/(x^2*Sqrt[1 - c^2*x^2])) + (3*c^2*(2/Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2))/(6*d^2) + (5*c^2*(-((a + b*ArcSin[c*x])/(x*(1 - c^2*x^2))) + (b*c*(2/Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2 + 3*c^2*(-1/2*b/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*(1 - c^2*x^2)) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x]]) - I*b*PolyLog[2, I*E^(I*ArcSin[c*x]])]/(2*c)))))/(3*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 52 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]`

rule 61 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((m + n + 2)/((b*c - a*d)*(m + 1)) Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

- rule 73 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)}*((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \text{LtQ}[-1, m, 0] \ \&\& \text{LeQ}[-1, n, 0] \ \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \text{NegQ}[a/b]$
- rule 241 $\text{Int}[(x_)*(a_.) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p+1)}/(2*b*(p+1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \text{NeQ}[p, -1]$
- rule 243 $\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m-1)/2)*(a + b*x)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \text{IntegerQ}[(m-1)/2]$
- rule 2715 $\text{Int}[\text{Log}[(a_.) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)(x_)))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^{(n)}], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \text{GtQ}[a, 0]$
- rule 2838 $\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \text{EqQ}[c*d, 1]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4669 $\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (f_.)(x_)]*((c_.) + (d_.)(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x)) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \text{IntegerQ}[2*k] \ \&\& \text{IGtQ}[m, 0]$

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p +
1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:= Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.34

method	result
derivativedivides	$c^3 \left(\frac{a \left(-\frac{1}{4(cx+1)} + \frac{5 \ln(cx+1)}{4} - \frac{1}{3c^3x^3} - \frac{2}{cx} - \frac{1}{4(cx-1)} - \frac{5 \ln(cx-1)}{4} \right)}{d^2} + \frac{b \left(-\frac{15c^4x^4 \arcsin(cx) - 2c^3x^3 \sqrt{-c^2x^2+1} - 10c^2x^2}{6c^3x^3(c^2x^2} \right)}{d^2} \right)$
default	$c^3 \left(\frac{a \left(-\frac{1}{4(cx+1)} + \frac{5 \ln(cx+1)}{4} - \frac{1}{3c^3x^3} - \frac{2}{cx} - \frac{1}{4(cx-1)} - \frac{5 \ln(cx-1)}{4} \right)}{d^2} + \frac{b \left(-\frac{15c^4x^4 \arcsin(cx) - 2c^3x^3 \sqrt{-c^2x^2+1} - 10c^2x^2}{6c^3x^3(c^2x^2} \right)}{d^2} \right)$
parts	$\frac{a \left(-\frac{c^3}{4(cx-1)} - \frac{5c^3 \ln(cx-1)}{4} - \frac{c^3}{4(cx+1)} + \frac{5c^3 \ln(cx+1)}{4} - \frac{1}{3x^3} - \frac{2c^2}{x} \right)}{d^2} + \frac{bc^3 \left(-\frac{15c^4x^4 \arcsin(cx) - 2c^3x^3 \sqrt{-c^2x^2+1} - 10c^2x^2}{6c^3x^3(c^2x^2} \right)}{d^2}$

input `int((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output `c^3*(a/d^2*(-1/4/(c*x+1)+5/4*ln(c*x+1)-1/3/c^3/x^3-2/c/x-1/4/(c*x-1)-5/4*ln(c*x-1))+b/d^2*(-1/6*(15*c^4*x^4*arcsin(c*x)-2*c^3*x^3*(-c^2*x^2+1)^(1/2)-10*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-2*arcsin(c*x))/c^3/x^3/(c^2*x^2-1)+13/6*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)-13/6*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-5/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))))+5/2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))+5/2*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))))-5/2*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{a}{c^4 x^8 - 2c^2 x^6 + x^4} dx + \int \frac{b \arcsin(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx$$

input `integrate((a+b*asin(c*x))/x**4/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b*asin(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^2 x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 - 10*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a + 1/12*(15*(c^5*x^5 - c^3*x^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 15*(c^5*x^5 - c^3*x^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(15*c^4*x^4 - 10*c^2*x^2 - 2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 12*(c^2*d^2*x^5 - d^2*x^3)*integrate(-1/12*(30*c^5*x^4 - 20*c^3*x^2 - 15*(c^6*x^5 - c^4*x^3)*log(c*x + 1) + 15*(c^6*x^5 - c^4*x^3)*log(-c*x + 1) - 4*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x))*b/(c^2*d^2*x^5 - d^2*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (d - c^2 dx^2)^2} dx$$

input `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^2),x)`

output `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^2} dx$$

$$= \frac{12 \left(\int \frac{\operatorname{asin}(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx \right) b c^2 x^5 - 12 \left(\int \frac{\operatorname{asin}(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx \right) b x^3 - 15 \log(c^2 x - c) a c^5 x^5 + 15 \log(c^2 x - c) a c^3}{12 d^2 x^3 (c^2 x^2 - 1)}$$

input `int((a+b*asin(c*x))/x^4/(-c^2*d*x^2+d)^2,x)`

output `(12*int(asin(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b*c**2*x**5 - 12*int(asin(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b*x**3 - 15*log(c**2*x - c)*a*c**5*x**5 + 15*log(c**2*x - c)*a*c**3*x**3 + 15*log(c**2*x + c)*a*c**5*x**5 - 15*log(c**2*x + c)*a*c**3*x**3 - 30*a*c**4*x**4 + 20*a*c**2*x**2 + 4*a)/(12*d**2*x**3*(c**2*x**2 - 1))`

3.46 $\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx$

Optimal result	580
Mathematica [B] (verified)	581
Rubi [A] (verified)	581
Maple [A] (verified)	585
Fricas [F]	586
Sympy [F]	586
Maxima [F]	587
Giac [F]	587
Mupad [F(-1)]	588
Reduce [F]	588

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx = -\frac{b}{12c^5d^3(1-c^2x^2)^{3/2}} + \frac{5b}{8c^5d^3\sqrt{1-c^2x^2}}$$

$$+ \frac{x^3(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \frac{3x(a+b \arcsin(cx))}{8c^4d^3(1-c^2x^2)}$$

$$- \frac{3i(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{4c^5d^3}$$

$$+ \frac{3ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{8c^5d^3}$$

$$- \frac{3ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{8c^5d^3}$$

output

```
-1/12*b/c^5/d^3/(-c^2*x^2+1)^(3/2)+5/8*b/c^5/d^3/(-c^2*x^2+1)^(1/2)+1/4*x^
3*(a+b*arcsin(c*x))/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b*arcsin(c*x))/c^4/d^3
/(-c^2*x^2+1)-3/4*I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^5
/d^3+3/8*I*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^3-3/8*I*b*poly
log(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 445 vs. $2(204) = 408$.

Time = 0.62 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.18

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= -\frac{2b\sqrt{1-c^2x^2}}{(-1+cx)^2} + \frac{bcx\sqrt{1-c^2x^2}}{(-1+cx)^2} - \frac{15b\sqrt{1-c^2x^2}}{-1+cx} - \frac{2b\sqrt{1-c^2x^2}}{(1+cx)^2} - \frac{bcx\sqrt{1-c^2x^2}}{(1+cx)^2} + \frac{15b\sqrt{1-c^2x^2}}{1+cx} + \frac{12acx}{(-1+c^2x^2)^2} + \frac{30acx}{-1+c^2x^2} - 9ib$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]
```

output

```
((-2*b*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 + (b*c*x*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 - (15*b*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (2*b*Sqrt[1 - c^2*x^2])/(1 + c*x)^2 - (b*c*x*Sqrt[1 - c^2*x^2])/(1 + c*x)^2 + (15*b*Sqrt[1 - c^2*x^2])/(1 + c*x) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (30*a*c*x)/(-1 + c^2*x^2) - (9*I)*b*Pi*ArcSin[c*x] + (3*b*ArcSin[c*x])/(-1 + c*x)^2 + (15*b*ArcSin[c*x])/(-1 + c*x) - (3*b*ArcSin[c*x])/(1 + c*x)^2 + (15*b*ArcSin[c*x])/(1 + c*x) + 9*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 18*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 9*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 18*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - 9*a*Log[1 - c*x] + 9*a*Log[1 + c*x] - 9*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - 9*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (18*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (18*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(48*c^5*d^3)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5206, 27, 243, 53, 2009, 5206, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx \\
& \quad \downarrow \text{5206} \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{d^2(1-c^2x^2)^2} dx}{4c^2d} - \frac{b \int \frac{x^3}{(1-c^2x^2)^{5/2}} dx}{4cd^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{27} \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \int \frac{x^3}{(1-c^2x^2)^{5/2}} dx}{4cd^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{243} \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \int \frac{x^2}{(1-c^2x^2)^{5/2}} dx^2}{8cd^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{53} \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \int \left(\frac{1}{c^2(1-c^2x^2)^{5/2}} - \frac{1}{c^2(1-c^2x^2)^{3/2}} \right) dx^2}{8cd^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{2009} \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{4c^2d^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} - \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3} \\
& \quad \downarrow \text{5206} \\
& \frac{3 \left(-\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{2c^2} - \frac{b \int \frac{x}{(1-c^2x^2)^{3/2}} dx}{2c} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{4c^2d^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3} \\
& \quad \downarrow \text{241} \\
& \frac{3 \left(-\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{2c^2} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b}{2c^3\sqrt{1-c^2x^2}} \right)}{4c^2d^3} + \frac{x^3(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3}
\end{aligned}$$

$$\begin{aligned}
& \downarrow 5164 \\
& \frac{3 \left(-\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c^3} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b}{2c^3\sqrt{1-c^2x^2}} \right)}{4c^2d^3} + \frac{x^3(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \\
& \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3} \\
& \downarrow 3042 \\
& \frac{3 \left(-\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c^3} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b}{2c^3\sqrt{1-c^2x^2}} \right)}{4c^2d^3} + \\
& \frac{x^3(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3} \\
& \downarrow 4669 \\
& \frac{3 \left(-\frac{b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c^3} \right)}{4c^2d^3} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \\
& \frac{x^3(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3} \\
& \downarrow 2715 \\
& \frac{3 \left(-\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c^3} \right)}{4c^2d^3} + \\
& \frac{x^3(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3} \\
& \downarrow 2838 \\
& \frac{3 \left(-\frac{2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c^3} \right)}{4c^2d^3} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b}{2c^3\sqrt{1-c^2x^2}} - \\
& \frac{x^3(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \frac{b \left(\frac{2}{3c^4(1-c^2x^2)^{3/2}} - \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{8cd^3}
\end{aligned}$$

input `Int[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]`

output

```
-1/8*(b*(2/(3*c^4*(1 - c^2*x^2)^(3/2)) - 2/(c^4*Sqrt[1 - c^2*x^2])))/(c*d^
3) + (x^3*(a + b*ArcSin[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*(-1/2*b/(c
^3*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*c^2*(1 - c^2*x^2)) - ((
-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*
E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c^3)))/(4*c^2
*d^3)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int
[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5164 `Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5206 `Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]`

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{a \left(\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{5}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right) b \left(-\frac{15c^3 x^3 \arcsin(cx) - 15c^2 x^2 \sqrt{-c^2 x^2 + 1} - 9c}{24(c^4 x^4 - 2c^2 x^2 + 1)} \right)}{d^3}$
default	$\frac{a \left(\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{5}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right) b \left(-\frac{15c^3 x^3 \arcsin(cx) - 15c^2 x^2 \sqrt{-c^2 x^2 + 1} - 9c}{24(c^4 x^4 - 2c^2 x^2 + 1)} \right)}{d^3}$
parts	$\frac{a \left(-\frac{1}{16c^5(cx-1)^2} - \frac{5}{16c^5(cx-1)} + \frac{3 \ln(cx-1)}{16c^5} + \frac{1}{16c^5(cx+1)^2} - \frac{5}{16c^5(cx+1)} - \frac{3 \ln(cx+1)}{16c^5} \right) b \left(-\frac{15c^3 x^3 \arcsin(cx) - 15c^2 x^2 \sqrt{-c^2 x^2 + 1} - 9c}{24(c^4 x^4 - 2c^2 x^2 + 1)} \right)}{d^3}$

input `int(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)`

output
$$\frac{1}{c^5} \left(-\frac{a}{d^3} \left(\frac{1}{16} \frac{1}{(cx+1)^2} - \frac{5}{16} \frac{1}{(cx+1)} - \frac{3}{16} \ln(cx+1) - \frac{1}{16} \frac{1}{(cx-1)^2} - \frac{5}{16} \frac{1}{(cx-1)} + \frac{3}{16} \ln(cx-1) \right) - \frac{b}{d^3} \left(-\frac{1}{24} (15c^3x^3 \arcsin(cx) - 15c^2x^2(-c^2x^2+1)^{1/2} - 9cx \arcsin(cx) + 13(-c^2x^2+1)^{1/2}) / (c^4x^4 - 2c^2x^2+1) + \frac{3}{8} \arcsin(cx) \ln(1+I*(I*cx+(-c^2x^2+1)^{1/2})) - \frac{3}{8} \arcsin(cx) \ln(1-I*(I*cx+(-c^2x^2+1)^{1/2})) - \frac{3}{8} I \operatorname{dilog}(1+I*(I*cx+(-c^2x^2+1)^{1/2})) + \frac{3}{8} I \operatorname{dilog}(1-I*(I*cx+(-c^2x^2+1)^{1/2})) \right) \right)$$

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^4}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b*x^4*arcsin(c*x) + a*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = -\int \frac{ax^4}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx + \int \frac{bx^4 \operatorname{asin}(cx)}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx$$

input `integrate(x**4*(a+b*asin(c*x))/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*x**4*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^4}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output

```
1/16*a*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3)) + 1/16*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) + 2*(5*c^3*x^3 - 3*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 16*(c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)*integrate(1/16*(10*c^3*x^3 - 6*c*x + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^10*d^3*x^6 - 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x))*b/(c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)
```

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^4}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output

```
integrate(-(b*arcsin(c*x) + a)*x^4/(c^2*d*x^2 - d)^3, x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^3} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3,x)`

output `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3, x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-16 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^9 x^4 + 32 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^7 x^2 - 16 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c}{}$$

input `int(x^4*(a+b*asin(c*x))/(-c^2*d*x^2+d)^3,x)`

output `(- 16*int((asin(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x) *b*c**9*x**4 + 32*int((asin(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**7*x**2 - 16*int((asin(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**5 - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c**2*x - c)*a*c**2*x**2 - 3*log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c**2*x**2 + 3*log(c**2*x + c)*a + 10*a*c**3*x**3 - 6*a*c*x)/(16*c**5*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))`

3.47 $\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx$

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Mathematica [A] (verified)	589
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Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2dx^2)^3} dx = -\frac{bx^3}{12cd^3(1 - c^2x^2)^{3/2}} + \frac{bx}{4c^3d^3\sqrt{1 - c^2x^2}} - \frac{b \arcsin(cx)}{4c^4d^3} + \frac{x^4(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2}$$

output `-1/12*b*x^3/c/d^3/(-c^2*x^2+1)^(3/2)+1/4*b*x/c^3/d^3/(-c^2*x^2+1)^(1/2)-1/4*b*arcsin(c*x)/c^4/d^3+1/4*x^4*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^2`

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2dx^2)^3} dx = \frac{bcx(3 - 4c^2x^2)\sqrt{1 - c^2x^2} + a(-3 + 6c^2x^2) + 3b(-1 + 2c^2x^2)\arcsin(cx)}{12c^4d^3(-1 + c^2x^2)^2}$$

input `Integrate[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]`

output

$$(b*c*x*(3 - 4*c^2*x^2)*\text{Sqrt}[1 - c^2*x^2] + a*(-3 + 6*c^2*x^2) + 3*b*(-1 + 2*c^2*x^2)*\text{ArcSin}[c*x])/(12*c^4*d^3*(-1 + c^2*x^2)^2)$$
Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5186, 252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$\downarrow 5186$$

$$\frac{x^4(a + b \arcsin(cx))}{4d^3(1 - c^2 x^2)^2} - \frac{bc \int \frac{x^4}{(1 - c^2 x^2)^{5/2}} dx}{4d^3}$$

$$\downarrow 252$$

$$\frac{x^4(a + b \arcsin(cx))}{4d^3(1 - c^2 x^2)^2} - \frac{bc \left(\frac{x^3}{3c^2(1 - c^2 x^2)^{3/2}} - \frac{\int \frac{x^2}{(1 - c^2 x^2)^{3/2}} dx}{c^2} \right)}{4d^3}$$

$$\downarrow 252$$

$$\frac{x^4(a + b \arcsin(cx))}{4d^3(1 - c^2 x^2)^2} - \frac{bc \left(\frac{x^3}{3c^2(1 - c^2 x^2)^{3/2}} - \frac{\frac{x}{c^2 \sqrt{1 - c^2 x^2}} - \frac{\int \frac{1}{\sqrt{1 - c^2 x^2}} dx}{c^2}}{c^2} \right)}{4d^3}$$

$$\downarrow 223$$

$$\frac{x^4(a + b \arcsin(cx))}{4d^3(1 - c^2 x^2)^2} - \frac{bc \left(\frac{x^3}{3c^2(1 - c^2 x^2)^{3/2}} - \frac{\frac{x}{c^2 \sqrt{1 - c^2 x^2}} - \frac{\arcsin(cx)}{c^3}}{c^2} \right)}{4d^3}$$

input

$$\text{Int}[(x^3*(a + b*\text{ArcSin}[c*x]))/(d - c^2*d*x^2)^3, x]$$

output

$$\frac{x^4(a + b\text{ArcSin}[c*x])}{4d^3(1 - c^2x^2)^2} - \frac{b*c*(x^3/(3*c^2*(1 - c^2*x^2)^{(3/2)}) - (x/(c^2*\text{Sqrt}[1 - c^2*x^2]) - \text{ArcSin}[c*x]/c^3)/c^2)}{4*d^3}$$
Defintions of rubi rules used

rule 223

$$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] \text{ ; FreeQ}\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$$

rule 252

$$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(2*b*(p+1))), x] - \text{Simp}[c^2*(m-1)/(2*b*(p+1)) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^{(p+1)}, x], x] \text{ ; FreeQ}\{a, b, c\}, x] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ !\text{ILtQ}[(m + 2*p + 3)/2, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$$

rule 5186

$$\text{Int}[((a_) + \text{ArcSin}[c_)*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] \text{ ; FreeQ}\{a, b, c, d, e, f, m, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{EqQ}[m + 2*p + 3, 0] \ \&\& \ \text{NeQ}[m, -1]$$
Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.76

method	result
orering	$\frac{(cx-1)(cx+1)(4c^4x^4+3c^2x^2-4)(a+b\arcsin(cx))}{4c^4(-c^2dx^2+d)^3} - \frac{(4c^2x^2-3)(cx-1)^2(cx+1)^2 \left(\frac{3x^2(a+b\arcsin(cx))}{(-c^2dx^2+d)^3} + \frac{\sqrt{-c^2x^2+1}}{12x^2c^4} \right)}{12x^2c^4}$
derivativedivides	$\frac{a \left(-\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{1}{16(cx-1)^2} - \frac{3}{16(cx-1)} \right)}{d^3} - \frac{b \left(-\frac{\arcsin(cx)}{16(cx+1)^2} + \frac{3\arcsin(cx)}{16(cx+1)} - \frac{\arcsin(cx)}{16(cx-1)^2} - \frac{3\arcsin(cx)}{16(cx-1)} + \frac{\sqrt{-(cx-1)^2}}{48(cx-1)} \right)}{c^4}$
default	$\frac{a \left(-\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{1}{16(cx-1)^2} - \frac{3}{16(cx-1)} \right)}{d^3} - \frac{b \left(-\frac{\arcsin(cx)}{16(cx+1)^2} + \frac{3\arcsin(cx)}{16(cx+1)} - \frac{\arcsin(cx)}{16(cx-1)^2} - \frac{3\arcsin(cx)}{16(cx-1)} + \frac{\sqrt{-(cx-1)^2}}{48(cx-1)} \right)}{c^4}$
parts	$\frac{a \left(-\frac{1}{16c^4(cx-1)^2} - \frac{3}{16c^4(cx-1)} - \frac{1}{16c^4(cx+1)^2} + \frac{3}{16c^4(cx+1)} \right)}{d^3} - \frac{b \left(-\frac{\arcsin(cx)}{16(cx+1)^2} + \frac{3\arcsin(cx)}{16(cx+1)} - \frac{\arcsin(cx)}{16(cx-1)^2} - \frac{3\arcsin(cx)}{16(cx-1)} \right)}{c^4}$

input

```
int(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/4*(c*x-1)*(c*x+1)*(4*c^4*x^4+3*c^2*x^2-4)/c^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3-1/12/x^2*(4*c^2*x^2-3)/c^4*(c*x-1)^2*(c*x+1)^2*(3*x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3+x^3*b*c/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^3+6*x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^4*c^2*d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2dx^2)^3} dx = \frac{3ac^4x^4 + 3(2bc^2x^2 - b)\arcsin(cx) - (4bc^3x^3 - 3bcx)\sqrt{-c^2x^2 + 1}}{12(c^8d^3x^4 - 2c^6d^3x^2 + c^4d^3)}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
1/12*(3*a*c^4*x^4 + 3*(2*b*c^2*x^2 - b)*arcsin(c*x) - (4*b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*x^2 + 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = -\int \frac{ax^3}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{bx^3 \arcsin(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx$$

input `integrate(x**3*(a+b*asin(c*x))/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*x**3*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^3}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `1/4*(2*c^2*x^2 - 1)*a/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 1/4*((2*c^2*x^2 - 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 4*(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)*integrate(1/4*(2*c^2*x^2 - 1)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))/(c^11*d^3*x^8 - 3*c^9*d^3*x^6 + 3*c^7*d^3*x^4 - c^5*d^3*x^2 + (c^9*d^3*x^6 - 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3)*e^(log(c*x + 1) + log(-c*x + 1))), x))*b/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

$$\begin{aligned} \int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx &= \frac{bx^4 \arcsin(cx)}{4(c^2 x^2 - 1)^2 d^3} + \frac{ax^4}{4(c^2 x^2 - 1)^2 d^3} \\ &+ \frac{bx^3}{12(c^2 x^2 - 1)\sqrt{-c^2 x^2 + 1}cd^3} \\ &+ \frac{bx}{4\sqrt{-c^2 x^2 + 1}c^3 d^3} - \frac{b \arcsin(cx)}{4c^4 d^3} - \frac{a}{4c^4 d^3} \end{aligned}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output $\frac{1}{4}bx^4\arcsin(cx)/((c^2x^2-1)^2d^3) + \frac{1}{4}ax^4/((c^2x^2-1)^2d^3) + \frac{1}{12}bx^3/((c^2x^2-1)\sqrt{-c^2x^2+1}cd^3) + \frac{1}{4}bx/(\sqrt{-c^2x^2+1}c^3d^3) - \frac{1}{4}b\arcsin(cx)/(c^4d^3) - \frac{1}{4}a/(c^4d^3)$

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^3} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3,x)`

output `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3, x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-4 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx \right) b c^4 x^4 + 8 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx \right) b c^2 x^2 - 4 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx \right) b + a}{4d^3 (c^4x^4 - 2c^2x^2 + 1)}$$

input `int(x^3*(a+b*asin(c*x))/(-c^2*d*x^2+d)^3,x)`

output `(- 4*int((asin(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)* b*c**4*x**4 + 8*int((asin(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**2*x**2 - 4*int((asin(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b + a*x**4)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))`

3.48
$$\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx$$

Optimal result	595
Mathematica [B] (verified)	596
Rubi [A] (verified)	596
Maple [A] (verified)	600
Fricas [F]	601
Sympy [F]	601
Maxima [F]	601
Giac [F]	602
Mupad [F(-1)]	602
Reduce [F]	603

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx = -\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}} + \frac{b}{8c^3d^3\sqrt{1-c^2x^2}}$$

$$+ \frac{x(a+b \arcsin(cx))}{4c^2d^3(1-c^2x^2)^2} - \frac{x(a+b \arcsin(cx))}{8c^2d^3(1-c^2x^2)}$$

$$+ \frac{i(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{4c^3d^3}$$

$$- \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{8c^3d^3} + \frac{ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{8c^3d^3}$$

output

```
-1/12*b/c^3/d^3/(-c^2*x^2+1)^(3/2)+1/8*b/c^3/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b*arcsin(c*x))/c^2/d^3/(-c^2*x^2+1)^2-1/8*x*(a+b*arcsin(c*x))/c^2/d^3/(-c^2*x^2+1)+1/4*I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^3/d^3-1/8*I*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^3+1/8*I*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^3
```


Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 445 vs. $2(202) = 404$.

Time = 0.77 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.20

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= -\frac{2b\sqrt{1-c^2x^2}}{(-1+cx)^2} + \frac{bcx\sqrt{1-c^2x^2}}{(-1+cx)^2} - \frac{3b\sqrt{1-c^2x^2}}{-1+cx} - \frac{2b\sqrt{1-c^2x^2}}{(1+cx)^2} - \frac{bcx\sqrt{1-c^2x^2}}{(1+cx)^2} + \frac{3b\sqrt{1-c^2x^2}}{1+cx} + \frac{12acx}{(-1+c^2x^2)^2} + \frac{6acx}{-1+c^2x^2} + 3ib\pi$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]
```

output

```
((-2*b*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 + (b*c*x*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 - (3*b*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (2*b*Sqrt[1 - c^2*x^2])/(1 + c*x)^2 - (b*c*x*Sqrt[1 - c^2*x^2])/(1 + c*x)^2 + (3*b*Sqrt[1 - c^2*x^2])/(1 + c*x) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (6*a*c*x)/(-1 + c^2*x^2) + (3*I)*b*Pi*ArcSin[c*x] + (3*b*ArcSin[c*x])/(-1 + c*x)^2 + (3*b*ArcSin[c*x])/(-1 + c*x) - (3*b*ArcSin[c*x])/(1 + c*x)^2 + (3*b*ArcSin[c*x])/(1 + c*x) - 3*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 6*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 3*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 6*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 3*a*Log[1 - c*x] - 3*a*Log[1 + c*x] + 3*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 3*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (6*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (6*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])]/(48*c^3*d^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5206, 27, 241, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx \\
& \quad \downarrow \text{5206} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{d^2(1-c^2x^2)^2} dx}{4c^2d} - \frac{b \int \frac{x}{(1-c^2x^2)^{5/2}} dx}{4cd^3} + \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{27} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \int \frac{x}{(1-c^2x^2)^{5/2}} dx}{4cd^3} + \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{241} \\
& -\frac{\int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{4c^2d^3} + \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} - \frac{b}{12c^3d^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{5162} \\
& -\frac{\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{4c^2d^3} + \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} - \\
& \quad \frac{b}{12c^3d^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{241} \\
& -\frac{\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3} + \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} - \frac{b}{12c^3d^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{5164} \\
& -\frac{\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3} + \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} - \\
& \quad \frac{b}{12c^3d^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{3042} \\
& -\frac{\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3} + \\
& \quad \frac{x(a + b \arcsin(cx))}{4c^2d^3(1 - c^2x^2)^2} - \frac{b}{12c^3d^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{4669}
\end{aligned}$$

$$\frac{-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a + b \arcsin(cx))}{2c} + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} -$$

$$\frac{x(a + b \arcsin(cx))}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{4c^2 d^3}{12c^3 d^3 (1 - c^2 x^2)^{3/2}}$$

↓ 2715

$$\frac{ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})(a + b \arcsin(cx))}{2c}$$

$$\frac{x(a + b \arcsin(cx))}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b}{12c^3 d^3 (1 - c^2 x^2)^{3/2}}$$

↓ 2838

$$\frac{-2i \arctan(e^{i \arcsin(cx)})(a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} - \frac{b}{2c\sqrt{1 - c^2 x^2}} +$$

$$\frac{x(a + b \arcsin(cx))}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{4c^2 d^3}{12c^3 d^3 (1 - c^2 x^2)^{3/2}}$$

input `Int[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]`

output `-1/12*b/(c^3*d^3*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcSin[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (-1/2*b/(c*sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*(1 - c^2*x^2)) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c))/(4*c^2*d^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^((n_))], x_Symbol]$
 $\text{:> Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] \text{:> Simp}[-\text{PolyLog}[2,$
 $(-c)*e*x^n/n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \text{:> Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinear}$
 $\text{Q}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol]$
 $\text{:> Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f)$
 $\text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 - E^{(I*k*Pi)*E^{(I*(e + f*x))}], x],$
 $x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^(m - 1)*\text{Log}[1 + E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x]) /;$
 $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5162 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol]$
 $\text{:> Simp}[(-x)*(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(2*d*(p + 1))), x] + (\text{Simp}[(2*p + 3)/(2*d*(p + 1)) \text{ Int}[(d + e*x^2)^(p + 1)*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x]) /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p,$
 $-1] \ \&\& \ \text{NeQ}[p, -3/2]$

rule 5164 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)/((d_) + (e_)*(x_)^2), x_Symbol]$
 $\text{:> Simp}[1/(c*d) \text{ Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /;$
 $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a \left(\frac{1}{16(cx+1)^2} - \frac{1}{16(cx+1)} + \frac{\ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{1}{16(cx-1)} - \frac{\ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(-\frac{3c^3 x^3 \arcsin(cx) - 3c^2 x^2 \sqrt{-c^2 x^2 + 1} + 3cx \arcsin(cx)}{24(c^4 x^4 - 2c^2 x^2 + 1)} \right)}{d^3}$
default	$\frac{a \left(\frac{1}{16(cx+1)^2} - \frac{1}{16(cx+1)} + \frac{\ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{1}{16(cx-1)} - \frac{\ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(-\frac{3c^3 x^3 \arcsin(cx) - 3c^2 x^2 \sqrt{-c^2 x^2 + 1} + 3cx \arcsin(cx)}{24(c^4 x^4 - 2c^2 x^2 + 1)} \right)}{d^3}$
parts	$\frac{a \left(-\frac{1}{16c^3(cx-1)^2} - \frac{1}{16c^3(cx-1)} - \frac{\ln(cx-1)}{16c^3} + \frac{1}{16c^3(cx+1)^2} - \frac{1}{16c^3(cx+1)} + \frac{\ln(cx+1)}{16c^3} \right)}{d^3} - \frac{b \left(-\frac{3c^3 x^3 \arcsin(cx) - 3c^2 x^2 \sqrt{-c^2 x^2 + 1} + 3cx \arcsin(cx)}{24(c^4 x^4 - 2c^2 x^2 + 1)} \right)}{d^3}$

input

```
int(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c^3*(-a/d^3*(1/16/(c*x+1)^2-1/16/(c*x+1)+1/16*ln(c*x+1)-1/16/(c*x-1)^2-1
/16/(c*x-1)-1/16*ln(c*x-1))-b/d^3*(-1/24*(3*c^3*x^3*arcsin(c*x)-3*c^2*x^2*
(-c^2*x^2+1)^(1/2)+3*c*x*arcsin(c*x)+(-c^2*x^2+1)^(1/2)))/(c^4*x^4-2*c^2*x^
2+1)-1/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/8*arcsin(c*x)*ln
(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)
))-1/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^2}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b*x^2*arcsin(c*x) + a*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = -\int \frac{ax^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{bx^2 \arcsin(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*x**2*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^2}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output

```
1/16*a*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x
+ 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) - 1/16*((c^4*x^4 - 2*c^2*x^2 + 1)
*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - (c^4*x^4 - 2*c^
2*x^2 + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(c
^3*x^3 + c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 16*(c^7*d^3*x^4
- 2*c^5*d^3*x^2 + c^3*d^3)*integrate(-1/16*(2*c^3*x^3 + 2*c*x - (c^4*x^4
- 2*c^2*x^2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x + 1))*s
qrt(c*x + 1)*sqrt(-c*x + 1)/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 -
c^2*d^3), x))*b/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)
```

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^2}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output

```
integrate(-(b*arcsin(c*x) + a)*x^2/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

input

```
int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3,x)
```

output

```
int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-16 \left(\int \frac{\arcsin(cx)x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^7 x^4 + 32 \left(\int \frac{\arcsin(cx)x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^5 x^2 - 16 \left(\int \frac{\arcsin(cx)x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c}{}$$

input

```
int(x^2*(a+b*asin(c*x))/(-c^2*d*x^2+d)^3,x)
```

output

```
( - 16*int((asin(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*b*c**7*x**4 + 32*int((asin(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x
**2 - 1),x)*b*c**5*x**2 - 16*int((asin(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4
+ 3*c**2*x**2 - 1),x)*b*c**3 + log(c**2*x - c)*a*c**4*x**4 - 2*log(c**2*x
- c)*a*c**2*x**2 + log(c**2*x - c)*a - log(c**2*x + c)*a*c**4*x**4 + 2*lo
g(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a + 2*a*c**3*x**3 + 2*a*c*x)/(
16*c**3*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```


3.49 $\int \frac{x(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx$

Optimal result	604
Mathematica [A] (verified)	604
Rubi [A] (verified)	605
Maple [B] (verified)	606
Fricas [A] (verification not implemented)	607
Sympy [F]	607
Maxima [F]	608
Giac [B] (verification not implemented)	608
Mupad [F(-1)]	609
Reduce [F]	609

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^3} dx = -\frac{bx}{12cd^3(1 - c^2x^2)^{3/2}} - \frac{bx}{6cd^3\sqrt{1 - c^2x^2}} + \frac{a + b \arcsin(cx)}{4c^2d^3(1 - c^2x^2)^2}$$

output

$$-1/12*b*x/c/d^3/(-c^2*x^2+1)^(3/2)-1/6*b*x/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a+b*arcsin(c*x))/c^2/d^3/(-c^2*x^2+1)^2$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^3} dx = \frac{bcx(-3+2c^2x^2)}{3(1-c^2x^2)^{3/2}} + \frac{a+b \arcsin(cx)}{(-1+c^2x^2)^2} \cdot \frac{1}{4c^2d^3}$$

input

$$\text{Integrate}[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]$$

output

$$((b*c*x*(-3 + 2*c^2*x^2))/(3*(1 - c^2*x^2)^(3/2)) + (a + b*ArcSin[c*x]))/(-1 + c^2*x^2)^2/(4*c^2*d^3)$$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5182, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$\downarrow \text{5182}$$

$$\frac{a + b \arcsin(cx)}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \int \frac{1}{(1 - c^2 x^2)^{5/2}} dx}{4cd^3}$$

$$\downarrow \text{209}$$

$$\frac{a + b \arcsin(cx)}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \left(\frac{2}{3} \int \frac{1}{(1 - c^2 x^2)^{3/2}} dx + \frac{x}{3(1 - c^2 x^2)^{3/2}} \right)}{4cd^3}$$

$$\downarrow \text{208}$$

$$\frac{a + b \arcsin(cx)}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \left(\frac{2x}{3\sqrt{1 - c^2 x^2}} + \frac{x}{3(1 - c^2 x^2)^{3/2}} \right)}{4cd^3}$$

input `Int[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]`

output `-1/4*(b*(x/(3*(1 - c^2*x^2)^(3/2)) + (2*x)/(3*sqrt[1 - c^2*x^2])))/(c*d^3) + (a + b*ArcSin[c*x])/(4*c^2*d^3*(1 - c^2*x^2)^2)`

Defintions of rubi rules used

```
rule 208 Int[((a_) + (b_)*(x_)^2)^(-3/2), x_Symbol] := Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

```
rule 209 Int[((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1)) Int[(a + b*x^2)^(p + 1)
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

```
rule 5182 Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(73) = 146.

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{a}{4d^3(c^2x^2-1)^2} - \frac{b\left(-\frac{\arcsin(cx)}{4(c^2x^2-1)^2} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{48(cx-1)^2} - \frac{\sqrt{-(cx-1)^2-2cx+2}}{12(cx-1)} - \frac{\sqrt{-(cx+1)^2+2cx+2}}{48(cx+1)^2} - \frac{\sqrt{-(cx+1)^2+2cx+2}}{12(cx+1)}\right)}{d^3}}{c^2}$
default	$\frac{\frac{a}{4d^3(c^2x^2-1)^2} - \frac{b\left(-\frac{\arcsin(cx)}{4(c^2x^2-1)^2} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{48(cx-1)^2} - \frac{\sqrt{-(cx-1)^2-2cx+2}}{12(cx-1)} - \frac{\sqrt{-(cx+1)^2+2cx+2}}{48(cx+1)^2} - \frac{\sqrt{-(cx+1)^2+2cx+2}}{12(cx+1)}\right)}{d^3}}{c^2}$
parts	$\frac{a}{4d^3c^2(c^2x^2-1)^2} - \frac{b\left(-\frac{\arcsin(cx)}{4(c^2x^2-1)^2} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{48(cx-1)^2} - \frac{\sqrt{-(cx-1)^2-2cx+2}}{12(cx-1)} - \frac{\sqrt{-(cx+1)^2+2cx+2}}{48(cx+1)^2} - \frac{\sqrt{-(cx+1)^2+2cx+2}}{12(cx+1)}\right)}{d^3c^2}$
oring	$\frac{(cx-1)(cx+1)(10c^4x^4-13c^2x^2-6)(a+b\arcsin(cx))}{12c^2(-c^2dx^2+d)^3} + \frac{(2c^2x^2-3)(cx-1)^2(cx+1)^2\left(\frac{a+b\arcsin(cx)}{(-c^2dx^2+d)^3} + \frac{xbc}{\sqrt{-c^2x^2+1}}(-c\right)}{12c^2}$

```
input int(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c^2*(1/4*a/d^3/(c^2*x^2-1)^2-b/d^3*(-1/4/(c^2*x^2-1)^2*arcsin(c*x)+1/48/
(c*x-1)^2*(-(c*x-1)^2-2*c*x+2)^(1/2)-1/12/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/
2)-1/48/(c*x+1)^2*(-(c*x+1)^2+2*c*x+2)^(1/2)-1/12/(c*x+1)*(-(c*x+1)^2+2*c*
x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= -\frac{3ac^4x^4 - 6ac^2x^2 - 3b \arcsin(cx) - (2bc^3x^3 - 3bcx)\sqrt{-c^2x^2 + 1}}{12(c^6d^3x^4 - 2c^4d^3x^2 + c^2d^3)}$$

input

```
integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
-1/12*(3*a*c^4*x^4 - 6*a*c^2*x^2 - 3*b*arcsin(c*x) - (2*b*c^3*x^3 - 3*b*c*
x)*sqrt(-c^2*x^2 + 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = -\int \frac{ax}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx + \int \frac{bx \arcsin(cx)}{c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1} dx$$

input

```
integrate(x*(a+b*asin(c*x))/(-c**2*d*x**2+d)**3,x)
```

output

```
-(Integral(a*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(
b*x*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `1/4*(4*(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)*integrate(1/4*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))/(c^9*d^3*x^8 - 3*c^7*d^3*x^6 + 3*c^5*d^3*x^4 - c^3*d^3*x^2 + (c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3)*e^(log(c*x + 1) + log(-c*x + 1))), x) + arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*b/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) + 1/4*a/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(72) = 144$.

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.07

$$\begin{aligned} \int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx &= \frac{bc^2 x^4 \arcsin(cx)}{4(c^2 x^2 - 1)^2 d^3} + \frac{ac^2 x^4}{4(c^2 x^2 - 1)^2 d^3} \\ &+ \frac{bcx^3}{12(c^2 x^2 - 1)\sqrt{-c^2 x^2 + 1}d^3} - \frac{bx^2 \arcsin(cx)}{2(c^2 x^2 - 1)d^3} \\ &- \frac{ax^2}{2(c^2 x^2 - 1)d^3} - \frac{bx}{4\sqrt{-c^2 x^2 + 1}cd^3} + \frac{b \arcsin(cx)}{4c^2 d^3} + \frac{a}{4c^2 d^3} \end{aligned}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `1/4*b*c^2*x^4*arcsin(c*x)/((c^2*x^2 - 1)^2*d^3) + 1/4*a*c^2*x^4/((c^2*x^2 - 1)^2*d^3) + 1/12*b*c*x^3/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*d^3) - 1/2*b*x^2*arcsin(c*x)/((c^2*x^2 - 1)*d^3) - 1/2*a*x^2/((c^2*x^2 - 1)*d^3) - 1/4*b*x/(sqrt(-c^2*x^2 + 1)*c*d^3) + 1/4*b*arcsin(c*x)/(c^2*d^3) + 1/4*a/(c^2*d^3)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^3} dx$$

input `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3,x)`output `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3, x)`**Reduce [F]**

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-4 \left(\int \frac{\operatorname{asin}(cx)x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^6 x^4 + 8 \left(\int \frac{\operatorname{asin}(cx)x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^4 x^2 - 4 \left(\int \frac{\operatorname{asin}(cx)x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^2 + a}{4c^2 d^3 (c^4 x^4 - 2c^2 x^2 + 1)}$$

input `int(x*(a+b*asin(c*x))/(-c^2*d*x^2+d)^3,x)`output `(- 4*int((asin(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**6*x**4 + 8*int((asin(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**4*x**2 - 4*int((asin(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**2 + a)/(4*c**2*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))`

3.50 $\int \frac{a+b \arcsin(cx)}{(d-c^2dx^2)^3} dx$

Optimal result	610
Mathematica [B] (verified)	611
Rubi [A] (verified)	611
Maple [A] (verified)	615
Fricas [F]	615
Sympy [F(-1)]	616
Maxima [F]	616
Giac [F(-2)]	617
Mupad [F(-1)]	617
Reduce [F]	617

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int \frac{a + b \arcsin(cx)}{(d - c^2dx^2)^3} dx = -\frac{b}{12cd^3(1 - c^2x^2)^{3/2}} - \frac{3b}{8cd^3\sqrt{1 - c^2x^2}} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2}$$

$$+ \frac{3x(a + b \arcsin(cx))}{8d^3(1 - c^2x^2)} - \frac{3i(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{4cd^3}$$

$$+ \frac{3ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{8cd^3} - \frac{3ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{8cd^3}$$

output

```
-1/12*b/c/d^3/(-c^2*x^2+1)^(3/2)-3/8*b/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b
*arcsin(c*x))/d^3/(-c^2*x^2+1)^2+3/8*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)-
3/4*I*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c/d^3+3/8*I*b*pol
ylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^3-3/8*I*b*polylog(2,I*(I*c*x+(-c
^2*x^2+1)^(1/2)))/c/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 501 vs. $2(196) = 392$.

Time = 0.88 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.56

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx = \frac{2b\sqrt{1-c^2x^2}}{3c(-1+cx)^2} - \frac{bx\sqrt{1-c^2x^2}}{3(-1+cx)^2} + \frac{2b\sqrt{1-c^2x^2}}{3c(1+cx)^2} + \frac{bx\sqrt{1-c^2x^2}}{3(1+cx)^2} + \frac{3b\sqrt{1-c^2x^2}}{c-c^2x} + \frac{3b\sqrt{1-c^2x^2}}{c+c^2x} - \frac{4ax}{(-1+c^2x^2)^2} + \frac{6ax}{-1+c^2x^2} + \frac{3ib\pi \arcsin(cx)}{c}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^3,x]
```

output

```
-1/16*((2*b*Sqrt[1 - c^2*x^2])/(3*c*(-1 + c*x)^2) - (b*x*Sqrt[1 - c^2*x^2])/(3*(-1 + c*x)^2) + (2*b*Sqrt[1 - c^2*x^2])/(3*c*(1 + c*x)^2) + (b*x*Sqrt[1 - c^2*x^2])/(3*(1 + c*x)^2) + (3*b*Sqrt[1 - c^2*x^2])/(c - c^2*x) + (3*b*Sqrt[1 - c^2*x^2])/(c + c^2*x) - (4*a*x)/(-1 + c^2*x^2)^2 + (6*a*x)/(-1 + c^2*x^2) + ((3*I)*b*Pi*ArcSin[c*x])/c - (b*ArcSin[c*x])/(c*(-1 + c*x)^2) + (b*ArcSin[c*x])/(c*(1 + c*x)^2) - (3*b*ArcSin[c*x])/(c - c^2*x) + (3*b*ArcSin[c*x])/(c + c^2*x) - (3*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])])/c - (6*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])])/c - (3*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])])/c + (6*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c + (3*a*Log[1 - c*x])/c - (3*a*Log[1 + c*x])/c + (3*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]])/c + (3*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]])/c - ((6*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/c + ((6*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/c)/d^3
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5162, 27, 241, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx \\
& \quad \downarrow \text{5162} \\
& \frac{3 \int \frac{a+b \arcsin(cx)}{d^2(1-c^2x^2)^2} dx}{4d} - \frac{bc \int \frac{x}{(1-c^2x^2)^{5/2}} dx}{4d^3} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{27} \\
& \frac{3 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{4d^3} - \frac{bc \int \frac{x}{(1-c^2x^2)^{5/2}} dx}{4d^3} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} \\
& \quad \downarrow \text{241} \\
& \frac{3 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{4d^3} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} - \frac{b}{12cd^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{5162} \\
& \frac{3 \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2} bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{4d^3} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} - \\
& \quad \frac{b}{12cd^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{241} \\
& \frac{3 \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{4d^3} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} - \\
& \quad \frac{b}{12cd^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{5164} \\
& \frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{4d^3} + \frac{x(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} - \\
& \quad \frac{b}{12cd^3(1 - c^2x^2)^{3/2}} \\
& \quad \downarrow \text{3042}
\end{aligned}$$

$$3 \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) +$$

$$\frac{x(a+b \arcsin(cx))}{4d^3(1-c^2x^2)^2} - \frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

↓ 4669

$$3 \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) +$$

$$\frac{x(a+b \arcsin(cx))}{4d^3(1-c^2x^2)^2} - \frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

↓ 2715

$$3 \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} \right) +$$

$$\frac{x(a+b \arcsin(cx))}{4d^3(1-c^2x^2)^2} - \frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

↓ 2838

$$3 \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) +$$

$$\frac{x(a+b \arcsin(cx))}{4d^3(1-c^2x^2)^2} - \frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

input

```
Int[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^3,x]
```

output

```
-1/12*b/(c*d^3*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcSin[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (3*(-1/2*b/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*(1 - c^2*x^2)) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])]/(2*c)))/(4*d^3)
```

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 241 $\text{Int}[(x_*)((a_) + (b_*)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p + 1)}/(2*b*(p + 1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_*)((F_)^{((e_*)((c_) + (d_*)(x_)))})^{(n_)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2838 $\text{Int}[\text{Log}[(c_*)((d_) + (e_*)(x_)^{(n_)}]/(x_)), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_*)(x_)]*((c_) + (d_*)(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$
- rule 5162 $\text{Int}[(a_) + \text{ArcSin}[(c_*)(x_)]*(b_))^{(n_)*((d_) + (e_*)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(-x)*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*d*(p + 1))), x] + (\text{Simp}[(2*p + 3)/(2*d*(p + 1)) \text{ Int}[(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[x*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
  /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.34

method	result
derivativedivides	$-\frac{a \left(\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} + \frac{3}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(\frac{9c^3 x^3 \arcsin(cx) - 9c^2 x^2 \sqrt{-c^2 x^2 + 1} - 15cx a}{24c^4 x^4 - 48c^2 x^2 + 24} \right)}{d^3}$
default	$-\frac{a \left(\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} + \frac{3}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(\frac{9c^3 x^3 \arcsin(cx) - 9c^2 x^2 \sqrt{-c^2 x^2 + 1} - 15cx a}{24c^4 x^4 - 48c^2 x^2 + 24} \right)}{d^3}$
parts	$-\frac{a \left(-\frac{1}{16c(cx-1)^2} + \frac{3}{16c(cx-1)} + \frac{3 \ln(cx-1)}{16c} + \frac{1}{16c(cx+1)^2} + \frac{3}{16c(cx+1)} - \frac{3 \ln(cx+1)}{16c} \right)}{d^3} - \frac{b \left(\frac{9c^3 x^3 \arcsin(cx) - 9c^2 x^2 \sqrt{-c^2 x^2 + 1} - 15cx a}{24c^4 x^4 - 48c^2 x^2 + 24} \right)}{d^3}$

input

```
int((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c*(-a/d^3*(1/16/(c*x+1)^2+3/16/(c*x+1)-3/16*ln(c*x+1)-1/16/(c*x-1)^2+3/16/(c*x-1)+3/16*ln(c*x-1))-b/d^3*(1/24*(9*c^3*x^3*arcsin(c*x)-9*c^2*x^2*(-c^2*x^2+1)^(1/2)-15*c*x*arcsin(c*x)+11*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x^2+1)+3/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output `integral(-(b*arcsin(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx = \text{Timed out}$$

input `integrate((a+b*asin(c*x))/(-c**2*d*x**2+d)**3,x)`

output Timed out

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/16*a*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) + 1/16*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(3*c^3*x^3 - 5*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 16*(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)*integrate(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x))*b/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{(d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))/(d - c^2*d*x^2)^3,x)`

output `int((a + b*asin(c*x))/(d - c^2*d*x^2)^3, x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-16 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^5 x^4 + 32 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c^3 x^2 - 16 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) b c}{1}$$

input `int((a+b*asin(c*x))/(-c^2*d*x^2+d)^3,x)`

output

```
( - 16*int(asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**5
*x**4 + 32*int(asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
c**3*x**2 - 16*int(asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x
)*b*c - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c**2*x - c)*a*c**2*x**2 - 3*
log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c*
*2*x**2 + 3*log(c**2*x + c)*a - 6*a*c**3*x**3 + 10*a*c*x)/(16*c*d**3*(c**4
*x**4 - 2*c**2*x**2 + 1))
```

3.51 $\int \frac{a+b \arcsin(cx)}{x(d-c^2dx^2)^3} dx$

Optimal result	619
Mathematica [B] (verified)	620
Rubi [A] (verified)	620
Maple [A] (verified)	624
Fricas [F]	625
Sympy [F(-1)]	625
Maxima [F]	626
Giac [F(-2)]	626
Mupad [F(-1)]	626
Reduce [F]	627

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2dx^2)^3} dx = -\frac{bcx}{12d^3(1 - c^2x^2)^{3/2}} - \frac{2bcx}{3d^3\sqrt{1 - c^2x^2}} + \frac{a + b \arcsin(cx)}{4d^3(1 - c^2x^2)^2} + \frac{a + b \arcsin(cx)}{2d^3(1 - c^2x^2)} - \frac{2(a + b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^3} + \frac{ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2d^3} - \frac{ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{2d^3}$$

output

```
-1/12*b*c*x/d^3/(-c^2*x^2+1)^(3/2)-2/3*b*c*x/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a
+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^2+1/2*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)-
2*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3+1/2*I*b*poly
log(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-1/2*I*b*polylog(2,(I*c*x+(-c^2*x^
2+1)^(1/2))^2)/d^3
```


Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 524 vs. $2(173) = 346$.

Time = 0.76 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.03

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^3} dx$$

$$= \frac{-\frac{b\sqrt{1-c^2x^2}}{6(-1+cx)^2} + \frac{bcx\sqrt{1-c^2x^2}}{12(-1+cx)^2} + \frac{b\sqrt{1-c^2x^2}}{6(1+cx)^2} + \frac{bcx\sqrt{1-c^2x^2}}{12(1+cx)^2} + \frac{5b\sqrt{1-c^2x^2}}{-4+4cx} + \frac{5b\sqrt{1-c^2x^2}}{4+4cx} + \frac{a}{(-1+c^2x^2)^2} - \frac{2a}{-1+c^2x^2} - 4ib\pi \arcsin(cx)}{1}$$

input `Integrate[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^3),x]`

output

```
(-1/6*(b*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 + (b*c*x*Sqrt[1 - c^2*x^2])/(12*(-1 + c*x)^2) + (b*Sqrt[1 - c^2*x^2])/(6*(1 + c*x)^2) + (b*c*x*Sqrt[1 - c^2*x^2])/(12*(1 + c*x)^2) + (5*b*Sqrt[1 - c^2*x^2])/(-4 + 4*c*x) + (5*b*Sqrt[1 - c^2*x^2])/(4 + 4*c*x) + a/(-1 + c^2*x^2)^2 - (2*a)/(-1 + c^2*x^2) - (4*I)*b*Pi*ArcSin[c*x] + (5*b*ArcSin[c*x])/(4 - 4*c*x) + (b*ArcSin[c*x])/(4*(-1 + c*x)^2) + (b*ArcSin[c*x])/(4*(1 + c*x)^2) + (5*b*ArcSin[c*x])/(4 + 4*c*x) - 8*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 2*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 4*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 4*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 4*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 4*a*Log[x] - 2*a*Log[1 - c^2*x^2] + 8*b*Pi*Log[Cos[ArcSin[c*x]/2]] - 2*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (4*I)*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*b*PolyLog[2, I*E^(I*ArcSin[c*x])] - (2*I)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(4*d^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5208, 27, 209, 208, 5208, 208, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^3} dx \\
& \quad \downarrow \text{5208} \\
& \frac{\int \frac{a+b \arcsin(cx)}{d^2 x(1-c^2 x^2)^2} dx}{d} - \frac{bc \int \frac{1}{(1-c^2 x^2)^{5/2}} dx}{4d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{27} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^2} dx}{d^3} - \frac{bc \int \frac{1}{(1-c^2 x^2)^{5/2}} dx}{4d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{209} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^2} dx}{d^3} - \frac{bc \left(\frac{2}{3} \int \frac{1}{(1-c^2 x^2)^{3/2}} dx + \frac{x}{3(1-c^2 x^2)^{3/2}} \right)}{4d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{208} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^2} dx}{d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2 x^2}} + \frac{x}{3(1-c^2 x^2)^{3/2}} \right)}{4d^3} \\
& \quad \downarrow \text{5208} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx - \frac{1}{2} bc \int \frac{1}{(1-c^2 x^2)^{3/2}} dx + \frac{a+b \arcsin(cx)}{2(1-c^2 x^2)}}{d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} - \\
& \quad \frac{bc \left(\frac{2x}{3\sqrt{1-c^2 x^2}} + \frac{x}{3(1-c^2 x^2)^{3/2}} \right)}{4d^3} \\
& \quad \downarrow \text{208} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx + \frac{a+b \arcsin(cx)}{2(1-c^2 x^2)} - \frac{bcx}{2\sqrt{1-c^2 x^2}}}{d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2 x^2}} + \frac{x}{3(1-c^2 x^2)^{3/2}} \right)}{4d^3} \\
& \quad \downarrow \text{5184} \\
& \frac{\int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2 x^2}} dx + \frac{a+b \arcsin(cx)}{2(1-c^2 x^2)} - \frac{bcx}{2\sqrt{1-c^2 x^2}}}{d^3} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2 x^2)^2} - \\
& \quad \frac{bc \left(\frac{2x}{3\sqrt{1-c^2 x^2}} + \frac{x}{3(1-c^2 x^2)^{3/2}} \right)}{4d^3} \\
& \quad \downarrow \text{4919}
\end{aligned}$$

$$\frac{2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) d^3}{4d^3}}{d^3} +$$

↓ 3042

$$\frac{2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) d^3}{4d^3}}{d^3} +$$

↓ 4671

$$\frac{2 \left(-\frac{1}{2} b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2} b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))) \right) + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) d^3}{4d^3}}{d^3}$$

↓ 2715

$$\frac{2 \left(\frac{1}{4} ib \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4} ib \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - (\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))) \right) + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) d^3}{4d^3}}{d^3}$$

↓ 2838

$$\frac{2 \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))) + \frac{1}{4} ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4} ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right) + \frac{a + b \arcsin(cx)}{4d^3 (1 - c^2x^2)^2} - \frac{bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) d^3}{4d^3}}{d^3}$$

input `Int[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^3),x]`

output

$$-1/4*(b*c*(x/(3*(1 - c^2*x^2)^{(3/2)}) + (2*x)/(3*sqrt[1 - c^2*x^2])))/d^3 + (a + b*ArcSin[c*x])/(4*d^3*(1 - c^2*x^2)^2) + (-1/2*(b*c*x)/sqrt[1 - c^2*x^2] + (a + b*ArcSin[c*x])/(2*(1 - c^2*x^2)) + 2*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])])) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/d^3$$
Defintions of rubi rules used

rule 27

$$\text{Int}[(a_)*(F x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F x, (b_)*(G x_)] /; \text{FreeQ}[b, x]$$

rule 208

$$\text{Int}[(a_) + (b_)*(x_)^2)^{-3/2}, x_Symbol] \rightarrow \text{Simp}[x/(a*sqrt[a + b*x^2]), x] /; \text{FreeQ}[\{a, b\}, x]$$

rule 209

$$\text{Int}[(a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(-x)*((a + b*x^2)^{(p + 1)}/(2*a*(p + 1))), x] + \text{Simp}[(2*p + 3)/(2*a*(p + 1)) \text{ Int}[(a + b*x^2)^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{ILtQ}[p + 3/2, 0]$$

rule 2715

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^((n_))], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$$

```
rule 4671 Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

```
rule 4919 Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

```
rule 5184 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSi
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

```
rule 5208 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{a\left(-\frac{1}{16(cx-1)^2} + \frac{5}{16(cx-1)} + \frac{\ln(cx-1)}{2} - \frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} + \frac{\ln(cx+1)}{2} - \ln(x)\right)}{d^3} - \frac{b\left(\frac{8ic^4x^4 - 8c^3x^3\sqrt{-c^2x^2+1} + 6c^2}{d^3}\right)}{d^3}$
derivativedivides	$-\frac{a\left(-\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} + \frac{\ln(cx+1)}{2} - \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{5}{16(cx-1)} + \frac{\ln(cx-1)}{2}\right)}{d^3} - \frac{b\left(\frac{8ic^4x^4 - 8c^3x^3\sqrt{-c^2x^2+1} + 6c^2}{d^3}\right)}{d^3}$
default	$-\frac{a\left(-\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} + \frac{\ln(cx+1)}{2} - \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{5}{16(cx-1)} + \frac{\ln(cx-1)}{2}\right)}{d^3} - \frac{b\left(\frac{8ic^4x^4 - 8c^3x^3\sqrt{-c^2x^2+1} + 6c^2}{d^3}\right)}{d^3}$

input `int((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)`

output `-a/d^3*(-1/16/(c*x-1)^2+5/16/(c*x-1)+1/2*ln(c*x-1)-1/16/(c*x+1)^2-5/16/(c*x+1)+1/2*ln(c*x+1)-ln(x))-b/d^3*(1/12*(8*I*c^4*x^4-8*c^3*x^3*(-c^2*x^2+1)^(1/2)+6*c^2*x^2*arcsin(c*x)-16*I*c^2*x^2+9*c*x*(-c^2*x^2+1)^(1/2)-9*arcsin(c*x)+8*I)/(c^4*x^4-2*c^2*x^2+1)+arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2)))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x (d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x (d - c^2 dx^2)^3} dx = \text{Timed out}$$

input `integrate((a+b*asin(c*x))/x/(-c**2*d*x**2+d)**3,x)`

output `Timed out`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/4*a*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x + 1)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x(d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^3),x)`

output `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^3), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^3} dx$$

$$= \frac{-4 \left(\int \frac{\arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx \right) b c^4 x^4 + 8 \left(\int \frac{\arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx \right) b c^2 x^2 - 4 \left(\int \frac{\arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx \right) b - 2 \log(x) a - a c^4 x^4 + 2a}{(4d^3(c^4 x^4 - 2c^2 x^2 + 1))}$$

input

```
int((a+b*asin(c*x))/x/(-c^2*d*x^2+d)^3,x)
```

output

```
( - 4*int(asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b*c**4*x**4 + 8*int(asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b*c**2*x**2 - 4*int(asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b - 2*log(c**2*x - c)*a*c**4*x**4 + 4*log(c**2*x - c)*a*c**2*x**2 - 2*log(c**2*x - c)*a - 2*log(c**2*x + c)*a*c**4*x**4 + 4*log(c**2*x + c)*a*c**2*x**2 - 2*log(c**2*x + c)*a + 4*log(x)*a*c**4*x**4 - 8*log(x)*a*c**2*x**2 + 4*log(x)*a - a*c**4*x**4 + 2*a)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```


3.52 $\int \frac{a+b \arcsin(cx)}{x^2(d-c^2dx^2)^3} dx$

Optimal result	628
Mathematica [B] (verified)	629
Rubi [A] (verified)	629
Maple [A] (verified)	635
Fricas [F]	635
Sympy [F]	636
Maxima [F]	636
Giac [F(-2)]	637
Mupad [F(-1)]	637
Reduce [F]	637

Optimal result

Integrand size = 25, antiderivative size = 230

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx = -\frac{bc}{12d^3 (1 - c^2 x^2)^{3/2}} - \frac{7bc}{8d^3 \sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{d^3 x} + \frac{c^2 x (a + b \arcsin(cx))}{4d^3 (1 - c^2 x^2)^2} + \frac{7c^2 x (a + b \arcsin(cx))}{8d^3 (1 - c^2 x^2)} - \frac{15ic(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{4d^3} - \frac{bc \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{d^3} + \frac{15ibc \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{8d^3} - \frac{15ibc \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{8d^3}$$

output

```
-1/12*b*c/d^3/(-c^2*x^2+1)^(3/2)-7/8*b*c/d^3/(-c^2*x^2+1)^(1/2)-(a+b*arcsin(c*x))/d^3/x+1/4*c^2*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^2+7/8*c^2*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)-15/4*I*c*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^3-b*c*arctanh((-c^2*x^2+1)^(1/2))/d^3+15/8*I*b*c*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3-15/8*I*b*c*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 512 vs. $2(230) = 460$.

Time = 1.45 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.23

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx =$$

$$\frac{16a}{x} + \frac{2bc\sqrt{1-c^2x^2}}{3(-1+cx)^2} - \frac{bc^2x\sqrt{1-c^2x^2}}{3(-1+cx)^2} - \frac{7bc\sqrt{1-c^2x^2}}{-1+cx} + \frac{2bc\sqrt{1-c^2x^2}}{3(1+cx)^2} + \frac{bc^2x\sqrt{1-c^2x^2}}{3(1+cx)^2} + \frac{7bc\sqrt{1-c^2x^2}}{1+cx} - \frac{4ac^2x}{(-1+c^2x^2)^2} + \frac{14ac}{-1+c^2x^2}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

output

```
-1/16*((16*a)/x + (2*b*c*Sqrt[1 - c^2*x^2])/(3*(-1 + c*x)^2) - (b*c^2*x*Sqrt[1 - c^2*x^2])/(3*(-1 + c*x)^2) - (7*b*c*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (2*b*c*Sqrt[1 - c^2*x^2])/(3*(1 + c*x)^2) + (b*c^2*x*Sqrt[1 - c^2*x^2])/(3*(1 + c*x)^2) + (7*b*c*Sqrt[1 - c^2*x^2])/(1 + c*x) - (4*a*c^2*x)/(-1 + c^2*x^2)^2 + (14*a*c^2*x)/(-1 + c^2*x^2) + (15*I)*b*c*Pi*ArcSin[c*x] + (16*b*ArcSin[c*x])/x - (b*c*ArcSin[c*x])/(-1 + c*x)^2 + (7*b*c*ArcSin[c*x])/(-1 + c*x) + (b*c*ArcSin[c*x])/(1 + c*x)^2 + (7*b*c*ArcSin[c*x])/(1 + c*x) + 16*b*c*ArcTanh[Sqrt[1 - c^2*x^2]] - 15*b*c*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 30*b*c*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 15*b*c*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 30*b*c*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 15*a*c*Log[1 - c*x] - 15*a*c*Log[1 + c*x] + 15*b*c*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 15*b*c*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (30*I)*b*c*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (30*I)*b*c*PolyLog[2, I*E^(I*ArcSin[c*x])])/d^3
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5204, 27, 243, 61, 61, 73, 221, 5162, 241, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 x^2)^3} dx \\
& \quad \downarrow \text{5204} \\
& 5c^2 \int \frac{a + b \arcsin(cx)}{d^3 (1 - c^2 x^2)^3} dx + \frac{bc \int \frac{1}{x(1-c^2 x^2)^{5/2}} dx}{d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{27} \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^3} dx}{d^3} + \frac{bc \int \frac{1}{x(1-c^2 x^2)^{5/2}} dx}{d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{243} \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^3} dx}{d^3} + \frac{bc \int \frac{1}{x^2(1-c^2 x^2)^{5/2}} dx^2}{2d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{61} \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^3} dx}{d^3} + \frac{bc \left(\int \frac{1}{x^2(1-c^2 x^2)^{3/2}} dx^2 + \frac{2}{3(1-c^2 x^2)^{3/2}} \right)}{2d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{61} \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^3} dx}{d^3} + \frac{bc \left(\int \frac{1}{x^2 \sqrt{1-c^2 x^2}} dx^2 + \frac{2}{\sqrt{1-c^2 x^2}} + \frac{2}{3(1-c^2 x^2)^{3/2}} \right)}{2d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{73} \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^3} dx}{d^3} + \frac{bc \left(-\frac{2 \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d\sqrt{1-c^2 x^2}}{c^2} + \frac{2}{\sqrt{1-c^2 x^2}} + \frac{2}{3(1-c^2 x^2)^{3/2}} \right)}{2d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} \\
& \quad \downarrow \text{221} \\
& \frac{5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^3} dx}{d^3} - \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^2} + \\
& \frac{bc \left(-2 \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) + \frac{2}{\sqrt{1-c^2 x^2}} + \frac{2}{3(1-c^2 x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{5162}
\end{aligned}$$

$$\begin{aligned}
& \frac{5c^2 \left(\frac{3}{4} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx - \frac{1}{4} bc \int \frac{x}{(1-c^2x^2)^{5/2}} dx + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} \right)}{d^3} - \frac{a+b \arcsin(cx)}{d^3x(1-c^2x^2)^2} + \\
& \frac{bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{241} \\
& \frac{5c^2 \left(\frac{3}{4} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right)}{d^3} - \frac{a+b \arcsin(cx)}{d^3x(1-c^2x^2)^2} + \\
& \frac{bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{5162} \\
& \frac{5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2} bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right)}{d^3} \\
& \frac{a+b \arcsin(cx)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{241} \\
& \frac{5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right)}{d^3} - \\
& \frac{a+b \arcsin(cx)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{5164} \\
& \frac{5c^2 \left(\frac{3}{4} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right)}{d^3} \\
& \frac{a+b \arcsin(cx)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{3042}
\end{aligned}$$

$$\frac{5c^2 \left(\frac{3}{4} \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh} \left(\sqrt{1-c^2x^2} \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 4669

$$\frac{5c^2 \left(\frac{3}{4} \left(\frac{-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh} \left(\sqrt{1-c^2x^2} \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 2715

$$\frac{5c^2 \left(\frac{3}{4} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh} \left(\sqrt{1-c^2x^2} \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 2838

$$\frac{5c^2 \left(\frac{3}{4} \left(\frac{-2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{d^3x(1-c^2x^2)^2} + \frac{bc \left(-2 \operatorname{arctanh} \left(\sqrt{1-c^2x^2} \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

output

```

-((a + b*ArcSin[c*x])/(d^3*x*(1 - c^2*x^2)^2)) + (b*c*(2/(3*(1 - c^2*x^2)^(3/2)) + 2/Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(2*d^3) + (5*c^2*(-1/12*b/(c*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcSin[c*x]))/(4*(1 - c^2*x^2)^2) + (3*(-1/2*b/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*(1 - c^2*x^2)) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c)))/4)/d^3
    
```

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 61 $\text{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_))^n, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*((c + d*x)^{n+1}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{ Int}[(a + b*x)^{m+1}*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 73 $\text{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_))^n, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \ \text{Subst}[\text{Int}[x^{p*(m+1)-1}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$
- rule 241 $\text{Int}[(x_)*((a_.) + (b_.)*(x_)^2)^p, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{p+1}/(2*b*(p + 1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 243 $\text{Int}[(x_)^m*((a_.) + (b_.)*(x_)^2)^p, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m-1)/2}*(a + b*x)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2715 $\text{Int}[\text{Log}[(a_.) + (b_.)*((F_)^e*((c_.) + (d_.)*(x_)))]^n, x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{e*(c + d*x)})^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*(d_)+(e_)*(x_)^{(n_)}]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_)+\text{Pi}*(k_)+(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c+d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e+f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \ \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1-E^{(I*k*Pi)*E^{(I*(e+f*x))}], x], x] + \text{Simp}[d*(m/f) \ \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1+E^{(I*k*Pi)*E^{(I*(e+f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5162 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(-x)*(d+e*x^2)^{(p+1)}*((a+b*\text{ArcSin}[c*x])^n/(2*d*(p+1))), x] + (\text{Simp}[(2*p+3)/(2*d*(p+1)) \ \text{Int}[(d+e*x^2)^{(p+1)}*(a+b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*(p+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \ \text{Int}[x*(1-c^2*x^2)^{(p+1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

rule 5164 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*((d_)+(e_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \ \text{Subst}[\text{Int}[(a+b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5204 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*((f_)*(x_))^{(m_)}*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d+e*x^2)^{(p+1)}*((a+b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1))) \ \text{Int}[(f*x)^{(m+2)}*(d+e*x^2)^p*(a+b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \ \text{Int}[(f*x)^{(m+1)}*(1-c^2*x^2)^{(p+1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.42

method	result
derivativedivides	$c \left(-\frac{a \left(\frac{1}{16(cx+1)^2} + \frac{7}{16(cx+1)} - \frac{15 \ln(cx+1)}{16} + \frac{1}{cx} - \frac{1}{16(cx-1)^2} + \frac{7}{16(cx-1)} + \frac{15 \ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(\frac{45c^4 x^4 \arcsin(cx) - 21c^3 x^3}{d^3} \right)}{d^3} \right)$
default	$c \left(-\frac{a \left(\frac{1}{16(cx+1)^2} + \frac{7}{16(cx+1)} - \frac{15 \ln(cx+1)}{16} + \frac{1}{cx} - \frac{1}{16(cx-1)^2} + \frac{7}{16(cx-1)} + \frac{15 \ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(\frac{45c^4 x^4 \arcsin(cx) - 21c^3 x^3}{d^3} \right)}{d^3} \right)$
parts	$-\frac{a \left(-\frac{c}{16(cx-1)^2} + \frac{7c}{16(cx-1)} + \frac{15c \ln(cx-1)}{16} + \frac{c}{16(cx+1)^2} + \frac{7c}{16(cx+1)} - \frac{15c \ln(cx+1)}{16} + \frac{1}{x} \right)}{d^3} - \frac{bc \left(\frac{45c^4 x^4 \arcsin(cx) - 21c^3 x^3}{d^3} \right)}{d^3}$

input `int((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)`

output `c*(-a/d^3*(1/16/(c*x+1)^2+7/16/(c*x+1)-15/16*ln(c*x+1)+1/c/x-1/16/(c*x-1)^2+7/16/(c*x-1)+15/16*ln(c*x-1))-b/d^3*(1/24*(45*c^4*x^4*arcsin(c*x)-21*c^3*x^3*(-c^2*x^2+1)^(1/2)-75*c^2*x^2*arcsin(c*x)+23*c*x*(-c^2*x^2+1)^(1/2)+24*arcsin(c*x))/c/x/(c^4*x^4-2*c^2*x^2+1)-ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+15/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))))-15/8*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-15/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+15/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx = - \int \frac{\frac{a}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2}}{d^3} dx + \int \frac{b \arcsin(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx$$

input `integrate((a+b*asin(c*x))/x**2/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral(b*asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x))/d**3`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx = \int - \frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/16*a*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) + 1/16*(15*(c^5*x^5 - 2*c^3*x^3 + c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(c*x + 1) - 15*(c^5*x^5 - 2*c^3*x^3 + c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(15*c^4*x^4 - 25*c^2*x^2 + 8)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + 16*(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x)*integrate(-1/16*(30*c^5*x^4 - 50*c^3*x^2 - 15*(c^6*x^5 - 2*c^4*x^3 + c^2*x)*log(c*x + 1) + 15*(c^6*x^5 - 2*c^4*x^3 + c^2*x)*log(-c*x + 1) + 16*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)) *b/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^3),x)`

output `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^3), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^3} dx$$

$$= \frac{-16 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx \right) b c^4 x^5 + 32 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx \right) b c^2 x^3 - 16 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx \right)}{1}$$

input `int((a+b*asin(c*x))/x^2/(-c^2*d*x^2+d)^3,x)`

output

```
( - 16*int(asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*b*c
**4*x**5 + 32*int(asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2)
,x)*b*c**2*x**3 - 16*int(asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4
- x**2),x)*b*x - 15*log(c**2*x - c)*a*c**5*x**5 + 30*log(c**2*x - c)*a*c**
3*x**3 - 15*log(c**2*x - c)*a*c*x + 15*log(c**2*x + c)*a*c**5*x**5 - 30*lo
g(c**2*x + c)*a*c**3*x**3 + 15*log(c**2*x + c)*a*c*x - 30*a*c**4*x**4 + 50
*a*c**2*x**2 - 16*a)/(16*d**3*x*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.53 $\int \frac{a+b \arcsin(cx)}{x^3(d-c^2dx^2)^3} dx$

Optimal result	639
Mathematica [B] (verified)	640
Rubi [A] (verified)	640
Maple [A] (verified)	646
Fricas [F]	646
Sympy [F]	647
Maxima [F]	647
Giac [F(-2)]	648
Mupad [F(-1)]	648
Reduce [F]	648

Optimal result

Integrand size = 25, antiderivative size = 231

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx = -\frac{bc^3 x}{12d^3 (1 - c^2 x^2)^{3/2}} - \frac{7bc^3 x}{6d^3 \sqrt{1 - c^2 x^2}}$$

$$-\frac{bc\sqrt{1 - c^2 x^2}}{2d^3 x} - \frac{bc^2 \arcsin(cx)}{d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2}$$

$$+ \frac{c^2(3 - 2c^2 x^2)^2 (a + b \arcsin(cx))}{4d^3 (1 - c^2 x^2)^2}$$

$$-\frac{6c^2(a + b \arcsin(cx)) \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^3}$$

$$+ \frac{3ibc^2 \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{2d^3}$$

$$-\frac{3ibc^2 \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{2d^3}$$

output

```
-1/12*b*c^3*x/d^3/(-c^2*x^2+1)^(3/2)-7/6*b*c^3*x/d^3/(-c^2*x^2+1)^(1/2)-1/
2*b*c*(-c^2*x^2+1)^(1/2)/d^3/x-b*c^2*arcsin(c*x)/d^3-1/2*(a+b*arcsin(c*x))
/d^3/x^2+1/4*c^2*(-2*c^2*x^2+3)^2*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^2-6*c
^2*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3+3/2*I*b*c^2
*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-3/2*I*b*c^2*polylog(2,(I*c*x
+(-c^2*x^2+1)^(1/2))^2)/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 568 vs. $2(231) = 462$.

Time = 1.22 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.46

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx$$

$$= \frac{-\frac{2a}{x^2} + \frac{ac^2}{(-1+c^2x^2)^2} - \frac{4ac^2}{-1+c^2x^2} + \frac{9bc^2(\sqrt{1-c^2x^2}-\arcsin(cx))}{-4+4cx} + \frac{9bc^2(\sqrt{1-c^2x^2}+\arcsin(cx))}{4+4cx} - \frac{2b(cx\sqrt{1-c^2x^2}+\arcsin(cx))}{x^2} + \dots}{(d - c^2 dx^2)^3}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^3),x]`

output

```
((-2*a)/x^2 + (a*c^2)/(-1 + c^2*x^2)^2 - (4*a*c^2)/(-1 + c^2*x^2) + (9*b*c^2*(Sqrt[1 - c^2*x^2] - ArcSin[c*x]))/(-4 + 4*c*x) + (9*b*c^2*(Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/(4 + 4*c*x) - (2*b*(c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/x^2 + (b*c^2*((-2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(12*(-1 + c*x)^2) + (b*c^2*((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(12*(1 + c*x)^2) + 12*a*c^2*Log[x] - 6*a*c^2*Log[1 - c^2*x^2] + 3*b*c^2*(I*ArcSin[c*x]^2 + ArcSin[c*x]*((-3*I)*Pi - 4*Log[1 + I*E^(I*ArcSin[c*x])]) + 2*Pi*(-2*Log[1 + E^((-I)*ArcSin[c*x])]) + Log[1 + I*E^(I*ArcSin[c*x])]) + 2*Log[Cos[ArcSin[c*x]/2]] - Log[-Cos[(Pi + 2*ArcSin[c*x])/4]]) + (4*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + 3*b*c^2*(I*ArcSin[c*x]^2 + ArcSin[c*x]*((-I)*Pi - 4*Log[1 - I*E^(I*ArcSin[c*x])]) + 2*Pi*(-2*Log[1 + E^((-I)*ArcSin[c*x])]) - Log[1 - I*E^(I*ArcSin[c*x])]) + 2*Log[Cos[ArcSin[c*x]/2]] + Log[Sin[(Pi + 2*ArcSin[c*x])/4]]) + (4*I)*PolyLog[2, I*E^(I*ArcSin[c*x])] + 12*b*c^2*(ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - (I/2)*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])])))/(4*d^3)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.26, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5204, 27, 245, 209, 208, 5208, 209, 208, 5208, 208, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the

transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5204} \\
 & 3c^2 \int \frac{a + b \arcsin(cx)}{d^3 x (1 - c^2 x^2)^3} dx + \frac{bc \int \frac{1}{x^2 (1 - c^2 x^2)^{5/2}} dx}{2d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & \frac{3c^2 \int \frac{a + b \arcsin(cx)}{x (1 - c^2 x^2)^3} dx}{d^3} + \frac{bc \int \frac{1}{x^2 (1 - c^2 x^2)^{5/2}} dx}{2d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{245} \\
 & \frac{3c^2 \int \frac{a + b \arcsin(cx)}{x (1 - c^2 x^2)^3} dx}{d^3} + \frac{bc \left(4c^2 \int \frac{1}{(1 - c^2 x^2)^{5/2}} dx - \frac{1}{x (1 - c^2 x^2)^{3/2}} \right)}{2d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{209} \\
 & \frac{3c^2 \int \frac{a + b \arcsin(cx)}{x (1 - c^2 x^2)^3} dx}{d^3} + \frac{bc \left(4c^2 \left(\frac{2}{3} \int \frac{1}{(1 - c^2 x^2)^{3/2}} dx + \frac{x}{3(1 - c^2 x^2)^{3/2}} \right) - \frac{1}{x (1 - c^2 x^2)^{3/2}} \right)}{2d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{208} \\
 & \frac{3c^2 \int \frac{a + b \arcsin(cx)}{x (1 - c^2 x^2)^3} dx}{d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2 (1 - c^2 x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1 - c^2 x^2}} + \frac{x}{3(1 - c^2 x^2)^{3/2}} \right) - \frac{1}{x (1 - c^2 x^2)^{3/2}} \right)}{2d^3} \\
 & \quad \downarrow \text{5208} \\
 & \frac{3c^2 \left(\int \frac{a + b \arcsin(cx)}{x (1 - c^2 x^2)^2} dx - \frac{1}{4} bc \int \frac{1}{(1 - c^2 x^2)^{5/2}} dx + \frac{a + b \arcsin(cx)}{4(1 - c^2 x^2)^2} \right)}{d^3} - \frac{a + b \arcsin(cx)}{2d^3 x^2 (1 - c^2 x^2)^2} + \\
 & \quad \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1 - c^2 x^2}} + \frac{x}{3(1 - c^2 x^2)^{3/2}} \right) - \frac{1}{x (1 - c^2 x^2)^{3/2}} \right)}{2d^3} \\
 & \quad \downarrow \text{209}
 \end{aligned}$$

$$\begin{aligned}
& \frac{3c^2 \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{1}{4}bc \left(\frac{2}{3} \int \frac{1}{(1-c^2x^2)^{3/2}} dx + \frac{x}{3(1-c^2x^2)^{3/2}} \right) + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} \right)}{d^3} - \\
& \frac{a+b \arcsin(cx)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{208} \\
& \frac{3c^2 \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} - \frac{1}{4}bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) \right)}{d^3} - \frac{a+b \arcsin(cx)}{2d^3x^2(1-c^2x^2)^2} + \\
& \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{5208} \\
& \frac{3c^2 \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{1}{2}bc \int \frac{1}{(1-c^2x^2)^{3/2}} dx + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} - \frac{1}{4}bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) \right)}{d^3} - \\
& \frac{a+b \arcsin(cx)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{208} \\
& \frac{3c^2 \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} - \frac{bcx}{2\sqrt{1-c^2x^2}} - \frac{1}{4}bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) \right)}{d^3} - \\
& \frac{a+b \arcsin(cx)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{5184} \\
& \frac{3c^2 \left(\int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2x^2}} d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} - \frac{bcx}{2\sqrt{1-c^2x^2}} - \frac{1}{4}bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) \right)}{d^3} - \\
& \frac{a+b \arcsin(cx)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3} \\
& \quad \downarrow \text{4919}
\end{aligned}$$

$$\frac{3c^2 \left(2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} - \frac{bcx}{2\sqrt{1-c^2x^2}} - \frac{1}{4}bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} \right) \right)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 3042

$$\frac{3c^2 \left(2 \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} + \frac{a+b \arcsin(cx)}{4(1-c^2x^2)^2} - \frac{bcx}{2\sqrt{1-c^2x^2}} - \frac{1}{4}bc \left(\frac{2x}{3\sqrt{1-c^2x^2}} \right) \right)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 4671

$$\frac{3c^2 \left(2 \left(-\frac{1}{2}b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})) \right) \right)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 2715

$$\frac{3c^2 \left(2 \left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right) \right)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

↓ 2838

$$\frac{3c^2 \left(2 \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)})) (a + b \arcsin(cx)) \right) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right)}{2d^3x^2(1-c^2x^2)^2} + \frac{bc \left(4c^2 \left(\frac{2x}{3\sqrt{1-c^2x^2}} + \frac{x}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x(1-c^2x^2)^{3/2}} \right)}{2d^3}$$

input `Int[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^3), x]`

output

$$\frac{(b*c*(-1/(x*(1 - c^2*x^2)^{(3/2)})) + 4*c^2*(x/(3*(1 - c^2*x^2)^{(3/2)})) + (2*x)/(3*sqrt[1 - c^2*x^2]))/(2*d^3) - (a + b*ArcSin[c*x])/(2*d^3*x^2*(1 - c^2*x^2)^2) + (3*c^2*(-1/2*(b*c*x)/sqrt[1 - c^2*x^2] - (b*c*(x/(3*(1 - c^2*x^2)^{(3/2)})) + (2*x)/(3*sqrt[1 - c^2*x^2])))/4 + (a + b*ArcSin[c*x])/(4*(1 - c^2*x^2)^2) + (a + b*ArcSin[c*x])/(2*(1 - c^2*x^2)) + 2*(-((a + b*ArcSin[c*x])*ArcTanh[E^{((2*I)*ArcSin[c*x])}] + (I/4)*b*PolyLog[2, -E^{((2*I)*ArcSin[c*x])}]) - (I/4)*b*PolyLog[2, E^{((2*I)*ArcSin[c*x])}])))/d^3$$

Definitions of rubi rules used

rule 27

$$\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] \text{ ; FreeQ}[b, x]$$

rule 208

$$\text{Int}[(a_*) + (b_*)(x_)^2)^{-3/2}, x_Symbol] \rightarrow \text{Simp}[x/(a*sqrt[a + b*x^2]), x] \text{ ; FreeQ}[\{a, b\}, x]$$

rule 209

$$\text{Int}[(a_*) + (b_*)(x_)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(-x)*((a + b*x^2)^{p+1}/(2*a*(p+1))), x] + \text{Simp}[(2*p+3)/(2*a*(p+1)) \text{ Int}[(a + b*x^2)^{p+1}], x], x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{ILtQ}[p+3/2, 0]$$

rule 245

$$\text{Int}[(x_)^m*((a_*) + (b_*)(x_)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[x^{m+1}*((a + b*x^2)^{p+1}/(a*(m+1))), x] - \text{Simp}[b*((m+2*(p+1)+1)/(a*(m+1))) \text{ Int}[x^{m+2}*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{ILtQ}[\text{Simplify}[(m+1)/2 + p + 1], 0] \ \&\& \ \text{NeQ}[m, -1]$$

rule 2715

$$\text{Int}[\text{Log}[(a_*) + (b_*)(F_)^{(e_)*((c_*) + (d_*)(x_))})^{n_}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n)], x] \text{ ; FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838

$$\text{Int}[\text{Log}[(c_)*((d_*) + (e_*)(x_)^{n_})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] \text{ ; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 4919 `Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]`

rule 5184 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5204 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]`

rule 5208 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1)) Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])`

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.67

method	result
derivativedivides	$c^2 \left(-\frac{a \left(-\frac{1}{16(cx+1)^2} - \frac{9}{16(cx+1)} + \frac{3 \ln(cx+1)}{2} + \frac{1}{2c^2 x^2} - 3 \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{9}{16(cx-1)} + \frac{3 \ln(cx-1)}{2} \right)}{d^3} - \frac{b \left(\frac{8ic^6 x^6 - 8c^6}{d^3} \right)}{d^3} \right)$
default	$c^2 \left(-\frac{a \left(-\frac{1}{16(cx+1)^2} - \frac{9}{16(cx+1)} + \frac{3 \ln(cx+1)}{2} + \frac{1}{2c^2 x^2} - 3 \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{9}{16(cx-1)} + \frac{3 \ln(cx-1)}{2} \right)}{d^3} - \frac{b \left(\frac{8ic^6 x^6 - 8c^6}{d^3} \right)}{d^3} \right)$
parts	$-\frac{a \left(-\frac{c^2}{16(cx-1)^2} + \frac{9c^2}{16(cx-1)} + \frac{3c^2 \ln(cx-1)}{2} - \frac{c^2}{16(cx+1)^2} - \frac{9c^2}{16(cx+1)} + \frac{3c^2 \ln(cx+1)}{2} + \frac{1}{2x^2} - 3c^2 \ln(x) \right)}{d^3} - \frac{bc^2 \left(\frac{8ic^6 x^6 - 8c^6}{d^3} \right)}{d^3}$

```
input int((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
output c^2*(-a/d^3*(-1/16/(c*x+1)^2-9/16/(c*x+1)+3/2*ln(c*x+1)+1/2/c^2/x^2-3*ln(c*x)-1/16/(c*x-1)^2+9/16/(c*x-1)+3/2*ln(c*x-1))-b/d^3*(1/12/(c^4*x^4-2*c^2*x^2+1)/c^2/x^2*(8*I*c^6*x^6-8*c^5*x^5*(-c^2*x^2+1)^(1/2)+18*c^4*x^4*arcsin(c*x)-16*I*c^4*x^4+3*c^3*x^3*(-c^2*x^2+1)^(1/2)-27*c^2*x^2*arcsin(c*x)+8*I*c^2*x^2+6*c*x*(-c^2*x^2+1)^(1/2)+6*arcsin(c*x))+3*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-3/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-3*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+3*I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-3*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+3*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x^3} dx$$

```
input integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output `integral(-(b*arcsin(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx = -\int \frac{a}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx + \int \frac{b \arcsin(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx$$

input `integrate((a+b*asin(c*x))/x**3/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral(b*asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x))/d**3`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/4*a*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2) + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) - b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^3),x)`

output `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^3), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^3} dx$$

$$= \frac{-4 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx \right) b c^4 x^6 + 8 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx \right) b c^2 x^4 - 4 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx \right) b x^2}{1}$$

input `int((a+b*asin(c*x))/x^3/(-c^2*d*x^2+d)^3,x)`

output

```
( - 4*int(asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*b*c*
*4*x**6 + 8*int(asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x
)*b*c**2*x**4 - 4*int(asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x
**3),x)*b*x**2 - 6*log(c**2*x - c)*a*c**6*x**6 + 12*log(c**2*x - c)*a*c**4
*x**4 - 6*log(c**2*x - c)*a*c**2*x**2 - 6*log(c**2*x + c)*a*c**6*x**6 + 12
*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c**2*x**2 + 12*log(x)*a
*c**6*x**6 - 24*log(x)*a*c**4*x**4 + 12*log(x)*a*c**2*x**2 - 3*a*c**6*x**6
+ 6*a*c**2*x**2 - 2*a)/(4*d**3*x**2*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.54 $\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)^3} dx$

Optimal result	650
Mathematica [B] (verified)	651
Rubi [A] (verified)	651
Maple [A] (verified)	659
Fricas [F]	660
Sympy [F]	660
Maxima [F]	661
Giac [F(-2)]	661
Mupad [F(-1)]	662
Reduce [F]	662

Optimal result

Integrand size = 25, antiderivative size = 291

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx = -\frac{bc^3}{12d^3 (1 - c^2 x^2)^{3/2}} - \frac{11bc^3}{8d^3 \sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{1 - c^2 x^2}}{6d^3 x^2} - \frac{a + b \arcsin(cx)}{3d^3 x^3} - \frac{3c^2(a + b \arcsin(cx))}{d^3 x} + \frac{c^4 x(a + b \arcsin(cx))}{4d^3 (1 - c^2 x^2)^2} + \frac{11c^4 x(a + b \arcsin(cx))}{8d^3 (1 - c^2 x^2)} - \frac{35ic^3(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{4d^3} - \frac{19bc^3 \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{6d^3} + \frac{35ibc^3 \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{8d^3} - \frac{35ibc^3 \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{8d^3}$$

output

```
-1/12*b*c^3/d^3/(-c^2*x^2+1)^(3/2)-11/8*b*c^3/d^3/(-c^2*x^2+1)^(1/2)-1/6*b*c*(-c^2*x^2+1)^(1/2)/d^3/x^2-1/3*(a+b*arcsin(c*x))/d^3/x^3-3*c^2*(a+b*arcsin(c*x))/d^3/x+1/4*c^4*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^2+11/8*c^4*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)-35/4*I*c^3*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^3-19/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))/d^3+35/8*I*b*c^3*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3-35/8*I*b*c^3*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 587 vs. $2(291) = 582$.

Time = 1.52 (sec) , antiderivative size = 587, normalized size of antiderivative = 2.02

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx =$$

$$\frac{16a}{x^3} + \frac{144ac^2}{x} + \frac{8bc\sqrt{1-c^2x^2}}{x^2} + \frac{2bc^3\sqrt{1-c^2x^2}}{(-1+cx)^2} - \frac{bc^4x\sqrt{1-c^2x^2}}{(-1+cx)^2} - \frac{33bc^3\sqrt{1-c^2x^2}}{-1+cx} + \frac{2bc^3\sqrt{1-c^2x^2}}{(1+cx)^2} + \frac{bc^4x\sqrt{1-c^2x^2}}{(1+cx)^2} + \frac{33bc^3\sqrt{1-c^2x^2}}{1+cx}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^3),x]`

output

```
-1/48*((16*a)/x^3 + (144*a*c^2)/x + (8*b*c*Sqrt[1 - c^2*x^2])/x^2 + (2*b*c^3*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 - (b*c^4*x*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 - (33*b*c^3*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (2*b*c^3*Sqrt[1 - c^2*x^2])/((1 + c*x)^2 + (b*c^4*x*Sqrt[1 - c^2*x^2]))/(1 + c*x) - (12*a*c^4*x)/(-1 + c^2*x^2)^2 + (66*a*c^4*x)/(-1 + c^2*x^2) + (105*I)*b*c^3*Pi*ArcSin[c*x] + (16*b*ArcSin[c*x])/x^3 + (144*b*c^2*ArcSin[c*x])/x - (3*b*c^3*ArcSin[c*x])/(-1 + c*x)^2 + (33*b*c^3*ArcSin[c*x])/(-1 + c*x) + (3*b*c^3*ArcSin[c*x])/((1 + c*x)^2 + (33*b*c^3*ArcSin[c*x]))/(1 + c*x) + 152*b*c^3*ArcTanh[Sqrt[1 - c^2*x^2]] - 105*b*c^3*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 210*b*c^3*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 105*b*c^3*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 210*b*c^3*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 105*a*c^3*Log[1 - c*x] - 105*a*c^3*Log[1 + c*x] + 105*b*c^3*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 105*b*c^3*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (210*I)*b*c^3*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (210*I)*b*c^3*PolyLog[2, I*E^(I*ArcSin[c*x])])/d^3
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.34, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {5204, 27, 243, 52, 61, 61, 73, 221, 5204, 243, 61, 61, 73, 221, 5162, 241, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5204} \\
 & \frac{7c^2}{3} \int \frac{a + b \arcsin(cx)}{d^3 x^2 (1 - c^2 x^2)^3} dx + \frac{bc \int \frac{1}{x^3 (1 - c^2 x^2)^{5/2}} dx}{3d^3} - \frac{a + b \arcsin(cx)}{3d^3 x^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & \frac{7c^2 \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^3} dx}{3d^3} + \frac{bc \int \frac{1}{x^3 (1 - c^2 x^2)^{5/2}} dx}{3d^3} - \frac{a + b \arcsin(cx)}{3d^3 x^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{243} \\
 & \frac{7c^2 \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^3} dx}{3d^3} + \frac{bc \int \frac{1}{x^4 (1 - c^2 x^2)^{5/2}} dx^2}{6d^3} - \frac{a + b \arcsin(cx)}{3d^3 x^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{52} \\
 & \frac{7c^2 \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^3} dx}{3d^3} + \frac{bc \left(\frac{5}{2} c^2 \int \frac{1}{x^2 (1 - c^2 x^2)^{5/2}} dx^2 - \frac{1}{x^2 (1 - c^2 x^2)^{3/2}} \right)}{6d^3} - \frac{a + b \arcsin(cx)}{3d^3 x^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{61} \\
 & \frac{7c^2 \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^3} dx}{3d^3} + \frac{bc \left(\frac{5}{2} c^2 \left(\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2}} dx^2 + \frac{2}{3(1 - c^2 x^2)^{3/2}} \right) - \frac{1}{x^2 (1 - c^2 x^2)^{3/2}} \right)}{6d^3} - \frac{a + b \arcsin(cx)}{3d^3 x^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{61} \\
 & \frac{7c^2 \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^3} dx}{3d^3} + \frac{bc \left(\frac{5}{2} c^2 \left(\int \frac{1}{x^2 \sqrt{1 - c^2 x^2}} dx^2 + \frac{2}{\sqrt{1 - c^2 x^2}} + \frac{2}{3(1 - c^2 x^2)^{3/2}} \right) - \frac{1}{x^2 (1 - c^2 x^2)^{3/2}} \right)}{6d^3} - \frac{a + b \arcsin(cx)}{3d^3 x^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{73}
 \end{aligned}$$

$$\frac{7c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^3} dx + bc \left(\frac{\frac{5}{2}c^2 \left(-\frac{2 \int \frac{1}{c^2 - \frac{x^4}{c^2}} d\sqrt{1-c^2x^2}}{c^2} + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3} - \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2}}{6d^3}$$

↓ 221

$$\frac{7c^2 \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^3} dx - \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{6d^3}$$

↓ 5204

$$\frac{7c^2 \left(5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^3} dx + bc \int \frac{1}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} \right) - \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{3d^3}$$

↓ 243

$$\frac{7c^2 \left(5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^3} dx + \frac{1}{2}bc \int \frac{1}{x^2(1-c^2x^2)^{5/2}} dx^2 - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} \right) - \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{3d^3}$$

↓ 61

$$\frac{7c^2 \left(5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^3} dx + \frac{1}{2}bc \left(\int \frac{1}{x^2(1-c^2x^2)^{3/2}} dx^2 + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} \right) - \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{3d^3}$$

↓ 61

$$\frac{7c^2 \left(5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^3} dx + \frac{1}{2} bc \left(\int \frac{1}{x^2 \sqrt{1-c^2x^2}} dx^2 + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} \right)}{3d^3} - \frac{a+b \arcsin(cx)}{3d^3 x^3 (1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2} c^2 \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 73

$$\frac{7c^2 \left(5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^3} dx + \frac{1}{2} bc \left(-\frac{2 \int \frac{1}{\frac{1}{c^2} - x^4} d\sqrt{1-c^2x^2}}{c^2} + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} \right)}{3d^3} - \frac{a+b \arcsin(cx)}{3d^3 x^3 (1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2} c^2 \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 221

$$\frac{7c^2 \left(5c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^3} dx - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} + \frac{1}{2} bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) \right)}{3d^3} - \frac{a+b \arcsin(cx)}{3d^3 x^3 (1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2} c^2 \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 5162

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx - \frac{1}{4} bc \int \frac{x}{(1-c^2x^2)^{5/2}} dx + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} + \frac{1}{2} bc \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^3} - \frac{a+b \arcsin(cx)}{3d^3 x^3 (1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2} c^2 \left(-2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 241

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) \right)}{3d^3} + \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 5162

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2}}{3d^3} + \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 241

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2}}{3d^3} + \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 5164

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)^{3/2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2}}{3d^3} + \frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + \frac{bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) \right) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}}}{6d^3}$$

↓ 3042

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) + \frac{x(a+b \arcsin(cx))}{4(1-c^2x^2)^2} - \frac{b}{12c(1-c^2x^2)} \right) \right)}{3d^3}$$

$$\frac{\frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{4669}$$

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) \right) \right)}{3d^3}$$

$$\frac{\frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{2715}$$

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{12c(1-c^2x^2)} \right) \right) \right)}{3d^3}$$

$$\frac{\frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}{2838}$$

$$\frac{7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{-2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{12c(1-c^2x^2)} \right) \right) \right)}{3d^3}$$

$$\frac{\frac{a+b \arcsin(cx)}{3d^3x^3(1-c^2x^2)^2} + bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) + \frac{2}{\sqrt{1-c^2x^2}} + \frac{2}{3(1-c^2x^2)^{3/2}} \right) - \frac{1}{x^2(1-c^2x^2)^{3/2}} \right)}{6d^3}}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^3), x]
```

output

$$\begin{aligned}
& -1/3*(a + b*\text{ArcSin}[c*x])/(d^3*x^3*(1 - c^2*x^2)^2) + (b*c*(-(1/(x^2*(1 - c^2*x^2)^{3/2}))) + (5*c^2*(2/(3*(1 - c^2*x^2)^{3/2})) + 2/\text{Sqrt}[1 - c^2*x^2] \\
& - 2*\text{ArcTanh}[\text{Sqrt}[1 - c^2*x^2]]))/2)/(6*d^3) + (7*c^2*(-((a + b*\text{ArcSin}[c*x])/(x*(1 - c^2*x^2)^2) + (b*c*(2/(3*(1 - c^2*x^2)^{3/2})) + 2/\text{Sqrt}[1 - c^2 \\
& *x^2] - 2*\text{ArcTanh}[\text{Sqrt}[1 - c^2*x^2]]))/2 + 5*c^2*(-1/12*b/(c*(1 - c^2*x^2)^{3/2}) + (x*(a + b*\text{ArcSin}[c*x]))/(4*(1 - c^2*x^2)^2) + (3*(-1/2*b/(c*\text{Sqrt}[\\
& 1 - c^2*x^2]) + (x*(a + b*\text{ArcSin}[c*x]))/(2*(1 - c^2*x^2)) + ((-2*I)*(a + b*\text{ArcSin}[c*x])*\text{ArcTan}[E^{(I*\text{ArcSin}[c*x])}] + I*b*\text{PolyLog}[2, (-I)*E^{(I*\text{ArcSin}[\\
& c*x]])] - I*b*\text{PolyLog}[2, I*E^{(I*\text{ArcSin}[c*x])}])/(2*c)))/4))/(3*d^3)
\end{aligned}$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(F_x_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[F_x, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_) /; \text{FreeQ}[b, x]]$$

rule 52

$$\text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{ILtQ}[m, -1] \ \&\& \ \text{FractionQ}[n] \ \&\& \ \text{LtQ}[n, 0]$$

rule 61

$$\text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*((c + d*x)^{(n + 1)}/((b*c - a*d)*(m + 1))), x] - \text{Simp}[d*((m + n + 2)/((b*c - a*d)*(m + 1))) \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !(\text{LtQ}[n, -1] \ \&\& \ (\text{EqQ}[a, 0] \ || \ (\text{NeQ}[c, 0] \ \&\& \ \text{LtQ}[m - n, 0] \ \&\& \ \text{IntegerQ}[n]))) \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 73

$$\text{Int}[((a_.) + (b_.)*(x_))^{(m_)}*((c_.) + (d_.)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

rule 221 $\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_ \text{Symbol}] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) \cdot \text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

rule 241 $\text{Int}[(x_) \cdot ((a_) + (b_ \cdot)(x_)^2)^{p_ }, x_ \text{Symbol}] \rightarrow \text{Simp}[(a + b \cdot x^2)^{p + 1} / (2 \cdot b \cdot (p + 1)), x] \text{ ; FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$

rule 243 $\text{Int}[(x_)^{m_ } \cdot ((a_) + (b_ \cdot)(x_)^2)^{p_ }, x_ \text{Symbol}] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m - 1)/2} \cdot (a + b \cdot x)^p, x], x, x^2], x] \text{ ; FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_ \cdot)(F_)^{(e_) \cdot ((c_) + (d_) \cdot (x_))})^{n_ }], x_ \text{Symbol}] \rightarrow \text{Simp}[1/(d \cdot e \cdot n \cdot \text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x]/x, x], x, (F^{e \cdot (c + d \cdot x)})^n], x] \text{ ; FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_) \cdot ((d_) + (e_) \cdot (x_)^{n_ })]/(x_), x_ \text{Symbol}] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n/n, x] \text{ ; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c \cdot d, 1]$

rule 3042 $\text{Int}[u_ , x_ \text{Symbol}] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi} \cdot (k_) + (f_) \cdot (x_)] \cdot ((c_) + (d_) \cdot (x_))^{m_ }, x_ \text{Symbol}] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{I \cdot k \cdot \text{Pi}}] \cdot E^{I \cdot (e + f \cdot x)})/f, x] + (-\text{Simp}[d \cdot (m/f) \ \text{Int}[(c + d \cdot x)^{m - 1} \cdot \text{Log}[1 - E^{I \cdot k \cdot \text{Pi}}] \cdot E^{I \cdot (e + f \cdot x)}], x], x] + \text{Simp}[d \cdot (m/f) \ \text{Int}[(c + d \cdot x)^{m - 1} \cdot \text{Log}[1 + E^{I \cdot k \cdot \text{Pi}}] \cdot E^{I \cdot (e + f \cdot x)}], x], x) \text{ ; FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:= Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x]
+ (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x]
+ Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:= Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:= Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(d*f*(m + 1))), x]
+ (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1)) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x]
- Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.28

method	result
derivativedivides	$c^3 \left(-\frac{a \left(\frac{1}{16(cx+1)^2} + \frac{11}{16(cx+1)} - \frac{35 \ln(cx+1)}{16} + \frac{1}{3c^3x^3} + \frac{3}{cx} - \frac{1}{16(cx-1)^2} + \frac{11}{16(cx-1)} + \frac{35 \ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(\frac{105 \arcsin(cx)c^6}{16} \right)}{d^3} \right)$
default	$c^3 \left(-\frac{a \left(\frac{1}{16(cx+1)^2} + \frac{11}{16(cx+1)} - \frac{35 \ln(cx+1)}{16} + \frac{1}{3c^3x^3} + \frac{3}{cx} - \frac{1}{16(cx-1)^2} + \frac{11}{16(cx-1)} + \frac{35 \ln(cx-1)}{16} \right)}{d^3} - \frac{b \left(\frac{105 \arcsin(cx)c^6}{16} \right)}{d^3} \right)$
parts	$-\frac{a \left(-\frac{c^3}{16(cx-1)^2} + \frac{11c^3}{16(cx-1)} + \frac{35c^3 \ln(cx-1)}{16} + \frac{c^3}{16(cx+1)^2} + \frac{11c^3}{16(cx+1)} - \frac{35c^3 \ln(cx+1)}{16} + \frac{1}{3x^3} + \frac{3e^2}{x} \right)}{d^3} - \frac{bc^3 \left(\frac{105 \arcsin(cx)c^6}{16} \right)}{d^3}$

input `int((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)`

output `c^3*(-a/d^3*(1/16/(c*x+1)^2+11/16/(c*x+1)-35/16*ln(c*x+1)+1/3/c^3/x^3+3/c/x-1/16/(c*x-1)^2+11/16/(c*x-1)+35/16*ln(c*x-1))-b/d^3*(1/24*(105*arcsin(c*x)*c^6*x^6-29*c^5*x^5*(-c^2*x^2+1)^(1/2)-175*c^4*x^4*arcsin(c*x)+27*c^3*x^3*(-c^2*x^2+1)^(1/2)+56*c^2*x^2*arcsin(c*x)+4*c*x*(-c^2*x^2+1)^(1/2)+8*arcsin(c*x))/(c^4*x^4-2*c^2*x^2+1)/c^3/x^3-19/6*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)+19/6*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+35/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))))-35/8*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-35/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+35/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx = -\int \frac{a}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx + \int \frac{b \arcsin(cx)}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx$$

input `integrate((a+b*asin(c*x))/x**4/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Integral(b*asin(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x))/d**3`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx = \int -\frac{b \arcsin(cx) + a}{(c^2 dx^2 - d)^3 x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `1/48*a*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3)) + 1/48*(105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(c*x + 1) - 105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(105*c^6*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + 48*(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3)*integrate(-1/48*(210*c^7*x^6 - 350*c^5*x^4 + 112*c^3*x^2 - 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3)*log(c*x + 1) + 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3)*log(-c*x + 1) + 16*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x))*b/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^3),x)`

output `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^3), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^3} dx$$

$$= \frac{-48 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx \right) b c^4 x^7 + 96 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx \right) b c^2 x^5 - 48 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx \right)}{1}$$

input `int((a+b*asin(c*x))/x^4/(-c^2*d*x^2+d)^3,x)`

output `(- 48*int(asin(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b*c**4*x**7 + 96*int(asin(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b*c**2*x**5 - 48*int(asin(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b*x**3 - 105*log(c**2*x - c)*a*c**7*x**7 + 210*log(c**2*x - c)*a*c**5*x**5 - 105*log(c**2*x - c)*a*c**3*x**3 + 105*log(c**2*x + c)*a*c**7*x**7 - 210*log(c**2*x + c)*a*c**5*x**5 + 105*log(c**2*x + c)*a*c**3*x**3 - 210*a*c**6*x**6 + 350*a*c**4*x**4 - 112*a*c**2*x**2 - 16*a)/(48*d**3*x**3*(c**4*x**4 - 2*c**2*x**2 + 1))`

3.55 $\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$

Optimal result	663
Mathematica [A] (verified)	664
Rubi [A] (verified)	664
Maple [C] (verified)	667
Fricas [F]	668
Sympy [F]	668
Maxima [F]	668
Giac [A] (verification not implemented)	669
Mupad [F(-1)]	669
Reduce [F]	670

Optimal result

Integrand size = 27, antiderivative size = 262

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{bx^2 \sqrt{d - c^2 dx^2}}{32c^3 \sqrt{1 - c^2 x^2}} + \frac{bx^4 \sqrt{d - c^2 dx^2}}{96c \sqrt{1 - c^2 x^2}} - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36 \sqrt{1 - c^2 x^2}} - \frac{x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{16c^4} - \frac{x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{24c^2} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{32bc^5 \sqrt{1 - c^2 x^2}}$$

output

```
1/32*b*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/96*b*x^4*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/36*b*c*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4-1/24*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2+1/6*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+1/32*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c^5/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d - c^2 dx^2} (9a^2 + b^2 c^2 x^2 (9 + 3c^2 x^2 - 8c^4 x^4)) + 6abcx \sqrt{1 - c^2 x^2} (-3 - 2c^2 x^2 + 8c^4 x^4) + 6b(3a + bcx \sqrt{1 - c^2 x^2})}{288bc^5 \sqrt{1 - c^2 x^2}}$$

input `Integrate[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(Sqrt[d - c^2*d*x^2]*(9*a^2 + b^2*c^2*x^2*(9 + 3*c^2*x^2 - 8*c^4*x^4) + 6*a*b*c*x*Sqrt[1 - c^2*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4) + 6*b*(3*a + b*c*x*Sqrt[1 - c^2*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4))*ArcSin[c*x] + 9*b^2*ArcSin[c*x]^2))/(288*b*c^5*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5198, 15, 5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5198$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{x^4 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{6\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^5 dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))$$

$$\downarrow 15$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{x^4 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}}$$

$$\downarrow 5210$$

$$\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}}$$

↓ 15

$$\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}}$$

↓ 5210

$$\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}}$$

↓ 15

$$\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} - \frac{x \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}}$$

↓ 5152

$$\frac{\frac{1}{6} x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \sqrt{d - c^2 dx^2} \left(-\frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{3 \left(\frac{(a+b \arcsin(cx))^2}{4bc^3} - \frac{x \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}}$$

input `Int[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `-1/36*(b*c*x^6*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/6 + (Sqrt[d - c^2*d*x^2]*((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2)))/(6*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5210 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.57

method	result
default	$-\frac{ax^3(-c^2dx^2+d)^{\frac{3}{2}}}{6c^2d} - \frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{8c^4d} + \frac{ax\sqrt{-c^2dx^2+d}}{16c^4} + \frac{ad \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{16c^4\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2}}{32c^5(c^2x^2-1)}\right)$
parts	$-\frac{ax^3(-c^2dx^2+d)^{\frac{3}{2}}}{6c^2d} - \frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{8c^4d} + \frac{ax\sqrt{-c^2dx^2+d}}{16c^4} + \frac{ad \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{16c^4\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2}}{32c^5(c^2x^2-1)}\right)$

input `int(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output

```
-1/6*a*x^3*(-c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^4*x*(-c^2*d*x^2+d)^(3/2)/d+1/16*a/c^4*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^4*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/(c^2*x^2-1)*arcsin(c*x)^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*(I+6*arcsin(c*x))/c^5/(c^2*x^2-1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))/c^5/(c^2*x^2-1)+1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(7*I+48*arcsin(c*x))*cos(5*arcsin(c*x))/c^5/(c^2*x^2-1)-1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(11*I+24*arcsin(c*x))*sin(5*arcsin(c*x))/c^5/(c^2*x^2-1)-3/512*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*cos(3*arcsin(c*x))/c^5/(c^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(8*arcsin(c*x)+I)*sin(3*arcsin(c*x))/c^5/(c^2*x^2-1))
```


Fricas [F]

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) x^4 dx$$

input `integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((b*x^4*arcsin(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d), x)`

Sympy [F]

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^4 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx)) dx$$

input `integrate(x**4*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)`

output `Integral(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)`

Maxima [F]

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) x^4 dx$$

input `integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) - 1/48*(8*(-c^2*d*x^2 + d)^(3/2)*x^3/(c^2*d) - 3*sqrt(-c^2*d*x^2 + d)*x/c^4 + 6*(-c^2*d*x^2 + d)^(3/2)*x/(c^4*d) - 3*sqrt(d)*arcsin(c*x)/c^5)*a`

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.93

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{1}{6} \sqrt{-c^2 dx^2 + d} ax^5 - \frac{\sqrt{-c^2 dx^2 + d} ax^3}{24 c^2}$$

$$+ \frac{\left(384 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} x \arcsin(cx) - 672 (-c^2 x^2 + 1)^{\frac{3}{2}} x \arcsin(cx) + 144 \sqrt{-c^2 x^2 + 1} x \arcsin \right)}{2304 c^4}$$

$$- \frac{\sqrt{-c^2 dx^2 + d} ax}{16 c^4} - \frac{ad \log(|-c\sqrt{-dx} + \sqrt{c^2 x^2 - 1}\sqrt{-d}|)}{16 c^5 \sqrt{-d}}$$

input `integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`output `1/6*sqrt(-c^2*d*x^2 + d)*a*x^5 - 1/24*sqrt(-c^2*d*x^2 + d)*a*x^3/c^2 + 1/2304*(384*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*x*arcsin(c*x) - 672*(-c^2*x^2 + 1)^(3/2)*x*arcsin(c*x) + 144*sqrt(-c^2*x^2 + 1)*x*arcsin(c*x) - 64*(c^2*x^2 - 1)^3/c - 168*(c^2*x^2 - 1)^2/c + 72*arcsin(c*x)^2/c - 72*(c^2*x^2 - 1)/c + 7/c)*b*sqrt(d)/c^4 - 1/16*sqrt(-c^2*d*x^2 + d)*a*x/c^4 - 1/16*a*d*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^5*sqrt(-d))`**Mupad [F(-1)]**

Timed out.

$$\int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^4 (a + b \arcsin(cx)) \sqrt{d - c^2 dx^2} dx$$

input `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)`output `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int x^4 \sqrt{d - c^2 x^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d} (3a \sin(cx) a + 8\sqrt{-c^2 x^2 + 1} a c^5 x^5 - 2\sqrt{-c^2 x^2 + 1} a c^3 x^3 - 3\sqrt{-c^2 x^2 + 1} a c x + 48 \int \sqrt{-c^2 x^2 + 1} a dx)}{48c^5}$$

input `int(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*(3*asin(c*x)*a + 8*sqrt(-c**2*x**2 + 1)*a*c**5*x**5 - 2*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt(-c**2*x**2 + 1)*a*c*x + 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*b*c**5))/(48*c**5)`

3.56 $\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$

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Giac [A] (verification not implemented)	676
Mupad [F(-1)]	677
Reduce [F]	677

Optimal result

Integrand size = 27, antiderivative size = 189

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{bx^2 \sqrt{d - c^2 dx^2}}{16c \sqrt{1 - c^2 x^2}} - \frac{bcx^4 \sqrt{d - c^2 dx^2}}{16 \sqrt{1 - c^2 x^2}} - \frac{x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{8c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{16bc^3 \sqrt{1 - c^2 x^2}}$$

output

```
1/16*b*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/16*b*c*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2+1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+1/16*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d - c^2 dx^2} (a^2 + b^2 c^2 x^2 (1 - c^2 x^2) + 2abcx \sqrt{1 - c^2 x^2} (-1 + 2c^2 x^2) + 2b(a + bcx \sqrt{1 - c^2 x^2} (-1 + 2c^2 x^2)))}{16bc^3 \sqrt{1 - c^2 x^2}}$$

input `Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(Sqrt[d - c^2*d*x^2]*(a^2 + b^2*c^2*x^2*(1 - c^2*x^2) + 2*a*b*c*x*Sqrt[1 - c^2*x^2]*(-1 + 2*c^2*x^2) + 2*b*(a + b*c*x*Sqrt[1 - c^2*x^2]*(-1 + 2*c^2*x^2))*ArcSin[c*x] + b^2*ArcSin[c*x]^2))/(16*b*c^3*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5198, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5198}$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^3 dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))$$

$$\downarrow \text{15}$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^4 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}}$$

$$\downarrow \text{5210}$$

$$\begin{aligned}
& \frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a + \\
& \quad b\arcsin(cx)) - \frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \\
& \quad \downarrow 15 \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a + \\
& \quad b\arcsin(cx)) - \frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \\
& \quad \downarrow 5152 \\
& \frac{\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) + \sqrt{d-c^2dx^2} \left(\frac{(a+b\arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4\sqrt{1-c^2x^2}} - \frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `-1/16*(b*c*x^4*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/4 + (Sqrt[d - c^2*d*x^2]*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)]^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.94

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d} + \frac{ax\sqrt{-c^2dx^2+d}}{8c^2} + \frac{ad \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{16c^3(c^2x^2-1)} + \frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)}{16c^3(c^2x^2-1)}\right)$
parts	$-\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d} + \frac{ax\sqrt{-c^2dx^2+d}}{8c^2} + \frac{ad \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{16c^3(c^2x^2-1)} + \frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)}{16c^3(c^2x^2-1)}\right)$

input

```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-1/4*a*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^2+1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-I+4*arcsin(c*x))/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) x^2 dx$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arcsin(c*x) + a*x^2), x)
```

Sympy [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^2 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx)) dx$$

input

```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)
```

output

```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)
```


Maxima [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + 1/8*a*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*arcsin(c*x)/c^3)`

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{1}{4} \sqrt{-c^2 dx^2 + d} a x^3$$

$$- \frac{\left(32 (-c^2 x^2 + 1)^{\frac{3}{2}} x \arcsin(cx) - 16 \sqrt{-c^2 x^2 + 1} x \arcsin(cx) + \frac{8 (c^2 x^2 - 1)^2}{c} - \frac{8 \arcsin(cx)^2}{c} + \frac{8 (c^2 x^2 - 1)}{c} + \frac{1}{c} \right) \sqrt{-c^2 dx^2 + d} - \frac{128 c^2}{8 c^3 \sqrt{-d}} \operatorname{ad} \log \left(\left| -c \sqrt{-d} x + \sqrt{c^2 x^2 - 1} \sqrt{-d} \right| \right)}{8 c^2}$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `1/4*sqrt(-c^2*d*x^2 + d)*a*x^3 - 1/128*(32*(-c^2*x^2 + 1)^(3/2)*x*arcsin(c*x) - 16*sqrt(-c^2*x^2 + 1)*x*arcsin(c*x) + 8*(c^2*x^2 - 1)^2/c - 8*arcsin(c*x)^2/c + 8*(c^2*x^2 - 1)/c + 1/c)*b*sqrt(d)/c^2 - 1/8*sqrt(-c^2*d*x^2 + d)*a*x/c^2 - 1/8*a*d*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d))`

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^2 (a + b \arcsin(cx)) \sqrt{d - c^2 dx^2} dx$$

input `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)`

output `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d} (a \arcsin(cx) a + 2\sqrt{-c^2 x^2 + 1} a c^3 x^3 - \sqrt{-c^2 x^2 + 1} a c x + 8 \int \sqrt{-c^2 x^2 + 1} a \arcsin(cx) x^2 dx) b c^3}{8c^3}$$

input `int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*(asin(c*x)*a + 2*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 - sqrt(-c**2*x**2 + 1)*a*c*x + 8*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*b*c**3)/(8*c**3)`

3.57 $\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$

Optimal result	678
Mathematica [A] (verified)	678
Rubi [A] (verified)	679
Maple [C] (verified)	680
Fricas [F]	681
Sympy [F]	681
Maxima [F]	682
Giac [F(-2)]	682
Mupad [F(-1)]	682
Reduce [F]	683

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= -\frac{bcx^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))$$

$$+ \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{4bc\sqrt{1 - c^2 x^2}}$$

output

$$-1/4*b*c*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*\arcsin(c*x))+1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*\arcsin(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d - c^2 dx^2} (a^2 - b^2 c^2 x^2 + 2abcx\sqrt{1 - c^2 x^2} + 2b(a + bcx\sqrt{1 - c^2 x^2}) \arcsin(cx) + b^2 \arcsin(cx)^2)}{4bc\sqrt{1 - c^2 x^2}}$$

input `Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(Sqrt[d - c^2*d*x^2]*(a^2 - b^2*c^2*x^2 + 2*a*b*c*x*Sqrt[1 - c^2*x^2] + 2*b*(a + b*c*x*Sqrt[1 - c^2*x^2])*ArcSin[c*x] + b^2*ArcSin[c*x]^2))/(4*b*c*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5156$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))$$

$$\downarrow 15$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

$$\downarrow 5152$$

$$\frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{4bc\sqrt{1 - c^2 x^2}} - \frac{bcx^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

input `Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `-1/4*(b*c*x^2*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] /; \text{FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_. + \text{ArcSin}[c_.](x_)]*(b_.))^{(n_.)}/\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5156 $\text{Int}[(a_. + \text{ArcSin}[c_.](x_)]*(b_.))^{(n_.)*\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^{n/2}), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.41

method	result
default	$\frac{ax\sqrt{-c^2dx^2+d}}{2} + \frac{ad \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + b \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{4(c^2x^2-1)c} + \frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2-1})}{4(c^2x^2-1)c} \right)$
parts	$\frac{ax\sqrt{-c^2dx^2+d}}{2} + \frac{ad \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + b \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{4(c^2x^2-1)c} + \frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2-1})}{4(c^2x^2-1)c} \right)$

input $\text{int}((-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x)), x, \text{method}=_RETURNVERBOSE)$

output

```
1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/
(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c
^2*x^2-1)/c*arcsin(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(
1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(I+2*arcsin(c*x))/(c^2*
x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3
*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))/(c^2*x^2-1)/c
```

Fricas [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) dx$$

input

```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx)) dx$$

input

```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)
```

output

```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a`

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) \sqrt{d - c^2 dx^2} dx$$

input `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)`

output `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \sqrt{d - c^2 x^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d} (a \sin(cx) a + \sqrt{-c^2 x^2 + 1} a c x + 2 (\int \sqrt{-c^2 x^2 + 1} a \sin(cx) dx) b c)}{2c}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*(asin(c*x)*a + sqrt(-c**2*x**2 + 1)*a*c*x + 2*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*b*c))/(2*c)`

3.58 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^2} dx$

Optimal result	684
Mathematica [A] (verified)	685
Rubi [A] (verified)	685
Maple [C] (verified)	687
Fricas [F]	687
Sympy [F]	688
Maxima [F]	688
Giac [F(-2)]	688
Mupad [F(-1)]	689
Reduce [F]	689

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^2} dx = -\frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x} - \frac{c\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2b\sqrt{1-c^2x^2}} + \frac{bc\sqrt{d-c^2dx^2} \log(x)}{\sqrt{1-c^2x^2}}$$

output

```

-((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)))/x-1/2*c*(-c^2*d*x^2+d)^(1/2)*(a+b*
arcsin(c*x))^2/b/(-c^2*x^2+1)^(1/2)+b*c*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x
^2+1)^(1/2)
    
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx$$

$$= -\frac{a\sqrt{-d(-1 + c^2 x^2)}}{x} + ac\sqrt{d} \arctan\left(\frac{cx\sqrt{-d(-1 + c^2 x^2)}}{\sqrt{d}(-1 + c^2 x^2)}\right)$$

$$- \frac{bc\sqrt{d(1 - c^2 x^2)}\left(\frac{2\sqrt{1 - c^2 x^2} \arcsin(cx)}{cx} + \arcsin(cx)^2 - 2 \log(cx)\right)}{2\sqrt{1 - c^2 x^2}}$$

input `Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^2,x]`

output `-((a*Sqrt[-(d*(-1 + c^2*x^2))])/x) + a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))] - (b*c*Sqrt[d*(1 - c^2*x^2)]*((2*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*x) + ArcSin[c*x]^2 - 2*Log[c*x]))/(2*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5196, 14, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx$$

$$\downarrow 5196$$

$$-\frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} + \frac{bc \sqrt{d - c^2 dx^2} \int \frac{1}{x} dx}{\sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x}$$

$$\downarrow 14$$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.71

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{dx} - ac^2x\sqrt{-c^2dx^2+d} - \frac{ac^2d \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}} + b\left(\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{2c^2x^2-2}\right)$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{dx} - ac^2x\sqrt{-c^2dx^2+d} - \frac{ac^2d \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}} + b\left(\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{2c^2x^2-2}\right)$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)`

output `-a/d/x*(-c^2*d*x^2+d)^(3/2)-a*c^2*x*(-c^2*d*x^2+d)^(1/2)-a*c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arcsin(c*x)^2*c+2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)*arcsin(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)/(c^2*x^2-1)/x-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c)`

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^2} dx = \int \frac{\sqrt{-c^2dx^2+d}(b \arcsin(cx)+a)}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2+d)*(b*arcsin(c*x)+a)/x^2,x)`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)}(a + b \arcsin(cx))}{x^2} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x**2,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x**2, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx = \int \frac{\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x^2, x) - (c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x^2} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^2,x)`output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^2, x)`**Reduce [F]**

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{\sqrt{d} \left(-a \sin^2(cx) b c x - 2 a \sin(cx) a c x - 2 \sqrt{-c^2 x^2 + 1} a + 2 \left(\int \frac{a \sin(cx)}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) b x \right)}{2x}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x^2,x)`output `(sqrt(d)*(-asin(c*x)**2*b*c*x - 2*asin(c*x)*a*c*x - 2*sqrt(-c**2*x**2 + 1)*a + 2*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**2),x)*b*x))/(2*x)`

3.59 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^4} dx$

Optimal result	690
Mathematica [A] (verified)	690
Rubi [A] (verified)	691
Maple [C] (verified)	692
Fricas [B] (verification not implemented)	693
Sympy [F]	693
Maxima [A] (verification not implemented)	694
Giac [F(-2)]	694
Mupad [F(-1)]	695
Reduce [F]	695

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^4} dx = -\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{3dx^3} - \frac{bc^3\sqrt{d-c^2dx^2} \log(x)}{3\sqrt{1-c^2x^2}}$$

output

```
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/d/x^3-1/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^4} dx = \frac{\sqrt{d-c^2dx^2}(bcx-3bc^3x^3+2a\sqrt{1-c^2x^2}-2ac^2x^2\sqrt{1-c^2x^2}+2b(1-c^2x^2)^{3/2} \arcsin(cx)+2bc^3x^3 \log(x))}{6x^3\sqrt{1-c^2x^2}}$$

input `Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x - 3*b*c^3*x^3 + 2*a*Sqrt[1 - c^2*x^2] - 2*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 2*b*(1 - c^2*x^2)^(3/2)*ArcSin[c*x] + 2*b*c^3*x^3*Log[x]))/(x^3*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5186, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^4} dx$$

$$\downarrow \text{5186}$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \frac{1 - c^2 x^2}{x^3} dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3dx^3}$$

$$\downarrow \text{244}$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \left(\frac{1}{x^3} - \frac{c^2}{x}\right) dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3dx^3}$$

$$\downarrow \text{2009}$$

$$\frac{bc\sqrt{d - c^2 dx^2} \left(c^2(-\log(x)) - \frac{1}{2x^2}\right)}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3dx^3}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(d*x^3) + (b*c*Sqrt[d - c^2*d*x^2]*(-1/2*1/x^2 - c^2*Log[x]))/(3*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

rule 244 $\text{Int}[\text{((c_)}*(x_))^{\text{(m_)}* \text{((a_)} + \text{(b_)}*(x_)^2)^{\text{(p_)}}, x_Symbol] \text{ :> Int[Expand Integrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ /; FreeQ}\{a, b, c, m\}, x\} \ \&\& \ \text{IGtQ}\{p, 0\}$

rule 2009 $\text{Int}[u_, x_Symbol] \text{ :> Simp[IntSum}[u, x], x] \text{ /; SumQ}[u]$

rule 5186 $\text{Int}[\text{((a_)} + \text{ArcSin}[\text{(c_)}*(x_)]*(b_))^{\text{(n_)}* \text{((f_)}*(x_))^{\text{(m_)}* \text{((d_)} + \text{(e_)}*(x_)^2)^{\text{(p_)}}, x_Symbol] \text{ :> Simp}[(f*x)^{\text{(m + 1)}}*(d + e*x^2)^{\text{(p + 1)}}*(a + b*\text{ArcSin}[c*x])^{\text{n}}/(d*f*(m + 1))), x] - \text{Simp}[b*c*(n/(f*(m + 1)))*\text{Simp}[(d + e*x^2)^{\text{p}}/(1 - c^2*x^2)^{\text{p}}] \text{Int}[(f*x)^{\text{(m + 1)}}*(1 - c^2*x^2)^{\text{(p + 1/2)}}*(a + b*\text{ArcSin}[c*x])^{\text{(n - 1)}}], x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, m, p\}, x\} \ \&\& \ \text{EqQ}\{c^2*d + e, 0\} \ \&\& \ \text{GtQ}\{n, 0\} \ \&\& \ \text{EqQ}\{m + 2*p + 3, 0\} \ \&\& \ \text{NeQ}\{m, -1\}$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.42

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3dx^3} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\left(2i\arcsin(cx)x^3c^3-2\ln\left(\left(icx+\sqrt{-c^2x^2+1}\right)^2-1\right)x^3c^3+2\arcsin(cx)\sqrt{-c^2x^2+1}\right)}{6x^3(c^2x^2-1)}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3dx^3} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\left(2i\arcsin(cx)x^3c^3-2\ln\left(\left(icx+\sqrt{-c^2x^2+1}\right)^2-1\right)x^3c^3+2\arcsin(cx)\sqrt{-c^2x^2+1}\right)}{6x^3(c^2x^2-1)}$

input $\text{int}((-c^2*d*x^2+d)^{\text{(1/2)}}*(a+b*\text{arcsin}(c*x))/x^4, x, \text{method}=_RETURNVERBOSE)$

output
$$-1/3*a/d/x^3*(-c^2*d*x^2+d)^{\text{(3/2)}}-1/6*b*(-d*(c^2*x^2-1))^{\text{(1/2)}}*(-c^2*x^2+1)^{\text{(1/2)}}*(2*I*\text{arcsin}(c*x)*x^3*c^3-2*\ln((I*c*x+(-c^2*x^2+1)^{\text{(1/2)}})^2-1)*x^3*c^3+2*\text{arcsin}(c*x)*(-c^2*x^2+1)^{\text{(1/2)}}*c^2*x^2-2*\text{arcsin}(c*x)*(-c^2*x^2+1)^{\text{(1/2)}}-c*x)/x^3/(c^2*x^2-1)$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(95) = 190$.

Time = 0.14 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.74

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^4} dx$$

$$= \left[\frac{(bc^5 x^5 - bc^3 x^3) \sqrt{d} \log\left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 + \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^4 - 1) \sqrt{d - d}}{c^2 x^4 - x^2}\right) - \sqrt{-c^2 dx^2 + d} (bcx^3 - bcx) \sqrt{-c^2 x^2 + 1}}{6(c^2 x^5 - x^3)} \right.$$

$$\left. - \frac{2(bc^5 x^5 - bc^3 x^3) \sqrt{-d} \arctan\left(\frac{\sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^2 - 1) \sqrt{-d}}{c^2 dx^4 + (c^2 - 1) dx^2 - d}\right) + \sqrt{-c^2 dx^2 + d} (bcx^3 - bcx) \sqrt{-c^2 x^2 + 1}}{6(c^2 x^5 - x^3)} \right]$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `[1/6*((b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(-c^2*x^2 + 1) + 2*(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3), -1/6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(-c^2*x^2 + 1) - 2*(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)]`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^4} dx = \int \frac{\sqrt{-d}(cx - 1)(cx + 1)(a + b \arcsin(cx))}{x^4} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x**4,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x**4, x)`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{\left((-1)^{-2c^2 dx^2 + 2d} c^2 d^{\frac{3}{2}} \log\left(-2c^2 d + \frac{2d}{x^2}\right) + c^2 d^{\frac{3}{2}} \log\left(x^2 - \frac{1}{c^2}\right) - \frac{\sqrt{c^4 dx^4 - 2c^2 dx^2 + dd}}{x^2} \right) bc}{6d}$$

$$- \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} b \arcsin(cx)}{3 dx^3} - \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} a}{3 dx^3}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output `1/6*((-1)^(-2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + c^2*d^(3/2)*log(x^2 - 1/c^2) - sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*d/x^2)*b*c/d - 1/3*(-c^2*d*x^2 + d)^(3/2)*b*arcsin(c*x)/(d*x^3) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(d*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x^4} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^4,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^4, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{\sqrt{d} \left(\sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a + 3 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^4} dx \right) b x^3 \right)}{3x^3}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x^4,x)`

output `(sqrt(d)*(sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - sqrt(-c**2*x**2 + 1)*a + 3*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**4,x)*b*x**3))/(3*x**3)`

3.60 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^6} dx$

Optimal result	696
Mathematica [A] (verified)	697
Rubi [A] (verified)	697
Maple [C] (verified)	699
Fricas [A] (verification not implemented)	700
Sympy [F]	701
Maxima [A] (verification not implemented)	701
Giac [F(-2)]	702
Mupad [F(-1)]	702
Reduce [F]	703

Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^6} dx = -\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{1-c^2x^2}} + \frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{5dx^5} - \frac{2c^2(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{15dx^3} - \frac{2bc^5\sqrt{d-c^2dx^2} \log(x)}{15\sqrt{1-c^2x^2}}$$

output

```
-1/20*b*c*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+1/30*b*c^3*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/d/x^5-2/15*c^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/d/x^3-2/15*b*c^5*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^6} dx$$

$$= \frac{\sqrt{d - c^2 dx^2}(-9bcx + 6bc^3x^3 + 50bc^5x^5 - 36a\sqrt{1 - c^2x^2} + 12ac^2x^2\sqrt{1 - c^2x^2} + 24ac^4x^4\sqrt{1 - c^2x^2} + 12c^5x^5\sqrt{1 - c^2x^2})}{180x^5\sqrt{1 - c^2x^2}}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^6,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(-9*b*c*x + 6*b*c^3*x^3 + 50*b*c^5*x^5 - 36*a*Sqrt[1 - c^2*x^2] + 12*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 12*b*Sqrt[1 - c^2*x^2]*(-3 + c^2*x^2 + 2*c^4*x^4)*ArcSin[c*x] - 24*b*c^5*x^5*Log[x]))/(180*x^5*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^6} dx$$

$$\downarrow 5194$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \frac{-2c^4 x^4 - c^2 x^2 + 3}{15x^5} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2}(a + b \arcsin(cx))}{5dx^5}$$

$$\frac{2c^2(d - c^2 dx^2)^{3/2}(a + b \arcsin(cx))}{15dx^3}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{-2c^4x^4-c^2x^2+3}{x^5} dx}{15\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{5dx^5} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{15dx^3} \\
& \quad \downarrow \text{1433} \\
& \frac{bc\sqrt{d-c^2dx^2} \int \left(-\frac{2c^4}{x} - \frac{c^2}{x^3} + \frac{3}{x^5}\right) dx}{15\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{5dx^5} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{15dx^3} \\
& \quad \downarrow \text{2009} \\
& -\frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{5dx^5} - \frac{2c^2(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{15dx^3} + \\
& \quad \frac{bc\sqrt{d-c^2dx^2} \left(-2c^4 \log(x) + \frac{c^2}{2x^2} - \frac{3}{4x^4}\right)}{15\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^6,x]`

output `-1/5*((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(d*x^5) - (2*c^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(15*d*x^3) + (b*c*Sqrt[d - c^2*d*x^2]*(-3/(4*x^4) + c^2/(2*x^2) - 2*c^4*Log[x]))/(15*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1433 `Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || !IntegerQ[(m + 1)/2])`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Simp
plifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 1903, normalized size of antiderivative = 10.18

method	result	size
default	Expression too large to display	1903
parts	Expression too large to display	1903

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^6,x,method=_RETURNVERBOSE)
```


output

```

a*(-1/5/d/x^5*(-c^2*d*x^2+d)^(3/2)-2/15*c^2/d/x^3*(-c^2*d*x^2+d)^(3/2))+9/
5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^5/(c^2*x^
2-1)*arcsin(c*x)+2/15*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2
-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c^5+1/4*b*(-d*(c^2*x^2-1))^(1/2)/(1
5*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c^2*x^2-1)*c^5*(-c^2*x^2+1)^(1/2)-2*I*b
*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^2/(c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^7+2*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*
x^6-5*c^4*x^4-15*c^2*x^2+9)*x^6/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)
*c^11-2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x
^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^9-3/10*I*b*(-d*(c^2*x^2-1)
)^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3/(c^2*x^2-1)*(-c^2*x^2+1)*c
^8-2/15*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5
/(c^2*x^2-1)*(-c^2*x^2+1)*c^10+6/5*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-
5*c^4*x^4-15*c^2*x^2+9)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^5+3/1
0*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x/(c^2*x^
2-1)*(-c^2*x^2+1)*c^6+2/15*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^
4-15*c^2*x^2+9)*x^7/(c^2*x^2-1)*(-c^2*x^2+1)*c^12-5/3*b*(-d*(c^2*x^2-1))^(
1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5/(c^2*x^2-1)*arcsin(c*x)*c^10-
1/2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c^2*
x^2-1)*(-c^2*x^2+1)^(1/2)*c^9-17/3*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6...

```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^6} dx$$

$$= \frac{4(bc^7 x^7 - bc^5 x^5) \sqrt{d} \log\left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 + \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^4 - 1) \sqrt{d - d}}{c^2 x^4 - x^2}\right) - (2bc^3 x^3 - (2bc^3 - 3bc)x^5 - 3bcx) \sqrt{-d} \arctan\left(\frac{\sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^2 - 1) \sqrt{d - d}}{c^2 dx^4 + (c^2 - 1) dx^2 - d}\right) + (2bc^3 x^3 - (2bc^3 - 3bc)x^5 - 3bcx) \sqrt{-d}}{1}$$

input

```

integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="fricas"
)

```

output

```
[1/60*(4*(b*c^7*x^7 - b*c^5*x^5)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + (2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*arcsin(c*x) + 3*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), -1/60*(8*(b*c^7*x^7 - b*c^5*x^5)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + (2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*arcsin(c*x) + 3*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^6} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)}(a + b \arcsin(cx))}{x^6} dx$$

input

```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x**6,x)
```

output

```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.75

$$\begin{aligned} & \int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^6} dx \\ &= -\frac{1}{60} \left(8c^4 \sqrt{d} \log(x) - \frac{2c^2 \sqrt{dx^2 - 3\sqrt{d}}}{x^4} \right) bc \\ & \quad - \frac{1}{15} b \left(\frac{2(-c^2 dx^2 + d)^{\frac{3}{2}} c^2}{dx^3} + \frac{3(-c^2 dx^2 + d)^{\frac{3}{2}}}{dx^5} \right) \arcsin(cx) \\ & \quad - \frac{1}{15} a \left(\frac{2(-c^2 dx^2 + d)^{\frac{3}{2}} c^2}{dx^3} + \frac{3(-c^2 dx^2 + d)^{\frac{3}{2}}}{dx^5} \right) \end{aligned}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="maxima")`

output `-1/60*(8*c^4*sqrt(d)*log(x) - (2*c^2*sqrt(d)*x^2 - 3*sqrt(d))/x^4)*b*c - 1/15*b*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(3/2)/(d*x^5))*arcsin(c*x) - 1/15*a*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(3/2)/(d*x^5))`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^6} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^6} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x^6} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^6,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^6, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 x^2} (a + b \arcsin(cx))}{x^6} dx$$

$$= \frac{\sqrt{d} \left(2\sqrt{-c^2 x^2 + 1} a c^4 x^4 + \sqrt{-c^2 x^2 + 1} a c^2 x^2 - 3\sqrt{-c^2 x^2 + 1} a + 15 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^6} dx \right) b x^5 \right)}{15x^5}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x^6,x)`

output `(sqrt(d)*(2*sqrt(-c**2*x**2+1)*a*c**4*x**4+sqrt(-c**2*x**2+1)*a*c**2*x**2-3*sqrt(-c**2*x**2+1)*a+15*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**6,x)*b*x**5))/(15*x**5)`

3.61 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^8} dx$

Optimal result	704
Mathematica [A] (verified)	705
Rubi [A] (verified)	705
Maple [C] (verified)	707
Fricas [A] (verification not implemented)	708
Sympy [F]	709
Maxima [A] (verification not implemented)	709
Giac [F(-2)]	710
Mupad [F(-1)]	710
Reduce [F]	711

Optimal result

Integrand size = 27, antiderivative size = 263

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^8} dx = -\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{1-c^2x^2}} + \frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{1-c^2x^2}} + \frac{2bc^5\sqrt{d-c^2dx^2}}{105x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{7dx^7} - \frac{4c^2(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{35dx^5} - \frac{8c^4(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{105dx^3} - \frac{8bc^7\sqrt{d-c^2dx^2} \log(x)}{105\sqrt{1-c^2x^2}}$$

output

```
-1/42*b*c*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+1/140*b*c^3*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+2/105*b*c^5*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/d/x^7-4/35*c^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/d/x^5-8/105*c^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/d/x^3-8/105*b*c^7*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^8} dx$$

$$= \frac{\sqrt{d - c^2 dx^2}(-50bcx + 15bc^3x^3 + 40bc^5x^5 + 392bc^7x^7 - 300a\sqrt{1 - c^2x^2} + 60ac^2x^2\sqrt{1 - c^2x^2} + 80ac^4x^4 - 2100a^2x^7)}{2100x^7\sqrt{1 - c^2x^2}}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^8,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(-50*b*c*x + 15*b*c^3*x^3 + 40*b*c^5*x^5 + 392*b*c^7*x^7 - 300*a*Sqrt[1 - c^2*x^2] + 60*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 80*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 160*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 20*b*Sqrt[1 - c^2*x^2]*(-15 + 3*c^2*x^2 + 4*c^4*x^4 + 8*c^6*x^6)*ArcSin[c*x] - 160*b*c^7*x^7*Log[x]))/(2100*x^7*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^8} dx$$

↓ 5194

$$-\frac{bc\sqrt{d - c^2 dx^2} \int \frac{-8c^6 x^6 - 4c^4 x^4 - 3c^2 x^2 + 15}{105x^7} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{7dx^7} - \frac{4c^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{35dx^5} - \frac{8c^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{105dx^3}$$

↓ 27

$$\frac{bc\sqrt{d-c^2dx^2} \int \frac{-8c^6x^6-4c^4x^4-3c^2x^2+15}{x^7} dx}{105\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{7dx^7} - \frac{4c^2(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{35dx^5} - \frac{8c^4(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{105dx^3}$$

↓ 2010

$$\frac{bc\sqrt{d-c^2dx^2} \int \left(-\frac{8c^6}{x} - \frac{4c^4}{x^3} - \frac{3c^2}{x^5} + \frac{15}{x^7}\right) dx}{105\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{7dx^7} - \frac{4c^2(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{35dx^5} - \frac{8c^4(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{105dx^3}$$

↓ 2009

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{7dx^7} - \frac{4c^2(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{35dx^5} - \frac{8c^4(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{105dx^3} + \frac{bc\sqrt{d-c^2dx^2} \left(-8c^6 \log(x) + \frac{2c^4}{x^2} + \frac{3c^2}{4x^4} - \frac{5}{2x^6}\right)}{105\sqrt{1-c^2x^2}}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^8,x]`

output `-1/7*((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(d*x^7) - (4*c^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(35*d*x^5) - (8*c^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(105*d*x^3) + (b*c*Sqrt[d - c^2*d*x^2]*(-5/(2*x^6) + (3*c^2)/(4*x^4) + (2*c^4)/x^2 - 8*c^6*Log[x]))/(105*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Simp
plifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 2751, normalized size of antiderivative = 10.46

method	result	size
default	Expression too large to display	2751
parts	Expression too large to display	2751

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^8,x,method=_RETURNVERBOSE)
```


output

```

469/60*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^
2*x^2+225)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^9+3057/35*b*(-d*(c^2*x^2-1
))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x/(c^2*x^2-1
)*arcsin(c*x)*c^8+30/7*I*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6
-21*c^4*x^4-315*c^2*x^2+225)*x^3/(c^2*x^2-1)*c^10-594/35*b*(-d*(c^2*x^2-1
))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x/(c^2*x^2-1
)*arcsin(c*x)*c^6-71/28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-2
1*c^4*x^4-315*c^2*x^2+225)/x^2/(c^2*x^2-1)*c^5*(-c^2*x^2+1)^(1/2)+342/7*b*
(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225
)/x^3/(c^2*x^2-1)*arcsin(c*x)*c^4-255/28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8
*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^4/(c^2*x^2-1)*c^3*(-c^2*x^2
+1)^(1/2)-585/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x
^4-315*c^2*x^2+225)/x^5/(c^2*x^2-1)*arcsin(c*x)*c^2+75/14*b*(-d*(c^2*x^2-1
))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^6/(c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)*c+64/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^
6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9/(c^2*x^2-1)*arcsin(c*x)*c^16-40/21*I
*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+
225)*x^9/(c^2*x^2-1)*c^16-214/105*I*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-
105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c^2*x^2-1)*c^14+152/105*I*b*(
-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+2...

```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^8} dx$$

$$= \frac{16 (bc^9 x^9 - bc^7 x^7) \sqrt{d} \log \left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 + \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^4 - 1) \sqrt{d - d}}{c^2 x^4 - x^2} \right) - (8 bc^5 x^5 - (8 bc^5 + 3 bc^3 - 10 bc) x^7 + 32 (bc^9 x^9 - bc^7 x^7) \sqrt{-d} \arctan \left(\frac{\sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^2 - 1) \sqrt{-d}}{c^2 dx^4 + (c^2 - 1) dx^2 - d} \right) + (8 bc^5 x^5 - (8 bc^5 + 3 bc^3 - 10 bc) x^7 + \dots}{\dots}}{$$

input

```

integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="fricas"
)

```

output

```
[1/420*(16*(b*c^9*x^9 - b*c^7*x^7)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(8*a*c^8*x^8 - 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2 + (8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*arcsin(c*x) + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/420*(32*(b*c^9*x^9 - b*c^7*x^7)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 4*(8*a*c^8*x^8 - 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2 + (8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*arcsin(c*x) + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^8} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)}(a + b \arcsin(cx))}{x^8} dx$$

input

```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x**8,x)
```

output

```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.76

$$\begin{aligned} & \int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^8} dx \\ &= -\frac{1}{420} \left(32 c^6 \sqrt{d} \log(x) - \frac{8 c^4 \sqrt{d} x^4 + 3 c^2 \sqrt{d} x^2 - 10 \sqrt{d}}{x^6} \right) bc \\ & - \frac{1}{105} \left(\frac{8 (-c^2 dx^2 + d)^{\frac{3}{2}} c^4}{dx^3} + \frac{12 (-c^2 dx^2 + d)^{\frac{3}{2}} c^2}{dx^5} + \frac{15 (-c^2 dx^2 + d)^{\frac{3}{2}}}{dx^7} \right) b \arcsin(cx) \\ & - \frac{1}{105} \left(\frac{8 (-c^2 dx^2 + d)^{\frac{3}{2}} c^4}{dx^3} + \frac{12 (-c^2 dx^2 + d)^{\frac{3}{2}} c^2}{dx^5} + \frac{15 (-c^2 dx^2 + d)^{\frac{3}{2}}}{dx^7} \right) a \end{aligned}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="maxima")`

output `-1/420*(32*c^6*sqrt(d)*log(x) - (8*c^4*sqrt(d)*x^4 + 3*c^2*sqrt(d)*x^2 - 10*sqrt(d))/x^6)*b*c - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*b*arcsin(c*x) - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^8} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^8} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x^8} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^8,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^8, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 x^2} (a + b \arcsin(cx))}{x^8} dx$$

$$= \frac{\sqrt{d} \left(8\sqrt{-c^2 x^2 + 1} a c^6 x^6 + 4\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 3\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 15\sqrt{-c^2 x^2 + 1} a + 105 \int \frac{\sqrt{-c^2 x^2 + 1} a \arcsin(cx)}{x^8} dx \right)}{105 x^7}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x^8,x)`

output `(sqrt(d)*(8*sqrt(-c**2*x**2+1)*a*c**6*x**6+4*sqrt(-c**2*x**2+1)*a*c**4*x**4+3*sqrt(-c**2*x**2+1)*a*c**2*x**2-15*sqrt(-c**2*x**2+1)*a+105*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**8,x)*b*x**7))/(105*x**7)`

3.62 $\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$

Optimal result	712
Mathematica [A] (verified)	713
Rubi [A] (verified)	713
Maple [A] (verified)	715
Fricas [A] (verification not implemented)	715
Sympy [F]	716
Maxima [A] (verification not implemented)	716
Giac [F(-2)]	717
Mupad [F(-1)]	717
Reduce [F]	718

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{8bx\sqrt{d - c^2 dx^2}}{105c^5\sqrt{1 - c^2 x^2}} + \frac{4bx^3\sqrt{d - c^2 dx^2}}{315c^3\sqrt{1 - c^2 x^2}} + \frac{bx^5\sqrt{d - c^2 dx^2}}{175c\sqrt{1 - c^2 x^2}} - \frac{bcx^7\sqrt{d - c^2 dx^2}}{49\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^6 d} + \frac{2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^6 d^2} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^6 d^3}$$

output $8/105*b*x*(-c^2*d*x^2+d)^{(1/2)}/c^5/(-c^2*x^2+1)^{(1/2)}+4/315*b*x^3*(-c^2*d*x^2+d)^{(1/2)}/c^3/(-c^2*x^2+1)^{(1/2)}+1/175*b*x^5*(-c^2*d*x^2+d)^{(1/2)}/c/(-c^2*x^2+1)^{(1/2)}-1/49*b*c*x^7*(-c^2*d*x^2+d)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}-1/3*(-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x))/c^6/d+2/5*(-c^2*d*x^2+d)^{(5/2)}*(a+b*\arcsin(c*x))/c^6/d^2-1/7*(-c^2*d*x^2+d)^{(7/2)}*(a+b*\arcsin(c*x))/c^6/d^3$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.61

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d - c^2 dx^2} (bcx(840 + 140c^2 x^2 + 63c^4 x^4 - 225c^6 x^6) + 105a\sqrt{1 - c^2 x^2}(-8 - 4c^2 x^2 - 3c^4 x^4 + 15c^6 x^6) + 11025c^6 \sqrt{1 - c^2 x^2})}{11025c^6 \sqrt{1 - c^2 x^2}}$$

input `Integrate[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(Sqrt[d - c^2*d*x^2]*(b*c*x*(840 + 140*c^2*x^2 + 63*c^4*x^4 - 225*c^6*x^6) + 105*a*Sqrt[1 - c^2*x^2]*(-8 - 4*c^2*x^2 - 3*c^4*x^4 + 15*c^6*x^6) + 105*b*Sqrt[1 - c^2*x^2]*(-8 - 4*c^2*x^2 - 3*c^4*x^4 + 15*c^6*x^6)*ArcSin[c*x]))/(11025*c^6*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5194, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int \frac{-15c^6 x^6 + 3c^4 x^4 + 4c^2 x^2 + 8}{105c^6} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^6 d^3} +$$

$$\frac{2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^6 d^2} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^6 d}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{b\sqrt{d-c^2dx^2} \int (-15c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{105c^5\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d^3} + \\
& \frac{2(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^6d^2} - \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{3c^6d} \\
& \quad \downarrow \text{2009} \\
& - \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d^3} + \frac{2(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^6d^2} - \\
& \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))}{3c^6d} + \frac{b\left(-\frac{15}{7}c^6x^7 + \frac{3c^4x^5}{5} + \frac{4c^2x^3}{3} + 8x\right)\sqrt{d-c^2dx^2}}{105c^5\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(b*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)/7))/(105*c^5*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^6*d) + (2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^6*d^2) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^6*d^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.76

method	result
orering	$\frac{(2925c^8x^8 - 3393c^6x^6 - 630c^4x^4 - 4760c^2x^2 + 5040)\sqrt{-c^2dx^2+d}(a+b\arcsin(cx))}{11025c^6(c^2x^2-1)} - \frac{(225c^6x^6 - 63c^4x^4 - 140c^2x^2 - 840)(5x^4\sqrt{-c^2dx^2+d})}{11025c^6(c^2x^2-1)}$
default	$a \left(-\frac{x^4(-c^2dx^2+d)^{\frac{3}{2}}}{7c^2d} + \frac{-4x^2(-c^2dx^2+d)^{\frac{3}{2}}}{35c^2d} - \frac{8(-c^2dx^2+d)^{\frac{3}{2}}}{105dc^4} \right) + b \left(\frac{\sqrt{-d(c^2x^2-1)}(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2dx^2+d})}{11025c^6(c^2x^2-1)} \right)$
parts	$a \left(-\frac{x^4(-c^2dx^2+d)^{\frac{3}{2}}}{7c^2d} + \frac{-4x^2(-c^2dx^2+d)^{\frac{3}{2}}}{35c^2d} - \frac{8(-c^2dx^2+d)^{\frac{3}{2}}}{105dc^4} \right) + b \left(\frac{\sqrt{-d(c^2x^2-1)}(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2dx^2+d})}{11025c^6(c^2x^2-1)} \right)$

input `int(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{11025} \frac{(2925c^8x^8 - 3393c^6x^6 - 630c^4x^4 - 4760c^2x^2 + 5040)}{c^6} \frac{(-c^2dx^2+d)^{\frac{1}{2}}(a+b\arcsin(cx))}{(c^2x^2-1)} - \frac{1}{11025} \frac{(225c^6x^6 - 63c^4x^4 - 140c^2x^2 - 840)}{c^6} \frac{5x^4(-c^2dx^2+d)^{\frac{1}{2}}(a+b\arcsin(cx))}{(c^2x^2-1)} - \frac{x^6(-c^2dx^2+d)^{\frac{1}{2}}(a+b\arcsin(cx))}{11025c^6(c^2x^2-1)} + \frac{b(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2dx^2+d})}{11025c^6(c^2x^2-1)}$$

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.69

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{(225bc^7x^7 - 63bc^5x^5 - 140bc^3x^3 - 840bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1} + 105(15ac^8x^8 - 18ac^6x^6 - ac^4x^4 - 840c^2x^2 - 840d)}{11025(c^8x^2 - c^6x^2 + c^4x^2 - c^2x^2 + d)}$$

input `integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output

```
1/11025*((225*b*c^7*x^7 - 63*b*c^5*x^5 - 140*b*c^3*x^3 - 840*b*c*x)*sqrt(-
c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 105*(15*a*c^8*x^8 - 18*a*c^6*x^6 - a*c
^4*x^4 - 4*a*c^2*x^2 + (15*b*c^8*x^8 - 18*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*
x^2 + 8*b)*arcsin(c*x) + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F]

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^5 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx)) dx$$

input

```
integrate(x**5*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)
```

output

```
Integral(x**5*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$=$$

$$-\frac{1}{105} \left(\frac{15(-c^2 dx^2 + d)^{\frac{3}{2}} x^4}{c^2 d} + \frac{12(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^4 d} + \frac{8(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^6 d} \right) b \arcsin(cx)$$

$$-\frac{1}{105} \left(\frac{15(-c^2 dx^2 + d)^{\frac{3}{2}} x^4}{c^2 d} + \frac{12(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^4 d} + \frac{8(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^6 d} \right) a$$

$$-\frac{(225 c^6 \sqrt{dx^7} - 63 c^4 \sqrt{dx^5} - 140 c^2 \sqrt{dx^3} - 840 \sqrt{dx}) b}{11025 c^5}$$

input

```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima"
)
```

output

```
-1/105*(15*(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*b*arcsin(c*x) - 1/105*(15*
(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^4*d)
+ 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*a - 1/11025*(225*c^6*sqrt(d)*x^7 - 63
*c^4*sqrt(d)*x^5 - 140*c^2*sqrt(d)*x^3 - 840*sqrt(d)*x)*b/c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^5 (a + b \arcsin(cx)) \sqrt{d - c^2 dx^2} dx$$

input

```
int(x^5*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output

```
int(x^5*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^5 \sqrt{d - c^2 x^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d} (15\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 3\sqrt{-c^2 x^2 + 1} a c^4 x^4 - 4\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 8\sqrt{-c^2 x^2 + 1} a + 105 \int \sqrt{-c^2 x^2 + 1} dx)}{105c^6}$$

input `int(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*(15*sqrt(-c**2*x**2+1)*a*c**6*x**6-3*sqrt(-c**2*x**2+1)*a*c**4*x**4-4*sqrt(-c**2*x**2+1)*a*c**2*x**2-8*sqrt(-c**2*x**2+1)*a+105*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**5,x)*b*c**6))/(105*c**6)`

3.63 $\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$

Optimal result	719
Mathematica [A] (verified)	720
Rubi [A] (verified)	720
Maple [A] (verified)	722
Fricas [A] (verification not implemented)	722
Sympy [F]	723
Maxima [A] (verification not implemented)	723
Giac [F(-2)]	724
Mupad [F(-1)]	724
Reduce [F]	724

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{2bx\sqrt{d - c^2 dx^2}}{15c^3\sqrt{1 - c^2 x^2}} + \frac{bx^3\sqrt{d - c^2 dx^2}}{45c\sqrt{1 - c^2 x^2}} - \frac{bcx^5\sqrt{d - c^2 dx^2}}{25\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^4 d} + \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^4 d^2}$$

output

```
2/15*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/45*b*x^3*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/25*b*c*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/c^4/d+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{\sqrt{d - c^2 dx^2} (15a \sqrt{1 - c^2 x^2} (-2 - c^2 x^2 + 3c^4 x^4) + b(30cx + 5c^3 x^3 - 9c^5 x^5) + 15b \sqrt{1 - c^2 x^2} (-2 - c^2 x^2 + 3c^4 x^4) \operatorname{ArcSin}[cx])}{225c^4 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(15*a*Sqrt[1 - c^2*x^2]*(-2 - c^2*x^2 + 3*c^4*x^4) +
b*(30*c*x + 5*c^3*x^3 - 9*c^5*x^5) + 15*b*Sqrt[1 - c^2*x^2]*(-2 - c^2*x^2
+ 3*c^4*x^4)*ArcSin[c*x]))/(225*c^4*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5194, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int \frac{-3c^4 x^4 + c^2 x^2 + 2}{15c^4} dx}{\sqrt{1 - c^2 x^2}} + \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^4 d} -$$

$$\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^4 d}$$

$$\downarrow 27$$

$$\frac{b\sqrt{d - c^2 dx^2} \int (-3c^4 x^4 + c^2 x^2 + 2) dx}{15c^3 \sqrt{1 - c^2 x^2}} + \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^4 d} -$$

$$\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^4 d}$$

$$\begin{array}{c} \downarrow \text{2009} \\ \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^4 d^2} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^4 d} + \\ \frac{b \left(-\frac{3}{5} c^4 x^5 + \frac{c^2 x^3}{3} + 2x \right) \sqrt{d - c^2 dx^2}}{15c^3 \sqrt{1 - c^2 x^2}} \end{array}$$

input `Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(b*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^4*d) + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^4*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98

method	result
ordering	$\frac{(81c^6x^6 - 107c^4x^4 - 120c^2x^2 + 120)\sqrt{-c^2dx^2 + d}(a + b\arcsin(cx))}{225c^4(c^2x^2 - 1)} - \frac{(9c^4x^4 - 5c^2x^2 - 30)\left(3x^2\sqrt{-c^2dx^2 + d}(a + b\arcsin(cx)) - \frac{x^4}{225x^2c^4}\right)}{225x^2c^4}$
default	$a\left(-\frac{x^2(-c^2dx^2 + d)^{\frac{3}{2}}}{5c^2d} - \frac{2(-c^2dx^2 + d)^{\frac{3}{2}}}{15dc^4}\right) + b\left(\frac{\sqrt{-d(c^2x^2 - 1)}(16c^6x^6 - 28c^4x^4 - 16i\sqrt{-c^2x^2 + 1}x^5c^5 + 13c^2x^2 + 20i\sqrt{-c^2x^2 + 1}x^3c^3 - 15c^2x^2 + 15i\sqrt{-c^2x^2 + 1}x^3c^3 - 15c^2x^2 + 15i\sqrt{-c^2x^2 + 1}x^3c^3)}{800c^4(c^2x^2 - 1)}\right)$
parts	$a\left(-\frac{x^2(-c^2dx^2 + d)^{\frac{3}{2}}}{5c^2d} - \frac{2(-c^2dx^2 + d)^{\frac{3}{2}}}{15dc^4}\right) + b\left(\frac{\sqrt{-d(c^2x^2 - 1)}(16c^6x^6 - 28c^4x^4 - 16i\sqrt{-c^2x^2 + 1}x^5c^5 + 13c^2x^2 + 20i\sqrt{-c^2x^2 + 1}x^3c^3 - 15c^2x^2 + 15i\sqrt{-c^2x^2 + 1}x^3c^3 - 15c^2x^2 + 15i\sqrt{-c^2x^2 + 1}x^3c^3)}{800c^4(c^2x^2 - 1)}\right)$

input `int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{225}*(81*c^6*x^6 - 107*c^4*x^4 - 120*c^2*x^2 + 120)/c^4/(c^2*x^2 - 1)*(-c^2*d*x^2 + d)^{(1/2)}*(a + b*arcsin(c*x)) - \frac{1}{225}/x^2*(9*c^4*x^4 - 5*c^2*x^2 - 30)/c^4*(3*x^2*(-c^2*d*x^2 + d)^{(1/2)}*(a + b*arcsin(c*x)) - x^4/(-c^2*d*x^2 + d)^{(1/2)}*(a + b*arcsin(c*x))*c^2*d + b*c*x^3*(-c^2*d*x^2 + d)^{(1/2)}/(-c^2*x^2 + 1)^{(1/2)})$$

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{(9bc^5x^5 - 5bc^3x^3 - 30bcx)\sqrt{-c^2dx^2 + d}\sqrt{-c^2x^2 + 1} + 15(3ac^6x^6 - 4ac^4x^4 - ac^2x^2 + (3bc^6x^6 - 4bc^4x^4 - bc^2x^2 + 2b)*\arcsin(cx) + 2a)*\sqrt{-c^2dx^2 + d}}{225(c^6x^2 - c^4)}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output
$$\frac{1}{225}*((9*b*c^5*x^5 - 5*b*c^3*x^3 - 30*b*c*x)*\sqrt{-c^2*d*x^2 + d}*\sqrt{-c^2*x^2 + 1} + 15*(3*a*c^6*x^6 - 4*a*c^4*x^4 - a*c^2*x^2 + (3*b*c^6*x^6 - 4*b*c^4*x^4 - b*c^2*x^2 + 2*b)*\arcsin(c*x) + 2*a)*\sqrt{-c^2*d*x^2 + d})/(c^6*x^2 - c^4)$$

Sympy [F]

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^3 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx)) dx$$

input `integrate(x**3*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)`

output `Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\begin{aligned} & \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx \\ &= -\frac{1}{15} b \left(\frac{3(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^4 d} \right) \arcsin(cx) \\ & \quad - \frac{1}{15} a \left(\frac{3(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^4 d} \right) \\ & \quad - \frac{(9c^4 \sqrt{d} x^5 - 5c^2 \sqrt{d} x^3 - 30\sqrt{d} x) b}{225c^3} \end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/15*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsin(c*x) - 1/15*a*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(d)*x^5 - 5*c^2*sqrt(d)*x^3 - 30*sqrt(d)*x)*b/c^3`

Giac [F(-2)]

Exception generated.

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^3 (a + b \arcsin(cx)) \sqrt{d - c^2 dx^2} dx$$

input `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)`

output `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} (3\sqrt{-c^2 x^2 + 1} a c^4 x^4 - \sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a + 15 (\int \sqrt{-c^2 x^2 + 1} a \sin(cx) x^3 dx) b c^4)}{15c^4}$$

input `int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output

```
(sqrt(d)*(3*sqrt(-c**2*x**2+1)*a*c**4*x**4 - sqrt(-c**2*x**2+1)*a*
c**2*x**2 - 2*sqrt(-c**2*x**2+1)*a + 15*int(sqrt(-c**2*x**2+1)*asi
n(c*x)*x**3,x)*b*c**4))/(15*c**4)
```

3.64 $\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) dx$

Optimal result	726
Mathematica [A] (verified)	726
Rubi [A] (verified)	727
Maple [A] (verified)	728
Fricas [A] (verification not implemented)	729
Sympy [F]	729
Maxima [A] (verification not implemented)	729
Giac [F(-2)]	730
Mupad [F(-1)]	730
Reduce [F]	731

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) dx = \frac{bx\sqrt{d - c^2dx^2}}{3c\sqrt{1 - c^2x^2}} - \frac{bcx^3\sqrt{d - c^2dx^2}}{9\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{3c^2d}$$

output

$1/3*b*x*(-c^2*d*x^2+d)^{(1/2)}/c/(-c^2*x^2+1)^{(1/2)}-1/9*b*c*x^3*(-c^2*d*x^2+d)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}-1/3*(-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x))/c^2/d$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) dx = \frac{\sqrt{d - c^2dx^2} \left(\frac{bc \left(x - \frac{c^2x^3}{3} \right)}{\sqrt{1 - c^2x^2}} + (-1 + c^2x^2)(a + b \arcsin(cx)) \right)}{3c^2}$$

input

`Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output

$$\frac{(\text{Sqrt}[d - c^2*d*x^2]*((b*c*(x - (c^2*x^3)/3))/\text{Sqrt}[1 - c^2*x^2] + (-1 + c^2*x^2)*(a + b*\text{ArcSin}[c*x])))/(3*c^2)}$$
Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) dx$$

$$\downarrow 5182$$

$$\frac{b\sqrt{d - c^2dx^2} \int (1 - c^2x^2) dx}{3c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{3c^2d}$$

$$\downarrow 2009$$

$$\frac{b\left(x - \frac{c^2x^3}{3}\right)\sqrt{d - c^2dx^2}}{3c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{3c^2d}$$

input

$$\text{Int}[x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x]),x]$$

output

$$\frac{(b*\text{Sqrt}[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(3*c*\text{Sqrt}[1 - c^2*x^2]) - ((d - c^2*d*x^2)^{(3/2})*(a + b*\text{ArcSin}[c*x]))/(3*c^2*d)}$$

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

method	result
orering	$\frac{(5c^4x^4 - 13c^2x^2 + 6)\sqrt{-c^2dx^2 + d}(a + b \arcsin(cx))}{9c^2(c^2x^2 - 1)} - \frac{(c^2x^2 - 3)\left(\sqrt{-c^2dx^2 + d}(a + b \arcsin(cx)) - \frac{x^2(a + b \arcsin(cx))c^2d}{\sqrt{-c^2dx^2 + d}} + \frac{bcx\sqrt{-c^2dx^2 + d}}{\sqrt{-c^2dx^2 + d}}\right)}{9c^2}$
default	$-\frac{a(-c^2dx^2 + d)^{\frac{3}{2}}}{3c^2d} + b\left(\frac{\sqrt{-d(c^2x^2 - 1)}(4c^4x^4 - 5c^2x^2 - 4i\sqrt{-c^2x^2 + 1}x^3c^3 + 3i\sqrt{-c^2x^2 + 1}cx + 1)(i + 3 \arcsin(cx))}{72c^2(c^2x^2 - 1)} - \frac{\sqrt{-d(c^2x^2 - 1)}}{\sqrt{-d(c^2x^2 - 1)}}\right)$
parts	$-\frac{a(-c^2dx^2 + d)^{\frac{3}{2}}}{3c^2d} + b\left(\frac{\sqrt{-d(c^2x^2 - 1)}(4c^4x^4 - 5c^2x^2 - 4i\sqrt{-c^2x^2 + 1}x^3c^3 + 3i\sqrt{-c^2x^2 + 1}cx + 1)(i + 3 \arcsin(cx))}{72c^2(c^2x^2 - 1)} - \frac{\sqrt{-d(c^2x^2 - 1)}}{\sqrt{-d(c^2x^2 - 1)}}\right)$

input `int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `1/9*(5*c^4*x^4-13*c^2*x^2+6)/c^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-1/9*(c^2*x^2-3)/c^2*((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-x^2/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*c^2*d+b*c*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))dx$$

$$= \frac{(bc^3x^3 - 3bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1} + 3(ac^4x^4 - 2ac^2x^2 + (bc^4x^4 - 2bc^2x^2 + b)\arcsin(cx) + a)\sqrt{-c^2dx^2+d}}{9(c^4x^2 - c^2)}$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `1/9*((b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 3*(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)`

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))dx = \int x\sqrt{-d(cx-1)(cx+1)}(a+b\arcsin(cx))dx$$

input `integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)`

output `Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))dx = -\frac{(-c^2dx^2+d)^{\frac{3}{2}}b\arcsin(cx)}{3c^2d} - \frac{(c^2d^{\frac{3}{2}}x^3 - 3d^{\frac{3}{2}}x)b}{9cd} - \frac{(-c^2dx^2+d)^{\frac{3}{2}}a}{3c^2d}$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
-1/3*(-c^2*d*x^2 + d)^(3/2)*b*arcsin(c*x)/(c^2*d) - 1/9*(c^2*d^(3/2)*x^3 -
3*d^(3/2)*x)*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))dx = \int x(a+b\arcsin(cx))\sqrt{d-c^2x^2}dx$$

input

```
int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output

```
int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))dx$$

$$= \frac{\sqrt{d}(\sqrt{-c^2x^2+1}ac^2x^2 - \sqrt{-c^2x^2+1}a + 3(\int\sqrt{-c^2x^2+1}\arcsin(cx)xdx)bc^2)}{3c^2}$$

input `int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*(sqrt(-c**2*x**2+1)*a*c**2*x**2 - sqrt(-c**2*x**2+1)*a + 3*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*b*c**2))/(3*c**2)`

3.65 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x} dx$

Optimal result	732
Mathematica [A] (verified)	733
Rubi [A] (verified)	733
Maple [A] (verified)	736
Fricas [F]	736
Sympy [F]	737
Maxima [F]	737
Giac [F(-2)]	737
Mupad [F(-1)]	738
Reduce [F]	738

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x} dx$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b \arcsin(cx))$$

$$- \frac{2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$+ \frac{ib\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} - \frac{ib\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

output

```
-b*c*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx$$

$$= a\sqrt{d - c^2 dx^2} + a\sqrt{d} \log(x) - a\sqrt{d} \log\left(d + \sqrt{d}\sqrt{d - c^2 dx^2}\right)$$

$$+ \frac{b\sqrt{d - c^2 dx^2}(-cx + \sqrt{1 - c^2 x^2} \arcsin(cx) + \arcsin(cx) \log(1 - e^{i \arcsin(cx)}) - \arcsin(cx) \log(1 + e^{i \arcsin(cx)}))}{\sqrt{1 - c^2 x^2}}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x,x]
```

output

```
a*Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) - ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])]) + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])])/Sqrt[1 - c^2*x^2]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx$$

$$\downarrow 5198$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int 1 dx}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2}(a + b \arcsin(cx))$$

$$\downarrow 24$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}}$$

↓ 5218

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx)}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}}$$

↓ 3042

$$\frac{\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}}$$

↓ 4671

$$\frac{\sqrt{d - c^2 dx^2}(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}}$$

$$\sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}}$$

↓ 2715

$$\frac{\sqrt{d - c^2 dx^2}(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}}$$

$$\sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}}$$

↓ 2838

$$\frac{\sqrt{d - c^2 dx^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}}$$

$$\sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}}$$

input

```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x,x]
```

output

$$-\left(\frac{b c x \sqrt{d - c^2 d x^2}}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x]) + (\sqrt{d - c^2 d x^2} (-2(a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}[E^{(I \operatorname{ArcSin}[c x])}] + I b \operatorname{PolyLog}[2, -E^{(I \operatorname{ArcSin}[c x])}] - I b \operatorname{PolyLog}[2, E^{(I \operatorname{ArcSin}[c x])}]}))\right) / \sqrt{1 - c^2 x^2}$$

Defintions of rubi rules used

rule 24

$$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a x, x] /; \operatorname{FreeQ}[a, x]$$

rule 2715

$$\operatorname{Int}[\operatorname{Log}[(a_) + (b_) ((F_)^{((e_) ((c_) + (d_) (x_)))})^{(n_)}], x_Symbol] \rightarrow \operatorname{Simp}[1/(d e n \operatorname{Log}[F]) \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b x]/x, x], x, (F^{(e(c + d x))})^{(n)}], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \operatorname{GtQ}[a, 0]$$

rule 2838

$$\operatorname{Int}[\operatorname{Log}[(c_) ((d_) + (e_) (x_)^{(n_)}] / (x_), x_Symbol] \rightarrow \operatorname{Simp}[-\operatorname{PolyLog}[2, (-c) e x^n / n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x\} \&\& \operatorname{EqQ}[c d, 1]$$

rule 3042

$$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{DeactivateTrig}[u, x], x] /; \operatorname{FunctionOfTrigOfLinearQ}[u, x]$$

rule 4671

$$\operatorname{Int}[\operatorname{csc}[(e_) + (f_) (x_)] ((c_) + (d_) (x_))^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[-2(c + d x)^m (\operatorname{ArcTanh}[E^{(I(e + f x))}] / f), x] + (-\operatorname{Simp}[d(m/f) \operatorname{Int}[(c + d x)^{(m-1)} \operatorname{Log}[1 - E^{(I(e + f x))}], x], x] + \operatorname{Simp}[d(m/f) \operatorname{Int}[(c + d x)^{(m-1)} \operatorname{Log}[1 + E^{(I(e + f x))}], x], x]) /; \operatorname{FreeQ}\{c, d, e, f\}, x\} \&\& \operatorname{IGtQ}[m, 0]$$

rule 5198

$$\operatorname{Int}[(a_) + \operatorname{ArcSin}[(c_) (x_)] (b_)^{(n_)} ((f_) (x_))^{(m_)} \sqrt{(d_) + (e_) (x_)^2}, x_Symbol] \rightarrow \operatorname{Simp}[(f x)^{(m+1)} \sqrt{d + e x^2} ((a + b \operatorname{ArcSin}[c x])^n / (f(m+2))), x] + (\operatorname{Simp}[(1/(m+2)) \operatorname{Simp}[\sqrt{d + e x^2} / \sqrt{1 - c^2 x^2}] \operatorname{Int}[(f x)^m ((a + b \operatorname{ArcSin}[c x])^n / \sqrt{1 - c^2 x^2}), x], x] - \operatorname{Simp}[b c (n/(f(m+2))) \operatorname{Simp}[\sqrt{d + e x^2} / \sqrt{1 - c^2 x^2}] \operatorname{Int}[(f x)^{(m+1)} (a + b \operatorname{ArcSin}[c x])^{(n-1)}, x], x]) /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{GtQ}[n, 0] \&\& (\operatorname{IGtQ}[m, -2] \mid \mid \operatorname{EqQ}[n, 1])$$

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.55

method	result
default	$-\sqrt{d} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2d x^2+d}}{x}\right) a + \sqrt{-c^2d x^2+d} a + b\left(\frac{\sqrt{-d(c^2x^2-1)}(c^2x^2-i\sqrt{-c^2x^2+1}cx-1)(\arcsin(cx)+1)}{2c^2x^2-2}\right)$
parts	$-\sqrt{d} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2d x^2+d}}{x}\right) a + \sqrt{-c^2d x^2+d} a + b\left(\frac{\sqrt{-d(c^2x^2-1)}(c^2x^2-i\sqrt{-c^2x^2+1}cx-1)(\arcsin(cx)+1)}{2c^2x^2-2}\right)$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)
```

output

```
-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a+(-c^2*d*x^2+d)^(1/2)
*a+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arc
sin(c*x)+I)/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c
*x+c^2*x^2-1)*(arcsin(c*x)-I)/(c^2*x^2-1)-I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x
^2+1)^(1/2)/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*ar
csin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/
2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx = \int \frac{\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x,x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)}(a + b \arcsin(cx))}{x} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx = \int \frac{\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x) - (sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x} dx = \sqrt{d} \left(\sqrt{-c^2 x^2 + 1} a \right. \\ \left. + \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x} dx \right) b \right. \\ \left. + \log \left(\tan \left(\frac{\arcsin(cx)}{2} \right) \right) a - a \right)$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x,x)`

output `sqrt(d)*(sqrt(-c**2*x**2 + 1)*a + int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x,x)*b + log(tan(asin(c*x)/2))*a - a)`

3.66 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^3} dx$

Optimal result	739
Mathematica [A] (verified)	740
Rubi [A] (verified)	740
Maple [A] (verified)	743
Fricas [F]	743
Sympy [F]	744
Maxima [F]	744
Giac [F(-2)]	745
Mupad [F(-1)]	745
Reduce [F]	745

Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^3} dx$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{2x^2}$$

$$+ \frac{c^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{ibc^2\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{2\sqrt{1-c^2x^2}} + \frac{ibc^2\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{2\sqrt{1-c^2x^2}}$$

output

```
-1/2*b*c*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)-1/2*(-c^2*d*x^2+d)^(1/2)
)*(a+b*arcsin(c*x))/x^2+c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh
(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-1/2*I*b*c^2*(-c^2*d*x^2+d)^(
1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+1/2*I*b*c^2*(
-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```


Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{1}{8} \left(-\frac{4a\sqrt{d - c^2 dx^2}}{x^2} - 4ac^2\sqrt{d} \log(x) + 4ac^2\sqrt{d} \log\left(d + \sqrt{d}\sqrt{d - c^2 dx^2}\right) \right.$$

$$\left. + \frac{bc^2 d \sqrt{1 - c^2 x^2} \left(-2 \cot\left(\frac{1}{2} \arcsin(cx)\right) - \arcsin(cx) \csc^2\left(\frac{1}{2} \arcsin(cx)\right) - 4 \arcsin(cx) \log\left(1 - e^{i \arcsin(cx)}\right)\right)}{\sqrt{d - c^2 dx^2}} \right)$$

input `Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^3,x]`

output `((-4*a*Sqrt[d - c^2*d*x^2])/x^2 - 4*a*c^2*Sqrt[d]*Log[x] + 4*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*c^2*d*Sqrt[1 - c^2*x^2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) + 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])]) - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])] + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2]))/Sqrt[d - c^2*d*x^2])/8`

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5196, 15, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx$$

$$\downarrow 5196$$

$$-\frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x \sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{bc \sqrt{d - c^2 dx^2} \int \frac{1}{x^2} dx}{2\sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2x^2}$$

$$\downarrow 15$$

$$\begin{aligned}
& \frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x \sqrt{1 - c^2 x^2}} dx}{2 \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2x^2} - \frac{bc \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow 5218 \\
& \frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx)}{2 \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2x^2} - \frac{bc \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow 3042 \\
& \frac{c^2 \sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{2 \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2x^2} - \frac{bc \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow 4671 \\
& \frac{c^2 \sqrt{d - c^2 dx^2} (-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}))}{2 \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2x^2} - \frac{bc \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow 2715 \\
& \frac{c^2 \sqrt{d - c^2 dx^2} (ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2 \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2x^2} - \frac{bc \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}} \\
& \quad \downarrow 2838 \\
& \frac{c^2 \sqrt{d - c^2 dx^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{2 \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2x^2} - \frac{bc \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}}
\end{aligned}$$

input

```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/2*(b*c*Sqrt[d - c^2*d*x^2])/(x*Sqrt[1 - c^2*x^2]) - (Sqrt[d - c^2*d*x^2]
]*(a + b*ArcSin[c*x]))/(2*x^2) - (c^2*Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSi
n[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] -
I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/(2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5196

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x],
x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int
[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

method	result
default	$a \left(-\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{2 d x^2} - \frac{c^2 \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right)}{2} \right) + b \left(-\frac{(c^2 x^2 \arcsin(c x) - c x \sqrt{-c^2 x^2 + 1} - \arcsin(c x))}{2 (c^2 x^2 - 1) x^2} \right)$
parts	$a \left(-\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{2 d x^2} - \frac{c^2 \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right)}{2} \right) + b \left(-\frac{(c^2 x^2 \arcsin(c x) - c x \sqrt{-c^2 x^2 + 1} - \arcsin(c x))}{2 (c^2 x^2 - 1) x^2} \right)$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output

```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(3/2)-1/2*c^2*((-c^2*d*x^2+d)^(1/2)-d^(1/2))*1
n((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))+b*(-1/2*(c^2*x^2*arcsin(c*x)-c
*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2+
I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^
2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,-I*
c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c^2/(2*c^2*x^
2-2))
```

Fricas [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas"
)
```

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/x^3, x)`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)}(a + b \arcsin(cx))}{x^3} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x**3,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x**3, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx = \int \frac{\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x^3, x) + 1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^3,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^3, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{\sqrt{d} \left(-\sqrt{-c^2 x^2 + 1} a + 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^3} dx \right) b x^2 - \log \left(\tan \left(\frac{\arcsin(cx)}{2} \right) \right) a c^2 x^2 \right)}{2x^2}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x^3,x)`

output

```
(sqrt(d)*(-sqrt(-c**2*x**2 + 1)*a + 2*int((sqrt(-c**2*x**2 + 1)*asin
(c*x))/x**3,x)*b*x**2 - log(tan(asin(c*x)/2))*a*c**2*x**2))/(2*x**2)
```

3.67 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^5} dx$

Optimal result	747
Mathematica [A] (verified)	748
Rubi [A] (verified)	748
Maple [A] (verified)	752
Fricas [F]	752
Sympy [F]	753
Maxima [F]	753
Giac [F(-2)]	753
Mupad [F(-1)]	754
Reduce [F]	754

Optimal result

Integrand size = 27, antiderivative size = 301

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x^5} dx$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{1-c^2x^2}}$$

$$- \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{4x^4} + \frac{c^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{8x^2}$$

$$+ \frac{c^4\sqrt{d-c^2dx^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{4\sqrt{1-c^2x^2}}$$

$$- \frac{ibc^4\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{8\sqrt{1-c^2x^2}} + \frac{ibc^4\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{8\sqrt{1-c^2x^2}}$$

output

```
-1/12*b*c*(-c^2*d*x^2+d)^(1/2)/x^3/(-c^2*x^2+1)^(1/2)+1/8*b*c^3*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)-1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^4+1/8*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^2+1/4*c^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-1/8*I*b*c^4*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+1/8*I*b*c^4*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```


Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx$$

$$= \frac{a(-2 + c^2 x^2) \sqrt{d - c^2 dx^2}}{8x^4} - \frac{1}{8} ac^4 \sqrt{d} \log(x) + \frac{1}{8} ac^4 \sqrt{d} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right)$$

$$+ \frac{bc^4 \sqrt{d - c^2 dx^2} \left(8 \cot\left(\frac{1}{2} \arcsin(cx)\right) + 6 \arcsin(cx) \csc^2\left(\frac{1}{2} \arcsin(cx)\right) - cx \csc^4\left(\frac{1}{2} \arcsin(cx)\right) - 3 \arcsin(cx)\right)}{8x^4}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^5,x]
```

output

```
(a*(-2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/(8*x^4) - (a*c^4*Sqrt[d]*Log[x])/8
+ (a*c^4*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/8 + (b*c^4*Sqrt[d -
c^2*d*x^2]*(8*Cot[ArcSin[c*x]/2] + 6*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - c
*x*Csc[ArcSin[c*x]/2]^4 - 3*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^4 - 24*ArcSin[c
*x]*Log[1 - E^(I*ArcSin[c*x])] + 24*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])]
- (24*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (24*I)*PolyLog[2, E^(I*ArcSin[c
*x])] - 6*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 + 3*ArcSin[c*x]*Sec[ArcSin[c*x]
/2]^4 - (16*Sin[ArcSin[c*x]/2]^4)/(c^3*x^3) + 8*Tan[ArcSin[c*x]/2]))/(192*
Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5196, 15, 5204, 15, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx$$

↓ 5196

$$\begin{aligned}
& -\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arcsin(cx)}{x^3\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int\frac{1}{x^4}dx}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4} \\
& \quad \downarrow 15 \\
& -\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arcsin(cx)}{x^3\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
& \quad \downarrow 5204 \\
& -\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int\frac{1}{x^2}dx-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
& \quad \downarrow 15 \\
& -\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc}{2x}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
& \quad \downarrow 5218 \\
& -\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int\frac{a+b\arcsin(cx)}{cx}d\arcsin(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc}{2x}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
& \quad \downarrow 3042 \\
& -\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int(a+b\arcsin(cx))\csc(\arcsin(cx))d\arcsin(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc}{2x}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
& \quad \downarrow 4671 \\
& -\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2(-b\int\log(1-e^{i\arcsin(cx)})d\arcsin(cx)+b\int\log(1+e^{i\arcsin(cx)})d\arcsin(cx)-2\operatorname{arctanh}(e^i)\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{4x^4}-\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
& \quad \downarrow 2715
\end{aligned}$$

$$\frac{c^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{2} c^2 (ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)}) \right)}{4\sqrt{1 - c^2 x^2}}$$

$$\frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{4x^4} - \frac{bc\sqrt{d - c^2 dx^2}}{12x^3 \sqrt{1 - c^2 x^2}}$$

↓ 2838

$$\frac{c^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{2} c^2 (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})) \right)}{4\sqrt{1 - c^2 x^2}}$$

$$\frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{4x^4} - \frac{bc\sqrt{d - c^2 dx^2}}{12x^3 \sqrt{1 - c^2 x^2}}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/x^5,x]`

output `-1/12*(b*c*Sqrt[d - c^2*d*x^2])/(x^3*Sqrt[1 - c^2*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(4*x^4) - (c^2*Sqrt[d - c^2*d*x^2]*(-1/2*(b*c)/x - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*x^2) + (c^2*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/2))/(4*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)(x_)]*((c_.) + (d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) \text{ ; FreeQ}\{c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 5196 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((f_.)(x_))^{(m_.)*\text{Sqrt}[(d_.) + (e_.)(x_)^2]}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+1))), x] + (-\text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] + \text{Simp}[(c^2/(f^2*(m+1)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{Int}[(f*x)^{(m+2)}*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x]) \text{ ; FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

rule 5204 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((f_.)(x_))^{(m_.)*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1))) \text{Int}[(f*x)^{(m+2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{ILtQ}[m, -1]$

rule 5218 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*x^{(m_.)}/\text{Sqrt}[(d_.) + (e_.)(x_)^2]}, x_Symbol] \rightarrow \text{Simp}[(1/c^{(m+1)})*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]] \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]^m, x], x, \text{ArcSin}[c*x]], x] \text{ ; FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.16

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{4dx^4} - \frac{ac^2(-c^2dx^2+d)^{\frac{3}{2}}}{8dx^2} + \frac{ac^4\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{8} - \frac{ac^4\sqrt{-c^2dx^2+d}}{8} + b\left(\frac{3c^4x^4\arcsin(cx)}{8}\right)$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{4dx^4} - \frac{ac^2(-c^2dx^2+d)^{\frac{3}{2}}}{8dx^2} + \frac{ac^4\sqrt{d}\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{8} - \frac{ac^4\sqrt{-c^2dx^2+d}}{8} + b\left(\frac{3c^4x^4\arcsin(cx)}{8}\right)$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^5,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & -1/4*a/d/x^4*(-c^2*d*x^2+d)^(3/2)-1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/8 \\ & *a*c^4*d^(1/2)*\ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/8*a*c^4*(-c^2* \\ & d*x^2+d)^(1/2)+b*(1/24*(3*c^4*x^4*\arcsin(c*x)-3*c^3*x^3*(-c^2*x^2+1)^(1/2) \\ & -9*c^2*x^2*\arcsin(c*x)+2*c*x*(-c^2*x^2+1)^(1/2)+6*\arcsin(c*x))*(-d*(c^2*x^ \\ & 2-1))^(1/2)/(c^2*x^2-1)/x^4+I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I \\ & *\arcsin(c*x)*\ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*\arcsin(c*x)*\ln(1-I*c*x-(-c^2 \\ & *x^2+1)^(1/2))+\operatorname{polylog}(2,-I*c*x-(-c^2*x^2+1)^(1/2))-\operatorname{polylog}(2,I*c*x+(-c^2* \\ & x^2+1)^(1/2)))*c^4/(8*c^2*x^2-8) \end{aligned}$$

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x^5} dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arcsin(cx)+a)}{x^5} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/x^5, x)`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)}(a + b \arcsin(cx))}{x^5} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))/x**5,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))/x**5, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx = \int \frac{\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{x^5} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="maxima")`

output `b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)/x^5, x) + 1/8*(c^4*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d)*c^4 - (-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^2) - 2*(-c^2*d*x^2 + d)^(3/2)/(d*x^4))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx = \int \frac{(a + b \arcsin(cx)) \sqrt{d - c^2 dx^2}}{x^5} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^5,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2))/x^5, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{x^5} dx$$

$$= \frac{\sqrt{d} \left(\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a + 8 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^5} dx \right) b x^4 - \log \left(\tan \left(\frac{\arcsin(cx)}{2} \right) \right) a c^4 x^4 \right)}{8x^4}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))/x^5,x)`

output `(sqrt(d)*(sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)*a
+ 8*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**5,x)*b*x**4 - log(tan(asin(c
*x)/2))*a*c**4*x**4))/(8*x**4)`

3.68 $\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	755
Mathematica [A] (verified)	756
Rubi [A] (verified)	756
Maple [C] (verified)	761
Fricas [F]	762
Sympy [F(-1)]	762
Maxima [F]	762
Giac [A] (verification not implemented)	763
Mupad [F(-1)]	764
Reduce [F]	764

Optimal result

Integrand size = 27, antiderivative size = 340

$$\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{3bdx^2\sqrt{d - c^2dx^2}}{256c^3\sqrt{1 - c^2x^2}} + \frac{bdx^4\sqrt{d - c^2dx^2}}{256c\sqrt{1 - c^2x^2}} - \frac{bcdx^6\sqrt{d - c^2dx^2}}{32\sqrt{1 - c^2x^2}} + \frac{bc^3dx^8\sqrt{d - c^2dx^2}}{64\sqrt{1 - c^2x^2}} - \frac{3dx\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{128c^4} - \frac{dx^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{64c^2} + \frac{1}{16}dx^5\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{1}{8}x^5(d - c^2dx^2)^{3/2}(a + b \arcsin(cx)) + \frac{3d\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{256bc^5\sqrt{1 - c^2x^2}}$$

output

```
3/256*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/256*b*d*x^4*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/32*b*c*d*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/64*b*c^3*d*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/128*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4-1/64*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2+1/16*d*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+1/8*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))+3/256*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c^5/(-c^2*x^2+1)^(1/2)
```


Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.57

$$\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{d\sqrt{d - c^2 dx^2}(3a^2 + b^2 c^2 x^2(3 + c^2 x^2 - 8c^4 x^4 + 4c^6 x^6) - 2abcx\sqrt{1 - c^2 x^2}(3 + 2c^2 x^2 - 8c^4 x^4 + 4c^6 x^6)) + b \arcsin(cx)}{(256b^2 c^5 \sqrt{1 - c^2 x^2})}$$

input `Integrate[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `(d*Sqrt[d - c^2*d*x^2]*(3*a^2 + b^2*c^2*x^2*(3 + c^2*x^2 - 8*c^4*x^4 + 4*c^6*x^6) - 2*a*b*c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6) - 2*b*(-3*a + b*c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6))*ArcSin[c*x] + 3*b^2*ArcSin[c*x]^2))/(256*b*c^5*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5202, 244, 2009, 5198, 15, 5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5202$$

$$\frac{3}{8}d \int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2 dx^2} \int x^5 (1 - c^2 x^2) dx}{8\sqrt{1 - c^2 x^2}} + \frac{1}{8}x^5(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))$$

$$\downarrow 244$$

$$\begin{aligned}
& \frac{3}{8}d \int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd \sqrt{d - c^2 dx^2} \int (x^5 - c^2 x^7) dx}{8\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{2009} \\
& \frac{3}{8}d \int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx + \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \\
& \quad \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5198} \\
& \frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^4 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{6\sqrt{1 - c^2 x^2}} - \frac{bc \sqrt{d - c^2 dx^2} \int x^5 dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \\
& \quad \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{15} \\
& \frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^4 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}} \right) + \\
& \quad \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5210} \\
& \frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \int \frac{x^2 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{4c^2} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{15}
\end{aligned}$$

$$\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \right)$$

↓ 5210

$$\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \right)$$

↓ 15

$$\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} - \frac{x \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \right)$$

↓ 5152

$$\frac{1}{8}x^5(d - c^2dx^2)^{3/2}(a + b \arcsin(cx)) + \frac{3}{8}d \left(\frac{1}{6}x^5\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{\sqrt{d - c^2dx^2} \left(-\frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} + \frac{3 \left(\frac{(a+b \arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} \right)}{6\sqrt{1 - c^2x^2}} \right) + \frac{bcd \left(\frac{x^6}{6} - \frac{c^2x^8}{8} \right) \sqrt{d - c^2dx^2}}{8\sqrt{1 - c^2x^2}}$$

input `Int[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `-1/8*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^6/6 - (c^2*x^8)/8))/Sqrt[1 - c^2*x^2] + (x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/8 + (3*d*(-1/36*(b*c*x^6*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/6 + (Sqrt[d - c^2*d*x^2]*((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2)))/(6*Sqrt[1 - c^2*x^2])))/8`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.26

method	result
default	$-\frac{ax^3(-c^2dx^2+d)^{\frac{5}{2}}}{8c^2d} - \frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{16c^4d} + \frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{64c^4} + \frac{3adx\sqrt{-c^2dx^2+d}}{128c^4} + \frac{3ad^2 \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{128c^4\sqrt{c^2d}} + b\left(\right)$
parts	$-\frac{ax^3(-c^2dx^2+d)^{\frac{5}{2}}}{8c^2d} - \frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{16c^4d} + \frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{64c^4} + \frac{3adx\sqrt{-c^2dx^2+d}}{128c^4} + \frac{3ad^2 \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{128c^4\sqrt{c^2d}} + b\left(\right)$

input `int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & -1/8*a*x^3*(-c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a/c^4*x*(-c^2*d*x^2+d)^(5/2)/d+ \\ & 1/64*a/c^4*x*(-c^2*d*x^2+d)^(3/2)+3/128*a/c^4*d*x*(-c^2*d*x^2+d)^(1/2)+3/1 \\ & 28*a/c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b* \\ & (-3/256*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/(c^2*x^2-1)*arcsin(c \\ & *x)^2*d-1/16384*(-d*(c^2*x^2-1))^(1/2)*(-128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+ \\ & 128*c^9*x^9+256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7-160*I*(-c^2*x^2+1) \\ &)^(1/2)*x^4*c^4+272*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3-I*(\\ & -c^2*x^2+1)^(1/2)+8*c*x)*(8*arcsin(c*x)+I)*d/c^5/(c^2*x^2-1)+1/1024*(-d*(c \\ & ^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+ \\ & 1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))* \\ & d/c^5/(c^2*x^2-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^ \\ & 4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(\\ & 1/2)+4*c*x)*(-I+4*arcsin(c*x))*d/c^5/(c^2*x^2-1)-1/16384*(-d*(c^2*x^2-1))^(\\ & 1/2)*(128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9-256*I*(-c^2*x^2+1)^(1/ \\ & 2)*x^6*c^6-320*c^7*x^7+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5-32*I*(\\ & -c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+8*c*x)*(-I+8*arc \\ & sin(c*x))*d/c^5/(c^2*x^2-1) \end{aligned}$$

Fricas [F]

$$\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)x^4 dx$$

input `integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^6 - a*d*x^4 + (b*c^2*d*x^6 - b*d*x^4)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)`

Sympy [F(-1)]

Timed out.

$$\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**4*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)x^4 dx$$

input `integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
b*sqrt(d)*integrate(-(c^2*d*x^6 - d*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) - 1/128*(16*(-c^2*d*x^2 + d)^(5/2)*x^3/(c^2*d) - 2*(-c^2*d*x^2 + d)^(3/2)*x/c^4 + 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^4*d) - 3*sqrt(-c^2*d*x^2 + d)*d*x/c^4 - 3*d^(3/2)*arcsin(c*x)/c^5)*a
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04

$$\int x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = -\frac{1}{8} \sqrt{-c^2 dx^2 + d} ac^2 dx^7 + \frac{3}{16} \sqrt{-c^2 dx^2 + d} adx^5 - \frac{\sqrt{-c^2 dx^2 + d} adx^3}{64 c^2} - \frac{3 \sqrt{-c^2 dx^2 + d} adx}{128 c^4} - \frac{3 ad^2 \log(|-c\sqrt{-d}x + \sqrt{c^2 x^2 - 1}\sqrt{-d}|)}{128 c^5 \sqrt{-d}} - \frac{1024 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} bd^{\frac{3}{2}} x \arcsin(cx) + 1536 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bd^{\frac{3}{2}} x \arcsin(cx) - 128 (-c^2 x^2 + 1)^{3/2} b d^{3/2} x \arcsin(cx) - 128 (c^2 x^2 - 1)^4 b d^{3/2} / c - 192 \sqrt{-c^2 x^2 + 1} b d^{3/2} x \arcsin(cx) - 256 (c^2 x^2 - 1)^3 b d^{3/2} / c - 32 (c^2 x^2 - 1)^2 b d^{3/2} / c - 96 b d^{3/2} a \arcsin(cx)^2 / c + 96 (c^2 x^2 - 1) b d^{3/2} / c + 15 b d^{3/2} / c}{c^4}$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
-1/8*sqrt(-c^2*d*x^2 + d)*a*c^2*d*x^7 + 3/16*sqrt(-c^2*d*x^2 + d)*a*d*x^5 - 1/64*sqrt(-c^2*d*x^2 + d)*a*d*x^3/c^2 - 3/128*sqrt(-c^2*d*x^2 + d)*a*d*x/c^4 - 3/128*a*d^2*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^5*sqrt(-d)) - 1/8192*(1024*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^(3/2)*x*arcsin(c*x) + 1536*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^(3/2)*x*arcsin(c*x) - 128*(-c^2*x^2 + 1)^(3/2)*b*d^(3/2)*x*arcsin(c*x) - 128*(c^2*x^2 - 1)^4*b*d^(3/2)/c - 192*sqrt(-c^2*x^2 + 1)*b*d^(3/2)*x*arcsin(c*x) - 256*(c^2*x^2 - 1)^3*b*d^(3/2)/c - 32*(c^2*x^2 - 1)^2*b*d^(3/2)/c - 96*b*d^(3/2)*a*arcsin(c*x)^2/c + 96*(c^2*x^2 - 1)*b*d^(3/2)/c + 15*b*d^(3/2)/c)/c^4
```


Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{3/2} dx$$

input `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d (3 \operatorname{asin}(cx) a - 16 \sqrt{-c^2 x^2 + 1} a c^7 x^7 + 24 \sqrt{-c^2 x^2 + 1} a c^5 x^5 - 2 \sqrt{-c^2 x^2 + 1} a c^3 x^3 + b \arcsin(cx))}{(128 c^5)}$$

input `int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d*(3*asin(c*x)*a - 16*sqrt(-c**2*x**2 + 1)*a*c**7*x**7 + 24*sqrt(-c**2*x**2 + 1)*a*c**5*x**5 - 2*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt(-c**2*x**2 + 1)*a*c*x - 128*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**6,x)*b*c**7 + 128*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*b*c**5))/(128*c**5)`

3.69 $\int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	765
Mathematica [A] (verified)	766
Rubi [A] (verified)	766
Maple [C] (verified)	769
Fricas [F]	770
Sympy [F(-1)]	771
Maxima [F]	771
Giac [A] (verification not implemented)	771
Mupad [F(-1)]	772
Reduce [F]	772

Optimal result

Integrand size = 27, antiderivative size = 265

$$\int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{bdx^2\sqrt{d - c^2dx^2}}{32c\sqrt{1 - c^2x^2}} - \frac{7bcdx^4\sqrt{d - c^2dx^2}}{96\sqrt{1 - c^2x^2}} + \frac{bc^3dx^6\sqrt{d - c^2dx^2}}{36\sqrt{1 - c^2x^2}} - \frac{dx\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{16c^2} + \frac{1}{8}dx^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{1}{6}x^3(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{d\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{32bc^3\sqrt{1 - c^2x^2}}$$

output

```
1/32*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-7/96*b*c*d*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/36*b*c^3*d*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2+1/8*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))+1/32*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.64

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{d\sqrt{d - c^2 dx^2}(9a^2 + b^2 c^2 x^2(9 - 21c^2 x^2 + 8c^4 x^4) - 6abcx\sqrt{1 - c^2 x^2}(3 - 14c^2 x^2 + 8c^4 x^4) + 9b^2 \arcsin(cx))}{288bc^3\sqrt{1 - c^2 x^2}}$$

input `Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`output `(d*Sqrt[d - c^2*d*x^2]*(9*a^2 + b^2*c^2*x^2*(9 - 21*c^2*x^2 + 8*c^4*x^4) - 6*a*b*c*x*Sqrt[1 - c^2*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) + 6*b*(3*a + b*c*x*Sqrt[1 - c^2*x^2]*(-3 + 14*c^2*x^2 - 8*c^4*x^4))*ArcSin[c*x] + 9*b^2*ArcSin[c*x]^2))/(288*b*c^3*Sqrt[1 - c^2*x^2])`**Rubi [A] (verified)**Time = 0.92 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5202, 244, 2009, 5198, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5202}$$

$$\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2 dx^2} \int x^3(1 - c^2 x^2) dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))$$

$$\downarrow \text{244}$$

$$\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2 dx^2} \int (x^3 - c^2 x^5) dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))$$

$$\begin{aligned}
 & \downarrow \text{2009} \\
 & \frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx + \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \\
 & \quad \frac{bcd \left(\frac{x^4}{4} - \frac{c^2 x^6}{6} \right) \sqrt{d - c^2 dx^2}}{6\sqrt{1 - c^2 x^2}} \\
 & \downarrow \text{5198} \\
 & \frac{1}{2}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^3 dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \\
 & \quad \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^4}{4} - \frac{c^2 x^6}{6} \right) \sqrt{d - c^2 dx^2}}{6\sqrt{1 - c^2 x^2}} \\
 & \downarrow \text{15} \\
 & \frac{1}{2}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^4 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} \right) + \\
 & \quad \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^4}{4} - \frac{c^2 x^6}{6} \right) \sqrt{d - c^2 dx^2}}{6\sqrt{1 - c^2 x^2}} \\
 & \downarrow \text{5210} \\
 & \frac{1}{2}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^4 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} \right) + \\
 & \quad \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^4}{4} - \frac{c^2 x^6}{6} \right) \sqrt{d - c^2 dx^2}}{6\sqrt{1 - c^2 x^2}} \\
 & \downarrow \text{15} \\
 & \frac{1}{2}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^4 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} \right) + \\
 & \quad \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^4}{4} - \frac{c^2 x^6}{6} \right) \sqrt{d - c^2 dx^2}}{6\sqrt{1 - c^2 x^2}}
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow \text{5152} \\
 & \frac{1}{6}x^3(d - c^2dx^2)^{3/2}(a + b\arcsin(cx)) + \\
 & \frac{1}{2}d \left(\frac{1}{4}x^3\sqrt{d - c^2dx^2}(a + b\arcsin(cx)) + \frac{\sqrt{d - c^2dx^2} \left(\frac{(a+b\arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4\sqrt{1-c^2x^2}} - \frac{bcx^4\sqrt{d - c^2dx^2}}{16\sqrt{1-c^2x^2}} \right. \\
 & \left. + \frac{bcd \left(\frac{x^4}{4} - \frac{c^2x^6}{6} \right) \sqrt{d - c^2dx^2}}{6\sqrt{1-c^2x^2}} \right)
 \end{aligned}$$

input `Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `-1/6*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^4/4 - (c^2*x^6)/6))/Sqrt[1 - c^2*x^2] + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/6 + (d*(-1/16*(b*c*x^4*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/4 + (Sqrt[d - c^2*d*x^2]*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*Sqrt[1 - c^2*x^2])))`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.57

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6c^2d} + \frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{24c^2} + \frac{adx\sqrt{-c^2dx^2+d}}{16c^2} + \frac{a d^2 \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{16c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2}}{32c^3(c^2x^2-1)}\right)$
parts	$-\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6c^2d} + \frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{24c^2} + \frac{adx\sqrt{-c^2dx^2+d}}{16c^2} + \frac{a d^2 \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{16c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2}}{32c^3(c^2x^2-1)}\right)$

input `int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `-1/6*a*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16*a/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^2*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*(I+6*arcsin(c*x))*d/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))*d/c^3/(c^2*x^2-1)+1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(11*I+24*arcsin(c*x))*cos(5*arcsin(c*x))*d/c^3/(c^2*x^2-1)-1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(7*I+48*arcsin(c*x))*sin(5*arcsin(c*x))*d/c^3/(c^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(8*arcsin(c*x)+I)*cos(3*arcsin(c*x))*d/c^3/(c^2*x^2-1)+3/512*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*sin(3*arcsin(c*x))*d/c^3/(c^2*x^2-1))`

Fricas [F]

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^4 - a*d*x^2 + (b*c^2*d*x^4 - b*d*x^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)`

Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a) x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate(-(c^2*d*x^4 - d*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + 1/48*a*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3)`

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.04

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = -\frac{1}{6} \sqrt{-c^2 dx^2 + d} a c^2 dx^5 + \frac{7}{24} \sqrt{-c^2 dx^2 + d} a d x^3 - \frac{\sqrt{-c^2 dx^2 + d} a d x}{16 c^2} - \frac{a d^2 \log(|-c \sqrt{-d} x + \sqrt{c^2 x^2 - 1} \sqrt{-d}|)}{16 c^3 \sqrt{-d}} - \frac{384 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b d^{\frac{3}{2}} x \arcsin(cx) - 96 (-c^2 x^2 + 1)^{\frac{3}{2}} b d^{\frac{3}{2}} x \arcsin(cx) - 144 \sqrt{-c^2 x^2 + 1} b d^{\frac{3}{2}} x a}{2304 c^2}$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output
$$\begin{aligned} & -1/6*\sqrt{-c^2*d*x^2 + d}*a*c^2*d*x^5 + 7/24*\sqrt{-c^2*d*x^2 + d}*a*d*x^3 \\ & - 1/16*\sqrt{-c^2*d*x^2 + d}*a*d*x/c^2 - 1/16*a*d^2*\log(\text{abs}(-c*\sqrt{-d}*x + \\ & \sqrt{c^2*x^2 - 1}*\sqrt{-d}))/c^3*\sqrt{-d} - 1/2304*(384*(c^2*x^2 - 1)^2 \\ & *\sqrt{-c^2*x^2 + 1}*b*d^{(3/2)}*x*\arcsin(c*x) - 96*(-c^2*x^2 + 1)^{(3/2)}*b*d^{(3/2)} \\ & *x*\arcsin(c*x) - 144*\sqrt{-c^2*x^2 + 1}*b*d^{(3/2)}*x*\arcsin(c*x) - 64* \\ & (c^2*x^2 - 1)^3*b*d^{(3/2)}/c - 24*(c^2*x^2 - 1)^2*b*d^{(3/2)}/c - 72*b*d^{(3/2)} \\ & *\arcsin(c*x)^2/c + 72*(c^2*x^2 - 1)*b*d^{(3/2)}/c + 25*b*d^{(3/2)}/c/c^2 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^2 (a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2} dx$$

input `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d}d(3a \arcsin(cx) a - 8\sqrt{-c^2 x^2 + 1} a c^5 x^5 + 14\sqrt{-c^2 x^2 + 1} a c^3 x^3 - 3\sqrt{-c^2 x^2 + 1} a c x - 48c^3)}{48c^3}$$

input `int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output
$$\frac{(\sqrt{d}*d*(3*\arcsin(c*x)*a - 8*\sqrt{-c**2*x**2 + 1}*a*c**5*x**5 + 14*\sqrt{-c**2*x**2 + 1}*a*c**3*x**3 - 3*\sqrt{-c**2*x**2 + 1}*a*c*x - 48*\int(\sqrt{-c**2*x**2 + 1}*\arcsin(c*x)*x**4,x)*b*c**5 + 48*\int(\sqrt{-c**2*x**2 + 1}*\arcsin(c*x)*x**2,x)*b*c**3))/(48*c**3)}$$

3.70 $\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$

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Giac [F(-2)]	779
Mupad [F(-1)]	779
Reduce [F]	779

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = -\frac{3bcdx^2 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} + \frac{bd(1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2}}{16c} + \frac{3}{8} dx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{1}{4} x (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{3d\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{16bc\sqrt{1 - c^2 x^2}}$$

output

```
-3/16*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/16*b*d*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))+3/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{24bd\sqrt{d - c^2 dx^2} \arcsin(cx)^2 - 48ad^{3/2}\sqrt{1 - c^2 x^2} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d(-1 + c^2 x^2)}}\right) + d\sqrt{d - c^2 dx^2}}{1}$$

input

```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]
```

output

```
(24*b*d*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^2 - 48*a*d^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + d*Sqrt[d - c^2*d*x^2]*(16*a*c*x*(5 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + 16*b*Cos[2*ArcSin[c*x]] + b*Cos[4*ArcSin[c*x]]) + 4*b*d*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]*(8*Sin[2*ArcSin[c*x]] + Sin[4*ArcSin[c*x]])/(128*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5158}$$

$$\frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) dx}{4\sqrt{1 - c^2 x^2}} +$$

$$\frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))$$

$$\downarrow \text{244}$$

$$\frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd \sqrt{d - c^2 dx^2} \int (x - c^2 x^3) dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))$$

↓ 2009

$$\frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^2}{2} - \frac{c^2 x^4}{4} \right) \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

↓ 5156

$$\frac{3}{4}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} - \frac{bc \sqrt{d - c^2 dx^2} \int x dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2}x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^2}{2} - \frac{c^2 x^4}{4} \right) \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

↓ 15

$$\frac{3}{4}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2}x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} \right) + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^2}{2} - \frac{c^2 x^4}{4} \right) \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

↓ 5152

$$\frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4}d \left(\frac{1}{2}x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{4bc\sqrt{1 - c^2 x^2}} - \frac{bcx^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} \right) - \frac{bcd \left(\frac{x^2}{2} - \frac{c^2 x^4}{4} \right) \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

input `Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output

$$-1/4*(b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/\text{Sqrt}[1 - c^2*x^2] + (x*(d - c^2*d*x^2)^{(3/2)}*(a + b*\text{ArcSin}[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*\text{Sqrt}[d - c^2*d*x^2])/ \text{Sqrt}[1 - c^2*x^2] + (x*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x]))/2 + (\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(4*b*c*\text{Sqrt}[1 - c^2*x^2])))/4$$
Defintions of rubi rules used

rule 15

$$\text{Int}[(a_*)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 244

$$\text{Int}[(c_*)(x_)^{(m_.)}*((a_) + (b_*)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{Expand}[\text{Integrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5152

$$\text{Int}[(a_*) + \text{ArcSin}[(c_*)(x_)]*(b_*)^{(n_.)}/\text{Sqrt}[(d_) + (e_*)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5156

$$\text{Int}[(a_*) + \text{ArcSin}[(c_*)(x_)]*(b_*)^{(n_.)}*\text{Sqrt}[(d_) + (e_*)(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^{n/2}), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{ Int}[(a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{ Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$$

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x],
x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.59

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4} + \frac{3adx\sqrt{-c^2dx^2+d}}{8} + \frac{3ad^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}} + b\left(-\frac{3\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2d}{16(c^2x^2-1)c} - \dots\right)$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4} + \frac{3adx\sqrt{-c^2dx^2+d}}{8} + \frac{3ad^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}} + b\left(-\frac{3\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2d}{16(c^2x^2-1)c} - \dots\right)$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/16*(-d*(c^2*x
^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arcsin(c*x)^2*d-1/256*(-d*(c
^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+
1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))*
d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^
2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))*d/(c^2*x^2-1)/c
-1/256*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(17*I+2
8*arcsin(c*x))*cos(3*arcsin(c*x))*d/(c^2*x^2-1)/c+3/256*(-d*(c^2*x^2-1))^(
1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(5*I+12*arcsin(c*x))*sin(3*arcsi
n(c*x))*d/(c^2*x^2-1)/c
```

Fricas [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a) dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)`

Sympy [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx)) dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x)), x)`

Maxima [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a) dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate(-(c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*arcsin(c*x)/c)*a`

Giac [F(-2)]

Exception generated.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2} dx$$

input `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)`

output `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d (3a \sin(cx) a - 2\sqrt{-c^2 x^2 + 1} a c^3 x^3 + 5\sqrt{-c^2 x^2 + 1} a c x - 8 \int \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx)}{8c}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output

```
(sqrt(d)*d*(3*asin(c*x)*a - 2*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 + 5*sqrt(-c**2*x**2 + 1)*a*c*x - 8*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*b*c**3 + 8*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*b*c))/(8*c)
```

3.71 $\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx$

Optimal result	781
Mathematica [A] (verified)	782
Rubi [A] (verified)	782
Maple [C] (verified)	785
Fricas [F]	786
Sympy [F]	786
Maxima [F]	786
Giac [F(-2)]	787
Mupad [F(-1)]	787
Reduce [F]	787

Optimal result

Integrand size = 27, antiderivative size = 185

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \frac{bc^3 dx^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} - \frac{3}{2} c^2 dx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} - \frac{3cd\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{4b\sqrt{1 - c^2 x^2}} + \frac{bcd\sqrt{d - c^2 dx^2} \log(x)}{\sqrt{1 - c^2 x^2}}$$

output

```
1/4*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/2*c^2*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x-3/4*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/(-c^2*x^2+1)^(1/2)+b*c*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.20

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \left(-\frac{ad}{x} - \frac{1}{2} ac^2 dx \right) \sqrt{-d(-1 + c^2 x^2)}$$

$$+ \frac{3}{2} acd^{3/2} \arctan \left(\frac{cx \sqrt{-d(-1 + c^2 x^2)}}{\sqrt{d}(-1 + c^2 x^2)} \right)$$

$$- \frac{bcd \sqrt{d(1 - c^2 x^2)} \left(\frac{2\sqrt{1 - c^2 x^2} \arcsin(cx)}{cx} + \arcsin(cx)^2 - 2 \log(cx) \right)}{2\sqrt{1 - c^2 x^2}}$$

$$- \frac{bcd \sqrt{d(1 - c^2 x^2)} (\cos(2 \arcsin(cx)) + 2 \arcsin(cx) (\arcsin(cx) + \sin(2 \arcsin(cx))))}{8\sqrt{1 - c^2 x^2}}$$

input `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^2,x]`

output `(-((a*d)/x) - (a*c^2*d*x)/2)*Sqrt[-(d*(-1 + c^2*x^2))] + (3*a*c*d^(3/2)*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/2 - (b*c*d*Sqrt[d*(1 - c^2*x^2)]*((2*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*x) + ArcSin[c*x]^2 - 2*Log[c*x]))/(2*Sqrt[1 - c^2*x^2]) - (b*c*d*Sqrt[d*(1 - c^2*x^2)]*(Cos[2*ArcSin[c*x]] + 2*ArcSin[c*x]*(ArcSin[c*x] + Sin[2*ArcSin[c*x]])))/(8*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5200, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx$$

↓ 5200

$$\begin{aligned}
& -3c^2d \int \sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx + \frac{bcd\sqrt{d - c^2dx^2} \int \frac{1-c^2x^2}{x} dx}{\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} \\
& \quad \downarrow \text{244} \\
& -3c^2d \int \sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx + \frac{bcd\sqrt{d - c^2dx^2} \int (\frac{1}{x} - c^2x) dx}{\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} \\
& \quad \downarrow \text{2009} \\
& -3c^2d \int \sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx - \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} + \\
& \quad \frac{bcd\sqrt{d - c^2dx^2} \left(\log(x) - \frac{c^2x^2}{2} \right)}{\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{5156} \\
& -3c^2d \left(\frac{\sqrt{d - c^2dx^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1 - c^2x^2}} - \frac{bc\sqrt{d - c^2dx^2} \int x dx}{2\sqrt{1 - c^2x^2}} + \frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) \right) - \\
& \quad \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} + \frac{bcd\sqrt{d - c^2dx^2} \left(\log(x) - \frac{c^2x^2}{2} \right)}{\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{15} \\
& -3c^2d \left(\frac{\sqrt{d - c^2dx^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1 - c^2x^2}} + \frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) - \frac{bcx^2\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \right) - \\
& \quad \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} + \frac{bcd\sqrt{d - c^2dx^2} \left(\log(x) - \frac{c^2x^2}{2} \right)}{\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{5152} \\
& -3c^2d \left(\frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{4bc\sqrt{1 - c^2x^2}} - \frac{bcx^2\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \right) - \\
& \quad \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} + \frac{bcd\sqrt{d - c^2dx^2} \left(\log(x) - \frac{c^2x^2}{2} \right)}{\sqrt{1 - c^2x^2}}
\end{aligned}$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^2,x]`

output `-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x) - 3*c^2*d*(-1/4*(b*c*x^2*
Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Sin[c*x]))/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c*Sqrt[1 -
c^2*x^2])) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/2*(c^2*x^2) + Log[x])/Sqrt[1
- c^2*x^2]`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]`

rule 5156 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
) * Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n/(f*(m + 1)), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.34

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{dx} - ac^2x(-c^2dx^2+d)^{\frac{3}{2}} - \frac{3ac^2dx\sqrt{-c^2dx^2+d}}{2} - \frac{3ac^2d^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + \frac{b\sqrt{-d}(c^2x^2-d)^{\frac{3}{2}}}{2\sqrt{c^2d}}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{dx} - ac^2x(-c^2dx^2+d)^{\frac{3}{2}} - \frac{3ac^2dx\sqrt{-c^2dx^2+d}}{2} - \frac{3ac^2d^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + \frac{b\sqrt{-d}(c^2x^2-d)^{\frac{3}{2}}}{2\sqrt{c^2d}}$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output

```
-a/d/x*(-c^2*d*x^2+d)^(5/2)-a*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d*x*(-c
^2*d*x^2+d)^(1/2)-3/2*a*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))+1/8*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-
1)/x*(4*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-2*c^3*x^3+6*arcsin(c*x)^2*c
*x+8*I*arcsin(c*x)*x*c-8*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*x*c+8*arcsin(c
*x)*(-c^2*x^2+1)^(1/2)+c*x)*d
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))}{x^2} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**2,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))/x**2, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^2, x) - 1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*x^2 + d)^(3/2)/x)*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^2} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^2,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^2, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx = \frac{\sqrt{d} d (-a \sin(cx)^2 b c x - 3 a \sin(cx) a c x - \sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2 \dots)}{\dots}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^2,x)`

output

```
(sqrt(d)*d*( - asin(c*x)**2*b*c*x - 3*asin(c*x)*a*c*x - sqrt( - c**2*x**2
+ 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 2*int(asin(c*x)/(sqrt( - c
**2*x**2 + 1)*x**2),x)*b*x - 2*int(sqrt( - c**2*x**2 + 1)*asin(c*x),x)*b*c
**2*x))/(2*x)
```

3.72 $\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx$

Optimal result	789
Mathematica [A] (verified)	790
Rubi [A] (verified)	790
Maple [C] (verified)	793
Fricas [F]	794
Sympy [F]	794
Maxima [F]	794
Giac [F(-2)]	795
Mupad [F(-1)]	795
Reduce [F]	796

Optimal result

Integrand size = 27, antiderivative size = 191

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = -\frac{bcd\sqrt{d - c^2 dx^2}}{6x^2\sqrt{1 - c^2 x^2}} + \frac{c^2 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x^3} + \frac{c^3 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2b\sqrt{1 - c^2 x^2}} - \frac{4bc^3 d \sqrt{d - c^2 dx^2} \log(x)}{3\sqrt{1 - c^2 x^2}}$$

output

```
-1/6*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)+c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^3+1/2*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/(-c^2*x^2+1)^(1/2)-4/3*b*c^3*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \frac{bd(-1 + 4c^2 x^2) \sqrt{d - c^2 dx^2} \arcsin(cx)}{3x^3} + \frac{bc^3 d \sqrt{d - c^2 dx^2} \arcsin(cx)^2}{2\sqrt{1 - c^2 x^2}} - ac^3 d^{3/2} \arctan\left(\frac{cx \sqrt{d - c^2 dx^2}}{\sqrt{d}(-1 + c^2 x^2)}\right) - \frac{d \sqrt{d - c^2 dx^2} (bcx + 2a(1 - 4c^2 x^2) \sqrt{1 - c^2 x^2} + 8bc^3 x^3 \log(cx))}{6x^3 \sqrt{1 - c^2 x^2}}$$

input `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^4,x]`

output `(b*d*(-1 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2]*ArcSin[c*x])/(3*x^3) + (b*c^3*d*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^2)/(2*Sqrt[1 - c^2*x^2]) - a*c^3*d^(3/2)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (d*Sqrt[d - c^2*d*x^2]*(b*c*x + 2*a*(1 - 4*c^2*x^2)*Sqrt[1 - c^2*x^2] + 8*b*c^3*x^3*Log[c*x]))/(6*x^3*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5200, 244, 2009, 5196, 14, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx$$

↓ 5200

$$c^2(-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^2} dx + \frac{bcd \sqrt{d - c^2 dx^2} \int \frac{1 - c^2 x^2}{x^3} dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x^3}$$

$$\begin{aligned}
& \downarrow 244 \\
& c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x^2} dx + \frac{bcd\sqrt{d-c^2dx^2} \int \left(\frac{1}{x^3} - \frac{c^2}{x}\right) dx}{3\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3x^3} \\
& \downarrow 2009 \\
& c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x^2} dx - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3x^3} + \\
& \quad \frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x)) - \frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}} \\
& \downarrow 5196 \\
& c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{bc\sqrt{d-c^2dx^2} \int \frac{1}{x} dx}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3x^3} + \frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x)) - \frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}} \\
& \downarrow 14 \\
& c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} + \frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3x^3} + \frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x)) - \frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}} \\
& \downarrow 5152 \\
& c^2(-d) \left(-\frac{c\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2b\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} + \frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3x^3} + \frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x)) - \frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}}
\end{aligned}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^4,x]
```

output

$$-1/3*((d - c^2*d*x^2)^{(3/2)}*(a + b*\text{ArcSin}[c*x]))/x^3 + (b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(-1/2*1/x^2 - c^2*\text{Log}[x]))/(3*\text{Sqrt}[1 - c^2*x^2]) - c^2*d*(-(\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x]))/x) - (c*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(2*b*\text{Sqrt}[1 - c^2*x^2]) + (b*c*\text{Sqrt}[d - c^2*d*x^2]*\text{Log}[x])/\text{Sqrt}[1 - c^2*x^2]$$
Defintions of rubi rules used

rule 14

$$\text{Int}[(a_)/(x_), x_Symbol] \rightarrow \text{Simp}[a*\text{Log}[x], x] \text{ ; FreeQ}[a, x]$$

rule 244

$$\text{Int}[(c_)*(x_)]^{(m_)}*((a_)+(b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5152

$$\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}/\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5196

$$\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*((f_)*(x_)]^{(m_)}*\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+1))), x] + (-\text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{ Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] + \text{Simp}[(c^2/(f^2*(m+1)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{ Int}[(f*x)^{(m+2)}*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]$$

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n/(f*(m + 1)), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.45

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{3dx^3} + \frac{2ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx} + \frac{2ac^4x(-c^2dx^2+d)^{\frac{3}{2}}}{3} + ac^4dx\sqrt{-c^2dx^2+d} + \frac{ac^4d^2 \arctan\left(\frac{\sqrt{c^2d}}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{3dx^3} + \frac{2ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx} + \frac{2ac^4x(-c^2dx^2+d)^{\frac{3}{2}}}{3} + ac^4dx\sqrt{-c^2dx^2+d} + \frac{ac^4d^2 \arctan\left(\frac{\sqrt{c^2d}}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2/3*a
*c^4*x*(-c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a*c^4*d^2/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d*(c^2*x^2-1
))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x^3*(3*arcsin(c*x)^2*c^3*x^3+8*I*a
rcsin(c*x)*x^3*c^3-8*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*x^3*c^3+8*arcsin(c
*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)-c*x)*d
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))}{x^4} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**4,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))/x**4, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output

```
-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^4, x) + 1/3*(3*sqrt(-c^2*d*x^2 + d)*c
^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*x^2 + d)^(3/2)*c^2/x - (-c^
2*d*x^2 + d)^(5/2)/(d*x^3))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^4} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^4, x)
```


Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx = \frac{\sqrt{d} d (3 \arcsin(cx)^2 b c^3 x^3 + 6 \arcsin(cx) a c^3 x^3 + 8 \sqrt{-c^2 x^2 + 1} a c^2 x^2)}{x^4}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^4,x)`

output `(sqrt(d)*d*(3*asin(c*x)**2*b*c**3*x**3 + 6*asin(c*x)*a*c**3*x**3 + 8*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)*a - 6*int(asin(c*x)/sqrt(-c**2*x**2 + 1)*x**2),x)*b*c**2*x**3 + 6*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**4,x)*b*x**3)/(6*x**3)`

3.73 $\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx$

Optimal result	797
Mathematica [A] (verified)	797
Rubi [A] (verified)	798
Maple [C] (verified)	799
Fricas [A] (verification not implemented)	800
Sympy [F]	801
Maxima [A] (verification not implemented)	801
Giac [F(-2)]	802
Mupad [F(-1)]	802
Reduce [F]	803

Optimal result

Integrand size = 27, antiderivative size = 154

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = -\frac{bcd\sqrt{d - c^2 dx^2}}{20x^4\sqrt{1 - c^2 x^2}} + \frac{bc^3 d\sqrt{d - c^2 dx^2}}{5x^2\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5dx^5} + \frac{bc^5 d\sqrt{d - c^2 dx^2} \log(x)}{5\sqrt{1 - c^2 x^2}}$$

output

```
-1/20*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+1/5*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^5+1/5*b*c^5*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = \frac{d\sqrt{d - c^2 dx^2} (-3bcx + 12bc^3 x^3 - 25bc^5 x^5 - 12a\sqrt{1 - c^2 x^2} + 24a^2)}{x^6}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^6,x]
```

output

```
(d*Sqrt[d - c^2*d*x^2]*(-3*b*c*x + 12*b*c^3*x^3 - 25*b*c^5*x^5 - 12*a*Sqrt[1 - c^2*x^2] + 24*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 12*a*c^4*x^4*Sqrt[1 - c^2*x^2] - 12*b*(1 - c^2*x^2)^(5/2)*ArcSin[c*x] + 12*b*c^5*x^5*Log[x]))/(60*x^5*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5186, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx$$

$$\downarrow \text{5186}$$

$$\frac{bcd\sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x^5} dx}{5\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5dx^5}$$

$$\downarrow \text{243}$$

$$\frac{bcd\sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x^6} dx^2}{10\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5dx^5}$$

$$\downarrow \text{49}$$

$$\frac{bcd\sqrt{d - c^2 dx^2} \int \left(\frac{c^4}{x^2} - \frac{2c^2}{x^4} + \frac{1}{x^6} \right) dx^2}{10\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5dx^5}$$

$$\downarrow \text{2009}$$

$$\frac{bcd\sqrt{d - c^2 dx^2} \left(c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4} \right)}{10\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5dx^5}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^6,x]
```

output

$$-1/5*((d - c^2*d*x^2)^{(5/2)}*(a + b*\text{ArcSin}[c*x]))/(d*x^5) + (b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(-1/2*1/x^4 + (2*c^2)/x^2 + c^4*\text{Log}[x^2]))/(10*\text{Sqrt}[1 - c^2*x^2])$$
Defintions of rubi rules used

rule 49

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[m + n + 2, 0]$$

rule 243

$$\text{Int}(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x)^p}, x], x, x^2], x] /; \text{FreeQ}\{a, b, m, p, x\} \&\& \text{IntegerQ}[(m - 1)/2]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

rule 5186

$$\text{Int}[(a_.) + \text{ArcSin}[c_.)*(x_.)]*(b_.)^{(n_.)}*((f_.)*(x_.))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)}*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m + 1))), x] - \text{Simp}[b*c*(n/(f*(m + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m + 1)}*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{EqQ}[m + 2*p + 3, 0] \&\& \text{NeQ}[m, -1]$$
Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 2350, normalized size of antiderivative = 15.26

method	result	size
default	Expression too large to display	2350
parts	Expression too large to display	2350

input

$$\text{int}((-c^2*d*x^2+d)^{(3/2)}*(a+b*\text{arcsin}(c*x))/x^6, x, \text{method}=_RETURNVERBOSE)$$

output

```

-1/5*a/d/x^5*(-c^2*d*x^2+d)^(5/2)+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^
8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^5/(c^2*x^2-1)*arcsin(c*x)+3/2*b*(-d
*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c^2*x
^2-1)*c^5*(-c^2*x^2+1)^(1/2)-1/5*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/
2)/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*d*c^5+1/5*I*b*(-d*(c^2*x
^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^9/(c^2*x^2-
1)*c^14-13/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^
4-5*c^2*x^2+1)*x^7/(c^2*x^2-1)*c^12+3/4*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^
8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c^2*x^2-1)*c^10+9/4*b*(-d*(c
^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^4/(c^2*
x^2-1)*(-c^2*x^2+1)^(1/2)*c^9+14*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*
c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c^2*x^2-1)*arcsin(c*x)*c^8-5/2*b*(-d*
(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^2/(c^
2*x^2-1)*c^7*(-c^2*x^2+1)^(1/2)-56/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8
-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x/(c^2*x^2-1)*arcsin(c*x)*c^6+28/5*b*(
-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x/(c
^2*x^2-1)*arcsin(c*x)*c^4-9/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^
6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^2/(c^2*x^2-1)*c^3*(-c^2*x^2+1)^(1/2)+2*I*b
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*d*c^5/(5*c^2*x^2-5)
-8/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*...

```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 526, normalized size of antiderivative = 3.42

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = \left[\frac{2(bc^7 dx^7 - bc^5 dx^5) \sqrt{d} \log\left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 - \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1}}{c^2 x^4 - x^2}\right)}{\dots} \right]$$

input

```

integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="fricas"
)

```

output

```
[1/20*(2*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 -
d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^
2*x^4 - x^2)) - (4*b*c^3*d*x^3 - (4*b*c^3 - b*c)*d*x^5 - b*c*d*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 4*(a*c^6*d*x^6 - 3*a*c^4*d*x^4 + 3*a*c^2
*d*x^2 - a*d + (b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*arcsin(
c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), 1/20*(4*(b*c^7*d*x^7 - b*c^5*
d*x^5)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*s
qrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d) - (4*b*c^3*d*x^3 - (4*b*c^3 - b
*c)*d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 4*(a*c^6*d*
x^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d + (b*c^6*d*x^6 - 3*b*c^4*d*x^4 +
3*b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))}{x^6} dx$$

input

```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**6,x)
```

output

```
Integral((-d*(c*x - 1)*(c*x + 1))**3/2*(a + b*asin(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx =$$

$$\frac{\left(2(-1)^{-2c^2 dx^2 + 2d} c^4 d^{\frac{5}{2}} \log\left(-2c^2 d + \frac{2d}{x^2}\right) + 2c^4 d^{\frac{5}{2}} \log\left(x^2 - \frac{1}{c^2}\right) - \frac{3\sqrt{c^4 dx^4 - 2c^2 dx^2 + d} c^2 d^2}{x^2} + \frac{\sqrt{c^4 dx^4 - 2c^2 dx^2 + d}}{x^4}\right)}{20d}$$

$$- \frac{(-c^2 dx^2 + d)^{\frac{5}{2}} b \arcsin(cx)}{5 dx^5} - \frac{(-c^2 dx^2 + d)^{\frac{5}{2}} a}{5 dx^5}$$

input

```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="maxima"
)
```

output

```
-1/20*(2*(-1)^(-2*c^2*d*x^2 + 2*d)*c^4*d^(5/2)*log(-2*c^2*d + 2*d/x^2) + 2
*c^4*d^(5/2)*log(x^2 - 1/c^2) - 3*sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*c^2*d^
2/x^2 + sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*d^2/x^4)*b*c/d - 1/5*(-c^2*d*x^2
+ d)^(5/2)*b*arcsin(c*x)/(d*x^5) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a/(d*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^6} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^6,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^6} dx = \frac{\sqrt{d} d \left(-\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 2\sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a \right)}{5x^5} + \frac{b \arcsin(cx)}{5x^5} + \frac{b c^2 x^2}{5x^5} + \frac{b c^4 x^4}{5x^5} + \frac{b \arcsin(cx)}{5x^5}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^6,x)`

output `(sqrt(d)*d*(-sqrt(-c**2*x**2+1)*a*c**4*x**4+2*sqrt(-c**2*x**2+1)*a*c**2*x**2-sqrt(-c**2*x**2+1)*a)+5*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**6,x)*b*x**5-5*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**4,x)*b*c**2*x**5)/(5*x**5)`

3.74 $\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx$

Optimal result	804
Mathematica [A] (verified)	805
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Maxima [A] (verification not implemented)	809
Giac [F(-2)]	810
Mupad [F(-1)]	810
Reduce [F]	811

Optimal result

Integrand size = 27, antiderivative size = 231

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = -\frac{bcd\sqrt{d - c^2 dx^2}}{42x^6\sqrt{1 - c^2 x^2}} + \frac{2bc^3 d\sqrt{d - c^2 dx^2}}{35x^4\sqrt{1 - c^2 x^2}} - \frac{bc^5 d\sqrt{d - c^2 dx^2}}{70x^2\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{7dx^7} - \frac{2c^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{35dx^5} + \frac{2bc^7 d\sqrt{d - c^2 dx^2} \log(x)}{35\sqrt{1 - c^2 x^2}}$$

output

```
-1/42*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+2/35*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-1/70*b*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^7-2/35*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^5+2/35*b*c^7*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \frac{d\sqrt{d - c^2 dx^2} (25bcx - 60bc^3 x^3 + 15bc^5 x^5 + 147bc^7 x^7 + 150a\sqrt{1 - c^2 x^2} - 240ac^2 x^2 \sqrt{1 - c^2 x^2} + 30ac^4 x^4 - 1050x^7 \sqrt{1 - c^2 x^2})}{1050x^7 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^8,x]
```

output

```
-1/1050*(d*Sqrt[d - c^2*d*x^2]*(25*b*c*x - 60*b*c^3*x^3 + 15*b*c^5*x^5 + 147*b*c^7*x^7 + 150*a*Sqrt[1 - c^2*x^2] - 240*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 30*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 60*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 30*b*(1 - c^2*x^2)^(5/2)*(5 + 2*c^2*x^2)*ArcSin[c*x] - 60*b*c^7*x^7*Log[x]))/(x^7*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx$$

↓ 5194

$$\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d(1 - c^2 x^2)^2 (2c^2 x^2 + 5)}{35x^7} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{7dx^7} - \frac{2c^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{35dx^5}$$

↓ 27

$$\begin{aligned}
& \frac{bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(2c^2x^2+5)}{x^7} dx}{35\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{7dx^7} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{35dx^5} \\
& \quad \downarrow \text{354} \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(2c^2x^2+5)}{x^8} dx^2}{70\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{7dx^7} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{35dx^5} \\
& \quad \downarrow \text{85} \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \left(\frac{2c^6}{x^2} + \frac{c^4}{x^4} - \frac{8c^2}{x^6} + \frac{5}{x^8}\right) dx^2}{70\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{7dx^7} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{35dx^5} \\
& \quad \downarrow \text{2009} \\
& -\frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{7dx^7} - \frac{2c^2(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{35dx^5} + \\
& \quad \frac{bcd\sqrt{d-c^2dx^2} \left(2c^6 \log(x^2) - \frac{c^4}{x^2} + \frac{4c^2}{x^4} - \frac{5}{3x^6}\right)}{70\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^8,x]`

output `-1/7*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(d*x^7) - (2*c^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(35*d*x^5) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-5/(3*x^6) + (4*c^2)/x^4 - c^4/x^2 + 2*c^6*Log[x^2]))/(70*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 85 `Int[((d_)*(x_))^(n_)*((a_) + (b_)*(x_))*((e_) + (f_)*(x_))^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*f, 0] && (!IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p, 1])`

rule 354 `Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 3384, normalized size of antiderivative = 14.65

method	result	size
default	Expression too large to display	3384
parts	Expression too large to display	3384

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^8,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned}
 & -2/35*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/(c^2*x^2-1)*\ln((I*c*x+(- \\
 & c^2*x^2+1)^{(1/2)})^2-1)*d*c^7+a*(-1/7/d/x^7*(-c^2*d*x^2+d)^{(5/2)}-2/35*c^2/d \\
 & /x^5*(-c^2*d*x^2+d)^{(5/2)})+3*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c \\
 & ^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c^2*x^2-1)*\arcsin(c*x)* \\
 & c^{16}+12*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^6*x^6+154 \\
 & *c^4*x^4-105*c^2*x^2+25)*x^7/(c^2*x^2-1)*\arcsin(c*x)*c^{14}-5/2*b*(-d*(c^2*x \\
 & ^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2 \\
 & +25)*x^6/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*c^{13}-164/5*b*(-d*(c^2*x^2-1))^{(1/2)} \\
 &)*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5/(c \\
 & ^2*x^2-1)*\arcsin(c*x)*c^{12}+11/6*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-3 \\
 & 5*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^4/(c^2*x^2-1)*(-c^2*x^2 \\
 & +1)^{(1/2)}*c^{11}-2*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^ \\
 & 6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^{11}/(c^2*x^2-1)*\arcsin(c*x)*c^{18}+5/21*I \\
 & *b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^6*x^6+154*c^4*x^ \\
 & 4-105*c^2*x^2+25)*x/(c^2*x^2-1)*c^8-25/21*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35 \\
 & *c^{10}*x^{10}-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3/(c^2*x^2- \\
 & 1)*c^{10}+1/21*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^6* \\
 & x^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c^2*x^2-1)*c^{16}-142/105*I*b*(-d*(c^2* \\
 & x^2-1))^{(1/2)}*d/(35*c^{10}*x^{10}-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^ \\
 & 2+25)*x^7/(c^2*x^2-1)*c^{14}+72/35*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d/(35*c^{10}*...
 \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.60

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \left[\frac{6(bc^9 dx^9 - bc^7 dx^7) \sqrt{d} \log\left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 - \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1}}{c^2 x^4 - x^2}\right)}{\dots} \right]$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="fricas")`

output

```
[1/210*(6*(b*c^9*d*x^9 - b*c^7*d*x^7)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 -
d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c
^2*x^4 - x^2)) + (3*b*c^5*d*x^5 - (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*
b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 6*(2*a*
c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d + (2*b*c^
8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 5*b*d)*arcsin(c*x
))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), 1/210*(12*(b*c^9*d*x^9 - b*c^7*d
*x^7)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sq
rt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (3*b*c^5*d*x^5 - (3*b*c^5 - 12
*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(-c^2*x^2 + 1) - 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a*
c^2*d*x^2 - 5*a*d + (2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^
2*d*x^2 - 5*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**8,x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \frac{1}{210} \left(12 c^6 d^{3/2} \log(x) - \frac{3 c^4 d^{3/2} x^4 - 12 c^2 d^{3/2} x^2 + 5 d^{3/2}}{x^6} \right) bc$$

$$- \frac{1}{35} b \left(\frac{2(-c^2 dx^2 + d)^{5/2} c^2}{dx^5} + \frac{5(-c^2 dx^2 + d)^{5/2}}{dx^7} \right) \arcsin(cx)$$

$$- \frac{1}{35} a \left(\frac{2(-c^2 dx^2 + d)^{5/2} c^2}{dx^5} + \frac{5(-c^2 dx^2 + d)^{5/2}}{dx^7} \right)$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="maxima")`

output `1/210*(12*c^6*d^(3/2)*log(x) - (3*c^4*d^(3/2)*x^4 - 12*c^2*d^(3/2)*x^2 + 5*d^(3/2))/x^6)*b*c - 1/35*b*(2*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))*arcsin(c*x) - 1/35*a*(2*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^8} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^8,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^8, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^8} dx = \frac{\sqrt{d} d \left(-2\sqrt{-c^2 x^2 + 1} a c^6 x^6 - \sqrt{-c^2 x^2 + 1} a c^4 x^4 + 8\sqrt{-c^2 x^2 + 1} a c^2 x^2 + 8\sqrt{-c^2 x^2 + 1} a \right)}{35 x^7} + \frac{b \arcsin(cx)}{x^7} + \frac{b c^2 x^5}{35 x^7} + \frac{b c^4 x^3}{35 x^7} + \frac{b c^6 x}{35 x^7}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^8,x)`

output `(sqrt(d)*d*(- 2*sqrt(- c**2*x**2 + 1)*a*c**6*x**6 - sqrt(- c**2*x**2 + 1)*a*c**4*x**4 + 8*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - 5*sqrt(- c**2*x**2 + 1)*a + 35*int((sqrt(- c**2*x**2 + 1)*asin(c*x))/x**8,x)*b*x**7 - 35*int((sqrt(- c**2*x**2 + 1)*asin(c*x))/x**6,x)*b*c**2*x**7))/(35*x**7)`

$$3.75 \quad \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx$$

Optimal result	812
Mathematica [A] (verified)	813
Rubi [A] (verified)	813
Maple [C] (verified)	815
Fricas [A] (verification not implemented)	816
Sympy [F(-1)]	817
Maxima [A] (verification not implemented)	818
Giac [F(-2)]	818
Mupad [F(-1)]	819
Reduce [F]	819

Optimal result

Integrand size = 27, antiderivative size = 308

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = & -\frac{bcd\sqrt{d - c^2 dx^2}}{72x^8\sqrt{1 - c^2 x^2}} \\ & + \frac{5bc^3 d\sqrt{d - c^2 dx^2}}{189x^6\sqrt{1 - c^2 x^2}} - \frac{bc^5 d\sqrt{d - c^2 dx^2}}{420x^4\sqrt{1 - c^2 x^2}} - \frac{2bc^7 d\sqrt{d - c^2 dx^2}}{315x^2\sqrt{1 - c^2 x^2}} \\ & - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{9dx^9} - \frac{4c^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{63dx^7} \\ & - \frac{8c^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{315dx^5} + \frac{8bc^9 d\sqrt{d - c^2 dx^2} \log(x)}{315\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/72*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^8/(-c^2*x^2+1)^(1/2)+5/189*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)-1/420*b*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-2/315*b*c^7*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/9*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^9-4/63*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^7-8/315*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^5+8/315*b*c^9*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.77

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx =$$

$$d\sqrt{d - c^2 dx^2} \left(3675bcx - 7000bc^3x^3 + 630bc^5x^5 + 1680bc^7x^7 + 18264bc^9x^9 + 29400a\sqrt{1 - c^2x^2} - 42000 \log[x] \right) / (x^9 \sqrt{1 - c^2x^2})$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^10,x]
```

output

```
-1/264600*(d*Sqrt[d - c^2*d*x^2]*(3675*b*c*x - 7000*b*c^3*x^3 + 630*b*c^5*x^5 + 1680*b*c^7*x^7 + 18264*b*c^9*x^9 + 29400*a*Sqrt[1 - c^2*x^2] - 42000*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 2520*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 3360*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 6720*a*c^8*x^8*Sqrt[1 - c^2*x^2] + 840*b*(1 - c^2*x^2)^(5/2)*(35 + 20*c^2*x^2 + 8*c^4*x^4)*ArcSin[c*x] - 6720*b*c^9*x^9*Log[x]))/(x^9*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx$$

↓ 5194

$$\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d(1 - c^2 x^2)^2 (8c^4 x^4 + 20c^2 x^2 + 35)}{315x^9} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{9dx^9}$$

$$\frac{4c^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{63dx^7} - \frac{8c^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{315dx^5}$$

↓ 27

$$\begin{aligned}
& \frac{bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{x^9} dx}{315\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{9dx^9} - \\
& \frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{63dx^7} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{315dx^5} \\
& \quad \downarrow 1578 \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{x^{10}} dx^2}{630\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{9dx^9} - \\
& \frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{63dx^7} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{315dx^5} \\
& \quad \downarrow 1195 \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \left(\frac{8c^8}{x^2} + \frac{4c^6}{x^4} + \frac{3c^4}{x^6} - \frac{50c^2}{x^8} + \frac{35}{x^{10}}\right) dx^2}{630\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{9dx^9} - \\
& \frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{63dx^7} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{315dx^5} \\
& \quad \downarrow 2009 \\
& -\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{9dx^9} - \frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{63dx^7} - \\
& \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{315dx^5} + \frac{bcd\sqrt{d-c^2dx^2} \left(8c^8 \log(x^2) - \frac{4c^6}{x^2} - \frac{3c^4}{2x^4} + \frac{50c^2}{3x^6} - \frac{35}{4x^8}\right)}{630\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^10,x]`

output `-1/9*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(d*x^9) - (4*c^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(63*d*x^7) - (8*c^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(315*d*x^5) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-35/(4*x^8) + (50*c^2)/(3*x^6) - (3*c^4)/(2*x^4) - (4*c^6)/x^2 + 8*c^8*Log[x^2]))/(630*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1195 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 1578 `Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 4563, normalized size of antiderivative = 14.81

method	result	size
default	Expression too large to display	4563
parts	Expression too large to display	4563

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^10,x,method=_RETURNVERBOSE)`

output

```

104/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-
2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^11/(c^2*x^2-1)*arcsin(c*x)*
c^20+16/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*
x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^10/(c^2*x^2-1)*(-c^2*x^
2+1)^(1/2)*c^19-212/15*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*
x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^9/(c^2*x^2
-1)*arcsin(c*x)*c^18-4*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*
x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^8/(c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)*c^17-174520/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+122
5)/x^3/(c^2*x^2-1)*arcsin(c*x)*c^6+1285/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*
c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2
+1225)/x^4/(c^2*x^2-1)*c^5*(-c^2*x^2+1)^(1/2)+19540/9*b*(-d*(c^2*x^2-1))^(
1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-
4725*c^2*x^2+1225)/x^5/(c^2*x^2-1)*arcsin(c*x)*c^4-21175/216*b*(-d*(c^2*x^
2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c
^4*x^4-4725*c^2*x^2+1225)/x^6/(c^2*x^2-1)*c^3*(-c^2*x^2+1)^(1/2)-7700/9*b*
(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6
*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^7/(c^2*x^2-1)*arcsin(c*x)*c^2+1225/
72*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-...

```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.18

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = \left[\frac{96 (bc^{11} dx^{11} - bc^9 dx^9) \sqrt{d} \log \left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 - \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + d}}{c^2 x^4 - x^2} \right)}{\dots} \right]$$

input

```

integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^10,x, algorithm="fricas
")

```

output

```
[1/7560*(96*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) + (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 - 200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d + (8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x^4 + 85*b*c^2*d*x^2 - 35*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), 1/7560*(192*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 - 200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d + (8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x^4 + 85*b*c^2*d*x^2 - 35*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**10,x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.68

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = \frac{1}{7560} \left(192 c^8 d^{3/2} \log(x) - \frac{48 c^6 d^{3/2} x^6 + 18 c^4 d^{3/2} x^4 - 200 c^2 d^{3/2} x^2 + 105 d^{3/2}}{x^8} \right) b c - \frac{1}{315} b \left(\frac{8 (-c^2 dx^2 + d)^{5/2} c^4}{dx^5} + \frac{20 (-c^2 dx^2 + d)^{5/2} c^2}{dx^7} + \frac{35 (-c^2 dx^2 + d)^{5/2}}{dx^9} \right) \arcsin(cx) - \frac{1}{315} a \left(\frac{8 (-c^2 dx^2 + d)^{5/2} c^4}{dx^5} + \frac{20 (-c^2 dx^2 + d)^{5/2} c^2}{dx^7} + \frac{35 (-c^2 dx^2 + d)^{5/2}}{dx^9} \right)$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^10,x, algorithm="maxima")`

output `1/7560*(192*c^8*d^(3/2)*log(x) - (48*c^6*d^(3/2)*x^6 + 18*c^4*d^(3/2)*x^4 - 200*c^2*d^(3/2)*x^2 + 105*d^(3/2))/x^8)*b*c - 1/315*b*(8*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^2*d*x^2 + d)^(5/2)/(d*x^9))*arcsin(c*x) - 1/315*a*(8*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^2*d*x^2 + d)^(5/2)/(d*x^9))`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^10,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^{10}} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^10,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^10, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{10}} dx = \frac{\sqrt{d} d (-8\sqrt{-c^2 x^2 + 1} a c^8 x^8 - 4\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 3\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 50\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 35\sqrt{-c^2 x^2 + 1} a + 315 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx)) / x^{10}, x) * b * x^9 - 315 \int (\sqrt{-c^2 x^2 + 1} a \sin(cx)) / x^8, x) * b * c^2 * x^9)}{(315 * x^9)}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^10,x)`

output `(sqrt(d)*d*(-8*sqrt(-c**2*x**2+1)*a*c**8*x**8-4*sqrt(-c**2*x**2+1)*a*c**6*x**6-3*sqrt(-c**2*x**2+1)*a*c**4*x**4+50*sqrt(-c**2*x**2+1)*a*c**2*x**2-35*sqrt(-c**2*x**2+1)*a+315*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**10,x)*b*x**9-315*int((sqrt(-c**2*x**2+1)*a*sin(c*x))/x**8,x)*b*c**2*x**9))/(315*x**9)`

3.76
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx$$

Optimal result	820
Mathematica [A] (verified)	821
Rubi [A] (verified)	821
Maple [C] (verified)	824
Fricas [A] (verification not implemented)	824
Sympy [F(-1)]	825
Maxima [A] (verification not implemented)	826
Giac [F(-2)]	826
Mupad [F(-1)]	827
Reduce [F]	827

Optimal result

Integrand size = 27, antiderivative size = 385

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = & -\frac{bcd\sqrt{d - c^2 dx^2}}{110x^{10}\sqrt{1 - c^2 x^2}} \\ & + \frac{bc^3 d\sqrt{d - c^2 dx^2}}{66x^8\sqrt{1 - c^2 x^2}} - \frac{bc^5 d\sqrt{d - c^2 dx^2}}{1386x^6\sqrt{1 - c^2 x^2}} - \frac{bc^7 d\sqrt{d - c^2 dx^2}}{770x^4\sqrt{1 - c^2 x^2}} \\ & - \frac{4bc^9 d\sqrt{d - c^2 dx^2}}{1155x^2\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{11dx^{11}} \\ & - \frac{2c^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{33dx^9} - \frac{8c^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{231dx^7} \\ & - \frac{16c^6 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{1155dx^5} + \frac{16bc^{11} d\sqrt{d - c^2 dx^2} \log(x)}{1155\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/110*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^10/(-c^2*x^2+1)^(1/2)+1/66*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^8/(-c^2*x^2+1)^(1/2)-1/1386*b*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)-1/770*b*c^7*d*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-4/1155*b*c^9*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/11*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^11-2/33*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^9-8/231*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^7-16/1155*c^6*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/d/x^5+16/1155*b*c^11*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.72

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx =$$

$$d\sqrt{d - c^2 dx^2} \left(6615bcx - 11025bc^3x^3 + 525bc^5x^5 + 945bc^7x^7 + 2520bc^9x^9 + 29524bc^{11}x^{11} + 66150a\sqrt{1 - c^2x^2} \right)$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^12,x]
```

output

```
-1/727650*(d*Sqrt[d - c^2*d*x^2]*(6615*b*c*x - 11025*b*c^3*x^3 + 525*b*c^5*x^5 + 945*b*c^7*x^7 + 2520*b*c^9*x^9 + 29524*b*c^11*x^11 + 66150*a*Sqrt[1 - c^2*x^2] - 88200*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 3150*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 3780*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 5040*a*c^8*x^8*Sqrt[1 - c^2*x^2] + 10080*a*c^10*x^10*Sqrt[1 - c^2*x^2] + 630*b*(1 - c^2*x^2)^(5/2)*(105 + 70*c^2*x^2 + 40*c^4*x^4 + 16*c^6*x^6)*ArcSin[c*x] - 10080*b*c^11*x^11*Log[x]))/(x^11*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx$$

↓ 5194

$$\begin{aligned}
& \frac{bcd\sqrt{d-c^2dx^2} \int -\frac{d(1-c^2x^2)^2(16c^6x^6+40c^4x^4+70c^2x^2+105)}{1155x^{11}} dx}{\sqrt{1-c^2x^2}} - \\
& \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} - \\
& \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} \\
& \quad \downarrow \mathbf{27} \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(16c^6x^6+40c^4x^4+70c^2x^2+105)}{x^{11}} dx}{1155\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{11dx^{11}} - \\
& \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{33dx^9} - \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \\
& \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} \\
& \quad \downarrow \mathbf{2331} \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(16c^6x^6+40c^4x^4+70c^2x^2+105)}{x^{12}} dx^2}{2310\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{11dx^{11}} - \\
& \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{33dx^9} - \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \\
& \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} \\
& \quad \downarrow \mathbf{2123} \\
& \frac{bcd\sqrt{d-c^2dx^2} \int \left(\frac{16c^{10}}{x^2} + \frac{8c^8}{x^4} + \frac{6c^6}{x^6} + \frac{5c^4}{x^8} - \frac{140c^2}{x^{10}} + \frac{105}{x^{12}}\right) dx^2}{2310\sqrt{1-c^2x^2}} - \\
& \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{11dx^{11}} - \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} - \\
& \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} \\
& \quad \downarrow \mathbf{2009} \\
& \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} - \\
& \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{1155dx^5} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{231dx^7} + \\
& \frac{bcd\sqrt{d-c^2dx^2} \left(16c^{10} \log(x^2) - \frac{8c^8}{x^2} - \frac{3c^6}{x^4} - \frac{5c^4}{3x^6} + \frac{35c^2}{x^8} - \frac{21}{x^{10}}\right)}{2310\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^12,x]`

output `-1/11*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(d*x^11) - (2*c^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(33*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(231*d*x^7) - (16*c^6*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(1155*d*x^5) + (b*c*d*Sqrt[d - c^2*d*x^2]*(-21/x^10 + (35*c^2)/x^8 - (5*c^4)/(3*x^6) - (3*c^6)/x^4 - (8*c^8)/x^2 + 16*c^10*Log[x^2]))/(2310*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2123 `Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])`

rule 2331 `Int[(Pq_)*(x_)^((m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`

rule 5194 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 5886, normalized size of antiderivative = 15.29

method	result	size
default	Expression too large to display	5886
parts	Expression too large to display	5886

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^12,x,method=_RETURNVERBOSE)`

output `result too large to display`

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.93

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = \left[\frac{48 (bc^{13} dx^{13} - bc^{11} dx^{11}) \sqrt{d} \log \left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 - \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2}}{c^2 x^4 - x^2} \right)}{\right.$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^12,x, algorithm="fricas")`

output

```
[1/6930*(48*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) + (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 6*(16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d + (16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11), 1/6930*(96*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 6*(16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d + (16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**12,x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.70

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = \frac{1}{6930} \left(96 c^{10} d^{3/2} \log(x) - \frac{24 c^8 d^{3/2} x^8 + 9 c^6 d^{3/2} x^6 + 5 c^4 d^{3/2} x^4 - 105 c^2 d^{3/2} x^2 + 105 d^{3/2}}{x^{10}} \right) b \arcsin(cx) - \frac{1}{1155} \left(\frac{16 (-c^2 dx^2 + d)^{5/2} c^6}{dx^5} + \frac{40 (-c^2 dx^2 + d)^{5/2} c^4}{dx^7} + \frac{70 (-c^2 dx^2 + d)^{5/2} c^2}{dx^9} + \frac{105 (-c^2 dx^2 + d)^{5/2}}{dx^{11}} \right) b \arcsin(cx) - \frac{1}{1155} \left(\frac{16 (-c^2 dx^2 + d)^{5/2} c^6}{dx^5} + \frac{40 (-c^2 dx^2 + d)^{5/2} c^4}{dx^7} + \frac{70 (-c^2 dx^2 + d)^{5/2} c^2}{dx^9} + \frac{105 (-c^2 dx^2 + d)^{5/2}}{dx^{11}} \right) a$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^12,x, algorithm="maxima")`

output `1/6930*(96*c^10*d^(3/2)*log(x) - (24*c^8*d^(3/2)*x^8 + 9*c^6*d^(3/2)*x^6 + 5*c^4*d^(3/2)*x^4 - 105*c^2*d^(3/2)*x^2 + 105*d^(3/2))/x^10)*b*c - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^9) + 105*(-c^2*d*x^2 + d)^(5/2)/(d*x^11))*b*arcsin(c*x) - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^9) + 105*(-c^2*d*x^2 + d)^(5/2)/(d*x^11))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^12,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^{12}} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^12,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^12, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^{12}} dx = \frac{\sqrt{d} d \left(-16\sqrt{-c^2 x^2 + 1} a c^{10} x^{10} - 8\sqrt{-c^2 x^2 + 1} a c^8 x^8 - 6\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 5\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 140\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 105\sqrt{-c^2 x^2 + 1} a + 1155 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx)) / x^{12}, x \right) * b * x^{11} - 1155 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx)) / x^{10}, x * b * c^{11}}{(1155 * x^{11})}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^12,x)`

output `(sqrt(d)*d*(-16*sqrt(-c**2*x**2+1)*a*c**10*x**10-8*sqrt(-c**2*x**2+1)*a*c**8*x**8-6*sqrt(-c**2*x**2+1)*a*c**6*x**6-5*sqrt(-c**2*x**2+1)*a*c**4*x**4+140*sqrt(-c**2*x**2+1)*a*c**2*x**2-105*sqrt(-c**2*x**2+1)*a+1155*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**12,x)*b*x**11-1155*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**10,x)*b*c**2*x**11)/(1155*x**11)`

3.77 $\int x^7(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	828
Mathematica [A] (verified)	829
Rubi [A] (verified)	829
Maple [A] (verified)	831
Fricas [A] (verification not implemented)	832
Sympy [F(-1)]	832
Maxima [A] (verification not implemented)	833
Giac [F(-2)]	833
Mupad [F(-1)]	834
Reduce [F]	834

Optimal result

Integrand size = 27, antiderivative size = 375

$$\int x^7(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{16bdx\sqrt{d - c^2 dx^2}}{1155c^7\sqrt{1 - c^2 x^2}} + \frac{8bdx^3\sqrt{d - c^2 dx^2}}{3465c^5\sqrt{1 - c^2 x^2}} + \frac{2bdx^5\sqrt{d - c^2 dx^2}}{1925c^3\sqrt{1 - c^2 x^2}} + \frac{bdx^7\sqrt{d - c^2 dx^2}}{1617c\sqrt{1 - c^2 x^2}} - \frac{4bcdx^9\sqrt{d - c^2 dx^2}}{297\sqrt{1 - c^2 x^2}} + \frac{bc^3 dx^{11}\sqrt{d - c^2 dx^2}}{121\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^8 d} + \frac{3(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^8 d^2} - \frac{(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{3c^8 d^3} + \frac{(d - c^2 dx^2)^{11/2} (a + b \arcsin(cx))}{11c^8 d^4}$$

output

```
16/1155*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^7/(-c^2*x^2+1)^(1/2)+8/3465*b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+2/1925*b*d*x^5*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/1617*b*d*x^7*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-4/297*b*c*d*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/121*b*c^3*d*x^11*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/c^8/d+3/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/c^8/d^2-1/3*(-c^2*d*x^2+d)^(9/2)*(a+b*arcsin(c*x))/c^8/d^3+1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arcsin(c*x))/c^8/d^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.46

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{d\sqrt{d - c^2 dx^2} \left(-3465a(1 - c^2 x^2)^{5/2} (16 + 40c^2 x^2 + 70c^4 x^4 + 105c^6 x^6) + bcx(55440 + 9240c^2 x^2 + 4158c^4 x^4 + 2475c^6 x^6 - 53900c^8 x^8 + 33075c^{10} x^{10}) - 3465b(1 - c^2 x^2)^{5/2} (16 + 40c^2 x^2 + 70c^4 x^4 + 105c^6 x^6) \arcsin(cx) \right)}{4002075c^8 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]
```

output

```
(d*Sqrt[d - c^2*d*x^2]*(-3465*a*(1 - c^2*x^2)^(5/2)*(16 + 40*c^2*x^2 + 70*c^4*x^4 + 105*c^6*x^6) + b*c*x*(55440 + 9240*c^2*x^2 + 4158*c^4*x^4 + 2475*c^6*x^6 - 53900*c^8*x^8 + 33075*c^10*x^10) - 3465*b*(1 - c^2*x^2)^(5/2)*(16 + 40*c^2*x^2 + 70*c^4*x^4 + 105*c^6*x^6)*ArcSin[c*x]))/(4002075*c^8*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)Time = 0.66 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d(1 - c^2 x^2)^2 (105c^6 x^6 + 70c^4 x^4 + 40c^2 x^2 + 16)}{1155c^8} dx}{\sqrt{1 - c^2 x^2}} +$$

$$\frac{(d - c^2 dx^2)^{11/2} (a + b \arcsin(cx))}{3(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))} - \frac{(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))} +$$

$$\frac{11c^8 d^4}{7c^8 d^2} - \frac{3c^8 d^3}{5c^8 d}$$

$$\downarrow 27$$

$$\frac{bd\sqrt{d-c^2dx^2} \int (1-c^2x^2)^2 (105c^6x^6 + 70c^4x^4 + 40c^2x^2 + 16) dx}{1155c^7\sqrt{1-c^2x^2}} +$$

$$\frac{(d-c^2dx^2)^{11/2} (a+b\arcsin(cx))}{11c^8d^4} - \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{3c^8d^3} +$$

$$\frac{3(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^8d^2} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^8d}$$

↓ 2341

$$\frac{bd\sqrt{d-c^2dx^2} \int (105c^{10}x^{10} - 140c^8x^8 + 5c^6x^6 + 6c^4x^4 + 8c^2x^2 + 16) dx}{1155c^7\sqrt{1-c^2x^2}} +$$

$$\frac{(d-c^2dx^2)^{11/2} (a+b\arcsin(cx))}{11c^8d^4} - \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{3c^8d^3} +$$

$$\frac{3(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^8d^2} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^8d}$$

↓ 2009

$$\frac{(d-c^2dx^2)^{11/2} (a+b\arcsin(cx))}{11c^8d^4} - \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{3c^8d^3} +$$

$$\frac{3(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^8d^2} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^8d} +$$

$$\frac{bd\left(\frac{105c^{10}x^{11}}{11} - \frac{140c^8x^9}{9} + \frac{5c^6x^7}{7} + \frac{6c^4x^5}{5} + \frac{8c^2x^3}{3} + 16x\right) \sqrt{d-c^2dx^2}}{1155c^7\sqrt{1-c^2x^2}}$$

input `Int[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `(b*d*Sqrt[d - c^2*d*x^2]*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6*x^7)/7 - (140*c^8*x^9)/9 + (105*c^10*x^11)/11))/(1155*c^7*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^8*d) + (3*(d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^8*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcSin[c*x]))/(3*c^8*d^3) + ((d - c^2*d*x^2)^(11/2)*(a + b*ArcSin[c*x]))/(11*c^8*d^4)`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2341 `Int[(P_q)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[P_q*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[P_q, x] && IGtQ[p, -2]`

rule 5194 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.68

method	result
orering	$\frac{(694575x^{12}c^{12} - 1619450c^{10}x^{10} + 904475c^8x^8 + 27720c^6x^6 + 70224c^4x^4 + 517440c^2x^2 - 443520)(-c^2dx^2 + d)^{\frac{3}{2}}(a + b \arcsin(cx))}{4002075c^8(cx-1)(cx+1)(c^2x^2-1)}$
default	Expression too large to display
parts	Expression too large to display

input `int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output

```
1/4002075*(694575*c^12*x^12-1619450*c^10*x^10+904475*c^8*x^8+27720*c^6*x^6
+70224*c^4*x^4+517440*c^2*x^2-443520)/c^8/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^
2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-1/4002075/x^6*(33075*c^10*x^10-53900*c^
8*x^8+2475*c^6*x^6+4158*c^4*x^4+9240*c^2*x^2+55440)/c^8/(c*x-1)/(c*x+1)*(7
*x^6*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-3*x^8*(-c^2*d*x^2+d)^(1/2)*(a+
b*arcsin(c*x))*c^2*d+x^7*(-c^2*d*x^2+d)^(3/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.66

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx =$$

$$\frac{(33075 bc^{11} dx^{11} - 53900 bc^9 dx^9 + 2475 bc^7 dx^7 + 4158 bc^5 dx^5 + 9240 bc^3 dx^3 + 55440 bcdx) \sqrt{-c^2 dx^2 + d}}{\dots}$$

input

```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas"
)
```

output

```
-1/4002075*((33075*b*c^11*d*x^11 - 53900*b*c^9*d*x^9 + 2475*b*c^7*d*x^7 +
4158*b*c^5*d*x^5 + 9240*b*c^3*d*x^3 + 55440*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*
sqrt(-c^2*x^2 + 1) + 3465*(105*a*c^12*d*x^12 - 245*a*c^10*d*x^10 + 145*a*c
^8*d*x^8 + a*c^6*d*x^6 + 2*a*c^4*d*x^4 + 8*a*c^2*d*x^2 - 16*a*d + (105*b*c
^12*d*x^12 - 245*b*c^10*d*x^10 + 145*b*c^8*d*x^8 + b*c^6*d*x^6 + 2*b*c^4*d
*x^4 + 8*b*c^2*d*x^2 - 16*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^10*x^
2 - c^8)
```

Sympy [F(-1)]

Timed out.

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input

```
integrate(x**7*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)
```

output Timed out

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.71

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx =$$

$$-\frac{1}{1155} \left(\frac{105 (-c^2 dx^2 + d)^{5/2} x^6}{c^2 d} + \frac{70 (-c^2 dx^2 + d)^{5/2} x^4}{c^4 d} + \frac{40 (-c^2 dx^2 + d)^{5/2} x^2}{c^6 d} + \frac{16 (-c^2 dx^2 + d)^{5/2}}{c^8 d} \right) b \arcsin(cx)$$

$$-\frac{1}{1155} \left(\frac{105 (-c^2 dx^2 + d)^{5/2} x^6}{c^2 d} + \frac{70 (-c^2 dx^2 + d)^{5/2} x^4}{c^4 d} + \frac{40 (-c^2 dx^2 + d)^{5/2} x^2}{c^6 d} + \frac{16 (-c^2 dx^2 + d)^{5/2}}{c^8 d} \right) a$$

$$+ \frac{\left(33075 c^{10} d^{3/2} x^{11} - 53900 c^8 d^{3/2} x^9 + 2475 c^6 d^{3/2} x^7 + 4158 c^4 d^{3/2} x^5 + 9240 c^2 d^{3/2} x^3 + 55440 d^{3/2} x \right) b}{4002075 c^7}$$

input `integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(5/2)/(c^8*d))*b*arcsin(c*x) - 1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(5/2)/(c^8*d))*a + 1/4002075*(33075*c^10*d^(3/2)*x^11 - 53900*c^8*d^(3/2)*x^9 + 2475*c^6*d^(3/2)*x^7 + 4158*c^4*d^(3/2)*x^5 + 9240*c^2*d^(3/2)*x^3 + 55440*d^(3/2)*x)*b/c^7`

Giac [F(-2)]

Exception generated.

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^7 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{3/2} dx$$

input `int(x^7*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^7*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^7 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d (-105 \sqrt{-c^2 x^2 + 1} a c^{10} x^{10} + 140 \sqrt{-c^2 x^2 + 1} a c^8 x^8 - 5 \sqrt{-c^2 x^2 + 1} a c^6 x^6 - 6 \sqrt{-c^2 x^2 + 1} a c^4 x^4 - 8 \sqrt{-c^2 x^2 + 1} a c^2 x^2 - 16 \sqrt{-c^2 x^2 + 1} a - 1155 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(c x) x^9, x) * b c^{10} + 1155 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(c x) x^7, x) * b c^8)}{(1155 c^8)}$$

input `int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d*(- 105*sqrt(- c**2*x**2 + 1)*a*c**10*x**10 + 140*sqrt(- c**2
*x**2 + 1)*a*c**8*x**8 - 5*sqrt(- c**2*x**2 + 1)*a*c**6*x**6 - 6*sqrt(-
c**2*x**2 + 1)*a*c**4*x**4 - 8*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - 16*sq
rt(- c**2*x**2 + 1)*a - 1155*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**9,x)*
b*c**10 + 1155*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**7,x)*b*c**8))/(1155
*c**8)`

3.78 $\int x^5(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	835
Mathematica [A] (verified)	836
Rubi [A] (verified)	836
Maple [A] (verified)	838
Fricas [A] (verification not implemented)	839
Sympy [F(-1)]	839
Maxima [A] (verification not implemented)	840
Giac [F(-2)]	840
Mupad [F(-1)]	841
Reduce [F]	841

Optimal result

Integrand size = 27, antiderivative size = 301

$$\begin{aligned} \int x^5(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx &= \frac{8bdx\sqrt{d - c^2dx^2}}{315c^5\sqrt{1 - c^2x^2}} \\ &+ \frac{4bdx^3\sqrt{d - c^2dx^2}}{945c^3\sqrt{1 - c^2x^2}} + \frac{bdx^5\sqrt{d - c^2dx^2}}{525c\sqrt{1 - c^2x^2}} - \frac{10bcdx^7\sqrt{d - c^2dx^2}}{441\sqrt{1 - c^2x^2}} \\ &+ \frac{bc^3dx^9\sqrt{d - c^2dx^2}}{81\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{5c^6d} \\ &+ \frac{2(d - c^2dx^2)^{7/2} (a + b \arcsin(cx))}{7c^6d^2} - \frac{(d - c^2dx^2)^{9/2} (a + b \arcsin(cx))}{9c^6d^3} \end{aligned}$$

output

```
8/315*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+4/945*b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/525*b*d*x^5*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-10/441*b*c*d*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/81*b*c^3*d*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/c^6/d+2/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/c^6/d^2-1/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arcsin(c*x))/c^6/d^3
```


Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.50

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{d\sqrt{d - c^2 dx^2} \left(-315a(1 - c^2 x^2)^{5/2} (8 + 20c^2 x^2 + 35c^4 x^4) + bcx(2520 + 420c^2 x^2 + 189c^4 x^4) \right) + b \arcsin(cx)}{99225c^6 \sqrt{1 - c^2 x^2}}$$

input `Integrate[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `(d*Sqrt[d - c^2*d*x^2]*(-315*a*(1 - c^2*x^2)^(5/2)*(8 + 20*c^2*x^2 + 35*c^4*x^4) + b*c*x*(2520 + 420*c^2*x^2 + 189*c^4*x^4 - 2250*c^6*x^6 + 1225*c^8*x^8) - 315*b*(1 - c^2*x^2)^(5/2)*(8 + 20*c^2*x^2 + 35*c^4*x^4)*ArcSin[c*x]))/(99225*c^6*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d(1 - c^2 x^2)^2 (35c^4 x^4 + 20c^2 x^2 + 8)}{315c^6} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{9c^6 d^3} +$$

$$\frac{2(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^6 d^2} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^6 d}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bd\sqrt{d-c^2dx^2} \int (1-c^2x^2)^2 (35c^4x^4 + 20c^2x^2 + 8) dx}{315c^5\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^6d^3} + \\
& \frac{2(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d^2} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^6d} \\
& \quad \downarrow 1467 \\
& \frac{bd\sqrt{d-c^2dx^2} \int (35c^8x^8 - 50c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{315c^5\sqrt{1-c^2x^2}} - \\
& \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^6d^3} + \frac{2(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d^2} - \\
& \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^6d} \\
& \quad \downarrow 2009 \\
& -\frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^6d^3} + \frac{2(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d^2} - \\
& \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^6d} + \frac{bd\left(\frac{35c^8x^9}{9} - \frac{50c^6x^7}{7} + \frac{3c^4x^5}{5} + \frac{4c^2x^3}{3} + 8x\right)\sqrt{d-c^2dx^2}}{315c^5\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `(b*d*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (50*c^6*x^7)/7 + (35*c^8*x^9)/9))/(315*c^5*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^6*d) + (2*(d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^6*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcSin[c*x]))/(9*c^6*d^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 1467 `Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_) , x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

method	result
orering	$\frac{(20825c^{10}x^{10} - 50900c^8x^8 + 29457c^6x^6 + 2730c^4x^4 + 19320c^2x^2 - 15120)(-c^2dx^2 + d)^{\frac{3}{2}}(a + b \arcsin(cx))}{99225c^6(cx-1)(cx+1)(c^2x^2-1)} - \frac{(1225c^8x^8 - 2250c^6x^6 + \dots)}{\dots}$
default	Expression too large to display
parts	Expression too large to display

input `int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `1/99225*(20825*c^10*x^10-50900*c^8*x^8+29457*c^6*x^6+2730*c^4*x^4+19320*c^2*x^2-15120)/c^6/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-1/99225/x^4*(1225*c^8*x^8-2250*c^6*x^6+189*c^4*x^4+420*c^2*x^2+2520)/c^6/(c*x-1)/(c*x+1)*(5*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-3*x^6*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*c^2*d+x^5*(-c^2*d*x^2+d)^(3/2)*b*c/(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.73

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx =$$

$$\frac{(1225 bc^9 dx^9 - 2250 bc^7 dx^7 + 189 bc^5 dx^5 + 420 bc^3 dx^3 + 2520 bcdx) \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} + 315 (35$$

input `integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `-1/99225*((1225*b*c^9*d*x^9 - 2250*b*c^7*d*x^7 + 189*b*c^5*d*x^5 + 420*b*c^3*d*x^3 + 2520*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 315*(35*a*c^10*d*x^10 - 85*a*c^8*d*x^8 + 53*a*c^6*d*x^6 + a*c^4*d*x^4 + 4*a*c^2*d*x^2 - 8*a*d + (35*b*c^10*d*x^10 - 85*b*c^8*d*x^8 + 53*b*c^6*d*x^6 + b*c^4*d*x^4 + 4*b*c^2*d*x^2 - 8*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)`

Sympy [F(-1)]

Timed out.

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**5*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.69

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx =$$

$$-\frac{1}{315} \left(\frac{35 (-c^2 dx^2 + d)^{5/2} x^4}{c^2 d} + \frac{20 (-c^2 dx^2 + d)^{5/2} x^2}{c^4 d} + \frac{8 (-c^2 dx^2 + d)^{5/2}}{c^6 d} \right) b \arcsin(cx)$$

$$-\frac{1}{315} \left(\frac{35 (-c^2 dx^2 + d)^{5/2} x^4}{c^2 d} + \frac{20 (-c^2 dx^2 + d)^{5/2} x^2}{c^4 d} + \frac{8 (-c^2 dx^2 + d)^{5/2}}{c^6 d} \right) a$$

$$+ \frac{(1225 c^8 d^{3/2} x^9 - 2250 c^6 d^{3/2} x^7 + 189 c^4 d^{3/2} x^5 + 420 c^2 d^{3/2} x^3 + 2520 d^{3/2} x) b}{99225 c^5}$$

input `integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/315*(35*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*b*arcsin(c*x) - 1/315*(35*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*a + 1/99225*(1225*c^8*d^(3/2)*x^9 - 250*c^6*d^(3/2)*x^7 + 189*c^4*d^(3/2)*x^5 + 420*c^2*d^(3/2)*x^3 + 2520*d^(3/2)*x)*b/c^5`

Giac [F(-2)]

Exception generated.

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^5 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{3/2} dx$$

input `int(x^5*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^5*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d (-35\sqrt{-c^2 x^2 + 1} a c^8 x^8 + 50\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 3\sqrt{-c^2 x^2 + 1} a c^4 x^4 - 4\sqrt{-c^2 x^2 + 1} a c^2 x^2 + 4\sqrt{-c^2 x^2 + 1} a) + 315 b \arcsin(cx) x^5}{315 c^6}$$

input `int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d*(- 35*sqrt(- c**2*x**2 + 1)*a*c**8*x**8 + 50*sqrt(- c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt(- c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt(- c**2*x**2 + 1)*a - 315*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**7,x)*b*c**8 + 315*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**5,x)*b*c**6)/(315*c**6)`

3.79 $\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	842
Mathematica [A] (verified)	843
Rubi [A] (verified)	843
Maple [A] (verified)	845
Fricas [A] (verification not implemented)	845
Sympy [F]	846
Maxima [A] (verification not implemented)	846
Giac [F(-2)]	847
Mupad [F(-1)]	847
Reduce [F]	848

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{2bdx\sqrt{d - c^2 dx^2}}{35c^3\sqrt{1 - c^2 x^2}} + \frac{bdx^3\sqrt{d - c^2 dx^2}}{105c\sqrt{1 - c^2 x^2}} - \frac{8bcdx^5\sqrt{d - c^2 dx^2}}{175\sqrt{1 - c^2 x^2}} + \frac{bc^3 dx^7\sqrt{d - c^2 dx^2}}{49\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^4 d} + \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^4 d^2}$$

output

```
2/35*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/105*b*d*x^3*(-c^2
*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-8/175*b*c*d*x^5*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)+1/49*b*c^3*d*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2
)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/c^4/d+1/7*(-c^2*d*x^2+d)^(7/2
)*(a+b*arcsin(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.56

$$\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{d\sqrt{d - c^2 dx^2} \left(-105a(1 - c^2 x^2)^{5/2} (2 + 5c^2 x^2) + bcx(210 + 35c^2 x^2 - 168c^4 x^4 + 75c^6 x^6) \right) + b \arcsin(cx)}{3675c^4 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]
```

output

```
(d*Sqrt[d - c^2*d*x^2]*(-105*a*(1 - c^2*x^2)^(5/2)*(2 + 5*c^2*x^2) + b*c*x*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6) - 105*b*(1 - c^2*x^2)^(5/2)*(2 + 5*c^2*x^2)*ArcSin[c*x]))/(3675*c^4*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d(1 - c^2 x^2)^2(5c^2 x^2 + 2)}{35c^4} dx}{\sqrt{1 - c^2 x^2}} + \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^4 d^2}$$

$$\frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^4 d}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bd\sqrt{d-c^2dx^2} \int (1-c^2x^2)^2 (5c^2x^2+2) dx}{35c^3\sqrt{1-c^2x^2}} + \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^4d^2} - \\
& \quad \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^4d} \\
& \quad \downarrow \text{290} \\
& \frac{bd\sqrt{d-c^2dx^2} \int (5c^6x^6 - 8c^4x^4 + c^2x^2 + 2) dx}{35c^3\sqrt{1-c^2x^2}} + \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^4d^2} - \\
& \quad \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^4d} \\
& \quad \downarrow \text{2009} \\
& \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^4d^2} - \frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))}{5c^4d} + \\
& \quad \frac{bd\left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x\right) \sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `(b*d*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - (8*c^4*x^5)/5 + (5*c^6*x^7)/7))/(35*c^3*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^4*d) + ((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^4*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 290 `Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98

method	result
orering	$\frac{(325c^8x^8 - 866c^6x^6 + 553c^4x^4 + 420c^2x^2 - 280)(-c^2dx^2 + d)^{\frac{3}{2}}(a + b \arcsin(cx))}{1225c^4(cx-1)(cx+1)(c^2x^2-1)} - \frac{(75c^6x^6 - 168c^4x^4 + 35c^2x^2 + 210)}{3x^2(-c^2dx^2 + d)^{\frac{3}{2}}}$
default	$a \left(-\frac{x^2(-c^2dx^2 + d)^{\frac{5}{2}}}{7c^2d} - \frac{2(-c^2dx^2 + d)^{\frac{5}{2}}}{35dc^4} \right) + b \left(-\frac{\sqrt{-d(c^2x^2-1)}(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2x^2+1} + 104c^4x^4 + 112ic^2x^2 + 210)}{3675x^2} \right)$
parts	$a \left(-\frac{x^2(-c^2dx^2 + d)^{\frac{5}{2}}}{7c^2d} - \frac{2(-c^2dx^2 + d)^{\frac{5}{2}}}{35dc^4} \right) + b \left(-\frac{\sqrt{-d(c^2x^2-1)}(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2x^2+1} + 104c^4x^4 + 112ic^2x^2 + 210)}{3675x^2} \right)$

input

```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/1225*(325*c^8*x^8-866*c^6*x^6+553*c^4*x^4+420*c^2*x^2-280)/c^4/(c*x-1)/(
c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-1/3675/x^2*(75*c
^6*x^6-168*c^4*x^4+35*c^2*x^2+210)/c^4/(c*x-1)/(c*x+1)*(3*x^2*(-c^2*d*x^2+
d)^(3/2)*(a+b*arcsin(c*x))-3*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*c
^2*d+x^3*(-c^2*d*x^2+d)^(3/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int x^3(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{(75bc^7dx^7 - 168bc^5dx^5 + 35bc^3dx^3 + 210bcdx)\sqrt{-c^2dx^2 + d}\sqrt{-c^2x^2 + 1} + 105(5ac^8dx^8 - 13ac^6dx^6 + 3675c^6dx^4 - 3675c^4dx^2 + 210c^2d)}{3675(c^2dx^2 + d)^{3/2}}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `-1/3675*((75*b*c^7*d*x^7 - 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 + 210*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 105*(5*a*c^8*d*x^8 - 13*a*c^6*d*x^6 + 9*a*c^4*d*x^4 + a*c^2*d*x^2 - 2*a*d + (5*b*c^8*d*x^8 - 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 + b*c^2*d*x^2 - 2*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)`

Sympy [F]

$$\int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^3 (-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx)) dx$$

input `integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x)), x)`

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.66

$$\begin{aligned} & \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \\ & -\frac{1}{35} \left(\frac{5(-c^2 dx^2 + d)^{5/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{5/2}}{c^4 d} \right) b \arcsin(cx) \\ & -\frac{1}{35} \left(\frac{5(-c^2 dx^2 + d)^{5/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{5/2}}{c^4 d} \right) a \\ & + \frac{(75 c^6 d^{3/2} x^7 - 168 c^4 d^{3/2} x^5 + 35 c^2 d^{3/2} x^3 + 210 d^{3/2} x)}{3675 c^3} \end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
-1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*b*arcsin(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a + 1/3675*(75*c^6*d^(3/2)*x^7 - 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 + 210*d^(3/2)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^3 (a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2} dx$$

input

```
int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output

```
int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d (-5\sqrt{-c^2 x^2 + 1} a c^6 x^6 + 8\sqrt{-c^2 x^2 + 1} a c^4 x^4 - \sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a + b \arcsin(cx))}{35c^4}$$

input `int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d*(-5*sqrt(-c**2*x**2+1)*a*c**6*x**6+8*sqrt(-c**2*x**2+1)*a*c**4*x**4-sqrt(-c**2*x**2+1)*a*c**2*x**2-2*sqrt(-c**2*x**2+1)*a-35*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**5,x)*b*c**6+35*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*b*c**4)/(35*c**4)`

3.80 $\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	849
Mathematica [A] (verified)	849
Rubi [A] (verified)	850
Maple [A] (verified)	851
Fricas [A] (verification not implemented)	852
Sympy [F]	852
Maxima [A] (verification not implemented)	852
Giac [F(-2)]	853
Mupad [F(-1)]	853
Reduce [F]	854

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{bdx\sqrt{d - c^2 dx^2}}{5c\sqrt{1 - c^2 x^2}} - \frac{2bcdx^3\sqrt{d - c^2 dx^2}}{15\sqrt{1 - c^2 x^2}} + \frac{bc^3 dx^5\sqrt{d - c^2 dx^2}}{25\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5c^2 d}$$

```
output 1/5*b*d*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2/15*b*c*d*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/25*b*c^3*d*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{d\sqrt{d - c^2 dx^2} \left(\frac{bc \left(x - \frac{2c^2 x^3}{3} + \frac{c^4 x^5}{5} \right)}{\sqrt{1 - c^2 x^2}} - (-1 + c^2 x^2)^2 (a + b \arcsin(cx)) \right)}{5c^2}$$

```
input Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]
```

output

$$\frac{(d\sqrt{d - c^2dx^2} * ((b*c*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/\sqrt{1 - c^2*x^2} - (-1 + c^2*x^2)^2*(a + b*\text{ArcSin}[c*x]))}{(5*c^2)}$$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5182, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5182}$$

$$\frac{bd\sqrt{d - c^2dx^2} \int (1 - c^2x^2)^2 dx}{5c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{5c^2d}$$

$$\downarrow \text{210}$$

$$\frac{bd\sqrt{d - c^2dx^2} \int (c^4x^4 - 2c^2x^2 + 1) dx}{5c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{5c^2d}$$

$$\downarrow \text{2009}$$

$$\frac{bd\left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x\right)\sqrt{d - c^2dx^2}}{5c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{5c^2d}$$

input

$$\text{Int}[x*(d - c^2*d*x^2)^(3/2)*(a + b*\text{ArcSin}[c*x]),x]$$

output

$$\frac{(b*d*\sqrt{d - c^2*d*x^2}*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(5*c*\sqrt{1 - c^2*x^2}) - ((d - c^2*d*x^2)^(5/2)*(a + b*\text{ArcSin}[c*x]))/(5*c^2*d)}$$

Defintions of rubi rules used

rule 210 $\text{Int}[(a_) + (b_)*(x_)^2]^{(p_)} , x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^2)^p, x], x] /;$ $\text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{IGtQ}\{p, 0\}$

rule 2009 $\text{Int}[u_ , x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /;$ $\text{SumQ}[u]$

rule 5182 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)]^{(n_)}*(x_)*((d_) + (e_)*(x_)^2)^{(p_)} , x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, p\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}\{n, 0\} \ \&\& \ \text{NeQ}\{p, -1\}$

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

method	result
orering	$\frac{(27c^6x^6 - 88c^4x^4 + 115c^2x^2 - 30)(-c^2dx^2 + d)^{\frac{3}{2}}(a + b \arcsin(cx))}{75c^2(cx-1)(cx+1)(c^2x^2-1)} - \frac{(3c^4x^4 - 10c^2x^2 + 15) \left((-c^2dx^2 + d)^{\frac{3}{2}}(a + b \arcsin(cx)) - 3x^2 \sqrt{-c^2dx^2 + d} \right)}{75c^2(cx-1)(cx+1)}$
default	$-\frac{a(-c^2dx^2 + d)^{\frac{5}{2}}}{5c^2d} + b \left(-\frac{\sqrt{-d(c^2x^2-1)} \left(16c^6x^6 - 28c^4x^4 - 16i\sqrt{-c^2x^2+1}x^5c^5 + 13c^2x^2 + 20i\sqrt{-c^2x^2+1}x^3c^3 - 5i\sqrt{-c^2x^2+1} \right)}{800c^2(c^2x^2-1)} \right)$
parts	$-\frac{a(-c^2dx^2 + d)^{\frac{5}{2}}}{5c^2d} + b \left(-\frac{\sqrt{-d(c^2x^2-1)} \left(16c^6x^6 - 28c^4x^4 - 16i\sqrt{-c^2x^2+1}x^5c^5 + 13c^2x^2 + 20i\sqrt{-c^2x^2+1}x^3c^3 - 5i\sqrt{-c^2x^2+1} \right)}{800c^2(c^2x^2-1)} \right)$

input $\text{int}(x*(-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x)), x, \text{method}=_RETURNVERBOSE)$

output
$$\frac{1}{75}*(27*c^6*x^6-88*c^4*x^4+115*c^2*x^2-30)/c^2/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x))-1/75*(3*c^4*x^4-10*c^2*x^2+15)/c^2/(c*x-1)/(c*x+1)*((-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x))-3*x^2*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))*c^2*d+x*(-c^2*d*x^2+d)^{(3/2)}*b*c/(-c^2*x^2+1)^{(1/2)}$$

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{(3bc^5 dx^5 - 10bc^3 dx^3 + 15bcdx)\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1} + 15(ac^6 dx^6 - 3ac^4 dx^4 + 3ac^2 dx^2 - ad + b^2 d^2)\arcsin(cx)}{75(c^4 x^2 - c^2)}$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `-1/75*((3*b*c^5*d*x^5 - 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 15*(a*c^6*d*x^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d + (b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)`

Sympy [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx)) dx$$

input `integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x)), x)`

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.57

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = -\frac{(-c^2 dx^2 + d)^{5/2} b \arcsin(cx)}{5 c^2 d} - \frac{(-c^2 dx^2 + d)^{5/2} a}{5 c^2 d} + \frac{(3 c^4 d^{5/2} x^5 - 10 c^2 d^{5/2} x^3 + 15 d^{5/2} x) b}{75 cd}$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/5*(-c^2*d*x^2 + d)^(5/2)*b*arcsin(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a/(c^2*d) + 1/75*(3*c^4*d^(5/2)*x^5 - 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*b/(c*d)`

Giac [F(-2)]

Exception generated.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2} dx$$

input `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)`

output `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d (-\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 2\sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a - 5(\int \sqrt{-c^2 x^2 + 1} dx) b c^4 + 5 \int \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx) b c^4}{5c^2}$$

input `int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d*(-sqrt(-c**2*x**2+1)*a*c**4*x**4+2*sqrt(-c**2*x**2+1)*a*c**2*x**2-sqrt(-c**2*x**2+1)*a-5*int(sqrt(-c**2*x**2+1)*a*asin(c*x)*x**3,x)*b*c**4+5*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*b*c**2)/(5*c**2)`

3.81
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx$$

Optimal result	855
Mathematica [A] (verified)	856
Rubi [A] (verified)	856
Maple [A] (verified)	860
Fricas [F]	860
Sympy [F]	861
Maxima [F]	861
Giac [F(-2)]	861
Mupad [F(-1)]	862
Reduce [F]	862

Optimal result

Integrand size = 27, antiderivative size = 280

$$\begin{aligned} & \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \\ & -\frac{4bcdx\sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} + \frac{bc^3 dx^3 \sqrt{d - c^2 dx^2}}{9\sqrt{1 - c^2 x^2}} \\ & + \frac{1}{3}(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{d - c^2 dx^2}(ad + bd \arcsin(cx)) \\ & - \frac{2d\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ & + \frac{ibd\sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ & - \frac{ibd\sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-4/3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/9*b*c^3*d*x^3*(-c^2
*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*
x))+(-c^2*d*x^2+d)^(1/2)*(a*d+b*d*arcsin(c*x))-2*d*(-c^2*d*x^2+d)^(1/2)*(a
+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+I*b*d
*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1
/2)-I*b*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x
^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.99

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = -\frac{1}{3} ad(-4 + c^2 x^2) \sqrt{d - c^2 dx^2} + ad^{3/2} \log(x) - ad^{3/2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) + \frac{bd\sqrt{d - c^2 dx^2}(-cx + \sqrt{1 - c^2 x^2} \arcsin(cx) + \arcsin(cx)) \log(1 - e^{i \arcsin(cx)})}{36 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x,x]
```

output

```
-1/3*(a*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) - ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])]) + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])])/Sqrt[1 - c^2*x^2] - (b*d*Sqrt[d - c^2*d*x^2]*(9*c*x - 3*ArcSin[c*x]*(3*Sqrt[1 - c^2*x^2] + Cos[3*ArcSin[c*x]]) + Sin[3*ArcSin[c*x]]))/(36*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5202, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx$$

↓ 5202

$$d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} dx - \frac{bcd\sqrt{d - c^2 dx^2} \int (1 - c^2 x^2) dx}{3\sqrt{1 - c^2 x^2} b \arcsin(cx)} + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a +$$

↓ 2009

$$\begin{aligned}
& d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} dx + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \\
& \quad \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5198} \\
& d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int 1 dx}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{24} \\
& d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \right) + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5218} \\
& d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx)}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \right) + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{3042} \\
& d \left(\frac{\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \right) + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{4671} \\
& d \left(\frac{\sqrt{d - c^2 dx^2} (-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \right) + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}}
\end{aligned}$$

↓ 2715

$$d \left(\frac{\sqrt{d - c^2 dx^2} (ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \right. \\ \left. \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3}\right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \right.$$

↓ 2838

$$d \left(\frac{\sqrt{d - c^2 dx^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \right. \\ \left. \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3}\right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \right.$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x,x]
```

output

```
-1/3*(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/Sqrt[1 - c^2*x^2] + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 + d*(-((b*c*x*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]) + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838 $\text{Int}[\text{Log}[(c_)*(d_)+(e_)*(x_)^{(n_)}]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_)+(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c+d*x)^m*(\text{ArcTanh}[E^{(I*(e+f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1-E^{(I*(e+f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1+E^{(I*(e+f*x))}], x], x) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5198 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*((f_)*(x_))^{(m_)}*\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d+e*x^2]*((a+b*\text{ArcSin}[c*x])^n/(f*(m+2))), x] + (\text{Simp}[(1/(m+2))*\text{Simp}[\text{Sqrt}[d+e*x^2]/\text{Sqrt}[1-c^2*x^2]] \text{Int}[(f*x)^m*((a+b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1-c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m+2)))*\text{Simp}[\text{Sqrt}[d+e*x^2]/\text{Sqrt}[1-c^2*x^2]] \text{Int}[(f*x)^{(m+1)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])$

rule 5202 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*((f_)*(x_))^{(m_)}*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d+e*x^2)^p*((a+b*\text{ArcSin}[c*x])^n/(f*(m+2*p+1))), x] + (\text{Simp}[2*d*(p/(m+2*p+1)) \text{Int}[(f*x)^m*(d+e*x^2)^{(p-1)}*(a+b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+2*p+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1-c^2*x^2)^{(p-1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ !\text{LtQ}[m, -1]$

rule 5218 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*(x_)^{(m_)}*\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/c^{(m+1)})*\text{Simp}[\text{Sqrt}[1-c^2*x^2]/\text{Sqrt}[d+e*x^2]] \text{Subst}[\text{Int}[(a+b*x)^n*\text{Sin}[x]^m, x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.88

method	result
default	$\frac{(-c^2dx^2+d)^{\frac{3}{2}}a}{3} - ad^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right) + ad\sqrt{-c^2dx^2+d} - \frac{b\sqrt{-d(c^2x^2-1)}d\sqrt{-c^2x^2+1}x^3c^3}{9(c^2x^2-1)} + \frac{4b\sqrt{-d(c^2x^2-1)}d\sqrt{-c^2x^2+1}x^3c^3}{9(c^2x^2-1)}$
parts	$\frac{(-c^2dx^2+d)^{\frac{3}{2}}a}{3} - ad^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right) + ad\sqrt{-c^2dx^2+d} - \frac{b\sqrt{-d(c^2x^2-1)}d\sqrt{-c^2x^2+1}x^3c^3}{9(c^2x^2-1)} + \frac{4b\sqrt{-d(c^2x^2-1)}d\sqrt{-c^2x^2+1}x^3c^3}{9(c^2x^2-1)}$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output `1/3*(-c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*d*(-c^2*d*x^2+d)^(1/2)-1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3*c^3+4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*c+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*arcsin(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*arcsin(c*x)*x^2*c^2-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*arcsin(c*x)-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x,x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))}{x} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))/x, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x) - 1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x,x)`output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x, x)`**Reduce [F]**

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx = \frac{\sqrt{d} d \left(-\sqrt{-c^2 x^2 + 1} a c^2 x^2 + 4\sqrt{-c^2 x^2 + 1} a + 3 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x} dx \right) \right)}{3}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x,x)`output `(sqrt(d)*d*(-sqrt(-c**2*x**2 + 1)*a*c**2*x**2 + 4*sqrt(-c**2*x**2 + 1)*a + 3*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x,x)*b - 3*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x,x)*b*c**2 + 3*log(tan(asin(c*x)/2))*a - 4*a))/3`

3.82
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx$$

Optimal result	863
Mathematica [A] (verified)	864
Rubi [A] (verified)	864
Maple [A] (verified)	868
Fricas [F]	869
Sympy [F]	869
Maxima [F]	870
Giac [F(-2)]	870
Mupad [F(-1)]	870
Reduce [F]	871

Optimal result

Integrand size = 27, antiderivative size = 297

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx &= -\frac{bcd\sqrt{d - c^2 dx^2}}{2x\sqrt{1 - c^2 x^2}} + \frac{bc^3 dx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \\ &- \frac{3}{2}c^2 d\sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{2x^2} \\ &+ \frac{3c^2 d\sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ &- \frac{3ibc^2 d\sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{2\sqrt{1 - c^2 x^2}} \\ &+ \frac{3ibc^2 d\sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{2\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/2*b*c*d*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)+b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/2*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^2+3*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3/2*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+3/2*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.31

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = -\frac{ad(1 + 2c^2 x^2) \sqrt{d - c^2 dx^2}}{2x^2} - \frac{3}{2} ac^2 d^{3/2} \log(x) + \frac{3}{2} ac^2 d^{3/2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) + \frac{bc^2 d \sqrt{d - c^2 dx^2} (cx - \sqrt{1 - c^2 x^2} \arcsin(cx) - \arcsin(cx) \log(1 - e^{i \arcsin(cx)}))}{2}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/2*(a*d*(1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^2 - (3*a*c^2*d^(3/2)*Log[x])/2 + (3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/2 + (b*c^2*d*Sqrt[d - c^2*d*x^2]*(c*x - Sqrt[1 - c^2*x^2]*ArcSin[c*x] - ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - I*PolyLog[2, -E^(I*ArcSin[c*x])] + I*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2] + (b*c^2*d^2*Sqrt[1 - c^2*x^2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])] + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2]))/(8*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5200, 244, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx$$

↓ 5200

$$\begin{aligned}
& -\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} dx + \frac{bcd\sqrt{d-c^2dx^2} \int \frac{1-c^2x^2}{x^2} dx}{2\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{244} \\
& -\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} dx + \frac{bcd\sqrt{d-c^2dx^2} \int (\frac{1}{x^2} - c^2) dx}{2\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{2x^2} \\
& \quad \downarrow \text{2009} \\
& -\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} dx - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \\
& \quad \frac{bcd(c^2(-x) - \frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \\
& \quad \downarrow \text{5198} \\
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \int 1 dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x) - \frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \\
& \quad \downarrow \text{24} \\
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x) - \frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \\
& \quad \downarrow \text{5218} \\
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x) - \frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \\
& \quad \downarrow \text{3042}
\end{aligned}$$

$$\begin{aligned}
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b \arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right. \\
& \quad \left. \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x)-\frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow 4671 \\
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2}(-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} \right. \\
& \quad \left. \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x)-\frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow 2715 \\
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2}(ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{\sqrt{1-c^2x^2}} \right. \\
& \quad \left. \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x)-\frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow 2838 \\
& -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} \right. \\
& \quad \left. \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd(c^2(-x)-\frac{1}{x})\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right)
\end{aligned}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
(b*c*d*(-x^(-1) - c^2*x)*Sqrt[d - c^2*d*x^2])/(2*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(2*x^2) - (3*c^2*d*((b*c*x*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])) + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2])/2
```

Definitions of rubi rules used

- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 244 `Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[Expand Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`
- rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 5198 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.47

method	result
default	$a \left(-\frac{(-c^2 d x^2 + d)^{\frac{5}{2}}}{2 d x^2} - \frac{3 c^2 \left(\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{3} + d \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right) \right)}{2} \right) + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)}}{\dots} \right)$
parts	$a \left(-\frac{(-c^2 d x^2 + d)^{\frac{5}{2}}}{2 d x^2} - \frac{3 c^2 \left(\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{3} + d \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right) \right)}{2} \right) + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)}}{\dots} \right)$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output

```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(5/2)-3/2*c^2*(1/3*(-c^2*d*x^2+d)^(3/2)+d*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))))+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)*c^2*d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)*c^2*d/(c^2*x^2-1)-1/2*d*(c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2+3*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2)))+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))*c^2*d/(2*c^2*x^2-2))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")
```

output

```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))}{x^3} dx$$

input

```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**3,x)
```

output

```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^3, x) + 1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 + d)^(5/2)/(d*x^2))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{3/2}}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^3,x)`

output `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx = \frac{\sqrt{d} d \left(-8\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 4\sqrt{-c^2 x^2 + 1} a + 8 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^3} dx \right) \right)}{8x^2}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^3,x)`

output `(sqrt(d)*d*(- 8*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - 4*sqrt(- c**2*x**2 + 1)*a + 8*int((sqrt(- c**2*x**2 + 1)*asin(c*x))/x**3,x)*b*x**2 - 8*int((sqrt(- c**2*x**2 + 1)*asin(c*x))/x,x)*b*c**2*x**2 - 12*log(tan(asin(c*x)/2))*a*c**2*x**2 + 9*a*c**2*x**2))/(8*x**2)`

3.83 $\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^5} dx$

Optimal result	872
Mathematica [A] (verified)	873
Rubi [A] (verified)	873
Maple [A] (verified)	878
Fricas [F]	878
Sympy [F]	879
Maxima [F]	879
Giac [F(-2)]	879
Mupad [F(-1)]	880
Reduce [F]	880

Optimal result

Integrand size = 27, antiderivative size = 307

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{x^5} dx = -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{1-c^2x^2}} + \frac{3c^2d\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{8x^2} - \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{4x^4} - \frac{3c^4d\sqrt{d-c^2dx^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{4\sqrt{1-c^2x^2}} + \frac{3ibc^4d\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{8\sqrt{1-c^2x^2}} - \frac{3ibc^4d\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{8\sqrt{1-c^2x^2}}$$

output

```
-1/12*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^3/(-c^2*x^2+1)^(1/2)+5/8*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)+3/8*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^4-3/4*c^4*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+3/8*I*b*c^4*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3/8*I*b*c^4*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 4.20 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.61

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \frac{ad(-2 + 5c^2 x^2) \sqrt{d - c^2 dx^2}}{8x^4} + \frac{3}{8} ac^4 d^{3/2} \log(x) - \frac{3}{8} ac^4 d^{3/2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) - \frac{bc^4 d^2 \sqrt{1 - c^2 x^2} (-2 \cot\left(\frac{1}{2} \arcsin(cx)\right) - \arcsin(cx) \csc^2\left(\frac{1}{2} \arcsin(cx)\right))}{8x^4}$$

input `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^5,x]`

output `(a*d*(-2 + 5*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(8*x^4) + (3*a*c^4*d^(3/2)*Log[x])/8 - (3*a*c^4*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/8 - (b*c^4*d^2*Sqrt[1 - c^2*x^2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])]) - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])]) - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])]) + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2]))/(8*Sqrt[d - c^2*d*x^2]) + (b*c^4*d*Sqrt[d - c^2*d*x^2]*(8*Cot[ArcSin[c*x]/2] + 6*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - c*x*Csc[ArcSin[c*x]/2]^4 - 3*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^4 - 24*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 24*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (24*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (24*I)*PolyLog[2, E^(I*ArcSin[c*x])] - 6*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 + 3*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^4 - (16*Sin[ArcSin[c*x]/2]^4)/(c^3*x^3) + 8*Tan[ArcSin[c*x]/2]))/(192*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5200, 244, 2009, 5196, 15, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx$$

$$\begin{aligned}
& \downarrow 5200 \\
& -\frac{3}{4}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x^3} dx + \frac{bcd\sqrt{d-c^2dx^2} \int \frac{1-c^2x^2}{x^4} dx}{4\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4} \\
& \downarrow 244 \\
& -\frac{3}{4}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x^3} dx + \frac{bcd\sqrt{d-c^2dx^2} \int \left(\frac{1}{x^4} - \frac{c^2}{x^2}\right) dx}{4\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4} \\
& \downarrow 2009 \\
& -\frac{3}{4}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x^3} dx - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4} + \\
& \quad \frac{bcd\left(\frac{c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \\
& \downarrow 5196 \\
& -\frac{3}{4}c^2d \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{bc\sqrt{d-c^2dx^2} \int \frac{1}{x^2} dx}{2\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{2x^2} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd\left(\frac{c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \\
& \downarrow 15 \\
& -\frac{3}{4}c^2d \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{2x^2} - \frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}} \right) - \\
& \quad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd\left(\frac{c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \\
& \downarrow 5218
\end{aligned}$$

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arcsin(cx)}{cx}d\arcsin(cx)}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

↓ 3042

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}\int(a+b\arcsin(cx))\csc(\arcsin(cx))d\arcsin(cx)}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

↓ 4671

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}(-b\int\log(1-e^{i\arcsin(cx)})d\arcsin(cx)+b\int\log(1+e^{i\arcsin(cx)})d\arcsin(cx)-2\arctan\left(\frac{e^{i\arcsin(cx)}}{1+e^{i\arcsin(cx)}}\right))}{2\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

↓ 2715

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}(ib\int e^{-i\arcsin(cx)}\log(1-e^{i\arcsin(cx)})de^{i\arcsin(cx)}-ib\int e^{-i\arcsin(cx)}\log(1+e^{i\arcsin(cx)})de^{i\arcsin(cx)})}{2\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

↓ 2838

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}(-2\operatorname{arctanh}(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\operatorname{PolyLog}(2,-e^{i\arcsin(cx)})-ib\operatorname{PolyLog}(2,e^{i\arcsin(cx)}))}{2\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{4x^4}+\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x^5,x]`

output

```
(b*c*d*(-1/3*1/x^3 + c^2/x)*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (
(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(4*x^4) - (3*c^2*d*(-1/2*(b*c*S
qrt[d - c^2*d*x^2])/(x*Sqrt[1 - c^2*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*Ar
cSin[c*x])))/(2*x^2) - (c^2*Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])*Arc
Tanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog
[2, E^(I*ArcSin[c*x])])))/(2*Sqrt[1 - c^2*x^2])))/4
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5196

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x],
x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int
[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))] Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5218

```
Int((((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{4dx^4} + \frac{ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{8dx^2} + \frac{ac^4(-c^2dx^2+d)^{\frac{3}{2}}}{8} - \frac{3ac^4d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{8} + \frac{3ac^4d\sqrt{-c^2dx^2+d}}{8}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{4dx^4} + \frac{ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{8dx^2} + \frac{ac^4(-c^2dx^2+d)^{\frac{3}{2}}}{8} - \frac{3ac^4d^{\frac{3}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{8} + \frac{3ac^4d\sqrt{-c^2dx^2+d}}{8}$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^5,x,method=_RETURNVERBOSE)`

output `-1/4*a/d/x^4*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^4*(-c^2*d*x^2+d)^(3/2)-3/8*a*c^4*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+3/8*a*c^4*d*(-c^2*d*x^2+d)^(1/2)+b*(1/24*d*(15*c^4*x^4*arcsin(c*x)-15*c^3*x^3*(-c^2*x^2+1)^(1/2)-21*c^2*x^2*arcsin(c*x)+2*c*x*(-c^2*x^2+1)^(1/2)+6*arcsin(c*x))*(-d*(c^2*x^2-1)^(1/2)/(c^2*x^2-1)/x^4-3*I*(-d*(c^2*x^2-1)^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*d*c^4/(8*c^2*x^2-8))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^5} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))}{x^5} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))/x**5,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))/x**5, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)}{x^5} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="maxima")`

output `-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^5, x) - 1/8*(3*c^4*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2)*c^4 - 3*sqrt(-c^2*d*x^2 + d)*c^4*d - (-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^2) + 2*(-c^2*d*x^2 + d)^(5/2)/(d*x^4))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{3/2}}{x^5} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^5,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^5} dx = \frac{\sqrt{d} d (5\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a + 8 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx)}{x^5} dx \right))}{8x^4}$$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))/x^5,x)
```

output

```
(sqrt(d)*d*(5*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)
)*a + 8*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**5,x)*b*x**4 - 8*int((sqr
t(-c**2*x**2 + 1)*asin(c*x))/x**3,x)*b*c**2*x**4 + 3*log(tan(asin(c*x)/2
))*a*c**4*x**4)/(8*x**4)
```

3.84 $\int x^4(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	881
Mathematica [A] (verified)	882
Rubi [A] (verified)	882
Maple [C] (verified)	887
Fricas [F]	888
Sympy [F(-1)]	889
Maxima [F]	889
Giac [A] (verification not implemented)	889
Mupad [F(-1)]	890
Reduce [F]	890

Optimal result

Integrand size = 27, antiderivative size = 430

$$\begin{aligned}
 \int x^4(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) dx = & \frac{3bd^2x^2\sqrt{d - c^2dx^2}}{512c^3\sqrt{1 - c^2x^2}} \\
 + & \frac{bd^2x^4\sqrt{d - c^2dx^2}}{512c\sqrt{1 - c^2x^2}} - \frac{31bcd^2x^6\sqrt{d - c^2dx^2}}{960\sqrt{1 - c^2x^2}} + \frac{21bc^3d^2x^8\sqrt{d - c^2dx^2}}{640\sqrt{1 - c^2x^2}} \\
 - & \frac{bc^5d^2x^{10}\sqrt{d - c^2dx^2}}{100\sqrt{1 - c^2x^2}} - \frac{3d^2x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{256c^4} \\
 - & \frac{d^2x^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{128c^2} + \frac{1}{32}d^2x^5\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) \\
 + & \frac{1}{16}dx^5(d - c^2dx^2)^{3/2}(a + b \arcsin(cx)) \\
 + & \frac{1}{10}x^5(d - c^2dx^2)^{5/2}(a + b \arcsin(cx)) + \frac{3d^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{512bc^5\sqrt{1 - c^2x^2}}
 \end{aligned}$$

output

$$\begin{aligned} & 3/512*b*d^2*x^2*(-c^2*d*x^2+d)^{(1/2)}/c^3/(-c^2*x^2+1)^{(1/2)}+1/512*b*d^2*x^4 \\ & 4*(-c^2*d*x^2+d)^{(1/2)}/c/(-c^2*x^2+1)^{(1/2)}-31/960*b*c*d^2*x^6*(-c^2*d*x^2+d)^{(1/2)}/(-c^2*x^2+1)^{(1/2)} \\ & +21/640*b*c^3*d^2*x^8*(-c^2*d*x^2+d)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}-1/100*b*c^5*d^2*x^{10}*(-c^2*d*x^2+d)^{(1/2)}/(-c^2*x^2+1)^{(1/2)} \\ & -3/256*d^2*x*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/c^4-1/128*d^2*x^3*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/c^2 \\ & +1/32*d^2*x^5*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))+1/16*d*x^5*(-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x))+1/10*x^5 \\ & (-c^2*d*x^2+d)^{(5/2)}*(a+b*\arcsin(c*x))+3/512*d^2*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))^2/b/c^5/(-c^2*x^2+1)^{(1/2)} \end{aligned}$$
Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.51

$$\int x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 \sqrt{d - c^2 dx^2} (225a^2 + b^2 c^2 x^2 (225 + 75c^2 x^2 - 1240c^4 x^4 + 1260c^6 x^6 - 384c^8 x^8) + 30a b \arcsin(cx))}{(38400 b^2 c^5 \sqrt{1 - c^2 x^2})}$$

input

$$\text{Integrate}[x^4*(d - c^2*d*x^2)^{(5/2)}*(a + b*\text{ArcSin}[c*x]),x]$$

output

$$\begin{aligned} & (d^2*\text{Sqrt}[d - c^2*d*x^2]*(225*a^2 + b^2*c^2*x^2*(225 + 75*c^2*x^2 - 1240*c^4*x^4 \\ & + 1260*c^6*x^6 - 384*c^8*x^8) + 30*a*b*c*x*\text{Sqrt}[1 - c^2*x^2]*(-15 - 10*c^2*x^2 \\ & + 248*c^4*x^4 - 336*c^6*x^6 + 128*c^8*x^8) + 30*b*(15*a + b*c*x*\text{Sqrt}[1 - c^2*x^2] \\ & *(-15 - 10*c^2*x^2 + 248*c^4*x^4 - 336*c^6*x^6 + 128*c^8*x^8))*\text{ArcSin}[c*x] + 225*b^2*\text{ArcSin}[c*x]^2))/(38400*b*c^5*\text{Sqrt}[1 - c^2*x^2]) \end{aligned}$$
Rubi [A] (verified)Time = 1.67 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {5202, 243, 49, 2009, 5202, 244, 2009, 5198, 15, 5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx \\
& \quad \downarrow \text{5202} \\
& \frac{1}{2}d \int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int x^5(1 - c^2 x^2)^2 dx}{10\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{10}x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{243} \\
& \frac{1}{2}d \int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int x^4(1 - c^2 x^2)^2 dx^2}{20\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{10}x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{49} \\
& \frac{1}{2}d \int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int (c^4 x^8 - 2c^2 x^6 + x^4) dx^2}{20\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{10}x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{2009} \\
& \frac{1}{2}d \int x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{10}x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \\
& \quad \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5202} \\
& \frac{1}{2}d \left(\frac{3}{8}d \int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd \sqrt{d - c^2 dx^2} \int x^5(1 - c^2 x^2) dx}{8\sqrt{1 - c^2 x^2}} + \frac{1}{8}x^5(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{10}x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{244} \\
& \frac{1}{2}d \left(\frac{3}{8}d \int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd \sqrt{d - c^2 dx^2} \int (x^5 - c^2 x^7) dx}{8\sqrt{1 - c^2 x^2}} + \frac{1}{8}x^5(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{10}x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{2009}
\end{aligned}$$

$$\frac{1}{2}d \left(\frac{3}{8}d \int x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx + \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \sqrt{d - c^2 dx^2}}{8\sqrt{1 - c^2 x^2}} \right. \\ \left. - \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \right)$$

↓ 5198

$$\frac{1}{2}d \left(\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^4 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{6\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^5 dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \right. \\ \left. - \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \right)$$

↓ 15

$$\frac{1}{2}d \left(\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^4 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^6 \sqrt{d - c^2 dx^2}}{36\sqrt{1 - c^2 x^2}} \right) + \frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \right. \\ \left. - \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \right)$$

↓ 5210

$$\frac{1}{2}d \left(\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \int \frac{x^2 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{4c^2} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right. \right. \\ \left. \left. - \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \right)$$

↓ 15

$$\frac{1}{2}d \left(\frac{3}{8}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{3 \int \frac{x^2 (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right. \right. \\ \left. \left. - \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \right)$$

↓ 5210

$$\frac{1}{2}d \left(\frac{3}{8}d \frac{\left(\sqrt{d - c^2 dx^2} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x \sqrt{1-c^2 x^2} (a+b \arcsin(cx))}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1-c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} \right) + \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}}$$

↓ 15

$$\frac{1}{2}d \left(\frac{3}{8}d \frac{\left(\sqrt{d - c^2 dx^2} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} dx}{2c^2} - \frac{x \sqrt{1-c^2 x^2} (a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1-c^2 x^2}} + \frac{1}{6}x^5 \sqrt{d - c^2 dx^2} \right) + \frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}}$$

↓ 5152

$$\frac{1}{10}x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{1}{2}d \left(\frac{1}{8}x^5 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{8}d \left(\frac{1}{6}x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{\sqrt{d - c^2 dx^2} \left(-\frac{x^3 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right)}{6\sqrt{1-c^2 x^2}} \right) + \frac{bcd^2 \left(\frac{c^4 x^{10}}{5} - \frac{c^2 x^8}{2} + \frac{x^6}{3} \right) \sqrt{d - c^2 dx^2}}{20\sqrt{1 - c^2 x^2}} \right)$$

input

Int[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]

output

$$\begin{aligned}
& -1/20*(b*c*d^2*\text{Sqrt}[d - c^2*d*x^2]*(x^6/3 - (c^2*x^8)/2 + (c^4*x^{10})/5))/\text{Sqrt}[1 - c^2*x^2] \\
& + (x^5*(d - c^2*d*x^2)^{(5/2)}*(a + b*\text{ArcSin}[c*x]))/10 + (d*(-1/8*(b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(x^6/6 - (c^2*x^8)/8))/\text{Sqrt}[1 - c^2*x^2] \\
& + (x^5*(d - c^2*d*x^2)^{(3/2)}*(a + b*\text{ArcSin}[c*x]))/8 + (3*d*(-1/36*(b*c*x^6*\text{Sqrt}[d - c^2*d*x^2])/ \\
& \text{Sqrt}[1 - c^2*x^2] + (x^5*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x]))/6 + (\text{Sqrt}[d - c^2*d*x^2]*((b*x^4)/(16*c) - (x^3*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x]))/(2*c^2) + (a + b*\text{ArcSin}[c*x])^2/(4*b*c^3)))/(4*c^2)))/(6*\text{Sqrt}[1 - c^2*x^2])))/8)/2
\end{aligned}$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_.)*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] /; \text{FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 49

$$\text{Int}[((a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 243

$$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{((m-1)/2)}*(a + b*x)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$$

rule 244

$$\text{Int}[((c_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

rule 5152

$$\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_)])*(b_.)^{(n_.)}/\text{Sqrt}[(d_) + (e_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 1106, normalized size of antiderivative = 2.57

method	result	size
default	Expression too large to display	1106
parts	Expression too large to display	1106

input

```
int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```

-1/10*a*x^3*(-c^2*d*x^2+d)^(7/2)/c^2/d-3/80*a/c^4*x*(-c^2*d*x^2+d)^(7/2)/d
+1/160*a/c^4*x*(-c^2*d*x^2+d)^(5/2)+1/128*a/c^4*d*x*(-c^2*d*x^2+d)^(3/2)+3
/256*a/c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+3/256*a/c^4*d^3/(c^2*d)^(1/2)*arctan
((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/512*(-d*(c^2*x^2-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/c^5/(c^2*x^2-1)*arcsin(c*x)^2*d^2+1/102400*(-d*(c^2*x^2-1
))^1/2)*(-512*I*(-c^2*x^2+1)^(1/2)*x^10*c^10+512*c^11*x^11+1280*I*(-c^2*x
^2+1)^(1/2)*x^8*c^8-1536*c^9*x^9-1120*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+1696*c^
7*x^7+400*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-832*c^5*x^5-50*I*(-c^2*x^2+1)^(1/2)
*x^2*c^2+170*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-10*c*x)*(I+10*arcsin(c*x))*d^2/c
^5/(c^2*x^2-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c
^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))*d^2/c^5/(c^2*x
^2-1)-3/819200*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)
*(11*I+40*arcsin(c*x))*cos(9*arcsin(c*x))*d^2/c^5/(c^2*x^2-1)+1/819200*(-d
*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(17*I+280*arcsin(
c*x))*sin(9*arcsin(c*x))*d^2/c^5/(c^2*x^2-1)+1/98304*(-d*(c^2*x^2-1))^(1/2
)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(5*I+72*arcsin(c*x))*cos(7*arcsin(c
*x))*d^2/c^5/(c^2*x^2-1)-1/98304*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1
/2)*c*x+c^2*x^2-1)*(11*I+24*arcsin(c*x))*sin(7*arcsin(c*x))*d^2/c^5/(c^2*x
^2-1)+1/12288*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*
(7*I+18*arcsin(c*x))*cos(5*arcsin(c*x))*d^2/c^5/(c^2*x^2-1)-5/12288*(-d...

```

Fricas [F]

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a) x^4 dx$$

input

```

integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas"
)

```

output

```

integral((a*c^4*d^2*x^8 - 2*a*c^2*d^2*x^6 + a*d^2*x^4 + (b*c^4*d^2*x^8 - 2
*b*c^2*d^2*x^6 + b*d^2*x^4)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)

```

Sympy [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**4*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{5}{2}} (b \arcsin(cx) + a) x^4 dx$$

input `integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate((c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) - 1/1280*(128*(-c^2*d*x^2 + d)^(7/2)*x^3/(c^2*d) - 8*(-c^2*d*x^2 + d)^(5/2)*x/c^4 + 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^4*d) - 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^4 - 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^4 - 15*d^(5/2)*arcsin(c*x)/c^5)*a`

Giac [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.02

$$\begin{aligned} \int x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx &= \frac{1}{10} \sqrt{-c^2 dx^2 + d} a c^4 d^2 x^9 \\ &- \frac{21}{80} \sqrt{-c^2 dx^2 + d} a c^2 d^2 x^7 + \frac{31}{160} \sqrt{-c^2 dx^2 + d} a d^2 x^5 - \frac{\sqrt{-c^2 dx^2 + d} a d^2 x^3}{128 c^2} \\ &- \frac{3 \sqrt{-c^2 dx^2 + d} a d^2 x}{256 c^4} - \frac{3 a d^3 \log(|-c \sqrt{-d} x + \sqrt{c^2 x^2 - 1} \sqrt{-d}|)}{256 c^5 \sqrt{-d}} \\ &+ \frac{122880 (c^2 x^2 - 1)^4 \sqrt{-c^2 x^2 + 1} b d^{\frac{5}{2}} x \arcsin(cx) + 168960 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b d^{\frac{5}{2}} x \arcsin(cx) + 7680}{1} \end{aligned}$$

input `integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `1/10*sqrt(-c^2*d*x^2 + d)*a*c^4*d^2*x^9 - 21/80*sqrt(-c^2*d*x^2 + d)*a*c^2*d^2*x^7 + 31/160*sqrt(-c^2*d*x^2 + d)*a*d^2*x^5 - 1/128*sqrt(-c^2*d*x^2 + d)*a*d^2*x^3/c^2 - 3/256*sqrt(-c^2*d*x^2 + d)*a*d^2*x/c^4 - 3/256*a*d^3*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^5*sqrt(-d)) + 1/1228800*(122880*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arcsin(c*x) + 168960*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arcsin(c*x) + 7680*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arcsin(c*x) - 12288*(c^2*x^2 - 1)^5*b*d^(5/2)/c + 9600*(-c^2*x^2 + 1)^(3/2)*b*d^(5/2)*x*arcsin(c*x) - 21120*(c^2*x^2 - 1)^4*b*d^(5/2)/c + 14400*sqrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arcsin(c*x) - 1280*(c^2*x^2 - 1)^3*b*d^(5/2)/c + 2400*(c^2*x^2 - 1)^2*b*d^(5/2)/c + 7200*b*d^(5/2)*arcsin(c*x)^2/c - 7200*(c^2*x^2 - 1)*b*d^(5/2)/c - 2149*b*d^(5/2)/c)/c^4`

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^4*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d^2 (15 a \operatorname{asin}(cx) a + 128 \sqrt{-c^2 x^2 + 1} a c^9 x^9 - 336 \sqrt{-c^2 x^2 + 1} a c^7 x^7 + 248 \sqrt{-c^2 x^2 + 1} a c^5 x^5 - 128 \sqrt{-c^2 x^2 + 1} a c^3 x^3 + 128 \sqrt{-c^2 x^2 + 1} a c x + 128 \sqrt{-c^2 x^2 + 1} b c^9 x^9 - 336 \sqrt{-c^2 x^2 + 1} b c^7 x^7 + 248 \sqrt{-c^2 x^2 + 1} b c^5 x^5 - 128 \sqrt{-c^2 x^2 + 1} b c^3 x^3 + 128 \sqrt{-c^2 x^2 + 1} b c x)}{c^4}$$

input `int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)`

output

```
(sqrt(d)*d**2*(15*asin(c*x)*a + 128*sqrt(-c**2*x**2 + 1)*a*c**9*x**9 - 36*sqrt(-c**2*x**2 + 1)*a*c**7*x**7 + 248*sqrt(-c**2*x**2 + 1)*a*c**5*x**5 - 10*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 - 15*sqrt(-c**2*x**2 + 1)*a*c*x + 1280*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**8,x)*b*c**9 - 2560*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**6,x)*b*c**7 + 1280*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*b*c**5))/(1280*c**5)
```


3.85 $\int x^2(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	892
Mathematica [A] (verified)	893
Rubi [A] (verified)	893
Maple [C] (verified)	898
Fricas [F]	899
Sympy [F(-1)]	899
Maxima [F]	899
Giac [A] (verification not implemented)	900
Mupad [F(-1)]	901
Reduce [F]	901

Optimal result

Integrand size = 27, antiderivative size = 351

$$\int x^2(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{5bd^2x^2\sqrt{d - c^2dx^2}}{256c\sqrt{1 - c^2x^2}} - \frac{59bcd^2x^4\sqrt{d - c^2dx^2}}{768\sqrt{1 - c^2x^2}} + \frac{17bc^3d^2x^6\sqrt{d - c^2dx^2}}{288\sqrt{1 - c^2x^2}} - \frac{bc^5d^2x^8\sqrt{d - c^2dx^2}}{64\sqrt{1 - c^2x^2}} - \frac{5d^2x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{128c^2} + \frac{5}{64}d^2x^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{5}{48}dx^3(d - c^2dx^2)^{3/2}(a + b \arcsin(cx)) + \frac{1}{8}x^3(d - c^2dx^2)^{5/2}(a + b \arcsin(cx)) + \frac{5d^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{256bc^3\sqrt{1 - c^2x^2}}$$

output

```
5/256*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-59/768*b*c*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+17/288*b*c^3*d^2*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/64*b*c^5*d^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/128*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2+5/64*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))+1/8*x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))+5/256*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.56

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 \sqrt{d - c^2 dx^2} (45a^2 + b^2 c^2 x^2 (45 - 177c^2 x^2 + 136c^4 x^4 - 36c^6 x^6) + 6abcx \sqrt{1 - c^2 x^2} (-15 + 118c^2 x^2 - 136c^4 x^4 + 48c^6 x^6) + 6b(15a + b c x \sqrt{1 - c^2 x^2} (-15 + 118c^2 x^2 - 136c^4 x^4 + 48c^6 x^6)) \operatorname{ArcSin}[c x] + 45b^2 \operatorname{ArcSin}[c x]^2)}{(2304 b^3 c^3 \sqrt{1 - c^2 x^2})}$$

input `Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output `(d^2*Sqrt[d - c^2*d*x^2]*(45*a^2 + b^2*c^2*x^2*(45 - 177*c^2*x^2 + 136*c^4*x^4 - 36*c^6*x^6) + 6*a*b*c*x*Sqrt[1 - c^2*x^2]*(-15 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) + 6*b*(15*a + b*c*x*Sqrt[1 - c^2*x^2]*(-15 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6))*ArcSin[c*x] + 45*b^2*ArcSin[c*x]^2))/(2304*b*c^3*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5202, 243, 49, 2009, 5202, 244, 2009, 5198, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5202$$

$$\frac{5}{8} d \int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int x^3(1 - c^2 x^2)^2 dx}{8 \sqrt{1 - c^2 x^2}} + \frac{1}{8} x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))$$

$$\downarrow 243$$

$$\begin{aligned}
& \frac{5}{8}d \int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))dx - \frac{bcd^2\sqrt{d - c^2dx^2} \int x^2(1 - c^2x^2)^2 dx^2}{16\sqrt{1 - c^2x^2}} + \\
& \quad \frac{1}{8}x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{49} \\
& \frac{5}{8}d \int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))dx - \frac{bcd^2\sqrt{d - c^2dx^2} \int (c^4x^6 - 2c^2x^4 + x^2) dx^2}{16\sqrt{1 - c^2x^2}} + \\
& \quad \frac{1}{8}x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{2009} \\
& \frac{5}{8}d \int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))dx + \frac{1}{8}x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) - \\
& \quad \frac{bcd^2\left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2}\right)\sqrt{d - c^2dx^2}}{16\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{5202} \\
& \frac{5}{8}d \left(\frac{1}{2}d \int x^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx - \frac{bcd\sqrt{d - c^2dx^2} \int x^3(1 - c^2x^2) dx}{6\sqrt{1 - c^2x^2}} + \frac{1}{6}x^3(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{8}x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2\left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2}\right)\sqrt{d - c^2dx^2}}{16\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{244} \\
& \frac{5}{8}d \left(\frac{1}{2}d \int x^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx - \frac{bcd\sqrt{d - c^2dx^2} \int (x^3 - c^2x^5) dx}{6\sqrt{1 - c^2x^2}} + \frac{1}{6}x^3(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{8}x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2\left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2}\right)\sqrt{d - c^2dx^2}}{16\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{2009} \\
& \frac{5}{8}d \left(\frac{1}{2}d \int x^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx + \frac{1}{6}x^3(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd\left(\frac{x^4}{4} - \frac{c^2x^6}{6}\right)\sqrt{d - c^2dx^2}}{6\sqrt{1 - c^2x^2}} \right) \\
& \quad \frac{1}{8}x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2\left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2}\right)\sqrt{d - c^2dx^2}}{16\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{5198}
\end{aligned}$$

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \int x^3 dx}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right) + \frac{1}{6}x^3 \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2} \right) \sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \right)$$

↓ 15

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \right) + \frac{1}{6}x^3 \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2} \right) \sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \right)$$

↓ 5210

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2c^2} \right)}{4\sqrt{1-c^2x^2}} \right) + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2} \right) \sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \right)$$

↓ 15

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4\sqrt{1-c^2x^2}} \right) + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2} \right) \sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \right)$$

↓ 5152

$$\frac{1}{8}x^3(d - c^2dx^2)^{5/2}(a + b\arcsin(cx)) + \frac{5}{8}d\left(\frac{1}{6}x^3(d - c^2dx^2)^{3/2}(a + b\arcsin(cx)) + \frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{d - c^2dx^2}(a + b\arcsin(cx)) + \frac{\sqrt{d - c^2dx^2}\left(\frac{a+b\arcsin(cx)}{4bc^3}\right)}{4}\right)\right) + \frac{bcd^2\left(\frac{c^4x^8}{4} - \frac{2c^2x^6}{3} + \frac{x^4}{2}\right)\sqrt{d - c^2dx^2}}{16\sqrt{1 - c^2x^2}}$$

input `Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output `-1/16*(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^4/2 - (2*c^2*x^6)/3 + (c^4*x^8)/4))/Sqrt[1 - c^2*x^2] + (x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/8 + (5*d*(-1/6*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^4/4 - (c^2*x^6)/6)))/Sqrt[1 - c^2*x^2] + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/6 + (d*(-1/16*(b*c*x^4*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/4 + (Sqrt[d - c^2*d*x^2]*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/4*Sqrt[1 - c^2*x^2])))/2)/8`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 49 `Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 244 $\text{Int}[\text{((c_.)*(x_))}^{\text{(m_.)}* \text{((a_) + (b_.)*(x_)^2)}^{\text{(p_.)}, x_Symbol] \text{ :> Int[ExpandIntegrand}[\text{(c*x)}^{\text{m}}*\text{(a + b*x^2)}^{\text{p}}, \text{x}], \text{x}] \text{ /; FreeQ}[\{\text{a, b, c, m}\}, \text{x}] \ \&\& \ \text{IGtQ}[\text{p}, 0]$

rule 2009 $\text{Int}[\text{u_}, x_Symbol] \text{ :> Simp[IntSum}[\text{u}, \text{x}], \text{x}] \text{ /; SumQ}[\text{u}]$

rule 5152 $\text{Int}[\text{((a_.) + ArcSin}[\text{(c_.)*(x_)}] * \text{(b_.))}^{\text{(n_.)} / \text{Sqrt}[\text{(d_) + (e_.)*(x_)^2}], x_Symbol] \text{ :> Simp}[\text{(1/(b*c*(n + 1))) * Simp[Sqrt}[\text{1 - c^2*x^2}] / \text{Sqrt}[\text{d + e*x^2}]] * \text{(a + b * ArcSin}[\text{c*x}] \text{)}^{\text{(n + 1)}, \text{x}] \text{ /; FreeQ}[\{\text{a, b, c, d, e, n}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c^2*d + e}, 0] \ \&\& \ \text{NeQ}[\text{n}, -1]$

rule 5198 $\text{Int}[\text{((a_.) + ArcSin}[\text{(c_.)*(x_)}] * \text{(b_.))}^{\text{(n_.)} * \text{((f_.)*(x_))}^{\text{(m_)} * \text{Sqrt}[\text{(d_) + (e_.)*(x_)^2}], x_Symbol] \text{ :> Simp}[\text{(f*x)}^{\text{(m + 1)} * \text{Sqrt}[\text{d + e*x^2}] * \text{((a + b * ArcSin}[\text{c*x}])^{\text{n}} / \text{(f*(m + 2)))}, \text{x}] + \text{(Simp}[\text{(1/(m + 2)) * Simp[Sqrt}[\text{d + e*x^2}] / \text{Sqrt}[\text{1 - c^2*x^2}]] \text{Int}[\text{(f*x)}^{\text{m}} * \text{((a + b * ArcSin}[\text{c*x}])^{\text{n}} / \text{Sqrt}[\text{1 - c^2*x^2}]), \text{x}], \text{x}] - \text{Simp}[\text{b*c*(n/(f*(m + 2))) * Simp[Sqrt}[\text{d + e*x^2}] / \text{Sqrt}[\text{1 - c^2*x^2}]] \text{Int}[\text{(f*x)}^{\text{(m + 1)} * \text{(a + b * ArcSin}[\text{c*x}])^{\text{(n - 1)}, \text{x}], \text{x})} \text{ /; FreeQ}[\{\text{a, b, c, d, e, f, m}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c^2*d + e}, 0] \ \&\& \ \text{GtQ}[\text{n}, 0] \ \&\& \ (\text{IGtQ}[\text{m}, -2] \ \|\ \text{EqQ}[\text{n}, 1])$

rule 5202 $\text{Int}[\text{((a_.) + ArcSin}[\text{(c_.)*(x_)}] * \text{(b_.))}^{\text{(n_.)} * \text{((f_.)*(x_))}^{\text{(m_)} * \text{((d_) + (e_.)*(x_)^2)}^{\text{(p_.)}, x_Symbol] \text{ :> Simp}[\text{(f*x)}^{\text{(m + 1)} * \text{(d + e*x^2)}^{\text{p}} * \text{((a + b * ArcSin}[\text{c*x}])^{\text{n}} / \text{(f*(m + 2*p + 1)))}, \text{x}] + \text{(Simp}[\text{2*d*(p/(m + 2*p + 1))} \text{Int}[\text{(f*x)}^{\text{m}} * \text{(d + e*x^2)}^{\text{(p - 1)} * \text{(a + b * ArcSin}[\text{c*x}])^{\text{n}}, \text{x}], \text{x}] - \text{Simp}[\text{b*c*(n/(f*(m + 2*p + 1))) * Simp}[\text{(d + e*x^2)}^{\text{p}} / \text{(1 - c^2*x^2)}^{\text{p}}] \text{Int}[\text{(f*x)}^{\text{(m + 1)} * \text{(1 - c^2*x^2)}^{\text{(p - 1/2)} * \text{(a + b * ArcSin}[\text{c*x}])^{\text{(n - 1)}, \text{x}], \text{x})} \text{ /; FreeQ}[\{\text{a, b, c, d, e, f, m}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c^2*d + e}, 0] \ \&\& \ \text{GtQ}[\text{n}, 0] \ \&\& \ \text{GtQ}[\text{p}, 0] \ \&\& \ \text{!LtQ}[\text{m}, -1]$

rule 5210 $\text{Int}[\text{((a_.) + ArcSin}[\text{(c_.)*(x_)}] * \text{(b_.))}^{\text{(n_.)} * \text{((f_.)*(x_))}^{\text{(m_)} * \text{((d_) + (e_.)*(x_)^2)}^{\text{(p_)}, x_Symbol] \text{ :> Simp}[\text{f*(f*x)}^{\text{(m - 1)} * \text{(d + e*x^2)}^{\text{(p + 1)} * \text{((a + b * ArcSin}[\text{c*x}])^{\text{n}} / \text{(e*(m + 2*p + 1)))}, \text{x}] + \text{(Simp}[\text{f^2*((m - 1)/(c^2*(m + 2*p + 1))} \text{Int}[\text{(f*x)}^{\text{(m - 2)} * \text{(d + e*x^2)}^{\text{p}} * \text{(a + b * ArcSin}[\text{c*x}])^{\text{n}}, \text{x}], \text{x}] + \text{Simp}[\text{b*f*(n/(c*(m + 2*p + 1))) * Simp}[\text{(d + e*x^2)}^{\text{p}} / \text{(1 - c^2*x^2)}^{\text{p}}] \text{Int}[\text{(f*x)}^{\text{(m - 1)} * \text{(1 - c^2*x^2)}^{\text{(p + 1/2)} * \text{(a + b * ArcSin}[\text{c*x}])^{\text{(n - 1)}, \text{x}], \text{x})} \text{ /; FreeQ}[\{\text{a, b, c, d, e, f, p}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c^2*d + e}, 0] \ \&\& \ \text{GtQ}[\text{n}, 0] \ \&\& \ \text{IGtQ}[\text{m}, 1] \ \&\& \ \text{NeQ}[\text{m + 2*p + 1}, 0]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 907, normalized size of antiderivative = 2.58

method	result
default	$-\frac{ax(-c^2dx^2+d)^{\frac{7}{2}}}{8c^2d} + \frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{48c^2} + \frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{192c^2} + \frac{5ad^2x\sqrt{-c^2dx^2+d}}{128c^2} + \frac{5ad^3 \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{128c^2\sqrt{c^2d}} + b$
parts	$-\frac{ax(-c^2dx^2+d)^{\frac{7}{2}}}{8c^2d} + \frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{48c^2} + \frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{192c^2} + \frac{5ad^2x\sqrt{-c^2dx^2+d}}{128c^2} + \frac{5ad^3 \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{128c^2\sqrt{c^2d}} + b$

input `int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output

```
-1/8*a*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/19
2*a/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/
128*a/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b
*(-5/256*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(
c*x)^2*d^2+1/16384*(-d*(c^2*x^2-1))^(1/2)*(-128*I*(-c^2*x^2+1)^(1/2)*x^8*c
^8+128*c^9*x^9+256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7-160*I*(-c^2*x^
2+1)^(1/2)*x^4*c^4+272*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3-
I*(-c^2*x^2+1)^(1/2)+8*c*x)*(8*arcsin(c*x)+I)*d^2/c^3/(c^2*x^2-1)+1/256*(-
d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2
+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))*d^2/c^3/(c^2*x^2-1)+1/147456*(-d*(c^2*
x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(73*I+312*arcsin(c*x))*
cos(7*arcsin(c*x))*d^2/c^3/(c^2*x^2-1)-1/147456*(-d*(c^2*x^2-1))^(1/2)*(I*
(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(55*I+456*arcsin(c*x))*sin(7*arcsin(c*x)
)*d^2/c^3/(c^2*x^2-1)+1/9216*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x
^2+1)^(1/2)-I)*(13*I+12*arcsin(c*x))*cos(5*arcsin(c*x))*d^2/c^3/(c^2*x^2-1
)-5/9216*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(I+12
*arcsin(c*x))*sin(5*arcsin(c*x))*d^2/c^3/(c^2*x^2-1)-3/1024*(-d*(c^2*x^2-1
))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(I+4*arcsin(c*x))*cos(3*arcs
in(c*x))*d^2/c^3/(c^2*x^2-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)
^(1/2)*c*x+c^2*x^2-1)*(5*I+4*arcsin(c*x))*sin(3*arcsin(c*x))*d^2/c^3/(c...
```

Fricas [F]

$$\int x^2(d - c^2dx^2)^{5/2}(a + b \arcsin(cx)) dx = \int (-c^2dx^2 + d)^{5/2}(b \arcsin(cx) + a)x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((a*c^4*d^2*x^6 - 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 - 2*b*c^2*d^2*x^4 + b*d^2*x^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)`

Sympy [F(-1)]

Timed out.

$$\int x^2(d - c^2dx^2)^{5/2}(a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int x^2(d - c^2dx^2)^{5/2}(a + b \arcsin(cx)) dx = \int (-c^2dx^2 + d)^{5/2}(b \arcsin(cx) + a)x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
b*sqrt(d)*integrate((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + 1/384*(8*(
-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c
^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/
2)*arcsin(c*x)/c^3)*a
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{1}{8} \sqrt{-c^2 dx^2 + d} a c^4 d^2 x^7 - \frac{17}{48} \sqrt{-c^2 dx^2 + d} a c^2 d^2 x^5 + \frac{59}{192} \sqrt{-c^2 dx^2 + d} a d^2 x^3 - \frac{5 \sqrt{-c^2 dx^2 + d} a d^2 x}{128 c^2} - \frac{5 a d^3 \log(|-c \sqrt{-d} x + \sqrt{c^2 x^2 - 1} \sqrt{-d}|)}{128 c^3 \sqrt{-d}} + \frac{9216 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + d} b d^{5/2} x \arcsin(cx) + 1536 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + d} b d^{5/2} x \arcsin(cx) + 1920 (-c^2 x^2 + d)^{3/2} b d^{5/2} x \arcsin(cx) - 1152 (c^2 x^2 - 1)^4 b d^{5/2} / c + 2880 \sqrt{-c^2 x^2 + d} b d^{5/2} x \arcsin(cx) - 256 (c^2 x^2 - 1)^3 b d^{5/2} / c + 480 (c^2 x^2 - 1)^2 b d^{5/2} / c + 1440 b d^{5/2} \arcsin(cx)^2 / c - 1440 (c^2 x^2 - 1) b d^{5/2} / c - 665 b d^{5/2} / c}{c^2}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
1/8*sqrt(-c^2*d*x^2 + d)*a*c^4*d^2*x^7 - 17/48*sqrt(-c^2*d*x^2 + d)*a*c^2*
d^2*x^5 + 59/192*sqrt(-c^2*d*x^2 + d)*a*d^2*x^3 - 5/128*sqrt(-c^2*d*x^2 +
d)*a*d^2*x/c^2 - 5/128*a*d^3*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sq
r(-d)))/(c^3*sqrt(-d)) + 1/73728*(9216*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*
b*d^(5/2)*x*arcsin(c*x) + 1536*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^(5/2
)*x*arcsin(c*x) + 1920*(-c^2*x^2 + 1)^(3/2)*b*d^(5/2)*x*arcsin(c*x) - 1152
*(c^2*x^2 - 1)^4*b*d^(5/2)/c + 2880*sqrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arcsin(
c*x) - 256*(c^2*x^2 - 1)^3*b*d^(5/2)/c + 480*(c^2*x^2 - 1)^2*b*d^(5/2)/c +
1440*b*d^(5/2)*arcsin(c*x)^2/c - 1440*(c^2*x^2 - 1)*b*d^(5/2)/c - 665*b*d
^(5/2)/c)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int x^2 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^2*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d^2 (15 \operatorname{asin}(cx) a + 48 \sqrt{-c^2 x^2 + 1} a c^7 x^7 - 136 \sqrt{-c^2 x^2 + 1} a c^5 x^5 + 118 \sqrt{-c^2 x^2 + 1} a c^3 x^3 - 15 \sqrt{-c^2 x^2 + 1} a c x + 384 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) x^6, x) b c^7 - 768 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) x^4, x) b c^5 + 384 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) x^2, x) b c^3)}{(384 c^3)}$$

input `int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d**2*(15*asin(c*x)*a + 48*sqrt(-c**2*x**2 + 1)*a*c**7*x**7 - 136*sqrt(-c**2*x**2 + 1)*a*c**5*x**5 + 118*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 - 15*sqrt(-c**2*x**2 + 1)*a*c*x + 384*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**6,x)*b*c**7 - 768*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*b*c**5 + 384*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*b*c**3)/(384*c**3)`

3.86 $\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	902
Mathematica [A] (verified)	903
Rubi [A] (verified)	903
Maple [C] (verified)	906
Fricas [F]	907
Sympy [F(-1)]	907
Maxima [F]	908
Giac [F(-2)]	908
Mupad [F(-1)]	908
Reduce [F]	909

Optimal result

Integrand size = 24, antiderivative size = 262

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = -\frac{5bcd^2 x^2 \sqrt{d - c^2 dx^2}}{32\sqrt{1 - c^2 x^2}} + \frac{5bd^2(1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2}}{96c} + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} + \frac{5}{16}d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{5}{24}dx (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{1}{6}x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))$$

output

```
-5/32*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+5/96*b*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c+1/36*b*d^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+5/16*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))+1/6*x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))+5/32*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 \left(360b\sqrt{d - c^2 dx^2} \arcsin(cx)^2 - 720a\sqrt{d}\sqrt{1 - c^2 x^2} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d(-1 + c^2 x^2)}}\right) + \sqrt{d - c^2 dx^2} \right)}{2304c\sqrt{1 - c^2 x^2}}$$

input

```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]
```

output

```
(d^2*(360*b*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^2 - 720*a*Sqrt[d]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(1584*a*c*x*Sqrt[1 - c^2*x^2] - 1248*a*c^3*x^3*Sqrt[1 - c^2*x^2] + 384*a*c^5*x^5*Sqrt[1 - c^2*x^2] + 270*b*Cos[2*ArcSin[c*x]] + 27*b*Cos[4*ArcSin[c*x]] + 2*b*Cos[6*ArcSin[c*x]]) + 12*b*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]*(45*Sin[2*ArcSin[c*x]] + 9*Sin[4*ArcSin[c*x]] + Sin[6*ArcSin[c*x]]))/ (2304*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5158, 241, 5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5158$$

$$\frac{5}{6}d \int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2)^2 dx}{6\sqrt{1 - c^2 x^2}} + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))$$

$$\downarrow 241$$

$$\frac{5}{6}d \int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c}$$

↓ 5158

$$\frac{5}{6}d \left(\frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} \right)$$

↓ 244

$$\frac{5}{6}d \left(\frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2 dx^2} \int (x - c^2 x^3) dx}{4\sqrt{1 - c^2 x^2}} + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} \right)$$

↓ 2009

$$\frac{5}{6}d \left(\frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd\left(\frac{x^2}{2} - \frac{c^2 x^4}{4}\right) \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} \right)$$

↓ 5156

$$\frac{5}{6}d \left(\frac{3}{4}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2}x\sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} \right)$$

↓ 15

$$\frac{5}{6}d \left(\frac{3}{4}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2}x\sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{bcx^2\sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} \right) + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{1}{6}x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \frac{bd^2(1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2}}{36c} \right)$$

$$\begin{aligned} & \downarrow 5152 \\ & \frac{1}{6}x(d - c^2dx^2)^{5/2}(a + b \arcsin(cx)) + \\ \frac{5}{6}d \left(\frac{1}{4}x(d - c^2dx^2)^{3/2}(a + b \arcsin(cx)) + \frac{3}{4}d \left(\frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{4bc\sqrt{1 - c^2x^2}} \right. \right. \\ & \left. \left. + \frac{bd^2(1 - c^2x^2)^{5/2}\sqrt{d - c^2dx^2}}{36c} \right) \right) \end{aligned}$$

input `Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output `(b*d^2*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/6 + (5*d*(-1/4*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/Sqrt[1 - c^2*x^2] + (x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])))/4)/6`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1))/(m + 1), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5158 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.64

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6} + \frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{24} + \frac{5ad^2x\sqrt{-c^2dx^2+d}}{16} + \frac{5ad^3 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{16\sqrt{c^2d}} + b\left(-\frac{5\sqrt{-d(c^2x^2-1)}\sqrt{-d}}{32(c^2d+d)}\right)$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6} + \frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{24} + \frac{5ad^2x\sqrt{-c^2dx^2+d}}{16} + \frac{5ad^3 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{16\sqrt{c^2d}} + b\left(-\frac{5\sqrt{-d(c^2x^2-1)}\sqrt{-d}}{32(c^2d+d)}\right)$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output

```

1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*
(-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))+b*(-5/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x
^2-1)/c*arcsin(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1
)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I
*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*(I+6*ar
csin(c*x))*d^2/(c^2*x^2-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+
1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))*
d^2/(c^2*x^2-1)/c-1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1
)^(1/2)-I)*(29*I+96*arcsin(c*x))*cos(5*arcsin(c*x))*d^2/(c^2*x^2-1)/c+5/46
08*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(5*I+24*arc
sin(c*x))*sin(5*arcsin(c*x))*d^2/(c^2*x^2-1)/c-3/512*(-d*(c^2*x^2-1))^(1/2
)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(11*I+16*arcsin(c*x))*cos(3*arcsin(
c*x))*d^2/(c^2*x^2-1)/c+9/512*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2
)*c*x+c^2*x^2-1)*(3*I+8*arcsin(c*x))*sin(3*arcsin(c*x))*d^2/(c^2*x^2-1)/c

```

Fricas [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a) dx$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```

integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)

```

Sympy [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)
```

output

Timed out

Maxima [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a) dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a`

Giac [F(-2)]

Exception generated.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d^2 (15 a \sin(cx) a + 8 \sqrt{-c^2 x^2 + 1} a c^5 x^5 - 26 \sqrt{-c^2 x^2 + 1} a c^3 x^3 + 33 \sqrt{-c^2 x^2 + 1} a c x + 48 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4, x) * b * c^5 - 96 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) * b * c^3 + 48 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx), x) * b * c)}{48 * c}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d**2*(15*asin(c*x)*a + 8*sqrt(-c**2*x**2 + 1)*a*c**5*x**5 - 26*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 + 33*sqrt(-c**2*x**2 + 1)*a*c*x + 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*b*c**5 - 96*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*b*c**3 + 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*b*c))/(48*c)`

3.87
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx$$

Optimal result	910
Mathematica [A] (verified)	911
Rubi [A] (verified)	911
Maple [C] (verified)	915
Fricas [F]	916
Sympy [F(-1)]	916
Maxima [F]	916
Giac [F(-2)]	917
Mupad [F(-1)]	917
Reduce [F]	918

Optimal result

Integrand size = 27, antiderivative size = 306

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = -\frac{bc^3 d^2 x^2 \sqrt{d - c^2 dx^2}}{16\sqrt{1 - c^2 x^2}} + \frac{bc^5 d^2 x^4 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}}$$

$$- \frac{5}{16} bcd^2 (1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2} - \frac{15}{8} c^2 d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))$$

$$- \frac{5}{4} c^2 dx (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} - \frac{15cd^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{16b\sqrt{1 - c^2 x^2}}$$

output

```
-1/16*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/4*b*c^5*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/16*b*c*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)-15/8*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x-15/16*c*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/b/(-c^2*x^2+1)^(1/2)+b*c*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.84

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \frac{d^2 \left(-120bcx\sqrt{d - c^2 dx^2} \arcsin(cx)^2 + 240ac\sqrt{dx}\sqrt{1 - c^2 x^2} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d}(-1 + c^2 x^2)}\right) + \sqrt{d - c^2 dx^2} \left(-32bcx\cos(2\arcsin(cx)) - bcx\cos(4\arcsin(cx)) + 16(a\sqrt{1 - c^2 x^2}(-8 - 9c^2 x^2 + 2c^4 x^4) + 8bcx\log(cx)) - 4b\sqrt{d - c^2 dx^2}\arcsin(cx)(32\sqrt{1 - c^2 x^2} + 16cx\sin(2\arcsin(cx)) + cx\sin(4\arcsin(cx))) \right) \right)}{(128x\sqrt{1 - c^2 x^2})}$$

input `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^2,x]`

output `(d^2*(-120*b*c*x*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^2 + 240*a*c*Sqrt[d]*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(-32*b*c*x*Cos[2*ArcSin[c*x]] - b*c*x*Cos[4*ArcSin[c*x]]) + 16*(a*Sqrt[1 - c^2*x^2]*(-8 - 9*c^2*x^2 + 2*c^4*x^4) + 8*b*c*x*Log[c*x])) - 4*b*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]*(32*Sqrt[1 - c^2*x^2] + 16*c*x*Sin[2*ArcSin[c*x]] + c*x*Sin[4*ArcSin[c*x]])))/(128*x*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5200, 243, 49, 2009, 5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx$$

↓ 5200

$$-5c^2 d \int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x}$$

↓ 243

$$\begin{aligned}
& -5c^2d \int (d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{bcd^2\sqrt{d - c^2dx^2} \int \frac{(1-c^2x^2)^2}{x^2} dx^2}{2\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{x} \\
& \quad \downarrow 49 \\
& -5c^2d \int (d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{bcd^2\sqrt{d - c^2dx^2} \int (x^2c^4 - 2c^2 + \frac{1}{x^2}) dx^2}{2\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{x} \\
& \quad \downarrow 2009 \\
& -5c^2d \int (d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{x} + \\
& \quad \frac{bcd^2\sqrt{d - c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1 - c^2x^2}} \\
& \quad \downarrow 5158 \\
& -5c^2d \left(\frac{3}{4}d \int \sqrt{d - c^2dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2dx^2} \int x(1 - c^2x^2) dx}{4\sqrt{1 - c^2x^2}} + \frac{1}{4}x(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{x} + \frac{bcd^2\sqrt{d - c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1 - c^2x^2}} \\
& \quad \downarrow 244 \\
& -5c^2d \left(\frac{3}{4}d \int \sqrt{d - c^2dx^2} (a + b \arcsin(cx)) dx - \frac{bcd\sqrt{d - c^2dx^2} \int (x - c^2x^3) dx}{4\sqrt{1 - c^2x^2}} + \frac{1}{4}x(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{x} + \frac{bcd^2\sqrt{d - c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1 - c^2x^2}} \\
& \quad \downarrow 2009 \\
& -5c^2d \left(\frac{3}{4}d \int \sqrt{d - c^2dx^2} (a + b \arcsin(cx)) dx + \frac{1}{4}x(d - c^2dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(\frac{x^2}{2} - \frac{c^2x^4}{4} \right) \sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \right) \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{x} + \frac{bcd^2\sqrt{d - c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1 - c^2x^2}} \\
& \quad \downarrow 5156
\end{aligned}$$

$$-5c^2d \left(\frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \int x dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \right) + \frac{1}{4}x(d-c^2dx^2)^{5/2}(a+b \arcsin(cx)) + \frac{bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1-c^2x^2}} \right)$$

↓ 15

$$-5c^2d \left(\frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) - \frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \right) + \frac{1}{4}x(d-c^2dx^2)^{5/2}(a+b \arcsin(cx)) + \frac{bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1-c^2x^2}} \right)$$

↓ 5152

$$-\frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{x} - 5c^2d \left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b \arcsin(cx)) + \frac{3}{4}d \left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) + \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{4bc\sqrt{1-c^2x^2}} + \frac{bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log(x^2) \right)}{2\sqrt{1-c^2x^2}} \right) \right)$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^2,x]`

output `-(((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x) - 5*c^2*d*(-1/4*(b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/Sqrt[1 - c^2*x^2] + (x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/4 + (3*d*(-1/4*(b*c*x^2*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2]))) / 4 + (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-2*c^2*x^2 + (c^4*x^4)/2 + Log[x^2]))/(2*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}*((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$
- rule 243 $\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{((m-1)/2)*(a + b*x)^p}, x], x, x^2], x] \text{ ; FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$
- rule 244 $\text{Int}[((c_.)(x_))^{(m_.)}*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 5152 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}/\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$
- rule 5156 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^{n/2}), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x],
x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx} - ac^2x(-c^2dx^2+d)^{\frac{5}{2}} - \frac{5ac^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4} - \frac{15ac^2d^2x\sqrt{-c^2dx^2+d}}{8} - \frac{15ac^2d^3 \arctan\left(\frac{\sqrt{-c^2dx^2+d}}{c}\right)}{8\sqrt{c^2d}}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx} - ac^2x(-c^2dx^2+d)^{\frac{5}{2}} - \frac{5ac^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4} - \frac{15ac^2d^2x\sqrt{-c^2dx^2+d}}{8} - \frac{15ac^2d^3 \arctan\left(\frac{\sqrt{-c^2dx^2+d}}{c}\right)}{8\sqrt{c^2d}}$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output

```
-a/d/x*(-c^2*d*x^2+d)^(7/2)-a*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a*c^2*d*x*(-c
^2*d*x^2+d)^(3/2)-15/8*a*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a*c^2*d^3/(c^
2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/128*b*(-d*(c^2*x
^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/x/(c^2*x^2-1)*(32*arcsin(c*x)*(-c^2*x^2+1)
^(1/2)*c^4*x^4-8*c^5*x^5-144*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2+72*c^3
*x^3-120*arcsin(c*x)^2*c*x-128*I*arcsin(c*x)*x*c+128*ln((I*c*x+(-c^2*x^2+1)
)^(1/2))^2-1)*x*c-128*arcsin(c*x)*(-c^2*x^2+1)^(1/2)-33*c*x)*d^2
```


Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")`

output `integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**2,x)`

output `Timed out`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output

```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^2, x) - 1/8*(10*(-
c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x + 15*c*d^
(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^2} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx = \frac{\sqrt{d} d^2 (-4 \arcsin(cx)^2 bcx - 15 \arcsin(cx) acx + 2\sqrt{-c^2 x^2 + 1} a c^4 x^4}{x^2}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^2,x)`

output `(sqrt(d)*d**2*(- 4*asin(c*x)**2*b*c*x - 15*asin(c*x)*a*c*x + 2*sqrt(- c**2*x**2 + 1)*a*c**4*x**4 - 9*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt(- c**2*x**2 + 1)*a + 8*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*x**2),x)*b*x + 8*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**2,x)*b*c**4*x - 16*int(sqrt(- c**2*x**2 + 1)*asin(c*x),x)*b*c**2*x))/(8*x)`

3.88
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx$$

Optimal result	919
Mathematica [A] (verified)	920
Rubi [A] (verified)	920
Maple [C] (verified)	924
Fricas [F]	925
Sympy [F(-1)]	925
Maxima [F]	925
Giac [F(-2)]	926
Mupad [F(-1)]	926
Reduce [F]	927

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = -\frac{bcd^2 \sqrt{d - c^2 dx^2}}{6x^2 \sqrt{1 - c^2 x^2}} - \frac{bc^5 d^2 x^2 \sqrt{d - c^2 dx^2}}{4\sqrt{1 - c^2 x^2}} + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) + \frac{5c^2 d (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} + \frac{5c^3 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{4b\sqrt{1 - c^2 x^2}} - \frac{7bc^3 d^2 \sqrt{d - c^2 dx^2} \log(x)}{3\sqrt{1 - c^2 x^2}}$$

output

```
-1/6*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/4*b*c^5*d^2*x^2
*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+5/2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)
)*(a+b*arcsin(c*x))+5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x-1/3
*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^3+5/4*c^3*d^2*(-c^2*d*x^2+d)^(1/2)
*(a+b*arcsin(c*x))^2/b/(-c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)
*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.88

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \frac{1}{24} d^2 \left(\frac{4b\sqrt{d - c^2 dx^2}(-2 + 14c^2 x^2 + 3c^4 x^4) \arcsin(cx)}{x^3} \right. \\ \left. + \frac{30bc^3 \sqrt{d - c^2 dx^2} \arcsin(cx)^2}{\sqrt{1 - c^2 x^2}} - 60ac^3 \sqrt{d} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d}(-1 + c^2 x^2)}\right) \right. \\ \left. + \frac{\sqrt{d - c^2 dx^2}(4a\sqrt{1 - c^2 x^2}(-2 + 14c^2 x^2 + 3c^4 x^4) + b(-4cx + 3c^3 x^3 - 6c^5 x^5) - 56bc^3 x^3 \log(cx))}{x^3 \sqrt{1 - c^2 x^2}} \right)$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^4,x]
```

output

```
(d^2*((4*b*Sqrt[d - c^2*d*x^2]*(-2 + 14*c^2*x^2 + 3*c^4*x^4)*ArcSin[c*x])/x^3 + (30*b*c^3*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^2)/Sqrt[1 - c^2*x^2] - 60*a*c^3*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (Sqrt[d - c^2*d*x^2]*(4*a*Sqrt[1 - c^2*x^2]*(-2 + 14*c^2*x^2 + 3*c^4*x^4) + b*(-4*c*x + 3*c^3*x^3 - 6*c^5*x^5) - 56*b*c^3*x^3*Log[c*x]))/(x^3*Sqrt[1 - c^2*x^2])))/24
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5200, 243, 49, 2009, 5200, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx$$

↓ 5200

$$\begin{aligned}
& -\frac{5}{3}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx + \frac{bcd^2\sqrt{d - c^2dx^2} \int \frac{(1-c^2x^2)^2}{x^3} dx}{3\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{243} \\
& -\frac{5}{3}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx + \frac{bcd^2\sqrt{d - c^2dx^2} \int \frac{(1-c^2x^2)^2}{x^4} dx^2}{6\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{49} \\
& -\frac{5}{3}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx + \frac{bcd^2\sqrt{d - c^2dx^2} \int \left(c^4 - \frac{2c^2}{x^2} + \frac{1}{x^4}\right) dx^2}{6\sqrt{1 - c^2x^2}} - \\
& \quad \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} \\
& \quad \downarrow \text{2009} \\
& -\frac{5}{3}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx - \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} + \\
& \quad \frac{bcd^2\sqrt{d - c^2dx^2} \left(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2}\right)}{6\sqrt{1 - c^2x^2}} \\
& \quad \downarrow \text{5200} \\
& -\frac{5}{3}c^2d \left(-3c^2d \int \sqrt{d - c^2dx^2} (a + b \arcsin(cx)) dx + \frac{bcd\sqrt{d - c^2dx^2} \int \frac{1-c^2x^2}{x} dx}{\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} \right. \\
& \quad \left. \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} + \frac{bcd^2\sqrt{d - c^2dx^2} \left(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2}\right)}{6\sqrt{1 - c^2x^2}} \right) \\
& \quad \downarrow \text{244} \\
& -\frac{5}{3}c^2d \left(-3c^2d \int \sqrt{d - c^2dx^2} (a + b \arcsin(cx)) dx + \frac{bcd\sqrt{d - c^2dx^2} \int \left(\frac{1}{x} - c^2x\right) dx}{\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))}{x} \right. \\
& \quad \left. \frac{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))}{3x^3} + \frac{bcd^2\sqrt{d - c^2dx^2} \left(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2}\right)}{6\sqrt{1 - c^2x^2}} \right) \\
& \quad \downarrow \text{2009}
\end{aligned}$$

$$-\frac{5}{3}c^2d \left(-3c^2d \int \sqrt{d - c^2dx^2}(a + b \arcsin(cx))dx - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{x} + \frac{bcd\sqrt{d - c^2dx^2}(\log(x))}{\sqrt{1 - c^2x^2}} \right. \\ \left. \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{3x^3} + \frac{bcd^2\sqrt{d - c^2dx^2}(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2})}{6\sqrt{1 - c^2x^2}} \right)$$

↓ 5156

$$-\frac{5}{3}c^2d \left(-3c^2d \left(\frac{\sqrt{d - c^2dx^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1 - c^2x^2}} - \frac{bc\sqrt{d - c^2dx^2} \int x dx}{2\sqrt{1 - c^2x^2}} + \frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) \right) - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{x} \right. \\ \left. \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{3x^3} + \frac{bcd^2\sqrt{d - c^2dx^2}(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2})}{6\sqrt{1 - c^2x^2}} \right)$$

↓ 15

$$-\frac{5}{3}c^2d \left(-3c^2d \left(\frac{\sqrt{d - c^2dx^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1 - c^2x^2}} + \frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) - \frac{bcx^2\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \right) - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{x} \right. \\ \left. \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{3x^3} + \frac{bcd^2\sqrt{d - c^2dx^2}(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2})}{6\sqrt{1 - c^2x^2}} \right)$$

↓ 5152

$$-\frac{5}{3}c^2d \left(-3c^2d \left(\frac{1}{2}x\sqrt{d - c^2dx^2}(a + b \arcsin(cx)) + \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{4bc\sqrt{1 - c^2x^2}} - \frac{bcx^2\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \right) - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{x} \right. \\ \left. \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{3x^3} + \frac{bcd^2\sqrt{d - c^2dx^2}(c^4x^2 - 2c^2 \log(x^2) - \frac{1}{x^2})}{6\sqrt{1 - c^2x^2}} \right)$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^3 - (5*c^2*d*(-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x) - 3*c^2*d*(-1/4*(b*c*x^2*sqrt[d - c^2*d*x^2])/sqrt[1 - c^2*x^2] + (x*sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/2 + (sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c*sqrt[1 - c^2*x^2])) + (b*c*d*sqrt[d - c^2*d*x^2]*(-1/2*(c^2*x^2) + Log[x]))/sqrt[1 - c^2*x^2]))/3 + (b*c*d^2*sqrt[d - c^2*d*x^2]*(-x^(-2) + c^4*x^2 - 2*c^2*Log[x^2]))/(6*sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

- rule 15 $\text{Int}[(a_)*(x_)^{(m_)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] /; \text{FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 49 $\text{Int}[((a_)+(b_)*(x_))^{(m_)}*((c_)+(d_)*(x_))^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a+b*x)^m*(c+d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m+n+2, 0]$
- rule 243 $\text{Int}[(x_)^{(m_)}*((a_)+(b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{((m-1)/2)*(a+b*x)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$
- rule 244 $\text{Int}[((c_)*(x_))^{(m_)}*((a_)+(b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a+b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 5152 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)} / \text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1-c^2*x^2] / \text{Sqrt}[d+e*x^2]]*(a+b*\text{ArcSin}[c*x])^{(n+1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{NeQ}[n, -1]$
- rule 5156 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d+e*x^2]*((a+b*\text{ArcSin}[c*x])^{n/2}), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d+e*x^2] / \text{Sqrt}[1-c^2*x^2]] \ \text{Int}[(a+b*\text{ArcSin}[c*x])^n / \text{Sqrt}[1-c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d+e*x^2] / \text{Sqrt}[1-c^2*x^2]] \ \text{Int}[x*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0]$

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.24

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{3dx^3} + \frac{4ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{3dx} + \frac{4ac^4x(-c^2dx^2+d)^{\frac{5}{2}}}{3} + \frac{5ac^4dx(-c^2dx^2+d)^{\frac{3}{2}}}{3} + \frac{5ac^4d^2x\sqrt{-c^2dx^2+d}}{2} + \dots$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{3dx^3} + \frac{4ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{3dx} + \frac{4ac^4x(-c^2dx^2+d)^{\frac{5}{2}}}{3} + \frac{5ac^4dx(-c^2dx^2+d)^{\frac{3}{2}}}{3} + \frac{5ac^4d^2x\sqrt{-c^2dx^2+d}}{2} + \dots$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*a*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+4/3*a
*c^4*x*(-c^2*d*x^2+d)^(5/2)+5/3*a*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d
^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/24*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/
(c^2*x^2-1)/x^3*(12*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^4*x^4-6*c^5*x^5+56*I*
arcsin(c*x)*x^3*c^3+30*arcsin(c*x)^2*c^3*x^3-56*ln((I*c*x+(-c^2*x^2+1)^(1/
2))^2-1)*x^3*c^3+56*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2+3*c^3*x^3-8*arc
sin(c*x)*(-c^2*x^2+1)^(1/2)-4*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**4,x)`

output `Timed out`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output

```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^4, x) + 1/6*(10*(-
c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x + 15*c^3*
d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d*x^2 + d)^(
7/2)/(d*x^3))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^4} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^4} dx = \frac{\sqrt{d} d^2 (6 \operatorname{asin}(cx)^2 b c^3 x^3 + 15 \operatorname{asin}(cx) a c^3 x^3 + 3 \sqrt{-c^2 x^2 + 1} a c^4)}{x^4}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^4,x)`

output `(sqrt(d)*d**2*(6*asin(c*x)**2*b*c**3*x**3 + 15*asin(c*x)*a*c**3*x**3 + 3*sqrt(-c**2*x**2 + 1)*a*c**4*x**4 + 14*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)*a - 12*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**2),x)*b*c**2*x**3 + 6*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**4,x)*b*x**3 + 6*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*b*c**4*x**3))/(6*x**3)`

3.89
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx$$

Optimal result	928
Mathematica [A] (verified)	929
Rubi [A] (verified)	929
Maple [C] (verified)	933
Fricas [F]	934
Sympy [F]	935
Maxima [F]	935
Giac [F(-2)]	935
Mupad [F(-1)]	936
Reduce [F]	936

Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx &= -\frac{bcd^2 \sqrt{d - c^2 dx^2}}{20x^4 \sqrt{1 - c^2 x^2}} \\ &+ \frac{11bc^3 d^2 \sqrt{d - c^2 dx^2}}{30x^2 \sqrt{1 - c^2 x^2}} - \frac{c^4 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} \\ &+ \frac{c^2 d (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x^3} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} \\ &- \frac{c^5 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2b \sqrt{1 - c^2 x^2}} + \frac{23bc^5 d^2 \sqrt{d - c^2 dx^2} \log(x)}{15 \sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/20*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+11/30*b*c^3*d^2*
(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-c^4*d^2*(-c^2*d*x^2+d)^(1/2)*(
a+b*arcsin(c*x))/x+1/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^3-1/
5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^5-1/2*c^5*d^2*(-c^2*d*x^2+d)^(1
/2)*(a+b*arcsin(c*x))^2/b/(-c^2*x^2+1)^(1/2)+23/15*b*c^5*d^2*(-c^2*d*x^2+d
)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.84

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \frac{1}{60} d^2 \left(-\frac{4b\sqrt{d - c^2 dx^2}(3 - 11c^2 x^2 + 23c^4 x^4) \arcsin(cx)}{x^5} \right. \\ \left. - \frac{30bc^5 \sqrt{d - c^2 dx^2} \arcsin(cx)^2}{\sqrt{1 - c^2 x^2}} + 60ac^5 \sqrt{d} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d}(-1 + c^2 x^2)}\right) \right. \\ \left. + \frac{\sqrt{d - c^2 dx^2}(bcx(-3 + 22c^2 x^2) - 4a\sqrt{1 - c^2 x^2}(3 - 11c^2 x^2 + 23c^4 x^4) + 92bc^5 x^5 \log(cx))}{x^5 \sqrt{1 - c^2 x^2}} \right)$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^6,x]
```

output

```
(d^2*((-4*b*Sqrt[d - c^2*d*x^2]*(3 - 11*c^2*x^2 + 23*c^4*x^4)*ArcSin[c*x])
/x^5 - (30*b*c^5*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^2)/Sqrt[1 - c^2*x^2] + 60
*a*c^5*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]
+ (Sqrt[d - c^2*d*x^2]*(b*c*x*(-3 + 22*c^2*x^2) - 4*a*Sqrt[1 - c^2*x^2]*(3
- 11*c^2*x^2 + 23*c^4*x^4) + 92*b*c^5*x^5*Log[c*x]))/(x^5*Sqrt[1 - c^2*x^
2])))/60
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5200, 243, 49, 2009, 5200, 244, 2009, 5196, 14, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx$$

↓ 5200

$$\begin{aligned}
& -c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x^5} dx}{5\sqrt{1 - c^2 x^2}} - \\
& \quad \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} \\
& \quad \downarrow \text{243} \\
& -c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x^6} dx^2}{10\sqrt{1 - c^2 x^2}} - \\
& \quad \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} \\
& \quad \downarrow \text{49} \\
& -c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \left(\frac{c^4}{x^2} - \frac{2c^2}{x^4} + \frac{1}{x^6} \right) dx^2}{10\sqrt{1 - c^2 x^2}} - \\
& \quad \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} \\
& \quad \downarrow \text{2009} \\
& -c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^4} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} + \\
& \quad \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4} \right)}{10\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5200} \\
& -c^2 d \left(c^2 (-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^2} dx + \frac{bcd \sqrt{d - c^2 dx^2} \int \frac{1 - c^2 x^2}{x^3} dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x^3} \right. \\
& \quad \left. \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} + \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4} \right)}{10\sqrt{1 - c^2 x^2}} \right) \\
& \quad \downarrow \text{244} \\
& -c^2 d \left(c^2 (-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^2} dx + \frac{bcd \sqrt{d - c^2 dx^2} \int \left(\frac{1}{x^3} - \frac{c^2}{x} \right) dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x^3} \right. \\
& \quad \left. \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} + \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4} \right)}{10\sqrt{1 - c^2 x^2}} \right) \\
& \quad \downarrow \text{2009}
\end{aligned}$$

$$-c^2 d \left(c^2 (-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x^2} dx - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x^3} + \frac{bcd \sqrt{d - c^2 dx^2} (c^2 (-\log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4}))}{3\sqrt{1 - c^2 x^2}} \right. \\ \left. + \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} + \frac{bcd^2 \sqrt{d - c^2 dx^2} (c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4})}{10\sqrt{1 - c^2 x^2}} \right)$$

↓ 5196

$$-c^2 d \left(c^2 (-d) \left(-\frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} + \frac{bc \sqrt{d - c^2 dx^2} \int \frac{1}{x} dx}{\sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} \right) \right. \\ \left. + \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} + \frac{bcd^2 \sqrt{d - c^2 dx^2} (c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4})}{10\sqrt{1 - c^2 x^2}} \right)$$

↓ 14

$$-c^2 d \left(c^2 (-d) \left(-\frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} + \frac{bc \log(x) \sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \right) \right. \\ \left. + \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} + \frac{bcd^2 \sqrt{d - c^2 dx^2} (c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4})}{10\sqrt{1 - c^2 x^2}} \right)$$

↓ 5152

$$- \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{5x^5} - \\ c^2 d \left(c^2 (-d) \left(-\frac{c \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2b \sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} + \frac{bc \log(x) \sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \right) \right. \\ \left. + \frac{bcd^2 \sqrt{d - c^2 dx^2} (c^4 \log(x^2) + \frac{2c^2}{x^2} - \frac{1}{2x^4})}{10\sqrt{1 - c^2 x^2}} \right)$$

input

```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^6,x]
```


output

$$\begin{aligned}
& -1/5*((d - c^2*d*x^2)^{(5/2)}*(a + b*\text{ArcSin}[c*x]))/x^5 - c^2*d*(-1/3*((d - c \\
& ^2*d*x^2)^{(3/2)}*(a + b*\text{ArcSin}[c*x]))/x^3 + (b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(-1/ \\
& 2*1/x^2 - c^2*\text{Log}[x]))/(3*\text{Sqrt}[1 - c^2*x^2]) - c^2*d*(-((\text{Sqrt}[d - c^2*d*x^2] \\
& *(a + b*\text{ArcSin}[c*x]))/x) - (c*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x])^2) \\
& / (2*b*\text{Sqrt}[1 - c^2*x^2]) + (b*c*\text{Sqrt}[d - c^2*d*x^2]*\text{Log}[x])/\text{Sqrt}[1 - c^2*x \\
& ^2])) + (b*c*d^2*\text{Sqrt}[d - c^2*d*x^2]*(-1/2*1/x^4 + (2*c^2)/x^2 + c^4*\text{Log}[x \\
& ^2]))/(10*\text{Sqrt}[1 - c^2*x^2])
\end{aligned}$$

Defintions of rubi rules used

rule 14

$$\text{Int}[(a_)/(x_), x_Symbol] \rightarrow \text{Simp}[a*\text{Log}[x], x] \text{ ; FreeQ}[a, x]$$

rule 49

$$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 243

$$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^2)^{(p_)}), x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m-1)/2}*(a + b*x)^p, x], x, x^2], x] \text{ ; FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$$

rule 244

$$\text{Int}[(c_*(x_))^{(m_)}*((a_ + (b_)*(x_)^2)^{(p_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5152

$$\text{Int}[(a_ + \text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}/\text{Sqrt}[(d_ + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5196

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x],
x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int
[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))] Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 2615, normalized size of antiderivative = 9.44

method	result	size
default	Expression too large to display	2615
parts	Expression too large to display	2615

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output

```
5819/30*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c^2*x^2-1)*(-c^2*x^2+1)*c^12-7153/60*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5/(c^2*x^2-1)*(-c^2*x^2+1)*c^10+759/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c^2*x^2-1)*(-c^2*x^2+1)*c^8-69/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c^2*x^2-1)*(-c^2*x^2+1)*c^6-69/5*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^5-23/15*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*d^2*c^5+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arcsin(c*x)^2*d^2*c^5+175/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c^2*x^2-1)*c^5*(-c^2*x^2+1)^(1/2)-207/5*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c^2*x^2-1)*c^8+69/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c^2*x^2-1)*c^6-1587*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^9/(c^2*x^2-1)*arcsin(c*x)*c^14+3519*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c^2*x^2-1)*arcsin(c*x)*c^12-759/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)...
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^6} dx$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="fricas")
```

output

```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^6, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \int \frac{(-d(cx - 1)(cx + 1))^{5/2} (a + b \arcsin(cx))}{x^6} dx$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**6,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))/x**6, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^6} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="maxima")`

output `b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^6, x) - 1/15*(10*(-c^2*d*x^2 + d)^(3/2)*c^6*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^6*d^2*x + 15*c^5*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^4/x - 2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^5))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^6,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^6} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^6,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^6} dx = \frac{\sqrt{d} d^2 (-15 \arcsin(cx)^2 b c^5 x^5 - 30 \arcsin(cx) a c^5 x^5 - 46 \sqrt{-c^2 x^2 + 1})}{x^6}$$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^6,x)
```

output

```
(sqrt(d)*d**2*( - 15*asin(c*x)**2*b*c**5*x**5 - 30*asin(c*x)*a*c**5*x**5 -
46*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 22*sqrt( - c**2*x**2 + 1)*a*c**2*
x**2 - 6*sqrt( - c**2*x**2 + 1)*a + 30*int(asin(c*x)/(sqrt( - c**2*x**2 +
1)*x**2),x)*b*c**4*x**5 + 30*int((sqrt( - c**2*x**2 + 1)*asin(c*x))/x**6,x
)*b*x**5 - 60*int((sqrt( - c**2*x**2 + 1)*asin(c*x))/x**4,x)*b*c**2*x**5))
/(30*x**5)
```

3.90 $\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx$

Optimal result	937
Mathematica [A] (verified)	937
Rubi [A] (verified)	938
Maple [C] (verified)	939
Fricas [A] (verification not implemented)	940
Sympy [F(-1)]	941
Maxima [A] (verification not implemented)	941
Giac [F(-2)]	942
Mupad [F(-1)]	942
Reduce [F]	943

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = -\frac{bcd^2 \sqrt{d - c^2 dx^2}}{42x^6 \sqrt{1 - c^2 x^2}} + \frac{3bc^3 d^2 \sqrt{d - c^2 dx^2}}{28x^4 \sqrt{1 - c^2 x^2}} - \frac{3bc^5 d^2 \sqrt{d - c^2 dx^2}}{14x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7dx^7} - \frac{bc^7 d^2 \sqrt{d - c^2 dx^2} \log(x)}{7\sqrt{1 - c^2 x^2}}$$

output

```
-1/42*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+3/28*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-3/14*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/d/x^7-1/7*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \frac{d^2 \sqrt{d - c^2 dx^2} (10bcx - 45bc^3 x^3 + 90bc^5 x^5 - 147bc^7 x^7 + 60a\sqrt{1 - c^2 x^2} - 180ac^2 x^2 \sqrt{1 - c^2 x^2} + 180ac^4 x^4)}{420x^7 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^8,x]
```

output

```
-1/420*(d^2*Sqrt[d - c^2*d*x^2]*(10*b*c*x - 45*b*c^3*x^3 + 90*b*c^5*x^5 -
147*b*c^7*x^7 + 60*a*Sqrt[1 - c^2*x^2] - 180*a*c^2*x^2*Sqrt[1 - c^2*x^2] +
180*a*c^4*x^4*Sqrt[1 - c^2*x^2] - 60*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 60*b*(
1 - c^2*x^2)^(7/2)*ArcSin[c*x] + 60*b*c^7*x^7*Log[x]))/(x^7*Sqrt[1 - c^2*x
^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5186, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx \\
 & \quad \downarrow \text{5186} \\
 & \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^3}{x^7} dx}{7\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7dx^7} \\
 & \quad \downarrow \text{243} \\
 & \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^3}{x^8} dx^2}{14\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7dx^7} \\
 & \quad \downarrow \text{49} \\
 & \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \left(-\frac{c^6}{x^2} + \frac{3c^4}{x^4} - \frac{3c^2}{x^6} + \frac{1}{x^8} \right) dx^2}{14\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7dx^7} \\
 & \quad \downarrow \text{2009} \\
 & \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(c^6 (-\log(x^2)) - \frac{3c^4}{x^2} + \frac{3c^2}{2x^4} - \frac{1}{3x^6} \right)}{14\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7dx^7}
 \end{aligned}$$

input $\text{Int}[(d - c^2 d x^2)^{5/2} (a + b \text{ArcSin}[c x]) / x^8, x]$

output
$$-1/7 * ((d - c^2 d x^2)^{7/2} (a + b \text{ArcSin}[c x]) / (d x^7) + (b c d^2 \sqrt{d - c^2 d x^2} * (-1/3 * 1/x^6 + (3 c^2) / (2 x^4) - (3 c^4) / x^2 - c^6 \text{Log}[x^2])) / (14 \sqrt{1 - c^2 x^2})$$

Defintions of rubi rules used

rule 49 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[m + n + 2, 0]$

rule 243 $\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{Subst}[\text{Int}[x^{(m-1)/2} (a + b x)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, m, p\}, x \&\& \text{IntegerQ}[(m-1)/2]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

rule 5186 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)] * (b_.)^{(n_.)} * ((f_.)(x_)^{(m_.)} * ((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f x)^{m+1} (d + e x^2)^{p+1} ((a + b \text{ArcSin}[c x])^n / (d f (m+1))), x] - \text{Simp}[b c (n / (f (m+1))) * \text{Simp}[(d + e x^2)^p / (1 - c^2 x^2)^p] \text{Int}[(f x)^{m+1} (1 - c^2 x^2)^{p+1/2} (a + b \text{ArcSin}[c x])^{n-1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x \&\& \text{EqQ}[c^2 d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{EqQ}[m + 2 p + 3, 0] \&\& \text{NeQ}[m, -1]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 4031, normalized size of antiderivative = 19.86

method	result	size
default	Expression too large to display	4031
parts	Expression too large to display	4031

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^8,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned}
 & -73/42*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x^{10}+35*c^8*x^8 \\
 & -35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9/(c^2*x^2-1)*c^{16}+67/42*I*b*(-d*(c^2 \\
 & *x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4 \\
 & *x^4-7*c^2*x^2+1)*x^7/(c^2*x^2-1)*c^{14}-11/14*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2 \\
 & /((7*c^{12}*x^{12}-21*c^{10}*x^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)* \\
 & x^5/(c^2*x^2-1)*c^{12}+17/84*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21* \\
 & c^{10}*x^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c^2*x^2-1)*c^{10} \\
 & -I*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x^{10}+35*c^8*x^8-35* \\
 & c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)}*arcsin(\\
 & c*x)*c^9-3*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x^{10}+35*c^8 \\
 & *x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^{10}/(c^2*x^2-1)*(-c^2*x^2+1)^{(1/2)} \\
 &)*arcsin(c*x)*c^{17}+5*I*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x \\
 & ^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^8/(c^2*x^2-1)*(-c^2*x^2 \\
 & +1)^{(1/2)}*arcsin(c*x)*c^{15}+55/7*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12} \\
 & -21*c^{10}*x^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^3/(c^2*x^2-1) \\
 &)*arcsin(c*x)*c^4-23/84*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}* \\
 & x^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^4/(c^2*x^2-1)*c^3*(-c \\
 & ^2*x^2+1)^{(1/2)}-11/7*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x^1 \\
 & 0+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^5/(c^2*x^2-1)*arcsin(c*x) \\
 &)*c^2+1/42*b*(-d*(c^2*x^2-1))^{(1/2)}*d^2/(7*c^{12}*x^{12}-21*c^{10}*x^{10}+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)
 \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.23

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \frac{\left[6 (bc^9 d^2 x^9 - bc^7 d^2 x^7) \sqrt{d} \log \left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 + \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1}}{c^2 x^4 - x^2} \right) \right.}{12 (bc^9 d^2 x^9 - bc^7 d^2 x^7) \sqrt{-d} \arctan \left(\frac{\sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^2 - 1) \sqrt{-d}}{c^2 dx^4 + (c^2 - 1) dx^2 - d} \right) - (18 bc^5 d^2 x^5 - (18 bc^5 - 9 bc^3 + 2 bc)}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="fricas")`

output

```
[1/84*(6*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) + (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 12*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2 + (b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/84*(12*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 12*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2 + (b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**8,x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \frac{\left(6(-1)^{-2c^2 dx^2 + 2d} c^6 d^{7/2} \log\left(-2c^2 d + \frac{2d}{x^2}\right) + 6c^6 d^{7/2} \log\left(x^2 - \frac{1}{c^2}\right)\right)}{7 dx^7} - \frac{(-c^2 dx^2 + d)^{7/2} b \arcsin(cx)}{7 dx^7} - \frac{(-c^2 dx^2 + d)^{7/2} a}{7 dx^7}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="maxima")
```

output

```
1/84*(6*(-1)^(-2*c^2*d*x^2 + 2*d)*c^6*d^(7/2)*log(-2*c^2*d + 2*d/x^2) + 6*
c^6*d^(7/2)*log(x^2 - 1/c^2) - 11*sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*c^4*d^
3/x^2 + 7*sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*c^2*d^3/x^4 - 2*sqrt(c^4*d*x^4
- 2*c^2*d*x^2 + d)*d^3/x^6)*b*c/d - 1/7*(-c^2*d*x^2 + d)^(7/2)*b*arcsin(c
*x)/(d*x^7) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a/(d*x^7)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^8,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^8} dx$$

input

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^8,x)
```

output

```
int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^8} dx = \frac{\sqrt{d} d^2 \left(\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 3\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 3\sqrt{-c^2 x^2 + 1} \right)}{x^8}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^8,x)`

output `(sqrt(d)*d**2*(sqrt(-c**2*x**2+1)*a*c**6*x**6-3*sqrt(-c**2*x**2+1)*a*c**4*x**4+3*sqrt(-c**2*x**2+1)*a+7*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**8,x)*b*x**7-14*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**6,x)*b*c**2*x**7+7*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**4,x)*b*c**4*x**7))/(7*x**7)`

3.91 $\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx$

Optimal result	944
Mathematica [A] (verified)	945
Rubi [A] (verified)	945
Maple [C] (verified)	948
Fricas [A] (verification not implemented)	948
Sympy [F(-1)]	949
Maxima [A] (verification not implemented)	950
Giac [F(-2)]	950
Mupad [F(-1)]	951
Reduce [F]	951

Optimal result

Integrand size = 27, antiderivative size = 282

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx =$$

$$-\frac{bc^3 d^2 \sqrt{d - c^2 dx^2}}{189x^6 \sqrt{1 - c^2 x^2}} + \frac{bc^5 d^2 \sqrt{d - c^2 dx^2}}{42x^4 \sqrt{1 - c^2 x^2}} - \frac{bc^7 d^2 \sqrt{d - c^2 dx^2}}{21x^2 \sqrt{1 - c^2 x^2}}$$

$$-\frac{bcd^2 (1 - c^2 x^2)^{7/2} \sqrt{d - c^2 dx^2}}{72x^8} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{9dx^9}$$

$$-\frac{2c^2 (d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{63dx^7} - \frac{2bc^9 d^2 \sqrt{d - c^2 dx^2} \log(x)}{63\sqrt{1 - c^2 x^2}}$$

output

```
-1/189*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+1/42*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-1/21*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/72*b*c*d^2*(-c^2*x^2+1)^(7/2)*(-c^2*d*x^2+d)^(1/2)/x^8-1/9*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/d/x^9-2/63*c^2*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/d/x^7-2/63*b*c^9*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.82

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx = \frac{d^2 \sqrt{d - c^2 dx^2} (-735bcx + 2660bc^3x^3 - 3150bc^5x^5 + 420bc^7x^7 +$$

input `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^10,x]`

output $(d^2 \sqrt{d - c^2 dx^2} (-735bcx + 2660bc^3x^3 - 3150bc^5x^5 + 420bc^7x^7 + 4566bc^9x^9 - 5880a\sqrt{1 - c^2x^2} + 15960ac^2x^2\sqrt{1 - c^2x^2} - 12600ac^4x^4\sqrt{1 - c^2x^2} + 840ac^6x^6\sqrt{1 - c^2x^2} + 1680ac^8x^8\sqrt{1 - c^2x^2} - 840b(1 - c^2x^2)^{7/2}(7 + 2c^2x^2)\arcsin(cx) - 1680bc^9x^9\log[x]) / (52920x^9\sqrt{1 - c^2x^2}))$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5194, 27, 354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d^2(1 - c^2 x^2)^3(2c^2 x^2 + 7)}{63x^9} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{9dx^9}$$

$$\frac{2c^2 (d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{63dx^7}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^3(2c^2x^2+7)}{x^9} dx}{63\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{9dx^9} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{63dx^7} \\
& \quad \downarrow 354 \\
& \frac{bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^3(2c^2x^2+7)}{x^{10}} dx^2}{126\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{9dx^9} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{63dx^7} \\
& \quad \downarrow 87 \\
& \frac{bcd^2\sqrt{d-c^2dx^2} \left(2c^2 \int \frac{(1-c^2x^2)^3}{x^8} dx^2 - \frac{7(1-c^2x^2)^4}{4x^8} \right)}{126\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{9dx^9} - \\
& \quad \frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{63dx^7} \\
& \quad \downarrow 49 \\
& \frac{bcd^2\sqrt{d-c^2dx^2} \left(2c^2 \int \left(-\frac{c^6}{x^2} + \frac{3c^4}{x^4} - \frac{3c^2}{x^6} + \frac{1}{x^8} \right) dx^2 - \frac{7(1-c^2x^2)^4}{4x^8} \right)}{126\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{9dx^9} - \frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{63dx^7} \\
& \quad \downarrow 2009 \\
& -\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{9dx^9} - \frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{63dx^7} + \\
& \quad \frac{bcd^2\sqrt{d-c^2dx^2} \left(2c^2 \left(c^6(-\log(x^2)) - \frac{3c^4}{x^2} + \frac{3c^2}{2x^4} - \frac{1}{3x^6} \right) - \frac{7(1-c^2x^2)^4}{4x^8} \right)}{126\sqrt{1-c^2x^2}}
\end{aligned}$$

input

```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^10,x]
```

output

```
-1/9*((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(d*x^9) - (2*c^2*(d - c^2
*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(63*d*x^7) + (b*c*d^2*Sqrt[d - c^2*d*x^
2]*((-7*(1 - c^2*x^2)^4)/(4*x^8) + 2*c^2*(-1/3*1/x^6 + (3*c^2)/(2*x^4) - (
3*c^4)/x^2 - c^6*Log[x^2])))/(126*Sqrt[1 - c^2*x^2])
```

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 49 $\text{Int}[((a_.) + (b_.)(x_))^{(m_.)}((c_.) + (d_.)(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$
- rule 87 $\text{Int}[((a_.) + (b_.)(x_))*((c_.) + (d_.)(x_))^{(n_.)}((e_.) + (f_.)(x_))^{(p_.)}, x_] \rightarrow \text{Simp}[(-b*e - a*f)*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)}/(f*(p + 1)*(c*f - d*e)), x] - \text{Simp}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)) \text{ Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ (\ !\text{LtQ}[n, -1] \ || \ \text{IntegerQ}[p] \ || \ \!(\text{IntegerQ}[n] \ || \ \!(\text{EqQ}[e, 0] \ || \ \!(\text{EqQ}[c, 0] \ || \ \text{LtQ}[p, n])))$
- rule 354 $\text{Int}[(x_)^{(m_.)}((a_) + (b_.)(x_)^2)^{(p_.)}((c_) + (d_.)(x_)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m - 1)/2}*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 5194 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))*(x_)^{(m_.)}((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[x^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[d + e*x^2], x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p - 1/2] \ \&\& \ \text{NeQ}[p, -2^{(-1)}] \ \&\& \ (\text{IGtQ}[(m + 1)/2, 0] \ || \ \text{ILtQ}[(m + 2*p + 3)/2, 0])$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 5324, normalized size of antiderivative = 18.88

method	result	size
default	Expression too large to display	5324
parts	Expression too large to display	5324

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^10,x,method=_RETURNVERBOSE)`

output `result too large to display`

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.65

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx = \frac{24 (bc^{11} d^2 x^{11} - bc^9 d^2 x^9) \sqrt{d} \log \left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 + \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2}}{c^2 x^4 - x^2} \right) + 48 (bc^{11} d^2 x^{11} - bc^9 d^2 x^9) \sqrt{-d} \arctan \left(\frac{\sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^2 - 1) \sqrt{-d}}{c^2 dx^4 + (c^2 - 1) dx^2 - d} \right) + (12 bc^7 d^2 x^7 - 90 bc^5 d^2 x^5 - (12 bc^7$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^10,x, algorithm="fricas")`

output

```
[1/1512*(24*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 24*(2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^2 + (2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), -1/1512*(48*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 24*(2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^2 + (2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**10,x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.57

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx =$$

$$-\frac{1}{1512} \left(48 c^8 d^{5/2} \log(x) - \frac{12 c^6 d^{5/2} x^6 - 90 c^4 d^{5/2} x^4 + 76 c^2 d^{5/2} x^2 - 21 d^{5/2}}{x^8} \right) bc$$

$$-\frac{1}{63} b \left(\frac{2(-c^2 dx^2 + d)^{7/2} c^2}{dx^7} + \frac{7(-c^2 dx^2 + d)^{7/2}}{dx^9} \right) \arcsin(cx)$$

$$-\frac{1}{63} a \left(\frac{2(-c^2 dx^2 + d)^{7/2} c^2}{dx^7} + \frac{7(-c^2 dx^2 + d)^{7/2}}{dx^9} \right)$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^10,x, algorithm="maxima")`

output `-1/1512*(48*c^8*d^(5/2)*log(x) - (12*c^6*d^(5/2)*x^6 - 90*c^4*d^(5/2)*x^4 + 76*c^2*d^(5/2)*x^2 - 21*d^(5/2))/x^8)*b*c - 1/63*b*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^9))*arcsin(c*x) - 1/63*a*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^9))`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^10,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^{10}} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^10,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^10, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{10}} dx = \frac{\sqrt{d} d^2 \left(2\sqrt{-c^2 x^2 + 1} a c^8 x^8 + \sqrt{-c^2 x^2 + 1} a c^6 x^6 - 15\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 19\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 7\sqrt{-c^2 x^2 + 1} a + 63 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx)) / x^{10}, x \right) b x^9 - 126 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx)) / x^8, x \right) b c^2 x^9 + 63 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx)) / x^6, x \right) b c^4 x^9)}{(63 x^9)}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^10,x)`

output `(sqrt(d)*d**2*(2*sqrt(-c**2*x**2 + 1)*a*c**8*x**8 + sqrt(-c**2*x**2 + 1)*a*c**6*x**6 - 15*sqrt(-c**2*x**2 + 1)*a*c**4*x**4 + 19*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 7*sqrt(-c**2*x**2 + 1)*a + 63*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**10,x)*b*x**9 - 126*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**8,x)*b*c**2*x**9 + 63*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**6,x)*b*c**4*x**9))/(63*x**9)`

3.92
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx$$

Optimal result	952
Mathematica [A] (verified)	953
Rubi [A] (verified)	953
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Giac [F(-2)]	958
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Reduce [F]	958

Optimal result

Integrand size = 27, antiderivative size = 361

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx &= -\frac{bcd^2 \sqrt{d - c^2 dx^2}}{110x^{10} \sqrt{1 - c^2 x^2}} \\ &+ \frac{23bc^3 d^2 \sqrt{d - c^2 dx^2}}{792x^8 \sqrt{1 - c^2 x^2}} - \frac{113bc^5 d^2 \sqrt{d - c^2 dx^2}}{4158x^6 \sqrt{1 - c^2 x^2}} \\ &+ \frac{bc^7 d^2 \sqrt{d - c^2 dx^2}}{924x^4 \sqrt{1 - c^2 x^2}} + \frac{2bc^9 d^2 \sqrt{d - c^2 dx^2}}{693x^2 \sqrt{1 - c^2 x^2}} \\ &- \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{11dx^{11}} - \frac{4c^2 (d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{99dx^9} \\ &- \frac{8c^4 (d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{693dx^7} - \frac{8bc^{11} d^2 \sqrt{d - c^2 dx^2} \log(x)}{693 \sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/110*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^10/(-c^2*x^2+1)^(1/2)+23/792*b*c^3*d
^2*(-c^2*d*x^2+d)^(1/2)/x^8/(-c^2*x^2+1)^(1/2)-113/4158*b*c^5*d^2*(-c^2*d*
x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+1/924*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x
^4/(-c^2*x^2+1)^(1/2)+2/693*b*c^9*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1
)^(1/2)-1/11*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/d/x^11-4/99*c^2*(-c^2*
d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/d/x^9-8/693*c^4*(-c^2*d*x^2+d)^(7/2)*(a+b
*arcsin(c*x))/d/x^7-8/693*b*c^11*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+
1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.75

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx = \frac{d^2 \sqrt{d - c^2 dx^2} (-15876bcx + 50715bc^3x^3 - 47460bc^5x^5 + 1890bc^7x^7 + 5040b^2c^9x^9 + 59048b^2c^{11}x^{11} - 158760a\sqrt{1 - c^2x^2} + 405720a^2c^2x^2\sqrt{1 - c^2x^2} - 284760a^2c^4x^4\sqrt{1 - c^2x^2} + 7560a^2c^6x^6\sqrt{1 - c^2x^2} + 10080a^2c^8x^8\sqrt{1 - c^2x^2} + 20160a^2c^{10}x^{10}\sqrt{1 - c^2x^2} - 2520b(1 - c^2x^2)^{7/2}(63 + 28c^2x^2 + 8c^4x^4)\text{ArcSin}[cx] - 20160b^2c^{11}x^{11}\text{Log}[x])}{(1746360x^{11}\sqrt{1 - c^2x^2})}$$

input `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^12,x]`

output `(d^2*Sqrt[d - c^2*d*x^2]*(-15876*b*c*x + 50715*b*c^3*x^3 - 47460*b*c^5*x^5 + 1890*b*c^7*x^7 + 5040*b*c^9*x^9 + 59048*b*c^11*x^11 - 158760*a*Sqrt[1 - c^2*x^2] + 405720*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 284760*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 7560*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 10080*a*c^8*x^8*Sqrt[1 - c^2*x^2] + 20160*a*c^10*x^10*Sqrt[1 - c^2*x^2] - 2520*b*(1 - c^2*x^2)^(7/2)*(63 + 28*c^2*x^2 + 8*c^4*x^4)*ArcSin[c*x] - 20160*b*c^11*x^11*Log[x])/(1746360*x^11*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx$$

↓ 5194

$$\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d^2(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{693x^{11}} dx - (d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{\frac{4c^2(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{99dx^9} - \frac{8c^4(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{693dx^7} - \frac{\sqrt{1 - c^2x^2}}{11dx^{11}}}$$

↓ 27

$$\frac{bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{x^{11}} dx}{693\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{693dx^7}$$

↓ 1578

$$\frac{bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{x^{12}} dx^2}{1386\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{693dx^7}$$

↓ 1195

$$\frac{bcd^2\sqrt{d-c^2dx^2} \int \left(-\frac{8c^{10}}{x^2} - \frac{4c^8}{x^4} - \frac{3c^6}{x^6} + \frac{113c^4}{x^8} - \frac{161c^2}{x^{10}} + \frac{63}{x^{12}}\right) dx^2}{1386\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{693dx^7}$$

↓ 2009

$$\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))}{693dx^7} + \frac{bcd^2\sqrt{d-c^2dx^2} \left(-8c^{10} \log(x^2) + \frac{4c^8}{x^2} + \frac{3c^6}{2x^4} - \frac{113c^4}{3x^6} + \frac{161c^2}{4x^8} - \frac{63}{5x^{10}}\right)}{1386\sqrt{1-c^2x^2}}$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^12,x]`

output `-1/11*((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(d*x^11) - (4*c^2*(d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(99*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(693*d*x^7) + (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-63/(5*x^10) + (161*c^2)/(4*x^8) - (113*c^4)/(3*x^6) + (3*c^6)/(2*x^4) + (4*c^8)/x^2 - 8*c^10*Log[x^2]))/(1386*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1195 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 1578 `Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 6761, normalized size of antiderivative = 18.73

method	result	size
default	Expression too large to display	6761
parts	Expression too large to display	6761

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^12,x,method=_RETURNVERBOSE)`

output result too large to display

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.30

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx = \text{Too large to display}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^12,x, algorithm="fricas")`

output

```
[1/83160*(480*(b*c^13*d^2*x^13 - b*c^11*d^2*x^11)*sqrt(d)*log((c^2*d*x^6 +
c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt
t(d) - d)/(c^2*x^4 - x^2)) - (240*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 - (240*
b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^2*x^11 - 2260*b*c^
5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(
-c^2*x^2 + 1) + 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^10 - a*c^8*d^2*x^8
- 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*x^2 + 63*a*d^2 +
(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*b*c^6*d^2*x^6
+ 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*arcsin(c*x))*sqrt(-c^
2*d*x^2 + d))/(c^2*x^13 - x^11), -1/83160*(960*(b*c^13*d^2*x^13 - b*c^11*d
^2*x^11)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)
*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (240*b*c^9*d^2*x^9 + 90*b*c
^7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^
2*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^
10 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2
*x^2 + 63*a*d^2 + (8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 -
116*b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*arc
sin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**12,x)`

output `Timed out`

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.61

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx =$$

$$-\frac{1}{83160} \left(960 c^{10} d^{5/2} \log(x) - \frac{240 c^8 d^{5/2} x^8 + 90 c^6 d^{5/2} x^6 - 2260 c^4 d^{5/2} x^4 + 2415 c^2 d^{5/2} x^2 - 756 d^{5/2}}{x^{10}} \right) bc$$

$$-\frac{1}{693} b \left(\frac{8(-c^2 dx^2 + d)^{7/2} c^4}{dx^7} + \frac{28(-c^2 dx^2 + d)^{7/2} c^2}{dx^9} + \frac{63(-c^2 dx^2 + d)^{7/2}}{dx^{11}} \right) \arcsin(cx)$$

$$-\frac{1}{693} a \left(\frac{8(-c^2 dx^2 + d)^{7/2} c^4}{dx^7} + \frac{28(-c^2 dx^2 + d)^{7/2} c^2}{dx^9} + \frac{63(-c^2 dx^2 + d)^{7/2}}{dx^{11}} \right)$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^12,x, algorithm="maxima")`

output `-1/83160*(960*c^10*d^(5/2)*log(x) - (240*c^8*d^(5/2)*x^8 + 90*c^6*d^(5/2)*x^6 - 2260*c^4*d^(5/2)*x^4 + 2415*c^2*d^(5/2)*x^2 - 756*d^(5/2))/x^10)*b*c - 1/693*b*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))*arcsin(c*x) - 1/693*a*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^12,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^{12}} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^12,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^12, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^{12}} dx = \frac{\sqrt{d} d^2 \left(8\sqrt{-c^2 x^2 + 1} a c^{10} x^{10} + 4\sqrt{-c^2 x^2 + 1} a c^8 x^8 + 3\sqrt{-c^2 x^2 + 1} a c^6 x^6 + 2\sqrt{-c^2 x^2 + 1} a c^4 x^4 + \sqrt{-c^2 x^2 + 1} a c^2 x^2 + a \right)}{d^2}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^12,x)`

output

```
(sqrt(d)*d**2*(8*sqrt(-c**2*x**2+1)*a*c**10*x**10+4*sqrt(-c**2*x**2+1)*a*c**8*x**8+3*sqrt(-c**2*x**2+1)*a*c**6*x**6-113*sqrt(-c**2*x**2+1)*a*c**4*x**4+161*sqrt(-c**2*x**2+1)*a*c**2*x**2-63*sqrt(-c**2*x**2+1)*a+693*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**12,x)*b*x**11-1386*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**10,x)*b*c**2*x**11+693*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**8,x)*b*c**4*x**11))/(693*x**11)
```

3.93 $\int x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	960
Mathematica [A] (verified)	961
Rubi [A] (verified)	961
Maple [A] (verified)	963
Fricas [A] (verification not implemented)	964
Sympy [F(-1)]	964
Maxima [A] (verification not implemented)	965
Giac [F(-2)]	965
Mupad [F(-1)]	966
Reduce [F]	966

Optimal result

Integrand size = 27, antiderivative size = 354

$$\int x^5(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{8bd^2 x \sqrt{d - c^2 dx^2}}{693c^5 \sqrt{1 - c^2 x^2}} + \frac{4bd^2 x^3 \sqrt{d - c^2 dx^2}}{2079c^3 \sqrt{1 - c^2 x^2}}$$

$$+ \frac{bd^2 x^5 \sqrt{d - c^2 dx^2}}{1155c \sqrt{1 - c^2 x^2}} - \frac{113bcd^2 x^7 \sqrt{d - c^2 dx^2}}{4851 \sqrt{1 - c^2 x^2}} + \frac{23bc^3 d^2 x^9 \sqrt{d - c^2 dx^2}}{891 \sqrt{1 - c^2 x^2}}$$

$$- \frac{bc^5 d^2 x^{11} \sqrt{d - c^2 dx^2}}{121 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^6 d}$$

$$+ \frac{2(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{9c^6 d^2} - \frac{(d - c^2 dx^2)^{11/2} (a + b \arcsin(cx))}{11c^6 d^3}$$

output

```
8/693*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+4/2079*b*d^2*x^3
*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/1155*b*d^2*x^5*(-c^2*d*x^2+
d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-113/4851*b*c*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(-
c^2*x^2+1)^(1/2)+23/891*b*c^3*d^2*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1
/2)-1/121*b*c^5*d^2*x^11*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/7*(-c^2
*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/c^6/d+2/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arcs
in(c*x))/c^6/d^2-1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arcsin(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.45

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 \sqrt{d - c^2 dx^2} \left(3465a(1 - c^2 x^2)^{7/2} (8 + 28c^2 x^2 + 63c^4 x^4) + bcx(-27720 - 4620c^2 x^2 - 2079c^4 x^4 + 55935c^6 x^6 - 61985c^8 x^8 + 19845c^{10} x^{10}) + 3465b(1 - c^2 x^2)^{7/2} (8 + 28c^2 x^2 + 63c^4 x^4) \arcsin(cx) \right)}{2401245c^6 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]
```

output

```
-1/2401245*(d^2*Sqrt[d - c^2*d*x^2]*(3465*a*(1 - c^2*x^2)^(7/2)*(8 + 28*c^2*x^2 + 63*c^4*x^4) + b*c*x*(-27720 - 4620*c^2*x^2 - 2079*c^4*x^4 + 55935*c^6*x^6 - 61985*c^8*x^8 + 19845*c^10*x^10) + 3465*b*(1 - c^2*x^2)^(7/2)*(8 + 28*c^2*x^2 + 63*c^4*x^4)*ArcSin[c*x]))/(c^6*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d^2(1 - c^2 x^2)^3 (63c^4 x^4 + 28c^2 x^2 + 8)}{693c^6} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{11/2} (a + b \arcsin(cx))}{11c^6 d^3} +$$

$$\frac{2(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{9c^6 d^2} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^6 d}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bd^2\sqrt{d-c^2dx^2} \int (1-c^2x^2)^3 (63c^4x^4 + 28c^2x^2 + 8) dx}{693c^5\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{11/2} (a+b\arcsin(cx))}{11c^6d^3} + \\
& \frac{2(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^6d^2} - \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d} \\
& \quad \downarrow 1467 \\
& \frac{bd^2\sqrt{d-c^2dx^2} \int (-63c^{10}x^{10} + 161c^8x^8 - 113c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{693c^5\sqrt{1-c^2x^2}} - \\
& \frac{(d-c^2dx^2)^{11/2} (a+b\arcsin(cx))}{11c^6d^3} + \frac{2(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^6d^2} - \\
& \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d} \\
& \quad \downarrow 2009 \\
& -\frac{(d-c^2dx^2)^{11/2} (a+b\arcsin(cx))}{11c^6d^3} + \frac{2(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^6d^2} - \\
& \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^6d} + \\
& \frac{bd^2 \left(-\frac{63}{11}c^{10}x^{11} + \frac{161c^8x^9}{9} - \frac{113c^6x^7}{7} + \frac{3c^4x^5}{5} + \frac{4c^2x^3}{3} + 8x \right) \sqrt{d-c^2dx^2}}{693c^5\sqrt{1-c^2x^2}}
\end{aligned}$$

input

```
Int[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]
```

output

```
(b*d^2*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (113*c^6*x^7)/7 + (161*c^8*x^9)/9 - (63*c^10*x^11)/11))/(693*c^5*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^6*d) + (2*(d - c^2*d*x^2)^(9/2)*(a + b*ArcSin[c*x]))/(9*c^6*d^2) - ((d - c^2*d*x^2)^(11/2)*(a + b*ArcSin[c*x]))/(11*c^6*d^3)
```

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 1467 `Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.72

method	result
orering	$\frac{(83349x^{12}c^{12} - 299047c^{10}x^{10} + 363737c^8x^8 - 140481c^6x^6 - 7854c^4x^4 - 53592c^2x^2 + 33264)(-c^2dx^2 + d)^{\frac{5}{2}}(a + b \arcsin(cx))}{480249c^6(cx-1)^2(cx+1)^2(c^2x^2-1)}$ (198)
default	Expression too large to display
parts	Expression too large to display

input `int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output

```
1/480249*(83349*c^12*x^12-299047*c^10*x^10+363737*c^8*x^8-140481*c^6*x^6-7
854*c^4*x^4-53592*c^2*x^2+33264)/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2
*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))-1/2401245/x^4*(19845*c^10*x^10-61985*c^8
*x^8+55935*c^6*x^6-2079*c^4*x^4-4620*c^2*x^2-27720)/c^6/(c*x-1)^2/(c*x+1)^
2*(5*x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))-5*x^6*(-c^2*d*x^2+d)^(3/2)
*(a+b*arcsin(c*x))*c^2*d+x^5*(-c^2*d*x^2+d)^(5/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.82

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{(19845 bc^{11} d^2 x^{11} - 61985 bc^9 d^2 x^9 + 55935 bc^7 d^2 x^7 - 2079 bc^5 d^2 x^5 - 4620 bc^3 d^2 x^3 - 27720 bc d^2 x) \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} + 3465 (63 a c^{12} d^2 x^{12} - 224 a c^{10} d^2 x^{10} + 274 a c^8 d^2 x^8 - 116 a c^6 d^2 x^6 - a c^4 d^2 x^4 - 4 a c^2 d^2 x^2 + 8 a d^2 + (63 b c^{12} d^2 x^{12} - 224 b c^{10} d^2 x^{10} + 274 b c^8 d^2 x^8 - 116 b c^6 d^2 x^6 - b c^4 d^2 x^4 - 4 b c^2 d^2 x^2 + 8 b d^2) \arcsin(cx)) \sqrt{-c^2 dx^2 + d}}{(c^8 x^2 - c^6)}$$

input

```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas"
)
```

output

```
1/2401245*((19845*b*c^11*d^2*x^11 - 61985*b*c^9*d^2*x^9 + 55935*b*c^7*d^2*
x^7 - 2079*b*c^5*d^2*x^5 - 4620*b*c^3*d^2*x^3 - 27720*b*c*d^2*x)*sqrt(-c^2
*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 3465*(63*a*c^12*d^2*x^12 - 224*a*c^10*d^2
*x^10 + 274*a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 - a*c^4*d^2*x^4 - 4*a*c^2*d^
2*x^2 + 8*a*d^2 + (63*b*c^12*d^2*x^12 - 224*b*c^10*d^2*x^10 + 274*b*c^8*d^
2*x^8 - 116*b*c^6*d^2*x^6 - b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 8*b*d^2)*arc
sin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F(-1)]

Timed out.

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input

```
integrate(x**5*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)
```

output Timed out

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.62

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx =$$

$$-\frac{1}{693} \left(\frac{63 (-c^2 dx^2 + d)^{7/2} x^4}{c^2 d} + \frac{28 (-c^2 dx^2 + d)^{7/2} x^2}{c^4 d} + \frac{8 (-c^2 dx^2 + d)^{7/2}}{c^6 d} \right) b \arcsin(cx)$$

$$-\frac{1}{693} \left(\frac{63 (-c^2 dx^2 + d)^{7/2} x^4}{c^2 d} + \frac{28 (-c^2 dx^2 + d)^{7/2} x^2}{c^4 d} + \frac{8 (-c^2 dx^2 + d)^{7/2}}{c^6 d} \right) a$$

$$-\frac{(19845 c^{10} d^{5/2} x^{11} - 61985 c^8 d^{5/2} x^9 + 55935 c^6 d^{5/2} x^7 - 2079 c^4 d^{5/2} x^5 - 4620 c^2 d^{5/2} x^3 - 27720 d^{5/2} x) b}{2401245 c^5}$$

input `integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/693*(63*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*b*arcsin(c*x) - 1/693*(63*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*a - 1/2401245*(19845*c^10*d^(5/2)*x^11 - 61985*c^8*d^(5/2)*x^9 + 55935*c^6*d^(5/2)*x^7 - 2079*c^4*d^(5/2)*x^5 - 4620*c^2*d^(5/2)*x^3 - 27720*d^(5/2)*x)*b/c^5`

Giac [F(-2)]

Exception generated.

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int x^5 (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int(x^5*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^5*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int x^5 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d^2 (63\sqrt{-c^2 x^2 + 1} a c^{10} x^{10} - 161\sqrt{-c^2 x^2 + 1} a c^8 x^8 + 113\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 3\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 4\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 8\sqrt{-c^2 x^2 + 1} a + 693 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(c x) x^9, x) * b c^{10} - 1386 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(c x) x^7, x) * b c^8 + 693 \int (\sqrt{-c^2 x^2 + 1} \operatorname{asin}(c x) x^5, x) * b c^6)}{(693 c^6)}$$

input `int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d**2*(63*sqrt(-c**2*x**2 + 1)*a*c**10*x**10 - 161*sqrt(-c**2*x**2 + 1)*a*c**8*x**8 + 113*sqrt(-c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt(-c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt(-c**2*x**2 + 1)*a + 693*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**9,x)*b*c**10 - 1386*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**7,x)*b*c**8 + 693*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**5,x)*b*c**6)/(693*c**6)`

3.94 $\int x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	967
Mathematica [A] (verified)	968
Rubi [A] (verified)	968
Maple [A] (verified)	970
Fricas [A] (verification not implemented)	970
Sympy [F(-1)]	971
Maxima [A] (verification not implemented)	971
Giac [F(-2)]	972
Mupad [F(-1)]	972
Reduce [F]	973

Optimal result

Integrand size = 27, antiderivative size = 278

$$\int x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{2bd^2 x \sqrt{d - c^2 dx^2}}{63c^3 \sqrt{1 - c^2 x^2}} + \frac{bd^2 x^3 \sqrt{d - c^2 dx^2}}{189c \sqrt{1 - c^2 x^2}} - \frac{bcd^2 x^5 \sqrt{d - c^2 dx^2}}{21 \sqrt{1 - c^2 x^2}} + \frac{19bc^3 d^2 x^7 \sqrt{d - c^2 dx^2}}{441 \sqrt{1 - c^2 x^2}} - \frac{bc^5 d^2 x^9 \sqrt{d - c^2 dx^2}}{81 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^4 d} + \frac{(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{9c^4 d^2}$$

output

```
2/63*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/189*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+19/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/c^4/d+1/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arcsin(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.49

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 \sqrt{d - c^2 dx^2} \left(-63a(1 - c^2 x^2)^{7/2} (2 + 7c^2 x^2) + b(126cx + 21c^3 x^3 - 189c^5 x^5 + 171c^7 x^7 - 49c^9 x^9) - 63b(1 - c^2 x^2)^{7/2} (2 + 7c^2 x^2) \arcsin(cx) \right)}{3969c^4 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]
```

output

```
(d^2*Sqrt[d - c^2*d*x^2]*(-63*a*(1 - c^2*x^2)^(7/2)*(2 + 7*c^2*x^2) + b*(126*c*x + 21*c^3*x^3 - 189*c^5*x^5 + 171*c^7*x^7 - 49*c^9*x^9) - 63*b*(1 - c^2*x^2)^(7/2)*(2 + 7*c^2*x^2)*ArcSin[c*x]))/(3969*c^4*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{d^2(1 - c^2 x^2)^3(7c^2 x^2 + 2)}{63c^4} dx}{\sqrt{1 - c^2 x^2}} + \frac{(d - c^2 dx^2)^{9/2} (a + b \arcsin(cx))}{9c^4 d^2} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^4 d}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{bd^2\sqrt{d-c^2dx^2} \int (1-c^2x^2)^3 (7c^2x^2+2) dx}{63c^3\sqrt{1-c^2x^2}} + \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^4d^2} - \\
& \quad \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^4d} \\
& \quad \downarrow \text{290} \\
& \frac{bd^2\sqrt{d-c^2dx^2} \int (-7c^8x^8+19c^6x^6-15c^4x^4+c^2x^2+2) dx}{63c^3\sqrt{1-c^2x^2}} + \\
& \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^4d^2} - \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^4d} \\
& \quad \downarrow \text{2009} \\
& \frac{(d-c^2dx^2)^{9/2} (a+b\arcsin(cx))}{9c^4d^2} - \frac{(d-c^2dx^2)^{7/2} (a+b\arcsin(cx))}{7c^4d} + \\
& \frac{bd^2\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3} + 2x\right)\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output `(b*d^2*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - 3*c^4*x^5 + (19*c^6*x^7)/7 - (7*c^8*x^9)/9))/(63*c^3*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^4*d) + ((d - c^2*d*x^2)^(9/2)*(a + b*ArcSin[c*x]))/(9*c^4*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 290 `Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Simp
plifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.86

method	result
orering	$\frac{(833c^{10}x^{10} - 3153c^8x^8 + 4167c^6x^6 - 1743c^4x^4 - 1008c^2x^2 + 504)(-c^2dx^2 + d)^{\frac{5}{2}}(a + b \arcsin(cx))}{3969c^4(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(49c^8x^8 - 171c^6x^6 + 189c^4x^4 - 21c^2x^2 - 126)}{c^4(cx-1)^2(cx+1)^2(3x^2(-c^2dx^2+d)^{\frac{5}{2}}(a+b\arcsin(cx)) - 5x^4(-c^2dx^2+d)^{\frac{3}{2}}(a+b\arcsin(cx)) + c^2d+x^3(-c^2dx^2+d)^{\frac{5}{2}})bc/(-c^2x^2+1)^{\frac{1}{2}}}$
default	Expression too large to display
parts	Expression too large to display

input

```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/3969*(833*c^10*x^10-3153*c^8*x^8+4167*c^6*x^6-1743*c^4*x^4-1008*c^2*x^2+
504)/c^4/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(
c*x))-1/3969/x^2*(49*c^8*x^8-171*c^6*x^6+189*c^4*x^4-21*c^2*x^2-126)/c^4/(
c*x-1)^2/(c*x+1)^2*(3*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))-5*x^4*(-
^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))*c^2*d+x^3*(-c^2*d*x^2+d)^(5/2)*b*c/(-c
^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.92

$$\int x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{(49 bc^9 d^2 x^9 - 171 bc^7 d^2 x^7 + 189 bc^5 d^2 x^5 - 21 bc^3 d^2 x^3 - 126 bcd^2 x) \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 d x^2 + d}}{3969 c^4 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output
$$\frac{1}{3969} \left((49bc^9d^2x^9 - 171b^2c^7d^2x^7 + 189b^3c^5d^2x^5 - 21b^4c^3d^2x^3 - 126b^5c^2d^2x) \sqrt{-c^2dx^2 + d} \sqrt{-c^2x^2 + 1} + 63(7a^2c^{10}d^2x^{10} - 26a^2c^8d^2x^8 + 34a^2c^6d^2x^6 - 16a^2c^4d^2x^4 - a^2c^2d^2x^2 + 2a^2d^2 + (7b^2c^{10}d^2x^{10} - 26b^2c^8d^2x^8 + 34b^2c^6d^2x^6 - 16b^2c^4d^2x^4 - b^2c^2d^2x^2 + 2b^2d^2) \arcsin(cx) \right) \sqrt{-c^2dx^2 + d} / (c^6x^2 - c^4)$$

Sympy [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)`

output Timed out

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.58

$$\begin{aligned} & \int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \\ & -\frac{1}{63} \left(\frac{7(-c^2 dx^2 + d)^{7/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{7/2}}{c^4 d} \right) b \arcsin(cx) \\ & -\frac{1}{63} \left(\frac{7(-c^2 dx^2 + d)^{7/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{7/2}}{c^4 d} \right) a \\ & - \frac{(49c^8 d^{5/2} x^9 - 171c^6 d^{5/2} x^7 + 189c^4 d^{5/2} x^5 - 21c^2 d^{5/2} x^3 - 126d^{5/2} x) b}{3969c^3} \end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*b*arcsin(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*d^(5/2)*x^9 - 171*c^6*d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 - 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*b/c^3`

Giac [F(-2)]

Exception generated.

$$\int x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int x^3 (a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^3*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d^2 (7\sqrt{-c^2 x^2 + 1} a c^8 x^8 - 19\sqrt{-c^2 x^2 + 1} a c^6 x^6 + 15\sqrt{-c^2 x^2 + 1} a c^4 x^4 - \sqrt{-c^2 x^2 + 1} a c^2 x^2 + b \arcsin(cx))}{63 c^4}$$

input `int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)`

output `(sqrt(d)*d**2*(7*sqrt(-c**2*x**2+1)*a*c**8*x**8-19*sqrt(-c**2*x**2+1)*a*c**6*x**6+15*sqrt(-c**2*x**2+1)*a*c**4*x**4-sqrt(-c**2*x**2+1)*a*c**2*x**2-2*sqrt(-c**2*x**2+1)*a+63*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**7,x)*b*c**8-126*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**5,x)*b*c**6+63*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*b*c**4)/(63*c**4)`

3.95 $\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	974
Mathematica [A] (verified)	974
Rubi [A] (verified)	975
Maple [A] (verified)	976
Fricas [A] (verification not implemented)	977
Sympy [F(-1)]	977
Maxima [A] (verification not implemented)	977
Giac [F(-2)]	978
Mupad [F(-1)]	978
Reduce [F]	979

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{bd^2 x \sqrt{d - c^2 dx^2}}{7c \sqrt{1 - c^2 x^2}} - \frac{bcd^2 x^3 \sqrt{d - c^2 dx^2}}{7 \sqrt{1 - c^2 x^2}} + \frac{3bc^3 d^2 x^5 \sqrt{d - c^2 dx^2}}{35 \sqrt{1 - c^2 x^2}} - \frac{bc^5 d^2 x^7 \sqrt{d - c^2 dx^2}}{49 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))}{7c^2 d}$$

output

```
1/7*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/7*b*c*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+3/35*b*c^3*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/49*b*c^5*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.46

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 \sqrt{d - c^2 dx^2} \left(\frac{bc \left(x - c^2 x^3 + \frac{3c^4 x^5}{5} - \frac{c^6 x^7}{7} \right)}{\sqrt{1 - c^2 x^2}} + (-1 + c^2 x^2)^3 (a + b \arcsin(cx)) \right)}{7c^2}$$

input `Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output $(d^2\sqrt{d - c^2dx^2}*((b*c*(x - c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7)))/\sqrt{1 - c^2*x^2} + (-1 + c^2*x^2)^3*(a + b*ArcSin[c*x]))/(7*c^2)$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5182, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2dx^2)^{5/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5182$$

$$\frac{bd^2\sqrt{d - c^2dx^2} \int (1 - c^2x^2)^3 dx}{7c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{7/2} (a + b \arcsin(cx))}{7c^2d}$$

$$\downarrow 210$$

$$\frac{bd^2\sqrt{d - c^2dx^2} \int (-c^6x^6 + 3c^4x^4 - 3c^2x^2 + 1) dx}{7c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{7/2} (a + b \arcsin(cx))}{7c^2d}$$

$$\downarrow 2009$$

$$\frac{bd^2\left(-\frac{1}{7}c^6x^7 + \frac{3c^4x^5}{5} - c^2x^3 + x\right)\sqrt{d - c^2dx^2}}{7c\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{7/2} (a + b \arcsin(cx))}{7c^2d}$$

input `Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output $(b*d^2*\sqrt{d - c^2*d*x^2}*(x - c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/(7*c*\sqrt{1 - c^2*x^2}) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^2*d)$

Defintions of rubi rules used

```
rule 210 Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2)
]^(p, x), x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5182 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

method	result
orering	$\frac{(65c^8x^8 - 271c^6x^6 + 441c^4x^4 - 385c^2x^2 + 70)(-c^2dx^2 + d)^{\frac{5}{2}}(a + b \arcsin(cx))}{245c^2(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(5c^6x^6 - 21c^4x^4 + 35c^2x^2 - 35)\left((-c^2dx^2 + d)^{\frac{5}{2}}(a + b \arcsin(cx))\right)}{6272c^2(c^2x^2-1)}$
default	$-\frac{a(-c^2dx^2 + d)^{\frac{7}{2}}}{7c^2d} + b\left(\frac{\sqrt{-d(c^2x^2-1)}(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2x^2+1} + 104c^4x^4 + 112i\sqrt{-c^2x^2+1}x^5c^5 - 25c^2x^2 - 56ic^2d)}{6272c^2(c^2x^2-1)}\right)$
parts	$-\frac{a(-c^2dx^2 + d)^{\frac{7}{2}}}{7c^2d} + b\left(\frac{\sqrt{-d(c^2x^2-1)}(64c^8x^8 - 144c^6x^6 - 64ic^7x^7\sqrt{-c^2x^2+1} + 104c^4x^4 + 112i\sqrt{-c^2x^2+1}x^5c^5 - 25c^2x^2 - 56ic^2d)}{6272c^2(c^2x^2-1)}\right)$

```
input int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output 1/245*(65*c^8*x^8-271*c^6*x^6+441*c^4*x^4-385*c^2*x^2+70)/c^2/(c*x-1)^2/(c
*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))-1/245*(5*c^6*x^
6-21*c^4*x^4+35*c^2*x^2-35)/c^2/(c*x-1)^2/(c*x+1)^2*((-c^2*d*x^2+d)^(5/2)*
(a+b*arcsin(c*x))-5*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))*c^2*d*x*(-c
^2*d*x^2+d)^(5/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{(5bc^7 d^2 x^7 - 21bc^5 d^2 x^5 + 35bc^3 d^2 x^3 - 35bcd^2 x) \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} + 35(ac^8 d^2 + b \arcsin(cx))}{245cd}$$

input `integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `1/245*((5*b*c^7*d^2*x^7 - 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 - 35*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 35*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2 + (b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)`

Sympy [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = -\frac{(-c^2 dx^2 + d)^{7/2} b \arcsin(cx)}{7c^2 d} - \frac{(-c^2 dx^2 + d)^{7/2} a}{7c^2 d} - \frac{(5c^6 d^{7/2} x^7 - 21c^4 d^{7/2} x^5 + 35c^2 d^{7/2} x^3 - 35d^{7/2} x)b}{245cd}$$

input `integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-1/7*(-c^2*d*x^2 + d)^(7/2)*b*arcsin(c*x)/(c^2*d) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a/(c^2*d) - 1/245*(5*c^6*d^(7/2)*x^7 - 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 - 35*d^(7/2)*x)*b/(c*d)`

Giac [F(-2)]

Exception generated.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int x(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int(x*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{\sqrt{d} d^2 (\sqrt{-c^2 x^2 + 1} a c^6 x^6 - 3\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 3\sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a + 7 \int \sqrt{-c^2 x^2 + 1} a \sin(cx) x^5 dx + b c^6 - 14 \int \sqrt{-c^2 x^2 + 1} a \sin(cx) x^3 dx + 7 \int \sqrt{-c^2 x^2 + 1} a \sin(cx) x dx + b c^2)}{(7 c^2)}$$

input

```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)
```

output

```
(sqrt(d)*d**2*(sqrt(-c**2*x**2+1)*a*c**6*x**6-3*sqrt(-c**2*x**2+1)*a*c**4*x**4+3*sqrt(-c**2*x**2+1)*a*c**2*x**2-sqrt(-c**2*x**2+1)*a+7*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**5,x)*b*c**6-14*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*b*c**4+7*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*b*c**2))/(7*c**2)
```


3.96 $\int \frac{(d-c^2 dx^2)^{5/2}(a+b \arcsin(cx))}{x} dx$

Optimal result	980
Mathematica [A] (verified)	981
Rubi [A] (verified)	981
Maple [A] (verified)	986
Fricas [F]	986
Sympy [F(-1)]	987
Maxima [F]	987
Giac [F(-2)]	988
Mupad [F(-1)]	988
Reduce [F]	988

Optimal result

Integrand size = 27, antiderivative size = 365

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = -\frac{23bcd^2 x \sqrt{d - c^2 dx^2}}{15\sqrt{1 - c^2 x^2}} + \frac{11bc^3 d^2 x^3 \sqrt{d - c^2 dx^2}}{45\sqrt{1 - c^2 x^2}} - \frac{bc^5 d^2 x^5 \sqrt{d - c^2 dx^2}}{25\sqrt{1 - c^2 x^2}} + \frac{1}{3}d(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) + \frac{1}{5}(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) + \sqrt{d - c^2 dx^2} (ad^2 + bd^2 \arcsin(cx)) - \frac{2d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}}$$

output

```
-23/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+11/45*b*c^3*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))+(-c^2*d*x^2+d)^(1/2)*(a*d^2+b*d^2*arcsin(c*x))-2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+I*b*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-I*b*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.08

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \frac{1}{15} ad^2 \sqrt{d - c^2 dx^2} (23 - 11c^2 x^2 + 3c^4 x^4) + ad^{5/2} \log(x) - ad^{5/2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) + \frac{bd^2 \sqrt{d - c^2 dx^2} (-cx + \sqrt{1 - c^2 x^2} \arcsin(cx) + \arcsin(cx) \log(1 - e^{i \arcsin(cx)}))}{\sqrt{1 - c^2 x^2}}$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x,x]
```

output

```
(a*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 + a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) - ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2] - (b*d^2*Sqrt[d - c^2*d*x^2]*(9*c*x - 3*ArcSin[c*x]*(3*Sqrt[1 - c^2*x^2] + Cos[3*ArcSin[c*x]]) + Sin[3*ArcSin[c*x]]))/(18*Sqrt[1 - c^2*x^2]) + (b*d^2*Sqrt[d - c^2*d*x^2]*(450*c*x - 15*ArcSin[c*x]*(30*Sqrt[1 - c^2*x^2] + 5*Cos[3*ArcSin[c*x]] - 3*Cos[5*ArcSin[c*x]]) + 25*Sin[3*ArcSin[c*x]] - 9*Sin[5*ArcSin[c*x]]))/(3600*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5202, 210, 2009, 5202, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx$$

↓ 5202

$$\begin{aligned}
& d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int (1 - c^2 x^2)^2 dx}{5\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{210} \\
& d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int (c^4 x^4 - 2c^2 x^2 + 1) dx}{5\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) \\
& \quad \downarrow \text{2009} \\
& d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx + \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \\
& \quad \frac{bcd^2 \left(\frac{c^4 x^5}{5} - \frac{2c^2 x^3}{3} + x \right) \sqrt{d - c^2 dx^2}}{5\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5202} \\
& d \left(d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} dx - \frac{bcd \sqrt{d - c^2 dx^2} \int (1 - c^2 x^2) dx}{3\sqrt{1 - c^2 x^2}} + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^5}{5} - \frac{2c^2 x^3}{3} + x \right) \sqrt{d - c^2 dx^2}}{5\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{2009} \\
& d \left(d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} dx + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{bcd \left(x - \frac{c^2 x^3}{3} \right) \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} \right) + \\
& \quad \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^5}{5} - \frac{2c^2 x^3}{3} + x \right) \sqrt{d - c^2 dx^2}}{5\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5198} \\
& d \left(d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x \sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} - \frac{bc \sqrt{d - c^2 dx^2} \int 1 dx}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) + \frac{1}{3} (d - c^2 dx^2)^{3/2} \right) \\
& \quad \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4 x^5}{5} - \frac{2c^2 x^3}{3} + x \right) \sqrt{d - c^2 dx^2}}{5\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{24}
\end{aligned}$$

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}} \right)$$

↓ 5218

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}} \right)$$

↓ 3042

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}} \right)$$

↓ 4671

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} (-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}} \right)$$

↓ 2715

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} (ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b \arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}} \right)$$

↓ 2838

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} (-2\operatorname{arctanh}(e^{i\arcsin(cx)}) (a+b\arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i\arcsin(cx)})}{\sqrt{1-c^2x^2}} \right) - \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b\arcsin(cx)) - \frac{bcd^2 \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}} \right)$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x,x]`

output `-1/5*(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/Sqrt[1 - c^2*x^2] + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/5 + d*(-1/3*(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/Sqrt[1 - c^2*x^2] + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 + d*(-((b*c*x*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]) + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])))/Sqrt[1 - c^2*x^2]))`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 210 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5202 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]`

rule 5218 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]`

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.79

method	result
default	$\frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5} + \frac{ad(-c^2dx^2+d)^{\frac{3}{2}}}{3} - ad^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right) + ad^2\sqrt{-c^2dx^2+d} - \frac{23b\sqrt{-d(c^2x^2-1)}}{15(c^2x^2-1)}$
parts	$\frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5} + \frac{ad(-c^2dx^2+d)^{\frac{3}{2}}}{3} - ad^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right) + ad^2\sqrt{-c^2dx^2+d} - \frac{23b\sqrt{-d(c^2x^2-1)}}{15(c^2x^2-1)}$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned} & 1/5*(-c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(-c^2*d*x^2+d)^(3/2)-a*d^(5/2)*\ln((2*d+ \\ & 2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*d^2*(-c^2*d*x^2+d)^(1/2)-23/15*b*(-d* \\ & (c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*\arcsin(c*x)+1/5*b*(-d*(c^2*x^2-1))^(1/2) \\ & *d^2/(c^2*x^2-1)*\arcsin(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(\\ & c^2*x^2-1)*\arcsin(c*x)*x^4*c^4+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2 \\ & -1)*\arcsin(c*x)*x^2*c^2-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2 \\ & *x^2-1)*d^2*\operatorname{polylog}(2,-I*c*x+(-c^2*x^2+1)^(1/2))+I*b*(-d*(c^2*x^2-1))^(1/2) \\ & *(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d^2*\operatorname{polylog}(2,I*c*x+(-c^2*x^2+1)^(1/2))+1 \\ & /25*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5*c^5-11 \\ & /45*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3*c^3+23 \\ & /15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*c+b*(-d* \\ & (c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d^2*\arcsin(c*x)*\ln(1+I*c \\ & *x+(-c^2*x^2+1)^(1/2))-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^ \\ & 2-1)*d^2*\arcsin(c*x)*\ln(1-I*c*x+(-c^2*x^2+1)^(1/2)) \end{aligned}$$
Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x,x, algorithm="fricas")`

output

```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x,x)
```

output

Timed out

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x,x, algorithm="maxima")
```

output

```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x) - 1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2 + d)*d^2)*a
```


Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x} dx = \frac{\sqrt{d} d^2 \left(3\sqrt{-c^2 x^2 + 1} a c^4 x^4 - 11\sqrt{-c^2 x^2 + 1} a c^2 x^2 + 23\sqrt{-c^2 x^2} \right)}{x}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x,x)`

output

```
(sqrt(d)*d**2*(3*sqrt(-c**2*x**2+1)*a*c**4*x**4-11*sqrt(-c**2*x**2+1)*a*c**2*x**2+23*sqrt(-c**2*x**2+1)*a+15*int((sqrt(-c**2*x**2+1)*asin(c*x))/x,x)*b+15*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*b*c**4-30*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*b*c**2+15*log(tan(asin(c*x)/2))*a-23*a))/15
```

3.97
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx$$

Optimal result	990
Mathematica [A] (verified)	991
Rubi [A] (verified)	992
Maple [A] (verified)	996
Fricas [F]	997
Sympy [F(-1)]	997
Maxima [F]	998
Giac [F(-2)]	998
Mupad [F(-1)]	999
Reduce [F]	999

Optimal result

Integrand size = 27, antiderivative size = 386

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = & -\frac{bcd^2 \sqrt{d - c^2 dx^2}}{2x \sqrt{1 - c^2 x^2}} \\ & + \frac{7bc^3 d^2 x \sqrt{d - c^2 dx^2}}{3\sqrt{1 - c^2 x^2}} - \frac{bc^5 d^2 x^3 \sqrt{d - c^2 dx^2}}{9\sqrt{1 - c^2 x^2}} - \frac{5}{2} c^2 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \\ & - \frac{5}{6} c^2 d (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{2x^2} \\ & + \frac{5c^2 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ & - \frac{5ibc^2 d^2 \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{2\sqrt{1 - c^2 x^2}} \\ & + \frac{5ibc^2 d^2 \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{2\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/2*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/9*b*c^5*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))-1/2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^2+5*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-5/2*I*b*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+5/2*I*b*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.25

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \frac{-12ad^3(-1 + c^2x^2)(-3 - 14c^2x^2 + 2c^4x^4) - 180ac^2d^{5/2}x^2\sqrt{d - c^2x^2}}{x^3}$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
(-12*a*d^3*(-1 + c^2*x^2)*(-3 - 14*c^2*x^2 + 2*c^4*x^4) - 180*a*c^2*d^(5/2)*x^2*Sqrt[d - c^2*d*x^2]*Log[x] + 180*a*c^2*d^(5/2)*x^2*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + 144*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*(c*x - Sqrt[1 - c^2*x^2]*ArcSin[c*x] - ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x])]) - Log[1 + E^(I*ArcSin[c*x])]) - I*(PolyLog[2, -E^(I*ArcSin[c*x])] - PolyLog[2, E^(I*ArcSin[c*x])])) + 2*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*(9*c*x - 3*ArcSin[c*x]*(3*Sqrt[1 - c^2*x^2] + Cos[3*ArcSin[c*x]]) + Sin[3*ArcSin[c*x]]) - 9*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*(2*Cot[ArcSin[c*x]/2] + ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 + 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] - (4*I)*PolyLog[2, E^(I*ArcSin[c*x])] - ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 + 2*Tan[ArcSin[c*x]/2]))/(72*x^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5200, 244, 2009, 5202, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx \\
 & \quad \downarrow \text{5200} \\
 & -\frac{5}{2}c^2d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x^2} dx}{2\sqrt{1 - c^2 x^2}} - \\
 & \quad \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{2x^2} \\
 & \quad \downarrow \text{244} \\
 & -\frac{5}{2}c^2d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int (x^2 c^4 - 2c^2 + \frac{1}{x^2}) dx}{2\sqrt{1 - c^2 x^2}} - \\
 & \quad \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{2x^2} \\
 & \quad \downarrow \text{2009} \\
 & -\frac{5}{2}c^2d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \\
 & \quad \frac{bcd^2 \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \sqrt{d - c^2 dx^2}}{2\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{5202} \\
 & -\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x} dx - \frac{bcd \sqrt{d - c^2 dx^2} \int (1 - c^2 x^2) dx}{3\sqrt{1 - c^2 x^2}} + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) \right) \\
 & \quad \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{bcd^2 \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \sqrt{d - c^2 dx^2}}{2\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$-\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} dx + \frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\arcsin(cx)) - \frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}} \right. \\ \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right)$$

↓ 5198

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \int 1 dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right) + \frac{1}{3}(d-c^2dx^2)^3 \right. \\ \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right)$$

↓ 24

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3}(d-c^2dx^2)^3 \right. \\ \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right)$$

↓ 5218

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3}(d-c^2dx^2)^3 \right. \\ \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right)$$

↓ 3042

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) \csc(\arcsin(cx)) d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) + \frac{1}{3}(d-c^2dx^2)^3 \right. \\ \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{2x^2} + \frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right)$$

↓ 4671

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \arctanh)}{\sqrt{1-c^2x^2}} \right. \right. \\ \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd^2 \left(\frac{c^4x^3}{3} - 2c^2x - \frac{1}{x} \right) \sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right) \right.$$

↓ 2715

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) d)}{\sqrt{1-c^2x^2}} \right. \right. \\ \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd^2 \left(\frac{c^4x^3}{3} - 2c^2x - \frac{1}{x} \right) \sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right) \right.$$

↓ 2838

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(-2 \arctanh(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} \right. \right. \\ \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{2x^2} + \frac{bcd^2 \left(\frac{c^4x^3}{3} - 2c^2x - \frac{1}{x} \right) \sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}} \right) \right.$$

input

```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
(b*c*d^2*Sqrt[d - c^2*d*x^2]*(-x^(-1) - 2*c^2*x + (c^4*x^3)/3))/(2*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(2*x^2) - (5*c^2*d*(-1/3*(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/Sqrt[1 - c^2*x^2] + ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 + d*(-((b*c*x*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]) + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])))/Sqrt[1 - c^2*x^2]))/2
```

Definitions of rubi rules used

- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`
- rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.71

method	result
default	$a \left(-\frac{(-c^2 d x^2 + d)^{\frac{7}{2}}}{2 d x^2} - \frac{5 c^2 \left(\frac{(-c^2 d x^2 + d)^{\frac{5}{2}}}{5} + d \left(\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{3} + d \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right) \right) \right)}{2} \right) +$
parts	$a \left(-\frac{(-c^2 d x^2 + d)^{\frac{7}{2}}}{2 d x^2} - \frac{5 c^2 \left(\frac{(-c^2 d x^2 + d)^{\frac{5}{2}}}{5} + d \left(\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{3} + d \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right) \right) \right)}{2} \right) +$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output

```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(7/2)-5/2*c^2*(1/5*(-c^2*d*x^2+d)^(5/2)+d*(1/3*(-c^2*d*x^2+d)^(3/2)+d*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))))+b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*(I+3*arcsin(c*x))*c^2*d^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)*c^2*d^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)*c^2*d^2/(c^2*x^2-1)+1/72*(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*(-I+3*arcsin(c*x))*c^2*d^2/(c^2*x^2-1)-1/2*d^2*(c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2+5*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2)))+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))*c^2*d^2/(2*c^2*x^2-2))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")
```

output

```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**3,x)
```

output Timed out

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^3, x) + 1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^3,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx = \frac{\sqrt{d} d^2 \left(8\sqrt{-c^2 x^2 + 1} a c^4 x^4 - 56\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 12\sqrt{-c^2 x^2 + 1} a + 24 \int \frac{\sqrt{-c^2 x^2 + 1} a c^2 x^2}{x^3} dx - 48 \int \frac{\sqrt{-c^2 x^2 + 1} a c^2 x^2}{x} dx + 24 \int \sqrt{-c^2 x^2 + 1} a c^2 x^2 dx - 60 \log(\tan(\arcsin(cx)/2)) a c^2 x^2 + 65 a c^2 x^2 \right)}{(24 x^2)}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^3,x)`

output `(sqrt(d)*d**2*(8*sqrt(-c**2*x**2 + 1)*a*c**4*x**4 - 56*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - 12*sqrt(-c**2*x**2 + 1)*a + 24*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**3,x)*b*x**2 - 48*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x,x)*b*c**2*x**2 + 24*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x,x)*b*c**4*x**2 - 60*log(tan(asin(c*x)/2))*a*c**2*x**2 + 65*a*c**2*x**2)/(24*x**2)`

3.98 $\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx$

Optimal result	1000
Mathematica [A] (verified)	1001
Rubi [A] (verified)	1002
Maple [A] (verified)	1007
Fricas [F]	1007
Sympy [F(-1)]	1008
Maxima [F]	1008
Giac [F(-2)]	1009
Mupad [F(-1)]	1009
Reduce [F]	1009

Optimal result

Integrand size = 27, antiderivative size = 389

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = & -\frac{bcd^2 \sqrt{d - c^2 dx^2}}{12x^3 \sqrt{1 - c^2 x^2}} \\ & + \frac{9bc^3 d^2 \sqrt{d - c^2 dx^2}}{8x \sqrt{1 - c^2 x^2}} - \frac{bc^5 d^2 x \sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} + \frac{15}{8} c^4 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \\ & + \frac{5c^2 d (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{8x^2} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{4x^4} \\ & - \frac{15c^4 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{4\sqrt{1 - c^2 x^2}} \\ & + \frac{15ibc^4 d^2 \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{8\sqrt{1 - c^2 x^2}} \\ & - \frac{15ibc^4 d^2 \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{8\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```

-1/12*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^3/(-c^2*x^2+1)^(1/2)+9/8*b*c^3*d^2*(-
c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)-b*c^5*d^2*x*(-c^2*d*x^2+d)^(1/2)/(
-c^2*x^2+1)^(1/2)+15/8*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))+5/8*
c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(5/2)*
(a+b*arcsin(c*x))/x^4-15/4*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*
arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+15/8*I*b*c^4*d^2*(-c^
2*d*x^2+d)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-1
5/8*I*b*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-
c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.65

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \frac{ad^2 \sqrt{d - c^2 dx^2} (-2 + 9c^2 x^2 + 8c^4 x^4)}{8x^4}$$

$$+ \frac{15}{8} ac^4 d^{5/2} \log(x)$$

$$- \frac{15}{8} ac^4 d^{5/2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) + \frac{bc^4 d^2 \sqrt{d - c^2 dx^2} (-cx + \sqrt{1 - c^2 x^2} \arcsin(cx) + \arcsin(cx) \log(1 - \dots)}{8x^4}$$

input

```

Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/x^5,x]

```

output

```
(a*d^2*Sqrt[d - c^2*d*x^2]*(-2 + 9*c^2*x^2 + 8*c^4*x^4))/(8*x^4) + (15*a*c^4*d^(5/2)*Log[x])/8 - (15*a*c^4*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/8 + (b*c^4*d^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])])))/Sqrt[1 - c^2*x^2] - (b*c^4*d^3*Sqrt[1 - c^2*x^2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])] + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2])))/(4*Sqrt[d - c^2*d*x^2]) + (b*c^4*d^2*Sqrt[d - c^2*d*x^2]*(8*Cot[ArcSin[c*x]/2] + 6*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - c*x*Csc[ArcSin[c*x]/2]^4 - 3*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^4 - 24*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 24*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (24*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (24*I)*PolyLog[2, E^(I*ArcSin[c*x])] - 6*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 + 3*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^4 - (16*Sin[ArcSin[c*x]/2]^4)/(c^3*x^3) + 8*Tan[ArcSin[c*x]/2])))/(192*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {5200, 244, 2009, 5200, 244, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx$$

↓ 5200

$$-\frac{5}{4}c^2d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx + \frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2}{x^4} dx}{4\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{4x^4}$$

↓ 244

$$\begin{aligned}
 & -\frac{5}{4}c^2d \int \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{x^3} dx + \frac{bcd^2\sqrt{d - c^2dx^2} \int \left(c^4 - \frac{2c^2}{x^2} + \frac{1}{x^4}\right) dx}{4\sqrt{1 - c^2x^2}} - \\
 & \qquad \qquad \qquad \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{4x^4} \\
 & \qquad \qquad \qquad \downarrow \text{2009} \\
 & -\frac{5}{4}c^2d \int \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{x^3} dx - \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{4x^4} + \\
 & \qquad \qquad \qquad \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \\
 & \qquad \qquad \qquad \downarrow \text{5200} \\
 & -\frac{5}{4}c^2d \left(-\frac{3}{2}c^2d \int \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{x} dx + \frac{bcd\sqrt{d - c^2dx^2} \int \frac{1 - c^2x^2}{x^2} dx}{2\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right) \\
 & \qquad \qquad \qquad \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \\
 & \qquad \qquad \qquad \downarrow \text{244} \\
 & -\frac{5}{4}c^2d \left(-\frac{3}{2}c^2d \int \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{x} dx + \frac{bcd\sqrt{d - c^2dx^2} \int \left(\frac{1}{x^2} - c^2\right) dx}{2\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right) \\
 & \qquad \qquad \qquad \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \\
 & \qquad \qquad \qquad \downarrow \text{2009} \\
 & -\frac{5}{4}c^2d \left(-\frac{3}{2}c^2d \int \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{x} dx - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{2x^2} + \frac{bcd\left(c^2(-x) - \frac{1}{x}\right)\sqrt{d - c^2dx^2}}{2\sqrt{1 - c^2x^2}} \right) \\
 & \qquad \qquad \qquad \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}} \\
 & \qquad \qquad \qquad \downarrow \text{5198} \\
 & -\frac{5}{4}c^2d \left(-\frac{3}{2}c^2d \left(\frac{\sqrt{d - c^2dx^2} \int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2x^2}} dx}{\sqrt{1 - c^2x^2}} - \frac{bc\sqrt{d - c^2dx^2} \int 1 dx}{\sqrt{1 - c^2x^2}} + \sqrt{d - c^2dx^2}(a + b \arcsin(cx)) \right) - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right) \\
 & \qquad \qquad \qquad \frac{(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d - c^2dx^2}}{4\sqrt{1 - c^2x^2}}
 \end{aligned}$$

$$\begin{aligned}
& \downarrow 24 \\
-\frac{5}{4}c^2d & \left(-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \right) \\
& \downarrow 5218 \\
-\frac{5}{4}c^2d & \left(-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \right) \\
& \downarrow 3042 \\
-\frac{5}{4}c^2d & \left(-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) \csc(\arcsin(cx)) d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \right) \\
& \downarrow 4671 \\
-\frac{5}{4}c^2d & \left(-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} (-b \int \log(1-e^{i\arcsin(cx)}) d\arcsin(cx) + b \int \log(1+e^{i\arcsin(cx)}) d\arcsin(cx) - 2a \arcsin(cx))}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \right) \\
& \downarrow 2715 \\
-\frac{5}{4}c^2d & \left(-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} (ib \int e^{-i\arcsin(cx)} \log(1-e^{i\arcsin(cx)}) de^{i\arcsin(cx)} - ib \int e^{-i\arcsin(cx)} \log(1+e^{i\arcsin(cx)}) de^{i\arcsin(cx)})}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) - \frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))}{4x^4} + \frac{bcd^2\left(c^4x + \frac{2c^2}{x} - \frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} \right) \\
& \downarrow 2838
\end{aligned}$$

rule 2838 $\text{Int}[\text{Log}[(c_)*(d_)+(e_)*(x_)^{(n_)}]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_)+(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c+d*x)^m*(\text{ArcTanh}[E^{(I*(e+f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1-E^{(I*(e+f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1+E^{(I*(e+f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 5198 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*((f_)*(x_))^{(m_)}*\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d+e*x^2]*((a+b*\text{ArcSin}[c*x])^n/(f*(m+2))), x] + (\text{Simp}[(1/(m+2))*\text{Simp}[\text{Sqrt}[d+e*x^2]/\text{Sqrt}[1-c^2*x^2]] \text{Int}[(f*x)^m*((a+b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1-c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m+2)))*\text{Simp}[\text{Sqrt}[d+e*x^2]/\text{Sqrt}[1-c^2*x^2]] \text{Int}[(f*x)^{(m+1)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& \text{GtQ}[n, 0] \&\& (\text{IGtQ}[m, -2] || \text{EqQ}[n, 1])$

rule 5200 $\text{Int}[((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*((f_)*(x_))^{(m_)}*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d+e*x^2)^p*((a+b*\text{ArcSin}[c*x])^n/(f*(m+1))), x] + (-\text{Simp}[2*e*(p/(f^2*(m+1))) \text{Int}[(f*x)^{(m+2)}*(d+e*x^2)^{(p-1)}*(a+b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1-c^2*x^2)^{(p-1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& \text{LtQ}[m, -1]$

rule 5218 $\text{Int}[(((a_)+\text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}*(x_)^{(m_)})/\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/c^{(m+1)})*\text{Simp}[\text{Sqrt}[1-c^2*x^2]/\text{Sqrt}[d+e*x^2]] \text{Subst}[\text{Int}[(a+b*x)^n*\text{Sin}[x]^m, x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{4dx^4} + \frac{3ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{8dx^2} + \frac{3ac^4(-c^2dx^2+d)^{\frac{5}{2}}}{8} + \frac{5ac^4d(-c^2dx^2+d)^{\frac{3}{2}}}{8} - \frac{15ac^4d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2}}{x}\right)}{8}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{4dx^4} + \frac{3ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{8dx^2} + \frac{3ac^4(-c^2dx^2+d)^{\frac{5}{2}}}{8} + \frac{5ac^4d(-c^2dx^2+d)^{\frac{3}{2}}}{8} - \frac{15ac^4d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2}}{x}\right)}{8}$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^5,x,method=_RETURNVERBOSE)`

output

```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(7/2)+3/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(7/2)+3/8
*a*c^4*(-c^2*d*x^2+d)^(5/2)+5/8*a*c^4*d*(-c^2*d*x^2+d)^(3/2)-15/8*a*c^4*d
(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+15/8*a*c^4*d^2*(-c^2*d*x
2+d)^(1/2)+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)
-1)*(arcsin(c*x)+I)*c^4*d^2/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c
2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)*c^4*d^2/(c^2*x^2-1)+1/24*d^2
*(27*c^4*x^4*arcsin(c*x)-27*c^3*x^3*(-c^2*x^2+1)^(1/2)-33*c^2*x^2*arcsin(c
*x)+2*c*x*(-c^2*x^2+1)^(1/2)+6*arcsin(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x
2-1)/x^4-15*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcsin(c*x)*ln(
1+I*c*x+(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+p
olylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c
^4*d^2/(8*c^2*x^2-8))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^5} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="fricas")`

output `integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))/x**5,x)`

output Timed out

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)}{x^5} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="maxima")`

output `b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x^5, x) - 1/8*(15*c^4*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^2*d*x^2 + d)^(5/2)*c^4 - 5*(-c^2*d*x^2 + d)^(3/2)*c^4*d - 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^2) + 2*(-c^2*d*x^2 + d)^(7/2)/(d*x^4))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))/x^5,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \int \frac{(a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2}}{x^5} dx$$

input `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^5,x)`

output `int(((a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2))/x^5, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{x^5} dx = \frac{\sqrt{d} d^2 (8\sqrt{-c^2 x^2 + 1} a c^4 x^4 + 9\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a c^2 x^2 + \dots)}{x^5}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))/x^5,x)`

output

```
(sqrt(d)*d**2*(8*sqrt(-c**2*x**2+1)*a*c**4*x**4+9*sqrt(-c**2*x**2+1)*a*c**2*x**2-2*sqrt(-c**2*x**2+1)*a+8*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**5,x)*b*x**4-16*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**3,x)*b*c**2*x**4+8*int((sqrt(-c**2*x**2+1)*asin(c*x))/x,x)*b*c**4*x**4+15*log(tan(asin(c*x)/2))*a*c**4*x**4-10*a*c**4*x**4))/(8*x**4)
```

3.99 $\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result	1011
Mathematica [A] (verified)	1011
Rubi [A] (verified)	1012
Maple [A] (verified)	1013
Fricas [A] (verification not implemented)	1014
Sympy [A] (verification not implemented)	1014
Maxima [A] (verification not implemented)	1015
Giac [A] (verification not implemented)	1015
Mupad [F(-1)]	1016
Reduce [F]	1016

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{3x^2}{16a^3} + \frac{x^4}{16a} - \frac{3x\sqrt{1-a^2x^2} \arcsin(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{3 \arcsin(ax)^2}{16a^5}$$

output

$3/16*x^2/a^3+1/16*x^4/a-3/8*x*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)/a^4-1/4*x^3*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)/a^2+3/16*\arcsin(a*x)^2/a^5$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{a^2x^2(3+a^2x^2) - 2ax\sqrt{1-a^2x^2}(3+2a^2x^2) \arcsin(ax) + 3 \arcsin(ax)^2}{16a^5}$$

input

`Integrate[(x^4*ArcSin[a*x])/Sqrt[1 - a^2*x^2], x]`

output

$$(a^2 x^2 (3 + a^2 x^2) - 2 a x \sqrt{1 - a^2 x^2}) (3 + 2 a^2 x^2) \operatorname{ArcSin}[a x] + 3 \operatorname{ArcSin}[a x]^2 / (16 a^5)$$
Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx \\ & \quad \downarrow \text{5210} \\ & \frac{3 \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{\int x^3 dx}{4a} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} \\ & \quad \downarrow \text{15} \\ & \frac{3 \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \\ & \quad \downarrow \text{5210} \\ & \frac{3 \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x dx}{2a} - \frac{x \sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \\ & \quad \downarrow \text{15} \\ & \frac{3 \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x \sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \\ & \quad \downarrow \text{5152} \\ & -\frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{3 \left(\frac{\arcsin(ax)^2}{4a^3} - \frac{x \sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right)}{4a^2} + \frac{x^4}{16a} \end{aligned}$$

input `Int[(x^4*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]`

output
$$\frac{x^4}{16a} - \frac{(x^3\sqrt{1 - a^2x^2})\text{ArcSin}[ax]}{4a^2} + \frac{(3x^2/(4a) - (x\sqrt{1 - a^2x^2})\text{ArcSin}[ax])/(2a^2) + \text{ArcSin}[ax]^2/(4a^3))}{4a^2}$$

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5210 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]`

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{-16 \arcsin(ax)\sqrt{-a^2x^2+1} a^3x^3+4a^4x^4-24 \arcsin(ax)\sqrt{-a^2x^2+1} ax+12a^2x^2+12 \arcsin(ax)^2+9}{64a^5}$	76

input `int(x^4*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output $1/64*(-16*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}*a^3*x^3+4*a^4*x^4-24*\arcsin(a*x)*(-a^2*x^2+1)^{(1/2)}*a*x+12*a^2*x^2+12*\arcsin(a*x)^2+9)/a^5$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{a^4x^4 + 3a^2x^2 - 2(2a^3x^3 + 3ax)\sqrt{-a^2x^2 + 1} \arcsin(ax) + 3 \arcsin(ax)^2}{16a^5}$$

input `integrate(x^4*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output $1/16*(a^4*x^4 + 3*a^2*x^2 - 2*(2*a^3*x^3 + 3*a*x)*\sqrt{-a^2*x^2 + 1}*\arcsin(a*x) + 3*\arcsin(a*x)^2)/a^5$

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

$$= \begin{cases} \frac{x^4}{16a} - \frac{x^3\sqrt{-a^2x^2+1}\arcsin(ax)}{4a^2} + \frac{3x^2}{16a^3} - \frac{3x\sqrt{-a^2x^2+1}\arcsin(ax)}{8a^4} + \frac{3\arcsin^2(ax)}{16a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x**4*asin(a*x)/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((x**4/(16*a) - x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)/(4*a**2) + 3*x**2/(16*a**3) - 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)/(8*a**4) + 3*asin(a*x)**2/(16*a**5), Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{1}{16} \left(\frac{x^4}{a^2} + \frac{3x^2}{a^4} - \frac{3 \arcsin(ax)^2}{a^6} \right) a$$

$$- \frac{1}{8} \left(\frac{2\sqrt{-a^2x^2+1}x^3}{a^2} + \frac{3\sqrt{-a^2x^2+1}x}{a^4} - \frac{3 \arcsin(ax)}{a^5} \right) \arcsin(ax)$$

input `integrate(x^4*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `1/16*(x^4/a^2 + 3*x^2/a^4 - 3*arcsin(a*x)^2/a^6)*a - 1/8*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2 + 1)*x/a^4 - 3*arcsin(a*x)/a^5)*arcsin(a*x)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{(-a^2x^2+1)^{\frac{3}{2}}x \arcsin(ax)}{4a^4} - \frac{5\sqrt{-a^2x^2+1}x \arcsin(ax)}{8a^4}$$

$$+ \frac{(a^2x^2-1)^2}{16a^5} + \frac{3 \arcsin(ax)^2}{16a^5} + \frac{5(a^2x^2-1)}{16a^5} + \frac{17}{128a^5}$$

input `integrate(x^4*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `1/4*(-a^2*x^2 + 1)^(3/2)*x*arcsin(a*x)/a^4 - 5/8*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a^4 + 1/16*(a^2*x^2 - 1)^2/a^5 + 3/16*arcsin(a*x)^2/a^5 + 5/16*(a^2*x^2 - 1)/a^5 + 17/128/a^5`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^4 \operatorname{asin}(ax)}{\sqrt{1-a^2x^2}} dx$$

input `int((x^4*asin(a*x))/(1 - a^2*x^2)^(1/2),x)`

output `int((x^4*asin(a*x))/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax) x^4}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^4*asin(a*x)/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)*x**4)/sqrt(- a**2*x**2 + 1),x)`

3.100 $\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result	1017
Mathematica [A] (verified)	1017
Rubi [A] (verified)	1018
Maple [A] (verified)	1019
Fricas [A] (verification not implemented)	1020
Sympy [A] (verification not implemented)	1020
Maxima [A] (verification not implemented)	1021
Giac [F(-2)]	1021
Mupad [F(-1)]	1022
Reduce [F]	1022

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{2x}{3a^3} + \frac{x^3}{9a} - \frac{2\sqrt{1-a^2x^2} \arcsin(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2} \arcsin(ax)}{3a^2}$$

output

```
2/3*x/a^3+1/9*x^3/a-2/3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{ax(6+a^2x^2) - 3\sqrt{1-a^2x^2}(2+a^2x^2) \arcsin(ax)}{9a^4}$$

input

```
Integrate[(x^3*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]
```

output

```
(a*x*(6 + a^2*x^2) - 3*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcSin[a*x])/(9*a^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx \\
 & \quad \downarrow \text{5210} \\
 & \frac{2 \int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{3a^2} + \frac{\int x^2 dx}{3a} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} \\
 & \quad \downarrow \text{15} \\
 & \frac{2 \int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{3a^2} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} + \frac{x^3}{9a} \\
 & \quad \downarrow \text{5182} \\
 & \frac{2 \left(\frac{\int 1 dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right)}{3a^2} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} + \frac{x^3}{9a} \\
 & \quad \downarrow \text{24} \\
 & -\frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} + \frac{2 \left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right)}{3a^2} + \frac{x^3}{9a}
 \end{aligned}$$

input `Int[(x^3*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]`

output `x^3/(9*a) - (x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(3*a^2) + (2*(x/a - (Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a^2))/(3*a^2)`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

rule 5210 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]`

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

method	result	size
default	$-\frac{(3 \arcsin(ax)a^4x^4 + 3 \arcsin(ax)a^2x^2 + \sqrt{-a^2x^2+1}a^3x^3 - 6 \arcsin(ax) + 6\sqrt{-a^2x^2+1}xa)\sqrt{-a^2x^2+1}}{9a^4(a^2x^2-1)}$	95
orering	$\frac{(5a^4x^4 + 12a^2x^2 - 24) \arcsin(ax)}{9a^4\sqrt{-a^2x^2+1}} - \frac{(a^2x^2+6)(ax+1)(ax-1)\left(\frac{3x^2 \arcsin(ax)}{\sqrt{-a^2x^2+1}} + \frac{x^3a}{-a^2x^2+1} + \frac{x^4 \arcsin(ax)a^2}{(-a^2x^2+1)^{\frac{3}{2}}}\right)}{9x^2a^4}$	130

input `int(x^3*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output

```
-1/9/a^4*(3*arcsin(a*x)*a^4*x^4+3*arcsin(a*x)*a^2*x^2+(-a^2*x^2+1)^(1/2)*a^3*x^3-6*arcsin(a*x)+6*(-a^2*x^2+1)^(1/2)*x*a)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{a^3x^3 - 3(a^2x^2 + 2)\sqrt{-a^2x^2 + 1} \arcsin(ax) + 6ax}{9a^4}$$

input

```
integrate(x^3*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
1/9*(a^3*x^3 - 3*(a^2*x^2 + 2)*sqrt(-a^2*x^2 + 1)*arcsin(a*x) + 6*a*x)/a^4
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{x^3}{9a} - \frac{x^2\sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{3a^2} + \frac{2x}{3a^3} - \frac{2\sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{3a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input

```
integrate(x**3*asin(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output

```
Piecewise((x**3/(9*a) - x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)/(3*a**2) + 2*x/(3*a**3) - 2*sqrt(-a**2*x**2 + 1)*asin(a*x)/(3*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{1}{9} a \left(\frac{x^3}{a^2} + \frac{6x}{a^4} \right) - \frac{1}{3} \left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arcsin(ax)$$

input `integrate(x^3*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `1/9*a*(x^3/a^2 + 6*x/a^4) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arcsin(a*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^3 \operatorname{asin}(ax)}{\sqrt{1-a^2x^2}} dx$$

input `int((x^3*asin(a*x))/(1 - a^2*x^2)^(1/2),x)`output `int((x^3*asin(a*x))/(1 - a^2*x^2)^(1/2), x)`**Reduce [F]**

$$\int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax) x^3}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^3*asin(a*x)/(-a^2*x^2+1)^(1/2),x)`output `int((asin(a*x)*x**3)/sqrt(- a**2*x**2 + 1),x)`

3.101 $\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result	1023
Mathematica [A] (verified)	1023
Rubi [A] (verified)	1024
Maple [A] (verified)	1025
Fricas [A] (verification not implemented)	1025
Sympy [A] (verification not implemented)	1026
Maxima [A] (verification not implemented)	1026
Giac [A] (verification not implemented)	1027
Mupad [F(-1)]	1027
Reduce [F]	1027

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x^2}{4a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{\arcsin(ax)^2}{4a^3}$$

output

$$1/4*x^2/a-1/2*x*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)/a^2+1/4*\arcsin(a*x)^2/a^3$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{a^2x^2 - 2ax\sqrt{1-a^2x^2} \arcsin(ax) + \arcsin(ax)^2}{4a^3}$$

input

$$\text{Integrate}[(x^2*\text{ArcSin}[a*x])/Sqrt[1 - a^2*x^2],x]$$

output

$$(a^2*x^2 - 2*a*x*Sqrt[1 - a^2*x^2]*ArcSin[a*x] + ArcSin[a*x]^2)/(4*a^3)$$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

↓ 5210

$$\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2}$$

↓ 15

$$\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a}$$

↓ 5152

$$\frac{\arcsin(ax)^2}{4a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a}$$

input `Int[(x^2*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]`

output `x^2/(4*a) - (x*Sqrt[1 - a^2*x^2]*ArcSin[a*x])/(2*a^2) + ArcSin[a*x]^2/(4*a^3)`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^ (m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x]
+ (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x]
+ Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-2 \arcsin(ax) \sqrt{-a^2 x^2 + 1} ax + a^2 x^2 + \arcsin(ax)^2}{4a^3}$	40

input

```
int(x^2*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4*(-2*arcsin(a*x)*(-a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arcsin(a*x)^2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1 - a^2 x^2}} dx = \frac{a^2 x^2 - 2 \sqrt{-a^2 x^2 + 1} ax \arcsin(ax) + \arcsin(ax)^2}{4a^3}$$

input

```
integrate(x^2*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output `1/4*(a^2*x^2 - 2*sqrt(-a^2*x^2 + 1)*a*x*arcsin(a*x) + arcsin(a*x)^2)/a^3`

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{x^2}{4a} - \frac{x\sqrt{-a^2x^2+1}\arcsin(ax)}{2a^2} + \frac{\arcsin^2(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x**2*asin(a*x)/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((x**2/(4*a) - x*sqrt(-a**2*x**2 + 1)*asin(a*x)/(2*a**2) + asin(a*x)**2/(4*a**3), Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{1}{4} a \left(\frac{x^2}{a^2} - \frac{\arcsin(ax)^2}{a^4} \right) - \frac{1}{2} \left(\frac{\sqrt{-a^2x^2+1}x}{a^2} - \frac{\arcsin(ax)}{a^3} \right) \arcsin(ax)$$

input `integrate(x^2*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `1/4*a*(x^2/a^2 - arcsin(a*x)^2/a^4) - 1/2*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)*arcsin(a*x)`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{-a^2x^2+1}x \arcsin(ax)}{2a^2} + \frac{\arcsin(ax)^2}{4a^3} + \frac{a^2x^2-1}{4a^3} + \frac{1}{8a^3}$$

input `integrate(x^2*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `-1/2*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a^2 + 1/4*arcsin(a*x)^2/a^3 + 1/4*(a^2*x^2 - 1)/a^3 + 1/8/a^3`**Mupad [F(-1)]**

Timed out.

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \operatorname{asin}(ax)}{\sqrt{1-a^2x^2}} dx$$

input `int((x^2*asin(a*x))/(1 - a^2*x^2)^(1/2),x)`output `int((x^2*asin(a*x))/(1 - a^2*x^2)^(1/2), x)`**Reduce [F]**

$$\int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax) x^2}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^2*asin(a*x)/(-a^2*x^2+1)^(1/2),x)`output `int((asin(a*x)*x**2)/sqrt(- a**2*x**2 + 1),x)`

3.102 $\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result	1028
Mathematica [A] (verified)	1028
Rubi [A] (verified)	1029
Maple [B] (verified)	1030
Fricas [A] (verification not implemented)	1030
Sympy [A] (verification not implemented)	1031
Maxima [A] (verification not implemented)	1031
Giac [A] (verification not implemented)	1031
Mupad [F(-1)]	1032
Reduce [B] (verification not implemented)	1032

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2}$$

output `x/a-(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^2`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2}$$

input `Integrate[(x*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]`

output `x/a - (Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a^2`

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

$$\downarrow 5182$$

$$\frac{\int 1 dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2}$$

$$\downarrow 24$$

$$\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2}$$

input `Int[(x*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]`

output `x/a - (Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a^2`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_.*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1} \left(\arcsin(ax)a^2x^2 - \arcsin(ax) + \sqrt{-a^2x^2+1} xa \right)}{a^2(a^2x^2-1)}$	62
orering	$\frac{(a^2x^2-2) \arcsin(ax)}{a^2\sqrt{-a^2x^2+1}} - \frac{(ax+1)(ax-1) \left(\frac{\arcsin(ax)}{\sqrt{-a^2x^2+1}} + \frac{xa}{-a^2x^2+1} + \frac{x^2 \arcsin(ax)a^2}{(-a^2x^2+1)^{\frac{3}{2}}} \right)}{a^2}$	102

input `int(x*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output
$$-1/a^2*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(arcsin(a*x)*a^2*x^2-arcsin(a*x))+(-a^2*x^2+1)^(1/2)*x*a$$

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{ax - \sqrt{-a^2x^2+1} \arcsin(ax)}{a^2}$$

input `integrate(x*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output
$$(a*x - \text{sqrt}(-a^2*x^2 + 1)*\arcsin(a*x))/a^2$$

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{x}{a} - \frac{\sqrt{-a^2x^2+1} \arcsin(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x*asin(a*x)/(-a**2*x**2+1)**(1/2),x)`output `Piecewise((x/a - sqrt(-a**2*x**2 + 1)*asin(a*x)/a**2, Ne(a, 0)), (0, True))`**Maxima [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x}{a} - \frac{\sqrt{-a^2x^2+1} \arcsin(ax)}{a^2}$$

input `integrate(x*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `x/a - sqrt(-a^2*x^2 + 1)*arcsin(a*x)/a^2`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x}{a} - \frac{\sqrt{-a^2x^2+1} \arcsin(ax)}{a^2}$$

input `integrate(x*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `x/a - sqrt(-a^2*x^2 + 1)*arcsin(a*x)/a^2`

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x \operatorname{asin}(ax)}{\sqrt{1-a^2x^2}} dx$$

input `int((x*asin(a*x))/(1 - a^2*x^2)^(1/2), x)`output `int((x*asin(a*x))/(1 - a^2*x^2)^(1/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{-\sqrt{-a^2x^2+1} \operatorname{asin}(ax) + ax}{a^2}$$

input `int(x*asin(a*x)/(-a^2*x^2+1)^(1/2), x)`output `(- sqrt(- a**2*x**2 + 1)*asin(a*x) + a*x)/a**2`

3.103 $\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result	1033
Mathematica [A] (verified)	1033
Rubi [A] (verified)	1034
Maple [A] (verified)	1034
Fricas [A] (verification not implemented)	1035
Sympy [A] (verification not implemented)	1035
Maxima [A] (verification not implemented)	1036
Giac [A] (verification not implemented)	1036
Mupad [B] (verification not implemented)	1036
Reduce [B] (verification not implemented)	1037

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^2}{2a}$$

output

`1/2*arcsin(a*x)^2/a`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^2}{2a}$$

input

`Integrate[ArcSin[a*x]/Sqrt[1 - a^2*x^2],x]`

output

`ArcSin[a*x]^2/(2*a)`

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

↓ 5152

$$\frac{\arcsin(ax)^2}{2a}$$

input `Int[ArcSin[a*x]/Sqrt[1 - a^2*x^2],x]`

output `ArcSin[a*x]^2/(2*a)`

Defintions of rubi rules used

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\arcsin(ax)^2}{2a}$	12
default	$\frac{\arcsin(ax)^2}{2a}$	12

input `int(arcsin(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `1/2*arcsin(a*x)^2/a`

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^2}{2a}$$

input `integrate(arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `1/2*arcsin(a*x)^2/a`

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{\arcsin^2(ax)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(asin(a*x)/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((asin(a*x)**2/(2*a), Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^2}{2a}$$

input `integrate(arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `1/2*arcsin(a*x)^2/a`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^2}{2a}$$

input `integrate(arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `1/2*arcsin(a*x)^2/a`**Mupad [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{\operatorname{asin}(ax)^2}{2a}$$

input `int(asin(a*x)/(1 - a^2*x^2)^(1/2),x)`output `asin(a*x)^2/(2*a)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{a \sin(ax)^2}{2a}$$

input `int(asin(a*x)/(-a^2*x^2+1)^(1/2),x)`

output `asin(a*x)**2/(2*a)`

3.104 $\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx$

Optimal result	1038
Mathematica [A] (verified)	1038
Rubi [A] (verified)	1039
Maple [A] (verified)	1040
Fricas [F]	1041
Sympy [F]	1041
Maxima [F]	1041
Giac [F]	1042
Mupad [F(-1)]	1042
Reduce [F]	1042

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = -2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) + i \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(ax)})$$

output `-2*arcsin(a*x)*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))+I*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-I*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))`

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \arcsin(ax) (\log(1 - e^{i \arcsin(ax)}) - \log(1 + e^{i \arcsin(ax)})) + i \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(ax)})$$

input `Integrate[ArcSin[a*x]/(x*Sqrt[1 - a^2*x^2]),x]`

output `ArcSin[a*x]*(Log[1 - E^(I*ArcSin[a*x])] - Log[1 + E^(I*ArcSin[a*x])]) + I*PolyLog[2, -E^(I*ArcSin[a*x])] - I*PolyLog[2, E^(I*ArcSin[a*x])]`

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx \\
 & \quad \downarrow \text{5218} \\
 & \int \frac{\arcsin(ax)}{ax} d\arcsin(ax) \\
 & \quad \downarrow \text{3042} \\
 & \int \arcsin(ax) \csc(\arcsin(ax)) d\arcsin(ax) \\
 & \quad \downarrow \text{4671} \\
 & - \int \log(1 - e^{i\arcsin(ax)}) d\arcsin(ax) + \int \log(1 + e^{i\arcsin(ax)}) d\arcsin(ax) - \\
 & \quad 2\arcsin(ax)\operatorname{arctanh}(e^{i\arcsin(ax)}) \\
 & \quad \downarrow \text{2715} \\
 & i \int e^{-i\arcsin(ax)} \log(1 - e^{i\arcsin(ax)}) de^{i\arcsin(ax)} - \\
 & i \int e^{-i\arcsin(ax)} \log(1 + e^{i\arcsin(ax)}) de^{i\arcsin(ax)} - 2\arcsin(ax)\operatorname{arctanh}(e^{i\arcsin(ax)}) \\
 & \quad \downarrow \text{2838} \\
 & -2\arcsin(ax)\operatorname{arctanh}(e^{i\arcsin(ax)}) + i\operatorname{PolyLog}(2, -e^{i\arcsin(ax)}) - i\operatorname{PolyLog}(2, e^{i\arcsin(ax)})
 \end{aligned}$$

input `Int[ArcSin[a*x]/(x*Sqrt[1 - a^2*x^2]),x]`

output `-2*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x])] + I*PolyLog[2, -E^(I*ArcSin[a*x])] - I*PolyLog[2, E^(I*ArcSin[a*x])]`

Definitions of rubi rules used

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)
^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]`

rule 5218 `Int[(((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_))/Sqrt[(d_) + (e_)*
(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]`

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.98

method	result
default	$\arcsin(ax) \ln(1 - iax - \sqrt{-a^2x^2 + 1}) - \arcsin(ax) \ln(1 + iax + \sqrt{-a^2x^2 + 1}) + i \operatorname{dilog}(1 +$

input `int(arcsin(a*x)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output

```
arcsin(a*x)*ln(1-I*a*x-(-a^2*x^2+1)^(1/2))-arcsin(a*x)*ln(1+I*a*x+(-a^2*x^2+1)^(1/2))+I*dilog(1+I*a*x+(-a^2*x^2+1)^(1/2))-I*dilog(1-I*a*x-(-a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)}{\sqrt{-a^2x^2+1x}} dx$$

input

```
integrate(arcsin(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)/(a^2*x^3 -x), x)
```

Sympy [F]

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{x\sqrt{-(ax-1)(ax+1)}} dx$$

input

```
integrate(asin(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

output

```
Integral(asin(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)}{\sqrt{-a^2x^2+1x}} dx$$

input

```
integrate(arcsin(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output

```
integrate(arcsin(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)}{\sqrt{-a^2x^2+1}x} dx$$

input `integrate(arcsin(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{x\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{\sqrt{-a^2x^2+1}x} dx$$

input `int(asin(a*x)/x/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)/(sqrt(-a**2*x**2 + 1)*x),x)`

3.105 $\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx$

Optimal result	1043
Mathematica [A] (verified)	1043
Rubi [A] (verified)	1044
Maple [A] (verified)	1045
Fricas [A] (verification not implemented)	1045
Sympy [F]	1045
Maxima [A] (verification not implemented)	1046
Giac [B] (verification not implemented)	1046
Mupad [F(-1)]	1046
Reduce [F]	1047

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{1-a^2x^2} \arcsin(ax)}{x} + a \log(x)$$

output `-(-a^2*x^2+1)^(1/2)*arcsin(a*x)/x+a*ln(x)`

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{1-a^2x^2} \arcsin(ax)}{x} + a \log(x)$$

input `Integrate[ArcSin[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]`

output `-((Sqrt[1 - a^2*x^2]*ArcSin[a*x])/x) + a*Log[x]`

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5186, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx$$

$$\downarrow 5186$$

$$a \int \frac{1}{x} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{x}$$

$$\downarrow 14$$

$$a \log(x) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{x}$$

input `Int[ArcSin[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]`

output `-((Sqrt[1 - a^2*x^2]*ArcSin[a*x])/x) + a*Log[x]`

Defintions of rubi rules used

rule 14 `Int[(a_.)/(x_), x_Symbol] :>Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b *ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{\ln(ax)ax + \arcsin(ax)\sqrt{-a^2x^2+1}}{x}$	32

input `int(arcsin(a*x)/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`output `-(-ln(a*x)*a*x+arcsin(a*x)*(-a^2*x^2+1)^(1/2))/x`**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = \frac{ax \log(x) - \sqrt{-a^2x^2+1} \arcsin(ax)}{x}$$

input `integrate(arcsin(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`output `(a*x*log(x) - sqrt(-a^2*x^2 + 1)*arcsin(a*x))/x`**Sympy [F]**

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)`output `Integral(asin(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = a \log(x) - \frac{\sqrt{-a^2x^2+1} \arcsin(ax)}{x}$$

input `integrate(arcsin(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `a*log(x) - sqrt(-a^2*x^2 + 1)*arcsin(a*x)/x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = \frac{1}{2} \left(\frac{a^4x}{(\sqrt{-a^2x^2+1}|a|+a)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|} \right) \arcsin(ax) + a \log(|x|)$$

input `integrate(arcsin(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `1/2*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a)))*arcsin(a*x) + a*log(abs(x))`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{x^2\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)}{x^2 \sqrt{1 - a^2 x^2}} dx = \int \frac{a \sin(ax)}{\sqrt{-a^2 x^2 + 1} x^2} dx$$

input `int(asin(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)/(sqrt(-a**2*x**2+1)*x**2),x)`

3.106 $\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx$

Optimal result	1048
Mathematica [A] (verified)	1048
Rubi [A] (verified)	1049
Maple [A] (verified)	1051
Fricas [F]	1052
Sympy [F]	1052
Maxima [F]	1052
Giac [F]	1053
Mupad [F(-1)]	1053
Reduce [F]	1053

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = -\frac{a}{2x} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{2x^2} - a^2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) + \frac{1}{2}ia^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - \frac{1}{2}ia^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)})$$

output

```
-1/2*a/x-1/2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/x^2-a^2*arcsin(a*x)*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))+1/2*I*a^2*polylog(2,-I*a*x+(-a^2*x^2+1)^(1/2))-1/2*I*a^2*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \frac{1}{8}a^2 \left(-2 \cot \left(\frac{1}{2} \arcsin(ax) \right) - \arcsin(ax) \operatorname{csc}^2 \left(\frac{1}{2} \arcsin(ax) \right) + 4 \arcsin(ax) \log(1 - e^{i \arcsin(ax)}) - 4 \arcsin(ax) \log(1 + e^{i \arcsin(ax)}) + 4i \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - 4i \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) + \arcsin(ax) \operatorname{sec}^2 \left(\frac{1}{2} \arcsin(ax) \right) - 2 \tan \left(\frac{1}{2} \arcsin(ax) \right) \right)$$

input `Integrate[ArcSin[a*x]/(x^3*sqrt[1 - a^2*x^2]),x]`

output `(a^2*(-2*Cot[ArcSin[a*x]/2] - ArcSin[a*x]*Csc[ArcSin[a*x]/2]^2 + 4*ArcSin[a*x]*Log[1 - E^(I*ArcSin[a*x])] - 4*ArcSin[a*x]*Log[1 + E^(I*ArcSin[a*x])] + (4*I)*PolyLog[2, -E^(I*ArcSin[a*x])] - (4*I)*PolyLog[2, E^(I*ArcSin[a*x])]) + ArcSin[a*x]*Sec[ArcSin[a*x]/2]^2 - 2*Tan[ArcSin[a*x]/2])/8`

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5204, 15, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\arcsin(ax)}{x^3 \sqrt{1-a^2x^2}} dx \\
 & \quad \downarrow \text{5204} \\
 & \frac{1}{2}a^2 \int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx + \frac{1}{2}a \int \frac{1}{x^2} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{2x^2} \\
 & \quad \downarrow \text{15} \\
 & \frac{1}{2}a^2 \int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{2x^2} - \frac{a}{2x} \\
 & \quad \downarrow \text{5218} \\
 & \frac{1}{2}a^2 \int \frac{\arcsin(ax)}{ax} d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{2x^2} - \frac{a}{2x} \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2}a^2 \int \arcsin(ax) \csc(\arcsin(ax)) d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{2x^2} - \frac{a}{2x} \\
 & \quad \downarrow \text{4671}
 \end{aligned}$$

$$\frac{1}{2}a^2 \left(- \int \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + \int \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) \right) - \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)}{2x^2} - \frac{a}{2x}$$

↓ 2715

$$\frac{1}{2}a^2 \left(i \int e^{-i \arcsin(ax)} \log(1 - e^{i \arcsin(ax)}) de^{i \arcsin(ax)} - i \int e^{-i \arcsin(ax)} \log(1 + e^{i \arcsin(ax)}) de^{i \arcsin(ax)} - 2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) \right) - \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)}{2x^2} - \frac{a}{2x}$$

↓ 2838

$$\frac{1}{2}a^2 \left(-2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) + i \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) \right) - \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)}{2x^2} - \frac{a}{2x}$$

input `Int[ArcSin[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]`

output `-1/2*a/x - (Sqrt[1 - a^2*x^2]*ArcSin[a*x])/((2*x^2) + (a^2*(-2*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x]]) + I*PolyLog[2, -E^(I*ArcSin[a*x])] - I*PolyLog[2, E^(I*ArcSin[a*x]])]))/2`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5204 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]`

rule 5218 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]`

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{-a^2x^2+1} \left(\arcsin(ax)a^2x^2 - \sqrt{-a^2x^2+1}xa - \arcsin(ax) \right)}{2(a^2x^2-1)x^2} + \frac{ia^2 \left(i \arcsin(ax) \ln(1+iax+\sqrt{-a^2x^2+1}) - i \arcsin(ax) \ln(1-iax+\sqrt{-a^2x^2+1}) \right)}{2(a^2x^2-1)x^2}$

input `int(arcsin(a*x)/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output

```
-1/2*(-a^2*x^2+1)^(1/2)*(arcsin(a*x)*a^2*x^2-(-a^2*x^2+1)^(1/2)*x*a-arcsin
(a*x))/(a^2*x^2-1)/x^2+1/2*I*a^2*(I*arcsin(a*x)*ln(1+I*a*x+(-a^2*x^2+1)^(1
/2))-I*arcsin(a*x)*ln(1-I*a*x-(-a^2*x^2+1)^(1/2))+polylog(2,-I*a*x-(-a^2*x
^2+1)^(1/2))-polylog(2,I*a*x+(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)}{\sqrt{-a^2x^2+1}x^3} dx$$

input

```
integrate(arcsin(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}} dx$$

input

```
integrate(asin(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

output

```
Integral(asin(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)}{\sqrt{-a^2x^2+1}x^3} dx$$

input

```
integrate(arcsin(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output `integrate(arcsin(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)`

Giac [F]

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)}{\sqrt{-a^2x^2+1}x^3} dx$$

input `integrate(arcsin(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{x^3\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)}{\sqrt{-a^2x^2+1}x^3} dx$$

input `int(asin(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)/(sqrt(- a**2*x**2 + 1)*x**3),x)`

3.107 $\int \frac{x^5(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$

Optimal result	1054
Mathematica [A] (verified)	1055
Rubi [A] (verified)	1055
Maple [A] (verified)	1057
Fricas [A] (verification not implemented)	1058
Sympy [F]	1058
Maxima [A] (verification not implemented)	1059
Giac [F(-2)]	1059
Mupad [F(-1)]	1060
Reduce [F]	1060

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{x^5(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{8bx\sqrt{1-c^2x^2}}{15c^5\sqrt{d-c^2dx^2}} + \frac{4bx^3\sqrt{1-c^2x^2}}{45c^3\sqrt{d-c^2dx^2}} + \frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}} - \frac{8\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{15c^6d} - \frac{4x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{15c^4d} - \frac{x^4\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{5c^2d}$$

output

```
8/15*b*x*(-c^2*x^2+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)+4/45*b*x^3*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+1/25*b*x^5*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-8/15*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^6/d-4/15*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4/d-1/5*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.53

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{bcx\sqrt{1 - c^2 x^2}(120 + 20c^2 x^2 + 9c^4 x^4) + 15a(-8 + 4c^2 x^2 + c^4 x^4 + 3c^6 x^6) + 15b(-8 + 4c^2 x^2 + c^4 x^4 + 3c^6 x^6)}{225c^6 \sqrt{d - c^2 dx^2}}$$

input `Integrate[(x^5*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

output `(b*c*x*Sqrt[1 - c^2*x^2]*(120 + 20*c^2*x^2 + 9*c^4*x^4) + 15*a*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) + 15*b*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6))*ArcSin[c*x]/(225*c^6*Sqrt[d - c^2*d*x^2])`

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5210, 15, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5210$$

$$\frac{4 \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{5c^2} + \frac{b\sqrt{1-c^2 x^2} \int x^4 dx}{5c\sqrt{d-c^2 dx^2}} - \frac{x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{5c^2 d}$$

$$\downarrow 15$$

$$\frac{4 \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{5c^2} - \frac{x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{5c^2 d} + \frac{bx^5 \sqrt{1-c^2 x^2}}{25c\sqrt{d-c^2 dx^2}}$$

$$\downarrow 5210$$

$$\begin{aligned}
& 4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{3c^2} + \frac{b\sqrt{1-c^2 x^2} \int x^2 dx}{3c\sqrt{d-c^2 dx^2}} - \frac{x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{3c^2 d} \right) \\
& \frac{5c^2}{x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))} + \frac{bx^5 \sqrt{1-c^2 x^2}}{25c\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow 15 \\
& 4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{3c^2} - \frac{x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{3c^2 d} + \frac{bx^3 \sqrt{1-c^2 x^2}}{9c\sqrt{d-c^2 dx^2}} \right) \\
& \frac{5c^2}{x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))} + \frac{bx^5 \sqrt{1-c^2 x^2}}{25c\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow 5182 \\
& 4 \left(\frac{2 \left(\frac{b\sqrt{1-c^2 x^2} \int 1 dx}{c\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{c^2 d} \right)}{3c^2} - \frac{x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{3c^2 d} + \frac{bx^3 \sqrt{1-c^2 x^2}}{9c\sqrt{d-c^2 dx^2}} \right) \\
& \frac{5c^2}{x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))} + \frac{bx^5 \sqrt{1-c^2 x^2}}{25c\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow 24 \\
& - \frac{x^4 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{5c^2 d} + \\
& 4 \left(- \frac{x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{3c^2 d} + \frac{2 \left(\frac{bx \sqrt{1-c^2 x^2}}{c\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{c^2 d} \right)}{3c^2} + \frac{bx^3 \sqrt{1-c^2 x^2}}{9c\sqrt{d-c^2 dx^2}} \right) + \\
& \frac{5c^2}{bx^5 \sqrt{1-c^2 x^2}} \\
& \frac{bx^5 \sqrt{1-c^2 x^2}}{25c\sqrt{d-c^2 dx^2}}
\end{aligned}$$

input `Int[(x^5*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

output `(b*x^5*Sqrt[1 - c^2*x^2])/(25*c*Sqrt[d - c^2*d*x^2]) - (x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(5*c^2*d) + (4*((b*x^3*Sqrt[1 - c^2*x^2])/(9*c*Sqrt[d - c^2*d*x^2]) - (x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(3*c^2*d) + (2*((b*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(c^2*d)))/(3*c^2)))/(5*c^2)`

Defintions of rubi rules used

rule 15 $\text{Int}[(a_.)(x_)^(m_.), x_Symbol] \rightarrow \text{Simp}[a*(x^(m + 1)/(m + 1)), x] \text{ /; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 24 $\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ /; FreeQ}[a, x]$

rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)(x_)^2)^(p_.), x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x] \text{ /; FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5210 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)]*(b_.))^(n_.)*((f_.)(x_)^(m_)*((d_) + (e_.)(x_)^2)^(p_)), x_Symbol] \rightarrow \text{Simp}[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(e*(m + 2*p + 1))), x] + (\text{Simp}[f^2*((m - 1)/(c^2*(m + 2*p + 1))) \text{ Int}[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m + 2*p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x]) \text{ /; FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0]$

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.79

method	result
orering	$\frac{(81c^6x^6+50c^4x^4+440c^2x^2-720)(a+b \arcsin(cx))}{225c^6\sqrt{-c^2dx^2+d}} - \frac{(9c^4x^4+20c^2x^2+120)(cx-1)(cx+1) \left(\frac{5x^4(a+b \arcsin(cx))}{\sqrt{-c^2dx^2+d}} + \frac{x^5bc}{\sqrt{-c^2x^2+1}\sqrt{-c^2x^2-1}} \right)}{225x^4c^6}$
default	$a \left(-\frac{x^4\sqrt{-c^2dx^2+d}}{5c^2d} + \frac{-4x^2\sqrt{-c^2dx^2+d} - 8\sqrt{-c^2dx^2+d}}{15c^2d} \right) + b \left(\frac{5\sqrt{-d(c^2x^2-1)} \left(-2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1 \right) (i+3 \arcsin(cx))}{576c^6d(c^2x^2-1)} \right)$
parts	$a \left(-\frac{x^4\sqrt{-c^2dx^2+d}}{5c^2d} + \frac{-4x^2\sqrt{-c^2dx^2+d} - 8\sqrt{-c^2dx^2+d}}{15c^2d} \right) + b \left(\frac{5\sqrt{-d(c^2x^2-1)} \left(-2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1 \right) (i+3 \arcsin(cx))}{576c^6d(c^2x^2-1)} \right)$

input `int(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{225} \cdot (81c^6x^6 + 50c^4x^4 + 440c^2x^2 - 720) / c^6 \cdot (a + b \arcsin(cx)) / (-c^2dx^2 + d)^{1/2} - \frac{1}{225} / x^4 \cdot (9c^4x^4 + 20c^2x^2 + 120) / c^6 \cdot (cx - 1) \cdot (cx + 1) \cdot (5x^4 \cdot (a + b \arcsin(cx)) / (-c^2dx^2 + d)^{1/2} + x^5 \cdot bc / (-c^2x^2 + 1)^{1/2} / (-c^2dx^2 + d)^{1/2} + x^6 \cdot (a + b \arcsin(cx)) / (-c^2dx^2 + d)^{3/2} \cdot c^2d)$$

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx = \frac{(9bc^5x^5 + 20bc^3x^3 + 120bcx)\sqrt{-c^2dx^2 + d}\sqrt{-c^2x^2 + 1} + 15(3ac^6x^6 + ac^4x^4 + 4ac^2x^2 + (3bc^6x^6 + \dots))}{225(c^8dx^2 - c^6d)}$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output
$$\frac{-1/225 \cdot ((9bc^5x^5 + 20bc^3x^3 + 120bcx) \cdot \sqrt{-c^2dx^2 + d} \cdot \sqrt{-c^2x^2 + 1} + 15 \cdot (3ac^6x^6 + ac^4x^4 + 4ac^2x^2 + (3bc^6x^6 + bc^4x^4 + 4bc^2x^2 - 8b) \cdot \arcsin(cx) - 8a) \cdot \sqrt{-c^2dx^2 + d})}{(c^8dx^2 - c^6d)}$$

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx = \int \frac{x^5(a + b \arcsin(cx))}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate(x**5*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**5*(a + b*asin(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$= -\frac{1}{15} \left(\frac{3\sqrt{-c^2 dx^2 + dx^4}}{c^2 d} + \frac{4\sqrt{-c^2 dx^2 + dx^2}}{c^4 d} + \frac{8\sqrt{-c^2 dx^2 + d}}{c^6 d} \right) b \arcsin(cx)$$

$$- \frac{1}{15} \left(\frac{3\sqrt{-c^2 dx^2 + dx^4}}{c^2 d} + \frac{4\sqrt{-c^2 dx^2 + dx^2}}{c^4 d} + \frac{8\sqrt{-c^2 dx^2 + d}}{c^6 d} \right) a$$

$$+ \frac{(9c^4 x^5 + 20c^2 x^3 + 120x)b}{225c^5 \sqrt{d}}$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b*arcsin(c*x) - 1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*a + 1/225*(9*c^4*x^5 + 20*c^2*x^3 + 120*x)*b/(c^5*sqrt(d))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^5*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^5*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{-3\sqrt{-c^2 x^2 + 1} a c^4 x^4 - 4\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 8\sqrt{-c^2 x^2 + 1} a + 15 \left(\int \frac{\operatorname{asin}(cx)x^5}{\sqrt{-c^2 x^2 + 1}} dx \right) b c^6}{15\sqrt{d} c^6}$$

input `int(x^5*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output `(- 3*sqrt(- c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt(- c**2*x**2 + 1)*a + 15*int((asin(c*x)*x**5)/sqrt(- c**2*x**2 + 1),x)*b*c**6)/(15*sqrt(d)*c**6)`

3.108 $\int \frac{x^4(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$

Optimal result	1061
Mathematica [A] (verified)	1062
Rubi [A] (verified)	1062
Maple [B] (verified)	1064
Fricas [F]	1065
Sympy [F]	1065
Maxima [F]	1066
Giac [A] (verification not implemented)	1066
Mupad [F(-1)]	1067
Reduce [F]	1067

Optimal result

Integrand size = 27, antiderivative size = 200

$$\int \frac{x^4(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{3bx^2\sqrt{1-c^2x^2}}{16c^3\sqrt{d-c^2dx^2}} + \frac{bx^4\sqrt{1-c^2x^2}}{16c\sqrt{d-c^2dx^2}} - \frac{3x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{8c^4d} - \frac{x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{4c^2d} + \frac{3\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{16bc^5\sqrt{d-c^2dx^2}}$$

output

```
3/16*b*x^2*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+1/16*b*x^4*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-3/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4/d-1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2/d+3/16*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/b/c^5/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{-\frac{16acx(3+2c^2x^2)\sqrt{d-c^2dx^2}}{d} - \frac{48a \arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1-c^2x^2}(-16\cos(2\arcsin(cx))+\cos(4\arcsin(cx))+4\arcsin(cx))(6\arcsin(cx)+4\arcsin^2(cx))}{\sqrt{d-c^2dx^2}}}{128c^5}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output

```
((-16*a*c*x*(3 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/d - (48*a*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[1 - c^2*x^2]*(-16*Cos[2*ArcSin[c*x]] + Cos[4*ArcSin[c*x]] + 4*ArcSin[c*x]*(6*ArcSin[c*x] - 8*Sin[2*ArcSin[c*x]] + Sin[4*ArcSin[c*x]])))/Sqrt[d - c^2*d*x^2))/(128*c^5)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow \text{5210}$$

$$\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{4c^2} + \frac{b\sqrt{1 - c^2 x^2} \int x^3 dx}{4c\sqrt{d - c^2 dx^2}} - \frac{x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{4c^2 d}$$

$$\downarrow \text{15}$$

$$\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{4c^2} - \frac{x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{4c^2 d} + \frac{bx^4 \sqrt{1 - c^2 x^2}}{16c\sqrt{d - c^2 dx^2}}$$

$$\begin{aligned}
& \downarrow 5210 \\
& \frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} + \frac{b\sqrt{1-c^2 x^2} \int x dx}{2c\sqrt{d-c^2 dx^2}} - \frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{2c^2 d} \right)}{4c^2} - \\
& \frac{x^3 \sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{4c^2 d} + \frac{bx^4 \sqrt{1-c^2 x^2}}{16c\sqrt{d-c^2 dx^2}} \\
& \downarrow 15 \\
& \frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} - \frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{2c^2 d} + \frac{bx^2 \sqrt{1-c^2 x^2}}{4c\sqrt{d-c^2 dx^2}} \right)}{4c^2} - \\
& \frac{x^3 \sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{4c^2 d} + \frac{bx^4 \sqrt{1-c^2 x^2}}{16c\sqrt{d-c^2 dx^2}} \\
& \downarrow 5152 \\
& \frac{-\frac{x^3 \sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{4c^2 d} + \left(-\frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{2c^2 d} + \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))^2}{4bc^3 \sqrt{d-c^2 dx^2}} + \frac{bx^2 \sqrt{1-c^2 x^2}}{4c\sqrt{d-c^2 dx^2}} \right)}{4c^2} + \frac{bx^4 \sqrt{1-c^2 x^2}}{16c\sqrt{d-c^2 dx^2}}
\end{aligned}$$

input `Int[(x^4*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2], x]`

output `(b*x^4*Sqrt[1 - c^2*x^2])/(16*c*Sqrt[d - c^2*d*x^2]) - (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(4*c^2*d) + (3*((b*x^2*Sqrt[1 - c^2*x^2])/(4*c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(2*c^2*d) + (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2]))) / (4*c^2)`

Defintions of rubi rules used

rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m + 1)/(m + 1)}), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}/\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5210 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((f_.)(x_))^{(m_.)*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m - 1)}*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(e*(m + 2*p + 1))), x] + (\text{Simp}[f^2*((m - 1)/(c^2*(m + 2*p + 1)) \text{ Int}[(f*x)^{(m - 2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m + 2*p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m - 1)}*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x)] \text{ ; FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(174) = 348$.

Time = 0.32 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.88

method	result
default	$-\frac{ax^3\sqrt{-c^2dx^2+d}}{4c^2d} - \frac{3ax\sqrt{-c^2dx^2+d}}{8c^4d} + \frac{3a \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{8c^4\sqrt{c^2d}} + b\left(-\frac{3\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{16c^5d(c^2x^2-1)} - \frac{16}{16}\right)$
parts	$-\frac{ax^3\sqrt{-c^2dx^2+d}}{4c^2d} - \frac{3ax\sqrt{-c^2dx^2+d}}{8c^4d} + \frac{3a \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{8c^4\sqrt{c^2d}} + b\left(-\frac{3\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{16c^5d(c^2x^2-1)} - \frac{16}{16}\right)$

input $\text{int}(x^4*(a+b*\arcsin(c*x))/(-c^2*d*x^2+d)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

output

```
-1/4*a*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(-c^2*d*x^2+d)^(1/2)+3/8*a/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)^2-1/16/c^5/(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)+1/8*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*arcsin(c*x)*x-1/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*cos(5*arcsin(c*x))-1/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)*sin(5*arcsin(c*x))+15/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*cos(3*arcsin(c*x))+7/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)*sin(3*arcsin(c*x))
```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)x^4}{\sqrt{-c^2 dx^2 + d}} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output

```
integral(-(b*x^4*arcsin(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input

```
integrate(x**4*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output

```
Integral(x**4*(a + b*asin(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)x^4}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/8*a*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^4*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + b*integrate(x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{32(-c^2 x^2 + 1)^{\frac{3}{2}} b x \arcsin(cx) + 32(-c^2 x^2 + 1)^{\frac{3}{2}} a x - 80 \sqrt{-c^2 x^2 + 1} b x \arcsin(cx) - 80 \sqrt{-c^2 x^2 + 1} a x}{128 c^4 \sqrt{d}}$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `1/128*(32*(-c^2*x^2 + 1)^(3/2)*b*x*arcsin(c*x) + 32*(-c^2*x^2 + 1)^(3/2)*a*x - 80*sqrt(-c^2*x^2 + 1)*b*x*arcsin(c*x) - 80*sqrt(-c^2*x^2 + 1)*a*x + 8*(c^2*x^2 - 1)^2*b/c + 24*b*arcsin(c*x)^2/c + 40*(c^2*x^2 - 1)*b/c + 48*a*arcsin(c*x)/c + 17*b/c)/(c^4*sqrt(d))`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{3a \operatorname{asin}(cx) a - 2\sqrt{-c^2 x^2 + 1} a c^3 x^3 - 3\sqrt{-c^2 x^2 + 1} a c x + 8 \left(\int \frac{\operatorname{asin}(cx) x^4}{\sqrt{-c^2 x^2 + 1}} dx \right) b c^5}{8\sqrt{d} c^5}$$

input `int(x^4*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output `(3*asin(c*x)*a - 2*sqrt(-c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt(-c**2*x**2 + 1)*a*c*x + 8*int((asin(c*x)*x**4)/sqrt(-c**2*x**2 + 1),x)*b*c**5)/(8*sqrt(d)*c**5)`

3.109 $\int \frac{x^3(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$

Optimal result	1068
Mathematica [A] (verified)	1068
Rubi [A] (verified)	1069
Maple [A] (verified)	1071
Fricas [A] (verification not implemented)	1071
Sympy [F]	1072
Maxima [A] (verification not implemented)	1072
Giac [F(-2)]	1073
Mupad [F(-1)]	1073
Reduce [F]	1073

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^3(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{2bx\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}} - \frac{2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{3c^4d} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{3c^2d}$$

output

```
2/3*b*x*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+1/9*b*x^3*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4/d-1/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{x^3(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{bcx\sqrt{1-c^2x^2}(6+c^2x^2)+3a(-2+c^2x^2+c^4x^4)+3b(-2+c^2x^2+c^4x^4)\arcsin(cx)}{9c^4\sqrt{d-c^2dx^2}}$$

input `Integrate[(x^3*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

output `(b*c*x*Sqrt[1 - c^2*x^2]*(6 + c^2*x^2) + 3*a*(-2 + c^2*x^2 + c^4*x^4) + 3*b*(-2 + c^2*x^2 + c^4*x^4)*ArcSin[c*x])/(9*c^4*Sqrt[d - c^2*d*x^2])`

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx \\
 & \quad \downarrow \text{5210} \\
 & \frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{3c^2} + \frac{b\sqrt{1-c^2 x^2} \int x^2 dx}{3c\sqrt{d-c^2 dx^2}} - \frac{x^2 \sqrt{d-c^2 dx^2} (a + b \arcsin(cx))}{3c^2 d} \\
 & \quad \downarrow \text{15} \\
 & \frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{3c^2} - \frac{x^2 \sqrt{d-c^2 dx^2} (a + b \arcsin(cx))}{3c^2 d} + \frac{bx^3 \sqrt{1-c^2 x^2}}{9c\sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{5182} \\
 & \frac{2 \left(\frac{b\sqrt{1-c^2 x^2} \int 1 dx}{c\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{c^2 d} \right)}{3c^2} - \frac{x^2 \sqrt{d-c^2 dx^2} (a + b \arcsin(cx))}{3c^2 d} + \frac{bx^3 \sqrt{1-c^2 x^2}}{9c\sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{24} \\
 & - \frac{x^2 \sqrt{d-c^2 dx^2} (a + b \arcsin(cx))}{3c^2 d} + \frac{2 \left(\frac{bx\sqrt{1-c^2 x^2}}{c\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{c^2 d} \right)}{3c^2} + \frac{bx^3 \sqrt{1-c^2 x^2}}{9c\sqrt{d-c^2 dx^2}}
 \end{aligned}$$

input `Int[(x^3*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

output
$$\frac{(b^2 x^3 \sqrt{1 - c^2 x^2}) / (9 c \sqrt{d - c^2 d x^2}) - (x^2 \sqrt{d - c^2 d x^2}) * (a + b \operatorname{ArcSin}[c x]) / (3 c^2 d) + (2 * ((b x \sqrt{1 - c^2 x^2}) / (c \sqrt{d - c^2 d x^2}) - (\sqrt{d - c^2 d x^2}) * (a + b \operatorname{ArcSin}[c x]) / (c^2 d))) / (3 c^2)}$$

Defintions of rubi rules used

rule 15
$$\operatorname{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[a*(x^{(m+1)})/(m+1), x] /; \operatorname{FreeQ}[\{a, m\}, x] \ \&\& \operatorname{NeQ}[m, -1]$$

rule 24
$$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$$

rule 5182
$$\operatorname{Int}[(a_. + \operatorname{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}*(x_)*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d + e*x^2)^{(p+1)}*((a + b*\operatorname{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \operatorname{Simp}[b*(n/(2*c*(p+1)))*\operatorname{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \operatorname{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\operatorname{ArcSin}[c*x])^{(n-1)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \operatorname{EqQ}[c^2*d + e, 0] \ \&\& \operatorname{GtQ}[n, 0] \ \&\& \operatorname{NeQ}[p, -1]$$

rule 5210
$$\operatorname{Int}[(a_. + \operatorname{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\operatorname{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\operatorname{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \operatorname{Int}[(f*x)^{(m-2)}*(d + e*x^2)^p*(a + b*\operatorname{ArcSin}[c*x])^n, x], x] + \operatorname{Simp}[b*f*(n/(c*(m+2*p+1)))*\operatorname{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \operatorname{Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\operatorname{ArcSin}[c*x])^{(n-1)}, x], x]) /; \operatorname{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \operatorname{EqQ}[c^2*d + e, 0] \ \&\& \operatorname{GtQ}[n, 0] \ \&\& \operatorname{IGtQ}[m, 1] \ \&\& \operatorname{NeQ}[m + 2*p + 1, 0]$$

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08

method	result
ordering	$\frac{(5c^4x^4+12c^2x^2-24)(a+b\arcsin(cx))}{9c^4\sqrt{-c^2dx^2+d}} - \frac{(c^2x^2+6)(cx-1)(cx+1)\left(\frac{3x^2(a+b\arcsin(cx))}{\sqrt{-c^2dx^2+d}} + \frac{x^3bc}{\sqrt{-c^2x^2+1}\sqrt{-c^2dx^2+d}} + \frac{x^4(a+b\arcsin(cx))}{(-c^2dx^2+d)}\right)}{9x^2c^4}$
default	$a\left(-\frac{x^2\sqrt{-c^2dx^2+d}}{3c^2d} - \frac{2\sqrt{-c^2dx^2+d}}{3dc^4}\right) + b\left(\frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1)(i+3\arcsin(cx))}{144c^4d(c^2x^2-1)} - \frac{3\sqrt{-d(c^2x^2-1)}}{144c^4d(c^2x^2-1)}\right)$
parts	$a\left(-\frac{x^2\sqrt{-c^2dx^2+d}}{3c^2d} - \frac{2\sqrt{-c^2dx^2+d}}{3dc^4}\right) + b\left(\frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1)(i+3\arcsin(cx))}{144c^4d(c^2x^2-1)} - \frac{3\sqrt{-d(c^2x^2-1)}}{144c^4d(c^2x^2-1)}\right)$

input `int(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{9}*(5*c^4*x^4+12*c^2*x^2-24)/c^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2)-1/9/x^2*(c^2*x^2+6)/c^4*(c*x-1)*(c*x+1)*(3*x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2)+x^3*b*c/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2)*c^2*d)$$

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{x^3(a+b\arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{(bc^3x^3+6bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}+3(ac^4x^4+ac^2x^2+(bc^4x^4+bc^2x^2-2b)\arcsin(cx)-2a)}{9(c^6dx^2-c^4d)}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output
$$-1/9*((b*c^3*x^3+6*b*c*x)*sqrt(-c^2*d*x^2+d)*sqrt(-c^2*x^2+1)+3*(a*c^4*x^4+a*c^2*x^2+(b*c^4*x^4+b*c^2*x^2-2*b)*arcsin(c*x)-2*a)*sqrt(-c^2*d*x^2+d))/(c^6*d*x^2-c^4*d)$$

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{-d}(cx - 1)(cx + 1)} dx$$

input `integrate(x**3*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**3*(a + b*asin(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \frac{x^3(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = -\frac{1}{3} b \left(\frac{\sqrt{-c^2 dx^2 + dx^2}}{c^2 d} + \frac{2 \sqrt{-c^2 dx^2 + d}}{c^4 d} \right) \arcsin(cx) \\ - \frac{1}{3} a \left(\frac{\sqrt{-c^2 dx^2 + dx^2}}{c^2 d} + \frac{2 \sqrt{-c^2 dx^2 + d}}{c^4 d} \right) \\ + \frac{(c^2 x^3 + 6x)b}{9 c^3 \sqrt{d}}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/3*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) *arcsin(c*x) - 1/3*a*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) + 1/9*(c^2*x^3 + 6*x)*b/(c^3*sqrt(d))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx \\ &= \frac{-\sqrt{-c^2 x^2 + 1} a c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a + 3 \left(\int \frac{\operatorname{asin}(cx) x^3}{\sqrt{-c^2 x^2 + 1}} dx \right) b c^4}{3\sqrt{d} c^4} \end{aligned}$$

input `int(x^3*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output $(-\sqrt{-c^2x^2+1})ac^2x^2 - 2\sqrt{-c^2x^2+1}a + 3\int \frac{(\arcsin(cx)x^3/\sqrt{-c^2x^2+1}, x) * b * c^4}{(3\sqrt{d}) * c^4}$

3.110 $\int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$

Optimal result	1075
Mathematica [A] (verified)	1075
Rubi [A] (verified)	1076
Maple [B] (verified)	1077
Fricas [F]	1078
Sympy [F]	1078
Maxima [F]	1079
Giac [A] (verification not implemented)	1079
Mupad [F(-1)]	1080
Reduce [F]	1080

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}$$

output

$$\frac{1}{4}bx^2(-c^2x^2+1)^{(1/2)}/c/(-c^2d*x^2+d)^{(1/2)}-1/2*x*(-c^2d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/c^2/d+1/4*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^2/b/c^3/(-c^2d*x^2+d)^{(1/2)}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

$$\int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{4acx\sqrt{d-c^2dx^2}}{d} + \frac{4a \arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1-c^2x^2}(-2 \arcsin(cx)^2+\cos(2 \arcsin(cx))+2 \arcsin(cx) \sin(2 \arcsin(cx)))}{\sqrt{d-c^2dx^2}}$$

$8c^3$

input `Integrate[(x^2*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

output
$$-1/8*((4*a*c*x*\text{Sqrt}[d - c^2*d*x^2])/d + (4*a*\text{ArcTan}[(c*x*\text{Sqrt}[d - c^2*d*x^2])/(\text{Sqrt}[d]*(-1 + c^2*x^2))])/ \text{Sqrt}[d] + (b*\text{Sqrt}[1 - c^2*x^2]*(-2*\text{ArcSin}[c*x]^2 + \text{Cos}[2*\text{ArcSin}[c*x]] + 2*\text{ArcSin}[c*x]*\text{Sin}[2*\text{ArcSin}[c*x]]))/ \text{Sqrt}[d - c^2*d*x^2])/c^3$$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

↓ 5210

$$\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} + \frac{b\sqrt{1-c^2 x^2} \int x dx}{2c\sqrt{d-c^2 dx^2}} - \frac{x\sqrt{d-c^2 dx^2}(a + b \arcsin(cx))}{2c^2 d}$$

↓ 15

$$\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} - \frac{x\sqrt{d-c^2 dx^2}(a + b \arcsin(cx))}{2c^2 d} + \frac{bx^2\sqrt{1-c^2 x^2}}{4c\sqrt{d-c^2 dx^2}}$$

↓ 5152

$$-\frac{x\sqrt{d-c^2 dx^2}(a + b \arcsin(cx))}{2c^2 d} + \frac{\sqrt{1-c^2 x^2}(a + b \arcsin(cx))^2}{4bc^3\sqrt{d-c^2 dx^2}} + \frac{bx^2\sqrt{1-c^2 x^2}}{4c\sqrt{d-c^2 dx^2}}$$

input `Int[(x^2*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

```
output (b*x^2*Sqrt[1 - c^2*x^2])/(4*c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x^2]*
2*(a + b*ArcSin[c*x]))/(2*c^2*d) + (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])
^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

```
rule 15 Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

```
rule 5152 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

```
rule 5210 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x
)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(108) = 216.

Time = 0.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.16

method	result
default	$-\frac{ax\sqrt{-c^2dx^2+d}}{2c^2d} + \frac{a \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{2c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{4c^3d(c^2x^2-1)} - \frac{\sqrt{-c^2x^2+1}}{16c^3\sqrt{-d(c^2x^2-1)}} + \sqrt{-d(c^2x^2-1)}\right)$
parts	$-\frac{ax\sqrt{-c^2dx^2+d}}{2c^2d} + \frac{a \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{2c^2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{4c^3d(c^2x^2-1)} - \frac{\sqrt{-c^2x^2+1}}{16c^3\sqrt{-d(c^2x^2-1)}} + \sqrt{-d(c^2x^2-1)}\right)$

input `int(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/2*a*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arcsin(c*x)^2-1/16/c^3/(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)+1/8*(-d*(c^2*x^2-1))^(1/2)/c^2/d/(c^2*x^2-1)*arcsin(c*x)*x+1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*cos(3*arcsin(c*x))+1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*arcsin(c*x)*sin(3*arcsin(c*x))`

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)x^2}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arcsin(c*x) + a*x^2)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**2*(a + b*asin(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)x^2}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/2*a*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + b*integrate(x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{4\sqrt{-c^2 x^2 + 1}bx \arcsin(cx) + 4\sqrt{-c^2 x^2 + 1}ax - \frac{2b \arcsin(cx)^2}{c} - \frac{2(c^2 x^2 - 1)b}{c} - \frac{4a \arcsin(cx)}{c} - \frac{b}{c}}{8c^2\sqrt{d}}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `-1/8*(4*sqrt(-c^2*x^2 + 1)*b*x*arcsin(c*x) + 4*sqrt(-c^2*x^2 + 1)*a*x - 2*b*arcsin(c*x)^2/c - 2*(c^2*x^2 - 1)*b/c - 4*a*arcsin(c*x)/c - b/c)/(c^2*sqrt(d))`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2),x)`output `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2), x)`**Reduce [F]**

$$\int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{\arcsin(cx) a - \sqrt{-c^2 x^2 + 1} acx + 2 \left(\int \frac{\arcsin(cx) x^2}{\sqrt{-c^2 x^2 + 1}} dx \right) b c^3}{2\sqrt{d} c^3}$$

input `int(x^2*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`output `(asin(c*x)*a - sqrt(-c**2*x**2 + 1)*a*c*x + 2*int((asin(c*x)*x**2)/sqrt(-c**2*x**2 + 1),x)*b*c**3)/(2*sqrt(d)*c**3)`

3.111 $\int \frac{x(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$

Optimal result	1081
Mathematica [A] (verified)	1081
Rubi [A] (verified)	1082
Maple [B] (verified)	1083
Fricas [A] (verification not implemented)	1083
Sympy [F(-2)]	1084
Maxima [A] (verification not implemented)	1084
Giac [F(-2)]	1084
Mupad [F(-1)]	1085
Reduce [B] (verification not implemented)	1085

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx = \frac{bx\sqrt{1 - c^2x^2}}{c\sqrt{d - c^2dx^2}} - \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{c^2d}$$

output

```
b*x*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx = \frac{bcx\sqrt{1 - c^2x^2} + a(-1 + c^2x^2) + b(-1 + c^2x^2) \arcsin(cx)}{c^2\sqrt{d - c^2dx^2}}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output

```
(b*c*x*Sqrt[1 - c^2*x^2] + a*(-1 + c^2*x^2) + b*(-1 + c^2*x^2)*ArcSin[c*x])/(c^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5182$$

$$\frac{b\sqrt{1 - c^2 x^2} \int 1 dx}{c\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{c^2 d}$$

$$\downarrow 24$$

$$\frac{bx\sqrt{1 - c^2 x^2}}{c\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{c^2 d}$$

input `Int[(x*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]`

output `(b*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(c^2*d)`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_.*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(61) = 122.

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.97

method	result
orering	$\frac{(c^2x^2-2)(a+b\arcsin(cx))}{c^2\sqrt{-c^2dx^2+d}} - \frac{(cx-1)(cx+1)\left(\frac{a+b\arcsin(cx)}{\sqrt{-c^2dx^2+d}} + \frac{xbc}{\sqrt{-c^2x^2+1}\sqrt{-c^2dx^2+d}} + \frac{x^2(a+b\arcsin(cx))c^2d}{(-c^2dx^2+d)^{\frac{3}{2}}}\right)}{c^2}$
default	$-\frac{a\sqrt{-c^2dx^2+d}}{c^2d} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}(c^2x^2-i\sqrt{-c^2x^2+1}cx-1)(\arcsin(cx)+i)}{2c^2d(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}(i\sqrt{-c^2x^2+1}cx+c^2x^2-1)}{2c^2d(c^2x^2-1)}\right)$
parts	$-\frac{a\sqrt{-c^2dx^2+d}}{c^2d} + b\left(-\frac{\sqrt{-d(c^2x^2-1)}(c^2x^2-i\sqrt{-c^2x^2+1}cx-1)(\arcsin(cx)+i)}{2c^2d(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}(i\sqrt{-c^2x^2+1}cx+c^2x^2-1)}{2c^2d(c^2x^2-1)}\right)$

input `int(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{(c^2x^2-2)}{c^2} \frac{(a+b\arcsin(cx))}{(-c^2dx^2+d)^{1/2}} - \frac{1}{c^2} \frac{(cx-1)(cx+1)\left(\frac{a+b\arcsin(cx)}{(-c^2dx^2+d)^{1/2}} + \frac{xbc}{(-c^2x^2+1)^{1/2}} + \frac{x^2(a+b\arcsin(cx))c^2d}{(-c^2dx^2+d)^{3/2}}\right)}{c^2d}$$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx$$

$$= -\frac{\sqrt{-c^2dx^2 + d}\sqrt{-c^2x^2 + 1}bcx + (ac^2x^2 + (bc^2x^2 - b) \arcsin(cx) - a)\sqrt{-c^2dx^2 + d}}{c^4dx^2 - c^2d}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output
$$-\frac{(\sqrt{-c^2dx^2 + d})\sqrt{-c^2x^2 + 1}bcx + (ac^2x^2 + (bc^2x^2 - b)\arcsin(cx) - a)\sqrt{-c^2dx^2 + d}}{(c^4dx^2 - c^2d)}$$

Sympy [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output `Exception raised: TypeError >> Invalid comparison of non-real zoo`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{bx}{c\sqrt{d}} - \frac{\sqrt{-c^2 dx^2 + d} b \arcsin(cx)}{c^2 d} - \frac{\sqrt{-c^2 dx^2 + d} a}{c^2 d}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `b*x/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*b*arcsin(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a/(c^2*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2),x)`

output `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{x(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{d} (-\sqrt{-c^2 x^2 + 1} \arcsin(cx) b - \sqrt{-c^2 x^2 + 1} a + bcx)}{c^2 d}$$

input `int(x*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output `(sqrt(d)*(-sqrt(-c**2*x**2 + 1)*asin(c*x)*b - sqrt(-c**2*x**2 + 1)*a + b*c*x))/(c**2*d)`

3.112 $\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2dx^2}} dx$

Optimal result	1086
Mathematica [A] (verified)	1086
Rubi [A] (verified)	1087
Maple [A] (verified)	1087
Fricas [F]	1088
Sympy [F]	1088
Maxima [A] (verification not implemented)	1089
Giac [A] (verification not implemented)	1089
Mupad [F(-1)]	1089
Reduce [B] (verification not implemented)	1090

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2dx^2}} dx = \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2}{2bc\sqrt{d - c^2dx^2}}$$

output

```
1/2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/b/c/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2dx^2}} dx = \frac{\sqrt{1 - c^2x^2} \arcsin(cx)(2a + b \arcsin(cx))}{2c\sqrt{d - c^2dx^2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output

```
(Sqrt[1 - c^2*x^2]*ArcSin[c*x]*(2*a + b*ArcSin[c*x]))/(2*c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx$$

↓ 5152

$$\frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2}{2bc\sqrt{d - c^2 dx^2}}$$

input `Int[(a + b*ArcSin[c*x])/Sqrt[d - c^2*d*x^2], x]`

output `(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{a \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{\sqrt{c^2 d}} - \frac{b\sqrt{-d(c^2 x^2 - 1)}\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{2d(c^2 x^2 - 1)c}$	86
parts	$\frac{a \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{\sqrt{c^2 d}} - \frac{b\sqrt{-d(c^2 x^2 - 1)}\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{2d(c^2 x^2 - 1)c}$	86

input `int((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output `a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/c*arcsin(c*x)^2`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx = \int \frac{b \arcsin(cx) + a}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x,algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \arcsin(cx)}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral((a + b*asin(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx = \frac{b \arcsin(cx)^2}{2c\sqrt{d}} + \frac{a \arcsin(cx)}{c\sqrt{d}}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`output `1/2*b*arcsin(c*x)^2/(c*sqrt(d)) + a*arcsin(c*x)/(c*sqrt(d))`**Giac [A] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx = \frac{b \arcsin(cx)^2 + 2a \arcsin(cx)}{2c\sqrt{d}}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`output `1/2*(b*arcsin(c*x)^2 + 2*a*arcsin(c*x))/(c*sqrt(d))`**Mupad [F(-1)]**

Timed out.

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))/(d - c^2*d*x^2)^(1/2), x)`output `int((a + b*asin(c*x))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{a + b \arcsin(cx)}{\sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{d} \operatorname{asin}(cx) (\operatorname{asin}(cx) b + 2a)}{2cd}$$

input `int((a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output `(sqrt(d)*asin(c*x)*(asin(c*x)*b + 2*a))/(2*c*d)`

3.113 $\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2dx^2}} dx$

Optimal result	1091
Mathematica [A] (verified)	1092
Rubi [A] (verified)	1092
Maple [A] (verified)	1094
Fricas [F]	1095
Sympy [F]	1095
Maxima [F]	1095
Giac [F(-2)]	1096
Mupad [F(-1)]	1096
Reduce [F]	1096

Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2dx^2}} dx = -\frac{2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}} + \frac{ib\sqrt{1 - c^2x^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}} - \frac{ib\sqrt{1 - c^2x^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}}$$

output

```
-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/
(-c^2*d*x^2+d)^(1/2)+I*b*(-c^2*x^2+1)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-I*b*(-c^2*x^2+1)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)
```


Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \frac{a \log(x)}{\sqrt{d}} - \frac{a \log\left(d + \sqrt{d}\sqrt{-d(-1 + c^2 x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1 - c^2 x^2}(\arcsin(cx) (\log(1 - e^{i \arcsin(cx)}) - \log(1 + e^{i \arcsin(cx)})) + i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{\sqrt{d(1 - c^2 x^2)}}$$

input `Integrate[(a + b*ArcSin[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]`

output `(a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[1 - c^2*x^2]*(ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c*x]])] + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[d*(1 - c^2*x^2)]`

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx \\ & \quad \downarrow \text{5218} \\ & \frac{\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx)}{\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{3042} \\ & \frac{\sqrt{1 - c^2 x^2} \int (a + b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx)}{\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{4671} \end{aligned}$$

$$\frac{\sqrt{1-c^2x^2}(-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{\sqrt{1-c^2x^2}(ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) de^{i \arcsin(cx)} -}{\sqrt{d-c^2dx^2}}$$

↓ 2838

$$\frac{\sqrt{1-c^2x^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{\sqrt{d-c^2dx^2}}$$

input

```
Int[(a + b*ArcSin[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

output

```
(Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*
b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])/Sqr
t[d - c^2*d*x^2]
```

Defintions of rubi rules used

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.24

method	result
default	$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{\sqrt{d}} - \frac{ib\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(i \arcsin(cx) \ln(1+icx+\sqrt{-c^2x^2+1}) - i \arcsin(cx) \ln(1-icx-\sqrt{-c^2x^2+1})\right)}{d(c^2x^2-1)}$
parts	$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{\sqrt{d}} - \frac{ib\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(i \arcsin(cx) \ln(1+icx+\sqrt{-c^2x^2+1}) - i \arcsin(cx) \ln(1-icx-\sqrt{-c^2x^2+1})\right)}{d(c^2x^2-1)}$

input

```
int((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-a/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-I*b*(-c^2*x^2+1)^(1/
2)*(-d*(c^2*x^2-1))^(1/2)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*
arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,-I*c*x-(-c^2*x^2+1)^(
1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))/d/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \int \frac{b \arcsin(cx) + a}{\sqrt{-c^2 dx^2 + dx}} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^2*d*x^3 - d*x), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))/x/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral((a + b*asin(c*x))/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \int \frac{b \arcsin(cx) + a}{\sqrt{-c^2 dx^2 + dx}} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x), x)/sqrt(d) - a*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x\sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^(1/2)),x)`

output `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx = \frac{\left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} x} dx\right) b + \log\left(\tan\left(\frac{\operatorname{asin}(cx)}{2}\right)\right) a}{\sqrt{d}}$$

input `int((a+b*asin(c*x))/x/(-c^2*d*x^2+d)^(1/2),x)`

output `(int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x),x)*b + log(tan(asin(c*x)/2))*a)/sqrt(d)`

3.114 $\int \frac{a+b \arcsin(cx)}{x^2 \sqrt{d-c^2 dx^2}} dx$

Optimal result	1097
Mathematica [A] (verified)	1097
Rubi [A] (verified)	1098
Maple [C] (verified)	1099
Fricas [A] (verification not implemented)	1099
Sympy [F]	1100
Maxima [A] (verification not implemented)	1100
Giac [F(-2)]	1101
Mupad [F(-1)]	1101
Reduce [F]	1101

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = -\frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{dx} + \frac{bc\sqrt{1 - c^2 x^2} \log(x)}{\sqrt{d - c^2 dx^2}}$$

output

```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d/x+b*c*(-c^2*x^2+1)^(1/2)*ln(x)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{d - c^2 dx^2}(-a\sqrt{1 - c^2 x^2} - b\sqrt{1 - c^2 x^2} \arcsin(cx) + bcx \log(x))}{dx\sqrt{1 - c^2 x^2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(-(a*Sqrt[1 - c^2*x^2]) - b*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + b*c*x*Log[x]))/(d*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {5186, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5186$$

$$\frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{x} dx}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{dx}$$

$$\downarrow 14$$

$$\frac{bc\sqrt{1 - c^2 x^2} \log(x)}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{dx}$$

input `Int[(a + b*ArcSin[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]`

output `-((Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(d*x)) + (b*c*Sqrt[1 - c^2*x^2]*Log[x])/Sqrt[d - c^2*d*x^2]`

Defintions of rubi rules used

rule 14 `Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^n_.*((f_.)*(x_)^m_)*((d_) + (e_.)*(x_)^2)^p_, x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.09

method	result
default	$-\frac{a\sqrt{-c^2dx^2+d}}{dx} + b \left(\frac{2i\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)c}{d(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)} \left(i\sqrt{-c^2x^2+1} cx + c^2x^2 - 1 \right) \arcsin(cx)}{dx(c^2x^2-1)} \right)$
parts	$-\frac{a\sqrt{-c^2dx^2+d}}{dx} + b \left(\frac{2i\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)c}{d(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)} \left(i\sqrt{-c^2x^2+1} cx + c^2x^2 - 1 \right) \arcsin(cx)}{dx(c^2x^2-1)} \right)$

input `int((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output `-a/d/x*(-c^2*d*x^2+d)^(1/2)+b*(2*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*arcsin(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)/d/x/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c)`

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.32

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx$$

$$= \left[\frac{bc\sqrt{dx} \log \left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 - \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^4 - 1) \sqrt{d - d}}{c^2 x^4 - x^2} \right)}{2 dx}, \frac{bc\sqrt{-dx}}{2 dx} \right]$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output

```
[1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - 2*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a))/(d*x), (b*c*sqrt(-d)*x*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a))/(d*x)]
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \arcsin(cx)}{x^2 \sqrt{-d(cx - 1)(cx + 1)}} dx$$

input

```
integrate((a+b*asin(c*x))/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

output

```
Integral((a + b*asin(c*x))/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = - \frac{\left((-1)^{-2c^2 dx^2 + 2d} \sqrt{d} \log(-2c^2 d + \frac{2d}{x^2}) + \sqrt{d} \log(x^2 - \frac{1}{c^2}) \right) bc}{2d} - \frac{\sqrt{-c^2 dx^2 + d} b \arcsin(cx)}{dx} - \frac{\sqrt{-c^2 dx^2 + d} a}{dx}$$

input

```
integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output

```
-1/2*((-1)^(-2*c^2*d*x^2 + 2*d)*sqrt(d)*log(-2*c^2*d + 2*d/x^2) + sqrt(d)*log(x^2 - 1/c^2))*b*c/d - sqrt(-c^2*d*x^2 + d)*b*arcsin(c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a/(d*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)),x)`

output `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} a + \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) b x}{\sqrt{d} x}$$

input `int((a+b*asin(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(- sqrt(- c**2*x**2 + 1)*a + int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*x**2),x)*b*x)/(sqrt(d)*x)`

3.115 $\int \frac{a+b \arcsin(cx)}{x^3 \sqrt{d-c^2 dx^2}} dx$

Optimal result	1102
Mathematica [A] (verified)	1103
Rubi [A] (verified)	1103
Maple [A] (verified)	1106
Fricas [F]	1106
Sympy [F]	1107
Maxima [F]	1107
Giac [F(-2)]	1107
Mupad [F(-1)]	1108
Reduce [F]	1108

Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx = -\frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} - \frac{c^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{d - c^2 dx^2}} + \frac{ibc^2\sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{2\sqrt{d - c^2 dx^2}} - \frac{ibc^2\sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{2\sqrt{d - c^2 dx^2}}$$

output

```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/x/(-c^2*d*x^2+d)^(1/2)-1/2*(-c^2*d*x^2+d)^(1/2)
)*(a+b*arcsin(c*x))/d/x^2-c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh
(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)+1/2*I*b*c^2*(-c^2*x^2+1)^(
1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-1/2*I*b*c^2
*(-c^2*x^2+1)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx$$

$$= -\frac{4a\sqrt{d-c^2dx^2}}{x^2} + 4ac^2\sqrt{d}\log(x) - 4ac^2\sqrt{d}\log\left(d + \sqrt{d}\sqrt{d - c^2dx^2}\right) + \frac{bc^2d^2(1-c^2x^2)^{3/2}(-2\cot(\frac{1}{2}\arcsin(cx))-\arcsin(cx))}{d - c^2dx^2}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]`

output

```
((-4*a*Sqrt[d - c^2*d*x^2])/x^2 + 4*a*c^2*Sqrt[d]*Log[x] - 4*a*c^2*Sqrt[d]
*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*c^2*d^2*(1 - c^2*x^2)^(3/2)*(-2
*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 + 4*ArcSin[c*x]*Log
[1 - E^(I*ArcSin[c*x])] - 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + (4*I)
*PolyLog[2, -E^(I*ArcSin[c*x])] - (4*I)*PolyLog[2, E^(I*ArcSin[c*x])] + Ar
cSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2]))/(d - c^2*d*x^2)^(3
/2))/(8*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5204, 15, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5204$$

$$\frac{1}{2}c^2 \int \frac{a + b \arcsin(cx)}{x \sqrt{d - c^2 dx^2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{x^2} dx}{2\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2}$$

$$\downarrow 15$$

$$\frac{1}{2}c^2 \int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} - \frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}}$$

↓ 5218

$$\frac{c^2\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx)}{2\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} - \frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}}$$

↓ 3042

$$\frac{c^2\sqrt{1 - c^2 x^2} \int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{2\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} -$$

$$\frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}}$$

↓ 4671

$$\frac{c^2\sqrt{1 - c^2 x^2}(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}))}{2\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} - \frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}}$$

↓ 2715

$$\frac{c^2\sqrt{1 - c^2 x^2}(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} - \frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}}$$

↓ 2838

$$\frac{c^2\sqrt{1 - c^2 x^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{2\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{2dx^2} - \frac{bc\sqrt{1 - c^2 x^2}}{2x\sqrt{d - c^2 dx^2}}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^3*sqrt[d - c^2*d*x^2]),x]
```

output

$$-1/2*(b*c*\text{Sqrt}[1 - c^2*x^2])/(x*\text{Sqrt}[d - c^2*d*x^2]) - (\text{Sqrt}[d - c^2*d*x^2] * (a + b*\text{ArcSin}[c*x]))/(2*d*x^2) + (c^2*\text{Sqrt}[1 - c^2*x^2]*(-2*(a + b*\text{ArcSin}[c*x])*\text{ArcTanh}[E^{(I*\text{ArcSin}[c*x])}] + I*b*\text{PolyLog}[2, -E^{(I*\text{ArcSin}[c*x])}] - I*b*\text{PolyLog}[2, E^{(I*\text{ArcSin}[c*x])}]])/(2*\text{Sqrt}[d - c^2*d*x^2])$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_*)*(x_)^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 2715

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_*)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] \text{ ; FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_*)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] \text{ ; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$$

rule 4671

$$\text{Int}[\text{csc}[(e_*) + (f_)*(x_)]*((c_*) + (d_)*(x_))^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) \text{ ; FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$$

rule 5204

$$\text{Int}[(a_*) + \text{ArcSin}[(c_)*(x_)]*(b_*)^{(n_*)}*((f_)*(x_))^{(m_*)}*((d_*) + (e_*)*(x_)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1)) \ \text{Int}[(f*x)^{(m+2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$$

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a\sqrt{-c^2dx^2+d}}{2dx^2} - \frac{ac^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{2\sqrt{d}} + b\left(-\frac{(c^2x^2 \arcsin(cx) - cx\sqrt{-c^2x^2+1} - \arcsin(cx))\sqrt{-d(c^2x^2-1)}}{2x^2d(c^2x^2-1)} - \dots\right)$
parts	$-\frac{a\sqrt{-c^2dx^2+d}}{2dx^2} - \frac{ac^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{2\sqrt{d}} + b\left(-\frac{(c^2x^2 \arcsin(cx) - cx\sqrt{-c^2x^2+1} - \arcsin(cx))\sqrt{-d(c^2x^2-1)}}{2x^2d(c^2x^2-1)} - \dots\right)$

input

```
int((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*a/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^
2*d*x^2+d)^(1/2))/x)+b*(-1/2*(c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-a
rcsin(c*x))*(-d*(c^2*x^2-1))^(1/2)/x^2/d/(c^2*x^2-1)-1/2*I*(-c^2*x^2+1)^(1
/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x
^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,-I*c*x
-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c^2)
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx = \int \frac{b \arcsin(cx) + a}{\sqrt{-c^2 dx^2 + dx^3}} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output

```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 x^2}} dx = \int \frac{a + b \arcsin(cx)}{x^3 \sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))/x**3/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral((a + b*asin(c*x))/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 x^2}} dx = \int \frac{b \arcsin(cx) + a}{\sqrt{-c^2 dx^2 + dx^3}} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) + sqrt(-c^2*d*x^2 + d)/(d*x^2))*a + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)),x)`

output `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{a + b \arcsin(cx)}{x^3 \sqrt{d - c^2 dx^2}} dx \\ &= \frac{-\sqrt{-c^2 x^2 + 1} a + 2 \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} x^3} dx \right) b x^2 + \log \left(\tan \left(\frac{\operatorname{asin}(cx)}{2} \right) \right) a c^2 x^2}{2\sqrt{d} x^2} \end{aligned}$$

input `int((a+b*asin(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x)`

output `(-sqrt(-c**2*x**2 + 1)*a + 2*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**3),x)*b*x**2 + log(tan(asin(c*x)/2))*a*c**2*x**2)/(2*sqrt(d)*x**2)`

3.116 $\int \frac{a+b \arcsin(cx)}{x^4 \sqrt{d-c^2 dx^2}} dx$

Optimal result	1109
Mathematica [A] (verified)	1109
Rubi [A] (verified)	1110
Maple [C] (verified)	1112
Fricas [A] (verification not implemented)	1113
Sympy [F]	1113
Maxima [A] (verification not implemented)	1114
Giac [F(-2)]	1114
Mupad [F(-1)]	1115
Reduce [F]	1115

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx = -\frac{bc\sqrt{1 - c^2 x^2}}{6x^2 \sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3dx^3} - \frac{2c^2 \sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3dx} + \frac{2bc^3 \sqrt{1 - c^2 x^2} \log(x)}{3\sqrt{d - c^2 dx^2}}$$

output

```
-1/6*b*c*(-c^2*x^2+1)^(1/2)/x^2/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d/x^3-2/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d/x+2/3*b*c^3*(-c^2*x^2+1)^(1/2)*ln(x)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{d - c^2 dx^2}(bcx + 6bc^3 x^3 + 2a\sqrt{1 - c^2 x^2} + 4ac^2 x^2 \sqrt{1 - c^2 x^2} + 2b\sqrt{1 - c^2 x^2}(1 + 2c^2 x^2) \arcsin(cx) - 6dx^3 \sqrt{1 - c^2 x^2})}{6dx^3 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output

$$-1/6*(\text{Sqrt}[d - c^2*d*x^2]*(b*c*x + 6*b*c^3*x^3 + 2*a*\text{Sqrt}[1 - c^2*x^2] + 4*a*c^2*x^2*\text{Sqrt}[1 - c^2*x^2] + 2*b*\text{Sqrt}[1 - c^2*x^2]*(1 + 2*c^2*x^2)*\text{ArcSin}[c*x] - 4*b*c^3*x^3*\text{Log}[x]))/(d*x^3*\text{Sqrt}[1 - c^2*x^2])$$
Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5204, 15, 5186, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5204$$

$$\frac{2}{3}c^2 \int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{x^3} dx}{3\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3dx^3}$$

$$\downarrow 15$$

$$\frac{2}{3}c^2 \int \frac{a + b \arcsin(cx)}{x^2 \sqrt{d - c^2 dx^2}} dx - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3dx^3} - \frac{bc\sqrt{1 - c^2 x^2}}{6x^2 \sqrt{d - c^2 dx^2}}$$

$$\downarrow 5186$$

$$\frac{2}{3}c^2 \left(\frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{x} dx}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{dx} \right) - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3dx^3} - \frac{bc\sqrt{1 - c^2 x^2}}{6x^2 \sqrt{d - c^2 dx^2}}$$

$$\downarrow 14$$

$$\frac{2}{3}c^2 \left(\frac{bc\sqrt{1 - c^2 x^2} \log(x)}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{dx} \right) - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3dx^3} - \frac{bc\sqrt{1 - c^2 x^2}}{6x^2 \sqrt{d - c^2 dx^2}}$$

input

$$\text{Int}[(a + b*\text{ArcSin}[c*x])/(x^4*\text{Sqrt}[d - c^2*d*x^2]),x]$$

output

$$-1/6*(b*c*\sqrt{1 - c^2*x^2})/(x^2*\sqrt{d - c^2*d*x^2}) - (\sqrt{d - c^2*d*x^2}*(a + b*\text{ArcSin}[c*x]))/(3*d*x^3) + (2*c^2*(-((\sqrt{d - c^2*d*x^2}*(a + b*\text{ArcSin}[c*x]))/(d*x)) + (b*c*\sqrt{1 - c^2*x^2}*\text{Log}[x])/(\sqrt{d - c^2*d*x^2}))) / 3$$

Defintions of rubi rules used

rule 14

$$\text{Int}[(a_)/(x_), x_Symbol] \rightarrow \text{Simp}[a*\text{Log}[x], x] \text{ /; FreeQ}[a, x]$$

rule 15

$$\text{Int}[(a_)*(x_)^(m_), x_Symbol] \rightarrow \text{Simp}[a*(x^(m + 1)/(m + 1)), x] \text{ /; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 5186

$$\text{Int}[(a_ + \text{ArcSin}[c_*(x_)]*(b_))^(n_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] \rightarrow \text{Simp}[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m + 1))), x] - \text{Simp}[b*c*(n/(f*(m + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x] \text{ /; FreeQ}[\{a, b, c, d, e, f, m, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{EqQ}[m + 2*p + 3, 0] \ \&\& \ \text{NeQ}[m, -1]$$

rule 5204

$$\text{Int}[(a_ + \text{ArcSin}[c_*(x_)]*(b_))^(n_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] \rightarrow \text{Simp}[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m + 1))), x] + (\text{Simp}[c^2*((m + 2*p + 3)/(f^2*(m + 1)))] \text{ Int}[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x] \text{ /; FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 850, normalized size of antiderivative = 5.78

method	result
default	$a \left(-\frac{\sqrt{-c^2 d x^2 + d}}{3 d x^3} - \frac{2 c^2 \sqrt{-c^2 d x^2 + d}}{3 d x} \right) - \frac{i b \sqrt{-d(c^2 x^2 - 1)} x (-c^2 x^2 + 1) c^4}{3(3 c^4 x^4 - 2 c^2 x^2 - 1) d} - \frac{2 i b \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(c x) c^3}{3(3 c^4 x^4 - 2 c^2 x^2 - 1) d}$
parts	$a \left(-\frac{\sqrt{-c^2 d x^2 + d}}{3 d x^3} - \frac{2 c^2 \sqrt{-c^2 d x^2 + d}}{3 d x} \right) - \frac{i b \sqrt{-d(c^2 x^2 - 1)} x (-c^2 x^2 + 1) c^4}{3(3 c^4 x^4 - 2 c^2 x^2 - 1) d} - \frac{2 i b \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(c x) c^3}{3(3 c^4 x^4 - 2 c^2 x^2 - 1) d}$

input `int((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output

```
a*(-1/3/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*c^2/d/x*(-c^2*d*x^2+d)^(1/2))-1/3*I
*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x*(-c^2*x^2+1)*c^4-2/3
*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*(-c^2*x^2+1)^(1/2)*a
rcsin(c*x)*c^3-2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^
5*c^8-2*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^2*(-c^2*x^2
+1)^(1/2)*arcsin(c*x)*c^5-2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-
1)/d*x^3*arcsin(c*x)*c^6-2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x
^2-1)/d*x^3*(-c^2*x^2+1)*c^6+1/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c
^2*x^2-1)/d*x*c^4+1/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d
*x^3*c^6+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x*arcsin(c
*x)*c^4+4/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*ar
csin(c*x)*c^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*c^3*(
-c^2*x^2+1)^(1/2)+4/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d/x
*arcsin(c*x)*c^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d/x^
2*(-c^2*x^2+1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1
)/d/x^3*arcsin(c*x)-2/3*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2
*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.95

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx$$

$$= \frac{2(bc^5 x^5 - bc^3 x^3) \sqrt{d} \log\left(\frac{c^2 dx^6 + c^2 dx^2 - dx^4 - \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} (x^4 - 1) \sqrt{d-d}}{c^2 x^4 - x^2}\right) - \sqrt{-c^2 dx^2 + d} (bcx^3 - bcx) \sqrt{-d}}{6(c^2 dx^5 - dx^3)}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output

```
[1/6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(-c^2*x^2 + 1) - 2*(2*a*c^4*x^4 - a*c^2*x^2 + (2*b*c^4*x^4 - b*c^2*x^2 - b)*arcsin(c*x) - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3), 1/6*(4*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(-c^2*x^2 + 1) - 2*(2*a*c^4*x^4 - a*c^2*x^2 + (2*b*c^4*x^4 - b*c^2*x^2 - b)*arcsin(c*x) - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3)]
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 \sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))/x**4/(-c**2*d*x**2+d)**(1/2),x)`

output

```
Integral((a + b*asin(c*x))/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx = \frac{1}{6} \left(\frac{4c^2 \log(x)}{\sqrt{d}} - \frac{1}{\sqrt{dx^2}} \right) bc$$

$$- \frac{1}{3} b \left(\frac{2\sqrt{-c^2 dx^2 + dc^2}}{dx} + \frac{\sqrt{-c^2 dx^2 + d}}{dx^3} \right) \arcsin(cx)$$

$$- \frac{1}{3} a \left(\frac{2\sqrt{-c^2 dx^2 + dc^2}}{dx} + \frac{\sqrt{-c^2 dx^2 + d}}{dx^3} \right)$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `1/6*(4*c^2*log(x)/sqrt(d) - 1/(sqrt(d)*x^2))*b*c - 1/3*b*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arcsin(c*x) - 1/3*a*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)),x)`

output `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{a + b \arcsin(cx)}{x^4 \sqrt{d - c^2 dx^2}} dx \\ &= \frac{-2\sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a + 3 \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} x^4} dx \right) b x^3}{3\sqrt{d} x^3} \end{aligned}$$

input `int((a+b*asin(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x)`

output `(-2*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 - sqrt(-c**2*x**2 + 1)*a + 3*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**4),x)*b*x**3)/(3*sqrt(d)*x**3)`

3.117
$$\int \frac{x^5(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx$$

Optimal result	1116
Mathematica [C] (verified)	1117
Rubi [A] (verified)	1117
Maple [C] (verified)	1119
Fricas [A] (verification not implemented)	1120
Sympy [F]	1120
Maxima [F]	1121
Giac [F(-2)]	1121
Mupad [F(-1)]	1122
Reduce [F]	1122

Optimal result

Integrand size = 27, antiderivative size = 221

$$\int \frac{x^5(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = -\frac{5bx\sqrt{1-c^2x^2}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{bx^3\sqrt{1-c^2x^2}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{c^6d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{c^6d^2} - \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))}{3c^6d^3} - \frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

output

```
-5/3*b*x*(-c^2*x^2+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(-c^2*x^2+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arcsin(c*x))/c^6/d/(-c^2*d*x^2+d)^(1/2)+2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^6/d^2-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/c^6/d^3-b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/c^6/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.75

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{\sqrt{d - c^2 dx^2}(\sqrt{-c^2}(bcx\sqrt{1 - c^2 x^2}(15 + c^2 x^2) + 3a(-8 + 4c^2 x^2 + c^4 x^4) + 3b($$

 $9c^6\sqrt{-c^2}$

input `Integrate[(x^5*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]`

output `(Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(b*c*x*Sqrt[1 - c^2*x^2]*(15 + c^2*x^2) + 3*a*(-8 + 4*c^2*x^2 + c^4*x^4) + 3*b*(-8 + 4*c^2*x^2 + c^4*x^4)*ArcSin[c*x]) - (9*I)*b*c*Sqrt[1 - c^2*x^2]*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]))/(9*c^6*Sqrt[-c^2]*d^2*(-1 + c^2*x^2))`

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

↓ 5194

$$-\frac{bc\sqrt{d - c^2 dx^2} \int \frac{-c^4 x^4 - 4c^2 x^2 + 8}{3c^6 d^2 (1 - c^2 x^2)} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3c^6 d^3} +$$

$$\frac{2\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{c^6 d^2} + \frac{a + b \arcsin(cx)}{c^6 d \sqrt{d - c^2 dx^2}}$$

↓ 27

$$\begin{aligned}
& -\frac{b\sqrt{d-c^2dx^2} \int \frac{-c^4x^4-4c^2x^2+8}{1-c^2x^2} dx}{3c^5d^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3c^6d^3} + \\
& \quad \frac{2\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^2} + \frac{a+b\arcsin(cx)}{c^6d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow 1467 \\
& -\frac{b\sqrt{d-c^2dx^2} \int \left(c^2x^2 + \frac{3}{1-c^2x^2} + 5\right) dx}{3c^5d^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3c^6d^3} + \\
& \quad \frac{2\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^2} + \frac{a+b\arcsin(cx)}{c^6d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow 2009 \\
& -\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))}{3c^6d^3} + \frac{2\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^2} + \frac{a+b\arcsin(cx)}{c^6d\sqrt{d-c^2dx^2}} - \\
& \quad \frac{b\left(\frac{3\operatorname{arctanh}(cx)}{c} + \frac{c^2x^3}{3} + 5x\right)\sqrt{d-c^2dx^2}}{3c^5d^2\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[(x^5*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]`

output `(a + b*ArcSin[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(c^6*d^2) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^6*d^3) - (b*Sqrt[d - c^2*d*x^2]*(5*x + (c^2*x^3)/3 + (3*ArcTanh[c*x])/c))/(3*c^5*d^2*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1467 `Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_) , x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.92

method	result
default	$a \left(-\frac{x^4}{3c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{-\frac{4x^2}{3c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{8}{3d c^4 \sqrt{-c^2 d x^2 + d}}}{c^2} \right) + \frac{5b \sqrt{-d(c^2 x^2 - 1)} \arcsin(cx) x^2}{3d^2 c^4 (c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sin(\arcsin(cx))}{72d^2 c^6 (c^2 x^2 - 1)}$
parts	$a \left(-\frac{x^4}{3c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{-\frac{4x^2}{3c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{8}{3d c^4 \sqrt{-c^2 d x^2 + d}}}{c^2} \right) + \frac{5b \sqrt{-d(c^2 x^2 - 1)} \arcsin(cx) x^2}{3d^2 c^4 (c^2 x^2 - 1)} - \frac{b \sqrt{-d(c^2 x^2 - 1)} \sin(\arcsin(cx))}{72d^2 c^6 (c^2 x^2 - 1)}$

input `int(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output `a*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+5/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcsin(c*x)*x^2-1/72*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*sin(4*arcsin(c*x))+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^6/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^6/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I)-65/24*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)+31/18*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+1/24*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)*cos(4*arcsin(c*x))`

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.00

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{\left[9(bc^2 x^2 - b)\sqrt{d} \log\left(-\frac{c^6 dx^6 + 5c^4 dx^4 - 5c^2 dx^2 + 4(c^3 x^3 + cx)\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}\sqrt{d-d}}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1}\right) \right.}{18(c^8 d^2 x^2 - c^6 d^2)}$$

$$\left. - \frac{9(bc^2 x^2 - b)\sqrt{-d} \arctan\left(\frac{2\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}c\sqrt{-dx}}{c^4 dx^4 - d}\right) - 2(bc^3 x^3 + 15bcx)\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1} - 6}{18(c^8 d^2 x^2 - c^6 d^2)} \right]$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `[1/36*(9*(b*c^2*x^2 - b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) + 4*(b*c^3*x^3 + 15*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 12*(a*c^4*x^4 + 4*a*c^2*x^2 + (b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*arcsin(c*x) - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*x^2 - c^6*d^2), -1/18*(9*(b*c^2*x^2 - b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 2*(b*c^3*x^3 + 15*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 6*(a*c^4*x^4 + 4*a*c^2*x^2 + (b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*arcsin(c*x) - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*x^2 - c^6*d^2)]`

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^5(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate(x**5*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral(x**5*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^5}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `-1/3*a*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^4*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) - 1/3*(3*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^6*d^2*integrate(1/3*(c^4*x^6 + 4*c^2*x^4 - 8*x^2)/(c^7*d^2*x^4 - c^5*d^2*x^2 + (c^5*d^2*x^2 - c^3*d^2)*e^(log(c*x + 1) + log(-c*x + 1))), x) + (c^4*x^4 + 4*c^2*x^2 - 8)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*b/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^6*d^(3/2))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^5*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^5*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{-3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x^5}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) b c^6 - a c^4 x^4 - 4a c^2 x^2 + 8a}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} c^6 d}$$

input `int(x^5*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 3*sqrt(- c**2*x**2 + 1)*int((asin(c*x)*x**5)/(sqrt(- c**2*x**2 + 1)*
c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b*c**6 - a*c**4*x**4 - 4*a*c**2*x**
2 + 8*a)/(3*sqrt(d)*sqrt(- c**2*x**2 + 1)*c**6*d)`

3.118 $\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx$

Optimal result	1123
Mathematica [A] (verified)	1124
Rubi [A] (verified)	1124
Maple [C] (verified)	1127
Fricas [F]	1128
Sympy [F]	1128
Maxima [F]	1128
Giac [F(-2)]	1129
Mupad [F(-1)]	1129
Reduce [F]	1130

Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = -\frac{bx^2\sqrt{1-c^2x^2}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3(a+b \arcsin(cx))}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{2c^4d^2} - \frac{3\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{4bc^5d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2} \log(1-c^2x^2)}{2c^5d\sqrt{d-c^2dx^2}}$$

output

```
-1/4*b*x^2*(-c^2*x^2+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*arcsin(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4/d^2-3/4*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/b/c^5/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c^5/d/(-c^2*d*x^2+d)^(1/2)
```


Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.81

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{-4ac\sqrt{dx}(-3 + c^2 x^2) + 12a\sqrt{d - c^2 dx^2} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d(-1 + c^2 x^2)}}\right) + b\sqrt{d}(8cx \arcsin(cx))}{(d - c^2 dx^2)^{3/2}}$$

input `Integrate[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2), x]`

output `(-4*a*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*Sqrt[d]*(8*c*x*ArcSin[c*x] + Sqrt[1 - c^2*x^2]*(-6*ArcSin[c*x]^2 + Cos[2*ArcSin[c*x]] + 4*Log[1 - c^2*x^2] + 2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*x^2])`

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5206, 243, 49, 2009, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx \\ & \quad \downarrow \text{5206} \\ & -\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1 - c^2 x^2} \int \frac{x^3}{1 - c^2 x^2} dx}{cd\sqrt{d - c^2 dx^2}} + \frac{x^3(a + b \arcsin(cx))}{c^2 d\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{243} \\ & -\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1 - c^2 x^2} \int \frac{x^2}{1 - c^2 x^2} dx^2}{2cd\sqrt{d - c^2 dx^2}} + \frac{x^3(a + b \arcsin(cx))}{c^2 d\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{49} \end{aligned}$$

$$\begin{aligned}
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \left(-\frac{1}{c^2} - \frac{1}{c^2(c^2 x^2 - 1)}\right) dx^2}{2cd\sqrt{d-c^2 dx^2}} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{2009} \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} - \frac{b\sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4}\right)}{2cd\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{5210} \\
& \frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} + \frac{b\sqrt{1-c^2 x^2} \int x dx}{2c\sqrt{d-c^2 dx^2}} - \frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{2c^2 d} \right)}{c^2 d} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} - \\
& \quad \frac{b\sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4}\right)}{2cd\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{15} \\
& \frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} - \frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{2c^2 d} + \frac{bx^2\sqrt{1-c^2 x^2}}{4c\sqrt{d-c^2 dx^2}} \right)}{c^2 d} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} - \\
& \quad \frac{b\sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4}\right)}{2cd\sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{5152} \\
& \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} - \frac{3 \left(-\frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))}{2c^2 d} + \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))^2}{4bc^3\sqrt{d-c^2 dx^2}} + \frac{bx^2\sqrt{1-c^2 x^2}}{4c\sqrt{d-c^2 dx^2}} \right)}{c^2 d} \\
& \quad \frac{b\sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4}\right)}{2cd\sqrt{d-c^2 dx^2}}
\end{aligned}$$

input

```
Int[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output

```
(x^3*(a + b*ArcSin[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (3*((b*x^2*Sqrt[1 - c^2*x^2])/(4*c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(2*c^2*d) + (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])))/(c^2*d) - (b*Sqrt[1 - c^2*x^2]*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Definitions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 49 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)}*((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$
- rule 243 $\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{((m-1)/2)*(a + b*x)^p}, x], x, x^2], x] \text{ ; FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 5152 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}/\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$
- rule 5206 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}*((f_.)(x_)^{(m_.)}*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^{(n)/(2*e*(p+1))}), x] + (-\text{Simp}[f^2*((m-1)/(2*e*(p+1)) \ \text{Int}[(f*x)^{(m-2)}*(d + e*x^2)^{(p+1)}*(a + b*\text{ArcSin}[c*x])^{(n)}, x] + \text{Simp}[b*f*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p \ \text{Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IGtQ}[m, 1]$

rule 5210

```

Int[((a._) + ArcSin[(c._)*(x_)])*(b._))^(n._)*((f._)*(x_)^(m._)*((d._) + (e._)
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]

```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.02

method	result
default	$-\frac{ax^3}{2c^2d\sqrt{-c^2dx^2+d}} + \frac{3ax}{2c^4d\sqrt{-c^2dx^2+d}} - \frac{3a \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2c^4d\sqrt{c^2d}} + \frac{3b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{4d^2c^5(c^2x^2-1)} + \frac{ib\sqrt{-d}}{4d^2c^5(c^2x^2-1)}$
parts	$-\frac{ax^3}{2c^2d\sqrt{-c^2dx^2+d}} + \frac{3ax}{2c^4d\sqrt{-c^2dx^2+d}} - \frac{3a \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2c^4d\sqrt{c^2d}} + \frac{3b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^2}{4d^2c^5(c^2x^2-1)} + \frac{ib\sqrt{-d}}{4d^2c^5(c^2x^2-1)}$

input

```
int(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2), x, method=_RETURNVERBOSE)
```

output

```

-1/2*a*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(-c^2*d*x^2+d)^(1/2)-3
/2*a/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/4*
b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcsin(c*x
)^2+I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcs
in(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*ln
(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/16*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^
2*x^2-1)*(-c^2*x^2+1)^(1/2)-9/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-
1)*arcsin(c*x)*x-1/16*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*cos(3*a
rcsin(c*x))-1/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arcsin(c*x)*s
in(3*arcsin(c*x))

```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral((b*x^4*arcsin(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^4(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate(x**4*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral(x**4*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output

```
-1/2*a*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*d)
) + 3*arcsin(c*x)/(c^5*d^(3/2))) - b*integrate(x^4*arctan2(c*x, sqrt(c*x +
1)*sqrt(-c*x + 1))/((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sq
r
t(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

input

```
int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output

```
int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2x^2)^{3/2}} dx = \frac{-3\sqrt{-c^2x^2 + 1} \arcsin(cx)^2 b - 6\sqrt{-c^2x^2 + 1} \arcsin(cx) a - 2\arcsin(cx) b c^3 x^3 + 2a^2}{(d - c^2x^2)^{3/2}}$$

input `int(x^4*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 3*sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b - 6*sqrt(- c**2*x**2 + 1)*asin(c*x)*a - 2*asin(c*x)*b*c**3*x**3 + 2*asin(c*x)*b*c*x - 4*sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b*c - sqrt(- c**2*x**2 + 1)*b*c**2*x**2 + sqrt(- c**2*x**2 + 1)*b - 2*a*c**3*x**3 + 6*a*c*x)/(4*sqrt(d)*sqrt(- c**2*x**2 + 1)*c**5*d)`

3.119 $\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx$

Optimal result	1131
Mathematica [C] (verified)	1131
Rubi [A] (verified)	1132
Maple [C] (verified)	1134
Fricas [A] (verification not implemented)	1134
Sympy [F]	1135
Maxima [A] (verification not implemented)	1135
Giac [F(-2)]	1136
Mupad [F(-1)]	1136
Reduce [F]	1137

Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = -\frac{bx\sqrt{1-c^2x^2}}{c^3d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{c^4d^2} - \frac{b\sqrt{1-c^2x^2}\operatorname{arctanh}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

output

```
-b*x*(-c^2*x^2+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arcsin(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^4/d^2-b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/c^4/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = \frac{\sqrt{d-c^2dx^2}(\sqrt{-c^2}(-2a+ac^2x^2+bcx\sqrt{1-c^2x^2}+b(-2+c^2x^2)\arcsin(cx)))}{c^4\sqrt{-c^2}d^2(-1+c^2x^2)}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```


output

```
(Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(-2*a + a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2] + b*(-2 + c^2*x^2)*ArcSin[c*x]) - I*b*c*Sqrt[1 - c^2*x^2]*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]))/(c^4*Sqrt[-c^2]*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int \frac{2 - c^2 x^2}{c^4 d^2 (1 - c^2 x^2)} dx}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{c^4 d^2} + \frac{a + b \arcsin(cx)}{c^4 d \sqrt{d - c^2 dx^2}}$$

$$\downarrow 27$$

$$-\frac{b\sqrt{d - c^2 dx^2} \int \frac{2 - c^2 x^2}{1 - c^2 x^2} dx}{c^3 d^2 \sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{c^4 d^2} + \frac{a + b \arcsin(cx)}{c^4 d \sqrt{d - c^2 dx^2}}$$

$$\downarrow 299$$

$$-\frac{b\sqrt{d - c^2 dx^2} \left(\int \frac{1}{1 - c^2 x^2} dx + x \right)}{c^3 d^2 \sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{c^4 d^2} + \frac{a + b \arcsin(cx)}{c^4 d \sqrt{d - c^2 dx^2}}$$

$$\downarrow 219$$

$$\frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{c^4 d^2} + \frac{a + b \arcsin(cx)}{c^4 d \sqrt{d - c^2 dx^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right) \sqrt{d - c^2 dx^2}}{c^3 d^2 \sqrt{1 - c^2 x^2}}$$

input

```
Int[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output

$$\frac{(a + b \operatorname{ArcSin}[c x]) / (c^4 d \sqrt{d - c^2 d x^2}) + (\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])) / (c^4 d^2) - (b \sqrt{d - c^2 d x^2} (x + \operatorname{ArcTanh}[c x] / c)) / (c^3 d^2 \sqrt{1 - c^2 x^2})}{1}$$
Defintions of rubi rules used

rule 27

$$\operatorname{Int}[(a_*)(F x_), x_Symbol] \rightarrow \operatorname{Simp}[a \operatorname{Int}[F x, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[F x, (b_*)(G x_)] /; \operatorname{FreeQ}[b, x]$$

rule 219

$$\operatorname{Int}[((a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2])) * \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] * (x / \operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a / b] \&\& (\operatorname{GtQ}[a, 0] \operatorname{||} \operatorname{LtQ}[b, 0])$$

rule 299

$$\operatorname{Int}[((a_*) + (b_*)(x_)^2)^{p_*) * ((c_*) + (d_*)(x_)^2), x_Symbol] \rightarrow \operatorname{Simp}[d * x * ((a + b * x^2)^{p+1} / (b * (2 * p + 3))), x] - \operatorname{Simp}[(a * d - b * c * (2 * p + 3)) / (b * (2 * p + 3)) \operatorname{Int}[(a + b * x^2)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b * c - a * d, 0] \&\& \operatorname{NeQ}[2 * p + 3, 0]$$

rule 5194

$$\operatorname{Int}[((a_*) + \operatorname{ArcSin}[(c_*)(x_)] * (b_*) * (x_)^{m_*) * ((d_*) + (e_*)(x_)^2)^{p_*)}, x_Symbol] \rightarrow \operatorname{With}[\{u = \operatorname{IntHide}[x^m * (d + e * x^2)^p, x]\}, \operatorname{Simp}[(a + b * \operatorname{ArcSin}[c * x]) u, x] - \operatorname{Simp}[b * c * \operatorname{Simp}[\sqrt{d + e * x^2} / \sqrt{1 - c^2 * x^2}] \operatorname{Int}[\operatorname{SimplifyIntegrand}[u / \sqrt{d + e * x^2}], x], x]] /; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \&\& \operatorname{EqQ}[c^2 * d + e, 0] \&\& \operatorname{IntegerQ}[p - 1/2] \&\& \operatorname{NeQ}[p, -2^{-1}] \&\& (\operatorname{IGtQ}[(m + 1) / 2, 0] \operatorname{||} \operatorname{ILtQ}[(m + 2 * p + 3) / 2, 0])$$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.16

method	result
default	$a \left(-\frac{x^2}{c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{2}{d c^4 \sqrt{-c^2 d x^2 + d}} \right) + \frac{b \sqrt{-d(c^2 x^2 - 1)} \arcsin(cx) x^2}{d^2 c^2 (c^2 x^2 - 1)} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} x}{d^2 c^3 (c^2 x^2 - 1)} - \frac{2b \sqrt{-d(c^2 x^2 - 1)}}{d^2 c^2}$
parts	$a \left(-\frac{x^2}{c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{2}{d c^4 \sqrt{-c^2 d x^2 + d}} \right) + \frac{b \sqrt{-d(c^2 x^2 - 1)} \arcsin(cx) x^2}{d^2 c^2 (c^2 x^2 - 1)} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} x}{d^2 c^3 (c^2 x^2 - 1)} - \frac{2b \sqrt{-d(c^2 x^2 - 1)}}{d^2 c^2}$

input `int(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output `a*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arcsin(c*x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x-2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcsin(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^4/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I)+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^4/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)`

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.69

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \left[\frac{4 \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} bcx + (bc^2 x^2 - b) \sqrt{d} \log \left(-\frac{c^6 dx^6 + 5 c^4 dx^4 - 5 c^2 dx^2 + 4}{c^6 x^6} \right)}{4} \right]$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output

```
[1/4*(4*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x + (b*c^2*x^2 - b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) + 4*(a*c^2*x^2 + (b*c^2*x^2 - 2*b)*arcsin(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d)/(c^6*d^2*x^2 - c^4*d^2), 1/2*(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - (b*c^2*x^2 - b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) + 2*(a*c^2*x^2 + (b*c^2*x^2 - 2*b)*arcsin(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d)/(c^6*d^2*x^2 - c^4*d^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^3(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input

```
integrate(x**3*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output

```
Integral(x**3*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\begin{aligned} \int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx &= -\frac{1}{2} bc \left(\frac{2x}{c^4 d^{3/2}} + \frac{\log(cx + 1)}{c^5 d^{3/2}} - \frac{\log(cx - 1)}{c^5 d^{3/2}} \right) \\ &- b \left(\frac{x^2}{\sqrt{-c^2 dx^2 + dc^2 d}} - \frac{2}{\sqrt{-c^2 dx^2 + dc^4 d}} \right) \arcsin(cx) \\ &- a \left(\frac{x^2}{\sqrt{-c^2 dx^2 + dc^2 d}} - \frac{2}{\sqrt{-c^2 dx^2 + dc^4 d}} \right) \end{aligned}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output

```
-1/2*b*c*(2*x/(c^4*d^(3/2)) + log(c*x + 1)/(c^5*d^(3/2)) - log(c*x - 1)/(c^5*d^(3/2))) - b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d))*arcsin(c*x) - a*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

input

```
int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output

```
int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)x^3}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) b c^4 - a c^2 x^2 + 2a}{\sqrt{d} \sqrt{-c^2 x^2 + 1} c^4 d}$$

input `int(x^3*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(-sqrt(-c**2*x**2+1)*int((asin(c*x)*x**3)/(sqrt(-c**2*x**2+1)*c**2*x**2-sqrt(-c**2*x**2+1)),x)*b*c**4-a*c**2*x**2+2*a)/(sqrt(d)*sqrt(-c**2*x**2+1)*c**4*d)`

3.120
$$\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx$$

Optimal result	1138
Mathematica [A] (verified)	1138
Rubi [A] (verified)	1139
Maple [C] (verified)	1140
Fricas [F]	1141
Sympy [F]	1141
Maxima [F]	1142
Giac [F(-2)]	1142
Mupad [F(-1)]	1142
Reduce [F]	1143

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = \frac{x(a+b \arcsin(cx))}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2bc^3d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2} \log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}$$

output

```
x*(a+b*arcsin(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/b/c^3/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19

$$\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = -\frac{ax\sqrt{-d(-1+c^2x^2)}}{c^2d^2(-1+c^2x^2)} + \frac{a \arctan\left(\frac{cx\sqrt{-d(-1+c^2x^2)}}{\sqrt{d(-1+c^2x^2)}}\right)}{c^3d^{3/2}} + \frac{b(2cx \arcsin(cx) - \sqrt{1-c^2x^2}(\arcsin(cx)^2 - 2 \log(\sqrt{1-c^2x^2})))}{2c^3d\sqrt{d(1-c^2x^2)}}$$

input `Integrate[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]`

output `-((a*x*Sqrt[-(d*(-1 + c^2*x^2))])/(c^2*d^2*(-1 + c^2*x^2))) + (a*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(c^3*d^(3/2)) + (b*(2*c*x*ArcSin[c*x] - Sqrt[1 - c^2*x^2]*(ArcSin[c*x]^2 - 2*Log[Sqrt[1 - c^2*x^2]])))/(2*c^3*d*Sqrt[d*(1 - c^2*x^2)])`

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5206, 240, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx \\
 & \quad \downarrow \text{5206} \\
 & -\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x}{1-c^2 x^2} dx}{cd\sqrt{d-c^2 dx^2}} + \frac{x(a + b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{240} \\
 & -\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} + \frac{x(a + b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} + \frac{b\sqrt{1-c^2 x^2} \log(1-c^2 x^2)}{2c^3 d\sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{5152} \\
 & \frac{x(a + b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} - \frac{\sqrt{1-c^2 x^2}(a + b \arcsin(cx))^2}{2bc^3 d\sqrt{d-c^2 dx^2}} + \frac{b\sqrt{1-c^2 x^2} \log(1-c^2 x^2)}{2c^3 d\sqrt{d-c^2 dx^2}}
 \end{aligned}$$

input `Int[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]`


```
output (x*(a + b*ArcSin[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[1 - c^2*x^2]*(
a + b*ArcSin[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^
2]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

```
rule 240 Int[(x_)/((a_) + (b_)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

```
rule 5152 Int[((a_) + ArcSin[(c_)*(x_)]*(b_.))^n_/Sqrt[(d_) + (e_)*(x_)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

```
rule 5206 Int[((a_) + ArcSin[(c_)*(x_)]*(b_.))^n_*((f_)*(x_)^m)*((d_) + (e_
)*(x_)^2)^p_, x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.03

method	result
default	$\frac{ax}{c^2 d \sqrt{-c^2 d x^2 + d}} - \frac{a \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{c^2 d \sqrt{c^2 d}} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{2 d^2 c^3 (c^2 x^2 - 1)} + \frac{ib \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{d^2 c^3 (c^2 x^2 - 1)}$
parts	$\frac{ax}{c^2 d \sqrt{-c^2 d x^2 + d}} - \frac{a \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{c^2 d \sqrt{c^2 d}} + \frac{b \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{2 d^2 c^3 (c^2 x^2 - 1)} + \frac{ib \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{d^2 c^3 (c^2 x^2 - 1)}$

input `int(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output `a*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*
x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^
2/c^3/(c^2*x^2-1)*arcsin(c*x)^2+I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/d^2/c^3/(c^2*x^2-1)*arcsin(c*x)-b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*
x^2-1)*arcsin(c*x)*x-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(
c^2*x^2-1)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)`

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arcsin(c*x) + a*x^2)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral(x**2*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `a*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) - b*integrate(x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) a - 2\sqrt{-c^2 x^2 + 1} \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1}} dx \right)}{2\sqrt{d} \sqrt{-c^2 x^2 + 1} c^3 d}$$

input `int(x^2*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(- sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a - 2*sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b*c + 2*a*c*x)/(2*sqrt(d)*sqrt(- c**2*x**2 + 1)*c**3*d)`

3.121 $\int \frac{x(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx$

Optimal result	1144
Mathematica [A] (verified)	1144
Rubi [A] (verified)	1145
Maple [C] (verified)	1146
Fricas [A] (verification not implemented)	1146
Sympy [F]	1147
Maxima [F]	1147
Giac [F(-2)]	1148
Mupad [F(-1)]	1148
Reduce [F]	1148

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^{3/2}} dx = \frac{a + b \arcsin(cx)}{c^2d\sqrt{d - c^2dx^2}} - \frac{b\sqrt{1 - c^2x^2}\operatorname{arctanh}(cx)}{c^2d\sqrt{d - c^2dx^2}}$$

output

$$(a+b*\arcsin(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(-c^2*x^2+1)^(1/2)*\operatorname{arctanh}(c*x)/c^2/d/(-c^2*d*x^2+d)^(1/2)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^{3/2}} dx = \frac{a + b \arcsin(cx) - b\sqrt{1 - c^2x^2}\operatorname{arctanh}(cx)}{c^2d\sqrt{d - c^2dx^2}}$$

input

$$\operatorname{Integrate}[(x*(a + b*\operatorname{ArcSin}[c*x]))/(d - c^2*d*x^2)^(3/2),x]$$

output

$$(a + b*\operatorname{ArcSin}[c*x] - b*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{ArcTanh}[c*x])/(c^2*d*\operatorname{Sqrt}[d - c^2*d*x^2])$$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5182, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow \text{5182}$$

$$\frac{a + b \arcsin(cx)}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{b \sqrt{1 - c^2 x^2} \int \frac{1}{1 - c^2 x^2} dx}{cd \sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{219}$$

$$\frac{a + b \arcsin(cx)}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{b \sqrt{1 - c^2 x^2} \operatorname{arctanh}(cx)}{c^2 d \sqrt{d - c^2 dx^2}}$$

input `Int[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]`

output `(a + b*ArcSin[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.66

method	result
default	$\frac{a}{c^2 d \sqrt{-c^2 d x^2 + d}} + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \arcsin(cx)}{d^2 c^2 (c^2 x^2 - 1)} + \frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \ln\left(\frac{icx + \sqrt{-c^2 x^2 + 1} + i}{icx + \sqrt{-c^2 x^2 + 1} - i}\right)}{d^2 c^2 (c^2 x^2 - 1)} - \frac{\sqrt{-d(c^2 x^2 - 1)}}{d^2 c^2 (c^2 x^2 - 1)} \right)$
parts	$\frac{a}{c^2 d \sqrt{-c^2 d x^2 + d}} + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \arcsin(cx)}{d^2 c^2 (c^2 x^2 - 1)} + \frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \ln\left(\frac{icx + \sqrt{-c^2 x^2 + 1} + i}{icx + \sqrt{-c^2 x^2 + 1} - i}\right)}{d^2 c^2 (c^2 x^2 - 1)} - \frac{\sqrt{-d(c^2 x^2 - 1)}}{d^2 c^2 (c^2 x^2 - 1)} \right)$

input `int(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output
$$\frac{a}{c^2 d} \sqrt{-c^2 d x^2 + d} + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \arcsin(cx)}{d^2 c^2 (c^2 x^2 - 1)} + \frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \ln\left(\frac{icx + \sqrt{-c^2 x^2 + 1} + i}{icx + \sqrt{-c^2 x^2 + 1} - i}\right)}{d^2 c^2 (c^2 x^2 - 1)} - \frac{\sqrt{-d(c^2 x^2 - 1)}}{d^2 c^2 (c^2 x^2 - 1)} \right)$$

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.82

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \left[\frac{(bc^2 x^2 - b)\sqrt{d} \log\left(-\frac{c^6 dx^6 + 5c^4 dx^4 - 5c^2 dx^2 + 4(c^3 x^3 + cx)\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}\sqrt{d-d}}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1}\right)}{4(c^4 d^2 x^2 - c^2 d^2)} - \frac{(bc^2 x^2 - b)\sqrt{-d} \arctan\left(\frac{2\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}c\sqrt{-dx}}{c^4 dx^4 - d}\right) + 2\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{2(c^4 d^2 x^2 - c^2 d^2)} \right]$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output

```
[1/4*((b*c^2*x^2 - b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a))/(c^4*d^2*x^2 - c^2*d^2), -1/2*((b*c^2*x^2 - b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) + 2*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a))/(c^4*d^2*x^2 - c^2*d^2)]
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input

```
integrate(x*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output

```
Integral(x*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input

```
integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output

```
(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^3*d^2*integrate(x^2/(c^4*d^2*x^4 - c^2*d^2*x^2 + (c^2*d^2*x^2 - d^2)*e^(log(c*x + 1) + log(-c*x + 1))), x) + arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*b/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2*d^(3/2)) + a/(sqrt(-c^2*d*x^2 + d)*c^2*d)
```


Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2),x)`

output `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) b c^2 + a}{\sqrt{d} \sqrt{-c^2 x^2 + 1} c^2 d}$$

input `int(x*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(- sqrt(- c**2*x**2 + 1)*int((asin(c*x)*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b*c**2 + a)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*c**2*d)`

3.122 $\int \frac{a+b \arcsin(cx)}{(d-c^2dx^2)^{3/2}} dx$

Optimal result	1149
Mathematica [A] (verified)	1149
Rubi [A] (verified)	1150
Maple [C] (verified)	1151
Fricas [F]	1152
Sympy [F]	1152
Maxima [A] (verification not implemented)	1152
Giac [F(-2)]	1153
Mupad [F(-1)]	1153
Reduce [F]	1153

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{a + b \arcsin(cx)}{(d - c^2dx^2)^{3/2}} dx = \frac{x(a + b \arcsin(cx))}{d\sqrt{d - c^2dx^2}} + \frac{b\sqrt{1 - c^2x^2} \log(1 - c^2x^2)}{2cd\sqrt{d - c^2dx^2}}$$

output

```
x*(a+b*arcsin(c*x))/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{a + b \arcsin(cx)}{(d - c^2dx^2)^{3/2}} dx = \frac{\sqrt{d - c^2dx^2}(2acx + 2bcx \arcsin(cx) + b\sqrt{1 - c^2x^2} \log(-1 + c^2x^2))}{2cd^2(-1 + c^2x^2)}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output

$$-1/2*(\text{Sqrt}[d - c^2*d*x^2]*(2*a*c*x + 2*b*c*x*\text{ArcSin}[c*x] + b*\text{Sqrt}[1 - c^2*x^2]*\text{Log}[-1 + c^2*x^2]))/(c*d^2*(-1 + c^2*x^2))$$
Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5160, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow \text{5160}$$

$$\frac{x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{x}{1 - c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{240}$$

$$\frac{x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} + \frac{b\sqrt{1 - c^2 x^2} \log(1 - c^2 x^2)}{2cd\sqrt{d - c^2 dx^2}}$$

input

$$\text{Int}[(a + b*\text{ArcSin}[c*x])/(d - c^2*d*x^2)^(3/2),x]$$

output

$$(x*(a + b*\text{ArcSin}[c*x]))/(d*\text{Sqrt}[d - c^2*d*x^2]) + (b*\text{Sqrt}[1 - c^2*x^2]*\text{Log}[1 - c^2*x^2])/(2*c*d*\text{Sqrt}[d - c^2*d*x^2])$$

Definitions of rubi rules used

rule 240 $\text{Int}[(x_+)/((a_) + (b_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^2, x]]/(2*b), x] /; \text{FreeQ}[\{a, b\}, x]$

rule 5160 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)]^{(n_)} / ((d_) + (e_)*(x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[x*((a + b*\text{ArcSin}[c*x])^n / (d*\text{Sqrt}[d + e*x^2])), x] - \text{Simp}[b*c*(n/d)*\text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] \text{Int}[x*((a + b*\text{ArcSin}[c*x])^{n-1} / (1 - c^2*x^2)), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

method	result
default	$\frac{ax}{d\sqrt{-c^2dx^2+d}} + \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\arcsin(cx)}{cd^2(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\arcsin(cx)x}{d^2(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\ln(1)}{cd^2(c^2x^2-1)}$
parts	$\frac{ax}{d\sqrt{-c^2dx^2+d}} + \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\arcsin(cx)}{cd^2(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\arcsin(cx)x}{d^2(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\ln(1)}{cd^2(c^2x^2-1)}$

input $\text{int}((a+b*\arcsin(c*x))/(-c^2*d*x^2+d)^{(3/2)}, x, \text{method}=_RETURNVERBOSE)$

output $a*x/d/(-c^2*d*x^2+d)^{(1/2)} + I*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/d^2/(c^2*x^2-1)*\arcsin(c*x) - b*(-d*(c^2*x^2-1))^{(1/2)}*\arcsin(c*x)/d^2/(c^2*x^2-1)*x - b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/d^2/(c^2*x^2-1)*\ln(1+(I*c*x+(-c^2*x^2+1)^{(1/2}))^2)$

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \arcsin(cx)}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx = \frac{bx \arcsin(cx)}{\sqrt{-c^2 dx^2 + dd}} + \frac{ax}{\sqrt{-c^2 dx^2 + dd}} - \frac{b \log(x^2 - \frac{1}{c^2})}{2cd^{\frac{3}{2}}}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `b*x*arcsin(c*x)/(sqrt(-c^2*d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/2*b*log(x^2 - 1/c^2)/(c*d^(3/2))`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((a + b*asin(c*x))/(d - c^2*d*x^2)^(3/2),x)`

output `int((a + b*asin(c*x))/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) b + ax}{\sqrt{d} \sqrt{-c^2 x^2 + 1} d}$$

input `int((a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(-sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**2*x**2 - sqrt(-c**2*x**2 + 1)),x)*b + a*x)/(sqrt(d)*sqrt(-c**2*x**2 + 1)*d)`

$$3.123 \quad \int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{3/2}} dx$$

Optimal result	1154
Mathematica [A] (verified)	1155
Rubi [A] (verified)	1155
Maple [A] (verified)	1158
Fricas [F]	1158
Sympy [F]	1159
Maxima [F]	1159
Giac [F(-2)]	1159
Mupad [F(-1)]	1160
Reduce [F]	1160

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{3/2}} dx = \frac{\frac{a}{d} + \frac{b \arcsin(cx)}{d}}{\sqrt{d-c^2 dx^2}} - \frac{2\sqrt{1-c^2 x^2}(a+b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{d\sqrt{d-c^2 dx^2}} - \frac{b\sqrt{1-c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2 dx^2}} + \frac{ib\sqrt{1-c^2 x^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d\sqrt{d-c^2 dx^2}} - \frac{ib\sqrt{1-c^2 x^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d\sqrt{d-c^2 dx^2}}$$

output

```
(a/d+b*arcsin(c*x)/d)/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/d/(-c^2*d*x^2+d)^(1/2)+I*b*(-c^2*x^2+1)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-I*b*(-c^2*x^2+1)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.34

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx = \frac{-\frac{a\sqrt{d-c^2 dx^2}}{-1+c^2 x^2} + a\sqrt{d} \log(x) - a\sqrt{d} \log\left(d + \sqrt{d}\sqrt{d - c^2 dx^2}\right) + \frac{bd(\arcsin(cx) + \sqrt{1-c^2 x^2} a}{x(d - c^2 dx^2)^{3/2}}}{1}$$

input `Integrate[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]`

output

```
(-((a*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)) + a*Sqrt[d]*Log[x] - a*Sqrt[d]*
Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*(ArcSin[c*x] + Sqrt[1 - c^2*x^
2]*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) - Sqrt[1 - c^2*x^2]*ArcSin[c*x]*
Log[1 + E^(I*ArcSin[c*x])] + Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Si
n[ArcSin[c*x]/2]] - Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[
c*x]/2]]) + I*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcSin[c*x])] - I*Sqrt[1 -
c^2*x^2]*PolyLog[2, E^(I*ArcSin[c*x])])/Sqrt[d - c^2*d*x^2])/d^2
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5208, 219, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow 5208$$

$$\frac{\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2 dx^2}} dx}{d} - \frac{bc\sqrt{1-c^2 x^2} \int \frac{1}{1-c^2 x^2} dx}{d\sqrt{d-c^2 dx^2}} + \frac{a + b \arcsin(cx)}{d\sqrt{d-c^2 dx^2}}$$

$$\downarrow 219$$

$$\frac{\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2 dx^2}} dx}{d} + \frac{a + b \arcsin(cx)}{d\sqrt{d-c^2 dx^2}} - \frac{b\sqrt{1-c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2 dx^2}}$$

$$\begin{aligned}
& \downarrow 5218 \\
& \frac{\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} \\
& \downarrow 3042 \\
& \frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \\
& \quad \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} \\
& \downarrow 4671 \\
& \frac{\sqrt{1-c^2x^2} (-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) d \arcsin(cx))}{d\sqrt{d-c^2dx^2}} \\
& \quad \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} \\
& \downarrow 2715 \\
& \frac{\sqrt{1-c^2x^2} (ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\
& \quad \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} \\
& \downarrow 2838 \\
& \frac{\sqrt{1-c^2x^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \\
& \quad \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output

```
(a + b*ArcSin[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2])
```

Definitions of rubi rules used

- rule 219 $\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$
- rule 2715 $\text{Int}[\text{Log}[(a_ + (b_ \cdot)((F_)^{(e_ \cdot)((c_ \cdot) + (d_ \cdot)(x_))})^{(n_ \cdot)}], x_Symbol] \rightarrow \text{Simp}[1/(d \cdot e \cdot n \cdot \text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x]/x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$
- rule 2838 $\text{Int}[\text{Log}[(c_ \cdot)((d_ + (e_ \cdot)(x_)^{(n_ \cdot)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n/n, x] /; \text{FreeQ}\{c, d, e, n\}, x \ \&\& \ \text{EqQ}[c \cdot d, 1]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4671 $\text{Int}[\text{csc}[(e_ \cdot) + (f_ \cdot)(x_)] \cdot ((c_ \cdot) + (d_ \cdot)(x_))^{(m_ \cdot)}, x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{(I \cdot (e + f \cdot x))}]/f), x] + (-\text{Simp}[d \cdot (m/f) \ \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 - E^{(I \cdot (e + f \cdot x))}], x], x] + \text{Simp}[d \cdot (m/f) \ \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 + E^{(I \cdot (e + f \cdot x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0]$
- rule 5208 $\text{Int}[(a_ \cdot + \text{ArcSin}[(c_ \cdot)(x_)] \cdot (b_ \cdot))^{(n_ \cdot)} \cdot ((f_ \cdot)(x_))^{(m_ \cdot)} \cdot ((d_ + (e_ \cdot)(x_)^2)^{(p_ \cdot)}, x_Symbol] \rightarrow \text{Simp}[(-f \cdot x)^{(m+1)} \cdot (d + e \cdot x^2)^{(p+1)} \cdot ((a + b \cdot \text{ArcSin}[c \cdot x])^n / (2 \cdot d \cdot f \cdot (p+1))), x] + (\text{Simp}[(m + 2 \cdot p + 3) / (2 \cdot d \cdot (p+1)) \ \text{Int}[(f \cdot x)^m \cdot (d + e \cdot x^2)^{(p+1)} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n], x] + \text{Simp}[b \cdot c \cdot (n / (2 \cdot f \cdot (p+1))) \cdot \text{Simp}[(d + e \cdot x^2)^p / (1 - c^2 \cdot x^2)^p] \ \text{Int}[(f \cdot x)^{(m+1)} \cdot (1 - c^2 \cdot x^2)^{(p+1/2)} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{EqQ}[c^2 \cdot d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !\text{GtQ}[m, 1] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{EqQ}[n, 1])$

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

method	result
default	$\frac{a}{d\sqrt{-c^2dx^2+d}} - \frac{a \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)} \arcsin(cx)}{d^2(c^2x^2-1)} + \frac{\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)} (\arcsin(cx) \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right) - \arcsin(cx))}{d^2(c^2x^2-1)}\right)$
parts	$\frac{a}{d\sqrt{-c^2dx^2+d}} - \frac{a \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}} + b\left(-\frac{\sqrt{-d(c^2x^2-1)} \arcsin(cx)}{d^2(c^2x^2-1)} + \frac{\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)} (\arcsin(cx) \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right) - \arcsin(cx))}{d^2(c^2x^2-1)}\right)$

input

```
int((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
a/d/(-c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))
/x)+b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arcsin(c*x)+(-c^2*x^2+1)^(1
/2)*(-d*(c^2*x^2-1))^(1/2)*(arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I
*arctan(I*c*x+(-c^2*x^2+1)^(1/2))-I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*di
log(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2dx^2 + d)^{\frac{3}{2}}x} dx$$

input

```
integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2
*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \arcsin(cx)}{x(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate((a+b*asin(c*x))/x/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{3/2} x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `-a*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(sqrt(-c^2*d*x^2 + d)*d)) - b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^2*d*x^3 - d*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x (d - c^2 dx^2)^{3/2}} dx$$

input `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^(3/2)),x)`

output `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^(3/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x (d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^3 - \sqrt{-c^2 x^2 + 1} x} dx \right) b + \sqrt{-c^2 x^2 + 1} \log \left(\tan \left(\frac{\operatorname{asin}(cx)}{2} \right) \right)}{\sqrt{d} \sqrt{-c^2 x^2 + 1} d}$$

input `int((a+b*asin(c*x))/x/(-c^2*d*x^2+d)^(3/2),x)`

output `(- sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**3 - sqrt(- c**2*x**2 + 1)*x),x)*b + sqrt(- c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a - sqrt(- c**2*x**2 + 1)*a + a)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*d)`

3.124
$$\int \frac{a+b \arcsin(cx)}{x^2(d-c^2dx^2)^{3/2}} dx$$

Optimal result	1161
Mathematica [A] (verified)	1161
Rubi [A] (verified)	1162
Maple [C] (verified)	1164
Fricas [F]	1164
Sympy [F]	1165
Maxima [A] (verification not implemented)	1165
Giac [F(-2)]	1166
Mupad [F(-1)]	1166
Reduce [F]	1166

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \frac{a + b \arcsin(cx)}{dx \sqrt{d - c^2 dx^2}} - \frac{2\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{d^2 x} + \frac{bc\sqrt{1 - c^2 x^2} \log(x)}{d\sqrt{d - c^2 dx^2}} + \frac{bc\sqrt{1 - c^2 x^2} \log(1 - c^2 x^2)}{2d\sqrt{d - c^2 dx^2}}$$

output

```
(a+b*arcsin(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d^2/x+b*c*(-c^2*x^2+1)^(1/2)*ln(x)/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*c*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \frac{\sqrt{d - c^2 dx^2}(-2a\sqrt{1 - c^2 x^2} + 4ac^2 x^2 \sqrt{1 - c^2 x^2} + 2b\sqrt{1 - c^2 x^2}(-1 + 2c^2 x^2) \arcsin(cx))}{2d^2 x (d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(-2*a*Sqrt[1 - c^2*x^2] + 4*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 2*b*Sqrt[1 - c^2*x^2]*(-1 + 2*c^2*x^2)*ArcSin[c*x] + b*c*x*(-1 + c^2*x^2)*Log[1 - 1/(c^2*x^2)] + 2*b*c*x*Log[1 - c^2*x^2] - 2*b*c^3*x^3*Log[1 - c^2*x^2]))/(2*d^2*x*(1 - c^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5194, 25, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{1-2c^2x^2}{d^2x(1-c^2x^2)} dx}{\sqrt{1 - c^2x^2}} + \frac{2c^2x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{dx\sqrt{d - c^2 dx^2}}$$

$$\downarrow 25$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \frac{1-2c^2x^2}{d^2x(1-c^2x^2)} dx}{\sqrt{1 - c^2x^2}} + \frac{2c^2x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{dx\sqrt{d - c^2 dx^2}}$$

$$\downarrow 27$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \frac{1-2c^2x^2}{x(1-c^2x^2)} dx}{d^2\sqrt{1 - c^2x^2}} + \frac{2c^2x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{dx\sqrt{d - c^2 dx^2}}$$

$$\downarrow 354$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \frac{1-2c^2x^2}{x^2(1-c^2x^2)} dx^2}{2d^2\sqrt{1 - c^2x^2}} + \frac{2c^2x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{dx\sqrt{d - c^2 dx^2}}$$

$$\downarrow 86$$

$$\frac{bc\sqrt{d - c^2 dx^2} \int \left(\frac{c^2}{c^2x^2-1} + \frac{1}{x^2}\right) dx^2}{2d^2\sqrt{1 - c^2x^2}} + \frac{2c^2x(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{dx\sqrt{d - c^2 dx^2}}$$

$$\downarrow 2009$$

$$\frac{2c^2x(a + b \arcsin(cx))}{d\sqrt{d - c^2dx^2}} - \frac{a + b \arcsin(cx)}{dx\sqrt{d - c^2dx^2}} + \frac{bc\sqrt{d - c^2dx^2}(\log(1 - c^2x^2) + \log(x^2))}{2d^2\sqrt{1 - c^2x^2}}$$

input `Int[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]`

output `-((a + b*ArcSin[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcSin[c*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(Log[x^2] + Log[1 - c^2*x^2]))/(2*d^2*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 86 `Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194

```
Int[((a._) + ArcSin[(c._)*(x_)])*(b._)*(x_)^(m_)*((d_) + (e._)*(x_)^2)^(p_)
, x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Simp
plifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

method	result
default	$a \left(-\frac{1}{dx\sqrt{-c^2dx^2+d}} + \frac{2c^2x}{d\sqrt{-c^2dx^2+d}} \right) + b \left(\frac{4i\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\arcsin(cx)c}{d^2(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}(2i\sqrt{-c^2x^2+1}cx)}{(c^2x^2-1)d^2} \right)$
parts	$a \left(-\frac{1}{dx\sqrt{-c^2dx^2+d}} + \frac{2c^2x}{d\sqrt{-c^2dx^2+d}} \right) + b \left(\frac{4i\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\arcsin(cx)c}{d^2(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}(2i\sqrt{-c^2x^2+1}cx)}{(c^2x^2-1)d^2} \right)$

input

```
int((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
a*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))+b*(4*I*(-d*
(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*arcsin(c*x)*c-(-d*(c
^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*arcsin(c*x)/(c^2
*x^2-1)/d^2/x-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*ln
((I*c*x+(-c^2*x^2+1)^(1/2))^4-1)*c)
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^2} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \arcsin(cx)}{x^2 (-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate((a+b*asin(c*x))/x**2/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\begin{aligned} \int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx &= \frac{1}{2} bc \left(\frac{\log(cx + 1)}{d^{3/2}} + \frac{\log(cx - 1)}{d^{3/2}} + \frac{2 \log(x)}{d^{3/2}} \right) \\ &+ \left(\frac{2c^2x}{\sqrt{-c^2dx^2 + dd}} - \frac{1}{\sqrt{-c^2dx^2 + dd}} \right) b \arcsin(cx) \\ &+ \left(\frac{2c^2x}{\sqrt{-c^2dx^2 + dd}} - \frac{1}{\sqrt{-c^2dx^2 + dd}} \right) a \end{aligned}$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `1/2*b*c*(log(c*x + 1)/d^(3/2) + log(c*x - 1)/d^(3/2) + 2*log(x)/d^(3/2)) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*x))*b*arcsin(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*x))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx$$

input `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)),x)`

output `int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^4 - \sqrt{-c^2 x^2 + 1} x^2} dx \right) b x + 2a c^2 x^2 - a}{\sqrt{d} \sqrt{-c^2 x^2 + 1} dx}$$

input `int((a+b*asin(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(- sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**4 - sqrt(- c**2*x**2 + 1)*x**2),x)*b*x + 2*a*c**2*x**2 - a)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*d*x)`

3.125 $\int \frac{a+b \arcsin(cx)}{x^3(d-c^2dx^2)^{3/2}} dx$

Optimal result	1167
Mathematica [A] (verified)	1168
Rubi [A] (verified)	1168
Maple [A] (verified)	1172
Fricas [F]	1173
Sympy [F]	1173
Maxima [F]	1174
Giac [F(-2)]	1174
Mupad [F(-1)]	1174
Reduce [F]	1175

Optimal result

Integrand size = 27, antiderivative size = 316

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = -\frac{bc\sqrt{1 - c^2x^2}}{2dx\sqrt{d - c^2dx^2}} + \frac{3c^2(a + b \arcsin(cx))}{2d\sqrt{d - c^2dx^2}} - \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2dx^2}} - \frac{3c^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{d\sqrt{d - c^2dx^2}} - \frac{bc^2\sqrt{1 - c^2x^2}\operatorname{arctanh}(cx)}{d\sqrt{d - c^2dx^2}} + \frac{3ibc^2\sqrt{1 - c^2x^2}\operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{2d\sqrt{d - c^2dx^2}} - \frac{3ibc^2\sqrt{1 - c^2x^2}\operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{2d\sqrt{d - c^2dx^2}}$$

output

```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d/x/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(a+b*arcsin(c*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsin(c*x))/d/x^2/(-c^2*d*x^2+d)^(1/2)-3*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-b*c^2*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/d/(-c^2*d*x^2+d)^(1/2)+3/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-3/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.28

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \frac{4a\sqrt{d}(-1+3c^2x^2)}{x^2\sqrt{d-c^2dx^2}} + 12ac^2 \log(x) - 12ac^2 \log\left(d + \sqrt{d}\sqrt{d - c^2dx^2}\right) + \frac{b\sqrt{d}(2 \arcsin(cx) - \dots)}{\dots}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]`

output

```
((4*a*sqrt[d]*(-1 + 3*c^2*x^2))/(x^2*sqrt[d - c^2*d*x^2]) + 12*a*c^2*Log[x] - 12*a*c^2*Log[d + sqrt[d]*sqrt[d - c^2*d*x^2]] + (b*sqrt[d]*(2*ArcSin[c*x] - 6*ArcSin[c*x]*Cos[2*ArcSin[c*x]] - 3*ArcSin[c*x]*Cos[3*ArcSin[c*x]])*Log[1 - E^(I*ArcSin[c*x])] + 3*ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 + E^(I*ArcSin[c*x])] - 2*cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] + sqrt[1 - c^2*x^2]*(3*ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x]]) - Log[1 + E^(I*ArcSin[c*x]])] + 2*(Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]) - Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2])) + 2*cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] - 2*Sin[2*ArcSin[c*x]] + (6*I)*c*x*PolyLog[2, -E^(I*ArcSin[c*x])] * Sin[2*ArcSin[c*x]] - (6*I)*c*x*PolyLog[2, E^(I*ArcSin[c*x])] * Sin[2*ArcSin[c*x]]))/(x^2*sqrt[d - c^2*d*x^2]))/(8*d^(3/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5204, 264, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx$$

↓ 5204

$$\begin{aligned}
& \frac{3}{2}c^2 \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{x^2(1 - c^2 x^2)} dx}{2d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{264} \\
& \frac{3}{2}c^2 \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \left(c^2 \int \frac{1}{1 - c^2 x^2} dx - \frac{1}{x} \right)}{2d\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{219} \\
& \frac{3}{2}c^2 \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx - \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} + \frac{bc\sqrt{1 - c^2 x^2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right)}{2d\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5208} \\
& \frac{3}{2}c^2 \left(\frac{\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx}{d} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{1 - c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{d\sqrt{d - c^2 dx^2}} \right) - \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} + \\
& \quad \frac{bc\sqrt{1 - c^2 x^2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right)}{2d\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{219} \\
& \frac{3}{2}c^2 \left(\frac{\int \frac{a + b \arcsin(cx)}{x\sqrt{d - c^2 dx^2}} dx}{d} + \frac{a + b \arcsin(cx)}{d\sqrt{d - c^2 dx^2}} - \frac{b\sqrt{1 - c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d - c^2 dx^2}} \right) - \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} + \\
& \quad \frac{bc\sqrt{1 - c^2 x^2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right)}{2d\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5218} \\
& \frac{3}{2}c^2 \left(\frac{\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx)}{d\sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{d\sqrt{d - c^2 dx^2}} - \frac{b\sqrt{1 - c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d - c^2 dx^2}} \right) - \\
& \quad \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} + \frac{bc\sqrt{1 - c^2 x^2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right)}{2d\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{3042} \\
& \frac{3}{2}c^2 \left(\frac{\sqrt{1 - c^2 x^2} \int (a + b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{d\sqrt{d - c^2 dx^2}} - \frac{b\sqrt{1 - c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d - c^2 dx^2}} \right) - \\
& \quad \frac{a + b \arcsin(cx)}{2dx^2\sqrt{d - c^2 dx^2}} + \frac{bc\sqrt{1 - c^2 x^2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right)}{2d\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{4671}
\end{aligned}$$

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2}(-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \right. \\ \left. + \frac{a+b \arcsin(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{1-c^2x^2}(\operatorname{arctanh}(cx) - \frac{1}{x})}{2d\sqrt{d-c^2dx^2}} \right)$$

↓ 2715

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2}(ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \right. \\ \left. + \frac{a+b \arcsin(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{1-c^2x^2}(\operatorname{arctanh}(cx) - \frac{1}{x})}{2d\sqrt{d-c^2dx^2}} \right)$$

↓ 2838

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \right. \\ \left. + \frac{a+b \arcsin(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{1-c^2x^2}(\operatorname{arctanh}(cx) - \frac{1}{x})}{2d\sqrt{d-c^2dx^2}} \right)$$

input `Int[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]`

output `-1/2*(a + b*ArcSin[c*x])/(d*x^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*(-x^(-1) + c*ArcTanh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) + (3*c^2*((a + b*ArcSin[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x]])))/(d*Sqrt[d - c^2*d*x^2]))) / 2`

Defintions of rubi rules used

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

- rule 264 $\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^2)^p, x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^2)^{p+1} / (a \cdot c \cdot (m+1)), x] - \text{Simp}[b \cdot (m+2 \cdot p+3) / (a \cdot c^2 \cdot (m+1)) \text{Int}[(c \cdot x)^{m+2} \cdot (a + b \cdot x^2)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
- rule 2715 $\text{Int}[\text{Log}[a + b \cdot x] \cdot (F)^{e \cdot (c + d \cdot x)^n}, x_Symbol] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F)^{e \cdot (c + d \cdot x)^n}], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
- rule 2838 $\text{Int}[\text{Log}[c \cdot (d + e \cdot x)^n] / (x), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n / n, x] / n, x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c \cdot d, 1]
- rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ FunctionOfTrigOfLinearQ[u, x]
- rule 4671 $\text{Int}[\text{csc}[e + f \cdot x] \cdot (c + d \cdot x)^m, x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{I \cdot (e + f \cdot x)}] / f), x] + (-\text{Simp}[d \cdot (m/f) \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 - E^{I \cdot (e + f \cdot x)}], x], x] + \text{Simp}[d \cdot (m/f) \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + E^{I \cdot (e + f \cdot x)}], x], x]) /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
- rule 5204 $\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x_Symbol] \rightarrow \text{Simp}[(f \cdot x)^{m+1} \cdot (d + e \cdot x^2)^{p+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n / (d \cdot f \cdot (m+1)), x] + (\text{Simp}[c^2 \cdot ((m+2 \cdot p+3) / (f^2 \cdot (m+1))) \text{Int}[(f \cdot x)^{m+2} \cdot (d + e \cdot x^2)^p \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n, x], x] - \text{Simp}[b \cdot c \cdot (n / (f \cdot (m+1))) \cdot \text{Simp}[(d + e \cdot x^2)^p / (1 - c^2 \cdot x^2)^p] \text{Int}[(f \cdot x)^{m+1} \cdot (1 - c^2 \cdot x^2)^{p+1/2} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n-1}, x], x]) /;$ FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2 \cdot d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int((((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

method	result
default	$a \left(-\frac{1}{2dx^2\sqrt{-c^2dx^2+d}} + \frac{3c^2 \left(\frac{1}{d\sqrt{-c^2dx^2+d}} - \frac{\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2} \right) - \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} (3i \arcsin(cx))}{1}$
parts	$a \left(-\frac{1}{2dx^2\sqrt{-c^2dx^2+d}} + \frac{3c^2 \left(\frac{1}{d\sqrt{-c^2dx^2+d}} - \frac{\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2} \right) - \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} (3i \arcsin(cx))}{1}$

input

```
int((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
a*(-1/2/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(1/d/(-c^2*d*x^2+d)^(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(3*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^4*c^4+4*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*I*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^2*x^2-3*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^2*c^2+I*x^3*c^3-4*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-3*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-3*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-I*(-c^2*x^2+1)^(1/2)*arcsin(c*x)-I*c*x)/d^2/(c^4*x^4-2*c^2*x^2+1)/x^2
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^3} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input

```
integrate((a+b*asin(c*x))/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

output

```
Integral((a + b*asin(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `-1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a - b *integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^2*d*x^5 - d*x^3) *sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx$$

input `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)),x)`

output `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{3/2}} dx = \frac{-8\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^5 - \sqrt{-c^2 x^2 + 1} x^3} dx \right) b x^2 + 12\sqrt{-c^2 x^2 + 1} \log\left(\tan\left(\frac{\arcsin(cx)}{2}\right)\right) a c^2 x^2 - 9\sqrt{-c^2 x^2 + 1} a c^2 x^2 + 12 a c^2 x^2 - 4a}{8\sqrt{d} \sqrt{-c^2 x^2 + 1} d x^2}$$

input `int((a+b*asin(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 8*sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**5 - sqrt(- c**2*x**2 + 1)*x**3),x)*b*x**2 + 12*sqrt(- c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 9*sqrt(- c**2*x**2 + 1)*a*c**2*x**2 + 12*a*c**2*x**2 - 4*a)/(8*sqrt(d)*sqrt(- c**2*x**2 + 1)*d*x**2)`

3.126 $\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)^{3/2}} dx$

Optimal result	1176
Mathematica [A] (verified)	1177
Rubi [A] (verified)	1177
Maple [C] (verified)	1179
Fricas [F]	1180
Sympy [F]	1181
Maxima [F]	1181
Giac [F(-2)]	1181
Mupad [F(-1)]	1182
Reduce [F]	1182

Optimal result

Integrand size = 27, antiderivative size = 234

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = -\frac{bc\sqrt{1 - c^2x^2}}{6dx^2\sqrt{d - c^2dx^2}} + \frac{a + b \arcsin(cx)}{dx^3\sqrt{d - c^2dx^2}}$$

$$- \frac{4\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{3d^2x^3} - \frac{8c^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{3d^2x}$$

$$+ \frac{5bc^3\sqrt{1 - c^2x^2} \log(x)}{3d\sqrt{d - c^2dx^2}} + \frac{bc^3\sqrt{1 - c^2x^2} \log(1 - c^2x^2)}{2d\sqrt{d - c^2dx^2}}$$

output

```
-1/6*b*c*(-c^2*x^2+1)^(1/2)/d/x^2/(-c^2*d*x^2+d)^(1/2)+(a+b*arcsin(c*x))/d
/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d^2/x
^3-8/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d^2/x+5/3*b*c^3*(-c^2*x^
2+1)^(1/2)*ln(x)/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*c^3*(-c^2*x^2+1)^(1/2)*ln(-c
^2*x^2+1)/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \frac{\sqrt{d - c^2 dx^2} (bcx - bc^3 x^3 + 2a\sqrt{1 - c^2 x^2} + 8ac^2 x^2 \sqrt{1 - c^2 x^2} - 16ac^4 x^4 \sqrt{1 - c^2 x^2} - 2b\sqrt{1 - c^2 x^2}(-1 - c^2 x^2))}{6d^2 x^3 (d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output

```
-1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x - b*c^3*x^3 + 2*a*Sqrt[1 - c^2*x^2] + 8*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 16*a*c^4*x^4*Sqrt[1 - c^2*x^2] - 2*b*Sqrt[1 - c^2*x^2]*(-1 - 4*c^2*x^2 + 8*c^4*x^4)*ArcSin[c*x] - 5*b*c^3*x^3*(-1 + c^2*x^2)*Log[1 - 1/(c^2*x^2)] - 8*b*c^3*x^3*Log[1 - c^2*x^2] + 8*b*c^5*x^5*Log[1 - c^2*x^2]))/(d^2*x^3*(1 - c^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx$$

↓ 5194

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{8c^4 x^4 + 4c^2 x^2 + 1}{3d^2 x^3 (1 - c^2 x^2)} dx}{\sqrt{1 - c^2 x^2}} - \frac{4c^2 (a + b \arcsin(cx))}{3dx\sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{3dx^3\sqrt{d - c^2 dx^2}} + \frac{8c^4 x (a + b \arcsin(cx))}{3d\sqrt{d - c^2 dx^2}}$$

↓ 27

$$\begin{aligned}
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{-8c^4x^4+4c^2x^2+1}{x^3(1-c^2x^2)} dx}{3d^2\sqrt{1-c^2x^2}} - \frac{4c^2(a+b\arcsin(cx))}{3dx\sqrt{d-c^2dx^2}} - \frac{a+b\arcsin(cx)}{3dx^3\sqrt{d-c^2dx^2}} + \\
& \quad \frac{8c^4x(a+b\arcsin(cx))}{3d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow 1578 \\
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{-8c^4x^4+4c^2x^2+1}{x^4(1-c^2x^2)} dx^2}{6d^2\sqrt{1-c^2x^2}} - \frac{4c^2(a+b\arcsin(cx))}{3dx\sqrt{d-c^2dx^2}} - \frac{a+b\arcsin(cx)}{3dx^3\sqrt{d-c^2dx^2}} + \\
& \quad \frac{8c^4x(a+b\arcsin(cx))}{3d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow 1195 \\
& \frac{bc\sqrt{d-c^2dx^2} \int \left(\frac{3c^4}{c^2x^2-1} + \frac{5c^2}{x^2} + \frac{1}{x^4}\right) dx^2}{6d^2\sqrt{1-c^2x^2}} - \frac{4c^2(a+b\arcsin(cx))}{3dx\sqrt{d-c^2dx^2}} - \frac{a+b\arcsin(cx)}{3dx^3\sqrt{d-c^2dx^2}} + \\
& \quad \frac{8c^4x(a+b\arcsin(cx))}{3d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow 2009 \\
& -\frac{4c^2(a+b\arcsin(cx))}{3dx\sqrt{d-c^2dx^2}} - \frac{a+b\arcsin(cx)}{3dx^3\sqrt{d-c^2dx^2}} + \frac{8c^4x(a+b\arcsin(cx))}{3d\sqrt{d-c^2dx^2}} + \\
& \quad \frac{bc\sqrt{d-c^2dx^2}(5c^2\log(x^2) + 3c^2\log(1-c^2x^2) - \frac{1}{x^2})}{6d^2\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]`

output `-1/3*(a + b*ArcSin[c*x])/(d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcSin[c*x]))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcSin[c*x]))/(3*d*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(-x^(-2) + 5*c^2*Log[x^2] + 3*c^2*Log[1 - c^2*x^2]))/(6*d^2*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1195 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 1578 `Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 1048, normalized size of antiderivative = 4.48

method	result	size
default	Expression too large to display	1048
parts	Expression too large to display	1048

input `int((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output

```

a*(-1/3/d/x^3/(-c^2*d*x^2+d)^(1/2)+4/3*c^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*
c^2/d*x/(-c^2*d*x^2+d)^(1/2)))+4/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7
*c^2*x^2-1)/d^2*x*c^4-16/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2
-1)/d^2*x^3*(-c^2*x^2+1)*c^6-8/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c
^2*x^2-1)/d^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^3+32/3*I*b*(-d*(c^2*x^2-1))
^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^5*(-c^2*x^2+1)*c^8+4*I*b*(-d*(c^2*x^2
-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*c^6+32/3*I*b*(-d*(c^2*x^2-1))^(
1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^7*c^10+16/3*I*b*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*arcsin(c*x)*c^3-64/3*b*(-d*(c^2*x^2-1))
^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*arcsin(c*x)*c^6-4/3*I*b*(-d*(c^2*x^
2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*(-c^2*x^2+1)*c^4-16*I*b*(-d*(c^2
*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^5*c^8-64/3*I*b*(-d*(c^2*x^2-1
))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^
5+8*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*arcsin(c*x)*c^4
+4/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*c^3*(-c^2*x^2+1)
^(1/2)+4*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x*arcsin(c*x
)*c^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^2*(-c^2*x
^2+1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^3
*arcsin(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*l
n(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)*c^3-5/3*b*(-d*(c^2*x^2-1))^(1/2)*(-c^...

```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{3/2} x^4} dx$$

input

```

integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)

```

output

```

integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2
*x^6 + d^2*x^4), x)

```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))/x**4/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^4} dx$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) - 1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a - b*integrate(arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/((c^2*d*x^6 - d*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx$$

input

```
int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

output

```
int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}} dx = \frac{-3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^6 - \sqrt{-c^2 x^2 + 1} x^4} dx \right) b x^3 + 8a c^4 x^4 - 4a c^2 x^2 - a}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} d x^3}$$

input

```
int((a+b*asin(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
( - 3*sqrt( - c**2*x**2 + 1)*int(asin(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*6 - sqrt( - c**2*x**2 + 1)*x**4),x)*b*x**3 + 8*a*c**4*x**4 - 4*a*c**2*x**
2 - a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**3)
```

3.127
$$\int \frac{x^6(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1183
Mathematica [A] (verified)	1184
Rubi [A] (verified)	1184
Maple [C] (verified)	1188
Fricas [F]	1189
Sympy [F]	1189
Maxima [F]	1190
Giac [F(-2)]	1190
Mupad [F(-1)]	1191
Reduce [F]	1191

Optimal result

Integrand size = 27, antiderivative size = 293

$$\int \frac{x^6(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx = -\frac{b}{6c^7d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{1-c^2x^2}}{4c^5d^2\sqrt{d-c^2dx^2}}$$

$$+ \frac{x^5(a+b \arcsin(cx))}{3c^2d(d-c^2dx^2)^{3/2}} - \frac{5x^3(a+b \arcsin(cx))}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{5x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{2c^6d^3}$$

$$+ \frac{5\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{4bc^7d^2\sqrt{d-c^2dx^2}} - \frac{7b\sqrt{1-c^2x^2} \log(1-c^2x^2)}{6c^7d^2\sqrt{d-c^2dx^2}}$$

output

```
-1/6*b/c^7/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/4*b*x^2*(-c^2*x^2+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*x^5*(a+b*arcsin(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)-5/3*x^3*(a+b*arcsin(c*x))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^6/d^3+5/4*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/b/c^7/d^2/(-c^2*d*x^2+d)^(1/2)-7/6*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c^7/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.86

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{4bc\sqrt{d}x(15 - 20c^2x^2 + 3c^4x^4) \arcsin(cx) - 30b\sqrt{d}(1 - c^2x^2)^{3/2} \arcsin(cx)^2 -$$

input `Integrate[(x^6*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2), x]`

output $(4*b*c*\text{Sqrt}[d]*x*(15 - 20*c^2*x^2 + 3*c^4*x^4)*\text{ArcSin}[c*x] - 30*b*\text{Sqrt}[d]*(1 - c^2*x^2)^{(3/2)}*\text{ArcSin}[c*x]^2 - 60*a*(-1 + c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2]*\text{ArcTan}[(c*x*\text{Sqrt}[d - c^2*d*x^2])/(\text{Sqrt}[d]*(-1 + c^2*x^2))] + \text{Sqrt}[d]*(4*a*c*x*(15 - 20*c^2*x^2 + 3*c^4*x^4) + b*\text{Sqrt}[1 - c^2*x^2]*(7 - 9*c^2*x^2 + 6*c^4*x^4) + 28*b*(1 - c^2*x^2)^{(3/2)}*\text{Log}[1 - c^2*x^2])/(24*c^7*d^{(5/2)}*(-1 + c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2])$

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5206, 243, 49, 2009, 5206, 243, 49, 2009, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx \\ & \quad \downarrow \text{5206} \\ & -\frac{5 \int \frac{x^4(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{3c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x^5}{(1-c^2 x^2)^2} dx}{3cd^2 \sqrt{d-c^2 dx^2}} + \frac{x^5(a + b \arcsin(cx))}{3c^2 d (d - c^2 dx^2)^{3/2}} \\ & \quad \downarrow \text{243} \\ & -\frac{5 \int \frac{x^4(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{3c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x^4}{(1-c^2 x^2)^2} dx^2}{6cd^2 \sqrt{d-c^2 dx^2}} + \frac{x^5(a + b \arcsin(cx))}{3c^2 d (d - c^2 dx^2)^{3/2}} \end{aligned}$$

$$\begin{aligned}
& \downarrow 49 \\
& \frac{5 \int \frac{x^4(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{3c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \left(\frac{1}{c^4} + \frac{2}{c^4(c^2 x^2-1)} + \frac{1}{c^4(c^2 x^2-1)^2} \right) dx^2}{6cd^2 \sqrt{d-c^2 dx^2}} + \\
& \frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} \\
& \downarrow 2009 \\
& \frac{5 \int \frac{x^4(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{3c^2 d} + \frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \frac{b\sqrt{1-c^2 x^2} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}{6cd^2 \sqrt{d-c^2 dx^2}} \\
& \downarrow 5206 \\
& \frac{5 \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x^3}{1-c^2 x^2} dx}{cd\sqrt{d-c^2 dx^2}} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} \right)}{3c^2 d} + \frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \\
& \frac{b\sqrt{1-c^2 x^2} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}{6cd^2 \sqrt{d-c^2 dx^2}} \\
& \downarrow 243 \\
& \frac{5 \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x^2}{1-c^2 x^2} dx^2}{2cd\sqrt{d-c^2 dx^2}} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} \right)}{3c^2 d} + \frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \\
& \frac{b\sqrt{1-c^2 x^2} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}{6cd^2 \sqrt{d-c^2 dx^2}} \\
& \downarrow 49 \\
& \frac{5 \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \left(-\frac{1}{c^2} - \frac{1}{c^2(c^2 x^2-1)} \right) dx^2}{2cd\sqrt{d-c^2 dx^2}} + \frac{x^3(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} \right)}{3c^2 d} + \\
& \frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \frac{b\sqrt{1-c^2 x^2} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}{6cd^2 \sqrt{d-c^2 dx^2}} \\
& \downarrow 2009
\end{aligned}$$

$$\frac{5 \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx)) dx}{\sqrt{d-c^2 dx^2}}}{c^2 d} + \frac{x^3(a+b \arcsin(cx))}{c^2 d \sqrt{d-c^2 dx^2}} - \frac{b \sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4} \right)}{2cd \sqrt{d-c^2 dx^2}} \right)}{\frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \frac{3c^2 d}{6cd^2 \sqrt{d-c^2 dx^2}} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)} +$$

5210

$$\frac{5 \left(-\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} + \frac{b \sqrt{1-c^2 x^2} \int x dx}{2c \sqrt{d-c^2 dx^2}} - \frac{x \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{2c^2 d} \right)}{c^2 d} + \frac{x^3(a+b \arcsin(cx))}{c^2 d \sqrt{d-c^2 dx^2}} - \frac{b \sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4} \right)}{2cd \sqrt{d-c^2 dx^2}} \right)}{\frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \frac{3c^2 d}{6cd^2 \sqrt{d-c^2 dx^2}} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}$$

15

$$\frac{5 \left(-\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{2c^2} - \frac{x \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{2c^2 d} + \frac{bx^2 \sqrt{1-c^2 x^2}}{4c \sqrt{d-c^2 dx^2}} \right)}{c^2 d} + \frac{x^3(a+b \arcsin(cx))}{c^2 d \sqrt{d-c^2 dx^2}} - \frac{b \sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4} \right)}{2cd \sqrt{d-c^2 dx^2}} \right)}{\frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \frac{3c^2 d}{6cd^2 \sqrt{d-c^2 dx^2}} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}$$

5152

$$\frac{x^5(a+b \arcsin(cx))}{3c^2 d (d-c^2 dx^2)^{3/2}} - \frac{5 \left(\frac{x^3(a+b \arcsin(cx))}{c^2 d \sqrt{d-c^2 dx^2}} - \frac{3 \left(-\frac{x \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{2c^2 d} + \frac{\sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{4bc^3 \sqrt{d-c^2 dx^2}} + \frac{bx^2 \sqrt{1-c^2 x^2}}{4c \sqrt{d-c^2 dx^2}} \right)}{c^2 d} - \frac{b \sqrt{1-c^2 x^2} \left(-\frac{x^2}{c^2} - \frac{\log(1-c^2 x^2)}{c^4} \right)}{2cd \sqrt{d-c^2 dx^2}} \right)}{\frac{b \sqrt{1-c^2 x^2} \left(\frac{x^2}{c^4} + \frac{1}{c^6(1-c^2 x^2)} + \frac{2 \log(1-c^2 x^2)}{c^6} \right)}{6cd^2 \sqrt{d-c^2 dx^2}}}$$

input `Int[(x^6*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]`

output

$$\begin{aligned} & (x^5(a + b\text{ArcSin}[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^{(3/2)}) - (b*\text{Sqrt}[1 - c^2*x^2]*(x^2/c^4 + 1/(c^6*(1 - c^2*x^2)) + (2*\text{Log}[1 - c^2*x^2])/c^6))/(6*c*d^2*\text{Sqrt}[d - c^2*d*x^2]) - (5*(x^3*(a + b\text{ArcSin}[c*x]))/(c^2*d*\text{Sqrt}[d - c^2*d*x^2]) - (3*((b*x^2*\text{Sqrt}[1 - c^2*x^2])/(4*c*\text{Sqrt}[d - c^2*d*x^2]) - (x*\text{Sqrt}[d - c^2*d*x^2]*(a + b\text{ArcSin}[c*x]))/(2*c^2*d) + (\text{Sqrt}[1 - c^2*x^2]*(a + b\text{ArcSin}[c*x])^2)/(4*b*c^3*\text{Sqrt}[d - c^2*d*x^2])))/(c^2*d) - (b*\text{Sqrt}[1 - c^2*x^2]*(-x^2/c^2) - \text{Log}[1 - c^2*x^2]/c^4))/(2*c*d*\text{Sqrt}[d - c^2*d*x^2]))/(3*c^2*d) \end{aligned}$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_.)*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 49

$$\text{Int}[((a_.) + (b_.)*(x_)^{(m_.)})*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$$

rule 243

$$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m-1)/2}*(a + b*x)^p, x], x, x^2], x] \text{ ; FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5152

$$\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^{(n_.)}/\text{Sqrt}[(d_) + (e_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.45

method	result
default	$-\frac{ax^5}{2c^2d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{5ax^3}{6c^4d(-c^2dx^2+d)^{\frac{3}{2}}} - \frac{5ax}{2c^6d^2\sqrt{-c^2dx^2+d}} + \frac{5a \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2c^6d^2\sqrt{c^2d}} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2-d}}{2c^6d^2\sqrt{c^2d}}$
parts	$-\frac{ax^5}{2c^2d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{5ax^3}{6c^4d(-c^2dx^2+d)^{\frac{3}{2}}} - \frac{5ax}{2c^6d^2\sqrt{-c^2dx^2+d}} + \frac{5a \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2c^6d^2\sqrt{c^2d}} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2-d}}{2c^6d^2\sqrt{c^2d}}$

input

```
int(x^6*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2), x, method=_RETURNVERBOSE)
```

output

```
-1/2*a*x^5/c^2/d/(-c^2*d*x^2+d)^(3/2)+5/6*a/c^4*x^3/d/(-c^2*d*x^2+d)^(3/2)
-5/2*a/c^6/d^2*x/(-c^2*d*x^2+d)^(1/2)+5/2*a/c^6/d^2/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/24*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x
^2+1)^(1/2)*(-12*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^5*c^5+6*c^6*x^6+30*arcsi
n(c*x)^2*x^4*c^4+56*I*arcsin(c*x)*x^4*c^4-56*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2)
))^2)*x^4*c^4+80*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^3*c^3-15*c^4*x^4-60*arcs
in(c*x)^2*x^2*c^2-112*I*arcsin(c*x)*x^2*c^2+112*ln(1+(I*c*x+(-c^2*x^2+1)^(
1/2))^2)*x^2*c^2-60*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+16*c^2*x^2+30*arcsi
n(c*x)^2+56*I*arcsin(c*x)-56*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-7)/d^3/(c^
6*x^6-3*c^4*x^4+3*c^2*x^2-1)/c^7
```

Fricas [F]

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^6}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```
integrate(x^6*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output

```
integral(-(b*x^6*arcsin(c*x) + a*x^6)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^6(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input

```
integrate(x**6*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output

```
Integral(x**6*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^6}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^6*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `-1/6*a*(3*x^5/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 5*x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))/c^2 + 5*x/(sqrt(-c^2*d*x^2 + d)*c^6*d^2) - 15*arcsin(c*x)/(c^7*d^(5/2))) + b*integrate(x^6*arctan(2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^6*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^6(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x^6*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)`

output `int((x^6*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x^6(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{15\sqrt{-c^2x^2 + 1} \operatorname{asin}(cx) a c^2 x^2 - 15\sqrt{-c^2x^2 + 1} \operatorname{asin}(cx) a + 6\sqrt{-c^2x^2 + 1}}{(d - c^2 dx^2)^{5/2}}$$

input `int(x^6*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `(15*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*c**2*x**2 - 15*sqrt(-c**2*x**2 + 1)*asin(c*x)*a + 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**6)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b*c**9*x**2 - 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**6)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b*c**7 + 3*a*c**5*x**5 - 20*a*c**3*x**3 + 15*a*c*x)/(6*sqrt(d)*sqrt(-c**2*x**2 + 1)*c**7*d**2*(c**2*x**2 - 1))`

3.128
$$\int \frac{x^5(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1192
Mathematica [C] (verified)	1193
Rubi [A] (verified)	1193
Maple [C] (verified)	1196
Fricas [A] (verification not implemented)	1196
Sympy [F]	1197
Maxima [F]	1197
Giac [F(-2)]	1198
Mupad [F(-1)]	1198
Reduce [F]	1199

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{x^5(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx = -\frac{bx}{6c^5d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{1-c^2x^2}}{c^5d^2\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{3c^6d(d-c^2dx^2)^{3/2}} - \frac{2(a+b \arcsin(cx))}{c^6d^2\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{c^6d^3} + \frac{11b\sqrt{1-c^2x^2}\operatorname{arctanh}(cx)}{6c^6d^2\sqrt{d-c^2dx^2}}$$

output

```
-1/6*b*x/c^5/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+b*x*(-c^2*x^2+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x))/c^6/d/(-c^2*d*x^2+d)^(3/2)-2*(a+b*arcsin(c*x))/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^6/d^3+11/6*b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.77

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{\sqrt{d - c^2 dx^2} \left(\sqrt{-c^2} (bcx \sqrt{1 - c^2 x^2} (-5 + 6c^2 x^2) + 2a(8 - 12c^2 x^2 + 3c^4 x^4) + 2bx(8 - 12c^2 x^2 + 3c^4 x^4) \arcsin(cx)) + (11I) * b * c * (1 - c^2 x^2)^{3/2} * \text{EllipticF}[I * \text{ArcSinh}[\text{Sqrt}[-c^2] * x], 1] \right)}{6c^4 (-c^2 dx^2)^{3/2}}$$

```
input Integrate[(x^5*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2), x]
```

```
output (Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(b*c*x*Sqrt[1 - c^2*x^2]*(-5 + 6*c^2*x^2) + 2*a*(8 - 12*c^2*x^2 + 3*c^4*x^4) + 2*b*(8 - 12*c^2*x^2 + 3*c^4*x^4)*ArcSin[c*x]) + (11*I)*b*c*(1 - c^2*x^2)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]))/(6*c^4*(-c^2)^(3/2)*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5194, 27, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

↓ 5194

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{3c^4 x^4 - 12c^2 x^2 + 8}{3c^6 d^3 (1 - c^2 x^2)^2} dx}{\sqrt{1 - c^2 x^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{c^6 d^3} - \frac{2(a + b \arcsin(cx))}{c^6 d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3c^6 d (d - c^2 dx^2)^{3/2}}$$

↓ 27

$$\begin{aligned}
 & \frac{b\sqrt{d-c^2dx^2} \int \frac{3c^4x^4-12c^2x^2+8}{(1-c^2x^2)^2} dx}{3c^5d^3\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^3} - \frac{2(a+b\arcsin(cx))}{c^6d^2\sqrt{d-c^2dx^2}} + \\
 & \qquad \qquad \qquad \frac{a+b\arcsin(cx)}{3c^6d(d-c^2dx^2)^{3/2}} \\
 & \qquad \qquad \qquad \downarrow 1471 \\
 & \frac{b\sqrt{d-c^2dx^2} \left(-\frac{1}{2} \int -\frac{17-6c^2x^2}{1-c^2x^2} dx - \frac{x}{2(1-c^2x^2)} \right)}{3c^5d^3\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^3} - \\
 & \qquad \qquad \qquad \frac{2(a+b\arcsin(cx))}{c^6d^2\sqrt{d-c^2dx^2}} + \frac{a+b\arcsin(cx)}{3c^6d(d-c^2dx^2)^{3/2}} \\
 & \qquad \qquad \qquad \downarrow 25 \\
 & \frac{b\sqrt{d-c^2dx^2} \left(\frac{1}{2} \int \frac{17-6c^2x^2}{1-c^2x^2} dx - \frac{x}{2(1-c^2x^2)} \right)}{3c^5d^3\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^3} - \\
 & \qquad \qquad \qquad \frac{2(a+b\arcsin(cx))}{c^6d^2\sqrt{d-c^2dx^2}} + \frac{a+b\arcsin(cx)}{3c^6d(d-c^2dx^2)^{3/2}} \\
 & \qquad \qquad \qquad \downarrow 299 \\
 & \frac{b\sqrt{d-c^2dx^2} \left(\frac{1}{2} \left(11 \int \frac{1}{1-c^2x^2} dx + 6x \right) - \frac{x}{2(1-c^2x^2)} \right)}{3c^5d^3\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^3} - \\
 & \qquad \qquad \qquad \frac{2(a+b\arcsin(cx))}{c^6d^2\sqrt{d-c^2dx^2}} + \frac{a+b\arcsin(cx)}{3c^6d(d-c^2dx^2)^{3/2}} \\
 & \qquad \qquad \qquad \downarrow 219 \\
 & -\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{c^6d^3} - \frac{2(a+b\arcsin(cx))}{c^6d^2\sqrt{d-c^2dx^2}} + \frac{a+b\arcsin(cx)}{3c^6d(d-c^2dx^2)^{3/2}} + \\
 & \qquad \qquad \qquad \frac{b \left(\frac{1}{2} \left(\frac{11\operatorname{arctanh}(cx)}{c} + 6x \right) - \frac{x}{2(1-c^2x^2)} \right) \sqrt{d-c^2dx^2}}{3c^5d^3\sqrt{1-c^2x^2}}
 \end{aligned}$$

input `Int[(x^5*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]`

output `(a + b*ArcSin[c*x])/(3*c^6*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcSin[c*x]))/(c^6*d^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(c^6*d^3) + (b*Sqrt[d - c^2*d*x^2]*(-1/2*x/(1 - c^2*x^2) + (6*x + (11*ArcTanh[c*x])/c)/2))/(3*c^5*d^3*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

- rule 25 $\text{Int}[-(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[\text{Fx}, \text{x}], \text{x}]$
- rule 27 $\text{Int}[(\text{a}_)*(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{a} \quad \text{Int}[\text{Fx}, \text{x}], \text{x}] \text{ ; FreeQ}[\text{a}, \text{x}] \ \&\& \ \text{!MatchQ}[\text{Fx}, (\text{b}_)*(\text{Gx}_)] \text{ ; FreeQ}[\text{b}, \text{x}]$
- rule 219 $\text{Int}[(\text{a}_) + (\text{b}_.)*(x_)^2)^{-1}, \text{x_Symbol}] \rightarrow \text{Simp}[(1/(\text{Rt}[\text{a}, 2]*\text{Rt}[-\text{b}, 2]))*\text{ArcTanh}[\text{Rt}[-\text{b}, 2]*(\text{x}/\text{Rt}[\text{a}, 2])], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}\}, \text{x}] \ \&\& \ \text{NegQ}[\text{a}/\text{b}] \ \&\& \ (\text{GtQ}[\text{a}, 0] \ || \ \text{LtQ}[\text{b}, 0])$
- rule 299 $\text{Int}[(\text{a}_) + (\text{b}_.)*(x_)^2)^{(\text{p}_)}*(\text{c}_) + (\text{d}_.)*(x_)^2), \text{x_Symbol}] \rightarrow \text{Simp}[\text{d}*x*((\text{a} + \text{b}*x^2)^{(\text{p} + 1)}/(\text{b}*(2*\text{p} + 3))), \text{x}] - \text{Simp}[(\text{a}*d - \text{b}*c*(2*\text{p} + 3))/(\text{b}*(2*\text{p} + 3)) \quad \text{Int}[(\text{a} + \text{b}*x^2)^{\text{p}}, \text{x}], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{b}*c - \text{a}*d, 0] \ \&\& \ \text{NeQ}[2*\text{p} + 3, 0]$
- rule 1471 $\text{Int}[(\text{d}_) + (\text{e}_.)*(x_)^2)^{(\text{q}_)}*(\text{a}_) + (\text{b}_.)*(x_)^2 + (\text{c}_.)*(x_)^4)^{(\text{p}_)}, \text{x_Symbol}] \rightarrow \text{With}[\{\text{Qx} = \text{PolynomialQuotient}[(\text{a} + \text{b}*x^2 + \text{c}*x^4)^{\text{p}}, \text{d} + \text{e}*x^2, \text{x}], \text{R} = \text{Coeff}[\text{PolynomialRemainder}[(\text{a} + \text{b}*x^2 + \text{c}*x^4)^{\text{p}}, \text{d} + \text{e}*x^2, \text{x}], \text{x}, 0]\}, \text{Simp}[(-\text{R})*x*((\text{d} + \text{e}*x^2)^{(\text{q} + 1)}/(2*d*(\text{q} + 1))), \text{x}] + \text{Simp}[1/(2*d*(\text{q} + 1)) \quad \text{Int}[(\text{d} + \text{e}*x^2)^{(\text{q} + 1)}*\text{ExpandToSum}[2*d*(\text{q} + 1)*\text{Qx} + \text{R}*(2*\text{q} + 3), \text{x}], \text{x}], \text{x}]] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{b}^2 - 4*\text{a}*c, 0] \ \&\& \ \text{NeQ}[\text{c}*d^2 - \text{b}*d*\text{e} + \text{a}*e^2, 0] \ \&\& \ \text{IGtQ}[\text{p}, 0] \ \&\& \ \text{LtQ}[\text{q}, -1]$
- rule 5194 $\text{Int}[(\text{a}_.) + \text{ArcSin}[(\text{c}_.)*(x_)]*(\text{b}_.))*(\text{x}_)^{(\text{m}_)}*(\text{d}_) + (\text{e}_.)*(x_)^2)^{(\text{p}_)}, \text{x_Symbol}] \rightarrow \text{With}[\{\text{u} = \text{IntHide}[x^m*(\text{d} + \text{e}*x^2)^{\text{p}}, \text{x}]\}, \text{Simp}[(\text{a} + \text{b}*\text{ArcSin}[\text{c}*x]) \quad \text{u}, \text{x}] - \text{Simp}[\text{b}*c*\text{Simp}[\text{Sqrt}[\text{d} + \text{e}*x^2]/\text{Sqrt}[1 - \text{c}^2*x^2]] \quad \text{Int}[\text{SimplifyIntegrand}[\text{u}/\text{Sqrt}[\text{d} + \text{e}*x^2], \text{x}], \text{x}], \text{x}]] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c}^2*\text{d} + \text{e}, 0] \ \&\& \ \text{IntegerQ}[\text{p} - 1/2] \ \&\& \ \text{NeQ}[\text{p}, -2^{(-1)}] \ \&\& \ (\text{IGtQ}[(\text{m} + 1)/2, 0] \ || \ \text{ILtQ}[(\text{m} + 2*\text{p} + 3)/2, 0])$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.89

method	result
default	$a \left(-\frac{x^4}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} + \frac{\frac{4x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{8}{3 d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}}}{c^2} \right) + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} (c^2 x^2 - i\sqrt{-c^2 x^2 + 1} c x - 1) (\arcsin(c x) + I)}{2 d^3 c^6 (c^2 x^2 - 1)} \right)$
parts	$a \left(-\frac{x^4}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} + \frac{\frac{4x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{8}{3 d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}}}{c^2} \right) + b \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} (c^2 x^2 - i\sqrt{-c^2 x^2 + 1} c x - 1) (\arcsin(c x) - I)}{2 d^3 c^6 (c^2 x^2 - 1)} \right)$

input `int(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output `a*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)/d^3/c^6/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)/d^3/c^6/(c^2*x^2-1)+1/6*(-d*(c^2*x^2-1))^(1/2)*(12*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-10*arcsin(c*x))/c^6/(c^2*x^2-1)^2/d^3-11/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^6/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)+11/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^6/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I))`

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.20

$$\int \frac{x^5 (a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \left[\frac{11 (bc^4 x^4 - 2bc^2 x^2 + b) \sqrt{d} \log \left(-\frac{c^6 dx^6 + 5c^4 dx^4 - 5c^2 dx^2 - 4(c^3 x^3 + cx) \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 dx^2 - 1}}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} \right)}{\dots} \right]$$

input `integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output

```
[1/24*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 8*(3*a*c^4*x^4 - 12*a*c^2*x^2 + (3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*arcsin(c*x) + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3), 1/12*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 2*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 4*(3*a*c^4*x^4 - 12*a*c^2*x^2 + (3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*arcsin(c*x) + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3)]
```

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^5(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input

```
integrate(x**5*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output

```
Integral(x**5*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^5}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```
integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output

```
-1/3*a*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2 + d)^(3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d)) + 1/3*(3*(c^8*d^3*x^2 - c^6*d^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*integrate(1/3*(3*c^4*x^6 - 12*c^2*x^4 + 8*x^2)/(c^9*d^3*x^6 - 2*c^7*d^3*x^4 + c^5*d^3*x^2 + (c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)*e^(log(c*x + 1) + log(-c*x + 1))), x) + (3*c^4*x^4 - 12*c^2*x^2 + 8)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))) * b/((c^8*d^3*x^2 - c^6*d^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((x^5*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((x^5*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)x^5}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b c^8 x^2 - 3\sqrt{-c^2 x^2 + 1}}{3\sqrt{d} \sqrt{-c^2 x^2 + 1}}$$

input `int(x^5*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `(3*sqrt(-c**2*x**2+1)*int((asin(c*x)*x**5)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b*c**8*x**2-3*sqrt(-c**2*x**2+1)*int((asin(c*x)*x**5)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b*c**6+3*a*c**4*x**4-12*a*c**2*x**2+8*a)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*c**6*d**2*(c**2*x**2-1))`

3.129
$$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1200
Mathematica [A] (verified)	1200
Rubi [A] (verified)	1201
Maple [C] (verified)	1204
Fricas [F]	1204
Sympy [F]	1205
Maxima [F]	1205
Giac [F(-2)]	1205
Mupad [F(-1)]	1206
Reduce [F]	1206

Optimal result

Integrand size = 27, antiderivative size = 212

$$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx = -\frac{b}{6c^5d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b \arcsin(cx))}{3c^2d(d-c^2dx^2)^{3/2}} - \frac{x(a+b \arcsin(cx))}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2bc^5d^2\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \log(1-c^2x^2)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

output

$$-1/6*b/c^5/d^2/(-c^2*x^2+1)^{(1/2)}/(-c^2*d*x^2+d)^{(1/2)}+1/3*x^3*(a+b*\arcsin(c*x))/c^2/d/(-c^2*d*x^2+d)^{(3/2)}-x*(a+b*\arcsin(c*x))/c^4/d^2/(-c^2*d*x^2+d)^{(1/2)}+1/2*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^2/b/c^5/d^2/(-c^2*d*x^2+d)^{(1/2)}-2/3*b*(-c^2*x^2+1)^{(1/2)}*\ln(-c^2*x^2+1)/c^5/d^2/(-c^2*d*x^2+d)^{(1/2)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx = \frac{-3b\sqrt{d}(1-c^2x^2)^{3/2} \arcsin(cx)^2 - 6a(-1+c^2x^2)\sqrt{d-c^2dx^2} \arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{(d-c^2dx^2)^{5/2}}$$

input `Integrate[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]`

output $(-3*b*\sqrt{d}*(1 - c^2*x^2)^{(3/2)}*ArcSin[c*x]^2 - 6*a*(-1 + c^2*x^2)*\sqrt{d - c^2*d*x^2}*ArcTan[(c*x*\sqrt{d - c^2*d*x^2})/(\sqrt{d}*(-1 + c^2*x^2))] + \sqrt{d}*(6*a*c*x - 8*a*c^3*x^3 + b*\sqrt{1 - c^2*x^2}) + 4*b*(1 - c^2*x^2)^{(3/2)}*\text{Log}[1 - c^2*x^2] + 2*b*\sqrt{d}*ArcSin[c*x]*\text{Sin}[3*ArcSin[c*x]])/(6*c^5*d^{(5/2)}*(-1 + c^2*x^2)*\sqrt{d - c^2*d*x^2})$

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5206, 243, 49, 2009, 5206, 240, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx \\
 & \quad \downarrow \text{5206} \\
 & -\frac{\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x^3}{(1-c^2 x^2)^2} dx}{3c^2 d \sqrt{d-c^2 dx^2}} + \frac{x^3(a + b \arcsin(cx))}{3c^2 d (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{243} \\
 & -\frac{\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x^2}{(1-c^2 x^2)^2} dx^2}{6c^2 d \sqrt{d-c^2 dx^2}} + \frac{x^3(a + b \arcsin(cx))}{3c^2 d (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{49} \\
 & -\frac{\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \left(\frac{1}{c^2(c^2 x^2-1)} + \frac{1}{c^2(c^2 x^2-1)^2} \right) dx^2}{6c^2 d \sqrt{d-c^2 dx^2}} + \frac{x^3(a + b \arcsin(cx))}{3c^2 d (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{2009} \\
 & -\frac{\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2 dx^2)^{3/2}} dx}{c^2 d} + \frac{x^3(a + b \arcsin(cx))}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{c^4(1-c^2 x^2)} + \frac{\log(1-c^2 x^2)}{c^4} \right)}{6c^2 d \sqrt{d-c^2 dx^2}}
 \end{aligned}$$

$$\begin{aligned}
& \downarrow 5206 \\
& -\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} - \frac{b\sqrt{1-c^2 x^2} \int \frac{x}{1-c^2 x^2} dx}{cd\sqrt{d-c^2 dx^2}} + \frac{x(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} + \frac{x^3(a+b \arcsin(cx))}{3c^2 d(d-c^2 dx^2)^{3/2}} - \\
& \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{c^4(1-c^2 x^2)} + \frac{\log(1-c^2 x^2)}{c^4} \right)}{6cd^2\sqrt{d-c^2 dx^2}} \\
& \downarrow 240 \\
& -\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{d-c^2 dx^2}} dx}{c^2 d} + \frac{x(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} + \frac{b\sqrt{1-c^2 x^2} \log(1-c^2 x^2)}{2c^3 d\sqrt{d-c^2 dx^2}} + \frac{x^3(a+b \arcsin(cx))}{3c^2 d(d-c^2 dx^2)^{3/2}} - \\
& \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{c^4(1-c^2 x^2)} + \frac{\log(1-c^2 x^2)}{c^4} \right)}{6cd^2\sqrt{d-c^2 dx^2}} \\
& \downarrow 5152 \\
& \frac{x^3(a+b \arcsin(cx))}{3c^2 d(d-c^2 dx^2)^{3/2}} - \frac{\frac{x(a+b \arcsin(cx))}{c^2 d\sqrt{d-c^2 dx^2}} - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))^2}{2bc^3 d\sqrt{d-c^2 dx^2}} + \frac{b\sqrt{1-c^2 x^2} \log(1-c^2 x^2)}{2c^3 d\sqrt{d-c^2 dx^2}}}{c^2 d} - \\
& \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{c^4(1-c^2 x^2)} + \frac{\log(1-c^2 x^2)}{c^4} \right)}{6cd^2\sqrt{d-c^2 dx^2}}
\end{aligned}$$

input `Int[(x^4*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]`

output `(x^3*(a + b*ArcSin[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (b*sqrt[1 - c^2*x^2]*(1/(c^4*(1 - c^2*x^2)) + Log[1 - c^2*x^2]/c^4))/(6*c*d^2*sqrt[d - c^2*d*x^2]) - ((x*(a + b*ArcSin[c*x]))/(c^2*d*sqrt[d - c^2*d*x^2]) - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/(2*b*c^3*d*sqrt[d - c^2*d*x^2]) + (b*sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2])/(2*c^3*d*sqrt[d - c^2*d*x^2]))/(c^2*d)`

Definitions of rubi rules used

- rule 49 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)}((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[m + n + 2, 0]$
- rule 240 $\text{Int}[(x_)/((a_) + (b_.)(x_)^2), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^2, x]]/(2*b), x] /; \text{FreeQ}\{a, b\}, x]$
- rule 243 $\text{Int}[(x_)^{(m_.)}((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x)^p}, x], x, x^2], x] /; \text{FreeQ}\{a, b, m, p\}, x \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 5152 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}/\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$
- rule 5206 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}((f_.)(x_)^{(m_.)}((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m - 1)}*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + (-\text{Simp}[f^2*((m - 1)/(2*e*(p + 1)) \ \text{Int}[(f*x)^{(m - 2)}*(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p \ \text{Int}[(f*x)^{(m - 1)}*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IGtQ}[m, 1]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.69

method	result
default	$\frac{ax^3}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}} - \frac{ax}{c^4d^2\sqrt{-c^2dx^2+d}} + \frac{a \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{c^4d^2\sqrt{c^2d}} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}}{3\arcsin(cx)^2x^4c^4+8i\arcsin(cx)}$
parts	$\frac{ax^3}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}} - \frac{ax}{c^4d^2\sqrt{-c^2dx^2+d}} + \frac{a \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{c^4d^2\sqrt{c^2d}} - \frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}}{3\arcsin(cx)^2x^4c^4+8i\arcsin(cx)}$

input `int(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{3}ax^3/c^2/d/(-c^2*d*x^2+d)^{(3/2)}-a/c^4/d^2*x/(-c^2*d*x^2+d)^{(1/2)}+a/c^4/d^2/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})-1/6*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*(3*\arcsin(c*x)^2*x^4*c^4+8*I*\arcsin(c*x)*x^4*c^4-8*\ln(1+(I*c*x+(-c^2*x^2+1)^{(1/2}))^2)*x^4*c^4+8*(-c^2*x^2+1)^{(1/2)}*\arcsin(c*x)*x^3*c^3-6*\arcsin(c*x)^2*x^2*c^2-16*I*\arcsin(c*x)*x^2*c^2+16*\ln(1+(I*c*x+(-c^2*x^2+1)^{(1/2}))^2)*x^2*c^2-6*\arcsin(c*x)*(-c^2*x^2+1)^{(1/2)}*c*x+c^2*x^2+3*\arcsin(c*x)^2+8*I*\arcsin(c*x)-8*\ln(1+(I*c*x+(-c^2*x^2+1)^{(1/2}))^2)-1)/d^3/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/c^5$$

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-(b*x^4*arcsin(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^4(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{\frac{5}{2}}} dx$$

input `integrate(x**4*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral(x**4*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(-c^2 dx^2 + d)^{\frac{5}{2}}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a + b*integrate(x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((x^4*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) a c^2 x^2 - 3\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) a + 3\sqrt{-c^2 x^2 + 1} \left(\int \frac{x^4(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx \right)}{(d - c^2 dx^2)^{5/2}}$$

input

```
int(x^4*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(3*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*c**2*x**2 - 3*sqrt(-c**2*x**2 + 1)
*asin(c*x)*a + 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**4)/(sqrt(-c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x
**2 + 1)),x)*b*c**7*x**2 - 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**4)/(
sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sq
rt(-c**2*x**2 + 1)),x)*b*c**5 - 4*a*c**3*x**3 + 3*a*c*x)/(3*sqrt(d)*sqrt
(-c**2*x**2 + 1)*c**5*d**2*(c**2*x**2 - 1))
```

3.130 $\int \frac{x^3(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$

Optimal result	1207
Mathematica [C] (verified)	1207
Rubi [A] (verified)	1208
Maple [C] (verified)	1210
Fricas [A] (verification not implemented)	1210
Sympy [F]	1211
Maxima [A] (verification not implemented)	1211
Giac [F(-2)]	1212
Mupad [F(-1)]	1212
Reduce [F]	1213

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = -\frac{bx}{6c^3d^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}} + \frac{a + b \arcsin(cx)}{3c^4d(d - c^2dx^2)^{3/2}} - \frac{a + b \arcsin(cx)}{c^4d^2\sqrt{d - c^2dx^2}} + \frac{5b\sqrt{1 - c^2x^2}\operatorname{arctanh}(cx)}{6c^4d^2\sqrt{d - c^2dx^2}}$$

output

```
-1/6*b*x/c^3/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x))/c^4/d/(-c^2*d*x^2+d)^(3/2)-(a+b*arcsin(c*x))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+5/6*b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/c^4/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = \frac{\sqrt{d - c^2dx^2}(\sqrt{-c^2}(-4a + 6ac^2x^2 - bcx\sqrt{1 - c^2x^2} + 2b(-2 + 3c^2x^2) \arcsin(cx))}{6c^4\sqrt{-c^2}d^3(-1 + c^2x^2)}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(-4*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 - c^2*x^2] + 2*b*(-2 + 3*c^2*x^2)*ArcSin[c*x]) - (5*I)*b*c*(1 - c^2*x^2)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]))/(6*c^4*Sqrt[-c^2]*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5194, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow 5194$$

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{2-3c^2 x^2}{3c^4 d^3 (1-c^2 x^2)^2} dx}{\sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{c^4 d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3c^4 d (d - c^2 dx^2)^{3/2}}$$

$$\downarrow 27$$

$$\frac{b\sqrt{d - c^2 dx^2} \int \frac{2-3c^2 x^2}{(1-c^2 x^2)^2} dx}{3c^3 d^3 \sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{c^4 d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3c^4 d (d - c^2 dx^2)^{3/2}}$$

$$\downarrow 298$$

$$\frac{b\sqrt{d - c^2 dx^2} \left(\frac{5}{2} \int \frac{1}{1-c^2 x^2} dx - \frac{x}{2(1-c^2 x^2)} \right)}{3c^3 d^3 \sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{c^4 d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3c^4 d (d - c^2 dx^2)^{3/2}}$$

$$\downarrow 219$$

$$-\frac{a + b \arcsin(cx)}{c^4 d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3c^4 d (d - c^2 dx^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2 x^2)} \right) \sqrt{d - c^2 dx^2}}{3c^3 d^3 \sqrt{1 - c^2 x^2}}$$

input

```
Int[(x^3*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
$$\frac{(a + b \operatorname{ArcSin}[c x]) / (3 c^4 d (d - c^2 d x^2)^{3/2}) - (a + b \operatorname{ArcSin}[c x]) / (c^4 d^2 \sqrt{d - c^2 d x^2}) + (b \sqrt{d - c^2 d x^2} (-1/2 x / (1 - c^2 x^2) + (5 \operatorname{ArcTanh}[c x]) / (2 c))) / (3 c^3 d^3 \sqrt{1 - c^2 x^2})}{1}$$

Defintions of rubi rules used

rule 27
$$\operatorname{Int}[(a_*)(F x_), x_Symbol] \rightarrow \operatorname{Simp}[a \operatorname{Int}[F x, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[F x, (b_*)(G x_)] /; \operatorname{FreeQ}[b, x]$$

rule 219
$$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 / (\operatorname{Rt}[a, 2] \operatorname{Rt}[-b, 2])) * \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] * (x / \operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \operatorname{||} \operatorname{LtQ}[b, 0])$$

rule 298
$$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{p_*) * ((c_*) + (d_*)(x_)^2), x_Symbol] \rightarrow \operatorname{Simp}[(- (b * c - a * d)) * x * ((a + b * x^2)^{p+1} / (2 * a * b * (p+1))), x] - \operatorname{Simp}[(a * d - b * c * (2 * p + 3)) / (2 * a * b * (p+1)) \operatorname{Int}[(a + b * x^2)^{p+1}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, p\}, x] \&\& \operatorname{NeQ}[b * c - a * d, 0] \&\& (\operatorname{LtQ}[p, -1] \operatorname{||} \operatorname{ILtQ}[1/2 + p, 0])$$

rule 5194
$$\operatorname{Int}[(a_*) + \operatorname{ArcSin}[(c_*)(x_)] * (b_*) * (x_)^{m_*) * ((d_*) + (e_*)(x_)^2)^{p_*)}, x_Symbol] \rightarrow \operatorname{With}[\{u = \operatorname{IntHide}[x^m * (d + e * x^2)^p, x]\}, \operatorname{Simp}[(a + b * \operatorname{ArcSin}[c * x]) u, x] - \operatorname{Simp}[b * c * \operatorname{Simp}[\sqrt{d + e * x^2} / \sqrt{1 - c^2 * x^2}] \operatorname{Int}[\operatorname{SimplifyIntegrand}[u / \sqrt{d + e * x^2}, x], x], x]] /; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \&\& \operatorname{EqQ}[c^2 * d + e, 0] \&\& \operatorname{IntegerQ}[p - 1/2] \&\& \operatorname{NeQ}[p, -2^{-1}] \&\& (\operatorname{IGtQ}[(m + 1)/2, 0] \operatorname{||} \operatorname{ILtQ}[(m + 2 * p + 3)/2, 0])$$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.68

method	result
default	$a \left(\frac{x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{2}{3 d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}} \right) + b \left(\frac{\sqrt{-d(c^2 x^2 - 1)} (6 c^2 x^2 \arcsin(cx) - cx \sqrt{-c^2 x^2 + 1} - 4 \arcsin(cx))}{6 (c^2 x^2 - 1)^2 d^3 c^4} - \frac{5 \sqrt{-d(c^2 x^2 - 1)}}{6 (c^2 x^2 - 1)^2 d^3 c^4} \right)$
parts	$a \left(\frac{x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{2}{3 d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}} \right) + b \left(\frac{\sqrt{-d(c^2 x^2 - 1)} (6 c^2 x^2 \arcsin(cx) - cx \sqrt{-c^2 x^2 + 1} - 4 \arcsin(cx))}{6 (c^2 x^2 - 1)^2 d^3 c^4} - \frac{5 \sqrt{-d(c^2 x^2 - 1)}}{6 (c^2 x^2 - 1)^2 d^3 c^4} \right)$

input `int(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output `a*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(6*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-4*arcsin(c*x))/(c^2*x^2-1)^2/d^3/c^4-5/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)+5/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I))`

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.81

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \left[-\frac{4 \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} b c x - 5 (b c^4 x^4 - 2 b c^2 x^2 + b) \sqrt{d} \log \left(-\frac{c^6 dx^6 + 5}{d - c^2 dx^2} \right)}{12 (c^8 d^3 x^4 - 2 c^6 d^3 x^2 + c^4 d^3)} \right. \\ \left. - \frac{2 \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} b c x - 5 (b c^4 x^4 - 2 b c^2 x^2 + b) \sqrt{-d} \arctan \left(\frac{2 \sqrt{-c^2 dx^2 + d} \sqrt{-c^2 x^2 + 1} c \sqrt{-dx}}{c^4 dx^4 - d} \right) - 4 (3 a + b \arcsin(cx)) \sqrt{-d}}{12 (c^8 d^3 x^4 - 2 c^6 d^3 x^2 + c^4 d^3)} \right]$$

input `integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output

```
[-1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - 5*(b*c^4*x^4 - 2
*b*c^2*x^2 + b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c
^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^
6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 8*(3*a*c^2*x^2 + (3*b*c^2*x^2 - 2*b)*arc
sin(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d
^3), -1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - 5*(b*c^4*x^4
- 2*b*c^2*x^2 + b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 +
1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 4*(3*a*c^2*x^2 + (3*b*c^2*x^2 - 2*b)*a
rcsin(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4
*d^3)]
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^3(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input

```
integrate(x**3*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output

```
Integral(x**3*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{1}{12} bc \left(\frac{2x}{c^6 d^{5/2} x^2 - c^4 d^{5/2}} + \frac{5 \log(cx + 1)}{c^5 d^{5/2}} - \frac{5 \log(cx - 1)}{c^5 d^{5/2}} \right) + \frac{1}{3} b \left(\frac{3x^2}{(-c^2 dx^2 + d)^{3/2} c^2 d} - \frac{2}{(-c^2 dx^2 + d)^{3/2} c^4 d} \right) \arcsin(cx) + \frac{1}{3} a \left(\frac{3x^2}{(-c^2 dx^2 + d)^{3/2} c^2 d} - \frac{2}{(-c^2 dx^2 + d)^{3/2} c^4 d} \right)$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```


output

```
1/12*b*c*(2*x/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) + 5*log(c*x + 1)/(c^5*d^(5/2)) - 5*log(c*x - 1)/(c^5*d^(5/2))) + 1/3*b*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arcsin(c*x) + 1/3*a*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((x^3*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)x^3}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b c^6 x^2 - 3\sqrt{-c^2 x^2 + 1} c^4 d^2}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} c^4 d^2}$$

input `int(x^3*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `(3*sqrt(-c**2*x**2+1)*int((asin(c*x)*x**3)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b*c**6*x**2-3*sqrt(-c**2*x**2+1)*int((asin(c*x)*x**3)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b*c**4-3*a*c**2*x**2+2*a)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*c**4*d**2*(c**2*x**2-1))`

3.131 $\int \frac{x^2(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$

Optimal result	1214
Mathematica [A] (verified)	1214
Rubi [A] (verified)	1215
Maple [C] (verified)	1216
Fricas [F]	1217
Sympy [F]	1218
Maxima [A] (verification not implemented)	1218
Giac [F(-2)]	1219
Mupad [F(-1)]	1219
Reduce [F]	1219

Optimal result

Integrand size = 27, antiderivative size = 125

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = -\frac{b}{6c^3d^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}} + \frac{x^3(a + b \arcsin(cx))}{3d(d - c^2dx^2)^{3/2}} - \frac{b\sqrt{1 - c^2x^2} \log(1 - c^2x^2)}{6c^3d^2\sqrt{d - c^2dx^2}}$$

output

$$-1/6*b/c^3/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*\arcsin(c*x))/d/(-c^2*d*x^2+d)^(3/2)-1/6*b*(-c^2*x^2+1)^(1/2)*\ln(-c^2*x^2+1)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = \frac{\sqrt{d - c^2dx^2} (2ac^3x^3 - b\sqrt{1 - c^2x^2} + 2bc^3x^3 \arcsin(cx) - b(1 - c^2x^2)^{3/2} \log(-))}{6c^3d^3(-1 + c^2x^2)^2}$$

input

$$\text{Integrate}[(x^2*(a + b*\text{ArcSin}[c*x]))/(d - c^2*d*x^2)^(5/2), x]$$

output

```
(Sqrt[d - c^2*d*x^2]*(2*a*c^3*x^3 - b*Sqrt[1 - c^2*x^2] + 2*b*c^3*x^3*ArcSin[c*x] - b*(1 - c^2*x^2)^(3/2)*Log[-1 + c^2*x^2]))/(6*c^3*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5186, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow \text{5186}$$

$$\frac{x^3(a + b \arcsin(cx))}{3d(d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{x^3}{(1 - c^2 x^2)^2} dx}{3d^2\sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{243}$$

$$\frac{x^3(a + b \arcsin(cx))}{3d(d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{x^2}{(1 - c^2 x^2)^2} dx^2}{6d^2\sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{49}$$

$$\frac{x^3(a + b \arcsin(cx))}{3d(d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} \int \left(\frac{1}{c^2(c^2 x^2 - 1)} + \frac{1}{c^2(c^2 x^2 - 1)^2} \right) dx^2}{6d^2\sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{2009}$$

$$\frac{x^3(a + b \arcsin(cx))}{3d(d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{1}{c^4(1 - c^2 x^2)} + \frac{\log(1 - c^2 x^2)}{c^4} \right)}{6d^2\sqrt{d - c^2 dx^2}}$$

input

```
Int[(x^2*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2), x]
```

output

$$\frac{(x^3(a + b\text{ArcSin}[c*x]))/(3*d*(d - c^2*d*x^2)^{(3/2)}) - (b*c*\text{Sqrt}[1 - c^2*x^2]*(1/(c^4*(1 - c^2*x^2)) + \text{Log}[1 - c^2*x^2]/c^4))/(6*d^2*\text{Sqrt}[d - c^2*d*x^2])}{}$$
Defintions of rubi rules used

rule 49

$$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)*((c_.) + (d_.)*(x_.)^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[m + n + 2, 0]$$

rule 243

$$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x)^p}, x], x, x^2], x] /; \text{FreeQ}\{a, b, m, p\}, x] \&\& \text{IntegerQ}[(m - 1)/2]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

rule 5186

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)*((f_.)*(x_.))^{(m_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)}*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m + 1))), x] - \text{Simp}[b*c*(n/(f*(m + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m + 1)}*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{EqQ}[m + 2*p + 3, 0] \&\& \text{NeQ}[m, -1]$$
Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 1242, normalized size of antiderivative = 9.94

method	result	size
default	Expression too large to display	1242
parts	Expression too large to display	1242

input

$$\text{int}(x^2*(a+b*\text{arcsin}(c*x))/(-c^2*d*x^2+d)^{(5/2)}, x, \text{method}=_RETURNVERBOSE)$$

output

```

a*(1/2*x/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/2/c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+
2/3/d^2*x/(-c^2*d*x^2+d)^(1/2)))-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8
*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4*x^7-1/3*I*b*(-d*(c^2*x^2-1))^(1
/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3*arcsin(c*x)*(-c^2
*x^2+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4
-5*c^2*x^2+1)*c^4*arcsin(c*x)*x^7+1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^
8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*(-c^2*x^2+1)*x^3+4/3*I*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c*arcsin(c*
x)*(-c^2*x^2+1)^(1/2)*x^2-2*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^
6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*x^4-b*(-d*(
c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2*arc
sin(c*x)*x^5-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*
x^4-5*c^2*x^2+1)*c*(-c^2*x^2+1)^(1/2)*x^4-2/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*arcsin(c*x)+I*b*(-d*(c^2*x^2-1))^(1/
2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3*arcsin(c*x)*(-c^2*
x^2+1)^(1/2)*x^6+1/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*c^2*x^5+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8
-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*arcsin(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(
1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c*(-c^2*x^2+1)^(1/2
)*x^2-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^...

```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```

integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)

```

output

```

integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arcsin(c*x) + a*x^2)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)

```

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^2(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral(x**2*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**5/2, x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\begin{aligned} \int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx &= \frac{1}{6} bc \left(\frac{1}{c^6 d^{5/2} x^2 - c^4 d^{5/2}} - \frac{\log(cx + 1)}{c^4 d^{5/2}} - \frac{\log(cx - 1)}{c^4 d^{5/2}} \right) \\ &- \frac{1}{3} b \left(\frac{x}{\sqrt{-c^2 dx^2 + d} c^2 d^2} - \frac{x}{(-c^2 dx^2 + d)^{3/2} c^2 d} \right) \arcsin(cx) \\ &- \frac{1}{3} a \left(\frac{x}{\sqrt{-c^2 dx^2 + d} c^2 d^2} - \frac{x}{(-c^2 dx^2 + d)^{3/2} c^2 d} \right) \end{aligned}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/6*b*c*(1/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) - log(c*x + 1)/(c^4*d^(5/2)) - log(c*x - 1)/(c^4*d^(5/2))) - 1/3*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arcsin(c*x) - 1/3*a*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)`

output `int((x^2*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)x^2}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b c^2 x^2 - 3\sqrt{-c^2 x^2 + 1}}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} d^2 (c^2 x^2)}$$

input `int(x^2*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output

```
(3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**2)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b*c**2*x**2 - 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**2)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b - a*x**3)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.132
$$\int \frac{x(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1221
Mathematica [A] (verified)	1221
Rubi [A] (verified)	1222
Maple [C] (verified)	1223
Fricas [A] (verification not implemented)	1224
Sympy [F]	1225
Maxima [F]	1225
Giac [F(-2)]	1225
Mupad [F(-1)]	1226
Reduce [F]	1226

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = -\frac{bx}{6cd^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}} + \frac{a + b \arcsin(cx)}{3c^2d(d - c^2dx^2)^{3/2}} - \frac{b\sqrt{1 - c^2x^2}\operatorname{arctanh}(cx)}{6c^2d^2\sqrt{d - c^2dx^2}}$$

output

```
-1/6*b*x/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x
))/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/6*b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/c^2/d^
2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = \frac{-2a + bcx\sqrt{1 - c^2x^2} - 2b \arcsin(cx) + b(1 - c^2x^2)^{3/2} \operatorname{arctanh}(cx)}{6c^2d^2(-1 + c^2x^2)\sqrt{d - c^2dx^2}}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output

$$\frac{(-2*a + b*c*x*\text{Sqrt}[1 - c^2*x^2] - 2*b*\text{ArcSin}[c*x] + b*(1 - c^2*x^2)^{(3/2)}*\text{ArcTanh}[c*x])}{(6*c^2*d^2*(-1 + c^2*x^2)*\text{Sqrt}[d - c^2*d*x^2])}$$
Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5182, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow \text{5182}$$

$$\frac{a + b \arcsin(cx)}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{b\sqrt{1 - c^2 x^2} \int \frac{1}{(1 - c^2 x^2)^2} dx}{3cd^2 \sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{215}$$

$$\frac{a + b \arcsin(cx)}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{b\sqrt{1 - c^2 x^2} \left(\frac{1}{2} \int \frac{1}{1 - c^2 x^2} dx + \frac{x}{2(1 - c^2 x^2)} \right)}{3cd^2 \sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{219}$$

$$\frac{a + b \arcsin(cx)}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{b\sqrt{1 - c^2 x^2} \left(\frac{\text{arctanh}(cx)}{2c} + \frac{x}{2(1 - c^2 x^2)} \right)}{3cd^2 \sqrt{d - c^2 dx^2}}$$

input

$$\text{Int}[(x*(a + b*\text{ArcSin}[c*x]))/(d - c^2*d*x^2)^{(5/2)}, x]$$

output

$$\frac{(a + b*\text{ArcSin}[c*x])}{(3*c^2*d*(d - c^2*d*x^2)^{(3/2)})} - \frac{(b*\text{Sqrt}[1 - c^2*x^2]*\frac{x}{2*(1 - c^2*x^2)} + \text{ArcTanh}[c*x]/(2*c))}{(3*c*d^2*\text{Sqrt}[d - c^2*d*x^2])}$$

Definitions of rubi rules used

rule 215 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*((a + b*x^2)^(p + 1) / (2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1)) Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.87

method	result
default	$\frac{a}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}} + b \left(\frac{\sqrt{-d(c^2x^2-1)}(-cx\sqrt{-c^2x^2+1}+2\arcsin(cx))}{6d^3(c^4x^4-2c^2x^2+1)c^2} + \frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\ln\left(icx+\sqrt{-c^2x^2+1}+i\right)}{6d^3c^2(c^2x^2-1)} \right)$
parts	$\frac{a}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}} + b \left(\frac{\sqrt{-d(c^2x^2-1)}(-cx\sqrt{-c^2x^2+1}+2\arcsin(cx))}{6d^3(c^4x^4-2c^2x^2+1)c^2} + \frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\ln\left(icx+\sqrt{-c^2x^2+1}+i\right)}{6d^3c^2(c^2x^2-1)} \right)$

input `int(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```
1/3*a/c^2/d/(-c^2*d*x^2+d)^(3/2)+b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(-c*x*(-c^2*x^2+1)^(1/2)+2*arcsin(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/c^2+1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)-1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.14

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \left[-\frac{4\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}bcx - (bc^4 x^4 - 2bc^2 x^2 + b)\sqrt{d} \log\left(-\frac{c^6 dx^6 + 5c^4 dx^4 - 5c^2 dx^2 + 4(c^3 x^3 + cx)\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}\sqrt{d} - d}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1}\right) - 8\sqrt{-c^2 dx^2 + d}(b \arcsin(cx) + a)}{24(c^6 d^3 x^4 - 2c^4 d^3 x^2 + c^2 d^3)} \right. \\ \left. - \frac{2\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}bcx + (bc^4 x^4 - 2bc^2 x^2 + b)\sqrt{-d} \arctan\left(\frac{2\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1}c\sqrt{-dx}}{c^4 dx^4 - d}\right) - 4\sqrt{-c^2 dx^2 + d}}{12(c^6 d^3 x^4 - 2c^4 d^3 x^2 + c^2 d^3)} \right]$$

input

```
integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output

```
[-1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 8*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3), -1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 4*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)]
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(-d(cx - 1)(cx + 1))^{\frac{5}{2}}} dx$$

input `integrate(x*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral(x*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)`

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x}{(-c^2 dx^2 + d)^{\frac{5}{2}}} dx$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `b*integrate(x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d) + 1/3*a/((-c^2*d*x^2 + d)^(3/2)*c^2*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)`

output `int((x*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b c^4 x^2 - 3\sqrt{-c^2 x^2 + 1}}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} c^2 d^2 (c^2 x^2 - 1)}$$

input `int(x*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `(3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b*c**4*x**2 - 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b*c**2 - a)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*c**2*d**2*(c**2*x**2 - 1))`

3.133 $\int \frac{a+b \arcsin(cx)}{(d-c^2dx^2)^{5/2}} dx$

Optimal result	1227
Mathematica [A] (verified)	1227
Rubi [A] (verified)	1228
Maple [C] (verified)	1230
Fricas [F]	1231
Sympy [F]	1231
Maxima [A] (verification not implemented)	1231
Giac [F(-2)]	1232
Mupad [F(-1)]	1232
Reduce [F]	1233

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{a + b \arcsin(cx)}{(d - c^2dx^2)^{5/2}} dx = -\frac{b}{6cd^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}} + \frac{x(a + b \arcsin(cx))}{3d(d - c^2dx^2)^{3/2}} + \frac{2x(a + b \arcsin(cx))}{3d^2\sqrt{d - c^2dx^2}} + \frac{b\sqrt{1 - c^2x^2} \log(1 - c^2x^2)}{3cd^2\sqrt{d - c^2dx^2}}$$

output

```
-1/6*b/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arcsin(c*x
))/d/(-c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arcsin(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)
+1/3*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{a + b \arcsin(cx)}{(d - c^2dx^2)^{5/2}} dx = \frac{\sqrt{d - c^2dx^2} \left(-6acx + 4ac^3x^3 + b\sqrt{1 - c^2x^2} + 2bcx(-3 + 2c^2x^2) \arcsin(cx) - 2b(1 - c^2x^2)^{3/2} \log(-1 + \sqrt{1 - c^2x^2}) \right)}{6cd^3(-1 + c^2x^2)^2}$$

input `Integrate[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^(5/2),x]`

output `-1/6*(Sqrt[d - c^2*d*x^2]*(-6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] + 2*b*c*x*(-3 + 2*c^2*x^2)*ArcSin[c*x] - 2*b*(1 - c^2*x^2)^(3/2)*Log[-1 + c^2*x^2]))/(c*d^3*(-1 + c^2*x^2)^2)`

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5162, 241, 5160, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx \\
 & \quad \downarrow \text{5162} \\
 & \frac{2 \int \frac{a+b \arcsin(cx)}{(d-c^2 dx^2)^{3/2}} dx}{3d} - \frac{bc\sqrt{1-c^2 x^2} \int \frac{x}{(1-c^2 x^2)^2} dx}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{x(a+b \arcsin(cx))}{3d(d-c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{241} \\
 & \frac{2 \int \frac{a+b \arcsin(cx)}{(d-c^2 dx^2)^{3/2}} dx}{3d} + \frac{x(a+b \arcsin(cx))}{3d(d-c^2 dx^2)^{3/2}} - \frac{b}{6cd^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{5160} \\
 & \frac{2 \left(\frac{x(a+b \arcsin(cx))}{d\sqrt{d-c^2 dx^2}} - \frac{bc\sqrt{1-c^2 x^2} \int \frac{x}{1-c^2 x^2} dx}{d\sqrt{d-c^2 dx^2}} \right)}{3d} + \frac{x(a+b \arcsin(cx))}{3d(d-c^2 dx^2)^{3/2}} - \frac{b}{6cd^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{240} \\
 & \frac{x(a+b \arcsin(cx))}{3d(d-c^2 dx^2)^{3/2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))}{d\sqrt{d-c^2 dx^2}} + \frac{b\sqrt{1-c^2 x^2} \log(1-c^2 x^2)}{2cd\sqrt{d-c^2 dx^2}} \right)}{3d} - \frac{b}{6cd^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 dx^2}}
 \end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(d - c^2*d*x^2)^(5/2),x]`

output `-1/6*b/(c*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcSin[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*((x*(a + b*ArcSin[c*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2]))) / (3*d)`

Defintions of rubi rules used

rule 240 `Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 5160 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*((a + b*ArcSin[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5162 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 1072, normalized size of antiderivative = 6.96

method	result	size
default	Expression too large to display	1072
parts	Expression too large to display	1072

input `int((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```

a*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))-I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*x+8/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*x^3+14/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^2-2*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^4-2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4*arcsin(c*x)*x^5-8/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*(-c^2*x^2+1)*x-7/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4*x^5-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c*(-c^2*x^2+1)^(1/2)*x^2+17/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*arcsin(c*x)*x^3+4/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^3/(c^2*x^2-1)*arcsin(c*x)+2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6*x^7+2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4*(-c^2*x^2+1)*x^5+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c*(-c^2*x^2+1)^(1/2)-4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*arcsin(c*x)*x-5/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*(-c^2*x^2+1)*x^...

```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\begin{aligned} \int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx &= \frac{1}{6} bc \left(\frac{1}{c^4 d^{5/2} x^2 - c^2 d^{5/2}} + \frac{2 \log(cx + 1)}{c^2 d^{5/2}} + \frac{2 \log(cx - 1)}{c^2 d^{5/2}} \right) \\ &+ \frac{1}{3} b \left(\frac{2x}{\sqrt{-c^2 dx^2 + dd^2}} + \frac{x}{(-c^2 dx^2 + d)^{3/2} d} \right) \arcsin(cx) \\ &+ \frac{1}{3} a \left(\frac{2x}{\sqrt{-c^2 dx^2 + dd^2}} + \frac{x}{(-c^2 dx^2 + d)^{3/2} d} \right) \end{aligned}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output $\frac{1}{6}bc\left(\frac{1}{c^4d^{5/2}}x^2 - c^2d^{5/2}\right) + 2\log(cx + 1)/(c^2d^{5/2}) + 2\log(cx - 1)/(c^2d^{5/2}) + \frac{1}{3}b\left(\frac{2x}{\sqrt{-c^2dx^2 + d}}d^2 + x/((-c^2dx^2 + d)^{3/2}d)\right) + \frac{1}{3}a\left(\frac{2x}{\sqrt{-c^2dx^2 + d}}d^2 + x/((-c^2dx^2 + d)^{3/2}d)\right)$

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((a + b*asin(c*x))/(d - c^2*d*x^2)^(5/2),x)`

output `int((a + b*asin(c*x))/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b c^2 x^2 - 3\sqrt{-c^2 x^2 + 1}}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} d^2 (c^2 x^2 - 1)}$$

input `int((a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `(3*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b*c**2*x**2-3*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b+2*a*c**2*x**3-3*a*x)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*d**2*(c**2*x**2-1))`

3.134 $\int \frac{a+b \arcsin(cx)}{x(d-c^2dx^2)^{5/2}} dx$

Optimal result	1234
Mathematica [A] (verified)	1235
Rubi [A] (verified)	1235
Maple [A] (verified)	1239
Fricas [F]	1240
Sympy [F]	1240
Maxima [F]	1241
Giac [F(-2)]	1241
Mupad [F(-1)]	1241
Reduce [F]	1242

Optimal result

Integrand size = 27, antiderivative size = 295

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2dx^2)^{5/2}} dx = -\frac{bcx}{6d^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}} + \frac{a + b \arcsin(cx)}{3d(d - c^2dx^2)^{3/2}}$$

$$+ \frac{\frac{a}{d^2} + \frac{b \arcsin(cx)}{d^2}}{\sqrt{d - c^2dx^2}} - \frac{2\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{d^2\sqrt{d - c^2dx^2}}$$

$$- \frac{7b\sqrt{1 - c^2x^2} \operatorname{arctanh}(cx)}{6d^2\sqrt{d - c^2dx^2}} + \frac{ib\sqrt{1 - c^2x^2} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d^2\sqrt{d - c^2dx^2}}$$

$$- \frac{ib\sqrt{1 - c^2x^2} \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d^2\sqrt{d - c^2dx^2}}$$

```
output -1/6*b*c*x/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x
))/d/(-c^2*d*x^2+d)^(3/2)+(a/d^2+b*arcsin(c*x)/d^2)/(-c^2*d*x^2+d)^(1/2)-2
*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d^
2/(-c^2*d*x^2+d)^(1/2)-7/6*b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/d^2/(-c^2*d*x
^2+d)^(1/2)+I*b*(-c^2*x^2+1)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d^
2/(-c^2*d*x^2+d)^(1/2)-I*b*(-c^2*x^2+1)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)
^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.55

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = -\frac{a(-4 + 3c^2 x^2) \sqrt{d - c^2 dx^2}}{3d^3 (-1 + c^2 x^2)^2} + \frac{a \log(x)}{d^{5/2}} - \frac{a \log(d + \sqrt{d} \sqrt{d - c^2 dx^2})}{d^{5/2}} + \frac{b(20 \arcsin(cx) + 12 \arcsin(cx) \cos(2 \arcsin(cx)) + 18\sqrt{1 - c^2 x^2} \arcsin(cx) \log(1 - e^{i \arcsin(cx)}) + 6 \arcsin(cx))}{d^{5/2}}$$

input `Integrate[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]`

output

```
-1/3*(a*(-4 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^3*(-1 + c^2*x^2)^2) + (a*Log[x])/d^(5/2) - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(5/2) + (b*(20*ArcSin[c*x] + 12*ArcSin[c*x]*Cos[2*ArcSin[c*x]] + 18*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 6*ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 - E^(I*ArcSin[c*x])] - 18*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - 6*ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 + E^(I*ArcSin[c*x])] + 21*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] + 7*Cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - 21*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] - 7*Cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] + (24*I)*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^(I*ArcSin[c*x])] - (24*I)*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^(I*ArcSin[c*x])] - 2*Sin[2*ArcSin[c*x]])/(24*d*(d - c^2*d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5208, 215, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx \\
& \quad \downarrow \text{5208} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{3/2}} dx}{d} - \frac{bc\sqrt{1-c^2 x^2} \int \frac{1}{(1-c^2 x^2)^2} dx}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{3d(d-c^2 dx^2)^{3/2}} \\
& \quad \downarrow \text{215} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{3/2}} dx}{d} - \frac{bc\sqrt{1-c^2 x^2} \left(\frac{1}{2} \int \frac{1}{1-c^2 x^2} dx + \frac{x}{2(1-c^2 x^2)} \right)}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{3d(d-c^2 dx^2)^{3/2}} \\
& \quad \downarrow \text{219} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x(d-c^2 dx^2)^{3/2}} dx}{d} + \frac{a+b \arcsin(cx)}{3d(d-c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2 x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2 x^2)} \right)}{3d^2 \sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{5208} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2 dx^2}} dx}{d} - \frac{bc\sqrt{1-c^2 x^2} \int \frac{1}{1-c^2 x^2} dx}{d\sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{3d(d-c^2 dx^2)^{3/2}} - \\
& \quad \frac{bc\sqrt{1-c^2 x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2 x^2)} \right)}{3d^2 \sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{219} \\
& \frac{\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2 dx^2}} dx}{d} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2 dx^2}} - \frac{b\sqrt{1-c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{3d(d-c^2 dx^2)^{3/2}} - \\
& \quad \frac{bc\sqrt{1-c^2 x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2 x^2)} \right)}{3d^2 \sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{5218} \\
& \frac{\sqrt{1-c^2 x^2} \int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx)}{d\sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2 dx^2}} - \frac{b\sqrt{1-c^2 x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2 dx^2}} + \frac{a+b \arcsin(cx)}{3d(d-c^2 dx^2)^{3/2}} - \\
& \quad \frac{bc\sqrt{1-c^2 x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2 x^2)} \right)}{3d^2 \sqrt{d-c^2 dx^2}} \\
& \quad \downarrow \text{3042}
\end{aligned}$$

$$\frac{\frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{3d(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

↓ 4671

$$\frac{\frac{\sqrt{1-c^2x^2} \left(-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})(a+b \arcsin(cx)) \right)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}}}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{\frac{\sqrt{1-c^2x^2} \left(ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)})(a+b \arcsin(cx)) \right)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}}}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

↓ 2838

$$\frac{\frac{\sqrt{1-c^2x^2} \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2}}{d}}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

input `Int[(a + b*ArcSin[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]`

output `(a + b*ArcSin[c*x])/(3*d*(d - c^2*d*x^2)^(3/2)) - (b*c*Sqrt[1 - c^2*x^2]*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (a + b*ArcSin[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2])/d`

Definitions of rubi rules used

- rule 215 $\text{Int}[(a_ + (b_ \cdot x_)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(-x) \cdot ((a + b \cdot x^2)^{p+1}) / (2 \cdot a \cdot (p+1)), x] + \text{Simp}[(2 \cdot p + 3) / (2 \cdot a \cdot (p+1)) \text{Int}[(a + b \cdot x^2)^{p+1}], x], x] /;$ FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6*p])
- rule 219 $\text{Int}[(a_ + (b_ \cdot x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x / \text{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
- rule 2715 $\text{Int}[\text{Log}[(a_ + (b_ \cdot (F_)^{(e_ \cdot ((c_ + (d_ \cdot x_))))^{n_})}], x_Symbol] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))^{n}})], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
- rule 2838 $\text{Int}[\text{Log}[(c_ \cdot ((d_ + (e_ \cdot x_)^{n_})) / (x_)], x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n / n], x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ FunctionOfTrigOfLinearQ[u, x]
- rule 4671 $\text{Int}[\text{csc}[(e_ + (f_ \cdot x_)] \cdot ((c_ + (d_ \cdot x_))^{m_}), x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{(I \cdot (e + f \cdot x))}] / f), x] + (-\text{Simp}[d \cdot (m/f) \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 - E^{(I \cdot (e + f \cdot x))}], x], x] + \text{Simp}[d \cdot (m/f) \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + E^{(I \cdot (e + f \cdot x))}], x], x]) /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.74

method	result
default	$\frac{a}{3d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{a}{d^2\sqrt{-c^2dx^2+d}} - \frac{a \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}} - \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\left(-8i\sqrt{-c^2x^2+1}\arcsin(cx)\right)}{d^{\frac{5}{2}}}$
parts	$\frac{a}{3d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{a}{d^2\sqrt{-c^2dx^2+d}} - \frac{a \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}} - \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\left(-8i\sqrt{-c^2x^2+1}\arcsin(cx)\right)}{d^{\frac{5}{2}}}$

input

```
int((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*a/d/(-c^2*d*x^2+d)^(3/2)+a/d^2/(-c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+
2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^
2+1)^(1/2)*(-8*I*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+6*dilog(1+I*c*x+(-c^2*x^2+
1)^(1/2))*c^4*x^4+6*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+14*arctan(I*c*
x+(-c^2*x^2+1)^(1/2))*c^4*x^4+6*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2
)))-I*x^3*c^3-12*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^2*c^2-12*di
log(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-12*dilog(I*c*x+(-c^2*x^2+1)^(1/2))
*c^2*x^2-28*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2+6*I*arcsin(c*x)*(-c^2
*x^2+1)^(1/2)*x^2*c^2+6*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^4*c
^4+I*c*x+6*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))+6*dilog(I*c*x+(-c^2*x^2+1)^(1
/2))+14*arctan(I*c*x+(-c^2*x^2+1)^(1/2)))/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/
d^3
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2} x} dx$$

input

```
integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^
3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \arcsin(cx)}{x(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input

```
integrate((a+b*asin(c*x))/x/(-c**2*d*x**2+d)**(5/2),x)
```

output

```
Integral((a + b*asin(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2} x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `-1/3*a*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2) - 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx$$

input `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^(5/2)),x)`

output `int((a + b*asin(c*x))/(x*(d - c^2*d*x^2)^(5/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^5 - 2\sqrt{-c^2 x^2 + 1} c^2 x^3 + \sqrt{-c^2 x^2 + 1} x} dx \right) b c^2 x^2 - 3\sqrt{-c^2 x^2 + 1}}$$

input `int((a+b*asin(c*x))/x/(-c^2*d*x^2+d)^(5/2),x)`

output `(3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**5 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**3 + sqrt(-c**2*x**2 + 1)*x),x)*b*c**2*x**2 - 3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**5 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**3 + sqrt(-c**2*x**2 + 1)*x),x)*b + 3*sqrt(-c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 3*sqrt(-c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a - 4*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 + 4*sqrt(-c**2*x**2 + 1)*a + 3*a*c**2*x**2 - 4*a)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))`

3.135
$$\int \frac{a+b \arcsin(cx)}{x^2(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1243
Mathematica [A] (verified)	1244
Rubi [A] (verified)	1244
Maple [C] (verified)	1246
Fricas [F]	1247
Sympy [F]	1248
Maxima [F]	1248
Giac [F(-2)]	1248
Mupad [F(-1)]	1249
Reduce [F]	1249

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{a + b \arcsin(cx)}{x^2(d - c^2dx^2)^{5/2}} dx = -\frac{bc}{6d^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}} + \frac{a + b \arcsin(cx)}{3dx(d - c^2dx^2)^{3/2}}$$

$$+ \frac{4(a + b \arcsin(cx))}{3d^2x\sqrt{d - c^2dx^2}} - \frac{8\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{3d^3x}$$

$$+ \frac{bc\sqrt{1 - c^2x^2} \log(x)}{d^2\sqrt{d - c^2dx^2}} + \frac{5bc\sqrt{1 - c^2x^2} \log(1 - c^2x^2)}{6d^2\sqrt{d - c^2dx^2}}$$

output

```
-1/6*b*c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x))
/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*(a+b*arcsin(c*x))/d^2/x/(-c^2*d*x^2+d)^(1/2)
-8/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d^3/x+b*c*(-c^2*x^2+1)^(1/2)*l
n(x)/d^2/(-c^2*d*x^2+d)^(1/2)+5/6*b*c*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/d^
2/(-c^2*d*x^2+d)^(1/2)
```


Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \frac{\sqrt{d - c^2 dx^2} (bcx - bc^3 x^3 + 6a\sqrt{1 - c^2 x^2} - 24ac^2 x^2 \sqrt{1 - c^2 x^2} + 16ac^4 x^4 \sqrt{1 - c^2 x^2} + 2b\sqrt{1 - c^2 x^2} (3 - 12c^2 x^2 + 8c^4 x^4) \operatorname{ArcSin}[cx] + 3bcx(-1 + c^2 x^2)^2 \operatorname{Log}[1 - 1/(c^2 x^2)] - 8bcx \operatorname{Log}[1 - c^2 x^2] + 16bc^3 x^3 \operatorname{Log}[1 - c^2 x^2] - 8bc^5 x^5 \operatorname{Log}[1 - c^2 x^2])}{d^3 x (1 - c^2 x^2)^{5/2}}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]`

output `-1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x - b*c^3*x^3 + 6*a*Sqrt[1 - c^2*x^2] - 24*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 16*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 2*b*Sqrt[1 - c^2*x^2]*(3 - 12*c^2*x^2 + 8*c^4*x^4)*ArcSin[c*x] + 3*b*c*x*(-1 + c^2*x^2)^2*Log[1 - 1/(c^2*x^2)] - 8*b*c*x*Log[1 - c^2*x^2] + 16*b*c^3*x^3*Log[1 - c^2*x^2] - 8*b*c^5*x^5*Log[1 - c^2*x^2]))/(d^3*x*(1 - c^2*x^2)^(5/2))`

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx$$

↓ 5194

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{8c^4 x^4 - 12c^2 x^2 + 3}{3d^3 x(1 - c^2 x^2)^2} dx}{\sqrt{1 - c^2 x^2}} + \frac{8c^2 x(a + b \arcsin(cx))}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{4c^2 x(a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}} - \frac{a + b \arcsin(cx)}{dx (d - c^2 dx^2)^{3/2}}$$

↓ 27

$$\begin{aligned}
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{8c^4x^4-12c^2x^2+3}{x(1-c^2x^2)^2} dx}{3d^3\sqrt{1-c^2x^2}} + \frac{8c^2x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{4c^2x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} - \\
& \quad \frac{a+b\arcsin(cx)}{dx(d-c^2dx^2)^{3/2}} \\
& \quad \downarrow \text{1578} \\
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{8c^4x^4-12c^2x^2+3}{x^2(1-c^2x^2)^2} dx^2}{6d^3\sqrt{1-c^2x^2}} + \frac{8c^2x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{4c^2x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} - \\
& \quad \frac{a+b\arcsin(cx)}{dx(d-c^2dx^2)^{3/2}} \\
& \quad \downarrow \text{1195} \\
& \frac{bc\sqrt{d-c^2dx^2} \int \left(\frac{5c^2}{c^2x^2-1} - \frac{c^2}{(c^2x^2-1)^2} + \frac{3}{x^2} \right) dx^2}{6d^3\sqrt{1-c^2x^2}} + \frac{8c^2x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \\
& \quad \frac{4c^2x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} - \frac{a+b\arcsin(cx)}{dx(d-c^2dx^2)^{3/2}} \\
& \quad \downarrow \text{2009} \\
& \frac{8c^2x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{4c^2x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} - \frac{a+b\arcsin(cx)}{dx(d-c^2dx^2)^{3/2}} + \\
& \quad \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{c^2x^2-1} + 5\log(1-c^2x^2) + 3\log(x^2) \right)}{6d^3\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]`

output `-((a + b*ArcSin[c*x])/(d*x*(d - c^2*d*x^2)^(3/2))) + (4*c^2*x*(a + b*ArcSin[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcSin[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((-1 + c^2*x^2)^(-1) + 3*Log[x^2] + 5*Log[1 - c^2*x^2]))/(6*d^3*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 1195 `Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

rule 1578 `Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 1346, normalized size of antiderivative = 6.01

method	result	size
default	Expression too large to display	1346
parts	Expression too large to display	1346

input `int((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```

a*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2
*x/(-c^2*d*x^2+d)^(1/2))-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x
^4+26*c^2*x^2-9)/d^3*x^2*(-c^2*x^2+1)^(1/2)*c^3+56*b*(-d*(c^2*x^2-1))^(1/2
)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^3*arcsin(c*x)*c^4-44*b*(-d*(c^
2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*arcsin(c*x)*c^2-
b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)*ln((I*c*x+(-c^
2*x^2+1)^(1/2))^2-1)*c+20*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4
+26*c^2*x^2-9)/d^3*x^3*(-c^2*x^2+1)*c^4-4*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^
6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*(-c^2*x^2+1)*c^2+32/3*I*b*(-d*(c^2*x^
2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^9*c^10-64/3*I*b*(-d*
(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^4*arcsin(c*x)
*(-c^2*x^2+1)^(1/2)*c^5-24*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^
4+26*c^2*x^2-9)/d^3*x^3*c^4+4*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4
*x^4+26*c^2*x^2-9)/d^3*x*c^2-80/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25
*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*(-c^2*x^2+1)*c^6-112/3*I*b*(-d*(c^2*x^2-1))
^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^7*c^8-24*I*b*(-d*(c^2*x^2
-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*arcsin(c*x)*(-c^2*x^2+1
)^(1/2)*c+136/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)/d^3*x^2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^3+32/3*I*b*(-d*(c^2*x^2-1))^(
1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^7*(-c^2*x^2+1)*c^8+9*b*...

```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2} x^2} dx$$

input

```

integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)

```

output

```

integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)

```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (-d(cx - 1)(cx + 1))^{\frac{5}{2}}} dx$$

input `integrate((a+b*asin(c*x))/x**2/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{\frac{5}{2}} x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/3*a*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/2)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx$$

input

```
int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

output

```
int((a + b*asin(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^6 - 2\sqrt{-c^2 x^2 + 1} c^2 x^4 + \sqrt{-c^2 x^2 + 1} x^2} dx \right) b c^2 x^3 - 3\sqrt{-c^2 x^2 + 1}}{3\sqrt{d} \sqrt{-c^2 x^2 + 1}}$$

input

```
int((a+b*asin(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**6
- 2*sqrt(-c**2*x**2 + 1)*c**2*x**4 + sqrt(-c**2*x**2 + 1)*x**2),x)*b*c
**2*x**3 - 3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*
c**4*x**6 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**4 + sqrt(-c**2*x**2 + 1)*x*
*2),x)*b*x + 8*a*c**4*x**4 - 12*a*c**2*x**2 + 3*a)/(3*sqrt(d)*sqrt(-c**2
*x**2 + 1)*d**2*x*(c**2*x**2 - 1))
```

3.136 $\int \frac{a+b \arcsin(cx)}{x^3(d-c^2dx^2)^{5/2}} dx$

Optimal result	1250
Mathematica [A] (verified)	1251
Rubi [A] (verified)	1252
Maple [A] (verified)	1258
Fricas [F]	1258
Sympy [F]	1259
Maxima [F]	1259
Giac [F(-2)]	1260
Mupad [F(-1)]	1260
Reduce [F]	1260

Optimal result

Integrand size = 27, antiderivative size = 392

$$\begin{aligned} \int \frac{a+b \arcsin(cx)}{x^3(d-c^2dx^2)^{5/2}} dx = & -\frac{bc^3x}{6d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{1-c^2x^2}}{2d^2x\sqrt{d-c^2dx^2}} \\ & + \frac{5c^2(a+b \arcsin(cx))}{6d(d-c^2dx^2)^{3/2}} - \frac{a+b \arcsin(cx)}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{5c^2(a+b \arcsin(cx))}{2d^2\sqrt{d-c^2dx^2}} \\ & - \frac{5c^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)})}{d^2\sqrt{d-c^2dx^2}} \\ & - \frac{13bc^2\sqrt{1-c^2x^2}\operatorname{arctanh}(cx)}{6d^2\sqrt{d-c^2dx^2}} + \frac{5ibc^2\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,-e^{i \arcsin(cx)})}{2d^2\sqrt{d-c^2dx^2}} \\ & - \frac{5ibc^2\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,e^{i \arcsin(cx)})}{2d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
-1/6*b*c^3*x/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*c*(-c^2*x^2+1)^(1/2)/d^2/x/(-c^2*d*x^2+d)^(1/2)+5/6*c^2*(a+b*arcsin(c*x))/d/(-c^2*d*x^2+d)^(3/2)-1/2*(a+b*arcsin(c*x))/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/2*c^2*(a+b*arcsin(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-5*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-13/6*b*c^2*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/d^2/(-c^2*d*x^2+d)^(1/2)+5/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 7.30 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.37

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \sqrt{-d(-1 + c^2 x^2)} \left(-\frac{a}{2d^3 x^2} + \frac{ac^2}{3d^3 (-1 + c^2 x^2)^2} - \frac{2ac^2}{d^3 (-1 + c^2 x^2)} \right) + \frac{5ac^2 \log(x)}{2d^{5/2}} - \frac{5ac^2 \log\left(d + \sqrt{d} \sqrt{-d(-1 + c^2 x^2)}\right)}{2d^{5/2}} + \frac{bc^2 \sqrt{1 - c^2 x^2} \left(-\frac{2(-1 + \arcsin(cx))}{-1 + cx} + 52 \arcsin(cx) - 6 \cot\left(\frac{1}{2} \arcsin(cx)\right) - 3 \arcsin(cx) \csc^2\left(\frac{1}{2} \arcsin(cx)\right) \right)}{2d^{5/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```


output

```

Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a/(d^3*x^2) + (a*c^2)/(3*d^3*(-1 + c^2*x^2)^2) - (2*a*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a*c^2*Log[x])/(2*d^(5/2)) - (5*a*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) + (b*c^2*Sqrt[1 - c^2*x^2]*((-2*(-1 + ArcSin[c*x])))/(-1 + c*x) + 52*ArcSin[c*x] - 6*Cos[ArcSin[c*x]/2] - 3*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 + 60*ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c*x])]) + 52*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - 52*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] + (60*I)*(PolyLog[2, -E^(I*ArcSin[c*x])] - PolyLog[2, E^(I*ArcSin[c*x])]) + 3*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 + (4*ArcSin[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2])^3 + (52*ArcSin[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]) - (4*ArcSin[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2])^3 + (2*(1 + ArcSin[c*x]))/(Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2])^2 - (52*ArcSin[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]) - 6*Tan[ArcSin[c*x]/2])/(24*d^2*Sqrt[d*(1 - c^2*x^2)])

```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {5204, 253, 264, 219, 5208, 215, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx \\
& \quad \downarrow \text{5204} \\
& \frac{5}{2} c^2 \int \frac{a + b \arcsin(cx)}{x (d - c^2 dx^2)^{5/2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{x^2 (1 - c^2 x^2)^2} dx}{2d^2 \sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} \\
& \quad \downarrow \text{253} \\
& \frac{5}{2} c^2 \int \frac{a + b \arcsin(cx)}{x (d - c^2 dx^2)^{5/2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{3}{2} \int \frac{1}{x^2 (1 - c^2 x^2)} dx + \frac{1}{2x(1 - c^2 x^2)} \right)}{2d^2 \sqrt{d - c^2 dx^2}} - \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} \\
& \quad \downarrow \text{264}
\end{aligned}$$

$$\begin{aligned}
& \frac{5}{2}c^2 \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{3}{2} \left(c^2 \int \frac{1}{1 - c^2 x^2} dx - \frac{1}{x} \right) + \frac{1}{2x(1 - c^2 x^2)} \right)}{2d^2 \sqrt{d - c^2 dx^2}} - \\
& \quad \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} \\
& \quad \downarrow \text{219} \\
& \frac{5}{2}c^2 \int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{5/2}} dx - \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} + \\
& \quad \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1 - c^2 x^2)} \right)}{2d^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5208} \\
& \frac{5}{2}c^2 \left(\frac{\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx}{d} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{1}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3d(d - c^2 dx^2)^{3/2}} \right) - \\
& \quad \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} + \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1 - c^2 x^2)} \right)}{2d^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{215} \\
& \frac{5}{2}c^2 \left(\frac{\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx}{d} - \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{1}{2} \int \frac{1}{1 - c^2 x^2} dx + \frac{x}{2(1 - c^2 x^2)} \right)}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{a + b \arcsin(cx)}{3d(d - c^2 dx^2)^{3/2}} \right) - \\
& \quad \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} + \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1 - c^2 x^2)} \right)}{2d^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{219} \\
& \frac{5}{2}c^2 \left(\frac{\int \frac{a + b \arcsin(cx)}{x(d - c^2 dx^2)^{3/2}} dx}{d} + \frac{a + b \arcsin(cx)}{3d(d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1 - c^2 x^2)} \right)}{3d^2 \sqrt{d - c^2 dx^2}} \right) - \\
& \quad \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2 dx^2)^{3/2}} + \frac{bc\sqrt{1 - c^2 x^2} \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1 - c^2 x^2)} \right)}{2d^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5208}
\end{aligned}$$

$$\frac{5}{2}c^2 \left(\frac{\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2dx^2}} dx - \frac{bc\sqrt{1-c^2x^2} \int \frac{1}{1-c^2x^2} dx}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}}}{d} + \frac{a+b \arcsin(cx)}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{1}{2x} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \\ + \frac{a+b \arcsin(cx)}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{1-c^2x^2} \left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d-c^2dx^2}}$$

↓ 219

$$\frac{5}{2}c^2 \left(\frac{\int \frac{a+b \arcsin(cx)}{x\sqrt{d-c^2dx^2}} dx + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d} + \frac{a+b \arcsin(cx)}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{1}{2x} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \\ + \frac{a+b \arcsin(cx)}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{1-c^2x^2} \left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d-c^2dx^2}}$$

↓ 5218

$$\frac{5}{2}c^2 \left(\frac{\frac{\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d} + \frac{a+b \arcsin(cx)}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{1}{2x} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \\ + \frac{a+b \arcsin(cx)}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{1-c^2x^2} \left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d-c^2dx^2}}$$

↓ 3042

$$\frac{5}{2}c^2 \left(\frac{\frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2} \operatorname{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d} + \frac{a+b \arcsin(cx)}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{1}{2x} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \\ + \frac{a+b \arcsin(cx)}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{1-c^2x^2} \left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d-c^2dx^2}}$$

↓ 4671

$$\frac{5}{2}c^2 \left(\frac{\frac{\sqrt{1-c^2x^2} \left(-b \int \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) \right)}{d\sqrt{d-c^2dx^2}}}{d} + \frac{a+b \arcsin(cx)}{3d(d-c^2dx^2)^{3/2}} - \frac{bc\sqrt{1-c^2x^2} \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{1}{2x} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \\ + \frac{a+b \arcsin(cx)}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{1-c^2x^2} \left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{5}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} \left(ib \int e^{-i \arcsin(cx)} \log(1-e^{i \arcsin(cx)}) dx e^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+e^{i \arcsin(cx)}) dx e^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right)}{d\sqrt{d-c^2dx^2}} \right) + \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2dx^2)^{3/2}} + \frac{bc\sqrt{1 - c^2x^2} \left(\frac{3}{2} (\operatorname{arctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d - c^2dx^2}}$$

↓ 2838

$$\frac{5}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)}{d\sqrt{d-c^2dx^2}} \right) + \frac{a + b \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{a + b \arcsin(cx)}{2dx^2 (d - c^2dx^2)^{3/2}} + \frac{bc\sqrt{1 - c^2x^2} \left(\frac{3}{2} (\operatorname{arctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right)}{2d^2\sqrt{d - c^2dx^2}}$$

input `Int[(a + b*ArcSin[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]`

output `-1/2*(a + b*ArcSin[c*x])/(d*x^2*(d - c^2*d*x^2)^(3/2)) + (b*c*Sqrt[1 - c^2*x^2]*(1/(2*x*(1 - c^2*x^2)) + (3*(-x^(-1) + c*ArcTanh[c*x]))/2))/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^2*((a + b*ArcSin[c*x])/(3*d*(d - c^2*d*x^2)^(3/2)) - (b*c*Sqrt[1 - c^2*x^2]*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((a + b*ArcSin[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2]))/d)/2`

Definitions of rubi rules used

rule 215 $\text{Int}[(a_ + (b_ \cdot x)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(-x) \cdot ((a + b \cdot x^2)^{p+1} / (2 \cdot a \cdot (p+1))), x] + \text{Simp}[(2 \cdot p + 3) / (2 \cdot a \cdot (p+1)) \text{Int}[(a + b \cdot x^2)^{p+1}, x], x] /;$ FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6*p])

rule 219 $\text{Int}[(a_ + (b_ \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x / \text{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

rule 253 $\text{Int}[(c_ \cdot x)^{m_} \cdot (a_ + (b_ \cdot x)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(-c \cdot x)^{m+1} \cdot ((a + b \cdot x^2)^{p+1} / (2 \cdot a \cdot c \cdot (p+1))), x] + \text{Simp}[(m + 2 \cdot p + 3) / (2 \cdot a \cdot (p+1)) \text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^2)^{p+1}, x], x] /;$ FreeQ[{a, b, c, m}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]

rule 264 $\text{Int}[(c_ \cdot x)^{m_} \cdot (a_ + (b_ \cdot x)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m+1} \cdot ((a + b \cdot x^2)^{p+1} / (a \cdot c \cdot (m+1))), x] - \text{Simp}[b \cdot ((m + 2 \cdot p + 3) / (a \cdot c^{2 \cdot (m+1)})) \text{Int}[(c \cdot x)^{m+2} \cdot (a + b \cdot x^2)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]

rule 2715 $\text{Int}[\text{Log}[(a_ + (b_ \cdot x)^2)^{n_}], x_Symbol] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{e \cdot (c + d \cdot x)})^n], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

rule 2838 $\text{Int}[\text{Log}[(c_ \cdot x)^{n_}], x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n] / n, x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ FunctionOfTrigOfLinearQ[u, x]

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.54

method	result
default	$-\frac{a}{2dx^2(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{5ac^2}{6d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{5ac^2}{2d^2\sqrt{-c^2dx^2+d}} - \frac{5ac^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{2d^{\frac{5}{2}}} - \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2d}}{2d^{\frac{5}{2}}}$
parts	$-\frac{a}{2dx^2(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{5ac^2}{6d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{5ac^2}{2d^2\sqrt{-c^2dx^2+d}} - \frac{5ac^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{2d^{\frac{5}{2}}} - \frac{ib\sqrt{-d(c^2x^2-1)}\sqrt{-c^2d}}{2d^{\frac{5}{2}}}$

input `int((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```

-1/2*a/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2*a*c^2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(-30*I*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*arcsin(c*x)*x^4*c^4+15*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^6*x^6+15*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^6*x^6+26*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^6*x^6-5*I*x^3*c^3+2*I*x^5*c^5+15*I*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*arcsin(c*x)*x^2*c^2-30*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4-30*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4-52*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*I*(-c^2*x^2+1)^(1/2)*arcsin(c*x)+15*I*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*x^4*c^4-20*I*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*x^2*c^2+15*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2+15*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2+26*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2+15*I*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*arcsin(c*x)*x^6*c^6+3*I*c*x/d^3/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/x^2
    
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2} x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \arcsin(cx)}{x^3 (-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))/x**3/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2} x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `-1/6*a*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/((c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx$$

input `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)),x)`

output `int((a + b*asin(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d - c^2 dx^2)^{5/2}} dx = \frac{24\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^7 - 2\sqrt{-c^2 x^2 + 1} c^2 x^5 + \sqrt{-c^2 x^2 + 1} x^3} dx \right) b c^2 x^4 - 24\sqrt{-c^2 x^2 + 1}}$$

input `int((a+b*asin(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x)`

output

```
(24*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**7
- 2*sqrt(-c**2*x**2 + 1)*c**2*x**5 + sqrt(-c**2*x**2 + 1)*x**3),x)*b*
c**2*x**4 - 24*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)
)*c**4*x**7 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**5 + sqrt(-c**2*x**2 + 1)*
x**3),x)*b*x**2 + 60*sqrt(-c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**4*x
**4 - 60*sqrt(-c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 65*sq
rt(-c**2*x**2 + 1)*a*c**4*x**4 + 65*sqrt(-c**2*x**2 + 1)*a*c**2*x**2 +
60*a*c**4*x**4 - 80*a*c**2*x**2 + 12*a)/(24*sqrt(d)*sqrt(-c**2*x**2 + 1)
*d**2*x**2*(c**2*x**2 - 1))
```

3.137
$$\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1262
Mathematica [A] (verified)	1263
Rubi [A] (verified)	1263
Maple [C] (verified)	1265
Fricas [F]	1266
Sympy [F(-1)]	1267
Maxima [A] (verification not implemented)	1267
Giac [F(-2)]	1268
Mupad [F(-1)]	1268
Reduce [F]	1268

Optimal result

Integrand size = 27, antiderivative size = 308

$$\int \frac{a+b \arcsin(cx)}{x^4(d-c^2dx^2)^{5/2}} dx = -\frac{bc^3}{6d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{1-c^2x^2}}{6d^2x^2\sqrt{d-c^2dx^2}} + \frac{a+b \arcsin(cx)}{3dx^3(d-c^2dx^2)^{3/2}} + \frac{2(a+b \arcsin(cx))}{d^2x^3\sqrt{d-c^2dx^2}} - \frac{8\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{3d^3x^3} - \frac{16c^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{3d^3x} + \frac{8bc^3\sqrt{1-c^2x^2} \log(x)}{3d^2\sqrt{d-c^2dx^2}} + \frac{4bc^3\sqrt{1-c^2x^2} \log(1-c^2x^2)}{3d^2\sqrt{d-c^2dx^2}}$$

output

```
-1/6*b*c^3/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/6*b*c*(-c^2*x^2+1)^(1/2)/d^2/x^2/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x))/d/x^3/(-c^2*d*x^2+d)^(3/2)+2*(a+b*arcsin(c*x))/d^2/x^3/(-c^2*d*x^2+d)^(1/2)-8/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d^3/x^3-16/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/d^3/x+8/3*b*c^3*(-c^2*x^2+1)^(1/2)*ln(x)/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*b*c^3*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.90

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx =$$

$$\frac{\sqrt{d - c^2 dx^2} (bcx - bc^3 x^3 + 2a\sqrt{1 - c^2 x^2} + 12ac^2 x^2 \sqrt{1 - c^2 x^2} - 48ac^4 x^4 \sqrt{1 - c^2 x^2} + 32ac^6 x^6 \sqrt{1 - c^2 x^2})}{d^3 x^3 (1 - c^2 x^2)^{5/2}}$$

input `Integrate[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]`

output `-1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x - b*c^3*x^3 + 2*a*Sqrt[1 - c^2*x^2] + 12*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 48*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 32*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 2*b*Sqrt[1 - c^2*x^2]*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6)*ArcSin[c*x] + 8*b*c^3*x^3*(-1 + c^2*x^2)^2*Log[1 - 1/(c^2*x^2)] - 16*b*c^3*x^3*Log[1 - c^2*x^2] + 32*b*c^5*x^5*Log[1 - c^2*x^2] - 16*b*c^7*x^7*Log[1 - c^2*x^2]))/(d^3*x^3*(1 - c^2*x^2)^(5/2))`

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5194, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx$$

↓ 5194

$$-\frac{bc\sqrt{d - c^2 dx^2} \int -\frac{16c^6 x^6 - 24c^4 x^4 + 6c^2 x^2 + 1}{3d^3 x^3 (1 - c^2 x^2)^2} dx}{\sqrt{1 - c^2 x^2}} - \frac{2c^2(a + b \arcsin(cx))}{dx (d - c^2 dx^2)^{3/2}} - \frac{a + b \arcsin(cx)}{3dx^3 (d - c^2 dx^2)^{3/2}} +$$

$$\frac{16c^4 x(a + b \arcsin(cx))}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{8c^4 x(a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}}$$

↓ 27

$$\begin{aligned}
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{x^3(1-c^2x^2)^2} dx}{3d^3\sqrt{1-c^2x^2}} - \frac{2c^2(a+b\arcsin(cx))}{dx(d-c^2dx^2)^{3/2}} - \frac{a+b\arcsin(cx)}{3dx^3(d-c^2dx^2)^{3/2}} + \\
& \frac{16c^4x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{8c^4x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} \\
& \quad \downarrow \text{2331} \\
& \frac{bc\sqrt{d-c^2dx^2} \int \frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{x^4(1-c^2x^2)^2} dx^2}{6d^3\sqrt{1-c^2x^2}} - \frac{2c^2(a+b\arcsin(cx))}{dx(d-c^2dx^2)^{3/2}} - \frac{a+b\arcsin(cx)}{3dx^3(d-c^2dx^2)^{3/2}} + \\
& \frac{16c^4x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{8c^4x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} \\
& \quad \downarrow \text{2123} \\
& \frac{bc\sqrt{d-c^2dx^2} \int \left(\frac{8c^4}{c^2x^2-1} - \frac{c^4}{(c^2x^2-1)^2} + \frac{8c^2}{x^2} + \frac{1}{x^4} \right) dx^2}{6d^3\sqrt{1-c^2x^2}} - \frac{2c^2(a+b\arcsin(cx))}{dx(d-c^2dx^2)^{3/2}} - \\
& \frac{a+b\arcsin(cx)}{3dx^3(d-c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{8c^4x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} \\
& \quad \downarrow \text{2009} \\
& -\frac{2c^2(a+b\arcsin(cx))}{dx(d-c^2dx^2)^{3/2}} - \frac{a+b\arcsin(cx)}{3dx^3(d-c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}} + \\
& \frac{8c^4x(a+b\arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} + \frac{bc\sqrt{d-c^2dx^2} \left(-\frac{c^2}{1-c^2x^2} + 8c^2 \log(x^2) + 8c^2 \log(1-c^2x^2) - \frac{1}{x^2} \right)}{6d^3\sqrt{1-c^2x^2}}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output

```
-1/3*(a + b*ArcSin[c*x])/(d*x^3*(d - c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*ArcSin[c*x]))/(d*x*(d - c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcSin[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcSin[c*x]))/(3*d^2*sqrt[d - c^2*d*x^2]) + (b*c*sqrt[d - c^2*d*x^2]*(-x^(-2) - c^2/(1 - c^2*x^2) + 8*c^2*Log[x^2] + 8*c^2*Log[1 - c^2*x^2]))/(6*d^3*sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2123 `Int[(Px_)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2])`

rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`

rule 5194 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 1877, normalized size of antiderivative = 6.09

method	result	size
default	Expression too large to display	1877
parts	Expression too large to display	1877

input `int((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```

-320/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2
*x^2-1)*x^7/d^3*(-c^2*x^2+1)*c^10+128/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8
*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^9/d^3*(-c^2*x^2+1)*c^12-8/3*I*b
*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x/
d^3*(-c^2*x^2+1)*c^4-16/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^
6+35*c^4*x^4-10*c^2*x^2-1)/d^3*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^3+32/3*I*b
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)*arcsin(c*x)*c^3
+80*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^
2-1)*x^5/d^3*(-c^2*x^2+1)*c^8-40/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-
36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^3/d^3*(-c^2*x^2+1)*c^6+128*I*b*(-d*(
c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^4/d^3*
(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^7-176/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^
8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^2/d^3*(-c^2*x^2+1)^(1/2)*arcsi
n(c*x)*c^5-64*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4
-10*c^2*x^2-1)*x^6/d^3*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^9-344/3*b*(-d*(c^2
*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^3/d^3*arc
sin(c*x)*c^6-2*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-
10*c^2*x^2-1)*x^2/d^3*c^5*(-c^2*x^2+1)^(1/2)-8/3*b*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^4-1)*c^3+
128/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c...

```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx = \int \frac{b \arcsin(cx) + a}{(-c^2 dx^2 + d)^{5/2} x^4} dx$$

input

```

integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)

```

output

```

integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d
^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx = \text{Timed out}$$

input `integrate((a+b*asin(c*x))/x**4/(-c**2*d*x**2+d)**(5/2),x)`

output Timed out

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.83

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx = \frac{1}{6} bc \left(\frac{8 c^2 \log(cx + 1)}{d^{5/2}} + \frac{8 c^2 \log(cx - 1)}{d^{5/2}} + \frac{16 c^2 \log(x)}{d^{5/2}} + \frac{1}{c^2 d^{5/2} x^4 - d^{5/2} x^2} \right) \\ + \frac{1}{3} \left(\frac{16 c^4 x}{\sqrt{-c^2 dx^2 + dd^2}} + \frac{8 c^4 x}{(-c^2 dx^2 + d)^{3/2} d} - \frac{6 c^2}{(-c^2 dx^2 + d)^{3/2} dx} - \frac{1}{(-c^2 dx^2 + d)^{3/2} dx^3} \right) b \arcsin(cx) \\ + \frac{1}{3} \left(\frac{16 c^4 x}{\sqrt{-c^2 dx^2 + dd^2}} + \frac{8 c^4 x}{(-c^2 dx^2 + d)^{3/2} d} - \frac{6 c^2}{(-c^2 dx^2 + d)^{3/2} dx} - \frac{1}{(-c^2 dx^2 + d)^{3/2} dx^3} \right) a$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/6*b*c*(8*c^2*log(c*x + 1)/d^(5/2) + 8*c^2*log(c*x - 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2*d^(5/2)*x^4 - d^(5/2)*x^2)) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*b*arcsin(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*a`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx$$

input `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)),x)`

output `int((a + b*asin(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^8 - 2\sqrt{-c^2 x^2 + 1} c^2 x^6 + \sqrt{-c^2 x^2 + 1} x^4} dx \right) b c^2 x^5 - 3\sqrt{-c^2 x^2 + 1}}{3\sqrt{d} \sqrt{-c^2 x^2 + 1}}$$

input `int((a+b*asin(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x)`

output

```
(3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**8
- 2*sqrt(-c**2*x**2 + 1)*c**2*x**6 + sqrt(-c**2*x**2 + 1)*x**4),x)*b*c
**2*x**5 - 3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*
c**4*x**8 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**6 + sqrt(-c**2*x**2 + 1)*x*
**4),x)*b*x**3 + 16*a*c**6*x**6 - 24*a*c**4*x**4 + 6*a*c**2*x**2 + a)/(3*sq
rt(d)*sqrt(-c**2*x**2 + 1)*d**2*x**3*(c**2*x**2 - 1))
```

3.138
$$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx$$

Optimal result	1270
Mathematica [A] (verified)	1270
Rubi [A] (verified)	1271
Maple [F]	1272
Fricas [F]	1272
Sympy [F]	1273
Maxima [F]	1273
Giac [F]	1273
Mupad [F(-1)]	1274
Reduce [F]	1274

Optimal result

Integrand size = 30, antiderivative size = 79

$$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx = \frac{2(fx)^{5/2}(a+b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2x^2\right)}{5f} - \frac{4bc(fx)^{7/2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}; \frac{9}{4}, \frac{11}{4}; c^2x^2\right)}{35f^2}$$

output

```
2/5*(f*x)^(5/2)*(a+b*arcsin(c*x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/f-4/35*b*c*(f*x)^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)/f^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx = \frac{2}{35}x(fx)^{3/2} \left(7(a+b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2x^2\right) - 2 \right)$$

input

```
Integrate[((f*x)^(3/2)*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2],x]
```

output

```
(2*x*(f*x)^(3/2)*(7*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c
^2*x^2] - 2*b*c*x*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2]))
/35
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx$$

↓ 5220

$$\frac{2(fx)^{5/2} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2x^2\right) (a + b \arcsin(cx))}{5f} - \frac{4bc(fx)^{7/2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}; c^2x^2\right)}{35f^2}$$

input

```
Int[((f*x)^(3/2)*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2],x]
```

output

```
(2*(f*x)^(5/2)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^
2])/(5*f) - (4*b*c*(f*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4
}, c^2*x^2])/(35*f^2)
```

Definitions of rubi rules used

rule 5220

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}}(a + b \arcsin(cx))}{\sqrt{-c^2x^2 + 1}} dx$$

input

```
int((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2),x)
```

output

```
int((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(b \arcsin(cx) + a)}{\sqrt{-c^2x^2 + 1}} dx$$

input

```
integrate((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*(b*f*x*arcsin(c*x) + a*f*x)*sqrt(f*x)/(c^2*x^
2 - 1), x)
```

Sympy [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(a + b \arcsin(cx))}{\sqrt{-(cx - 1)(cx + 1)}} dx$$

input `integrate((f*x)**(3/2)*(a+b*asin(c*x))/(-c**2*x**2+1)**(1/2),x)`

output `Integral((f*x)**(3/2)*(a + b*asin(c*x))/sqrt(-(c*x - 1)*(c*x + 1)), x)`

Maxima [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(b \arcsin(cx) + a)}{\sqrt{-c^2x^2 + 1}} dx$$

input `integrate((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate((f*x)^(3/2)*(b*arcsin(c*x) + a)/sqrt(-c^2*x^2 + 1), x)`

Giac [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(b \arcsin(cx) + a)}{\sqrt{-c^2x^2 + 1}} dx$$

input `integrate((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate((f*x)^(3/2)*(b*arcsin(c*x) + a)/sqrt(-c^2*x^2 + 1), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx = \int \frac{(a + b \arcsin(cx)) (fx)^{3/2}}{\sqrt{1 - c^2x^2}} dx$$

input `int(((a + b*asin(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2),x)`

output `int(((a + b*asin(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx = \frac{\sqrt{f} f \left(-6\sqrt{x} \sqrt{-c^2x^2 + 1} \arcsin(cx) b - 6\sqrt{x} \sqrt{-c^2x^2 + 1} a + 4\sqrt{x} bcx - \right)}{9c^2}$$

input `int((f*x)^(3/2)*(a+b*asin(c*x))/(-c^2*x^2+1)^(1/2),x)`

output `(sqrt(f)*f*(- 6*sqrt(x)*sqrt(- c**2*x**2 + 1)*asin(c*x)*b - 6*sqrt(x)*sqrt(- c**2*x**2 + 1)*a + 4*sqrt(x)*b*c*x - 3*int((sqrt(x)*sqrt(- c**2*x**2 + 1)*asin(c*x))/(c**2*x**3 - x),x)*b - 3*int((sqrt(x)*sqrt(- c**2*x**2 + 1))/(c**2*x**3 - x),x)*a)/(9*c**2)`

3.139
$$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$$

Optimal result	1275
Mathematica [A] (verified)	1275
Rubi [A] (verified)	1276
Maple [F]	1277
Fricas [F]	1277
Sympy [F(-1)]	1278
Maxima [F]	1278
Giac [F]	1278
Mupad [F(-1)]	1279
Reduce [F]	1279

Optimal result

Integrand size = 31, antiderivative size = 137

$$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{2(fx)^{5/2}\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2x^2\right)}{5f\sqrt{d-c^2dx^2}} - \frac{4bc(fx)^{7/2}\sqrt{1-c^2x^2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}; c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}$$

output

```
2/5*(f*x)^(5/2)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*hypergeom([1/2, 5/4], [9/4], c^2*x^2)/f/(-c^2*d*x^2+d)^(1/2)-4/35*b*c*(f*x)^(7/2)*(-c^2*x^2+1)^(1/2)*hypergeom([1, 7/4, 7/4], [9/4, 11/4], c^2*x^2)/f^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

$$\int \frac{(fx)^{3/2}(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx = \frac{2x(fx)^{3/2}\sqrt{1-c^2x^2}(-7(a+b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2x^2\right) + 2bcx {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}; c^2x^2\right))}{35\sqrt{d-c^2dx^2}}$$

input

```
Integrate[((f*x)^(3/2)*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2], x]
```


output

```
(-2*x*(f*x)^(3/2)*Sqrt[1 - c^2*x^2]*(-7*(a + b*ArcSin[c*x])*Hypergeometric
2F1[1/2, 5/4, 9/4, c^2*x^2] + 2*b*c*x*HypergeometricPFQ[{1, 7/4, 7/4}, {9/
4, 11/4}, c^2*x^2]))/(35*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

↓ 5220

$$\frac{2\sqrt{1 - c^2 x^2}(fx)^{5/2} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right) (a + b \arcsin(cx))}{5f\sqrt{d - c^2 dx^2}} - \frac{4bc\sqrt{1 - c^2 x^2}(fx)^{7/2} {}_3F_2\left(1, \frac{7}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}; c^2 x^2\right)}{35f^2\sqrt{d - c^2 dx^2}}$$

input

```
Int[((f*x)^(3/2)*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output

```
(2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) - (4*b*c*(f*x)^(7/2)*Sqrt[
1 - c^2*x^2]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(35*f
^2*Sqrt[d - c^2*d*x^2])
```

Definitions of rubi rules used

rule 5220

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}}(a + b \arcsin(cx))}{\sqrt{-c^2dx^2 + d}} dx$$

input

```
int((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output

```
int((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(b \arcsin(cx) + a)}{\sqrt{-c^2dx^2 + d}} dx$$

input

```
integrate((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"fricas")
```

output

```
integral(-sqrt(-c^2*d*x^2 + d)*(b*f*x*arcsin(c*x) + a*f*x)*sqrt(f*x)/(c^2*
d*x^2 - d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \text{Timed out}$$

input `integrate((f*x)**(3/2)*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output Timed out

Maxima [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(b \arcsin(cx) + a)}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `integrate((f*x)^(3/2)*(b*arcsin(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)`

Giac [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(fx)^{\frac{3}{2}}(b \arcsin(cx) + a)}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate((f*x)^(3/2)*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `integrate((f*x)^(3/2)*(b*arcsin(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \arcsin(cx)) (fx)^{3/2}}{\sqrt{d - c^2 dx^2}} dx$$

input `int(((a + b*asin(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2),x)`

output `int(((a + b*asin(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{(fx)^{3/2}(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{f} \sqrt{d} f \left(-6\sqrt{x} \sqrt{-c^2 x^2 + 1} \arcsin(cx) b - 6\sqrt{x} \sqrt{-c^2 x^2 + 1} a + 4\sqrt{x} b c a \right)}{9c^2 d}$$

input `int((f*x)^(3/2)*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output `(sqrt(f)*sqrt(d)*f*(- 6*sqrt(x)*sqrt(- c**2*x**2 + 1)*asin(c*x)*b - 6*sqrt(x)*sqrt(- c**2*x**2 + 1)*a + 4*sqrt(x)*b*c*x - 3*int((sqrt(x)*sqrt(- c**2*x**2 + 1)*asin(c*x))/(c**2*x**3 - x),x)*b - 3*int((sqrt(x)*sqrt(- c**2*x**2 + 1))/(c**2*x**3 - x),x)*a)/(9*c**2*d)`

3.140 $\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	1280
Mathematica [A] (verified)	1281
Rubi [A] (verified)	1281
Maple [F]	1285
Fricas [F]	1286
Sympy [F]	1286
Maxima [F]	1287
Giac [F]	1287
Mupad [F(-1)]	1288
Reduce [F]	1288

Optimal result

Integrand size = 25, antiderivative size = 315

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= -\frac{bcd^3(2271 + 1329m + 284m^2 + 27m^3 + m^4) x^{2+m} \sqrt{1 - c^2 x^2}}{(3 + m)^2(5 + m)^2(7 + m)^2}$$

$$+ \frac{bc^3 d^3(9 + m)(13 + 2m)x^{4+m} \sqrt{1 - c^2 x^2}}{(5 + m)^2(7 + m)^2} - \frac{bc^5 d^3 x^{6+m} \sqrt{1 - c^2 x^2}}{(7 + m)^2}$$

$$+ \frac{d^3 x^{1+m} (a + b \arcsin(cx))}{1 + m} - \frac{3c^2 d^3 x^{3+m} (a + b \arcsin(cx))}{3 + m}$$

$$+ \frac{3c^4 d^3 x^{5+m} (a + b \arcsin(cx))}{5 + m} - \frac{c^6 d^3 x^{7+m} (a + b \arcsin(cx))}{7 + m}$$

$$- \frac{3bcd^3(2161 + 1813m + 455m^2 + 35m^3) x^{2+m} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{(1 + m)(2 + m)(3 + m)^2(5 + m)^2(7 + m)^2}$$

output

```
-b*c*d^3*(m^4+27*m^3+284*m^2+1329*m+2271)*x^(2+m)*(-c^2*x^2+1)^(1/2)/(3+m)
^2/(5+m)^2/(7+m)^2+b*c^3*d^3*(9+m)*(13+2*m)*x^(4+m)*(-c^2*x^2+1)^(1/2)/(5+
m)^2/(7+m)^2-b*c^5*d^3*x^(6+m)*(-c^2*x^2+1)^(1/2)/(7+m)^2+d^3*x^(1+m)*(a+b
*arcsin(c*x))/(1+m)-3*c^2*d^3*x^(3+m)*(a+b*arcsin(c*x))/(3+m)+3*c^4*d^3*x^
(5+m)*(a+b*arcsin(c*x))/(5+m)-c^6*d^3*x^(7+m)*(a+b*arcsin(c*x))/(7+m)-3*b*
c*d^3*(35*m^3+455*m^2+1813*m+2161)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2
*m],c^2*x^2)/(1+m)/(2+m)/(3+m)^2/(5+m)^2/(7+m)^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.81

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$= x^{1+m} \left((d - c^2 dx^2)^3 (a + b \arcsin(cx)) - \frac{bcd^3 x \operatorname{Hypergeometric2F1}\left(-\frac{5}{2}, 1 + \frac{m}{2}, 2 + \frac{m}{2}, c^2 x^2\right)}{2+m} + \frac{6d \left((d - c^2 dx^2)^2 (a + b \arcsin(cx)) \right)}{2+m} \right)$$

input

```
Integrate[x^m*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(x^(1 + m)*((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]) - (b*c*d^3*x*Hypergeometric2F1[-5/2, 1 + m/2, 2 + m/2, c^2*x^2])/(2 + m) + (6*d*((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2 + m/2, c^2*x^2])/(2 + m) - (4*d^2*((2 + m)*(-3 + c^2*x^2 + m*(-1 + c^2*x^2))*(a + b*ArcSin[c*x]) + b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, c^2*x^2] + 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2]))/(1 + m)*(2 + m)*(3 + m))))/(5 + m))/(7 + m)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5192, 27, 2340, 25, 1590, 25, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{d^3 x^{m+1} \left(-\frac{c^6 x^6}{m+7} + \frac{3c^4 x^4}{m+5} - \frac{3c^2 x^2}{m+3} + \frac{1}{m+1} \right)}{\sqrt{1 - c^2 x^2}} dx - \frac{c^6 d^3 x^{m+7} (a + b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{3c^2 d^3 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^3 x^{m+1} (a + b \arcsin(cx))}{m+1}$$

$$\begin{aligned}
 & \downarrow 27 \\
 & -bcd^3 \int \frac{x^{m+1} \left(-\frac{c^6 x^6}{m+7} + \frac{3c^4 x^4}{m+5} - \frac{3c^2 x^2}{m+3} + \frac{1}{m+1} \right)}{\sqrt{1-c^2 x^2}} dx - \frac{c^6 d^3 x^{m+7} (a + b \arcsin(cx))}{m+7} + \\
 & \frac{3c^4 d^3 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{3c^2 d^3 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^3 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \downarrow 2340 \\
 & -bcd^3 \left(\frac{c^4 \sqrt{1-c^2 x^2} x^{m+6}}{(m+7)^2} - \frac{\int -\frac{x^{m+1} \left(\frac{(m+9)(2m+13)x^4 c^6}{(m+5)(m+7)} - \frac{3(m+7)x^2 c^4}{m+3} + \frac{(m+7)c^2}{m+1} \right)}{\sqrt{1-c^2 x^2}} dx}{c^2(m+7)} \right) - \\
 & \frac{c^6 d^3 x^{m+7} (a + b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{3c^2 d^3 x^{m+3} (a + b \arcsin(cx))}{m+3} + \\
 & \frac{d^3 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \downarrow 25 \\
 & -bcd^3 \left(\frac{\int \frac{x^{m+1} \left(\frac{(m+9)(2m+13)x^4 c^6}{(m+5)(m+7)} - \frac{3(m+7)x^2 c^4}{m+3} + \frac{(m+7)c^2}{m+1} \right)}{\sqrt{1-c^2 x^2}} dx}{c^2(m+7)} + \frac{c^4 \sqrt{1-c^2 x^2} x^{m+6}}{(m+7)^2} \right) - \\
 & \frac{c^6 d^3 x^{m+7} (a + b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{3c^2 d^3 x^{m+3} (a + b \arcsin(cx))}{m+3} + \\
 & \frac{d^3 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \downarrow 1590 \\
 & -bcd^3 \left(\frac{\int -\frac{c^4 x^{m+1} \left(\frac{(m+5)(m+7)}{m+1} - \frac{c^2 (m^4 + 27m^3 + 284m^2 + 1329m + 2271)x^2}{(m+3)(m+5)(m+7)} \right)}{\sqrt{1-c^2 x^2}} dx}{c^2(m+5)} - \frac{c^4 (m+9)(2m+13)\sqrt{1-c^2 x^2} x^{m+4}}{(m+5)^2(m+7)} + \frac{c^4 \sqrt{1-c^2 x^2} x^{m+6}}{(m+7)^2} \right) + \\
 & \frac{c^6 d^3 x^{m+7} (a + b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{3c^2 d^3 x^{m+3} (a + b \arcsin(cx))}{m+3} + \\
 & \frac{d^3 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \downarrow 25
 \end{aligned}$$

$$\begin{aligned}
 & -bcd^3 \left(\frac{\int \frac{c^4 x^{m+1} \left(\frac{(m+5)(m+7)}{m+1} - \frac{c^2(m^4+27m^3+284m^2+1329m+2271)x^2}{(m+3)(m+5)(m+7)} \right)}{\sqrt{1-c^2x^2}} dx - \frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)} + \frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2} \right. \\
 & \left. \frac{c^6 d^3 x^{m+7}(a+b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5}(a+b \arcsin(cx))}{\frac{m+5}{d^3 x^{m+1}(a+b \arcsin(cx))}} - \frac{3c^2 d^3 x^{m+3}(a+b \arcsin(cx))}{m+3} + \right. \\
 & \left. \downarrow 27 \right.
 \end{aligned}$$

$$\begin{aligned}
 & -bcd^3 \left(\frac{c^2 \int \frac{x^{m+1} \left(\frac{(m+5)(m+7)}{m+1} - \frac{c^2(m^4+27m^3+284m^2+1329m+2271)x^2}{(m+3)(m+5)(m+7)} \right)}{\sqrt{1-c^2x^2}} dx - \frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)} + \frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2} \right. \\
 & \left. \frac{c^6 d^3 x^{m+7}(a+b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5}(a+b \arcsin(cx))}{\frac{m+5}{d^3 x^{m+1}(a+b \arcsin(cx))}} - \frac{3c^2 d^3 x^{m+3}(a+b \arcsin(cx))}{m+3} + \right. \\
 & \left. \downarrow 363 \right.
 \end{aligned}$$

$$\begin{aligned}
 & -bcd^3 \left(\frac{c^2 \left(\frac{3(35m^3+455m^2+1813m+2161) \int \frac{x^{m+1}}{\sqrt{1-c^2x^2}} dx + \frac{(m^4+27m^3+284m^2+1329m+2271)\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2(m+5)(m+7)} \right)}{m+5} - \frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)} \right. \\
 & \left. \frac{c^6 d^3 x^{m+7}(a+b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5}(a+b \arcsin(cx))}{\frac{m+5}{d^3 x^{m+1}(a+b \arcsin(cx))}} - \frac{3c^2 d^3 x^{m+3}(a+b \arcsin(cx))}{m+3} + \right. \\
 & \left. \downarrow 278 \right.
 \end{aligned}$$

$$\begin{aligned}
& -\frac{c^6 d^3 x^{m+7} (a + b \arcsin(cx))}{m+7} + \frac{3c^4 d^3 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{3c^2 d^3 x^{m+3} (a + b \arcsin(cx))}{m+3} + \\
& \frac{d^3 x^{m+1} (a + b \arcsin(cx))}{m+1} - \\
bcd^3 & \left(\frac{c^2 \left(\frac{3(35m^3 + 455m^2 + 1813m + 2161)x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)(m+7)} + \frac{(m^4 + 27m^3 + 284m^2 + 1329m + 2271)\sqrt{1-c^2 x^2} x^{m+2}}{(m+3)^2(m+5)(m+7)} \right)}{m+5} - \frac{c^4}{c^2(m+7)} \right)
\end{aligned}$$

input `Int[x^m*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `(d^3*x^(1+m)*(a + b*ArcSin[c*x]))/(1+m) - (3*c^2*d^3*x^(3+m)*(a + b*ArcSin[c*x]))/(3+m) + (3*c^4*d^3*x^(5+m)*(a + b*ArcSin[c*x]))/(5+m) - (c^6*d^3*x^(7+m)*(a + b*ArcSin[c*x]))/(7+m) - b*c*d^3*((c^4*x^(6+m))*Sqrt[1 - c^2*x^2])/(7+m)^2 + (-((c^4*(9+m)*(13+2*m)*x^(4+m))*Sqrt[1 - c^2*x^2])/((5+m)^2*(7+m))) + (c^2*((2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*x^(2+m))*Sqrt[1 - c^2*x^2])/((3+m)^2*(5+m)*(7+m)) + (3*(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2+m))*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/((1+m)*(2+m)*(3+m)^2*(5+m)*(7+m)))/(5+m)/(c^2*(7+m))`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 278 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((c*x)^(m+1)/(c*(m+1)))*Hypergeometric2F1[-p, (m+1)/2, (m+1)/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

rule 363

```
Int[((e._)*(x._))^(m._)*((a._) + (b._)*(x._)^2)^(p._)*((c._) + (d._)*(x._)^2), x
_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int(((f._)*(x._))^(m._)*((d._) + (e._)*(x._)^2)^(q._)*((a._) + (b._)*(x._)^2 + (
c._)*(x._)^4)^(p._), x_Symbol] := Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
+ 1)) Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340

```
Int[(Pq)*((c._)*(x._))^(m._)*((a._) + (b._)*(x._)^2)^(p._), x_Symbol] := With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1)) Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5192

```
Int(((a._) + ArcSin[(c._)*(x._)]*(b._))*((f._)*(x._))^(m._)*((d._) + (e._)*(x._)
^2)^(p._), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [F]

$$\int x^m (-c^2 dx^2 + d)^3 (a + b \arcsin(cx)) dx$$

input

```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x)
```

output `int(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x)`

Fricas [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arcsin(c*x))*x^m, x)`

Sympy [F]

$$\begin{aligned} \int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = & -d^3 \left(\int (-ax^m) dx + \int (-bx^m \operatorname{asin}(cx)) dx \right. \\ & + \int 3ac^2 x^2 x^m dx + \int (-3ac^4 x^4 x^m) dx \\ & + \int ac^6 x^6 x^m dx + \int 3bc^2 x^2 x^m \operatorname{asin}(cx) dx \\ & + \int (-3bc^4 x^4 x^m \operatorname{asin}(cx)) dx \\ & \left. + \int bc^6 x^6 x^m \operatorname{asin}(cx) dx \right) \end{aligned}$$

input `integrate(x**m*(-c**2*d*x**2+d)**3*(a+b*asin(c*x)),x)`

output `-d**3*(Integral(-a*x**m, x) + Integral(-b*x**m*asin(c*x), x) + Integral(3*a*c**2*x**2*x**m, x) + Integral(-3*a*c**4*x**4*x**m, x) + Integral(a*c**6*x**6*x**m, x) + Integral(3*b*c**2*x**2*x**m*asin(c*x), x) + Integral(-3*b*c**4*x**4*x**m*asin(c*x), x) + Integral(b*c**6*x**6*x**m*asin(c*x), x))`

Maxima [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-a*c^6*d^3*x^(m + 7)/(m + 7) + 3*a*c^4*d^3*x^(m + 5)/(m + 5) - 3*a*c^2*d^3*x^(m + 3)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) - (((b*c^6*d^3*m^3 + 9*b*c^6*d^3*m^2 + 23*b*c^6*d^3*m + 15*b*c^6*d^3)*x^7 - 3*(b*c^4*d^3*m^3 + 11*b*c^4*d^3*m^2 + 31*b*c^4*d^3*m + 21*b*c^4*d^3)*x^5 + 3*(b*c^2*d^3*m^3 + 13*b*c^2*d^3*m^2 + 47*b*c^2*d^3*m + 35*b*c^2*d^3)*x^3 - (b*d^3*m^3 + 15*b*d^3*m^2 + 71*b*d^3*m + 105*b*d^3)*x)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrate(-((b*c^7*d^3*m^3 + 9*b*c^7*d^3*m^2 + 23*b*c^7*d^3*m + 15*b*c^7*d^3)*x^7 - 3*(b*c^5*d^3*m^3 + 11*b*c^5*d^3*m^2 + 31*b*c^5*d^3*m + 21*b*c^5*d^3)*x^5 + 3*(b*c^3*d^3*m^3 + 13*b*c^3*d^3*m^2 + 47*b*c^3*d^3*m + 35*b*c^3*d^3)*x^3 - (b*c*d^3*m^3 + 15*b*c*d^3*m^2 + 71*b*c*d^3*m + 105*b*c*d^3)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)`

Giac [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)^3*(b*arcsin(c*x) + a)*x^m, x)`

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \int x^m (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^3 dx$$

input `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^3,x)`output `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^3, x)`**Reduce [F]**

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx)) dx = \text{Too large to display}$$

input `int(x^m*(-c^2*d*x^2+d)^3*(a+b*asin(c*x)),x)`

output

```
(d**3*( - x**m*a*c**6*m**3*x**7 - 9*x**m*a*c**6*m**2*x**7 - 23*x**m*a*c**6
*m*x**7 - 15*x**m*a*c**6*x**7 + 3*x**m*a*c**4*m**3*x**5 + 33*x**m*a*c**4*m
**2*x**5 + 93*x**m*a*c**4*m*x**5 + 63*x**m*a*c**4*x**5 - 3*x**m*a*c**2*m**
3*x**3 - 39*x**m*a*c**2*m**2*x**3 - 141*x**m*a*c**2*m*x**3 - 105*x**m*a*c
*2*x**3 + x**m*a*m**3*x + 15*x**m*a*m**2*x + 71*x**m*a*m*x + 105*x**m*a*x
- int(x**m*asin(c*x)*x**6,x)*b*c**6*m**4 - 16*int(x**m*asin(c*x)*x**6,x)*b
*c**6*m**3 - 86*int(x**m*asin(c*x)*x**6,x)*b*c**6*m**2 - 176*int(x**m*asin
(c*x)*x**6,x)*b*c**6*m - 105*int(x**m*asin(c*x)*x**6,x)*b*c**6 + 3*int(x**
m*asin(c*x)*x**4,x)*b*c**4*m**4 + 48*int(x**m*asin(c*x)*x**4,x)*b*c**4*m**
3 + 258*int(x**m*asin(c*x)*x**4,x)*b*c**4*m**2 + 528*int(x**m*asin(c*x)*x
**4,x)*b*c**4*m + 315*int(x**m*asin(c*x)*x**4,x)*b*c**4 - 3*int(x**m*asin(c
*x)*x**2,x)*b*c**2*m**4 - 48*int(x**m*asin(c*x)*x**2,x)*b*c**2*m**3 - 258*
int(x**m*asin(c*x)*x**2,x)*b*c**2*m**2 - 528*int(x**m*asin(c*x)*x**2,x)*b*
c**2*m - 315*int(x**m*asin(c*x)*x**2,x)*b*c**2 + int(x**m*asin(c*x),x)*b*m
**4 + 16*int(x**m*asin(c*x),x)*b*m**3 + 86*int(x**m*asin(c*x),x)*b*m**2 +
176*int(x**m*asin(c*x),x)*b*m + 105*int(x**m*asin(c*x),x)*b))/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105)
```

3.141 $\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	1289
Mathematica [A] (verified)	1290
Rubi [A] (verified)	1290
Maple [F]	1293
Fricas [F]	1293
Sympy [F]	1294
Maxima [F]	1294
Giac [F]	1295
Mupad [F(-1)]	1295
Reduce [F]	1295

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= -\frac{bcd^2(38 + 13m + m^2) x^{2+m} \sqrt{1 - c^2 x^2}}{(3 + m)^2(5 + m)^2}$$

$$+ \frac{bc^3 d^2 x^{4+m} \sqrt{1 - c^2 x^2}}{(5 + m)^2} + \frac{d^2 x^{1+m} (a + b \arcsin(cx))}{1 + m}$$

$$- \frac{2c^2 d^2 x^{3+m} (a + b \arcsin(cx))}{3 + m} + \frac{c^4 d^2 x^{5+m} (a + b \arcsin(cx))}{5 + m}$$

$$- \frac{bcd^2(149 + 100m + 15m^2) x^{2+m} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{(1 + m)(2 + m)(3 + m)^2(5 + m)^2}$$

output

```
-b*c*d^2*(m^2+13*m+38)*x^(2+m)*(-c^2*x^2+1)^(1/2)/(3+m)^2/(5+m)^2+b*c^3*d^2*x^(4+m)*(-c^2*x^2+1)^(1/2)/(5+m)^2+d^2*x^(1+m)*(a+b*arcsin(c*x))/(1+m)-2*c^2*d^2*x^(3+m)*(a+b*arcsin(c*x))/(3+m)+c^4*d^2*x^(5+m)*(a+b*arcsin(c*x))/(5+m)-b*c*d^2*(15*m^2+100*m+149)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(1+m)/(2+m)/(3+m)^2/(5+m)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.86

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{x^{1+m} \left((d - c^2 dx^2)^2 (a + b \arcsin(cx)) - \frac{bcd^2 x \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, 1 + \frac{m}{2}, 2 + \frac{m}{2}, c^2 x^2\right)}{2+m} - \frac{4d^2((2+m)(-3+c^2 x^2+m(-1+c^2 x^2))}{5+m} \right)}{5+m}$$

input

```
Integrate[x^m*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(x^(1 + m)*((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]) - (b*c*d^2*x*Hypergeometric2F1[-3/2, 1 + m/2, 2 + m/2, c^2*x^2])/(2 + m) - (4*d^2*((2 + m)*(-3 + c^2*x^2 + m*(-1 + c^2*x^2))*(a + b*ArcSin[c*x]) + b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, c^2*x^2] + 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)*(3 + m)))/(5 + m)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5192, 27, 1590, 25, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5192$$

$$-bc \int \frac{d^2 x^{m+1} \left(\frac{c^4 x^4}{m+5} - \frac{2c^2 x^2}{m+3} + \frac{1}{m+1} \right)}{\sqrt{1 - c^2 x^2}} dx + \frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+1}$$

$$\downarrow 27$$

$$\begin{aligned}
 & -bcd^2 \int \frac{x^{m+1} \left(\frac{c^4 x^4}{m+5} - \frac{2c^2 x^2}{m+3} + \frac{1}{m+1} \right)}{\sqrt{1-c^2 x^2}} dx + \frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \\
 & \quad \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{1590} \\
 & -bcd^2 \left(\frac{\int -\frac{c^2 x^{m+1} \left(\frac{m+5}{m+1} - \frac{c^2 (m^2+13m+38)x^2}{(m+3)(m+5)} \right)}{\sqrt{1-c^2 x^2}} dx}{c^2(m+5)} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+4}}{(m+5)^2} \right) + \\
 & \quad \frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{25} \\
 & -bcd^2 \left(\frac{\int \frac{c^2 x^{m+1} \left(\frac{m+5}{m+1} - \frac{c^2 (m^2+13m+38)x^2}{(m+3)(m+5)} \right)}{\sqrt{1-c^2 x^2}} dx}{c^2(m+5)} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+4}}{(m+5)^2} \right) + \\
 & \quad \frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{27} \\
 & -bcd^2 \left(\frac{\int \frac{x^{m+1} \left(\frac{m+5}{m+1} - \frac{c^2 (m^2+13m+38)x^2}{(m+3)(m+5)} \right)}{\sqrt{1-c^2 x^2}} dx}{m+5} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+4}}{(m+5)^2} \right) + \\
 & \quad \frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{363} \\
 & -bcd^2 \left(\frac{\frac{(15m^2+100m+149) \int \frac{x^{m+1}}{\sqrt{1-c^2 x^2}} dx}{(m+1)(m+3)^2(m+5)} + \frac{(m^2+13m+38) \sqrt{1-c^2 x^2} x^{m+2}}{(m+3)^2(m+5)}}{m+5} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+4}}{(m+5)^2} \right) + \\
 & \quad \frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{278}
 \end{aligned}$$

$$\frac{c^4 d^2 x^{m+5} (a + b \arcsin(cx))}{m+5} - \frac{2c^2 d^2 x^{m+3} (a + b \arcsin(cx))}{m+5} + \frac{d^2 x^{m+1} (a + b \arcsin(cx))}{m+5} - bcd^2 \left(\frac{\frac{(15m^2+100m+149)x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)} + \frac{(m^2+13m+38)\sqrt{1-c^2 x^2} x^{m+2}}{(m+3)^2(m+5)}}{m+5} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+4}}{(m+5)^2} \right)$$

input `Int[x^m*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `(d^2*x^(1+m)*(a + b*ArcSin[c*x]))/(1+m) - (2*c^2*d^2*x^(3+m)*(a + b*ArcSin[c*x]))/(3+m) + (c^4*d^2*x^(5+m)*(a + b*ArcSin[c*x]))/(5+m) - b*c*d^2*(-((c^2*x^(4+m)*Sqrt[1 - c^2*x^2])/(5+m)^2) + (((38 + 13*m + m^2)*x^(2+m)*Sqrt[1 - c^2*x^2])/((3+m)^2*(5+m)) + ((149 + 100*m + 15*m^2)*x^(2+m)*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/((1+m)*(2+m)*(3+m)^2*(5+m)))/(5+m)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 278 `Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((c*x)^(m+1)/(c*(m+1)))*Hypergeometric2F1[-p, (m+1)/2, (m+1)/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

rule 363 `Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] := Simp[d*(e*x)^(m+1)*((a + b*x^2)^(p+1)/(b*e*(m+2*p+3))), x] - Simp[(a*d*(m+1) - b*c*(m+2*p+3))/(b*(m+2*p+3)) Int[(e*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m+2*p+3, 0]`

rule 1590

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
+ 1)) Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [F]

$$\int x^m (-c^2 dx^2 + d)^2 (a + b \arcsin(cx)) dx$$

input

```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x)
```

output

```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a) x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arcsin(c*x))*x^m, x)
```

Sympy [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = d^2 \left(\int ax^m dx + \int bx^m \arcsin(cx) dx \right. \\ \left. + \int (-2ac^2 x^2 x^m) dx + \int ac^4 x^4 x^m dx \right. \\ \left. + \int (-2bc^2 x^2 x^m \arcsin(cx)) dx \right. \\ \left. + \int bc^4 x^4 x^m \arcsin(cx) dx \right)$$

input `integrate(x**m*(-c**2*d*x**2+d)**2*(a+b*asin(c*x)),x)`

output `d**2*(Integral(a*x**m, x) + Integral(b*x**m*asin(c*x), x) + Integral(-2*a*c**2*x**2*x**m, x) + Integral(a*c**4*x**4*x**m, x) + Integral(-2*b*c**2*x**2*x**m*asin(c*x), x) + Integral(b*c**4*x**4*x**m*asin(c*x), x))`

Maxima [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `a*c^4*d^2*x^(m + 5)/(m + 5) - 2*a*c^2*d^2*x^(m + 3)/(m + 3) + a*d^2*x^(m + 1)/(m + 1) + (((b*c^4*d^2*m^2 + 4*b*c^4*d^2*m + 3*b*c^4*d^2)*x^5 - 2*(b*c^2*d^2*m^2 + 6*b*c^2*d^2*m + 5*b*c^2*d^2)*x^3 + (b*d^2*m^2 + 8*b*d^2*m + 15*b*d^2)*x)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (m^3 + 9*m^2 + 23*m + 15)*integrate(-((b*c^5*d^2*m^2 + 4*b*c^5*d^2*m + 3*b*c^5*d^2)*x^5 - 2*(b*c^3*d^2*m^2 + 6*b*c^3*d^2*m + 5*b*c^3*d^2)*x^3 + (b*c*d^2*m^2 + 8*b*c*d^2*m + 15*b*c*d^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)`

Giac [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate((c^2*d*x^2 - d)^2*(b*arcsin(c*x) + a)*x^m, x)`

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx = \int x^m (a + b \arcsin(cx)) (d - c^2 dx^2)^2 dx$$

input `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^2,x)`

output `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{d^2 (x^m a c^4 m^2 x^5 + 4x^m a c^4 m x^5 + 3x^m a c^4 x^5 - 2x^m a c^2 m^2 x^3 - 12x^m a c^2 m x^3 - 10x^m a c^2 x^3 + x^m a m^2 x - \dots}{\dots}$$

input `int(x^m*(-c^2*d*x^2+d)^2*(a+b*asin(c*x)),x)`

output

```
(d**2*(x**m*a*c**4*m**2*x**5 + 4*x**m*a*c**4*m*x**5 + 3*x**m*a*c**4*x**5 -
  2*x**m*a*c**2*m**2*x**3 - 12*x**m*a*c**2*m*x**3 - 10*x**m*a*c**2*x**3 + x
**m*a*m**2*x + 8*x**m*a*m*x + 15*x**m*a*x + int(x**m*asin(c*x)*x**4,x)*b*c
**4*m**3 + 9*int(x**m*asin(c*x)*x**4,x)*b*c**4*m**2 + 23*int(x**m*asin(c*x
)*x**4,x)*b*c**4*m + 15*int(x**m*asin(c*x)*x**4,x)*b*c**4 - 2*int(x**m*asi
n(c*x)*x**2,x)*b*c**2*m**3 - 18*int(x**m*asin(c*x)*x**2,x)*b*c**2*m**2 - 4
6*int(x**m*asin(c*x)*x**2,x)*b*c**2*m - 30*int(x**m*asin(c*x)*x**2,x)*b*c*
**2 + int(x**m*asin(c*x),x)*b*m**3 + 9*int(x**m*asin(c*x),x)*b*m**2 + 23*in
t(x**m*asin(c*x),x)*b*m + 15*int(x**m*asin(c*x),x)*b))/(m**3 + 9*m**2 + 23
*m + 15)
```

3.142 $\int x^m(d - c^2 dx^2) (a + b \arcsin(cx)) dx$

Optimal result	1297
Mathematica [A] (verified)	1297
Rubi [A] (verified)	1298
Maple [F]	1300
Fricas [F]	1300
Sympy [F]	1300
Maxima [F]	1301
Giac [F]	1301
Mupad [F(-1)]	1301
Reduce [F]	1302

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int x^m(d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= -\frac{bcdx^{2+m}\sqrt{1 - c^2x^2}}{(3 + m)^2} + \frac{dx^{1+m}(a + b \arcsin(cx))}{1 + m} - \frac{c^2dx^{3+m}(a + b \arcsin(cx))}{3 + m}$$

$$- \frac{bcd(7 + 3m)x^{2+m} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{(1 + m)(2 + m)(3 + m)^2}$$

output

```
-b*c*d*x^(2+m)*(-c^2*x^2+1)^(1/2)/(3+m)^2+d*x^(1+m)*(a+b*arcsin(c*x))/(1+m)
-c^2*d*x^(3+m)*(a+b*arcsin(c*x))/(3+m)-b*c*d*(7+3*m)*x^(2+m)*hypergeom([1
/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(1+m)/(2+m)/(3+m)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int x^m(d - c^2 dx^2) (a + b \arcsin(cx)) dx =$$

$$-\frac{dx^{1+m}((2 + m)(-3 + c^2x^2 + m(-1 + c^2x^2))(a + b \arcsin(cx)) + bc(1 + m)x \operatorname{Hypergeometric2F1}(-1, 1 + m, 2 + m, c^2x^2))}{(1 + m)(2 + m)(3 + m)}$$

input `Integrate[x^m*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]`

output `-((d*x^(1 + m)*((2 + m)*(-3 + c^2*x^2 + m*(-1 + c^2*x^2))*(a + b*ArcSin[c*x]) + b*c*(1 + m)*x*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, c^2*x^2] + 2*b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)*(3 + m)))`

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5192, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx \\
 & \quad \downarrow \text{5192} \\
 & -bc \int \frac{dx^{m+1} \left(\frac{1}{m+1} - \frac{c^2 x^2}{m+3} \right)}{\sqrt{1 - c^2 x^2}} dx - \frac{c^2 dx^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{dx^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{27} \\
 & -bcd \int \frac{x^{m+1} \left(\frac{1}{m+1} - \frac{c^2 x^2}{m+3} \right)}{\sqrt{1 - c^2 x^2}} dx - \frac{c^2 dx^{m+3} (a + b \arcsin(cx))}{m+3} + \frac{dx^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{363} \\
 & -bcd \left(\frac{(3m+7) \int \frac{x^{m+1}}{\sqrt{1 - c^2 x^2}} dx}{(m+1)(m+3)^2} + \frac{\sqrt{1 - c^2 x^2} x^{m+2}}{(m+3)^2} \right) - \frac{c^2 dx^{m+3} (a + b \arcsin(cx))}{m+3} + \\
 & \quad \frac{dx^{m+1} (a + b \arcsin(cx))}{m+1} \\
 & \quad \downarrow \text{278}
 \end{aligned}$$

$$bcd \left(\frac{-\frac{c^2 dx^{m+3}(a + b \arcsin(cx))}{m+3} + \frac{dx^{m+1}(a + b \arcsin(cx))}{m+1}}{(3m+7)x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right) + \frac{\sqrt{1-c^2 x^2} x^{m+2}}{(m+3)^2}} \right)$$

input `Int[x^m*(d - c^2*d*x^2)*(a + b*ArcSin[c*x]),x]`

output `(d*x^(1+m)*(a + b*ArcSin[c*x]))/(1+m) - (c^2*d*x^(3+m)*(a + b*ArcSin[c*x]))/(3+m) - b*c*d*((x^(2+m)*Sqrt[1 - c^2*x^2])/(3+m)^2 + ((7+3*m)*x^(2+m)*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/((1+m)*(2+m)*(3+m)^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 278 `Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((c*x)^(m+1)/(c*(m+1)))*Hypergeometric2F1[-p, (m+1)/2, (m+1)/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

rule 363 `Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] := Simp[d*(e*x)^(m+1)*((a + b*x^2)^(p+1)/(b*e*(m+2*p+3))), x] - Simp[(a*d*(m+1) - b*c*(m+2*p+3))/(b*(m+2*p+3)) Int[(e*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m+2*p+3, 0]`

rule 5192 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

Maple [F]

$$\int x^m (-c^2 d x^2 + d) (a + b \arcsin(cx)) dx$$

input `int(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x)`

output `int(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x)`

Fricas [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int -(c^2 dx^2 - d) (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*x^m, x)`

Sympy [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx = -d \left(\int (-ax^m) dx + \int (-bx^m \arcsin(cx)) dx \right) + \int ac^2 x^2 x^m dx + \int bc^2 x^2 x^m \arcsin(cx) dx$$

input `integrate(x**m*(-c**2*d*x**2+d)*(a+b*asin(c*x)),x)`

output `-d*(Integral(-a*x**m, x) + Integral(-b*x**m*asin(c*x), x) + Integral(a*c**2*x**2*x**m, x) + Integral(b*c**2*x**2*x**m*asin(c*x), x))`

Maxima [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int -(c^2 dx^2 - d) (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `-a*c^2*d*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) - (((b*c^2*d*m + b*c^2*d)*x^3 - (b*d*m + 3*b*d)*x)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (m^2 + 4*m + 3)*integrate(((b*c^3*d*m + b*c^3*d)*x^3 - (b*c*d*m + 3*b*c*d)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)`

Giac [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int -(c^2 dx^2 - d) (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)*(b*arcsin(c*x) + a)*x^m, x)`

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx = \int x^m (a + b \arcsin(cx)) (d - c^2 dx^2) dx$$

input `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2),x)`

output `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2), x)`

Reduce [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx)) dx$$

$$= \frac{d(-x^m a c^2 m x^3 - x^m a c^2 x^3 + x^m a m x + 3x^m a x - (\int x^m a \sin(cx) x^2 dx) b c^2 m^2 - 4(\int x^m a \sin(cx) x^2 dx))}{m^2 + 4m + 3}$$

input `int(x^m*(-c^2*d*x^2+d)*(a+b*asin(c*x)),x)`

output `(d*(- x**m*a*c**2*m*x**3 - x**m*a*c**2*x**3 + x**m*a*m*x + 3*x**m*a*x - i
nt(x**m*asin(c*x)*x**2,x)*b*c**2*m**2 - 4*int(x**m*asin(c*x)*x**2,x)*b*c**
2*m - 3*int(x**m*asin(c*x)*x**2,x)*b*c**2 + int(x**m*asin(c*x),x)*b*m**2 +
4*int(x**m*asin(c*x),x)*b*m + 3*int(x**m*asin(c*x),x)*b))/(m**2 + 4*m + 3
)`

3.143 $\int \frac{x^m(a+b \arcsin(cx))}{d-c^2dx^2} dx$

Optimal result	1303
Mathematica [N/A]	1303
Rubi [N/A]	1304
Maple [N/A]	1304
Fricas [N/A]	1305
Sympy [N/A]	1305
Maxima [N/A]	1305
Giac [F(-2)]	1306
Mupad [N/A]	1306
Reduce [N/A]	1307

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x^m(a+b \arcsin(cx))}{d-c^2dx^2} dx = \text{Int}\left(\frac{x^m(a+b \arcsin(cx))}{d-c^2dx^2}, x\right)$$

output `Defer(Int)(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d), x)`

Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b \arcsin(cx))}{d-c^2dx^2} dx = \int \frac{x^m(a+b \arcsin(cx))}{d-c^2dx^2} dx$$

input `Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2), x]`

output `Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2), x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

↓ 5234

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx$$

input `Int[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a + b \arcsin(cx))}{-c^2 d x^2 + d} dx$$

input `int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x)`

output `int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^m}{c^2 dx^2 - d} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)*x^m/(c^2*d*x^2 - d), x)`

Sympy [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx = -\frac{\int \frac{ax^m}{c^2 x^2 - 1} dx + \int \frac{bx^m \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x**m*(a+b*asin(c*x))/(-c**2*d*x**2+d),x)`

output `-(Integral(a*x**m/(c**2*x**2 - 1), x) + Integral(b*x**m*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)x^m}{c^2 dx^2 - d} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-integrate((b*arcsin(c*x) + a)*x^m/(c^2*d*x^2 - d), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \int \frac{x^m(a + b \operatorname{asin}(cx))}{d - c^2 dx^2} dx$$

input `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2),x)`

output `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{x^m(a + b \arcsin(cx))}{d - c^2 dx^2} dx = \frac{-\left(\int \frac{x^m}{c^2 x^2 - 1} dx\right) a - \left(\int \frac{x^m \arcsin(cx)}{c^2 x^2 - 1} dx\right) b}{d}$$

input `int(x^m*(a+b*asin(c*x))/(-c^2*d*x^2+d),x)`

output `(- (int(x**m/(c**2*x**2 - 1),x)*a + int((x**m*asin(c*x))/(c**2*x**2 - 1),x)*b))/d`

3.144 $\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx$

Optimal result	1308
Mathematica [N/A]	1308
Rubi [N/A]	1309
Maple [N/A]	1310
Fricas [N/A]	1310
Sympy [N/A]	1310
Maxima [N/A]	1311
Giac [F(-2)]	1311
Mupad [N/A]	1312
Reduce [N/A]	1312

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx = \text{Int}\left(\frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^2}, x\right)$$

output `Defer(Int)(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x)`

Mathematica [N/A]

Not integrable

Time = 4.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx = \int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^2} dx$$

input `Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]`

output `Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2, x]`

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx \\
 & \quad \downarrow \text{5208} \\
 & \frac{(1 - m) \int \frac{x^m(a + b \arcsin(cx))}{d(1 - c^2 x^2)} dx}{2d} - \frac{bc \int \frac{x^{m+1}}{(1 - c^2 x^2)^{3/2}} dx}{2d^2} + \frac{x^{m+1}(a + b \arcsin(cx))}{2d^2(1 - c^2 x^2)} \\
 & \quad \downarrow \text{27} \\
 & \frac{(1 - m) \int \frac{x^m(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{2d^2} - \frac{bc \int \frac{x^{m+1}}{(1 - c^2 x^2)^{3/2}} dx}{2d^2} + \frac{x^{m+1}(a + b \arcsin(cx))}{2d^2(1 - c^2 x^2)} \\
 & \quad \downarrow \text{278} \\
 & \frac{(1 - m) \int \frac{x^m(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{2d^2} + \frac{x^{m+1}(a + b \arcsin(cx))}{2d^2(1 - c^2 x^2)} - \\
 & \quad \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{2d^2(m+2)} \\
 & \quad \downarrow \text{5234} \\
 & \frac{(1 - m) \int \frac{x^m(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{2d^2} + \frac{x^{m+1}(a + b \arcsin(cx))}{2d^2(1 - c^2 x^2)} - \\
 & \quad \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{2d^2(m+2)}
 \end{aligned}$$

input

```
Int[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^2,x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))}{(-c^2 d x^2 + d)^2} dx$$

input `int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x)`

output `int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{x^m (a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^m}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)*x^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [N/A]

Not integrable

Time = 16.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{x^m (a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{ax^m}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{bx^m \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input `integrate(x**m*(a+b*asin(c*x))/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a*x**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**m*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^m}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)*x^m/(c^2*d*x^2 - d)^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage20OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \int \frac{x^m(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^2} dx$$

input `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2,x)`

output `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^2, x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^2} dx = \frac{\left(\int \frac{x^m}{c^4 x^4 - 2c^2 x^2 + 1} dx\right) a + \left(\int \frac{x^m \operatorname{asin}(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx\right) b}{d^2}$$

input `int(x^m*(a+b*asin(c*x))/(-c^2*d*x^2+d)^2,x)`

output `(int(x**m/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a + int((x**m*asin(c*x))/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b)/d**2`

$$3.145 \quad \int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx$$

Optimal result	1313
Mathematica [N/A]	1313
Rubi [N/A]	1314
Maple [N/A]	1315
Fricas [N/A]	1316
Sympy [F(-1)]	1316
Maxima [N/A]	1316
Giac [F(-2)]	1317
Mupad [N/A]	1317
Reduce [N/A]	1317

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx = \text{Int}\left(\frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^3}, x\right)$$

output `Defer(Int)(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x)`

Mathematica [N/A]

Not integrable

Time = 5.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx = \int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^3} dx$$

input `Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]`

output `Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3, x]`

Rubi [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5208} \\
 & \frac{(3 - m) \int \frac{x^m(a+b \arcsin(cx))}{d^2(1-c^2x^2)^2} dx}{4d} - \frac{bc \int \frac{x^{m+1}}{(1-c^2x^2)^{5/2}} dx}{4d^3} + \frac{x^{m+1}(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & \frac{(3 - m) \int \frac{x^m(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{4d^3} - \frac{bc \int \frac{x^{m+1}}{(1-c^2x^2)^{5/2}} dx}{4d^3} + \frac{x^{m+1}(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} \\
 & \quad \downarrow \text{278} \\
 & \frac{(3 - m) \int \frac{x^m(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{4d^3} + \frac{x^{m+1}(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} - \\
 & \quad \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{4d^3(m+2)} \\
 & \quad \downarrow \text{5208} \\
 & \frac{(3 - m) \left(\frac{1}{2}(1 - m) \int \frac{x^m(a+b \arcsin(cx))}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x^{m+1}}{(1-c^2x^2)^{3/2}} dx + \frac{x^{m+1}(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{4d^3} + \\
 & \quad \frac{x^{m+1}(a + b \arcsin(cx))}{4d^3(1 - c^2x^2)^2} - \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{4d^3(m+2)} \\
 & \quad \downarrow \text{278}
 \end{aligned}$$

$$(3-m) \left(\frac{\frac{1}{2}(1-m) \int \frac{x^m(a+b \arcsin(cx))}{1-c^2x^2} dx + \frac{x^{m+1}(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{2(m+2)}}{4d^3} \right) +$$

$$\frac{x^{m+1}(a+b \arcsin(cx))}{4d^3(1-c^2x^2)^2} - \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{4d^3(m+2)}$$

↓ 5234

$$(3-m) \left(\frac{\frac{1}{2}(1-m) \int \frac{x^m(a+b \arcsin(cx))}{1-c^2x^2} dx + \frac{x^{m+1}(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{2(m+2)}}{4d^3} \right) +$$

$$\frac{x^{m+1}(a+b \arcsin(cx))}{4d^3(1-c^2x^2)^2} - \frac{bcx^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{4d^3(m+2)}$$

input

```
Int[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^3,x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b \arcsin(cx))}{(-c^2dx^2+d)^3} dx$$

input

```
int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x)
```

output

```
int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x)
```


Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^m}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b*arcsin(c*x) + a)*x^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \text{Timed out}$$

input `integrate(x**m*(a+b*asin(c*x))/(-c**2*d*x**2+d)**3,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)x^m}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-integrate((b*arcsin(c*x) + a)*x^m/(c^2*d*x^2 - d)^3, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \int \frac{x^m(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^3} dx$$

input `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3,x)`

output `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^3, x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^3} dx = \frac{-\left(\int \frac{x^m}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx\right) a - \left(\int \frac{x^m \operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx\right) b}{d^3}$$

input `int(x^m*(a+b*asin(c*x))/(-c^2*d*x^2+d)^3,x)`

output $(- (\text{int}(x^m/(c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1),x)*a + \text{int}(x^m$
 $*\text{asin}(c*x))/(c^6x^6 - 3c^4x^4 + 3c^2x^2 - 1),x)*b))/d^3$

3.146 $\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx$

Optimal result	1319
Mathematica [A] (verified)	1320
Rubi [A] (verified)	1321
Maple [F]	1324
Fricas [F]	1325
Sympy [F(-1)]	1325
Maxima [F]	1325
Giac [F(-2)]	1326
Mupad [F(-1)]	1326
Reduce [F]	1326

Optimal result

Integrand size = 27, antiderivative size = 635

$$\begin{aligned}
 \int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = & -\frac{15bcd^2 x^{2+m} \sqrt{d - c^2 dx^2}}{(2 + m)^2 (4 + m) (6 + m) \sqrt{1 - c^2 x^2}} \\
 & - \frac{5bcd^2 x^{2+m} \sqrt{d - c^2 dx^2}}{(6 + m) (8 + 6m + m^2) \sqrt{1 - c^2 x^2}} - \frac{bcd^2 x^{2+m} \sqrt{d - c^2 dx^2}}{(12 + 8m + m^2) \sqrt{1 - c^2 x^2}} \\
 & + \frac{5bc^3 d^2 x^{4+m} \sqrt{d - c^2 dx^2}}{(4 + m)^2 (6 + m) \sqrt{1 - c^2 x^2}} + \frac{2bc^3 d^2 x^{4+m} \sqrt{d - c^2 dx^2}}{(4 + m) (6 + m) \sqrt{1 - c^2 x^2}} \\
 & - \frac{bc^5 d^2 x^{6+m} \sqrt{d - c^2 dx^2}}{(6 + m)^2 \sqrt{1 - c^2 x^2}} + \frac{15d^2 x^{1+m} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{(6 + m) (8 + 6m + m^2)} \\
 & + \frac{5dx^{1+m} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{(4 + m) (6 + m)} + \frac{x^{1+m} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{6 + m} \\
 & + \frac{15d^2 x^{1+m} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right)}{(6 + m) (8 + 14m + 7m^2 + m^3) \sqrt{1 - c^2 x^2}} \\
 & - \frac{15bcd^2 x^{2+m} \sqrt{d - c^2 dx^2} {}_3F_2\left(1, 1 + \frac{m}{2}, 1 + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}; c^2 x^2\right)}{(1 + m) (2 + m)^2 (4 + m) (6 + m) \sqrt{1 - c^2 x^2}}
 \end{aligned}$$

output

```

-15*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(6+m)/(-c^2*x^2+1)^(
1/2)-5*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)/(-c^2*x^2+1
)^(1/2)-b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(m^2+8*m+12)/(-c^2*x^2+1)^(1/
2)+5*b*c^3*d^2*x^(4+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)/(-c^2*x^2+1)^(1/
2)+2*b*c^3*d^2*x^(4+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)/(6+m)/(-c^2*x^2+1)^(1/2)
-b*c^5*d^2*x^(6+m)*(-c^2*d*x^2+d)^(1/2)/(6+m)^2/(-c^2*x^2+1)^(1/2)+15*d^2*x
^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(6+m)/(m^2+6*m+8)+5*d*x^(1+m)
*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))/(4+m)/(6+m)+x^(1+m)*(-c^2*d*x^2+d)
^(5/2)*(a+b*arcsin(c*x))/(6+m)+15*d^2*x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*
arcsin(c*x))*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2*x^2)/(6+m)/(m^3+7*
m^2+14*m+8)/(-c^2*x^2+1)^(1/2)-15*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*hyp
ergeom([1, 1+1/2*m, 1+1/2*m], [2+1/2*m, 3/2+1/2*m], c^2*x^2)/(1+m)/(2+m)^2/(
4+m)/(6+m)/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.53

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \frac{d^2 x^{1+m} \sqrt{d - c^2 dx^2} \left(-bc(1+m)(2+m)(4+m)x((4+m)(6+m) - 2c^2(2+m)(6+m)) + (4+m)(6+m) - 2c^2(2+m)(6+m) \right)}{((1+m)(2+m)^2(4+m)^2(6+m)(1 - c^2 x^2)^{5/2}(a + b \arcsin[cx]) - 5(6+m)(b*c*(1+m)*(2+m)*x*(4+m - c^2*(2+m)*x^2) - (1+m)*(2+m)^2*(4+m)*(1 - c^2*x^2)^{3/2}(a + b*ArcSin[c*x]) + 3*(4+m)*(b*c*(1+m)*x - (1+m)*(2+m)*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) - (2+m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2] + b*c*x*HypergeometricPFQ[{1, 1+m/2, 1+m/2}, {3/2+m/2, 2+m/2}, c^2*x^2])}$$

input

```
Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]
```

output

```

(d^2*x^(1+m)*Sqrt[d - c^2*d*x^2]*(-(b*c*(1+m)*(2+m)*(4+m)*x*((4+m)
*(6+m) - 2*c^2*(2+m)*(6+m)*x^2 + c^4*(2+m)*(4+m)*x^4)) + (1+m)
*(2+m)^2*(4+m)^2*(6+m)*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]) - 5
*(6+m)*(b*c*(1+m)*(2+m)*x*(4+m - c^2*(2+m)*x^2) - (1+m)*(2+m)
)^2*(4+m)*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]) + 3*(4+m)*(b*c*(1+
m)*x - (1+m)*(2+m)*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) - (2+m)*(a
+ b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2] + b
*c*x*HypergeometricPFQ[{1, 1+m/2, 1+m/2}, {3/2+m/2, 2+m/2}, c^2*x^
2])))/((1+m)*(2+m)^2*(4+m)^2*(6+m)^2*Sqrt[1 - c^2*x^2])

```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5202, 244, 2009, 5202, 244, 2009, 5198, 15, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx \\
 & \quad \downarrow \text{5202} \\
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx}{m+6} - \frac{bcd^2 \sqrt{d - c^2 dx^2} \int x^{m+1} (1 - c^2 x^2)^2 dx}{(m+6)\sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{m+6} \\
 & \quad \downarrow \text{244} \\
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx}{m+6} - \\
 & \frac{bcd^2 \sqrt{d - c^2 dx^2} \int (x^{m+1} - 2c^2 x^{m+3} + c^4 x^{m+5}) dx}{(m+6)\sqrt{1 - c^2 x^2}} + \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{m+6} \\
 & \quad \downarrow \text{2009} \\
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx}{m+6} + \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{m+6} - \\
 & \quad \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{c^4 x^{m+6}}{m+6} - \frac{2c^2 x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{5202} \\
 & 5d \left(\frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx}{m+4} - \frac{bcd \sqrt{d - c^2 dx^2} \int x^{m+1} (1 - c^2 x^2) dx}{(m+4)\sqrt{1 - c^2 x^2}} + \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m+4} \right) + \\
 & \quad \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))}{m+6} - \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{c^4 x^{m+6}}{m+6} - \frac{2c^2 x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{244}
 \end{aligned}$$

$$5d \left(\frac{3d \int x^m \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx}{m+4} - \frac{bcd \sqrt{d-c^2 dx^2} \int (x^{m+1} - c^2 x^{m+3}) dx}{(m+4)\sqrt{1-c^2 x^2}} + \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))}{m+4} \right) +$$

$$\frac{x^{m+1} (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{m+6} - \frac{bcd^2 \sqrt{d-c^2 dx^2} \left(\frac{c^4 x^{m+6}}{m+6} - \frac{2c^2 x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

↓ 2009

$$5d \left(\frac{3d \int x^m \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx}{m+4} + \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))}{m+4} - \frac{bcd \sqrt{d-c^2 dx^2} \left(\frac{x^{m+2}}{m+2} - \frac{c^2 x^{m+4}}{m+4} \right)}{(m+4)\sqrt{1-c^2 x^2}} \right) +$$

$$\frac{x^{m+1} (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{m+6} - \frac{bcd^2 \sqrt{d-c^2 dx^2} \left(\frac{c^4 x^{m+6}}{m+6} - \frac{2c^2 x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

↓ 5198

$$5d \left(\frac{3d \left(\frac{\sqrt{d-c^2 dx^2} \int \frac{x^m (a+b \arcsin(cx)) dx}{\sqrt{1-c^2 x^2}} - \frac{bc \sqrt{d-c^2 dx^2} \int x^{m+1} dx}{(m+2)\sqrt{1-c^2 x^2}} + \frac{x^{m+1} \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{m+2} \right)}{m+4} + \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))}{m+4} \right) +$$

$$\frac{x^{m+1} (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{m+6} - \frac{bcd^2 \sqrt{d-c^2 dx^2} \left(\frac{c^4 x^{m+6}}{m+6} - \frac{2c^2 x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

↓ 15

$$5d \left(\frac{3d \left(\frac{\sqrt{d-c^2 dx^2} \int \frac{x^m (a+b \arcsin(cx)) dx}{\sqrt{1-c^2 x^2}} + \frac{x^{m+1} \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+2} \sqrt{d-c^2 dx^2}}{(m+2)^2 \sqrt{1-c^2 x^2}} \right)}{m+4} + \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))}{m+4} \right) +$$

$$\frac{x^{m+1} (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))}{m+6} - \frac{bcd^2 \sqrt{d-c^2 dx^2} \left(\frac{c^4 x^{m+6}}{m+6} - \frac{2c^2 x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

↓ 5220

$$5d \left(\frac{3d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{x^{m+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right) (a+b \arcsin(cx))}{m+1} - \frac{bcx^{m+2} {}_3F_2\left(1, \frac{m}{2}+1, \frac{m}{2}+1; \frac{m}{2}+\frac{3}{2}, \frac{m}{2}+2; c^2x^2\right)}{m^2+3m+2} \right)}{(m+2)\sqrt{1-c^2x^2}} + \frac{x^{m+1}\sqrt{d-c^2d}}{\dots} \right)}{m+4} \right)$$

$$\frac{x^{m+1}(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))}{m+6} - \frac{bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^{m+6}}{m+6} - \frac{2c^2x^{m+4}}{m+4} + \frac{x^{m+2}}{m+2} \right)}{(m+6)\sqrt{1-c^2x^2}} \quad m+6$$

input `Int[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]),x]`

output `-((b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^(2 + m)/(2 + m) - (2*c^2*x^(4 + m))/(4 + m) + (c^4*x^(6 + m))/(6 + m)))/((6 + m)*Sqrt[1 - c^2*x^2])) + (x^(1 + m)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(6 + m) + (5*d*(-((b*c*d*Sqrt[d - c^2*d*x^2]*(x^(2 + m)/(2 + m) - (c^2*x^(4 + m))/(4 + m)))/((4 + m)*Sqrt[1 - c^2*x^2])) + (x^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(4 + m) + (3*d*(-((b*c*x^(2 + m)*Sqrt[d - c^2*d*x^2])/((2 + m)^2*Sqrt[1 - c^2*x^2])) + (x^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]))/(2 + m) + (Sqrt[d - c^2*d*x^2]*((x^(1 + m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(1 + m) - (b*c*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + 3*m + m^2)))/((2 + m)*Sqrt[1 - c^2*x^2])))/(4 + m)))/(6 + m)`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5220

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int x^m (-c^2 d x^2 + d)^{\frac{5}{2}} (a + b \arcsin(cx)) dx$$

input

```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x)
```

output

```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)`

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**m*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx)) dx = \int x^m (a + b \arcsin(cx)) (d - c^2 dx^2)^{5/2} dx$$

input `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\begin{aligned} & \int x^m (d - c^2 dx^2)^{5/2} (a \\ & + b \arcsin(cx)) dx = \sqrt{d} d^2 \left(\left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4 dx \right) b c^4 \right. \\ & - 2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2 dx \right) b c^2 \\ & + \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx \right) b + \left(\int x^m \sqrt{-c^2 x^2 + 1} x^4 dx \right) a c^4 \\ & \left. - 2 \left(\int x^m \sqrt{-c^2 x^2 + 1} x^2 dx \right) a c^2 + \left(\int x^m \sqrt{-c^2 x^2 + 1} dx \right) a \right) \end{aligned}$$

input `int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x)),x)`

output `sqrt(d)*d**2*(int(x**m*sqrt(-c**2*x**2+1)*asin(c*x)*x**4,x)*b*c**4 - 2
*int(x**m*sqrt(-c**2*x**2+1)*asin(c*x)*x**2,x)*b*c**2 + int(x**m*sqrt(-
c**2*x**2+1)*asin(c*x),x)*b + int(x**m*sqrt(-c**2*x**2+1)*x**4,x)
*a*c**4 - 2*int(x**m*sqrt(-c**2*x**2+1)*x**2,x)*a*c**2 + int(x**m*sqrt(
-c**2*x**2+1),x)*a)`

3.147 $\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$

Optimal result	1328
Mathematica [A] (verified)	1329
Rubi [A] (verified)	1329
Maple [F]	1332
Fricas [F]	1332
Sympy [F(-1)]	1333
Maxima [F]	1333
Giac [F(-2)]	1333
Mupad [F(-1)]	1334
Reduce [F]	1334

Optimal result

Integrand size = 27, antiderivative size = 399

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = -\frac{3bcdx^{2+m}\sqrt{d - c^2 dx^2}}{(2 + m)^2(4 + m)\sqrt{1 - c^2 x^2}} - \frac{bcdx^{2+m}\sqrt{d - c^2 dx^2}}{(8 + 6m + m^2)\sqrt{1 - c^2 x^2}} + \frac{bc^3 dx^{4+m}\sqrt{d - c^2 dx^2}}{(4 + m)^2\sqrt{1 - c^2 x^2}} + \frac{3dx^{1+m}\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{8 + 6m + m^2} + \frac{x^{1+m}(d - c^2 dx^2)^{3/2}(a + b \arcsin(cx))}{4 + m} + \frac{3dx^{1+m}\sqrt{d - c^2 dx^2}(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right)}{(8 + 14m + 7m^2 + m^3)\sqrt{1 - c^2 x^2}} - \frac{3bcdx^{2+m}\sqrt{d - c^2 dx^2} {}_3F_2\left(1, 1 + \frac{m}{2}, 1 + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}; c^2 x^2\right)}{(1 + m)(2 + m)^2(4 + m)\sqrt{1 - c^2 x^2}}$$

output

```
-3*b*c*d*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(-c^2*x^2+1)^(1/2)-b*c
*d*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(m^2+6*m+8)/(-c^2*x^2+1)^(1/2)+b*c^3*d*x^(
4+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^2/(-c^2*x^2+1)^(1/2)+3*d*x^(1+m)*(-c^2*d*x
^2+d)^(1/2)*(a+b*arcsin(c*x))/(m^2+6*m+8)+x^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+
b*arcsin(c*x))/(4+m)+3*d*x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*hy
pergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2*x^2)/(m^3+7*m^2+14*m+8)/(-c^2*x^2
+1)^(1/2)-3*b*c*d*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/
2*m], [2+1/2*m, 3/2+1/2*m], c^2*x^2)/(1+m)/(2+m)^2/(4+m)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.59

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \frac{dx^{1+m} \sqrt{d - c^2 dx^2} \left(-\frac{bcx(4+m-c^2(2+m)x^2)}{(2+m)(4+m)\sqrt{1-c^2x^2}} + (1 - c^2x^2)(a + b \arcsin(cx)) - \frac{3(bc(1+m)x - (1-c^2x^2)^{3/2}}{(2+m)(4+m)\sqrt{1-c^2x^2}} \right)}{(2+m)(4+m)\sqrt{1-c^2x^2}}$$

input `Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output `(d*x^(1 + m)*Sqrt[d - c^2*d*x^2]*(-(b*c*x*(4 + m - c^2*(2 + m)*x^2))/((2 + m)*(4 + m)*Sqrt[1 - c^2*x^2])) + (1 - c^2*x^2)*(a + b*ArcSin[c*x]) - (3*(b*c*(1 + m)*x - (1 + m)*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) - (2 + m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 + m)^2*Sqrt[1 - c^2*x^2]))/(4 + m)`

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5202, 244, 2009, 5198, 15, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5202$$

$$\frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx}{m + 4} - \frac{bcd \sqrt{d - c^2 dx^2} \int x^{m+1} (1 - c^2 x^2) dx}{(m + 4) \sqrt{1 - c^2 x^2}} +$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m + 4}$$

$$\downarrow 244$$

$$\begin{aligned}
 & \frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx}{m + 4} - \frac{bcd \sqrt{d - c^2 dx^2} \int (x^{m+1} - c^2 x^{m+3}) dx}{(m + 4) \sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m + 4} \\
 & \quad \downarrow \text{2009} \\
 & \frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx}{m + 4} + \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m + 4} - \\
 & \quad \frac{bcd \sqrt{d - c^2 dx^2} \left(\frac{x^{m+2}}{m+2} - \frac{c^2 x^{m+4}}{m+4} \right)}{(m + 4) \sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{5198} \\
 & \frac{3d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^m (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{(m+2) \sqrt{1 - c^2 x^2}} - \frac{bc \sqrt{d - c^2 dx^2} \int x^{m+1} dx}{(m+2) \sqrt{1 - c^2 x^2}} + \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{m+2} \right)}{m + 4} + \\
 & \quad \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m + 4} - \frac{bcd \sqrt{d - c^2 dx^2} \left(\frac{x^{m+2}}{m+2} - \frac{c^2 x^{m+4}}{m+4} \right)}{(m + 4) \sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{15} \\
 & \frac{3d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^m (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{(m+2) \sqrt{1 - c^2 x^2}} + \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{m+2} - \frac{bc x^{m+2} \sqrt{d - c^2 dx^2}}{(m+2)^2 \sqrt{1 - c^2 x^2}} \right)}{m + 4} + \\
 & \quad \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m + 4} - \frac{bcd \sqrt{d - c^2 dx^2} \left(\frac{x^{m+2}}{m+2} - \frac{c^2 x^{m+4}}{m+4} \right)}{(m + 4) \sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{5220} \\
 & \frac{3d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{x^{m+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2 x^2\right) (a + b \arcsin(cx))}{m+1} - \frac{bc x^{m+2} {}_3F_2\left(1, \frac{m}{2} + 1, \frac{m}{2} + 1; \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + 2; c^2 x^2\right)}{m^2 + 3m + 2} \right)}{(m+2) \sqrt{1 - c^2 x^2}} \right)}{m + 4} + \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{m + 4} \\
 & \quad \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{m + 4} - \frac{bcd \sqrt{d - c^2 dx^2} \left(\frac{x^{m+2}}{m+2} - \frac{c^2 x^{m+4}}{m+4} \right)}{(m + 4) \sqrt{1 - c^2 x^2}}
 \end{aligned}$$

input `Int[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]),x]`

output

$$\begin{aligned}
& -((b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(x^{(2 + m)/(2 + m)} - (c^2*x^{(4 + m))/(4 + m)}) \\
&)/((4 + m)*\text{Sqrt}[1 - c^2*x^2])) + (x^{(1 + m)}*(d - c^2*d*x^2)^{(3/2)}*(a + b* \\
& \text{ArcSin}[c*x]))/(4 + m) + (3*d*(-((b*c*x^{(2 + m)}*\text{Sqrt}[d - c^2*d*x^2])/((2 + m) \\
&)^2*\text{Sqrt}[1 - c^2*x^2])) + (x^{(1 + m)}*\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x \\
&]))/(2 + m) + (\text{Sqrt}[d - c^2*d*x^2]*((x^{(1 + m)}*(a + b*\text{ArcSin}[c*x])* \\
& \text{Hypergeometric2F1}[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(1 + m) - (b*c*x^{(2 + m)}* \\
& \text{HypergeometricPFQ}[\{1, 1 + m/2, 1 + m/2\}, \{3/2 + m/2, 2 + m/2\}, c^2*x^2])/(2 \\
& + 3*m + m^2)))/((2 + m)*\text{Sqrt}[1 - c^2*x^2]))/(4 + m)
\end{aligned}$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_.)*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m + 1)/(m + 1)}), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 244

$$\text{Int}(((c_.)*(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5198

$$\begin{aligned}
& \text{Int}(((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^{(n_.)}*((f_.)*(x_)^{(m_)})*\text{Sqrt}[(d_) + \\
& (e_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcS} \\
& \text{in}[c*x])^n/(f*(m + 2))), x] + (\text{Simp}[(1/(m + 2))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 \\
& - c^2*x^2]] \text{ Int}[(f*x)^m*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x \\
&] - \text{Simp}[b*c*(n/(f*(m + 2)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{ Int} \\
& (f*x)^{(m + 1)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, \\
& f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])
\end{aligned}$$

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5220

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int x^m (-c^2 d x^2 + d)^{\frac{3}{2}} (a + b \arcsin(cx)) dx$$

input

```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x)
```

output

```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a) x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="fricas"
)
```

output `integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)`

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate(x**m*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \int x^m (a + b \operatorname{asin}(cx)) (d - c^2 dx^2)^{3/2} dx$$

input

```
int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output

```
int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\begin{aligned} & \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx)) dx = \sqrt{d} d \left(- \left(\int x^m \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) x^2 dx \right) b c^2 \right. \\ & + \left(\int x^m \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) dx \right) b - \left(\int x^m \sqrt{-c^2 x^2 + 1} x^2 dx \right) a c^2 \\ & \left. + \left(\int x^m \sqrt{-c^2 x^2 + 1} dx \right) a \right) \end{aligned}$$

input

```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x)),x)
```

output

```
sqrt(d)*d*( - int(x**m*sqrt( - c**2*x**2 + 1)*asin(c*x)*x**2,x)*b*c**2 + i
nt(x**m*sqrt( - c**2*x**2 + 1)*asin(c*x),x)*b - int(x**m*sqrt( - c**2*x**2
+ 1)*x**2,x)*a*c**2 + int(x**m*sqrt( - c**2*x**2 + 1),x)*a)
```

3.148 $\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$

Optimal result	1335
Mathematica [A] (verified)	1336
Rubi [A] (verified)	1336
Maple [F]	1338
Fricas [F]	1338
Sympy [F]	1339
Maxima [F]	1339
Giac [F(-2)]	1339
Mupad [F(-1)]	1340
Reduce [F]	1340

Optimal result

Integrand size = 27, antiderivative size = 245

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= -\frac{bcx^{2+m} \sqrt{d - c^2 dx^2}}{(2 + m)^2 \sqrt{1 - c^2 x^2}} + \frac{x^{1+m} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2 + m}$$

$$+ \frac{x^{1+m} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right)}{(2 + 3m + m^2) \sqrt{1 - c^2 x^2}}$$

$$- \frac{bcx^{2+m} \sqrt{d - c^2 dx^2} {}_3F_2\left(1, 1 + \frac{m}{2}, 1 + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}; c^2 x^2\right)}{(1 + m)(2 + m)^2 \sqrt{1 - c^2 x^2}}$$

output

```
-b*c*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(2+m)^2/(-c^2*x^2+1)^(1/2)+x^(1+m)*(-c^2
*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(2+m)+x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*
arcsin(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/(m^2+3*m+2)/(-
c^2*x^2+1)^(1/2)-b*c*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*hypergeom([1, 1+1/2*m,
1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/(1+m)/(2+m)^2/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.74

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$= \frac{x^{1+m} \sqrt{d - c^2 dx^2} ((1+m)(-bcx + a(2+m)\sqrt{1 - c^2 x^2} + b(2+m)\sqrt{1 - c^2 x^2} \arcsin(cx)) + (2+m)(a - bcx) \sqrt{d - c^2 dx^2}}{(1+m)(2+m)\sqrt{1 - c^2 x^2}}$$

input `Integrate[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]),x]`

output `(x^(1+m)*Sqrt[d - c^2*d*x^2]*((1+m)*(-b*c*x) + a*(2+m)*Sqrt[1 - c^2*x^2] + b*(2+m)*Sqrt[1 - c^2*x^2]*ArcSin[c*x]) + (2+m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2] - b*c*x*HypergeometricPFQ[{1, 1+m/2, 1+m/2}, {3/2+m/2, 2+m/2}, c^2*x^2])/((1+m)*(2+m)^2*Sqrt[1 - c^2*x^2])`

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5198, 15, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx$$

$$\downarrow 5198$$

$$\frac{\sqrt{d - c^2 dx^2} \int \frac{x^m (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx}{(m+2)\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^{m+1} dx}{(m+2)\sqrt{1 - c^2 x^2}} +$$

$$\frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{m+2}$$

$$\downarrow 15$$

$$\frac{\sqrt{d-c^2x^2} \int \frac{x^{m(a+b\arcsin(cx))}}{\sqrt{1-c^2x^2}} dx}{(m+2)\sqrt{1-c^2x^2}} + \frac{x^{m+1}\sqrt{d-c^2x^2}(a+b\arcsin(cx))}{m+2} - \frac{bcx^{m+2}\sqrt{d-c^2x^2}}{(m+2)^2\sqrt{1-c^2x^2}}$$

↓ 5220

$$\frac{\sqrt{d-c^2x^2} \left(\frac{x^{m+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right)(a+b\arcsin(cx))}{m+1} - \frac{bcx^{m+2} {}_3F_2\left(1, \frac{m}{2}+1, \frac{m}{2}+1; \frac{m}{2}+\frac{3}{2}, \frac{m}{2}+2; c^2x^2\right)}{m^2+3m+2} \right)}{(m+2)\sqrt{1-c^2x^2}} + \frac{x^{m+1}\sqrt{d-c^2x^2}(a+b\arcsin(cx))}{m+2} - \frac{bcx^{m+2}\sqrt{d-c^2x^2}}{(m+2)^2\sqrt{1-c^2x^2}}$$

input `Int [x^m*sqrt [d - c^2*d*x^2]*(a + b*ArcSin [c*x]), x]`

output `-((b*c*x^(2 + m)*sqrt [d - c^2*d*x^2])/((2 + m)^2*sqrt [1 - c^2*x^2])) + (x^(1 + m)*sqrt [d - c^2*d*x^2]*(a + b*ArcSin [c*x]))/(2 + m) + (sqrt [d - c^2*d*x^2]*((x^(1 + m)*(a + b*ArcSin [c*x])*Hypergeometric2F1 [1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(1 + m) - (b*c*x^(2 + m)*HypergeometricPFQ [{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + 3*m + m^2)))/((2 + m)*sqrt [1 - c^2*x^2])`

Defintions of rubi rules used

rule 15 `Int [(a_.)*(x_)^(m_.), x_Symbol] := Simp [a*(x^(m + 1))/(m + 1), x] /; FreeQ [a, m], x] && NeQ [m, -1]`

rule 5198 `Int [((a_.) + ArcSin [(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_)*sqrt [(d_) + (e_.)*(x_)^2], x_Symbol] := Simp [(f*x)^(m + 1)*sqrt [d + e*x^2]*((a + b*ArcSin [c*x])^n/(f*(m + 2))), x] + (Simp [(1/(m + 2))*Simp [sqrt [d + e*x^2]/sqrt [1 - c^2*x^2]] Int [(f*x)^m*((a + b*ArcSin [c*x])^n/sqrt [1 - c^2*x^2]), x], x] - Simp [b*c*(n/(f*(m + 2)))*Simp [sqrt [d + e*x^2]/sqrt [1 - c^2*x^2]] Int [(f*x)^(m + 1)*(a + b*ArcSin [c*x])^(n - 1), x], x]) /; FreeQ [{a, b, c, d, e, f, m}, x] && EqQ [c^2*d + e, 0] && GtQ [n, 0] && (IGtQ [m, -2] || EqQ [n, 1])`

rule 5220

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_)^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int x^m \sqrt{-c^2 d x^2 + d} (a + b \arcsin(cx)) dx$$

input

```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x)
```

output

```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int x^m \sqrt{d - c^2 d x^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 d x^2 + d} (b \arcsin(cx) + a) x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="fricas"
)
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)*x^m, x)
```

Sympy [F]

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^m \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx)) dx$$

input `integrate(x**m*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x)),x)`

output `Integral(x**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x)), x)`

Maxima [F]

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a) x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \int x^m (a + b \arcsin(cx)) \sqrt{d - c^2 dx^2} dx$$

input `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2),x)`

output `int(x^m*(a + b*asin(c*x))*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) dx = \sqrt{d} \left(\left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx \right) b \right. \\ \left. + \left(\int x^m \sqrt{-c^2 x^2 + 1} dx \right) a \right)$$

input `int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x)),x)`

output `sqrt(d)*(int(x**m*sqrt(-c**2*x**2 + 1)*asin(c*x),x)*b + int(x**m*sqrt(-c**2*x**2 + 1),x)*a)`

3.149 $\int \frac{x^m(a+b \arcsin(cx))}{\sqrt{d-c^2dx^2}} dx$

Optimal result 1341
 Mathematica [A] (verified) 1342
 Rubi [A] (verified) 1342
 Maple [F] 1343
 Fracas [F] 1343
 Sympy [F] 1344
 Maxima [F] 1344
 Giac [F(-2)] 1344
 Mupad [F(-1)] 1345
 Reduce [F] 1345

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx$$

$$= \frac{x^{1+m}\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right)}{(1 + m)\sqrt{d - c^2dx^2}}$$

$$- \frac{bcx^{2+m}\sqrt{1 - c^2x^2} {}_3F_2\left(1, 1 + \frac{m}{2}, 1 + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}; c^2x^2\right)}{(2 + 3m + m^2)\sqrt{d - c^2dx^2}}$$

output

```
x^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2*x^2)/(1+m)/(-c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*(-c^2*x^2+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m], [2+1/2*m, 3/2+1/2*m], c^2*x^2)/(m^2+3*m+2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79

$$\int \frac{x^m (a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{x^{1+m} \sqrt{1 - c^2 x^2} ((2 + m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right) - bcx {}_3F_2\left(1, 1 + \frac{m}{2}, \frac{3+m}{2}, 2 + \frac{m}{2}, c^2 x^2\right))}{(1 + m)(2 + m)\sqrt{d - c^2 dx^2}}$$

input

```
Integrate[(x^m*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output

```
(x^(1 + m)*Sqrt[1 - c^2*x^2]*((2 + m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5220$$

$$\frac{\sqrt{1 - c^2 x^2} x^{m+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2 x^2\right) (a + b \arcsin(cx))}{(m + 1)\sqrt{d - c^2 dx^2}} - \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} {}_3F_2\left(1, \frac{m}{2} + 1, \frac{m}{2} + 1; \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + 2; c^2 x^2\right)}{(m^2 + 3m + 2)\sqrt{d - c^2 dx^2}}$$

input

```
Int[(x^m*(a + b*ArcSin[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output $(x^{(1+m)}\sqrt{1-c^2x^2}(a+b\text{ArcSin}[c*x])\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2x^2])/((1+m)\sqrt{d-c^2dx^2}) - (b*c*x^{(2+m)}\sqrt{1-c^2x^2}\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2x^2])/((2+3m+m^2)\sqrt{d-c^2dx^2})$

Defintions of rubi rules used

rule 5220 $\text{Int}[(((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^{(m_.)})/\text{Sqrt}[(d_.) + (e_.)*(x_.)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}/(f*(m+1))]*\text{Simp}[\text{Sqrt}[1-c^2*x^2]/\text{Sqrt}[d+e*x^2]]*(a+b*\text{ArcSin}[c*x])*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2], x] - \text{Simp}[b*c*((f*x)^{(m+2)}/(f^2*(m+1)*(m+2)))*\text{Simp}[\text{Sqrt}[1-c^2*x^2]/\text{Sqrt}[d+e*x^2]]*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2*x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& !\text{IntegerQ}[m]$

Maple [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{-c^2dx^2 + d}} dx$$

input $\text{int}(x^m*(a+b*\arcsin(c*x))/(-c^2*d*x^2+d)^{(1/2)}, x)$

output $\text{int}(x^m*(a+b*\arcsin(c*x))/(-c^2*d*x^2+d)^{(1/2)}, x)$

Fricas [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2dx^2}} dx = \int \frac{(b \arcsin(cx) + a)x^m}{\sqrt{-c^2dx^2 + d}} dx$$

input $\text{integrate}(x^m*(a+b*\arcsin(c*x))/(-c^2*d*x^2+d)^{(1/2)}, x, \text{algorithm}=\text{"fricas"})$

output $\text{integral}(-\text{sqrt}(-c^2*d*x^2 + d)*(b*\arcsin(c*x) + a)*x^m/(c^2*d*x^2 - d), x)$

Sympy [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^m(a + b \arcsin(cx))}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate(x**m*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**m*(a + b*asin(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)x^m}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)*x^m/sqrt(-c^2*d*x^2 + d), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^m(a + b \operatorname{asin}(cx))}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx = \frac{\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1}} dx\right) a + \left(\int \frac{x^m \operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1}} dx\right) b}{\sqrt{d}}$$

input `int(x^m*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(1/2),x)`

output `(int(x**m/sqrt(-c**2*x**2 + 1),x)*a + int((x**m*asin(c*x))/sqrt(-c**2*x**2 + 1),x)*b)/sqrt(d)`

3.150
$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx$$

Optimal result	1346
Mathematica [A] (verified)	1347
Rubi [A] (verified)	1347
Maple [F]	1349
Fricas [F]	1349
Sympy [F]	1350
Maxima [F]	1350
Giac [F(-2)]	1350
Mupad [F(-1)]	1351
Reduce [F]	1351

Optimal result

Integrand size = 27, antiderivative size = 272

$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^{3/2}} dx = \frac{x^{1+m}(a+b \arcsin(cx))}{d\sqrt{d-c^2dx^2}} - \frac{mx^{1+m}\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right)}{d(1+m)\sqrt{d-c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1-c^2x^2} \operatorname{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{d(2+m)\sqrt{d-c^2dx^2}} + \frac{bcmx^{2+m}\sqrt{1-c^2x^2} {}_3F_2\left(1, 1+\frac{m}{2}, 1+\frac{m}{2}; \frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}; c^2x^2\right)}{d(2+3m+m^2)\sqrt{d-c^2dx^2}}$$

output

```
x^(1+m)*(a+b*arcsin(c*x))/d/(-c^2*d*x^2+d)^(1/2)-m*x^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2*x^2)/d/(1+m)/(-c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*(-c^2*x^2+1)^(1/2)*hypergeom([1, 1+1/2*m], [2+1/2*m], c^2*x^2)/d/(2+m)/(-c^2*d*x^2+d)^(1/2)+b*c*m*x^(2+m)*(-c^2*x^2+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m], [2+1/2*m, 3/2+1/2*m], c^2*x^2)/d/(m^2+3*m+2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.76

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{x^{1+m}(-m(2+m)\sqrt{1-c^2x^2}(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right) + (1+m)((2+m)(a + b \arcsin(cx)) - bcx\sqrt{1-c^2x^2}) \operatorname{Hypergeometric2F1}\left(1, 1+\frac{m}{2}, 2+\frac{m}{2}, c^2x^2\right) + bc m x \sqrt{1-c^2x^2} \operatorname{HypergeometricPFQ}\left[\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\}, \{3/2+\frac{m}{2}, 2+\frac{m}{2}\}, c^2x^2\right])}{d(1+m)(2+m)\sqrt{d-c^2dx^2}}$$

input

```
Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output

```
(x^(1 + m)*(-(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) + (1 + m)*((2 + m)*(a + b*ArcSin[c*x]) - b*c*x*Sqrt[1 - c^2*x^2])*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, c^2*x^2]) + b*c*m*x*Sqrt[1 - c^2*x^2])*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2))/(d*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5208, 278, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx \\ & \quad \downarrow \text{5208} \\ & -\frac{m \int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{d} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{x^{m+1}}{1 - c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}} + \frac{x^{m+1}(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{278} \\ & -\frac{m \int \frac{x^m(a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{d} + \frac{x^{m+1}(a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \\ & \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(1, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{d(m+2)\sqrt{d - c^2 dx^2}} \end{aligned}$$

↓ 5220

$$m \left(\frac{\sqrt{1-c^2x^2} x^{m+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right) (a+b \arcsin(cx))}{(m+1)\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{1-c^2x^2} x^{m+2} {}_3F_2\left(1, \frac{m}{2}+1, \frac{m}{2}+1; \frac{m}{2}+\frac{3}{2}, \frac{m}{2}+2; c^2x^2\right)}{(m^2+3m+2)\sqrt{d-c^2dx^2}} \right) - \frac{x^{m+1}(a+b \arcsin(cx))}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{1-c^2x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(1, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{d(m+2)\sqrt{d-c^2dx^2}}$$

input `Int[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(3/2),x]`

output `(x^(1 + m)*(a + b*ArcSin[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d*(2 + m)*Sqrt[d - c^2*d*x^2]) - (m*((x^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(1 + m)*Sqrt[d - c^2*d*x^2]) - (b*c*x^(2 + m)*Sqrt[1 - c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + 3*m + m^2)*Sqrt[d - c^2*d*x^2]))/d`

Defintions of rubi rules used

rule 278 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

rule 5208 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1)) * Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])`

rule 5220

```

Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_)]/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]

```

Maple [F]

$$\int \frac{x^m (a + b \arcsin(cx))}{(-c^2 d x^2 + d)^{\frac{3}{2}}} dx$$

input

```
int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^m (a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^m}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input

```
integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)*x^m/(c^4*d^2*x^4 - 2*c^2
*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^m(a + b \arcsin(cx))}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate(x**m*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral(x**m*(a + b*asin(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)x^m}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)*x^m/(-c^2*d*x^2 + d)^(3/2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^m(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx\right) a - \left(\int \frac{x^m \operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx\right) b}{\sqrt{d} d}$$

input `int(x^m*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(3/2),x)`

output `(- (int(x**m/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)), x)*a + int((x**m*asin(c*x))/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b))/(sqrt(d)*d)`

3.151
$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	1352
Mathematica [A] (verified)	1353
Rubi [A] (verified)	1353
Maple [F]	1356
Fricas [F]	1356
Sympy [F(-1)]	1356
Maxima [F]	1357
Giac [F(-2)]	1357
Mupad [F(-1)]	1357
Reduce [F]	1358

Optimal result

Integrand size = 27, antiderivative size = 408

$$\int \frac{x^m(a+b \arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx = \frac{x^{1+m}(a+b \arcsin(cx))}{3d(d-c^2dx^2)^{3/2}} + \frac{(2-m)x^{1+m}(a+b \arcsin(cx))}{3d^2\sqrt{d-c^2dx^2}}$$

$$- \frac{(2-m)mx^{1+m}\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right)}{3d^2(1+m)\sqrt{d-c^2dx^2}}$$

$$- \frac{bc(2-m)x^{2+m}\sqrt{1-c^2x^2} \operatorname{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{3d^2(2+m)\sqrt{d-c^2dx^2}}$$

$$- \frac{bcx^{2+m}\sqrt{1-c^2x^2} \operatorname{Hypergeometric2F1}\left(2, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{3d^2(2+m)\sqrt{d-c^2dx^2}}$$

$$+ \frac{bc(2-m)mx^{2+m}\sqrt{1-c^2x^2} {}_3F_2\left(1, 1+\frac{m}{2}, 1+\frac{m}{2}; \frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}; c^2x^2\right)}{3d^2(2+3m+m^2)\sqrt{d-c^2dx^2}}$$

output

$$\frac{1}{3}x^{1+m}(a+b\arcsin(cx))/d/(-c^2dx^2+d)^{3/2}+1/3(2-m)x^{1+m}(a+b\arcsin(cx))/d^2/(-c^2dx^2+d)^{1/2}-1/3(2-m)m x^{1+m}(-c^2x^2+1)^{(1/2)}(a+b\arcsin(cx))*\text{hypergeom}([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2x^2)/d^2/(1+m)/(-c^2dx^2+d)^{1/2}-1/3b*c*(2-m)x^{2+m}(-c^2x^2+1)^{(1/2)}\text{hypergeom}([1, 1+1/2*m], [2+1/2*m], c^2x^2)/d^2/(2+m)/(-c^2dx^2+d)^{1/2}-1/3b*c*x^{2+m}(-c^2x^2+1)^{(1/2)}\text{hypergeom}([2, 1+1/2*m], [2+1/2*m], c^2x^2)/d^2/(2+m)/(-c^2dx^2+d)^{1/2}+1/3b*c*(2-m)m x^{2+m}(-c^2x^2+1)^{(1/2)}\text{hypergeom}([1, 1+1/2*m, 1+1/2*m], [2+1/2*m, 3/2+1/2*m], c^2x^2)/d^2/(m^2+3m+2)/(-c^2dx^2+d)^{1/2}$$
Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.68

$$\int \frac{x^m(a+b\arcsin(cx))}{(d-c^2dx^2)^{5/2}} dx = \frac{x^{1+m}(d(1+m)(2+m)(a+b\arcsin(cx)) - bcd(1+m)x(1-c^2x^2)^{3/2} \text{Hypergeom}([1/2, 1/2+1/2*m], [3/2+1/2*m], c^2x^2)/d^2/(1+m) - 1/3b*c*(2-m)x^{2+m}(-c^2x^2+1)^{(1/2)}\text{hypergeom}([1, 1+1/2*m], [2+1/2*m], c^2x^2)/d^2/(2+m) - 1/3b*c*x^{2+m}(-c^2x^2+1)^{(1/2)}\text{hypergeom}([2, 1+1/2*m], [2+1/2*m], c^2x^2)/d^2/(2+m) + 1/3b*c*(2-m)m x^{2+m}(-c^2x^2+1)^{(1/2)}\text{hypergeom}([1, 1+1/2*m, 1+1/2*m], [2+1/2*m, 3/2+1/2*m], c^2x^2)/d^2/(m^2+3m+2)}}{(d-c^2dx^2)^{5/2}}$$

input

`Integrate[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2), x]`

output

$$\frac{(x^{1+m}(d(1+m)(2+m)(a+b\text{ArcSin}[c*x]) - b*c*d*(1+m)*x*(1-c^2*x^2)^{3/2})\text{Hypergeometric2F1}[2, 1+m/2, 2+m/2, c^2*x^2] + (2-m)*(d-c^2*d*x^2)*((1+m)(2+m)(a+b\text{ArcSin}[c*x]) - b*c*(1+m)*x*\text{Sqrt}[1-c^2*x^2])\text{Hypergeometric2F1}[1, 1+m/2, 2+m/2, c^2*x^2] - m*\text{Sqrt}[1-c^2*x^2]*((2+m)(a+b\text{ArcSin}[c*x])\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2] - b*c*x*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2*x^2])))/(3*d^2*(1+m)(2+m)(d-c^2*d*x^2)^{3/2})$$
Rubi [A] (verified)Time = 0.92 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5208, 278, 5208, 278, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^m (a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx \\
& \quad \downarrow \text{5208} \\
& \frac{(2 - m) \int \frac{x^m (a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx}{3d} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{x^{m+1}}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{x^{m+1} (a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}} \\
& \quad \downarrow \text{278} \\
& \frac{(2 - m) \int \frac{x^m (a + b \arcsin(cx))}{(d - c^2 dx^2)^{3/2}} dx}{3d} + \frac{x^{m+1} (a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}} - \\
& \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(2, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{3d^2 (m+2) \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5208} \\
& (2 - m) \left(-\frac{m \int \frac{x^m (a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{d} - \frac{bc\sqrt{1 - c^2 x^2} \int \frac{x^{m+1}}{1 - c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}} + \frac{x^{m+1} (a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} \right) \\
& \quad \downarrow \text{278} \\
& \frac{x^{m+1} (a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(2, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{3d^2 (m+2) \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5220} \\
& (2 - m) \left(-\frac{m \int \frac{x^m (a + b \arcsin(cx))}{\sqrt{d - c^2 dx^2}} dx}{d} + \frac{x^{m+1} (a + b \arcsin(cx))}{d\sqrt{d - c^2 dx^2}} - \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(1, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{d(m+2) \sqrt{d - c^2 dx^2}} \right) \\
& \quad \downarrow \text{5220} \\
& \frac{x^{m+1} (a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(2, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{3d^2 (m+2) \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5220} \\
& (2 - m) \left(-\frac{m \left(\frac{\sqrt{1 - c^2 x^2} x^{m+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2 x^2\right) (a + b \arcsin(cx))}{(m+1) \sqrt{d - c^2 dx^2}} - \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} {}_3F_2\left(1, \frac{m}{2} + 1, \frac{m}{2} + 1; \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + 2; c^2 x^2\right)}{(m^2 + 3m + 2) \sqrt{d - c^2 dx^2}} \right)}{d} \right) \\
& \quad \downarrow \text{5220} \\
& \frac{x^{m+1} (a + b \arcsin(cx))}{3d (d - c^2 dx^2)^{3/2}} - \frac{bc\sqrt{1 - c^2 x^2} x^{m+2} \operatorname{Hypergeometric2F1}\left(2, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right)}{3d^2 (m+2) \sqrt{d - c^2 dx^2}}
\end{aligned}$$

input

```
Int[(x^m*(a + b*ArcSin[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output

$$\begin{aligned} & (x^{(1+m)}(a + b \operatorname{ArcSin}[c x])) / (3 d (d - c^2 d x^2)^{(3/2)}) - (b c x^{(2+m)} \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Hypergeometric2F1}[2, (2+m)/2, (4+m)/2, c^2 x^2]) / \\ & (3 d^2 (2+m) \operatorname{Sqrt}[d - c^2 d x^2]) + ((2-m) (x^{(1+m)}(a + b \operatorname{ArcSin}[c x])) / (d \operatorname{Sqrt}[d - c^2 d x^2]) - (b c x^{(2+m)} \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Hypergeom} \\ & \operatorname{etric2F1}[1, (2+m)/2, (4+m)/2, c^2 x^2]) / (d (2+m) \operatorname{Sqrt}[d - c^2 d x^2]) - (m (x^{(1+m)} \operatorname{Sqrt}[1 - c^2 x^2] (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1} \\ & [1/2, (1+m)/2, (3+m)/2, c^2 x^2]) / ((1+m) \operatorname{Sqrt}[d - c^2 d x^2]) - (b c x^{(2+m)} \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2 \\ & + m/2, 2+m/2\}, c^2 x^2]) / ((2+3m+m^2) \operatorname{Sqrt}[d - c^2 d x^2])) / (3 d) \end{aligned}$$

Defintions of rubi rules used

rule 278

$$\operatorname{Int}[\{(c \cdot) (x) \}^{(m \cdot)} \{(a \cdot) + (b \cdot) (x)^2 \}^{(p \cdot)}, x_Symbol] \rightarrow \operatorname{Simp}[a^p (c x)^{(m+1)} / (c (m+1)) \operatorname{Hypergeometric2F1}[-p, (m+1)/2, (m+1)/2 + 1, (-b) (x^2/a)], x] /; \operatorname{FreeQ}[\{a, b, c, m, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \mid \mid \operatorname{GtQ}[a, 0])$$

rule 5208

$$\begin{aligned} & \operatorname{Int}[\{(a \cdot) + \operatorname{ArcSin}[c \cdot] (x) \} (b \cdot) \}^{(n \cdot)} \{(f \cdot) (x) \}^{(m \cdot)} \{(d \cdot) + (e \cdot) \\ & \} (x)^2 \}^{(p \cdot)}, x_Symbol] \rightarrow \operatorname{Simp}[(-f x)^{(m+1)} (d + e x^2)^{(p+1)} \{(a + b \operatorname{ArcSin}[c x])^n / (2 d f (p+1))\}, x] + (\operatorname{Simp}[(m+2p+3) / (2 d (p+1)) \\ & \operatorname{Int}[(f x)^m (d + e x^2)^{(p+1)} (a + b \operatorname{ArcSin}[c x])^n, x], x] + \operatorname{Simp}[b c \\ & \cdot (n / (2 f (p+1))) \operatorname{Simp}[(d + e x^2)^p / (1 - c^2 x^2)^p] \operatorname{Int}[(f x)^{(m+1)} \\ & (1 - c^2 x^2)^{(p+1/2)} (a + b \operatorname{ArcSin}[c x])^{(n-1)}, x], x]) /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{LtQ}[p, -1] \&\& !\operatorname{GtQ}[m, 1] \&\& (\operatorname{IntegerQ}[m] \mid \mid \operatorname{IntegerQ}[p] \mid \mid \operatorname{EqQ}[n, 1]) \end{aligned}$$

rule 5220

$$\begin{aligned} & \operatorname{Int}[\{(a \cdot) + \operatorname{ArcSin}[c \cdot] (x) \} (b \cdot) \} \{(f \cdot) (x) \}^{(m \cdot)} / \operatorname{Sqrt}[(d \cdot) + (e \cdot) \\ & \} (x)^2], x_Symbol] \rightarrow \operatorname{Simp}[\{(f x)^{(m+1)} / (f (m+1))\} \operatorname{Simp}[\operatorname{Sqrt}[1 - c^2 x^2] / \operatorname{Sqrt}[d + e x^2]] \cdot (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}[1/2, (1+m)/2, \\ & (3+m)/2, c^2 x^2], x] - \operatorname{Simp}[b c \cdot \{(f x)^{(m+2)} / (f^2 (m+1) (m+2))\} \operatorname{Simp}[\operatorname{Sqrt}[1 - c^2 x^2] / \operatorname{Sqrt}[d + e x^2]] \cdot \operatorname{HypergeometricPFQ}[\{1, 1+m/2, 1+m \\ & /2\}, \{3/2 + m/2, 2+m/2\}, c^2 x^2], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{EqQ}[c^2 d + e, 0] \&\& !\operatorname{IntegerQ}[m] \end{aligned}$$

Maple [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(-c^2dx^2 + d)^{\frac{5}{2}}} dx$$

input `int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `int(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

Fricas [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^m}{(-c^2dx^2 + d)^{\frac{5}{2}}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)*x^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2dx^2)^{5/2}} dx = \text{Timed out}$$

input `integrate(x**m*(a+b*asin(c*x))/(-c**2*d*x**2+d)**(5/2),x)`

output `Timed out`

Maxima [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)x^m}{(-c^2 dx^2 + d)^{\frac{5}{2}}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)*x^m/(-c^2*d*x^2 + d)^(5/2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^m(a + b \operatorname{asin}(cx))}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2),x)`

output `int((x^m*(a + b*asin(c*x)))/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x^m(a + b \arcsin(cx))}{(d - c^2 dx^2)^{5/2}} dx = \frac{\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) a + \left(\int \frac{x^m \arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b}{\sqrt{d} d^2}$$

input `int(x^m*(a+b*asin(c*x))/(-c^2*d*x^2+d)^(5/2),x)`

output `(int(x**m/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*a+int((x**m*asin(c*x))/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b)/(sqrt(d)*d**2)`

3.152 $\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal result	1359
Mathematica [A] (verified)	1359
Rubi [A] (verified)	1360
Maple [F]	1361
Fricas [F]	1361
Sympy [F]	1362
Maxima [F]	1362
Giac [F]	1362
Mupad [F(-1)]	1363
Reduce [F]	1363

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x^{1+m} \arcsin(ax) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right)}{1+m} - \frac{ax^{2+m} {}_3F_2\left(1, 1 + \frac{m}{2}, 1 + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}; a^2x^2\right)}{2+3m+m^2}$$

output

```
x^(1+m)*arcsin(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)-
a*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],a^2*x^2)/(m
^2+3*m+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \frac{x^{1+m} \left((2+m) \arcsin(ax) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right) - ax {}_3F_2\left(1, 1 + \frac{m}{2}, 1 + \frac{m}{2}; \frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}; a^2x^2\right) \right)}{(1+m)(2+m)}$$

input

```
Integrate[(x^m*ArcSin[a*x])/Sqrt[1 - a^2*x^2],x]
```

output $(x^{(1+m)}((2+m)\text{ArcSin}[a*x]*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, a^2*x^2] - a*x*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, a^2*x^2]))/(1+m)*(2+m)$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx$$

↓ 5220

$$\frac{x^{m+1} \arcsin(ax) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, a^2x^2\right)}{ax^{m+2} {}_3F_2\left(1, \frac{m}{2}+1, \frac{m}{2}+1; \frac{m}{2}+\frac{3}{2}, \frac{m}{2}+2; a^2x^2\right)} - \frac{m+1}{m^2+3m+2}$$

input $\text{Int}[(x^m*\text{ArcSin}[a*x])/Sqrt[1 - a^2*x^2],x]$

output $(x^{(1+m)}*\text{ArcSin}[a*x]*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, a^2*x^2])/(1+m) - (a*x^{(2+m)}*\text{HypergeometricPFQ}[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, a^2*x^2])/(2+3*m+m^2)$

Definitions of rubi rules used

rule 5220

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int \frac{x^m \arcsin(ax)}{\sqrt{-a^2x^2 + 1}} dx$$

input `int(x^m*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x)`

output `int(x^m*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x)`

Fricas [F]

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1 - a^2x^2}} dx = \int \frac{x^m \arcsin(ax)}{\sqrt{-a^2x^2 + 1}} dx$$

input `integrate(x^m*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^m*arcsin(a*x)/(a^2*x^2 - 1), x)`

Sympy [F]

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)}{\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(x**m*asin(a*x)/(-a**2*x**2+1)**(1/2),x)`

output `Integral(x**m*asin(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)`

Maxima [F]

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^m*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(x^m*arcsin(a*x)/sqrt(-a^2*x^2 + 1), x)`

Giac [F]

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^m*arcsin(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(x^m*arcsin(a*x)/sqrt(-a^2*x^2 + 1), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)}{\sqrt{1-a^2x^2}} dx$$

input `int((x^m*asin(a*x))/(1 - a^2*x^2)^(1/2),x)`output `int((x^m*asin(a*x))/(1 - a^2*x^2)^(1/2), x)`**Reduce [F]**

$$\int \frac{x^m \arcsin(ax)}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^m*asin(a*x)/(-a^2*x^2+1)^(1/2),x)`output `int((x**m*asin(a*x))/sqrt(- a**2*x**2 + 1),x)`

3.153 $\int x^4(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$

Optimal result	1364
Mathematica [A] (verified)	1365
Rubi [A] (verified)	1365
Maple [A] (verified)	1370
Fricas [A] (verification not implemented)	1371
Sympy [A] (verification not implemented)	1371
Maxima [A] (verification not implemented)	1372
Giac [A] (verification not implemented)	1374
Mupad [F(-1)]	1375
Reduce [F]	1375

Optimal result

Integrand size = 25, antiderivative size = 290

$$\begin{aligned}
 & \int x^4(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx \\
 &= -\frac{304b^2 dx}{3675c^4} - \frac{152b^2 dx^3}{11025c^2} - \frac{38b^2 dx^5}{6125} + \frac{2}{343} b^2 c^2 dx^7 \\
 &+ \frac{32bd\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{525c^5} + \frac{16bdx^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{525c^3} \\
 &+ \frac{4bdx^4\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{175c} + \frac{2bd(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{21c^5} \\
 &- \frac{4bd(1 - c^2 x^2)^{5/2}(a + b \arcsin(cx))}{35c^5} + \frac{2bd(1 - c^2 x^2)^{7/2}(a + b \arcsin(cx))}{49c^5} \\
 &+ \frac{2}{35} dx^5(a + b \arcsin(cx))^2 + \frac{1}{7} dx^5(1 - c^2 x^2)(a + b \arcsin(cx))^2
 \end{aligned}$$

output

```

-304/3675*b^2*d*x/c^4-152/11025*b^2*d*x^3/c^2-38/6125*b^2*d*x^5+2/343*b^2*
c^2*d*x^7+32/525*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^5+16/525*b*d*x
^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+4/175*b*d*x^4*(-c^2*x^2+1)^(1/
2)*(a+b*arcsin(c*x))/c+2/21*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c^5-4
/35*b*d*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c^5+2/49*b*d*(-c^2*x^2+1)^(7/
2)*(a+b*arcsin(c*x))/c^5+2/35*d*x^5*(a+b*arcsin(c*x))^2+1/7*d*x^5*(-c^2*x^
2+1)*(a+b*arcsin(c*x))^2

```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.70

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \frac{d(11025a^2c^5x^5(-7 + 5c^2x^2) + 210ab\sqrt{1 - c^2x^2}(-152 - 76c^2x^2 - 57c^4x^4 + 75c^6x^6) + b^2(31920cx + 5$$

input

```
Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/385875*(d*(11025*a^2*c^5*x^5*(-7 + 5*c^2*x^2) + 210*a*b*Sqrt[1 - c^2*x^2]*(-152 - 76*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x + 5320*c^3*x^3 + 2394*c^5*x^5 - 2250*c^7*x^7) + 210*b*(105*a*c^5*x^5*(-7 + 5*c^2*x^2) + b*Sqrt[1 - c^2*x^2]*(-152 - 76*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6))*ArcSin[c*x] + 11025*b^2*c^5*x^5*(-7 + 5*c^2*x^2)*ArcSin[c*x]^2))/c^5
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5202, 5138, 5194, 27, 2009, 5210, 15, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5202}$$

$$-\frac{2}{7}bcd \int x^5 \sqrt{1 - c^2x^2} (a + b \arcsin(cx)) dx + \frac{2}{7}d \int x^4 (a + b \arcsin(cx))^2 dx + \frac{1}{7}dx^5 (1 - c^2x^2) (a + b \arcsin(cx))^2$$

$$\downarrow \text{5138}$$

$$\begin{aligned}
& \frac{2}{7}d\left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) - \frac{2}{7}bcd \int x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{1}{7}dx^5(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{5194} \\
& \frac{2}{7}d\left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) - \\
& \frac{2}{7}bcd\left(-bc \int -\frac{-15c^6x^6+3c^4x^4+4c^2x^2+8}{105c^6}dx - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} + \frac{2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6}\right. \\
& \quad \left. + \frac{1}{7}dx^5(1-c^2x^2)(a+b\arcsin(cx))^2\right) \\
& \quad \downarrow \text{27} \\
& \frac{2}{7}d\left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) - \\
& \frac{2}{7}bcd\left(\frac{b \int (-15c^6x^6+3c^4x^4+4c^2x^2+8)dx}{105c^5} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} + \frac{2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6}\right. \\
& \quad \left. + \frac{1}{7}dx^5(1-c^2x^2)(a+b\arcsin(cx))^2\right) \\
& \quad \downarrow \text{2009} \\
& \frac{2}{7}d\left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) + \frac{1}{7}dx^5(1-c^2x^2)(a+b\arcsin(cx))^2 - \\
& \frac{2}{7}bcd\left(-\frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} + \frac{2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^6}\right) \\
& \quad \downarrow \text{5210} \\
& \frac{2}{7}d\left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc\left(\frac{4 \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2} + \frac{b \int x^4dx}{5c} - \frac{x^4\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{5c^2}\right)\right) + \\
& \quad \frac{1}{7}dx^5(1-c^2x^2)(a+b\arcsin(cx))^2 - \\
& \frac{2}{7}bcd\left(-\frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} + \frac{2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^6}\right) \\
& \quad \downarrow \text{15}
\end{aligned}$$

$$\frac{2}{7}d \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{5c^2} - \frac{x^4\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{5c^2} + \frac{bx^5}{25c} \right) \right) +$$

$$\frac{1}{7}dx^5(1 - c^2x^2)(a + b \arcsin(cx))^2 -$$

$$\frac{2}{7}bcd \left(-\frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} + \frac{2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} - \frac{(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^6} \right)$$

↓ 5210

$$\frac{2}{7}d \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{b \int x^2 dx}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} \right)}{5c^2} - \frac{x^4\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{5c^2} \right) \right) +$$

$$\frac{1}{7}dx^5(1 - c^2x^2)(a + b \arcsin(cx))^2 -$$

$$\frac{2}{7}bcd \left(-\frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} + \frac{2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} - \frac{(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^6} \right)$$

↓ 15

$$\frac{2}{7}d \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{5c^2} - \frac{x^4\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{5c^2} \right) \right) +$$

$$\frac{1}{7}dx^5(1 - c^2x^2)(a + b \arcsin(cx))^2 -$$

$$\frac{2}{7}bcd \left(-\frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} + \frac{2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} - \frac{(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^6} \right)$$

↓ 5182

$$\begin{aligned}
 & \frac{2}{7}d \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \left(\frac{b \int 1 dx - \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{5c^2} - \frac{x^4 \sqrt{1-c^2x^2}}{5c^2} \right) \right. \\
 & \qquad \qquad \qquad \left. \frac{1}{7}dx^5(1 - c^2x^2)(a + b \arcsin(cx))^2 - \right. \\
 & \left. \frac{2}{7}bcd \left(-\frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} + \frac{2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} - \frac{(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^6} \right) \right) \\
 & \qquad \qquad \qquad \downarrow 24 \\
 & \qquad \qquad \qquad \frac{1}{7}dx^5(1 - c^2x^2)(a + b \arcsin(cx))^2 + \\
 & \frac{2}{7}d \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(-\frac{x^4 \sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{5c^2} + \frac{4 \left(-\frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{2 \left(\frac{bx}{c} - \sqrt{1-c^2x^2} \right)}{5c^2} \right)}{5c^2} \right) \right. \\
 & \left. \frac{2}{7}bcd \left(-\frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} + \frac{2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} - \frac{(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^6} \right) \right)
 \end{aligned}$$

input `Int[x^4*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]`

output `(d*x^5*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/7 - (2*b*c*d*((b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)/7))/(105*c^5) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^6) + (2*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^6) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^6))/7 + (2*d*((x^5*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*((b*x^5)/(25*c) - (x^4*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(5*c^2) + (4*((b*x^3)/(9*c) - (x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(3*c^2) + (2*((b*x)/c - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/(3*c^2)))/(5*c^2))/5)/7`

Defintions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 24 $\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$
- rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] \text{ ; FreeQ}[b, x]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 5138 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]], x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5182 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*(x_)*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Simp}[b*c*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] \text{ ; FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$
- rule 5194 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))*(x_)^{(m_.)}*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[x^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[d + e*x^2], x], x], x]] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p - 1/2] \ \&\& \ \text{NeQ}[p, -2^{(-1)}] \ \&\& \ (\text{IGtQ}[(m+1)/2, 0] \ || \ \text{ILtQ}[(m+2*p+3)/2, 0])$

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x
)^m*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

method	result
parts	$-da^2\left(\frac{1}{7}c^2x^7 - \frac{1}{5}x^5\right) - \frac{db^2\left(-\frac{\arcsin(cx)^2c^5x^5}{5} - \frac{2\arcsin(cx)(3c^4x^4+4c^2x^2+8)\sqrt{-c^2x^2+1}}{75} + \frac{38c^5x^5}{6125} + \frac{152c^3x^3}{11025} + \frac{304cx}{3675} + a\right)}{c^5}$
derivativedivides	$-da^2\left(\frac{1}{7}c^7x^7 - \frac{1}{5}c^5x^5\right) - db^2\left(-\frac{\arcsin(cx)^2c^5x^5}{5} - \frac{2\arcsin(cx)(3c^4x^4+4c^2x^2+8)\sqrt{-c^2x^2+1}}{75} + \frac{38c^5x^5}{6125} + \frac{152c^3x^3}{11025} + \frac{304cx}{3675} + a\right)$
default	$-da^2\left(\frac{1}{7}c^7x^7 - \frac{1}{5}c^5x^5\right) - db^2\left(-\frac{\arcsin(cx)^2c^5x^5}{5} - \frac{2\arcsin(cx)(3c^4x^4+4c^2x^2+8)\sqrt{-c^2x^2+1}}{75} + \frac{38c^5x^5}{6125} + \frac{152c^3x^3}{11025} + \frac{304cx}{3675} + a\right)$
orering	$\frac{(142875c^{10}x^{10} - 346302c^8x^8 + 107235c^6x^6 - 505400c^4x^4 + 872480c^2x^2 - 383040)(-c^2dx^2 + d)(a + b\arcsin(cx))^2}{385875xc^6(c^2x^2 - 1)^2}$

input

```
int(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-d*a^2*(1/7*c^2*x^7-1/5*x^5)-d*b^2/c^5*(-1/5*arcsin(c*x)^2*c^5*x^5-2/75*arcsin(c*x)*(3*c^4*x^4+4*c^2*x^2+8)*(-c^2*x^2+1)^(1/2)+38/6125*c^5*x^5+152/1025*c^3*x^3+304/3675*c*x+1/7*arcsin(c*x)^2*c^7*x^7+2/245*arcsin(c*x)*(5*c^6*x^6+6*c^4*x^4+8*c^2*x^2+16)*(-c^2*x^2+1)^(1/2)-2/343*c^7*x^7)-2*d*a*b/c^5*(1/7*arcsin(c*x)*c^7*x^7-1/5*c^5*x^5*arcsin(c*x)-19/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)-76/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)-152/3675*(-c^2*x^2+1)^(1/2))+1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.79

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx =$$

$$\frac{1125 (49 a^2 - 2 b^2) c^7 dx^7 - 63 (1225 a^2 - 38 b^2) c^5 dx^5 + 5320 b^2 c^3 dx^3 + 31920 b^2 c dx + 11025 (5 b^2 c^7 dx^7$$

input

```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
-1/385875*(1125*(49*a^2 - 2*b^2)*c^7*d*x^7 - 63*(1225*a^2 - 38*b^2)*c^5*d*x^5 + 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 - 7*b^2*c^5*d*x^5)*arcsin(c*x)^2 + 22050*(5*a*b*c^7*d*x^7 - 7*a*b*c^5*d*x^5)*arcsin(c*x) + 210*(75*a*b*c^6*d*x^6 - 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 - 152*a*b*d + (75*b^2*c^6*d*x^6 - 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 - 152*b^2*d)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.34

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^2 dx^7}{7} + \frac{a^2 dx^5}{5} - \frac{2abc^2 dx^7 \arcsin(cx)}{7} - \frac{2abcdx^6 \sqrt{-c^2 x^2 + 1}}{49} + \frac{2abdx^5 \arcsin(cx)}{5} + \frac{38abdx^4 \sqrt{-c^2 x^2 + 1}}{1225c} + \frac{152abdx^2 \sqrt{-c^2 x^2 + 1}}{3675c^3} \\ \frac{a^2 dx^5}{5} \end{cases}$$

input

```
integrate(x**4*(-c**2*d*x**2+d)*(a+b*asin(c*x))**2,x)
```


output

```
Piecewise((-a**2*c**2*d*x**7/7 + a**2*d*x**5/5 - 2*a*b*c**2*d*x**7*asin(c*x)/7 - 2*a*b*c*d*x**6*sqrt(-c**2*x**2 + 1)/49 + 2*a*b*d*x**5*asin(c*x)/5 + 38*a*b*d*x**4*sqrt(-c**2*x**2 + 1)/(1225*c) + 152*a*b*d*x**2*sqrt(-c**2*x**2 + 1)/(3675*c**3) + 304*a*b*d*sqrt(-c**2*x**2 + 1)/(3675*c**5) - b**2*c**2*d*x**7*asin(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 - 2*b**2*c*d*x**6*sqrt(-c**2*x**2 + 1)*asin(c*x)/49 + b**2*d*x**5*asin(c*x)**2/5 - 38*b**2*d*x**5/6125 + 38*b**2*d*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)/(1225*c) - 152*b**2*d*x**3/(11025*c**2) + 152*b**2*d*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(3675*c**3) - 304*b**2*d*x/(3675*c**4) + 304*b**2*d*sqrt(-c**2*x**2 + 1)*asin(c*x)/(3675*c**5), Ne(c, 0)), (a**2*d*x**5/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.56

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= -\frac{1}{7} b^2 c^2 dx^7 \arcsin(cx)^2 - \frac{1}{7} a^2 c^2 dx^7 + \frac{1}{5} b^2 dx^5 \arcsin(cx)^2 + \frac{1}{5} a^2 dx^5$$

$$- \frac{2}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right)$$

$$- \frac{2}{25725} \left(105 \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) c \arcsin(cx) \right)$$

$$+ \frac{2}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) abd$$

$$+ \frac{2}{1125} \left(15 \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \arcsin(cx) - \frac{9 c^4 x^5 + 20 c^2 x^3 + 1}{c^4} \right)$$

input

```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```

-1/7*b^2*c^2*d*x^7*arcsin(c*x)^2 - 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcsi
n(c*x)^2 + 1/5*a^2*d*x^5 - 2/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 +
1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 +
16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d - 2/25725*(105*(5*sqrt(-c^2*x^2 +
1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6
+ 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arcsin(c*x) - (75*c^6*x^7 + 126*c^4*x^5 + 2
80*c^2*x^3 + 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arcsin(c*x) + (3*sqrt(-
c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)
/c^6)*c)*a*b*d + 2/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^
2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arcsin(c*x) - (9*c^4*x^5 + 20
*c^2*x^3 + 120*x)/c^4)*b^2*d

```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.71

$$\begin{aligned}
\int x^4(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx = & -\frac{1}{7} a^2 c^2 dx^7 + \frac{1}{5} a^2 dx^5 \\
& - \frac{(c^2 x^2 - 1)^3 b^2 dx \arcsin(cx)^2}{7 c^4} \\
& - \frac{2(c^2 x^2 - 1)^3 ab dx \arcsin(cx)}{7 c^4} \\
& - \frac{8(c^2 x^2 - 1)^2 b^2 dx \arcsin(cx)^2}{35 c^4} \\
& + \frac{2(c^2 x^2 - 1)^3 b^2 dx}{343 c^4} \\
& - \frac{16(c^2 x^2 - 1)^2 ab dx \arcsin(cx)}{35 c^4} \\
& - \frac{(c^2 x^2 - 1) b^2 dx \arcsin(cx)^2}{35 c^4} \\
& - \frac{2(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b^2 d \arcsin(cx)}{49 c^5} \\
& + \frac{484(c^2 x^2 - 1)^2 b^2 dx}{42875 c^4} \\
& - \frac{2(c^2 x^2 - 1) ab dx \arcsin(cx)}{35 c^4} \\
& + \frac{2 b^2 dx \arcsin(cx)^2}{35 c^4} \\
& - \frac{2(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} abd}{49 c^5} \\
& - \frac{16(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d \arcsin(cx)}{175 c^5} \\
& - \frac{3358(c^2 x^2 - 1) b^2 dx}{385875 c^4} + \frac{4 ab dx \arcsin(cx)}{35 c^4} \\
& - \frac{16(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} abd}{175 c^5} \\
& + \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d \arcsin(cx)}{105 c^5} \\
& - \frac{37384 b^2 dx}{385875 c^4} + \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} abd}{105 c^5} \\
& + \frac{4 \sqrt{-c^2 x^2 + 1} b^2 d \arcsin(cx)}{35 c^5} \\
& + \frac{4 \sqrt{-c^2 x^2 + 1} abd}{35 c^5}
\end{aligned}$$

input `integrate(x^4*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output
$$\begin{aligned} & -1/7*a^2*c^2*d*x^7 + 1/5*a^2*d*x^5 - 1/7*(c^2*x^2 - 1)^3*b^2*d*x*arcsin(c*x)^2/c^4 - 2/7*(c^2*x^2 - 1)^3*a*b*d*x*arcsin(c*x)/c^4 - 8/35*(c^2*x^2 - 1)^2*b^2*d*x*arcsin(c*x)^2/c^4 + 2/343*(c^2*x^2 - 1)^3*b^2*d*x/c^4 - 16/35*(c^2*x^2 - 1)^2*a*b*d*x*arcsin(c*x)/c^4 - 1/35*(c^2*x^2 - 1)*b^2*d*x*arcsin(c*x)^2/c^4 - 2/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2*d*arcsin(c*x)/c^5 + 484/42875*(c^2*x^2 - 1)^2*b^2*d*x/c^4 - 2/35*(c^2*x^2 - 1)*a*b*d*x*arcsin(c*x)/c^4 + 2/35*b^2*d*x*arcsin(c*x)^2/c^4 - 2/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d/c^5 - 16/175*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d*arcsin(c*x)/c^5 - 3358/385875*(c^2*x^2 - 1)*b^2*d*x/c^4 + 4/35*a*b*d*x*arcsin(c*x)/c^4 - 16/175*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d/c^5 + 2/105*(-c^2*x^2 + 1)^(3/2)*b^2*d*arcsin(c*x)/c^5 - 37384/385875*b^2*d*x/c^4 + 2/105*(-c^2*x^2 + 1)^(3/2)*a*b*d/c^5 + 4/35*sqrt(-c^2*x^2 + 1)*b^2*d*arcsin(c*x)/c^5 + 4/35*sqrt(-c^2*x^2 + 1)*a*b*d/c^5 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int x^4 (a + b \arcsin(cx))^2 (d - c^2 dx^2) dx$$

input `int(x^4*(a + b*asin(c*x))^2*(d - c^2*d*x^2),x)`

output `int(x^4*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`

Reduce [F]

$$\begin{aligned} & \int x^4 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx \\ & = \frac{d(-1050a \arcsin(cx) ab c^7 x^7 + 1470a \arcsin(cx) ab c^5 x^5 - 150\sqrt{-c^2 x^2 + 1} ab c^6 x^6 + 114\sqrt{-c^2 x^2 + 1} ab c^4 x^4 + \dots}{\dots} \end{aligned}$$

input `int(x^4*(-c^2*d*x^2+d)*(a+b*asin(c*x))^2,x)`

output

```
(d*( - 1050*asin(c*x)*a*b*c**7*x**7 + 1470*asin(c*x)*a*b*c**5*x**5 - 150*sqrt( - c**2*x**2 + 1)*a*b*c**6*x**6 + 114*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 + 152*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2 + 304*sqrt( - c**2*x**2 + 1)*a*b - 3675*int(asin(c*x)**2*x**6,x)*b**2*c**7 + 3675*int(asin(c*x)**2*x**4,x)*b**2*c**5 - 525*a**2*c**7*x**7 + 735*a**2*c**5*x**5))/(3675*c**5)
```

3.154 $\int x^3(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$

Optimal result	1377
Mathematica [A] (verified)	1378
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Fricas [A] (verification not implemented)	1383
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Optimal result

Integrand size = 25, antiderivative size = 202

$$\int x^3(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = -\frac{b^2 dx^2}{24c^2} - \frac{1}{72} b^2 dx^4 + \frac{1}{108} b^2 c^2 dx^6 + \frac{bdx\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{12c^3} + \frac{bdx^3\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{18c} - \frac{1}{18} bcdx^5\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) - \frac{d(a + b \arcsin(cx))^2}{24c^4} + \frac{1}{12} dx^4(a + b \arcsin(cx))^2 + \frac{1}{6} dx^4(1 - c^2 x^2) (a + b \arcsin(cx))^2$$

output

```
-1/24*b^2*d*x^2/c^2-1/72*b^2*d*x^4+1/108*b^2*c^2*d*x^6+1/12*b*d*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+1/18*b*d*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c-1/18*b*c*d*x^5*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-1/24*d*(a+b*arcsin(c*x))^2/c^4+1/12*d*x^4*(a+b*arcsin(c*x))^2+1/6*d*x^4*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2
```


$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{2}bc \int \frac{x^4(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx\right) - \frac{1}{3}bcd \int x^4\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{1}{6}dx^4(1-c^2x^2)(a+b\arcsin(cx))^2$$

↓ 5198

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{2}bc \int \frac{x^4(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx\right) - \frac{1}{3}bcd\left(\frac{1}{6} \int \frac{x^4(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx - \frac{1}{6}bc \int x^5 dx + \frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right) + \frac{1}{6}dx^4(1-c^2x^2)(a+b\arcsin(cx))^2$$

↓ 15

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{2}bc \int \frac{x^4(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx\right) - \frac{1}{3}bcd\left(\frac{1}{6} \int \frac{x^4(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx + \frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{36}bcx^6\right) + \frac{1}{6}dx^4(1-c^2x^2)(a+b\arcsin(cx))^2$$

↓ 5210

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{2}bc\left(\frac{3 \int \frac{x^2(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{4c^2}\right)\right) - \frac{1}{3}bcd\left(\frac{1}{6}\left(\frac{3 \int \frac{x^2(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{4c^2}\right) + \frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right) + \frac{1}{6}dx^4(1-c^2x^2)(a+b\arcsin(cx))^2$$

↓ 15

$$\frac{1}{3}d\left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{2}bc\left(\frac{3 \int \frac{x^2(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{4c^2} + \frac{bx^4}{16c}\right)\right) - \frac{1}{3}bcd\left(\frac{1}{6}\left(\frac{3 \int \frac{x^2(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{4c^2} + \frac{bx^4}{16c}\right) + \frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right) + \frac{1}{6}dx^4(1-c^2x^2)(a+b\arcsin(cx))^2$$

↓ 5210

$$\frac{1}{3}d \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} \right) \right.$$

$$\left. \frac{1}{3}bcd \left(\frac{1}{6} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right) + \frac{1}{6} \right) \right.$$

$$\left. \frac{1}{6}dx^4(1-c^2x^2)(a+b \arcsin(cx))^2 \right)$$

↓ 15

$$\frac{1}{3}d \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} \right) \right.$$

$$\left. \frac{1}{3}bcd \left(\frac{1}{6} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right) + \frac{1}{6}x^5 \right) \right.$$

$$\left. \frac{1}{6}dx^4(1-c^2x^2)(a+b \arcsin(cx))^2 \right)$$

↓ 5152

$$\frac{1}{6}dx^4(1-c^2x^2)(a+b \arcsin(cx))^2 +$$

$$\frac{1}{3}d \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(-\frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} + \frac{3 \left(\frac{(a+b \arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} \right) \right.$$

$$\left. \frac{1}{3}bcd \left(\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b \arcsin(cx)) + \frac{1}{6} \left(-\frac{x^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{4c^2} + \frac{3 \left(\frac{(a+b \arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} \right) \right) \right)$$

input `Int [x^3*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]`

output

$$\begin{aligned} & (d*x^4*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/6 - (b*c*d*(-1/36*(b*c*x^6) + \\ & (x^5*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/6 + ((b*x^4)/(16*c) - (x^3*sqrt[\\ & t[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*sqrt[\\ & 1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3 \\ &))/(4*c^2))/6))/3 + (d*((x^4*(a + b*ArcSin[c*x])^2)/4 - (b*c*((b*x^4)/(16 \\ & *c) - (x^3*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4 \\ & *c) - (x*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c* \\ & x])^2/(4*b*c^3)))/(4*c^2))/2))/3 \end{aligned}$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_.)*(x_)^(m_.), x_Symbol] \rightarrow \text{Simp}[a*(x^(m + 1)/(m + 1)), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 5138

$$\begin{aligned} & \text{Int}[(a_.) + \text{ArcSin}[c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] \\ & \rightarrow \text{Simp}[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - \text{Simp}[b*c*(n \\ & /((d*(m + 1))) \ \text{Int}[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/sqrt[1 - c^2 \\ & *x^2]), x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1] \end{aligned}$$

rule 5152

$$\begin{aligned} & \text{Int}[(a_.) + \text{ArcSin}[c_.)*(x_)]*(b_.))^(n_.)/sqrt[(d_) + (e_.)*(x_)^2], x_S \\ & \text{ymbol] } \rightarrow \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[sqrt[1 - c^2*x^2]/sqrt[d + e*x^2]]*(a \\ & + b*ArcSin[c*x])^(n + 1), x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d \\ & + e, 0] \ \&\& \ \text{NeQ}[n, -1] \end{aligned}$$

rule 5198

$$\begin{aligned} & \text{Int}[(a_.) + \text{ArcSin}[c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*sqrt[(d_) + \\ & (e_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^(m + 1)*sqrt[d + e*x^2]*((a + b*ArcS \\ & in[c*x])^n/(f*(m + 2))), x] + (\text{Simp}[(1/(m + 2))*\text{Simp}[sqrt[d + e*x^2]/sqrt[1 \\ & - c^2*x^2]] \ \text{Int}[(f*x)^m*((a + b*ArcSin[c*x])^n/sqrt[1 - c^2*x^2]), x], x \\ &] - \text{Simp}[b*c*(n/(f*(m + 2)))*\text{Simp}[sqrt[d + e*x^2]/sqrt[1 - c^2*x^2]] \ \text{Int}[\\ & (f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, \\ & f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1]) \end{aligned}$$

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.63

method	result
parts	$-d a^2 \left(\frac{1}{6} c^2 x^6 - \frac{1}{4} x^4 \right) - \frac{d b^2 \left(-\frac{\arcsin(cx)^2 x^4 c^4}{4} + \frac{\arcsin(cx) (-2c^3 x^3 \sqrt{-c^2 x^2 + 1} - 3cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx))}{16} - \frac{\arcsin(cx)}{24} \right)}{16}$
derivativedivides	$-d a^2 \left(\frac{1}{6} c^6 x^6 - \frac{1}{4} c^4 x^4 \right) - d b^2 \left(-\frac{\arcsin(cx)^2 x^4 c^4}{4} + \frac{\arcsin(cx) (-2c^3 x^3 \sqrt{-c^2 x^2 + 1} - 3cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx))}{16} - \frac{\arcsin(cx)}{24} \right)$
default	$-d a^2 \left(\frac{1}{6} c^6 x^6 - \frac{1}{4} c^4 x^4 \right) - d b^2 \left(-\frac{\arcsin(cx)^2 x^4 c^4}{4} + \frac{\arcsin(cx) (-2c^3 x^3 \sqrt{-c^2 x^2 + 1} - 3cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx))}{16} - \frac{\arcsin(cx)}{24} \right)$
orering	$\frac{(182c^8 x^8 - 473c^6 x^6 + 42c^4 x^4 + 369c^2 x^2 - 180)(-c^2 d x^2 + d)(a + b \arcsin(cx))^2}{432c^4 (c^2 x^2 - 1)^2} - \frac{(10c^6 x^6 - 21c^4 x^4 - 23c^2 x^2 + 24)(3x^2)}{16}$

input

```
int(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-d*a^2*(1/6*c^2*x^6-1/4*x^4)-d*b^2/c^4*(-1/4*arcsin(c*x)^2*x^4*c^4+1/16*arcsin(c*x)*(-2*c^3*x^3*(-c^2*x^2+1)^(1/2)-3*c*x*(-c^2*x^2+1)^(1/2)+3*arcsin(c*x))-1/24*arcsin(c*x)^2+1/128*(2*c^2*x^2+3)^2+1/6*arcsin(c*x)^2*c^6*x^6-1/144*arcsin(c*x)*(-8*c^5*x^5*(-c^2*x^2+1)^(1/2)-10*c^3*x^3*(-c^2*x^2+1)^(1/2)-15*c*x*(-c^2*x^2+1)^(1/2)+15*arcsin(c*x))-1/108*(c^2*x^2-1)^3-13/288*(c^2*x^2-1)^2-11/96*c^2*x^2+11/96)-2*d*a*b/c^4*(1/6*arcsin(c*x)*c^6*x^6-1/4*c^4*x^4*arcsin(c*x)-1/36*c^3*x^3*(-c^2*x^2+1)^(1/2)-1/24*c*x*(-c^2*x^2+1)^(1/2)+1/24*arcsin(c*x)+1/36*c^5*x^5*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx =$$

$$\frac{-2(18a^2 - b^2)c^6 dx^6 - 3(18a^2 - b^2)c^4 dx^4 + 9b^2 c^2 dx^2 + 9(4b^2 c^6 dx^6 - 6b^2 c^4 dx^4 + b^2 d) \arcsin(cx)^2 + \dots}{\dots}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
-1/216*(2*(18*a^2 - b^2)*c^6*d*x^6 - 3*(18*a^2 - b^2)*c^4*d*x^4 + 9*b^2*c^2*d*x^2 + 9*(4*b^2*c^6*d*x^6 - 6*b^2*c^4*d*x^4 + b^2*d)*arcsin(c*x)^2 + 18*(4*a*b*c^6*d*x^6 - 6*a*b*c^4*d*x^4 + a*b*d)*arcsin(c*x) + 6*(2*a*b*c^5*d*x^5 - 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x + (2*b^2*c^5*d*x^5 - 2*b^2*c^3*d*x^3 - 3*b^2*c*d*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.64

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^2 dx^6}{6} + \frac{a^2 dx^4}{4} - \frac{abc^2 dx^6 \arcsin(cx)}{3} - \frac{abcdx^5 \sqrt{-c^2 x^2 + 1}}{18} + \frac{abdx^4 \arcsin(cx)}{2} + \frac{abdx^3 \sqrt{-c^2 x^2 + 1}}{18c} + \frac{abdx \sqrt{-c^2 x^2 + 1}}{12c^3} - \frac{abd \arcsin(cx)}{12c^3} \\ \frac{a^2 dx^4}{4} \end{cases}$$

input

```
integrate(x**3*(-c**2*d*x**2+d)*(a+b*asin(c*x))**2,x)
```

output

```
Piecewise((-a**2*c**2*d*x**6/6 + a**2*d*x**4/4 - a*b*c**2*d*x**6*asin(c*x)
/3 - a*b*c*d*x**5*sqrt(-c**2*x**2 + 1)/18 + a*b*d*x**4*asin(c*x)/2 + a*b*d
*x**3*sqrt(-c**2*x**2 + 1)/(18*c) + a*b*d*x*sqrt(-c**2*x**2 + 1)/(12*c**3)
- a*b*d*asin(c*x)/(12*c**4) - b**2*c**2*d*x**6*asin(c*x)**2/6 + b**2*c**2
*d*x**6/108 - b**2*c*d*x**5*sqrt(-c**2*x**2 + 1)*asin(c*x)/18 + b**2*d*x**
4*asin(c*x)**2/4 - b**2*d*x**4/72 + b**2*d*x**3*sqrt(-c**2*x**2 + 1)*asin(
c*x)/(18*c) - b**2*d*x**2/(24*c**2) + b**2*d*x*sqrt(-c**2*x**2 + 1)*asin(
c*x)/(12*c**3) - b**2*d*asin(c*x)**2/(24*c**4), Ne(c, 0)), (a**2*d*x**4/4,
True))
```

Maxima [F]

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)(b \arcsin(cx) + a)^2 x^3 dx$$

input

```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```
-1/6*a^2*c^2*d*x^6 + 1/4*a^2*d*x^4 - 1/144*(48*x^6*arcsin(c*x) + (8*sqrt(-
c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 +
1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^2*d + 1/16*(8*x^4*arcsin(c*x) + (2
*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c
^5)*c)*a*b*d - 1/12*(2*b^2*c^2*d*x^6 - 3*b^2*d*x^4)*arctan2(c*x, sqrt(c*x
+ 1)*sqrt(-c*x + 1))^2 - integrate(1/6*(2*b^2*c^3*d*x^6 - 3*b^2*c*d*x^4)*s
qrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^
2*x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(177) = 354$.

Time = 0.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.87

$$\int x^3(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx$$

$$= -\frac{1}{6}a^2c^2dx^6 + \frac{1}{4}a^2dx^4 - \frac{(c^2x^2 - 1)^2\sqrt{-c^2x^2 + 1}b^2dx \arcsin(cx)}{18c^3}$$

$$- \frac{(c^2x^2 - 1)^3b^2d \arcsin(cx)^2}{6c^4} - \frac{(c^2x^2 - 1)^2\sqrt{-c^2x^2 + 1}abd x}{18c^3}$$

$$+ \frac{(-c^2x^2 + 1)^{\frac{3}{2}}b^2dx \arcsin(cx)}{18c^3} - \frac{(c^2x^2 - 1)^3abd \arcsin(cx)}{3c^4}$$

$$- \frac{(c^2x^2 - 1)^2b^2d \arcsin(cx)^2}{4c^4} + \frac{(-c^2x^2 + 1)^{\frac{3}{2}}abd x}{18c^3} + \frac{\sqrt{-c^2x^2 + 1}b^2dx \arcsin(cx)}{12c^3}$$

$$+ \frac{(c^2x^2 - 1)^3b^2d}{108c^4} - \frac{(c^2x^2 - 1)^2abd \arcsin(cx)}{2c^4} + \frac{\sqrt{-c^2x^2 + 1}abd x}{12c^3}$$

$$+ \frac{(c^2x^2 - 1)^2b^2d}{72c^4} + \frac{b^2d \arcsin(cx)^2}{24c^4} - \frac{(c^2x^2 - 1)b^2d}{24c^4} + \frac{abd \arcsin(cx)}{12c^4} - \frac{5b^2d}{216c^4}$$

input `integrate(x^3*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `-1/6*a^2*c^2*d*x^6 + 1/4*a^2*d*x^4 - 1/18*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d*x*arcsin(c*x)/c^3 - 1/6*(c^2*x^2 - 1)^3*b^2*d*arcsin(c*x)^2/c^4 - 1/18*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d*x/c^3 + 1/18*(-c^2*x^2 + 1)^(3/2)*b^2*d*x*arcsin(c*x)/c^3 - 1/3*(c^2*x^2 - 1)^3*a*b*d*arcsin(c*x)/c^4 - 1/4*(c^2*x^2 - 1)^2*b^2*d*arcsin(c*x)^2/c^4 + 1/18*(-c^2*x^2 + 1)^(3/2)*a*b*d*x/c^3 + 1/12*sqrt(-c^2*x^2 + 1)*b^2*d*x*arcsin(c*x)/c^3 + 1/108*(c^2*x^2 - 1)^3*b^2*d/c^4 - 1/2*(c^2*x^2 - 1)^2*a*b*d*arcsin(c*x)/c^4 + 1/12*sqrt(-c^2*x^2 + 1)*a*b*d*x/c^3 + 1/72*(c^2*x^2 - 1)^2*b^2*d/c^4 + 1/24*b^2*d*arcsin(c*x)^2/c^4 - 1/24*(c^2*x^2 - 1)*b^2*d/c^4 + 1/12*a*b*d*arcsin(c*x)/c^4 - 5/216*b^2*d/c^4`

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int x^3 (a + b \arcsin(cx))^2 (d - c^2 dx^2) dx$$

input `int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`output `int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`**Reduce [F]**

$$\int x^3 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \frac{d(-12 \arcsin(cx) ab c^6 x^6 + 18 \arcsin(cx) ab c^4 x^4 - 3 \arcsin(cx) ab - 2\sqrt{-c^2 x^2 + 1} ab c^5 x^5 + 2\sqrt{-c^2 x^2 + 1} ab c^3 x^3)}{36}$$

input `int(x^3*(-c^2*d*x^2+d)*(a+b*asin(c*x))^2,x)`output `(d*(- 12*asin(c*x)*a*b*c**6*x**6 + 18*asin(c*x)*a*b*c**4*x**4 - 3*asin(c*x)*a*b - 2*sqrt(- c**2*x**2 + 1)*a*b*c**5*x**5 + 2*sqrt(- c**2*x**2 + 1)*a*b*c**3*x**3 + 3*sqrt(- c**2*x**2 + 1)*a*b*c*x - 36*int(asin(c*x)**2*x**5,x)*b**2*c**6 + 36*int(asin(c*x)**2*x**3,x)*b**2*c**4 - 6*a**2*c**6*x**6 + 9*a**2*c**4*x**4))/(36*c**4)`

3.155 $\int x^2(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$

Optimal result	1387
Mathematica [A] (verified)	1388
Rubi [A] (verified)	1388
Maple [A] (verified)	1392
Fricas [A] (verification not implemented)	1393
Sympy [A] (verification not implemented)	1393
Maxima [A] (verification not implemented)	1394
Giac [A] (verification not implemented)	1395
Mupad [F(-1)]	1396
Reduce [F]	1396

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int x^2(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = -\frac{52b^2 dx}{225c^2} - \frac{26}{675}b^2 dx^3 + \frac{2}{125}b^2 c^2 dx^5$$

$$+ \frac{8bd\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{45c^3}$$

$$+ \frac{4bdx^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{45c}$$

$$+ \frac{2bd(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{15c^3}$$

$$- \frac{2bd(1 - c^2 x^2)^{5/2}(a + b \arcsin(cx))}{25c^3}$$

$$+ \frac{2}{15}dx^3(a + b \arcsin(cx))^2$$

$$+ \frac{1}{5}dx^3(1 - c^2 x^2)(a + b \arcsin(cx))^2$$

output

```
-52/225*b^2*d*x/c^2-26/675*b^2*d*x^3+2/125*b^2*c^2*d*x^5+8/45*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+4/45*b*d*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+2/15*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c^3-2/25*b*d*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c^3+2/15*d*x^3*(a+b*arcsin(c*x))^2+1/5*d*x^3*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2
```


Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx =$$

$$\frac{d(225a^2c^3x^3(-5 + 3c^2x^2) + 30ab\sqrt{1 - c^2x^2}(-26 - 13c^2x^2 + 9c^4x^4) + b^2(780cx + 130c^3x^3 - 54c^5x^5))}{c^3}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/3375*(d*(225*a^2*c^3*x^3*(-5 + 3*c^2*x^2) + 30*a*b*Sqrt[1 - c^2*x^2]*(-26 - 13*c^2*x^2 + 9*c^4*x^4) + b^2*(780*c*x + 130*c^3*x^3 - 54*c^5*x^5) + 30*b*(15*a*c^3*x^3*(-5 + 3*c^2*x^2) + b*Sqrt[1 - c^2*x^2]*(-26 - 13*c^2*x^2 + 9*c^4*x^4))*ArcSin[c*x] + 225*b^2*c^3*x^3*(-5 + 3*c^2*x^2)*ArcSin[c*x]^2))/c^3
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5202, 5138, 5194, 27, 2009, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5202$$

$$-\frac{2}{5}bcd \int x^3 \sqrt{1 - c^2x^2} (a + b \arcsin(cx)) dx + \frac{2}{5}d \int x^2 (a + b \arcsin(cx))^2 dx +$$

$$\frac{1}{5}dx^3 (1 - c^2x^2) (a + b \arcsin(cx))^2$$

$$\downarrow 5138$$

$$\begin{aligned}
& \frac{2}{5}d\left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc\int\frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) - \frac{2}{5}bcd\int x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{1}{5}dx^3(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{5194} \\
& \frac{2}{5}d\left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc\int\frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) - \\
& \frac{2}{5}bcd\left(-bc\int-\frac{-3c^4x^4+c^2x^2+2}{15c^4}dx + \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^4}\right) + \\
& \quad \frac{1}{5}dx^3(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{27} \\
& \frac{2}{5}d\left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc\int\frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) - \\
& \frac{2}{5}bcd\left(\frac{b\int(-3c^4x^4+c^2x^2+2)dx}{15c^3} + \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^4}\right) + \\
& \quad \frac{1}{5}dx^3(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{2009} \\
& \frac{2}{5}d\left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc\int\frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx\right) + \frac{1}{5}dx^3(1-c^2x^2)(a+b\arcsin(cx))^2 - \\
& \frac{2}{5}bcd\left(\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^4} + \frac{b\left(-\frac{3}{5}c^4x^5 + \frac{c^2x^3}{3} + 2x\right)}{15c^3}\right) \\
& \quad \downarrow \text{5210} \\
& \frac{2}{5}d\left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc\left(\frac{2\int\frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2} + \frac{b\int x^2dx}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2}\right)\right) + \\
& \quad \frac{1}{5}dx^3(1-c^2x^2)(a+b\arcsin(cx))^2 - \\
& \frac{2}{5}bcd\left(\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^4} + \frac{b\left(-\frac{3}{5}c^4x^5 + \frac{c^2x^3}{3} + 2x\right)}{15c^3}\right) \\
& \quad \downarrow \text{15}
\end{aligned}$$

$$\frac{2}{5}d \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right) \right) + \frac{1}{5}dx^3(1-c^2x^2)(a + b \arcsin(cx))^2 - \frac{2}{5}bcd \left(\frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^4} + \frac{b(-\frac{3}{5}c^4x^5 + \frac{c^2x^3}{3} + 2x)}{15c^3} \right)$$

↓ 5182

$$\frac{2}{5}d \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right) \right) + \frac{1}{5}dx^3(1-c^2x^2)(a + b \arcsin(cx))^2 - \frac{2}{5}bcd \left(\frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^4} + \frac{b(-\frac{3}{5}c^4x^5 + \frac{c^2x^3}{3} + 2x)}{15c^3} \right)$$

↓ 24

$$\frac{1}{5}dx^3(1-c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{5}d \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(-\frac{x^2\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} + \frac{2 \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} + \frac{bx^3}{9c} \right) \right) + \frac{2}{5}bcd \left(\frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{3c^4} + \frac{b(-\frac{3}{5}c^4x^5 + \frac{c^2x^3}{3} + 2x)}{15c^3} \right)$$

input `Int[x^2*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]`

output `(d*x^3*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*d*((b*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^4) + ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^4))/5 + (2*d*((x^3*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*((b*x^3)/(9*c) - (x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(3*c^2) + (2*((b*x)/c - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/(3*c^2)))/3)/5`

Defintions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 24 $\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$
- rule 27 $\text{Int}[(a_)*(F_x_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[F_x, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] \text{ ; FreeQ}[b, x]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 5138 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1)))}, x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2])}, x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5182 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1)))}, x] + \text{Simp}[b*c*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(1 - c^2*x^2)^{(p+1/2)*((a + b*\text{ArcSin}[c*x])^{(n-1)}), x], x] \text{ ; FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$
- rule 5194 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(m_.)*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[x^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[d + e*x^2], x], x], x] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p - 1/2] \ \&\& \ \text{NeQ}[p, -2^{(-1)}] \ \&\& \ (\text{IGtQ}[(m+1)/2, 0] \ || \ \text{ILtQ}[(m+2*p+3)/2, 0])$

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x
)^m*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

method	result
parts	$-d a^2 \left(\frac{1}{5} c^2 x^5 - \frac{1}{3} x^3 \right) - \frac{d b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 3) cx}{3} + \frac{4cx}{15} - \frac{4 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1) \sqrt{-c^2 x^2 + 1}}{45} \right)}{1}$
derivativedivides	$-d a^2 \left(\frac{1}{5} c^5 x^5 - \frac{1}{3} c^3 x^3 \right) - d b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 3) cx}{3} + \frac{4cx}{15} - \frac{4 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1) \sqrt{-c^2 x^2 + 1}}{45} \right) - \frac{2}{1}$
default	$-d a^2 \left(\frac{1}{5} c^5 x^5 - \frac{1}{3} c^3 x^3 \right) - d b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 3) cx}{3} + \frac{4cx}{15} - \frac{4 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1) \sqrt{-c^2 x^2 + 1}}{45} \right) - \frac{2}{1}$
orering	$\frac{(1647c^8x^8 - 4862c^6x^6 - 4033c^4x^4 + 7800c^2x^2 - 3120)(-c^2dx^2 + d)(a + b \arcsin(cx))^2}{3375xc^4(c^2x^2 - 1)^2} - \frac{(324c^6x^6 - 893c^4x^4 - 2665c^2x^2 + 1200)(a + b \arcsin(cx))^2}{45}$

input

```
int(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-d*a^2*(1/5*c^2*x^5-1/3*x^3)-d*b^2/c^3*(1/3*arcsin(c*x)^2*(c^2*x^2-3)*c*x+
4/15*c*x-4/15*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+2/45*arcsin(c*x)*(c^2*x^2-1)*
(-c^2*x^2+1)^(1/2)-2/135*(c^2*x^2-3)*c*x+1/15*arcsin(c*x)^2*(3*c^4*x^4-10*
c^2*x^2+15)*c*x+2/25*arcsin(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)-2/375*(3
*c^4*x^4-10*c^2*x^2+15)*c*x)-2*d*a*b/c^3*(1/5*c^5*x^5*arcsin(c*x)-1/3*c^3*
x^3*arcsin(c*x)-13/225*c^2*x^2*(-c^2*x^2+1)^(1/2)-26/225*(-c^2*x^2+1)^(1/2)
)+1/25*c^4*x^4*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \frac{27(25a^2 - 2b^2)c^5 dx^5 - 5(225a^2 - 26b^2)c^3 dx^3 + 780b^2 c dx + 225(3b^2 c^5 dx^5 - 5b^2 c^3 dx^3) \arcsin(cx)^2}{c^3}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
-1/3375*(27*(25*a^2 - 2*b^2)*c^5*d*x^5 - 5*(225*a^2 - 26*b^2)*c^3*d*x^3 +
780*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 - 5*b^2*c^3*d*x^3)*arcsin(c*x)^2 + 45
0*(3*a*b*c^5*d*x^5 - 5*a*b*c^3*d*x^3)*arcsin(c*x) + 30*(9*a*b*c^4*d*x^4 -
13*a*b*c^2*d*x^2 - 26*a*b*d + (9*b^2*c^4*d*x^4 - 13*b^2*c^2*d*x^2 - 26*b^2
*d)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.48

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \begin{cases} -\frac{a^2 c^2 dx^5}{5} + \frac{a^2 dx^3}{3} - \frac{2abc^2 dx^5 \arcsin(cx)}{5} - \frac{2abcdx^4 \sqrt{-c^2 x^2 + 1}}{25} + \frac{2abdx^3 \arcsin(cx)}{3} + \frac{26abd x^2 \sqrt{-c^2 x^2 + 1}}{225c} + \frac{52abd \sqrt{-c^2 x^2 + 1}}{225c^3} \\ \frac{a^2 dx^3}{3} \end{cases}$$

input

```
integrate(x**2*(-c**2*d*x**2+d)*(a+b*asin(c*x))**2,x)
```

output

```
Piecewise((-a**2*c**2*d*x**5/5 + a**2*d*x**3/3 - 2*a*b*c**2*d*x**5*asin(c*x)/5 - 2*a*b*c*d*x**4*sqrt(-c**2*x**2 + 1)/25 + 2*a*b*d*x**3*asin(c*x)/3 + 26*a*b*d*x**2*sqrt(-c**2*x**2 + 1)/(225*c) + 52*a*b*d*sqrt(-c**2*x**2 + 1)/(225*c**3) - b**2*c**2*d*x**5*asin(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 - 2*b**2*c*d*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)/25 + b**2*d*x**3*asin(c*x)**2/3 - 26*b**2*d*x**3/675 + 26*b**2*d*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(225*c) - 52*b**2*d*x/(225*c**2) + 52*b**2*d*sqrt(-c**2*x**2 + 1)*asin(c*x)/(225*c**3), Ne(c, 0)), (a**2*d*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.68

$$\int x^2 (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= -\frac{1}{5} b^2 c^2 dx^5 \arcsin(cx)^2 - \frac{1}{5} a^2 c^2 dx^5 + \frac{1}{3} b^2 dx^3 \arcsin(cx)^2$$

$$- \frac{2}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) abc^2 d$$

$$- \frac{2}{1125} \left(15 \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \arcsin(cx) - \frac{9 c^4 x^5 + 20 c^2 x^3 + 15}{c^4} \right) abc^2 d$$

$$+ \frac{1}{3} a^2 dx^3 + \frac{2}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) abd$$

$$+ \frac{2}{27} \left(3 c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \arcsin(cx) - \frac{c^2 x^3 + 6 x}{c^2} \right) b^2 d$$

input

```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```

-1/5*b^2*c^2*d*x^5*arcsin(c*x)^2 - 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcsi
n(c*x)^2 - 2/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sq
rt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d - 2/1125
*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(-c^2*x^2 + 1)/c^6)*c*arcsin(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*
b^2*c^2*d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)
*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d + 2/27*(3*c*(sqrt(-c^2*x^2 + 1)
)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcsin(c*x) - (c^2*x^3 + 6*x)/c^2)*b
^2*d

```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.69

$$\begin{aligned}
& \int x^2(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx \\
&= -\frac{1}{5} a^2 c^2 dx^5 + \frac{1}{3} a^2 dx^3 - \frac{(c^2 x^2 - 1)^2 b^2 dx \arcsin(cx)^2}{5 c^2} - \frac{2(c^2 x^2 - 1)^2 ab dx \arcsin(cx)}{5 c^2} \\
&\quad - \frac{(c^2 x^2 - 1) b^2 dx \arcsin(cx)^2}{15 c^2} + \frac{2(c^2 x^2 - 1)^2 b^2 dx}{125 c^2} - \frac{2(c^2 x^2 - 1) ab dx \arcsin(cx)}{15 c^2} \\
&\quad + \frac{2 b^2 dx \arcsin(cx)^2}{15 c^2} - \frac{2(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d \arcsin(cx)}{25 c^3} - \frac{22(c^2 x^2 - 1) b^2 dx}{3375 c^2} \\
&\quad + \frac{4 ab dx \arcsin(cx)}{15 c^2} - \frac{2(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} abd}{25 c^3} + \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d \arcsin(cx)}{45 c^3} \\
&\quad - \frac{856 b^2 dx}{3375 c^2} + \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} abd}{45 c^3} + \frac{4 \sqrt{-c^2 x^2 + 1} b^2 d \arcsin(cx)}{15 c^3} + \frac{4 \sqrt{-c^2 x^2 + 1} abd}{15 c^3}
\end{aligned}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```


output

```
(d*( - 90*asin(c*x)*a*b*c**5*x**5 + 150*asin(c*x)*a*b*c**3*x**3 - 18*sqrt(
- c**2*x**2 + 1)*a*b*c**4*x**4 + 26*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2
+ 52*sqrt( - c**2*x**2 + 1)*a*b - 225*int(asin(c*x)**2*x**4,x)*b**2*c**5 +
225*int(asin(c*x)**2*x**2,x)*b**2*c**3 - 45*a**2*c**5*x**5 + 75*a**2*c**3
*x**3))/(225*c**3)
```

3.156 $\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$

Optimal result	1398
Mathematica [A] (verified)	1399
Rubi [A] (verified)	1399
Maple [A] (verified)	1402
Fricas [A] (verification not implemented)	1403
Sympy [A] (verification not implemented)	1403
Maxima [F]	1404
Giac [A] (verification not implemented)	1405
Mupad [F(-1)]	1406
Reduce [F]	1406

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = -\frac{3}{32} b^2 dx^2 + \frac{b^2 d(1 - c^2 x^2)^2}{32c^2} + \frac{3bdx\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{16c} + \frac{bdx(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{8c} + \frac{3d(a + b \arcsin(cx))^2}{32c^2} - \frac{d(1 - c^2 x^2)^2(a + b \arcsin(cx))^2}{4c^2}$$

output

```
-3/32*b^2*d*x^2+1/32*b^2*d*(-c^2*x^2+1)^2/c^2+3/16*b*d*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+1/8*b*d*x*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c+3/32*d*(a+b*arcsin(c*x))^2/c^2-1/4*d*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \frac{d(b^2 c^2 x^2 (5 - c^2 x^2) + 2abcx\sqrt{1 - c^2 x^2}(-5 + 2c^2 x^2) + a^2(5 - 16c^2 x^2 + 8c^4 x^4) + 2b(bcx\sqrt{1 - c^2 x^2}(-5 + 2c^2 x^2) + a^2(5 - 16c^2 x^2 + 8c^4 x^4))}{32c^2}$$

input

```
Integrate[x*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/32*(d*(b^2*c^2*x^2*(5 - c^2*x^2) + 2*a*b*c*x*Sqrt[1 - c^2*x^2]*(-5 + 2*c^2*x^2) + a^2*(5 - 16*c^2*x^2 + 8*c^4*x^4) + 2*b*(b*c*x*Sqrt[1 - c^2*x^2]*(-5 + 2*c^2*x^2) + a*(5 - 16*c^2*x^2 + 8*c^4*x^4))*ArcSin[c*x] + b^2*(5 - 16*c^2*x^2 + 8*c^4*x^4)*ArcSin[c*x]^2))/c^2
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5182, 5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5182$$

$$\frac{bd \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx}{2c} - \frac{d(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{4c^2}$$

$$\downarrow 5158$$

$$\frac{bd \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{4} bc \int x(1 - c^2 x^2) dx + \frac{1}{4} x(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right)}{2c} - \frac{d(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{4c^2}$$

↓ 244

$$\frac{bd\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx-\frac{1}{4}bc\int(x-c^2x^3)dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))\right)}{\frac{d(1-c^2x^2)^2(a+b\arcsin(cx))^2}{4c^2}}$$

↓ 2009

$$\frac{bd\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))-\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)}{\frac{d(1-c^2x^2)^2(a+b\arcsin(cx))^2}{4c^2}}$$

↓ 5156

$$\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))-\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)}{\frac{d(1-c^2x^2)^2(a+b\arcsin(cx))^2}{4c^2}}$$

↓ 15

$$\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))-\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)}{\frac{d(1-c^2x^2)^2(a+b\arcsin(cx))^2}{4c^2}}$$

↓ 5152

$$\frac{bd\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))+\frac{(a+b\arcsin(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)-\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)}{\frac{d(1-c^2x^2)^2(a+b\arcsin(cx))^2}{4c^2}}$$

input

```
Int[x*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

$$-1/4*(d*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/c^2 + (b*d*(-1/4*(b*c*(x^2/2 - (c^2*x^4)/4)) + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (x*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)))/4)/(2*c)$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_.)*(x_)^(m_.), x_Symbol] \rightarrow \text{Simp}[a*(x^(m + 1)/(m + 1)), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 244

$$\text{Int}[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5152

$$\text{Int}[(a_. + \text{ArcSin}[(c_.)*(x_)]*(b_.))^(n_.)/\text{Sqrt}[(d_) + (e_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5156

$$\text{Int}[(a_. + \text{ArcSin}[(c_.)*(x_)]*(b_.))^(n_.)*\text{Sqrt}[(d_) + (e_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(a + b*ArcSin[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$$

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1))
Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.31

method	result
derivativedivides	$-\frac{d a^2 (c^2 x^2 - 1)^2}{4} - d b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^2}{4} - \frac{\arcsin(cx) (-2c^3 x^3 \sqrt{-c^2 x^2 + 1} + 5cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx))}{16} + \frac{3 \arcsin(cx)^2}{32} \right) \frac{1}{c^2}$
default	$-\frac{d a^2 (c^2 x^2 - 1)^2}{4} - d b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^2}{4} - \frac{\arcsin(cx) (-2c^3 x^3 \sqrt{-c^2 x^2 + 1} + 5cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx))}{16} + \frac{3 \arcsin(cx)^2}{32} \right) \frac{1}{c^2}$
parts	$-\frac{d a^2 (c^2 x^2 - 1)^2}{4c^2} - \frac{d b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^2}{4} - \frac{\arcsin(cx) (-2c^3 x^3 \sqrt{-c^2 x^2 + 1} + 5cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx))}{16} + \frac{3 \arcsin(cx)^2}{32} \right)}{c^2}$
orering	$\frac{(37c^6 x^6 - 144c^4 x^4 + 113c^2 x^2 - 30)(-c^2 d x^2 + d)(a + b \arcsin(cx))^2}{64c^2 (c^2 x^2 - 1)^2} - \frac{(9c^4 x^4 - 41c^2 x^2 + 20) \left((-c^2 d x^2 + d)(a + b \arcsin(cx)) \right)}{64c^2 (c^2 x^2 - 1)^2}$

input

```
int(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/c^2*(-1/4*d*a^2*(c^2*x^2-1)^2-d*b^2*(1/4*arcsin(c*x)^2*(c^2*x^2-1)^2-1/16*arcsin(c*x)*(-2*c^3*x^3*(-c^2*x^2+1)^(1/2)+5*c*x*(-c^2*x^2+1)^(1/2)+3*arcsin(c*x))+3/32*arcsin(c*x)^2-1/128*(2*c^2*x^2-5)^2)-2*d*a*b*(1/4*c^4*x^4*arcsin(c*x)-1/2*c^2*x^2*arcsin(c*x)+5/32*arcsin(c*x)+1/16*c^3*x^3*(-c^2*x^2+1)^(1/2)-5/32*c*x*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.20

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx =$$

$$\frac{(8a^2 - b^2)c^4 dx^4 - (16a^2 - 5b^2)c^2 dx^2 + (8b^2 c^4 dx^4 - 16b^2 c^2 dx^2 + 5b^2 d) \arcsin(cx)^2 + 2(8abc^4 dx^4 - 16abc^2 dx^2 + 5abd) \arcsin(cx)}{c^2}$$

input

```
integrate(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
-1/32*((8*a^2 - b^2)*c^4*d*x^4 - (16*a^2 - 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d*x^4 - 16*b^2*c^2*d*x^2 + 5*b^2*d)*arcsin(c*x)^2 + 2*(8*a*b*c^4*d*x^4 - 16*a*b*c^2*d*x^2 + 5*a*b*d)*arcsin(c*x) + 2*(2*a*b*c^3*d*x^3 - 5*a*b*c*d*x + (2*b^2*c^3*d*x^3 - 5*b^2*c*d*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.83

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^2 dx^4}{4} + \frac{a^2 dx^2}{2} - \frac{abc^2 dx^4 \arcsin(cx)}{2} - \frac{abcdx^3 \sqrt{-c^2 x^2 + 1}}{8} + abdx^2 \arcsin(cx) + \frac{5abdx \sqrt{-c^2 x^2 + 1}}{16c} - \frac{5abd \arcsin(cx)}{16c^2} - \frac{b^2 c}{16c^2} \\ \frac{a^2 dx^2}{2} \end{cases}$$

input

```
integrate(x*(-c**2*d*x**2+d)*(a+b*asin(c*x))**2,x)
```


output

```
Piecewise((-a**2*c**2*d*x**4/4 + a**2*d*x**2/2 - a*b*c**2*d*x**4*asin(c*x)
/2 - a*b*c*d*x**3*sqrt(-c**2*x**2 + 1)/8 + a*b*d*x**2*asin(c*x) + 5*a*b*d*
x*sqrt(-c**2*x**2 + 1)/(16*c) - 5*a*b*d*asin(c*x)/(16*c**2) - b**2*c**2*d*
x**4*asin(c*x)**2/4 + b**2*c**2*d*x**4/32 - b**2*c*d*x**3*sqrt(-c**2*x**2
+ 1)*asin(c*x)/8 + b**2*d*x**2*asin(c*x)**2/2 - 5*b**2*d*x**2/32 + 5*b**2*
d*x*sqrt(-c**2*x**2 + 1)*asin(c*x)/(16*c) - 5*b**2*d*asin(c*x)**2/(32*c**2
), Ne(c, 0)), (a**2*d*x**2/2, True))
```

Maxima [F]

$$\int x(d - c^2 dx^2)(a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)(b \arcsin(cx) + a)^2 x dx$$

input

```
integrate(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```
-1/4*a^2*c^2*d*x^4 - 1/16*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c
^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*a*b*c^2*d + 1/2*a^
2*d*x^2 + 1/2*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*
x)/c^3))*a*b*d - 1/4*(b^2*c^2*d*x^4 - 2*b^2*d*x^2)*arctan2(c*x, sqrt(c*x +
1)*sqrt(-c*x + 1))^2 - integrate(1/2*(b^2*c^3*d*x^4 - 2*b^2*c*d*x^2)*sqrt
(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x
^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.69

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = -\frac{1}{4} a^2 c^2 dx^4 + \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 dx \arcsin(cx)}{8c} - \frac{(c^2 x^2 - 1)^2 b^2 d \arcsin(cx)^2}{4c^2} + \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} ab dx}{8c} + \frac{3\sqrt{-c^2 x^2 + 1} b^2 dx \arcsin(cx)}{16c} - \frac{(c^2 x^2 - 1)^2 abd \arcsin(cx)}{2c^2} + \frac{3\sqrt{-c^2 x^2 + 1} ab dx}{16c} + \frac{(c^2 x^2 - 1)^2 b^2 d}{32c^2} + \frac{3b^2 d \arcsin(cx)^2}{32c^2} + \frac{(c^2 x^2 - 1)a^2 d}{2c^2} - \frac{3(c^2 x^2 - 1)b^2 d}{32c^2} + \frac{3abd \arcsin(cx)}{16c^2} - \frac{15b^2 d}{256c^2}$$

input `integrate(x*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `-1/4*a^2*c^2*d*x^4 + 1/8*(-c^2*x^2 + 1)^(3/2)*b^2*d*x*arcsin(c*x)/c - 1/4*(c^2*x^2 - 1)^2*b^2*d*arcsin(c*x)^2/c^2 + 1/8*(-c^2*x^2 + 1)^(3/2)*a*b*d*x/c + 3/16*sqrt(-c^2*x^2 + 1)*b^2*d*x*arcsin(c*x)/c - 1/2*(c^2*x^2 - 1)^2*a*b*d*arcsin(c*x)/c^2 + 3/16*sqrt(-c^2*x^2 + 1)*a*b*d*x/c + 1/32*(c^2*x^2 - 1)^2*b^2*d/c^2 + 3/32*b^2*d*arcsin(c*x)^2/c^2 + 1/2*(c^2*x^2 - 1)*a^2*d/c^2 - 3/32*(c^2*x^2 - 1)*b^2*d/c^2 + 3/16*a*b*d*arcsin(c*x)/c^2 - 15/256*b^2*d/c^2`

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int x(a + b \arcsin(cx))^2 (d - c^2 dx^2) dx$$

input `int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`

output `int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`

Reduce [F]

$$\int x(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \frac{d(8 \arcsin(cx)^2 b^2 c^2 x^2 - 4 \arcsin(cx)^2 b^2 + 8 \sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 cx - 8 \arcsin(cx) ab c^4 x^4 + 16 \arcsin(cx) ab c^2 x^2)}{16 c^2}$$

input `int(x*(-c^2*d*x^2+d)*(a+b*asin(c*x))^2,x)`

output `(d*(8*asin(c*x)**2*b**2*c**2*x**2 - 4*asin(c*x)**2*b**2 + 8*sqrt(-c**2*x**2 + 1)*asin(c*x)*b**2*c*x - 8*asin(c*x)*a*b*c**4*x**4 + 16*asin(c*x)*a*b*c**2*x**2 - 5*asin(c*x)*a*b - 2*sqrt(-c**2*x**2 + 1)*a*b*c**3*x**3 + 5*sqrt(-c**2*x**2 + 1)*a*b*c*x - 16*int(asin(c*x)**2*x**3,x)*b**2*c**4 - 4*a**2*c**4*x**4 + 8*a**2*c**2*x**2 - 4*b**2*c**2*x**2))/(16*c**2)`

3.157 $\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$

Optimal result	1407
Mathematica [A] (verified)	1408
Rubi [A] (verified)	1408
Maple [A] (verified)	1410
Fricas [A] (verification not implemented)	1411
Sympy [A] (verification not implemented)	1412
Maxima [B] (verification not implemented)	1412
Giac [A] (verification not implemented)	1413
Mupad [F(-1)]	1414
Reduce [F]	1414

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = -\frac{14}{9}b^2 dx + \frac{2}{27}b^2 c^2 dx^3 + \frac{4bd\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{3c} + \frac{2bd(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{9c} + \frac{2}{3}dx(a + b \arcsin(cx))^2 + \frac{1}{3}dx(1 - c^2 x^2)(a + b \arcsin(cx))^2$$

output

```
-14/9*b^2*d*x+2/27*b^2*c^2*d*x^3+4/3*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+2/9*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c+2/3*d*x*(a+b*arcsin(c*x))^2+1/3*d*x*(1-c^2*x^2)*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \frac{d(-2b^2 cx(-21 + c^2 x^2) + 6ab\sqrt{1 - c^2 x^2}(-7 + c^2 x^2) + 9a^2 cx(-3 + c^2 x^2) + 6b(b\sqrt{1 - c^2 x^2}(-7 + c^2 x^2) + 3acx(-3 + c^2 x^2))\arcsin(cx) + 9b^2 cx(-3 + c^2 x^2)\arcsin(cx)^2)/c}{27c}$$

input

```
Integrate[(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/27*(d*(-2*b^2*c*x*(-21 + c^2*x^2) + 6*a*b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + 9*a^2*c*x*(-3 + c^2*x^2) + 6*b*(b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + 3*a*c*x*(-3 + c^2*x^2))*ArcSin[c*x] + 9*b^2*c*x*(-3 + c^2*x^2)*ArcSin[c*x]^2))/c
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5158, 5130, 5182, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5158}$$

$$-\frac{2}{3}bcd \int x\sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{2}{3}d \int (a + b \arcsin(cx))^2 dx + \frac{1}{3}dx(1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\downarrow \text{5130}$$

$$\frac{2}{3}d \left(x(a + b \arcsin(cx))^2 - 2bc \int \frac{x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx \right) - \frac{2}{3}bcd \int x\sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{3}dx(1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\begin{aligned}
& \downarrow 5182 \\
& \frac{2}{3}d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) - \\
& \frac{2}{3}bcd \left(\frac{b \int (1 - c^2 x^2) dx}{3c} - \frac{(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{3c^2} \right) + \frac{1}{3}dx(1 - c^2 x^2)(a + \\
& \qquad \qquad \qquad b \arcsin(cx))^2 \\
& \downarrow 24 \\
& -\frac{2}{3}bcd \left(\frac{b \int (1 - c^2 x^2) dx}{3c} - \frac{(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{3c^2} \right) + \frac{1}{3}dx(1 - c^2 x^2)(a + \\
& b \arcsin(cx))^2 + \frac{2}{3}d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \\
& \downarrow 2009 \\
& \frac{1}{3}dx(1 - c^2 x^2)(a + b \arcsin(cx))^2 + \\
& \frac{2}{3}d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) - \\
& \frac{2}{3}bcd \left(\frac{b \left(x - \frac{c^2 x^3}{3} \right)}{3c} - \frac{(1 - c^2 x^2)^{3/2}(a + b \arcsin(cx))}{3c^2} \right)
\end{aligned}$$

input `Int[(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]`

output `(d*x*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*d*((b*(x - (c^2*x^3)/3))/(3*c) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^2)))/3 + (2*d*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/3`

Definitions of rubi rules used

rule 24 $\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$

rule 5130 $\text{Int}[(a_. + \text{ArcSin}[c_.](x_.)](b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Simp}[b*c*n \text{ Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] \text{ ; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$

rule 5158 $\text{Int}[(a_. + \text{ArcSin}[c_.](x_.)](b_.))^{(n_.)}*((d_.) + (e_.)(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[x*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^{n/(2*p + 1)}, x] + (\text{Simp}[2*d*(p/(2*p + 1)) \text{ Int}[(d + e*x^2)^{(p-1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(2*p + 1))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[x*(1 - c^2*x^2)^{(p-1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0]$

rule 5182 $\text{Int}[(a_. + \text{ArcSin}[c_.](x_.)](b_.))^{(n_.)}*(x_.)*((d_.) + (e_.)(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)}*(a + b*\text{ArcSin}[c*x])^{n/(2*e*(p+1))}, x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] \text{ ; FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

method	result
derivativedivides	$-da^2\left(\frac{1}{3}c^3x^3-cx\right)-db^2\left(\frac{\arcsin(cx)^2(c^2x^2-3)cx}{3}+\frac{4cx}{3}-\frac{4\arcsin(cx)\sqrt{-c^2x^2+1}}{3}+\frac{2\arcsin(cx)(c^2x^2-1)\sqrt{-c^2x^2+1}}{9}-\frac{2(c^2x^2-1)}{9}\right)$
default	$-da^2\left(\frac{1}{3}c^3x^3-cx\right)-db^2\left(\frac{\arcsin(cx)^2(c^2x^2-3)cx}{3}+\frac{4cx}{3}-\frac{4\arcsin(cx)\sqrt{-c^2x^2+1}}{3}+\frac{2\arcsin(cx)(c^2x^2-1)\sqrt{-c^2x^2+1}}{9}-\frac{2(c^2x^2-1)}{9}\right)$
parts	$-da^2\left(\frac{1}{3}c^2x^3-x\right)-\frac{db^2\left(\frac{\arcsin(cx)^2(c^2x^2-3)cx}{3}+\frac{4cx}{3}-\frac{4\arcsin(cx)\sqrt{-c^2x^2+1}}{3}+\frac{2\arcsin(cx)(c^2x^2-1)\sqrt{-c^2x^2+1}}{9}\right)}{c}$
orering	$\frac{x(19c^4x^4-166c^2x^2+27)(-c^2dx^2+d)(a+b\arcsin(cx))^2}{27(c^2x^2-1)^2}-\frac{(2c^4x^4-29c^2x^2+7)\left(-2c^2dx(a+b\arcsin(cx))^2+\frac{2(-c^2dx^2+d)}{9}\right)}{9c^2(c^2x^2-1)}$

input `int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output `1/c*(-d*a^2*(1/3*c^3*x^3-c*x)-d*b^2*(1/3*arcsin(c*x)^2*(c^2*x^2-3)*c*x+4/3*c*x-4/3*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+2/9*arcsin(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-2/27*(c^2*x^2-3)*c*x)-2*d*a*b*(1/3*c^3*x^3*arcsin(c*x)-c*x*arcsin(c*x)+1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-7/9*(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \frac{(9a^2 - 2b^2)c^3 dx^3 - 3(9a^2 - 14b^2)cdx + 9(b^2c^3 dx^3 - 3b^2cdx) \arcsin(cx)^2 + 18(abc^3 dx^3 - 3abcdx)}{27c}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `-1/27*((9*a^2 - 2*b^2)*c^3*d*x^3 - 3*(9*a^2 - 14*b^2)*c*d*x + 9*(b^2*c^3*d*x^3 - 3*b^2*c*d*x)*arcsin(c*x)^2 + 18*(a*b*c^3*d*x^3 - 3*a*b*c*d*x)*arcsin(c*x) + 6*(a*b*c^2*d*x^2 - 7*a*b*d + (b^2*c^2*d*x^2 - 7*b^2*d)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c`

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.75

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^2 dx^3}{3} + a^2 dx - \frac{2abc^2 dx^3 \arcsin(cx)}{3} - \frac{2abcdx^2 \sqrt{-c^2 x^2 + 1}}{9} + 2abdx \arcsin(cx) + \frac{14abd \sqrt{-c^2 x^2 + 1}}{9c} - \frac{b^2 c^2 dx^3 \arcsin^2(cx)}{3} \\ a^2 dx \end{cases}$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))**2,x)`

output

```
Piecewise((-a**2*c**2*d*x**3/3 + a**2*d*x - 2*a*b*c**2*d*x**3*asin(c*x)/3
- 2*a*b*c*d*x**2*sqrt(-c**2*x**2 + 1)/9 + 2*a*b*d*x*asin(c*x) + 14*a*b*d*s
qrt(-c**2*x**2 + 1)/(9*c) - b**2*c**2*d*x**3*asin(c*x)**2/3 + 2*b**2*c**2*
d*x**3/27 - 2*b**2*c*d*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/9 + b**2*d*x*as
in(c*x)**2 - 14*b**2*d*x/9 + 14*b**2*d*sqrt(-c**2*x**2 + 1)*asin(c*x)/(9*c
), Ne(c, 0)), (a**2*d*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(111) = 222.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.82

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= -\frac{1}{3} b^2 c^2 dx^3 \arcsin^2(cx) - \frac{1}{3} a^2 c^2 dx^3$$

$$- \frac{2}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) abc^2 d$$

$$- \frac{2}{27} \left(3c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \arcsin(cx) - \frac{c^2 x^3 + 6x}{c^2} \right) b^2 c^2 d$$

$$+ b^2 dx \arcsin^2(cx) - 2b^2 d \left(x - \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{c} \right)$$

$$+ a^2 dx + \frac{2(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1})abd}{c}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-1/3*b^2*c^2*d*x^3*arcsin(c*x)^2 - 1/3*a^2*c^2*d*x^3 - 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1))*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*a*b*c^2*d - 2/27*(3*c*(sqrt(-c^2*x^2 + 1))*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcsin(c*x) - (c^2*x^3 + 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arcsin(c*x)^2 - 2*b^2*d*(x - sqrt(-c^2*x^2 + 1))*arcsin(c*x)/c + a^2*d*x + 2*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a*b*d/c`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.53

$$\begin{aligned} & \int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx \\ &= -\frac{1}{3} a^2 c^2 dx^3 - \frac{1}{3} (c^2 x^2 - 1) b^2 dx \arcsin(cx)^2 - \frac{2}{3} (c^2 x^2 - 1) ab dx \arcsin(cx) \\ & \quad + \frac{2}{3} b^2 dx \arcsin(cx)^2 + \frac{2}{27} (c^2 x^2 - 1) b^2 dx + \frac{4}{3} ab dx \arcsin(cx) \\ & \quad + \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d \arcsin(cx)}{9c} + a^2 dx - \frac{40}{27} b^2 dx + \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} abd}{9c} \\ & \quad + \frac{4\sqrt{-c^2 x^2 + 1} b^2 d \arcsin(cx)}{3c} + \frac{4\sqrt{-c^2 x^2 + 1} abd}{3c} \end{aligned}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `-1/3*a^2*c^2*d*x^3 - 1/3*(c^2*x^2 - 1)*b^2*d*x*arcsin(c*x)^2 - 2/3*(c^2*x^2 - 1)*a*b*d*x*arcsin(c*x) + 2/3*b^2*d*x*arcsin(c*x)^2 + 2/27*(c^2*x^2 - 1)*b^2*d*x + 4/3*a*b*d*x*arcsin(c*x) + 2/9*(-c^2*x^2 + 1)^(3/2)*b^2*d*arcsin(c*x)/c + a^2*d*x - 40/27*b^2*d*x + 2/9*(-c^2*x^2 + 1)^(3/2)*a*b*d/c + 4/3*sqrt(-c^2*x^2 + 1)*b^2*d*arcsin(c*x)/c + 4/3*sqrt(-c^2*x^2 + 1)*a*b*d/c`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 (d - c^2 dx^2) dx$$

input `int((a + b*asin(c*x))^2*(d - c^2*d*x^2),x)`

output `int((a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`

Reduce [F]

$$\int (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \frac{d(9 \arcsin(cx)^2 b^2 cx + 18\sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 - 6 \arcsin(cx) ab c^3 x^3 + 18 \arcsin(cx) ab cx - 2\sqrt{-c^2 x^2 + 1} a^2)}{9c}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x))^2,x)`

output `(d*(9*asin(c*x)**2*b**2*c*x + 18*sqrt(-c**2*x**2 + 1)*asin(c*x)*b**2 - 6*asin(c*x)*a*b*c**3*x**3 + 18*asin(c*x)*a*b*c*x - 2*sqrt(-c**2*x**2 + 1)*a*b*c**2*x**2 + 14*sqrt(-c**2*x**2 + 1)*a*b - 9*int(asin(c*x)**2*x**2,x))*b**2*c**3 - 3*a**2*c**3*x**3 + 9*a**2*c*x - 18*b**2*c*x)/(9*c)`

3.158 $\int \frac{(d-c^2 dx^2)(a+b \arcsin(cx))^2}{x} dx$

Optimal result	1415
Mathematica [A] (verified)	1416
Rubi [A] (verified)	1417
Maple [A] (verified)	1421
Fricas [F]	1422
Sympy [F]	1422
Maxima [F]	1423
Giac [F(-2)]	1423
Mupad [F(-1)]	1424
Reduce [F]	1424

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx = \frac{1}{4}b^2c^2 dx^2 - \frac{1}{2}bcdx\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) - \frac{1}{4}d(a + b \arcsin(cx))^2 + \frac{1}{2}d(1 - c^2x^2)(a + b \arcsin(cx))^2 - \frac{id(a + b \arcsin(cx))^3}{3b} + d(a + b \arcsin(cx))^2 \log(1 - e^{2i \arcsin(cx)}) - ibd(a + b \arcsin(cx)) \text{PolyLog}(2, e^{2i \arcsin(cx)}) + \frac{1}{2}b^2d \text{PolyLog}(3, e^{2i \arcsin(cx)})$$

output

```
1/4*b^2*c^2*d*x^2-1/2*b*c*d*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-1/4*d*(a+b*arcsin(c*x))^2+1/2*d*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2-1/3*I*d*(a+b*arcsin(c*x))^3/b+d*(a+b*arcsin(c*x))^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*b*d*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*d*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.44

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx = \frac{1}{2}d \left(-a^2 c^2 x^2 - 2abc^2 x^2 \arcsin(cx) - ab \left(cx \sqrt{1 - c^2 x^2} - 2 \arctan \left(\frac{cx}{-1 + \sqrt{1 - c^2 x^2}} \right) \right) + \frac{1}{4}b^2(-1 + 2 \arcsin(cx))^2 \cos(2 \arcsin(cx)) + 4ab \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) + 2a^2 \log(x) - 2iab(\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)})) + \frac{1}{12}b^2(-i\pi^3 + 8i \arcsin(cx))^3 + 24 \arcsin(cx)^2 \log(1 - e^{-2i \arcsin(cx)}) + 24i \arcsin(cx) \text{PolyLog}(2, e^{-2i \arcsin(cx)}) + 12 \text{PolyLog}(3, e^{-2i \arcsin(cx)}) - \frac{1}{2}b^2 \arcsin(cx) \sin(2 \arcsin(cx)) \right)$$

input `Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x,x]`output `(d*(-(a^2*c^2*x^2) - 2*a*b*c^2*x^2*ArcSin[c*x] - a*b*(c*x*Sqrt[1 - c^2*x^2] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])])) + (b^2*(-1 + 2*ArcSin[c*x]^2)*Cos[2*ArcSin[c*x]])/4 + 4*a*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 2*a^2*Log[x] - (2*I)*a*b*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])]) + (b^2*((-I)*Pi^3 + (8*I)*ArcSin[c*x]^3 + 24*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] + (24*I)*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcSin[c*x])]))/12 - (b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])/2)/2`

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5202, 5136, 3042, 25, 4200, 25, 2620, 3011, 2720, 5156, 15, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx$$

$$\downarrow 5202$$

$$-bcd \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + d \int \frac{(a + b \arcsin(cx))^2}{x} dx + \frac{1}{2} d (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\downarrow 5136$$

$$-bcd \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + d \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2}{cx} d \arcsin(cx) + \frac{1}{2} d (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\downarrow 3042$$

$$-bcd \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + d \int -(a + b \arcsin(cx))^2 \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx) + \frac{1}{2} d (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\downarrow 25$$

$$-bcd \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - d \int (a + b \arcsin(cx))^2 \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx) + \frac{1}{2} d (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\downarrow 4200$$

$$-bcd \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + d \left(2i \int -\frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))^2}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^3}{3b} \right) + \frac{1}{2} d (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$\downarrow 25$$

$$\begin{aligned}
& -bcd \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \\
& d\left(-2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))^2}{1-e^{2i\arcsin(cx)}}d\arcsin(cx) - \frac{i(a+b\arcsin(cx))^3}{3b}\right) + \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{2620} \\
& -bcd \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \\
& d\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))^2 - ib \int (a+b\arcsin(cx))\log(1-e^{2i\arcsin(cx)})d\arcsin(cx)\right) + \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{3011} \\
& -bcd \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \\
& d\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))^2 - ib\left(\frac{1}{2}i\text{PolyLog}\left(2, e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx)) - \frac{1}{2}ib \int\right)\right) + \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{2720} \\
& -bcd \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \\
& d\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))^2 - ib\left(\frac{1}{2}i\text{PolyLog}\left(2, e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx)) - \frac{1}{4}b \int\right)\right) + \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{5156} \\
& -bcd\left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx - \frac{1}{2}bc \int xdx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right) + \\
& d\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))^2 - ib\left(\frac{1}{2}i\text{PolyLog}\left(2, e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx)) - \frac{1}{4}b \int\right)\right) + \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{15} \\
& -bcd\left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{4}bcx^2\right) + \\
& d\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))^2 - ib\left(\frac{1}{2}i\text{PolyLog}\left(2, e^{2i\arcsin(cx)}\right)(a+b\arcsin(cx)) - \frac{1}{4}b \int\right)\right) + \\
& \frac{1}{2}d(1-c^2x^2)(a+b\arcsin(cx))^2
\end{aligned}$$

↓ 5152

$$d \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{4} b \int \right) \right. \right. \\ \left. \left. \frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx))^2 - \right. \right. \\ \left. \left. bcd \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) \right)$$

↓ 7143

$$\frac{1}{2} d(1 - c^2 x^2) (a + b \arcsin(cx))^2 - \\ bcd \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) + \\ d \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog} \right) \right) \right)$$

input `Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x,x]`

output `(d*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/2 - b*c*d*(-1/4*(b*c*x^2) + (x*Sqr
t[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)) + d
*(((-1/3*I)*(a + b*ArcSin[c*x])^3)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])^2*
Log[1 - E^((2*I)*ArcSin[c*x])] - I*b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2,
E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])/4)))`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]`

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n * Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^(m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.07

method	result
parts	$-d a^2 \left(\frac{c^2 x^2}{2} - \ln(x) \right) - d b^2 \left(\frac{i \arcsin(cx)^3}{3} - \arcsin(cx)^2 \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) + 2 \right)$
derivativedivides	$-d a^2 \left(\frac{c^2 x^2}{2} - \ln(cx) \right) - d b^2 \left(\frac{i \arcsin(cx)^3}{3} - \arcsin(cx)^2 \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) + 2 \right)$
default	$-d a^2 \left(\frac{c^2 x^2}{2} - \ln(cx) \right) - d b^2 \left(\frac{i \arcsin(cx)^3}{3} - \arcsin(cx)^2 \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) + 2 \right)$

input

```
int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output

```
-d*a^2*(1/2*c^2*x^2-ln(x))-d*b^2*(1/3*I*arcsin(c*x)^3-arcsin(c*x)^2*ln(1+I
*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/
2))-2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-(-c^2*
x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*polylo
g(3,I*c*x+(-c^2*x^2+1)^(1/2))-1/8*(2*arcsin(c*x)^2-1)*cos(2*arcsin(c*x))+1
/4*arcsin(c*x)*sin(2*arcsin(c*x))-2*d*a*b*(1/2*I*arcsin(c*x)^2-arcsin(c*x)
)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-arc
sin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1
/2))-1/4*arcsin(c*x)*cos(2*arcsin(c*x))+1/8*sin(2*arcsin(c*x)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x,x, algorithm="fricas")
```

output

```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 +
2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx = -d \left(\int \left(-\frac{a^2}{x} \right) dx + \int a^2 c^2 x dx \right. \\ \left. + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x} \right) dx \right. \\ \left. + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x} \right) dx \right. \\ \left. + \int b^2 c^2 x \operatorname{asin}^2(cx) dx \right. \\ \left. + \int 2abc^2 x \operatorname{asin}(cx) dx \right)$$

input

```
integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))**2/x,x)
```

output

```
-d*(Integral(-a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(-b**2*asin(
c*x)**2/x, x) + Integral(-2*a*b*asin(c*x)/x, x) + Integral(b**2*c**2*x*asi
n(c*x)**2, x) + Integral(2*a*b*c**2*x*asin(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")
```

output

```
-1/2*a^2*c^2*d*x^2 + a^2*d*log(x) - integrate(((b^2*c^2*d*x^2 - b^2*d)*arc
tan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arct
an2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```


3.159 $\int \frac{(d-c^2 dx^2)(a+b \arcsin(cx))^2}{x^2} dx$

Optimal result	1425
Mathematica [A] (verified)	1426
Rubi [A] (verified)	1426
Maple [A] (verified)	1431
Fricas [F]	1431
Sympy [F]	1432
Maxima [F]	1432
Giac [F(-2)]	1433
Mupad [F(-1)]	1433
Reduce [F]	1433

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx = 2b^2c^2 dx - 2bcd\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) - 2c^2 dx(a + b \arcsin(cx))^2 - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))^2}{x} - 4bcd(a + b \arcsin(cx))\operatorname{arctanh}(e^{i \arcsin(cx)}) + 2ib^2cd \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - 2ib^2cd \operatorname{PolyLog}(2, e^{i \arcsin(cx)})$$

output

```
2*b^2*c^2*d*x-2*b*c*d*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-2*c^2*d*x*(a+b*arcsin(c*x))^2-d*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/x-4*b*c*d*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))+2*I*b^2*c*d*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*b^2*c*d*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.36

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx =$$

$$\frac{d(a^2 + a^2 c^2 x^2 + 2abcx(\sqrt{1 - c^2 x^2} + cx \arcsin(cx)) + b^2 cx(2\sqrt{1 - c^2 x^2} \arcsin(cx) + cx(-2 + \arcsin(cx)))}{x^2}$$

input

```
Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```
-((d*(a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(Sqrt[1 - c^2*x^2] + c*x*ArcSin[c*x])
+ b^2*c*x*(2*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + c*x*(-2 + ArcSin[c*x]^2)) + 2
*a*b*(ArcSin[c*x] + c*x*ArcTanh[Sqrt[1 - c^2*x^2]])) - I*b^2*(I*ArcSin[c*x]
*(ArcSin[c*x] + 2*c*x*(-Log[1 - E^(I*ArcSin[c*x]])) + Log[1 + E^(I*ArcSin[c
*x]]))) + 2*c*x*PolyLog[2, -E^(I*ArcSin[c*x])] - 2*c*x*PolyLog[2, E^(I*Arc
Sin[c*x])])))/x)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5200, 5130, 5182, 24, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx$$

$$\downarrow \text{5200}$$

$$2bcd \int \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{x} dx - 2c^2 d \int \frac{(a + b \arcsin(cx))^2 dx}{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}$$

$$\downarrow \text{5130}$$

$$\begin{aligned}
& -2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \int \frac{x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx \right) + \\
& 2bcd \int \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{x} dx - \frac{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}{x} \\
& \quad \downarrow \text{5182} \\
& -2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) + \\
& 2bcd \int \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{x} dx - \frac{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}{x} \\
& \quad \downarrow \text{24} \\
& 2bcd \int \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{x} dx - \\
& 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) - \\
& \quad \frac{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}{x} \\
& \quad \downarrow \text{5198} \\
& 2bcd \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2 x^2}} dx - bc \int 1 dx + \sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \right) - \\
& 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) - \\
& \quad \frac{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}{x} \\
& \quad \downarrow \text{24} \\
& 2bcd \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1 - c^2 x^2}} dx + \sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) - bcx \right) - \\
& 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) - \\
& \quad \frac{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}{x} \\
& \quad \downarrow \text{5218} \\
& 2bcd \left(\int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx) + \sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) - bcx \right) - \\
& 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{c^2} \right) \right) - \\
& \quad \frac{d(1 - c^2 x^2)(a + b \arcsin(cx))^2}{x}
\end{aligned}$$

↓ 3042

$$2bcd \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) - bcx \right) - 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{c^2} \right) \right) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x}$$

↓ 4671

$$2bcd \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) \right) - 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{c^2} \right) \right) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x}$$

↓ 2715

$$2bcd \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) d e^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) d e^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) \right) - 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{c^2} \right) \right) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x}$$

↓ 2838

$$2bcd \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \right) - 2c^2 d \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{c^2} \right) \right) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x}$$

input

```
Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```

-((d*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/x) - 2*c^2*d*(x*(a + b*ArcSin[c*
x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2)) + 2
*b*c*d*(-(b*c*x) + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) - 2*(a + b*ArcSin
[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I
*b*PolyLog[2, E^(I*ArcSin[c*x])])

```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5130

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^n, x_Symbol] := Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Simp[b*c*n Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x)) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))] Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.68

method	result
derivativedivides	$c\left(-d a^2\left(cx + \frac{1}{cx}\right) - 2db^2 \arcsin(cx) \sqrt{-c^2x^2 + 1} - db^2 \arcsin(cx)^2 cx + 2b^2cdx - \frac{d}{c}\right)$
default	$c\left(-d a^2\left(cx + \frac{1}{cx}\right) - 2db^2 \arcsin(cx) \sqrt{-c^2x^2 + 1} - db^2 \arcsin(cx)^2 cx + 2b^2cdx - \frac{d}{c}\right)$
parts	$-d a^2\left(c^2x + \frac{1}{x}\right) - 2db^2c \arcsin(cx) \sqrt{-c^2x^2 + 1} - db^2c^2 \arcsin(cx)^2 x + 2b^2c^2dx - \frac{d}{c}$

input `int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)`

output `c*(-d*a^2*(c*x+1/c/x)-2*d*b^2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)-d*b^2*arcsin(c*x)^2*c*x+2*b^2*c*d*x-d*b^2/c/x*arcsin(c*x)^2+2*d*b^2*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*d*b^2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*d*b^2*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+2*I*d*b^2*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*d*a*b*(c*x*arcsin(c*x)+arcsin(c*x)/c/x+arctanh(1/(-c^2*x^2+1)^(1/2))+(-c^2*x^2+1)^(1/2))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")`

output `integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))/x^2, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx = -d \left(\int a^2 c^2 dx + \int \left(-\frac{a^2}{x^2} \right) dx \right. \\ \left. + \int b^2 c^2 \operatorname{asin}^2(cx) dx + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x^2} \right) dx \right. \\ \left. + \int 2abc^2 \operatorname{asin}(cx) dx \right. \\ \left. + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x^2} \right) dx \right)$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))**2/x**2,x)`

output `-d*(Integral(a**2*c**2, x) + Integral(-a**2/x**2, x) + Integral(b**2*c**2*asin(c*x)**2, x) + Integral(-b**2*asin(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asin(c*x), x) + Integral(-2*a*b*asin(c*x)/x**2, x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="maxima")`

output `-b^2*c^2*d*x*arcsin(c*x)^2 + 2*b^2*c^2*d*(x - sqrt(-c^2*x^2 + 1))*arcsin(c*x)/c - a^2*c^2*d*x - 2*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a*b*c*d - 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*a*b*d - (2*c*x*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)/(c^2*x^3 - x), x) + arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2)*b^2*d/x - a^2*d/x`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)}{x^2} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2))/x^2,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2))/x^2, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^2} dx$$

$$= \frac{d(-a \sin(cx))^2 b^2 c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a \sin(cx) b^2 c x - 2a \sin(cx) a b c^2 x^2 - 2a \sin(cx) a b - 2\sqrt{-c^2 x^2 + 1} a}{x}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x))^2/x^2,x)`

output

```
(d*( - asin(c*x)**2*b**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*asin(c*x)*b*  
*2*c*x - 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b - 2*sqrt( - c**2*x**2  
+ 1)*a*b*c*x + int(asin(c*x)**2/x**2,x)*b**2*x + 2*log(tan(asin(c*x)/2))*  
a*b*c*x - a**2*c**2*x**2 - a**2 + 2*b**2*c**2*x**2))/x
```

3.160 $\int \frac{(d-c^2 dx^2)(a+b \arcsin(cx))^2}{x^3} dx$

Optimal result	1435
Mathematica [A] (verified)	1436
Rubi [A] (verified)	1436
Maple [B] (verified)	1441
Fricas [F]	1442
Sympy [F]	1442
Maxima [F]	1443
Giac [F(-2)]	1443
Mupad [F(-1)]	1444
Reduce [F]	1444

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx = -\frac{bcd\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{x} - \frac{1}{2}c^2d(a + b \arcsin(cx))^2 - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))^2}{2x^2} + \frac{ic^2d(a + b \arcsin(cx))^3}{3b} - c^2d(a + b \arcsin(cx))^2 \log(1 - e^{2i \arcsin(cx)}) + b^2c^2d \log(x) + ibc^2d(a + b \arcsin(cx)) \text{PolyLog}(2, e^{2i \arcsin(cx)}) - \frac{1}{2}b^2c^2d \text{PolyLog}(3, e^{2i \arcsin(cx)})$$

output

```
-b*c*d*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/x-1/2*c^2*d*(a+b*arcsin(c*x))^2-1/2*d*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/x^2+1/3*I*c^2*d*(a+b*arcsin(c*x))^3/b-c^2*d*(a+b*arcsin(c*x))^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+b^2*c^2*d*ln(x)+I*b*c^2*d*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*b^2*c^2*d*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```


Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx$$

$$= \frac{1}{2}d \left(-\frac{a^2}{x^2} - \frac{2ab(cx\sqrt{1-c^2x^2} + \arcsin(cx))}{x^2} - 2a^2c^2 \log(x) \right. \\ \left. - \frac{b^2(2cx\sqrt{1-c^2x^2} \arcsin(cx) + \arcsin(cx)^2 - 2c^2x^2 \log(cx))}{x^2} \right. \\ \left. + 2iabc^2(\arcsin(cx)(\arcsin(cx) + 2i \log(1 - e^{2i \arcsin(cx)})) + \text{PolyLog}(2, e^{2i \arcsin(cx)})) \right. \\ \left. + \frac{1}{12}ib^2c^2(\pi^3 - 8 \arcsin(cx)^3 + 24i \arcsin(cx)^2 \log(1 - e^{-2i \arcsin(cx)}) \right. \\ \left. - 24 \arcsin(cx) \text{PolyLog}(2, e^{-2i \arcsin(cx)}) + 12i \text{PolyLog}(3, e^{-2i \arcsin(cx)}) \right)$$

input `Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x^3,x]`

output `(d*(-(a^2/x^2) - (2*a*b*(c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/x^2 - 2*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 + (2*I)*a*b*c^2*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*Log[1 - E^((2*I)*ArcSin[c*x])]) + PolyLog[2, E^((2*I)*ArcSin[c*x])]) + (I/12)*b^2*c^2*(Pi^3 - 8*ArcSin[c*x]^3 + (24*I)*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] - 24*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcSin[c*x])])))/2`

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5200, 5136, 3042, 25, 4200, 25, 2620, 3011, 2720, 5196, 14, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2) (a + b \arcsin(cx))^2}{x^3} dx \\
& \quad \downarrow \text{5200} \\
& bcd \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x^2} dx + c^2(-d) \int \frac{(a + b \arcsin(cx))^2}{x} dx - \\
& \quad \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{5136} \\
& c^2(-d) \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2}{cx} d \arcsin(cx) + bcd \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x^2} dx - \\
& \quad \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{3042} \\
& bcd \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x^2} dx + c^2(-d) \int -(a + \\
& b \arcsin(cx))^2 \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{25} \\
& bcd \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x^2} dx + c^2 d \int (a + \\
& b \arcsin(cx))^2 \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{4200} \\
& bcd \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x^2} dx + \\
& c^2(-d) \left(2i \int -\frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))^2}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^3}{3b} \right) - \\
& \quad \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{25} \\
& bcd \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x^2} dx + \\
& c^2(-d) \left(-2i \int \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))^2}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^3}{3b} \right) - \\
& \quad \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2}
\end{aligned}$$

$$\begin{aligned} & \downarrow 2620 \\ & bcd \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x^2} dx + \\ c^2(-d) & \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \int (a+b\arcsin(cx)) \log(1-e^{2i\arcsin(cx)}) d\arcsin \right. \right. \\ & \left. \left. \frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{2x^2} \right) \right) \end{aligned}$$

$$\downarrow 3011$$

$$\begin{aligned} c^2(-d) & \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \right. \right. \right. \\ & \left. \left. bcd \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x^2} dx - \frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{2x^2} \right) \right) \end{aligned}$$

$$\downarrow 2720$$

$$\begin{aligned} c^2(-d) & \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \right. \right. \right. \\ & \left. \left. bcd \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x^2} dx - \frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{2x^2} \right) \right) \end{aligned}$$

$$\downarrow 5196$$

$$\begin{aligned} c^2(-d) & \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \right. \right. \right. \\ & \left. \left. bcd \left(c^2 \left(- \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx \right) + bc \int \frac{1}{x} dx - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} \right) - \right. \right. \\ & \left. \left. \frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{2x^2} \right) \right) \end{aligned}$$

$$\downarrow 14$$

$$\begin{aligned} c^2(-d) & \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \right. \right. \right. \\ & \left. \left. bcd \left(c^2 \left(- \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx \right) - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} + bc \log(x) \right) - \right. \right. \\ & \left. \left. \frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{2x^2} \right) \right) \end{aligned}$$

$$\downarrow 5152$$

$$c^2(-d) \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} + bcd \left(-\frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} - \frac{c(a + b \arcsin(cx))^2}{2b} + bc \log(x) \right) \right) \right)$$

↓ 7143

$$c^2(-d) \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{2x^2} + bcd \left(-\frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} - \frac{c(a + b \arcsin(cx))^2}{2b} + bc \log(x) \right) \right) \right)$$

input `Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x^3,x]`

output `-1/2*(d*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/x^2 + b*c*d*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/x) - (c*(a + b*ArcSin[c*x])^2)/(2*b) + b*c*Log[x]) - c^2*d*(((1/3*I)*(a + b*ArcSin[c*x])^3)/b - (2*I)*((1/2)*(a + b*ArcSin[c*x])^2*Log[1 - E^((2*I)*ArcSin[c*x])] - I*b*((1/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])]/4)))`

Defintions of rubi rules used

rule 14 `Int[(a.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 25 `Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5196

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x],
x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int
[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))] Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(211) = 422$.

Time = 0.46 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.37

method	result
derivativedivides	$c^2 \left(-d a^2 \left(\ln(cx) + \frac{1}{2c^2 x^2} \right) - d b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \frac{\arcsin(cx) \left(-2ic^2 x^2 + 2cx\sqrt{-c^2 x^2 + 1} + \arcsin(cx) \right)}{2c^2 x^2} \right) \right)$
default	$c^2 \left(-d a^2 \left(\ln(cx) + \frac{1}{2c^2 x^2} \right) - d b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \frac{\arcsin(cx) \left(-2ic^2 x^2 + 2cx\sqrt{-c^2 x^2 + 1} + \arcsin(cx) \right)}{2c^2 x^2} \right) \right)$
parts	$-d a^2 c^2 \ln(x) - \frac{d a^2}{2x^2} - d b^2 c^2 \left(-\frac{i \arcsin(cx)^3}{3} + \frac{\arcsin(cx) \left(-2ic^2 x^2 + 2cx\sqrt{-c^2 x^2 + 1} + \arcsin(cx) \right)}{2c^2 x^2} \right) + \dots$

input

```
int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output

```
c^2*(-d*a^2*(ln(c*x)+1/2/c^2/x^2)-d*b^2*(-1/3*I*arcsin(c*x)^3+1/2*arcsin(c*x)*(-2*I*c^2*x^2+2*c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(3,-I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2*ln(1-I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+2*ln(I*c*x+(-c^2*x^2+1)^(1/2))-ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)-ln(1+I*c*x+(-c^2*x^2+1)^(1/2)))-2*d*a*b*(-1/2*I*arcsin(c*x)^2+1/2*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)*ln(1-I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="fricas")
```

output

```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))/x^3, x)
```

Sympy [F]

$$\begin{aligned} \int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx = & -d \left(\int \left(-\frac{a^2}{x^3} \right) dx + \int \frac{a^2 c^2}{x} dx \right. \\ & + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x^3} \right) dx \\ & + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x^3} \right) dx \\ & \left. + \int \frac{b^2 c^2 \operatorname{asin}^2(cx)}{x} dx + \int \frac{2abc^2 \operatorname{asin}(cx)}{x} dx \right) \end{aligned}$$

input

```
integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))**2/x**3,x)
```

output

```
-d*(Integral(-a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(-b**2*asin(c*x)**2/x**3, x) + Integral(-2*a*b*asin(c*x)/x**3, x) + Integral(b**2*c**2*asin(c*x)**2/x, x) + Integral(2*a*b*c**2*asin(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="maxima")
```

output

```
-a^2*c^2*d*log(x) - a*b*d*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) - 1/2*a^2*d/x^2 - integrate((2*a*b*c^2*d*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + (b^2*c^2*d*x^2 - b^2*d)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```


Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)}{x^3} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2))/x^3,x)`output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2))/x^3, x)`**Reduce [F]**

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^3} dx$$

$$= \frac{d \left(-2a \arcsin(cx) ab - 2\sqrt{-c^2 x^2 + 1} abcx - 4 \left(\int \frac{\arcsin(cx)}{x} dx \right) ab c^2 x^2 + 2 \left(\int \frac{\arcsin(cx)^2}{x^3} dx \right) b^2 x^2 - 2 \left(\int \frac{\arcsin(cx)^2}{x} dx \right) a}{2x^2}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x))^2/x^3,x)`output `(d*(- 2*asin(c*x)*a*b - 2*sqrt(- c**2*x**2 + 1)*a*b*c*x - 4*int(asin(c*x)/x,x)*a*b*c**2*x**2 + 2*int(asin(c*x)**2/x**3,x)*b**2*x**2 - 2*int(asin(c*x)**2/x,x)*b**2*c**2*x**2 - 2*log(x)*a**2*c**2*x**2 - a**2))/(2*x**2)`

$$3.161 \quad \int \frac{(d-c^2 dx^2)(a+b \arcsin(cx))^2}{x^4} dx$$

Optimal result	1445
Mathematica [A] (verified)	1446
Rubi [A] (verified)	1446
Maple [A] (verified)	1450
Fricas [F]	1451
Sympy [F]	1451
Maxima [F]	1452
Giac [F(-2)]	1452
Mupad [F(-1)]	1453
Reduce [F]	1453

Optimal result

Integrand size = 25, antiderivative size = 176

$$\begin{aligned} \int \frac{(d-c^2 dx^2)(a+b \arcsin(cx))^2}{x^4} dx = & -\frac{b^2 c^2 d}{3x} - \frac{bcd\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{3x^2} \\ & + \frac{2c^2 d(a+b \arcsin(cx))^2}{3x} \\ & - \frac{d(1-c^2 x^2)(a+b \arcsin(cx))^2}{3x^3} \\ & + \frac{10}{3} bc^3 d(a+b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)}) \\ & - \frac{5}{3} ib^2 c^3 d \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \\ & + \frac{5}{3} ib^2 c^3 d \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \end{aligned}$$

output

```
-1/3*b^2*c^2*d/x-1/3*b*c*d*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/x^2+2/3*c^2*d*(a+b*arcsin(c*x))^2/x-1/3*d*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/x^3+10/3*b*c^3*d*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))-5/3*I*b^2*c^3*d*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+5/3*I*b^2*c^3*d*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.51

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx$$

$$= \frac{d(-a^2 + 3a^2c^2x^2 - b^2c^2x^2 - abcx\sqrt{1 - c^2x^2} - 2ab \arcsin(cx) + 6abc^2x^2 \arcsin(cx) - b^2cx\sqrt{1 - c^2x^2} \arcsin(cx) + b^2c^3x^3 \operatorname{ArcTanh}[\sqrt{1 - c^2x^2}] - 5b^2c^3x^3 \arcsin(cx) \operatorname{Log}[1 - E^{(I \arcsin(cx))}] + 5b^2c^3x^3 \arcsin(cx) \operatorname{Log}[1 + E^{(I \arcsin(cx))}] - (5I)b^2c^3x^3 \operatorname{PolyLog}[2, -E^{(I \arcsin(cx))}] + (5I)b^2c^3x^3 \operatorname{PolyLog}[2, E^{(I \arcsin(cx))}])}{3x^3}$$

input

```
Integrate[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

```
(d*(-a^2 + 3*a^2*c^2*x^2 - b^2*c^2*x^2 - a*b*c*x*Sqrt[1 - c^2*x^2] - 2*a*b*ArcSin[c*x] + 6*a*b*c^2*x^2*ArcSin[c*x] - b^2*c*x*Sqrt[1 - c^2*x^2]*ArcSin[c*x] - b^2*ArcSin[c*x]^2 + 3*b^2*c^2*x^2*ArcSin[c*x]^2 + 5*a*b*c^3*x^3*ArcTanh[Sqrt[1 - c^2*x^2]] - 5*b^2*c^3*x^3*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) + 5*b^2*c^3*x^3*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (5*I)*b^2*c^3*x^3*PolyLog[2, -E^(I*ArcSin[c*x])] + (5*I)*b^2*c^3*x^3*PolyLog[2, E^(I*ArcSin[c*x])])/(3*x^3)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5200, 5138, 5196, 15, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx$$

$$\downarrow \text{5200}$$

$$-\frac{2}{3}c^2d \int \frac{(a + b \arcsin(cx))^2}{x^2} dx + \frac{2}{3}bcd \int \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{x^3} dx - \frac{d(1 - c^2x^2)(a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{5138}$$

$$-\frac{2}{3}c^2d\left(2bc\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arcsin(cx))^2}{x}\right)+\frac{2}{3}bcd\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x^3}dx-\frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5196

$$-\frac{2}{3}c^2d\left(2bc\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arcsin(cx))^2}{x}\right)+\frac{2}{3}bcd\left(-\frac{1}{2}c^2\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int\frac{1}{x^2}dx-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}\right)-\frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{3x^3}$$

↓ 15

$$-\frac{2}{3}c^2d\left(2bc\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arcsin(cx))^2}{x}\right)+\frac{2}{3}bcd\left(-\frac{1}{2}c^2\int\frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc}{2x}\right)-\frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5218

$$\frac{2}{3}bcd\left(-\frac{1}{2}c^2\int\frac{a+b\arcsin(cx)}{cx}d\arcsin(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc}{2x}\right)-\frac{2}{3}c^2d\left(2bc\int\frac{a+b\arcsin(cx)}{cx}d\arcsin(cx)-\frac{(a+b\arcsin(cx))^2}{x}\right)-\frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{3x^3}$$

↓ 3042

$$\frac{2}{3}bcd\left(-\frac{1}{2}c^2\int(a+b\arcsin(cx))\csc(\arcsin(cx))d\arcsin(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{2x^2}-\frac{bc}{2x}\right)-\frac{2}{3}c^2d\left(2bc\int(a+b\arcsin(cx))\csc(\arcsin(cx))d\arcsin(cx)-\frac{(a+b\arcsin(cx))^2}{x}\right)-\frac{d(1-c^2x^2)(a+b\arcsin(cx))^2}{3x^3}$$

↓ 4671

$$\frac{2}{3}bcd \left(-\frac{1}{2}c^2 \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right) \right. \\ \left. \frac{2}{3}c^2 d \left(-\frac{(a + b \arcsin(cx))^2}{x} + 2bc \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) \right) \right) \right. \\ \left. \frac{d(1 - c^2x^2)(a + b \arcsin(cx))^2}{3x^3} \right) \\ \downarrow \text{2715}$$

$$\frac{2}{3}bcd \left(-\frac{1}{2}c^2 \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right) \right. \\ \left. \frac{2}{3}c^2 d \left(-\frac{(a + b \arcsin(cx))^2}{x} + 2bc \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right) \right) \right. \\ \left. \frac{d(1 - c^2x^2)(a + b \arcsin(cx))^2}{3x^3} \right) \\ \downarrow \text{2838}$$

$$\frac{2}{3}bcd \left(-\frac{1}{2}c^2 \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right) \right. \\ \left. \frac{2}{3}c^2 d \left(-\frac{(a + b \arcsin(cx))^2}{x} + 2bc \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right) \right) \right. \\ \left. \frac{d(1 - c^2x^2)(a + b \arcsin(cx))^2}{3x^3} \right)$$

input

```
Int[((d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

```
-1/3*(d*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/x^3 - (2*c^2*d*(-((a + b*ArcSin[c*x])^2/x) + 2*b*c*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])))/3 + (2*b*c*d*(-1/2*(b*c)/x - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*x^2) - (c^2*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])))/2))/3
```

Definitions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] /; \text{FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^{n}], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2838 $\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$
- rule 5138 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^{(n_.)*((d_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5196 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^{(n_.)*((f_.)*(x_))^{(m_.)*\text{Sqrt}[(d_) + (e_.)*(x_)^2]}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+1))), x] + (-\text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] + \text{Simp}[(c^2/(f^2*(m+1)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^{(m+2)}*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]$

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.48

method	result
parts	$-da^2\left(\frac{1}{3x^3} - \frac{c^2}{x}\right) - db^2c^3\left(-\frac{3\arcsin(cx)^2x^2c^2 - \arcsin(cx)\sqrt{-c^2x^2+1}cx - \arcsin(cx)^2 - c^2x^2}{3c^3x^3} + \frac{5\arcsin(cx)}{3c^3x^3}\right)$
derivativedivides	$c^3\left(-da^2\left(\frac{1}{3c^3x^3} - \frac{1}{cx}\right) - db^2\left(-\frac{3\arcsin(cx)^2x^2c^2 - \arcsin(cx)\sqrt{-c^2x^2+1}cx - \arcsin(cx)^2 - c^2x^2}{3c^3x^3} + \frac{5\arcsin(cx)}{3c^3x^3}\right)\right)$
default	$c^3\left(-da^2\left(\frac{1}{3c^3x^3} - \frac{1}{cx}\right) - db^2\left(-\frac{3\arcsin(cx)^2x^2c^2 - \arcsin(cx)\sqrt{-c^2x^2+1}cx - \arcsin(cx)^2 - c^2x^2}{3c^3x^3} + \frac{5\arcsin(cx)}{3c^3x^3}\right)\right)$

input

```
int((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output

```
-d*a^2*(1/3/x^3-c^2/x)-d*b^2*c^3*(-1/3*(3*arcsin(c*x)^2*x^2*c^2-arcsin(c*x)
)*(-c^2*x^2+1)^(1/2)*c*x-arcsin(c*x)^2-c^2*x^2)/c^3/x^3+5/3*arcsin(c*x)*ln
(1-I*c*x-(-c^2*x^2+1)^(1/2))-5/3*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-5/3
*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+5/3*I*polylog(2,-I*c*x-(-c^2*x
^2+1)^(1/2))-2*d*a*b*c^3*(1/3*arcsin(c*x)/c^3/x^3-arcsin(c*x)/c/x+1/6/c^2
/x^2*(-c^2*x^2+1)^(1/2)-5/6*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="fricas")`

output `integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))/x^4, x)`

Sympy [F]

$$\begin{aligned} \int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx = & -d \left(\int \left(-\frac{a^2}{x^4} \right) dx + \int \frac{a^2 c^2}{x^2} dx \right. \\ & + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x^4} \right) dx \\ & + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x^4} \right) dx \\ & \left. + \int \frac{b^2 c^2 \operatorname{asin}^2(cx)}{x^2} dx + \int \frac{2abc^2 \operatorname{asin}(cx)}{x^2} dx \right) \end{aligned}$$

input `integrate((-c**2*d*x**2+d)*(a+b*asin(c*x))**2/x**4,x)`

output `-d*(Integral(-a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(-b**2*asin(c*x)**2/x**4, x) + Integral(-2*a*b*asin(c*x)/x**4, x) + Integral(b**2*c**2*asin(c*x)**2/x**2, x) + Integral(2*a*b*c**2*asin(c*x)/x**2, x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx = \int -\frac{(c^2 dx^2 - d)(b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="maxima")`

output `2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*a*b*c^2*d - 1/3*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*a*b*d + a^2*c^2*d/x - 1/3*a^2*d/x^3 + 1/3*(3*x^3*integrate(2/3*(3*b^2*c^3*d*x^2 - b^2*c*d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^5 - x^3), x) + (3*b^2*c^2*d*x^2 - b^2*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/x^3`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)}{x^4} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2))/x^4,x)`output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2))/x^4, x)`**Reduce [F]**

$$\int \frac{(d - c^2 dx^2)(a + b \arcsin(cx))^2}{x^4} dx$$

$$= \frac{d \left(6 \arcsin(cx) ab c^2 x^2 - 2 \arcsin(cx) ab - \sqrt{-c^2 x^2 + 1} ab cx + 3 \left(\int \frac{\arcsin(cx)^2}{x^4} dx \right) b^2 x^3 - 3 \left(\int \frac{\arcsin(cx)^2}{x^2} dx \right) b^2 c^2 x \right)}{3x^3}$$

input `int((-c^2*d*x^2+d)*(a+b*asin(c*x))^2/x^4,x)`output `(d*(6*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b - sqrt(-c**2*x**2 + 1)*a*b*c*x + 3*int(asin(c*x)**2/x**4,x)*b**2*x**3 - 3*int(asin(c*x)**2/x**2,x)*b**2*c**2*x**3 - 5*log(tan(asin(c*x)/2))*a*b*c**3*x**3 + 3*a**2*c**2*x**2 - a**2))/(3*x**3)`

3.162 $\int x^4(d - c^2dx^2)^2 (a + b \arcsin(cx))^2 dx$

Optimal result	1454
Mathematica [A] (verified)	1455
Rubi [A] (verified)	1456
Maple [A] (verified)	1462
Fricas [A] (verification not implemented)	1463
Sympy [A] (verification not implemented)	1464
Maxima [B] (verification not implemented)	1464
Giac [B] (verification not implemented)	1465
Mupad [F(-1)]	1466
Reduce [F]	1467

Optimal result

Integrand size = 27, antiderivative size = 395

$$\begin{aligned} \int x^4(d - c^2dx^2)^2 (a + b \arcsin(cx))^2 dx = & -\frac{4208b^2d^2x}{99225c^4} - \frac{2104b^2d^2x^3}{297675c^2} - \frac{526b^2d^2x^5}{165375} \\ & + \frac{212b^2c^2d^2x^7}{27783} - \frac{2}{729}b^2c^4d^2x^9 + \frac{128bd^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{4725c^5} \\ & + \frac{64bd^2x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{4725c^3} + \frac{16bd^2x^4\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{1575c} \\ & + \frac{8bd^2(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{189c^5} - \frac{2bd^2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{315c^5} \\ & - \frac{20bd^2(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{441c^5} + \frac{2bd^2(1 - c^2x^2)^{9/2}(a + b \arcsin(cx))}{81c^5} \\ & + \frac{8}{315}d^2x^5(a + b \arcsin(cx))^2 + \frac{4}{63}d^2x^5(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{1}{9}d^2x^5(1 - c^2x^2)^2(a + b \arcsin(cx))^2 \end{aligned}$$

output

```
-4208/99225*b^2*d^2*x/c^4-2104/297675*b^2*d^2*x^3/c^2-526/165375*b^2*d^2*x^5+212/27783*b^2*c^2*d^2*x^7-2/729*b^2*c^4*d^2*x^9+128/4725*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^5+64/4725*b*d^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+16/1575*b*d^2*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+8/189*b*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c^5-2/315*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c^5-20/441*b*d^2*(-c^2*x^2+1)^(7/2)*(a+b*arcsin(c*x))/c^5+2/81*b*d^2*(-c^2*x^2+1)^(9/2)*(a+b*arcsin(c*x))/c^5+8/315*d^2*x^5*(a+b*arcsin(c*x))^2+4/63*d^2*x^5*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+1/9*d^2*x^5*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.64

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2 (99225a^2c^5x^5(63 - 90c^2x^2 + 35c^4x^4) + 630ab\sqrt{1 - c^2x^2}(2104 + 1052c^2x^2 + 789c^4x^4 - 2650c^6x^6 + 1225c^8x^8) - 2b^2cx(662760 + 110460c^2x^2 + 49707c^4x^4 - 119250c^6x^6 + 2875c^8x^8) + 630b(315ac^5x^5(63 - 90c^2x^2 + 35c^4x^4) + b\sqrt{1 - c^2x^2}(2104 + 1052c^2x^2 + 789c^4x^4 - 2650c^6x^6 + 1225c^8x^8))\arcsin[cx] + 99225b^2c^5x^5(63 - 90c^2x^2 + 35c^4x^4)\arcsin[cx]^2)}{(31255875c^5)}$$

input

```
Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*(99225*a^2*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) + 630*a*b*Sqrt[1 - c^2*x^2]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8) - 2*b^2*c*x*(662760 + 110460*c^2*x^2 + 49707*c^4*x^4 - 119250*c^6*x^6 + 2875*c^8*x^8) + 630*b*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8))*ArcSin[c*x] + 99225*b^2*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcSin[c*x]^2)/(31255875*c^5)
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {5202, 27, 5194, 27, 1467, 2009, 5202, 5138, 5194, 27, 2009, 5210, 15, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & -\frac{2}{9}bcd^2 \int x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{4}{9}d \int dx^4 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \\
 & \quad \frac{1}{9}d^2 x^5 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{27} \\
 & -\frac{2}{9}bcd^2 \int x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{4}{9}d^2 \int x^4 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \\
 & \quad \frac{1}{9}d^2 x^5 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5194} \\
 & \frac{4}{9}d^2 \int x^4 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx - \\
 & \frac{2}{9}bcd^2 \left(-bc \int -\frac{(1 - c^2 x^2)^2 (35c^4 x^4 + 20c^2 x^2 + 8)}{315c^6} dx - \frac{(1 - c^2 x^2)^{9/2} (a + b \arcsin(cx))}{9c^6} + \frac{2(1 - c^2 x^2)^{7/2} (a + b \arcsin(cx))}{7c^6} \right) \\
 & \quad \frac{1}{9}d^2 x^5 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{27} \\
 & \frac{4}{9}d^2 \int x^4 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx - \\
 & \frac{2}{9}bcd^2 \left(\frac{b \int (1 - c^2 x^2)^2 (35c^4 x^4 + 20c^2 x^2 + 8) dx}{315c^5} - \frac{(1 - c^2 x^2)^{9/2} (a + b \arcsin(cx))}{9c^6} + \frac{2(1 - c^2 x^2)^{7/2} (a + b \arcsin(cx))}{7c^6} \right) \\
 & \quad \frac{1}{9}d^2 x^5 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{1467}
 \end{aligned}$$

$$\frac{4}{9}d^2 \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{9}bcd^2 \left(\frac{b \int (35c^8x^8 - 50c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{315c^5} - \frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) + \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2$$

↓ 2009

$$\frac{4}{9}d^2 \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2 dx + \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 5202

$$\frac{4}{9}d^2 \left(-\frac{2}{7}bc \int x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{2}{7} \int x^4(a+b\arcsin(cx))^2 dx + \frac{1}{7}x^5(1-c^2x^2)(a+b\arcsin(cx)) \right) + \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 5138

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{7}bc \int x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{1}{7}x^5(1-c^2x^2)(a+b\arcsin(cx)) \right) + \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 5194

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{7}bc \left(-bc \int -\frac{-15c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8}{105c^6} dx + \frac{1}{7}x^5(1-c^2x^2)(a+b\arcsin(cx)) \right) \right) + \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 27

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a + b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{7}bc \left(\frac{b \int (-15c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{105c^5} \right. \right. \\ \left. \left. - \frac{1}{9}d^2x^5(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a + b \arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} \right) \right)$$

↓ 2009

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a + b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{7}x^5(1-c^2x^2)(a + b \arcsin(cx))^2 - \frac{2}{7}bc \left(\frac{1}{9}d^2x^5(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a + b \arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} \right) \right)$$

↓ 5210

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{5c^2} + \frac{b \int x^4 dx}{5c} - \frac{x^4 \sqrt{1-c^2x^2}(a + b \arcsin(cx))}{5c^2} \right) \right) \right) \\ \left. - \frac{1}{9}d^2x^5(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a + b \arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} \right) \right)$$

↓ 15

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{5c^2} - \frac{x^4 \sqrt{1-c^2x^2}(a + b \arcsin(cx))}{5c^2} + \frac{bx^5}{25c} \right) \right) \right) + \frac{1}{7} \\ \left. - \frac{1}{9}d^2x^5(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a + b \arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^6} \right) \right)$$

↓ 5210

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{b \int x^2 dx}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} \right)}{5c^2} \right) - \frac{x^4\sqrt{1-c^2x^2}}{5c^2} \right) - \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 15

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{5c^2} \right) - \frac{x^4\sqrt{1-c^2x^2}}{5c^2} \right) - \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 5182

$$\frac{4}{9}d^2 \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{5c^2} \right) - \frac{x^4\sqrt{1-c^2x^2}}{5c^2} \right) - \frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right)$$

↓ 24

$$\frac{1}{9}d^2x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 +$$

$$\frac{4}{9}d^2 \left(\frac{1}{7}x^5(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(-\frac{x^4\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{5c^2} + \right. \right. \right.$$

$$\left. \left. \frac{2}{9}bcd^2 \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} + \frac{2(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^6} \right) \right)$$

input `Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]`

output

```
(d^2*x^5*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/9 - (2*b*c*d^2*((b*(8*x +
(4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (50*c^6*x^7)/7 + (35*c^8*x^9)/9))/(315*c^5
) - ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^6) + (2*(1 - c^2*x^2)^(
7/2)*(a + b*ArcSin[c*x]))/(7*c^6) - ((1 - c^2*x^2)^(9/2)*(a + b*ArcSin[c*x
]))/(9*c^6))/9 + (4*d^2*((x^5*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/7 - (2
*b*c*((b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)/7))/(105*c^5)
- ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^6) + (2*(1 - c^2*x^2)^(5
/2)*(a + b*ArcSin[c*x]))/(5*c^6) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcSin[c*x]
))/(7*c^6))/7 + (2*((x^5*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*((b*x^5)/(25*c
) - (x^4*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(5*c^2) + (4*((b*x^3)/(9*c
) - (x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(3*c^2) + (2*((b*x)/c - (S
qrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/(3*c^2)))/(5*c^2))/5))/7))/9
```

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]`

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 1467 $\text{Int}[\{(d_)+(e_)(x_)^2\}^q\{(a_)+(b_)(x_)^2+(c_)(x_)^4\}^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d+e*x^2)^q*(a+b*x^2+c*x^4)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2-4*a*c, 0] \&\& \text{NeQ}[c*d^2-b*d*e+a*e^2, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, -2]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

rule 5138 $\text{Int}[\{(a_)+\text{ArcSin}[c_](x_)\}^n\{(d_)(x_)\}^m, x_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}\{(a+b*\text{ArcSin}[c*x])^n/(d*(m+1))\}, x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{Int}[(d*x)^{m+1}\{(a+b*\text{ArcSin}[c*x])^{n-1}/\text{Sqrt}[1-c^2*x^2]\}, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5182 $\text{Int}[\{(a_)+\text{ArcSin}[c_](x_)\}^n(x_)\{(d_)+(e_)(x_)^2\}^p, x_Symbol] \rightarrow \text{Simp}[(d+e*x^2)^{p+1}\{(a+b*\text{ArcSin}[c*x])^n/(2*e*(p+1))\}, x] + \text{Simp}[b*(n/(2*c*(p+1)))\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \text{Int}[(1-c^2*x^2)^{p+1/2}\{(a+b*\text{ArcSin}[c*x])^{n-1}\}, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[p, -1]$

rule 5194 $\text{Int}[\{(a_)+\text{ArcSin}[c_](x_)\}^n(x_)^m\{(d_)+(e_)(x_)^2\}^p, x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[x^m*(d+e*x^2)^p, x]\}, \text{Simp}[(a+b*\text{ArcSin}[c*x])^n u, x] - \text{Simp}[b*c*\text{Simp}[\text{Sqrt}[d+e*x^2]/\text{Sqrt}[1-c^2*x^2]] \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[d+e*x^2], x], x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& \text{IntegerQ}[p-1/2] \&\& \text{NeQ}[p, -2^{(-1)}] \&\& (\text{IGtQ}[(m+1)/2, 0] || \text{ILtQ}[(m+2*p+3)/2, 0])$

rule 5202 $\text{Int}[\{(a_)+\text{ArcSin}[c_](x_)\}^n\{(f_)(x_)\}^m\{(d_)+(e_)(x_)^2\}^p, x_Symbol] \rightarrow \text{Simp}[(f*x)^{m+1}(d+e*x^2)^p\{(a+b*\text{ArcSin}[c*x])^n/(f*(m+2*p+1))\}, x] + (\text{Simp}[2*d*(p/(m+2*p+1)) \text{Int}[(f*x)^m(d+e*x^2)^{p-1}\{(a+b*\text{ArcSin}[c*x])^n\}, x], x] - \text{Simp}[b*c*(n/(f*(m+2*p+1)))\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \text{Int}[(f*x)^{m+1}(1-c^2*x^2)^{p-1/2}\{(a+b*\text{ArcSin}[c*x])^{n-1}\}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[c^2*d+e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{GtQ}[p, 0] \&\& !\text{LtQ}[m, -1]$

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.34

method	result
parts	$d^2 a^2 \left(\frac{1}{9} c^4 x^9 - \frac{2}{7} c^2 x^7 + \frac{1}{5} x^5 \right) + \frac{d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{525} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15)}{78} \right)}{15}$
derivativedivides	$d^2 a^2 \left(\frac{1}{9} c^9 x^9 - \frac{2}{7} c^7 x^7 + \frac{1}{5} c^5 x^5 \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{525} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15)}{78} \right)$
default	$d^2 a^2 \left(\frac{1}{9} c^9 x^9 - \frac{2}{7} c^7 x^7 + \frac{1}{5} c^5 x^5 \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{525} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15)}{78} \right)$
orering	$\frac{(9303875x^{12}c^{12} - 34087625c^{10}x^{10} + 40400953c^8x^8 - 8418363c^6x^6 + 38661000c^4x^4 - 46835040c^2x^2 + 15906240)(-c^2dx^2 + d)^2}{31255875x^6(cx-1)(cx+1)(c^2x^2-1)^2}$

input

```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
d^2*a^2*(1/9*c^4*x^9-2/7*c^2*x^7+1/5*x^5)+d^2*b^2/c^5*(1/15*arcsin(c*x)^2*
(3*c^4*x^4-10*c^2*x^2+15)*c*x+2/525*arcsin(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)
^(1/2)-2/7875*(3*c^4*x^4-10*c^2*x^2+15)*c*x-8/945*arcsin(c*x)*(c^2*x^2-1)*
(-c^2*x^2+1)^(1/2)+8/2835*(c^2*x^2-3)*c*x-16/315*c*x+16/315*arcsin(c*x)*(-
c^2*x^2+1)^(1/2)+2/35*arcsin(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c
*x+20/441*arcsin(c*x)*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-4/3087*(5*c^6*x^6-2
1*c^4*x^4+35*c^2*x^2-35)*c*x+1/315*arcsin(c*x)^2*(35*c^8*x^8-180*c^6*x^6+3
78*c^4*x^4-420*c^2*x^2+315)*c*x+2/81*arcsin(c*x)*(c^2*x^2-1)^4*(-c^2*x^2+1)
^(1/2)-2/25515*(35*c^8*x^8-180*c^6*x^6+378*c^4*x^4-420*c^2*x^2+315)*c*x)+
2*d^2*a*b/c^5*(1/9*arcsin(c*x)*c^9*x^9-2/7*arcsin(c*x)*c^7*x^7+1/5*c^5*x^5
*arcsin(c*x)+263/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)+1052/99225*c^2*x^2*(-c^2
*x^2+1)^(1/2)+2104/99225*(-c^2*x^2+1)^(1/2)-106/3969*c^6*x^6*(-c^2*x^2+1)^(
1/2)+1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.85

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{42875 (81 a^2 - 2 b^2) c^9 d^2 x^9 - 2250 (3969 a^2 - 106 b^2) c^7 d^2 x^7 + 189 (33075 a^2 - 526 b^2) c^5 d^2 x^5 - 220920 b^2 c^3 d^2 x^3 - 1325520 b^2 c d^2 x + 99225 (35 b^2 c^9 d^2 x^9 - 90 b^2 c^7 d^2 x^7 + 63 b^2 c^5 d^2 x^5) \arcsin(cx)^2 + 198450 (35 a b c^9 d^2 x^9 - 90 a b c^7 d^2 x^7 + 63 a b c^5 d^2 x^5) \arcsin(cx) + 630 (1225 a b c^8 d^2 x^8 - 2650 a b c^6 d^2 x^6 + 789 a b c^4 d^2 x^4 + 1052 a b c^2 d^2 x^2 + 2104 a b d^2 + (1225 b^2 c^8 d^2 x^8 - 2650 b^2 c^6 d^2 x^6 + 789 b^2 c^4 d^2 x^4 + 1052 b^2 c^2 d^2 x^2 + 2104 b^2 d^2) \arcsin(cx)) \sqrt{-c^2 x^2 + 1}}{c^5}$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
1/31255875*(42875*(81*a^2 - 2*b^2)*c^9*d^2*x^9 - 2250*(3969*a^2 - 106*b^2)
*c^7*d^2*x^7 + 189*(33075*a^2 - 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*
x^3 - 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 - 90*b^2*c^7*d^2*x^7
+ 63*b^2*c^5*d^2*x^5)*arcsin(c*x)^2 + 198450*(35*a*b*c^9*d^2*x^9 - 90*a*b
*c^7*d^2*x^7 + 63*a*b*c^5*d^2*x^5)*arcsin(c*x) + 630*(1225*a*b*c^8*d^2*x^8
- 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 + 1052*a*b*c^2*d^2*x^2 + 210
4*a*b*d^2 + (1225*b^2*c^8*d^2*x^8 - 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2
*x^4 + 1052*b^2*c^2*d^2*x^2 + 2104*b^2*d^2)*arcsin(c*x))*sqrt(-c^2*x^2 + 1
))/c^5
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.43

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} \frac{a^2 c^4 d^2 x^9}{9} - \frac{2a^2 c^2 d^2 x^7}{7} + \frac{a^2 d^2 x^5}{5} + \frac{2abc^4 d^2 x^9 \arcsin(cx)}{9} + \frac{2abc^3 d^2 x^8 \sqrt{-c^2 x^2 + 1}}{81} - \frac{4abc^2 d^2 x^7 \arcsin(cx)}{7} - \frac{212abcd^2 x^6 \sqrt{-c^2 x^2 + 1}}{3969} \\ \frac{a^2 d^2 x^5}{5} \end{cases}$$

input `integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2,x)`

output `Piecewise((a**2*c**4*d**2*x**9/9 - 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**5/5 + 2*a*b*c**4*d**2*x**9*asin(c*x)/9 + 2*a*b*c**3*d**2*x**8*sqrt(-c**2*x**2 + 1)/81 - 4*a*b*c**2*d**2*x**7*asin(c*x)/7 - 212*a*b*c*d**2*x**6*sqrt(-c**2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*asin(c*x)/5 + 526*a*b*d**2*x**4*sqrt(-c**2*x**2 + 1)/(33075*c) + 2104*a*b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(9925*c**3) + 4208*a*b*d**2*sqrt(-c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**2*x**9*asin(c*x)**2/9 - 2*b**2*c**4*d**2*x**9/729 + 2*b**2*c**3*d**2*x**8*sqrt(-c**2*x**2 + 1)*asin(c*x)/81 - 2*b**2*c**2*d**2*x**7*asin(c*x)**2/7 + 212*b**2*c**2*d**2*x**7/27783 - 212*b**2*c*d**2*x**6*sqrt(-c**2*x**2 + 1)*asin(c*x)/3969 + b**2*d**2*x**5*asin(c*x)**2/5 - 526*b**2*d**2*x**5/165375 + 526*b**2*d**2*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)/(33075*c) - 2104*b**2*d**2*x**3/(297675*c**2) + 2104*b**2*d**2*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(99225*c**3) - 4208*b**2*d**2*x/(99225*c**4) + 4208*b**2*d**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(99225*c**5), Ne(c, 0)), (a**2*d**2*x**5/5, True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(349) = 698$.

Time = 0.15 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.98

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

1/9*b^2*c^4*d^2*x^9*arcsin(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 - 2/7*b^2*c^2*d^2*
x^7*arcsin(c*x)^2 - 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arcsin(c*x)^2 +
2/2835*(315*x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^
2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 + 2/893025*(315*(35
*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2
*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)
/c^10)*c*arcsin(c*x) - (1225*c^8*x^9 + 1800*c^6*x^7 + 3024*c^4*x^5 + 6720*
c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 - 4/245*(35*x^7*arcs
in(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8
*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 -
4/25725*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4
+ 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arcsin(c*x)
- (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^2*d^2 + 2/7
5*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 +
1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*d^2 + 2/1125*(15*(3*sqrt(-c^
2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c
^6)*c*arcsin(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*b^2*d^2

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(349) = 698$.

Time = 0.16 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.78

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```

1/9*a^2*c^4*d^2*x^9 - 2/7*a^2*c^2*d^2*x^7 + 1/5*a^2*d^2*x^5 + 1/9*(c^2*x^2
- 1)^4*b^2*d^2*x*arcsin(c*x)^2/c^4 + 2/9*(c^2*x^2 - 1)^4*a*b*d^2*x*arcsin
(c*x)/c^4 + 10/63*(c^2*x^2 - 1)^3*b^2*d^2*x*arcsin(c*x)^2/c^4 - 2/729*(c^2
*x^2 - 1)^4*b^2*d^2*x/c^4 + 20/63*(c^2*x^2 - 1)^3*a*b*d^2*x*arcsin(c*x)/c^
4 + 1/105*(c^2*x^2 - 1)^2*b^2*d^2*x*arcsin(c*x)^2/c^4 + 2/81*(c^2*x^2 - 1)
^4*sqrt(-c^2*x^2 + 1)*b^2*d^2*arcsin(c*x)/c^5 - 836/250047*(c^2*x^2 - 1)^3
*b^2*d^2*x/c^4 + 2/105*(c^2*x^2 - 1)^2*a*b*d^2*x*arcsin(c*x)/c^4 - 4/315*(
c^2*x^2 - 1)*b^2*d^2*x*arcsin(c*x)^2/c^4 + 2/81*(c^2*x^2 - 1)^4*sqrt(-c^2*
x^2 + 1)*a*b*d^2/c^5 + 20/441*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2*d^2*a
rcsin(c*x)/c^5 + 33862/10418625*(c^2*x^2 - 1)^2*b^2*d^2*x/c^4 - 8/315*(c^2
*x^2 - 1)*a*b*d^2*x*arcsin(c*x)/c^4 + 8/315*b^2*d^2*x*arcsin(c*x)^2/c^4 +
20/441*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^2/c^5 + 2/525*(c^2*x^2 - 1)
^2*sqrt(-c^2*x^2 + 1)*b^2*d^2*arcsin(c*x)/c^5 - 47248/31255875*(c^2*x^2 -
1)*b^2*d^2*x/c^4 + 16/315*a*b*d^2*x*arcsin(c*x)/c^4 + 2/525*(c^2*x^2 - 1)
^2*sqrt(-c^2*x^2 + 1)*a*b*d^2/c^5 + 8/945*(-c^2*x^2 + 1)^(3/2)*b^2*d^2*arc
sin(c*x)/c^5 - 1493104/31255875*b^2*d^2*x/c^4 + 8/945*(-c^2*x^2 + 1)^(3/2)
*a*b*d^2/c^5 + 16/315*sqrt(-c^2*x^2 + 1)*b^2*d^2*arcsin(c*x)/c^5 + 16/315*
sqrt(-c^2*x^2 + 1)*a*b*d^2/c^5

```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int x^4 (a + b \operatorname{asin}(cx))^2 (d - c^2 dx^2)^2 dx$$

input

```
int(x^4*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output

```
int(x^4*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2 (22050 \operatorname{asin}(cx) ab c^9 x^9 - 56700 \operatorname{asin}(cx) ab c^7 x^7 + 39690 \operatorname{asin}(cx) ab c^5 x^5 + 2450 \sqrt{-c^2 x^2 + 1} ab c^8 x^8 -$$

input `int(x^4*(-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2,x)`

output

```
(d**2*(22050*asin(c*x)*a*b*c**9*x**9 - 56700*asin(c*x)*a*b*c**7*x**7 + 39690*asin(c*x)*a*b*c**5*x**5 + 2450*sqrt(-c**2*x**2 + 1)*a*b*c**8*x**8 - 5300*sqrt(-c**2*x**2 + 1)*a*b*c**6*x**6 + 1578*sqrt(-c**2*x**2 + 1)*a*b*c**4*x**4 + 2104*sqrt(-c**2*x**2 + 1)*a*b*c**2*x**2 + 4208*sqrt(-c**2*x**2 + 1)*a*b + 99225*int(asin(c*x)**2*x**8,x)*b**2*c**9 - 198450*int(asin(c*x)**2*x**6,x)*b**2*c**7 + 99225*int(asin(c*x)**2*x**4,x)*b**2*c**5 + 11025*a**2*c**9*x**9 - 28350*a**2*c**7*x**7 + 19845*a**2*c**5*x**5))/(99225*c**5)
```


3.163 $\int x^3(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$

Optimal result	1468
Mathematica [A] (verified)	1469
Rubi [A] (verified)	1469
Maple [A] (verified)	1475
Fricas [A] (verification not implemented)	1476
Sympy [A] (verification not implemented)	1477
Maxima [F]	1477
Giac [A] (verification not implemented)	1479
Mupad [F(-1)]	1480
Reduce [F]	1480

Optimal result

Integrand size = 27, antiderivative size = 302

$$\begin{aligned}
 \int x^3(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = & -\frac{73b^2 d^2 x^2}{3072c^2} - \frac{73b^2 d^2 x^4}{9216} \\
 & + \frac{43b^2 c^2 d^2 x^6}{3456} - \frac{1}{256} b^2 c^4 d^2 x^8 \\
 & + \frac{73bd^2 x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{1536c^3} \\
 & + \frac{73bd^2 x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{2304c} \\
 & - \frac{25}{576} bcd^2 x^5 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \\
 & - \frac{1}{32} bcd^2 x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \\
 & - \frac{73d^2 (a + b \arcsin(cx))^2}{3072c^4} \\
 & + \frac{1}{24} d^2 x^4 (a + b \arcsin(cx))^2 \\
 & + \frac{1}{12} d^2 x^4 (1 - c^2 x^2) (a + b \arcsin(cx))^2 \\
 & + \frac{1}{8} d^2 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2
 \end{aligned}$$

output

```
-73/3072*b^2*d^2*x^2/c^2-73/9216*b^2*d^2*x^4+43/3456*b^2*c^2*d^2*x^6-1/256
*b^2*c^4*d^2*x^8+73/1536*b*d^2*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+
73/2304*b*d^2*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c-25/576*b*c*d^2*x^
5*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-1/32*b*c*d^2*x^5*(-c^2*x^2+1)^(3/2)
*(a+b*arcsin(c*x))-73/3072*d^2*(a+b*arcsin(c*x))^2/c^4+1/24*d^2*x^4*(a+b*a
rcsin(c*x))^2+1/12*d^2*x^4*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+1/8*d^2*x^4*(-
c^2*x^2+1)^2*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2 (cx(1152a^2c^3x^3(6 - 8c^2x^2 + 3c^4x^4) - b^2cx(657 + 219c^2x^2 - 344c^4x^4 + 108c^6x^6) + 6ab\sqrt{1 - c^2x^2}(219$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*(c*x*(1152*a^2*c^3*x^3*(6 - 8*c^2*x^2 + 3*c^4*x^4) - b^2*c*x*(657 + 2
19*c^2*x^2 - 344*c^4*x^4 + 108*c^6*x^6) + 6*a*b*Sqrt[1 - c^2*x^2]*(219 + 1
46*c^2*x^2 - 344*c^4*x^4 + 144*c^6*x^6)) + 6*b*(b*c*x*Sqrt[1 - c^2*x^2]*(2
19 + 146*c^2*x^2 - 344*c^4*x^4 + 144*c^6*x^6) + 3*a*(-73 + 768*c^4*x^4 - 1
024*c^6*x^6 + 384*c^8*x^8))*ArcSin[c*x] + 9*b^2*(-73 + 768*c^4*x^4 - 1024*
c^6*x^6 + 384*c^8*x^8)*ArcSin[c*x]^2))/(27648*c^4)
```

Rubi [A] (verified)

Time = 2.58 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {5202, 27, 5202, 244, 2009, 5138, 5198, 15, 5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
& \quad \downarrow \text{5202} \\
& -\frac{1}{4}bcd^2 \int x^4 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{2}d \int dx^3 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \\
& \quad \frac{1}{8}d^2 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{27} \\
& -\frac{1}{4}bcd^2 \int x^4 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{2}d^2 \int x^3 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \\
& \quad \frac{1}{8}d^2 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{5202} \\
& -\frac{1}{4}bcd^2 \left(\frac{3}{8} \int x^4 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{8}bc \int x^5 (1 - c^2 x^2) dx + \frac{1}{8}x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) \\
& \frac{1}{2}d^2 \left(-\frac{1}{3}bc \int x^4 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{3} \int x^3 (a + b \arcsin(cx))^2 dx + \frac{1}{6}x^4 (1 - c^2 x^2) (a + b \arcsin(cx)) \right) \\
& \quad \frac{1}{8}d^2 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{244} \\
& \frac{1}{2}d^2 \left(-\frac{1}{3}bc \int x^4 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{3} \int x^3 (a + b \arcsin(cx))^2 dx + \frac{1}{6}x^4 (1 - c^2 x^2) (a + b \arcsin(cx)) \right) \\
& \frac{1}{4}bcd^2 \left(\frac{3}{8} \int x^4 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{8}bc \int (x^5 - c^2 x^7) dx + \frac{1}{8}x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) + \\
& \quad \frac{1}{8}d^2 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{2009} \\
& \frac{1}{2}d^2 \left(-\frac{1}{3}bc \int x^4 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{3} \int x^3 (a + b \arcsin(cx))^2 dx + \frac{1}{6}x^4 (1 - c^2 x^2) (a + b \arcsin(cx)) \right) \\
& \frac{1}{4}bcd^2 \left(\frac{3}{8} \int x^4 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{8}x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) - \frac{1}{8}bc \left(\frac{x^6}{6} - \frac{c^2 x^8}{8} \right) \right) + \\
& \quad \frac{1}{8}d^2 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{5138}
\end{aligned}$$

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx \right) - \frac{1}{3}bc \int x^4\sqrt{1 - c^2x^2}(a + b \arcsin(cx))dx + \frac{1}{4}bcd^2 \left(\frac{3}{8} \int x^4\sqrt{1 - c^2x^2}(a + b \arcsin(cx))dx + \frac{1}{8}x^5(1 - c^2x^2)^{3/2}(a + b \arcsin(cx)) - \frac{1}{8}bc \left(\frac{x^6}{6} - \frac{c^2x^8}{8} \right) \right) + \frac{1}{8}d^2x^4(1 - c^2x^2)^2(a + b \arcsin(cx))^2 \right)$$

↓ 5198

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx \right) - \frac{1}{3}bc \left(\frac{1}{6} \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx - \frac{1}{6}bc \int \frac{1}{\sqrt{1 - c^2x^2}} dx \right) - \frac{1}{6}bc \int \frac{1}{\sqrt{1 - c^2x^2}} dx + \frac{1}{4}bcd^2 \left(\frac{3}{8} \left(\frac{1}{6} \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx - \frac{1}{6}bc \int x^5 dx + \frac{1}{6}x^5\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \right) + \frac{1}{8}x^5(1 - c^2x^2)^{3/2} \right) + \frac{1}{8}d^2x^4(1 - c^2x^2)^2(a + b \arcsin(cx))^2 \right)$$

↓ 15

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx \right) - \frac{1}{3}bc \left(\frac{1}{6} \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx + \frac{1}{6}x^5\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \right) + \frac{1}{4}bcd^2 \left(\frac{3}{8} \left(\frac{1}{6} \int \frac{x^4(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx + \frac{1}{6}x^5\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) - \frac{1}{36}bcx^6 \right) + \frac{1}{8}x^5(1 - c^2x^2)^{3/2}(a + b \arcsin(cx)) \right) + \frac{1}{8}d^2x^4(1 - c^2x^2)^2(a + b \arcsin(cx))^2 \right)$$

↓ 5210

$$-\frac{1}{4}bcd^2 \left(\frac{3}{8} \left(\frac{1}{6} \left(\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{4c^2} \right) + \frac{1}{6}x^5\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \right) \right)$$

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} dx}{4c^2} + \frac{b \int x^3 dx}{4c} - \frac{x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{4c^2} \right) \right) + \frac{1}{8}d^2x^4(1 - c^2x^2)^2(a + b \arcsin(cx))^2 \right)$$

↓ 15

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right) \right) - \frac{1}{3} \right) - \frac{1}{4}bcd^2 \left(\frac{3}{8} \left(\frac{1}{6} \left(\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right) + \frac{1}{6}x^5\sqrt{1-c^2x^2}(a + b \arcsin(cx)) \right) \right) - \frac{1}{8}d^2x^4(1-c^2x^2)^2(a + b \arcsin(cx))^2$$

↓ 5210

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} \right) - \frac{x^3\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{4c^2} \right) \right) - \frac{1}{4}bcd^2 \left(\frac{3}{8} \left(\frac{1}{6} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right) \right) \right) - \frac{1}{8}d^2x^4(1-c^2x^2)^2(a + b \arcsin(cx))^2$$

↓ 15

$$\frac{1}{2}d^2 \left(\frac{1}{3} \left(\frac{1}{4}x^4(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{4c^2} \right) \right) \right) - \frac{1}{4}bcd^2 \left(\frac{3}{8} \left(\frac{1}{6} \left(\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{4c^2} + \frac{bx^4}{16c} \right) \right) \right) - \frac{1}{8}d^2x^4(1-c^2x^2)^2(a + b \arcsin(cx))^2$$

↓ 5152

$$\frac{1}{8}d^2x^4(1-c^2x^2)^2(a+b\arcsin(cx))^2 +$$

$$\frac{1}{2}d^2\left(\frac{1}{6}x^4(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{1}{3}\left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{2}bc\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{4c^2}\right)\right)\right) +$$

$$\frac{1}{4}bcd^2\left(\frac{1}{8}x^5(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{8}\left(\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{1}{6}\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{4c^2}\right)\right)\right)$$

input `Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]`

output `(d^2*x^4*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/8 - (b*c*d^2*(-1/8*(b*c*(x^6/6 - (c^2*x^8)/8)) + (x^5*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/8 + (3*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/6 + ((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/8)/4 + (d^2*((x^4*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/6 - (b*c*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/6 + ((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/3 + ((x^4*(a + b*ArcSin[c*x])^2)/4 - (b*c*((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2)))/3))/2`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5138 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]`

rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5202 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
x^2)^(p - 1/2)(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]`

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.40

method	result
parts	$d^2 a^2 \left(\frac{1}{8} c^4 x^8 - \frac{1}{3} c^2 x^6 + \frac{1}{4} x^4 \right) + \frac{d^2 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^3}{6} + \frac{\arcsin(cx) (8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 26c^3 x^3 \sqrt{-c^2 x^2 + 1} + 33cx \sqrt{-c^2 x^2 + 1} - 3c}{144} \right)}{144}$
derivativedivides	$d^2 a^2 \left(\frac{1}{8} c^8 x^8 - \frac{1}{3} c^6 x^6 + \frac{1}{4} c^4 x^4 \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^3}{6} + \frac{\arcsin(cx) (8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 26c^3 x^3 \sqrt{-c^2 x^2 + 1} + 33cx \sqrt{-c^2 x^2 + 1} - 3c}{144} \right)$
default	$d^2 a^2 \left(\frac{1}{8} c^8 x^8 - \frac{1}{3} c^6 x^6 + \frac{1}{4} c^4 x^4 \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^3}{6} + \frac{\arcsin(cx) (8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 26c^3 x^3 \sqrt{-c^2 x^2 + 1} + 33cx \sqrt{-c^2 x^2 + 1} - 3c}{144} \right)$
oring	$\frac{(18252c^{10}x^{10} - 69716c^8x^8 + 87751c^6x^6 - 492c^4x^4 - 36135c^2x^2 + 13140)(-c^2dx^2 + d)^2(a + b \arcsin(cx))^2}{55296c^4(cx - 1)(cx + 1)(c^2x^2 - 1)^2} - \frac{(2268c^8x^8)}{144}$

input

```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```


output

```
d^2*a^2*(1/8*c^4*x^8-1/3*c^2*x^6+1/4*x^4)+d^2*b^2/c^4*(1/6*arcsin(c*x)^2*(c^2*x^2-1)^3+1/144*arcsin(c*x)*(8*c^5*x^5*(-c^2*x^2+1)^(1/2)-26*c^3*x^3*(-c^2*x^2+1)^(1/2)+33*c*x*(-c^2*x^2+1)^(1/2)+15*arcsin(c*x))-55/3072*arcsin(c*x)^2-11/3456*(c^2*x^2-1)^3+55/9216*(c^2*x^2-1)^2-55/3072*c^2*x^2+55/3072+1/8*arcsin(c*x)^2*(c^2*x^2-1)^4-1/1536*arcsin(c*x)*(-48*c^7*x^7*(-c^2*x^2+1)^(1/2)+200*c^5*x^5*(-c^2*x^2+1)^(1/2)-326*c^3*x^3*(-c^2*x^2+1)^(1/2)+279*c*x*(-c^2*x^2+1)^(1/2)+105*arcsin(c*x))-1/256*(c^2*x^2-1)^4)+2*d^2*a*b/c^4*(1/8*arcsin(c*x)*c^8*x^8-1/3*arcsin(c*x)*c^6*x^6+1/4*c^4*x^4*arcsin(c*x)+73/4608*c^3*x^3*(-c^2*x^2+1)^(1/2)+73/3072*c*x*(-c^2*x^2+1)^(1/2)-73/3072*arcsin(c*x)-43/1152*c^5*x^5*(-c^2*x^2+1)^(1/2)+1/64*c^7*x^7*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.06

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{108 (32 a^2 - b^2) c^8 d^2 x^8 - 8 (1152 a^2 - 43 b^2) c^6 d^2 x^6 + 3 (2304 a^2 - 73 b^2) c^4 d^2 x^4 - 657 b^2 c^2 d^2 x^2 + 9 (384 b^2 c^8 d^2 x^8 - 1024 b^2 c^6 d^2 x^6 + 768 b^2 c^4 d^2 x^4 - 73 b^2 d^2) \arcsin(c x)^2 + 18 (384 a b c^8 d^2 x^8 - 1024 a b c^6 d^2 x^6 + 768 a b c^4 d^2 x^4 - 73 a b d^2) \arcsin(c x) + 6 (144 a b c^7 d^2 x^7 - 344 a b c^5 d^2 x^5 + 146 a b c^3 d^2 x^3 + 219 a b c d^2 x + (144 b^2 c^7 d^2 x^7 - 344 b^2 c^5 d^2 x^5 + 146 b^2 c^3 d^2 x^3 + 219 b^2 c d^2 x) \arcsin(c x)) \sqrt{-c^2 x^2 + 1}}{c^4}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
1/27648*(108*(32*a^2 - b^2)*c^8*d^2*x^8 - 8*(1152*a^2 - 43*b^2)*c^6*d^2*x^6 + 3*(2304*a^2 - 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c^8*d^2*x^8 - 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*arcsin(c*x)^2 + 18*(384*a*b*c^8*d^2*x^8 - 1024*a*b*c^6*d^2*x^6 + 768*a*b*c^4*d^2*x^4 - 73*a*b*d^2)*arcsin(c*x) + 6*(144*a*b*c^7*d^2*x^7 - 344*a*b*c^5*d^2*x^5 + 146*a*b*c^3*d^2*x^3 + 219*a*b*c*d^2*x + (144*b^2*c^7*d^2*x^7 - 344*b^2*c^5*d^2*x^5 + 146*b^2*c^3*d^2*x^3 + 219*b^2*c*d^2*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1)/c^4
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.71

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \left\{ \begin{array}{l} \frac{a^2 c^4 d^2 x^8}{8} - \frac{a^2 c^2 d^2 x^6}{3} + \frac{a^2 d^2 x^4}{4} + \frac{abc^4 d^2 x^8 \arcsin(cx)}{4} + \frac{abc^3 d^2 x^7 \sqrt{-c^2 x^2 + 1}}{32} - \frac{2abc^2 d^2 x^6 \arcsin(cx)}{3} - \frac{43abcd^2 x^5 \sqrt{-c^2 x^2 + 1}}{576} + \dots \\ \frac{a^2 d^2 x^4}{4} \end{array} \right.$$

input `integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2,x)`

output `Piecewise((a**2*c**4*d**2*x**8/8 - a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/4 + a*b*c**4*d**2*x**8*asin(c*x)/4 + a*b*c**3*d**2*x**7*sqrt(-c**2*x**2 + 1)/32 - 2*a*b*c**2*d**2*x**6*asin(c*x)/3 - 43*a*b*c*d**2*x**5*sqrt(-c**2*x**2 + 1)/576 + a*b*d**2*x**4*asin(c*x)/2 + 73*a*b*d**2*x**3*sqrt(-c**2*x**2 + 1)/(2304*c) + 73*a*b*d**2*x*sqrt(-c**2*x**2 + 1)/(1536*c**3) - 73*a*b*d**2*asin(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*asin(c*x)**2/8 - b**2*c**4*d**2*x**8/256 + b**2*c**3*d**2*x**7*sqrt(-c**2*x**2 + 1)*asin(c*x)/32 - b**2*c**2*d**2*x**6*asin(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 - 43*b**2*c*d**2*x**5*sqrt(-c**2*x**2 + 1)*asin(c*x)/576 + b**2*d**2*x**4*asin(c*x)**2/4 - 73*b**2*d**2*x**4/9216 + 73*b**2*d**2*x**3*sqrt(-c**2*x**2 + 1)*asin(c*x)/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) + 73*b**2*d**2*x*sqrt(-c**2*x**2 + 1)*asin(c*x)/(1536*c**3) - 73*b**2*d**2*asin(c*x)**2/(3072*c**4) , Ne(c, 0)), (a**2*d**2*x**4/4, True))`

Maxima [F]

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2 x^3 dx$$

input `integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

1/8*a^2*c^4*d^2*x^8 - 1/3*a^2*c^2*d^2*x^6 + 1/1536*(384*x^8*arcsin(c*x) +
(48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-
c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)
*c)*a*b*c^4*d^2 + 1/4*a^2*d^2*x^4 - 1/72*(48*x^6*arcsin(c*x) + (8*sqrt(-c^
2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)
*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^2*d^2 + 1/16*(8*x^4*arcsin(c*x) + (2
*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c
^5)*c)*a*b*d^2 + 1/24*(3*b^2*c^4*d^2*x^8 - 8*b^2*c^2*d^2*x^6 + 6*b^2*d^2*x
^4)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + integrate(1/12*(3*b^2*c
^5*d^2*x^8 - 8*b^2*c^3*d^2*x^6 + 6*b^2*c*d^2*x^4)*sqrt(c*x + 1)*sqrt(-c*x
+ 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^2 - 1), x)

```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.73

$$\begin{aligned}
\int x^3(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = & \frac{1}{8} a^2 c^4 d^2 x^8 - \frac{1}{3} a^2 c^2 d^2 x^6 + \frac{1}{4} a^2 d^2 x^4 \\
& + \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b^2 d^2 x \arcsin(cx)}{32 c^3} \\
& + \frac{(c^2 x^2 - 1)^4 b^2 d^2 \arcsin(cx)^2}{8 c^4} \\
& + \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} a b d^2 x}{32 c^3} \\
& + \frac{11 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^2 x \arcsin(cx)}{576 c^3} \\
& + \frac{(c^2 x^2 - 1)^4 a b d^2 \arcsin(cx)}{4 c^4} \\
& + \frac{(c^2 x^2 - 1)^3 b^2 d^2 \arcsin(cx)^2}{6 c^4} \\
& + \frac{11 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^2 x}{576 c^3} \\
& + \frac{55 (-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d^2 x \arcsin(cx)}{2304 c^3} \\
& - \frac{(c^2 x^2 - 1)^4 b^2 d^2}{256 c^4} \\
& + \frac{(c^2 x^2 - 1)^3 a b d^2 \arcsin(cx)}{3 c^4} \\
& + \frac{55 (-c^2 x^2 + 1)^{\frac{3}{2}} a b d^2 x}{2304 c^3} \\
& + \frac{55 \sqrt{-c^2 x^2 + 1} b^2 d^2 x \arcsin(cx)}{1536 c^3} \\
& - \frac{11 (c^2 x^2 - 1)^3 b^2 d^2}{3456 c^4} \\
& + \frac{55 \sqrt{-c^2 x^2 + 1} a b d^2 x}{1536 c^3} + \frac{55 (c^2 x^2 - 1)^2 b^2 d^2}{9216 c^4} \\
& + \frac{55 b^2 d^2 \arcsin(cx)^2}{3072 c^4} - \frac{55 (c^2 x^2 - 1) b^2 d^2}{3072 c^4} \\
& + \frac{55 a b d^2 \arcsin(cx)}{1536 c^4} - \frac{9835 b^2 d^2}{884736 c^4}
\end{aligned}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
1/8*a^2*c^4*d^2*x^8 - 1/3*a^2*c^2*d^2*x^6 + 1/4*a^2*d^2*x^4 + 1/32*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2*d^2*x*arcsin(c*x)/c^3 + 1/8*(c^2*x^2 - 1)^4*b^2*d^2*arcsin(c*x)^2/c^4 + 1/32*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^2*x/c^3 + 11/576*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^2*x*arcsin(c*x)/c^3 + 1/4*(c^2*x^2 - 1)^4*a*b*d^2*arcsin(c*x)/c^4 + 1/6*(c^2*x^2 - 1)^3*b^2*d^2*arcsin(c*x)^2/c^4 + 11/576*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^2*x/c^3 + 55/2304*(-c^2*x^2 + 1)^(3/2)*b^2*d^2*x*arcsin(c*x)/c^3 - 1/256*(c^2*x^2 - 1)^4*b^2*d^2/c^4 + 1/3*(c^2*x^2 - 1)^3*a*b*d^2*arcsin(c*x)/c^4 + 55/2304*(-c^2*x^2 + 1)^(3/2)*a*b*d^2*x/c^3 + 55/1536*sqrt(-c^2*x^2 + 1)*b^2*d^2*x*arcsin(c*x)/c^3 - 11/3456*(c^2*x^2 - 1)^3*b^2*d^2/c^4 + 55/1536*sqrt(-c^2*x^2 + 1)*a*b*d^2*x/c^3 + 55/9216*(c^2*x^2 - 1)^2*b^2*d^2/c^4 + 55/3072*b^2*d^2*arcsin(c*x)^2/c^4 - 55/3072*(c^2*x^2 - 1)*b^2*d^2/c^4 + 55/1536*a*b*d^2*arcsin(c*x)/c^4 - 9835/884736*b^2*d^2/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int x^3 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^2 dx$$

input

```
int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output

```
int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^3 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2 (1152 a \sin(cx) a b c^8 x^8 - 3072 a \sin(cx) a b c^6 x^6 + 2304 a \sin(cx) a b c^4 x^4 - 219 a \sin(cx) a b + 144 \sqrt{-c^2 x^2}}$$

input

```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2,x)
```

output

```
(d**2*(1152*asin(c*x)*a*b*c**8*x**8 - 3072*asin(c*x)*a*b*c**6*x**6 + 2304*
asin(c*x)*a*b*c**4*x**4 - 219*asin(c*x)*a*b + 144*sqrt(-c**2*x**2 + 1)*a
*b*c**7*x**7 - 344*sqrt(-c**2*x**2 + 1)*a*b*c**5*x**5 + 146*sqrt(-c**2
*x**2 + 1)*a*b*c**3*x**3 + 219*sqrt(-c**2*x**2 + 1)*a*b*c*x + 4608*int(a
sin(c*x)**2*x**7,x)*b**2*c**8 - 9216*int(asin(c*x)**2*x**5,x)*b**2*c**6 +
4608*int(asin(c*x)**2*x**3,x)*b**2*c**4 + 576*a**2*c**8*x**8 - 1536*a**2*c
**6*x**6 + 1152*a**2*c**4*x**4))/(4608*c**4)
```

3.164 $\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$

Optimal result	1482
Mathematica [A] (verified)	1483
Rubi [A] (verified)	1483
Maple [A] (verified)	1489
Fricas [A] (verification not implemented)	1489
Sympy [A] (verification not implemented)	1490
Maxima [B] (verification not implemented)	1491
Giac [B] (verification not implemented)	1492
Mupad [F(-1)]	1494
Reduce [F]	1494

Optimal result

Integrand size = 27, antiderivative size = 310

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = -\frac{1636b^2 d^2 x}{11025c^2} - \frac{818b^2 d^2 x^3}{33075} + \frac{136b^2 c^2 d^2 x^5}{6125} - \frac{2}{343} b^2 c^4 d^2 x^7 + \frac{32bd^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{315c^3} + \frac{16bd^2 x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{315c} + \frac{8bd^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{105c^3} + \frac{2bd^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{175c^3} - \frac{2bd^2 (1 - c^2 x^2)^{7/2} (a + b \arcsin(cx))}{49c^3} + \frac{8}{105} d^2 x^3 (a + b \arcsin(cx))^2 + \frac{4}{35} d^2 x^3 (1 - c^2 x^2) (a + b \arcsin(cx))^2 + \frac{1}{7} d^2 x^3 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2$$

output

```
-1636/11025*b^2*d^2*x/c^2-818/33075*b^2*d^2*x^3+136/6125*b^2*c^2*d^2*x^5-2/343*b^2*c^4*d^2*x^7+32/315*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+16/315*b*d^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+8/105*b*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c^3+2/175*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c^3-2/49*b*d^2*(-c^2*x^2+1)^(7/2)*(a+b*arcsin(c*x))/c^3+8/105*d^2*x^3*(a+b*arcsin(c*x))^2+4/35*d^2*x^3*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+1/7*d^2*x^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.74

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2 (11025a^2 c^3 x^3 (35 - 42c^2 x^2 + 15c^4 x^4) + 210ab \sqrt{1 - c^2 x^2} (818 + 409c^2 x^2 - 612c^4 x^4 + 225c^6 x^6) - 2b^2 c a}{1157625c^3}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*(11025*a^2*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) + 210*a*b*Sqrt[1 - c^2*x^2]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) - 2*b^2*c*x*(85890 + 14315*c^2*x^2 - 12852*c^4*x^4 + 3375*c^6*x^6) + 210*b*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6))*ArcSin[c*x] + 11025*b^2*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4)*ArcSin[c*x]^2))/(1157625*c^3)
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5202, 27, 5194, 27, 290, 2009, 5202, 5138, 5194, 27, 2009, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5202$$

$$-\frac{2}{7}bcd^2 \int x^3 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{4}{7}d \int dx^2 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \frac{1}{7}d^2 x^3 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{4}{7}d^2 \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{7}bcd^2 \int x^3(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx + \\
& \quad \frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{5194} \\
& \frac{4}{7}d^2 \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \\
& \frac{2}{7}bcd^2 \left(-bc \int -\frac{(1-c^2x^2)^2(5c^2x^2+2)}{35c^4} dx + \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} \right. \\
& \quad \left. + \frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 \right) \\
& \quad \downarrow \text{27} \\
& \frac{4}{7}d^2 \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \\
& \frac{2}{7}bcd^2 \left(\frac{b \int (1-c^2x^2)^2(5c^2x^2+2) dx}{35c^3} + \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} \right) + \\
& \quad \frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{290} \\
& \frac{4}{7}d^2 \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \\
& \frac{2}{7}bcd^2 \left(\frac{b \int (5c^6x^6 - 8c^4x^4 + c^2x^2 + 2) dx}{35c^3} + \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} \right) \\
& \quad \frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow \text{2009} \\
& \frac{4}{7}d^2 \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx + \frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \\
& \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} + \frac{b\left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x\right)}{35c^3} \right) \\
& \quad \downarrow \text{5202} \\
& \frac{4}{7}d^2 \left(-\frac{2}{5}bc \int x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{2}{5} \int x^2(a+b\arcsin(cx))^2 dx + \frac{1}{5}x^3(1-c^2x^2)(a+b\arcsin(cx)) \right) \\
& \quad \frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \\
& \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^4} + \frac{b\left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x\right)}{35c^3} \right)
\end{aligned}$$

↓ 5138

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{5}bc \int x^3 \sqrt{1-c^2x^2}(a + b \arcsin(cx)) dx + \right. \\ \left. \frac{1}{7}d^2x^3(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \right. \\ \left. \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right) \right)$$

↓ 5194

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{5}bc \left(-bc \int -\frac{-3c^4x^4 + c^2x^2 + 2}{15c^4} dx + \frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right) \right)$$

↓ 27

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{5}bc \left(\frac{b \int (-3c^4x^4 + c^2x^2 + 2) dx}{15c^3} + \frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right) \right)$$

↓ 2009

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a + b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{5}x^3(1-c^2x^2)(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{b \int (-3c^4x^4 + c^2x^2 + 2) dx}{15c^3} + \frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right) \right)$$

↓ 5210

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{b \int x^2 dx}{3c} - \frac{x^2 \sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} \right) \right) \right) - \frac{1}{7}d^2x^3(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right)$$

↓ 15

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right) \right) \right) + \frac{1}{5}d^2x^3(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right)$$

↓ 5182

$$\frac{4}{7}d^2 \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} \right) \right) \right) + \frac{1}{5}d^2x^3(1-c^2x^2)^2(a + b \arcsin(cx))^2 - \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right)$$

↓ 24

$$\frac{1}{7}d^2x^3(1-c^2x^2)^2(a + b \arcsin(cx))^2 + \frac{4}{7}d^2 \left(\frac{1}{5}x^3(1-c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(-\frac{x^2 \sqrt{1-c^2x^2}(a + b \arcsin(cx))}{3c^2} \right) \right) \right) + \frac{2}{7}bcd^2 \left(\frac{(1-c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^4} - \frac{(1-c^2x^2)^{5/2}(a + b \arcsin(cx))}{5c^4} + \frac{b \left(\frac{5c^6x^7}{7} - \frac{8c^4x^5}{5} + \frac{c^2x^3}{3} + 2x \right)}{35c^3} \right)$$

input `Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]`

output

$$\begin{aligned} & (d^2 x^3 (1 - c^2 x^2)^2 (a + b \operatorname{ArcSin}[c x])^2) / 7 - (2 b c d^2 ((b (2 x + \\ & (c^2 x^3) / 3 - (8 c^4 x^5) / 5 + (5 c^6 x^7) / 7)) / (35 c^3) - ((1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcSin}[c x]) \\ &)) / (5 c^4) + ((1 - c^2 x^2)^{7/2} (a + b \operatorname{ArcSin}[c x]) \\ &)) / (7 c^4)) / 7 + (4 d^2 ((x^3 (1 - c^2 x^2) (a + b \operatorname{ArcSin}[c x])^2) / 5 - (2 b c \\ & ((b (2 x + (c^2 x^3) / 3 - (3 c^4 x^5) / 5)) / (15 c^3) - ((1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcSin}[c x]) \\ &)) / (3 c^4) + ((1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcSin}[c x]) \\ &)) / (5 c^4)) / 5 + (2 ((x^3 (a + b \operatorname{ArcSin}[c x])^2) / 3 - (2 b c ((b x^3) / (9 c) \\ & - (x^2 \operatorname{Sqrt}[1 - c^2 x^2] (a + b \operatorname{ArcSin}[c x])) / (3 c^2) + (2 ((b x) / c - (\operatorname{Sqrt}[1 - c^2 x^2] (a + b \operatorname{ArcSin}[c x])) / c^2)) / (3 c^2))) / 3) / 5) / 7 \end{aligned}$$

Defintions of rubi rules used

rule 15

$$\operatorname{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[a(x^{(m+1)}) / (m+1), x] \;/; \operatorname{FreeQ}[\{a, m\}, x] \ \&\& \operatorname{NeQ}[m, -1]$$

rule 24

$$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a x, x] \;/; \operatorname{FreeQ}[a, x]$$

rule 27

$$\operatorname{Int}[(a_)(F_x), x_Symbol] \rightarrow \operatorname{Simp}[a \operatorname{Int}[F_x, x], x] \;/; \operatorname{FreeQ}[a, x] \ \&\& \operatorname{!MatchQ}[F_x, (b_)(G_x)] \;/; \operatorname{FreeQ}[b, x]$$

rule 290

$$\operatorname{Int}[(a_) + (b_)(x_)^2)^{(p_.)((c_) + (d_)(x_)^2)^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b x^2)^p (c + d x^2)^q, x], x] \;/; \operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[b c - a d, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{IGtQ}[q, 0]$$

rule 2009

$$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{IntSum}[u, x], x] \;/; \operatorname{SumQ}[u]$$

rule 5138

$$\operatorname{Int}[(a_.) + \operatorname{ArcSin}[(c_)(x_)](b_.))^{(n_.)((d_)(x_))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d x)^{(m+1)} ((a + b \operatorname{ArcSin}[c x])^n / (d^{(m+1)})), x] - \operatorname{Simp}[b c (n / (d^{(m+1)})) \operatorname{Int}[(d x)^{(m+1)} ((a + b \operatorname{ArcSin}[c x])^{(n-1)} / \operatorname{Sqrt}[1 - c^2 x^2]), x], x] \;/; \operatorname{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{NeQ}[m, -1]$$

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_) , x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.29

method	result
parts	$d^2 a^2 \left(\frac{1}{7} c^4 x^7 - \frac{2}{5} c^2 x^5 + \frac{1}{3} x^3 \right) + \frac{d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{175} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15) c^3 x^3}{26} \right)}{26}$
derivativedivides	$d^2 a^2 \left(\frac{1}{7} c^7 x^7 - \frac{2}{5} c^5 x^5 + \frac{1}{3} c^3 x^3 \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{175} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15) c^3 x^3}{26} \right)$
default	$d^2 a^2 \left(\frac{1}{7} c^7 x^7 - \frac{2}{5} c^5 x^5 + \frac{1}{3} c^3 x^3 \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{175} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15) c^3 x^3}{26} \right)$
orering	$\frac{(428625c^{10}x^{10} - 1739907c^8x^8 + 2486259c^6x^6 + 2357383c^4x^4 - 2404920c^2x^2 + 687120)(-c^2dx^2 + d)^2(a + b \arcsin(cx))^2}{1157625x c^4 (cx - 1)(cx + 1)(c^2x^2 - 1)^2}$

```
input int(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

```
output d^2*a^2*(1/7*c^4*x^7-2/5*c^2*x^5+1/3*x^3)+d^2*b^2/c^3*(1/15*arcsin(c*x)^2*(3*c^4*x^4-10*c^2*x^2+15)*c*x+2/175*arcsin(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)-2/2625*(3*c^4*x^4-10*c^2*x^2+15)*c*x-8/315*arcsin(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+8/945*(c^2*x^2-3)*c*x-16/105*c*x+16/105*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+1/35*arcsin(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x+2/49*arcsin(c*x)*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-2/1715*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x)+2*d^2*a*b/c^3*(1/7*arcsin(c*x)*c^7*x^7-2/5*c^5*x^5*arcsin(c*x)+1/3*c^3*x^3*arcsin(c*x)+409/11025*c^2*x^2*(-c^2*x^2+1)^(1/2)+818/11025*(-c^2*x^2+1)^(1/2)-68/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)+1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.95

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{3375 (49 a^2 - 2 b^2) c^7 d^2 x^7 - 378 (1225 a^2 - 68 b^2) c^5 d^2 x^5 + 35 (11025 a^2 - 818 b^2) c^3 d^2 x^3 - 171780 b^2 c d^2 x}{1157625 x c^4 (cx - 1)(cx + 1)(c^2 x^2 - 1)^2}$$

input `integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output
$$\frac{1}{1157625} * (3375 * (49 * a^2 - 2 * b^2) * c^7 * d^2 * x^7 - 378 * (1225 * a^2 - 68 * b^2) * c^5 * d^2 * x^5 + 35 * (11025 * a^2 - 818 * b^2) * c^3 * d^2 * x^3 - 171780 * b^2 * c * d^2 * x + 11025 * (15 * b^2 * c^7 * d^2 * x^7 - 42 * b^2 * c^5 * d^2 * x^5 + 35 * b^2 * c^3 * d^2 * x^3) * \arcsin(c * x)^2 + 22050 * (15 * a * b * c^7 * d^2 * x^7 - 42 * a * b * c^5 * d^2 * x^5 + 35 * a * b * c^3 * d^2 * x^3) * \arcsin(c * x) + 210 * (225 * a * b * c^6 * d^2 * x^6 - 612 * a * b * c^4 * d^2 * x^4 + 409 * a * b * c^2 * d^2 * x^2 + 818 * a * b * d^2 + (225 * b^2 * c^6 * d^2 * x^6 - 612 * b^2 * c^4 * d^2 * x^4 + 409 * b^2 * c^2 * d^2 * x^2 + 818 * b^2 * d^2) * \arcsin(c * x)) * \sqrt{-c^2 * x^2 + 1}) / c^3$$

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.56

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} \frac{a^2 c^4 d^2 x^7}{7} - \frac{2 a^2 c^2 d^2 x^5}{5} + \frac{a^2 d^2 x^3}{3} + \frac{2 a b c^4 d^2 x^7 \arcsin(cx)}{7} + \frac{2 a b c^3 d^2 x^6 \sqrt{-c^2 x^2 + 1}}{49} - \frac{4 a b c^2 d^2 x^5 \arcsin(cx)}{5} - \frac{136 a b c d^2 x^4 \sqrt{-c^2 x^2 + 1}}{1225} \\ \frac{a^2 d^2 x^3}{3} \end{cases}$$

input `integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2,x)`

output
$$\text{Piecewise}((a**2*c**4*d**2*x**7/7 - 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**3/3 + 2*a*b*c**4*d**2*x**7*asin(c*x)/7 + 2*a*b*c**3*d**2*x**6*\sqrt{-c**2*x**2 + 1}/49 - 4*a*b*c**2*d**2*x**5*asin(c*x)/5 - 136*a*b*c*d**2*x**4*\sqrt{-c**2*x**2 + 1}/1225 + 2*a*b*d**2*x**3*asin(c*x)/3 + 818*a*b*d**2*x**2*\sqrt{-c**2*x**2 + 1}/(11025*c) + 1636*a*b*d**2*\sqrt{-c**2*x**2 + 1}/(11025*c**3) + b**2*c**4*d**2*x**7*asin(c*x)**2/7 - 2*b**2*c**4*d**2*x**7/343 + 2*b**2*c**3*d**2*x**6*\sqrt{-c**2*x**2 + 1}*asin(c*x)/49 - 2*b**2*c**2*d**2*x**5*asin(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 - 136*b**2*c*d**2*x**4*\sqrt{-c**2*x**2 + 1}*asin(c*x)/1225 + b**2*d**2*x**3*asin(c*x)**2/3 - 818*b**2*d**2*x**3/33075 + 818*b**2*d**2*x**2*\sqrt{-c**2*x**2 + 1}*asin(c*x)/(11025*c) - 1636*b**2*d**2*x/(11025*c**2) + 1636*b**2*d**2*\sqrt{-c**2*x**2 + 1}*asin(c*x)/(11025*c**3), Ne(c, 0)), (a**2*d**2*x**3/3, True))$$

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(274) = 548$.

Time = 0.15 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.05

$$\begin{aligned}
 & \int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
 &= \frac{1}{7} b^2 c^4 d^2 x^7 \arcsin(cx)^2 + \frac{1}{7} a^2 c^4 d^2 x^7 - \frac{2}{5} b^2 c^2 d^2 x^5 \arcsin(cx)^2 - \frac{2}{5} a^2 c^2 d^2 x^5 \\
 &+ \frac{2}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) \right) \\
 &+ \frac{2}{25725} \left(105 \left(\frac{5 \sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6 \sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16 \sqrt{-c^2 x^2 + 1}}{c^8} \right) c \arcsin(cx) \right. \\
 &+ \left. \frac{1}{3} b^2 d^2 x^3 \arcsin(cx)^2 \right. \\
 &- \left. \frac{4}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) abc^2 d^2 \right. \\
 &- \left. \frac{4}{1125} \left(15 \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \arcsin(cx) - \frac{9 c^4 x^5 + 20 c^2 x^3 + 15}{c^4} \right) \right. \\
 &+ \left. \frac{1}{3} a^2 d^2 x^3 + \frac{2}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) abd^2 \right. \\
 &+ \left. \frac{2}{27} \left(3 c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \arcsin(cx) - \frac{c^2 x^3 + 6 x}{c^2} \right) b^2 d^2 \right)
 \end{aligned}$$

input `integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

1/7*b^2*c^4*d^2*x^7*arcsin(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 - 2/5*b^2*c^2*d^2*
x^5*arcsin(c*x)^2 - 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arcsin(c*x) + (5*s
qrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2
+ 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 + 2/25725*(105*(
5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*
x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arcsin(c*x) - (75*c^6*x^7
+ 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*a
rcsin(c*x)^2 - 4/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 +
4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 -
4/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c*arcsin(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)
/c^4)*b^2*c^2*d^2 + 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^
2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d^2 + 2/27*(3*c*(sqrt(
-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcsin(c*x) - (c^2*x^3 +
6*x)/c^2)*b^2*d^2

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(274) = 548$.

Time = 0.16 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.78

$$\begin{aligned}
\int x^2 (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = & \frac{1}{7} a^2 c^4 d^2 x^7 - \frac{2}{5} a^2 c^2 d^2 x^5 \\
& + \frac{(c^2 x^2 - 1)^3 b^2 d^2 x \arcsin(cx)^2}{7 c^2} + \frac{1}{3} a^2 d^2 x^3 \\
& + \frac{2 (c^2 x^2 - 1)^3 a b d^2 x \arcsin(cx)}{7 c^2} \\
& + \frac{(c^2 x^2 - 1)^2 b^2 d^2 x \arcsin(cx)^2}{35 c^2} \\
& - \frac{2 (c^2 x^2 - 1)^3 b^2 d^2 x}{343 c^2} \\
& + \frac{2 (c^2 x^2 - 1)^2 a b d^2 x \arcsin(cx)}{35 c^2} \\
& - \frac{4 (c^2 x^2 - 1) b^2 d^2 x \arcsin(cx)^2}{105 c^2} \\
& + \frac{2 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b^2 d^2 \arcsin(cx)}{49 c^3} \\
& + \frac{202 (c^2 x^2 - 1)^2 b^2 d^2 x}{42875 c^2} \\
& - \frac{8 (c^2 x^2 - 1) a b d^2 x \arcsin(cx)}{105 c^2} \\
& + \frac{8 b^2 d^2 x \arcsin(cx)^2}{105 c^2} \\
& + \frac{2 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} a b d^2}{49 c^3} \\
& + \frac{2 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^2 \arcsin(cx)}{175 c^3} \\
& + \frac{2528 (c^2 x^2 - 1) b^2 d^2 x}{1157625 c^2} \\
& + \frac{16 a b d^2 x \arcsin(cx)}{105 c^2} \\
& + \frac{2 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^2}{175 c^3} \\
& + \frac{8 (-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d^2 \arcsin(cx)}{315 c^3} \\
& - \frac{181456 b^2 d^2 x}{1157625 c^2} + \frac{8 (-c^2 x^2 + 1)^{\frac{3}{2}} a b d^2}{315 c^3} \\
& + \frac{16 \sqrt{-c^2 x^2 + 1} b^2 d^2 \arcsin(cx)}{105 c^3} \\
& + \frac{16 \sqrt{-c^2 x^2 + 1} a b d^2}{105 c^3}
\end{aligned}$$

input `integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output
$$\begin{aligned} & 1/7*a^2*c^4*d^2*x^7 - 2/5*a^2*c^2*d^2*x^5 + 1/7*(c^2*x^2 - 1)^3*b^2*d^2*x* \\ & \arcsin(c*x)^2/c^2 + 1/3*a^2*d^2*x^3 + 2/7*(c^2*x^2 - 1)^3*a*b*d^2*x*\arcsin \\ & (c*x)/c^2 + 1/35*(c^2*x^2 - 1)^2*b^2*d^2*x*\arcsin(c*x)^2/c^2 - 2/343*(c^2* \\ & x^2 - 1)^3*b^2*d^2*x/c^2 + 2/35*(c^2*x^2 - 1)^2*a*b*d^2*x*\arcsin(c*x)/c^2 \\ & - 4/105*(c^2*x^2 - 1)*b^2*d^2*x*\arcsin(c*x)^2/c^2 + 2/49*(c^2*x^2 - 1)^3*s \\ & \text{qrt}(-c^2*x^2 + 1)*b^2*d^2*\arcsin(c*x)/c^3 + 202/42875*(c^2*x^2 - 1)^2*b^2* \\ & d^2*x/c^2 - 8/105*(c^2*x^2 - 1)*a*b*d^2*x*\arcsin(c*x)/c^2 + 8/105*b^2*d^2* \\ & x*\arcsin(c*x)^2/c^2 + 2/49*(c^2*x^2 - 1)^3*\text{sqrt}(-c^2*x^2 + 1)*a*b*d^2/c^3 \\ & + 2/175*(c^2*x^2 - 1)^2*\text{sqrt}(-c^2*x^2 + 1)*b^2*d^2*\arcsin(c*x)/c^3 + 2528/ \\ & 1157625*(c^2*x^2 - 1)*b^2*d^2*x/c^2 + 16/105*a*b*d^2*x*\arcsin(c*x)/c^2 + 2 \\ & /175*(c^2*x^2 - 1)^2*\text{sqrt}(-c^2*x^2 + 1)*a*b*d^2/c^3 + 8/315*(-c^2*x^2 + 1) \\ & ^{(3/2)}*b^2*d^2*\arcsin(c*x)/c^3 - 181456/1157625*b^2*d^2*x/c^2 + 8/315*(-c^ \\ & 2*x^2 + 1)^{(3/2)}*a*b*d^2/c^3 + 16/105*\text{sqrt}(-c^2*x^2 + 1)*b^2*d^2*\arcsin(c* \\ & x)/c^3 + 16/105*\text{sqrt}(-c^2*x^2 + 1)*a*b*d^2/c^3 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int x^2 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^2 dx$$

input `int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2,x)`

output `int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int x^2(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2(3150a \arcsin(cx) ab c^7 x^7 - 8820a \arcsin(cx) ab c^5 x^5 + 7350a \arcsin(cx) ab c^3 x^3 + 450\sqrt{-c^2 x^2 + 1} ab c^6 x^6 - 122$$

input `int(x^2*(-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2,x)`

output `(d**2*(3150*asin(c*x)*a*b*c**7*x**7 - 8820*asin(c*x)*a*b*c**5*x**5 + 7350*asin(c*x)*a*b*c**3*x**3 + 450*sqrt(-c**2*x**2 + 1)*a*b*c**6*x**6 - 1224*sqrt(-c**2*x**2 + 1)*a*b*c**4*x**4 + 818*sqrt(-c**2*x**2 + 1)*a*b*c**2*x**2 + 1636*sqrt(-c**2*x**2 + 1)*a*b + 11025*int(asin(c*x)**2*x**6,x)*b**2*c**7 - 22050*int(asin(c*x)**2*x**4,x)*b**2*c**5 + 11025*int(asin(c*x)**2*x**2,x)*b**2*c**3 + 1575*a**2*c**7*x**7 - 4410*a**2*c**5*x**5 + 3675*a**2*c**3*x**3))/(11025*c**3)`

3.165 $\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$

Optimal result	1496
Mathematica [A] (verified)	1497
Rubi [A] (verified)	1497
Maple [A] (verified)	1501
Fricas [A] (verification not implemented)	1501
Sympy [B] (verification not implemented)	1502
Maxima [F]	1503
Giac [A] (verification not implemented)	1503
Mupad [F(-1)]	1504
Reduce [F]	1504

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = -\frac{5}{96} b^2 d^2 x^2 + \frac{5b^2 d^2 (1 - c^2 x^2)^2}{288c^2} + \frac{b^2 d^2 (1 - c^2 x^2)^3}{108c^2} + \frac{5bd^2 x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{48c} + \frac{5bd^2 x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{72c} + \frac{bd^2 x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{18c} + \frac{5d^2 (a + b \arcsin(cx))^2}{96c^2} - \frac{d^2 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}$$

output

```
-5/96*b^2*d^2*x^2+5/288*b^2*d^2*(-c^2*x^2+1)^2/c^2+1/108*b^2*d^2*(-c^2*x^2+1)^3/c^2+5/48*b*d^2*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+5/72*b*d^2*x*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c+1/18*b*d^2*x*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c+5/96*d^2*(a+b*arcsin(c*x))^2/c^2-1/6*d^2*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2(b^2 c^2 x^2(-99 + 39c^2 x^2 - 8c^4 x^4) + 6abcx\sqrt{1 - c^2 x^2}(33 - 26c^2 x^2 + 8c^4 x^4) + 9a^2(-11 + 48c^2 x^2 - 48c^4 x^4))}{864c^2}$$

input

```
Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*(b^2*c^2*x^2*(-99 + 39*c^2*x^2 - 8*c^4*x^4) + 6*a*b*c*x*Sqrt[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4) + 9*a^2*(-11 + 48*c^2*x^2 - 48*c^4*x^4 + 16*c^6*x^6) + 6*b*(b*c*x*Sqrt[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4) + 3*a*(-11 + 48*c^2*x^2 - 48*c^4*x^4 + 16*c^6*x^6))*ArcSin[c*x] + 9*b^2*(-11 + 48*c^2*x^2 - 48*c^4*x^4 + 16*c^6*x^6)*ArcSin[c*x]^2))/(864*c^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5182, 5158, 241, 5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5182$$

$$\frac{bd^2 \int (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx}{3c} - \frac{d^2 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}$$

$$\downarrow 5158$$

$$\frac{bd^2 \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{1}{6} bc \int x(1 - c^2 x^2)^2 dx + \frac{1}{6} x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) \right)}{\frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}} \quad \frac{3c}{6c^2}$$

↓ 241

$$\frac{bd^2 \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{6} x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{b(1 - c^2 x^2)^3}{36c} \right)}{\frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}} \quad \frac{3c}{6c^2}$$

↓ 5158

$$\frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{4} bc \int x(1 - c^2 x^2) dx + \frac{1}{4} x(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) + \frac{1}{6} x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) \right)}{\frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}} \quad \frac{3c}{6c^2}$$

↓ 244

$$\frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{4} bc \int (x - c^2 x^3) dx + \frac{1}{4} x(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) + \frac{1}{6} x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) \right)}{\frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}} \quad \frac{3c}{6c^2}$$

↓ 2009

$$\frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{4} x(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{x^2}{2} - \frac{c^2 x^4}{4} \right) \right) + \frac{1}{6} x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) \right)}{\frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}} \quad \frac{3c}{6c^2}$$

↓ 5156

$$\frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{2} bc \int x dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) + \frac{1}{4} x(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) \right)}{\frac{d^2(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{6c^2}} \quad \frac{3c}{6c^2}$$

↓ 15

$$\frac{bd^2 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) - \frac{1}{4} bcx^2 \right) + \frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) - \right)}{3c} \right.}{\left. \frac{d^2(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{6c^2} \right)}$$

↓ 5152

$$\frac{bd^2 \left(\frac{1}{6} x (1-c^2x^2)^{5/2} (a+b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) - \right) \right)}{3c} \right)}{\frac{d^2(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{6c^2}}$$

input `Int[x*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]`

output `-1/6*(d^2*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x])^2)/c^2 + (b*d^2*((b*(1 - c^2*x^2)^3)/(36*c) + (x*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/6 + (5*(-1/4*(b*c*(x^2/2 - (c^2*x^4)/4)) + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (x*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c))))/4)/6)/(3*c)`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_.))^n_/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_.))^n_*Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5158 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_.))^n_*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]`

rule 5182 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_.))^n*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{d^2 a^2 (c^2 x^2 - 1)^3}{6} + d^2 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^3}{6} + \frac{\arcsin(cx) (8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 26c^3 x^3 \sqrt{-c^2 x^2 + 1} + 33cx \sqrt{-c^2 x^2 + 1} + 15 \arcsin(cx))}{144} \right)$
default	$\frac{d^2 a^2 (c^2 x^2 - 1)^3}{6} + d^2 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^3}{6} + \frac{\arcsin(cx) (8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 26c^3 x^3 \sqrt{-c^2 x^2 + 1} + 33cx \sqrt{-c^2 x^2 + 1} + 15 \arcsin(cx))}{144} \right)$
parts	$\frac{d^2 a^2 (c^2 x^2 - 1)^3}{6c^2} + \frac{d^2 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^3}{6} + \frac{\arcsin(cx) (8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 26c^3 x^3 \sqrt{-c^2 x^2 + 1} + 33cx \sqrt{-c^2 x^2 + 1} + 15 \arcsin(cx))}{144} \right)}{c^2}$
orering	$\frac{(728c^8 x^8 - 3251c^6 x^6 + 6466c^4 x^4 - 3177c^2 x^2 + 594)(-c^2 d x^2 + d)^2 (a + b \arcsin(cx))^2}{1728c^2 (cx - 1)(cx + 1)(c^2 x^2 - 1)^2} - \frac{(120c^6 x^6 - 571c^4 x^4 + 1323c^2 x^2)}{c^2}$

```
input int(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

```
output 1/c^2*(1/6*d^2*a^2*(c^2*x^2-1)^3+d^2*b^2*(1/6*arcsin(c*x)^2*(c^2*x^2-1)^3+
1/144*arcsin(c*x)*(8*c^5*x^5*(-c^2*x^2+1)^(1/2)-26*c^3*x^3*(-c^2*x^2+1)^(1
/2)+33*c*x*(-c^2*x^2+1)^(1/2)+15*arcsin(c*x))-5/96*arcsin(c*x)^2-1/108*(c^
2*x^2-1)^3+5/288*(c^2*x^2-1)^2-5/96*c^2*x^2+5/96)+2*d^2*a*b*(1/6*arcsin(c*
x)*c^6*x^6-1/2*c^4*x^4*arcsin(c*x)+1/2*c^2*x^2*arcsin(c*x)-11/96*arcsin(c*
x)+1/36*c^5*x^5*(-c^2*x^2+1)^(1/2)-13/144*c^3*x^3*(-c^2*x^2+1)^(1/2)+11/96
*c*x*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.28

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{8(18a^2 - b^2)c^6 d^2 x^6 - 3(144a^2 - 13b^2)c^4 d^2 x^4 + 9(48a^2 - 11b^2)c^2 d^2 x^2 + 9(16b^2 c^6 d^2 x^6 - 48b^2 c^4 d^2 x^4 - 48b^2 c^2 d^2 x^2 + 16b^2 d^2)}{1728c^2 (cx - 1)(cx + 1)(c^2 x^2 - 1)^2}$$

```
input integrate(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
1/864*(8*(18*a^2 - b^2)*c^6*d^2*x^6 - 3*(144*a^2 - 13*b^2)*c^4*d^2*x^4 + 9
*(48*a^2 - 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 - 48*b^2*c^4*d^2*x^
4 + 48*b^2*c^2*d^2*x^2 - 11*b^2*d^2)*arcsin(c*x)^2 + 18*(16*a*b*c^6*d^2*x^
6 - 48*a*b*c^4*d^2*x^4 + 48*a*b*c^2*d^2*x^2 - 11*a*b*d^2)*arcsin(c*x) + 6*
(8*a*b*c^5*d^2*x^5 - 26*a*b*c^3*d^2*x^3 + 33*a*b*c*d^2*x + (8*b^2*c^5*d^2*
x^5 - 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1)
)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(202) = 404$.

Time = 0.72 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.97

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} \frac{a^2 c^4 d^2 x^6}{6} - \frac{a^2 c^2 d^2 x^4}{2} + \frac{a^2 d^2 x^2}{2} + \frac{abc^4 d^2 x^6 \arcsin(cx)}{3} + \frac{abc^3 d^2 x^5 \sqrt{-c^2 x^2 + 1}}{18} - abc^2 d^2 x^4 \arcsin(cx) - \frac{13abcd^2 x^3 \sqrt{-c^2 x^2 + 1}}{72} \\ \frac{a^2 d^2 x^2}{2} \end{cases}$$

input

```
integrate(x*(-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2,x)
```

output

```
Piecewise((a**2*c**4*d**2*x**6/6 - a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/
2 + a*b*c**4*d**2*x**6*asin(c*x)/3 + a*b*c**3*d**2*x**5*sqrt(-c**2*x**2 +
1)/18 - a*b*c**2*d**2*x**4*asin(c*x) - 13*a*b*c*d**2*x**3*sqrt(-c**2*x**2
+ 1)/72 + a*b*d**2*x**2*asin(c*x) + 11*a*b*d**2*x*sqrt(-c**2*x**2 + 1)/(48
*c) - 11*a*b*d**2*asin(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*asin(c*x)**2/6
- b**2*c**4*d**2*x**6/108 + b**2*c**3*d**2*x**5*sqrt(-c**2*x**2 + 1)*asin
(c*x)/18 - b**2*c**2*d**2*x**4*asin(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/288
- 13*b**2*c*d**2*x**3*sqrt(-c**2*x**2 + 1)*asin(c*x)/72 + b**2*d**2*x**2*
asin(c*x)**2/2 - 11*b**2*d**2*x**2/96 + 11*b**2*d**2*x*sqrt(-c**2*x**2 + 1)
*asin(c*x)/(48*c) - 11*b**2*d**2*asin(c*x)**2/(96*c**2), Ne(c, 0)), (a**2
*d**2*x**2/2, True))
```

Maxima [F]

$$\int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2 x dx$$

input `integrate(x*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
1/6*a^2*c^4*d^2*x^6 - 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arcsin(c*x) + (8
*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2
*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^4*d^2 - 1/8*(8*x^4*arcsin(c
*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsi
n(c*x)/c^5)*c)*a*b*c^2*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arcsin(c*x) + c*
(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*a*b*d^2 + 1/6*(b^2*c^4*d^2*x
^6 - 3*b^2*c^2*d^2*x^4 + 3*b^2*d^2*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c
*x + 1))^2 + integrate(1/3*(b^2*c^5*d^2*x^6 - 3*b^2*c^3*d^2*x^4 + 3*b^2*c*
d^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x
+ 1))/(c^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.76

$$\begin{aligned} & \int x(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\ &= \frac{1}{6} a^2 c^4 d^2 x^6 - \frac{1}{2} a^2 c^2 d^2 x^4 + \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^2 x \arcsin(cx)}{18 c} \\ &+ \frac{(c^2 x^2 - 1)^3 b^2 d^2 \arcsin(cx)^2}{6 c^2} + \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^2 x}{18 c} \\ &+ \frac{5(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d^2 x \arcsin(cx)}{72 c} + \frac{(c^2 x^2 - 1)^3 a b d^2 \arcsin(cx)}{3 c^2} \\ &+ \frac{5(-c^2 x^2 + 1)^{\frac{3}{2}} a b d^2 x}{72 c} + \frac{5 \sqrt{-c^2 x^2 + 1} b^2 d^2 x \arcsin(cx)}{48 c} - \frac{(c^2 x^2 - 1)^3 b^2 d^2}{108 c^2} \\ &+ \frac{5 \sqrt{-c^2 x^2 + 1} a b d^2 x}{48 c} + \frac{5(c^2 x^2 - 1)^2 b^2 d^2}{288 c^2} + \frac{5 b^2 d^2 \arcsin(cx)^2}{96 c^2} \\ &+ \frac{(c^2 x^2 - 1) a^2 d^2}{2 c^2} - \frac{5(c^2 x^2 - 1) b^2 d^2}{96 c^2} + \frac{5 a b d^2 \arcsin(cx)}{48 c^2} - \frac{245 b^2 d^2}{6912 c^2} \end{aligned}$$

output

```
(d**2*(72*asin(c*x)**2*b**2*c**2*x**2 - 36*asin(c*x)**2*b**2 + 72*sqrt(-
c**2*x**2 + 1)*asin(c*x)*b**2*c*x + 48*asin(c*x)*a*b*c**6*x**6 - 144*asin(
c*x)*a*b*c**4*x**4 + 144*asin(c*x)*a*b*c**2*x**2 - 33*asin(c*x)*a*b + 8*sq
rt(- c**2*x**2 + 1)*a*b*c**5*x**5 - 26*sqrt(- c**2*x**2 + 1)*a*b*c**3*x*
*3 + 33*sqrt(- c**2*x**2 + 1)*a*b*c*x + 144*int(asin(c*x)**2*x**5,x)*b**2
*c**6 - 288*int(asin(c*x)**2*x**3,x)*b**2*c**4 + 24*a**2*c**6*x**6 - 72*a*
*2*c**4*x**4 + 72*a**2*c**2*x**2 - 36*b**2*c**2*x**2))/(144*c**2)
```

3.166 $\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$

Optimal result	1506
Mathematica [A] (verified)	1507
Rubi [A] (verified)	1507
Maple [A] (verified)	1510
Fricas [A] (verification not implemented)	1511
Sympy [A] (verification not implemented)	1511
Maxima [B] (verification not implemented)	1512
Giac [A] (verification not implemented)	1514
Mupad [F(-1)]	1515
Reduce [F]	1515

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = -\frac{298}{225}b^2d^2x + \frac{76}{675}b^2c^2d^2x^3 - \frac{2}{125}b^2c^4d^2x^5 + \frac{16bd^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{15c} + \frac{8bd^2(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{45c} + \frac{2bd^2(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{25c} + \frac{8}{15}d^2x(a + b \arcsin(cx))^2 + \frac{4}{15}d^2x(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{1}{5}d^2x(1 - c^2x^2)^2(a + b \arcsin(cx))^2$$

output

```
-298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3-2/125*b^2*c^4*d^2*x^5+16/15*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+8/45*b*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c+2/25*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c+8/15*d^2*x*(a+b*arcsin(c*x))^2+4/15*d^2*x*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+1/5*d^2*x*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.88

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2(225a^2cx(15 - 10c^2x^2 + 3c^4x^4) + 30ab\sqrt{1 - c^2x^2}(149 - 38c^2x^2 + 9c^4x^4) - 2b^2cx(2235 - 190c^2x^2 + 27c^4x^4) + 30b^2(15acx(15 - 10c^2x^2 + 3c^4x^4) + b\sqrt{1 - c^2x^2}(149 - 38c^2x^2 + 9c^4x^4))\arcsin(cx) + 225b^2cx(15 - 10c^2x^2 + 3c^4x^4)\arcsin(cx)^2)}{(3375c)}$$

input

```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*(225*a^2*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 30*a*b*Sqrt[1 - c^2*x^2]
*(149 - 38*c^2*x^2 + 9*c^4*x^4) - 2*b^2*c*x*(2235 - 190*c^2*x^2 + 27*c^4*x
^4) + 30*b*(15*a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)+ b*Sqrt[1 - c^2*x^2]*(
149 - 38*c^2*x^2 + 9*c^4*x^4))*ArcSin[c*x] + 225*b^2*c*x*(15 - 10*c^2*x^2
+ 3*c^4*x^4)*ArcSin[c*x]^2))/(3375*c)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5158, 27, 5158, 5130, 5182, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5158}$$

$$-\frac{2}{5}bcd^2 \int x(1 - c^2x^2)^{3/2} (a + b \arcsin(cx))dx + \frac{4}{5}d \int d(1 - c^2x^2) (a + b \arcsin(cx))^2 dx + \frac{1}{5}d^2x(1 - c^2x^2)^2 (a + b \arcsin(cx))^2$$

$$\downarrow \text{27}$$

$$-\frac{2}{5}bcd^2 \int x(1 - c^2x^2)^{3/2} (a + b \arcsin(cx))dx + \frac{4}{5}d^2 \int (1 - c^2x^2) (a + b \arcsin(cx))^2 dx + \frac{1}{5}d^2x(1 - c^2x^2)^2 (a + b \arcsin(cx))^2$$

↓ 5158

$$-\frac{2}{5}bcd^2 \int x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx +$$

$$\frac{4}{5}d^2 \left(-\frac{2}{3}bc \int x\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{2}{3} \int (a+b\arcsin(cx))^2 dx + \frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 \right)$$

$$+\frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arcsin(cx))^2$$

↓ 5130

$$\frac{4}{5}d^2 \left(\frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2bc \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{3}bc \int x\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 \right)$$

$$+\frac{2}{5}bcd^2 \int x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx + \frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arcsin(cx))^2$$

↓ 5182

$$\frac{4}{5}d^2 \left(\frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2} \right) \right) - \frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^2} \right) \right)$$

$$+\frac{2}{5}bcd^2 \left(\frac{b \int (1-c^2x^2)^2 dx}{5c} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^2} \right) + \frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arcsin(cx))^2$$

↓ 24

$$\frac{4}{5}d^2 \left(-\frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^2} \right) + \frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2bc \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) \right)$$

$$+\frac{2}{5}bcd^2 \left(\frac{b \int (1-c^2x^2)^2 dx}{5c} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^2} \right) + \frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arcsin(cx))^2$$

↓ 210

$$\frac{4}{5}d^2 \left(-\frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^2} \right) + \frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2bc \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) \right)$$

$$+\frac{2}{5}bcd^2 \left(\frac{b \int (c^4x^4 - 2c^2x^2 + 1) dx}{5c} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^2} \right) + \frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arcsin(cx))^2$$

↓ 2009

$$\frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \frac{4}{5}d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3}\left(x(a+b\arcsin(cx))^2 - 2bc\left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2}\right)\right)\right) + \frac{2}{5}bcd^2\left(\frac{b\left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x\right)}{5c} - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{5c^2}\right)$$

input `Int[(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]`

output `(d^2*x*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*d^2*((b*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(5*c) - ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^2)))/5 + (4*d^2*((x*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*((b*(x - (c^2*x^3)/3))/(3*c) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2)))/3))/5`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 210 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5130 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^n, x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*(a + b*ArcSin[c*x])^(n-1)/Sqrt[1 - c^2*x^2]], x, x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x],
x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.26

method	result
derivativedivides	$d^2 a^2 \left(\frac{1}{5} c^5 x^5 - \frac{2}{3} c^3 x^3 + cx \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{25} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15)}{375} \right)$
default	$d^2 a^2 \left(\frac{1}{5} c^5 x^5 - \frac{2}{3} c^3 x^3 + cx \right) + d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{25} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15)}{375} \right)$
parts	$d^2 a^2 \left(\frac{1}{5} c^4 x^5 - \frac{2}{3} c^2 x^3 + x \right) + \frac{d^2 b^2 \left(\frac{\arcsin(cx)^2 (3c^4 x^4 - 10c^2 x^2 + 15) cx}{15} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}}{25} - \frac{2(3c^4 x^4 - 10c^2 x^2 + 15)}{375} \right)}{c}$
orering	$\frac{x(1647c^6 x^6 - 8677c^4 x^4 + 51845c^2 x^2 - 3375)(-c^2 d x^2 + d)^2 (a + b \arcsin(cx))^2}{3375(cx - 1)(cx + 1)(c^2 x^2 - 1)^2} - \frac{(324c^6 x^6 - 2035c^4 x^4 + 18450c^2 x^2 - 22500)}{3375}$

input

```
int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/c*(d^2*a^2*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b^2*(1/15*arcsin(c*x)^2*(3*c^4*x^4-10*c^2*x^2+15)*c*x+2/25*arcsin(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)-2/375*(3*c^4*x^4-10*c^2*x^2+15)*c*x-8/45*arcsin(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+8/135*(c^2*x^2-3)*c*x-16/15*c*x+16/15*arcsin(c*x)*(-c^2*x^2+1)^(1/2))+2*d^2*a*b*(1/5*c^5*x^5*arcsin(c*x)-2/3*c^3*x^3*arcsin(c*x)+c*x*arcsin(c*x)+149/225*(-c^2*x^2+1)^(1/2)-38/225*c^2*x^2*(-c^2*x^2+1)^(1/2)+1/25*c^4*x^4*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{27(25a^2 - 2b^2)c^5 d^2 x^5 - 10(225a^2 - 38b^2)c^3 d^2 x^3 + 15(225a^2 - 298b^2)cd^2 x + 225(3b^2 c^5 d^2 x^5 - 10b^2 c^3 d^2 x^3 + 15b^2 c d^2 x) \arcsin(cx) + 30(9a^2 b^2 c^4 d^2 x^4 - 38a^2 b^2 c^2 d^2 x^2 + 149a^2 b^2 d^2 + (9b^2 c^4 d^2 x^4 - 38b^2 c^2 d^2 x^2 + 149b^2 d^2) \arcsin(cx)) \sqrt{-c^2 x^2 + 1}}{c}$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
1/3375*(27*(25*a^2 - 2*b^2)*c^5*d^2*x^5 - 10*(225*a^2 - 38*b^2)*c^3*d^2*x^3 + 15*(225*a^2 - 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 - 10*b^2*c^3*d^2*x^3 + 15*b^2*c*d^2*x)*arcsin(c*x)^2 + 450*(3*a*b*c^5*d^2*x^5 - 10*a*b*c^3*d^2*x^3 + 15*a*b*c*d^2*x)*arcsin(c*x) + 30*(9*a*b*c^4*d^2*x^4 - 38*a*b*c^2*d^2*x^2 + 149*a*b*d^2 + (9*b^2*c^4*d^2*x^4 - 38*b^2*c^2*d^2*x^2 + 149*b^2*d^2)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.78

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} \frac{a^2 c^4 d^2 x^5}{5} - \frac{2a^2 c^2 d^2 x^3}{3} + a^2 d^2 x + \frac{2abc^4 d^2 x^5 \arcsin(cx)}{5} + \frac{2abc^3 d^2 x^4 \sqrt{-c^2 x^2 + 1}}{25} - \frac{4abc^2 d^2 x^3 \arcsin(cx)}{3} - \frac{76abcd^2 x^2 \sqrt{-c^2 x^2 + 1}}{225} \\ a^2 d^2 x \end{cases}$$

input

```
integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2,x)
```

output

```
Piecewise((a**2*c**4*d**2*x**5/5 - 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
2*a*b*c**4*d**2*x**5*asin(c*x)/5 + 2*a*b*c**3*d**2*x**4*sqrt(-c**2*x**2 +
1)/25 - 4*a*b*c**2*d**2*x**3*asin(c*x)/3 - 76*a*b*c*d**2*x**2*sqrt(-c**2*
x**2 + 1)/225 + 2*a*b*d**2*x*asin(c*x) + 298*a*b*d**2*sqrt(-c**2*x**2 + 1)
/(225*c) + b**2*c**4*d**2*x**5*asin(c*x)**2/5 - 2*b**2*c**4*d**2*x**5/125
+ 2*b**2*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)/25 - 2*b**2*c**2*d
**2*x**3*asin(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 - 76*b**2*c*d**2*x**2*
sqrt(-c**2*x**2 + 1)*asin(c*x)/225 + b**2*d**2*x*asin(c*x)**2 - 298*b**2*d
**2*x/225 + 298*b**2*d**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(225*c), Ne(c, 0)
), (a**2*d**2*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(193) = 386$.

Time = 0.13 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.12

$$\begin{aligned}
& \int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
&= \frac{1}{5} b^2 c^4 d^2 x^5 \arcsin(cx)^2 + \frac{1}{5} a^2 c^4 d^2 x^5 - \frac{2}{3} b^2 c^2 d^2 x^3 \arcsin(cx)^2 \\
&+ \frac{2}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) abc^4 d^2 \\
&+ \frac{2}{1125} \left(15 \left(\frac{3 \sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4 \sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8 \sqrt{-c^2 x^2 + 1}}{c^6} \right) c \arcsin(cx) - \frac{9 c^4 x^5 + 20 c^2 x^3 +}{c^4} \right. \\
&- \frac{2}{3} a^2 c^2 d^2 x^3 - \frac{4}{9} \left(3 x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) abc^2 d^2 \\
&- \frac{4}{27} \left(3 c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2 \sqrt{-c^2 x^2 + 1}}{c^4} \right) \arcsin(cx) - \frac{c^2 x^3 + 6 x}{c^2} \right) b^2 c^2 d^2 \\
&+ b^2 d^2 x \arcsin(cx)^2 - 2 b^2 d^2 \left(x - \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{c} \right) \\
&+ a^2 d^2 x + \frac{2 (cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1}) abd^2}{c}
\end{aligned}$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

$$\begin{aligned}
& 1/5*b^2*c^4*d^2*x^5*\arcsin(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 - 2/3*b^2*c^2*d^2* \\
& x^3*\arcsin(c*x)^2 + 2/75*(15*x^5*\arcsin(c*x) + (3*\sqrt{-c^2*x^2 + 1})*x^4/c \\
& ^2 + 4*\sqrt{-c^2*x^2 + 1}*x^2/c^4 + 8*\sqrt{-c^2*x^2 + 1}/c^6)*c)*a*b*c^4*d \\
& ^2 + 2/1125*(15*(3*\sqrt{-c^2*x^2 + 1})*x^4/c^2 + 4*\sqrt{-c^2*x^2 + 1}*x^2/c \\
& ^4 + 8*\sqrt{-c^2*x^2 + 1}/c^6)*c*\arcsin(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 1 \\
& 20*x)/c^4)*b^2*c^4*d^2 - 2/3*a^2*c^2*d^2*x^3 - 4/9*(3*x^3*\arcsin(c*x) + c* \\
& (\sqrt{-c^2*x^2 + 1})*x^2/c^2 + 2*\sqrt{-c^2*x^2 + 1}/c^4))*a*b*c^2*d^2 - 4/2 \\
& 7*(3*c*(\sqrt{-c^2*x^2 + 1})*x^2/c^2 + 2*\sqrt{-c^2*x^2 + 1}/c^4)*\arcsin(c*x) \\
& - (c^2*x^3 + 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*\arcsin(c*x)^2 - 2*b^2*d^2* \\
& (x - \sqrt{-c^2*x^2 + 1})*\arcsin(c*x)/c + a^2*d^2*x + 2*(c*x*\arcsin(c*x) + \\
& \sqrt{-c^2*x^2 + 1})*a*b*d^2/c
\end{aligned}$$

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.71

$$\begin{aligned}
\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = & \frac{1}{5} a^2 c^4 d^2 x^5 - \frac{2}{3} a^2 c^2 d^2 x^3 \\
& + \frac{1}{5} (c^2 x^2 - 1)^2 b^2 d^2 x \arcsin(cx)^2 \\
& + \frac{2}{5} (c^2 x^2 - 1)^2 a b d^2 x \arcsin(cx) \\
& - \frac{4}{15} (c^2 x^2 - 1) b^2 d^2 x \arcsin(cx)^2 \\
& - \frac{2}{125} (c^2 x^2 - 1)^2 b^2 d^2 x \\
& - \frac{8}{15} (c^2 x^2 - 1) a b d^2 x \arcsin(cx) \\
& + \frac{8}{15} b^2 d^2 x \arcsin(cx)^2 \\
& + \frac{2(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^2 \arcsin(cx)}{25 c} \\
& + \frac{272}{3375} (c^2 x^2 - 1) b^2 d^2 x + \frac{16}{15} a b d^2 x \arcsin(cx) \\
& + \frac{2(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^2}{25 c} \\
& + \frac{8(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d^2 \arcsin(cx)}{45 c} + a^2 d^2 x \\
& - \frac{4144}{3375} b^2 d^2 x + \frac{8(-c^2 x^2 + 1)^{\frac{3}{2}} a b d^2}{45 c} \\
& + \frac{16 \sqrt{-c^2 x^2 + 1} b^2 d^2 \arcsin(cx)}{15 c} \\
& + \frac{16 \sqrt{-c^2 x^2 + 1} a b d^2}{15 c}
\end{aligned}$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
1/5*a^2*c^4*d^2*x^5 - 2/3*a^2*c^2*d^2*x^3 + 1/5*(c^2*x^2 - 1)^2*b^2*d^2*x*
arcsin(c*x)^2 + 2/5*(c^2*x^2 - 1)^2*a*b*d^2*x*arcsin(c*x) - 4/15*(c^2*x^2
- 1)*b^2*d^2*x*arcsin(c*x)^2 - 2/125*(c^2*x^2 - 1)^2*b^2*d^2*x - 8/15*(c^2
*x^2 - 1)*a*b*d^2*x*arcsin(c*x) + 8/15*b^2*d^2*x*arcsin(c*x)^2 + 2/25*(c^2
*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^2*arcsin(c*x)/c + 272/3375*(c^2*x^2 -
1)*b^2*d^2*x + 16/15*a*b*d^2*x*arcsin(c*x) + 2/25*(c^2*x^2 - 1)^2*sqrt(-c
^2*x^2 + 1)*a*b*d^2/c + 8/45*(-c^2*x^2 + 1)^(3/2)*b^2*d^2*arcsin(c*x)/c +
a^2*d^2*x - 4144/3375*b^2*d^2*x + 8/45*(-c^2*x^2 + 1)^(3/2)*a*b*d^2/c + 16
/15*sqrt(-c^2*x^2 + 1)*b^2*d^2*arcsin(c*x)/c + 16/15*sqrt(-c^2*x^2 + 1)*a*
b*d^2/c
```

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 (d - c^2 dx^2)^2 dx$$

input

```
int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output

```
int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^2 (225 a \sin(cx)^2 b^2 cx + 450 \sqrt{-c^2 x^2 + 1} a \sin(cx) b^2 + 90 a \sin(cx) a b c^5 x^5 - 300 a \sin(cx) a b c^3 x^3 + 450 a \sin(cx) a b c x + 450 a^2 b^2 c^3 x^3 + 450 a^2 b^2 c x + 450 a^2 b^2 c^3 x^3 + 450 a^2 b^2 c x)}{c^5}$$

input

```
int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2,x)
```


output

```
(d**2*(225*asin(c*x)**2*b**2*c*x + 450*sqrt(-c**2*x**2 + 1)*asin(c*x)*b*  
*2 + 90*asin(c*x)*a*b*c**5*x**5 - 300*asin(c*x)*a*b*c**3*x**3 + 450*asin(c  
*x)*a*b*c*x + 18*sqrt(-c**2*x**2 + 1)*a*b*c**4*x**4 - 76*sqrt(-c**2*x*  
*2 + 1)*a*b*c**2*x**2 + 298*sqrt(-c**2*x**2 + 1)*a*b + 225*int(asin(c*x)  
**2*x**4,x)*b**2*c**5 - 450*int(asin(c*x)**2*x**2,x)*b**2*c**3 + 45*a**2*c  
**5*x**5 - 150*a**2*c**3*x**3 + 225*a**2*c*x - 450*b**2*c*x))/(225*c)
```

$$3.167 \quad \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx$$

Optimal result	1517
Mathematica [A] (verified)	1518
Rubi [A] (verified)	1519
Maple [A] (verified)	1526
Fricas [F]	1527
Sympy [F]	1528
Maxima [F]	1528
Giac [F(-2)]	1529
Mupad [F(-1)]	1529
Reduce [F]	1529

Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = & \frac{11}{32} b^2 c^2 d^2 x^2 - \frac{1}{32} b^2 d^2 (1 - c^2 x^2)^2 \\ & - \frac{11}{16} b c d^2 x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \\ & - \frac{1}{8} b c d^2 x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \\ & - \frac{11}{32} d^2 (a + b \arcsin(cx))^2 \\ & + \frac{1}{2} d^2 (1 - c^2 x^2) (a + b \arcsin(cx))^2 \\ & + \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\ & - \frac{d^2 (a + b \arcsin(cx))^3}{3b} \\ & + d^2 (a + b \arcsin(cx))^2 \log(1 - e^{2i \arcsin(cx)}) \\ & - i b d^2 (a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \\ & + \frac{1}{2} b^2 d^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)}) \end{aligned}$$

output

```

11/32*b^2*c^2*d^2*x^2-1/32*b^2*d^2*(-c^2*x^2+1)^2-11/16*b*c*d^2*x*(-c^2*x^
2+1)^(1/2)*(a+b*arcsin(c*x))-1/8*b*c*d^2*x*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(
c*x))-11/32*d^2*(a+b*arcsin(c*x))^2+1/2*d^2*(-c^2*x^2+1)*(a+b*arcsin(c*x))
^2+1/4*d^2*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2-1/3*I*d^2*(a+b*arcsin(c*x))^
3/b+d^2*(a+b*arcsin(c*x))^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*b*d^2*(a+
b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*d^2*polylog
(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)

```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.34

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = \frac{1}{768} d^2 \left(-32ib^2\pi^3 - 768a^2c^2x^2 + 192a^2c^4x^4 \right. \\
- 624abcx\sqrt{1 - c^2x^2} + 96abc^3x^3\sqrt{1 - c^2x^2} \\
- 1536abc^2x^2 \arcsin(cx) + 384abc^4x^4 \arcsin(cx) \\
- 768iab \arcsin(cx)^2 + 256ib^2 \arcsin(cx)^3 \\
+ 1248ab \arctan\left(\frac{cx}{-1 + \sqrt{1 - c^2x^2}}\right) \\
- 144b^2 \cos(2 \arcsin(cx)) \\
+ 288b^2 \arcsin(cx)^2 \cos(2 \arcsin(cx)) \\
- 3b^2 \cos(4 \arcsin(cx)) \\
+ 24b^2 \arcsin(cx)^2 \cos(4 \arcsin(cx)) \\
+ 768b^2 \arcsin(cx)^2 \log(1 - e^{-2i \arcsin(cx)}) \\
+ 1536ab \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) \\
+ 768a^2 \log(cx) \\
+ 768ib^2 \arcsin(cx) \text{PolyLog}(2, e^{-2i \arcsin(cx)}) \\
- 768iab \text{PolyLog}(2, e^{2i \arcsin(cx)}) \\
+ 384b^2 \text{PolyLog}(3, e^{-2i \arcsin(cx)}) \\
- 288b^2 \arcsin(cx) \sin(2 \arcsin(cx)) \\
\left. - 12b^2 \arcsin(cx) \sin(4 \arcsin(cx)) \right)$$

input

```

Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x,x]

```

output

```
(d^2*((-32*I)*b^2*Pi^3 - 768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 - 624*a*b*c*x*Sqrt[1 - c^2*x^2] + 96*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 1536*a*b*c^2*x^2*ArcSin[c*x] + 384*a*b*c^4*x^4*ArcSin[c*x] - (768*I)*a*b*ArcSin[c*x]^2 + (256*I)*b^2*ArcSin[c*x]^3 + 1248*a*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) - 144*b^2*Cos[2*ArcSin[c*x]] + 288*b^2*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] - 3*b^2*Cos[4*ArcSin[c*x]] + 24*b^2*ArcSin[c*x]^2*Cos[4*ArcSin[c*x]] + 768*b^2*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] + 1536*a*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 768*a^2*Log[c*x] + (768*I)*b^2*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] - (768*I)*a*b*PolyLog[2, E^((2*I)*ArcSin[c*x])] + 384*b^2*PolyLog[3, E^((-2*I)*ArcSin[c*x])] - 288*b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]] - 12*b^2*ArcSin[c*x]*Sin[4*ArcSin[c*x]]))/768
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.25, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {5202, 27, 5158, 244, 2009, 5156, 15, 5152, 5202, 5136, 3042, 25, 4200, 25, 2620, 3011, 2720, 5156, 15, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx$$

$$\downarrow \text{5202}$$

$$-\frac{1}{2}bcd^2 \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + d \int \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx + \frac{1}{4}d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2$$

$$\downarrow \text{27}$$

$$-\frac{1}{2}bcd^2 \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + d^2 \int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx + \frac{1}{4}d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2$$

$$\downarrow \text{5158}$$

$$\begin{aligned}
& -\frac{1}{2}bcd^2\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx-\frac{1}{4}bc\int x(1-c^2x^2)dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))\right)+ \\
& \quad d^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow 244 \\
& \quad d^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x}dx- \\
& \frac{1}{2}bcd^2\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx-\frac{1}{4}bc\int(x-c^2x^3)dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))\right)+ \\
& \quad \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow 2009 \\
& \quad d^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x}dx- \\
& \frac{1}{2}bcd^2\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))-\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)+ \\
& \quad \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow 5156 \\
& \quad d^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x}dx- \\
& \frac{1}{2}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx-\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))\right)+ \\
& \quad \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow 15 \\
& \quad d^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x}dx- \\
& \frac{1}{2}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))-\frac{1}{4}bcx^2\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))\right)+ \\
& \quad \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 \\
& \quad \downarrow 5152 \\
& \quad d^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2- \\
& \frac{1}{2}bcd^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))+\frac{(a+b\arcsin(cx))^2}{4bc}-\frac{1}{4}bcx^2\right)\right) \\
& \quad \downarrow 5202
\end{aligned}$$

$$d^2 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \int \frac{(a+b\arcsin(cx))^2}{x} dx + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))^2 \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bcd^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4}bcx^2 \right) \right)$$

↓ 5136

$$d^2 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{cx} d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))^2 \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bcd^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4}bcx^2 \right) \right)$$

↓ 3042

$$d^2 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \int -(a+b\arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))^2 \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bcd^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4}bcx^2 \right) \right)$$

↓ 25

$$d^2 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - \int (a+b\arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))^2 \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bcd^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4}bcx^2 \right) \right)$$

↓ 4200

$$d^2 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + 2i \int -\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))^2}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))^2 \right) + \frac{1}{4}d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bcd^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4}bcx^2 \right) \right)$$

↓ 25

$$\begin{aligned}
& d^2 \left(-bc \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx - 2i \int \frac{e^{2i \arcsin(cx)} (a+b \arcsin(cx))^2}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2} (1-c^2x^2) (a+b \arcsin(cx))^2 - \right. \\
& \quad \left. \frac{1}{4} d^2 (1-c^2x^2)^2 (a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) + \frac{(a+b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcd^2 \right) \right) \right) \\
& \quad \downarrow \text{2620}
\end{aligned}$$

$$\begin{aligned}
& d^2 \left(-bc \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a+b \arcsin(cx))^2 - ib \int (a+b \arcsin(cx))^2 dx \right) - \right. \\
& \quad \left. \frac{1}{4} d^2 (1-c^2x^2)^2 (a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) + \frac{(a+b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcd^2 \right) \right) \right) \\
& \quad \downarrow \text{3011}
\end{aligned}$$

$$\begin{aligned}
& d^2 \left(-bc \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a+b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \text{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) \right) \right) - \right. \\
& \quad \left. \frac{1}{4} d^2 (1-c^2x^2)^2 (a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) + \frac{(a+b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcd^2 \right) \right) \right) \\
& \quad \downarrow \text{2720}
\end{aligned}$$

$$\begin{aligned}
& d^2 \left(-bc \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a+b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \text{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) \right) \right) - \right. \\
& \quad \left. \frac{1}{4} d^2 (1-c^2x^2)^2 (a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) + \frac{(a+b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcd^2 \right) \right) \right) \\
& \quad \downarrow \text{5156}
\end{aligned}$$

$$\begin{aligned}
& d^2 \left(-bc \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2} bc \int x dx + \frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) \right) - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a+b \arcsin(cx))^2 - \right. \right. \\
& \quad \left. \left. \frac{1}{4} d^2 (1-c^2x^2)^2 (a+b \arcsin(cx))^2 - \right. \right. \\
& \left. \left. \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1-c^2x^2)^{3/2} (a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) + \frac{(a+b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcd^2 \right) \right) \right) \right) \\
& \quad \downarrow \text{15}
\end{aligned}$$

$$d^2 \left(-bc \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) - \frac{1}{4} bcx^2 \right) - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 - \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) \right) \right)$$

↓ 5152

$$d^2 \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{4} b \right) \right) - \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 - \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) \right)$$

↓ 7143

$$d^2 \left(\frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx))^2 - bc \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - \frac{1}{4} d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 - \frac{1}{2} bcd^2 \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) \right) \right)$$

input `Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x,x]`

output `(d^2*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2/4 - (b*c*d^2*(-1/4*(b*c*(x^2/2 - (c^2*x^4)/4)) + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)))/4)/2 + d^2*(((1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/2 - ((I/3)*(a + b*ArcSin[c*x])^3)/b - b*c*(-1/4*(b*c*x^2) + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)) - (2*I)*((I/2)*(a + b*ArcSin[c*x])^2*Log[1 - E^((2*I)*ArcSin[c*x])] - I*b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, E^((2*I)*ArcSin[c*x]]))/4)))/4))`

Defintions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 25 $\text{Int}[-(Fx_), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \ \text{Int}[Fx, x], x]$
- rule 27 $\text{Int}[(a_)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[Fx, (b_)(Gx_)] \text{ ; FreeQ}[b, x]$
- rule 244 $\text{Int}[((c_.)(x_))^{(m_.)}*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 2620 $\text{Int}[(((F_)^{((g_.)((e_.) + (f_.)(x_)))})^{(n_.)}*((c_.) + (d_.)(x_))^{(m_.)})/((a_) + (b_.)((F_)^{((g_.)((e_.) + (f_.)(x_)))})^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m/(b*f*g*n*\text{Log}[F])]*\text{Log}[1 + b*((F^{(g*(e + f*x)))})^n/a], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \ \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + b*((F^{(g*(e + f*x)))})^n/a], x], x] \text{ ; FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \ \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] \text{ ; FunctionOfExponentialQ}[u, x] \ \&\& \ \text{!MatchQ}[u, (w_)*((a_.)(v_)^{(n_)})^{(m_)}] \text{ ; FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ \text{!MatchQ}[u, E^{((c_.)((a_.) + (b_.)*x))}*(F_)[v_]] \text{ ; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_.)((F_)^{((c_.)((a_.) + (b_.)(x_)))})^{(n_.)}]]*((f_.) + (g_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \ \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n], x], x] \text{ ; FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5158 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]`

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.55

method	result
parts	$d^2 a^2 \left(\frac{c^4 x^4}{4} - c^2 x^2 + \ln(x) \right) + d^2 b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \arcsin(cx)^2 \ln(1 + icx + \sqrt{-c^2 x^2 - 1}) \right)$
derivativedivides	$d^2 a^2 \left(\frac{c^4 x^4}{4} - c^2 x^2 + \ln(cx) \right) + d^2 b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \arcsin(cx)^2 \ln(1 + icx + \sqrt{-c^2 x^2 - 1}) \right)$
default	$d^2 a^2 \left(\frac{c^4 x^4}{4} - c^2 x^2 + \ln(cx) \right) + d^2 b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \arcsin(cx)^2 \ln(1 + icx + \sqrt{-c^2 x^2 - 1}) \right)$

input

```
int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output

```
d^2*a^2*(1/4*c^4*x^4-c^2*x^2+ln(x))+d^2*b^2*(-1/3*I*arcsin(c*x)^3+arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+1/256*(8*arcsin(c*x)^2-1)*cos(4*arcsin(c*x))-1/64*arcsin(c*x)*sin(4*arcsin(c*x))+3/16*(2*arcsin(c*x)^2-1)*cos(2*arcsin(c*x))-3/8*arcsin(c*x)*sin(2*arcsin(c*x)))+2*d^2*a*b*(-1/2*I*arcsin(c*x)^2+arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+1/32*arcsin(c*x)*cos(4*arcsin(c*x))-1/128*sin(4*arcsin(c*x))+3/8*arcsin(c*x)*cos(2*arcsin(c*x))-3/16*sin(2*arcsin(c*x)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x,x, algorithm="fricas")
```

output

```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = d^2 \left(\int \frac{a^2}{x} dx + \int (-2a^2 c^2 x) dx + \int a^2 c^4 x^3 dx \right. \\ \left. + \int \frac{b^2 \arcsin^2(cx)}{x} dx + \int \frac{2ab \arcsin(cx)}{x} dx \right. \\ \left. + \int (-2b^2 c^2 x \arcsin^2(cx)) dx \right. \\ \left. + \int b^2 c^4 x^3 \arcsin^2(cx) dx \right. \\ \left. + \int (-4abc^2 x \arcsin(cx)) dx \right. \\ \left. + \int 2abc^4 x^3 \arcsin(cx) dx \right)$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2/x,x)`

output `d**2*(Integral(a**2/x, x) + Integral(-2*a**2*c**2*x, x) + Integral(a**2*c**4*x**3, x) + Integral(b**2*asin(c*x)**2/x, x) + Integral(2*a*b*asin(c*x)/x, x) + Integral(-2*b**2*c**2*x*asin(c*x)**2, x) + Integral(b**2*c**4*x**3*asin(c*x)**2, x) + Integral(-4*a*b*c**2*x*asin(c*x), x) + Integral(2*a*b*c**4*x**3*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output `1/4*a^2*c^4*d^2*x^4 - a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^2}{x} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x} dx$$

$$= \frac{d^2 \left(-16 a \sin(cx)^2 b^2 c^2 x^2 + 8 a \sin(cx)^2 b^2 - 16 \sqrt{-c^2 x^2 + 1} a \sin(cx) b^2 c x + 8 a \sin(cx) a b c^4 x^4 - 32 a \sin(cx) \right)}{x^2}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2/x,x)`

output

```
(d**2*( - 16*asin(c*x)**2*b**2*c**2*x**2 + 8*asin(c*x)**2*b**2 - 16*sqrt(
- c**2*x**2 + 1)*asin(c*x)*b**2*c*x + 8*asin(c*x)*a*b*c**4*x**4 - 32*asin(
c*x)*a*b*c**2*x**2 + 13*asin(c*x)*a*b + 2*sqrt( - c**2*x**2 + 1)*a*b*c**3*
x**3 - 13*sqrt( - c**2*x**2 + 1)*a*b*c*x + 32*int(asin(c*x)/x,x)*a*b + 16*
int(asin(c*x)**2/x,x)*b**2 + 16*int(asin(c*x)**2*x**3,x)*b**2*c**4 + 16*log(x)*a**2 + 4*a**2*c**4*x**4 - 16*a**2*c**2*x**2 + 8*b**2*c**2*x**2))/16
```

3.168 $\int \frac{(d-c^2 dx^2)^2 (a+b \arcsin(cx))^2}{x^2} dx$

Optimal result	1531
Mathematica [A] (verified)	1532
Rubi [A] (verified)	1533
Maple [A] (verified)	1539
Fricas [F]	1539
Sympy [F]	1540
Maxima [F]	1540
Giac [F(-2)]	1541
Mupad [F(-1)]	1541
Reduce [F]	1542

Optimal result

Integrand size = 27, antiderivative size = 249

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx = \frac{32}{9} b^2 c^2 d^2 x - \frac{2}{27} b^2 c^4 d^2 x^3$$

$$- \frac{10}{3} bcd^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))$$

$$- \frac{2}{9} bcd^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))$$

$$- \frac{8}{3} c^2 d^2 x (a + b \arcsin(cx))^2$$

$$- \frac{4}{3} c^2 d^2 x (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$- \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x}$$

$$- 4bcd^2 (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})$$

$$+ 2ib^2 cd^2 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})$$

$$- 2ib^2 cd^2 \operatorname{PolyLog}(2, e^{i \arcsin(cx)})$$

output

```
32/9*b^2*c^2*d^2*x-2/27*b^2*c^4*d^2*x^3-10/3*b*c*d^2*(-c^2*x^2+1)^(1/2)*(a
+b*arcsin(c*x))-2/9*b*c*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))-8/3*c^2*d
^2*x*(a+b*arcsin(c*x))^2-4/3*c^2*d^2*x*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2-d
^2*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/x-4*b*c*d^2*(a+b*arcsin(c*x))*arctanh
(I*c*x+(-c^2*x^2+1)^(1/2))+2*I*b^2*c*d^2*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/
2))-2*I*b^2*c*d^2*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.29

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx$$

$$= \frac{1}{54} d^2 \left(-\frac{54a^2}{x} - 108a^2 c^2 x + 18a^2 c^4 x^3 + 12abc \sqrt{1 - c^2 x^2} (2 + c^2 x^2) \right. \\ \left. + 36abc^4 x^3 \arcsin(cx) - 189b^2 c \sqrt{1 - c^2 x^2} \arcsin(cx) \right. \\ \left. - 216abc \left(\sqrt{1 - c^2 x^2} + cx \arcsin(cx) \right) - 108b^2 c^2 x (-2 + \arcsin(cx)^2) \right. \\ \left. + 2b^2 c^2 x (-2(6 + c^2 x^2) + 9c^2 x^2 \arcsin(cx)^2) \right. \\ \left. - \frac{108ab(\arcsin(cx) + cx \operatorname{arctanh}(\sqrt{1 - c^2 x^2}))}{x} - 3b^2 c \arcsin(cx) \cos(3 \arcsin(cx)) \right. \\ \left. - \frac{54b^2 \arcsin(cx) (\arcsin(cx) + 2cx (-\log(1 - e^{i \arcsin(cx)}) + \log(1 + e^{i \arcsin(cx)})))}{x} \right. \\ \left. + 108ib^2 c \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - 108ib^2 c \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)$$

input

```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```
(d^2*((-54*a^2)/x - 108*a^2*c^2*x + 18*a^2*c^4*x^3 + 12*a*b*c*Sqrt[1 - c^2
*x^2]*(2 + c^2*x^2) + 36*a*b*c^4*x^3*ArcSin[c*x] - 189*b^2*c*Sqrt[1 - c^2*
x^2]*ArcSin[c*x] - 216*a*b*c*(Sqrt[1 - c^2*x^2] + c*x*ArcSin[c*x]) - 108*b
^2*c^2*x*(-2 + ArcSin[c*x]^2) + 2*b^2*c^2*x*(-2*(6 + c^2*x^2) + 9*c^2*x^2*
ArcSin[c*x]^2) - (108*a*b*(ArcSin[c*x] + c*x*ArcTanh[Sqrt[1 - c^2*x^2]]))/
x - 3*b^2*c*ArcSin[c*x]*Cos[3*ArcSin[c*x]] - (54*b^2*ArcSin[c*x]*(ArcSin[c
*x] + 2*c*x*(-Log[1 - E^(I*ArcSin[c*x]])] + Log[1 + E^(I*ArcSin[c*x]])))/x
+ (108*I)*b^2*c*PolyLog[2, -E^(I*ArcSin[c*x])] - (108*I)*b^2*c*PolyLog[2,
E^(I*ArcSin[c*x])])/54
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.26, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {5200, 27, 5158, 5130, 5182, 24, 2009, 5202, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx$$

$$\downarrow \text{5200}$$

$$2bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - 4c^2 d \int d(1 - c^2 x^2) (a + b \arcsin(cx))^2 dx - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{27}$$

$$2bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - 4c^2 d^2 \int (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5158}$$

$$2bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - 4c^2 d^2 \left(-\frac{2}{3} bc \int x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{2}{3} \int (a + b \arcsin(cx))^2 dx + \frac{1}{3} x(1 - c^2 x^2) (a + b \arcsin(cx))^2 \right) - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5130}$$

$$-4c^2 d^2 \left(\frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \int \frac{x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx \right) - \frac{2}{3} bc \int x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{3} \int (a + b \arcsin(cx))^2 dx \right) - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5182}$$

$$-4c^2d^2\left(\frac{2}{3}\left(x(a+b\arcsin(cx))^2-2bc\left(\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2}\right)\right)\right)-\frac{2}{3}bc\left(\frac{b\int(1-c^2x^2)dx}{3c}-\frac{2bcd^2\int\frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x}dx-\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x}}{3c}\right)$$

↓ 24

$$-4c^2d^2\left(-\frac{2}{3}bc\left(\frac{b\int(1-c^2x^2)dx}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^2}\right)+\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2+\frac{2}{3}\left(\frac{b\int(1-c^2x^2)dx}{3c}-\frac{2bcd^2\int\frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x}dx-\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x}}{3c}\right)\right)$$

↓ 2009

$$2bcd^2\int\frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x}dx-\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x}-4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2+\frac{2}{3}\left(x(a+b\arcsin(cx))^2-2bc\left(\frac{bx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2}\right)\right)\right)$$

↓ 5202

$$2bcd^2\left(\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x}dx-\frac{1}{3}bc\int(1-c^2x^2)dx+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))\right)-\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x}-4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2+\frac{2}{3}\left(x(a+b\arcsin(cx))^2-2bc\left(\frac{bx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2}\right)\right)\right)$$

↓ 2009

$$2bcd^2\left(\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x}dx+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))-\frac{1}{3}bc\left(x-\frac{c^2x^3}{3}\right)\right)-\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x}$$

$$4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2+\frac{2}{3}\left(x(a+b\arcsin(cx))^2-2bc\left(\frac{bx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2}\right)\right)\right)$$

↓ 5198

$$2bcd^2 \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int 1 dx + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - \frac{1}{3} \frac{d^2(1-c^2x^2)^2 (a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right) \right)$$

↓ 24

$$2bcd^2 \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - \frac{1}{3}bc \left(x - \frac{c^2x}{3} \right) \frac{d^2(1-c^2x^2)^2 (a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right) \right)$$

↓ 5218

$$2bcd^2 \left(\int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx) + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - \frac{1}{3}bc \frac{d^2(1-c^2x^2)^2 (a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right) \right)$$

↓ 3042

$$2bcd^2 \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - \frac{1}{3}bc \frac{d^2(1-c^2x^2)^2 (a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right) \right)$$

↓ 4671

$$2bcd^2 \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right) \frac{d^2(1 - c^2x^2)^2(a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right) \downarrow 2715$$

$$2bcd^2 \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right) \frac{d^2(1 - c^2x^2)^2(a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right) \downarrow 2838$$

$$2bcd^2 \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{3}(1 - c^2x^2)^{3/2}(a + b \arcsin(cx)) + \sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \right) \frac{d^2(1 - c^2x^2)^2(a + b \arcsin(cx))^2}{x} - 4c^2d^2 \left(\frac{1}{3}x(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{c^2} \right) \right) \right)$$

input `Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x^2,x]`

output `-((d^2*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/x) - 4*c^2*d^2*((x*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*((b*(x - (c^2*x^3)/3))/(3*c) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])/c^2)))/3 + 2*b*c*d^2*(-(b*c*x) - (b*c*(x - (c^2*x^3)/3)))/3 + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) + ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])`

Defintions of rubi rules used

- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`
- rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 5130 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*(a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1))
  Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]
  Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]
  Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))
  Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))
  Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.49

method	result
derivativedivides	$c \left(d^2 a^2 \left(\frac{c^3 x^3}{3} - 2cx - \frac{1}{cx} \right) - \frac{7d^2 b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{2} - \frac{7d^2 b^2 \arcsin(cx)^2 cx}{4} + \frac{7d^2 b^2 cx}{2} - \frac{d^2 b^2 \arcsin(cx)}{c} \right)$
default	$c \left(d^2 a^2 \left(\frac{c^3 x^3}{3} - 2cx - \frac{1}{cx} \right) - \frac{7d^2 b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{2} - \frac{7d^2 b^2 \arcsin(cx)^2 cx}{4} + \frac{7d^2 b^2 cx}{2} - \frac{d^2 b^2 \arcsin(cx)}{c} \right)$
parts	$d^2 a^2 \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) - \frac{7d^2 b^2 c \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{2} - \frac{7d^2 b^2 c^2 \arcsin(cx)^2 x}{4} + \frac{7b^2 c^2 d^2 x}{2} - \frac{d^2 b^2 \arcsin(cx)}{c}$

input

```
int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output

```
c*(d^2*a^2*(1/3*c^3*x^3-2*c*x-1/c/x)-7/2*d^2*b^2*arcsin(c*x)*(-c^2*x^2+1)^(
1/2)-7/4*d^2*b^2*arcsin(c*x)^2*c*x+7/2*d^2*b^2*c*x-d^2*b^2/c/x*arcsin(c*x
)^2+2*d^2*b^2*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*d^2*b^2*arcsin(
c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*d^2*b^2*polylog(2,I*c*x+(-c^2*x^2+
1)^(1/2))+2*I*d^2*b^2*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-1/18*d^2*b^2*ar
csin(c*x)*cos(3*arcsin(c*x))-1/12*d^2*b^2*arcsin(c*x)^2*sin(3*arcsin(c*x))
+1/54*d^2*b^2*sin(3*arcsin(c*x))+2*d^2*a*b*(1/3*c^3*x^3*arcsin(c*x)-2*c*x*
arcsin(c*x)-arcsin(c*x)/c/x+1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/9*(-c^2*x^2+
1)^(1/2)-arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x^2} dx$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")
```


output `integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))/x^2, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx = d^2 \left(\int (-2a^2 c^2) dx + \int \frac{a^2}{x^2} dx + \int a^2 c^4 x^2 dx + \int (-2b^2 c^2 \arcsin^2(cx)) dx + \int \frac{b^2 \arcsin^2(cx)}{x^2} dx + \int (-4abc^2 \arcsin(cx)) dx + \int \frac{2ab \arcsin(cx)}{x^2} dx + \int b^2 c^4 x^2 \arcsin^2(cx) dx + \int 2abc^4 x^2 \arcsin(cx) dx \right)$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2/x**2,x)`

output `d**2*(Integral(-2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c**4*x**2, x) + Integral(-2*b**2*c**2*asin(c*x)**2, x) + Integral(b**2*asin(c*x)**2/x**2, x) + Integral(-4*a*b*c**2*asin(c*x), x) + Integral(2*a*b*asin(c*x)/x**2, x) + Integral(b**2*c**4*x**2*asin(c*x)**2, x) + Integral(2*a*b*c**4*x**2*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^2,x, algorithm="maxima")`

output

```
1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1))*x^2/c
^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 - 2*b^2*c^2*d^2*x*arcsin(c*x)^
2 + 4*b^2*c^2*d^2*(x - sqrt(-c^2*x^2 + 1)*arcsin(c*x)/c) - 2*a^2*c^2*d^2*x
- 4*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a*b*c*d^2 - 2*(c*log(2*sqrt(-c
^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*
((b^2*c^4*d^2*x^4 - 3*b^2*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^
2 + 3*x*integrate(2/3*(b^2*c^5*d^2*x^4 - 3*b^2*c*d^2)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^3 - x), x))/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^2,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^2}{x^2} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x^2,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^2} dx$$

$$= \frac{d^2 \left(-18 \operatorname{asin}(cx)^2 b^2 c^2 x^2 - 36 \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) b^2 cx + 6 \operatorname{asin}(cx) ab c^4 x^4 - 36 \operatorname{asin}(cx) ab c^2 x^2 - 18 \operatorname{asin}(cx)^2 ab^2 c^2 x^2 + 18 ab^2 c^2 x^2 \right)}{9x^3}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2/x^2,x)`

output `(d**2*(- 18*asin(c*x)**2*b**2*c**2*x**2 - 36*sqrt(- c**2*x**2 + 1)*asin(c*x)*b**2*c*x + 6*asin(c*x)*a*b*c**4*x**4 - 36*asin(c*x)*a*b*c**2*x**2 - 18*asin(c*x)*a*b + 2*sqrt(- c**2*x**2 + 1)*a*b*c**3*x**3 - 32*sqrt(- c**2*x**2 + 1)*a*b*c*x + 9*int(asin(c*x)**2/x**2,x)*b**2*x + 9*int(asin(c*x)**2*x**2,x)*b**2*c**4*x + 18*log(tan(asin(c*x)/2))*a*b*c*x + 3*a**2*c**4*x**4 - 18*a**2*c**2*x**2 - 9*a**2 + 36*b**2*c**2*x**2))/(9*x)`

3.169 $\int \frac{(d-c^2 dx^2)^2 (a+b \arcsin(cx))^2}{x^3} dx$

Optimal result	1543
Mathematica [A] (verified)	1544
Rubi [A] (verified)	1545
Maple [B] (verified)	1552
Fricas [F]	1553
Sympy [F]	1554
Maxima [F]	1554
Giac [F(-2)]	1555
Mupad [F(-1)]	1555
Reduce [F]	1555

Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx$$

$$= -\frac{1}{4} b^2 c^4 d^2 x^2 - \frac{1}{2} b c^3 d^2 x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))$$

$$- \frac{b c d^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x}$$

$$- \frac{1}{4} c^2 d^2 (a + b \arcsin(cx))^2 - c^2 d^2 (1 - c^2 x^2) (a + b \arcsin(cx))^2$$

$$- \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \frac{2ic^2 d^2 (a + b \arcsin(cx))^3}{3b}$$

$$- 2c^2 d^2 (a + b \arcsin(cx))^2 \log(1 - e^{2i \arcsin(cx)}) + b^2 c^2 d^2 \log(x)$$

$$+ 2ibc^2 d^2 (a + b \arcsin(cx)) \text{PolyLog}(2, e^{2i \arcsin(cx)}) - b^2 c^2 d^2 \text{PolyLog}(3, e^{2i \arcsin(cx)})$$

output

```
-1/4*b^2*c^4*d^2*x^2-1/2*b*c^3*d^2*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-
b*c*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/x-1/4*c^2*d^2*(a+b*arcsin(c*x)
)^2-c^2*d^2*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2-1/2*d^2*(-c^2*x^2+1)^2*(a+b*
arcsin(c*x))^2/x^2+2/3*I*c^2*d^2*(a+b*arcsin(c*x))^3/b-2*c^2*d^2*(a+b*arcs
in(c*x))^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^2*ln(x)+2*I*b*c^2*
d^2*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)-b^2*c^2*d^2*
polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.26

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx \\
&= \frac{1}{2} d^2 \left(-\frac{a^2}{x^2} + a^2 c^4 x^2 + 2abc^4 x^2 \arcsin(cx) - \frac{2ab(cx\sqrt{1-c^2x^2} + \arcsin(cx))}{x^2} \right. \\
&\quad \left. + abc^2 \left(cx\sqrt{1-c^2x^2} - 2 \arctan \left(\frac{cx}{-1 + \sqrt{1-c^2x^2}} \right) \right) \right. \\
&\quad - \frac{1}{4} b^2 c^2 (-1 + 2 \arcsin(cx))^2 \cos(2 \arcsin(cx)) - 8abc^2 \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) \\
&\quad - 4a^2 c^2 \log(x) - \frac{b^2 (2cx\sqrt{1-c^2x^2} \arcsin(cx) + \arcsin(cx)^2 - 2c^2 x^2 \log(cx))}{x^2} \\
&\quad \left. + 4iabc^2 (\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)})) \right. \\
&\quad \left. + \frac{1}{6} ib^2 c^2 (\pi^3 - 8 \arcsin(cx)^3 + 24i \arcsin(cx)^2 \log(1 - e^{-2i \arcsin(cx)}) \right. \\
&\quad \left. - 24 \arcsin(cx) \text{PolyLog}(2, e^{-2i \arcsin(cx)}) + 12i \text{PolyLog}(3, e^{-2i \arcsin(cx)}) \right) \\
&\quad \left. + \frac{1}{2} b^2 c^2 \arcsin(cx) \sin(2 \arcsin(cx)) \right)
\end{aligned}$$

input `Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x^3,x]`output `(d^2*(-(a^2/x^2) + a^2*c^4*x^2 + 2*a*b*c^4*x^2*ArcSin[c*x] - (2*a*b*(c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x])/x^2 + a*b*c^2*(c*x*Sqrt[1 - c^2*x^2] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) - (b^2*c^2*(-1 + 2*ArcSin[c*x]^2)*Cos[2*ArcSin[c*x]])/4 - 8*a*b*c^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])]) - 4*a^2*c^2*Log[x] - (b^2*(2*c*x*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]^2 - 2*c^2*x^2*Log[c*x]))/x^2 + (4*I)*a*b*c^2*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])]) + (I/6)*b^2*c^2*(Pi^3 - 8*ArcSin[c*x]^3 + (24*I)*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])]) - 24*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])]) + (12*I)*PolyLog[3, E^((-2*I)*ArcSin[c*x])]) + (b^2*c^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])/2)`

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.20, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {5200, 27, 5200, 244, 2009, 5156, 15, 5152, 5202, 5136, 3042, 25, 4200, 25, 2620, 3011, 2720, 5156, 15, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx$$

$$\downarrow 5200$$

$$bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx - 2c^2 d \int \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2}$$

$$\downarrow 27$$

$$bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x^2} dx - 2c^2 d^2 \int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2}$$

$$\downarrow 5200$$

$$bcd^2 \left(-3c^2 \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + bc \int \frac{1 - c^2 x^2}{x} dx - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right) - 2c^2 d^2 \int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2}$$

$$\downarrow 244$$

$$bcd^2 \left(-3c^2 \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + bc \int \left(\frac{1}{x} - c^2 x \right) dx - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right) - 2c^2 d^2 \int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx - \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2}$$

$$\downarrow 2009$$

$$\begin{aligned}
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx + \\
bcd^2 & \left(-3c^2 \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x} + bc \left(\log(x) - \frac{c^2x^2}{2} \right) \right) - \\
& \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{5156} \\
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx + \\
bcd^2 & \left(-3c^2 \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2} bc \int x dx + \frac{1}{2} x \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x} \right) - \\
& \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{15} \\
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx + \\
bcd^2 & \left(-3c^2 \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{4} bcx^2 \right) - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x} \right) - \\
& \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{2x^2} \\
& \quad \downarrow \text{5152} \\
& -2c^2 d^2 \int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{2x^2} + \\
bcd^2 & \left(-3c^2 \left(\frac{1}{2} x \sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x} \right) - \\
& \quad \downarrow \text{5202} \\
& -2c^2 d^2 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \int \frac{(a+b\arcsin(cx))^2}{x} dx + \frac{1}{2} (1-c^2x^2)(a+b\arcsin(cx))^2 \right) - \\
& \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{2x^2} + \\
bcd^2 & \left(-3c^2 \left(\frac{1}{2} x \sqrt{1-c^2x^2}(a+b\arcsin(cx)) + \frac{(a+b\arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x} \right) - \\
& \quad \downarrow \text{5136}
\end{aligned}$$

$$\begin{aligned}
& -2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2}{cx} d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\
& \quad \left. + \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} \right) + \\
& bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)
\end{aligned}$$

↓ 3042

$$\begin{aligned}
& -2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \int -(a + b \arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\
& \quad \left. + \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} \right) + \\
& bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)
\end{aligned}$$

↓ 25

$$\begin{aligned}
& -2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \int (a + b \arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\
& \quad \left. + \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} \right) + \\
& bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)
\end{aligned}$$

↓ 4200

$$\begin{aligned}
& -2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + 2i \int -\frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))^2}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx)) \right. \\
& \quad \left. + \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} \right) + \\
& bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)
\end{aligned}$$

↓ 25

$$-2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - 2i \int \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))^2}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2} (1 - c^2 x^2) \right. \\ \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right.$$

$$bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)$$

↓ 2620

$$-2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - 2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 - ib \int (a + b \arcsin(cx)) \right. \right. \\ \left. \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \right.$$

$$bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)$$

↓ 3011

$$-2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - 2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \text{PolyL} \right. \right. \right. \\ \left. \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \right.$$

$$bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)$$

↓ 2720

$$-2c^2 d^2 \left(-bc \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - 2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \text{PolyL} \right. \right. \right. \\ \left. \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \right.$$

$$bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right)$$

↓ 5156

$$-2c^2 d^2 \left(-bc \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{2} bc \int x dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) \right. \right. \\ \left. \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \right. \\ \left. \left. bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right) \right)$$

↓ 15

$$-2c^2 d^2 \left(-bc \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) - \frac{1}{4} bcx^2 \right) - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) \right. \right. \\ \left. \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \right. \\ \left. \left. bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right) \right)$$

↓ 5152

$$-2c^2 d^2 \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx))^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) \right) \right. \right. \\ \left. \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \right. \\ \left. \left. bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right) \right)$$

↓ 7143

$$-2c^2 d^2 \left(\frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx))^2 - bc \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) \right. \\ \left. \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{2x^2} + \right. \\ \left. bcd^2 \left(-3c^2 \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{(a + b \arcsin(cx))^2}{4bc} - \frac{1}{4} bcx^2 \right) - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} \right) \right)$$

input `Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x^3,x]`

output

```
-1/2*(d^2*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/x^2 + b*c*d^2*(-(((1 - c^
2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/x) - 3*c^2*(-1/4*(b*c*x^2) + (x*Sqrt[1 -
c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)) + b*c*(-
1/2*(c^2*x^2) + Log[x])) - 2*c^2*d^2*(((1 - c^2*x^2)*(a + b*ArcSin[c*x])^2
)/2 - ((I/3)*(a + b*ArcSin[c*x])^3)/b - b*c*(-1/4*(b*c*x^2) + (x*Sqrt[1 -
c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)) - (2*I)*
((I/2)*(a + b*ArcSin[c*x])^2*Log[1 - E^((2*I)*ArcSin[c*x])] - I*b*((I/2)*(a
+ b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, E^((2*
I)*ArcSin[c*x])])/4)))
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 244

```
Int[(((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n * Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(301) = 602$.

Time = 0.66 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.23

method	result
parts	$d^2 a^2 \left(\frac{c^4 x^2}{2} - 2c^2 \ln(x) - \frac{1}{2x^2} \right) + d^2 b^2 c^2 \left(\frac{2i \arcsin(cx)^3}{3} + \frac{(2i \arcsin(cx) + 2 \arcsin(cx)^2 - 1)(-2i\sqrt{-c}}$
derivativedivides	$c^2 \left(d^2 a^2 \left(\frac{c^2 x^2}{2} - 2 \ln(cx) - \frac{1}{2c^2 x^2} \right) + d^2 b^2 \left(\frac{2i \arcsin(cx)^3}{3} + \frac{(2i \arcsin(cx) + 2 \arcsin(cx)^2 - 1)(-2i\sqrt{-c}}$
default	$c^2 \left(d^2 a^2 \left(\frac{c^2 x^2}{2} - 2 \ln(cx) - \frac{1}{2c^2 x^2} \right) + d^2 b^2 \left(\frac{2i \arcsin(cx)^3}{3} + \frac{(2i \arcsin(cx) + 2 \arcsin(cx)^2 - 1)(-2i\sqrt{-c}}$

input

```
int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output

```

d^2*a^2*(1/2*c^4*x^2-2*c^2*ln(x)-1/2/x^2)+d^2*b^2*c^2*(2/3*I*arcsin(c*x)^3
+1/16*(2*I*arcsin(c*x)+2*arcsin(c*x)^2-1)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c
^2*x^2-1)+1/16*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arcsin(c*x)^2-1
-2*I*arcsin(c*x))-1/2*arcsin(c*x)*(-2*I*c^2*x^2+2*c*x*(-c^2*x^2+1)^(1/2)+a
rcsin(c*x))/c^2/x^2-2*ln(I*c*x+(-c^2*x^2+1)^(1/2))+ln(I*c*x+(-c^2*x^2+1)^(
1/2)-1)+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^
2+1)^(1/2))+4*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-4*polylog
(3,-I*c*x-(-c^2*x^2+1)^(1/2))-2*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2
))+4*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-4*polylog(3,I*c*x+(
-c^2*x^2+1)^(1/2))+2*d^2*a*b*c^2*(I*arcsin(c*x)^2+1/16*(I+2*arcsin(c*x))*
(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)+1/16*(2*I*(-c^2*x^2+1)^(1/2)*c*x
+2*c^2*x^2-1)*(-I+2*arcsin(c*x))-1/2*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+a
rcsin(c*x))/c^2/x^2-2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*arcsin(c
*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))
+2*I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^3,x, algorithm="fricas")
```

output

```

integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))/x^3, x)

```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx = d^2 \left(\int \frac{a^2}{x^3} dx + \int \left(-\frac{2a^2 c^2}{x} \right) dx + \int a^2 c^4 x dx \right. \\ \left. + \int \frac{b^2 \operatorname{asin}^2(cx)}{x^3} dx + \int \frac{2ab \operatorname{asin}(cx)}{x^3} dx \right. \\ \left. + \int \left(-\frac{2b^2 c^2 \operatorname{asin}^2(cx)}{x} \right) dx \right. \\ \left. + \int b^2 c^4 x \operatorname{asin}^2(cx) dx \right. \\ \left. + \int \left(-\frac{4abc^2 \operatorname{asin}(cx)}{x} \right) dx \right. \\ \left. + \int 2abc^4 x \operatorname{asin}(cx) dx \right)$$

input `integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2/x**3,x)`

output `d**2*(Integral(a**2/x**3, x) + Integral(-2*a**2*c**2/x, x) + Integral(a**2*c**4*x, x) + Integral(b**2*asin(c*x)**2/x**3, x) + Integral(2*a*b*asin(c*x)/x**3, x) + Integral(-2*b**2*c**2*asin(c*x)**2/x, x) + Integral(b**2*c**4*x*asin(c*x)**2, x) + Integral(-4*a*b*c**2*asin(c*x)/x, x) + Integral(2*a*b*c**4*x*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^3,x, algorithm="maxima")`

output `1/2*a^2*c^4*d^2*x^2 - 2*a^2*c^2*d^2*log(x) - a*b*d^2*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/x^3, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^2}{x^3} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x^3,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^3} dx$$

$$= \frac{d^2 \left(2a \sin(cx)^2 b^2 c^4 x^4 - a \sin(cx)^2 b^2 c^2 x^2 + 2\sqrt{-c^2 x^2 + 1} a \sin(cx) b^2 c^3 x^3 + 4a \sin(cx) a b c^4 x^4 - 2a \sin(cx) \right)}{x^3}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2/x^3,x)`

output

```
(d**2*(2*asin(c*x)**2*b**2*c**4*x**4 - asin(c*x)**2*b**2*c**2*x**2 + 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*b**2*c**3*x**3 + 4*asin(c*x)*a*b*c**4*x**4 - 2*asin(c*x)*a*b*c**2*x**2 - 4*asin(c*x)*a*b + 2*sqrt(-c**2*x**2 + 1)*a*b*c**3*x**3 - 4*sqrt(-c**2*x**2 + 1)*a*b*c*x - 16*int(asin(c*x)/x,x)*a*b*c**2*x**2 + 4*int(asin(c*x)**2/x**3,x)*b**2*x**2 - 8*int(asin(c*x)**2/x,x)*b**2*c**2*x**2 - 8*log(x)*a**2*c**2*x**2 + 2*a**2*c**4*x**4 - 2*a**2 - b**2*c**4*x**4))/(4*x**2)
```

$$3.170 \quad \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx$$

Optimal result	1557
Mathematica [A] (verified)	1558
Rubi [A] (verified)	1558
Maple [A] (verified)	1565
Fricas [F]	1565
Sympy [F]	1566
Maxima [F]	1566
Giac [F(-1)]	1567
Mupad [F(-1)]	1567
Reduce [F]	1568

Optimal result

Integrand size = 27, antiderivative size = 268

$$\begin{aligned}
 \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx = & -\frac{b^2 c^2 d^2}{3x} - 2b^2 c^4 d^2 x \\
 & + \frac{5}{3} b c^3 d^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \\
 & - \frac{b c d^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{3x^2} \\
 & + \frac{8}{3} c^4 d^2 x (a + b \arcsin(cx))^2 \\
 & + \frac{4c^2 d^2 (1 - c^2 x^2) (a + b \arcsin(cx))^2}{3x} \\
 & - \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{3x^3} \\
 & + \frac{22}{3} b c^3 d^2 (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)}) \\
 & - \frac{11}{3} i b^2 c^3 d^2 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \\
 & + \frac{11}{3} i b^2 c^3 d^2 \operatorname{PolyLog}(2, e^{i \arcsin(cx)})
 \end{aligned}$$

output

$$\begin{aligned}
& -1/3*b^2*c^2*d^2/x-2*b^2*c^4*d^2*x+5/3*b*c^3*d^2*(-c^2*x^2+1)^{(1/2)}*(a+b*arcsin(c*x))-1/3*b*c*d^2*(-c^2*x^2+1)^{(3/2)}*(a+b*arcsin(c*x))/x^2+8/3*c^4*d^2*x*(a+b*arcsin(c*x))^2+4/3*c^2*d^2*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/x-1/3*d^2*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/x^3+22/3*b*c^3*d^2*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^{(1/2)})-11/3*I*b^2*c^3*d^2*polylog(2,-I*c*x-(-c^2*x^2+1)^{(1/2)})+11/3*I*b^2*c^3*d^2*polylog(2,I*c*x+(-c^2*x^2+1)^{(1/2)})
\end{aligned}$$
Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.40

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx \\
& = \frac{d^2(-a^2 + 6a^2c^2x^2 - b^2c^2x^2 + 3a^2c^4x^4 - 6b^2c^4x^4 - abcx\sqrt{1 - c^2x^2} + 6abc^3x^3\sqrt{1 - c^2x^2} - 2ab \arcsin(cx))}{x^4}
\end{aligned}$$

input

`Integrate[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x^4,x]`

output

$$\begin{aligned}
& (d^2*(-a^2 + 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 - 6*b^2*c^4*x^4 - a*b*c*x*sqrt[1 - c^2*x^2] + 6*a*b*c^3*x^3*sqrt[1 - c^2*x^2] - 2*a*b*ArcSin[c*x] + 12*a*b*c^2*x^2*ArcSin[c*x] + 6*a*b*c^4*x^4*ArcSin[c*x] - b^2*c*x*sqrt[1 - c^2*x^2]*ArcSin[c*x] + 6*b^2*c^3*x^3*sqrt[1 - c^2*x^2]*ArcSin[c*x] - b^2*ArcSin[c*x]^2 + 6*b^2*c^2*x^2*ArcSin[c*x]^2 + 3*b^2*c^4*x^4*ArcSin[c*x]^2 + 11*a*b*c^3*x^3*ArcTanh[sqrt[1 - c^2*x^2]] - 11*b^2*c^3*x^3*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 11*b^2*c^3*x^3*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (11*I)*b^2*c^3*x^3*PolyLog[2, -E^(I*ArcSin[c*x])] + (11*I)*b^2*c^3*x^3*PolyLog[2, E^(I*ArcSin[c*x])]))/(3*x^3)
\end{aligned}$$
Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5200, 27, 5200, 244, 2009, 5130, 5182, 24, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx \\
& \quad \downarrow \text{5200} \\
& \frac{2}{3}bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx - \frac{4}{3}c^2 d \int \frac{d(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x^2} dx - \\
& \quad \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{3x^3} \\
& \quad \downarrow \text{27} \\
& -\frac{4}{3}c^2 d^2 \int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x^2} dx + \frac{2}{3}bcd^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x^3} dx - \\
& \quad \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{3x^3} \\
& \quad \downarrow \text{5200} \\
& \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} dx + \frac{1}{2}bc \int \frac{1 - c^2 x^2}{x^2} dx - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{2x^2} \right) - \\
& \frac{4}{3}c^2 d^2 \left(2bc \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} dx - 2c^2 \int (a + b \arcsin(cx))^2 dx - \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} \right) - \\
& \quad \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{3x^3} \\
& \quad \downarrow \text{244} \\
& \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} dx + \frac{1}{2}bc \int \left(\frac{1}{x^2} - c^2 \right) dx - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{2x^2} \right) - \\
& \frac{4}{3}c^2 d^2 \left(2bc \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} dx - 2c^2 \int (a + b \arcsin(cx))^2 dx - \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} \right) - \\
& \quad \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{3x^3} \\
& \quad \downarrow \text{2009}
\end{aligned}$$

$$\frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(c^2(-x) - \frac{1}{x} \right) \right) - \frac{4}{3}c^2d^2 \left(2bc \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - 2c^2 \int (a+b\arcsin(cx))^2 dx - \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} \right) - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5130

$$-\frac{4}{3}c^2d^2 \left(-2c^2 \left(x(a+b\arcsin(cx))^2 - 2bc \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) + 2bc \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx \right) - \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(c^2(-x) - \frac{1}{x} \right) \right) - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5182

$$-\frac{4}{3}c^2d^2 \left(-2c^2 \left(x(a+b\arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2} \right) \right) \right) + 2bc \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(c^2(-x) - \frac{1}{x} \right) \right) - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}$$

↓ 24

$$-\frac{4}{3}c^2d^2 \left(2bc \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - 2c^2 \left(x(a+b\arcsin(cx))^2 - 2bc \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2} \right) \right) \right) - \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(c^2(-x) - \frac{1}{x} \right) \right) - \frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5198

$$-\frac{4}{3}c^2 d^2 \left(2bc \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int 1 dx + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) \right) - 2c^2 \left(x(a + b \arcsin(cx))^2 - \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int 1 dx + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) \right) - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right. \right. \\ \left. \left. \frac{d^2(1-c^2x^2)^2(a + b \arcsin(cx))^2}{3x^3} \right) \right)$$

↓ 24

$$-\frac{4}{3}c^2 d^2 \left(2bc \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - bcx \right) - 2c^2 \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - bcx \right) - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right. \right. \right. \\ \left. \left. \frac{d^2(1-c^2x^2)^2(a + b \arcsin(cx))^2}{3x^3} \right) \right)$$

↓ 5218

$$-\frac{4}{3}c^2 d^2 \left(2bc \left(\int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - bcx \right) - 2c^2 \left(x(a + b \arcsin(cx))^2 - \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \left(\int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - bcx \right) - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right. \right. \\ \left. \left. \frac{d^2(1-c^2x^2)^2(a + b \arcsin(cx))^2}{3x^3} \right) \right)$$

↓ 3042

$$-\frac{4}{3}c^2 d^2 \left(2bc \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - bcx \right) - 2c^2 \left(x(a + b \arcsin(cx))^2 - \frac{2}{3}bcd^2 \left(-\frac{3}{2}c^2 \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \sqrt{1-c^2x^2}(a + b \arcsin(cx)) - bcx \right) - \frac{(1-c^2x^2)^{3/2}(a + b \arcsin(cx))}{2x^2} \right. \right. \\ \left. \left. \frac{d^2(1-c^2x^2)^2(a + b \arcsin(cx))^2}{3x^3} \right) \right)$$

↓ 4671

$$-\frac{4}{3}c^2d^2\left(2bc\left(-b\int\log\left(1-e^{i\arcsin(cx)}\right)d\arcsin(cx)+b\int\log\left(1+e^{i\arcsin(cx)}\right)d\arcsin(cx)-2\operatorname{arctanh}\left(e^{i\arcsin(cx)}\right)\right)\right. \\ \left.\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(-b\int\log\left(1-e^{i\arcsin(cx)}\right)d\arcsin(cx)+b\int\log\left(1+e^{i\arcsin(cx)}\right)d\arcsin(cx)-2\operatorname{arctanh}\left(e^{i\arcsin(cx)}\right)\right)\right)\right. \\ \left.\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}\right)$$

↓ 2715

$$-\frac{4}{3}c^2d^2\left(2bc\left(ib\int e^{-i\arcsin(cx)}\log\left(1-e^{i\arcsin(cx)}\right)de^{i\arcsin(cx)}-ib\int e^{-i\arcsin(cx)}\log\left(1+e^{i\arcsin(cx)}\right)de^{i\arcsin(cx)}\right)\right. \\ \left.\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(ib\int e^{-i\arcsin(cx)}\log\left(1-e^{i\arcsin(cx)}\right)de^{i\arcsin(cx)}-ib\int e^{-i\arcsin(cx)}\log\left(1+e^{i\arcsin(cx)}\right)de^{i\arcsin(cx)}\right)\right)\right. \\ \left.\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}\right)$$

↓ 2838

$$-\frac{4}{3}c^2d^2\left(2bc\left(-2\operatorname{arctanh}\left(e^{i\arcsin(cx)}\right)(a+b\arcsin(cx))+\sqrt{1-c^2x^2}(a+b\arcsin(cx))+ib\operatorname{PolyLog}\left(2,-e^{i\arcsin(cx)}\right)\right)\right. \\ \left.\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(-2\operatorname{arctanh}\left(e^{i\arcsin(cx)}\right)(a+b\arcsin(cx))+\sqrt{1-c^2x^2}(a+b\arcsin(cx))+ib\operatorname{PolyLog}\left(2,-e^{i\arcsin(cx)}\right)\right)\right)\right. \\ \left.\frac{d^2(1-c^2x^2)^2(a+b\arcsin(cx))^2}{3x^3}\right)$$

input

```
Int[((d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

```
-1/3*(d^2*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/x^3 - (4*c^2*d^2*(-(((1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/x) - 2*c^2*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2)) + 2*b*c*(-(b*c*x) + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])))/3 + (2*b*c*d^2*((b*c*(-x^(-1) - c^2*x))/2 - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(2*x^2) - (3*c^2*(-(b*c*x) + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])))/2))/3
```

Definitions of rubi rules used

- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 27 `Int[(a_)*(F_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 244 `Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`
- rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 5130 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*(a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))] Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5218

```
Int((((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.41

method	result
derivativedivides	$c^3 \left(d^2 a^2 \left(cx - \frac{1}{3c^3 x^3} + \frac{2}{cx} \right) + 2d^2 b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1} + d^2 b^2 \arcsin(cx)^2 cx - 2d^2 b^2 \arcsin(cx) \right)$
default	$c^3 \left(d^2 a^2 \left(cx - \frac{1}{3c^3 x^3} + \frac{2}{cx} \right) + 2d^2 b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1} + d^2 b^2 \arcsin(cx)^2 cx - 2d^2 b^2 \arcsin(cx) \right)$
parts	$d^2 a^2 \left(c^4 x - \frac{1}{3x^3} + \frac{2c^2}{x} \right) + 2d^2 b^2 c^3 \sqrt{-c^2 x^2 + 1} \arcsin(cx) + d^2 b^2 c^4 \arcsin(cx)^2 x - 2b^2 d^2 \arcsin(cx)$

input `int((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^4,x,method=_RETURNVERBOSE)`

output $c^3 \left(d^2 a^2 \left(cx - \frac{1}{3c^3 x^3} + \frac{2}{cx} \right) + 2d^2 b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1} + d^2 b^2 \arcsin(cx)^2 cx - 2d^2 b^2 \arcsin(cx) \right)$

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^4,x, algorithm="fricas")`

output

```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx = d^2 \left(\int a^2 c^4 dx + \int \frac{a^2}{x^4} dx + \int \left(-\frac{2a^2 c^2}{x^2} \right) dx \right. \\ \left. + \int b^2 c^4 \operatorname{asin}^2(cx) dx + \int \frac{b^2 \operatorname{asin}^2(cx)}{x^4} dx \right. \\ \left. + \int 2abc^4 \operatorname{asin}(cx) dx + \int \frac{2ab \operatorname{asin}(cx)}{x^4} dx \right. \\ \left. + \int \left(-\frac{2b^2 c^2 \operatorname{asin}^2(cx)}{x^2} \right) dx \right. \\ \left. + \int \left(-\frac{4abc^2 \operatorname{asin}(cx)}{x^2} \right) dx \right)$$

input

```
integrate((-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2/x**4,x)
```

output

```
d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(-2*a**2*c
**2/x**2, x) + Integral(b**2*c**4*asin(c*x)**2, x) + Integral(b**2*asin(c*
x)**2/x**4, x) + Integral(2*a*b*c**4*asin(c*x), x) + Integral(2*a*b*asin(c
*x)/x**4, x) + Integral(-2*b**2*c**2*asin(c*x)**2/x**2, x) + Integral(-4*a
*b*c**2*asin(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2}{x^4} dx$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^4,x, algorithm="maxima")
```

output

```

b^2*c^4*d^2*x*arcsin(c*x)^2 - 2*b^2*c^4*d^2*(x - sqrt(-c^2*x^2 + 1))*arcsin
(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a*b*c^
3*d^2 + 4*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*
a*b*c^2*d^2 - 1/3*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt
(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*a*b*d^2 + 2*a^2*c^2*d^2/x - 1/3
*a^2*d^2/x^3 + 1/3*(3*x^3*integrate(2/3*(6*b^2*c^3*d^2*x^2 - b^2*c*d^2)*sq
rt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2
*x^5 - x^3), x) + (6*b^2*c^2*d^2*x^2 - b^2*d^2)*arctan2(c*x, sqrt(c*x + 1)
*sqrt(-c*x + 1))^2)/x^3

```

Giac [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx = \text{Timed out}$$

input

```
integrate((-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2/x^4,x, algorithm="giac")
```

output

Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^2}{x^4} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x^4,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2 (a + b \arcsin(cx))^2}{x^4} dx$$

$$= \frac{d^2 \left(3 \arcsin(cx)^2 b^2 c^4 x^4 + 6 \sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 c^3 x^3 + 6 \arcsin(cx) a b c^4 x^4 + 12 \arcsin(cx) a b c^2 x^2 - 2 \arcsin(cx) a^2 \right)}{3 x^3}$$

input `int((-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2/x^4,x)`

output `(d**2*(3*asin(c*x)**2*b**2*c**4*x**4 + 6*sqrt(-c**2*x**2 + 1)*asin(c*x)*b**2*c**3*x**3 + 6*asin(c*x)*a*b*c**4*x**4 + 12*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + 6*sqrt(-c**2*x**2 + 1)*a*b*c**3*x**3 - sqrt(-c**2*x**2 + 1)*a*b*c*x + 3*int(asin(c*x)**2/x**4,x)*b**2*x**3 - 6*int(asin(c*x)**2/x**2,x)*b**2*c**2*x**3 - 11*log(tan(asin(c*x)/2))*a*b*c**3*x**3 + 3*a**2*c**4*x**4 + 6*a**2*c**2*x**2 - a**2 - 6*b**2*c**4*x**4))/(3*x**3)`

3.171 $\int x^4(d - c^2dx^2)^3 (a + b \arcsin(cx))^2 dx$

Optimal result	1569
Mathematica [A] (verified)	1570
Rubi [A] (verified)	1571
Maple [A] (verified)	1579
Fricas [A] (verification not implemented)	1580
Sympy [A] (verification not implemented)	1580
Maxima [B] (verification not implemented)	1581
Giac [B] (verification not implemented)	1582
Mupad [F(-1)]	1583
Reduce [F]	1584

Optimal result

Integrand size = 27, antiderivative size = 476

$$\int x^4(d - c^2dx^2)^3 (a + b \arcsin(cx))^2 dx = -\frac{100976b^2d^3x}{4002075c^4} - \frac{50488b^2d^3x^3}{12006225c^2} - \frac{12622b^2d^3x^5}{6670125}$$

$$+ \frac{9410b^2c^2d^3x^7}{1120581} - \frac{182b^2c^4d^3x^9}{29403} + \frac{2b^2c^6d^3x^{11}}{1331} + \frac{256bd^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{17325c^5}$$

$$+ \frac{128bd^3x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{17325c^3} + \frac{32bd^3x^4\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{5775c}$$

$$+ \frac{16bd^3(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{693c^5} - \frac{4bd^3(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{1155c^5}$$

$$+ \frac{2bd^3(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{1617c^5} - \frac{8bd^3(1 - c^2x^2)^{9/2}(a + b \arcsin(cx))}{297c^5}$$

$$+ \frac{2bd^3(1 - c^2x^2)^{11/2}(a + b \arcsin(cx))}{121c^5} + \frac{16d^3x^5(a + b \arcsin(cx))^2}{1155}$$

$$+ \frac{8}{231}d^3x^5(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{33}d^3x^5(1 - c^2x^2)^2(a + b \arcsin(cx))^2 + \frac{1}{11}d^3x^5(1 - c^2x^2)^3(a + b \arcsin(cx))^2$$

output

```
-100976/4002075*b^2*d^3*x/c^4-50488/12006225*b^2*d^3*x^3/c^2-12622/6670125
*b^2*d^3*x^5+9410/1120581*b^2*c^2*d^3*x^7-182/29403*b^2*c^4*d^3*x^9+2/1331
*b^2*c^6*d^3*x^11+256/17325*b*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^5
+128/17325*b*d^3*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+32/5775*b*d^
3*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+16/693*b*d^3*(-c^2*x^2+1)^(3/
2)*(a+b*arcsin(c*x))/c^5-4/1155*b*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))
/c^5+2/1617*b*d^3*(-c^2*x^2+1)^(7/2)*(a+b*arcsin(c*x))/c^5-8/297*b*d^3*(-c
^2*x^2+1)^(9/2)*(a+b*arcsin(c*x))/c^5+2/121*b*d^3*(-c^2*x^2+1)^(11/2)*(a+b
*arcsin(c*x))/c^5+16/1155*d^3*x^5*(a+b*arcsin(c*x))^2+8/231*d^3*x^5*(-c^2*
x^2+1)*(a+b*arcsin(c*x))^2+2/33*d^3*x^5*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2
+1/11*d^3*x^5*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.63

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx =$$

$$\frac{d^3 (12006225 a^2 c^5 x^5 (-231 + 495 c^2 x^2 - 385 c^4 x^4 + 105 c^6 x^6) + 6930 ab \sqrt{1 - c^2 x^2} (-50488 - 25244 c^2 x^2$$

input

```
Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/13867189875*(d^3*(12006225*a^2*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^
4 + 105*c^6*x^6) + 6930*a*b*Sqrt[1 - c^2*x^2]*(-50488 - 25244*c^2*x^2 - 18
933*c^4*x^4 + 117625*c^6*x^6 - 111475*c^8*x^8 + 33075*c^10*x^10) + b^2*(34
9881840*c*x + 58313640*c^3*x^3 + 26241138*c^5*x^5 - 116448750*c^7*x^7 + 85
835750*c^9*x^9 - 20837250*c^11*x^11) + 6930*b*(3465*a*c^5*x^5*(-231 + 495*
c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-50488 - 25244
*c^2*x^2 - 18933*c^4*x^4 + 117625*c^6*x^6 - 111475*c^8*x^8 + 33075*c^10*x^
10))*ArcSin[c*x] + 12006225*b^2*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4
+ 105*c^6*x^6)*ArcSin[c*x]^2))/c^5
```

Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.49, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.815$, Rules used = {5202, 27, 5194, 27, 1467, 2009, 5202, 5194, 27, 1467, 2009, 5202, 5138, 5194, 27, 2009, 5210, 15, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & -\frac{2}{11}bcd^3 \int x^5 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{6}{11}d \int d^2 x^4 (1 - c^2 x^2)^2 (a + \\
 & \quad b \arcsin(cx))^2 dx + \frac{1}{11}d^3 x^5 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{27} \\
 & -\frac{2}{11}bcd^3 \int x^5 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{6}{11}d^3 \int x^4 (1 - c^2 x^2)^2 (a + \\
 & \quad b \arcsin(cx))^2 dx + \frac{1}{11}d^3 x^5 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5194} \\
 & \frac{6}{11}d^3 \int x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx - \\
 & \frac{2}{11}bcd^3 \left(-bc \int -\frac{(1 - c^2 x^2)^3 (63c^4 x^4 + 28c^2 x^2 + 8)}{693c^6} dx - \frac{(1 - c^2 x^2)^{11/2} (a + b \arcsin(cx))}{11c^6} + \frac{2(1 - c^2 x^2)^{9/2} (a + b \arcsin(cx))}{9c^6} \right. \\
 & \quad \left. + \frac{1}{11}d^3 x^5 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 \right) \\
 & \quad \downarrow \text{27} \\
 & \frac{6}{11}d^3 \int x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx - \\
 & \frac{2}{11}bcd^3 \left(\frac{b \int (1 - c^2 x^2)^3 (63c^4 x^4 + 28c^2 x^2 + 8) dx}{693c^5} - \frac{(1 - c^2 x^2)^{11/2} (a + b \arcsin(cx))}{11c^6} + \frac{2(1 - c^2 x^2)^{9/2} (a + b \arcsin(cx))}{9c^6} \right. \\
 & \quad \left. + \frac{1}{11}d^3 x^5 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 \right) \\
 & \quad \downarrow \text{1467}
 \end{aligned}$$

$$\frac{6}{11}d^3 \int x^4(1-c^2x^2)^2(a+b\arcsin(cx))^2dx - \frac{2}{11}bcd^3 \left(\frac{b \int (-63c^{10}x^{10} + 161c^8x^8 - 113c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{693c^5} - \frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) + \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2$$

↓ 2009

$$\frac{6}{11}d^3 \int x^4(1-c^2x^2)^2(a+b\arcsin(cx))^2dx + \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right)$$

↓ 5202

$$\frac{6}{11}d^3 \left(-\frac{2}{9}bc \int x^5(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx + \frac{4}{9} \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2dx + \frac{1}{9}x^5(1-c^2x^2)^5 \right) + \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right)$$

↓ 5194

$$\frac{6}{11}d^3 \left(\frac{4}{9} \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2dx - \frac{2}{9}bc \left(-bc \int -\frac{(1-c^2x^2)^2(35c^4x^4 + 20c^2x^2 + 8)}{315c^6} dx - \frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} \right) + \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right)$$

↓ 27

$$\frac{6}{11}d^3 \left(\frac{4}{9} \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2dx - \frac{2}{9}bc \left(\frac{b \int (1-c^2x^2)^2(35c^4x^4 + 20c^2x^2 + 8) dx}{315c^5} - \frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} \right) + \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right)$$

↓ 1467

$$\frac{6}{11}d^3 \left(\frac{4}{9} \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{9}bc \left(\frac{b \int (35c^8x^8 - 50c^6x^6 + 3c^4x^4 + 4c^2x^2 + 8) dx}{315c^5} - \frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} \right) - \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right)$$

↓ 2009

$$\frac{6}{11}d^3 \left(\frac{4}{9} \int x^4(1-c^2x^2)(a+b\arcsin(cx))^2 dx + \frac{1}{9}x^5(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{9}bc \left(-\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right) \right)$$

↓ 5202

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(-\frac{2}{7}bc \int x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{2}{7} \int x^4(a+b\arcsin(cx))^2 dx + \frac{1}{7}x^5(1-c^2x^2)(a+b\arcsin(cx))^2 \right) - \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right)$$

↓ 5138

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{7}bc \int x^5\sqrt{1-c^2x^2}(a+b\arcsin(cx))dx \right) - \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right)$$

↓ 5194

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{7}bc \left(-bc \int -\frac{-15c^6x^6 + 3c^4x^4 + 4c^2}{105c^6} \right. \right. \right. \\ \left. \left. \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \right. \\ \left. \left. \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right. \right. \right. \\ \left. \left. \left. \downarrow 27 \right. \right. \right.$$

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{7}bc \left(\frac{b \int (-15c^6x^6 + 3c^4x^4 + 4c^2x^2 + 4c^2)}{105c^5} \right. \right. \right. \\ \left. \left. \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \right. \\ \left. \left. \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right. \right. \right. \\ \left. \left. \left. \downarrow 2009 \right. \right. \right.$$

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \int \frac{x^5(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{7}x^5(1-c^2x^2)(a+b\arcsin(cx))^2 - \frac{2}{7}bc \int \frac{-15c^6x^6 + 3c^4x^4 + 4c^2x^2 + 4c^2}{105c^5} \right. \right. \\ \left. \left. \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \right. \\ \left. \left. \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right. \right. \right. \\ \left. \left. \left. \downarrow 5210 \right. \right. \right.$$

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{5c^2} + \frac{b \int x^4 dx}{5c} - \frac{x^4\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{5c^2} \right) \right. \right. \right. \\ \left. \left. \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \right. \\ \left. \left. \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right. \right. \right. \\ \left. \left. \left. \downarrow 15 \right. \right. \right.$$

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{5c^2} - \frac{x^4\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{5c^2} + \frac{bx^5}{25c} \right) \right) \right) \right. \\ \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \\ \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right)$$

↓ 5210

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{b \int x^2 dx}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} \right)}{5c^2} \right) \right) \right) \right. \\ \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \\ \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right)$$

↓ 15

$$\frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{4 \left(\frac{2 \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{5c^2} \right) \right) \right) \right. \\ \left. \frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \\ \left. \frac{2}{11}bcd^3 \left(-\frac{(1-c^2x^2)^{11/2}(a+b\arcsin(cx))}{11c^6} + \frac{2(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^6} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^6} \right) \right)$$

↓ 5182

$$\begin{aligned}
 & \frac{6}{11}d^3 \left(\frac{4}{9} \left(\frac{2}{7} \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{2 \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{5c^2} \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \frac{1}{11}d^3x^5(1 - c^2x^2)^3(a + b \arcsin(cx))^2 - \right. \\
 & \left. \frac{2}{11}bcd^3 \left(-\frac{(1 - c^2x^2)^{11/2}(a + b \arcsin(cx))}{11c^6} + \frac{2(1 - c^2x^2)^{9/2}(a + b \arcsin(cx))}{9c^6} - \frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} \right) \right) \\
 & \qquad \qquad \qquad \downarrow 24 \\
 & \qquad \qquad \qquad \frac{1}{11}d^3x^5(1 - c^2x^2)^3(a + b \arcsin(cx))^2 + \\
 & \frac{6}{11}d^3 \left(\frac{1}{9}x^5(1 - c^2x^2)^2(a + b \arcsin(cx))^2 + \frac{4}{9} \left(\frac{1}{7}x^5(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{7} \left(\frac{1}{5}x^5(a + b \arcsin(cx))^2 \right. \right. \right. \\
 & \left. \left. \frac{2}{11}bcd^3 \left(-\frac{(1 - c^2x^2)^{11/2}(a + b \arcsin(cx))}{11c^6} + \frac{2(1 - c^2x^2)^{9/2}(a + b \arcsin(cx))}{9c^6} - \frac{(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{7c^6} \right) \right) \right)
 \end{aligned}$$

input `Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]`

output

$$\begin{aligned} & (d^3 x^5 (1 - c^2 x^2)^3 (a + b \operatorname{ArcSin}[c x])^2) / 11 - (2 b c d^3 ((b (8 x + \\ & (4 c^2 x^3) / 3 + (3 c^4 x^5) / 5 - (113 c^6 x^7) / 7 + (161 c^8 x^9) / 9 - (63 c \\ & ^{10} x^{11}) / 11)) / (693 c^5) - ((1 - c^2 x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])) / (7 c^6) \\ & + (2 (1 - c^2 x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])) / (9 c^6) - ((1 - c^2 x^2)^{11/2} (a + b \operatorname{ArcSin}[c x])) / (11 c^6)) / 11 + (6 d^3 ((x^5 (1 - c^2 x^2)^2 (a + b \operatorname{ArcSin}[c x])^2) / 9 - (2 b c ((b (8 x + (4 c^2 x^3) / 3 + (3 c^4 x^5) / 5 - (50 c^6 x^7) / 7 + (35 c^8 x^9) / 9)) / (315 c^5) - ((1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])) / (5 c^6) + (2 (1 - c^2 x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])) / (7 c^6) - ((1 - c^2 x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])) / (9 c^6)) / 9 + (4 ((x^5 (1 - c^2 x^2) (a + b \operatorname{ArcSin}[c x])^2) / 7 - (2 b c ((b (8 x + (4 c^2 x^3) / 3 + (3 c^4 x^5) / 5 - (15 c^6 x^7) / 7)) / (105 c^5) - ((1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])) / (3 c^6) + (2 (1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])) / (5 c^6) - ((1 - c^2 x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])) / (7 c^6)) / 7 + (2 ((x^5 (a + b \operatorname{ArcSin}[c x])^2) / 5 - (2 b c ((b x^5) / (25 c) - (x^4 \operatorname{Sqrt}[1 - c^2 x^2] (a + b \operatorname{ArcSin}[c x])) / (5 c^2) + (4 ((b x^3) / (9 c) - (x^2 \operatorname{Sqrt}[1 - c^2 x^2] (a + b \operatorname{ArcSin}[c x])) / (3 c^2) + (2 ((b x) / c - (\operatorname{Sqrt}[1 - c^2 x^2] (a + b \operatorname{ArcSin}[c x])) / c^2)) / (3 c^2)) / (5 c^2)) / 5) / 7) / 9) / 11 \end{aligned}$$

Defintions of rubi rules used

rule 15

$$\operatorname{Int}[(a_)(x_)^{(m_)}, x_Symbol] \rightarrow \operatorname{Simp}[a(x^{(m+1)}) / (m+1), x] /; \operatorname{FreeQ}\{a, m\}, x \ \&\& \operatorname{NeQ}[m, -1]$$

rule 24

$$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a x, x] /; \operatorname{FreeQ}[a, x]$$

rule 27

$$\operatorname{Int}[(a_)(F x_), x_Symbol] \rightarrow \operatorname{Simp}[a \operatorname{Int}[F x, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \operatorname{!MatchQ}[F x, (b_)(G x_)] /; \operatorname{FreeQ}[b, x]$$

rule 1467

$$\operatorname{Int}[(d_ + (e_)(x_)^2)^{(q_)}((a_ + (b_)(x_)^2 + (c_)(x_)^4)^{(p_)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e x^2)^q (a + b x^2 + c x^4)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{NeQ}[b^2 - 4 a c, 0] \ \&\& \operatorname{NeQ}[c d^2 - b d e + a e^2, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{IGtQ}[q, -2]$$

rule 2009

$$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{IntSum}[u, x], x] /; \operatorname{SumQ}[u]$$

rule 5138

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Simp
lifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.30

method	result
ordering	$\frac{(3448564875c^{14}x^{14}-16454567500c^{12}x^{12}+29885660250c^{10}x^{10}-23335495700c^8x^8+3719665587c^6x^6-16269505560c^4x^4+15161546400c^2x^2-4198582080)/x/c^6/(cx-1)^2/(cx+1)^2/(c^2x^2-1)^2(-c^2dx^2+d)^3(a+b\arcsin(cx))^2}{13867189875xc^6(cx-1)^2(cx+1)^2(c^2x^2-1)^2}$
parts	$-d^3a^2\left(\frac{1}{11}c^6x^{11}-\frac{1}{3}c^4x^9+\frac{3}{7}c^2x^7-\frac{1}{5}x^5\right)-\frac{d^3b^2\left(\frac{4(3c^4x^4-10c^2x^2+15)cx}{28875}-\frac{4\arcsin(cx)(c^2x^2-1)^2\sqrt{-c^2x^2+1}}{1925}\right)}{13867189875xc^6(cx-1)^2(cx+1)^2(c^2x^2-1)^2}$
derivativedivides	$-d^3a^2\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b^2\left(\frac{4(3c^4x^4-10c^2x^2+15)cx}{28875}-\frac{4\arcsin(cx)(c^2x^2-1)^2\sqrt{-c^2x^2+1}}{1925}-\frac{32\arcsin(cx)}{13867189875}\right)$
default	$-d^3a^2\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b^2\left(\frac{4(3c^4x^4-10c^2x^2+15)cx}{28875}-\frac{4\arcsin(cx)(c^2x^2-1)^2\sqrt{-c^2x^2+1}}{1925}-\frac{32\arcsin(cx)}{13867189875}\right)$

```
input int(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

```
output 1/13867189875*(3448564875*c^14*x^14-16454567500*c^12*x^12+29885660250*c^10*x^10-23335495700*c^8*x^8+3719665587*c^6*x^6-16269505560*c^4*x^4+15161546400*c^2*x^2-4198582080)/x/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2-1/13867189875*(312558750*c^12*x^12-1399654375*c^10*x^10+2243437625*c^8*x^8-1188259281*c^6*x^6-470882643*c^4*x^4-3178093380*c^2*x^2+1574468280)/x^4/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(4*x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2-6*x^5*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2*c^2*d+2*x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/13867189875/x^3*(10418625*c^10*x^10-42917875*c^8*x^8+58224375*c^6*x^6-13120569*c^4*x^4-29156820*c^2*x^2-174940920)/c^6/(c*x-1)^2/(c*x+1)^2*(12*x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2-54*x^4*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2*c^2*d+16*x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2))+24*x^6*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2*c^4*d^2-24*x^5*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))*c^3*d*b/(-c^2*x^2+1)^(1/2)+2*x^4*(-c^2*d*x^2+d)^3*b^2*c^2/(-c^2*x^2+1)+2*x^5*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))
```


Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.87

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx =$$

$$\frac{10418625 (121 a^2 - 2 b^2) c^{11} d^3 x^{11} - 471625 (9801 a^2 - 182 b^2) c^9 d^3 x^9 + 12375 (480249 a^2 - 9410 b^2) c^7 d^3 x^7 - 2079 (1334025 a^2 - 12622 b^2) c^5 d^3 x^5 + 58313640 b^2 c^3 d^3 x^3 + 34988 1840 b^2 c d^3 x + 12006225 (105 b^2 c^{11} d^3 x^{11} - 385 b^2 c^9 d^3 x^9 + 495 b^2 c^7 d^3 x^7 - 231 b^2 c^5 d^3 x^5) \arcsin(cx)^2 + 24012450 (105 a b c^{11} d^3 x^{11} - 385 a b c^9 d^3 x^9 + 495 a b c^7 d^3 x^7 - 231 a b c^5 d^3 x^5) \arcsin(cx) + 6930 (33075 a b c^{10} d^3 x^{10} - 111475 a b c^8 d^3 x^8 + 117625 a b c^6 d^3 x^6 - 18933 a b c^4 d^3 x^4 - 25244 a b c^2 d^3 x^2 - 50488 a b d^3 + (33075 b^2 c^{10} d^3 x^{10} - 111475 b^2 c^8 d^3 x^8 + 117625 b^2 c^6 d^3 x^6 - 18933 b^2 c^4 d^3 x^4 - 25244 b^2 c^2 d^3 x^2 - 50488 b^2 d^3) \arcsin(cx)) \sqrt{-c^2 x^2 + 1}}{c^5}$$

input `integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`output `-1/13867189875*(10418625*(121*a^2 - 2*b^2)*c^11*d^3*x^11 - 471625*(9801*a^2 - 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 - 9410*b^2)*c^7*d^3*x^7 - 2079*(1334025*a^2 - 12622*b^2)*c^5*d^3*x^5 + 58313640*b^2*c^3*d^3*x^3 + 349881840*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 - 385*b^2*c^9*d^3*x^9 + 495*b^2*c^7*d^3*x^7 - 231*b^2*c^5*d^3*x^5)*arcsin(c*x)^2 + 24012450*(105*a*b*c^11*d^3*x^11 - 385*a*b*c^9*d^3*x^9 + 495*a*b*c^7*d^3*x^7 - 231*a*b*c^5*d^3*x^5)*arcsin(c*x) + 6930*(33075*a*b*c^10*d^3*x^10 - 111475*a*b*c^8*d^3*x^8 + 117625*a*b*c^6*d^3*x^6 - 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3*x^2 - 50488*a*b*d^3 + (33075*b^2*c^10*d^3*x^10 - 111475*b^2*c^8*d^3*x^8 + 117625*b^2*c^6*d^3*x^6 - 18933*b^2*c^4*d^3*x^4 - 25244*b^2*c^2*d^3*x^2 - 50488*b^2*d^3)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c^5`**Sympy [A] (verification not implemented)**

Time = 3.16 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.47

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2,x)`

output

```
Piecewise((-a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 - 3*a**2*c**2*
d**3*x**7/7 + a**2*d**3*x**5/5 - 2*a*b*c**6*d**3*x**11*asin(c*x)/11 - 2*a*
b*c**5*d**3*x**10*sqrt(-c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*asin(c*x
)/3 + 182*a*b*c**3*d**3*x**8*sqrt(-c**2*x**2 + 1)/3267 - 6*a*b*c**2*d**3*x
**7*asin(c*x)/7 - 9410*a*b*c*d**3*x**6*sqrt(-c**2*x**2 + 1)/160083 + 2*a*b
*d**3*x**5*asin(c*x)/5 + 12622*a*b*d**3*x**4*sqrt(-c**2*x**2 + 1)/(1334025
*c) + 50488*a*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(4002075*c**3) + 100976*a*b
*d**3*sqrt(-c**2*x**2 + 1)/(4002075*c**5) - b**2*c**6*d**3*x**11*asin(c*x)
**2/11 + 2*b**2*c**6*d**3*x**11/1331 - 2*b**2*c**5*d**3*x**10*sqrt(-c**2*x
**2 + 1)*asin(c*x)/121 + b**2*c**4*d**3*x**9*asin(c*x)**2/3 - 182*b**2*c**
4*d**3*x**9/29403 + 182*b**2*c**3*d**3*x**8*sqrt(-c**2*x**2 + 1)*asin(c*x)
/3267 - 3*b**2*c**2*d**3*x**7*asin(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/11
20581 - 9410*b**2*c*d**3*x**6*sqrt(-c**2*x**2 + 1)*asin(c*x)/160083 + b**2
*d**3*x**5*asin(c*x)**2/5 - 12622*b**2*d**3*x**5/6670125 + 12622*b**2*d**3
*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(1
2006225*c**2) + 50488*b**2*d**3*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(40020
75*c**3) - 100976*b**2*d**3*x/(4002075*c**4) + 100976*b**2*d**3*sqrt(-c**2
*x**2 + 1)*asin(c*x)/(4002075*c**5), Ne(c, 0)), (a**2*d**3*x**5/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(421) = 842$.

Time = 0.18 (sec) , antiderivative size = 1141, normalized size of antiderivative = 2.40

$$\int x^4(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```

-1/11*b^2*c^6*d^3*x^11*arcsin(c*x)^2 - 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4
*d^3*x^9*arcsin(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 - 3/7*b^2*c^2*d^3*x^7*arcsin(
c*x)^2 - 3/7*a^2*c^2*d^3*x^7 - 2/7623*(693*x^11*arcsin(c*x) + (63*sqrt(-c^
2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(-c^2*x^2 + 1
)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(-c^2*x^2 + 1)*x^2/c^1
0 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 - 2/26413695*(3465*(63*sq
rt(-c^2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(-c^2*x^
2 + 1)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(-c^2*x^2 + 1)*x^
2/c^10 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c*arcsin(c*x) - (19845*c^10*x^11 + 2
6950*c^8*x^9 + 39600*c^6*x^7 + 66528*c^4*x^5 + 147840*c^2*x^3 + 887040*x)/
c^10)*b^2*c^6*d^3 + 1/5*b^2*d^3*x^5*arcsin(c*x)^2 + 2/945*(315*x^9*arcsin(
c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48
*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^
2*x^2 + 1)/c^10)*c)*a*b*c^4*d^3 + 2/297675*(315*(35*sqrt(-c^2*x^2 + 1)*x^8
/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*
sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c*arcsin(c*x) -
(1225*c^8*x^9 + 1800*c^6*x^7 + 3024*c^4*x^5 + 6720*c^2*x^3 + 40320*x)/c^8)
*b^2*c^4*d^3 + 1/5*a^2*d^3*x^5 - 6/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*
x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2
/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^3 - 2/8575*(105*(5*sqrt(...

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(421) = 842$.

Time = 0.18 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.82

$$\int x^4(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```

-1/11*a^2*c^6*d^3*x^11 + 1/3*a^2*c^4*d^3*x^9 - 3/7*a^2*c^2*d^3*x^7 + 1/5*a
^2*d^3*x^5 - 1/11*(c^2*x^2 - 1)^5*b^2*d^3*x*arcsin(c*x)^2/c^4 - 2/11*(c^2*
x^2 - 1)^5*a*b*d^3*x*arcsin(c*x)/c^4 - 4/33*(c^2*x^2 - 1)^4*b^2*d^3*x*arcs
in(c*x)^2/c^4 + 2/1331*(c^2*x^2 - 1)^5*b^2*d^3*x/c^4 - 8/33*(c^2*x^2 - 1)^
4*a*b*d^3*x*arcsin(c*x)/c^4 - 1/231*(c^2*x^2 - 1)^3*b^2*d^3*x*arcsin(c*x)^
2/c^4 - 2/121*(c^2*x^2 - 1)^5*sqrt(-c^2*x^2 + 1)*b^2*d^3*arcsin(c*x)/c^5 +
428/323433*(c^2*x^2 - 1)^4*b^2*d^3*x/c^4 - 2/231*(c^2*x^2 - 1)^3*a*b*d^3*
x*arcsin(c*x)/c^4 + 2/385*(c^2*x^2 - 1)^2*b^2*d^3*x*arcsin(c*x)^2/c^4 - 2/
121*(c^2*x^2 - 1)^5*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^5 - 8/297*(c^2*x^2 - 1)^4
*sqrt(-c^2*x^2 + 1)*b^2*d^3*arcsin(c*x)/c^5 - 148174/110937519*(c^2*x^2 -
1)^3*b^2*d^3*x/c^4 + 4/385*(c^2*x^2 - 1)^2*a*b*d^3*x*arcsin(c*x)/c^4 - 8/1
155*(c^2*x^2 - 1)*b^2*d^3*x*arcsin(c*x)^2/c^4 - 8/297*(c^2*x^2 - 1)^4*sqrt
(-c^2*x^2 + 1)*a*b*d^3/c^5 - 2/1617*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2
*d^3*arcsin(c*x)/c^5 + 5487704/4622396625*(c^2*x^2 - 1)^2*b^2*d^3*x/c^4 -
16/1155*(c^2*x^2 - 1)*a*b*d^3*x*arcsin(c*x)/c^4 + 16/1155*b^2*d^3*x*arcsin
(c*x)^2/c^4 - 2/1617*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^5 + 4/19
25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^3*arcsin(c*x)/c^5 - 606416/138
67189875*(c^2*x^2 - 1)*b^2*d^3*x/c^4 + 32/1155*a*b*d^3*x*arcsin(c*x)/c^4 +
4/1925*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^5 + 16/3465*(-c^2*x^2
+ 1)^(3/2)*b^2*d^3*arcsin(c*x)/c^5 - 382986368/13867189875*b^2*d^3*x/c...

```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int x^4 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^3 dx$$

input

```
int(x^4*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output

```
int(x^4*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^4 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^3 (-727650 \operatorname{asin}(cx) ab c^{11} x^{11} + 2668050 \operatorname{asin}(cx) ab c^9 x^9 - 3430350 \operatorname{asin}(cx) ab c^7 x^7 + 1600830 \operatorname{asin}(cx) ab c^5 x^5 - 6150 \sqrt{-c^2 x^2 + 1} ab c^{10} x^{10} + 222950 \sqrt{-c^2 x^2 + 1} ab c^8 x^8 - 235250 \sqrt{-c^2 x^2 + 1} ab c^6 x^6 + 37866 \sqrt{-c^2 x^2 + 1} ab c^4 x^4 + 50488 \sqrt{-c^2 x^2 + 1} ab c^2 x^2 + 100976 \sqrt{-c^2 x^2 + 1} ab - 4002075 \int (\operatorname{asin}(cx))^2 x^{10}, x) b^2 c^{11} + 12006225 \int (\operatorname{asin}(cx))^2 x^8, x) b^2 c^9 - 12006225 \int (\operatorname{asin}(cx))^2 x^6, x) b^2 c^7 + 4002075 \int (\operatorname{asin}(cx))^2 x^4, x) b^2 c^5 - 363825 a^2 c^{11} x^{11} + 1334025 a^2 c^9 x^9 - 1715175 a^2 c^7 x^7 + 800415 a^2 c^5 x^5)}{(4002075 c^5)}$$

input `int(x^4*(-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2,x)`

output `(d**3*(- 727650*asin(c*x)*a*b*c**11*x**11 + 2668050*asin(c*x)*a*b*c**9*x**9 - 3430350*asin(c*x)*a*b*c**7*x**7 + 1600830*asin(c*x)*a*b*c**5*x**5 - 6150*sqrt(-c**2*x**2 + 1)*a*b*c**10*x**10 + 222950*sqrt(-c**2*x**2 + 1)*a*b*c**8*x**8 - 235250*sqrt(-c**2*x**2 + 1)*a*b*c**6*x**6 + 37866*sqrt(-c**2*x**2 + 1)*a*b*c**4*x**4 + 50488*sqrt(-c**2*x**2 + 1)*a*b*c**2*x**2 + 100976*sqrt(-c**2*x**2 + 1)*a*b - 4002075*int(asin(c*x)**2*x**10,x)*b**2*c**11 + 12006225*int(asin(c*x)**2*x**8,x)*b**2*c**9 - 12006225*int(asin(c*x)**2*x**6,x)*b**2*c**7 + 4002075*int(asin(c*x)**2*x**4,x)*b**2*c**5 - 363825*a**2*c**11*x**11 + 1334025*a**2*c**9*x**9 - 1715175*a**2*c**7*x**7 + 800415*a**2*c**5*x**5))/(4002075*c**5)`

3.172 $\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$

Optimal result	1585
Mathematica [A] (verified)	1586
Rubi [B] (verified)	1587
Maple [A] (verified)	1593
Fricas [A] (verification not implemented)	1594
Sympy [A] (verification not implemented)	1595
Maxima [F]	1595
Giac [A] (verification not implemented)	1596
Mupad [F(-1)]	1597
Reduce [F]	1597

Optimal result

Integrand size = 27, antiderivative size = 384

$$\begin{aligned}
 \int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = & -\frac{79b^2 d^3 x^2}{5120c^2} - \frac{79b^2 d^3 x^4}{15360} + \frac{401b^2 c^2 d^3 x^6}{28800} \\
 & - \frac{57b^2 c^4 d^3 x^8}{6400} + \frac{1}{500} b^2 c^6 d^3 x^{10} \\
 & + \frac{79bd^3 x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{2560c^3} \\
 & + \frac{79bd^3 x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{3840c} \\
 & - \frac{31}{960} bcd^3 x^5 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \\
 & - \frac{1}{32} bcd^3 x^5 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \\
 & - \frac{1}{50} bcd^3 x^5 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) \\
 & - \frac{79d^3 (a + b \arcsin(cx))^2}{5120c^4} \\
 & + \frac{1}{40} d^3 x^4 (a + b \arcsin(cx))^2 \\
 & + \frac{1}{20} d^3 x^4 (1 - c^2 x^2) (a + b \arcsin(cx))^2 \\
 & + \frac{3}{40} d^3 x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 \\
 & + \frac{1}{10} d^3 x^4 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2
 \end{aligned}$$

output

```
-79/5120*b^2*d^3*x^2/c^2-79/15360*b^2*d^3*x^4+401/28800*b^2*c^2*d^3*x^6-57/6400*b^2*c^4*d^3*x^8+1/500*b^2*c^6*d^3*x^10+79/2560*b*d^3*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+79/3840*b*d^3*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c-31/960*b*c*d^3*x^5*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-1/32*b*c*d^3*x^5*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))-1/50*b*c*d^3*x^5*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))-79/5120*d^3*(a+b*arcsin(c*x))^2/c^4+1/40*d^3*x^4*(a+b*arcsin(c*x))^2+1/20*d^3*x^4*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+3/40*d^3*x^4*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2+1/10*d^3*x^4*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \frac{d^3 (cx(28800a^2c^3x^3(-10 + 20c^2x^2 - 15c^4x^4 + 4c^6x^6) + 30ab\sqrt{1 - c^2x^2}(-1185 - 790c^2x^2 + 3208c^4x^4$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/1152000*(d^3*(c*x*(28800*a^2*c^3*x^3*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6) + 30*a*b*Sqrt[1 - c^2*x^2]*(-1185 - 790*c^2*x^2 + 3208*c^4*x^4 - 2736*c^6*x^6 + 768*c^8*x^8) + b^2*(17775*c*x + 5925*c^3*x^3 - 16040*c^5*x^5 + 10260*c^7*x^7 - 2304*c^9*x^9)) + 30*b*(b*c*x*Sqrt[1 - c^2*x^2]*(-1185 - 790*c^2*x^2 + 3208*c^4*x^4 - 2736*c^6*x^6 + 768*c^8*x^8) + 15*a*(79 - 1280*c^4*x^4 + 2560*c^6*x^6 - 1920*c^8*x^8 + 512*c^10*x^10))*ArcSin[c*x] + 225*b^2*(79 - 1280*c^4*x^4 + 2560*c^6*x^6 - 1920*c^8*x^8 + 512*c^10*x^10)*ArcSin[c*x]^2))/c^4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 916 vs. $2(384) = 768$.

Time = 4.15 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.39, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {5202, 27, 5202, 243, 49, 2009, 5202, 244, 2009, 5138, 5198, 15, 5210, 15, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5202}$$

$$-\frac{1}{5}bcd^3 \int x^4 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{3}{5}d \int d^2 x^3 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx + \frac{1}{10}d^3 x^4 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

$$\downarrow \text{27}$$

$$-\frac{1}{5}bcd^3 \int x^4 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{3}{5}d^3 \int x^3 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx + \frac{1}{10}d^3 x^4 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

$$\downarrow \text{5202}$$

$$-\frac{1}{5}bcd^3 \left(\frac{1}{2} \int x^4 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{1}{10}bc \int x^5 (1 - c^2 x^2)^2 dx + \frac{1}{10}x^5 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2 \right) + \frac{3}{5}d^3 \left(-\frac{1}{4}bc \int x^4 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{2} \int x^3 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \frac{1}{8}x^4 (1 - c^2 x^2)^2 \right) + \frac{1}{10}d^3 x^4 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

$$\downarrow \text{243}$$

$$-\frac{1}{5}bcd^3 \left(\frac{1}{2} \int x^4 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{1}{20}bc \int x^4 (1 - c^2 x^2)^2 dx + \frac{1}{10}x^5 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2 \right) + \frac{3}{5}d^3 \left(-\frac{1}{4}bc \int x^4 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{2} \int x^3 (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + \frac{1}{8}x^4 (1 - c^2 x^2)^2 \right) + \frac{1}{10}d^3 x^4 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

↓ 49

$$\begin{aligned} & \frac{3}{5}d^3 \left(-\frac{1}{4}bc \int x^4(1-c^2x^2)^{3/2} (a+b \arcsin(cx))dx + \frac{1}{2} \int x^3(1-c^2x^2) (a+b \arcsin(cx))^2dx + \frac{1}{8}x^4(1-c^2x^2)^2 \right. \\ & \left. \frac{1}{5}bcd^3 \left(\frac{1}{2} \int x^4(1-c^2x^2)^{3/2} (a+b \arcsin(cx))dx - \frac{1}{20}bc \int (c^4x^8 - 2c^2x^6 + x^4) dx^2 + \frac{1}{10}x^5(1-c^2x^2)^{5/2} (a+b \right. \right. \\ & \left. \left. \frac{1}{10}d^3x^4(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \right) \end{aligned}$$

↓ 2009

$$\begin{aligned} & \frac{3}{5}d^3 \left(-\frac{1}{4}bc \int x^4(1-c^2x^2)^{3/2} (a+b \arcsin(cx))dx + \frac{1}{2} \int x^3(1-c^2x^2) (a+b \arcsin(cx))^2dx + \frac{1}{8}x^4(1-c^2x^2)^2 \right. \\ & \left. \frac{1}{5}bcd^3 \left(\frac{1}{2} \int x^4(1-c^2x^2)^{3/2} (a+b \arcsin(cx))dx + \frac{1}{10}x^5(1-c^2x^2)^{5/2} (a+b \arcsin(cx)) - \frac{1}{20}bc \left(\frac{c^4x^{10}}{5} - \frac{c^2x^8}{2} \right) \right. \right. \\ & \left. \left. \frac{1}{10}d^3x^4(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \right) \end{aligned}$$

↓ 5202

$$\begin{aligned} & \frac{3}{5}d^3 \left(-\frac{1}{4}bc \left(\frac{3}{8} \int x^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))dx - \frac{1}{8}bc \int x^5(1-c^2x^2) dx + \frac{1}{8}x^5(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right. \\ & \left. \frac{1}{5}bcd^3 \left(\frac{1}{2} \left(\frac{3}{8} \int x^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))dx - \frac{1}{8}bc \int x^5(1-c^2x^2) dx + \frac{1}{8}x^5(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right. \right. \\ & \left. \left. \frac{1}{10}d^3x^4(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \right) \end{aligned}$$

↓ 244

$$\begin{aligned} & \frac{3}{5}d^3 \left(\frac{1}{2} \left(-\frac{1}{3}bc \int x^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))dx + \frac{1}{3} \int x^3(a+b \arcsin(cx))^2dx + \frac{1}{6}x^4(1-c^2x^2) (a+b \arcsin(cx)) \right) \right. \\ & \left. \frac{1}{5}bcd^3 \left(\frac{1}{2} \left(\frac{3}{8} \int x^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))dx - \frac{1}{8}bc \int (x^5 - c^2x^7) dx + \frac{1}{8}x^5(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right. \right. \\ & \left. \left. \frac{1}{10}d^3x^4(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \right) \end{aligned}$$

↓ 2009

$$\begin{aligned} & \frac{3}{5}d^3 \left(\frac{1}{2} \left(-\frac{1}{3}bc \int x^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))dx + \frac{1}{3} \int x^3(a+b \arcsin(cx))^2dx + \frac{1}{6}x^4(1-c^2x^2) (a+b \arcsin(cx)) \right) \right. \\ & \left. \frac{1}{5}bcd^3 \left(\frac{1}{2} \left(\frac{3}{8} \int x^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))dx + \frac{1}{8}x^5(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{8}bc \left(\frac{x^6}{6} - \frac{c^2x^8}{8} \right) \right) \right) \right. \\ & \left. \frac{1}{10}d^3x^4(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \end{aligned}$$

↓ 5138

↓ 5210

$$\frac{1}{10}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2x^4 -$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}(1-c^2x^2)^{5/2}(a+b\arcsin(cx))x^5 - \frac{1}{20}bc\left(\frac{c^4x^{10}}{5} - \frac{c^2x^8}{2} + \frac{x^6}{3}\right) + \frac{1}{2}\left(\frac{1}{8}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))x^5 - \frac{1}{8}bc\left(\frac{x^6}{6} - \frac{c^2x^8}{8}\right) + \frac{3}{8}\right)\right)$$

↓ 15

$$\frac{1}{10}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2x^4 -$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}(1-c^2x^2)^{5/2}(a+b\arcsin(cx))x^5 - \frac{1}{20}bc\left(\frac{c^4x^{10}}{5} - \frac{c^2x^8}{2} + \frac{x^6}{3}\right) + \frac{1}{2}\left(\frac{1}{8}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))x^5 - \frac{1}{8}bc\left(\frac{x^6}{6} - \frac{c^2x^8}{8}\right) + \frac{3}{8}\right)\right)$$

↓ 5152

$$\frac{1}{10}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2x^4 -$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}(1-c^2x^2)^{5/2}(a+b\arcsin(cx))x^5 - \frac{1}{20}bc\left(\frac{c^4x^{10}}{5} - \frac{c^2x^8}{2} + \frac{x^6}{3}\right) + \frac{1}{2}\left(\frac{1}{8}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))x^5 - \frac{1}{8}bc\left(\frac{x^6}{6} - \frac{c^2x^8}{8}\right) + \frac{3}{8}\right)\right)$$

input

```
Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^3*x^4*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x])^2)/10 - (b*c*d^3*(-1/20*(b*c*
(x^6/3 - (c^2*x^8)/2 + (c^4*x^10)/5)) + (x^5*(1 - c^2*x^2)^(5/2)*(a + b*Ar
cSin[c*x]))/10 + (-1/8*(b*c*(x^6/6 - (c^2*x^8)/8)) + (x^5*(1 - c^2*x^2)^(3
/2)*(a + b*ArcSin[c*x]))/8 + (3*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 - c^2*x^2]*
(a + b*ArcSin[c*x]))/6 + ((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*Ar
cSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*Arc
Sin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2))/6)/8)/2))
/5 + (3*d^3*((x^4*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/8 - (b*c*(-1/8*(b
*c*(x^6/6 - (c^2*x^8)/8)) + (x^5*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/
8 + (3*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/6 +
((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(4*c^2) + (3
*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a +
b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2))/6)/8))/4 + ((x^4*(1 - c^2*x^2)*(a
+ b*ArcSin[c*x])^2)/6 - (b*c*(-1/36*(b*c*x^6) + (x^5*Sqrt[1 - c^2*x^2]*(a
+ b*ArcSin[c*x]))/6 + ((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcS
in[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin
[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/3 + ((x^4
*(a + b*ArcSin[c*x])^2)/4 - (b*c*((b*x^4)/(16*c) - (x^3*Sqrt[1 - c^2*x^2]*
(a + b*ArcSin[c*x]))/(4*c^2) + (3*((b*x^2)/(4*c) - (x*Sqrt[1 - c^2*x^2]*(a
+ b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/(4*c^2))...
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5138 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.35

method	result
parts	$-d^3 a^2 \left(\frac{1}{10} c^6 x^{10} - \frac{3}{8} c^4 x^8 + \frac{1}{2} c^2 x^6 - \frac{1}{4} x^4 \right) - \frac{d^3 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^4}{8} - \arcsin(cx) (-48 c^7 x^7 \sqrt{-c^2 x^2 + 1} + 200 c^5 x^5 \sqrt{-c^2 x^2 + 1} - 48 c^3 x^3 \sqrt{-c^2 x^2 + 1} + 48 c x \sqrt{-c^2 x^2 + 1}) \right)}{8}$
derivativedivides	$-d^3 a^2 \left(\frac{1}{10} c^{10} x^{10} - \frac{3}{8} c^8 x^8 + \frac{1}{2} c^6 x^6 - \frac{1}{4} c^4 x^4 \right) - d^3 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^4}{8} - \arcsin(cx) (-48 c^7 x^7 \sqrt{-c^2 x^2 + 1} + 200 c^5 x^5 \sqrt{-c^2 x^2 + 1} - 48 c^3 x^3 \sqrt{-c^2 x^2 + 1} + 48 c x \sqrt{-c^2 x^2 + 1}) \right)$
default	$-d^3 a^2 \left(\frac{1}{10} c^{10} x^{10} - \frac{3}{8} c^8 x^8 + \frac{1}{2} c^6 x^6 - \frac{1}{4} c^4 x^4 \right) - d^3 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^4}{8} - \arcsin(cx) (-48 c^7 x^7 \sqrt{-c^2 x^2 + 1} + 200 c^5 x^5 \sqrt{-c^2 x^2 + 1} - 48 c^3 x^3 \sqrt{-c^2 x^2 + 1} + 48 c x \sqrt{-c^2 x^2 + 1}) \right)$
orering	$\frac{(208128 x^{12} c^{12} - 1019388 c^{10} x^{10} + 1928796 c^8 x^8 - 1587835 c^6 x^6 - 38650 c^4 x^4 + 408825 c^2 x^2 - 118500) (-c^2 d x^2 + d)^3 (a + b \arcsin(cx))^2}{768000 c^4 (cx - 1)^2 (cx + 1)^2 (c^2 x^2 - 1)^2}$

input

```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-d^3*a^2*(1/10*c^6*x^10-3/8*c^4*x^8+1/2*c^2*x^6-1/4*x^4)-d^3*b^2/c^4*(1/8*
arcsin(c*x)^2*(c^2*x^2-1)^4-1/1536*arcsin(c*x)*(-48*c^7*x^7*(-c^2*x^2+1)^(
1/2)+200*c^5*x^5*(-c^2*x^2+1)^(1/2)-326*c^3*x^3*(-c^2*x^2+1)^(1/2)+279*c*x
*(-c^2*x^2+1)^(1/2)+105*arcsin(c*x))+49/5120*arcsin(c*x)^2-7/6400*(c^2*x^2
-1)^4+49/28800*(c^2*x^2-1)^3-49/15360*(c^2*x^2-1)^2+49/5120*c^2*x^2-49/512
0+1/10*arcsin(c*x)^2*(c^2*x^2-1)^5+1/6400*arcsin(c*x)*(128*c^9*x^9*(-c^2*x
^2+1)^(1/2)-656*c^7*x^7*(-c^2*x^2+1)^(1/2)+1368*c^5*x^5*(-c^2*x^2+1)^(1/2)
-1490*c^3*x^3*(-c^2*x^2+1)^(1/2)+965*c*x*(-c^2*x^2+1)^(1/2)+315*arcsin(c*x
))-1/500*(c^2*x^2-1)^5)-2*d^3*a*b/c^4*(1/10*arcsin(c*x)*c^10*x^10-3/8*arcs
in(c*x)*c^8*x^8+1/2*arcsin(c*x)*c^6*x^6-1/4*c^4*x^4*arcsin(c*x)-79/7680*c^
3*x^3*(-c^2*x^2+1)^(1/2)-79/5120*c*x*(-c^2*x^2+1)^(1/2)+79/5120*arcsin(c*x
)+401/9600*c^5*x^5*(-c^2*x^2+1)^(1/2)-57/1600*c^7*x^7*(-c^2*x^2+1)^(1/2)+1
/100*c^9*x^9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.03

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx =$$

$$\frac{2304 (50 a^2 - b^2) c^{10} d^3 x^{10} - 540 (800 a^2 - 19 b^2) c^8 d^3 x^8 + 40 (14400 a^2 - 401 b^2) c^6 d^3 x^6 - 75 (3840 a^2 -$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
-1/1152000*(2304*(50*a^2 - b^2)*c^10*d^3*x^10 - 540*(800*a^2 - 19*b^2)*c^8
*d^3*x^8 + 40*(14400*a^2 - 401*b^2)*c^6*d^3*x^6 - 75*(3840*a^2 - 79*b^2)*c
^4*d^3*x^4 + 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 - 1920*b^2
*c^8*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 - 1280*b^2*c^4*d^3*x^4 + 79*b^2*d^3)*a
rcsin(c*x)^2 + 450*(512*a*b*c^10*d^3*x^10 - 1920*a*b*c^8*d^3*x^8 + 2560*a*
b*c^6*d^3*x^6 - 1280*a*b*c^4*d^3*x^4 + 79*a*b*d^3)*arcsin(c*x) + 30*(768*a
*b*c^9*d^3*x^9 - 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^3*x^5 - 790*a*b*c^3
*d^3*x^3 - 1185*a*b*c*d^3*x + (768*b^2*c^9*d^3*x^9 - 2736*b^2*c^7*d^3*x^7
+ 3208*b^2*c^5*d^3*x^5 - 790*b^2*c^3*d^3*x^3 - 1185*b^2*c*d^3*x)*arcsin(c*
x))*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.70

$$\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2,x)`

output

```
Piecewise((-a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 - a**2*c**2*
d**3*x**6/2 + a**2*d**3*x**4/4 - a*b*c**6*d**3*x**10*asin(c*x)/5 - a*b*c**
5*d**3*x**9*sqrt(-c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*asin(c*x)/4 + 5
7*a*b*c**3*d**3*x**7*sqrt(-c**2*x**2 + 1)/800 - a*b*c**2*d**3*x**6*asin(c*
x) - 401*a*b*c*d**3*x**5*sqrt(-c**2*x**2 + 1)/4800 + a*b*d**3*x**4*asin(c*
x)/2 + 79*a*b*d**3*x**3*sqrt(-c**2*x**2 + 1)/(3840*c) + 79*a*b*d**3*x*sqrt
(-c**2*x**2 + 1)/(2560*c**3) - 79*a*b*d**3*asin(c*x)/(2560*c**4) - b**2*c*
**6*d**3*x**10*asin(c*x)**2/10 + b**2*c**6*d**3*x**10/500 - b**2*c**5*d**3*
x**9*sqrt(-c**2*x**2 + 1)*asin(c*x)/50 + 3*b**2*c**4*d**3*x**8*asin(c*x)**
2/8 - 57*b**2*c**4*d**3*x**8/6400 + 57*b**2*c**3*d**3*x**7*sqrt(-c**2*x**2
+ 1)*asin(c*x)/800 - b**2*c**2*d**3*x**6*asin(c*x)**2/2 + 401*b**2*c**2*d
**3*x**6/28800 - 401*b**2*c*d**3*x**5*sqrt(-c**2*x**2 + 1)*asin(c*x)/4800
+ b**2*d**3*x**4*asin(c*x)**2/4 - 79*b**2*d**3*x**4/15360 + 79*b**2*d**3*x
**3*sqrt(-c**2*x**2 + 1)*asin(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c**2
) + 79*b**2*d**3*x*sqrt(-c**2*x**2 + 1)*asin(c*x)/(2560*c**3) - 79*b**2*d*
**3*asin(c*x)**2/(5120*c**4), Ne(c, 0)), (a**2*d**3*x**4/4, True))
```

Maxima [F]

$$\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2 x^3 dx$$

input `integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

-1/10*a^2*c^6*d^3*x^10 + 3/8*a^2*c^4*d^3*x^8 - 1/2*a^2*c^2*d^3*x^6 - 1/640
0*(1280*x^10*arcsin(c*x) + (128*sqrt(-c^2*x^2 + 1)*x^9/c^2 + 144*sqrt(-c^2
*x^2 + 1)*x^7/c^4 + 168*sqrt(-c^2*x^2 + 1)*x^5/c^6 + 210*sqrt(-c^2*x^2 + 1
)*x^3/c^8 + 315*sqrt(-c^2*x^2 + 1)*x/c^10 - 315*arcsin(c*x)/c^11)*c)*a*b*c
^6*d^3 + 1/512*(384*x^8*arcsin(c*x) + (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*
sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2
*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*a*b*c^4*d^3 + 1/4*a^2*d^3*x^4 -
1/48*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^
2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*
c^2*d^3 + 1/16*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt
(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*a*b*d^3 - 1/40*(4*b^2*c^6*d^3
*x^10 - 15*b^2*c^4*d^3*x^8 + 20*b^2*c^2*d^3*x^6 - 10*b^2*d^3*x^4)*arctan2(
c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 - integrate(1/20*(4*b^2*c^7*d^3*x^10
- 15*b^2*c^5*d^3*x^8 + 20*b^2*c^3*d^3*x^6 - 10*b^2*c*d^3*x^4)*sqrt(c*x + 1
)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^2 - 1),
x)

```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.64

$$\int x^3(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
-1/10*a^2*c^6*d^3*x^10 + 3/8*a^2*c^4*d^3*x^8 - 1/2*a^2*c^2*d^3*x^6 + 1/4*a^2*d^3*x^4 - 1/50*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c^3 - 1/10*(c^2*x^2 - 1)^5*b^2*d^3*arcsin(c*x)^2/c^4 - 1/50*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c^3 - 7/800*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c^3 - 1/5*(c^2*x^2 - 1)^5*a*b*d^3*arcsin(c*x)/c^4 - 1/8*(c^2*x^2 - 1)^4*b^2*d^3*arcsin(c*x)^2/c^4 - 7/800*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c^3 + 49/4800*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c^3 + 1/500*(c^2*x^2 - 1)^5*b^2*d^3/c^4 - 1/4*(c^2*x^2 - 1)^4*a*b*d^3*arcsin(c*x)/c^4 + 49/4800*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c^3 + 49/3840*(-c^2*x^2 + 1)^(3/2)*b^2*d^3*x*arcsin(c*x)/c^3 + 7/6400*(c^2*x^2 - 1)^4*b^2*d^3/c^4 + 49/3840*(-c^2*x^2 + 1)^(3/2)*a*b*d^3*x/c^3 + 49/2560*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c^3 - 49/28800*(c^2*x^2 - 1)^3*b^2*d^3/c^4 + 49/2560*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c^3 + 49/15360*(c^2*x^2 - 1)^2*b^2*d^3/c^4 + 49/5120*b^2*d^3*arcsin(c*x)^2/c^4 - 49/5120*(c^2*x^2 - 1)*b^2*d^3/c^4 + 49/2560*a*b*d^3*arcsin(c*x)/c^4 - 232981/36864000*b^2*d^3/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int x^3 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^3 dx$$

input

```
int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output

```
int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^3 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^3 (-7680 \arcsin(cx) ab c^{10} x^{10} + 28800 \arcsin(cx) ab c^8 x^8 - 38400 \arcsin(cx) ab c^6 x^6 + 19200 \arcsin(cx) ab c^4 x^4 - \dots}{\dots}$$

input

```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2,x)
```

output

```
(d**3*( - 7680*asin(c*x)*a*b*c**10*x**10 + 28800*asin(c*x)*a*b*c**8*x**8 -
 38400*asin(c*x)*a*b*c**6*x**6 + 19200*asin(c*x)*a*b*c**4*x**4 - 1185*asin
(c*x)*a*b - 768*sqrt( - c**2*x**2 + 1)*a*b*c**9*x**9 + 2736*sqrt( - c**2*x
**2 + 1)*a*b*c**7*x**7 - 3208*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 + 790*s
qrt( - c**2*x**2 + 1)*a*b*c**3*x**3 + 1185*sqrt( - c**2*x**2 + 1)*a*b*c*x
- 38400*int(asin(c*x)**2*x**9,x)*b**2*c**10 + 115200*int(asin(c*x)**2*x**7
,x)*b**2*c**8 - 115200*int(asin(c*x)**2*x**5,x)*b**2*c**6 + 38400*int(asin
(c*x)**2*x**3,x)*b**2*c**4 - 3840*a**2*c**10*x**10 + 14400*a**2*c**8*x**8
- 19200*a**2*c**6*x**6 + 9600*a**2*c**4*x**4))/(38400*c**4)
```

3.173 $\int x^2(d - c^2dx^2)^3 (a + b \arcsin(cx))^2 dx$

Optimal result	1599
Mathematica [A] (verified)	1600
Rubi [A] (verified)	1600
Maple [A] (verified)	1607
Fricas [A] (verification not implemented)	1608
Sympy [A] (verification not implemented)	1609
Maxima [B] (verification not implemented)	1609
Giac [B] (verification not implemented)	1610
Mupad [F(-1)]	1611
Reduce [F]	1612

Optimal result

Integrand size = 27, antiderivative size = 391

$$\int x^2(d - c^2dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= -\frac{10516b^2d^3x}{99225c^2} - \frac{5258b^2d^3x^3}{297675} + \frac{4198b^2c^2d^3x^5}{165375} - \frac{374b^2c^4d^3x^7}{27783} + \frac{2}{729}b^2c^6d^3x^9$$

$$+ \frac{64bd^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{945c^3} + \frac{32bd^3x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{945c}$$

$$+ \frac{16bd^3(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{315c^3} + \frac{4bd^3(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{525c^3}$$

$$+ \frac{2bd^3(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{441c^3} - \frac{2bd^3(1 - c^2x^2)^{9/2}(a + b \arcsin(cx))}{81c^3}$$

$$+ \frac{16}{315}d^3x^3(a + b \arcsin(cx))^2 + \frac{8}{105}d^3x^3(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{2}{21}d^3x^3(1 - c^2x^2)^2(a + b \arcsin(cx))^2$$

output

```
-10516/99225*b^2*d^3*x/c^2-5258/297675*b^2*d^3*x^3+4198/165375*b^2*c^2*d^3
*x^5-374/27783*b^2*c^4*d^3*x^7+2/729*b^2*c^6*d^3*x^9+64/945*b*d^3*(-c^2*x^
2+1)^(1/2)*(a+b*arcsin(c*x))/c^3+32/945*b*d^3*x^2*(-c^2*x^2+1)^(1/2)*(a+b*
arcsin(c*x))/c+16/315*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c^3+4/525
*b*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c^3+2/441*b*d^3*(-c^2*x^2+1)^(
7/2)*(a+b*arcsin(c*x))/c^3-2/81*b*d^3*(-c^2*x^2+1)^(9/2)*(a+b*arcsin(c*x))
/c^3+16/315*d^3*x^3*(a+b*arcsin(c*x))^2+8/105*d^3*x^3*(-c^2*x^2+1)*(a+b*ar
csin(c*x))^2+2/21*d^3*x^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2+1/9*d^3*x^3*(
-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.71

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx =$$

$$\frac{d^3 (99225a^2 c^3 x^3 (-105 + 189c^2 x^2 - 135c^4 x^4 + 35c^6 x^6) + 630ab\sqrt{1 - c^2 x^2} (-5258 - 2629c^2 x^2 + 6297c$$

input

```
Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/31255875*(d^3*(99225*a^2*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35
*c^6*x^6) + 630*a*b*Sqrt[1 - c^2*x^2]*(-5258 - 2629*c^2*x^2 + 6297*c^4*x^4
- 4675*c^6*x^6 + 1225*c^8*x^8) + b^2*(3312540*c*x + 552090*c^3*x^3 - 7934
22*c^5*x^5 + 420750*c^7*x^7 - 85750*c^9*x^9) + 630*b*(315*a*c^3*x^3*(-105
+ 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-5258 - 2
629*c^2*x^2 + 6297*c^4*x^4 - 4675*c^6*x^6 + 1225*c^8*x^8))*ArcSin[c*x] + 9
9225*b^2*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6)*ArcSin[c*
x]^2))/c^3
```

Rubi [A] (verified)

Time = 3.21 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.37, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.741$, Rules used = {5202, 27, 5194, 27, 290, 2009, 5202, 5194, 27, 290, 2009, 5202, 5138, 5194, 27, 2009, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5202$$

$$-\frac{2}{9}bcd^3 \int x^3 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{2}{3}d \int d^2 x^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx + \frac{1}{9}d^3 x^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

$$\begin{aligned}
& \downarrow 27 \\
& \frac{2}{3}d^3 \int x^2(1-c^2x^2)^2(a+b\arcsin(cx))^2dx - \frac{2}{9}bcd^3 \int x^3(1-c^2x^2)^{5/2}(a+b\arcsin(cx))dx + \\
& \quad \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 \\
& \downarrow 5194 \\
& \frac{2}{3}d^3 \int x^2(1-c^2x^2)^2(a+b\arcsin(cx))^2dx - \\
& \frac{2}{9}bcd^3 \left(-bc \int -\frac{(1-c^2x^2)^3(7c^2x^2+2)}{63c^4}dx + \frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right. \\
& \quad \left. + \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 \right) \\
& \downarrow 27 \\
& \frac{2}{3}d^3 \int x^2(1-c^2x^2)^2(a+b\arcsin(cx))^2dx - \\
& \frac{2}{9}bcd^3 \left(\frac{b \int (1-c^2x^2)^3(7c^2x^2+2)dx}{63c^3} + \frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right) + \\
& \quad \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 \\
& \downarrow 290 \\
& \frac{2}{3}d^3 \int x^2(1-c^2x^2)^2(a+b\arcsin(cx))^2dx - \\
& \frac{2}{9}bcd^3 \left(\frac{b \int (-7c^8x^8+19c^6x^6-15c^4x^4+c^2x^2+2)dx}{63c^3} + \frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right) + \\
& \quad \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 \\
& \downarrow 2009 \\
& \frac{2}{3}d^3 \int x^2(1-c^2x^2)^2(a+b\arcsin(cx))^2dx + \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \\
& \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \\
& \downarrow 5202
\end{aligned}$$

$$\frac{2}{3}d^3 \left(\frac{4}{7} \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{7}bc \int x^3(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx + \frac{1}{7}x^3(1-c^2x^2)^2(a+b\arcsin(cx)) \right. \\ \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right)$$

↓ 5194

$$\frac{2}{3}d^3 \left(\frac{4}{7} \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{7}bc \left(-bc \int -\frac{(1-c^2x^2)^2(5c^2x^2+2)}{35c^4} dx + \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right) \right. \\ \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right)$$

↓ 27

$$\frac{2}{3}d^3 \left(\frac{4}{7} \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{7}bc \left(\frac{b \int (1-c^2x^2)^2(5c^2x^2+2) dx}{35c^3} + \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right) \right. \\ \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right)$$

↓ 290

$$\frac{2}{3}d^3 \left(\frac{4}{7} \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx - \frac{2}{7}bc \left(\frac{b \int (5c^6x^6 - 8c^4x^4 + c^2x^2 + 2) dx}{35c^3} + \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right) \right. \\ \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right)$$

↓ 2009

$$\begin{aligned} & \frac{2}{3}d^3 \left(\frac{4}{7} \int x^2(1-c^2x^2)(a+b\arcsin(cx))^2 dx + \frac{1}{7}x^3(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{2}{7}bc \left(\frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} \right. \right. \\ & \quad \left. \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right) \\ & \quad \downarrow 5202 \end{aligned}$$

$$\begin{aligned} & \frac{2}{3}d^3 \left(\frac{4}{7} \left(-\frac{2}{5}bc \int x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{2}{5} \int x^2(a+b\arcsin(cx))^2 dx + \frac{1}{5}x^3(1-c^2x^2)(a+b\arcsin(cx))^2 \right. \right. \\ & \quad \left. \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right) \\ & \quad \downarrow 5138 \end{aligned}$$

$$\begin{aligned} & \frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{5}bc \int x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx \right. \right. \\ & \quad \left. \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right) \\ & \quad \downarrow 5194 \end{aligned}$$

$$\begin{aligned} & \frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{5}bc \left(-bc \int -\frac{-3c^4x^4 + c^2x^2 + 2}{15c^4} dx + \right. \right. \right. \\ & \quad \left. \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b\arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b\left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3}\right)}{63c^3} \right) \right) \\ & \quad \downarrow 27 \end{aligned}$$

$$\frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{5}bc \left(\frac{b \int (-3c^4x^4 + c^2x^2 + 2) dx}{15c^3} + (1 - c^2x^2)^{9/2} (a+b\arcsin(cx)) - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3})}{63c^3} \right) \right)$$

↓ 2009

$$\frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc \int \frac{x^3(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{5}x^3(1-c^2x^2)(a+b\arcsin(cx))^2 - \frac{2}{5}bc \left(\frac{b \int (-3c^4x^4 + c^2x^2 + 2) dx}{15c^3} + (1 - c^2x^2)^{9/2} (a+b\arcsin(cx)) - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3})}{63c^3} \right) \right)$$

↓ 5210

$$\frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{b \int x^2 dx}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} \right) - \frac{2}{5}bc \left(\frac{b \int (-3c^4x^4 + c^2x^2 + 2) dx}{15c^3} + (1 - c^2x^2)^{9/2} (a+b\arcsin(cx)) - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3})}{63c^3} \right) \right)$$

↓ 15

$$\frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a+b\arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right) - \frac{2}{5}bc \left(\frac{b \int (-3c^4x^4 + c^2x^2 + 2) dx}{15c^3} + (1 - c^2x^2)^{9/2} (a+b\arcsin(cx)) - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{(1-c^2x^2)^{7/2}(a+b\arcsin(cx))}{7c^4} + \frac{b(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3})}{63c^3} \right) \right)$$

↓ 5182

$$\begin{aligned}
& \frac{2}{3}d^3 \left(\frac{4}{7} \left(\frac{2}{5} \left(\frac{1}{3}x^3(a + b \arcsin(cx))^2 - \frac{2}{3}bc \left(\frac{2 \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} \right. \right. \right. \\
& \qquad \qquad \qquad \left. \left. \left. - \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 - \right. \right. \right. \\
& \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b \arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b \arcsin(cx))}{7c^4} + \frac{b \left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3} \right)}{63c^3} \right. \\
& \qquad \qquad \qquad \downarrow 24 \\
& \frac{1}{9}d^3x^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 + \\
& \frac{2}{3}d^3 \left(\frac{1}{7}x^3(1-c^2x^2)^2(a+b \arcsin(cx))^2 + \frac{4}{7} \left(\frac{1}{5}x^3(1-c^2x^2)(a+b \arcsin(cx))^2 + \frac{2}{5} \left(\frac{1}{3}x^3(a+b \arcsin(cx))^2 - \right. \right. \right. \\
& \left. \left. \left. \frac{2}{9}bcd^3 \left(\frac{(1-c^2x^2)^{9/2}(a+b \arcsin(cx))}{9c^4} - \frac{(1-c^2x^2)^{7/2}(a+b \arcsin(cx))}{7c^4} + \frac{b \left(-\frac{7}{9}c^8x^9 + \frac{19c^6x^7}{7} - 3c^4x^5 + \frac{c^2x^3}{3} \right)}{63c^3} \right) \right. \right.
\end{aligned}$$

input `Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]`

output `(d^3*x^3*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x])^2)/9 - (2*b*c*d^3*((b*(2*x + (c^2*x^3)/3 - 3*c^4*x^5 + (19*c^6*x^7)/7 - (7*c^8*x^9)/9))/(63*c^3) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^4) + ((1 - c^2*x^2)^(9/2)*(a + b*ArcSin[c*x]))/(9*c^4))/9 + (2*d^3*((x^3*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/7 - (2*b*c*((b*(2*x + (c^2*x^3)/3 - (8*c^4*x^5)/5 + (5*c^6*x^7)/7))/(35*c^3) - ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^4) + ((1 - c^2*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^4)))/7 + (4*((x^3*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*((b*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)/5))/(15*c^3) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^4) + ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^4)))/5 + (2*((x^3*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*((b*x^3)/(9*c) - (x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(3*c^2) + (2*((b*x)/c - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/(3*c^2)))/3))/5)/7)/3`

Defintions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 24 $\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$
- rule 27 $\text{Int}[(a_)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[Fx, (b_)(Gx_)] \text{ ; FreeQ}[b, x]$
- rule 290 $\text{Int}[(a_) + (b_.)(x_)^2)^{(p_.)}*((c_) + (d_.)(x_)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^2)^p*(c + d*x^2)^q, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IGtQ}[q, 0]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 5138 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*(x_)*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] \text{ ; FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$
- rule 5194 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))*(x_)^{(m_.)}*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[x^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[d + e*x^2], x], x], x]] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p - 1/2] \ \&\& \ \text{NeQ}[p, -2^{(-1)}] \ \&\& \ (\text{IGtQ}[(m+1)/2, 0] \ || \ \text{ILtQ}[(m+2*p+3)/2, 0])$

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1)), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.34

method	result
parts	$-d^3 a^2 \left(\frac{1}{9} x^9 c^6 - \frac{3}{7} c^4 x^7 + \frac{3}{5} c^2 x^5 - \frac{1}{3} x^3 \right) - \frac{d^3 b^2 \left(\frac{\arcsin(cx)^2 (5c^6 x^6 - 21c^4 x^4 + 35c^2 x^2 - 35) cx}{35} + \frac{2 \arcsin(cx) (c^2 x^2 - 1) \sqrt{-c^2 d + e}}{441} \right)}{3125875 x c^4 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)^2}$
derivativedivides	$-d^3 a^2 \left(\frac{1}{9} c^9 x^9 - \frac{3}{7} c^7 x^7 + \frac{3}{5} c^5 x^5 - \frac{1}{3} c^3 x^3 \right) - d^3 b^2 \left(\frac{\arcsin(cx)^2 (5c^6 x^6 - 21c^4 x^4 + 35c^2 x^2 - 35) cx}{35} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^3 \sqrt{-c^2 d + e}}{441} \right)$
default	$-d^3 a^2 \left(\frac{1}{9} c^9 x^9 - \frac{3}{7} c^7 x^7 + \frac{3}{5} c^5 x^5 - \frac{1}{3} c^3 x^3 \right) - d^3 b^2 \left(\frac{\arcsin(cx)^2 (5c^6 x^6 - 21c^4 x^4 + 35c^2 x^2 - 35) cx}{35} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^3 \sqrt{-c^2 d + e}}{441} \right)$
orering	$\frac{(9303875 x^{12} c^{12} - 47172500 c^{10} x^{10} + 95052594 c^8 x^8 - 88615068 c^6 x^6 - 86474829 c^4 x^4 + 59625720 c^2 x^2 - 13250160) (-c^2 d + e)}{3125875 x c^4 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)^2}$

input

```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-d^3*a^2*(1/9*x^9*c^6-3/7*c^4*x^7+3/5*c^2*x^5-1/3*x^3)-d^3*b^2/c^3*(1/35*arcsin(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x+2/441*arcsin(c*x)*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-2/15435*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-4/525*arcsin(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)+4/7875*(3*c^4*x^4-10*c^2*x^2+15)*c*x+16/945*arcsin(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/2835*(c^2*x^2-3)*c*x+32/315*c*x-32/315*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+1/315*arcsin(c*x)^2*(35*c^8*x^8-180*c^6*x^6+378*c^4*x^4-420*c^2*x^2+315)*c*x+2/81*arcsin(c*x)*(c^2*x^2-1)^4*(-c^2*x^2+1)^(1/2)-2/25515*(35*c^8*x^8-180*c^6*x^6+378*c^4*x^4-420*c^2*x^2+315)*c*x)-2*d^3*a*b/c^3*(1/9*arcsin(c*x)*c^9*x^9-3/7*arcsin(c*x)*c^7*x^7+3/5*c^5*x^5*arcsin(c*x)-1/3*c^3*x^3*arcsin(c*x)-2629/99225*c^2*x^2*(-c^2*x^2+1)^(1/2)-5258/99225*(-c^2*x^2+1)^(1/2)+2099/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)-187/3969*c^6*x^6*(-c^2*x^2+1)^(1/2)+1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.95

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx =$$

$$\frac{42875(81a^2 - 2b^2)c^9d^3x^9 - 1125(11907a^2 - 374b^2)c^7d^3x^7 + 189(99225a^2 - 4198b^2)c^5d^3x^5 - 105(99225a^2 - 5258b^2)c^3d^3x^3 + 3312540b^2c^2d^3x + 99225(35b^2c^9d^3x^9 - 135b^2c^7d^3x^7 + 189b^2c^5d^3x^5 - 105b^2c^3d^3x^3)*\arcsin(cx)^2 + 198450(35a*b*c^9d^3x^9 - 135a*b*c^7d^3x^7 + 189a*b*c^5d^3x^5 - 105a*b*c^3d^3x^3)*\arcsin(cx) + 630(1225a*b*c^8d^3x^8 - 4675a*b*c^6d^3x^6 + 6297a*b*c^4d^3x^4 - 2629a*b*c^2d^3x^2 - 5258a*b*d^3 + (1225b^2c^8d^3x^8 - 4675b^2c^6d^3x^6 + 6297b^2c^4d^3x^4 - 2629b^2c^2d^3x^2 - 5258b^2d^3)*\arcsin(cx))*\sqrt{-c^2x^2 + 1}}{c^3}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
-1/31255875*(42875*(81*a^2 - 2*b^2)*c^9*d^3*x^9 - 1125*(11907*a^2 - 374*b^2)*c^7*d^3*x^7 + 189*(99225*a^2 - 4198*b^2)*c^5*d^3*x^5 - 105*(99225*a^2 - 5258*b^2)*c^3*d^3*x^3 + 3312540*b^2*c^2*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 - 135*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 - 105*b^2*c^3*d^3*x^3)*arcsin(c*x)^2 + 198450*(35*a*b*c^9*d^3*x^9 - 135*a*b*c^7*d^3*x^7 + 189*a*b*c^5*d^3*x^5 - 105*a*b*c^3*d^3*x^3)*arcsin(c*x) + 630*(1225*a*b*c^8*d^3*x^8 - 4675*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 - 2629*a*b*c^2*d^3*x^2 - 5258*a*b*d^3 + (1225*b^2*c^8*d^3*x^8 - 4675*b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4 - 2629*b^2*c^2*d^3*x^2 - 5258*b^2*d^3)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.60

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^6 d^3 x^9}{9} + \frac{3a^2 c^4 d^3 x^7}{7} - \frac{3a^2 c^2 d^3 x^5}{5} + \frac{a^2 d^3 x^3}{3} - \frac{2abc^6 d^3 x^9 \arcsin(cx)}{9} - \frac{2abc^5 d^3 x^8 \sqrt{-c^2 x^2 + 1}}{81} + \frac{6abc^4 d^3 x^7 \arcsin(cx)}{7} + \frac{374abc^3 d^3 x^6 \sqrt{-c^2 x^2 + 1}}{3969} - \frac{6abc^2 d^3 x^5 \arcsin(cx)}{5} - \frac{4198abc d^3 x^4 \sqrt{-c^2 x^2 + 1}}{33075} + \frac{2abd^3 x^3 \arcsin(cx)}{3} + \frac{5258abd^3 x^2 \sqrt{-c^2 x^2 + 1}}{(99225c)} + \frac{10516abd^3 \sqrt{-c^2 x^2 + 1}}{(99225c^2)} - \frac{b^2 c^6 d^3 x^9 \arcsin(cx)^2}{9} + \frac{2b^2 c^6 d^3 x^9}{729} - \frac{2b^2 c^5 d^3 x^8 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{81} + \frac{3b^2 c^4 d^3 x^7 \arcsin(cx)^2}{7} - \frac{374b^2 c^4 d^3 x^7}{27783} + \frac{374b^2 c^3 d^3 x^6 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{3969} - \frac{3b^2 c^2 d^3 x^5 \arcsin(cx)^2}{5} + \frac{4198b^2 c^2 d^3 x^5}{165375} - \frac{4198b^2 c d^3 x^4 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{33075} + \frac{b^2 d^3 x^3 \arcsin(cx)^2}{3} - \frac{5258b^2 d^3 x^3}{297675} + \frac{5258b^2 d^3 x^2 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{(99225c)} - \frac{10516b^2 d^3 x}{(99225c^2)} + \frac{10516b^2 d^3 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{(99225c^3)}, \text{Ne}(c, 0) \end{cases}$$

input

```
integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2,x)
```

output

```
Piecewise((-a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 - 3*a**2*c**2*d**3*x**5/5 + a**2*d**3*x**3/3 - 2*a*b*c**6*d**3*x**9*asin(c*x)/9 - 2*a*b*c**5*d**3*x**8*sqrt(-c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*asin(c*x)/7 + 374*a*b*c**3*d**3*x**6*sqrt(-c**2*x**2 + 1)/3969 - 6*a*b*c**2*d**3*x**5*asin(c*x)/5 - 4198*a*b*c*d**3*x**4*sqrt(-c**2*x**2 + 1)/33075 + 2*a*b*d**3*x**3*asin(c*x)/3 + 5258*a*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(99225*c) + 10516*a*b*d**3*sqrt(-c**2*x**2 + 1)/(99225*c**2) - b**2*c**6*d**3*x**9*asin(c*x)**2/9 + 2*b**2*c**6*d**3*x**9/729 - 2*b**2*c**5*d**3*x**8*sqrt(-c**2*x**2 + 1)*asin(c*x)/81 + 3*b**2*c**4*d**3*x**7*asin(c*x)**2/7 - 374*b**2*c**4*d**3*x**7/27783 + 374*b**2*c**3*d**3*x**6*sqrt(-c**2*x**2 + 1)*asin(c*x)/3969 - 3*b**2*c**2*d**3*x**5*asin(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/165375 - 4198*b**2*c*d**3*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)/33075 + b**2*d**3*x**3*asin(c*x)**2/3 - 5258*b**2*d**3*x**3/297675 + 5258*b**2*d**3*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c**2) + 10516*b**2*d**3*sqrt(-c**2*x**2 + 1)*asin(c*x)/(99225*c**3), Ne(c, 0)), (a**2*d**3*x**3/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. 2(346) = 692.

Time = 0.17 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.42

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```

-1/9*b^2*c^6*d^3*x^9*arcsin(c*x)^2 - 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3
*x^7*arcsin(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 - 3/5*b^2*c^2*d^3*x^5*arcsin(c*x)
^2 - 2/2835*(315*x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt
(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2
+ 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 - 2/893025*(31
5*(35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt
(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2
+ 1)/c^10)*c*arcsin(c*x) - (1225*c^8*x^9 + 1800*c^6*x^7 + 3024*c^4*x^5 +
6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^6*d^3 - 3/5*a^2*c^2*d^3*x^5 + 6/245*(35
*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^
4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c
^4*d^3 + 2/8575*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*
x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arcs
in(c*x) - (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^4*d
^3 + 1/3*b^2*d^3*x^3*arcsin(c*x)^2 - 2/25*(15*x^5*arcsin(c*x) + (3*sqrt(-c
^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/
c^6)*c)*a*b*c^2*d^3 - 2/375*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^
2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arcsin(c*x) - (9*c^4*x^5
+ 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcs
in(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a...

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(346) = 692$.

Time = 0.17 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.83

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```

-1/9*a^2*c^6*d^3*x^9 + 3/7*a^2*c^4*d^3*x^7 - 3/5*a^2*c^2*d^3*x^5 - 1/9*(c^
2*x^2 - 1)^4*b^2*d^3*x*arcsin(c*x)^2/c^2 - 2/9*(c^2*x^2 - 1)^4*a*b*d^3*x*a
rcsin(c*x)/c^2 - 1/63*(c^2*x^2 - 1)^3*b^2*d^3*x*arcsin(c*x)^2/c^2 + 2/729*
(c^2*x^2 - 1)^4*b^2*d^3*x/c^2 + 1/3*a^2*d^3*x^3 - 2/63*(c^2*x^2 - 1)^3*a*b
*d^3*x*arcsin(c*x)/c^2 + 2/105*(c^2*x^2 - 1)^2*b^2*d^3*x*arcsin(c*x)^2/c^2
- 2/81*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b^2*d^3*arcsin(c*x)/c^3 - 622/2
50047*(c^2*x^2 - 1)^3*b^2*d^3*x/c^2 + 4/105*(c^2*x^2 - 1)^2*a*b*d^3*x*arcs
in(c*x)/c^2 - 8/315*(c^2*x^2 - 1)*b^2*d^3*x*arcsin(c*x)^2/c^2 - 2/81*(c^2*
x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^3 - 2/441*(c^2*x^2 - 1)^3*sqrt(-c^
2*x^2 + 1)*b^2*d^3*arcsin(c*x)/c^3 + 15224/10418625*(c^2*x^2 - 1)^2*b^2*d^
3*x/c^2 - 16/315*(c^2*x^2 - 1)*a*b*d^3*x*arcsin(c*x)/c^2 + 16/315*b^2*d^3*
x*arcsin(c*x)^2/c^2 - 2/441*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^3
+ 4/525*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^3*arcsin(c*x)/c^3 + 1155
04/31255875*(c^2*x^2 - 1)*b^2*d^3*x/c^2 + 32/315*a*b*d^3*x*arcsin(c*x)/c^2
+ 4/525*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^3 + 16/945*(-c^2*x^2
+ 1)^(3/2)*b^2*d^3*arcsin(c*x)/c^3 - 3406208/31255875*b^2*d^3*x/c^2 + 16/
945*(-c^2*x^2 + 1)^(3/2)*a*b*d^3/c^3 + 32/315*sqrt(-c^2*x^2 + 1)*b^2*d^3*a
rcsin(c*x)/c^3 + 32/315*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^3

```

Mupad [F(-1)]

Timed out.

$$\int x^2(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int x^2 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^3 dx$$

input

```
int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output

```
int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3, x)
```


Reduce [F]

$$\int x^2 (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^3 (-22050 \arcsin(cx) ab c^9 x^9 + 85050 \arcsin(cx) ab c^7 x^7 - 119070 \arcsin(cx) ab c^5 x^5 + 66150 \arcsin(cx) ab c^3 x^3 - \dots}{\dots}$$

input `int(x^2*(-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2,x)`

output

```
(d**3*( - 22050*asin(c*x)*a*b*c**9*x**9 + 85050*asin(c*x)*a*b*c**7*x**7 -
119070*asin(c*x)*a*b*c**5*x**5 + 66150*asin(c*x)*a*b*c**3*x**3 - 2450*sqrt
( - c**2*x**2 + 1)*a*b*c**8*x**8 + 9350*sqrt( - c**2*x**2 + 1)*a*b*c**6*x*
*6 - 12594*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 + 5258*sqrt( - c**2*x**2 +
1)*a*b*c**2*x**2 + 10516*sqrt( - c**2*x**2 + 1)*a*b - 99225*int(asin(c*x)
**2*x**8,x)*b**2*c**9 + 297675*int(asin(c*x)**2*x**6,x)*b**2*c**7 - 297675
*int(asin(c*x)**2*x**4,x)*b**2*c**5 + 99225*int(asin(c*x)**2*x**2,x)*b**2*
c**3 - 11025*a**2*c**9*x**9 + 42525*a**2*c**7*x**7 - 59535*a**2*c**5*x**5
+ 33075*a**2*c**3*x**3))/(99225*c**3)
```

3.174 $\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$

Optimal result	1613
Mathematica [A] (verified)	1614
Rubi [A] (verified)	1614
Maple [A] (verified)	1618
Fricas [A] (verification not implemented)	1619
Sympy [B] (verification not implemented)	1620
Maxima [F]	1620
Giac [B] (verification not implemented)	1621
Mupad [F(-1)]	1623
Reduce [F]	1623

Optimal result

Integrand size = 25, antiderivative size = 277

$$\begin{aligned}
 \int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = & -\frac{35b^2 d^3 x^2}{1024} + \frac{35b^2 d^3 (1 - c^2 x^2)^2}{3072c^2} \\
 & + \frac{7b^2 d^3 (1 - c^2 x^2)^3}{1152c^2} + \frac{b^2 d^3 (1 - c^2 x^2)^4}{256c^2} \\
 & + \frac{35bd^3 x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{512c} \\
 & + \frac{35bd^3 x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{768c} \\
 & + \frac{7bd^3 x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{192c} \\
 & + \frac{bd^3 x (1 - c^2 x^2)^{7/2} (a + b \arcsin(cx))}{32c} \\
 & + \frac{35d^3 (a + b \arcsin(cx))^2}{1024c^2} \\
 & - \frac{d^3 (1 - c^2 x^2)^4 (a + b \arcsin(cx))^2}{8c^2}
 \end{aligned}$$

output

```
-35/1024*b^2*d^3*x^2+35/3072*b^2*d^3*(-c^2*x^2+1)^2/c^2+7/1152*b^2*d^3*(-c^2*x^2+1)^3/c^2+1/256*b^2*d^3*(-c^2*x^2+1)^4/c^2+35/512*b*d^3*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+35/768*b*d^3*x*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c+7/192*b*d^3*x*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/c+1/32*b*d^3*x*(-c^2*x^2+1)^(7/2)*(a+b*arcsin(c*x))/c+35/1024*d^3*(a+b*arcsin(c*x))^2/c^2-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arcsin(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.93

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx =$$

$$\frac{d^3 (cx(b^2 cx(837 - 489c^2 x^2 + 200c^4 x^4 - 36c^6 x^6) + 1152a^2 cx(-4 + 6c^2 x^2 - 4c^4 x^4 + c^6 x^6) + 6ab\sqrt{1 - c^2 x^2}))}{c^2}$$

input

```
Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/9216*(d^3*(c*x*(b^2*c*x*(837 - 489*c^2*x^2 + 200*c^4*x^4 - 36*c^6*x^6) + 1152*a^2*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6) + 6*a*b*Sqrt[1 - c^2*x^2]*(-279 + 326*c^2*x^2 - 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*(b*c*x*Sqrt[1 - c^2*x^2]*(-279 + 326*c^2*x^2 - 200*c^4*x^4 + 48*c^6*x^6) + 3*a*(93 - 512*c^2*x^2 + 768*c^4*x^4 - 512*c^6*x^6 + 128*c^8*x^8))*ArcSin[c*x] + 9*b^2*(93 - 512*c^2*x^2 + 768*c^4*x^4 - 512*c^6*x^6 + 128*c^8*x^8)*ArcSin[c*x]^2))/c^2
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5182, 5158, 241, 5158, 241, 5158, 244, 2009, 5156, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2x^2)^3 (a + b \arcsin(cx))^2 dx$$

↓ 5182

$$\frac{bd^3 \int (1 - c^2x^2)^{7/2} (a + b \arcsin(cx)) dx}{4c} - \frac{d^3(1 - c^2x^2)^4 (a + b \arcsin(cx))^2}{8c^2}$$

↓ 5158

$$\frac{bd^3 \left(\frac{7}{8} \int (1 - c^2x^2)^{5/2} (a + b \arcsin(cx)) dx - \frac{1}{8} bc \int x(1 - c^2x^2)^3 dx + \frac{1}{8} x(1 - c^2x^2)^{7/2} (a + b \arcsin(cx)) \right)}{4c}$$

$$\frac{d^3(1 - c^2x^2)^4 (a + b \arcsin(cx))^2}{8c^2}$$

↓ 241

$$\frac{bd^3 \left(\frac{7}{8} \int (1 - c^2x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{1}{8} x(1 - c^2x^2)^{7/2} (a + b \arcsin(cx)) + \frac{b(1 - c^2x^2)^4}{64c} \right)}{4c}$$

$$\frac{d^3(1 - c^2x^2)^4 (a + b \arcsin(cx))^2}{8c^2}$$

↓ 5158

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \int (1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{1}{6} bc \int x(1 - c^2x^2)^2 dx + \frac{1}{6} x(1 - c^2x^2)^{5/2} (a + b \arcsin(cx)) \right) \right)}{4c}$$

$$\frac{d^3(1 - c^2x^2)^4 (a + b \arcsin(cx))^2}{8c^2}$$

↓ 241

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \int (1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{6} x(1 - c^2x^2)^{5/2} (a + b \arcsin(cx)) + \frac{b(1 - c^2x^2)^3}{36c} \right) + \frac{1}{8} x(1 - c^2x^2)^{7/2} (a + b \arcsin(cx)) \right)}{4c}$$

$$\frac{d^3(1 - c^2x^2)^4 (a + b \arcsin(cx))^2}{8c^2}$$

↓ 5158

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx - \frac{1}{4} bc \int x(1-c^2x^2) dx + \frac{1}{4} x(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right) + \frac{1}{6} x \right)}{4c}$$

$$\frac{d^3(1-c^2x^2)^4 (a+b \arcsin(cx))^2}{8c^2}$$

↓ 244

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx - \frac{1}{4} bc \int (x-c^2x^3) dx + \frac{1}{4} x(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right) + \frac{1}{6} x \right)}{4c}$$

$$\frac{d^3(1-c^2x^2)^4 (a+b \arcsin(cx))^2}{8c^2}$$

↓ 2009

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx + \frac{1}{4} x(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{x^2}{2} - \frac{c^2x^4}{4} \right) \right) \right) + \frac{1}{6} x(1-c^2x^2) \right)}{4c}$$

$$\frac{d^3(1-c^2x^2)^4 (a+b \arcsin(cx))^2}{8c^2}$$

↓ 5156

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2} bc \int x dx + \frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) \right) \right) + \frac{1}{4} x(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right)}{4c}$$

$$\frac{d^3(1-c^2x^2)^4 (a+b \arcsin(cx))^2}{8c^2}$$

↓ 15

$$\frac{bd^3 \left(\frac{7}{8} \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1-c^2x^2} (a+b \arcsin(cx)) - \frac{1}{4} bcx^2 \right) \right) + \frac{1}{4} x(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) \right) \right)}{4c}$$

$$\frac{d^3(1-c^2x^2)^4 (a+b \arcsin(cx))^2}{8c^2}$$

↓ 5152

$$bd^3 \left(\frac{1}{8}x(1-c^2x^2)^{7/2}(a+b\arcsin(cx)) + \frac{7}{8} \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{d^3(1-c^2x^2)^4(a+b\arcsin(cx))^2}{8c^2} \right) \right) \right)$$

input `Int[x*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]`

output `-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcSin[c*x])^2)/c^2 + (b*d^3*((b*(1 - c^2*x^2)^4)/(64*c) + (x*(1 - c^2*x^2)^(7/2)*(a + b*ArcSin[c*x]))/8 + (7*((b*(1 - c^2*x^2)^3)/(36*c) + (x*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/6 + (5*(-1/4*(b*c*(x^2/2 - (c^2*x^4)/4)) + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/4 + (3*(-1/4*(b*c*x^2) + (x*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/2 + (a + b*ArcSin[c*x])^2/(4*b*c)))/4))/8))/(4*c)`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 244 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p, 0]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.21

method	result
derivativedivides	$-\frac{d^3 a^2 (c^2 x^2 - 1)^4}{8} - d^3 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^4}{8} - \frac{\arcsin(cx) (-48c^7 x^7 \sqrt{-c^2 x^2 + 1} + 200c^5 x^5 \sqrt{-c^2 x^2 + 1} - 326c^3 x^3 \sqrt{-c^2 x^2 + 1})}{1536} \right)$
default	$-\frac{d^3 a^2 (c^2 x^2 - 1)^4}{8} - d^3 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^4}{8} - \frac{\arcsin(cx) (-48c^7 x^7 \sqrt{-c^2 x^2 + 1} + 200c^5 x^5 \sqrt{-c^2 x^2 + 1} - 326c^3 x^3 \sqrt{-c^2 x^2 + 1})}{1536} \right)$
parts	$-\frac{d^3 a^2 (c^2 x^2 - 1)^4}{8c^2} - \frac{d^3 b^2 \left(\frac{\arcsin(cx)^2 (c^2 x^2 - 1)^4}{8} - \frac{\arcsin(cx) (-48c^7 x^7 \sqrt{-c^2 x^2 + 1} + 200c^5 x^5 \sqrt{-c^2 x^2 + 1} - 326c^3 x^3 \sqrt{-c^2 x^2 + 1})}{1536} \right)}{c^2}$
orering	$\frac{(6084c^{10}x^{10} - 32348c^8x^8 + 72453c^6x^6 - 97420c^4x^4 + 34749c^2x^2 - 5022)(-c^2dx^2 + d)^3(a + b\arcsin(cx))^2}{18432c^2(cx - 1)^2(cx + 1)^2(c^2x^2 - 1)^2} - \frac{(756c^8x^8 - \dots)}{\dots}$

input `int(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & 1/c^2*(-1/8*d^3*a^2*(c^2*x^2-1)^4-d^3*b^2*(1/8*arcsin(c*x))^2*(c^2*x^2-1)^4 \\ & -1/1536*arcsin(c*x)*(-48*c^7*x^7*(-c^2*x^2+1)^{(1/2)}+200*c^5*x^5*(-c^2*x^2+ \\ & 1)^{(1/2)}-326*c^3*x^3*(-c^2*x^2+1)^{(1/2)}+279*c*x*(-c^2*x^2+1)^{(1/2)}+105*arc \\ & sin(c*x))+35/1024*arcsin(c*x)^2-1/256*(c^2*x^2-1)^4+7/1152*(c^2*x^2-1)^3-3 \\ & 5/3072*(c^2*x^2-1)^2+35/1024*c^2*x^2-35/1024)-2*d^3*a*b*(1/8*arcsin(c*x)*c \\ & ^8*x^8-1/2*arcsin(c*x)*c^6*x^6+3/4*c^4*x^4*arcsin(c*x)-1/2*c^2*x^2*arcsin(\\ & c*x)+93/1024*arcsin(c*x)+1/64*c^7*x^7*(-c^2*x^2+1)^{(1/2)}-25/384*c^5*x^5*(- \\ & c^2*x^2+1)^{(1/2)}+163/1536*c^3*x^3*(-c^2*x^2+1)^{(1/2)}-93/1024*c*x*(-c^2*x^2 \\ & +1)^{(1/2})) \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.28

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \frac{36(32a^2 - b^2)c^8 d^3 x^8 - 8(576a^2 - 25b^2)c^6 d^3 x^6 + 3(2304a^2 - 163b^2)c^4 d^3 x^4 - 9(512a^2 - 93b^2)c^2 d^3 x^2 + 9(128b^2 c^8 d^3 x^8 - 512b^2 c^6 d^3 x^6 + 768b^2 c^4 d^3 x^4 - 512b^2 c^2 d^3 x^2 + 93b^2 d^3) \arcsin(cx)^2 + 18(128a b c^8 d^3 x^8 - 512a b c^6 d^3 x^6 + 768a b c^4 d^3 x^4 - 512a b c^2 d^3 x^2 + 93a b d^3) \arcsin(cx) + 6(48a b c^7 d^3 x^7 - 200a b c^5 d^3 x^5 + 326a b c^3 d^3 x^3 - 279a b c d^3 x + (48b^2 c^7 d^3 x^7 - 200b^2 c^5 d^3 x^5 + 326b^2 c^3 d^3 x^3 - 279b^2 c d^3 x) \arcsin(cx)) \sqrt{-c^2 x^2 + 1}}{c^2}$$

input `integrate(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output
$$\begin{aligned} & -1/9216*(36*(32*a^2 - b^2)*c^8*d^3*x^8 - 8*(576*a^2 - 25*b^2)*c^6*d^3*x^6 \\ & + 3*(2304*a^2 - 163*b^2)*c^4*d^3*x^4 - 9*(512*a^2 - 93*b^2)*c^2*d^3*x^2 + \\ & 9*(128*b^2*c^8*d^3*x^8 - 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 - 512*b \\ & ^2*c^2*d^3*x^2 + 93*b^2*d^3)*arcsin(c*x)^2 + 18*(128*a*b*c^8*d^3*x^8 - 512 \\ & *a*b*c^6*d^3*x^6 + 768*a*b*c^4*d^3*x^4 - 512*a*b*c^2*d^3*x^2 + 93*a*b*d^3) \\ & *arcsin(c*x) + 6*(48*a*b*c^7*d^3*x^7 - 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d \\ & ^3*x^3 - 279*a*b*c*d^3*x + (48*b^2*c^7*d^3*x^7 - 200*b^2*c^5*d^3*x^5 + 326 \\ & *b^2*c^3*d^3*x^3 - 279*b^2*c*d^3*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1)/c^2 \end{aligned}$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(258) = 516$.

Time = 1.28 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.07

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^6 d^3 x^8}{8} + \frac{a^2 c^4 d^3 x^6}{2} - \frac{3a^2 c^2 d^3 x^4}{4} + \frac{a^2 d^3 x^2}{2} - \frac{abc^6 d^3 x^8 \arcsin(cx)}{4} - \frac{abc^5 d^3 x^7 \sqrt{-c^2 x^2 + 1}}{32} + abc^4 d^3 x^6 \arcsin(cx) + \frac{25abc^3 d^3 x^5 \sqrt{-c^2 x^2 + 1}}{192} - \frac{3abc^2 d^3 x^4 \arcsin(cx)}{2} - \frac{163abc d^3 x^3 \sqrt{-c^2 x^2 + 1}}{768} + \frac{abd^3 x^2 \arcsin(cx)}{512} + \frac{93abd^3 x \sqrt{-c^2 x^2 + 1}}{(512c)} - \frac{93abd^3 \arcsin(cx)}{(512c^2)} - \frac{b^2 c^6 d^3 x^8 \arcsin(cx)^2}{8} + \frac{b^2 c^6 d^3 x^8}{256} - \frac{b^2 c^5 d^3 x^7 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{32} + \frac{b^2 c^4 d^3 x^6 \arcsin(cx)^2}{2} - \frac{25b^2 c^4 d^3 x^6}{1152} + \frac{25b^2 c^3 d^3 x^5 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{192} - \frac{3b^2 c^2 d^3 x^4 \arcsin(cx)^2}{4} + \frac{163b^2 c^2 d^3 x^4}{3072} - \frac{163b^2 c d^3 x^3 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{768} + \frac{b^2 d^3 x^2 \arcsin(cx)^2}{2} - \frac{93b^2 d^3 x^2}{1024} + \frac{93b^2 d^3 x \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{(512c)} - \frac{93b^2 d^3 \arcsin(cx)^2}{(1024c^2)}, \text{Ne}(c, 0), (a^2 d^3 x^2 / 2, \text{True}) \end{cases}$$

input `integrate(x*(-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2,x)`

output

```
Piecewise((-a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 - 3*a**2*c**2*d**3*x**4/4 + a**2*d**3*x**2/2 - a*b*c**6*d**3*x**8*asin(c*x)/4 - a*b*c**5*d**3*x**7*sqrt(-c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*asin(c*x) + 25*a*b*c**3*d**3*x**5*sqrt(-c**2*x**2 + 1)/192 - 3*a*b*c**2*d**3*x**4*asin(c*x)/2 - 163*a*b*c*d**3*x**3*sqrt(-c**2*x**2 + 1)/768 + a*b*d**3*x**2*asin(c*x) + 93*a*b*d**3*x*sqrt(-c**2*x**2 + 1)/(512*c) - 93*a*b*d**3*asin(c*x)/(512*c**2) - b**2*c**6*d**3*x**8*asin(c*x)**2/8 + b**2*c**6*d**3*x**8/256 - b**2*c**5*d**3*x**7*sqrt(-c**2*x**2 + 1)*asin(c*x)/32 + b**2*c**4*d**3*x**6*asin(c*x)**2/2 - 25*b**2*c**4*d**3*x**6/1152 + 25*b**2*c**3*d**3*x**5*sqrt(-c**2*x**2 + 1)*asin(c*x)/192 - 3*b**2*c**2*d**3*x**4*asin(c*x)**2/4 + 163*b**2*c**2*d**3*x**4/3072 - 163*b**2*c*d**3*x**3*sqrt(-c**2*x**2 + 1)*asin(c*x)/768 + b**2*d**3*x**2*asin(c*x)**2/2 - 93*b**2*d**3*x**2/1024 + 93*b**2*d**3*x*sqrt(-c**2*x**2 + 1)*asin(c*x)/(512*c) - 93*b**2*d**3*asin(c*x)**2/(1024*c**2), Ne(c, 0)), (a**2*d**3*x**2/2, True))
```

Maxima [F]

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2 x dx$$

input `integrate(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

-1/8*a^2*c^6*d^3*x^8 + 1/2*a^2*c^4*d^3*x^6 - 1/1536*(384*x^8*arcsin(c*x) +
(48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(
-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9
)*c)*a*b*c^6*d^3 - 3/4*a^2*c^2*d^3*x^4 + 1/48*(48*x^6*arcsin(c*x) + (8*sq
rt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2
+ 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^4*d^3 - 3/16*(8*x^4*arcsin(c*x)
+ (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c
*x)/c^5)*c)*a*b*c^2*d^3 + 1/2*a^2*d^3*x^2 + 1/2*(2*x^2*arcsin(c*x) + c*(sq
rt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*a*b*d^3 - 1/8*(b^2*c^6*d^3*x^8
- 4*b^2*c^4*d^3*x^6 + 6*b^2*c^2*d^3*x^4 - 4*b^2*d^3*x^2)*arctan2(c*x, sqrt
(c*x + 1)*sqrt(-c*x + 1))^2 - integrate(1/4*(b^2*c^7*d^3*x^8 - 4*b^2*c^5*d
^3*x^6 + 6*b^2*c^3*d^3*x^4 - 4*b^2*c*d^3*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)
*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^2 - 1), x)

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(245) = 490$.

Time = 0.17 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.78

$$\begin{aligned}
 \int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = & -\frac{1}{8} a^2 c^6 d^3 x^8 + \frac{1}{2} a^2 c^4 d^3 x^6 - \frac{3}{4} a^2 c^2 d^3 x^4 \\
 & - \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b^2 d^3 x \arcsin(cx)}{32 c} \\
 & - \frac{(c^2 x^2 - 1)^4 b^2 d^3 \arcsin(cx)^2}{8 c^2} \\
 & - \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} a b d^3 x}{32 c} \\
 & + \frac{7 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^3 x \arcsin(cx)}{192 c} \\
 & - \frac{(c^2 x^2 - 1)^4 a b d^3 \arcsin(cx)}{4 c^2} \\
 & + \frac{7 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^3 x}{192 c} \\
 & + \frac{35 (-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d^3 x \arcsin(cx)}{768 c} \\
 & + \frac{(c^2 x^2 - 1)^4 b^2 d^3}{256 c^2} + \frac{35 (-c^2 x^2 + 1)^{\frac{3}{2}} a b d^3 x}{768 c} \\
 & + \frac{35 \sqrt{-c^2 x^2 + 1} b^2 d^3 x \arcsin(cx)}{512 c} \\
 & - \frac{7 (c^2 x^2 - 1)^3 b^2 d^3}{1152 c^2} + \frac{35 \sqrt{-c^2 x^2 + 1} a b d^3 x}{512 c} \\
 & + \frac{35 (c^2 x^2 - 1)^2 b^2 d^3}{3072 c^2} + \frac{35 b^2 d^3 \arcsin(cx)^2}{1024 c^2} \\
 & + \frac{(c^2 x^2 - 1) a^2 d^3}{2 c^2} - \frac{35 (c^2 x^2 - 1) b^2 d^3}{1024 c^2} \\
 & + \frac{35 a b d^3 \arcsin(cx)}{512 c^2} - \frac{7175 b^2 d^3}{294912 c^2}
 \end{aligned}$$

input

```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
-1/8*a^2*c^6*d^3*x^8 + 1/2*a^2*c^4*d^3*x^6 - 3/4*a^2*c^2*d^3*x^4 - 1/32*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c - 1/8*(c^2*x^2 - 1)^4*b^2*d^3*arcsin(c*x)^2/c^2 - 1/32*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c + 7/192*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c - 1/4*(c^2*x^2 - 1)^4*a*b*d^3*arcsin(c*x)/c^2 + 7/192*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c + 35/768*(-c^2*x^2 + 1)^(3/2)*b^2*d^3*x*arcsin(c*x)/c + 1/256*(c^2*x^2 - 1)^4*b^2*d^3/c^2 + 35/768*(-c^2*x^2 + 1)^(3/2)*a*b*d^3*x/c + 35/512*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arcsin(c*x)/c - 7/1152*(c^2*x^2 - 1)^3*b^2*d^3/c^2 + 35/512*sqrt(-c^2*x^2 + 1)*a*b*d^3*x/c + 35/3072*(c^2*x^2 - 1)^2*b^2*d^3/c^2 + 35/1024*b^2*d^3*arcsin(c*x)^2/c^2 + 1/2*(c^2*x^2 - 1)*a^2*d^3/c^2 - 35/1024*(c^2*x^2 - 1)*b^2*d^3/c^2 + 35/512*a*b*d^3*arcsin(c*x)/c^2 - 7175/294912*b^2*d^3/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int x(a + b \operatorname{asin}(cx))^2 (d - c^2 dx^2)^3 dx$$

input

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \frac{d^3(768 \operatorname{asin}(cx)^2 b^2 c^2 x^2 - 384 \operatorname{asin}(cx)^2 b^2 + 768 \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) b^2 cx - 384 \operatorname{asin}(cx) ab c^8 x^8 + 1536$$

input

```
int(x*(-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2,x)
```

output

```
(d**3*(768*asin(c*x)**2*b**2*c**2*x**2 - 384*asin(c*x)**2*b**2 + 768*sqrt(
- c**2*x**2 + 1)*asin(c*x)*b**2*c*x - 384*asin(c*x)*a*b*c**8*x**8 + 1536*
asin(c*x)*a*b*c**6*x**6 - 2304*asin(c*x)*a*b*c**4*x**4 + 1536*asin(c*x)*a*
b*c**2*x**2 - 279*asin(c*x)*a*b - 48*sqrt(- c**2*x**2 + 1)*a*b*c**7*x**7
+ 200*sqrt(- c**2*x**2 + 1)*a*b*c**5*x**5 - 326*sqrt(- c**2*x**2 + 1)*a*
b*c**3*x**3 + 279*sqrt(- c**2*x**2 + 1)*a*b*c*x - 1536*int(asin(c*x)**2*x
**7,x)*b**2*c**8 + 4608*int(asin(c*x)**2*x**5,x)*b**2*c**6 - 4608*int(asin
(c*x)**2*x**3,x)*b**2*c**4 - 192*a**2*c**8*x**8 + 768*a**2*c**6*x**6 - 115
2*a**2*c**4*x**4 + 768*a**2*c**2*x**2 - 384*b**2*c**2*x**2))/(1536*c**2)
```

3.175 $\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$

Optimal result	1625
Mathematica [A] (verified)	1626
Rubi [A] (verified)	1626
Maple [A] (verified)	1629
Fricas [A] (verification not implemented)	1630
Sympy [A] (verification not implemented)	1631
Maxima [B] (verification not implemented)	1632
Giac [B] (verification not implemented)	1633
Mupad [F(-1)]	1635
Reduce [F]	1635

Optimal result

Integrand size = 24, antiderivative size = 298

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= -\frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} - \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7$$

$$+ \frac{32bd^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{35c} + \frac{16bd^3(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))}{105c}$$

$$+ \frac{12bd^3(1 - c^2x^2)^{5/2}(a + b \arcsin(cx))}{175c} + \frac{2bd^3(1 - c^2x^2)^{7/2}(a + b \arcsin(cx))}{49c}$$

$$+ \frac{16}{35}d^3x(a + b \arcsin(cx))^2 + \frac{8}{35}d^3x(1 - c^2x^2)(a + b \arcsin(cx))^2 + \frac{6}{35}d^3x(1 - c^2x^2)^2(a + b \arcsin(cx))^2 + \frac{1}{7}$$

output

```
-4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3-234/6125*b^2*c^4*d^3*x^5+2
/343*b^2*c^6*d^3*x^7+32/35*b*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+16
/105*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/c+12/175*b*d^3*(-c^2*x^2+1
)^(5/2)*(a+b*arcsin(c*x))/c+2/49*b*d^3*(-c^2*x^2+1)^(7/2)*(a+b*arcsin(c*x
))/c+16/35*d^3*x*(a+b*arcsin(c*x))^2+8/35*d^3*x*(-c^2*x^2+1)*(a+b*arcsin(c*
x))^2+6/35*d^3*x*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2+1/7*d^3*x*(-c^2*x^2+1
)^3*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.81

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \frac{d^3 (2b^2 cx(226905 - 26495c^2 x^2 + 7371c^4 x^4 - 1125c^6 x^6) + 11025a^2 cx(-35 + 35c^2 x^2 - 21c^4 x^4 + 5c^6 x^6))}{c}$$

input

```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/385875*(d^3*(2*b^2*c*x*(226905 - 26495*c^2*x^2 + 7371*c^4*x^4 - 1125*c^6*x^6) + 11025*a^2*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 210*a*b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6) + 210*b*(105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6))*ArcSin[c*x] + 11025*b^2*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcSin[c*x]^2))/c
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5158, 27, 5158, 5158, 5130, 5182, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5158$$

$$-\frac{2}{7}bcd^3 \int x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + \frac{6}{7}d \int d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx + \frac{1}{7}d^3 x(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

$$\downarrow 27$$

$$\begin{aligned}
& -\frac{2}{7}bcd^3 \int x(1-c^2x^2)^{5/2} (a+b \arcsin(cx))dx + \frac{6}{7}d^3 \int (1-c^2x^2)^2 (a+b \arcsin(cx))^2 dx + \\
& \quad \frac{1}{7}d^3 x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \\
& \quad \downarrow \text{5158} \\
& -\frac{2}{7}bcd^3 \int x(1-c^2x^2)^{5/2} (a+b \arcsin(cx))dx + \\
& \frac{6}{7}d^3 \left(-\frac{2}{5}bc \int x(1-c^2x^2)^{3/2} (a+b \arcsin(cx))dx + \frac{4}{5} \int (1-c^2x^2) (a+b \arcsin(cx))^2 dx + \frac{1}{5}x(1-c^2x^2)^2 (a+b \arcsin(cx))^2 \right. \\
& \quad \left. + \frac{1}{7}d^3 x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \\
& \quad \downarrow \text{5158} \\
& -\frac{2}{7}bcd^3 \int x(1-c^2x^2)^{5/2} (a+b \arcsin(cx))dx + \\
& \frac{6}{7}d^3 \left(-\frac{2}{5}bc \int x(1-c^2x^2)^{3/2} (a+b \arcsin(cx))dx + \frac{4}{5} \left(-\frac{2}{3}bc \int x\sqrt{1-c^2x^2} (a+b \arcsin(cx))dx + \frac{2}{3} \int (a+b \arcsin(cx))^2 dx \right) \right. \\
& \quad \left. + \frac{1}{7}d^3 x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \\
& \quad \downarrow \text{5130} \\
& \frac{6}{7}d^3 \left(\frac{4}{5} \left(\frac{2}{3} \left(x(a+b \arcsin(cx))^2 - 2bc \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{3}bc \int x\sqrt{1-c^2x^2} (a+b \arcsin(cx))dx + \frac{1}{3} \int (a+b \arcsin(cx))^2 dx \right) \right. \\
& \quad \left. + \frac{2}{7}bcd^3 \int x(1-c^2x^2)^{5/2} (a+b \arcsin(cx))dx + \frac{1}{7}d^3 x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \\
& \quad \downarrow \text{5182} \\
& \frac{6}{7}d^3 \left(\frac{4}{5} \left(\frac{2}{3} \left(x(a+b \arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2} (a+b \arcsin(cx))}{c^2} \right) \right) \right) - \frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{1}{3} \int (a+b \arcsin(cx))^2 dx \right) \right. \\
& \quad \left. + \frac{2}{7}bcd^3 \left(\frac{b \int (1-c^2x^2)^3 dx}{7c} - \frac{(1-c^2x^2)^{7/2} (a+b \arcsin(cx))}{7c^2} \right) + \frac{1}{7}d^3 x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \\
& \quad \downarrow \text{24} \\
& \frac{6}{7}d^3 \left(\frac{4}{5} \left(-\frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2} (a+b \arcsin(cx))}{3c^2} \right) + \frac{1}{3}x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \int (a+b \arcsin(cx))^2 dx \right) \right. \\
& \quad \left. + \frac{2}{7}bcd^3 \left(\frac{b \int (1-c^2x^2)^3 dx}{7c} - \frac{(1-c^2x^2)^{7/2} (a+b \arcsin(cx))}{7c^2} \right) + \frac{1}{7}d^3 x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right)
\end{aligned}$$

↓ 210

$$\frac{6}{7}d^3 \left(\frac{4}{5} \left(-\frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2} (a+b \arcsin(cx))}{3c^2} \right) + \frac{1}{3}x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \left(\frac{2}{7}bcd^3 \left(\frac{b \int (-c^6x^6 + 3c^4x^4 - 3c^2x^2 + 1) dx}{7c} - \frac{(1-c^2x^2)^{7/2} (a+b \arcsin(cx))}{7c^2} \right) + \frac{1}{7}d^3x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 \right) \right)$$

↓ 2009

$$\frac{1}{7}d^3x(1-c^2x^2)^3 (a+b \arcsin(cx))^2 + \frac{6}{7}d^3 \left(\frac{1}{5}x(1-c^2x^2)^2 (a+b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \left(x(a+b \arcsin(cx))^2 - 2bc \frac{2}{7}bcd^3 \left(\frac{b \left(-\frac{1}{7}c^6x^7 + \frac{3c^4x^5}{5} - c^2x^3 + x \right)}{7c} - \frac{(1-c^2x^2)^{7/2} (a+b \arcsin(cx))}{7c^2} \right) \right) \right)$$

input `Int[(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]`

output `(d^3*x*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x])^2)/7 - (2*b*c*d^3*((b*(x - c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/(7*c) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcSin[c*x]))/(7*c^2))/7 + (6*d^3*((x*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*((b*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(5*c) - ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^2))/5 + (4*((x*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*((b*(x - (c^2*x^3)/3))/(3*c) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^2))/3 + (2*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/3))/5))/7`

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 210 $\text{Int}[(a_.) + (b_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^2)^p, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[p, 0]

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /;$ SumQ[u]

rule 5130 $\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Simp}[b*c*n \text{ Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ FreeQ[{a, b, c}, x] && GtQ[n, 0]

rule 5158 $\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)^{(n_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[x*(d + e*x^2)^p*((a + b*\text{ArcSin}[c*x])^n/(2*p + 1)), x] + (\text{Simp}[2*d*(p/(2*p + 1)) \text{ Int}[(d + e*x^2)^{(p-1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(2*p + 1))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[x*(1 - c^2*x^2)^{(p-1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]

rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_.)]*(b_.)^{(n_.)}*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-d^3 a^2 \left(\frac{1}{7} c^7 x^7 - \frac{3}{5} c^5 x^5 + c^3 x^3 - cx \right) - d^3 b^2 \left(\frac{\arcsin(cx)^2 (5c^6 x^6 - 21c^4 x^4 + 35c^2 x^2 - 35) cx}{35} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1}}{49} \right)$
default	$-d^3 a^2 \left(\frac{1}{7} c^7 x^7 - \frac{3}{5} c^5 x^5 + c^3 x^3 - cx \right) - d^3 b^2 \left(\frac{\arcsin(cx)^2 (5c^6 x^6 - 21c^4 x^4 + 35c^2 x^2 - 35) cx}{35} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1}}{49} \right)$
parts	$-a^2 d^3 \left(\frac{1}{7} x^7 c^6 - \frac{3}{5} c^4 x^5 + c^2 x^3 - x \right) - \frac{b^2 d^3 \left(\frac{\arcsin(cx)^2 (5c^6 x^6 - 21c^4 x^4 + 35c^2 x^2 - 35) cx}{35} + \frac{2 \arcsin(cx) (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1}}{49} \right)}{128625 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)^2}$
oring	$\frac{x(47625c^8 x^8 - 271212c^6 x^6 + 741678c^4 x^4 - 3539900c^2 x^2 + 128625)(-c^2 d x^2 + d)^3 (a + b \arcsin(cx))^2}{128625 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)^2} - \frac{(20250c^8 x^8 - 128625c^6 x^6 + 3539900c^4 x^4 - 128625c^2 x^2 + 128625)}{128625 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)^2}$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output `1/c*(-d^3*a^2*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b^2*(1/35*arcsin(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x+2/49*arcsin(c*x)*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-2/1715*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-12/175*arcsin(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)+4/875*(3*c^4*x^4-10*c^2*x^2+15)*c*x+16/105*arcsin(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/315*(c^2*x^2-3)*c*x+32/35*c*x-32/35*arcsin(c*x)*(-c^2*x^2+1)^(1/2))-2*d^3*a*b*(1/7*a*arcsin(c*x)*c^7*x^7-3/5*c^5*x^5*arcsin(c*x)+c^3*x^3*arcsin(c*x)-c*x*arcsin(c*x)-2161/3675*(-c^2*x^2+1)^(1/2)+757/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)-117/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)+1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.08

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \frac{1125 (49 a^2 - 2 b^2) c^7 d^3 x^7 - 189 (1225 a^2 - 78 b^2) c^5 d^3 x^5 + 35 (11025 a^2 - 1514 b^2) c^3 d^3 x^3 - 105 (3675 a^2 - 252 b^2) c d^3 x - 105 (3675 a^2 - 252 b^2) c d^3 x^3 - 105 (3675 a^2 - 252 b^2) c d^3 x^5 - 105 (3675 a^2 - 252 b^2) c d^3 x^7}{128625 (cx-1)^2 (cx+1)^2 (c^2 x^2 - 1)^2}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output

```
-1/385875*(1125*(49*a^2 - 2*b^2)*c^7*d^3*x^7 - 189*(1225*a^2 - 78*b^2)*c^5
*d^3*x^5 + 35*(11025*a^2 - 1514*b^2)*c^3*d^3*x^3 - 105*(3675*a^2 - 4322*b^
2)*c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 - 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^
3*x^3 - 35*b^2*c*d^3*x)*arcsin(c*x)^2 + 22050*(5*a*b*c^7*d^3*x^7 - 21*a*b*
c^5*d^3*x^5 + 35*a*b*c^3*d^3*x^3 - 35*a*b*c*d^3*x)*arcsin(c*x) + 210*(75*a
*b*c^6*d^3*x^6 - 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 - 2161*a*b*d^3
+ (75*b^2*c^6*d^3*x^6 - 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 - 2161*b
^2*d^3)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.76

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$$

$$= \begin{cases} -\frac{a^2 c^6 d^3 x^7}{7} + \frac{3a^2 c^4 d^3 x^5}{5} - a^2 c^2 d^3 x^3 + a^2 d^3 x - \frac{2abc^6 d^3 x^7 \arcsin(cx)}{7} - \frac{2abc^5 d^3 x^6 \sqrt{-c^2 x^2 + 1}}{49} + \frac{6abc^4 d^3 x^5 \arcsin(cx)}{5} + \frac{234a^2 d^3 x}{5} \\ a^2 d^3 x \end{cases}$$

input

```
integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2,x)
```

output

```
Piecewise((-a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 - a**2*c**2*d**
3*x**3 + a**2*d**3*x - 2*a*b*c**6*d**3*x**7*asin(c*x)/7 - 2*a*b*c**5*d**3
*x**6*sqrt(-c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*asin(c*x)/5 + 234*a*b
*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)/1225 - 2*a*b*c**2*d**3*x**3*asin(c*x)
- 1514*a*b*c*d**3*x**2*sqrt(-c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*asin(c*x)
+ 4322*a*b*d**3*sqrt(-c**2*x**2 + 1)/(3675*c) - b**2*c**6*d**3*x**7*asin(
c*x)**2/7 + 2*b**2*c**6*d**3*x**7/343 - 2*b**2*c**5*d**3*x**6*sqrt(-c**2*x
**2 + 1)*asin(c*x)/49 + 3*b**2*c**4*d**3*x**5*asin(c*x)**2/5 - 234*b**2*c
**4*d**3*x**5/6125 + 234*b**2*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)*asin(c*x)
/1225 - b**2*c**2*d**3*x**3*asin(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025
- 1514*b**2*c*d**3*x**2*sqrt(-c**2*x**2 + 1)*asin(c*x)/3675 + b**2*d**3*x
asin(c*x)**2 - 4322*b**2*d**3*x/3675 + 4322*b**2*d**3*sqrt(-c**2*x**2 + 1)
*asin(c*x)/(3675*c), Ne(c, 0)), (a**2*d**3*x, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(263) = 526$.

Time = 0.16 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.45

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
-1/7*b^2*c^6*d^3*x^7*arcsin(c*x)^2 - 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3
*x^5*arcsin(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 - 2/245*(35*x^7*arcsin(c*x) + (5*
sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^
2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 - 2/25725*(105*
(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2
*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arcsin(c*x) - (75*c^6*x^7
+ 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^6*d^3 - b^2*c^2*d^3*x^3*
arcsin(c*x)^2 + 2/25*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 +
4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 +
2/375*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c*arcsin(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)
/c^4)*b^2*c^4*d^3 - a^2*c^2*d^3*x^3 - 2/3*(3*x^3*arcsin(c*x) + c*(sqrt(-c^
2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 - 2/9*(3*c*(sq
rt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcsin(c*x) - (c^2*x^
3 + 6*x)/c^2)*b^2*c^2*d^3 + b^2*d^3*x*arcsin(c*x)^2 - 2*b^2*d^3*(x - sqrt(
-c^2*x^2 + 1)*arcsin(c*x)/c) + a^2*d^3*x + 2*(c*x*arcsin(c*x) + sqrt(-c^2*
x^2 + 1))*a*b*d^3/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(263) = 526$.

Time = 0.17 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.77

$$\begin{aligned}
\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = & -\frac{1}{7} a^2 c^6 d^3 x^7 + \frac{3}{5} a^2 c^4 d^3 x^5 \\
& - \frac{1}{7} (c^2 x^2 - 1)^3 b^2 d^3 x \arcsin(cx)^2 \\
& - a^2 c^2 d^3 x^3 - \frac{2}{7} (c^2 x^2 - 1)^3 a b d^3 x \arcsin(cx) \\
& + \frac{6}{35} (c^2 x^2 - 1)^2 b^2 d^3 x \arcsin(cx)^2 \\
& + \frac{2}{343} (c^2 x^2 - 1)^3 b^2 d^3 x \\
& + \frac{12}{35} (c^2 x^2 - 1)^2 a b d^3 x \arcsin(cx) \\
& - \frac{8}{35} (c^2 x^2 - 1) b^2 d^3 x \arcsin(cx)^2 \\
& - \frac{2 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} b^2 d^3 \arcsin(cx)}{49 c} \\
& - \frac{888}{42875} (c^2 x^2 - 1)^2 b^2 d^3 x \\
& - \frac{16}{35} (c^2 x^2 - 1) a b d^3 x \arcsin(cx) \\
& + \frac{16}{35} b^2 d^3 x \arcsin(cx)^2 \\
& - \frac{2 (c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} a b d^3}{49 c} \\
& + \frac{12 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^3 \arcsin(cx)}{175 c} \\
& + \frac{30256}{385875} (c^2 x^2 - 1) b^2 d^3 x \\
& + \frac{32}{35} a b d^3 x \arcsin(cx) \\
& + \frac{12 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^3}{175 c} \\
& + \frac{16 (-c^2 x^2 + 1)^{\frac{3}{2}} b^2 d^3 \arcsin(cx)}{105 c} + a^2 d^3 x \\
& - \frac{413312}{385875} b^2 d^3 x + \frac{16 (-c^2 x^2 + 1)^{\frac{3}{2}} a b d^3}{105 c} \\
& + \frac{32 \sqrt{-c^2 x^2 + 1} b^2 d^3 \arcsin(cx)}{35 c} \\
& + \frac{32 \sqrt{-c^2 x^2 + 1} a b d^3}{35 c}
\end{aligned}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output
$$\begin{aligned} & -1/7*a^2*c^6*d^3*x^7 + 3/5*a^2*c^4*d^3*x^5 - 1/7*(c^2*x^2 - 1)^3*b^2*d^3*x \\ & *arcsin(c*x)^2 - a^2*c^2*d^3*x^3 - 2/7*(c^2*x^2 - 1)^3*a*b*d^3*x*arcsin(c*x) \\ & + 6/35*(c^2*x^2 - 1)^2*b^2*d^3*x*arcsin(c*x)^2 + 2/343*(c^2*x^2 - 1)^3* \\ & b^2*d^3*x + 12/35*(c^2*x^2 - 1)^2*a*b*d^3*x*arcsin(c*x) - 8/35*(c^2*x^2 - \\ & 1)*b^2*d^3*x*arcsin(c*x)^2 - 2/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b^2*d \\ & ^3*arcsin(c*x)/c - 888/42875*(c^2*x^2 - 1)^2*b^2*d^3*x - 16/35*(c^2*x^2 - \\ & 1)*a*b*d^3*x*arcsin(c*x) + 16/35*b^2*d^3*x*arcsin(c*x)^2 - 2/49*(c^2*x^2 - \\ & 1)^3*sqrt(-c^2*x^2 + 1)*a*b*d^3/c + 12/175*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 \\ & + 1)*b^2*d^3*arcsin(c*x)/c + 30256/385875*(c^2*x^2 - 1)*b^2*d^3*x + 32/35* \\ & a*b*d^3*x*arcsin(c*x) + 12/175*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^3/ \\ & c + 16/105*(-c^2*x^2 + 1)^(3/2)*b^2*d^3*arcsin(c*x)/c + a^2*d^3*x - 413312 \\ & /385875*b^2*d^3*x + 16/105*(-c^2*x^2 + 1)^(3/2)*a*b*d^3/c + 32/35*sqrt(-c^2 \\ & *x^2 + 1)*b^2*d^3*arcsin(c*x)/c + 32/35*sqrt(-c^2*x^2 + 1)*a*b*d^3/c \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 (d - c^2 dx^2)^3 dx$$

input `int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3,x)`

output `int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3, x)`

Reduce [F]

$$\begin{aligned} & \int (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx \\ & = \frac{d^3 (3675 \arcsin(cx)^2 b^2 cx + 7350 \sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 - 1050 \arcsin(cx) a b c^7 x^7 + 4410 \arcsin(cx) a b c^5 x^5 - \dots}{\dots} \end{aligned}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2,x)`

output

```
(d**3*(3675*asin(c*x)**2*b**2*c*x + 7350*sqrt(-c**2*x**2 + 1)*asin(c*x)*
b**2 - 1050*asin(c*x)*a*b*c**7*x**7 + 4410*asin(c*x)*a*b*c**5*x**5 - 7350*
asin(c*x)*a*b*c**3*x**3 + 7350*asin(c*x)*a*b*c*x - 150*sqrt(-c**2*x**2 +
1)*a*b*c**6*x**6 + 702*sqrt(-c**2*x**2 + 1)*a*b*c**4*x**4 - 1514*sqrt(
-c**2*x**2 + 1)*a*b*c**2*x**2 + 4322*sqrt(-c**2*x**2 + 1)*a*b - 3675*in
t(asin(c*x)**2*x**6,x)*b**2*c**7 + 11025*int(asin(c*x)**2*x**4,x)*b**2*c**
5 - 11025*int(asin(c*x)**2*x**2,x)*b**2*c**3 - 525*a**2*c**7*x**7 + 2205*a
**2*c**5*x**5 - 3675*a**2*c**3*x**3 + 3675*a**2*c*x - 7350*b**2*c*x))/(367
5*c)
```

3.176
$$\int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))^2}{x} dx$$

Optimal result	1637
Mathematica [A] (verified)	1638
Rubi [A] (verified)	1639
Maple [A] (verified)	1649
Fricas [F]	1649
Sympy [F]	1650
Maxima [F]	1651
Giac [F(-2)]	1651
Mupad [F(-1)]	1651
Reduce [F]	1652

Optimal result

Integrand size = 27, antiderivative size = 360

$$\begin{aligned} \int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))^2}{x} dx = & \frac{19}{48} b^2 c^2 d^3 x^2 - \frac{7}{144} b^2 d^3 (1-c^2 x^2)^2 \\ & - \frac{1}{108} b^2 d^3 (1-c^2 x^2)^3 \\ & - \frac{19}{24} b c d^3 x \sqrt{1-c^2 x^2} (a+b \arcsin(cx)) \\ & - \frac{7}{36} b c d^3 x (1-c^2 x^2)^{3/2} (a+b \arcsin(cx)) \\ & - \frac{1}{18} b c d^3 x (1-c^2 x^2)^{5/2} (a+b \arcsin(cx)) \\ & - \frac{19}{48} d^3 (a+b \arcsin(cx))^2 \\ & + \frac{1}{2} d^3 (1-c^2 x^2) (a+b \arcsin(cx))^2 \\ & + \frac{1}{4} d^3 (1-c^2 x^2)^2 (a+b \arcsin(cx))^2 \\ & + \frac{1}{6} d^3 (1-c^2 x^2)^3 (a+b \arcsin(cx))^2 \\ & - \frac{d^3 (a+b \arcsin(cx))^3}{3b} \\ & + d^3 (a+b \arcsin(cx))^2 \log(1-e^{2i \arcsin(cx)}) \\ & - i b d^3 (a+b \arcsin(cx)) \text{PolyLog}(2, e^{2i \arcsin(cx)}) \\ & + \frac{1}{2} b^2 d^3 \text{PolyLog}(3, e^{2i \arcsin(cx)}) \end{aligned}$$

output

```

19/48*b^2*c^2*d^3*x^2-7/144*b^2*d^3*(-c^2*x^2+1)^2-1/108*b^2*d^3*(-c^2*x^2+1)^3-19/24*b*c*d^3*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-7/36*b*c*d^3*x*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))-1/18*b*c*d^3*x*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))-19/48*d^3*(a+b*arcsin(c*x))^2+1/2*d^3*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+1/4*d^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2+1/6*d^3*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2-1/3*I*d^3*(a+b*arcsin(c*x))^3/b+d^3*(a+b*arcsin(c*x))^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*b*d^3*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*d^3*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)

```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.29

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx$$

$$= \frac{d^3 \left(-144ib^2\pi^3 - 5184a^2c^2x^2 + 2592a^2c^4x^4 - 576a^2c^6x^6 - 3600abcx\sqrt{1 - c^2x^2} + 1056abc^3x^3\sqrt{1 - c^2x^2} \right)}{x}$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x,x]
```

output

```

(d^3*((-144*I)*b^2*Pi^3 - 5184*a^2*c^2*x^2 + 2592*a^2*c^4*x^4 - 576*a^2*c^6*x^6 - 3600*a*b*c*x*Sqrt[1 - c^2*x^2] + 1056*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 192*a*b*c^5*x^5*Sqrt[1 - c^2*x^2] - 10368*a*b*c^2*x^2*ArcSin[c*x] + 5184*a*b*c^4*x^4*ArcSin[c*x] - 1152*a*b*c^6*x^6*ArcSin[c*x] - (3456*I)*a*b*ArcSin[c*x]^2 + (1152*I)*b^2*ArcSin[c*x]^3 + 7200*a*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) - 783*b^2*Cos[2*ArcSin[c*x]] + 1566*b^2*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] - 27*b^2*Cos[4*ArcSin[c*x]] + 216*b^2*ArcSin[c*x]^2*Cos[4*ArcSin[c*x]] - b^2*Cos[6*ArcSin[c*x]] + 18*b^2*ArcSin[c*x]^2*Cos[6*ArcSin[c*x]] + 3456*b^2*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] + 6912*a*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 3456*a^2*Log[c*x] + (3456*I)*b^2*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] - (3456*I)*a*b*PolyLog[2, E^((2*I)*ArcSin[c*x])] + 1728*b^2*PolyLog[3, E^((-2*I)*ArcSin[c*x])] - 1566*b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]] - 108*b^2*ArcSin[c*x]*Sin[4*ArcSin[c*x]] - 6*b^2*ArcSin[c*x]*Sin[6*ArcSin[c*x]]))/3456

```

Rubi [A] (verified)

Time = 3.91 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.51, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {5202, 27, 5158, 241, 5158, 244, 2009, 5156, 15, 5152, 5202, 5158, 244, 2009, 5156, 15, 5152, 5202, 5136, 3042, 25, 4200, 25, 2620, 3011, 2720, 5156, 15, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx$$

↓ 5202

$$-\frac{1}{3}bcd^3 \int (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + d \int \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx + \frac{1}{6}d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

↓ 27

$$-\frac{1}{3}bcd^3 \int (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx + d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx + \frac{1}{6}d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

↓ 5158

$$-\frac{1}{3}bcd^3 \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx - \frac{1}{6}bc \int x(1 - c^2 x^2)^2 dx + \frac{1}{6}x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) \right) + d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx + \frac{1}{6}d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

↓ 241

$$-\frac{1}{3}bcd^3 \left(\frac{5}{6} \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{6}x(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{b(1 - c^2 x^2)^3}{36c} \right) + d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx + \frac{1}{6}d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2$$

↓ 5158

$$-\frac{1}{3}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - \frac{1}{4}bc \int x(1-c^2x^2)dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right. \\ \left. d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx + \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 \right)$$

↓ 244

$$d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx - \\ \frac{1}{3}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - \frac{1}{4}bc \int (x-c^2x^3)dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right) + \\ \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2$$

↓ 2009

$$d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx - \\ \frac{1}{3}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{x^2}{2} - \frac{c^2x^4}{4} \right) \right) \right) + \frac{1}{6}d^3 \\ \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2$$

↓ 5156

$$d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx - \\ \frac{1}{3}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \int x dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) + \\ \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2$$

↓ 15

$$d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx - \\ \frac{1}{3}bcd^3 \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{4}bcx^2 \right) \right) + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) + \\ \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2$$

↓ 5152

$$d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx + \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 -$$

$$\frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 5202

$$d^3 \left(-\frac{1}{2} bc \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx + \frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 - \right.$$

$$\left. \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \right.$$

$$\frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 5158

$$d^3 \left(-\frac{1}{2} bc \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{4} bc \int x (1 - c^2 x^2) dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) - \right.$$

$$\left. \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \right.$$

$$\frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 244

$$d^3 \left(\int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx - \frac{1}{2} bc \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx - \frac{1}{4} bc \int (x - c^2 x^3) dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) - \right.$$

$$\left. \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \right.$$

$$\frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 2009

$$d^3 \left(\int \frac{(1 - c^2 x^2) (a + b \arcsin(cx))^2}{x} dx - \frac{1}{2} bc \left(\frac{3}{4} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) - \right.$$

$$\left. \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \right.$$

$$\frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 5156

$$\begin{aligned}
& d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{1}{2}bc \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \int x dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right. \right. \\
& \quad \left. \left. - \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right) \\
& \quad \downarrow 15
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{1}{2}bc \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right. \right. \\
& \quad \left. \left. - \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right) \\
& \quad \downarrow 5152
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right. \\
& \quad \left. - \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right) \\
& \quad \downarrow 5202
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \int \frac{(a+b\arcsin(cx))^2}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \frac{1}{2}(1-c^2x^2) \right. \\
& \quad \left. - \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right) \\
& \quad \downarrow 5136
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b \arcsin(cx))dx + \int \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{cx} d \arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b \arcsin(cx)) \right. \\
& \qquad \qquad \qquad \left. + \frac{1}{6}d^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \right) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{3042}
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b \arcsin(cx))dx + \int -(a+b \arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b \arcsin(cx)) \right. \\
& \qquad \qquad \qquad \left. + \frac{1}{6}d^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \right) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{25}
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b \arcsin(cx))dx - \int (a+b \arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b \arcsin(cx)) \right. \\
& \qquad \qquad \qquad \left. + \frac{1}{6}d^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \right) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{4200}
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b \arcsin(cx))dx + 2i \int -\frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))^2}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b \arcsin(cx)) \right. \\
& \qquad \qquad \qquad \left. + \frac{1}{6}d^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \right) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{25}
\end{aligned}$$

$$\begin{aligned}
& d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b \arcsin(cx))dx - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))^2}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b \arcsin(cx)) \right. \\
& \qquad \qquad \qquad \left. + \frac{1}{6}d^3(1-c^2x^2)^3(a+b \arcsin(cx))^2 - \right. \\
& \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \right) \right) \right) \right)
\end{aligned}$$

↓ 2620

$$d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \int (a+b\arcsin(cx)) \right. \right. \\ \left. \left. \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \\ \left. \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right)$$

↓ 3011

$$d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \text{PolyLog} \left(2, \frac{1}{2}e^{2i\arcsin(cx)} \right) \right) \right. \right. \\ \left. \left. \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \\ \left. \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right)$$

↓ 2720

$$d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \text{PolyLog} \left(2, \frac{1}{2}e^{2i\arcsin(cx)} \right) \right) \right. \right. \\ \left. \left. \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \\ \left. \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right)$$

↓ 5156

$$d^3 \left(-bc \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}dx - \frac{1}{2}bc \int xdx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right. \right. \\ \left. \left. \frac{1}{6}d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2 - \right. \right. \\ \left. \left. \frac{1}{3}bcd^3 \left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \right)$$

↓ 15

$$d^3 \left(-bc \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) - \frac{1}{4} bcx^2 \right) - 2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) \right)^2 - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 5152

$$d^3 \left(-2i \left(\frac{1}{2} i \log \left(1 - e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) \right)^2 - ib \left(\frac{1}{2} i \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right) (a + b \arcsin(cx)) - \frac{1}{4} b \right) \right) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

↓ 7143

$$d^3 \left(\frac{1}{4} (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 + \frac{1}{2} (1 - c^2 x^2) (a + b \arcsin(cx))^2 - bc \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) + \frac{1}{4} bcx^2 \right) \right) - \frac{1}{6} d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2 - \frac{1}{3} bcd^3 \left(\frac{1}{6} x (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) + \frac{5}{6} \left(\frac{1}{4} x (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2} x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right) \right)$$

input

$\operatorname{Int}[(d - c^2 d x^2)^3 (a + b \operatorname{ArcSin}[c x])^2 / x, x]$

output $(d^3(1 - c^2x^2)^3(a + b\text{ArcSin}[cx])^2)/6 - (b^3cd^3((b(1 - c^2x^2)^3)/(36c) + (x(1 - c^2x^2)^{5/2}(a + b\text{ArcSin}[cx]))/6 + (5(-1/4(bcx^2/2 - (c^2x^4)/4)) + (x(1 - c^2x^2)^{3/2}(a + b\text{ArcSin}[cx]))/4 + (3(-1/4(bcx^2) + (x\text{Sqrt}[1 - c^2x^2](a + b\text{ArcSin}[cx]))/2 + (a + b\text{ArcSin}[cx])^2/(4bc)))/4)/6)/3 + d^3(((1 - c^2x^2)(a + b\text{ArcSin}[cx])^2)/2 + ((1 - c^2x^2)^2(a + b\text{ArcSin}[cx])^2)/4 - ((I/3)(a + b\text{ArcSin}[cx])^3)/b - bc(-1/4(bcx^2) + (x\text{Sqrt}[1 - c^2x^2](a + b\text{ArcSin}[cx]))/2 + (a + b\text{ArcSin}[cx])^2/(4bc)) - (bc(-1/4(bcx^2/2 - (c^2x^4)/4)) + (x(1 - c^2x^2)^{3/2}(a + b\text{ArcSin}[cx]))/4 + (3(-1/4(bcx^2) + (x\text{Sqrt}[1 - c^2x^2](a + b\text{ArcSin}[cx]))/2 + (a + b\text{ArcSin}[cx])^2/(4bc)))/4)/2 - (2I)((I/2)(a + b\text{ArcSin}[cx])^2\text{Log}[1 - E^((2I)\text{ArcSin}[cx])]) - I*b((I/2)(a + b\text{ArcSin}[cx])\text{PolyLog}[2, E^((2I)\text{ArcSin}[cx])]) - (b\text{PolyLog}[3, E^((2I)\text{ArcSin}[cx])])/4))$

Defintions of rubi rules used

rule 15 $\text{Int}[(a_*)(x_)^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] /; \text{FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$

rule 25 $\text{Int}[-(Fx_), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \ \text{Int}[Fx, x], x]$

rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$

rule 241 $\text{Int}[(x_*)((a_) + (b_*)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p+1)}/(2*b*(p+1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$

rule 244 $\text{Int}[((c_*)(x_))^{(m_*)((a_) + (b_*)(x_)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.38

method	result
parts	$-d^3 a^2 \left(\frac{c^6 x^6}{6} - \frac{3c^4 x^4}{4} + \frac{3c^2 x^2}{2} - \ln(x) \right) - d^3 b^2 \left(\frac{i \arcsin(cx)^3}{3} - \arcsin(cx)^2 \ln(1 + icx + \sqrt{1 - c^2 x^2}) \right)$
derivativedivides	$-d^3 a^2 \left(\frac{c^6 x^6}{6} - \frac{3c^4 x^4}{4} + \frac{3c^2 x^2}{2} - \ln(cx) \right) - d^3 b^2 \left(\frac{i \arcsin(cx)^3}{3} - \arcsin(cx)^2 \ln(1 + icx + \sqrt{1 - c^2 x^2}) \right)$
default	$-d^3 a^2 \left(\frac{c^6 x^6}{6} - \frac{3c^4 x^4}{4} + \frac{3c^2 x^2}{2} - \ln(cx) \right) - d^3 b^2 \left(\frac{i \arcsin(cx)^3}{3} - \arcsin(cx)^2 \ln(1 + icx + \sqrt{1 - c^2 x^2}) \right)$

input

```
int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output

```
-d^3*a^2*(1/6*c^6*x^6-3/4*c^4*x^4+3/2*c^2*x^2-ln(x))-d^3*b^2*(1/3*I*arcsin
(c*x)^3-arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polyl
og(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-arc
sin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,I*c*x+
(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))-1/3456*(18*arcsi
n(c*x)^2-1)*cos(6*arcsin(c*x))+1/576*arcsin(c*x)*sin(6*arcsin(c*x))-1/128*
(8*arcsin(c*x)^2-1)*cos(4*arcsin(c*x))+1/32*arcsin(c*x)*sin(4*arcsin(c*x))
-29/128*(2*arcsin(c*x)^2-1)*cos(2*arcsin(c*x))+29/64*arcsin(c*x)*sin(2*arc
sin(c*x))-2*d^3*a*b*(1/2*I*arcsin(c*x)^2-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2
+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1+I*c*x+(-
c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-1/192*arcsin(c*x)
*cos(6*arcsin(c*x))+1/1152*sin(6*arcsin(c*x))-1/16*arcsin(c*x)*cos(4*arcsi
n(c*x))+1/64*sin(4*arcsin(c*x))-29/64*arcsin(c*x)*cos(2*arcsin(c*x))+29/12
8*sin(2*arcsin(c*x)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x} dx$$

input

```
integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x,x, algorithm="fricas")
```

output

```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arcsin(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arcsin(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx = -d^3 \left(\int \left(-\frac{a^2}{x} \right) dx + \int 3a^2 c^2 x dx + \int (-3a^2 c^4 x^3) dx + \int a^2 c^6 x^5 dx + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x} \right) dx + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x} \right) dx + \int 3b^2 c^2 x \operatorname{asin}^2(cx) dx + \int (-3b^2 c^4 x^3 \operatorname{asin}^2(cx)) dx + \int b^2 c^6 x^5 \operatorname{asin}^2(cx) dx + \int 6abc^2 x \operatorname{asin}(cx) dx + \int (-6abc^4 x^3 \operatorname{asin}(cx)) dx + \int 2abc^6 x^5 \operatorname{asin}(cx) dx \right)$$

input

```
integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2/x,x)
```

output

```
-d**3*(Integral(-a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(-3*a**2*c**4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(-b**2*asin(c*x)**2/x, x) + Integral(-2*a*b*asin(c*x)/x, x) + Integral(3*b**2*c**2*x*asin(c*x)**2, x) + Integral(-3*b**2*c**4*x**3*asin(c*x)**2, x) + Integral(b**2*c**6*x**5*asin(c*x)**2, x) + Integral(6*a*b*c**2*x*asin(c*x), x) + Integral(-6*a*b*c**4*x**3*asin(c*x), x) + Integral(2*a*b*c**6*x**5*asin(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output `-1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 - 3/2*a^2*c^2*d^3*x^2 + a^2*d^3*log(x) - integrate(((b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^3}{x} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x} dx$$

$$= \frac{d^3 \left(-108 \arcsin(cx)^2 b^2 c^2 x^2 + 54 \arcsin(cx)^2 b^2 - 108 \sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 cx - 24 \arcsin(cx) ab c^6 x^6 + 108 a \right)}{72}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2/x,x)`

output `(d**3*(- 108*asin(c*x)**2*b**2*c**2*x**2 + 54*asin(c*x)**2*b**2 - 108*sqrt(- c**2*x**2 + 1)*asin(c*x)*b**2*c*x - 24*asin(c*x)*a*b*c**6*x**6 + 108*asin(c*x)*a*b*c**4*x**4 - 216*asin(c*x)*a*b*c**2*x**2 + 75*asin(c*x)*a*b - 4*sqrt(- c**2*x**2 + 1)*a*b*c**5*x**5 + 22*sqrt(- c**2*x**2 + 1)*a*b*c**3*x**3 - 75*sqrt(- c**2*x**2 + 1)*a*b*c*x + 144*int(asin(c*x)/x,x)*a*b + 72*int(asin(c*x)**2/x,x)*b**2 - 72*int(asin(c*x)**2*x**5,x)*b**2*c**6 + 216*int(asin(c*x)**2*x**3,x)*b**2*c**4 + 72*log(x)*a**2 - 12*a**2*c**6*x**6 + 54*a**2*c**4*x**4 - 108*a**2*c**2*x**2 + 54*b**2*c**2*x**2))/72`

3.177
$$\int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))^2}{x^2} dx$$

Optimal result	1653
Mathematica [A] (verified)	1655
Rubi [A] (verified)	1656
Maple [A] (verified)	1664
Fricas [F]	1665
Sympy [F]	1665
Maxima [F]	1666
Giac [F(-1)]	1666
Mupad [F(-1)]	1667
Reduce [F]	1667

Optimal result

Integrand size = 27, antiderivative size = 329

$$\begin{aligned} \int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))^2}{x^2} dx = & \frac{122}{25} b^2 c^2 d^3 x - \frac{14}{75} b^2 c^4 d^3 x^3 + \frac{2}{125} b^2 c^6 d^3 x^5 \\ & - \frac{22}{5} bcd^3 \sqrt{1-c^2 x^2} (a+b \arcsin(cx)) \\ & - \frac{2}{5} bcd^3 (1-c^2 x^2)^{3/2} (a+b \arcsin(cx)) \\ & - \frac{2}{25} bcd^3 (1-c^2 x^2)^{5/2} (a+b \arcsin(cx)) \\ & - \frac{16}{5} c^2 d^3 x (a+b \arcsin(cx))^2 \\ & - \frac{8}{5} c^2 d^3 x (1-c^2 x^2) (a+b \arcsin(cx))^2 \\ & - \frac{6}{5} c^2 d^3 x (1-c^2 x^2)^2 (a+b \arcsin(cx))^2 \\ & - \frac{d^3 (1-c^2 x^2)^3 (a+b \arcsin(cx))^2}{x} \\ & - 4bcd^3 (a+b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)}) \\ & + 2ib^2 cd^3 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \\ & - 2ib^2 cd^3 \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \end{aligned}$$

output

```
122/25*b^2*c^2*d^3*x-14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-22/5*b*c*
d^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-2/5*b*c*d^3*(-c^2*x^2+1)^(3/2)*(a
+b*arcsin(c*x))-2/25*b*c*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))-16/5*c^2
*d^3*x*(a+b*arcsin(c*x))^2-8/5*c^2*d^3*x*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2-
6/5*c^2*d^3*x*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2-d^3*(-c^2*x^2+1)^3*(a+b*a
rcsin(c*x))^2/x-4*b*c*d^3*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/
2))+2*I*b^2*c*d^3*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*b^2*c*d^3*polyl
og(2,I*c*x+(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.47

$$\begin{aligned}
\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx = & \frac{1}{720} d^3 \left(-\frac{720a^2}{x} - 2160a^2 c^2 x + 3460b^2 c^2 x \right. \\
& + 720a^2 c^4 x^3 - 144a^2 c^6 x^5 - \frac{17568}{5} abc \sqrt{1 - c^2 x^2} \\
& + \frac{2016}{5} abc^3 x^2 \sqrt{1 - c^2 x^2} - \frac{288}{5} abc^5 x^4 \sqrt{1 - c^2 x^2} \\
& - \frac{1440ab \arcsin(cx)}{x} - 4320abc^2 x \arcsin(cx) \\
& + 1440abc^4 x^3 \arcsin(cx) - 288abc^6 x^5 \arcsin(cx) \\
& \quad - 3420b^2 c \sqrt{1 - c^2 x^2} \arcsin(cx) \\
& - \frac{720b^2 \arcsin(cx)^2}{x} - 1890b^2 c^2 x \arcsin(cx)^2 \\
& \quad - 1440abc \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) \\
& \quad + 80b^2 c^2 x \cos(2 \arcsin(cx)) \\
& - 360b^2 c^2 x \arcsin(cx)^2 \cos(2 \arcsin(cx)) \\
& \quad - 90b^2 c \arcsin(cx) \cos(3 \arcsin(cx)) \\
& \quad - \frac{18}{5} b^2 c \arcsin(cx) \cos(5 \arcsin(cx)) \\
& + 1440b^2 c \arcsin(cx) \log(1 - e^{i \arcsin(cx)}) \\
& - 1440b^2 c \arcsin(cx) \log(1 + e^{i \arcsin(cx)}) \\
& \quad + 1440ib^2 c \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \\
& \quad - 1440ib^2 c \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \\
& \quad - 10b^2 c \sin(3 \arcsin(cx)) \\
& + 45b^2 c \arcsin(cx)^2 \sin(3 \arcsin(cx)) \\
& \quad + \frac{18}{25} b^2 c \sin(5 \arcsin(cx)) \\
& \left. - 9b^2 c \arcsin(cx)^2 \sin(5 \arcsin(cx)) \right)
\end{aligned}$$

input `Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x^2,x]`

output

```
(d^3*((-720*a^2)/x - 2160*a^2*c^2*x + 3460*b^2*c^2*x + 720*a^2*c^4*x^3 - 144*a^2*c^6*x^5 - (17568*a*b*c*Sqrt[1 - c^2*x^2])/5 + (2016*a*b*c^3*x^2*Sqrt[1 - c^2*x^2])/5 - (288*a*b*c^5*x^4*Sqrt[1 - c^2*x^2])/5 - (1440*a*b*ArcSin[c*x])/x - 4320*a*b*c^2*x*ArcSin[c*x] + 1440*a*b*c^4*x^3*ArcSin[c*x] - 288*a*b*c^6*x^5*ArcSin[c*x] - 3420*b^2*c*Sqrt[1 - c^2*x^2]*ArcSin[c*x] - (720*b^2*ArcSin[c*x]^2)/x - 1890*b^2*c^2*x*ArcSin[c*x]^2 - 1440*a*b*c*ArcTanh[Sqrt[1 - c^2*x^2]] + 80*b^2*c^2*x*Cos[2*ArcSin[c*x]] - 360*b^2*c^2*x*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] - 90*b^2*c*ArcSin[c*x]*Cos[3*ArcSin[c*x]] - (18*b^2*c*ArcSin[c*x]*Cos[5*ArcSin[c*x]])/5 + 1440*b^2*c*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - 1440*b^2*c*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + (1440*I)*b^2*c*PolyLog[2, -E^(I*ArcSin[c*x])] - (1440*I)*b^2*c*PolyLog[2, E^(I*ArcSin[c*x])] - 10*b^2*c*Sin[3*ArcSin[c*x]] + 45*b^2*c*ArcSin[c*x]^2*Sin[3*ArcSin[c*x]] + (18*b^2*c*Sin[5*ArcSin[c*x]])/25 - 9*b^2*c*ArcSin[c*x]^2*Sin[5*ArcSin[c*x]]))/720
```

Rubi [A] (verified)

Time = 3.71 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.41, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {5200, 27, 5158, 5158, 5130, 5182, 24, 210, 2009, 5202, 210, 2009, 5202, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx$$

$$\downarrow 5200$$

$$2bcd^3 \int \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x} dx - 6c^2 d \int d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx - \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{x}$$

$$\downarrow 27$$

$$2bcd^3 \int \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x} dx - 6c^2 d^3 \int (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx - \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{x}$$

$$\begin{aligned} & \downarrow 5158 \\ & 2bcd^3 \int \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} dx - \\ & 6c^2d^3 \left(-\frac{2}{5}bc \int x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx + \frac{4}{5} \int (1-c^2x^2)(a+b\arcsin(cx))^2 dx + \frac{1}{5}x(1-c^2x^2)^2(a+b\arcsin(cx)) \right) \\ & \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} \end{aligned}$$

$$\begin{aligned} & \downarrow 5158 \\ & 2bcd^3 \int \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} dx - \\ & 6c^2d^3 \left(-\frac{2}{5}bc \int x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx + \frac{4}{5} \left(-\frac{2}{3}bc \int x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{2}{3} \int (a+b\arcsin(cx))^2 dx \right) \right) \\ & \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} \end{aligned}$$

$$\begin{aligned} & \downarrow 5130 \\ & -6c^2d^3 \left(\frac{4}{5} \left(\frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2bc \int \frac{x(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} dx \right) - \frac{2}{3}bc \int x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx \right) \right) \\ & 2bcd^3 \int \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} dx - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} \end{aligned}$$

$$\begin{aligned} & \downarrow 5182 \\ & -6c^2d^3 \left(\frac{4}{5} \left(\frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{c^2} \right) \right) \right) - \frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} \right) \right) \\ & 2bcd^3 \int \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} dx - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} \end{aligned}$$

$$\begin{aligned} & \downarrow 24 \\ & -6c^2d^3 \left(\frac{4}{5} \left(-\frac{2}{3}bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{3c^2} \right) + \frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \right) \right) \\ & 2bcd^3 \int \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} dx - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} \end{aligned}$$

$$\downarrow 210$$

$$\begin{aligned}
& 2bcd^3 \int \frac{(1-c^2x^2)^{5/2} (a+b \arcsin(cx))}{x} dx - \\
6c^2d^3 & \left(\frac{4}{5} \left(-\frac{2}{3} bc \left(\frac{b \int (1-c^2x^2) dx}{3c} - \frac{(1-c^2x^2)^{3/2} (a+b \arcsin(cx))}{3c^2} \right) + \frac{1}{3} x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \right. \right. \\
& \left. \left. \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} \right) \right)
\end{aligned}$$

↓ 2009

$$\begin{aligned}
& 2bcd^3 \int \frac{(1-c^2x^2)^{5/2} (a+b \arcsin(cx))}{x} dx - \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} - \\
6c^2d^3 & \left(\frac{1}{5} x(1-c^2x^2)^2 (a+b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3} x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \left(x(a+b \arcsin(cx))^2 - 2b \right. \right. \right. \\
& \left. \left. \left. \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} \right) \right) \right)
\end{aligned}$$

↓ 5202

$$\begin{aligned}
& 2bcd^3 \left(\int \frac{(1-c^2x^2)^{3/2} (a+b \arcsin(cx))}{x} dx - \frac{1}{5} bc \int (1-c^2x^2)^2 dx + \frac{1}{5} (1-c^2x^2)^{5/2} (a+b \arcsin(cx)) \right) - \\
& \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} -
\end{aligned}$$

$$\begin{aligned}
6c^2d^3 & \left(\frac{1}{5} x(1-c^2x^2)^2 (a+b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3} x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \left(x(a+b \arcsin(cx))^2 - 2b \right. \right. \right. \\
& \left. \left. \left. \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} \right) \right) \right)
\end{aligned}$$

↓ 210

$$\begin{aligned}
& 2bcd^3 \left(\int \frac{(1-c^2x^2)^{3/2} (a+b \arcsin(cx))}{x} dx - \frac{1}{5} bc \int (c^4x^4 - 2c^2x^2 + 1) dx + \frac{1}{5} (1-c^2x^2)^{5/2} (a+b \arcsin(cx)) \right) - \\
& \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} -
\end{aligned}$$

$$\begin{aligned}
6c^2d^3 & \left(\frac{1}{5} x(1-c^2x^2)^2 (a+b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3} x(1-c^2x^2) (a+b \arcsin(cx))^2 + \frac{2}{3} \left(x(a+b \arcsin(cx))^2 - 2b \right. \right. \right. \\
& \left. \left. \left. \frac{d^3(1-c^2x^2)^3 (a+b \arcsin(cx))^2}{x} \right) \right) \right)
\end{aligned}$$

↓ 2009

$$2bcd^3 \left(\int \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}{x} dx + \frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) - \frac{1}{5}bc \left(\frac{c^4x^5}{5} - \frac{2c^2x^3}{3} + x \right) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} - 6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2b \right) \right) \right) \downarrow 5202$$

$$2bcd^3 \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx - \frac{1}{3}bc \int (1-c^2x^2) dx + \frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} - 6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2b \right) \right) \right) \downarrow 2009$$

$$2bcd^3 \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx + \frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} - 6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2b \right) \right) \right) \downarrow 5198$$

$$2bcd^3 \left(\int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int 1 dx + \frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arcsin(cx)) + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) - \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{x} - 6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2)(a+b\arcsin(cx))^2 + \frac{2}{3} \left(x(a+b\arcsin(cx))^2 - 2b \right) \right) \right) \downarrow 24$$

$$2bcd^3 \left(\int \frac{a + b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{1}{5}(1-c^2x^2)^{5/2} (a + b \arcsin(cx)) + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2} \right) - \frac{d^3(1-c^2x^2)^3 (a + b \arcsin(cx))^2}{x}$$

$$6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2 (a + b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2b \right) \right) \right)$$

↓ 5218

$$2bcd^3 \left(\int \frac{a + b \arcsin(cx)}{cx} d \arcsin(cx) + \frac{1}{5}(1-c^2x^2)^{5/2} (a + b \arcsin(cx)) + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2} \right) - \frac{d^3(1-c^2x^2)^3 (a + b \arcsin(cx))^2}{x}$$

$$6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2 (a + b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2b \right) \right) \right)$$

↓ 3042

$$2bcd^3 \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \frac{1}{5}(1-c^2x^2)^{5/2} (a + b \arcsin(cx)) + \frac{1}{3}(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1-c^2x^2} \right) - \frac{d^3(1-c^2x^2)^3 (a + b \arcsin(cx))^2}{x}$$

$$6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2 (a + b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2b \right) \right) \right)$$

↓ 4671

$$2bcd^3 \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right) - \frac{d^3(1-c^2x^2)^3 (a + b \arcsin(cx))^2}{x}$$

$$6c^2d^3 \left(\frac{1}{5}x(1-c^2x^2)^2 (a + b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1-c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2b \right) \right) \right)$$

↓ 2715

$$2bcd^3 \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \frac{d^3(1 - c^2x^2)^3 (a + b \arcsin(cx))^2}{x} \right)$$

$$6c^2d^3 \left(\frac{1}{5}x(1 - c^2x^2)^2 (a + b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1 - c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2b \right) \right) \right)$$

↓ 2838

$$2bcd^3 \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{5}(1 - c^2x^2)^{5/2} (a + b \arcsin(cx)) + \frac{1}{3}(1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) - 2 \frac{d^3(1 - c^2x^2)^3 (a + b \arcsin(cx))^2}{x} \right)$$

$$6c^2d^3 \left(\frac{1}{5}x(1 - c^2x^2)^2 (a + b \arcsin(cx))^2 + \frac{4}{5} \left(\frac{1}{3}x(1 - c^2x^2) (a + b \arcsin(cx))^2 + \frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2b \right) \right) \right)$$

input `Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x^2,x]`

output `-((d^3*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x])^2)/x) - 6*c^2*d^3*((x*(1 - c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*((b*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(5*c) - ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(5*c^2)))/5 + (4*((x*(1 - c^2*x^2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*((b*(x - (c^2*x^3)/3))/(3*c) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/3)/5) + 2*b*c*d^3*(-(b*c*x) - (b*c*(x - (c^2*x^3)/3))/3 - (b*c*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/5 + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) + ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 + ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/5 - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])`

Definitions of rubi rules used

- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 210 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`
- rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 5130 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1))
  Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]
  Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]
  Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))]
  Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:= Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))
  Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.26

method	result
parts	$-d^3 a^2 \left(\frac{c^6 x^5}{5} - c^4 x^3 + 3c^2 x + \frac{1}{x} \right) - d^3 b^2 c \left(\frac{19(-i\sqrt{-c^2 x^2 + 1} + cx) (\arcsin(cx)^2 - 2 + 2i \arcsin(cx))}{16} + \dots \right)$
derivativedivides	$c \left(-d^3 a^2 \left(\frac{c^5 x^5}{5} - c^3 x^3 + 3cx + \frac{1}{cx} \right) - d^3 b^2 \left(\frac{19(-i\sqrt{-c^2 x^2 + 1} + cx) (\arcsin(cx)^2 - 2 + 2i \arcsin(cx))}{16} \right) \right) - \dots$
default	$c \left(-d^3 a^2 \left(\frac{c^5 x^5}{5} - c^3 x^3 + 3cx + \frac{1}{cx} \right) - d^3 b^2 \left(\frac{19(-i\sqrt{-c^2 x^2 + 1} + cx) (\arcsin(cx)^2 - 2 + 2i \arcsin(cx))}{16} \right) \right) - \dots$

input

```
int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output

```
-d^3*a^2*(1/5*c^6*x^5-c^4*x^3+3*c^2*x+1/x)-d^3*b^2*c*(19/16*(-I*(-c^2*x^2+
1)^(1/2)+c*x)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))+19/16*(I*(-c^2*x^2+1)^(1/2
)+c*x)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))+1/c/x*arcsin(c*x)^2-2*arcsin(c*x)
*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2
))+2*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*I*polylog(2,-I*c*x-(-c^2*x^2+
1)^(1/2))+1/200*arcsin(c*x)*cos(5*arcsin(c*x))+1/2000*(25*arcsin(c*x)^2-2)
*sin(5*arcsin(c*x))+1/8*arcsin(c*x)*cos(3*arcsin(c*x))+1/48*(9*arcsin(c*x)
^2-2)*sin(3*arcsin(c*x))-2*d^3*a*b*c*(1/5*c^5*x^5*arcsin(c*x)-c^3*x^3*arc
sin(c*x)+3*c*x*arcsin(c*x)+arcsin(c*x)/c/x+1/25*c^4*x^4*(-c^2*x^2+1)^(1/2)
-7/25*c^2*x^2*(-c^2*x^2+1)^(1/2)+61/25*(-c^2*x^2+1)^(1/2)+arctanh(1/(-c^2*
x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")`

output `integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arcsin(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arcsin(c*x))/x^2, x)`

Sympy [F]

$$\begin{aligned} & \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx \\ &= -d^3 \left(\int 3a^2 c^2 dx + \int \left(-\frac{a^2}{x^2} \right) dx + \int (-3a^2 c^4 x^2) dx + \int a^2 c^6 x^4 dx \right. \\ & \quad \left. + \int 3b^2 c^2 \operatorname{asin}^2(cx) dx + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x^2} \right) dx + \int 6abc^2 \operatorname{asin}(cx) dx \right. \\ & \quad \left. + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x^2} \right) dx + \int (-3b^2 c^4 x^2 \operatorname{asin}^2(cx)) dx + \int b^2 c^6 x^4 \operatorname{asin}^2(cx) dx \right. \\ & \quad \left. + \int (-6abc^4 x^2 \operatorname{asin}(cx)) dx + \int 2abc^6 x^4 \operatorname{asin}(cx) dx \right) \end{aligned}$$

input `integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2/x**2,x)`

output `-d**3*(Integral(3*a**2*c**2, x) + Integral(-a**2/x**2, x) + Integral(-3*a**2*c**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*asin(c*x)**2, x) + Integral(-b**2*asin(c*x)**2/x**2, x) + Integral(6*a*b*c**2*asin(c*x), x) + Integral(-2*a*b*asin(c*x)/x**2, x) + Integral(-3*b**2*c**4*x**2*asin(c*x)**2, x) + Integral(b**2*c**6*x**4*asin(c*x)**2, x) + Integral(-6*a*b*c**4*x**2*asin(c*x), x) + Integral(2*a*b*c**6*x**4*asin(c*x), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^2,x, algorithm="maxima")`

output `-1/5*a^2*c^6*d^3*x^5 - 2/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1))*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^6*d^3 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1))*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 - 3*b^2*c^2*d^3*x*arcsin(c*x)^2 + 6*b^2*c^2*d^3*(x - sqrt(-c^2*x^2 + 1)*arcsin(c*x)/c) - 3*a^2*c^2*d^3*x - 6*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a*b*c*d^3 - 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*a*b*d^3 - a^2*d^3/x - 1/5*((b^2*c^6*d^3*x^6 - 5*b^2*c^4*d^3*x^4 + 5*b^2*d^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 5*x*integrate(2/5*(b^2*c^7*d^3*x^6 - 5*b^2*c^5*d^3*x^4 + 5*b^2*c*d^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^3 - x), x))/x`

Giac [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx = \text{Timed out}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^2,x, algorithm="giac")`

output `Timed out`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^3}{x^2} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x^2,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x^2, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^2} dx$$

$$= \frac{d^3 \left(-75 \arcsin(cx)^2 b^2 c^2 x^2 - 150 \sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 cx - 10 \arcsin(cx) ab c^6 x^6 + 50 \arcsin(cx) ab c^4 x^4 - 15 \right)}{25x}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2/x^2,x)`

output `(d**3*(- 75*asin(c*x)**2*b**2*c**2*x**2 - 150*sqrt(- c**2*x**2 + 1)*asin(c*x)*b**2*c*x - 10*asin(c*x)*a*b*c**6*x**6 + 50*asin(c*x)*a*b*c**4*x**4 - 150*asin(c*x)*a*b*c**2*x**2 - 50*asin(c*x)*a*b - 2*sqrt(- c**2*x**2 + 1)*a*b*c**5*x**5 + 14*sqrt(- c**2*x**2 + 1)*a*b*c**3*x**3 - 122*sqrt(- c**2*x**2 + 1)*a*b*c*x + 25*int(asin(c*x)**2/x**2,x)*b**2*x - 25*int(asin(c*x)**2*x**4,x)*b**2*c**6*x + 75*int(asin(c*x)**2*x**2,x)*b**2*c**4*x + 50*log(tan(asin(c*x)/2))*a*b*c*x - 5*a**2*c**6*x**6 + 25*a**2*c**4*x**4 - 75*a**2*c**2*x**2 - 25*a**2 + 150*b**2*c**2*x**2))/(25*x)`

$$3.178 \quad \int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))^2}{x^3} dx$$

Optimal result	1668
Mathematica [A] (verified)	1669
Rubi [F]	1670
Maple [A] (verified)	1677
Fricas [F]	1678
Sympy [F]	1678
Maxima [F]	1679
Giac [F(-2)]	1679
Mupad [F(-1)]	1680
Reduce [F]	1680

Optimal result

Integrand size = 27, antiderivative size = 396

$$\begin{aligned}
& \int \frac{(d-c^2 dx^2)^3 (a+b \arcsin(cx))^2}{x^3} dx \\
&= -\frac{35}{32} b^2 c^4 d^3 x^2 + \frac{1}{4} b^2 c^6 d^3 x^4 - \frac{7}{32} b^2 c^2 d^3 (1-c^2 x^2)^2 \\
&\quad + \frac{3}{16} b c^3 d^3 x \sqrt{1-c^2 x^2} (a+b \arcsin(cx)) \\
&\quad - \frac{7}{8} b c^3 d^3 x (1-c^2 x^2)^{3/2} (a+b \arcsin(cx)) - \frac{b c d^3 (1-c^2 x^2)^{5/2} (a+b \arcsin(cx))}{x} \\
&\quad\quad + \frac{3}{32} c^2 d^3 (a+b \arcsin(cx))^2 - \frac{3}{2} c^2 d^3 (1-c^2 x^2) (a+b \arcsin(cx))^2 \\
&\quad - \frac{3}{4} c^2 d^3 (1-c^2 x^2)^2 (a+b \arcsin(cx))^2 - \frac{d^3 (1-c^2 x^2)^3 (a+b \arcsin(cx))^2}{2x^2} \\
&\quad + \frac{i c^2 d^3 (a+b \arcsin(cx))^3}{b} - 3 c^2 d^3 (a+b \arcsin(cx))^2 \log(1-e^{2i \arcsin(cx)}) \\
&\quad\quad + b^2 c^2 d^3 \log(x) + 3 i b c^2 d^3 (a+b \arcsin(cx)) \text{PolyLog}(2, e^{2i \arcsin(cx)}) \\
&\quad\quad\quad - \frac{3}{2} b^2 c^2 d^3 \text{PolyLog}(3, e^{2i \arcsin(cx)})
\end{aligned}$$

output

```
-35/32*b^2*c^4*d^3*x^2+1/4*b^2*c^6*d^3*x^4-7/32*b^2*c^2*d^3*(-c^2*x^2+1)^2
+3/16*b*c^3*d^3*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-7/8*b*c^3*d^3*x*(-c
^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))-b*c*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c
*x))/x+3/32*c^2*d^3*(a+b*arcsin(c*x))^2-3/2*c^2*d^3*(-c^2*x^2+1)*(a+b*arcs
in(c*x))^2-3/4*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2-1/2*d^3*(-c^2*x^
2+1)^3*(a+b*arcsin(c*x))^2/x^2+I*c^2*d^3*(a+b*arcsin(c*x))^3/b-3*c^2*d^3*(
a+b*arcsin(c*x))^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^3*ln(x)+3*
I*b*c^2*d^3*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)-3/2*
b^2*c^2*d^3*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.40

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx =$$

$$\frac{d^3 \left(128a^2 - 32ib^2c^2\pi^3x^2 - 384a^2c^4x^4 + 64a^2c^6x^6 + 256abcx\sqrt{1 - c^2x^2} - 336abc^3x^3\sqrt{1 - c^2x^2} + 32ab \right)}{x^3}$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x^3,x]
```

output

```
-1/256*(d^3*(128*a^2 - (32*I)*b^2*c^2*Pi^3*x^2 - 384*a^2*c^4*x^4 + 64*a^2*c
^6*x^6 + 256*a*b*c*x*sqrt[1 - c^2*x^2] - 336*a*b*c^3*x^3*sqrt[1 - c^2*x^2
] + 32*a*b*c^5*x^5*sqrt[1 - c^2*x^2] + 256*a*b*ArcSin[c*x] - 768*a*b*c^4*x
^4*ArcSin[c*x] + 128*a*b*c^6*x^6*ArcSin[c*x] + 256*b^2*c*x*sqrt[1 - c^2*x^
2]*ArcSin[c*x] + 128*b^2*ArcSin[c*x]^2 - (768*I)*a*b*c^2*x^2*ArcSin[c*x]^2
+ (256*I)*b^2*c^2*x^2*ArcSin[c*x]^3 + 672*a*b*c^2*x^2*ArcTan[(c*x)/(-1 +
sqrt[1 - c^2*x^2]]) - 80*b^2*c^2*x^2*cos[2*ArcSin[c*x]] + 160*b^2*c^2*x^2*
ArcSin[c*x]^2*cos[2*ArcSin[c*x]] - b^2*c^2*x^2*cos[4*ArcSin[c*x]] + 8*b^2*
c^2*x^2*ArcSin[c*x]^2*cos[4*ArcSin[c*x]] + 768*b^2*c^2*x^2*ArcSin[c*x]^2*L
og[1 - E^((-2*I)*ArcSin[c*x])] + 1536*a*b*c^2*x^2*ArcSin[c*x]*Log[1 - E^((
2*I)*ArcSin[c*x])] + 768*a^2*c^2*x^2*Log[x] - 256*b^2*c^2*x^2*Log[c*x] + (
768*I)*b^2*c^2*x^2*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] - (768*I
)*a*b*c^2*x^2*PolyLog[2, E^((2*I)*ArcSin[c*x])] + 384*b^2*c^2*x^2*PolyLog[
3, E^((-2*I)*ArcSin[c*x])] - 160*b^2*c^2*x^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]
] - 4*b^2*c^2*x^2*ArcSin[c*x]*Sin[4*ArcSin[c*x]]))/x^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx \\
 & \quad \downarrow \text{5200} \\
 & bcd^3 \int \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx - 3c^2 d \int \frac{d^2 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx - \\
 & \quad \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{27} \\
 & bcd^3 \int \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x^2} dx - 3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx - \\
 & \quad \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{5200} \\
 & bcd^3 \left(-5c^2 \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + bc \int \frac{(1 - c^2 x^2)^2}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x} \right) - \\
 & \quad 3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx - \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{243} \\
 & bcd^3 \left(-5c^2 \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx + \frac{1}{2} bc \int \frac{(1 - c^2 x^2)^2}{x^2} dx^2 - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x} \right) - \\
 & \quad 3c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} dx - \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{49}
 \end{aligned}$$

$$\begin{aligned}
& -3c^2d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx + \\
bcd^3 & \left(-5c^2 \int (1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx + \frac{1}{2}bc \int \left(x^2c^4 - 2c^2 + \frac{1}{x^2} \right) dx^2 - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) \\
& \quad \downarrow \text{2009} \\
& -3c^2d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx + \\
bcd^3 & \left(-5c^2 \int (1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx - \frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} + \frac{1}{2}bc \left(\frac{c^4x^4}{2} - 2c^2x^2 + \log \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) \\
& \quad \downarrow \text{5158} \\
& -3c^2d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx + \\
bcd^3 & \left(-5c^2 \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx - \frac{1}{4}bc \int x(1-c^2x^2) dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) \\
& \quad \downarrow \text{244} \\
& -3c^2d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx + \\
bcd^3 & \left(-5c^2 \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx - \frac{1}{4}bc \int (x-c^2x^3) dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) \\
& \quad \downarrow \text{2009} \\
& -3c^2d^3 \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))^2}{x} dx + \\
bcd^3 & \left(-5c^2 \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{x^2}{2} - \frac{c^2x^4}{4} \right) \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) \\
& \quad \downarrow \text{5156}
\end{aligned}$$

$$-3c^2d^3 \int \frac{(1-c^2x^2)^2 (a+b\arcsin(cx))^2}{x} dx +$$

$$bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \int x dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) + \frac{d^3(1-c^2x^2)^3 (a+b\arcsin(cx))^2}{2x^2}$$

↓ 15

$$-3c^2d^3 \int \frac{(1-c^2x^2)^2 (a+b\arcsin(cx))^2}{x} dx +$$

$$bcd^3 \left(-5c^2 \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{4}bcx^2 \right) \right) + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) + \frac{d^3(1-c^2x^2)^3 (a+b\arcsin(cx))^2}{2x^2}$$

↓ 5152

$$-3c^2d^3 \int \frac{(1-c^2x^2)^2 (a+b\arcsin(cx))^2}{x} dx - \frac{d^3(1-c^2x^2)^3 (a+b\arcsin(cx))^2}{2x^2} +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 5202

$$-3c^2d^3 \left(-\frac{1}{2}bc \int (1-c^2x^2)^{3/2}(a+b\arcsin(cx)) dx + \int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx)) \right) + \frac{d^3(1-c^2x^2)^3 (a+b\arcsin(cx))^2}{2x^2} +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 5158

$$-3c^2d^3 \left(-\frac{1}{2}bc \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx - \frac{1}{4}bc \int x(1-c^2x^2) dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right) + \frac{d^3(1-c^2x^2)^3 (a+b\arcsin(cx))^2}{2x^2} +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 244

$$-3c^2 d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{1}{2}bc \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx - \frac{1}{4}bc \int (x-c^2x^3) dx \right) \right. \\ \left. + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 2009

$$-3c^2 d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{1}{2}bc \left(\frac{3}{4} \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right) \right. \\ \left. + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 5156

$$-3c^2 d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{1}{2}bc \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \int x dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right. \\ \left. + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 15

$$-3c^2 d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx - \frac{1}{2}bc \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right. \\ \left. + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) +$$

$$bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right)$$

↓ 5152

$$\begin{aligned}
& -3c^2 d^3 \left(\int \frac{(1-c^2x^2)(a+b\arcsin(cx))^2}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) \right. \right. \\
& \qquad \qquad \qquad \left. \left. + \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) + \right. \\
& bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{5202}
\end{aligned}$$

$$\begin{aligned}
& -3c^2 d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \int \frac{(a+b\arcsin(cx))^2}{x} dx + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))^2 + \right. \\
& \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) + \\
& bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{5136}
\end{aligned}$$

$$\begin{aligned}
& -3c^2 d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{cx} d\arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))^2 \right. \\
& \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) + \\
& bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{3042}
\end{aligned}$$

$$\begin{aligned}
& -3c^2 d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx)) dx + \int -(a+b\arcsin(cx))^2 \tan \left(\arcsin(cx) + \frac{\pi}{2} \right) d\arcsin(cx) + \frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))^2 \right. \\
& \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} \right) + \\
& bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right) \\
& \qquad \qquad \qquad \downarrow \text{25}
\end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - \int (a+b\arcsin(cx))^2 \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d\arcsin(cx) + \frac{1}{4}(1-c^2x^2) \right. \\
 & \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow 4200 \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx + 2i \int -\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))^2}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) + \frac{1}{4}(1-c^2x^2) \right. \\
 & \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow 25 \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))^2}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) + \frac{1}{4}(1-c^2x^2) \right. \\
 & \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow 2620 \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \left(\frac{1}{2}i \log\left(1-e^{2i\arcsin(cx)}\right) (a+b\arcsin(cx))^2 - ib \int (a+b\arcsin(cx)) \right. \right. \\
 & \qquad \qquad \qquad \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow 3011 \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \text{PolyL} \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow \text{2720} \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \int \sqrt{1-c^2x^2}(a+b\arcsin(cx))dx - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx))^2 - ib \left(\frac{1}{2}i \text{PolyL} \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow \text{5156} \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \int x dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \left. \downarrow \text{15} \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & -3c^2d^3 \left(-bc \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{4}bcx^2 \right) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right. \right. \\
 & \qquad \qquad \qquad \left. \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{2x^2} + \right. \right. \\
 & bcd^3 \left(-\frac{(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}{x} - 5c^2 \left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \frac{3}{4} \left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right. \right. \right.
 \end{aligned}$$

input `Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x^3,x]`

output `$Aborted`

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.79

method	result
derivativedivides	$c^2 \left(-d^3 a^2 \left(\frac{c^4 x^4}{4} - \frac{3c^2 x^2}{2} + 3 \ln(cx) + \frac{1}{2c^2 x^2} \right) - d^3 b^2 \left(-i \arcsin(cx)^3 - \frac{5(2i \arcsin(cx) + 2 \arcsin^3(cx))}{3} \right) \right)$
default	$c^2 \left(-d^3 a^2 \left(\frac{c^4 x^4}{4} - \frac{3c^2 x^2}{2} + 3 \ln(cx) + \frac{1}{2c^2 x^2} \right) - d^3 b^2 \left(-i \arcsin(cx)^3 - \frac{5(2i \arcsin(cx) + 2 \arcsin^3(cx))}{3} \right) \right)$
parts	$-d^3 a^2 \left(\frac{c^6 x^4}{4} - \frac{3c^4 x^2}{2} + 3c^2 \ln(x) + \frac{1}{2x^2} \right) - d^3 b^2 c^2 \left(-i \arcsin(cx)^3 - \frac{5(2i \arcsin(cx) + 2 \arcsin^3(cx))}{3} \right)$

input

```
int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output

```
c^2*(-d^3*a^2*(1/4*c^4*x^4-3/2*c^2*x^2+3*ln(c*x)+1/2/c^2/x^2)-d^3*b^2*(-I*arcsin(c*x)^3-5/32*(2*I*arcsin(c*x)+2*arcsin(c*x)^2-1)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)-5/32*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))+1/2*arcsin(c*x)*(-2*I*c^2*x^2+2*c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+2*ln(I*c*x+(-c^2*x^2+1)^(1/2))-ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)-ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-6*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+3*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-6*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+6*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))+3*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+6*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+1/256*(8*arcsin(c*x)^2-1)*cos(4*arcsin(c*x))-1/64*arcsin(c*x)*sin(4*arcsin(c*x))-2*d^3*a*b*(-3/2*I*arcsin(c*x)^2-5/32*(I+2*arcsin(c*x))*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)-5/32*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(-I+2*arcsin(c*x))+1/2*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+3*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+3*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-3*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-3*I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+1/32*arcsin(c*x)*cos(4*arcsin(c*x))-1/128*sin(4*arcsin(c*x))))
```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^3,x, algorithm="fricas")`

output `integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arcsin(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arcsin(c*x))/x^3, x)`

Sympy [F]

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx = & -d^3 \left(\int \left(-\frac{a^2}{x^3} \right) dx + \int \frac{3a^2 c^2}{x} dx \right. \\ & + \int (-3a^2 c^4 x) dx + \int a^2 c^6 x^3 dx \\ & + \int \left(-\frac{b^2 \operatorname{asin}^2(cx)}{x^3} \right) dx \\ & + \int \left(-\frac{2ab \operatorname{asin}(cx)}{x^3} \right) dx \\ & + \int \frac{3b^2 c^2 \operatorname{asin}^2(cx)}{x} dx \\ & + \int (-3b^2 c^4 x \operatorname{asin}^2(cx)) dx \\ & + \int b^2 c^6 x^3 \operatorname{asin}^2(cx) dx + \int \frac{6abc^2 \operatorname{asin}(cx)}{x} dx \\ & + \int (-6abc^4 x \operatorname{asin}(cx)) dx \\ & \left. + \int 2abc^6 x^3 \operatorname{asin}(cx) dx \right) \end{aligned}$$

input `integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2/x**3,x)`

output

```
-d**3*(Integral(-a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(-3*
a**2*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(-b**2*asin(c*x)**
2/x**3, x) + Integral(-2*a*b*asin(c*x)/x**3, x) + Integral(3*b**2*c**2*asi
n(c*x)**2/x, x) + Integral(-3*b**2*c**4*x*asin(c*x)**2, x) + Integral(b**2
*c**6*x**3*asin(c*x)**2, x) + Integral(6*a*b*c**2*asin(c*x)/x, x) + Integr
al(-6*a*b*c**4*x*asin(c*x), x) + Integral(2*a*b*c**6*x**3*asin(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x^3} dx$$

input

```
integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^3,x, algorithm="maxima")
```

output

```
-1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 - 3*a^2*c^2*d^3*log(x) - a*b*d^
3*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) - 1/2*a^2*d^3/x^2 - integrate
(((b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arct
an2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*
d^3*x^4 + 3*a*b*c^2*d^3*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/x
^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^3,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^3}{x^3} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x^3,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^3} dx$$

$$= \frac{d^3 \left(24 \operatorname{asin}(cx)^2 b^2 c^4 x^4 - 12 \operatorname{asin}(cx)^2 b^2 c^2 x^2 + 24 \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) b^2 c^3 x^3 - 8 \operatorname{asin}(cx) a b c^6 x^6 + 48 a^2 \right)}{16 x^2}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2/x^3,x)`

output `(d**3*(24*asin(c*x)**2*b**2*c**4*x**4 - 12*asin(c*x)**2*b**2*c**2*x**2 + 24*sqrt(-c**2*x**2 + 1)*asin(c*x)*b**2*c**3*x**3 - 8*asin(c*x)*a*b*c**6*x**6 + 48*asin(c*x)*a*b*c**4*x**4 - 21*asin(c*x)*a*b*c**2*x**2 - 16*asin(c*x)*a*b - 2*sqrt(-c**2*x**2 + 1)*a*b*c**5*x**5 + 21*sqrt(-c**2*x**2 + 1)*a*b*c**3*x**3 - 16*sqrt(-c**2*x**2 + 1)*a*b*c*x - 96*int(asin(c*x)/x,x)*a*b*c**2*x**2 + 16*int(asin(c*x)**2/x**3,x)*b**2*x**2 - 48*int(asin(c*x)**2/x,x)*b**2*c**2*x**2 - 16*int(asin(c*x)**2*x**3,x)*b**2*c**6*x**2 - 48*log(x)*a**2*c**2*x**2 - 4*a**2*c**6*x**6 + 24*a**2*c**4*x**4 - 8*a**2 - 12*b**2*c**4*x**4)/(16*x**2)`

3.179
$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx$$

Optimal result	1681
Mathematica [A] (verified)	1682
Rubi [A] (verified)	1683
Maple [A] (verified)	1691
Fricas [F]	1691
Sympy [F]	1692
Maxima [F]	1693
Giac [F(-1)]	1693
Mupad [F(-1)]	1694
Reduce [F]	1694

Optimal result

Integrand size = 27, antiderivative size = 348

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx = & -\frac{b^2 c^2 d^3}{3x} - \frac{50}{9} b^2 c^4 d^3 x + \frac{2}{27} b^2 c^6 d^3 x^3 \\ & + 5bc^3 d^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \\ & - \frac{1}{9} bc^3 d^3 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \\ & - \frac{bcd^3 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{3x^2} \\ & + \frac{16}{3} c^4 d^3 x (a + b \arcsin(cx))^2 \\ & + \frac{8}{3} c^4 d^3 x (1 - c^2 x^2) (a + b \arcsin(cx))^2 \\ & + \frac{2c^2 d^3 (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} \\ & - \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3} \\ & + \frac{34}{3} bc^3 d^3 (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)}) \\ & - \frac{17}{3} ib^2 c^3 d^3 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \\ & + \frac{17}{3} ib^2 c^3 d^3 \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \end{aligned}$$

output

```
-1/3*b^2*c^2*d^3/x-50/9*b^2*c^4*d^3*x+2/27*b^2*c^6*d^3*x^3+5*b*c^3*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-1/9*b*c^3*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))-1/3*b*c*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/x^2+16/3*c^4*d^3*x*(a+b*arcsin(c*x))^2+8/3*c^4*d^3*x*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+2*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/x-1/3*d^3*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2/x^3+34/3*b*c^3*d^3*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))-17/3*I*b^2*c^3*d^3*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))+17/3*I*b^2*c^3*d^3*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.38

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx =$$

$$\frac{d^3(9a^2 - 81a^2c^2x^2 + 9b^2c^2x^2 - 81a^2c^4x^4 + 150b^2c^4x^4 + 9a^2c^6x^6 - 2b^2c^6x^6 + 9abcx\sqrt{1 - c^2x^2} - 150a$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

```
-1/27*(d^3*(9*a^2 - 81*a^2*c^2*x^2 + 9*b^2*c^2*x^2 - 81*a^2*c^4*x^4 + 150*b^2*c^4*x^4 + 9*a^2*c^6*x^6 - 2*b^2*c^6*x^6 + 9*a*b*c*x*Sqrt[1 - c^2*x^2] - 150*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 6*a*b*c^5*x^5*Sqrt[1 - c^2*x^2] + 18*a*b*ArcSin[c*x] - 162*a*b*c^2*x^2*ArcSin[c*x] - 162*a*b*c^4*x^4*ArcSin[c*x] + 18*a*b*c^6*x^6*ArcSin[c*x] + 9*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcSin[c*x] - 150*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + 6*b^2*c^5*x^5*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + 9*b^2*ArcSin[c*x]^2 - 81*b^2*c^2*x^2*ArcSin[c*x]^2 - 81*b^2*c^4*x^4*ArcSin[c*x]^2 + 9*b^2*c^6*x^6*ArcSin[c*x]^2 - 153*a*b*c^3*x^3*ArcTanH[Sqrt[1 - c^2*x^2]] + 153*b^2*c^3*x^3*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - 153*b^2*c^3*x^3*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + (153*I)*b^2*c^3*x^3*PolyLog[2, -E^(I*ArcSin[c*x])] - (153*I)*b^2*c^3*x^3*PolyLog[2, E^(I*ArcSin[c*x])]))/x^3
```

Rubi [A] (verified)

Time = 3.39 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.58, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {5200, 27, 5200, 244, 2009, 5158, 5130, 5182, 24, 2009, 5202, 2009, 5198, 24, 5218, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx$$

$$\downarrow \text{5200}$$

$$\frac{2}{3}bcd^3 \int \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx - 2c^2 d \int \frac{d^2(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x^2} dx - \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{27}$$

$$-2c^2 d^3 \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x^2} dx + \frac{2}{3}bcd^3 \int \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{x^3} dx - \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{5200}$$

$$\frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx + \frac{1}{2}bc \int \frac{(1 - c^2 x^2)^2}{x^2} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} \right)$$

$$2c^2 d^3 \left(-4c^2 \int (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + 2bc \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} \right) - \frac{d^3(1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{244}$$

$$\begin{aligned}
& -2c^2 d^3 \left(-4c^2 \int (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + 2bc \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx + \frac{1}{2} bc \int \left(x^2 c^4 - 2c^2 + \frac{1}{x^2} \right) dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 2009

$$\begin{aligned}
& -2c^2 d^3 \left(-4c^2 \int (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx + 2bc \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bc \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 5158

$$\begin{aligned}
& -2c^2 d^3 \left(-4c^2 \left(-\frac{2}{3} bc \int x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx + \frac{2}{3} \int (a + b \arcsin(cx))^2 dx + \frac{1}{3} x (1 - c^2 x^2) (a + b \arcsin(cx)) \right) \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bc \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 5130

$$\begin{aligned}
& -2c^2 d^3 \left(-4c^2 \left(\frac{2}{3} \left(x (a + b \arcsin(cx))^2 - 2bc \int \frac{x (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} dx \right) - \frac{2}{3} bc \int x \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx \right) \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bc \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 5182

$$\begin{aligned}
& -2c^2 d^3 \left(-4c^2 \left(\frac{2}{3} \left(x(a + b \arcsin(cx))^2 - 2bc \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{c^2} \right) \right) \right) - \frac{2}{3} bc \left(\frac{b \int (1 - c^2 x^2)}{3c} \right) \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bc \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 24

$$\begin{aligned}
& -2c^2 d^3 \left(-4c^2 \left(-\frac{2}{3} bc \left(\frac{b \int (1 - c^2 x^2) dx}{3c} - \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{3c^2} \right) \right) + \frac{1}{3} x(1 - c^2 x^2) (a + b \arcsin(cx)) \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bc \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 2009

$$\begin{aligned}
& -2c^2 d^3 \left(2bc \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))^2}{x} - 4c^2 \left(\frac{1}{3} x(1 - c^2 x^2) (a + b \arcsin(cx)) \right) \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \int \frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))}{x} dx - \frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))}{2x^2} + \frac{1}{2} bc \left(\frac{c^4 x^3}{3} - 2c^2 x - \frac{1}{x} \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 5202

$$\begin{aligned}
& -2c^2 d^3 \left(2bc \left(\int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} dx - \frac{1}{3} bc \int (1 - c^2 x^2) dx + \frac{1}{3} (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) \right) \\
& \frac{2}{3} bcd^3 \left(-\frac{5}{2} c^2 \left(\int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{x} dx - \frac{1}{3} bc \int (1 - c^2 x^2) dx + \frac{1}{3} (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) \right) \right) \\
& \frac{d^3 (1 - c^2 x^2)^3 (a + b \arcsin(cx))^2}{3x^3}
\end{aligned}$$

↓ 2009

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) - \frac{1}{3}bc \left(x - \frac{c^2x^3}{3} \right) \right) - \frac{1}{3} \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{x} dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) - \frac{1}{3}bc \left(x - \frac{c^2x^3}{3} \right) \right) - \frac{1}{3} \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{3x^3} \right) \\
& \quad \downarrow \text{5198}
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(\int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int 1 dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(\int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int 1 dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{3x^3} \right) \\
& \quad \downarrow \text{24}
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(\int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{3}bc \left(x - \frac{c^2x^3}{3} \right) \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(\int \frac{a+b\arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) - \frac{1}{3}bc \left(x - \frac{c^2x^3}{3} \right) \right) \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{3x^3} \right) \\
& \quad \downarrow \text{5218}
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(\int \frac{a+b\arcsin(cx)}{cx} d\arcsin(cx) + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(\int \frac{a+b\arcsin(cx)}{cx} d\arcsin(cx) + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx)) + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right) \right) \right. \\
& \left. \frac{d^3(1-c^2x^2)^3(a+b\arcsin(cx))^2}{3x^3} \right) \\
& \quad \downarrow \text{3042}
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \frac{1}{3}(1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1 - c^2x^2} \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(\int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \frac{1}{3}(1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1 - c^2x^2} \right) \right. \right. \\
& \qquad \qquad \qquad \left. \left. \frac{d^3(1 - c^2x^2)^3 (a + b \arcsin(cx))^2}{3x^3} \right) \right. \\
& \qquad \qquad \qquad \downarrow 4671
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) \right) \right. \right. \\
& \qquad \qquad \qquad \left. \left. \frac{d^3(1 - c^2x^2)^3 (a + b \arcsin(cx))^2}{3x^3} \right) \right. \\
& \qquad \qquad \qquad \downarrow 2715
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right) \right. \right. \\
& \qquad \qquad \qquad \left. \left. \frac{d^3(1 - c^2x^2)^3 (a + b \arcsin(cx))^2}{3x^3} \right) \right. \\
& \qquad \qquad \qquad \downarrow 2838
\end{aligned}$$

$$\begin{aligned}
& -2c^2d^3 \left(2bc \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{3}(1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1 - c^2x^2} (a + b \arcsin(cx)) \right) \right. \\
& \left. \frac{2}{3}bcd^3 \left(-\frac{5}{2}c^2 \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{3}(1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) + \sqrt{1 - c^2x^2} (a + b \arcsin(cx)) \right) \right. \right. \\
& \qquad \qquad \qquad \left. \left. \frac{d^3(1 - c^2x^2)^3 (a + b \arcsin(cx))^2}{3x^3} \right) \right.
\end{aligned}$$

input

```
Int[((d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

$$\begin{aligned}
& -1/3*(d^3*(1 - c^2*x^2)^3*(a + b*ArcSin[c*x])^2)/x^3 - 2*c^2*d^3*(-(((1 - \\
& c^2*x^2)^2*(a + b*ArcSin[c*x])^2)/x) - 4*c^2*((x*(1 - c^2*x^2)*(a + b*ArcS \\
& in[c*x])^2)/3 - (2*b*c*((b*(x - (c^2*x^3)/3))/(3*c) - ((1 - c^2*x^2)^(3/2) \\
& *(a + b*ArcSin[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcSin[c*x])^2 - 2*b*c*((\\
& b*x)/c - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/3 + 2*b*c*(-(b*c* \\
& x) - (b*c*(x - (c^2*x^3)/3))/3 + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) + (\\
& (1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 - 2*(a + b*ArcSin[c*x])*ArcTanh \\
& [E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, \\
& E^(I*ArcSin[c*x])]) + (2*b*c*d^3*((b*c*(-x^(-1) - 2*c^2*x + (c^4*x^3)/3)) \\
& /2 - ((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x]))/(2*x^2) - (5*c^2*(-(b*c*x) \\
& - (b*c*(x - (c^2*x^3)/3)))/3 + Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]) + ((1 \\
& - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x]))/3 - 2*(a + b*ArcSin[c*x])*ArcTanh[E^ \\
& (I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(\\
& I*ArcSin[c*x])]))/2))/3
\end{aligned}$$

Defintions of rubi rules used

rule 24

$$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$$

rule 27

$$\text{Int}[(a_)*(F_x_), x_Symbol] \text{ :> } \text{Simp}[a \text{ Int}[F_x, x], x] \text{ /; } \text{FreeQ}[a, x] \ \&\& \ !\text{Ma} \\
\text{tchQ}[F_x, (b_)*(G_x_) \text{ /; } \text{FreeQ}[b, x]$$

rule 244

$$\text{Int}[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] \text{ :> } \text{Int}[\text{Expand} \\
\text{Integrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ /; } \text{FreeQ}\{a, b, c, m\}, x \ \&\& \ \text{IGtQ}[p, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Simp}[\text{IntSum}[u, x], x] \text{ /; } \text{SumQ}[u]$$

rule 2715

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))]^(n_)], x_Symbol] \\
\text{ :> } \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x) \\
))^(n)], x] \text{ /; } \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838 $\text{Int}[\text{Log}[(c_)*(d_)+(e_)*(x_)^{(n_)}]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_)+(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c+d*x)^m*(\text{ArcTanh}[E^{(I*(e+f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \ \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1-E^{(I*(e+f*x))}], x], x] + \text{Simp}[d*(m/f) \ \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1+E^{(I*(e+f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5130 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[x*(a+b*\text{ArcSin}[c*x])^n, x] - \text{Simp}[b*c*n \ \text{Int}[x*(a+b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1-c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$

rule 5158 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[x*(d+e*x^2)^p*(a+b*\text{ArcSin}[c*x])^n/(2*p+1), x] + (\text{Simp}[2*d*(p/(2*p+1)) \ \text{Int}[(d+e*x^2)^{(p-1)}*(a+b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(2*p+1))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \ \text{Int}[x*(1-c^2*x^2)^{(p-1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0]$

rule 5182 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*(x_)*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d+e*x^2)^{(p+1)}*(a+b*\text{ArcSin}[c*x])^n/(2*e*(p+1)), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \ \text{Int}[(1-c^2*x^2)^{(p+1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.40

method	result
derivativedivides	$c^3 \left(-d^3 a^2 \left(\frac{c^3 x^3}{3} - 3cx + \frac{1}{3c^3 x^3} - \frac{3}{cx} \right) - \frac{50d^3 b^2 cx}{9} + \frac{2d^3 b^2 c^3 x^3}{27} - \frac{d^3 b^2}{3cx} + \frac{17d^3 b^2 \arcsin(cx) \ln(1+\sqrt{-c^2 x^2 + 1})}{3} \right)$
default	$c^3 \left(-d^3 a^2 \left(\frac{c^3 x^3}{3} - 3cx + \frac{1}{3c^3 x^3} - \frac{3}{cx} \right) - \frac{50d^3 b^2 cx}{9} + \frac{2d^3 b^2 c^3 x^3}{27} - \frac{d^3 b^2}{3cx} + \frac{17d^3 b^2 \arcsin(cx) \ln(1+\sqrt{-c^2 x^2 + 1})}{3} \right)$
parts	$-d^3 a^2 \left(\frac{c^6 x^3}{3} - 3c^4 x + \frac{1}{3x^3} - \frac{3c^2}{x} \right) - \frac{50b^2 c^4 d^3 x}{9} - \frac{2d^3 b^2 c^5 \sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2}{9} + \frac{2b^2 c^6 d^3 x^3}{27} - \frac{d^3 b^2}{3cx} + \frac{17d^3 b^2 \arcsin(cx) \ln(1+\sqrt{-c^2 x^2 + 1})}{3}$

input `int((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^4,x,method=_RETURNVERBOSE)`

output `c^3*(-d^3*a^2*(1/3*c^3*x^3-3*c*x+1/3/c^3/x^3-3/c/x)-50/9*d^3*b^2*c*x+2/27*d^3*b^2*c^3*x^3-1/3*d^3*b^2/c/x+17/3*d^3*b^2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-17/3*d^3*b^2*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2/9*d^3*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^2*x^2+3*d^3*b^2/c/x*arcsin(c*x)^2-1/3*d^3*b^2/c^3/x^3*arcsin(c*x)^2-1/3*d^3*b^2*arcsin(c*x)^2*c^3*x^3+3*d^3*b^2*arcsin(c*x)^2*c*x+17/3*I*d^3*b^2*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-17/3*I*d^3*b^2*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-1/3*d^3*b^2/c^2/x^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)+50/9*d^3*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)-2*d^3*a*b*(1/3*c^3*x^3*arcsin(c*x)-3*c*x*arcsin(c*x)+1/3*arcsin(c*x)/c^3/x^3-3*arcsin(c*x)/c/x+1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)-17/6*arctanh(1/(-c^2*x^2+1)^(1/2))+1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-25/9*(-c^2*x^2+1)^(1/2))`

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^4,x, algorithm="fricas")`

output

```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arcsin(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arcsin(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx = -d^3 \left(\int (-3a^2 c^4) dx + \int \left(-\frac{a^2}{x^4} \right) dx + \int \frac{3a^2 c^2}{x^2} dx + \int a^2 c^6 x^2 dx + \int (-3b^2 c^4 \arcsin^2(cx)) dx + \int \left(-\frac{b^2 \arcsin^2(cx)}{x^4} \right) dx + \int (-6abc^4 \arcsin(cx)) dx + \int \left(-\frac{2ab \arcsin(cx)}{x^4} \right) dx + \int \frac{3b^2 c^2 \arcsin^2(cx)}{x^2} dx + \int b^2 c^6 x^2 \arcsin^2(cx) dx + \int \frac{6abc^2 \arcsin(cx)}{x^2} dx + \int 2abc^6 x^2 \arcsin(cx) dx \right)$$

input

```
integrate((-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2/x**4,x)
```

output

```
-d**3*(Integral(-3*a**2*c**4, x) + Integral(-a**2/x**4, x) + Integral(3*a**2*c**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(-3*b**2*c**4*asin(c*x)**2, x) + Integral(-b**2*asin(c*x)**2/x**4, x) + Integral(-6*a*b*c**4*asin(c*x), x) + Integral(-2*a*b*asin(c*x)/x**4, x) + Integral(3*b**2*c**2*asin(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*asin(c*x)**2, x) + Integral(6*a*b*c**2*asin(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*asin(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx = \int -\frac{(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^4,x, algorithm="maxima")`

output

```
-1/3*a^2*c^6*d^3*x^3 - 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/
c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arcsin(c*x)
^2 - 6*b^2*c^4*d^3*(x - sqrt(-c^2*x^2 + 1)*arcsin(c*x)/c) + 3*a^2*c^4*d^3*
x + 6*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*a*b*c^3*d^3 + 6*(c*log(2*sqrt
(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*a*b*c^2*d^3 - 1/3*((c^2
*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c +
2*arcsin(c*x)/x^3)*a*b*d^3 + 3*a^2*c^2*d^3/x - 1/3*a^2*d^3/x^3 - 1/3*(3*x
^3*integrate(2/3*(b^2*c^7*d^3*x^6 - 9*b^2*c^3*d^3*x^2 + b^2*c*d^3)*sqrt(c*
x + 1)*sqrt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*x^5
- x^3), x) + (b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 + b^2*d^3)*arctan2(c*x,
sqrt(c*x + 1)*sqrt(-c*x + 1))^2)/x^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx = \text{Timed out}$$

input `integrate((-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2/x^4,x, algorithm="giac")`

output Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^3}{x^4} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x^4,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^3)/x^4, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^3 (a + b \arcsin(cx))^2}{x^4} dx$$

$$= \frac{d^3 \left(27 \operatorname{asin}(cx)^2 b^2 c^4 x^4 + 54 \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) b^2 c^3 x^3 - 6 \operatorname{asin}(cx) a b c^6 x^6 + 54 \operatorname{asin}(cx) a b c^4 x^4 + 54 \operatorname{asin}(cx) a^2 b^2 c^2 x^2 - 6 \operatorname{asin}(cx) a^2 b^2 c^2 x^2 - 2 \sqrt{-c^2 x^2 + 1} a^2 b^2 c^2 x^2 + 50 \sqrt{-c^2 x^2 + 1} a^2 b^2 c^2 x^2 - 3 \sqrt{-c^2 x^2 + 1} a^2 b^2 c^2 x^2 + 9 \int \operatorname{asin}(cx)^2 / x^4, x \right) b^2 c^2 x^3 - 27 \int \operatorname{asin}(cx)^2 / x^2, x \right) b^2 c^2 x^3 - 9 \int \operatorname{asin}(cx)^2 / x^2, x \right) b^2 c^2 x^3 - 9 \int \operatorname{asin}(cx)^2 / x^2, x \right) b^2 c^2 x^3 - 51 \log(\tan(\operatorname{asin}(cx)/2)) a^2 b^2 c^2 x^3 - 3 a^2 b^2 c^2 x^3 + 27 a^2 b^2 c^2 x^3 + 27 a^2 b^2 c^2 x^3 - 3 a^2 b^2 c^2 x^3 - 54 b^2 c^2 x^3)}{(9 x^3)}$$

input `int((-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2/x^4,x)`

output `(d**3*(27*asin(c*x)**2*b**2*c**4*x**4 + 54*sqrt(-c**2*x**2 + 1)*asin(c*x)*b**2*c**3*x**3 - 6*asin(c*x)*a*b*c**6*x**6 + 54*asin(c*x)*a*b*c**4*x**4 + 54*asin(c*x)*a*b*c**2*x**2 - 6*asin(c*x)*a*b - 2*sqrt(-c**2*x**2 + 1)*a*b*c**5*x**5 + 50*sqrt(-c**2*x**2 + 1)*a*b*c**3*x**3 - 3*sqrt(-c**2*x**2 + 1)*a*b*c*x + 9*int(asin(c*x)**2/x**4,x)*b**2*x**3 - 27*int(asin(c*x)**2/x**2,x)*b**2*c**2*x**3 - 9*int(asin(c*x)**2/x**2,x)*b**2*c**6*x**3 - 51*log(tan(asin(c*x)/2))*a*b*c**3*x**3 - 3*a**2*c**6*x**6 + 27*a**2*c**4*x**4 + 27*a**2*c**2*x**2 - 3*a**2 - 54*b**2*c**4*x**4)/(9*x**3)`

3.180 $\int \frac{x^4(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$

Optimal result	1695
Mathematica [A] (verified)	1696
Rubi [A] (verified)	1697
Maple [A] (verified)	1701
Fricas [F]	1702
Sympy [F]	1702
Maxima [F]	1703
Giac [F]	1703
Mupad [F(-1)]	1704
Reduce [F]	1704

Optimal result

Integrand size = 27, antiderivative size = 297

$$\int \frac{x^4(a+b \arcsin(cx))^2}{d-c^2dx^2} dx = \frac{22b^2x}{9c^4d} + \frac{2b^2x^3}{27c^2d} - \frac{22b\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{9c^5d}$$

$$- \frac{2bx^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{9c^3d}$$

$$- \frac{x(a+b \arcsin(cx))^2}{c^4d} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

$$- \frac{2i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{c^5d}$$

$$+ \frac{2ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^5d}$$

$$- \frac{2ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^5d}$$

$$- \frac{2b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{c^5d}$$

$$+ \frac{2b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{c^5d}$$

output

```
22/9*b^2*x/c^4/d+2/27*b^2*x^3/c^2/d-22/9*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^5/d-2/9*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/d-x*(a+b*arcsin(c*x))^2/c^4/d-1/3*x^3*(a+b*arcsin(c*x))^2/c^2/d-2*I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^5/d+2*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d-2*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d-2*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d+2*b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.71

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \frac{108a^2 cx - 270b^2 cx + 36a^2 c^3 x^3 + 264ab\sqrt{1 - c^2 x^2} + 24abc^2 x^2 \sqrt{1 - c^2 x^2} + 108iab\pi \arcsin(cx) + 216a^2 b \arcsin^2(cx)}{d - c^2 dx^2}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]
```

output

```
-1/108*(108*a^2*c*x - 270*b^2*c*x + 36*a^2*c^3*x^3 + 264*a*b*Sqrt[1 - c^2*x^2] + 24*a*b*c^2*x^2*Sqrt[1 - c^2*x^2] + (108*I)*a*b*Pi*ArcSin[c*x] + 216*a*b*c*x*ArcSin[c*x] + 72*a*b*c^3*x^3*ArcSin[c*x] + 270*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + 135*b^2*c*x*ArcSin[c*x]^2 - 6*b^2*ArcSin[c*x]*Cos[3*ArcSin[c*x]] - 108*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 216*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 108*b^2*ArcSin[c*x]^2*Log[1 - I*E^(I*ArcSin[c*x])] - 108*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 216*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 108*b^2*ArcSin[c*x]^2*Log[1 + I*E^(I*ArcSin[c*x])] + 54*a^2*Log[1 - c*x] - 54*a^2*Log[1 + c*x] + 108*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 108*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (216*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (216*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] + 216*b^2*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 216*b^2*PolyLog[3, I*E^(I*ArcSin[c*x])] + 2*b^2*Sin[3*ArcSin[c*x]] - 9*b^2*ArcSin[c*x]^2*Sin[3*ArcSin[c*x]])/(c^5*d)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5210, 27, 5210, 15, 5164, 3042, 4669, 3011, 2720, 5182, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^4(a+b \arcsin(cx))^2}{d-c^2x^2} dx \\
 & \quad \downarrow \text{5210} \\
 & \frac{\int \frac{x^2(a+b \arcsin(cx))^2}{d(1-c^2x^2)} dx}{c^2} + \frac{2b \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d} \\
 & \quad \downarrow \text{27} \\
 & \frac{\int \frac{x^2(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2d} + \frac{2b \int \frac{x^3(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d} \\
 & \quad \downarrow \text{5210} \\
 & \frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{b \int x^2 dx}{3c} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))}{3c^2} \right)}{3cd} + \\
 & \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))^2}{c^2} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d} \\
 & \quad \downarrow \text{15} \\
 & \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))^2}{c^2} + \\
 & \frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d} \\
 & \quad \downarrow \text{5164}
 \end{aligned}$$

$$\frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} + \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))^2}{c^2}}{c^2d} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

↓ 3042

$$\frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} + \frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^3} + \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))^2}{c^2}}{c^2d} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

↓ 4669

$$\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} - \frac{c^2d}{c^2d}$$

$$\frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

↓ 3011

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) d \arcsin(cx))}{c^3} - \frac{c^2d}{c^2d}$$

$$\frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

↓ 2720

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{c^3} - \frac{c^2d}{c^2d}$$

$$\frac{2b \left(\frac{2 \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

↓ 5182

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)))}{c^3}$$

$$\frac{2b \left(\frac{2 \left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{bx^3}{9c} \right)}{3cd} - \frac{x^3(a+b \arcsin(cx))^2}{3c^2d}$$

↓ 24

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)))}{c^3}$$

$$\frac{x^3(a+b \arcsin(cx))^2}{3c^2d} + \frac{2b \left(-\frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{2 \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} + \frac{bx^3}{9c} \right)}{3cd}$$

↓ 7143

$$\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)))}{c^3}$$

$$\frac{x^3(a+b \arcsin(cx))^2}{3c^2d} + \frac{2b \left(-\frac{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^2} + \frac{2 \left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2} \right)}{3c^2} + \frac{bx^3}{9c} \right)}{3cd}$$

c^2d

input `Int[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]`

output `-1/3*(x^3*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*((b*x^3)/(9*c) - (x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(3*c^2) + (2*((b*x)/c - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/(3*c^2)))/(3*c*d) + (-((x*(a + b*ArcSin[c*x]))^2)/c^2) + (2*b*((b*x)/c - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/c^2))/c + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])]) + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])])/c^3)/(c^2*d)`

Defintions of rubi rules used

- rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`
- rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`
- rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`
- rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4669 `Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

```
rule 5164 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
  /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

```
rule 5182 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.),
  x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x]
  + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*
  (a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

```
rule 5210 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.),
  x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x]
  + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x]
  + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*
  (a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

```
rule 7143 Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol]
  := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{5(-i\sqrt{-c^2x^2+1+cx})(2ib^2 \arcsin(cx)+\arcsin(cx)^2b^2+2iab+2 \arcsin(cx)ab+a^2-2b^2)}{8d} - \frac{5(i\sqrt{-c^2x^2+1+cx})(-2ib^2 \arcsin(cx)+\arcsin(cx)^2b^2+2iab+2 \arcsin(cx)ab+a^2-2b^2)}{8d}$
default	$\frac{5(-i\sqrt{-c^2x^2+1+cx})(2ib^2 \arcsin(cx)+\arcsin(cx)^2b^2+2iab+2 \arcsin(cx)ab+a^2-2b^2)}{8d} - \frac{5(i\sqrt{-c^2x^2+1+cx})(-2ib^2 \arcsin(cx)+\arcsin(cx)^2b^2+2iab+2 \arcsin(cx)ab+a^2-2b^2)}{8d}$

```
input int(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d), x, method=_RETURNVERBOSE)
```

output

```
1/c^5*(-5/8*(-I*(-c^2*x^2+1)^(1/2)+c*x)*(2*I*b^2*arcsin(c*x)+arcsin(c*x)^2
*b^2+2*I*a*b+2*arcsin(c*x)*a*b+a^2-2*b^2)/d-5/8*(I*(-c^2*x^2+1)^(1/2)+c*x)
*(-2*I*b^2*arcsin(c*x)+arcsin(c*x)^2*b^2-2*I*b*a+2*arcsin(c*x)*a*b+a^2-2*b
^2)/d+1/d*b^2*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*b^2*a
rcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2/d*b^2*polylog(3,I*(I*
c*x+(-c^2*x^2+1)^(1/2)))-1/d*b^2*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)
^(1/2)))+2*I/d*b^2*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2/d
*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2/d*a*b*arcsin(c*x)*ln(1-I*(
I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*a*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)
))-2/d*a*b*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*I/d*a*b*polylo
g(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*a^2*arctan(I*c*x+(-c^2*x^2+1)^(1/
2))+1/18*b*(a+b*arcsin(c*x))/d*cos(3*arcsin(c*x))+1/108*(9*arcsin(c*x)^2*b
^2+18*arcsin(c*x)*a*b+9*a^2-2*b^2)/d*sin(3*arcsin(c*x))
```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^4}{c^2 dx^2 - d} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output

```
integral(-(b^2*x^4*arcsin(c*x)^2 + 2*a*b*x^4*arcsin(c*x) + a^2*x^4)/(c^2*d
*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\frac{\int \frac{a^2 x^4}{c^2 x^2 - 1} dx + \int \frac{b^2 x^4 \operatorname{asin}^2(cx)}{c^2 x^2 - 1} dx + \int \frac{2abx^4 \operatorname{asin}(cx)}{c^2 x^2 - 1} dx}{d}$$

input

```
integrate(x**4*(a+b*asin(c*x))**2/(-c**2*d*x**2+d),x)
```

output $-(\text{Integral}(a^{**2}x^{**4}/(c^{**2}x^{**2} - 1), x) + \text{Integral}(b^{**2}x^{**4}*\text{asin}(cx)^{**2}/(c^{**2}x^{**2} - 1), x) + \text{Integral}(2*a*b*x^{**4}*\text{asin}(cx)/(c^{**2}x^{**2} - 1), x))/d$

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^4}{c^2 dx^2 - d} dx$$

input `integrate(x^4*(a+b*arcsin(cx))^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output $-1/6*a^2*(2*(c^2*x^3 + 3*x)/(c^4*d) - 3*\log(cx + 1)/(c^5*d) + 3*\log(cx - 1)/(c^5*d)) + 1/6*(6*c^5*d*\text{integrate}(-1/3*(6*a*b*c^4*x^4*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1)) - (3*b^2*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1))*\log(cx + 1) - 3*b^2*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1))*\log(-cx + 1) - 2*(b^2*c^3*x^3 + 3*b^2*c*x)*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1)))*\text{sqrt}(cx + 1)*\text{sqrt}(-cx + 1))/(c^6*d*x^2 - c^4*d), x) + 3*b^2*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1))^2*\log(cx + 1) - 3*b^2*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1))^2*\log(-cx + 1) - 2*(b^2*c^3*x^3 + 3*b^2*c*x)*\text{arctan2}(cx, \text{sqrt}(cx + 1))*\text{sqrt}(-cx + 1))^2)/(c^5*d)$

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^4}{c^2 dx^2 - d} dx$$

input `integrate(x^4*(a+b*arcsin(cx))^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output `integrate(-(b*arcsin(cx) + a)^2*x^4/(c^2*d*x^2 - d), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))^2}{d - c^2 dx^2} dx$$

input `int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2),x)`

output `int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2), x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$= \frac{-12 \left(\int \frac{a \operatorname{asin}(cx) x^4}{c^2 x^2 - 1} dx \right) a b c^5 - 6 \left(\int \frac{a \operatorname{asin}(cx)^2 x^4}{c^2 x^2 - 1} dx \right) b^2 c^5 - 3 \log(c^2 x - c) a^2 + 3 \log(c^2 x + c) a^2 - 2 a^2 c^3 x^3 - 6 c^5 d}{6 c^5 d}$$

input `int(x^4*(a+b*asin(c*x))^2/(-c^2*d*x^2+d),x)`

output `(- 12*int((asin(c*x)*x**4)/(c**2*x**2 - 1),x)*a*b*c**5 - 6*int((asin(c*x)**2*x**4)/(c**2*x**2 - 1),x)*b**2*c**5 - 3*log(c**2*x - c)*a**2 + 3*log(c**2*x + c)*a**2 - 2*a**2*c**3*x**3 - 6*a**2*c*x)/(6*c**5*d)`

3.181 $\int \frac{x^3(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$

Optimal result	1705
Mathematica [B] (verified)	1706
Rubi [A] (verified)	1706
Maple [A] (verified)	1711
Fricas [F]	1711
Sympy [F]	1712
Maxima [F]	1712
Giac [F(-2)]	1713
Mupad [F(-1)]	1713
Reduce [F]	1713

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{x^3(a+b \arcsin(cx))^2}{d-c^2dx^2} dx = \frac{b^2x^2}{4c^2d} - \frac{bx\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^3d} + \frac{(a+b \arcsin(cx))^2}{4c^4d} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} + \frac{i(a+b \arcsin(cx))^3}{3bc^4d} - \frac{(a+b \arcsin(cx))^2 \log(1+e^{2i \arcsin(cx)})}{c^4d} + \frac{ib(a+b \arcsin(cx)) \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{c^4d} - \frac{b^2 \text{PolyLog}(3, -e^{2i \arcsin(cx)})}{2c^4d}$$

output

```
1/4*b^2*x^2/c^2/d-1/2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/d+1/4*(
a+b*arcsin(c*x))^2/c^4/d-1/2*x^2*(a+b*arcsin(c*x))^2/c^2/d+1/3*I*(a+b*arcs
in(c*x))^3/b/c^4/d-(a+b*arcsin(c*x))^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/
c^4/d+I*b*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d
-1/2*b^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 459 vs. $2(210) = 420$.

Time = 0.52 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.19

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 x^2} dx = \frac{12a^2 c^2 x^2 + 12abcx\sqrt{1 - c^2 x^2} + 48iab\pi \arcsin(cx) + 24abc^2 x^2 \arcsin(cx) - 24iab \arcsin(cx)^2 - 8ib^2 \arcsin^3(cx)}{d - c^2 x^2}$$

input `Integrate[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]`

output `-1/24*(12*a^2*c^2*x^2 + 12*a*b*c*x*Sqrt[1 - c^2*x^2] + (48*I)*a*b*Pi*ArcSin[c*x] + 24*a*b*c^2*x^2*ArcSin[c*x] - (24*I)*a*b*ArcSin[c*x]^2 - (8*I)*b^2*ArcSin[c*x]^3 - 24*a*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] + 3*b^2*Cos[2*ArcSin[c*x]] - 6*b^2*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] + 96*a*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] + 24*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 48*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 24*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 48*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 24*b^2*ArcSin[c*x]^2*Log[1 + E^((2*I)*ArcSin[c*x])] + 12*a^2*Log[1 - c^2*x^2] - 96*a*b*Pi*Log[Cos[ArcSin[c*x]/2]] + 24*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - 24*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (48*I)*a*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (48*I)*a*b*PolyLog[2, I*E^(I*ArcSin[c*x])] - (24*I)*b^2*ArcSin[c*x]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] + 12*b^2*PolyLog[3, -E^((2*I)*ArcSin[c*x])] + 6*b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])/(c^4*d)`

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5210, 27, 5180, 3042, 4202, 2620, 3011, 2720, 5210, 15, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^3(a+b \arcsin(cx))^2}{d-c^2x^2} dx \\
& \quad \downarrow \text{5210} \\
& \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} + \frac{\int \frac{x(a+b \arcsin(cx))^2}{d(1-c^2x^2)} dx}{c^2} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{27} \\
& \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} + \frac{\int \frac{x(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2d} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{5180} \\
& \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} + \frac{\int \frac{cx(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^4d} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{3042} \\
& \frac{\int (a+b \arcsin(cx))^2 \tan(\arcsin(cx)) d \arcsin(cx)}{c^4d} + \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \\
& \quad \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{4202} \\
& \frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))^2}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^4d} + \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \\
& \quad \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{2620} \\
& \frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib \int (a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2}i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)))}{c^4d}}{c^4d} \\
& \quad \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{3011} \\
& \frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a+b \arcsin(cx))) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx))}{c^4d}}{c^4d} \\
& \quad \frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2d} \\
& \quad \downarrow \text{2720}
\end{aligned}$$

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx)}{c^4 d}}{\frac{b \int \frac{x^2(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d}}$$

↓ 5210

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx)}{c^4 d}}{\frac{b \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x dx}{2c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right)}{cd} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d}}$$

↓ 15

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx)}{c^4 d}}{\frac{b \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{cd} - \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d}}$$

↓ 5152

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx)}{c^4 d}}{\frac{x^2(a+b \arcsin(cx))^2}{2c^2 d} + \frac{b \left(\frac{(a+b \arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{cd}}$$

↓ 7143

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)}) - \frac{1}{2}i \log)}{c^4 d}}{\frac{x^2(a+b \arcsin(cx))^2}{2c^2 d} + \frac{b \left(\frac{(a+b \arcsin(cx))^2}{4bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} + \frac{bx^2}{4c} \right)}{cd}}$$

input

$\operatorname{Int}[(x^3*(a + b*\operatorname{ArcSin}[c*x])^2)/(d - c^2*d*x^2), x]$

output

```
-1/2*(x^2*(a + b*ArcSin[c*x])^2)/(c^2*d) + (b*((b*x^2)/(4*c) - (x*Sqrt[1 -
c^2*x^2]*(a + b*ArcSin[c*x]))/(2*c^2) + (a + b*ArcSin[c*x])^2/(4*b*c^3)))/
/(c*d) + (((I/3)*(a + b*ArcSin[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[
c*x])^2*Log[1 + E^((2*I)*ArcSin[c*x])] + I*b*((I/2)*(a + b*ArcSin[c*x])*Po
lyLog[2, -E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])]/
4)))/(c^4*d)
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 2620

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4202 `Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x)))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5180 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*(x_))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5210 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.)), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]`

rule 7143 `Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{a^2 \left(\frac{c^2 x^2}{2} + \frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} \right)}{d} + \frac{iab \operatorname{polylog} \left(2, - \left(icx + \sqrt{-c^2 x^2 + 1} \right)^2 \right)}{d} - \frac{b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1} cx}{2d} - \frac{b^2 \arcsin(cx)^2 c^2 x^2}{2d} +$
default	$-\frac{a^2 \left(\frac{c^2 x^2}{2} + \frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} \right)}{d} + \frac{iab \operatorname{polylog} \left(2, - \left(icx + \sqrt{-c^2 x^2 + 1} \right)^2 \right)}{d} - \frac{b^2 \arcsin(cx) \sqrt{-c^2 x^2 + 1} cx}{2d} - \frac{b^2 \arcsin(cx)^2 c^2 x^2}{2d} +$
parts	$-\frac{a^2 x^2}{2d c^2} - \frac{a^2 \ln(c^2 x^2 - 1)}{2d c^4} + \frac{iab \operatorname{polylog} \left(2, - \left(icx + \sqrt{-c^2 x^2 + 1} \right)^2 \right)}{d c^4} - \frac{b^2 \sqrt{-c^2 x^2 + 1} \arcsin(cx) x}{2d c^3} - \frac{b^2 \arcsin(cx)^2 c^2 x^2}{2d c^2}$

input `int(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output

```
1/c^4*(-a^2/d*(1/2*c^2*x^2+1/2*ln(c*x-1)+1/2*ln(c*x+1))+I*a*b/d*polylog(2,
-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*b^2/d*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*
x-1/2*b^2/d*arcsin(c*x)^2*c^2*x^2+1/4*b^2/d*arcsin(c*x)^2+1/4*b^2/d*c^2*x^
2-1/8*b^2/d-b^2/d*arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/3*I*b
^2/d*arcsin(c*x)^3-1/2*b^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+I*b
^2/d*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*a*b/d*(-c^2*x
^2+1)^(1/2)*c*x-a*b/d*arcsin(c*x)*c^2*x^2+1/2*a*b/d*arcsin(c*x)-2*a*b/d*ar
csin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+I*a*b/d*arcsin(c*x)^2)
```

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^3}{c^2 dx^2 - d} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")`

output

```
integral(-(b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3)/(c^2*d
*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\int \frac{\frac{a^2 x^3}{c^2 x^2 - 1}}{d} dx + \int \frac{\frac{b^2 x^3 \arcsin^2(cx)}{c^2 x^2 - 1}}{d} dx + \int \frac{\frac{2abx^3 \arcsin(cx)}{c^2 x^2 - 1}}{d} dx$$

input `integrate(x**3*(a+b*asin(c*x))**2/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2*x**3/(c**2*x**2 - 1), x) + Integral(b**2*x**3*asin(c*x)**2/(c**2*x**2 - 1), x) + Integral(2*a*b*x**3*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^3}{c^2 dx^2 - d} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*a^2*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) - 1/2*(b^2*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*c^4*d*integrate((2*a*b*c^3*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (b^2*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) + b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^5*d*x^2 - c^3*d), x) + b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) + b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1))/(c^4*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{d - c^2 dx^2} dx$$

input `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2),x)`

output `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2), x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$= \frac{-2 \operatorname{asin}(cx)^2 b^2 c^2 x^2 + \operatorname{asin}(cx)^2 b^2 - 2 \sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) b^2 cx - 4 \operatorname{asin}(cx) ab c^2 x^2 + 2 \operatorname{asin}(cx) ab - 2}{d - c^2 dx^2}$$

input `int(x^3*(a+b*asin(c*x))^2/(-c^2*d*x^2+d),x)`

output

```
( - 2*asin(c*x)**2*b**2*c**2*x**2 + asin(c*x)**2*b**2 - 2*sqrt( - c**2*x**2 + 1)*asin(c*x)*b**2*c*x - 4*asin(c*x)*a*b*c**2*x**2 + 2*asin(c*x)*a*b - 2*sqrt( - c**2*x**2 + 1)*a*b*c*x - 8*int((asin(c*x)*x)/(c**2*x**2 - 1),x)*a*b*c**2 - 4*int((asin(c*x)**2*x)/(c**2*x**2 - 1),x)*b**2*c**2 - 2*log(c**2*x - c)*a**2 - 2*log(c**2*x + c)*a**2 - 2*a**2*c**2*x**2 + b**2*c**2*x**2 - b**2)/(4*c**4*d)
```

3.182 $\int \frac{x^2(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$

Optimal result	1715
Mathematica [A] (verified)	1716
Rubi [A] (verified)	1716
Maple [B] (verified)	1720
Fricas [F]	1721
Sympy [F]	1721
Maxima [F]	1721
Giac [F]	1722
Mupad [F(-1)]	1722
Reduce [F]	1723

Optimal result

Integrand size = 27, antiderivative size = 218

$$\int \frac{x^2(a+b \arcsin(cx))^2}{d-c^2dx^2} dx = \frac{2b^2x}{c^2d} - \frac{2b\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^3d} - \frac{x(a+b \arcsin(cx))^2}{c^2d} - \frac{2i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{c^3d} + \frac{2ib(a+b \arcsin(cx)) \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^3d} - \frac{2ib(a+b \arcsin(cx)) \text{PolyLog}(2, ie^{i \arcsin(cx)})}{c^3d} - \frac{2b^2 \text{PolyLog}(3, -ie^{i \arcsin(cx)})}{c^3d} + \frac{2b^2 \text{PolyLog}(3, ie^{i \arcsin(cx)})}{c^3d}$$

output

```
2*b^2*x/c^2/d-2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/d-x*(a+b*arcsin
(c*x))^2/c^2/d-2*I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^
3/d+2*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d
-2*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d-2*b
^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d+2*b^2*polylog(3,I*(I*c*x
+(-c^2*x^2+1)^(1/2)))/c^3/d
```


Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.89

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \frac{2a^2 cx - 4b^2 cx + 4ab\sqrt{1 - c^2 x^2} + 2iab\pi \arcsin(cx) + 4abcx \arcsin(cx) + 4b^2\sqrt{1 - c^2 x^2} \arcsin(cx) + 2$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]
```

output

```
-1/2*(2*a^2*c*x - 4*b^2*c*x + 4*a*b*Sqrt[1 - c^2*x^2] + (2*I)*a*b*Pi*ArcSin[c*x] + 4*a*b*c*x*ArcSin[c*x] + 4*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + 2*b^2*c*x*ArcSin[c*x]^2 - 2*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 4*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 2*b^2*ArcSin[c*x]^2*Log[1 - I*E^(I*ArcSin[c*x])] - 2*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 4*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b^2*ArcSin[c*x]^2*Log[1 + I*E^(I*ArcSin[c*x])] + a^2*Log[1 - c*x] - a^2*Log[1 + c*x] + 2*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (4*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] + 4*b^2*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 4*b^2*PolyLog[3, I*E^(I*ArcSin[c*x])])/(c^3*d)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5210, 27, 5164, 3042, 4669, 3011, 2720, 5182, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$\downarrow \text{5210}$$

$$\frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx}{cd} + \frac{\int \frac{(a+b \arcsin(cx))^2}{d(1-c^2 x^2)} dx}{c^2} - \frac{x(a + b \arcsin(cx))^2}{c^2 d}$$

$$\begin{aligned}
& \downarrow 27 \\
& \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} + \frac{\int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2d} - \frac{x(a+b \arcsin(cx))^2}{c^2d} \\
& \downarrow 5164 \\
& \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3d} - \frac{x(a+b \arcsin(cx))^2}{c^2d} \\
& \downarrow 3042 \\
& \frac{\int (a+b \arcsin(cx))^2 \csc\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx)}{c^3d} + \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \\
& \quad \frac{x(a+b \arcsin(cx))^2}{c^2d} \\
& \downarrow 4669 \\
& \frac{-2b \int (a+b \arcsin(cx)) \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx)}{c^3d} \\
& \quad \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x(a+b \arcsin(cx))^2}{c^2d} \\
& \downarrow 3011 \\
& \frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx))}{c^3d} \\
& \quad \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x(a+b \arcsin(cx))^2}{c^2d} \\
& \downarrow 2720 \\
& \frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{c^3d} \\
& \quad \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{cd} - \frac{x(a+b \arcsin(cx))^2}{c^2d} \\
& \downarrow 5182 \\
& \frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{c^3d} \\
& \quad \frac{2b\left(\frac{b \int 1 dx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2}\right)}{cd} - \frac{x(a+b \arcsin(cx))^2}{c^2d}
\end{aligned}$$

↓ 24

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)))}{cd - \frac{x(a + b \arcsin(cx))^2}{c^2 d}}$$

$$\frac{2b\left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2}\right)}{cd} - \frac{x(a + b \arcsin(cx))^2}{c^2 d}$$

↓ 7143

$$\frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)))}{c^3 d} - \frac{2b\left(\frac{bx}{c} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^2}\right)}{cd} - \frac{x(a + b \arcsin(cx))^2}{c^2 d}$$

input

```
Int[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]
```

output

```

-((x*(a + b*ArcSin[c*x])^2)/(c^2*d)) + (2*b*((b*x)/c - (Sqrt[1 - c^2*x^2]*
(a + b*ArcSin[c*x]))/c^2))/(c*d) + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^
(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[
c*x]]) - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x]])] - 2*b*(I*(a + b*ArcSin[c*x]
)*PolyLog[2, I*E^(I*ArcSin[c*x]])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])]))/(c
^3*d)

```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 2720

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

rule 3011 $\text{Int}[\text{Log}[1 + (e_.) * ((F_)^{(c_.) * (a_.) + (b_.) * (x_))})^{(n_.)}] * ((f_.) + (g_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n] / (b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^{(m - 1)} * \text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_.) + \text{Pi} * (k_.) + (f_.) * (x_)] * ((c_.) + (d_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2 * (c + d*x)^m * (\text{ArcTanh}[E^{(I*k*Pi)} * E^{(I*(e + f*x))}] / f), x] + (-\text{Simp}[d * (m/f) \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 - E^{(I*k*Pi)} * E^{(I*(e + f*x))}], x], x] + \text{Simp}[d * (m/f) \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 + E^{(I*k*Pi)} * E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)]^{(n_.)} / ((d_.) + (e_.) * (x_)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \text{Subst}[\text{Int}[(a + b*x)^n * \text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)]^{(n_.)} * (x_.) * ((d_.) + (e_.) * (x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)} * ((a + b*\text{ArcSin}[c*x])^n / (2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1))) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{(p + 1/2)} * (a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[p, -1]$

rule 5210 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)]^{(n_.)} * ((f_.) * (x_))^{(m_.)} * ((d_.) + (e_.) * (x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[f * (f*x)^{(m - 1)} * (d + e*x^2)^{(p + 1)} * ((a + b*\text{ArcSin}[c*x])^n / (e*(m + 2*p + 1))), x] + (\text{Simp}[f^2 * ((m - 1) / (c^2 * (m + 2*p + 1))) \text{Int}[(f*x)^{(m - 2)} * (d + e*x^2)^p * (a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m + 2*p + 1))) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m - 1)} * (1 - c^2*x^2)^{(p + 1/2)} * (a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{IGtQ}[m, 1] \&\& \text{NeQ}[m + 2*p + 1, 0]$

rule 7143

```
Int [PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp [PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(249) = 498$.

Time = 0.80 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.33

method	result
derivativedivides	$\frac{-\left(-i\sqrt{-c^2x^2+1+cx}\right)\left(2ib^2\arcsin(cx)+\arcsin(cx)^2b^2+2iab+2\arcsin(cx)ab+a^2-2b^2\right)}{2d} - \frac{\left(i\sqrt{-c^2x^2+1+cx}\right)\left(-2ib^2\arcsin(cx)+\arcsin(cx)^2b^2+2iab+2\arcsin(cx)ab+a^2-2b^2\right)}{2d}$
default	$\frac{-\left(-i\sqrt{-c^2x^2+1+cx}\right)\left(2ib^2\arcsin(cx)+\arcsin(cx)^2b^2+2iab+2\arcsin(cx)ab+a^2-2b^2\right)}{2d} - \frac{\left(i\sqrt{-c^2x^2+1+cx}\right)\left(-2ib^2\arcsin(cx)+\arcsin(cx)^2b^2+2iab+2\arcsin(cx)ab+a^2-2b^2\right)}{2d}$

input

```
int(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
1/c^3*(-1/2*(-I*(-c^2*x^2+1)^(1/2)+c*x)*(2*I*b^2*arcsin(c*x)+arcsin(c*x)^2
*b^2+2*I*a*b+2*arcsin(c*x)*a*b+a^2-2*b^2)/d-1/2*(I*(-c^2*x^2+1)^(1/2)+c*x)
*(-2*I*b^2*arcsin(c*x)+arcsin(c*x)^2*b^2-2*I*b*a+2*arcsin(c*x)*a*b+a^2-2*b
^2)/d+1/d*b^2*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*b^2*a
rcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2/d*b^2*polylog(3,I*(I*
c*x+(-c^2*x^2+1)^(1/2)))-1/d*b^2*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)
^(1/2)))+2*I/d*b^2*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2/d
*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2/d*a*b*arcsin(c*x)*ln(1-I*(
I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*a*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)
))-2/d*a*b*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*I/d*a*b*polylo
g(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*a^2*arctan(I*c*x+(-c^2*x^2+1)^(1/
2)))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^2}{c^2 dx^2 - d} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\frac{\int \frac{a^2 x^2}{c^2 x^2 - 1} dx + \int \frac{b^2 x^2 \arcsin^2(cx)}{c^2 x^2 - 1} dx + \int \frac{2abx^2 \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x**2*(a+b*asin(c*x))**2/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2*x**2/(c**2*x**2 - 1), x) + Integral(b**2*x**2*asin(c*x)**2/(c**2*x**2 - 1), x) + Integral(2*a*b*x**2*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^2}{c^2 dx^2 - d} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output

```
-1/2*a^2*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d)) - 1/2
*(2*b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 - 2*c^3*d*integrate(
-(2*a*b*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (2*b^2*c*x
*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - b^2*arctan2(c*x, sqrt(c*x +
1)*sqrt(-c*x + 1))*log(c*x + 1) + b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x
+ 1))*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d*x^2 - c^2*d), x
) - b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) + b^2*ar
ctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1))/(c^3*d)
```

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^2}{c^2 dx^2 - d} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output

```
integrate(-(b*arcsin(c*x) + a)^2*x^2/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

input

```
int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2),x)
```

output

```
int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$= \frac{-2 \arcsin(cx)^2 b^2 cx - 4\sqrt{-c^2 x^2 + 1} \arcsin(cx) b^2 - 4 \arcsin(cx) abcx - 4\sqrt{-c^2 x^2 + 1} ab - 4 \left(\int \frac{\arcsin(cx)}{c^2 x^2 - 1} dx \right) abc}{2c^3 d}$$

input `int(x^2*(a+b*asin(c*x))^2/(-c^2*d*x^2+d),x)`

output `(- 2*asin(c*x)**2*b**2*c*x - 4*sqrt(- c**2*x**2 + 1)*asin(c*x)*b**2 - 4*asin(c*x)*a*b*c*x - 4*sqrt(- c**2*x**2 + 1)*a*b - 4*int(asin(c*x)/(c**2*x**2 - 1),x)*a*b*c - 2*int(asin(c*x)**2/(c**2*x**2 - 1),x)*b**2*c - log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2 - 2*a**2*c*x + 4*b**2*c*x)/(2*c**3*d)`

3.183 $\int \frac{x(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$

Optimal result	1724
Mathematica [B] (verified)	1725
Rubi [A] (verified)	1725
Maple [A] (verified)	1728
Fricas [F]	1728
Sympy [F]	1729
Maxima [F]	1729
Giac [F(-2)]	1730
Mupad [F(-1)]	1730
Reduce [F]	1730

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{x(a+b \arcsin(cx))^2}{d-c^2dx^2} dx = \frac{i(a+b \arcsin(cx))^3}{3bc^2d} - \frac{(a+b \arcsin(cx))^2 \log(1+e^{2i \arcsin(cx)})}{c^2d} + \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{c^2d} - \frac{b^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{2c^2d}$$

output

```
1/3*I*(a+b*arcsin(c*x))^3/b/c^2/d-(a+b*arcsin(c*x))^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^2/d+I*b*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^2/d-1/2*b^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^2/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 342 vs. $2(117) = 234$.

Time = 0.57 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.92

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$= \frac{-12iab\pi \arcsin(cx) + 6iab \arcsin(cx)^2 + 2ib^2 \arcsin(cx)^3 - 24ab\pi \log(1 + e^{-i \arcsin(cx)}) - 6ab\pi \log(1 - e^{-i \arcsin(cx)})}{d - c^2 dx^2}$$

input `Integrate[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]`

output

```
((-12*I)*a*b*Pi*ArcSin[c*x] + (6*I)*a*b*ArcSin[c*x]^2 + (2*I)*b^2*ArcSin[c*x]^3 - 24*a*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 6*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 12*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 6*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 12*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - 6*b^2*ArcSin[c*x]^2*Log[1 + E^((2*I)*ArcSin[c*x])] - 3*a^2*Log[1 - c^2*x^2] + 24*a*b*Pi*Log[Cos[ArcSin[c*x]/2]] - 6*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 6*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (12*I)*a*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (12*I)*a*b*PolyLog[2, I*E^(I*ArcSin[c*x])] + (6*I)*b^2*ArcSin[c*x]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - 3*b^2*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/(6*c^2*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5180, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

↓ 5180

$$\frac{\int \frac{cx(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^2d}$$

↓ 3042

$$\frac{\int (a+b \arcsin(cx))^2 \tan(\arcsin(cx)) d \arcsin(cx)}{c^2d}$$

↓ 4202

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))^2}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^2d}$$

↓ 2620

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib \int (a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2}i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)))}{c^2d}}$$

↓ 3011

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx))}{c^2d}}$$

↓ 2720

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx))}{c^2d}}$$

↓ 7143

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(3, -e^{2i \arcsin(cx)})) - \frac{1}{2}i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)))}{c^2d}}$$

input

```
Int[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2), x]
```

output

```
((I/3)*(a + b*ArcSin[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])^2*Log[1 + E^((2*I)*ArcSin[c*x])] + I*b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4)))/(c^2*d)
```

Definitions of rubi rules used

rule 2620 $\text{Int}[(((F_)^{(g_.)*(e_.) + (f_.)*(x_.)})^{(n_.)*((c_.) + (d_.)*(x_.))^{(m_.)}) / ((a_.) + (b_.)*((F_)^{(g_.)*(e_.) + (f_.)*(x_.)})^{(n_.)}), x_Symbol] \rightarrow \text{Simp} [((c + d*x)^m / (b*f*g*n*\text{Log}[F]))*\text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a)], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a)], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_)*((a_.)*(v_)^{(n_)})^{(m_)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))}*(F_)^{v_}] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

rule 3011 $\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))^{(n_.)}}] * ((f_.) + (g_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n] / (b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\text{tan}[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[I * ((c + d*x)^{(m + 1)} / (d*(m + 1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * (E^{(2*I*(e + f*x))} / (1 + E^{(2*I*(e + f*x))})), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 5180 $\text{Int}[(((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}*(x_.)) / ((d_.) + (e_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[-e^{(-1)} \text{Subst}[\text{Int}[(a + b*x)^n * \text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.76

method	result
parts	$-\frac{a^2 \ln(c^2 x^2 - 1)}{2d c^2} - \frac{b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \arcsin(cx)^2 \ln \left(1 + (icx + \sqrt{-c^2 x^2 + 1})^2 \right) - i \arcsin(cx) \operatorname{polylog} \left(2, - (icx + \sqrt{-c^2 x^2 + 1}) \right) \right)}{d c^2}$
derivativedivides	$-\frac{a^2 \left(\frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} \right)}{d} - \frac{b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \arcsin(cx)^2 \ln \left(1 + (icx + \sqrt{-c^2 x^2 + 1})^2 \right) - i \arcsin(cx) \operatorname{polylog} \left(2, - (icx + \sqrt{-c^2 x^2 + 1}) \right) \right)}{d}$
default	$-\frac{a^2 \left(\frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} \right)}{d} - \frac{b^2 \left(-\frac{i \arcsin(cx)^3}{3} + \arcsin(cx)^2 \ln \left(1 + (icx + \sqrt{-c^2 x^2 + 1})^2 \right) - i \arcsin(cx) \operatorname{polylog} \left(2, - (icx + \sqrt{-c^2 x^2 + 1}) \right) \right)}{d}$

input

```
int(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
-1/2*a^2/d/c^2*ln(c^2*x^2-1)-b^2/d/c^2*(-1/3*I*arcsin(c*x)^3+arcsin(c*x)^2
*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*
x^2+1)^(1/2))^2)+1/2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))-2*a*b/d/c^2
*(-1/2*I*arcsin(c*x)^2+arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*
I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x}{c^2 dx^2 - d} dx$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output `integral(-(b^2*x*arcsin(c*x))^2 + 2*a*b*x*arcsin(c*x) + a^2*x)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\frac{\int \frac{a^2 x}{c^2 x^2 - 1} dx + \int \frac{b^2 x \arcsin^2(cx)}{c^2 x^2 - 1} dx + \int \frac{2abx \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x*(a+b*asin(c*x))**2/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2*x/(c**2*x**2 - 1), x) + Integral(b**2*x*asin(c*x)**2/(c**2*x**2 - 1), x) + Integral(2*a*b*x*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x}{c^2 dx^2 - d} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*a^2*log(c^2*d*x^2 - d)/(c^2*d) - 1/2*(b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*log(c*x + 1) + b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*log(-c*x + 1) + 2*c^2*d*integrate((2*a*b*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + (b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(c*x + 1) + b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^3*d*x^2 - c*d), x)/(c^2*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

input `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2),x)`

output `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \frac{-4 \left(\int \frac{a \arcsin(cx) x}{c^2 x^2 - 1} dx \right) ab c^2 - 2 \left(\int \frac{a \arcsin(cx)^2 x}{c^2 x^2 - 1} dx \right) b^2 c^2 - \log(c^2 x - c) a^2 - \log(c^2 x + c) a^2}{2c^2 d}$$

input `int(x*(a+b*asin(c*x))^2/(-c^2*d*x^2+d),x)`

output

```
( - 4*int((asin(c*x)*x)/(c**2*x**2 - 1),x)*a*b*c**2 - 2*int((asin(c*x)**2*x)/(c**2*x**2 - 1),x)*b**2*c**2 - log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2)/(2*c**2*d)
```


3.184 $\int \frac{(a+b \arcsin(cx))^2}{d-c^2 dx^2} dx$

Optimal result	1732
Mathematica [A] (verified)	1733
Rubi [A] (verified)	1733
Maple [A] (verified)	1735
Fricas [F]	1736
Sympy [F]	1736
Maxima [F]	1737
Giac [F(-2)]	1737
Mupad [F(-1)]	1738
Reduce [F]	1738

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\frac{2i(a + b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{cd} + \frac{2ib(a + b \arcsin(cx)) \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{cd} - \frac{2ib(a + b \arcsin(cx)) \text{PolyLog}(2, ie^{i \arcsin(cx)})}{cd} - \frac{2b^2 \text{PolyLog}(3, -ie^{i \arcsin(cx)})}{cd} + \frac{2b^2 \text{PolyLog}(3, ie^{i \arcsin(cx)})}{cd}$$

output

```
-2*I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c/d+2*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d-2*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d-2*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d+2*b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.94

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \frac{2iab\pi \arcsin(cx) + 4ib^2 \arcsin(cx)^2 \arctan(e^{i \arcsin(cx)}) - 2ab\pi \log(1 - ie^{i \arcsin(cx)}) - 4ab \arcsin(cx) \log(1 - ie^{i \arcsin(cx)})}{cd}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2),x]
```

output

```
-1/2*((2*I)*a*b*Pi*ArcSin[c*x] + (4*I)*b^2*ArcSin[c*x]^2*ArcTan[E^(I*ArcSin[c*x])]) - 2*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 4*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 2*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 4*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + a^2*Log[1 - c*x] - a^2*Log[1 + c*x] + 2*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (4*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] + 4*b^2*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 4*b^2*PolyLog[3, I*E^(I*ArcSin[c*x])])/(c*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5164, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$\downarrow 5164$$

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} d \arcsin(cx)$$

$$\frac{\quad}{cd}$$

$$\downarrow 3042$$

$$\frac{\int (a + b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{cd}$$

↓ 4669

$$\frac{-2b \int (a + b \arcsin(cx)) \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a + b \arcsin(cx)) \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx)}{cd}$$

↓ 3011

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) d \arcsin(cx))}{cd}$$

↓ 2720

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{cd}$$

↓ 7143

$$\frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx))) - 2i \arctan(e^{-i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(3, ie^{i \arcsin(cx)}) (a + b \arcsin(cx)))}{cd}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2), x]
```

output

```
((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])]))/(c*d)
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011 $\text{Int}[\text{Log}[1 + (e_)*((F_)^((c_)*((a_)+(b_)*(x_)))^n)]*((f_)+(g_)*(x_))^m, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^(m - 1)*\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_)+\text{Pi}*(k_)+(f_)*(x_)]*((c_)+(d_)*(x_))^m, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^n/((d_)+(e_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 7143 $\text{Int}[\text{PolyLog}[n_, (c_)*((a_)+(b_)*(x_))]^p/((d_)+(e_)*(x_)), x_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{b^2 \arcsin(cx)^2 \ln(1 - i(icx + \sqrt{-c^2x^2 + 1}))}{d} - \frac{2ib^2 \arcsin(cx) \text{polylog}(2, i(icx + \sqrt{-c^2x^2 + 1}))}{d} + \frac{2b^2 \text{polylog}(3, i(icx + \sqrt{-c^2x^2 + 1}))}{d}$
default	$\frac{b^2 \arcsin(cx)^2 \ln(1 - i(icx + \sqrt{-c^2x^2 + 1}))}{d} - \frac{2ib^2 \arcsin(cx) \text{polylog}(2, i(icx + \sqrt{-c^2x^2 + 1}))}{d} + \frac{2b^2 \text{polylog}(3, i(icx + \sqrt{-c^2x^2 + 1}))}{d}$

input $\text{int}((a+b*\arcsin(c*x))^2/(-c^2*d*x^2+d), x, \text{method}=_RETURNVERBOSE)$

output

```
1/c*(1/d*b^2*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*b^2*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2/d*b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/d*b^2*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*I/d*b^2*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2/d*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2/d*a*b*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*a*b*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2/d*a*b*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*I/d*a*b*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I/d*a^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2}{c^2 dx^2 - d} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output

```
integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\frac{\int \frac{a^2}{c^2 x^2 - 1} dx + \int \frac{b^2 \arcsin^2(cx)}{c^2 x^2 - 1} dx + \int \frac{2ab \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input

```
integrate((a+b*asin(c*x))**2/(-c**2*d*x**2+d),x)
```

output

```
-(Integral(a**2/(c**2*x**2 - 1), x) + Integral(b**2*asin(c*x)**2/(c**2*x**2 - 1), x) + Integral(2*a*b*asin(c*x)/(c**2*x**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2}{c^2 dx^2 - d} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `1/2*a^2*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) + 1/2*(b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) + 2*c*d*integrate(-(2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*d*x^2 - d), x))/(c*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{d - c^2 dx^2} dx$$

input `int((a + b*asin(c*x))^2/(d - c^2*d*x^2),x)`output `int((a + b*asin(c*x))^2/(d - c^2*d*x^2), x)`**Reduce [F]**

$$\int \frac{(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$= \frac{-4 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^2 - 1} dx \right) abc - 2 \left(\int \frac{\operatorname{asin}(cx)^2}{c^2 x^2 - 1} dx \right) b^2 c - \log(c^2 x - c) a^2 + \log(c^2 x + c) a^2}{2cd}$$

input `int((a+b*asin(c*x))^2/(-c^2*d*x^2+d),x)`output `(- 4*int(asin(c*x)/(c**2*x**2 - 1),x)*a*b*c - 2*int(asin(c*x)**2/(c**2*x**2 - 1),x)*b**2*c - log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2)/(2*c*d)`

3.185 $\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)} dx$

Optimal result	1739
Mathematica [B] (verified)	1740
Rubi [A] (verified)	1740
Maple [B] (verified)	1743
Fricas [F]	1743
Sympy [F]	1744
Maxima [F]	1744
Giac [F(-2)]	1745
Mupad [F(-1)]	1745
Reduce [F]	1745

Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2dx^2)} dx = -\frac{2(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d} + \frac{ib(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d} - \frac{ib(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d} - \frac{b^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{2d} + \frac{b^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)})}{2d}$$

output

```
-2*(a+b*arcsin(c*x))^2*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+I*b*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d-I*b*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d-1/2*b^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d
```


Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 453 vs. $2(131) = 262$.

Time = 0.40 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.46

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx$$

$$= \frac{-ib^2\pi^3 - 48iab\pi \arcsin(cx) + 16ib^2 \arcsin(cx)^3 - 96ab\pi \log(1 + e^{-i \arcsin(cx)}) - 24ab\pi \log(1 - ie^{i \arcsin(cx)})}{24d}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)),x]`

output

```
((-I)*b^2*Pi^3 - (48*I)*a*b*Pi*ArcSin[c*x] + (16*I)*b^2*ArcSin[c*x]^3 - 96
*a*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 24*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*
x])] - 48*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 24*a*b*Pi*Log[1 +
I*E^(I*ArcSin[c*x])] - 48*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] +
24*b^2*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] + 48*a*b*ArcSin[c*x]*
Log[1 - E^((2*I)*ArcSin[c*x])] - 24*b^2*ArcSin[c*x]^2*Log[1 + E^((2*I)*Arc
Sin[c*x])] + 24*a^2*Log[c*x] - 12*a^2*Log[1 - c^2*x^2] + 96*a*b*Pi*Log[Cos
[ArcSin[c*x]/2]] - 24*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 24*a*b*Pi
*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (48*I)*a*b*PolyLog[2, (-I)*E^(I*ArcSin
[c*x])] + (48*I)*a*b*PolyLog[2, I*E^(I*ArcSin[c*x])] + (24*I)*b^2*ArcSin[c
*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] + (24*I)*b^2*ArcSin[c*x]*PolyLog[2,
-E^((2*I)*ArcSin[c*x])] - (24*I)*a*b*PolyLog[2, E^((2*I)*ArcSin[c*x])] +
12*b^2*PolyLog[3, E^((-2*I)*ArcSin[c*x])] - 12*b^2*PolyLog[3, -E^((2*I)*Ar
cSin[c*x])])/(24*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5184, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx \\
 & \quad \downarrow 5184 \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{cx\sqrt{1-c^2x^2}} d \arcsin(cx)}{d} \\
 & \quad \downarrow 4919 \\
 & \frac{2 \int (a + b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx)}{d} \\
 & \quad \downarrow 3042 \\
 & \frac{2 \int (a + b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx)}{d} \\
 & \quad \downarrow 4671 \\
 & \frac{2(-b \int (a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + b \int (a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx))}{d} \\
 & \quad \downarrow 3011 \\
 & \frac{2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx)) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, e^{2i \arcsin(cx)}) d \arcsin(cx))}{d} \\
 & \quad \downarrow 2720 \\
 & \frac{2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{2i \arcsin(cx)} \text{PolyLog}(2, e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)})}{d} \\
 & \quad \downarrow 7143 \\
 & \frac{2(-\text{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 + b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(3, -E^{((2*I)*\text{ArcSin}[c*x])}) - (b*\text{PolyLog}[3, -E^{((2*I)*\text{ArcSin}[c*x])}]/4)))/d}
 \end{aligned}$$

input `Int[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)),x]`

output `(2*(-((a + b*ArcSin[c*x])^2*ArcTanh[E^((2*I)*ArcSin[c*x])]) + b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4) - b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])/4)))/d`

Definitions of rubi rules used

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 4919 `Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]`

rule 5184 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_.) + (e_.)*(x_)^2)), x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 7143 `Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(175) = 350$.

Time = 0.46 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.45

method	result
parts	$-\frac{a^2 \left(\frac{\ln(cx-1)}{2} + \frac{\ln(cx+1)}{2} - \ln(x) \right)}{d} - \frac{b^2 \left(-\arcsin(cx)^2 \ln(1+icx+\sqrt{-c^2x^2+1}) + 2i \arcsin(cx) \operatorname{polylog}(2, -icx-\sqrt{-c^2x^2+1}) \right)}{d}$
derivativedivides	$-\frac{a^2 \left(\frac{\ln(cx+1)}{2} - \ln(cx) + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b^2 \left(-\arcsin(cx)^2 \ln(1+icx+\sqrt{-c^2x^2+1}) + 2i \arcsin(cx) \operatorname{polylog}(2, -icx-\sqrt{-c^2x^2+1}) \right)}{d}$
default	$-\frac{a^2 \left(\frac{\ln(cx+1)}{2} - \ln(cx) + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b^2 \left(-\arcsin(cx)^2 \ln(1+icx+\sqrt{-c^2x^2+1}) + 2i \arcsin(cx) \operatorname{polylog}(2, -icx-\sqrt{-c^2x^2+1}) \right)}{d}$

input `int((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output `-a^2/d*(1/2*ln(c*x-1)+1/2*ln(c*x+1)-ln(x))-b^2/d*(-arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))-2*a*b/d*(arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))`

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^3 - d*x), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx = -\int \frac{a^2}{c^2 x^3 - x} dx + \int \frac{b^2 \arcsin^2(cx)}{c^2 x^3 - x} dx + \int \frac{2ab \arcsin(cx)}{c^2 x^3 - x} dx$$

input `integrate((a+b*asin(c*x))**2/x/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2/(c**2*x**3 - x), x) + Integral(b**2*asin(c*x)**2/(c**2*x**3 - x), x) + Integral(2*a*b*asin(c*x)/(c**2*x**3 - x), x))/d`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*a^2*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/(c^2*d*x^3 - d*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)} dx = \frac{-4 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^3 - x} dx \right) ab - 2 \left(\int \frac{\operatorname{asin}(cx)^2}{c^2 x^3 - x} dx \right) b^2 - \log(c^2 x - c) a^2 - \log(c^2 x + c) a^2 + 2 \log(x) a^2}{2d}$$

input `int((a+b*asin(c*x))^2/x/(-c^2*d*x^2+d),x)`

output

```
( - 4*int(asin(c*x)/(c**2*x**3 - x),x)*a*b - 2*int(asin(c*x)**2/(c**2*x**3
- x),x)*b**2 - log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2 + 2*log(x)*a**
2)/(2*d)
```

3.186 $\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)} dx$

Optimal result	1747
Mathematica [B] (verified)	1748
Rubi [A] (verified)	1749
Maple [A] (verified)	1753
Fricas [F]	1754
Sympy [F]	1754
Maxima [F]	1755
Giac [F(-2)]	1755
Mupad [F(-1)]	1756
Reduce [F]	1756

Optimal result

Integrand size = 27, antiderivative size = 238

$$\begin{aligned}
 \int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)} dx = & -\frac{(a+b \arcsin(cx))^2}{dx} \\
 & -\frac{2ic(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{d} \\
 & -\frac{4bc(a+b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{d} \\
 & +\frac{2ib^2c \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d} \\
 & +\frac{2ibc(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d} \\
 & -\frac{2ibc(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d} \\
 & -\frac{2ib^2c \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d} \\
 & -\frac{2b^2c \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{d} \\
 & +\frac{2b^2c \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{d}
 \end{aligned}$$

output

```

-(a+b*arcsin(c*x))^2/d/x-2*I*c*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+
1)^(1/2))/d-4*b*c*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d+2*
I*b^2*c*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d+2*I*b*c*(a+b*arcsin(c*x))*p
olylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d-2*I*b*c*(a+b*arcsin(c*x))*polylo
g(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d-2*I*b^2*c*polylog(2,I*c*x+(-c^2*x^2+1)
^(1/2))/d-2*b^2*c*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d+2*b^2*c*polyl
og(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d

```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 525 vs. $2(238) = 476$.

Time = 0.75 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.21

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx =$$

$$\frac{2a^2 + 4ab \arcsin(cx) + 2iabc\pi x \arcsin(cx) + 2b^2 \arcsin(cx)^2 + 4abcx \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) - 4b^2 cx \arcsin(cx)}{d}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)),x]
```

output

```

-1/2*(2*a^2 + 4*a*b*ArcSin[c*x] + (2*I)*a*b*c*Pi*x*ArcSin[c*x] + 2*b^2*Arc
Sin[c*x]^2 + 4*a*b*c*x*ArcTanh[Sqrt[1 - c^2*x^2]] - 4*b^2*c*x*ArcSin[c*x]*
Log[1 - E^(I*ArcSin[c*x])] - 2*a*b*c*Pi*x*Log[1 - I*E^(I*ArcSin[c*x])] - 4
*a*b*c*x*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 2*b^2*c*x*ArcSin[c*x]^
2*Log[1 - I*E^(I*ArcSin[c*x])] - 2*a*b*c*Pi*x*Log[1 + I*E^(I*ArcSin[c*x])]
+ 4*a*b*c*x*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b^2*c*x*ArcSin[c
*x]^2*Log[1 + I*E^(I*ArcSin[c*x])] + 4*b^2*c*x*ArcSin[c*x]*Log[1 + E^(I*Ar
cSin[c*x])] + a^2*c*x*Log[1 - c*x] - a^2*c*x*Log[1 + c*x] + 2*a*b*c*Pi*x*L
og[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*a*b*c*Pi*x*Log[Sin[(Pi + 2*ArcSin[c*x
])/4]] - (4*I)*b^2*c*x*PolyLog[2, -E^(I*ArcSin[c*x])] - (4*I)*b*c*x*(a + b
*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*a*b*c*x*PolyLog[2
, I*E^(I*ArcSin[c*x])] + (4*I)*b^2*c*x*ArcSin[c*x]*PolyLog[2, I*E^(I*ArcSi
n[c*x])] + (4*I)*b^2*c*x*PolyLog[2, E^(I*ArcSin[c*x])] + 4*b^2*c*x*PolyLog
[3, (-I)*E^(I*ArcSin[c*x])] - 4*b^2*c*x*PolyLog[3, I*E^(I*ArcSin[c*x])]/(
d*x)

```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {5204, 27, 5164, 3042, 4669, 3011, 2720, 5218, 3042, 4671, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx \\
 & \quad \downarrow \text{5204} \\
 & c^2 \int \frac{(a + b \arcsin(cx))^2}{d(1 - c^2 x^2)} dx + \frac{2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{dx} \\
 & \quad \downarrow \text{27} \\
 & \frac{c^2 \int \frac{(a+b \arcsin(cx))^2}{1-c^2 x^2} dx}{d} + \frac{2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{dx} \\
 & \quad \downarrow \text{5164} \\
 & \frac{2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2 x^2}} dx}{d} + \frac{c \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} d \arcsin(cx)}{d} - \frac{(a + b \arcsin(cx))^2}{dx} \\
 & \quad \downarrow \text{3042} \\
 & \frac{2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2 x^2}} dx}{d} + \frac{c \int (a + b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{(a + b \arcsin(cx))^2 dx} - \\
 & \quad \downarrow \text{4669} \\
 & \frac{c(-2b \int (a + b \arcsin(cx)) \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a + b \arcsin(cx)) \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx))}{d} \\
 & \quad \downarrow \text{3011} \\
 & \frac{2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{dx}
 \end{aligned}$$

↓ 2838

$$\frac{c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{(a + b \arcsin(cx))^2} dx$$

↓ 7143

$$\frac{c(-2i \operatorname{arctan}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(3, 2b(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{(a + b \arcsin(cx))^2} dx$$

input `Int[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)),x]`

output `-((a + b*ArcSin[c*x])^2/(d*x)) + (2*b*c*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])/d + (c*((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])])))/d`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.99

method	result
parts	$-\frac{a^2 \left(\frac{c \ln(cx-1)}{2} - \frac{c \ln(cx+1)}{2} + \frac{1}{x} \right)}{d} - \frac{b^2 c \left(\frac{\arcsin(cx)^2}{cx} - 2i \operatorname{dilog}(icx + \sqrt{-c^2 x^2 + 1}) + 2 \arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) \right)}{d}$
derivativedivides	$c \left(-\frac{a^2 \left(-\frac{\ln(cx+1)}{2} + \frac{\ln(cx-1)}{2} + \frac{1}{cx} \right)}{d} - \frac{b^2 \left(\frac{\arcsin(cx)^2}{cx} - 2i \operatorname{dilog}(icx + \sqrt{-c^2 x^2 + 1}) + 2 \arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) \right)}{d} \right)$
default	$c \left(-\frac{a^2 \left(-\frac{\ln(cx+1)}{2} + \frac{\ln(cx-1)}{2} + \frac{1}{cx} \right)}{d} - \frac{b^2 \left(\frac{\arcsin(cx)^2}{cx} - 2i \operatorname{dilog}(icx + \sqrt{-c^2 x^2 + 1}) + 2 \arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) \right)}{d} \right)$

input

```
int((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
-a^2/d*(1/2*c*ln(c*x-1)-1/2*c*ln(c*x+1)+1/x)-b^2/d*c*(1/c/x*arcsin(c*x)^2-
2*I*dilog(I*c*x+(-c^2*x^2+1)^(1/2))+2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(
1/2))-2*I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*(I*c*x+(-
c^2*x^2+1)^(1/2)))+2*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))
)-2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+arcsin(c*x)^2*ln(1+I*(I*c*x+(-
c^2*x^2+1)^(1/2)))-2*I*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)
))+2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*a*b/d*c*(arcsin(c*x)/c/x-a
rcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+arcsin(c*x)*ln(1+I*(I*c*x+(-
c^2*x^2+1)^(1/2)))+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-ln(I*c*x+(-c^2*x^2+1)^(1
/2)-1)-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^
2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x^2} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output

```
integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^4 - d*x^2
), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx = -\frac{\int \frac{a^2}{c^2 x^4 - x^2} dx + \int \frac{b^2 \arcsin^2(cx)}{c^2 x^4 - x^2} dx + \int \frac{2ab \arcsin(cx)}{c^2 x^4 - x^2} dx}{d}$$

input

```
integrate((a+b*asin(c*x))**2/x**2/(-c**2*d*x**2+d),x)
```

output

```
-(Integral(a**2/(c**2*x**4 - x**2), x) + Integral(b**2*asin(c*x)**2/(c**2*
x**4 - x**2), x) + Integral(2*a*b*asin(c*x)/(c**2*x**4 - x**2), x))/d
```

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output

```
1/2*a^2*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x)) + 1/2*(b^2*c*x*arc
tan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - b^2*c*x*arctan2(c
*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*b^2*arctan2(c*x, sqr
t(c*x + 1)*sqrt(-c*x + 1))^2 + 2*d*x*integrate(-(2*a*b*arctan2(c*x, sqrt(c
*x + 1)*sqrt(-c*x + 1)) - (b^2*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*
x + 1))*log(c*x + 1) - b^2*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x +
1))*log(-c*x + 1) - 2*b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*
sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*d*x^4 - d*x^2), x))/(d*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```


Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 (d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)} dx$$

$$= \frac{-4 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^4 - x^2} dx \right) abx - 2 \left(\int \frac{\operatorname{asin}(cx)^2}{c^2 x^4 - x^2} dx \right) b^2 x - \log(c^2 x - c) a^2 cx + \log(c^2 x + c) a^2 cx - 2a^2}{2dx}$$

input `int((a+b*asin(c*x))^2/x^2/(-c^2*d*x^2+d),x)`

output `(- 4*int(asin(c*x)/(c**2*x**4 - x**2),x)*a*b*x - 2*int(asin(c*x)**2/(c**2*x**4 - x**2),x)*b**2*x - log(c**2*x - c)*a**2*c*x + log(c**2*x + c)*a**2*c*x - 2*a**2)/(2*d*x)`

3.187 $\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)} dx$

Optimal result	1757
Mathematica [B] (verified)	1758
Rubi [A] (verified)	1759
Maple [B] (verified)	1763
Fricas [F]	1764
Sympy [F]	1764
Maxima [F]	1764
Giac [F(-2)]	1765
Mupad [F(-1)]	1765
Reduce [F]	1765

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)} dx = -\frac{bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{dx} - \frac{(a+b \arcsin(cx))^2}{2dx^2} - \frac{2c^2(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d} + \frac{b^2c^2 \log(x)}{d} + \frac{ibc^2(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d} - \frac{ibc^2(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d} - \frac{b^2c^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{2d} + \frac{b^2c^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)})}{2d}$$

output

```
-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/d/x-1/2*(a+b*arcsin(c*x))^2/d/x^2-2*c^2*(a+b*arcsin(c*x))^2*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+b^2*c^2*ln(x)/d+I*b*c^2*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d-I*b*c^2*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d-1/2*b^2*c^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*c^2*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 614 vs. $2(210) = 420$.

Time = 0.81 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.92

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx =$$

$$-\frac{1}{12} i b^2 c^2 \pi^3 + \frac{a^2}{x^2} + \frac{2abc\sqrt{1-c^2x^2}}{x} + 4iabc^2\pi \arcsin(cx) + \frac{2ab \arcsin(cx)}{x^2} + \frac{2b^2c\sqrt{1-c^2x^2} \arcsin(cx)}{x} + \frac{b^2 \arcsin(cx)^2}{x^2} - \frac{4}{3}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)),x]`

output

```
-1/2*((I/12)*b^2*c^2*Pi^3 + a^2/x^2 + (2*a*b*c*Sqrt[1 - c^2*x^2])/x + (4*I
)*a*b*c^2*Pi*ArcSin[c*x] + (2*a*b*ArcSin[c*x])/x^2 + (2*b^2*c*Sqrt[1 - c^2
*x^2]*ArcSin[c*x])/x + (b^2*ArcSin[c*x]^2)/x^2 - ((4*I)/3)*b^2*c^2*ArcSin[
c*x]^3 + 8*a*b*c^2*Pi*Log[1 + E^((-I)*ArcSin[c*x])] + 2*a*b*c^2*Pi*Log[1 -
I*E^(I*ArcSin[c*x])] + 4*a*b*c^2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])]
- 2*a*b*c^2*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 4*a*b*c^2*ArcSin[c*x]*Log[1
+ I*E^(I*ArcSin[c*x])] - 2*b^2*c^2*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin
[c*x])] - 4*a*b*c^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 2*b^2*c^2
*ArcSin[c*x]^2*Log[1 + E^((2*I)*ArcSin[c*x])] - 2*a^2*c^2*Log[x] - 2*b^2*c
^2*Log[c*x] + a^2*c^2*Log[1 - c^2*x^2] - 8*a*b*c^2*Pi*Log[Cos[ArcSin[c*x]/
2]] + 2*a*b*c^2*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - 2*a*b*c^2*Pi*Log[Si
n[(Pi + 2*ArcSin[c*x])/4]] - (4*I)*a*b*c^2*PolyLog[2, (-I)*E^(I*ArcSin[c*x
])] - (4*I)*a*b*c^2*PolyLog[2, I*E^(I*ArcSin[c*x])] - (2*I)*b^2*c^2*ArcSin
[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] - (2*I)*b^2*c^2*ArcSin[c*x]*PolyL
og[2, -E^((2*I)*ArcSin[c*x])] + (2*I)*a*b*c^2*PolyLog[2, E^((2*I)*ArcSin[c
*x])] - b^2*c^2*PolyLog[3, E^((-2*I)*ArcSin[c*x])] + b^2*c^2*PolyLog[3, -E
^((2*I)*ArcSin[c*x])]/d
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5204, 27, 5184, 4919, 3042, 4671, 3011, 2720, 5186, 14, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx \\
 & \quad \downarrow \text{5204} \\
 & c^2 \int \frac{(a + b \arcsin(cx))^2}{dx (1 - c^2 x^2)} dx + \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2} \\
 & \quad \downarrow \text{27} \\
 & \frac{c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} dx}{d} + \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2} \\
 & \quad \downarrow \text{5184} \\
 & \frac{c^2 \int \frac{(a+b \arcsin(cx))^2}{cx \sqrt{1-c^2 x^2}} d \arcsin(cx)}{d} + \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2} \\
 & \quad \downarrow \text{4919} \\
 & \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} + \frac{2c^2 \int (a + b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx)}{(a + b \arcsin(cx))^2} \frac{d}{2dx^2} - \\
 & \quad \downarrow \text{3042} \\
 & \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} + \frac{2c^2 \int (a + b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx)}{(a + b \arcsin(cx))^2} \frac{d}{2dx^2} - \\
 & \quad \downarrow \text{4671}
 \end{aligned}$$

$$\frac{2c^2(-b \int (a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + b \int (a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx))}{d} \\ \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2}$$

↓ 3011

$$\frac{2c^2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx)) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, e^{2i \arcsin(cx)}) d \arcsin(cx))}{d}}{d} \\ \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2}$$

↓ 2720

$$\frac{2c^2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{2i \arcsin(cx)} \text{PolyLog}(2, e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)})}{d}}{d} \\ \frac{bc \int \frac{a+b \arcsin(cx)}{x^2 \sqrt{1-c^2 x^2}} dx}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2}$$

↓ 5186

$$\frac{2c^2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{2i \arcsin(cx)} \text{PolyLog}(2, e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)})}{d}}{d} \\ \frac{bc \left(bc \int \frac{1}{x} dx - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{x} \right)}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2}$$

↓ 14

$$\frac{2c^2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{2i \arcsin(cx)} \text{PolyLog}(2, e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)})}{d}}{d} \\ \frac{bc \left(bc \log(x) - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{x} \right)}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2}$$

↓ 7143

$$\frac{2c^2(-\text{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 + b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx))) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)))}{d}}{d} \\ \frac{bc \left(bc \log(x) - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{x} \right)}{d} - \frac{(a + b \arcsin(cx))^2}{2dx^2}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)),x]`

output `-1/2*(a + b*ArcSin[c*x])^2/(d*x^2) + (b*c*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/x) + b*c*Log[x]))/d + (2*c^2*(-((a + b*ArcSin[c*x])^2*ArcTanh[E^((2*I)*ArcSin[c*x])]) + b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4) - b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])/4))/d`

Defintions of rubi rules used

rule 14 `Int[(a_)/(x_), x_Symbol] := Simp[a*Log[x], x] /; FreeQ[a, x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_)^(m_)), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)(x_)]*((c_.) + (d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)(x_)]^{(n_.)}*((c_.) + (d_.)(x_))^{(m_.)}*\text{Sec}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{RationalQ}[m]$

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}/((x_)*((d_.) + (e_.)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + b*x)^n/(\text{Cos}[x]*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5186 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{EqQ}[m + 2*p + 3, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m + 2*p + 3)/(f^2*(m + 1))) \text{Int}[(f*x)^{(m+2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$

rule 7143 $\text{Int}[\text{PolyLog}[n, (c_.)*((a_.) + (b_.)(x_))^{(p_.)}]/((d_.) + (e_.)(x_)), x_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x \ \&\& \ \text{EqQ}[b*d, a*e]$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(250) = 500$.

Time = 0.64 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.90

method	result
derivativedivides	$c^2 \left(-\frac{a^2 \left(\frac{\ln(cx+1)}{2} + \frac{1}{2c^2x^2} - \ln(cx) + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b^2 \left(\frac{\arcsin(cx) (-2ic^2x^2 + 2cx\sqrt{-c^2x^2+1} + \arcsin(cx))}{2c^2x^2} + 2\ln(icx + \sqrt{-c^2x^2+1}) \right)}{d} \right)$
default	$c^2 \left(-\frac{a^2 \left(\frac{\ln(cx+1)}{2} + \frac{1}{2c^2x^2} - \ln(cx) + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b^2 \left(\frac{\arcsin(cx) (-2ic^2x^2 + 2cx\sqrt{-c^2x^2+1} + \arcsin(cx))}{2c^2x^2} + 2\ln(icx + \sqrt{-c^2x^2+1}) \right)}{d} \right)$
parts	$-\frac{a^2 \left(\frac{c^2 \ln(cx-1)}{2} + \frac{c^2 \ln(cx+1)}{2} + \frac{1}{2x^2} - c^2 \ln(x) \right)}{d} - \frac{b^2 c^2 \left(\frac{\arcsin(cx) (-2ic^2x^2 + 2cx\sqrt{-c^2x^2+1} + \arcsin(cx))}{2c^2x^2} + 2\ln(icx + \sqrt{-c^2x^2+1}) \right)}{d}$

```
input int((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d), x, method=_RETURNVERBOSE)
```

```
output c^2*(-a^2/d*(1/2*ln(c*x+1)+1/2/c^2/x^2-ln(c*x)+1/2*ln(c*x-1))-b^2/d*(1/2*arcsin(c*x)*(-2*I*c^2*x^2+2*c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+2*ln(I*c*x+(-c^2*x^2+1)^(1/2))-ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)-ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))-2*a*b/d*(1/2*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2+arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))))
```


Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^5 - d*x^3), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx = -\frac{\int \frac{a^2}{c^2 x^5 - x^3} dx + \int \frac{b^2 \arcsin^2(cx)}{c^2 x^5 - x^3} dx + \int \frac{2ab \arcsin(cx)}{c^2 x^5 - x^3} dx}{d}$$

input `integrate((a+b*asin(c*x))**2/x**3/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2/(c**2*x**5 - x**3), x) + Integral(b**2*asin(c*x)**2/(c**2*x**5 - x**3), x) + Integral(2*a*b*asin(c*x)/(c**2*x**5 - x**3), x))/d`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2))*a^2 - integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/(c^2*d*x^5 - d*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d),x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^3 (d - c^2 dx^2)} dx$$

input `int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)),x)`

output `int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)} dx$$

$$= \frac{-4 \left(\int \frac{\operatorname{asin}(cx)}{c^2 x^5 - x^3} dx \right) ab x^2 - 2 \left(\int \frac{\operatorname{asin}(cx)^2}{c^2 x^5 - x^3} dx \right) b^2 x^2 - \log(c^2 x - c) a^2 c^2 x^2 - \log(c^2 x + c) a^2 c^2 x^2 + 2 \log(x) a^2}{2d x^2}$$

input `int((a+b*asin(c*x))^2/x^3/(-c^2*d*x^2+d),x)`

output

```
( - 4*int(asin(c*x)/(c**2*x**5 - x**3),x)*a*b*x**2 - 2*int(asin(c*x)**2/(c
**2*x**5 - x**3),x)*b**2*x**2 - log(c**2*x - c)*a**2*c**2*x**2 - log(c**2*
x + c)*a**2*c**2*x**2 + 2*log(x)*a**2*c**2*x**2 - a**2)/(2*d*x**2)
```

3.188 $\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)} dx$

Optimal result	1767
Mathematica [B] (verified)	1768
Rubi [A] (verified)	1769
Maple [A] (verified)	1775
Fricas [F]	1776
Sympy [F]	1776
Maxima [F]	1776
Giac [F(-2)]	1777
Mupad [F(-1)]	1777
Reduce [F]	1778

Optimal result

Integrand size = 27, antiderivative size = 333

$$\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)} dx = -\frac{b^2c^2}{3dx} - \frac{bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3dx^2}$$

$$-\frac{(a+b \arcsin(cx))^2}{3dx^3} - \frac{c^2(a+b \arcsin(cx))^2}{dx}$$

$$-\frac{2ic^3(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{d}$$

$$-\frac{14bc^3(a+b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{3d}$$

$$+\frac{7ib^2c^3 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{3d}$$

$$+\frac{2ibc^3(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d}$$

$$-\frac{2ibc^3(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d}$$

$$-\frac{7ib^2c^3 \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{3d}$$

$$-\frac{2b^2c^3 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{d}$$

$$+\frac{2b^2c^3 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{d}$$

output

```
-1/3*b^2*c^2/d/x-1/3*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/d/x^2-1/3*(a
+b*arcsin(c*x))^2/d/x^3-c^2*(a+b*arcsin(c*x))^2/d/x-2*I*c^3*(a+b*arcsin(c*
x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d-14/3*b*c^3*(a+b*arcsin(c*x))*arct
anh(I*c*x+(-c^2*x^2+1)^(1/2))/d+7/3*I*b^2*c^3*polylog(2,-I*c*x+(-c^2*x^2+1
)^(1/2))/d+2*I*b*c^3*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1
/2)))/d-2*I*b*c^3*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))
)/d-7/3*I*b^2*c^3*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d-2*b^2*c^3*polylog(
3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d+2*b^2*c^3*polylog(3,I*(I*c*x+(-c^2*x^2+
1)^(1/2)))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 849 vs. $2(333) = 666$.

Time = 7.34 (sec) , antiderivative size = 849, normalized size of antiderivative = 2.55

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)),x]
```

output

```

-1/3*a^2/(d*x^3) - (a^2*c^2)/(d*x) - (a^2*c^3*Log[1 - c*x])/(2*d) + (a^2*c
^3*Log[1 + c*x])/(2*d) - (2*a*b*(-(c^2*(-(ArcSin[c*x]/x) - c*ArcTanh[Sqrt[
1 - c^2*x^2]])) + (c*x*Sqrt[1 - c^2*x^2] + 2*ArcSin[c*x] + c^3*x^3*ArcTanh
[Sqrt[1 - c^2*x^2]])/(6*x^3) + (c^4*(((3*I)/2)*Pi*ArcSin[c*x])/c - ((I/2)
*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c - (Pi*Log[1 + I
*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c -
(2*Pi*Log[Cos[ArcSin[c*x]/2]])/c + (Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]])/
c - ((2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/c)/2 - (c^4*(((I/2)*Pi*Arc
Sin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x]
)])/c + (Pi*Log[1 - I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 - I*E^(I
*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c - (Pi*Log[Sin[(Pi + 2
*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, I*E^(I*ArcSin[c*x])])/c)/2)/d -
(b^2*c^3*(4*Cot[ArcSin[c*x]/2] + 14*ArcSin[c*x]^2*Cot[ArcSin[c*x]/2] + 2*
ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 + (c*x*ArcSin[c*x]^2*Csc[ArcSin[c*x]/2]^4
)/2 - 56*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - 24*ArcSin[c*x]^2*Log[1 -
I*E^(I*ArcSin[c*x])] + 24*ArcSin[c*x]^2*Log[1 + I*E^(I*ArcSin[c*x])] + 56
*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (56*I)*PolyLog[2, -E^(I*ArcSin[c
*x])] - (48*I)*ArcSin[c*x]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (48*I)*Arc
Sin[c*x]*PolyLog[2, I*E^(I*ArcSin[c*x])] + (56*I)*PolyLog[2, E^(I*ArcSin[c
*x])] + 48*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 48*PolyLog[3, I*E^(I*Ar...

```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5204, 27, 5204, 15, 5164, 3042, 4669, 3011, 2720, 5218, 3042, 4671, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)} dx$$

$$\downarrow 5204$$

$$c^2 \int \frac{(a + b \arcsin(cx))^2}{dx^2 (1 - c^2 x^2)} dx + \frac{2bc \int \frac{a+b \arcsin(cx)}{x^3 \sqrt{1-c^2 x^2}} dx}{3d} - \frac{(a + b \arcsin(cx))^2}{3dx^3}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{c^2 \int \frac{(a+b \arcsin(cx))^2}{x^2(1-c^2x^2)} dx}{d} + \frac{2bc \int \frac{a+b \arcsin(cx)}{x^3\sqrt{1-c^2x^2}} dx}{3d} - \frac{(a+b \arcsin(cx))^2}{3dx^3} \\
& \quad \downarrow 5204 \\
& \frac{c^2 \left(c^2 \int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx + 2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - \frac{(a+b \arcsin(cx))^2}{x} \right)}{d} + \\
& \frac{2bc \left(\frac{1}{2}c^2 \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{1}{2}bc \int \frac{1}{x^2} dx - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2x^2} \right)}{3d} - \frac{(a+b \arcsin(cx))^2}{3dx^3} \\
& \quad \downarrow 15 \\
& \frac{c^2 \left(c^2 \int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx + 2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - \frac{(a+b \arcsin(cx))^2}{x} \right)}{d} + \\
& \frac{2bc \left(\frac{1}{2}c^2 \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a+b \arcsin(cx))^2}{3dx^3} \\
& \quad \downarrow 5164 \\
& \frac{c^2 \left(2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + c \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx) - \frac{(a+b \arcsin(cx))^2}{x} \right)}{d} + \\
& \frac{2bc \left(\frac{1}{2}c^2 \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a+b \arcsin(cx))^2}{3dx^3} \\
& \quad \downarrow 3042 \\
& \frac{2bc \left(\frac{1}{2}c^2 \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} + \\
& \frac{c^2 \left(2bc \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + c \int (a+b \arcsin(cx))^2 \csc \left(\arcsin(cx) + \frac{\pi}{2} \right) d \arcsin(cx) - \frac{(a+b \arcsin(cx))^2}{x} \right)}{d} \\
& \quad \downarrow 4669 \\
& \frac{c^2 \left(c(-2b \int (a+b \arcsin(cx)) \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) \right)}{d} \\
& \frac{2bc \left(\frac{1}{2}c^2 \int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a+b \arcsin(cx))^2}{3dx^3} \\
& \quad \downarrow 3011
\end{aligned}$$

$$c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) \right)$$

$$\frac{2bc \left(\frac{1}{2} c^2 \int \frac{a+b \arcsin(cx)}{x \sqrt{1-c^2 x^2}} dx - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a + b \arcsin(cx))^2}{3dx^3}$$

↓ 2720

$$c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) \right)$$

$$\frac{2bc \left(\frac{1}{2} c^2 \int \frac{a+b \arcsin(cx)}{x \sqrt{1-c^2 x^2}} dx - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a + b \arcsin(cx))^2}{3dx^3}$$

↓ 5218

$$c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) \right)$$

$$\frac{2bc \left(\frac{1}{2} c^2 \int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx) - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a + b \arcsin(cx))^2}{3dx^3}$$

↓ 3042

$$c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) \right)$$

$$\frac{2bc \left(\frac{1}{2} c^2 \int (a + b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) - \frac{\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{2x^2} - \frac{bc}{2x} \right)}{3d} - \frac{(a + b \arcsin(cx))^2}{3dx^3}$$

↓ 4671

$$c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) \right)$$

$$2bc \left(\frac{1}{2} c^2 (-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{3dx^3}$$

↓ 2715

$$\frac{c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right) - 2bc \left(\frac{1}{2} c^2 (ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2a \right)}{3d} \frac{(a + b \arcsin(cx))^2}{3dx^3} \downarrow 2838$$

$$\frac{c^2 \left(c(2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right) - 2bc \left(\frac{1}{2} c^2 (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})) \right)}{3d} \frac{(a + b \arcsin(cx))^2}{3dx^3} \downarrow 7143$$

$$\frac{c^2 \left(c(-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) \right) - 2bc \left(\frac{1}{2} c^2 (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)})) \right)}{3d} \frac{(a + b \arcsin(cx))^2}{3dx^3}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)),x]`

output `-1/3*(a + b*ArcSin[c*x])^2/(d*x^3) + (2*b*c*(-1/2*(b*c)/x - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x]))/(2*x^2) + (c^2*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])]))/2)/(3*d) + (c^2*(-((a + b*ArcSin[c*x])^2/x) + 2*b*c*(-2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])]) - I*b*PolyLog[2, E^(I*ArcSin[c*x])]) + c*((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])])))/d`

Definitions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 27 $\text{Int}[(a_)*(Fx_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[Fx, (b_)*(Gx_)] \text{ ; FreeQ}[b, x]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^{(n)}], x] \text{ ; FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \ \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] \text{ ; FunctionOfExponentialQ}[u, x] \ \&\& \ \text{!MatchQ}[u, (w_)*((a_.)*(v_)^{(n_)})^{(m_)}] \text{ ; FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ \text{!MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))}*(F_)^{(v_)}] \text{ ; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$
- rule 2838 $\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] \text{ ; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_)))})^{(n_.)}]*((f_.) + (g_.)*(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \ \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n], x], x] \text{ ; FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_.) + \text{Pi}(k_.) + (f_.)(x_.)]*((c_.) + (d_.)(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)]*((c_.) + (d_.)(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_.)]*(b_.))^{(n_.)} / ((d_.) + (e_.)(x_.)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_.)]*(b_.))^{(n_.)}*((f_.)(x_.))^{(m_.)}*((d_.) + (e_.)(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1))) \text{Int}[(f*x)^{(m+2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1))) * \text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$

rule 5218 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_.)]*(b_.))^{(n_.)}*(x_.)^{(m_.)} / \text{Sqrt}[(d_.) + (e_.)(x_.)^2], x_Symbol] \rightarrow \text{Simp}[(1/c^{(m+1)}) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sin}[x]^m, x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

rule 7143 $\text{Int}[\text{PolyLog}[n, (c_.)*((a_.) + (b_.)(x_.))^{(p_.)}] / ((d_.) + (e_.)(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x \ \&\& \ \text{EqQ}[b*d, a*e]$

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.68

method	result
derivativedivides	$c^3 \left(-\frac{a^2 \left(-\frac{\ln(cx+1)}{2} + \frac{1}{3c^3x^3} + \frac{1}{cx} + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b^2 \left(\frac{3 \arcsin(cx)^2 x^2 c^2 + \arcsin(cx) \sqrt{-c^2 x^2 + 1} cx + \arcsin(cx)^2 + c^2 x^2}{3c^3 x^3} \right)}{d} \right)$
default	$c^3 \left(-\frac{a^2 \left(-\frac{\ln(cx+1)}{2} + \frac{1}{3c^3x^3} + \frac{1}{cx} + \frac{\ln(cx-1)}{2} \right)}{d} - \frac{b^2 \left(\frac{3 \arcsin(cx)^2 x^2 c^2 + \arcsin(cx) \sqrt{-c^2 x^2 + 1} cx + \arcsin(cx)^2 + c^2 x^2}{3c^3 x^3} \right)}{d} \right)$
parts	$-\frac{a^2 \left(\frac{c^3 \ln(cx-1)}{2} - \frac{c^3 \ln(cx+1)}{2} + \frac{1}{3x^3} + \frac{c^2}{x} \right)}{d} - \frac{b^2 c^3 \left(\frac{3 \arcsin(cx)^2 x^2 c^2 + \arcsin(cx) \sqrt{-c^2 x^2 + 1} cx + \arcsin(cx)^2 + c^2 x^2}{3c^3 x^3} \right)}{d}$

input `int((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)`

output `c^3*(-a^2/d*(-1/2*ln(c*x+1)+1/3/c^3/x^3+1/c/x+1/2*ln(c*x-1))-b^2/d*(1/3*(3*arcsin(c*x)^2*x^2*c^2+arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+arcsin(c*x)^2+c^2*x^2)/c^3/x^3-7/3*I*dilog(I*c*x+(-c^2*x^2+1)^(1/2))+7/3*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-7/3*I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2))))-2*a*b/d*(1/6*(6*c^2*x^2*arcsin(c*x)+c*x*(-c^2*x^2+1)^(1/2)+2*arcsin(c*x))/c^3/x^3-7/6*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)+7/6*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^6 - d*x^4), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)} dx = -\frac{\int \frac{a^2}{c^2 x^6 - x^4} dx + \int \frac{b^2 \arcsin^2(cx)}{c^2 x^6 - x^4} dx + \int \frac{2ab \arcsin(cx)}{c^2 x^6 - x^4} dx}{d}$$

input `integrate((a+b*asin(c*x))**2/x**4/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2/(c**2*x**6 - x**4), x) + Integral(b**2*asin(c*x)**2/(c**2*x**6 - x**4), x) + Integral(2*a*b*asin(c*x)/(c**2*x**6 - x**4), x))/d`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")`

output

```
1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^
3))*a^2 + 1/6*(3*b^2*c^3*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*
log(c*x + 1) - 3*b^2*c^3*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*
log(-c*x + 1) + 6*d*x^3*integrate(-1/3*(6*a*b*arctan2(c*x, sqrt(c*x + 1))*s
qrt(-c*x + 1)) - (3*b^2*c^4*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))
*log(c*x + 1) - 3*b^2*c^4*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*l
og(-c*x + 1) - 2*(3*b^2*c^3*x^3 + b^2*c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt
(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^2*d*x^6 - d*x^4), x) - 2*(3*
b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4(d - c^2 dx^2)} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4(d - c^2 dx^2)} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^4(d - c^2 dx^2)} dx$$

input

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)} dx$$

$$= \frac{-12 \left(\int \frac{\arcsin(cx)}{c^2 x^6 - x^4} dx \right) ab x^3 - 6 \left(\int \frac{\arcsin(cx)^2}{c^2 x^6 - x^4} dx \right) b^2 x^3 - 3 \log(c^2 x - c) a^2 c^3 x^3 + 3 \log(c^2 x + c) a^2 c^3 x^3 - 6 a^2 c^2}{6 d x^3}$$

input `int((a+b*asin(c*x))^2/x^4/(-c^2*d*x^2+d),x)`

output `(- 12*int(asin(c*x)/(c**2*x**6 - x**4),x)*a*b*x**3 - 6*int(asin(c*x)**2/(c**2*x**6 - x**4),x)*b**2*x**3 - 3*log(c**2*x - c)*a**2*c**3*x**3 + 3*log(c**2*x + c)*a**2*c**3*x**3 - 6*a**2*c**2*x**2 - 2*a**2)/(6*d*x**3)`

3.189
$$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$$

Optimal result	1779
Mathematica [B] (verified)	1780
Rubi [A] (verified)	1781
Maple [A] (verified)	1787
Fricas [F]	1788
Sympy [F]	1788
Maxima [F]	1789
Giac [F]	1789
Mupad [F(-1)]	1790
Reduce [F]	1790

Optimal result

Integrand size = 27, antiderivative size = 300

$$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx = -\frac{2b^2x}{c^4d^2} - \frac{b(a+b \arcsin(cx))}{c^5d^2\sqrt{1-c^2x^2}}$$

$$+ \frac{2b\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^5d^2}$$

$$+ \frac{3x(a+b \arcsin(cx))^2}{2c^4d^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$+ \frac{3i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{c^5d^2}$$

$$+ \frac{b^2 \operatorname{arctanh}(cx)}{c^5d^2}$$

$$- \frac{3ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^5d^2}$$

$$+ \frac{3ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^5d^2}$$

$$+ \frac{3b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{c^5d^2}$$

$$- \frac{3b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{c^5d^2}$$

output

```

-2*b^2*x/c^4/d^2-b*(a+b*arcsin(c*x))/c^5/d^2/(-c^2*x^2+1)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^5/d^2+3/2*x*(a+b*arcsin(c*x))^2/c^4/d^2+1/2*x^3*(a+b*arcsin(c*x))^2/c^2/d^2/(-c^2*x^2+1)+3*I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^5/d^2+b^2*arctanh(c*x)/c^5/d^2-3*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^2+3*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^2+3*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^2-3*b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^2

```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 614 vs. $2(300) = 600$.

Time = 2.35 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.05

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{4a^2 cx + \frac{8b^2 c^3 x^3}{1-c^2 x^2} + 8ab\sqrt{1-c^2 x^2} + \frac{2ab\sqrt{1-c^2 x^2}}{-1+cx} - \frac{2ab\sqrt{1-c^2 x^2}}{1+cx} - \frac{2a^2 cx}{-1+c^2 x^2} + \frac{8b^2 cx}{-1+c^2 x^2} + 4b^2 \coth^{-1}(cx) + 6iabn}{}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output

```
(4*a^2*c*x + (8*b^2*c^3*x^3)/(1 - c^2*x^2) + 8*a*b*Sqrt[1 - c^2*x^2] + (2*
a*b*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (2*a*b*Sqrt[1 - c^2*x^2))/(1 + c*x) -
(2*a^2*c*x)/(-1 + c^2*x^2) + (8*b^2*c*x)/(-1 + c^2*x^2) + 4*b^2*ArcCoth[c*
x] + (6*I)*a*b*Pi*ArcSin[c*x] + 8*a*b*c*x*ArcSin[c*x] - (2*a*b*ArcSin[c*x]
)/(-1 + c*x) - (2*a*b*ArcSin[c*x))/(1 + c*x) + (2*b^2*ArcSin[c*x])/Sqrt[1
- c^2*x^2] - (6*b^2*c^2*x^2*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + 2*b^2*Sqrt[1
- c^2*x^2]*ArcSin[c*x] + (6*b^2*c*x*ArcSin[c*x]^2)/(1 - c^2*x^2) + (4*b^2*
c^3*x^3*ArcSin[c*x]^2)/(-1 + c^2*x^2) + (12*I)*b^2*ArcSin[c*x]^2*ArcTan[E^
(I*ArcSin[c*x])] - 6*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 12*a*b*ArcSin[c
*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 6*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] +
12*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 3*a^2*Log[1 - c*x] - 3*
a^2*Log[1 + c*x] + 6*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 6*a*b*Pi*L
og[Sin[(Pi + 2*ArcSin[c*x])/4]] - (12*I)*b*(a + b*ArcSin[c*x])*PolyLog[2,
(-I)*E^(I*ArcSin[c*x])] + (12*I)*b*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*
ArcSin[c*x])] + 12*b^2*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 12*b^2*PolyLog[
3, I*E^(I*ArcSin[c*x])])/(4*c^5*d^2)
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5206, 27, 5194, 27, 299, 219, 5210, 5164, 3042, 4669, 3011, 2720, 5182, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$\downarrow 5206$$

$$-\frac{b \int \frac{x^3(a + b \arcsin(cx))}{(1 - c^2 x^2)^{3/2}} dx}{cd^2} - \frac{3 \int \frac{x^2(a + b \arcsin(cx))^2}{d(1 - c^2 x^2)} dx}{2c^2 d} + \frac{x^3(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)}$$

$$\downarrow 27$$

$$-\frac{3 \int \frac{x^2(a + b \arcsin(cx))^2}{1 - c^2 x^2} dx}{2c^2 d^2} - \frac{b \int \frac{x^3(a + b \arcsin(cx))}{(1 - c^2 x^2)^{3/2}} dx}{cd^2} + \frac{x^3(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)}$$

$$\begin{aligned}
& \downarrow 5194 \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \left(-bc \int \frac{2-c^2x^2}{c^4(1-c^2x^2)} dx + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} \right)}{cd^2} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 27 \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \left(-\frac{b \int \frac{2-c^2x^2}{1-c^2x^2} dx}{c^3} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} \right)}{cd^2} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 299 \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \left(-\frac{b \left(\int \frac{1}{1-c^2x^2} dx + x \right)}{c^3} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} \right)}{cd^2} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 219 \\
& \frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{2c^2d^2} - \frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 5210 \\
& \frac{3 \left(\frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))^2}{c^2} \right)}{2c^2d^2} - \\
& \quad \frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \downarrow 5164
\end{aligned}$$

$$\begin{aligned}
 & \frac{3 \left(\frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))^2}{c^2} \right)}{2c^2d^2} \\
 & \frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
 & \quad \downarrow \text{3042} \\
 & \frac{3 \left(\frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^3} + \frac{2b \int \frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))^2}{c^2} \right)}{2c^2d^2} \\
 & \frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
 & \quad \downarrow \text{4669} \\
 & \frac{3 \left(\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} \right)}{2c^2d^2} \\
 & \frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
 & \quad \downarrow \text{3011} \\
 & \frac{3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} \right)}{2c^2d^2} \\
 & \frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
 & \quad \downarrow \text{2720}
 \end{aligned}$$

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{c^3} \right)$$

$$\frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 5182

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{c^3} \right)$$

$$\frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 24

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{c^3} \right)$$

$$\frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 7143

$$3 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{c^3} \right)$$

$2c^2d^2$

$$\frac{b \left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^4} + \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{c} + x \right)}{c^3} \right)}{cd^2} + \frac{x^3(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

input `Int[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]`

output

$$\begin{aligned} & (x^3(a + b\text{ArcSin}[c*x])^2)/(2*c^2*d^2*(1 - c^2*x^2)) - (b*((a + b\text{ArcSin}[c*x])/ \\ & (c^4*\text{Sqrt}[1 - c^2*x^2]) + (\text{Sqrt}[1 - c^2*x^2]*(a + b\text{ArcSin}[c*x]))/c^4 - \\ & (b*(x + \text{ArcTanh}[c*x]/c))/c^3))/c^3)/c^2) - (3*(-((x*(a + b\text{ArcSin}[c*x])^2)/c^2) + \\ & (2*b*((b*x)/c - (\text{Sqrt}[1 - c^2*x^2]*(a + b\text{ArcSin}[c*x]))/c^2))/c + \\ & ((-2*I)*(a + b\text{ArcSin}[c*x])^2*\text{ArcTan}[E^(I*\text{ArcSin}[c*x])] + 2*b*(I*(a + b\text{ArcSin}[c*x])* \\ & \text{PolyLog}[2, (-I)*E^(I*\text{ArcSin}[c*x])] - b*\text{PolyLog}[3, (-I)*E^(I*\text{ArcSin}[c*x])]) - \\ & 2*b*(I*(a + b\text{ArcSin}[c*x])* \text{PolyLog}[2, I*E^(I*\text{ArcSin}[c*x])] - b*\text{PolyLog}[3, I*E^(I*\text{ArcSin}[c*x])])))/c^3))/ \\ & (2*c^2*d^2) \end{aligned}$$

Defintions of rubi rules used

rule 24

$$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$$

rule 27

$$\text{Int}[(a_)*(F_x_), x_Symbol] \text{ :> } \text{Simp}[a \text{ Int}[F_x, x], x] \text{ /; } \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_)*(G_x_)] \text{ /; } \text{FreeQ}[b, x]$$

rule 219

$$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \text{ :> } \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] \text{ /; } \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

rule 299

$$\text{Int}[((a_) + (b_.)*(x_)^2)^{(p_)*((c_) + (d_.)*(x_)^2)}, x_Symbol] \text{ :> } \text{Simp}[d*x*((a + b*x^2)^{(p + 1)}/(b*(2*p + 3))), x] - \text{Simp}[(a*d - b*c*(2*p + 3))/(b*(2*p + 3)) \text{ Int}[(a + b*x^2)^p, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[2*p + 3, 0]$$

rule 2720

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{ Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] \text{ /; } \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_.)*(v_)^{(n_)})^{(m_)}] \text{ /; } \text{FreeQ}\{a, m, n\}, x \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))}*(F_)[v_]] \text{ /; } \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{InverseFunctionQ}[F[x]]$$

rule 3011 $\text{Int}[\text{Log}[1 + (e_)*(F_)^{\((c_)*(a_)\ + (b_)*(x_)\)}\]^{\(n_)\}]*(f_)\ + (g_)*(x_)^{\(m_)\}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^{m - 1}*\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_)\ + \text{Pi}*(k_)\ + (f_)*(x_)]*\((c_)\ + (d_)*(x_))^{\(m_)\}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m - 1}*\text{Log}[1 - E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m - 1}*\text{Log}[1 + E^{(I*k*Pi)*E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[\((a_)\ + \text{ArcSin}[(c_)*(x_)]*(b_))^{\(n_)\}/((d_)\ + (e_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[\((a_)\ + \text{ArcSin}[(c_)*(x_)]*(b_))^{\(n_)\}*(x_)*((d_)\ + (e_)*(x_)^2)^{\(p_)\}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{p + 1}*\((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{p + 1/2}*(a + b*\text{ArcSin}[c*x])^{n - 1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[p, -1]$

rule 5194 $\text{Int}[\((a_)\ + \text{ArcSin}[(c_)*(x_)]*(b_))*\((x_)^{\(m_)\}*((d_)\ + (e_)*(x_)^2)^{\(p_)\}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[x^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) u, x] - \text{Simp}[b*c*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[d + e*x^2], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[p - 1/2] \&\& \text{NeQ}[p, -2^{(-1)}] \&\& (\text{IGtQ}[(m + 1)/2, 0] || \text{ILtQ}[(m + 2*p + 3)/2, 0])$

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{a^2 \left(cx - \frac{1}{4(cx+1)} - \frac{3 \ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{3 \ln(cx-1)}{4} \right)}{d^2} + \frac{2b^2 \arcsin(cx) \sqrt{-c^2x^2+1}}{d^2} + \frac{b^2 \arcsin(cx)^2 cx}{d^2} - \frac{2b^2 cx}{d^2} - \frac{b^2 \arcsin(cx)^2 cx}{2d^2(c^2x^2-1)}$
default	$\frac{a^2 \left(cx - \frac{1}{4(cx+1)} - \frac{3 \ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{3 \ln(cx-1)}{4} \right)}{d^2} + \frac{2b^2 \arcsin(cx) \sqrt{-c^2x^2+1}}{d^2} + \frac{b^2 \arcsin(cx)^2 cx}{d^2} - \frac{2b^2 cx}{d^2} - \frac{b^2 \arcsin(cx)^2 cx}{2d^2(c^2x^2-1)}$
parts	$\frac{a^2 \left(\frac{x}{c^4} - \frac{1}{4c^5(cx-1)} + \frac{3 \ln(cx-1)}{4c^5} - \frac{1}{4c^5(cx+1)} - \frac{3 \ln(cx+1)}{4c^5} \right)}{d^2} + \frac{2b^2 \sqrt{-c^2x^2+1} \arcsin(cx)}{d^2 c^5} + \frac{b^2 \arcsin(cx)^2 x}{d^2 c^4} - \frac{2b^2 x}{c^4 d^2}$

input

```
int(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```


output

```

1/c^5*(a^2/d^2*(c*x-1/4/(c*x+1)-3/4*ln(c*x+1)-1/4/(c*x-1)+3/4*ln(c*x-1))+
*b^2/d^2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+b^2/d^2*arcsin(c*x)^2*c*x-2*b^2/d^
2*c*x-1/2*b^2/d^2/(c^2*x^2-1)*arcsin(c*x)^2*c*x+b^2/d^2/(c^2*x^2-1)*arcsin
(c*x)*(-c^2*x^2+1)^(1/2)-3/2*b^2/d^2*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2
+1)^(1/2)))-2*I*b^2/d^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))-3*b^2/d^2*polylog
(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/2*b^2/d^2*arcsin(c*x)^2*ln(1+I*(I*c*x+(
-c^2*x^2+1)^(1/2)))+3*I*b^2/d^2*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1
)^(1/2)))+3*b^2/d^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3*I*b^2/d^2*a
rcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*a*b/d^2*(-c^2*x^2+1)
^(1/2)+2*a*b/d^2*arcsin(c*x)*c*x-a*b/d^2/(c^2*x^2-1)*arcsin(c*x)*c*x+a*b/d
^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+3*a*b/d^2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^
2*x^2+1)^(1/2)))-3*a*b/d^2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-
3*I*a*b/d^2*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3*I*a*b/d^2*dilog(1-I*(I
*c*x+(-c^2*x^2+1)^(1/2)))

```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b^2*x^4*arcsin(c*x)^2 + 2*a*b*x^4*arcsin(c*x) + a^2*x^4)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{a^2 x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b^2 x^4 \arcsin^2(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{2abx^4 \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input

```
integrate(x**4*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output

```
(Integral(a**2*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**4
*asin(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*asin
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output

```
-1/4*a^2*(2*x/(c^6*d^2*x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^
5*d^2) - 3*log(c*x - 1)/(c^5*d^2)) - 1/4*(3*(b^2*c^2*x^2 - b^2)*arctan2(c*
x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - 3*(b^2*c^2*x^2 - b^2)*ar
ctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*(2*b^2*c^3*x^
3 - 3*b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 4*(c^7*d^2*x
^2 - c^5*d^2)*integrate(-1/2*(4*a*b*c^4*x^4*arctan2(c*x, sqrt(c*x + 1)*sq
r(-c*x + 1)) - (3*(b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x
+ 1))*log(c*x + 1) - 3*(b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sq
r(-c*x + 1))*log(-c*x + 1) - 2*(2*b^2*c^3*x^3 - 3*b^2*c*x)*arctan2(c*x, sq
rt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^8*d^2*x^4 -
2*c^6*d^2*x^2 + c^4*d^2), x))/(c^7*d^2*x^2 - c^5*d^2)
```

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

```
integrate((b*arcsin(c*x) + a)^2*x^4/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^2} dx$$

input `int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2,x)`

output `int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{8 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^7 x^2 - 8 \left(\int \frac{\operatorname{asin}(cx)x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^5 + 4 \left(\int \frac{\operatorname{asin}(cx)^2 x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2 c^7 x^2 - 4 \left(\int \frac{\operatorname{asin}(cx)^2 x^4}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^5}{4c^5}$$

input `int(x^4*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

output `(8*int((asin(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**7*x**2 - 8*int((asin(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**5 + 4*int((asin(c*x)**2*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**7*x**2 - 4*int((asin(c*x)**2*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**5 + 3*log(c**2*x - c)*a**2*c**2*x**2 - 3*log(c**2*x - c)*a**2 - 3*log(c**2*x + c)*a**2*c**2*x**2 + 3*log(c**2*x + c)*a**2 + 4*a**2*c**3*x**3 - 6*a**2*c*x)/(4*c**5*d**2*(c**2*x**2 - 1))`

3.190
$$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$$

Optimal result	1791
Mathematica [B] (verified)	1792
Rubi [A] (verified)	1792
Maple [A] (verified)	1797
Fricas [F]	1798
Sympy [F]	1798
Maxima [F]	1798
Giac [F(-2)]	1799
Mupad [F(-1)]	1799
Reduce [F]	1800

Optimal result

Integrand size = 27, antiderivative size = 227

$$\begin{aligned} \int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx = & -\frac{bx(a+b \arcsin(cx))}{c^3d^2\sqrt{1-c^2x^2}} + \frac{(a+b \arcsin(cx))^2}{2c^4d^2} \\ & + \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} - \frac{i(a+b \arcsin(cx))^3}{3bc^4d^2} \\ & + \frac{(a+b \arcsin(cx))^2 \log(1+e^{2i \arcsin(cx)})}{c^4d^2} \\ & - \frac{b^2 \log(1-c^2x^2)}{2c^4d^2} \\ & - \frac{ib(a+b \arcsin(cx)) \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{c^4d^2} \\ & + \frac{b^2 \text{PolyLog}(3, -e^{2i \arcsin(cx)})}{2c^4d^2} \end{aligned}$$

output

```
-b*x*(a+b*arcsin(c*x))/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arcsin(c*x))^2/c^4/d^2+1/2*x^2*(a+b*arcsin(c*x))^2/c^2/d^2/(-c^2*x^2+1)-1/3*I*(a+b*arcsin(c*x))^3/b/c^4/d^2+(a+b*arcsin(c*x))^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*b^2*ln(-c^2*x^2+1)/c^4/d^2-I*b*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d^2+1/2*b^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^4/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 502 vs. $2(227) = 454$.

Time = 1.02 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.21

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{3ab\sqrt{1-c^2x^2}}{-1+cx} + \frac{3ab\sqrt{1-c^2x^2}}{1+cx} - \frac{3a^2}{-1+c^2x^2} + 12iab\pi \arcsin(cx) - \frac{3ab \arcsin(cx)}{-1+cx} + \frac{3ab \arcsin(cx)}{1+cx} - \frac{6b^2cx \arcsin(cx)}{\sqrt{1-c^2x^2}} - 6iab$$

input `Integrate[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]`

output
$$\frac{((3*a*b*\text{Sqrt}[1 - c^2*x^2])/(-1 + c*x) + (3*a*b*\text{Sqrt}[1 - c^2*x^2])/(1 + c*x) - (3*a^2)/(-1 + c^2*x^2) + (12*I)*a*b*\text{Pi}*ArcSin[c*x] - (3*a*b*ArcSin[c*x])/(-1 + c*x) + (3*a*b*ArcSin[c*x])/(1 + c*x) - (6*b^2*c*x*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - (6*I)*a*b*ArcSin[c*x]^2 + (3*b^2*ArcSin[c*x]^2)/(1 - c^2*x^2) - (2*I)*b^2*ArcSin[c*x]^3 + 24*a*b*\text{Pi}*Log[1 + E^((-I)*ArcSin[c*x])] + 6*a*b*\text{Pi}*Log[1 - I*E^(I*ArcSin[c*x])] + 12*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 6*a*b*\text{Pi}*Log[1 + I*E^(I*ArcSin[c*x])] + 12*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 6*b^2*ArcSin[c*x]^2*Log[1 + E^((2*I)*ArcSin[c*x])] + 3*a^2*Log[1 - c^2*x^2] - 3*b^2*Log[1 - c^2*x^2] - 24*a*b*\text{Pi}*Log[Cos[ArcSin[c*x]/2]] + 6*a*b*\text{Pi}*Log[-Cos[(\text{Pi} + 2*ArcSin[c*x])/4]] - 6*a*b*\text{Pi}*Log[\text{Sin}[(\text{Pi} + 2*ArcSin[c*x])/4]] - (12*I)*a*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (12*I)*a*b*PolyLog[2, I*E^(I*ArcSin[c*x])] - (6*I)*b^2*ArcSin[c*x]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] + 3*b^2*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/(6*c^4*d^2)$$

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5206, 27, 5180, 3042, 4202, 2620, 3011, 2720, 5206, 240, 5152, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx \\
& \quad \downarrow \text{5206} \\
& -\frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} - \frac{\int \frac{x(a+b \arcsin(cx))^2}{d(1-c^2x^2)} dx}{c^2d} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow \text{27} \\
& -\frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} - \frac{\int \frac{x(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{c^2d^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow \text{5180} \\
& -\frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} - \frac{\int \frac{cx(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^4d^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow \text{3042} \\
& -\frac{\int (a+b \arcsin(cx))^2 \tan(\arcsin(cx)) d \arcsin(cx)}{c^4d^2} - \frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \\
& \quad \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow \text{4202} \\
& -\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))^2}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^4d^2} - \frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \\
& \quad \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow \text{2620} \\
& -\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib \int (a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2}i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)))}{c^4d^2}}{c^4d^2} \\
& \quad \frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} \\
& \quad \downarrow \text{3011}
\end{aligned}$$

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{2}ib \int \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx}{c^4 d^2}}{\frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d^2(1-c^2x^2)}}{\downarrow 2720}$$

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx}{c^4 d^2}}{\frac{b \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d^2(1-c^2x^2)}}{\downarrow 5206}$$

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx}{c^4 d^2}}{\frac{b \left(-\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{c^2} - \frac{b \int \frac{x}{1-c^2x^2} dx}{c} + \frac{x(a+b \arcsin(cx))}{c^2 \sqrt{1-c^2x^2}} \right)}{cd^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d^2(1-c^2x^2)}}{\downarrow 240}$$

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx}{c^4 d^2}}{\frac{b \left(-\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{c^2} + \frac{x(a+b \arcsin(cx))}{c^2 \sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c^3} \right)}{cd^2} + \frac{x^2(a+b \arcsin(cx))^2}{2c^2 d^2(1-c^2x^2)}}{\downarrow 5152}$$

$$\frac{\frac{i(a+b \arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) dx}{c^4 d^2}}{\frac{x^2(a+b \arcsin(cx))^2}{2c^2 d^2(1-c^2x^2)} - \frac{b \left(-\frac{(a+b \arcsin(cx))^2}{2bc^3} + \frac{x(a+b \arcsin(cx))}{c^2 \sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c^3} \right)}{cd^2}}{\downarrow 7143}$$

$$\frac{\frac{i(a+b\arcsin(cx))^3}{3b} - 2i(ib(\frac{1}{2}i\text{PolyLog}(2, -e^{2i\arcsin(cx)})(a+b\arcsin(cx)) - \frac{1}{4}b\text{PolyLog}(3, -e^{2i\arcsin(cx)})) - \frac{1}{2}i\log(1-c^2x^2))}{2c^2d^2(1-c^2x^2)} - \frac{b\left(-\frac{(a+b\arcsin(cx))^2}{2bc^3} + \frac{x(a+b\arcsin(cx))}{c^2\sqrt{1-c^2x^2}} + \frac{b\log(1-c^2x^2)}{2c^3}\right)}{cd^2}$$

input `Int[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]`

output `(x^2*(a + b*ArcSin[c*x])^2)/(2*c^2*d^2*(1 - c^2*x^2)) - (b*((x*(a + b*ArcSin[c*x]))/(c^2*sqrt[1 - c^2*x^2]) - (a + b*ArcSin[c*x])^2/(2*b*c^3) + (b*log[1 - c^2*x^2])/(2*c^3)))/(c*d^2) - (((I/3)*(a + b*ArcSin[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])^2*log[1 + E^((2*I)*ArcSin[c*x])] + I*b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4)))/(c^4*d^2)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 240 `Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`

rule 2620 `Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 $\text{Int}[\text{Log}[1 + (e_)*((F_)^((c_)*((a_)+(b_)*(x_)))^n)]*((f_)+(g_)*(x_))^m, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^(m - 1)*\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}(((c_)+(d_)*(x_))^m*\text{tan}[(e_)+(f_)*(x_)], x_Symbol) \rightarrow \text{Simp}[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m*(E^(2*I*(e + f*x))/(1 + E^(2*I*(e + f*x))))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 5152 $\text{Int}(((a_)+\text{ArcSin}[c_*(x_)]*(b_))^n/\text{Sqrt}[(d_)+(e_)*(x_)^2], x_Symbol) \rightarrow \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^(n + 1), x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[n, -1]$

rule 5180 $\text{Int}(((a_)+\text{ArcSin}[c_*(x_)]*(b_))^n*(x_)/((d_)+(e_)*(x_)^2), x_Symbol) \rightarrow \text{Simp}[-e^(-1) \text{Subst}[\text{Int}[(a + b*x)^n*\text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5206 $\text{Int}(((a_)+\text{ArcSin}[c_*(x_)]*(b_))^n*((f_)*(x_))^m*((d_)+(e_)*(x_)^2)^p, x_Symbol) \rightarrow \text{Simp}[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + (-\text{Simp}[f^2*((m - 1)/(2*e*(p + 1))) \text{Int}[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p \text{Int}[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IGtQ}[m, 1]$

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{a^2 \left(\frac{1}{4cx+4} + \frac{\ln(cx+1)}{2} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{2} \right)}{d^2} + \frac{b^2 \left(-\frac{i \arcsin(cx)^3}{3} - \frac{(2ic^2x^2 - 2cx\sqrt{-c^2x^2+1} + \arcsin(cx) - 2i) \arcsin(cx)}{2(c^2x^2-1)} + \arcsin(cx) \right)}{2(c^2x^2-1)}$
default	$\frac{a^2 \left(\frac{1}{4cx+4} + \frac{\ln(cx+1)}{2} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{2} \right)}{d^2} + \frac{b^2 \left(-\frac{i \arcsin(cx)^3}{3} - \frac{(2ic^2x^2 - 2cx\sqrt{-c^2x^2+1} + \arcsin(cx) - 2i) \arcsin(cx)}{2(c^2x^2-1)} + \arcsin(cx) \right)}{2(c^2x^2-1)}$
parts	$\frac{a^2 \left(-\frac{1}{4c^4(cx-1)} + \frac{\ln(cx-1)}{2c^4} + \frac{1}{4c^4(cx+1)} + \frac{\ln(cx+1)}{2c^4} \right)}{d^2} + \frac{b^2 \left(-\frac{i \arcsin(cx)^3}{3} - \frac{(2ic^2x^2 - 2cx\sqrt{-c^2x^2+1} + \arcsin(cx) - 2i) \arcsin(cx)}{2(c^2x^2-1)} + \arcsin(cx) \right)}{2(c^2x^2-1)}$

input

```
int(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c^4*(a^2/d^2*(1/4/(c*x+1)+1/2*ln(c*x+1)-1/4/(c*x-1)+1/2*ln(c*x-1))+b^2/d
^2*(-1/3*I*arcsin(c*x)^3-1/2*(2*I*c^2*x^2-2*c*x*(-c^2*x^2+1)^(1/2)+arcsin(
c*x)-2*I)*arcsin(c*x)/(c^2*x^2-1)+arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(
1/2))^2)-I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*polylo
g(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*ln(I*c*x+(-c^2*x^2+1)^(1/2))-ln(1+(I*
c*x+(-c^2*x^2+1)^(1/2))^2))+2*a*b/d^2*(-1/2*I*arcsin(c*x)^2-1/2*(I*c^2*x^2
-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x)-I)/(c^2*x^2-1)+arcsin(c*x)*ln(1+(I*c*x
+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^3}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{a^2 x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b^2 x^3 \arcsin^2(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{2abx^3 \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input `integrate(x**3*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a**2*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**3*asin(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^3}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output

```
-1/2*a^2*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2)) - 1/2*(b
^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 - (b^2*c^2*x^2 - b^2)*arct
an2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - (b^2*c^2*x^2 - b^2
)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*(c^6*d^2*
x^2 - c^4*d^2)*integrate((2*a*b*c^3*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c
*x + 1)) - (b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (b^2*c^2*x^2
- b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - (b^2*c^2*
x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*sqrt(
c*x + 1)*sqrt(-c*x + 1))/(c^7*d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2), x))/(c^6
*d^2*x^2 - c^4*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^2} dx$$

input

```
int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2,x)
```

output

```
int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^6 x^2 - 4 \left(\int \frac{\arcsin(cx)x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^4 + 2 \left(\int \frac{\arcsin(cx)^2 x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2 c^6 x^2 - 2 \left(\int \frac{\arcsin(cx)^2 x^3}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^4}{2c^4 d^2 (c^2 x^2 - 1)}$$

input

```
int(x^3*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output

```
(4*int((asin(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**6*x**2 - 4
*int((asin(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**4 + 2*int((a
sin(c*x)**2*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**6*x**2 - 2*int(
(asin(c*x)**2*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**4 + log(c**2*
x - c)*a**2*c**2*x**2 - log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2*c**2*x
**2 - log(c**2*x + c)*a**2 - a**2*c**2*x**2)/(2*c**4*d**2*(c**2*x**2 - 1))
```

3.191 $\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$

Optimal result	1801
Mathematica [A] (verified)	1802
Rubi [A] (verified)	1802
Maple [A] (verified)	1806
Fricas [F]	1807
Sympy [F]	1807
Maxima [F]	1808
Giac [F]	1808
Mupad [F(-1)]	1809
Reduce [F]	1809

Optimal result

Integrand size = 27, antiderivative size = 233

$$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx = -\frac{b(a+b \arcsin(cx))}{c^3d^2\sqrt{1-c^2x^2}} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)} + \frac{i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{c^3d^2} + \frac{b^2 \operatorname{arctanh}(cx)}{c^3d^2} - \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^3d^2} + \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^3d^2} + \frac{b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{c^3d^2} - \frac{b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{c^3d^2}$$

output

```
-b*(a+b*arcsin(c*x))/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsin(c*x))^2/c^2/d^2/(-c^2*x^2+1)+I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^3/d^2+b^2*arctanh(c*x)/c^3/d^2-I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^2+I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^2+b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^2-b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^2
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.96

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{2ab\sqrt{1-c^2x^2}}{-1+cx} - \frac{2ab\sqrt{1-c^2x^2}}{1+cx} - \frac{2a^2cx}{-1+c^2x^2} + 4b^2 \coth^{-1}(cx) + 2iab\pi \arcsin(cx) - \frac{2ab \arcsin(cx)}{-1+cx} - \frac{2ab \arcsin(cx)}{1+cx} - \frac{4b^2 a}{\sqrt{1-c^2x^2}}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output

```
((2*a*b*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (2*a*b*Sqrt[1 - c^2*x^2])/(1 + c*x)
) - (2*a^2*c*x)/(-1 + c^2*x^2) + 4*b^2*ArcCoth[c*x] + (2*I)*a*b*Pi*ArcSin[
c*x] - (2*a*b*ArcSin[c*x])/(-1 + c*x) - (2*a*b*ArcSin[c*x])/(1 + c*x) - (4
*b^2*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + (2*b^2*c*x*ArcSin[c*x]^2)/(1 - c^2*x
^2) + (4*I)*b^2*ArcSin[c*x]^2*ArcTan[E^(I*ArcSin[c*x])] - 2*a*b*Pi*Log[1 -
I*E^(I*ArcSin[c*x])] - 4*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 2
*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 4*a*b*ArcSin[c*x]*Log[1 + I*E^(I*Ar
cSin[c*x])] + a^2*Log[1 - c*x] - a^2*Log[1 + c*x] + 2*a*b*Pi*Log[-Cos[(Pi
+ 2*ArcSin[c*x])/4]] + 2*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (4*I)*b
*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*b*(a + b*A
rcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] + 4*b^2*PolyLog[3, (-I)*E^(I*A
rcSin[c*x])] - 4*b^2*PolyLog[3, I*E^(I*ArcSin[c*x])])]/(4*c^3*d^2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5206, 27, 5164, 3042, 4669, 3011, 2720, 5182, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

↓ 5206

$$-\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} - \frac{\int \frac{(a+b \arcsin(cx))^2}{d(1-c^2x^2)} dx}{2c^2d} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 27

$$-\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} - \frac{\int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{2c^2d^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 5164

$$-\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} - \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c^3d^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 3042

$$-\frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c^3d^2} - \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 4669

$$-\frac{-2b \int (a+b \arcsin(cx)) \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx)}{2c^3d^2}$$

$$+\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 3011

$$-\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) d \arcsin(cx))}{2c^3d^2}$$

$$+\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 2720

$$-\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c^3d^2}$$

$$+\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{cd^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2d^2(1-c^2x^2)}$$

↓ 5182

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b \int \frac{b \left(\frac{a+b \arcsin(cx)}{c^2 \sqrt{1-c^2 x^2}} - \frac{b \int \frac{1}{1-c^2 x^2} dx}{c} \right)}{cd^2} + \frac{x(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)}}{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b \int \frac{b \left(\frac{a+b \arcsin(cx)}{c^2 \sqrt{1-c^2 x^2}} - \frac{\operatorname{barctanh}(cx)}{c^2} \right)}{cd^2} + \frac{x(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)}}{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(3, e^{i \arcsin(cx)})) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(3, e^{i \arcsin(cx)})} + \frac{b \left(\frac{a+b \arcsin(cx)}{c^2 \sqrt{1-c^2 x^2}} - \frac{\operatorname{barctanh}(cx)}{c^2} \right)}{cd^2} + \frac{x(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)}}$$

input

```
Int[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output

```
(x*(a + b*ArcSin[c*x])^2)/(2*c^2*d^2*(1 - c^2*x^2)) - (b*((a + b*ArcSin[c*x])/(c^2*Sqrt[1 - c^2*x^2]) - (b*ArcTanh[c*x])/c^2))/(c*d^2) - ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])]))/(2*c^3*d^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 219 $\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

rule 2720 $\text{Int}[u, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \ \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w) \cdot ((a \cdot v)^n)^m] /; \text{FreeQ}\{a, m, n\}, x \ \&\& \ \text{IntegerQ}[m \cdot n] \ \&\& \ !\text{MatchQ}[u, E^{(c \cdot v) \cdot (a \cdot v) \cdot x}] \cdot (F)[v] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{InverseFunctionQ}[F[x]]$

rule 3011 $\text{Int}[\text{Log}[1 + (e \cdot v)^{(c \cdot (a \cdot v) + (b \cdot x))}]^n] \cdot ((f \cdot v) + (g \cdot x))^m, x_Symbol] \rightarrow \text{Simp}[-(f + g \cdot x)^m \cdot (\text{PolyLog}[2, (-e) \cdot (F^{(c \cdot (a + b \cdot x))})^n] / (b \cdot c \cdot n \cdot \text{Log}[F]))], x] + \text{Simp}[g \cdot m / (b \cdot c \cdot n \cdot \text{Log}[F]) \ \text{Int}[(f + g \cdot x)^{m-1} \cdot \text{PolyLog}[2, (-e) \cdot (F^{(c \cdot (a + b \cdot x))})^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x \ \&\& \ \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}(e + \text{Pi} \cdot k + (f \cdot x) \cdot (c + (d \cdot x)^m), x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}]/f), x] + (-\text{Simp}[d \cdot m / f \ \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 - E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}], x], x] + \text{Simp}[d \cdot m / f \ \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IntegerQ}[2 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a + \text{ArcSin}(c \cdot x) \cdot (b \cdot x))^n / ((d + (e \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[1/(c \cdot d) \ \text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Sec}[x], x], x, \text{ArcSin}[c \cdot x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2 \cdot d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a + \text{ArcSin}(c \cdot x) \cdot (b \cdot x))^n \cdot (d + (e \cdot x)^2)^p, x_Symbol] \rightarrow \text{Simp}[(d + e \cdot x^2)^{p+1} \cdot ((a + b \cdot \text{ArcSin}[c \cdot x])^n / (2 \cdot e \cdot (p + 1))), x] + \text{Simp}[b \cdot n / (2 \cdot c \cdot (p + 1)) \cdot \text{Simp}[(d + e \cdot x^2)^p / (1 - c^2 \cdot x^2)^p] \ \text{Int}[(1 - c^2 \cdot x^2)^{p+1/2} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n-1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \ \text{EqQ}[c^2 \cdot d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{a^2 \left(-\frac{1}{4(cx+1)} - \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(cx \arcsin(cx) - 2\sqrt{-c^2x^2+1}}{2(c^2x^2-1)} - \frac{\arcsin(cx)^2 \ln(1-i(ix+\sqrt{-c^2x^2+1}))}{2} \right)}{d^2}$
default	$\frac{a^2 \left(-\frac{1}{4(cx+1)} - \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} + \frac{\ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(cx \arcsin(cx) - 2\sqrt{-c^2x^2+1}}{2(c^2x^2-1)} - \frac{\arcsin(cx)^2 \ln(1-i(ix+\sqrt{-c^2x^2+1}))}{2} \right)}{d^2}$
parts	$\frac{a^2 \left(-\frac{1}{4c^3(cx-1)} + \frac{\ln(cx-1)}{4c^3} - \frac{1}{4c^3(cx+1)} - \frac{\ln(cx+1)}{4c^3} \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(cx \arcsin(cx) - 2\sqrt{-c^2x^2+1}}{2(c^2x^2-1)} - \frac{\arcsin(cx)^2 \ln(1-i(ix+\sqrt{-c^2x^2+1}))}{2} \right)}{d^2}$

input

```
int(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c^3*(a^2/d^2*(-1/4/(c*x+1)-1/4*ln(c*x+1)-1/4/(c*x-1)+1/4*ln(c*x-1))+b^2/
d^2*(-1/2/(c^2*x^2-1)*arcsin(c*x)*(c*x*arcsin(c*x)-2*(-c^2*x^2+1)^(1/2))-1
/2*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*arcsin(c*x)*polylog(
2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/
2*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*arcsin(c*x)*polylog(2
,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2
*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2)))+2*a*b/d^2*(-1/2*(c*x*arcsin(c*x)-(-c^
2*x^2+1)^(1/2))/(c^2*x^2-1)+1/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/
2)))-1/2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/2*I*dilog(1+I*(I
*c*x+(-c^2*x^2+1)^(1/2)))+1/2*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{a^2 x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b^2 x^2 \arcsin^2(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{2abx^2 \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input

```
integrate(x**2*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output

```
(Integral(a**2*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**2
*asin(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*asin
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/4*a^2*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2)) - 1/4*(2*b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) + 4*(c^5*d^2*x^2 - c^3*d^2)*integrate(-1/2*(4*a*b*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (2*b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x))/(c^5*d^2*x^2 - c^3*d^2)`

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)^2*x^2/(c^2*d*x^2 - d)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{x^2(a + b \sin(cx))^2}{(d - c^2 dx^2)^2} dx$$

input `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2,x)`

output `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{8 \left(\int \frac{\arcsin(cx)x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^5 x^2 - 8 \left(\int \frac{\arcsin(cx)x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^3 + 4 \left(\int \frac{\arcsin(cx)^2 x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2 c^5 x^2 - 4 \left(\int \frac{\arcsin(cx)^2 x^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2 c^3}{4c^3 d^2 (c^2 x^2 - d)}$$

input `int(x^2*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

output `(8*int((asin(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**5*x**2 - 8*int((asin(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**3 + 4*int((asin(c*x)**2*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**5*x**2 - 4*int((asin(c*x)**2*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**3 + log(c**2*x - c)*a**2*c**2*x**2 - log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2*c**2*x**2 + log(c**2*x + c)*a**2 - 2*a**2*c*x)/(4*c**3*d**2*(c**2*x**2 - 1))`

3.192 $\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$

Optimal result	1810
Mathematica [A] (verified)	1810
Rubi [A] (verified)	1811
Maple [B] (verified)	1812
Fricas [A] (verification not implemented)	1813
Sympy [F]	1813
Maxima [B] (verification not implemented)	1814
Giac [B] (verification not implemented)	1814
Mupad [F(-1)]	1815
Reduce [F]	1815

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2dx^2)^2} dx = -\frac{bx(a + b \arcsin(cx))}{cd^2\sqrt{1 - c^2x^2}} + \frac{(a + b \arcsin(cx))^2}{2c^2d^2(1 - c^2x^2)} - \frac{b^2 \log(1 - c^2x^2)}{2c^2d^2}$$

output `-b*x*(a+b*arcsin(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arcsin(c*x))^2/c^2/d^2/(-c^2*x^2+1)-1/2*b^2*ln(-c^2*x^2+1)/c^2/d^2`

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2dx^2)^2} dx = -\frac{\frac{2bcx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{(a+b \arcsin(cx))^2}{-1+c^2x^2} + b^2 \log(1 - c^2x^2)}{2c^2d^2}$$

input `Integrate[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]`

output `-1/2*((2*b*c*x*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (a + b*ArcSin[c*x])^2/(-1 + c^2*x^2) + b^2*Log[1 - c^2*x^2])/(c^2*d^2)`

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5182, 5160, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$\downarrow \text{5182}$$

$$\frac{(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b \int \frac{a + b \arcsin(cx)}{(1 - c^2 x^2)^{3/2}} dx}{cd^2}$$

$$\downarrow \text{5160}$$

$$\frac{(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b \left(\frac{x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} - bc \int \frac{x}{1 - c^2 x^2} dx \right)}{cd^2}$$

$$\downarrow \text{240}$$

$$\frac{(a + b \arcsin(cx))^2}{2c^2 d^2 (1 - c^2 x^2)} - \frac{b \left(\frac{x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} + \frac{b \log(1 - c^2 x^2)}{2c} \right)}{cd^2}$$

input

```
Int[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output

```
(a + b*ArcSin[c*x])^2/(2*c^2*d^2*(1 - c^2*x^2)) - (b*((x*(a + b*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)))/(c*d^2)
```


Defintions of rubi rules used

rule 240 `Int[(x_)/((a_) + (b_)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`

rule 5160 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2)^(3/2), x_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*((a + b*ArcSin[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5182 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

method	result
derivativedivides	$-\frac{a^2}{2d^2(c^2x^2-1)} + \frac{b^2 \left(-\frac{\arcsin(cx)^2}{2(c^2x^2-1)} + \frac{\sqrt{-c^2x^2+1} cx \arcsin(cx)}{c^2x^2-1} - \frac{\ln(-c^2x^2+1)}{2} \right)}{d^2} + \frac{2ab \left(-\frac{\arcsin(cx)}{2(c^2x^2-1)} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{4cx-4} \right)}{d^2} + \frac{c^2}{d^2}$
default	$-\frac{a^2}{2d^2(c^2x^2-1)} + \frac{b^2 \left(-\frac{\arcsin(cx)^2}{2(c^2x^2-1)} + \frac{\sqrt{-c^2x^2+1} cx \arcsin(cx)}{c^2x^2-1} - \frac{\ln(-c^2x^2+1)}{2} \right)}{d^2} + \frac{2ab \left(-\frac{\arcsin(cx)}{2(c^2x^2-1)} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{4cx-4} \right)}{d^2} + \frac{c^2}{d^2}$
parts	$-\frac{a^2}{2d^2c^2(c^2x^2-1)} + \frac{b^2 \left(-\frac{\arcsin(cx)^2}{2(c^2x^2-1)} + \frac{\sqrt{-c^2x^2+1} cx \arcsin(cx)}{c^2x^2-1} - \frac{\ln(-c^2x^2+1)}{2} \right)}{d^2c^2} + \frac{2ab \left(-\frac{\arcsin(cx)}{2(c^2x^2-1)} + \frac{\sqrt{-(cx-1)^2-2cx+2}}{4cx-4} \right)}{d^2c^2}$

input `int(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output

```
1/c^2*(-1/2*a^2/d^2/(c^2*x^2-1)+b^2/d^2*(-1/2*arcsin(c*x)^2/(c^2*x^2-1)+(-
c^2*x^2+1)^(1/2)/(c^2*x^2-1)*c*x*arcsin(c*x)-1/2*ln(-c^2*x^2+1))+2*a*b/d^2
*(-1/2/(c^2*x^2-1)*arcsin(c*x)+1/4/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)+1/4/
(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \frac{b^2 \arcsin(cx)^2 + 2ab \arcsin(cx) + a^2 + (b^2 c^2 x^2 - b^2) \log(c^2 x^2 - 1) - 2(b^2 cx \arcsin(cx) + abcx) \sqrt{-c^2 x^2 + 1}}{2(c^4 d^2 x^2 - c^2 d^2)}$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output

```
-1/2*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2 + (b^2*c^2*x^2 - b^2)*lo
g(c^2*x^2 - 1) - 2*(b^2*c*x*arcsin(c*x) + a*b*c*x)*sqrt(-c^2*x^2 + 1))/(c^
4*d^2*x^2 - c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{a^2 x}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b^2 x \arcsin^2(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{2abx \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input

```
integrate(x*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output

```
(Integral(a**2*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x*asin(
c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*asin(c*x)/(c*
**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(82) = 164$.

Time = 0.17 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{1}{2} \left(\left(\frac{\sqrt{-c^2 x^2 + 1} c^2 d^2}{c^7 d^4 x + c^6 d^4} + \frac{\sqrt{-c^2 x^2 + 1} c^2 d^2}{c^7 d^4 x - c^6 d^4} \right) c^2 - \frac{2 \arcsin(cx)}{c^4 d^2 x^2 - c^2 d^2} \right) ab$$

$$- \frac{1}{2} \left(c^3 \left(\frac{\log(cx + 1)}{c^5 d^2} + \frac{\log(cx - 1)}{c^5 d^2} \right) - \left(\frac{\sqrt{-c^2 x^2 + 1} c^2 d^2}{c^7 d^4 x + c^6 d^4} + \frac{\sqrt{-c^2 x^2 + 1} c^2 d^2}{c^7 d^4 x - c^6 d^4} \right) c^2 \arcsin(cx) \right) b^2$$

$$- \frac{b^2 \arcsin(cx)^2}{2(c^4 d^2 x^2 - c^2 d^2)} - \frac{a^2}{2(c^4 d^2 x^2 - c^2 d^2)}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `1/2*((sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 - 2*arcsin(c*x)/(c^4*d^2*x^2 - c^2*d^2))*a*b - 1/2*(c^3*(log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) - (sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2*arcsin(c*x))*b^2 - 1/2*b^2*arcsin(c*x)^2/(c^4*d^2*x^2 - c^2*d^2) - 1/2*a^2/(c^4*d^2*x^2 - c^2*d^2)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(82) = 164$.

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = -\frac{b^2 x^2 \arcsin(cx)^2}{2(c^2 x^2 - 1)d^2} - \frac{abx^2 \arcsin(cx)}{(c^2 x^2 - 1)d^2} - \frac{a^2 x^2}{2(c^2 x^2 - 1)d^2}$$

$$- \frac{b^2 x \arcsin(cx)}{\sqrt{-c^2 x^2 + 1} cd^2} + \frac{b^2 \arcsin(cx)^2}{2c^2 d^2} - \frac{abx}{\sqrt{-c^2 x^2 + 1} cd^2}$$

$$+ \frac{ab \arcsin(cx)}{c^2 d^2} - \frac{b^2 \log(2)}{c^2 d^2} - \frac{b^2 \log(|-c^2 x^2 + 1|)}{2c^2 d^2} + \frac{a^2}{2c^2 d^2}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output
$$-1/2*b^2*x^2*arcsin(c*x)^2/((c^2*x^2 - 1)*d^2) - a*b*x^2*arcsin(c*x)/((c^2*x^2 - 1)*d^2) - 1/2*a^2*x^2/((c^2*x^2 - 1)*d^2) - b^2*x*arcsin(c*x)/(sqrt(-c^2*x^2 + 1)*c*d^2) + 1/2*b^2*arcsin(c*x)^2/(c^2*d^2) - a*b*x/(sqrt(-c^2*x^2 + 1)*c*d^2) + a*b*arcsin(c*x)/(c^2*d^2) - b^2*log(2)/(c^2*d^2) - 1/2*b^2*log(abs(-c^2*x^2 + 1))/(c^2*d^2) + 1/2*a^2/(c^2*d^2)$$

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{x(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^2} dx$$

input `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2,x)`

output `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2, x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \frac{4 \left(\int \frac{\operatorname{asin}(cx)x}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^2 x^2 - 4 \left(\int \frac{\operatorname{asin}(cx)x}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab + 2 \left(\int \frac{\operatorname{asin}(cx)^2 x}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2 c^2 x^2 - 2 \left(\int \frac{\operatorname{asin}(cx)^2 x}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) d}{2d^2 (c^2 x^2 - 1)}$$

input `int(x*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

output
$$(4*\operatorname{int}((\operatorname{asin}(c*x)*x)/(c^{**4}*x^{**4} - 2*c^{**2}*x^{**2} + 1),x)*a*b*c^{**2}*x^{**2} - 4*\operatorname{int}((\operatorname{asin}(c*x)*x)/(c^{**4}*x^{**4} - 2*c^{**2}*x^{**2} + 1),x)*a*b + 2*\operatorname{int}((\operatorname{asin}(c*x)**2*x)/(c^{**4}*x^{**4} - 2*c^{**2}*x^{**2} + 1),x)*b^{**2}*c^{**2}*x^{**2} - 2*\operatorname{int}((\operatorname{asin}(c*x)**2*x)/(c^{**4}*x^{**4} - 2*c^{**2}*x^{**2} + 1),x)*b^{**2} - a^{**2}*x^{**2})/(2*d^{**2}*(c^{**2}*x^{**2} - 1))$$

3.193 $\int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^2} dx$

Optimal result	1816
Mathematica [A] (verified)	1817
Rubi [A] (verified)	1817
Maple [A] (verified)	1821
Fricas [F]	1822
Sympy [F]	1822
Maxima [F]	1823
Giac [F(-2)]	1823
Mupad [F(-1)]	1824
Reduce [F]	1824

Optimal result

Integrand size = 24, antiderivative size = 230

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2dx^2)^2} dx = -\frac{b(a + b \arcsin(cx))}{cd^2\sqrt{1 - c^2x^2}} + \frac{x(a + b \arcsin(cx))^2}{2d^2(1 - c^2x^2)} - \frac{i(a + b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{cd^2} + \frac{b^2 \operatorname{arctanh}(cx)}{cd^2} + \frac{ib(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{cd^2} - \frac{ib(a + b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{cd^2} - \frac{b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{cd^2} + \frac{b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{cd^2}$$

output

```
-b*(a+b*arcsin(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arcsin(c*x))^2/d^2/(-c^2*x^2+1)-I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c/d^2+b^2*arctanh(c*x)/c/d^2+I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^2-I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^2-b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^2+b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^2
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.99

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= -\frac{2a^2 x}{-1+c^2 x^2} - \frac{a^2 \log(1-cx)}{c} + \frac{a^2 \log(1+cx)}{c} + \frac{2ab \left(\frac{\sqrt{1-c^2 x^2}}{-1+cx} - \frac{\sqrt{1-c^2 x^2}}{1+cx} - i\pi \arcsin(cx) + \frac{\arcsin(cx)}{1-cx} - \frac{\arcsin(cx)}{1+cx} + \pi \log(1 - i e^{i \arcsin(cx)}) \right)}{4d^2}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^2,x]`

output

```
((-2*a^2*x)/(-1 + c^2*x^2) - (a^2*Log[1 - c*x])/c + (a^2*Log[1 + c*x])/c +
(2*a*b*(Sqrt[1 - c^2*x^2]/(-1 + c*x) - Sqrt[1 - c^2*x^2]/(1 + c*x) - I*Pi
*ArcSin[c*x] + ArcSin[c*x]/(1 - c*x) - ArcSin[c*x]/(1 + c*x) + Pi*Log[1 -
I*E^(I*ArcSin[c*x])]) + 2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + Pi*Log
[1 + I*E^(I*ArcSin[c*x])] - 2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - P
i*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]]
+ (2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (2*I)*PolyLog[2, I*E^(I*ArcSi
n[c*x])]))/c + (4*b^2*(ArcCoth[c*x] - ArcSin[c*x]/Sqrt[1 - c^2*x^2] + (c*x
*ArcSin[c*x]^2)/(2 - 2*c^2*x^2) - I*ArcSin[c*x]^2*ArcTan[E^(I*ArcSin[c*x])
] + I*ArcSin[c*x]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*ArcSin[c*x]*PolyL
og[2, I*E^(I*ArcSin[c*x])] - PolyLog[3, (-I)*E^(I*ArcSin[c*x])] + PolyLog[
3, I*E^(I*ArcSin[c*x])]))/c)/(4*d^2)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5162, 27, 5164, 3042, 4669, 3011, 2720, 5182, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

↓ 5162

$$\begin{aligned}
& -\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{\int \frac{(a+b \arcsin(cx))^2}{d(1-c^2x^2)} dx}{2d} + \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 27 \\
& -\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{\int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx}{2d^2} + \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 5164 \\
& -\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2cd^2} + \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 3042 \\
& -\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2cd^2} + \\
& \quad \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 4669 \\
& \frac{-2b \int (a+b \arcsin(cx)) \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx)}{2cd^2} \\
& \quad \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 3011 \\
& \frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) d \arcsin(cx))}{2cd^2} \\
& \quad \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 2720 \\
& \frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2cd^2} \\
& \quad \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} \\
& \quad \downarrow 5182
\end{aligned}$$

rule 219 $\text{Int}[(a + (b \cdot x^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /;$ $\text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

rule 2720 $\text{Int}[u, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$ $\text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w) \cdot ((a) \cdot (v)^{(n)})^{(m)}] /;$ $\text{FreeQ}\{a, m, n, x\} \ \&\& \ \text{IntegerQ}[m \cdot n] \ \&\& \ !\text{MatchQ}[u, E^{((c) \cdot ((a) + (b) \cdot x))} \cdot (F)[v]] /;$ $\text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{InverseFunctionQ}[F[x]]$

rule 3011 $\text{Int}[\text{Log}[1 + (e) \cdot ((F)^{((c) \cdot ((a) + (b) \cdot x))})^{(n)}] \cdot ((f) + (g) \cdot (x))^{(m)}, x_Symbol] \rightarrow \text{Simp}[(-f + g \cdot x)^m \cdot (\text{PolyLog}[2, (-e) \cdot (F^{(c \cdot (a + b \cdot x))})^n] / (b \cdot c \cdot n \cdot \text{Log}[F]))], x] + \text{Simp}[g \cdot m / (b \cdot c \cdot n \cdot \text{Log}[F]) \text{Int}[(f + g \cdot x)^{(m-1)} \cdot \text{PolyLog}[2, (-e) \cdot (F^{(c \cdot (a + b \cdot x))})^n], x], x] /;$ $\text{FreeQ}\{F, a, b, c, e, f, g, n, x\} \ \&\& \ \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e) + \text{Pi} \cdot (k) + (f) \cdot (x)] \cdot ((c) + (d) \cdot (x))^{(m)}, x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}]/f), x] + (-\text{Simp}[d \cdot m / f \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 - E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}], x], x] + \text{Simp}[d \cdot m / f \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 + E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}], x], x]) /;$ $\text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IntegerQ}[2 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5162 $\text{Int}[(a + \text{ArcSin}[c \cdot (x)] \cdot (b))^{(n)} \cdot ((d) + (e) \cdot (x^2)^{(p)}), x_Symbol] \rightarrow \text{Simp}[(-x) \cdot (d + e \cdot x^2)^{(p+1)} \cdot ((a + b \cdot \text{ArcSin}[c \cdot x])^n / (2 \cdot d \cdot (p+1))), x] + (\text{Simp}[(2 \cdot p + 3) / (2 \cdot d \cdot (p+1)) \text{Int}[(d + e \cdot x^2)^{(p+1)} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n, x], x] + \text{Simp}[b \cdot c \cdot n / (2 \cdot (p+1)) \cdot \text{Simp}[(d + e \cdot x^2)^p / (1 - c^2 \cdot x^2)^p \text{Int}[x \cdot (1 - c^2 \cdot x^2)^{(p+1/2)} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{(n-1)}, x], x]) /;$ $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[c^2 \cdot d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

```
rule 5164 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
  /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]

rule 5182 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.),
  x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x]
  + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*
  (a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]

rule 7143 Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol]
  := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{a^2 \left(-\frac{1}{4(cx+1)} + \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(cx \arcsin(cx) - 2\sqrt{-c^2x^2+1})}{2(c^2x^2-1)} + \frac{\arcsin(cx)^2 \ln(1-i(cx+\sqrt{-c^2x^2+1})/2)}{2} \right)}{d^2}$
default	$\frac{a^2 \left(-\frac{1}{4(cx+1)} + \frac{\ln(cx+1)}{4} - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(cx \arcsin(cx) - 2\sqrt{-c^2x^2+1})}{2(c^2x^2-1)} + \frac{\arcsin(cx)^2 \ln(1-i(cx+\sqrt{-c^2x^2+1})/2)}{2} \right)}{d^2}$
parts	$\frac{a^2 \left(-\frac{1}{4c(cx-1)} - \frac{\ln(cx-1)}{4c} - \frac{1}{4c(cx+1)} + \frac{\ln(cx+1)}{4c} \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(cx \arcsin(cx) - 2\sqrt{-c^2x^2+1})}{2(c^2x^2-1)} + \frac{\arcsin(cx)^2 \ln(1-i(cx+\sqrt{-c^2x^2+1})/2)}{2} \right)}{d^2}$

```
input int((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c*(a^2/d^2*(-1/4/(c*x+1)+1/4*ln(c*x+1)-1/4/(c*x-1)-1/4*ln(c*x-1))+b^2/d^
2*(-1/2/(c^2*x^2-1)*arcsin(c*x)*(c*x*arcsin(c*x)-2*(-c^2*x^2+1)^(1/2))+1/2
*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*arcsin(c*x)*polylog(2,
I*(I*c*x+(-c^2*x^2+1)^(1/2)))+polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/2*
arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*arcsin(c*x)*polylog(2,-
I*(I*c*x+(-c^2*x^2+1)^(1/2)))-polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I
*arctan(I*c*x+(-c^2*x^2+1)^(1/2))+2*a*b/d^2*(-1/2*(c*x*arcsin(c*x)-(-c^2*
x^2+1)^(1/2))/(c^2*x^2-1)-1/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)
))+1/2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/2*I*dilog(1+I*(I*c
*x+(-c^2*x^2+1)^(1/2)))-1/2*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^4 - 2*c^
2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{a^2}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b^2 \operatorname{asin}^2(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{2ab \operatorname{asin}(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input

```
integrate((a+b*asin(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output

```
(Integral(a**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*asin(c*x)
**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*asin(c*x)/(c**4*x**
4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/4*a^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2)) - 1/4*(2*b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 - (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) + (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) - 4*(c^3*d^2*x^2 - c*d^2)*integrate(1/2*(4*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (2*b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) + (b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x))/(c^3*d^2*x^2 - c*d^2)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^2} dx$$

input `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^2,x)`output `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^2, x)`**Reduce [F]**

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{8 \left(\int \frac{\operatorname{asin}(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab c^3 x^2 - 8 \left(\int \frac{\operatorname{asin}(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) abc + 4 \left(\int \frac{\operatorname{asin}(cx)^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2 c^3 x^2 - 4 \left(\int \frac{\operatorname{asin}(cx)^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) abc}{4c d^2 (c^2 x^2 - d)}$$

input `int((a+b*asin(c*x))^2/(-c^2*d*x^2+d)^2,x)`output `(8*int(asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**3*x**2 - 8*int(asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c + 4*int(asin(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**3*x**2 - 4*int(asin(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c - log(c**2*x - c)*a**2*c**2*x**2 + log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2*c**2*x**2 - log(c**2*x + c)*a**2 - 2*a**2*c*x)/(4*c*d**2*(c**2*x**2 - 1))`

3.194 $\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^2} dx$

Optimal result	1825
Mathematica [B] (verified)	1826
Rubi [A] (verified)	1827
Maple [B] (verified)	1831
Fricas [F]	1832
Sympy [F]	1832
Maxima [F]	1832
Giac [F(-2)]	1833
Mupad [F(-1)]	1833
Reduce [F]	1834

Optimal result

Integrand size = 27, antiderivative size = 211

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2dx^2)^2} dx = -\frac{bcx(a + b \arcsin(cx))}{d^2\sqrt{1 - c^2x^2}} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2x^2)} - \frac{2(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^2} - \frac{b^2 \log(1 - c^2x^2)}{2d^2} + \frac{ib(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d^2} - \frac{ib(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d^2} - \frac{b^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{2d^2} + \frac{b^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)})}{2d^2}$$

```
output -b*c*x*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arcsin(c*x))^2/d^2/(-c^2*x^2+1)-2*(a+b*arcsin(c*x))^2*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-1/2*b^2*ln(-c^2*x^2+1)/d^2+I*b*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-I*b*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-1/2*b^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2+1/2*b^2*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 612 vs. $2(211) = 422$.

Time = 1.62 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.90

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx$$

$$= \frac{-\frac{1}{12}ib^2\pi^3 + \frac{a^2}{1-c^2x^2} + \frac{ab\sqrt{1-c^2x^2}}{-1+cx} + \frac{ab\sqrt{1-c^2x^2}}{1+cx} - 4iab\pi \arcsin(cx) + \frac{ab \arcsin(cx)}{1-cx} + \frac{ab \arcsin(cx)}{1+cx} - \frac{2b^2cx \arcsin(cx)}{\sqrt{1-c^2x^2}}}{1}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^2),x]`

output

```
((-1/12*I)*b^2*Pi^3 + a^2/(1 - c^2*x^2) + (a*b*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (a*b*Sqrt[1 - c^2*x^2])/(1 + c*x) - (4*I)*a*b*Pi*ArcSin[c*x] + (a*b*ArcSin[c*x])/(1 - c*x) + (a*b*ArcSin[c*x])/(1 + c*x) - (2*b^2*c*x*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + (b^2*ArcSin[c*x]^2)/(1 - c^2*x^2) + ((4*I)/3)*b^2*ArcSin[c*x]^3 - 8*a*b*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 2*a*b*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 4*a*b*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 2*a*b*Pi*Log[1 + I*E^(I*ArcSin[c*x])] - 4*a*b*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b^2*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] + 4*a*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - 2*b^2*ArcSin[c*x]^2*Log[1 + E^((2*I)*ArcSin[c*x])] + 2*a^2*Log[c*x] - a^2*Log[1 - c^2*x^2] - b^2*Log[1 - c^2*x^2] + 8*a*b*Pi*Log[Cos[ArcSin[c*x]/2]] - 2*a*b*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*a*b*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] + (4*I)*a*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*a*b*PolyLog[2, I*E^(I*ArcSin[c*x])] + (2*I)*b^2*ArcSin[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] + (2*I)*b^2*ArcSin[c*x]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (2*I)*a*b*PolyLog[2, E^((2*I)*ArcSin[c*x])] + b^2*PolyLog[3, E^((-2*I)*ArcSin[c*x])] - b^2*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/(2*d^2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5208, 27, 5160, 240, 5184, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx \\
 & \quad \downarrow \text{5208} \\
 & -\frac{bc \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^{3/2}} dx}{d^2} + \frac{\int \frac{(a+b \arcsin(cx))^2}{dx(1-c^2 x^2)} dx}{d} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2 x^2)} \\
 & \quad \downarrow \text{27} \\
 & -\frac{bc \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^{3/2}} dx}{d^2} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} dx}{d^2} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2 x^2)} \\
 & \quad \downarrow \text{5160} \\
 & -\frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} - bc \int \frac{x}{1-c^2 x^2} dx \right)}{d^2} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} dx}{d^2} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2 x^2)} \\
 & \quad \downarrow \text{240} \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} dx}{d^2} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2 x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} + \frac{b \log(1-c^2 x^2)}{2c} \right)}{d^2} \\
 & \quad \downarrow \text{5184} \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{cx\sqrt{1-c^2 x^2}} d \arcsin(cx)}{d^2} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2 x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} + \frac{b \log(1-c^2 x^2)}{2c} \right)}{d^2} \\
 & \quad \downarrow \text{4919} \\
 & \frac{2 \int (a + b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx)}{d^2} + \frac{(a + b \arcsin(cx))^2}{2d^2(1 - c^2 x^2)} - \\
 & \quad \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} + \frac{b \log(1-c^2 x^2)}{2c} \right)}{d^2} \\
 & \quad \downarrow \text{3042}
 \end{aligned}$$

$$\frac{2 \int (a + b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx)}{d^2} + \frac{(a + b \arcsin(cx))^2}{2d^2 (1 - c^2x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^2}$$

↓ 4671

$$\frac{2(-b \int (a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + b \int (a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx))}{d^2}$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 (1 - c^2x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^2}$$

↓ 3011

$$\frac{2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx)) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, e^{2i \arcsin(cx)}) d \arcsin(cx))}{d^2}$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 (1 - c^2x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^2}$$

↓ 2720

$$\frac{2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{2i \arcsin(cx)} \text{PolyLog}(2, e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)})}{d^2}$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 (1 - c^2x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^2}$$

↓ 7143

$$\frac{2(-\text{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 + b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(3, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(3, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(3, e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{2i \arcsin(cx)} \text{PolyLog}(3, e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)})}{d^2}$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 (1 - c^2x^2)} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^2}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^2),x]
```

output

```
(a + b*ArcSin[c*x])^2/(2*d^2*(1 - c^2*x^2)) - (b*c*((x*(a + b*ArcSin[c*x])
)/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)))/d^2 + (2*(-((a + b*ArcS
in[c*x])^2*ArcTanh[E^((2*I)*ArcSin[c*x])]) + b*((I/2)*(a + b*ArcSin[c*x])*
PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])
)/4) - b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])]) - (b
*PolyLog[3, E^((2*I)*ArcSin[c*x])])/(4)))/d^2
```

Defintions of rubi rules used

rule 277

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 240

```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)(x_)]*((c_.) + (d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)(x_)]^{(n_.)}*((c_.) + (d_.)(x_))^{(m_.)}*\text{Sec}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \text{IntegerQ}[n] \ \&\& \text{RationalQ}[m]$

rule 5160 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}/((d_.) + (e_.)(x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[x*((a + b*\text{ArcSin}[c*x])^n/(d*\text{Sqrt}[d + e*x^2])), x] - \text{Simp}[b*c*(n/d)*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]] \text{Int}[x*((a + b*\text{ArcSin}[c*x])^{(n-1)})/(1 - c^2*x^2)], x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{GtQ}[n, 0]$

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}/((x_)*((d_.) + (e_.)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + b*x)^n/(\text{Cos}[x]*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{IGtQ}[n, 0]$

rule 5208 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(-f*x)^{(m+1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*d*f*(p+1))), x] + (\text{Simp}[(m + 2*p + 3)/(2*d*(p+1)) \text{Int}[(f*x)^m*(d + e*x^2)^{(p+1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*f*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{GtQ}[n, 0] \ \&\& \text{LtQ}[p, -1] \ \&\& \text{!GtQ}[m, 1] \ \&\& (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{EqQ}[n, 1])$

rule 7143 $\text{Int}[\text{PolyLog}[n, (c_.)*((a_.) + (b_.)(x_))^{(p_.)}]/((d_.) + (e_.)(x_)), x_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x \ \&\& \text{EqQ}[b*d, a*e]$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(249) = 498$.

Time = 0.58 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.86

method	result
parts	$\frac{a^2 \left(-\frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{2} + \frac{1}{4cx+4} - \frac{\ln(cx+1)}{2} + \ln(x) \right)}{d^2} + \frac{b^2 \left(-\frac{(2ic^2x^2 - 2cx\sqrt{-c^2x^2+1} + \arcsin(cx) - 2i) \arcsin(cx)}{2(c^2x^2-1)} + 2\ln(i) \right)}{d^2}$
derivativedivides	$\frac{a^2 \left(\frac{1}{4cx+4} - \frac{\ln(cx+1)}{2} + \ln(cx) - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{2} \right)}{d^2} + \frac{b^2 \left(-\frac{(2ic^2x^2 - 2cx\sqrt{-c^2x^2+1} + \arcsin(cx) - 2i) \arcsin(cx)}{2(c^2x^2-1)} + 2\ln(i) \right)}{d^2}$
default	$\frac{a^2 \left(\frac{1}{4cx+4} - \frac{\ln(cx+1)}{2} + \ln(cx) - \frac{1}{4(cx-1)} - \frac{\ln(cx-1)}{2} \right)}{d^2} + \frac{b^2 \left(-\frac{(2ic^2x^2 - 2cx\sqrt{-c^2x^2+1} + \arcsin(cx) - 2i) \arcsin(cx)}{2(c^2x^2-1)} + 2\ln(i) \right)}{d^2}$

input

```
int((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
a^2/d^2*(-1/4/(c*x-1)-1/2*ln(c*x-1)+1/4/(c*x+1)-1/2*ln(c*x+1)+ln(x))+b^2/d^2*(-1/2*(2*I*c^2*x^2-2*c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x)-2*I)*arcsin(c*x)/(c^2*x^2-1)+2*ln(I*c*x+(-c^2*x^2+1)^(1/2))-ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))+2*a*b/d^2*(-1/2*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x)-I)/(c^2*x^2-1)-arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx = \int \frac{a^2}{c^4 x^5 - 2c^2 x^3 + x} dx + \int \frac{b^2 \arcsin^2(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx + \int \frac{2ab \arcsin(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx$$

input `integrate((a+b*asin(c*x))**2/x/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a**2/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b**2*asin(c*x)**2/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(2*a*b*asin(c*x)/(c**4*x**5 - 2*c**2*x**3 + x), x))/d**2`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output

```
-1/2*a^2*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*
log(x)/d^2) + integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2
+ 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/(c^4*d^2*x^5 - 2*c^2*d
^2*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(d - c^2 dx^2)^2} dx$$

input

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^2),x)
```

output

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^2} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx \right) ab c^2 x^2 - 4 \left(\int \frac{\arcsin(cx)}{c^4 x^5 - 2c^2 x^3 + x} dx \right) ab + 2 \left(\int \frac{\arcsin(cx)^2}{c^4 x^5 - 2c^2 x^3 + x} dx \right) b^2 c^2 x^2 - 2 \left(\int \frac{\arcsin(cx)^2}{c^4 x^5 - 2c^2 x^3 + x} dx \right) ab}{1}$$

input

```
int((a+b*asin(c*x))^2/x/(-c^2*d*x^2+d)^2,x)
```

output

```
(4*int(asin(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*a*b*c**2*x**2 - 4*int(asin(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*a*b + 2*int(asin(c*x)**2/(c**4*x**5 - 2*c**2*x**3 + x),x)*b**2*c**2*x**2 - 2*int(asin(c*x)**2/(c**4*x**5 - 2*c**2*x**3 + x),x)*b**2 - log(c**2*x - c)*a**2*c**2*x**2 + log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2*c**2*x**2 + log(c**2*x + c)*a**2 + 2*log(x)*a**2*c**2*x**2 - 2*log(x)*a**2 - a**2*c**2*x**2)/(2*d**2*(c**2*x**2 - 1))
```

3.195 $\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^2} dx$

Optimal result	1835
Mathematica [B] (warning: unable to verify)	1836
Rubi [A] (verified)	1837
Maple [A] (verified)	1845
Fricas [F]	1846
Sympy [F]	1846
Maxima [F]	1846
Giac [F(-2)]	1847
Mupad [F(-1)]	1847
Reduce [F]	1848

Optimal result

Integrand size = 27, antiderivative size = 324

$$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^2} dx = -\frac{bc(a+b \arcsin(cx))}{d^2\sqrt{1-c^2x^2}} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)} + \frac{3c^2x(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} - \frac{3ic(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{d^2} - \frac{4bc(a+b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{d^2} + \frac{b^2c \operatorname{arctanh}(cx)}{d^2} + \frac{2ib^2c \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d^2} + \frac{3ibc(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d^2} - \frac{3ibc(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d^2} - \frac{2ib^2c \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d^2} - \frac{3b^2c \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{d^2} + \frac{3b^2c \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{d^2}$$

output

```
-b*c*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)^(1/2)-(a+b*arcsin(c*x))^2/d^2/x/(-
c^2*x^2+1)+3/2*c^2*x*(a+b*arcsin(c*x))^2/d^2/(-c^2*x^2+1)-3*I*c*(a+b*arcsi
n(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^2-4*b*c*(a+b*arcsin(c*x))*arc
tanh(I*c*x+(-c^2*x^2+1)^(1/2))/d^2+b^2*c*arctanh(c*x)/d^2+2*I*b^2*c*polylo
g(2,-I*c*x+(-c^2*x^2+1)^(1/2))/d^2+3*I*b*c*(a+b*arcsin(c*x))*polylog(2,-I*
(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2-3*I*b*c*(a+b*arcsin(c*x))*polylog(2,I*(I*c
*x+(-c^2*x^2+1)^(1/2)))/d^2-2*I*b^2*c*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/
d^2-3*b^2*c*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2+3*b^2*c*polylog(3
,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1161 vs. $2(324) = 648$.

Time = 8.84 (sec) , antiderivative size = 1161, normalized size of antiderivative = 3.58

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^2),x]
```

output

```

-(a^2/(d^2*x)) - (a^2*c^2*x)/(2*d^2*(-1 + c^2*x^2)) - (3*a^2*c*Log[1 - c*x
])/ (4*d^2) + (3*a^2*c*Log[1 + c*x])/ (4*d^2) + (2*a*b*c*((Sqrt[1 - c^2*x^2]
- ArcSin[c*x])/ (4*(-1 + c*x)) - ArcSin[c*x]/(c*x) - (Sqrt[1 - c^2*x^2] +
ArcSin[c*x])/ (4*(1 + c*x)) - ArcTanh[Sqrt[1 - c^2*x^2]] - (3*((3*I)/2)*Pi
*ArcSin[c*x] - (I/2)*ArcSin[c*x]^2 + 2*Pi*Log[1 + E^((-I)*ArcSin[c*x])]) -
Pi*Log[1 + I*E^(I*ArcSin[c*x])]) + 2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x]
)]) - 2*Pi*Log[Cos[ArcSin[c*x]/2]] + Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] -
(2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])]) /4 + (3*((I/2)*Pi*ArcSin[c*x] -
(I/2)*ArcSin[c*x]^2 + 2*Pi*Log[1 + E^((-I)*ArcSin[c*x])]) + Pi*Log[1 - I*E
^(I*ArcSin[c*x])]) + 2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])]) - 2*Pi*Log[
Cos[ArcSin[c*x]/2]] - Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*PolyLog[
2, I*E^(I*ArcSin[c*x])]) /4) /d^2 + (b^2*c*(-4*ArcSin[c*x] - 2*ArcSin[c*x]
^2*Cot[ArcSin[c*x]/2] + 8*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 6*ArcSi
n[c*x]^2*Log[1 - I*E^(I*ArcSin[c*x])]) + 6*Pi*ArcSin[c*x]*Log[(-1)^(1/4)*(
1 - I*E^(I*ArcSin[c*x])]) / (2*E^((I/2)*ArcSin[c*x])]) - 6*ArcSin[c*x]^2*Log
[1 + I*E^(I*ArcSin[c*x])]) - 6*ArcSin[c*x]^2*Log[((1/2 + I/2)*(-I + E^(I*Ar
cSin[c*x])) / E^((I/2)*ArcSin[c*x])]) + 6*Pi*ArcSin[c*x]*Log[-1/2*((-1)^(1/4
))*(-I + E^(I*ArcSin[c*x])) / E^((I/2)*ArcSin[c*x])]) - 8*ArcSin[c*x]*Log[1 +
E^(I*ArcSin[c*x])]) + 6*ArcSin[c*x]^2*Log[((1 + I) + (1 - I)*E^(I*ArcSin[c
*x])) / (2*E^((I/2)*ArcSin[c*x]))]) - 6*Pi*ArcSin[c*x]*Log[-Cos[(Pi + 2*Ar...

```

Rubi [A] (verified)

Time = 3.21 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5204, 27, 5162, 5164, 3042, 4669, 3011, 2720, 5182, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx$$

$$\downarrow 5204$$

$$3c^2 \int \frac{(a + b \arcsin(cx))^2}{d^2 (1 - c^2 x^2)^2} dx + \frac{2bc \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^{3/2}} dx}{d^2} - \frac{(a + b \arcsin(cx))^2}{d^2 x (1 - c^2 x^2)}$$

$$\downarrow 27$$

$$\frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}}{d^2}$$

↓ 5162

$$\frac{3c^2 \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right) + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}}{d^2}$$

↓ 5164

$$\frac{3c^2 \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right) + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}}{d^2}$$

↓ 3042

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} + \frac{3c^2 \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right) - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}}{d^2}$$

↓ 4669

$$\frac{3c^2 \left(\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx))}{2c} \right) + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}}{d^2}$$

↓ 3011

$$\frac{3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) d \arcsin(cx))}{2c} \right) + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}}{d^2}$$

↓ 2720

$$3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))(a+b \arcsin(cx))}{2c} \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}$$

↓ 5182

$$3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))(a+b \arcsin(cx))}{2c} \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}$$

↓ 219

$$3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))(a+b \arcsin(cx))}{2c} \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}$$

↓ 5208

$$3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))(a+b \arcsin(cx))}{2c} \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int \frac{1}{1-c^2x^2} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}$$

↓ 219

$$3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))(a+b \arcsin(cx))}{2c} \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} - b \operatorname{arctanh}(cx) \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{d^2x(1-c^2x^2)}$$

↓ 5218

$$\frac{3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c} \right)}{d^2} - \frac{2bc \left(\int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} - b \operatorname{arctanh}(cx) \right) (a+b \arcsin(cx))^2}{d^2 x (1-c^2x^2)}$$

↓ 3042

$$\frac{3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c} \right)}{d^2} - \frac{2bc \left(\int (a+b \arcsin(cx)) \csc(\arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} - b \operatorname{arctanh}(cx) \right) (a+b \arcsin(cx))^2}{d^2 x (1-c^2x^2)}$$

↓ 4671

$$\frac{3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c} \right)}{d^2} - \frac{2bc \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) \right) (a+b \arcsin(cx))^2}{d^2 x (1-c^2x^2)}$$

↓ 2715

$$\frac{3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c} \right)}{d^2} - \frac{2bc \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) \right) (a+b \arcsin(cx))^2}{d^2 x (1-c^2x^2)}$$

↓ 2838

$$\frac{3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c} \right)}{2bc \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)}$$

$$\frac{(a + b \arcsin(cx))^2}{d^2 x (1 - c^2 x^2)}$$

↓ 7143

$$\frac{3c^2 \left(\frac{-2i \operatorname{arctan}(e^{i \arcsin(cx)}) (a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, ie^{i \arcsin(cx)}))}{2c} \right)}{2bc \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)}$$

$$\frac{(a + b \arcsin(cx))^2}{d^2 x (1 - c^2 x^2)}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^2),x]`

output `-((a + b*ArcSin[c*x])^2/(d^2*x*(1 - c^2*x^2))) + (2*b*c*((a + b*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])]) - b*ArcTanh[c*x] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])]/d^2 + (3*c^2*((x*(a + b*ArcSin[c*x])^2)/(2*(1 - c^2*x^2)) - b*c*((a + b*ArcSin[c*x])/(c^2*Sqrt[1 - c^2*x^2]) - (b*ArcTanh[c*x])/c^2) + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])]) + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])]))/(2*c))/d^2`

Definitions of rubi rules used

- rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 219 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`
- rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]`
- rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`
- rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_)^(m_)), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F]))], x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
  := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[
  d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
  x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
  )]], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
  2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
  d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)
  ^^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IG
  tQ[m, 0]
```

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
  Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p +
  1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
  cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
  *x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
  ]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
  -1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
  /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
  .), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
  1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
  nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
  b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```


rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.87

method	result
derivativedivides	$c \left(\frac{a^2 \left(-\frac{1}{4(cx+1)} + \frac{3 \ln(cx+1)}{4} - \frac{1}{cx} - \frac{1}{4(cx-1)} - \frac{3 \ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{(3c^2 x^2 \arcsin(cx) - 2cx \sqrt{-c^2 x^2 + 1} - 2 \arcsin(cx)) \arcsin(cx)}{2cx(c^2 x^2 - 1)} \right)}{d^2} \right)$
default	$c \left(\frac{a^2 \left(-\frac{1}{4(cx+1)} + \frac{3 \ln(cx+1)}{4} - \frac{1}{cx} - \frac{1}{4(cx-1)} - \frac{3 \ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{(3c^2 x^2 \arcsin(cx) - 2cx \sqrt{-c^2 x^2 + 1} - 2 \arcsin(cx)) \arcsin(cx)}{2cx(c^2 x^2 - 1)} \right)}{d^2} \right)$
parts	$\frac{a^2 \left(-\frac{c}{4(cx-1)} - \frac{3c \ln(cx-1)}{4} - \frac{c}{4(cx+1)} + \frac{3c \ln(cx+1)}{4} - \frac{1}{x} \right)}{d^2} + \frac{b^2 c \left(-\frac{(3c^2 x^2 \arcsin(cx) - 2cx \sqrt{-c^2 x^2 + 1} - 2 \arcsin(cx)) \arcsin(cx)}{2cx(c^2 x^2 - 1)} \right)}{d^2}$

input `int((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output `c*(a^2/d^2*(-1/4/(c*x+1)+3/4*ln(c*x+1)-1/c/x-1/4/(c*x-1)-3/4*ln(c*x-1))+b^2/d^2*(-1/2/c/x/(c^2*x^2-1)*(3*c^2*x^2*arcsin(c*x)-2*c*x*(-c^2*x^2+1)^(1/2)-2*arcsin(c*x))*arcsin(c*x)+3/2*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*dilog(I*c*x+(-c^2*x^2+1)^(1/2))+2*I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2))-3/2*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3*I*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2))))+2*a*b/d^2*(-1/2*(3*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-2*arcsin(c*x))/c/x/(c^2*x^2-1)+ln(I*c*x+(-c^2*x^2+1)^(1/2))-1)-ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-3/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/2*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/2*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/2*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))`

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{a^2}{c^4 x^6 - 2c^2 x^4 + x^2} dx + \int \frac{b^2 \arcsin^2(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx + \int \frac{2ab \arcsin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx$$

input `integrate((a+b*asin(c*x))**2/x**2/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a**2/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b**2*asin(c*x)**2/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(2*a*b*asin(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output

```
-1/4*a^2*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x + 1)/d^2 +
3*c*log(c*x - 1)/d^2) + 1/4*(3*(b^2*c^3*x^3 - b^2*c*x)*arctan2(c*x, sqrt(
c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - 3*(b^2*c^3*x^3 - b^2*c*x)*arctan
2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*(3*b^2*c^2*x^2 -
2*b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 4*(c^2*d^2*x^3 - d^2
*x)*integrate(1/2*(4*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (3*(
b^2*c^4*x^4 - b^2*c^2*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(
c*x + 1) - 3*(b^2*c^4*x^4 - b^2*c^2*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-
c*x + 1))*log(-c*x + 1) - 2*(3*b^2*c^3*x^3 - 2*b^2*c*x)*arctan2(c*x, sqrt(
c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d^2*x^6 - 2*c
^2*d^2*x^4 + d^2*x^2), x))/(c^2*d^2*x^3 - d^2*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 (d - c^2 dx^2)^2} dx$$

input

```
int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^2),x)
```

output

```
int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^2} dx$$

$$= \frac{8 \left(\int \frac{\arcsin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx \right) ab c^2 x^3 - 8 \left(\int \frac{\arcsin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx \right) abx + 4 \left(\int \frac{\arcsin(cx)^2}{c^4 x^6 - 2c^2 x^4 + x^2} dx \right) b^2 c^2 x^3 - 4 \left(\int \frac{\arcsin(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} dx \right)}$$

input

```
int((a+b*asin(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x)
```

output

```
(8*int(asin(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*a*b*c**2*x**3 - 8*int(
asin(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*a*b*x + 4*int(asin(c*x)**2/
(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b**2*c**2*x**3 - 4*int(asin(c*x)**2/(c
**4*x**6 - 2*c**2*x**4 + x**2),x)*b**2*x - 3*log(c**2*x - c)*a**2*c**3*x**
3 + 3*log(c**2*x - c)*a**2*c*x + 3*log(c**2*x + c)*a**2*c**3*x**3 - 3*log(
c**2*x + c)*a**2*c*x - 6*a**2*c**2*x**2 + 4*a**2)/(4*d**2*x*(c**2*x**2 - 1
))
```

3.196 $\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^2} dx$

Optimal result	1849
Mathematica [B] (verified)	1850
Rubi [A] (verified)	1851
Maple [B] (verified)	1857
Fricas [F]	1858
Sympy [F]	1859
Maxima [F]	1859
Giac [F(-2)]	1860
Mupad [F(-1)]	1860
Reduce [F]	1860

Optimal result

Integrand size = 27, antiderivative size = 261

$$\int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^2} dx = -\frac{bc(a+b \arcsin(cx))}{d^2x\sqrt{1-c^2x^2}} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2} + \frac{c^2(a+b \arcsin(cx))^2}{2d^2(1-c^2x^2)} - \frac{4c^2(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^2} + \frac{b^2c^2 \log(x)}{d^2} - \frac{b^2c^2 \log(1-c^2x^2)}{2d^2} + \frac{2ibc^2(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d^2} - \frac{2ibc^2(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d^2} - \frac{b^2c^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{d^2} + \frac{b^2c^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)})}{d^2}$$

output

```
-b*c*(a+b*arcsin(c*x))/d^2/x/(-c^2*x^2+1)^(1/2)-1/2*(a+b*arcsin(c*x))^2/d^2/x^2+1/2*c^2*(a+b*arcsin(c*x))^2/d^2/(-c^2*x^2+1)-4*c^2*(a+b*arcsin(c*x))^2*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2+b^2*c^2*ln(x)/d^2-1/2*b^2*c^2*ln(-c^2*x^2+1)/d^2+2*I*b*c^2*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-2*I*b*c^2*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2-b^2*c^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2+b^2*c^2*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 759 vs. $2(261) = 522$.

Time = 1.50 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.91

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^2} dx$$

$$= \frac{-\frac{a^2}{x^2} + \frac{a^2 c^2}{1 - c^2 x^2} - \frac{2abc\sqrt{1 - c^2 x^2}}{x} + \frac{abc^2\sqrt{1 - c^2 x^2}}{-1 + cx} + \frac{abc^2\sqrt{1 - c^2 x^2}}{1 + cx} - 8iabc^2\pi \arcsin(cx) - \frac{2ab \arcsin(cx)}{x^2} + \frac{abc^2 \arcsin(cx)}{1 - cx}}{d^2}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^2),x]
```

output

```
(-(a^2/x^2) + (a^2*c^2)/(1 - c^2*x^2) - (2*a*b*c*Sqrt[1 - c^2*x^2])/x + (a
*b*c^2*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (a*b*c^2*Sqrt[1 - c^2*x^2])/(1 + c*
x) - (8*I)*a*b*c^2*Pi*ArcSin[c*x] - (2*a*b*ArcSin[c*x])/x^2 + (a*b*c^2*Arc
Sin[c*x])/(1 - c*x) + (a*b*c^2*ArcSin[c*x])/(1 + c*x) - (2*b^2*c^3*x*ArcSi
n[c*x])/Sqrt[1 - c^2*x^2] - (2*b^2*c*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/x - (b
^2*ArcSin[c*x]^2)/x^2 + (b^2*c^2*ArcSin[c*x]^2)/(1 - c^2*x^2) - 16*a*b*c^2
*Pi*Log[1 + E^((-I)*ArcSin[c*x])] - 4*a*b*c^2*Pi*Log[1 - I*E^(I*ArcSin[c*x
])] - 8*a*b*c^2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 4*a*b*c^2*Pi*Lo
g[1 + I*E^(I*ArcSin[c*x])] - 8*a*b*c^2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c
*x])] + 8*a*b*c^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 4*b^2*c^2*A
rcSin[c*x]^2*Log[1 - E^((2*I)*ArcSin[c*x])] - 4*b^2*c^2*ArcSin[c*x]^2*Log[
1 + E^((2*I)*ArcSin[c*x])] + 4*a^2*c^2*Log[x] + 2*b^2*c^2*Log[(c*x)/Sqrt[1
- c^2*x^2]] - 2*a^2*c^2*Log[1 - c^2*x^2] + 16*a*b*c^2*Pi*Log[Cos[ArcSin[c
*x]/2]] - 4*a*b*c^2*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 4*a*b*c^2*Pi*Lo
g[Sin[(Pi + 2*ArcSin[c*x])/4]] + (8*I)*a*b*c^2*PolyLog[2, (-I)*E^(I*ArcSin
[c*x])] + (8*I)*a*b*c^2*PolyLog[2, I*E^(I*ArcSin[c*x])] + (4*I)*b^2*c^2*Ar
cSin[c*x]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (4*I)*a*b*c^2*PolyLog[2, E^
((2*I)*ArcSin[c*x])] - (4*I)*b^2*c^2*ArcSin[c*x]*PolyLog[2, E^((2*I)*ArcSi
n[c*x])] - 2*b^2*c^2*PolyLog[3, -E^((2*I)*ArcSin[c*x])] + 2*b^2*c^2*PolyLo
g[3, E^((2*I)*ArcSin[c*x])])/(2*d^2)
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {5204, 27, 5194, 25, 354, 86, 2009, 5208, 5160, 240, 5184, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^2} dx$$

$$\downarrow 5204$$

$$2c^2 \int \frac{(a + b \arcsin(cx))^2}{d^2 x (1 - c^2 x^2)^2} dx + \frac{bc \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^{3/2}} dx}{d^2} - \frac{(a + b \arcsin(cx))^2}{2d^2 x^2 (1 - c^2 x^2)}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{2c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^2} + \frac{bc \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^{3/2}} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} \\
& \quad \downarrow \text{5194} \\
& \frac{2c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^2} + \frac{bc \left(-bc \int -\frac{1-2c^2x^2}{x(1-c^2x^2)} dx + \frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} \right)}{d^2} - \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} \\
& \quad \downarrow \text{25} \\
& \frac{2c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^2} + \frac{bc \left(bc \int \frac{1-2c^2x^2}{x(1-c^2x^2)} dx + \frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} \right)}{d^2} - \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} \\
& \quad \downarrow \text{354} \\
& \frac{2c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^2} + \frac{bc \left(\frac{1}{2}bc \int \frac{1-2c^2x^2}{x^2(1-c^2x^2)} dx^2 + \frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} \right)}{d^2} - \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} \\
& \quad \downarrow \text{86} \\
& \frac{2c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^2} + \frac{bc \left(\frac{1}{2}bc \int \left(\frac{c^2}{c^2x^2-1} + \frac{1}{x^2} \right) dx^2 + \frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} \right)}{d^2} - \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} \\
& \quad \downarrow \text{2009} \\
& \frac{2c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \\
& \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2} \\
& \quad \downarrow \text{5208} \\
& \frac{2c^2 \left(-bc \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \\
& \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2} \\
& \quad \downarrow \text{5160}
\end{aligned}$$

$$\frac{2c^2 \left(-bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - bc \int \frac{x}{1-c^2x^2} dx \right) + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2}$$

↓ 240

$$\frac{2c^2 \left(\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2}$$

↓ 5184

$$\frac{2c^2 \left(\int \frac{(a+b \arcsin(cx))^2}{cx\sqrt{1-c^2x^2}} d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2}$$

↓ 4919

$$\frac{2c^2 \left(2 \int (a+b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2}$$

↓ 3042

$$\frac{2c^2 \left(2 \int (a+b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2}$$

↓ 4671

$$\frac{2c^2 \left(2(-b \int (a+b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + b \int (a+b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) \right)}{d^2} - \frac{(a+b \arcsin(cx))^2}{2d^2x^2(1-c^2x^2)} + \frac{bc \left(\frac{2c^2x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\log(1-c^2x^2) + \log(x^2)) \right)}{d^2}$$

↓ 3011

$$2c^2 \left(2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx)) - b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 x^2 (1 - c^2 x^2)} + \frac{bc \left(\frac{2c^2 x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{x \sqrt{1 - c^2 x^2}} + \frac{1}{2}bc(\log(1 - c^2 x^2) + \log(x^2)) \right)}{d^2}$$

↓ 2720

$$2c^2 \left(2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right)$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 x^2 (1 - c^2 x^2)} + \frac{bc \left(\frac{2c^2 x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{x \sqrt{1 - c^2 x^2}} + \frac{1}{2}bc(\log(1 - c^2 x^2) + \log(x^2)) \right)}{d^2}$$

↓ 7143

$$2c^2 \left(-\text{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 + b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{2d^2 x^2 (1 - c^2 x^2)} + \frac{bc \left(\frac{2c^2 x(a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} - \frac{a + b \arcsin(cx)}{x \sqrt{1 - c^2 x^2}} + \frac{1}{2}bc(\log(1 - c^2 x^2) + \log(x^2)) \right)}{d^2}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^2),x]`

output `-1/2*(a + b*ArcSin[c*x])^2/(d^2*x^2*(1 - c^2*x^2)) + (b*c*(-((a + b*ArcSin[c*x])/sqrt[1 - c^2*x^2])) + (2*c^2*x*(a + b*ArcSin[c*x]))/sqrt[1 - c^2*x^2] + (b*c*(Log[x^2] + Log[1 - c^2*x^2])/2))/d^2 + (2*c^2*((a + b*ArcSin[c*x])^2/(2*(1 - c^2*x^2)) - b*c*((x*(a + b*ArcSin[c*x]))/sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)) + 2*(-((a + b*ArcSin[c*x])^2*ArcTanh[E^((2*I)*ArcSin[c*x])]) + b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4) - b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])/4)))/d^2`

Defintions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 86 `Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`
- rule 240 `Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`
- rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 $\text{Int}[\text{Log}[1 + (e_.) * ((F_)^((c_.) * (a_.) + (b_.) * (x_)))^{(n_)}] * ((f_.) + (g_.) * (x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n]) / (b*c*n*\text{Log}[F]), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^{m-1} * \text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.) * (x_)] * ((c_.) + (d_.) * (x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m * (\text{ArcTanh}[E^{(I*(e + f*x))}] / f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.) * (x_)]^{(n_)} * ((c_.) + (d_.) * (x_))^{(m_)} * \text{Sec}[(a_.) + (b_.) * (x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m * \text{Csc}[2*a + 2*b*x]^n, x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IntegerQ}[n] \&\& \text{RationalQ}[m]$

rule 5160 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)]^{(n_)} / ((d_.) + (e_.) * (x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[x * ((a + b*\text{ArcSin}[c*x])^n / (d*\text{Sqrt}[d + e*x^2])), x] - \text{Simp}[b*c*(n/d)*\text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] \text{Int}[x * ((a + b*\text{ArcSin}[c*x])^{n-1} / (1 - c^2*x^2)), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)]^{(n_)} / ((x_.) * ((d_.) + (e_.) * (x_)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + b*x)^n / (\text{Cos}[x] * \text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Simp
plifyIntegrand[u/Sqrt[d + e*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.
)*(x_)^2)^(p_)), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.
)*(x_)^2)^(p_)), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(301) = 602$.

Time = 0.74 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.53

method	result
derivativedivides	$c^2 \left(\frac{a^2 \left(\frac{1}{4cx+4} - \ln(cx+1) - \frac{1}{2c^2x^2} + 2 \ln(cx) - \frac{1}{4(cx-1)} - \ln(cx-1) \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(2c^2x^2 \arcsin(cx) - 2cx\sqrt{-c^2x^2+1}}{2c^2x^2(c^2x^2-1)} \right)}{d^2} \right)$
default	$c^2 \left(\frac{a^2 \left(\frac{1}{4cx+4} - \ln(cx+1) - \frac{1}{2c^2x^2} + 2 \ln(cx) - \frac{1}{4(cx-1)} - \ln(cx-1) \right)}{d^2} + \frac{b^2 \left(-\frac{\arcsin(cx)(2c^2x^2 \arcsin(cx) - 2cx\sqrt{-c^2x^2+1}}{2c^2x^2(c^2x^2-1)} \right)}{d^2} \right)$
parts	$\frac{a^2 \left(-\frac{c^2}{4(cx-1)} - c^2 \ln(cx-1) + \frac{c^2}{4cx+4} - c^2 \ln(cx+1) - \frac{1}{2x^2} + 2c^2 \ln(x) \right)}{d^2} + \frac{b^2 c^2 \left(-\frac{\arcsin(cx)(2c^2x^2 \arcsin(cx) - 2cx\sqrt{-c^2x^2+1}}{2c^2x^2(c^2x^2-1)} \right)}{d^2}$

```
input int((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
output c^2*(a^2/d^2*(1/4/(c*x+1)-ln(c*x+1)-1/2/c^2/x^2+2*ln(c*x)-1/4/(c*x-1)-ln(c*x-1))+b^2/d^2*(-1/2/c^2/x^2/(c^2*x^2-1)*arcsin(c*x)*(2*c^2*x^2*arcsin(c*x)-2*c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))-ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-4*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+4*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))+2*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-4*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+4*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))-2*arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))+2*a*b/d^2*(-1/2*(2*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))/c^2/x^2/(c^2*x^2-1)-2*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*arcsin(c*x)*ln(1-I*c*x+(-c^2*x^2+1)^(1/2))-2*I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x^3} dx$$

```
input integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^2} dx = \int \frac{a^2}{c^4 x^7 - 2c^2 x^5 + x^3} dx + \int \frac{b^2 \arcsin^2(cx)}{c^4 x^7 - 2c^2 x^5 + x^3} dx + \int \frac{2ab \arcsin(cx)}{c^4 x^7 - 2c^2 x^5 + x^3} dx$$

input `integrate((a+b*asin(c*x))**2/x**3/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a**2/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b**2*asin(c*x)**2/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(2*a*b*asin(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `-1/2*a^2*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2 + (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)`

output

```
(4*int(asin(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*a*b*c**2*x**4 - 4*int
(asin(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*a*b*x**2 + 2*int(asin(c*x)*
**2/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b**2*c**2*x**4 - 2*int(asin(c*x)**2
/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b**2*x**2 - 2*log(c**2*x - c)*a**2*c*
*4*x**4 + 2*log(c**2*x - c)*a**2*c**2*x**2 - 2*log(c**2*x + c)*a**2*c**4*x
**4 + 2*log(c**2*x + c)*a**2*c**2*x**2 + 4*log(x)*a**2*c**4*x**4 - 4*log(x
)*a**2*c**2*x**2 - 2*a**2*c**4*x**4 + a**2)/(2*d**2*x**2*(c**2*x**2 - 1))
```

$$3.197 \quad \int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^2} dx$$

Optimal result	1863
Mathematica [B] (warning: unable to verify)	1864
Rubi [A] (verified)	1865
Maple [A] (verified)	1873
Fricas [F]	1874
Sympy [F]	1874
Maxima [F]	1874
Giac [F(-1)]	1875
Mupad [F(-1)]	1875
Reduce [F]	1876

Optimal result

Integrand size = 27, antiderivative size = 439

$$\begin{aligned}
 \int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = & -\frac{b^2 c^2}{3d^2 x} - \frac{2bc^3(a + b \arcsin(cx))}{3d^2 \sqrt{1 - c^2 x^2}} \\
 & - \frac{bc(a + b \arcsin(cx))}{3d^2 x^2 \sqrt{1 - c^2 x^2}} - \frac{(a + b \arcsin(cx))^2}{3d^2 x^3 (1 - c^2 x^2)} \\
 & - \frac{5c^2(a + b \arcsin(cx))^2}{3d^2 x (1 - c^2 x^2)} + \frac{5c^4 x (a + b \arcsin(cx))^2}{2d^2 (1 - c^2 x^2)} \\
 & - \frac{5ic^3(a + b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{d^2} \\
 & - \frac{26bc^3(a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{3d^2} \\
 & + \frac{b^2 c^3 \operatorname{arctanh}(cx)}{d^2} + \frac{13ib^2 c^3 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{3d^2} \\
 & + \frac{5ibc^3(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d^2} \\
 & - \frac{5ibc^3(a + b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d^2} \\
 & - \frac{13ib^2 c^3 \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{3d^2} \\
 & - \frac{5b^2 c^3 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{d^2} \\
 & + \frac{5b^2 c^3 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{d^2}
 \end{aligned}$$

output

```

-1/3*b^2*c^2/d^2/x-2/3*b*c^3*(a+b*arcsin(c*x))/d^2/(-c^2*x^2+1)^(1/2)-1/3*
b*c*(a+b*arcsin(c*x))/d^2/x^2/(-c^2*x^2+1)^(1/2)-1/3*(a+b*arcsin(c*x))^2/d
^2/x^3/(-c^2*x^2+1)-5/3*c^2*(a+b*arcsin(c*x))^2/d^2/x/(-c^2*x^2+1)+5/2*c^4
*x*(a+b*arcsin(c*x))^2/d^2/(-c^2*x^2+1)-5*I*c^3*(a+b*arcsin(c*x))^2*arctan
(I*c*x+(-c^2*x^2+1)^(1/2))/d^2-26/3*b*c^3*(a+b*arcsin(c*x))*arctanh(I*c*x+
(-c^2*x^2+1)^(1/2))/d^2+b^2*c^3*arctanh(c*x)/d^2+13/3*I*b^2*c^3*polylog(2,
-I*c*x-(-c^2*x^2+1)^(1/2))/d^2+5*I*b*c^3*(a+b*arcsin(c*x))*polylog(2,-I*(I
*c*x+(-c^2*x^2+1)^(1/2)))/d^2-5*I*b*c^3*(a+b*arcsin(c*x))*polylog(2,I*(I*c
*x+(-c^2*x^2+1)^(1/2)))/d^2-13/3*I*b^2*c^3*polylog(2,I*c*x+(-c^2*x^2+1)^(1
/2))/d^2-5*b^2*c^3*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2+5*b^2*c^3*
polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2

```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1390 vs. $2(439) = 878$.

Time = 10.73 (sec) , antiderivative size = 1390, normalized size of antiderivative = 3.17

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = \text{Too large to display}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^2),x]`

output

```
-1/3*a^2/(d^2*x^3) - (2*a^2*c^2)/(d^2*x) - (a^2*c^4*x)/(2*d^2*(-1 + c^2*x^2)) - (5*a^2*c^3*Log[1 - c*x])/(4*d^2) + (5*a^2*c^3*Log[1 + c*x])/(4*d^2) + (2*a*b*((c^3*(Sqrt[1 - c^2*x^2] - ArcSin[c*x]))/(4*(-1 + c*x)) - (c^4*(Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/(4*(c + c^2*x)) + 2*c^2*(-(ArcSin[c*x]/x) - c*ArcTanh[Sqrt[1 - c^2*x^2]])) - (c*x*Sqrt[1 - c^2*x^2] + 2*ArcSin[c*x] + c^3*x^3*ArcTanh[Sqrt[1 - c^2*x^2]])/(6*x^3) - (5*c^4*(((3*I)/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c - (Pi*Log[1 + I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c + (Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/c)/4 + (5*c^4*(((I/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c + (Pi*Log[1 - I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c - (Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, I*E^(I*ArcSin[c*x])])/c)/4)/d^2 + (b^2*c^3*(-24*ArcSin[c*x] - (6*ArcSin[c*x]^2)/(-1 + c*x) - 4*Cot[ArcSin[c*x]/2] - 26*ArcSin[c*x]^2*Cot[ArcSin[c*x]/2] - 2*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - (c*x*ArcSin[c*x]^2*Csc[ArcSin[c*x]/2]^4)/2 + 104*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 60*ArcSin[c*x]^2*Log[1 - I*E^(I*ArcSin[c*x])] + 60*Pi*ArcSin[c*x]*Log[(-1)^(1/4)*(1 - I*E^(I*ArcSin[c*x]))]/(2*E^((I/2)*ArcSin[c*x])) - 60*ArcSin[c*x]^2*Log[1 + I*E^(I*...
```

Rubi [A] (verified)

Time = 4.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.19, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {5204, 27, 5204, 264, 219, 5162, 5164, 3042, 4669, 3011, 2720, 5182, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx \\
 & \quad \downarrow \text{5204} \\
 & \frac{5}{3} c^2 \int \frac{(a + b \arcsin(cx))^2}{d^2 x^2 (1 - c^2 x^2)^2} dx + \frac{2bc \int \frac{a+b \arcsin(cx)}{x^3 (1-c^2 x^2)^{3/2}} dx}{3d^2} - \frac{(a + b \arcsin(cx))^2}{3d^2 x^3 (1 - c^2 x^2)} \\
 & \quad \downarrow \text{27} \\
 & \frac{5c^2 \int \frac{(a+b \arcsin(cx))^2}{x^2 (1-c^2 x^2)^2} dx}{3d^2} + \frac{2bc \int \frac{a+b \arcsin(cx)}{x^3 (1-c^2 x^2)^{3/2}} dx}{3d^2} - \frac{(a + b \arcsin(cx))^2}{3d^2 x^3 (1 - c^2 x^2)} \\
 & \quad \downarrow \text{5204} \\
 & \frac{5c^2 \left(3c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^2} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} \right)}{3d^2} + \\
 & \frac{2bc \left(\frac{3}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{3/2}} dx + \frac{1}{2} bc \int \frac{1}{x^2(1-c^2 x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2 \sqrt{1-c^2 x^2}} \right)}{3d^2} - \frac{(a + b \arcsin(cx))^2}{3d^2 x^3 (1 - c^2 x^2)} \\
 & \quad \downarrow \text{264} \\
 & \frac{5c^2 \left(3c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^2} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} \right)}{3d^2} + \\
 & \frac{2bc \left(\frac{3}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{3/2}} dx + \frac{1}{2} bc \left(c^2 \int \frac{1}{1-c^2 x^2} dx - \frac{1}{x} \right) - \frac{a+b \arcsin(cx)}{2x^2 \sqrt{1-c^2 x^2}} \right)}{3d^2} - \frac{(a + b \arcsin(cx))^2}{3d^2 x^3 (1 - c^2 x^2)} \\
 & \quad \downarrow \text{219} \\
 & \frac{2bc \left(\frac{3}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2 \sqrt{1-c^2 x^2}} + \frac{1}{2} bc (\operatorname{arctanh}(cx) - \frac{1}{x}) \right)}{3d^2} + \\
 & \frac{5c^2 \left(3c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^2} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2 x^2)} \right)}{3d^2} - \frac{(a + b \arcsin(cx))^2}{3d^2 x^3 (1 - c^2 x^2)}
 \end{aligned}$$

↓ 5162

$$\frac{2bc\left(\frac{3}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\operatorname{carctanh}(cx) - \frac{1}{x})\right)}{3d^2} +$$

$$\frac{5c^2\left(3c^2\left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right) + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))}{x(1-c^2x^2)}\right)}{3d^2}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 5164

$$\frac{2bc\left(\frac{3}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\operatorname{carctanh}(cx) - \frac{1}{x})\right)}{3d^2} +$$

$$5c^2\left(3c^2\left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right) + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))}{x(1-c^2x^2)}\right)$$

$$\frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 3042

$$\frac{2bc\left(\frac{3}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\operatorname{carctanh}(cx) - \frac{1}{x})\right)}{3d^2} +$$

$$5c^2\left(2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx + 3c^2\left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int (a+b \arcsin(cx))^2 \operatorname{csc}(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}\right)\right)$$

$$\frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 4669

$$5c^2\left(3c^2\left(\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{2c}\right)\right)$$

$$\frac{2bc\left(\frac{3}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\operatorname{carctanh}(cx) - \frac{1}{x})\right)}{3d^2} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 3011

$$5c^2\left(3c^2\left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^i \arcsin(cx)) d \arcsin(cx))}{2c}\right)\right)$$

$$\frac{2bc\left(\frac{3}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2}bc(\operatorname{carctanh}(cx) - \frac{1}{x})\right)}{3d^2} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 2720

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) \right)}{3d^2} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 5182

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) \right)}{3d^2} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 219

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) \right)}{3d^2} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 5208

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int \frac{1}{1-c^2x^2} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) \right)}{\frac{3d^2}{3d^2x^3(1-c^2x^2)}} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 219

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} - b \operatorname{arctanh}(cx) \right) - \frac{a+b \arcsin(cx)}{2x^2\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) \right)}{\frac{3d^2}{3d^2x^3(1-c^2x^2)}} - \frac{(a+b \arcsin(cx))^2}{3d^2x^3(1-c^2x^2)}$$

↓ 5218

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} - b \operatorname{arctanh}(cx) \right) - \frac{a+b \arcsin(cx)}{2x^2 \sqrt{1-c^2x^2}} + \frac{1}{2} bc (\operatorname{carctanh}(cx) - \frac{1}{x}) \right)}{\frac{3d^2}{3d^2x^3(1-c^2x^2)} (a+b \arcsin(cx))^2}$$

↓ 3042

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \left(\int (a+b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} - b \operatorname{arctanh}(cx) \right) - \frac{a+b \arcsin(cx)}{2x^2 \sqrt{1-c^2x^2}} + \frac{1}{2} bc (\operatorname{carctanh}(cx) - \frac{1}{x}) \right)}{\frac{3d^2}{3d^2x^3(1-c^2x^2)} (a+b \arcsin(cx))^2}$$

↓ 4671

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{\frac{3d^2}{3d^2x^3(1-c^2x^2)} (a+b \arcsin(cx))^2}$$

↓ 2715

$$5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\frac{3}{2} c^2 \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{\frac{3d^2}{3d^2x^3(1-c^2x^2)} (a+b \arcsin(cx))^2}$$

↓ 2838

$$\frac{5c^2 \left(3c^2 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right)}{3d^2} + \frac{2bc \left(\frac{3}{2} c^2 \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right) \right)}{3d^2} + \frac{(a+b \arcsin(cx))^2}{3d^2 x^3 (1-c^2x^2)}$$

↓ 7143

$$\frac{5c^2 \left(3c^2 \left(\frac{-2i \operatorname{arctan}(e^{i \arcsin(cx)}) (a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right)}{3d^2} + \frac{2bc \left(\frac{3}{2} c^2 \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right) \right)}{3d^2} + \frac{(a+b \arcsin(cx))^2}{3d^2 x^3 (1-c^2x^2)}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^2),x]
```

output

```
-1/3*(a + b*ArcSin[c*x])^2/(d^2*x^3*(1 - c^2*x^2)) + (2*b*c*(-1/2*(a + b*ArcSin[c*x]))/(x^2*sqrt[1 - c^2*x^2]) + (b*c*(-x^(-1) + c*ArcTanh[c*x]))/2 + (3*c^2*((a + b*ArcSin[c*x])/sqrt[1 - c^2*x^2] - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] - b*ArcTanh[c*x] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])/(2))/(3*d^2) + (5*c^2*(-((a + b*ArcSin[c*x])^2/(x*(1 - c^2*x^2))) + 2*b*c*((a + b*ArcSin[c*x])/sqrt[1 - c^2*x^2] - 2*(a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] - b*ArcTanh[c*x] + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])/(2))/(3*d^2) + (3*c^2*((x*(a + b*ArcSin[c*x])^2)/(2*(1 - c^2*x^2)) - b*c*((a + b*ArcSin[c*x])/sqrt[1 - c^2*x^2]) - (b*ArcTanh[c*x])/c^2) + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])]))/(2*c)))/(3*d^2)
```

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 219 $\text{Int}[((a_) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$
- rule 264 $\text{Int}(((c_*)(x_))^{(m_)*((a_) + (b_*)(x_)^2)^{(p_)}}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)*((a+b*x^2)^{(p+1})/(a*c*(m+1)))}, x] - \text{Simp}[b*((m+2*p+3)/(a*c^2*(m+1))) \text{ Int}[(c*x)^{(m+2)*((a+b*x^2)^p)}, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{(e_)*((c_) + (d_)*(x_))})^{(n_)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c+d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{ Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)}] /; \text{FreeQ}\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{(c_)*((a_) + (b_)*x)}*(F_)[v_] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$
- rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_)*((F_)^{(c_)*((a_) + (b_)*(x_))})^{(n_)}]*((f_) + (g_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^{(c*(a+b*x))})^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{ Int}[(f + g*x)^{(m-1)*\text{PolyLog}[2, (-e)*(F^{(c*(a+b*x))})^n]}, x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \text{ :> Int}[DeactivateTrig[u, x], x] \text{ ;/; FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)*E^{(I*(e + f*x))}}]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*k*Pi)*E^{(I*(e + f*x))}}], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*k*Pi)*E^{(I*(e + f*x))}}], x], x]) \text{ ;/; FreeQ}\{c, d, e, f\}, x \ \&\& \text{IntegerQ}[2*k] \ \&\& \text{IGtQ}[m, 0]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \text{ :> Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) \text{ ;/; FreeQ}\{c, d, e, f\}, x \ \&\& \text{IGtQ}[m, 0]$

rule 5162 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \text{ :> Simp}[-x*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*d*(p + 1))), x] + (\text{Simp}[(2*p + 3)/(2*d*(p + 1)) \text{ Int}[(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[x*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) \text{ ;/; FreeQ}\{a, b, c, d, e\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{GtQ}[n, 0] \ \&\& \text{LtQ}[p, -1] \ \&\& \text{NeQ}[p, -3/2]$

rule 5164 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}/((d_.) + (e_.)*(x_.)^2), x_Symbol] \text{ :> Simp}[1/(c*d) \text{ Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] \text{ ;/; FreeQ}\{a, b, c, d, e\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)]^{(n_.)}*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \text{ :> Simp}[(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] \text{ ;/; FreeQ}\{a, b, c, d, e, p\}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{GtQ}[n, 0] \ \&\& \text{NeQ}[p, -1]$

rule 5204

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]

```

rule 5208

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])

```

rule 5218

```

Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]

```

rule 7143

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.61

method	result
derivativedivides	$c^3 \left(\frac{a^2 \left(-\frac{1}{4(cx+1)} + \frac{5 \ln(cx+1)}{4} - \frac{1}{3c^3 x^3} - \frac{2}{cx} - \frac{1}{4(cx-1)} - \frac{5 \ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{15 \arcsin(cx)^2 x^4 c^4 - 4 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{d^2} \right)}{d^2} \right)$
default	$c^3 \left(\frac{a^2 \left(-\frac{1}{4(cx+1)} + \frac{5 \ln(cx+1)}{4} - \frac{1}{3c^3 x^3} - \frac{2}{cx} - \frac{1}{4(cx-1)} - \frac{5 \ln(cx-1)}{4} \right)}{d^2} + \frac{b^2 \left(-\frac{15 \arcsin(cx)^2 x^4 c^4 - 4 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{d^2} \right)}{d^2} \right)$
parts	$\frac{a^2 \left(-\frac{c^3}{4(cx-1)} - \frac{5c^3 \ln(cx-1)}{4} - \frac{c^3}{4(cx+1)} + \frac{5c^3 \ln(cx+1)}{4} - \frac{1}{3x^3} - \frac{2c^2}{x} \right)}{d^2} + \frac{b^2 c^3 \left(-\frac{15 \arcsin(cx)^2 x^4 c^4 - 4 \sqrt{-c^2 x^2 + 1} \arcsin(cx)}{d^2} \right)}{d^2}$

input `int((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)`

output

```

c^3*(a^2/d^2*(-1/4/(c*x+1)+5/4*ln(c*x+1)-1/3/c^3/x^3-2/c/x-1/4/(c*x-1)-5/4
*ln(c*x-1))+b^2/d^2*(-1/6*(15*arcsin(c*x)^2*x^4*c^4-4*(-c^2*x^2+1)^(1/2)*a
rcsin(c*x)*x^3*c^3-10*arcsin(c*x)^2*x^2*c^2+2*c^4*x^4-2*arcsin(c*x)*(-c^2*
x^2+1)^(1/2)*c*x-2*arcsin(c*x)^2-2*c^2*x^2)/(c^2*x^2-1)/c^3/x^3+5/2*arcsin
(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-5*I*arcsin(c*x)*polylog(2,I*(I*
c*x+(-c^2*x^2+1)^(1/2)))+5*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-13/3*ar
csin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+13/3*I*dilog(I*c*x+(-c^2*x^2+1)^(
1/2))+13/3*I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arctan(I*c*x+(-c^2*x^2+
1)^(1/2))-5/2*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+5*I*arcsin(
c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-5*polylog(3,-I*(I*c*x+(-c^2*
x^2+1)^(1/2)))+2*a*b/d^2*(-1/6*(15*c^4*x^4*arcsin(c*x)-2*c^3*x^3*(-c^2*x^
2+1)^(1/2)-10*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-2*arcsin(c*x))/(c
^2*x^2-1)/c^3/x^3+13/6*ln(I*c*x+(-c^2*x^2+1)^(1/2))-13/6*ln(1+I*c*x+(-c^
2*x^2+1)^(1/2))-5/2*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+5/2*arc
sin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+5/2*I*dilog(1+I*(I*c*x+(-c^2*x
^2+1)^(1/2)))-5/2*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
    
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{a^2}{c^4 x^8 - 2c^2 x^6 + x^4} dx + \int \frac{b^2 \arcsin^2(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx + \int \frac{2ab \arcsin(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx$$

input `integrate((a+b*asin(c*x))**2/x**4/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a**2/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b**2*asin(c*x)**2/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(2*a*b*asin(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^2 x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output

```
1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 -
10*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a^2 + 1/12*(15*(b^2*c^5*x^5 - b^2
*c^3*x^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - 15*(
b^2*c^5*x^5 - b^2*c^3*x^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*lo
g(-c*x + 1) - 2*(15*b^2*c^4*x^4 - 10*b^2*c^2*x^2 - 2*b^2)*arctan2(c*x, sqr
t(c*x + 1)*sqrt(-c*x + 1))^2 + 12*(c^2*d^2*x^5 - d^2*x^3)*integrate(1/6*(1
2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (15*(b^2*c^6*x^6 - b^2*
c^4*x^4)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 15*(b^2
*c^6*x^6 - b^2*c^4*x^4)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*
x + 1) - 2*(15*b^2*c^5*x^5 - 10*b^2*c^3*x^3 - 2*b^2*c*x)*arctan2(c*x, sqrt
(c*x + 1)*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d^2*x^8 - 2*
c^2*d^2*x^6 + d^2*x^4), x))/(c^2*d^2*x^5 - d^2*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = \text{Timed out}$$

input

```
integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output

Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx = \int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx$$

input

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^2), x)
```

output

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^2), x)
```


Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^2} dx$$

$$= 24 \left(\int \frac{\arcsin(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx \right) ab c^2 x^5 - 24 \left(\int \frac{\arcsin(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx \right) ab x^3 + 12 \left(\int \frac{\arcsin(cx)^2}{c^4 x^8 - 2c^2 x^6 + x^4} dx \right) b^2 c^2 x^5 - 12 \left(\int \frac{\arcsin(cx)}{c^4 x^8 - 2c^2 x^6 + x^4} dx \right) b^2 c^2 x^3$$

input

```
int((a+b*asin(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x)
```

output

```
(24*int(asin(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*a*b*c**2*x**5 - 24*int(asin(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*a*b*x**3 + 12*int(asin(c*x)**2/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b**2*c**2*x**5 - 12*int(asin(c*x)**2/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b**2*x**3 - 15*log(c**2*x - c)*a**2*c**5*x**5 + 15*log(c**2*x + c)*a**2*c**3*x**3 + 15*log(c**2*x + c)*a**2*c**5*x**5 - 15*log(c**2*x + c)*a**2*c**3*x**3 - 30*a**2*c**4*x**4 + 20*a**2*c**2*x**2 + 4*a**2)/(12*d**2*x**3*(c**2*x**2 - 1))
```

$$3.198 \quad \int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$$

Optimal result	1877
Mathematica [A] (verified)	1878
Rubi [A] (verified)	1879
Maple [A] (verified)	1886
Fricas [F]	1886
Sympy [F]	1887
Maxima [F]	1887
Giac [F]	1888
Mupad [F(-1)]	1888
Reduce [F]	1889

Optimal result

Integrand size = 27, antiderivative size = 343

$$\begin{aligned} \int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx = & \frac{b^2x}{12c^4d^3(1-c^2x^2)} - \frac{b(a+b \arcsin(cx))}{6c^5d^3(1-c^2x^2)^{3/2}} \\ & + \frac{5b(a+b \arcsin(cx))}{4c^5d^3\sqrt{1-c^2x^2}} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\ & - \frac{3x(a+b \arcsin(cx))^2}{8c^4d^3(1-c^2x^2)} \\ & - \frac{3i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{4c^5d^3} \\ & - \frac{7b^2 \operatorname{arctanh}(cx)}{6c^5d^3} \\ & + \frac{3ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{4c^5d^3} \\ & - \frac{3ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{4c^5d^3} \\ & - \frac{3b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{4c^5d^3} \\ & + \frac{3b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{4c^5d^3} \end{aligned}$$

output

```

1/12*b^2*x/c^4/d^3/(-c^2*x^2+1)-1/6*b*(a+b*arcsin(c*x))/c^5/d^3/(-c^2*x^2+
1)^(3/2)+5/4*b*(a+b*arcsin(c*x))/c^5/d^3/(-c^2*x^2+1)^(1/2)+1/4*x^3*(a+b*a
rcsin(c*x))^2/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b*arcsin(c*x))^2/c^4/d^3/(-c
^2*x^2+1)-3/4*I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^5/d
^3-7/6*b^2*arctanh(c*x)/c^5/d^3+3/4*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*
c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^3-3/4*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*
c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^3-3/4*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)
^(1/2)))/c^5/d^3+3/4*b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^5/d^3

```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.94

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{24a^2cx}{(-1+c^2x^2)^2} + \frac{60a^2cx}{-1+c^2x^2} - \frac{60ab(\sqrt{1-c^2x^2}-\arcsin(cx))}{-1+cx} + \frac{60ab(\sqrt{1-c^2x^2}+\arcsin(cx))}{1+cx} + \frac{4ab((-2+cx)\sqrt{1-c^2x^2}+3\arcsin(cx))}{(-1+cx)^2} - \dots$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output

```

((24*a^2*c*x)/(-1 + c^2*x^2)^2 + (60*a^2*c*x)/(-1 + c^2*x^2) - (60*a*b*(Sqrt[1 - c^2*x^2] - ArcSin[c*x]))/(-1 + c*x) + (60*a*b*(Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/(1 + c*x) + (4*a*b*((-2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(-1 + c*x)^2 - (4*a*b*((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(1 + c*x)^2 - 18*a^2*Log[1 - c*x] + 18*a^2*Log[1 + c*x] + 18*a*b*(I*ArcSin[c*x]^2 + ArcSin[c*x]*((-3*I)*Pi - 4*Log[1 + I*E^(I*ArcSin[c*x])])) + 2*Pi*(-2*Log[1 + E^((-I)*ArcSin[c*x])] + Log[1 + I*E^(I*ArcSin[c*x])]) + 2*Log[Cos[ArcSin[c*x]/2]] - Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + (4*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + 18*a*b*((-I)*ArcSin[c*x]^2 + ArcSin[c*x]*(I*Pi + 4*Log[1 - I*E^(I*ArcSin[c*x])])) + 2*Pi*(2*Log[1 + E^((-I)*ArcSin[c*x])] + Log[1 - I*E^(I*ArcSin[c*x])]) - 2*Log[Cos[ArcSin[c*x]/2]] - Log[Sin[(Pi + 2*ArcSin[c*x])/4]]) - (4*I)*PolyLog[2, I*E^(I*ArcSin[c*x])] - 8*b^2*(14*ArcCoth[c*x] + (9*I)*ArcSin[c*x]^2*ArcTan[E^(I*ArcSin[c*x])] - (9*I)*ArcSin[c*x]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (9*I)*ArcSin[c*x]*PolyLog[2, I*E^(I*ArcSin[c*x])] + 9*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 9*PolyLog[3, I*E^(I*ArcSin[c*x])]) + (b^2*(ArcSin[c*x]*(74*Sqrt[1 - c^2*x^2] + 30*Cos[3*ArcSin[c*x]]) + 3*ArcSin[c*x]^2*(3*c*x - 5*Sin[3*ArcSin[c*x]]) + 2*(c*x + Sin[3*ArcSin[c*x]])))/(-1 + c^2*x^2)^2)/(96*c^5*d^3)

```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5206, 27, 5194, 27, 298, 219, 5206, 5164, 3042, 4669, 3011, 2720, 5182, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx \\
 & \quad \downarrow 5206 \\
 & -\frac{b \int \frac{x^3(a + b \arcsin(cx))}{(1 - c^2 x^2)^{5/2}} dx}{2cd^3} - \frac{3 \int \frac{x^2(a + b \arcsin(cx))^2}{d^2(1 - c^2 x^2)^2} dx}{4c^2 d} + \frac{x^3(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow 27
 \end{aligned}$$

$$\begin{aligned}
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \int \frac{x^3(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{5194} \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \left(-bc \int -\frac{2-3c^2x^2}{3c^4(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} \right)}{2cd^3} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{27} \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \left(\frac{b \int \frac{2-3c^2x^2}{(1-c^2x^2)^2} dx}{3c^3} - \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} \right)}{2cd^3} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{298} \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \left(\frac{b \left(\frac{5}{2} \int \frac{1}{1-c^2x^2} dx - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} - \frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} \right)}{2cd^3} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{219} \\
& -\frac{3 \int \frac{x^2(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx}{4c^2d^3} - \frac{b \left(-\frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{\operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \\
& \quad \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{5206}
\end{aligned}$$

$$\begin{aligned}
 & \frac{3 \left(-\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c} - \frac{\int \frac{(a+b \arcsin(cx))^2 dx}{2c^2} + \frac{x(a+b \arcsin(cx))^2}{2c^2(1-c^2x^2)} \right)}{4c^2d^3} \\
 & \frac{b \left(-\frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \quad \downarrow 5164 \\
 & \frac{3 \left(-\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c} - \frac{\int \frac{(a+b \arcsin(cx))^2 d \arcsin(cx)}{\sqrt{1-c^2x^2} 2c^3} + \frac{x(a+b \arcsin(cx))^2}{2c^2(1-c^2x^2)} \right)}{4c^2d^3} \\
 & \frac{b \left(-\frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \quad \downarrow 3042 \\
 & \frac{3 \left(-\frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c^3} - \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c} + \frac{x(a+b \arcsin(cx))^2}{2c^2(1-c^2x^2)} \right)}{4c^2d^3} \\
 & \frac{b \left(-\frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \quad \downarrow 4669 \\
 & \frac{3 \left(-\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{2c^3} \right)}{4c^2d^3} \\
 & \frac{b \left(-\frac{a+b \arcsin(cx)}{c^4\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \quad \downarrow 3011
 \end{aligned}$$

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c^3} \right)$$

$$\frac{b \left(-\frac{a+b \arcsin(cx)}{c^4 \sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 2720

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c^3} \right)$$

$$\frac{b \left(-\frac{a+b \arcsin(cx)}{c^4 \sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 5182

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c^3} \right)$$

$$\frac{b \left(-\frac{a+b \arcsin(cx)}{c^4 \sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 219

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c^3} \right)$$

$$\frac{b \left(-\frac{a+b \arcsin(cx)}{c^4 \sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3c^4(1-c^2x^2)^{3/2}} + \frac{b \left(\frac{5 \operatorname{arctanh}(cx)}{2c} - \frac{x}{2(1-c^2x^2)} \right)}{3c^3} \right)}{2cd^3} + \frac{x^3(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

rule 298 $\text{Int}[(a + (b \cdot x^2)^p) \cdot (c + (d \cdot x^2)), x_Symbol] \rightarrow \text{Simp}[(- (b \cdot c - a \cdot d)) \cdot x \cdot (a + b \cdot x^2)^{p+1} / (2 \cdot a \cdot b \cdot (p+1)), x] - \text{Simp}[(a \cdot d - b \cdot c \cdot (2 \cdot p + 3)) / (2 \cdot a \cdot b \cdot (p+1)) \text{Int}[(a + b \cdot x^2)^{p+1}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, p\}, x\} \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& (\text{LtQ}[p, -1] \parallel \text{ILtQ}[1/2 + p, 0])$

rule 2720 $\text{Int}[u, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$ $\text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w \cdot (a \cdot v)^n)^m] /;$ $\text{FreeQ}\{a, m, n\}, x\} \&\& \text{IntegerQ}[m \cdot n] \&\& \text{!MatchQ}[u, E^{(c \cdot (a \cdot v) + (b \cdot v) \cdot x)} \cdot (F \cdot v) /;$ $\text{FreeQ}\{a, b, c\}, x\} \&\& \text{InverseFunctionQ}[F[x]]$

rule 3011 $\text{Int}[\text{Log}[1 + (e \cdot (F \cdot (c \cdot (a \cdot v) + (b \cdot v) \cdot x))^n] \cdot ((f \cdot v) + (g \cdot v) \cdot x)^m], x_Symbol] \rightarrow \text{Simp}[(- (f + g \cdot x)^m) \cdot (\text{PolyLog}[2, (-e) \cdot (F \cdot (c \cdot (a + b \cdot x)))^n] / (b \cdot c \cdot n \cdot \text{Log}[F])), x] + \text{Simp}[g \cdot m / (b \cdot c \cdot n \cdot \text{Log}[F]) \text{Int}[(f + g \cdot x)^{m-1} \cdot \text{PolyLog}[2, (-e) \cdot (F \cdot (c \cdot (a + b \cdot x)))^n], x], x] /;$ $\text{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e \cdot v) + \text{Pi} \cdot (k \cdot v) + (f \cdot v) \cdot x] \cdot (c \cdot v + (d \cdot v) \cdot x)^m], x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}] / f), x] + (- \text{Simp}[d \cdot (m/f) \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 - E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}], x], x] + \text{Simp}[d \cdot (m/f) \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + E^{(I \cdot k \cdot \text{Pi})} \cdot E^{(I \cdot (e + f \cdot x))}], x], x)) /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IntegerQ}[2 \cdot k] \&\& \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a \cdot v + \text{ArcSin}[c \cdot v] \cdot (b \cdot v))^n / ((d \cdot v) + (e \cdot v) \cdot x^2), x_Symbol] \rightarrow \text{Simp}[1/(c \cdot d) \text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Sec}[x], x], x, \text{ArcSin}[c \cdot x]], x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2 \cdot d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_) , x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[SimplifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.66

method	result
derivativedivides	$-\frac{a^2 \left(\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{5}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(-\frac{15 \arcsin(cx)^2 c^3 x^3 - 30 \arcsin(cx) \sqrt{-c^2}}{d^3} \right)}{d^3}$
default	$-\frac{a^2 \left(\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{5}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(-\frac{15 \arcsin(cx)^2 c^3 x^3 - 30 \arcsin(cx) \sqrt{-c^2}}{d^3} \right)}{d^3}$
parts	$-\frac{a^2 \left(-\frac{1}{16c^5(cx-1)^2} - \frac{5}{16c^5(cx-1)} + \frac{3 \ln(cx-1)}{16c^5} + \frac{1}{16c^5(cx+1)^2} - \frac{5}{16c^5(cx+1)} - \frac{3 \ln(cx+1)}{16c^5} \right)}{d^3} - \frac{b^2 \left(-\frac{15 \arcsin(cx)^2 c^3 x^3 - 30 \arcsin(cx) \sqrt{-c^2}}{d^3} \right)}{d^3}$

input `int(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned} & 1/c^5*(-a^2/d^3*(1/16/(c*x+1)^2-5/16/(c*x+1)-3/16*\ln(c*x+1)-1/16/(c*x-1)^2 \\ & -5/16/(c*x-1)+3/16*\ln(c*x-1))-b^2/d^3*(-1/24*(15*\arcsin(c*x)^2*c^3*x^3-30* \\ & \arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-9*\arcsin(c*x)^2*c*x-2*c^3*x^3+26*\ar \\ & \arcsin(c*x)*(-c^2*x^2+1)^(1/2)+2*c*x)/(c^4*x^4-2*c^2*x^2+1)+3/8*\arcsin(c*x)^ \\ & 2*\ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/4*I*\arcsin(c*x)*\text{polylog}(2,-I*(I*c*x \\ & +(-c^2*x^2+1)^(1/2)))+3/4*\text{polylog}(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*\ar \\ & \arcsin(c*x)^2*\ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/4*I*\arcsin(c*x)*\text{polylog}(2, \\ & I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/4*\text{polylog}(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))- \\ & 7/3*I*\arctan(I*c*x+(-c^2*x^2+1)^(1/2))-2*a*b/d^3*(-1/24*(15*c^3*x^3*\arcsi \\ & n(c*x)-15*c^2*x^2*(-c^2*x^2+1)^(1/2)-9*c*x*\arcsin(c*x)+13*(-c^2*x^2+1)^(1/ \\ & 2))/(c^4*x^4-2*c^2*x^2+1)+3/8*\arcsin(c*x)*\ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2) \\ &))-3/8*\arcsin(c*x)*\ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*I*\text{dilog}(1+I*(I*c \\ & *x+(-c^2*x^2+1)^(1/2)))+3/8*I*\text{dilog}(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))) \end{aligned}$$
Fricas [F]

$$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx)+a)^2 x^4}{(c^2 dx^2-d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b^2*x^4*arcsin(c*x))^2 + 2*a*b*x^4*arcsin(c*x) + a^2*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= - \frac{\int \frac{a^2 x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{b^2 x^4 \arcsin^2(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{2abx^4 \arcsin(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx}{d^3}$$

input `integrate(x**4*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b**2*x**4*asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(2*a*b*x**4*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int - \frac{(b \arcsin(cx) + a)^2 x^4}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output

```
1/16*a^2*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*
log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3)) + 1/16*(3*(b^2*c^4*x^4
- 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*
x + 1) - 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*
sqrt(-c*x + 1))^2*log(-c*x + 1) + 2*(5*b^2*c^3*x^3 - 3*b^2*c*x)*arctan2(c*
x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 16*(c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5
*d^3)*integrate(-1/8*(16*a*b*c^4*x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x
+ 1)) - (3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*
sqrt(-c*x + 1))*log(c*x + 1) - 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arcta
n2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) + 2*(5*b^2*c^3*x^3 - 3
*b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-
c*x + 1))/(c^10*d^3*x^6 - 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x))/(c
^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)
```

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^4}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output

```
integrate(-(b*arcsin(c*x) + a)^2*x^4/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^3} dx$$

input

```
int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output

```
int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= -32 \left(\int \frac{\arcsin(cx)x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^9 x^4 + 64 \left(\int \frac{\arcsin(cx)x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^7 x^2 - 32 \left(\int \frac{\arcsin(cx)x^4}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right)$$

input

```
int(x^4*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output

```
( - 32*int((asin(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*a*b*c**9*x**4 + 64*int((asin(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2
*x**2 - 1),x)*a*b*c**7*x**2 - 32*int((asin(c*x)*x**4)/(c**6*x**6 - 3*c**4*
x**4 + 3*c**2*x**2 - 1),x)*a*b*c**5 - 16*int((asin(c*x)**2*x**4)/(c**6*x**
6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**9*x**4 + 32*int((asin(c*x)**
2*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**7*x**2 - 16
*int((asin(c*x)**2*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
**2*c**5 - 3*log(c**2*x - c)*a**2*c**4*x**4 + 6*log(c**2*x - c)*a**2*c**2*x
**2 - 3*log(c**2*x - c)*a**2 + 3*log(c**2*x + c)*a**2*c**4*x**4 - 6*log(c
**2*x + c)*a**2*c**2*x**2 + 3*log(c**2*x + c)*a**2 + 10*a**2*c**3*x**3 - 6*
a**2*c*x)/(16*c**5*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

$$3.199 \quad \int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$$

Optimal result	1890
Mathematica [A] (verified)	1891
Rubi [A] (verified)	1891
Maple [C] (verified)	1894
Fricas [A] (verification not implemented)	1895
Sympy [F]	1896
Maxima [F]	1896
Giac [B] (verification not implemented)	1897
Mupad [F(-1)]	1898
Reduce [F]	1898

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx = \frac{b^2}{12c^4d^3(1-c^2x^2)} - \frac{bx^3(a+b \arcsin(cx))}{6cd^3(1-c^2x^2)^{3/2}} + \frac{bx(a+b \arcsin(cx))}{2c^3d^3\sqrt{1-c^2x^2}} - \frac{(a+b \arcsin(cx))^2}{4c^4d^3} + \frac{x^4(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{b^2 \log(1-c^2x^2)}{3c^4d^3}$$

output

```
1/12*b^2/c^4/d^3/(-c^2*x^2+1)-1/6*b*x^3*(a+b*arcsin(c*x))/c/d^3/(-c^2*x^2+
1)^(3/2)+1/2*b*x*(a+b*arcsin(c*x))/c^3/d^3/(-c^2*x^2+1)^(1/2)-1/4*(a+b*arc
sin(c*x))^2/c^4/d^3+1/4*x^4*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)^2+1/3*b^2
*ln(-c^2*x^2+1)/c^4/d^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-3a^2 + b^2 + 6a^2 c^2 x^2 - b^2 c^2 x^2 + 6abcx\sqrt{1 - c^2 x^2} - 8abc^3 x^3 \sqrt{1 - c^2 x^2} + 2b(bc x(3 - 4c^2 x^2) \sqrt{1 - c^2 x^2} + 12c^4 d^3 (-1 + c^2 x^2))}{12c^4 d^3 (-1 + c^2 x^2)}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output

```
(-3*a^2 + b^2 + 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 - c^2*x^2] - 8*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*(b*c*x*(3 - 4*c^2*x^2)*Sqrt[1 - c^2*x^2] + a*(-3 + 6*c^2*x^2))*ArcSin[c*x] + 3*b^2*(-1 + 2*c^2*x^2)*ArcSin[c*x]^2 + 4*b^2*(-1 + c^2*x^2)^2*Log[1 - c^2*x^2])/(12*c^4*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5186, 5206, 243, 49, 2009, 5206, 240, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$\downarrow 5186$$

$$\frac{x^4(a + b \arcsin(cx))^2}{4d^3(1 - c^2 x^2)^2} - \frac{bc \int \frac{x^4(a + b \arcsin(cx))}{(1 - c^2 x^2)^{5/2}} dx}{2d^3}$$

$$\downarrow 5206$$

$$\begin{aligned}
& \frac{x^4(a+b\arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc\left(-\frac{\int \frac{x^2(a+b\arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c^2} - \frac{b\int \frac{x^3}{(1-c^2x^2)^2} dx}{3c} + \frac{x^3(a+b\arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3} \\
& \quad \downarrow 243 \\
& \frac{x^4(a+b\arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc\left(-\frac{\int \frac{x^2(a+b\arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c^2} - \frac{b\int \frac{x^2}{(1-c^2x^2)^2} dx^2}{6c} + \frac{x^3(a+b\arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3} \\
& \quad \downarrow 49 \\
& \frac{x^4(a+b\arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc\left(-\frac{\int \frac{x^2(a+b\arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c^2} - \frac{b\int \left(\frac{1}{c^2(c^2x^2-1)} + \frac{1}{c^2(c^2x^2-1)^2}\right) dx^2}{6c} + \frac{x^3(a+b\arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3} \\
& \quad \downarrow 2009 \\
& \frac{x^4(a+b\arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc\left(-\frac{\int \frac{x^2(a+b\arcsin(cx))}{(1-c^2x^2)^{3/2}} dx}{c^2} + \frac{x^3(a+b\arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}} - \frac{b\left(\frac{1}{c^4(1-c^2x^2)} + \frac{\log(1-c^2x^2)}{c^4}\right)}{6c}\right)}{2d^3} \\
& \quad \downarrow 5206 \\
& \frac{x^4(a+b\arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc\left(-\frac{\int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{c^2} - \frac{b\int \frac{x}{1-c^2x^2} dx}{c} + \frac{x(a+b\arcsin(cx))}{c^2\sqrt{1-c^2x^2}} + \frac{x^3(a+b\arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}} - \frac{b\left(\frac{1}{c^4(1-c^2x^2)} + \frac{\log(1-c^2x^2)}{c^4}\right)}{6c}\right)}{2d^3} \\
& \quad \downarrow 240
\end{aligned}$$

$$\frac{x^4(a + b \arcsin(cx))^2}{4d^3(1 - c^2x^2)^2} - \frac{bc \left(-\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} dx}{c^2} + \frac{x(a+b \arcsin(cx))}{c^2\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c^3} + \frac{x^3(a+b \arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{1}{c^4(1-c^2x^2)} + \frac{\log(1-c^2x^2)}{c^4} \right)}{6c} \right)}{2d^3}$$

↓ 5152

$$\frac{x^4(a + b \arcsin(cx))^2}{4d^3(1 - c^2x^2)^2} - \frac{bc \left(\frac{x^3(a+b \arcsin(cx))}{3c^2(1-c^2x^2)^{3/2}} - \frac{(a+b \arcsin(cx))^2}{2bc^3} + \frac{x(a+b \arcsin(cx))}{c^2\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c^3} - \frac{b \left(\frac{1}{c^4(1-c^2x^2)} + \frac{\log(1-c^2x^2)}{c^4} \right)}{6c} \right)}{2d^3}$$

input `Int[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]`

output `(x^4*(a + b*ArcSin[c*x])^2)/(4*d^3*(1 - c^2*x^2)^2) - (b*c*((x^3*(a + b*ArcSin[c*x]))/(3*c^2*(1 - c^2*x^2)^(3/2)) - (b*(1/(c^4*(1 - c^2*x^2)) + Log[1 - c^2*x^2]/c^4))/(6*c) - ((x*(a + b*ArcSin[c*x]))/(c^2*sqrt[1 - c^2*x^2]) - (a + b*ArcSin[c*x])^2/(2*b*c^3) + (b*Log[1 - c^2*x^2])/(2*c^3))/c^2)/(2*d^3)`

Defintions of rubi rules used

rule 49 `Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 0] && IGtQ[m + n + 2, 0]`

rule 240 `Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^2)^p_, x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

rule 5206 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^2)^p_, x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.24

method	result
derivativedivides	$\frac{a^2 \left(-\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{1}{16(cx-1)^2} - \frac{3}{16(cx-1)} \right)}{d^3} - \frac{b^2 \left(\frac{4i \arcsin(cx)}{3} - 8i \arcsin(cx)x^4 c^4 - 8\sqrt{-c^2x^2+1} \arcsin(cx)x^3 c^3 + 6 \right)}{d^3}$
default	$\frac{a^2 \left(-\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{1}{16(cx-1)^2} - \frac{3}{16(cx-1)} \right)}{d^3} - \frac{b^2 \left(\frac{4i \arcsin(cx)}{3} - 8i \arcsin(cx)x^4 c^4 - 8\sqrt{-c^2x^2+1} \arcsin(cx)x^3 c^3 + 6 \right)}{d^3}$
parts	$\frac{a^2 \left(-\frac{1}{16c^4(cx-1)^2} - \frac{3}{16c^4(cx-1)} - \frac{1}{16c^4(cx+1)^2} + \frac{3}{16c^4(cx+1)} \right)}{d^3} - \frac{b^2 \left(\frac{4i \arcsin(cx)}{3} - 8i \arcsin(cx)x^4 c^4 - 8\sqrt{-c^2x^2+1} \arcsin(cx)x^3 c^3 + 6 \right)}{d^3}$

```
input int(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
output 1/c^4*(-a^2/d^3*(-1/16/(c*x+1)^2+3/16/(c*x+1)-1/16/(c*x-1)^2-3/16/(c*x-1))
-b^2/d^3*(4/3*I*arcsin(c*x)-1/12*(8*I*arcsin(c*x)*x^4*c^4-8*(-c^2*x^2+1)^(
1/2)*arcsin(c*x)*x^3*c^3+6*arcsin(c*x)^2*x^2*c^2-16*I*arcsin(c*x)*x^2*c^2+
6*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x-3*arcsin(c*x)^2+8*I*arcsin(c*x)-c^2*x
^2+1)/(c^4*x^4-2*c^2*x^2+1)-2/3*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2))-2*a*b/
d^3*(-1/16*arcsin(c*x)/(c*x+1)^2+3/16*arcsin(c*x)/(c*x+1)-1/16*arcsin(c*x)
/(c*x-1)^2-3/16*arcsin(c*x)/(c*x-1)+1/48/(c*x-1)^2*(-(c*x-1)^2-2*c*x+2)^(1
/2)+1/6/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)-1/48/(c*x+1)^2*(-(c*x+1)^2+2*c*
x+2)^(1/2)+1/6/(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.15

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2dx^2)^3} dx$$

$$= \frac{(6a^2 - b^2)c^2x^2 + 3(2b^2c^2x^2 - b^2) \arcsin(cx)^2 - 3a^2 + b^2 + 6(2abc^2x^2 - ab) \arcsin(cx) + 4(b^2c^4x^4 - 2c^6d^3)}{12(c^8d^3x^4 - 2c^6d^3)}$$

```
input integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
1/12*((6*a^2 - b^2)*c^2*x^2 + 3*(2*b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - 3*a^2 + b^2 + 6*(2*a*b*c^2*x^2 - a*b)*arcsin(c*x) + 4*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 - 1) - 2*(4*a*b*c^3*x^3 - 3*a*b*c*x + (4*b^2*c^3*x^3 - 3*b^2*c*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= -\frac{\int \frac{a^2 x^3}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{b^2 x^3 \arcsin^2(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{2abx^3 \arcsin(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx}{d^3}$$

input

```
integrate(x**3*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output

```
-(Integral(a**2*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b**2*x**3*asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(2*a*b*x**3*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^3}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output

```
1/4*(2*c^2*x^2 - 1)*a^2/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 1/4*((2*
b^2*c^2*x^2 - b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 4*(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)*integrate(-1/2*(4*a*b*c^3*x^3*arctan2(c*
x, sqrt(c*x + 1))*sqrt(-c*x + 1)) - (2*b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sq
rt(-c*x + 1)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/(c^9*d^3*x^6 - 3*c
^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3), x))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 +
c^4*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(154) = 308$.

Time = 0.20 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.85

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \frac{b^2 x^4 \arcsin(cx)^2}{4(c^2 x^2 - 1)^2 d^3} + \frac{abx^4 \arcsin(cx)}{2(c^2 x^2 - 1)^2 d^3}$$

$$+ \frac{a^2 x^4}{4(c^2 x^2 - 1)^2 d^3} + \frac{b^2 x^3 \arcsin(cx)}{6(c^2 x^2 - 1)\sqrt{-c^2 x^2 + 1}cd^3}$$

$$+ \frac{abx^3}{6(c^2 x^2 - 1)\sqrt{-c^2 x^2 + 1}cd^3} - \frac{b^2 x^2}{12(c^2 x^2 - 1)c^2 d^3}$$

$$+ \frac{b^2 x \arcsin(cx)}{2\sqrt{-c^2 x^2 + 1}c^3 d^3} - \frac{b^2 \arcsin(cx)^2}{4c^4 d^3}$$

$$+ \frac{abx}{2\sqrt{-c^2 x^2 + 1}c^3 d^3} - \frac{ab \arcsin(cx)}{2c^4 d^3} + \frac{2b^2 \log(2)}{3c^4 d^3}$$

$$+ \frac{b^2 \log(|-c^2 x^2 + 1|)}{3c^4 d^3} - \frac{a^2}{4c^4 d^3} + \frac{b^2}{12c^4 d^3}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output

```
1/4*b^2*x^4*arcsin(c*x)^2/((c^2*x^2 - 1)^2*d^3) + 1/2*a*b*x^4*arcsin(c*x)/
((c^2*x^2 - 1)^2*d^3) + 1/4*a^2*x^4/((c^2*x^2 - 1)^2*d^3) + 1/6*b^2*x^3*ar
csin(c*x)/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*c*d^3) + 1/6*a*b*x^3/((c^2*x^2
- 1)*sqrt(-c^2*x^2 + 1)*c*d^3) - 1/12*b^2*x^2/((c^2*x^2 - 1)*c^2*d^3) + 1
/2*b^2*x*arcsin(c*x)/(sqrt(-c^2*x^2 + 1)*c^3*d^3) - 1/4*b^2*arcsin(c*x)^2/
(c^4*d^3) + 1/2*a*b*x/(sqrt(-c^2*x^2 + 1)*c^3*d^3) - 1/2*a*b*arcsin(c*x)/(
c^4*d^3) + 2/3*b^2*log(2)/(c^4*d^3) + 1/3*b^2*log(abs(-c^2*x^2 + 1))/(c^4*
d^3) - 1/4*a^2/(c^4*d^3) + 1/12*b^2/(c^4*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^3} dx$$

input `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3,x)`output `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3, x)`**Reduce [F]**

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-8 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^4 x^4 + 16 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^2 x^2 - 8 \left(\int \frac{\operatorname{asin}(cx)x^3}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab}{4d^3 (c^4 x^2 - d)^3}$$

input `int(x^3*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^3,x)`output `(- 8*int((asin(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)* a*b*c**4*x**4 + 16*int((asin(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b*c**2*x**2 - 8*int((asin(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b - 4*int((asin(c*x)**2*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**4*x**4 + 8*int((asin(c*x)**2*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**2*x**2 - 4*int((asin(c*x)**2*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2 + a**2*x**4)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))`

$$3.200 \quad \int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$$

Optimal result	1899
Mathematica [A] (verified)	1900
Rubi [A] (verified)	1901
Maple [A] (verified)	1906
Fricas [F]	1907
Sympy [F]	1907
Maxima [F]	1908
Giac [F]	1908
Mupad [F(-1)]	1909
Reduce [F]	1909

Optimal result

Integrand size = 27, antiderivative size = 341

$$\begin{aligned} \int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx = & \frac{b^2x}{12c^2d^3(1-c^2x^2)} - \frac{b(a+b \arcsin(cx))}{6c^3d^3(1-c^2x^2)^{3/2}} \\ & + \frac{b(a+b \arcsin(cx))}{4c^3d^3\sqrt{1-c^2x^2}} \\ & + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} - \frac{x(a+b \arcsin(cx))^2}{8c^2d^3(1-c^2x^2)} \\ & + \frac{i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{4c^3d^3} - \frac{b^2 \operatorname{arctanh}(cx)}{6c^3d^3} \\ & - \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{4c^3d^3} \\ & + \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{4c^3d^3} \\ & + \frac{b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{4c^3d^3} \\ & - \frac{b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{4c^3d^3} \end{aligned}$$

output

```

1/12*b^2*x/c^2/d^3/(-c^2*x^2+1)-1/6*b*(a+b*arcsin(c*x))/c^3/d^3/(-c^2*x^2+
1)^(3/2)+1/4*b*(a+b*arcsin(c*x))/c^3/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b*arc
sin(c*x))^2/c^2/d^3/(-c^2*x^2+1)^2-1/8*x*(a+b*arcsin(c*x))^2/c^2/d^3/(-c^2
*x^2+1)+1/4*I*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^3/d^3
-1/6*b^2*arctanh(c*x)/c^3/d^3-1/4*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*
x+(-c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*
x+(-c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1
/2)))/c^3/d^3-1/4*b^2*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^3/d^3

```

Mathematica [A] (verified)

Time = 4.32 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.96

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{4a^2 cx}{(-1+c^2 x^2)^2} + \frac{2a^2 cx}{-1+c^2 x^2} - \frac{2ab(\sqrt{1-c^2 x^2}-\arcsin(cx))}{-1+cx} + \frac{2ab(\sqrt{1-c^2 x^2}+\arcsin(cx))}{1+cx} + \frac{2ab((-2+cx)\sqrt{1-c^2 x^2}+3\arcsin(cx))}{3(-1+cx)^2} - \frac{2ab((2+cx)\sqrt{1-c^2 x^2}-3\arcsin(cx))}{3(-1+cx)^2}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output

```

((4*a^2*c*x)/(-1 + c^2*x^2)^2 + (2*a^2*c*x)/(-1 + c^2*x^2) - (2*a*b*(Sqrt[
1 - c^2*x^2] - ArcSin[c*x]))/(-1 + c*x) + (2*a*b*(Sqrt[1 - c^2*x^2] + ArcS
in[c*x]))/(1 + c*x) + (2*a*b*((-2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]
))/(3*(-1 + c*x)^2) - (2*a*b*((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]
))/(3*(1 + c*x)^2) + a^2*Log[1 - c*x] - a^2*Log[1 + c*x] + a*b*((-I)*ArcSin
[c*x]^2 + ArcSin[c*x]*((3*I)*Pi + 4*Log[1 + I*E^(I*ArcSin[c*x])])) + 2*Pi*(
2*Log[1 + E^((-I)*ArcSin[c*x])] - Log[1 + I*E^(I*ArcSin[c*x])]) - 2*Log[Cos
[ArcSin[c*x]/2]] + Log[-Cos[(Pi + 2*ArcSin[c*x])/4]]) - (4*I)*PolyLog[2, (
-I)*E^(I*ArcSin[c*x])] + a*b*(I*ArcSin[c*x]^2 + ArcSin[c*x]*((-I)*Pi - 4*
Log[1 - I*E^(I*ArcSin[c*x])]) + 2*Pi*(-2*Log[1 + E^((-I)*ArcSin[c*x])]) - L
og[1 - I*E^(I*ArcSin[c*x])]) + 2*Log[Cos[ArcSin[c*x]/2]] + Log[Sin[(Pi + 2*
ArcSin[c*x])/4]]) + (4*I)*PolyLog[2, I*E^(I*ArcSin[c*x])] - (4*b^2*(2*Arc
Coth[c*x] - (3*I)*ArcSin[c*x]^2*ArcTan[E^(I*ArcSin[c*x])]) + (3*I)*ArcSin[c
*x]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (3*I)*ArcSin[c*x]*PolyLog[2, I*E^
(I*ArcSin[c*x])] - 3*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] + 3*PolyLog[3, I*E^
(I*ArcSin[c*x])]))/3 + (b^2*(2*ArcSin[c*x]*(Sqrt[1 - c^2*x^2] + 3*Cos[3*A
rcSin[c*x]]) - 3*ArcSin[c*x]^2*(-7*c*x + Sin[3*ArcSin[c*x]]) + 2*(c*x + Si
n[3*ArcSin[c*x]])))/(6*(-1 + c^2*x^2)^2)/(16*c^3*d^3)

```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5206, 27, 5162, 5164, 3042, 4669, 3011, 2720, 5182, 215, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5206} \\
 & -\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^{5/2}} dx}{2cd^3} - \frac{\int \frac{(a+b \arcsin(cx))^2}{d^2(1-c^2 x^2)^2} dx}{4c^2 d} + \frac{x(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & -\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^{5/2}} dx}{2cd^3} - \frac{\int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^2} dx}{4c^2 d^3} + \frac{x(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2}
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 5162 \\
 & \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} - \frac{-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}}{4c^2d^3} + \\
 & \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \downarrow 5164 \\
 & \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} - \\
 & \frac{-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}}{4c^2d^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \downarrow 3042 \\
 & \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} - \\
 & \frac{-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}}{4c^2d^3} + \\
 & \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \downarrow 4669 \\
 & \frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c}}{4c^2d^3} \\
 & \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \downarrow 3011 \\
 & \frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^i \arcsin(cx)) d \arcsin(cx))}{2c}}{4c^2d^3} \\
 & \frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\
 & \downarrow 2720
 \end{aligned}$$

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c}$$

$$\frac{b \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 5182

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c}$$

$$\frac{b \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \int \frac{1}{(1-c^2x^2)^2} dx}{3c} \right)}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 215

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c}$$

$$\frac{b \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{1}{2} \int \frac{1}{1-c^2x^2} dx + \frac{x}{2(1-c^2x^2)} \right)}{3c} \right)}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 219

$$\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2c}$$

$$\frac{b \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3c} \right)}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

↓ 7143

$$\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, ie^{i \arcsin(cx)}))}{2c}$$

$$\frac{b \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3c} \right)}{2cd^3} + \frac{x(a+b \arcsin(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

 $4c^2d^3$

input `Int[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]`

output `(x*(a + b*ArcSin[c*x])^2)/(4*c^2*d^3*(1 - c^2*x^2)^2) - (b*((a + b*ArcSin[c*x])/(3*c^2*(1 - c^2*x^2)^(3/2)) - (b*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c)))/(2*c*d^3) - ((x*(a + b*ArcSin[c*x])^2)/(2*(1 - c^2*x^2)) - b*c*((a + b*ArcSin[c*x])/(c^2*sqrt[1 - c^2*x^2]) - (b*ArcTanh[c*x])/c^2) + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])]) + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])]) - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])]) - b*PolyLog[3, I*E^(I*ArcSin[c*x])])/(2*c))/(4*c^2*d^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 215 `Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1)) Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*m/(b*c*n*Log[F]) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5162 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.67

method	result
derivativedivides	$-\frac{a^2 \left(\frac{1}{16(cx+1)^2} - \frac{1}{16(cx+1)} + \frac{\ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{1}{16(cx-1)} - \frac{\ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(-\frac{3 \arcsin(cx)^2 c^3 x^3 - 6 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{2} \right)}{d^3}$
default	$-\frac{a^2 \left(\frac{1}{16(cx+1)^2} - \frac{1}{16(cx+1)} + \frac{\ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} - \frac{1}{16(cx-1)} - \frac{\ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(-\frac{3 \arcsin(cx)^2 c^3 x^3 - 6 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{2} \right)}{d^3}$
parts	$-\frac{a^2 \left(-\frac{1}{16c^3(cx-1)^2} - \frac{1}{16c^3(cx-1)} - \frac{\ln(cx-1)}{16c^3} + \frac{1}{16c^3(cx+1)^2} - \frac{1}{16c^3(cx+1)} + \frac{\ln(cx+1)}{16c^3} \right)}{d^3} - \frac{b^2 \left(-\frac{3 \arcsin(cx)^2 c^3 x^3 - 6 \arcsin(cx) \sqrt{-c^2 x^2 + 1}}{2} \right)}{d^3}$

input

```
int(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c^3*(-a^2/d^3*(1/16/(c*x+1)^2-1/16/(c*x+1)+1/16*ln(c*x+1)-1/16/(c*x-1)^2-1/16/(c*x-1)-1/16*ln(c*x-1))-b^2/d^3*(-1/24*(3*arcsin(c*x)^2*c^3*x^3-6*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2+3*arcsin(c*x)^2*c*x-2*c^3*x^3+2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+2*c*x)/(c^4*x^4-2*c^2*x^2+1)+1/8*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/4*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/4*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/8*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/4*I*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/4*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/3*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2))-2*a*b/d^3*(-1/24*(3*c^3*x^3*arcsin(c*x)-3*c^2*x^2*(-c^2*x^2+1)^(1/2)+3*c*x*arcsin(c*x)+(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x^2+1)-1/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/8*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+1/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-1/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^2}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral(-(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= -\frac{\int \frac{a^2 x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{b^2 x^2 \operatorname{asin}^2(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{2abx^2 \operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx}{d^3}$$

input

```
integrate(x**2*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**3,x)
```


output

```
-(Integral(a**2*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b**2*x**2*asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(2*a*b*x**2*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^2}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output

```
1/16*a^2*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) - 1/16*((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*(b^2*c^3*x^3 + b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 16*(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)*integrate(1/8*(16*a*b*c^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + ((b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(b^2*c^3*x^3 + b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x)/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)
```

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^2}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output `integrate(-(b*arcsin(c*x) + a)^2*x^2/(c^2*d*x^2 - d)^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^3} dx$$

input `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3,x)`

output `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3, x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-32 \left(\int \frac{\operatorname{asin}(cx)x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^7 x^4 + 64 \left(\int \frac{\operatorname{asin}(cx)x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^5 x^2 - 32 \left(\int \frac{\operatorname{asin}(cx)x^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right)}{1}$$

input `int(x^2*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^3,x)`

output `(- 32*int((asin(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x) *a*b*c**7*x**4 + 64*int((asin(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b*c**5*x**2 - 32*int((asin(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b*c**3 - 16*int((asin(c*x)**2*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**7*x**4 + 32*int((asin(c*x)**2*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**5*x**2 - 16*int((asin(c*x)**2*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**3 + log(c**2*x - c)*a**2*c**4*x**4 - 2*log(c**2*x - c)*a**2*c**2*x**2 + log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2*c**4*x**4 + 2*log(c**2*x + c)*a**2*c**2*x**2 - log(c**2*x + c)*a**2 + 2*a**2*c**3*x**3 + 2*a**2*c*x)/(16*c**3*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))`

3.201
$$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^3} dx$$

Optimal result	1910
Mathematica [A] (verified)	1910
Rubi [A] (verified)	1911
Maple [A] (verified)	1913
Fricas [A] (verification not implemented)	1914
Sympy [F]	1914
Maxima [F]	1915
Giac [B] (verification not implemented)	1915
Mupad [F(-1)]	1916
Reduce [F]	1916

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2dx^2)^3} dx = \frac{b^2}{12c^2d^3(1 - c^2x^2)} - \frac{bx(a + b \arcsin(cx))}{6cd^3(1 - c^2x^2)^{3/2}} - \frac{bx(a + b \arcsin(cx))}{3cd^3\sqrt{1 - c^2x^2}} + \frac{(a + b \arcsin(cx))^2}{4c^2d^3(1 - c^2x^2)^2} - \frac{b^2 \log(1 - c^2x^2)}{6c^2d^3}$$

output

```
1/12*b^2/c^2/d^3/(-c^2*x^2+1)-1/6*b*x*(a+b*arcsin(c*x))/c/d^3/(-c^2*x^2+1)^(3/2)-1/3*b*x*(a+b*arcsin(c*x))/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a+b*arcsin(c*x))^2/c^2/d^3/(-c^2*x^2+1)^2-1/6*b^2*ln(-c^2*x^2+1)/c^2/d^3
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2dx^2)^3} dx = \frac{3a^2 + b^2 - b^2c^2x^2 - 6abcx\sqrt{1 - c^2x^2} + 4abc^3x^3\sqrt{1 - c^2x^2} + 2b(3a + bcx\sqrt{1 - c^2x^2}(-3 + 2c^2x^2)) \arcsin(cx)}{12c^2d^3(-1 + c^2x^2)^2}$$

input `Integrate[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]`

output `(3*a^2 + b^2 - b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 - c^2*x^2] + 4*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*(3*a + b*c*x*Sqrt[1 - c^2*x^2]*(-3 + 2*c^2*x^2))*ArcSin[c*x] + 3*b^2*ArcSin[c*x]^2 - 2*b^2*(-1 + c^2*x^2)^2*Log[1 - c^2*x^2])/(12*c^2*d^3*(-1 + c^2*x^2)^2)`

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5182, 5162, 241, 5160, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5182} \\
 & \frac{(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^{5/2}} dx}{2cd^3} \\
 & \quad \downarrow \text{5162} \\
 & \frac{(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \left(\frac{2}{3} \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^{3/2}} dx - \frac{1}{3} bc \int \frac{x}{(1-c^2 x^2)^2} dx + \frac{x(a+b \arcsin(cx))}{3(1-c^2 x^2)^{3/2}} \right)}{2cd^3} \\
 & \quad \downarrow \text{241} \\
 & \frac{(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \left(\frac{2}{3} \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{3(1-c^2 x^2)^{3/2}} - \frac{b}{6c(1-c^2 x^2)} \right)}{2cd^3} \\
 & \quad \downarrow \text{5160} \\
 & \frac{(a + b \arcsin(cx))^2}{4c^2 d^3 (1 - c^2 x^2)^2} - \frac{b \left(\frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} - bc \int \frac{x}{1-c^2 x^2} dx \right) + \frac{x(a+b \arcsin(cx))}{3(1-c^2 x^2)^{3/2}} - \frac{b}{6c(1-c^2 x^2)} \right)}{2cd^3} \\
 & \quad \downarrow \text{240}
 \end{aligned}$$

$$\frac{(a + b \arcsin(cx))^2}{4c^2d^3(1 - c^2x^2)^2} - \frac{b \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2cd^3}$$

input `Int[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]`

output `(a + b*ArcSin[c*x])^2/(4*c^2*d^3*(1 - c^2*x^2)^2) - (b*(-1/6*b/(c*(1 - c^2*x^2)) + (x*(a + b*ArcSin[c*x]))/(3*(1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c))))/3)/(2*c*d^3)`

Defintions of rubi rules used

rule 240 `Int[(x_)/((a_) + (b_)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]`

rule 241 `Int[(x_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 5160 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/((d_) + (e_)*(x_)^2)^(3/2), x_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*((a + b*ArcSin[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5162 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]`

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{a^2}{4d^3(c^2x^2-1)^2} - \frac{b^2 \left(-\frac{\arcsin(cx)^2}{4(c^2x^2-1)^2} + \frac{cx \arcsin(cx) \sqrt{-c^2x^2+1}}{6(c^2x^2-1)^2} + \frac{1}{12c^2x^2-12} - \frac{\sqrt{-c^2x^2+1} cx \arcsin(cx)}{3(c^2x^2-1)} + \frac{\ln(-c^2x^2+1)}{6} \right)}{d^3} - \frac{c^2}{2ab}$
default	$\frac{a^2}{4d^3(c^2x^2-1)^2} - \frac{b^2 \left(-\frac{\arcsin(cx)^2}{4(c^2x^2-1)^2} + \frac{cx \arcsin(cx) \sqrt{-c^2x^2+1}}{6(c^2x^2-1)^2} + \frac{1}{12c^2x^2-12} - \frac{\sqrt{-c^2x^2+1} cx \arcsin(cx)}{3(c^2x^2-1)} + \frac{\ln(-c^2x^2+1)}{6} \right)}{d^3} - \frac{c^2}{2ab}$
parts	$\frac{a^2}{4d^3c^2(c^2x^2-1)^2} - \frac{b^2 \left(-\frac{\arcsin(cx)^2}{4(c^2x^2-1)^2} + \frac{cx \arcsin(cx) \sqrt{-c^2x^2+1}}{6(c^2x^2-1)^2} + \frac{1}{12c^2x^2-12} - \frac{\sqrt{-c^2x^2+1} cx \arcsin(cx)}{3(c^2x^2-1)} + \frac{\ln(-c^2x^2+1)}{6} \right)}{d^3c^2}$

input

```
int(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c^2*(1/4*a^2/d^3/(c^2*x^2-1)^2-b^2/d^3*(-1/4*arcsin(c*x)^2/(c^2*x^2-1)^2+1/6*c*x/(c^2*x^2-1)^2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+1/12/(c^2*x^2-1)-1/3*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*c*x*arcsin(c*x)+1/6*ln(-c^2*x^2+1))-2*a*b/d^3*(-1/4/(c^2*x^2-1)^2*arcsin(c*x)+1/48/(c*x-1)^2*(-(c*x-1)^2-2*c*x+2)^(1/2)-1/12/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)-1/48/(c*x+1)^2*(-(c*x+1)^2+2*c*x+2)^(1/2)-1/12/(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \frac{b^2 c^2 x^2 - 3 b^2 \arcsin(cx)^2 - 6 ab \arcsin(cx) - 3 a^2 - b^2 + 2(b^2 c^4 x^4 - 2 b^2 c^2 x^2 + b^2) \log(c^2 x^2 - 1) - 2}{12(c^6 d^3 x^4 - 2 c^4 d^3 x^2 + c^2 d^3)}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `-1/12*(b^2*c^2*x^2 - 3*b^2*arcsin(c*x)^2 - 6*a*b*arcsin(c*x) - 3*a^2 - b^2 + 2*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c^2*x^2 - 1) - 2*(2*a*b*c^3*x^3 - 3*a*b*c*x + (2*b^2*c^3*x^3 - 3*b^2*c*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)`

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{a^2 x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{b^2 x \arcsin^2(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{2abx \arcsin(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx$$

input `integrate(x*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b**2*x*asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(2*a*b*x*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3`

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output

```
1/4*a^2/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) + 1/4*(b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 4*(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)*
integrate(-1/2*(4*a*b*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/
(c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3), x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(134) = 268.

Time = 0.19 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.63

$$\begin{aligned} \int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = & \frac{b^2 c^2 x^4 \arcsin(cx)^2}{4(c^2 x^2 - 1)^2 d^3} + \frac{abc^2 x^4 \arcsin(cx)}{2(c^2 x^2 - 1)^2 d^3} + \frac{a^2 c^2 x^4}{4(c^2 x^2 - 1)^2 d^3} \\ & + \frac{b^2 c x^3 \arcsin(cx)}{6(c^2 x^2 - 1)\sqrt{-c^2 x^2 + 1}d^3} - \frac{b^2 x^2 \arcsin(cx)^2}{2(c^2 x^2 - 1)d^3} \\ & + \frac{abcx^3}{6(c^2 x^2 - 1)\sqrt{-c^2 x^2 + 1}d^3} - \frac{abx^2 \arcsin(cx)}{(c^2 x^2 - 1)d^3} \\ & - \frac{a^2 x^2}{2(c^2 x^2 - 1)d^3} - \frac{b^2 x^2}{12(c^2 x^2 - 1)d^3} - \frac{b^2 x \arcsin(cx)}{2\sqrt{-c^2 x^2 + 1}cd^3} \\ & + \frac{b^2 \arcsin(cx)^2}{4c^2 d^3} - \frac{abx}{2\sqrt{-c^2 x^2 + 1}cd^3} + \frac{ab \arcsin(cx)}{2c^2 d^3} \\ & - \frac{b^2 \log(2)}{3c^2 d^3} - \frac{b^2 \log(|-c^2 x^2 + 1|)}{6c^2 d^3} + \frac{a^2}{4c^2 d^3} + \frac{b^2}{12c^2 d^3} \end{aligned}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output

```
1/4*b^2*c^2*x^4*arcsin(c*x)^2/((c^2*x^2 - 1)^2*d^3) + 1/2*a*b*c^2*x^4*arcsin(c*x)/((c^2*x^2 - 1)^2*d^3) + 1/6*b^2*c*x^3*arcsin(c*x)/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*d^3) - 1/2*b^2*x^2*arcsin(c*x)^2/((c^2*x^2 - 1)*d^3) + 1/6*a*b*c*x^3/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*d^3) - a*b*x^2*arcsin(c*x)/((c^2*x^2 - 1)*d^3) - 1/2*a^2*x^2/((c^2*x^2 - 1)*d^3) - 1/12*b^2*x^2/((c^2*x^2 - 1)*d^3) - 1/2*b^2*x*arcsin(c*x)/(sqrt(-c^2*x^2 + 1)*c*d^3) + 1/4*b^2*arcsin(c*x)^2/(c^2*d^3) - 1/2*a*b*x/(sqrt(-c^2*x^2 + 1)*c*d^3) + 1/2*a*b*arcsin(c*x)/(c^2*d^3) - 1/3*b^2*log(2)/(c^2*d^3) - 1/6*b^2*log(abs(-c^2*x^2 + 1))/(c^2*d^3) + 1/4*a^2/(c^2*d^3) + 1/12*b^2/(c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{x(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^3} dx$$

input

```
int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output

```
int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-8 \left(\int \frac{\operatorname{asin}(cx)x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^6 x^4 + 16 \left(\int \frac{\operatorname{asin}(cx)x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^4 x^2 - 8 \left(\int \frac{\operatorname{asin}(cx)x}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab}{4c^2 d^3 (c^4$$

input

```
int(x*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output

```
( - 8*int((asin(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b
*c**6*x**4 + 16*int((asin(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 -
1),x)*a*b*c**4*x**2 - 8*int((asin(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c*
**2*x**2 - 1),x)*a*b*c**2 - 4*int((asin(c*x)**2*x)/(c**6*x**6 - 3*c**4*x**4
+ 3*c**2*x**2 - 1),x)*b**2*c**6*x**4 + 8*int((asin(c*x)**2*x)/(c**6*x**6
- 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**4*x**2 - 4*int((asin(c*x)**2*x
)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**2 + a**2)/(4*c**2
*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

$$3.202 \quad \int \frac{(a+b \arcsin(cx))^2}{(d-c^2x^2)^3} dx$$

Optimal result	1918
Mathematica [B] (verified)	1919
Rubi [A] (verified)	1920
Maple [A] (verified)	1925
Fricas [F]	1926
Sympy [F(-1)]	1926
Maxima [F]	1927
Giac [F(-2)]	1927
Mupad [F(-1)]	1928
Reduce [F]	1928

Optimal result

Integrand size = 24, antiderivative size = 332

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{(d-c^2x^2)^3} dx = & \frac{b^2x}{12d^3(1-c^2x^2)} - \frac{b(a+b \arcsin(cx))}{6cd^3(1-c^2x^2)^{3/2}} - \frac{3b(a+b \arcsin(cx))}{4cd^3\sqrt{1-c^2x^2}} \\ & + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{3x(a+b \arcsin(cx))^2}{8d^3(1-c^2x^2)} \\ & - \frac{3i(a+b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{4cd^3} + \frac{5b^2 \operatorname{arctanh}(cx)}{6cd^3} \\ & + \frac{3ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{4cd^3} \\ & - \frac{3ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{4cd^3} \\ & - \frac{3b^2 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{4cd^3} \\ & + \frac{3b^2 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{4cd^3} \end{aligned}$$

output

```

1/12*b^2*x/d^3/(-c^2*x^2+1)-1/6*b*(a+b*arcsin(c*x))/c/d^3/(-c^2*x^2+1)^(3/
2)-3/4*b*(a+b*arcsin(c*x))/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b*arcsin(c*x)
)^2/d^3/(-c^2*x^2+1)^2+3/8*x*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)-3/4*I*(a
+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c/d^3+5/6*b^2*arctanh(c
*x)/c/d^3+3/4*I*b*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)
))/c/d^3-3/4*I*b*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))
/c/d^3-3/4*b^2*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^3+3/4*b^2*poly
log(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c/d^3

```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 666 vs. $2(332) = 664$.

Time = 3.57 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.01

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx =$$

$$-\frac{12a^2x}{(-1+c^2x^2)^2} + \frac{18a^2x}{-1+c^2x^2} + \frac{9a^2 \log(1-cx)}{c} - \frac{9a^2 \log(1+cx)}{c} + \frac{2ab \left(\frac{2\sqrt{1-c^2x^2}}{(-1+cx)^2} - \frac{cx\sqrt{1-c^2x^2}}{(-1+cx)^2} - \frac{9\sqrt{1-c^2x^2}}{-1+cx} + \frac{2\sqrt{1-c^2x^2}}{(1+cx)^2} + \frac{cx\sqrt{1-c^2x^2}}{(1+cx)^2} \right)}{c}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^3,x]
```

output

```

-1/48*((-12*a^2*x)/(-1 + c^2*x^2)^2 + (18*a^2*x)/(-1 + c^2*x^2) + (9*a^2*Log[1 - c*x])/c - (9*a^2*Log[1 + c*x])/c + (2*a*b*((2*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 - (c*x*Sqrt[1 - c^2*x^2])/(-1 + c*x)^2 - (9*Sqrt[1 - c^2*x^2])/(-1 + c*x) + (2*Sqrt[1 - c^2*x^2])/(1 + c*x)^2 + (c*x*Sqrt[1 - c^2*x^2])/(1 + c*x)^2 + (9*Sqrt[1 - c^2*x^2])/(1 + c*x) + (9*I)*Pi*ArcSin[c*x] - (3*ArcSin[c*x])/(-1 + c*x)^2 + (9*ArcSin[c*x])/(-1 + c*x) + (3*ArcSin[c*x])/(1 + c*x)^2 + (9*ArcSin[c*x])/(1 + c*x) - 9*Pi*Log[1 - I*E^(I*ArcSin[c*x])] - 18*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 9*Pi*Log[1 + I*E^(I*ArcSin[c*x])] + 18*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 9*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 9*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (18*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (18*I)*PolyLog[2, I*E^(I*ArcSin[c*x])])]/c + (2*b^2*((2*c*x)/(-1 + c^2*x^2) - 18*ArcCoth[c*x] + (4*ArcSin[c*x])/(1 - c^2*x^2)^(3/2) + (18*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - (6*c*x*ArcSin[c*x]^2)/(-1 + c^2*x^2)^2 + (9*c*x*ArcSin[c*x]^2)/(-1 + c^2*x^2) + (18*I)*ArcSin[c*x]^2*ArcTan[E^(I*ArcSin[c*x])] - 2*ArcTanh[c*x] - (18*I)*ArcSin[c*x]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (18*I)*ArcSin[c*x]*PolyLog[2, I*E^(I*ArcSin[c*x])] + 18*PolyLog[3, (-I)*E^(I*ArcSin[c*x])] - 18*PolyLog[3, I*E^(I*ArcSin[c*x])])]/c)/d^3

```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5162, 27, 5162, 5164, 3042, 4669, 3011, 2720, 5182, 215, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5162} \\
 & -\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \frac{3 \int \frac{(a+b \arcsin(cx))^2}{d^2(1-c^2x^2)^2} dx}{4d} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & -\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \frac{3 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx}{4d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}
 \end{aligned}$$

$$\begin{aligned} & \downarrow 5162 \\ & \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \\ & \frac{3 \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{1}{2} \int \frac{(a+b \arcsin(cx))}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right)}{4d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \end{aligned}$$

$$\begin{aligned} & \downarrow 5164 \\ & \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \\ & \frac{3 \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right)}{4d^3} + \\ & \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \end{aligned}$$

$$\begin{aligned} & \downarrow 3042 \\ & \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \\ & \frac{3 \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right)}{4d^3} + \\ & \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \end{aligned}$$

$$\begin{aligned} & \downarrow 4669 \\ & \frac{3 \left(\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx))}{2c} \right)}{4d^3} \\ & \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \end{aligned}$$

$$\begin{aligned} & \downarrow 3011 \\ & \frac{3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) (a+b \arcsin(cx)))}{2c} \right)}{4d^3} \\ & \frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \end{aligned}$$

$$\downarrow 2720$$

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) de^i \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^i \arcsin(cx)) de^i \arcsin(cx))}{2c} \right)$$

$$\frac{bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 5182

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) de^i \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^i \arcsin(cx)) de^i \arcsin(cx))}{2c} \right)$$

$$\frac{bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \int \frac{1}{(1-c^2x^2)^2} dx}{3c} \right)}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 215

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) de^i \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^i \arcsin(cx)) de^i \arcsin(cx))}{2c} \right)$$

$$\frac{bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{1}{2} \int \frac{1}{1-c^2x^2} dx + \frac{x}{2(1-c^2x^2)} \right)}{3c} \right)}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 219

$$3 \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) de^i \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^i \arcsin(cx)) de^i \arcsin(cx))}{2c} \right)$$

$$\frac{bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3c} \right)}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 7143

$$3 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, ie^{i \arcsin(cx)}))}{2c} \right) \frac{bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right)}{3c} \right)}{2d^3} + \frac{x(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

input `Int[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^3,x]`

output `(x*(a + b*ArcSin[c*x])^2)/(4*d^3*(1 - c^2*x^2)^2) - (b*c*((a + b*ArcSin[c*x])/(3*c^2*(1 - c^2*x^2)^(3/2)) - (b*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c)))/(2*d^3) + (3*((x*(a + b*ArcSin[c*x])^2)/(2*(1 - c^2*x^2)) - b*c*((a + b*ArcSin[c*x])/(c^2*Sqrt[1 - c^2*x^2]) - (b*ArcTanh[c*x])/c^2) + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])]) + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])]) - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])]) - b*PolyLog[3, I*E^(I*ArcSin[c*x])]))/(2*c)))/(4*d^3)`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 215 `Int[((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1)) Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6*p])`

rule 219 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*m/(b*c*n*Log[F]) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5162 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.72

method	result
derivativedivides	$-\frac{a^2 \left(\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} + \frac{3}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(\frac{9 \arcsin(cx)^2 c^3 x^3 - 18 \arcsin(cx) \sqrt{-c^2 x^2}}{d^3} \right)}{d^3}$
default	$-\frac{a^2 \left(\frac{1}{16(cx+1)^2} + \frac{3}{16(cx+1)} - \frac{3 \ln(cx+1)}{16} - \frac{1}{16(cx-1)^2} + \frac{3}{16(cx-1)} + \frac{3 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(\frac{9 \arcsin(cx)^2 c^3 x^3 - 18 \arcsin(cx) \sqrt{-c^2 x^2}}{d^3} \right)}{d^3}$
parts	$-\frac{a^2 \left(-\frac{1}{16c(cx-1)^2} + \frac{3}{16c(cx-1)} + \frac{3 \ln(cx-1)}{16c} + \frac{1}{16c(cx+1)^2} + \frac{3}{16c(cx+1)} - \frac{3 \ln(cx+1)}{16c} \right)}{d^3} - \frac{b^2 \left(\frac{9 \arcsin(cx)^2 c^3 x^3 - 18 \arcsin(cx) \sqrt{-c^2 x^2}}{d^3} \right)}{d^3}$

input

```
int((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c*(-a^2/d^3*(1/16/(c*x+1)^2+3/16/(c*x+1)-3/16*ln(c*x+1)-1/16/(c*x-1)^2+3/16/(c*x-1)+3/16*ln(c*x-1))-b^2/d^3*(1/24*(9*arcsin(c*x)^2*c^3*x^3-18*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-15*arcsin(c*x)^2*c*x+2*c^3*x^3+22*arcsin(c*x)*(-c^2*x^2+1)^(1/2)-2*c*x)/(c^4*x^4-2*c^2*x^2+1)+3/8*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/4*I*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/4*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/4*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/4*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+5/3*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2))-2*a*b/d^3*(1/24*(9*c^3*x^3*arcsin(c*x)-9*c^2*x^2*(-c^2*x^2+1)^(1/2)-15*c*x*arcsin(c*x)+11*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x^2+1)+3/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-3/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+3/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \text{Timed out}$$

input

```
integrate((a+b*asin(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/16*a^2*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) + 1/16*(3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*log(c*x + 1) - 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*(3*b^2*c^3*x^3 - 5*b^2*c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 16*(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)*integrate(-1/8*(16*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) - (3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(c*x + 1) - 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(3*b^2*c^3*x^3 - 5*b^2*c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^3,x)`output `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^3, x)`**Reduce [F]**

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-32 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^5 x^4 + 64 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right) ab c^3 x^2 - 32 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx \right)}$$

input `int((a+b*asin(c*x))^2/(-c^2*d*x^2+d)^3,x)`output `(- 32*int(asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b*c*
*5*x**4 + 64*int(asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*
a*b*c**3*x**2 - 32*int(asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 -
1),x)*a*b*c - 16*int(asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 -
1),x)*b**2*c**5*x**4 + 32*int(asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c
2*x2 - 1),x)*b**2*c**3*x**2 - 16*int(asin(c*x)**2/(c**6*x**6 - 3*c**4*
x**4 + 3*c**2*x**2 - 1),x)*b**2*c - 3*log(c**2*x - c)*a**2*c**4*x**4 + 6*1
og(c**2*x - c)*a**2*c**2*x**2 - 3*log(c**2*x - c)*a**2 + 3*log(c**2*x + c)
*a**2*c**4*x**4 - 6*log(c**2*x + c)*a**2*c**2*x**2 + 3*log(c**2*x + c)*a**
2 - 6*a**2*c**3*x**3 + 10*a**2*c*x)/(16*c*d**3*(c**4*x**4 - 2*c**2*x**2 +
1))`

$$3.203 \quad \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^3} dx$$

Optimal result	1929
Mathematica [B] (verified)	1930
Rubi [A] (verified)	1931
Maple [B] (verified)	1937
Fricas [F]	1938
Sympy [F]	1938
Maxima [F]	1938
Giac [F(-2)]	1939
Mupad [F(-1)]	1939
Reduce [F]	1940

Optimal result

Integrand size = 27, antiderivative size = 296

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^3} dx = & \frac{b^2}{12d^3(1-c^2x^2)} - \frac{bcx(a+b \arcsin(cx))}{6d^3(1-c^2x^2)^{3/2}} \\ & - \frac{4bcx(a+b \arcsin(cx))}{3d^3\sqrt{1-c^2x^2}} \\ & + \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{(a+b \arcsin(cx))^2}{2d^3(1-c^2x^2)} \\ & - \frac{2(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^3} \\ & - \frac{2b^2 \log(1-c^2x^2)}{3d^3} \\ & + \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d^3} \\ & - \frac{ib(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d^3} \\ & - \frac{b^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{2d^3} + \frac{b^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)})}{2d^3} \end{aligned}$$

output

```

1/12*b^2/d^3/(-c^2*x^2+1)-1/6*b*c*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(3/
2)-4/3*b*c*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a+b*arcsin(c*x)
)^2/d^3/(-c^2*x^2+1)^2+1/2*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)-2*(a+b*arc
sin(c*x))^2*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-2/3*b^2*ln(-c^2*x^2+
1)/d^3+I*b*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-
I*b*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-1/2*b^2*
polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3+1/2*b^2*polylog(3,(I*c*x+(-c^
2*x^2+1)^(1/2))^2)/d^3

```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 756 vs. $2(296) = 592$.

Time = 3.57 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.55

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^3),x]
```

output

```

((6*a^2)/(-1 + c^2*x^2)^2 - (12*a^2)/(-1 + c^2*x^2) + (15*a*b*(Sqrt[1 - c^
2*x^2] - ArcSin[c*x]))/(-1 + c*x) + (15*a*b*(Sqrt[1 - c^2*x^2] + ArcSin[c*
x]))/(1 + c*x) + (a*b*((-2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(-1
+ c*x)^2 + (a*b*((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(1 + c*x)^2
+ 48*a*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 24*a^2*Log[c*x] - 1
2*a^2*Log[1 - c^2*x^2] + 12*a*b*(I*ArcSin[c*x]^2 + ArcSin[c*x]*((-3*I)*Pi
- 4*Log[1 + I*E^(I*ArcSin[c*x])]) + 2*Pi*(-2*Log[1 + E^((-I)*ArcSin[c*x])]
+ Log[1 + I*E^(I*ArcSin[c*x])]) + 2*Log[Cos[ArcSin[c*x]/2]] - Log[-Cos[(Pi
+ 2*ArcSin[c*x])/4]]) + (4*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + 12*a*
b*(I*ArcSin[c*x]^2 + ArcSin[c*x]*((-I)*Pi - 4*Log[1 - I*E^(I*ArcSin[c*x])])
) + 2*Pi*(-2*Log[1 + E^((-I)*ArcSin[c*x])] - Log[1 - I*E^(I*ArcSin[c*x])])
+ 2*Log[Cos[ArcSin[c*x]/2]] + Log[Sin[(Pi + 2*ArcSin[c*x])/4]]) + (4*I)*Po
lyLog[2, I*E^(I*ArcSin[c*x])]) - (24*I)*a*b*(ArcSin[c*x]^2 + PolyLog[2, E^
((2*I)*ArcSin[c*x])]) - b^2*(I*Pi^3 + 2/(-1 + c^2*x^2) + (4*c*x*ArcSin[c*x
])/(-1 - c^2*x^2)^(3/2) + (32*c*x*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - (6*ArcSi
n[c*x]^2)/(-1 + c^2*x^2)^2 + (12*ArcSin[c*x]^2)/(-1 + c^2*x^2) - (16*I)*Ar
cSin[c*x]^3 - 24*ArcSin[c*x]^2*Log[1 - E^((-2*I)*ArcSin[c*x])] + 24*ArcSin
[c*x]^2*Log[1 + E^((2*I)*ArcSin[c*x])] + 16*Log[1 - c^2*x^2] - (24*I)*ArcS
in[c*x]*PolyLog[2, E^((-2*I)*ArcSin[c*x])] - (24*I)*ArcSin[c*x]*PolyLog[2,
-E^((2*I)*ArcSin[c*x])] - 12*PolyLog[3, E^((-2*I)*ArcSin[c*x])] + 12*P...

```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {5208, 27, 5162, 241, 5160, 240, 5208, 5160, 240, 5184, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx$$

$$\downarrow 5208$$

$$-\frac{bc \int \frac{a+b \arcsin(cx)}{(1-c^2 x^2)^{5/2}} dx}{2d^3} + \frac{\int \frac{(a+b \arcsin(cx))^2}{d^2 x(1-c^2 x^2)^2} dx}{d} + \frac{(a + b \arcsin(cx))^2}{4d^3(1 - c^2 x^2)^2}$$

$$\downarrow 27$$

$$\begin{aligned}
& -\frac{bc \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{5/2}} dx}{2d^3} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^3} + \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{5162} \\
& -\frac{bc \left(\frac{2}{3} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx - \frac{1}{3} bc \int \frac{x}{(1-c^2x^2)^2} dx + \frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} \right)}{2d^3} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^3} + \\
& \quad \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{241} \\
& -\frac{bc \left(\frac{2}{3} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^3} + \\
& \quad \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{5160} \\
& -\frac{bc \left(\frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - bc \int \frac{x}{1-c^2x^2} dx \right) + \frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} + \\
& \quad \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^3} + \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \\
& \quad \downarrow \text{240} \\
& \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx}{d^3} + \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \\
& \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow \text{5208} \\
& \frac{-bc \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)}}{d^3} + \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \\
& \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow \text{5160}
\end{aligned}$$

$$\begin{aligned}
& \frac{-bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - bc \int \frac{x}{1-c^2x^2} dx \right) + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)}}{d^3} + \\
& \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow 240 \\
& \frac{\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^3} + \\
& \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow 5184 \\
& \frac{\int \frac{(a+b \arcsin(cx))^2}{cx\sqrt{1-c^2x^2}} d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^3} + \\
& \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow 4919 \\
& \frac{2 \int (a+b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^3} + \\
& \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow 3042 \\
& \frac{2 \int (a+b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right)}{d^3} + \\
& \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3} \\
& \quad \downarrow 4671 \\
& \frac{2(-b \int (a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + b \int (a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx))}{d^3} + \\
& \frac{(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} - \frac{bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right)}{2d^3}
\end{aligned}$$

3011

$$2\left(b\left(\frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right)\right) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right) d \arcsin(cx)\right) - b\left(\frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right)\right) (a + b \arcsin(cx))$$

$$\frac{(a + b \arcsin(cx))^2}{4d^3 (1 - c^2x^2)^2} - \frac{bc\left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3}\left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c}\right) - \frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

2720

$$2\left(b\left(\frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right)\right) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right) de^{2i \arcsin(cx)}\right) - b\left(\frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right)\right) (a + b \arcsin(cx))$$

$$\frac{(a + b \arcsin(cx))^2}{4d^3 (1 - c^2x^2)^2} - \frac{bc\left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3}\left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c}\right) - \frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

7143

$$2\left(-\operatorname{arctanh}\left(e^{2i \arcsin(cx)}\right) (a + b \arcsin(cx))^2 + b\left(\frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right)\right) (a + b \arcsin(cx)) - \frac{1}{4}b \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right) (a + b \arcsin(cx))\right) - b\left(\frac{1}{2}i \operatorname{PolyLog}\left(2, -e^{2i \arcsin(cx)}\right)\right) (a + b \arcsin(cx))$$

$$\frac{(a + b \arcsin(cx))^2}{4d^3 (1 - c^2x^2)^2} - \frac{bc\left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3}\left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c}\right) - \frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^3),x]
```

output

```
(a + b*ArcSin[c*x])^2/(4*d^3*(1 - c^2*x^2)^2) - (b*c*(-1/6*b/(c*(1 - c^2*x^2)) + (x*(a + b*ArcSin[c*x]))/(3*(1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)))/3)/(2*d^3) + ((a + b*ArcSin[c*x])^2/(2*(1 - c^2*x^2)) - b*c*((x*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)) + 2*(-((a + b*ArcSin[c*x])^2 * ArcTanh[E^((2*I)*ArcSin[c*x])]) + b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4) - b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])] - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])/4)))/d^3
```

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 240 $\text{Int}[(x_)/((a_) + (b_*)(x_)^2), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^2, x]]/(2*b), x] /; \text{FreeQ}[\{a, b\}, x]$
- rule 241 $\text{Int}[(x_)*((a_) + (b_*)(x_)^2)^(p_), x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{ Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^((c_)*((a_) + (b_)*x)) * (F_) [v_] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n] / (b*c*n*\text{Log}[F]))], x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{ Int}[(f + g*x)^(m - 1)*\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4671 $\text{Int}[\text{csc}[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m * (\text{ArcTanh}[E^(I*(e + f*x))]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^(m - 1)*\text{Log}[1 - E^(I*(e + f*x))], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^(m - 1)*\text{Log}[1 + E^(I*(e + f*x))], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)(x_)]^{(n_.)}((c_.) + (d_.)(x_))^{(m_.)}\text{Sec}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + dx)^m \text{Csc}[2a + 2bx]^n, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{RationalQ}[m]$

rule 5160 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}((d_.) + (e_.)(x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[x((a + b\text{ArcSin}[cx])^n/(d\sqrt{d + ex^2}))], x] - \text{Simp}[b*c*(n/d)*\text{Simp}[\sqrt{1 - c^2x^2}/\sqrt{d + ex^2}] \text{Int}[x((a + b\text{ArcSin}[cx])^{(n-1)}(1 - c^2x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

rule 5162 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(-x)(d + ex^2)^{(p+1)}((a + b\text{ArcSin}[cx])^n/(2d*(p+1))), x] + (\text{Simp}[(2p+3)/(2d*(p+1)) \text{Int}[(d + ex^2)^{(p+1)}(a + b\text{ArcSin}[cx])^n, x], x] + \text{Simp}[b*c*(n/(2*(p+1)))*\text{Simp}[(d + ex^2)^p/(1 - c^2x^2)^p] \text{Int}[x*(1 - c^2x^2)^{(p+1/2)}(a + b\text{ArcSin}[cx])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}((x_)((d_.) + (e_.)(x_)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + bx)^n/(\text{Cos}[x]*\text{Sin}[x]), x], x, \text{ArcSin}[cx]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5208 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)}((f_.)(x_))^{(m_.)}((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(-f*x)^{(m+1)}(d + ex^2)^{(p+1)}((a + b\text{ArcSin}[cx])^n/(2d*f*(p+1))), x] + (\text{Simp}[(m+2p+3)/(2d*(p+1)) \text{Int}[(f*x)^m(d + ex^2)^{(p+1)}(a + b\text{ArcSin}[cx])^n, x], x] + \text{Simp}[b*c*(n/(2*f*(p+1)))*\text{Simp}[(d + ex^2)^p/(1 - c^2x^2)^p] \text{Int}[(f*x)^{(m+1)}(1 - c^2x^2)^{(p+1/2)}(a + b\text{ArcSin}[cx])^{(n-1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{EqQ}[c^2d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !\text{GtQ}[m, 1] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{EqQ}[n, 1])$

rule 7143 $\text{Int}[\text{PolyLog}[n, (c_.)((a_.) + (b_.)(x_))^{(p_.)}]/((d_.) + (e_.)(x_)), x_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n+1, c*(a + bx)^p/(e*p)], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x \ \&\& \ \text{EqQ}[b*d, a*e]$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(324) = 648$.

Time = 0.74 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.56

method	result
parts	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2} + \frac{5}{16(cx-1)} + \frac{\ln(cx-1)}{2} - \frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} + \frac{\ln(cx+1)}{2} - \ln(x)\right)}{d^3} - \frac{b^2\left(\frac{16i \arcsin(cx)x^4 c^4 - 16\sqrt{-c^2x}}{\dots}\right)}{\dots}$
derivativedivides	$-\frac{a^2\left(-\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} + \frac{\ln(cx+1)}{2} - \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{5}{16(cx-1)} + \frac{\ln(cx-1)}{2}\right)}{d^3} - \frac{b^2\left(\frac{16i \arcsin(cx)x^4 c^4 - 16\sqrt{-c^2x}}{\dots}\right)}{\dots}$
default	$-\frac{a^2\left(-\frac{1}{16(cx+1)^2} - \frac{5}{16(cx+1)} + \frac{\ln(cx+1)}{2} - \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{5}{16(cx-1)} + \frac{\ln(cx-1)}{2}\right)}{d^3} - \frac{b^2\left(\frac{16i \arcsin(cx)x^4 c^4 - 16\sqrt{-c^2x}}{\dots}\right)}{\dots}$

input

```
int((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```
-a^2/d^3*(-1/16/(c*x-1)^2+5/16/(c*x-1)+1/2*ln(c*x-1)-1/16/(c*x+1)^2-5/16/(c*x+1)+1/2*ln(c*x+1)-ln(x))-b^2/d^3*(1/12*(16*I*arcsin(c*x)*c^4*x^4-16*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^3*c^3+6*arcsin(c*x)^2*x^2*c^2-32*I*arcsin(c*x)*c^2*x^2+18*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x-9*arcsin(c*x)^2+16*I*arcsin(c*x)+c^2*x^2-1)/(c^4*x^4-2*c^2*x^2+1)-8/3*ln(I*c*x+(-c^2*x^2+1)^(1/2))+4/3*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))-2*a*b/d^3*(1/12*(8*I*c^4*x^4-8*c^3*x^3*(-c^2*x^2+1)^(1/2)+6*c^2*x^2*arcsin(c*x)-16*I*c^2*x^2+9*c*x*(-c^2*x^2+1)^(1/2)-9*arcsin(c*x)+8*I)/(c^4*x^4-2*c^2*x^2+1)+arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx$$

$$= -\frac{\int \frac{a^2}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx + \int \frac{b^2 \arcsin^2(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx + \int \frac{2ab \arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx}{d^3}$$

input `integrate((a+b*asin(c*x))**2/x/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral(b**2*asin(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral(2*a*b*asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x))/d**3`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output

```
-1/4*a^2*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x
+ 1)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - integrate((b^2*arctan2(c*x
, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(
-c*x + 1)))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(d - c^2 dx^2)^3} dx$$

input

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^3),x)
```

output

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^3), x)
```


Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^3} dx$$

$$= \frac{-8 \left(\int \frac{\arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx \right) ab c^4 x^4 + 16 \left(\int \frac{\arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx \right) ab c^2 x^2 - 8 \left(\int \frac{\arcsin(cx)}{c^6 x^7 - 3c^4 x^5 + 3c^2 x^3 - x} dx \right) ab}{1}$$

input

```
int((a+b*asin(c*x))^2/x/(-c^2*d*x^2+d)^3,x)
```

output

```
( - 8*int(asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*a*b*c**4*x**4 + 16*int(asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*a*b*c**2*x**2 - 8*int(asin(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*a*b - 4*int(asin(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b**2*c**4*x**4 + 8*int(asin(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b**2*c**2*x**2 - 4*int(asin(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b**2 - 2*log(c**2*x - c)*a**2*c**4*x**4 + 4*log(c**2*x - c)*a**2*c**2*x**2 - 2*log(c**2*x - c)*a**2 - 2*log(c**2*x + c)*a**2*c**4*x**4 + 4*log(c**2*x + c)*a**2*c**2*x**2 - 2*log(c**2*x + c)*a**2 + 4*log(x)*a**2*c**4*x**4 - 8*log(x)*a**2*c**2*x**2 + 4*log(x)*a**2 - a**2*c**4*x**4 + 2*a**2)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

$$3.204 \quad \int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^3} dx$$

Optimal result	1942
Mathematica [B] (warning: unable to verify)	1943
Rubi [A] (verified)	1944
Maple [A] (verified)	1953
Fricas [F]	1954
Sympy [F]	1955
Maxima [F]	1955
Giac [F(-2)]	1956
Mupad [F(-1)]	1956
Reduce [F]	1956

Optimal result

Integrand size = 27, antiderivative size = 429

$$\begin{aligned}
 \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx = & \frac{b^2 c^2 x}{12d^3 (1 - c^2 x^2)} - \frac{bc(a + b \arcsin(cx))}{6d^3 (1 - c^2 x^2)^{3/2}} \\
 & - \frac{7bc(a + b \arcsin(cx))}{4d^3 \sqrt{1 - c^2 x^2}} - \frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2} \\
 & + \frac{5c^2 x (a + b \arcsin(cx))^2}{4d^3 (1 - c^2 x^2)^2} + \frac{15c^2 x (a + b \arcsin(cx))^2}{8d^3 (1 - c^2 x^2)} \\
 & - \frac{15ic(a + b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{4d^3} \\
 & - \frac{4bc(a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{d^3} \\
 & + \frac{11b^2 c \operatorname{arctanh}(cx)}{6d^3} + \frac{2ib^2 c \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d^3} \\
 & + \frac{15ibc(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{4d^3} \\
 & - \frac{15ibc(a + b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{4d^3} \\
 & - \frac{2ib^2 c \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d^3} \\
 & - \frac{15b^2 c \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{4d^3} \\
 & + \frac{15b^2 c \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{4d^3}
 \end{aligned}$$

output

```

1/12*b^2*c^2*x/d^3/(-c^2*x^2+1)-1/6*b*c*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)
^(3/2)-7/4*b*c*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(1/2)-(a+b*arcsin(c*x))^
2/d^3/x/(-c^2*x^2+1)^2+5/4*c^2*x*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)^2+15
/8*c^2*x*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)-15/4*I*c*(a+b*arcsin(c*x))^2
*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^3-4*b*c*(a+b*arcsin(c*x))*arctanh(I*c*
x+(-c^2*x^2+1)^(1/2))/d^3+11/6*b^2*c*arctanh(c*x)/d^3+2*I*b^2*c*polylog(2,
-I*c*x-(-c^2*x^2+1)^(1/2))/d^3+15/4*I*b*c*(a+b*arcsin(c*x))*polylog(2,-I*(
I*c*x+(-c^2*x^2+1)^(1/2)))/d^3-15/4*I*b*c*(a+b*arcsin(c*x))*polylog(2,I*(I
*c*x+(-c^2*x^2+1)^(1/2)))/d^3-2*I*b^2*c*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)
)/d^3-15/4*b^2*c*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3+15/4*b^2*c*p
olylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3

```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1400 vs. $2(429) = 858$.

Time = 10.47 (sec) , antiderivative size = 1400, normalized size of antiderivative = 3.26

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^3),x]
```

output

```
-(a^2/(d^3*x)) + (a^2*c^2*x)/(4*d^3*(-1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d^3*(-1 + c^2*x^2)) - (15*a^2*c*Log[1 - c*x])/(16*d^3) + (15*a^2*c*Log[1 + c*x])/(16*d^3) - (2*a*b*c*((-7*(Sqrt[1 - c^2*x^2] - ArcSin[c*x]))/(16*(-1 + c*x)) + ArcSin[c*x]/(c*x) + (7*(Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/(16*(1 + c*x)) - ((-2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x])/(48*(-1 + c*x)^2) + ((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x])/(48*(1 + c*x)^2) + ArcTanh[Sqrt[1 - c^2*x^2]] + (15*(((3*I)/2)*Pi*ArcSin[c*x] - (I/2)*ArcSin[c*x]^2 + 2*Pi*Log[1 + E^((-I)*ArcSin[c*x])]) - Pi*Log[1 + I*E^(I*ArcSin[c*x])]) + 2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - 2*Pi*Log[Cos[ArcSin[c*x]/2]] + Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/16 - (15*(((I/2)*Pi*ArcSin[c*x] - (I/2)*ArcSin[c*x]^2 + 2*Pi*Log[1 + E^((-I)*ArcSin[c*x])]) + Pi*Log[1 - I*E^(I*ArcSin[c*x])] + 2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 2*Pi*Log[Cos[ArcSin[c*x]/2]] - Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*PolyLog[2, I*E^(I*ArcSin[c*x])]))/16)/d^3 - (b^2*c*((-2*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (44*ArcSin[c*x] + 15*ArcSin[c*x]^3 - 45*ArcSin[c*x]^2*Log[1 - I*E^(I*ArcSin[c*x])] - 45*Pi*ArcSin[c*x]*Log[(-1)^(1/4)*(1 - I*E^(I*ArcSin[c*x])])]/(2*E^((I/2)*ArcSin[c*x]))]) + 45*ArcSin[c*x]^2*Log[1 + I*E^(I*ArcSin[c*x])] + 45*ArcSin[c*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcSin[c*x])))/E^((I/2)*ArcSin[c*x])] - 45*Pi*ArcSin[c*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcSin[c*x])))/E^((I/2)*ArcSi...
```

Rubi [A] (verified)

Time = 4.26 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.17, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.852$, Rules used = {5204, 27, 5162, 5162, 5164, 3042, 4669, 3011, 2720, 5182, 215, 219, 5208, 215, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5204} \\
 & 5c^2 \int \frac{(a + b \arcsin(cx))^2}{d^3 (1 - c^2 x^2)^3} dx + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{5/2}} dx}{d^3} - \frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & \frac{5c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^3} dx}{d^3} + \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{5/2}} dx}{d^3} - \frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{5162} \\
 & \frac{5c^2 \left(-\frac{1}{2} bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^{5/2}} dx + \frac{3}{4} \int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^2} dx + \frac{x(a+b \arcsin(cx))^2}{4(1-c^2 x^2)^2} \right)}{d^3} + \\
 & \quad \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{5/2}} dx}{d^3} - \frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{5162} \\
 & \frac{5c^2 \left(-\frac{1}{2} bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^{5/2}} dx + \frac{3}{4} \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{1-c^2 x^2} dx + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2 x^2)} \right) + \frac{x(a+b \arcsin(cx))^2}{4(1-c^2 x^2)} \right)}{d^3} \\
 & \quad \frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)^{5/2}} dx}{d^3} - \frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{5164}
 \end{aligned}$$

$$5c^2 \left(-\frac{1}{2}bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx + \frac{3}{4} \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3 x (1-c^2x^2)^2}$$

↓ 3042

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} +$$

$$5c^2 \left(-\frac{1}{2}bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx + \frac{3}{4} \left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{d^3 x (1-c^2x^2)^2}$$

↓ 4669

$$5c^2 \left(\frac{3}{4} \left(\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{2c} \right) \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3 x (1-c^2x^2)^2}$$

↓ 3011

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, ie^i \arcsin(cx)) d \arcsin(cx))}{2c} \right) \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3 x (1-c^2x^2)^2}$$

↓ 2720

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) d e^i \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^i \arcsin(cx))(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^i \arcsin(cx)) d e^i \arcsin(cx))}{2c} \right) \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3 x (1-c^2x^2)^2}$$

↓ 5182

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 215

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 219

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 5208

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{1}{3}bc \int \frac{1}{(1-c^2x^2)^2} dx + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} \right)}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 215

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx - \frac{1}{3}bc \left(\frac{1}{2} \int \frac{1}{1-c^2x^2} dx + \frac{x}{2(1-c^2x^2)} \right) + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} \right)}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 219

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3}bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) \right)}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 5208

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int \frac{1}{1-c^2x^2} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3}bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) \right)}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 219

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$\frac{2bc \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3}bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) - b \operatorname{arctanh}(cx) \right)}{d^3} - \frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 5218

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$2bc \left(\int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3}bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) - b \operatorname{arctanh}(cx) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 3042

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$2bc \left(\int (a+b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3}bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 4671

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{2c} \right) \right)$$

$$2bc \left(-b \int \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) - 2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{d^3x(1-c^2x^2)^2}$$

↓ 2715

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) \right)$$

$$2bc \left(ib \int e^{-i \arcsin(cx)} \log(1 - e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + e^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2 \operatorname{arctan} \right)$$

$$\frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2}$$

↓ 2838

$$5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) \right)$$

$$2bc \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2 x^2)^{3/2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2}$$

↓ 7143

$$5c^2 \left(\frac{3}{4} \left(\frac{-2i \operatorname{arctan}(e^{i \arcsin(cx)})(a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) \right)$$

$$2bc \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2 x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2 x^2)^{3/2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^2}$$

input Int[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^3), x]

output

```

-((a + b*ArcSin[c*x])^2/(d^3*x*(1 - c^2*x^2)^2)) + (2*b*c*((a + b*ArcSin[c
*x]))/(3*(1 - c^2*x^2)^(3/2)) + (a + b*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - 2*(
a + b*ArcSin[c*x])*ArcTanh[E^(I*ArcSin[c*x])] - b*ArcTanh[c*x] - (b*c*(x/(
2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/3 + I*b*PolyLog[2, -E^(I*ArcSin[c*
x])] - I*b*PolyLog[2, E^(I*ArcSin[c*x])])/d^3 + (5*c^2*((x*(a + b*ArcSin[
c*x])^2)/(4*(1 - c^2*x^2)^2) - (b*c*((a + b*ArcSin[c*x]))/(3*c^2*(1 - c^2*x
^2)^(3/2)) - (b*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c)))/2 + (3
*((x*(a + b*ArcSin[c*x])^2)/(2*(1 - c^2*x^2)) - b*c*((a + b*ArcSin[c*x]))/(
c^2*Sqrt[1 - c^2*x^2]) - (b*ArcTanh[c*x])/c^2) + ((-2*I)*(a + b*ArcSin[c*x
])^2*ArcTan[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I
)*E^(I*ArcSin[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a +
b*ArcSin[c*x])*PolyLog[2, I*E^(I*ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcS
in[c*x])]))/(2*c))/4)/d^3

```

Defintions of rubi rules used

rule 27

```

Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]

```

rule 215

```

Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1)) Int[(a + b*x^2)^(p + 1)
], x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])

```

rule 219

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

rule 2715

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol]
:= Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p +
1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:= Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.70

method	result
derivativedivides	$c \left(-\frac{a^2 \left(\frac{1}{16(cx+1)^2} + \frac{7}{16(cx+1)} - \frac{15 \ln(cx+1)}{16} + \frac{1}{cx} - \frac{1}{16(cx-1)^2} + \frac{7}{16(cx-1)} + \frac{15 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(45 \arcsin(cx)^2 x^4 c^4 - 42 \sqrt{\dots} \right)}{\dots} \right)$
default	$c \left(-\frac{a^2 \left(\frac{1}{16(cx+1)^2} + \frac{7}{16(cx+1)} - \frac{15 \ln(cx+1)}{16} + \frac{1}{cx} - \frac{1}{16(cx-1)^2} + \frac{7}{16(cx-1)} + \frac{15 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(45 \arcsin(cx)^2 x^4 c^4 - 42 \sqrt{\dots} \right)}{\dots} \right)$
parts	$-\frac{a^2 \left(-\frac{c}{16(cx-1)^2} + \frac{7c}{16(cx-1)} + \frac{15c \ln(cx-1)}{16} + \frac{c}{16(cx+1)^2} + \frac{7c}{16(cx+1)} - \frac{15c \ln(cx+1)}{16} + \frac{1}{x} \right)}{d^3} - \frac{b^2 c \left(45 \arcsin(cx)^2 x^4 c^4 - 42 \sqrt{\dots} \right)}{\dots}$

input

```
int((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```

c*(-a^2/d^3*(1/16/(c*x+1)^2+7/16/(c*x+1)-15/16*ln(c*x+1)+1/c/x-1/16/(c*x-1)
)^2+7/16/(c*x-1)+15/16*ln(c*x-1))-b^2/d^3*(1/24*(45*arcsin(c*x)^2*x^4*c^4-
42*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^3*c^3-75*arcsin(c*x)^2*x^2*c^2+2*c^4*x
^4+46*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+24*arcsin(c*x)^2-2*c^2*x^2)/c/x/(
c^4*x^4-2*c^2*x^2+1)-15/8*arcsin(c*x)^2*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))
+15/4*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-15/4*polylog(3
,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2)
)-2*I*dilog(I*c*x+(-c^2*x^2+1)^(1/2))-2*I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2)
)+11/3*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2))+15/8*arcsin(c*x)^2*ln(1+I*(I*c*x
+(-c^2*x^2+1)^(1/2)))-15/4*I*arcsin(c*x)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(
1/2)))+15/4*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2))))-2*a*b/d^3*(1/24*(45
*c^4*x^4*arcsin(c*x)-21*c^3*x^3*(-c^2*x^2+1)^(1/2)-75*c^2*x^2*arcsin(c*x)+
23*c*x*(-c^2*x^2+1)^(1/2)+24*arcsin(c*x))/c/x/(c^4*x^4-2*c^2*x^2+1)-ln(I*c
*x+(-c^2*x^2+1)^(1/2))-1+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+15/8*arcsin(c*x)*l
n(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-15/8*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2
+1)^(1/2)))-15/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+15/8*I*dilog(1-I*
(I*c*x+(-c^2*x^2+1)^(1/2))))

```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x^2} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^8 - 3*c
^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx$$

$$= - \frac{\int \frac{a^2}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx + \int \frac{b^2 \arcsin^2(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx + \int \frac{2ab \arcsin(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx}{d^3}$$

input `integrate((a+b*asin(c*x))**2/x**2/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral(b**2*asin(c*x)**2/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral(2*a*b*asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x))/d**3`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx = \int - \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/16*a^2*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) + 1/16*(15*(b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*log(c*x + 1) - 15*(b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2*log(-c*x + 1) - 2*(15*b^2*c^4*x^4 - 25*b^2*c^2*x^2 + 8*b^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 16*(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x)*integrate(-1/8*(16*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) - (15*(b^2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(c*x + 1) - 15*(b^2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*log(-c*x + 1) - 2*(15*b^2*c^5*x^5 - 25*b^2*c^3*x^3 + 8*b^2*c*x)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x))/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 (d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^3),x)`

output `int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^3), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^3} dx$$

$$= \frac{-32 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx \right) ab c^4 x^5 + 64 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx \right) ab c^2 x^3 - 32 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^8 - 3c^4 x^6 + 3c^2 x^4 - x^2} dx \right)}{1}$$

input `int((a+b*asin(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x)`

output

```
( - 32*int(asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*a*b
*c**4*x**5 + 64*int(asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**
2),x)*a*b*c**2*x**3 - 32*int(asin(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x
**4 - x**2),x)*a*b*x - 16*int(asin(c*x)**2/(c**6*x**8 - 3*c**4*x**6 + 3*c
**2*x**4 - x**2),x)*b**2*c**4*x**5 + 32*int(asin(c*x)**2/(c**6*x**8 - 3*c
**4*x**6 + 3*c**2*x**4 - x**2),x)*b**2*c**2*x**3 - 16*int(asin(c*x)**2/(c**6
*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*b**2*x - 15*log(c**2*x - c)*a
**2*c**5*x**5 + 30*log(c**2*x - c)*a**2*c**3*x**3 - 15*log(c**2*x - c)*a**
2*c*x + 15*log(c**2*x + c)*a**2*c**5*x**5 - 30*log(c**2*x + c)*a**2*c**3*x
**3 + 15*log(c**2*x + c)*a**2*c*x - 30*a**2*c**4*x**4 + 50*a**2*c**2*x**2
- 16*a**2)/(16*d**3*x*(c**4*x**4 - 2*c**2*x**2 + 1))
```

$$3.205 \quad \int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^3} dx$$

Optimal result	1958
Mathematica [B] (verified)	1959
Rubi [A] (verified)	1960
Maple [B] (verified)	1969
Fricas [F]	1970
Sympy [F]	1971
Maxima [F]	1971
Giac [F(-2)]	1972
Mupad [F(-1)]	1972
Reduce [F]	1972

Optimal result

Integrand size = 27, antiderivative size = 403

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^3} dx = & \frac{b^2c^2}{12d^3(1-c^2x^2)} - \frac{bc(a+b \arcsin(cx))}{d^3x(1-c^2x^2)^{3/2}} \\ & + \frac{5bc^3x(a+b \arcsin(cx))}{6d^3(1-c^2x^2)^{3/2}} \\ & - \frac{4bc^3x(a+b \arcsin(cx))}{3d^3\sqrt{1-c^2x^2}} + \frac{3c^2(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2} \\ & - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + \frac{3c^2(a+b \arcsin(cx))^2}{2d^3(1-c^2x^2)} \\ & - \frac{6c^2(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^3} \\ & + \frac{b^2c^2 \log(x)}{d^3} - \frac{7b^2c^2 \log(1-c^2x^2)}{6d^3} \\ & + \frac{3ibc^2(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{d^3} \\ & - \frac{3ibc^2(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{d^3} \\ & - \frac{3b^2c^2 \operatorname{PolyLog}(3, -e^{2i \arcsin(cx)})}{2d^3} \\ & + \frac{3b^2c^2 \operatorname{PolyLog}(3, e^{2i \arcsin(cx)})}{2d^3} \end{aligned}$$

output

```

1/12*b^2*c^2/d^3/(-c^2*x^2+1)-b*c*(a+b*arcsin(c*x))/d^3/x/(-c^2*x^2+1)^(3/2)+5/6*b*c^3*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(3/2)-4/3*b*c^3*x*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(1/2)+3/4*c^2*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)^2-1/2*(a+b*arcsin(c*x))^2/d^3/x^2/(-c^2*x^2+1)^2+3/2*c^2*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)-6*c^2*(a+b*arcsin(c*x))^2*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3+b^2*c^2*ln(x)/d^3-7/6*b^2*c^2*ln(-c^2*x^2+1)/d^3+3*I*b*c^2*(a+b*arcsin(c*x))*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-3*I*b*c^2*(a+b*arcsin(c*x))*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3-3/2*b^2*c^2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3+3/2*b^2*c^2*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3

```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 989 vs. $2(403) = 806$.

Time = 7.59 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.45

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^3),x]
```

output

```

-1/2*a^2/(d^3*x^2) + (a^2*c^2)/(4*d^3*(-1 + c^2*x^2)^2) - (a^2*c^2)/(d^3*(-1 + c^2*x^2)) + (3*a^2*c^2*Log[x])/d^3 - (3*a^2*c^2*Log[1 - c^2*x^2])/(2*d^3) - (2*a*b*((c^2*((2 - c*x)*Sqrt[1 - c^2*x^2] - 3*ArcSin[c*x]))/(48*(-1 + c*x)^2) - (9*c^2*(Sqrt[1 - c^2*x^2] - ArcSin[c*x]))/(16*(-1 + c*x)) - (9*c^3*(Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/(16*(c + c^2*x)) + (c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x])/(2*x^2) - (c^2*((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[c*x]))/(48*(1 + c*x)^2) + (3*c^3*(((3*I)/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c - (Pi*Log[1 + I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c + (Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/c)/2 + (3*c^3*(((I/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c + (Pi*Log[1 - I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c - (Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, I*E^(I*ArcSin[c*x])])/c)/2 - 3*c^2*(ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])]) - (I/2)*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])]))/d^3 - (b^2*c^2*((I/8)*Pi^3 - 1/(12*(1 - c^2*x^2)) + (c*x*ArcSin[c*x])/(6*(1 - c^2*x^2)^(3/2)) + (7*c*x*ArcSin[c*x])/(3*Sqrt[1 - c^2*x^2]) + (Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*x) + ArcSin[c*x]^2/(2*c^2*x^2) - ArcSin[c*x]^2/(4*(1 - c^2*x^2)^2) - ArcSin[c*...

```

Rubi [A] (verified)

Time = 3.54 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.24, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.815$, Rules used = {5204, 27, 5194, 27, 1578, 1195, 2009, 5208, 5162, 241, 5160, 240, 5208, 5160, 240, 5184, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx$$

$$\downarrow 5204$$

$$3c^2 \int \frac{(a + b \arcsin(cx))^2}{d^3 x (1 - c^2 x^2)^3} dx + \frac{bc \int \frac{a + b \arcsin(cx)}{x^2 (1 - c^2 x^2)^{5/2}} dx}{d^3} - \frac{(a + b \arcsin(cx))^2}{2d^3 x^2 (1 - c^2 x^2)^2}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^3} dx}{d^3} + \frac{bc \int \frac{a+b \arcsin(cx)}{x^2(1-c^2x^2)^{5/2}} dx}{d^3} - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} \\
& \quad \downarrow \text{5194} \\
& \frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^3} dx}{d^3} + \\
& \frac{bc \left(-bc \int -\frac{8c^4x^4-12c^2x^2+3}{3x(1-c^2x^2)^2} dx + \frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} \right)}{d^3} \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} \\
& \quad \downarrow \text{27} \\
& \frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^3} dx}{d^3} + \\
& \frac{bc \left(\frac{1}{3}bc \int \frac{8c^4x^4-12c^2x^2+3}{x(1-c^2x^2)^2} dx + \frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} \right)}{d^3} \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} \\
& \quad \downarrow \text{1578} \\
& \frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^3} dx}{d^3} + \\
& \frac{bc \left(\frac{1}{6}bc \int \frac{8c^4x^4-12c^2x^2+3}{x^2(1-c^2x^2)^2} dx + \frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} \right)}{d^3} \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} \\
& \quad \downarrow \text{1195} \\
& \frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^3} dx}{d^3} + \\
& \frac{bc \left(\frac{1}{6}bc \int \left(\frac{5c^2}{c^2x^2-1} - \frac{c^2}{(c^2x^2-1)^2} + \frac{3}{x^2} \right) dx + \frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} \right)}{d^3} \\
& \quad \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} \\
& \quad \downarrow \text{2009}
\end{aligned}$$

$$\frac{3c^2 \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^3} dx - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 5208

$$\frac{3c^2 \left(-\frac{1}{2}bc \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{5/2}} dx + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} \right) - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 5162

$$\frac{3c^2 \left(-\frac{1}{2}bc \left(\frac{2}{3} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx - \frac{1}{3}bc \int \frac{x}{(1-c^2x^2)^2} dx + \frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} \right) + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} \right) - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 241

$$\frac{3c^2 \left(-\frac{1}{2}bc \left(\frac{2}{3} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{b}{6c(1-c^2x^2)} \right) + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} \right) - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 5160

$$\frac{3c^2 \left(-\frac{1}{2}bc \left(\frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - bc \int \frac{x}{1-c^2x^2} dx \right) + \frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{b}{6c(1-c^2x^2)} \right) + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} \right) - \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 240

$$\frac{3c^2 \left(\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - \frac{1}{2}bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)$$

↓ 5208

$$\frac{3c^2 \left(-bc \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^{3/2}} dx + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - \frac{1}{2}bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)$$

↓ 5160

$$\frac{3c^2 \left(-bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - bc \int \frac{x}{1-c^2x^2} dx \right) + \int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - \frac{1}{2}bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)$$

↓ 240

$$\frac{3c^2 \left(\int \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} dx + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{1}{2}bc \left(\frac{x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3} \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{b}{6c(1-c^2x^2)} \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)$$

↓ 5184

$$\frac{3c^2 \left(\int \frac{(a+b \arcsin(cx))^2}{cx\sqrt{1-c^2x^2}} d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{b \log(1-c^2x^2)}{2c} \right) - \frac{1}{2} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 4919

$$\frac{3c^2 \left(2 \int (a+b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 3042

$$\frac{3c^2 \left(2 \int (a+b \arcsin(cx))^2 \csc(2 \arcsin(cx)) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 4671

$$\frac{3c^2 \left((-b \int (a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + b \int (a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{2(1-c^2x^2)} + \frac{(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - bc \left(\frac{x(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} + \frac{1}{2} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right) \right)}{d^3} + \frac{(a+b \arcsin(cx))^2}{2d^3x^2(1-c^2x^2)^2} + bc \left(\frac{8c^2x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6} bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 3011

$$\frac{3c^2 \left(2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \text{PolyLog}(2, -e^{2i \arcsin(cx)}) d \arcsin(cx)) - b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) \right)}{d^3} + bc \left(\frac{8c^2 x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2 x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 2720

$$\frac{3c^2 \left(2(b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \text{PolyLog}(2, -e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) \right)}{d^3} + bc \left(\frac{8c^2 x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2 x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

↓ 7143

$$\frac{3c^2 \left(2(-\text{arctanh}(e^{2i \arcsin(cx)}) (a + b \arcsin(cx))^2 + b(\frac{1}{2}i \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(2, -e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) \right)}{d^3} + bc \left(\frac{8c^2 x(a+b \arcsin(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2 x(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} + \frac{1}{6}bc \left(\frac{1}{c^2x^2-1} + 5 \log(1-c^2x^2) + 3 \log(x^2) \right) \right)}{d^3}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^3),x]`

output

```

-1/2*(a + b*ArcSin[c*x])^2/(d^3*x^2*(1 - c^2*x^2)^2) + (b*c*(-((a + b*ArcSin[c*x])/(x*(1 - c^2*x^2)^(3/2))) + (4*c^2*x*(a + b*ArcSin[c*x]))/(3*(1 - c^2*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcSin[c*x]))/(3*Sqrt[1 - c^2*x^2]) + (b*c*((-1 + c^2*x^2)^(-1) + 3*Log[x^2] + 5*Log[1 - c^2*x^2]))/6)/d^3 + (3*c^2*((a + b*ArcSin[c*x])^2/(4*(1 - c^2*x^2)^2) + (a + b*ArcSin[c*x])^2/(2*(1 - c^2*x^2)) - b*c*((x*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)) - (b*c*(-1/6*b/(c*(1 - c^2*x^2)) + (x*(a + b*ArcSin[c*x]))/(3*(1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (b*Log[1 - c^2*x^2])/(2*c)))/3))/2 + 2*(-((a + b*ArcSin[c*x])^2*ArcTan h[E^((2*I)*ArcSin[c*x])]) + b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, -E^((2*I)*ArcSin[c*x])])/4) - b*((I/2)*(a + b*ArcSin[c*x])*PolyLog[2, E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[3, E^((2*I)*ArcSin[c*x])])/4))/d^3

```

Defintions of rubi rules used

rule 27

```

Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]

```

rule 240

```

Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]

```

rule 241

```

Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]

```

rule 1195

```

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

```

rule 1578

```

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

```

- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`
- rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 4919 `Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]`
- rule 5160 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2] Int[x*((a + b*ArcSin[c*x])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]`

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p +
1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

rule 5184

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSi
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5194

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] := With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin
[c*x]) u, x] - Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 920 vs. 2(427) = 854.

Time = 0.84 (sec) , antiderivative size = 921, normalized size of antiderivative = 2.29

method	result
derivativedivides	$c^2 \left(-\frac{a^2 \left(-\frac{1}{16(cx+1)^2} - \frac{9}{16(cx+1)} + \frac{3 \ln(cx+1)}{2} + \frac{1}{2c^2 x^2} - 3 \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{9}{16(cx-1)} + \frac{3 \ln(cx-1)}{2} \right)}{d^3} - \frac{b^2 \left(\frac{16i \arcsin(cx)}{d} \right)}{d^3} \right)$
default	$c^2 \left(-\frac{a^2 \left(-\frac{1}{16(cx+1)^2} - \frac{9}{16(cx+1)} + \frac{3 \ln(cx+1)}{2} + \frac{1}{2c^2 x^2} - 3 \ln(cx) - \frac{1}{16(cx-1)^2} + \frac{9}{16(cx-1)} + \frac{3 \ln(cx-1)}{2} \right)}{d^3} - \frac{b^2 \left(\frac{16i \arcsin(cx)}{d} \right)}{d^3} \right)$
parts	$-\frac{a^2 \left(-\frac{c^2}{16(cx-1)^2} + \frac{9c^2}{16(cx-1)} + \frac{3c^2 \ln(cx-1)}{2} - \frac{c^2}{16(cx+1)^2} - \frac{9c^2}{16(cx+1)} + \frac{3c^2 \ln(cx+1)}{2} + \frac{1}{2x^2} - 3c^2 \ln(x) \right)}{d^3} - \frac{b^2 c^2 \left(\frac{16i \arcsin(cx)}{d} \right)}{d^3}$

input

```
int((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output

```

c^2*(-a^2/d^3*(-1/16/(c*x+1)^2-9/16/(c*x+1)+3/2*ln(c*x+1)+1/2/c^2/x^2-3*ln
(c*x)-1/16/(c*x-1)^2+9/16/(c*x-1)+3/2*ln(c*x-1))-b^2/d^3*(1/12/(c^4*x^4-2*
c^2*x^2+1)/c^2/x^2*(16*I*arcsin(c*x)*c^6*x^6-16*(-c^2*x^2+1)^(1/2)*arcsin(
c*x)*x^5*c^5+18*arcsin(c*x)^2*x^4*c^4-32*I*arcsin(c*x)*c^4*x^4+6*(-c^2*x^2
+1)^(1/2)*arcsin(c*x)*x^3*c^3-27*arcsin(c*x)^2*x^2*c^2+16*I*arcsin(c*x)*c^
2*x^2+c^4*x^4+12*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+6*arcsin(c*x)^2-c^2*x^
2)-8/3*ln(I*c*x+(-c^2*x^2+1)^(1/2))+7/3*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)
-ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)-ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-3*arcsin(c*
x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+6*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2
*x^2+1)^(1/2))-6*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-3*arcsin(c*x)^2*ln(1
-I*c*x-(-c^2*x^2+1)^(1/2))+6*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1
/2))-6*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+3*arcsin(c*x)^2*ln(1+(I*c*x+(-c
^2*x^2+1)^(1/2))^2)-3*I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^
2)+3/2*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))-2*a*b/d^3*(1/12/(c^4*x^4-
2*c^2*x^2+1)/c^2/x^2*(8*I*c^6*x^6-8*c^5*x^5*(-c^2*x^2+1)^(1/2)+18*c^4*x^4*
arcsin(c*x)-16*I*c^4*x^4+3*c^3*x^3*(-c^2*x^2+1)^(1/2)-27*c^2*x^2*arcsin(c*
x)+8*I*c^2*x^2+6*c*x*(-c^2*x^2+1)^(1/2)+6*arcsin(c*x))+3*arcsin(c*x)*ln(1+
(I*c*x+(-c^2*x^2+1)^(1/2))^2)-3/2*I*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^
2)-3*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+3*I*polylog(2,-I*c*x-(-c^2
*x^2+1)^(1/2))-3*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+3*I*polylog...

```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x^3} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^9 - 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx$$

$$= - \frac{\int \frac{a^2}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx + \int \frac{b^2 \arcsin^2(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx + \int \frac{2ab \arcsin(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx}{d^3}$$

input `integrate((a+b*asin(c*x))**2/x**3/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral(b**2*asin(c*x)**2/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral(2*a*b*asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x))/d**3`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx = \int - \frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-1/4*a^2*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2) + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) - integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^3 (d - c^2 dx^2)^3} dx$$

input `int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^3),x)`

output `int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^3), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^3} dx$$

$$= \frac{-8 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx \right) ab c^4 x^6 + 16 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx \right) ab c^2 x^4 - 8 \left(\int \frac{\operatorname{asin}(cx)}{c^6 x^9 - 3c^4 x^7 + 3c^2 x^5 - x^3} dx \right)}{1}$$

input `int((a+b*asin(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x)`

output

```
( - 8*int(asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*a*b*
c**4*x**6 + 16*int(asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3
),x)*a*b*c**2*x**4 - 8*int(asin(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**
5 - x**3),x)*a*b*x**2 - 4*int(asin(c*x)**2/(c**6*x**9 - 3*c**4*x**7 + 3*c*
**2*x**5 - x**3),x)*b**2*c**4*x**6 + 8*int(asin(c*x)**2/(c**6*x**9 - 3*c**4
*x**7 + 3*c**2*x**5 - x**3),x)*b**2*c**2*x**4 - 4*int(asin(c*x)**2/(c**6*x
**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*b**2*x**2 - 6*log(c**2*x - c)*a
**2*c**6*x**6 + 12*log(c**2*x - c)*a**2*c**4*x**4 - 6*log(c**2*x - c)*a**2
*c**2*x**2 - 6*log(c**2*x + c)*a**2*c**6*x**6 + 12*log(c**2*x + c)*a**2*c*
**4*x**4 - 6*log(c**2*x + c)*a**2*c**2*x**2 + 12*log(x)*a**2*c**6*x**6 - 24
*log(x)*a**2*c**4*x**4 + 12*log(x)*a**2*c**2*x**2 - 3*a**2*c**6*x**6 + 6*a
**2*c**2*x**2 - 2*a**2)/(4*d**3*x**2*(c**4*x**4 - 2*c**2*x**2 + 1))
```

$$3.206 \quad \int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^3} dx$$

Optimal result	1975
Mathematica [B] (warning: unable to verify)	1976
Rubi [A] (verified)	1977
Maple [A] (verified)	1989
Fricas [F]	1990
Sympy [F]	1990
Maxima [F]	1990
Giac [F(-1)]	1991
Mupad [F(-1)]	1991
Reduce [F]	1992

Optimal result

Integrand size = 27, antiderivative size = 544

$$\begin{aligned}
 \int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx = & -\frac{b^2 c^2}{3d^3 x} + \frac{b^2 c^4 x}{12d^3 (1 - c^2 x^2)} + \frac{bc^3 (a + b \arcsin(cx))}{6d^3 (1 - c^2 x^2)^{3/2}} \\
 & - \frac{bc(a + b \arcsin(cx))}{3d^3 x^2 (1 - c^2 x^2)^{3/2}} - \frac{29bc^3 (a + b \arcsin(cx))}{12d^3 \sqrt{1 - c^2 x^2}} \\
 & - \frac{(a + b \arcsin(cx))^2}{3d^3 x^3 (1 - c^2 x^2)^2} - \frac{7c^2 (a + b \arcsin(cx))^2}{3d^3 x (1 - c^2 x^2)^2} \\
 & + \frac{35c^4 x (a + b \arcsin(cx))^2}{12d^3 (1 - c^2 x^2)^2} + \frac{35c^4 x (a + b \arcsin(cx))^2}{8d^3 (1 - c^2 x^2)} \\
 & - \frac{35ic^3 (a + b \arcsin(cx))^2 \arctan(e^{i \arcsin(cx)})}{4d^3} \\
 & - \frac{38bc^3 (a + b \arcsin(cx)) \operatorname{arctanh}(e^{i \arcsin(cx)})}{3d^3} \\
 & + \frac{17b^2 c^3 \operatorname{arctanh}(cx)}{6d^3} + \frac{19ib^2 c^3 \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{3d^3} \\
 & + \frac{35ibc^3 (a + b \arcsin(cx)) \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{4d^3} \\
 & - \frac{35ibc^3 (a + b \arcsin(cx)) \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{4d^3} \\
 & - \frac{19ib^2 c^3 \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{3d^3} \\
 & - \frac{35b^2 c^3 \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)})}{4d^3} \\
 & + \frac{35b^2 c^3 \operatorname{PolyLog}(3, ie^{i \arcsin(cx)})}{4d^3}
 \end{aligned}$$

output

```
-1/3*b^2*c^2/d^3/x+1/12*b^2*c^4*x/d^3/(-c^2*x^2+1)+1/6*b*c^3*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(3/2)-1/3*b*c*(a+b*arcsin(c*x))/d^3/x^2/(-c^2*x^2+1)^(3/2)-29/12*b*c^3*(a+b*arcsin(c*x))/d^3/(-c^2*x^2+1)^(1/2)-1/3*(a+b*arcsin(c*x))^2/d^3/x^3/(-c^2*x^2+1)^2-7/3*c^2*(a+b*arcsin(c*x))^2/d^3/x/(-c^2*x^2+1)^2+35/12*c^4*x*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)^2+35/8*c^4*x*(a+b*arcsin(c*x))^2/d^3/(-c^2*x^2+1)+19/3*I*b^2*c^3*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))/d^3-38/3*b*c^3*(a+b*arcsin(c*x))*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d^3+17/6*b^2*c^3*arctanh(c*x)/d^3-19/3*I*b^2*c^3*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d^3-35/4*I*c^3*(a+b*arcsin(c*x))^2*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^3-35/4*I*b*c^3*(a+b*arcsin(c*x))*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3+35/4*I*b*c^3*(a+b*arcsin(c*x))*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3-35/4*b^2*c^3*polylog(3,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3+35/4*b^2*c^3*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1657 vs. $2(544) = 1088$.

Time = 11.30 (sec) , antiderivative size = 1657, normalized size of antiderivative = 3.05

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^3),x]
```

output

```

-1/3*a^2/(d^3*x^3) - (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(-1 + c^2*x^
2)^2) - (11*a^2*c^4*x)/(8*d^3*(-1 + c^2*x^2)) - (35*a^2*c^3*Log[1 - c*x])/
(16*d^3) + (35*a^2*c^3*Log[1 + c*x])/(16*d^3) - (2*a*b*((c^3*((2 - c*x)*Sq
rt[1 - c^2*x^2] - 3*ArcSin[c*x]))/(48*(-1 + c*x)^2) - (11*c^3*(Sqrt[1 - c^
2*x^2] - ArcSin[c*x]))/(16*(-1 + c*x)) + (11*c^4*(Sqrt[1 - c^2*x^2] + ArcS
in[c*x]))/(16*(c + c^2*x)) + (c^3*((2 + c*x)*Sqrt[1 - c^2*x^2] + 3*ArcSin[
c*x]))/(48*(1 + c*x)^2) - 3*c^2*(-(ArcSin[c*x]/x) - c*ArcTanh[Sqrt[1 - c^2
*x^2]]) + (c*x*Sqrt[1 - c^2*x^2] + 2*ArcSin[c*x] + c^3*x^3*ArcTanh[Sqrt[1
- c^2*x^2]])/(6*x^3) + (35*c^4*(((3*I)/2)*Pi*ArcSin[c*x])/c - ((I/2)*ArcS
in[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])])/c - (Pi*Log[1 + I*E^(I
*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])])/c - (2*Pi
*Log[Cos[ArcSin[c*x]/2]])/c + (Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]])/c - (
(2*I)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])])/c)/16 - (35*c^4*(((I/2)*Pi*ArcS
in[c*x])/c - ((I/2)*ArcSin[c*x]^2)/c + (2*Pi*Log[1 + E^((-I)*ArcSin[c*x])
])/c + (Pi*Log[1 - I*E^(I*ArcSin[c*x])])/c + (2*ArcSin[c*x]*Log[1 - I*E^(I
*ArcSin[c*x])])/c - (2*Pi*Log[Cos[ArcSin[c*x]/2]])/c - (Pi*Log[Sin[(Pi + 2*
ArcSin[c*x])/4]])/c - ((2*I)*PolyLog[2, I*E^(I*ArcSin[c*x])])/c)/16)/d^3
- (b^2*c^3*(((19*I)/3)*PolyLog[2, -E^(I*ArcSin[c*x])] + ((19*I)/3)*PolyL
og[2, E^(I*ArcSin[c*x])] + (68*ArcSin[c*x] + 35*ArcSin[c*x]^3 - 105*ArcSin
[c*x]^2*Log[1 - I*E^(I*ArcSin[c*x])] - 105*Pi*ArcSin[c*x]*Log[(-1)^(1/...

```

Rubi [A] (verified)

Time = 6.45 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.42, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5204, 27, 5204, 253, 264, 219, 5162, 5162, 5164, 3042, 4669, 3011, 2720, 5182, 215, 219, 5208, 215, 219, 5208, 219, 5218, 3042, 4671, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx$$

$$\downarrow 5204$$

$$\frac{7}{3} c^2 \int \frac{(a + b \arcsin(cx))^2}{d^3 x^2 (1 - c^2 x^2)^3} dx + \frac{2bc \int \frac{a + b \arcsin(cx)}{x^3 (1 - c^2 x^2)^{5/2}} dx}{3d^3} - \frac{(a + b \arcsin(cx))^2}{3d^3 x^3 (1 - c^2 x^2)^2}$$

$$\downarrow 27$$

$$\begin{aligned}
& \frac{7c^2 \int \frac{(a+b \arcsin(cx))^2}{x^2(1-c^2x^2)^3} dx}{3d^3} + \frac{2bc \int \frac{a+b \arcsin(cx)}{x^3(1-c^2x^2)^{5/2}} dx}{3d^3} - \frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2} \\
& \quad \downarrow 5204 \\
& \frac{7c^2 \left(5c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^3} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} \right)}{3d^3} + \\
& \frac{2bc \left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx + \frac{1}{2}bc \int \frac{1}{x^2(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} \right)}{3d^3} - \frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2} \\
& \quad \downarrow 253 \\
& \frac{7c^2 \left(5c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^3} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} \right)}{3d^3} + \\
& \frac{2bc \left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx + \frac{1}{2}bc \left(\frac{3}{2} \int \frac{1}{x^2(1-c^2x^2)} dx + \frac{1}{2x(1-c^2x^2)} \right) - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} \right)}{3d^3} - \\
& \frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2} \\
& \quad \downarrow 264 \\
& \frac{7c^2 \left(5c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^3} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} \right)}{3d^3} + \\
& \frac{2bc \left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx + \frac{1}{2}bc \left(\frac{3}{2} \left(c^2 \int \frac{1}{1-c^2x^2} dx - \frac{1}{x} \right) + \frac{1}{2x(1-c^2x^2)} \right) - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} \right)}{3d^3} - \\
& \frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2} \\
& \quad \downarrow 219 \\
& \frac{2bc \left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2}bc \left(\frac{3}{2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) + \frac{1}{2x(1-c^2x^2)} \right) \right)}{3d^3} + \\
& \frac{7c^2 \left(5c^2 \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^3} dx + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} \right)}{3d^3} - \frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2} \\
& \quad \downarrow 5162 \\
& \frac{2bc \left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2}bc \left(\frac{3}{2} \left(\operatorname{carctanh}(cx) - \frac{1}{x} \right) + \frac{1}{2x(1-c^2x^2)} \right) \right)}{3d^3} + \\
& \frac{7c^2 \left(5c^2 \left(-\frac{1}{2}bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx + \frac{3}{4} \int \frac{(a+b \arcsin(cx))^2}{(1-c^2x^2)^2} dx + \frac{x(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} \right) + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{x(1-c^2x^2)^2} \right)}{3d^3} - \\
& \frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}
\end{aligned}$$

↓ 5162

$$\frac{2bc\left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2}bc\left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3} +$$

$$\frac{7c^2\left(5c^2\left(-\frac{1}{2}bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx + \frac{3}{4}\left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right) + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 5164

$$\frac{2bc\left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2}bc\left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3} +$$

$$\frac{7c^2\left(5c^2\left(-\frac{1}{2}bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx + \frac{3}{4}\left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right) + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 3042

$$\frac{2bc\left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2}bc\left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3} +$$

$$\frac{7c^2\left(2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx + 5c^2\left(-\frac{1}{2}bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{5/2}} dx + \frac{3}{4}\left(-bc \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^{3/2}} dx + \frac{\int (a+b \arcsin(cx))^2 \operatorname{csc}(\arcsin(cx))}{2c}\right) + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 4669

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(\frac{-2b \int (a+b \arcsin(cx)) \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{2c}\right) + \frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)}\right)}{3d^3}$$

$$\frac{2bc\left(\frac{5}{2}c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2}bc\left(\frac{3}{2}(\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 3011

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right) \right) \right)$$

$$2bc \left(\frac{5}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right) \right)$$

$$\frac{3d^3 (a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 2720

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right) \right) \right)$$

$$2bc \left(\frac{5}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right) \right)$$

$$\frac{3d^3 (a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 5182

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right) \right) \right)$$

$$2bc \left(\frac{5}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right) \right)$$

$$\frac{3d^3 (a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 215

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} \right) \right) \right)$$

$$2bc \left(\frac{5}{2} c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{5/2}} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx) - \frac{1}{x}) + \frac{1}{2x(1-c^2x^2)} \right) \right)$$

$$\frac{3d^3 (a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 219

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right) \right)$$

$$\frac{2bc \left(\frac{5}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^{3/2}} dx + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3} bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) \right) - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx)) \right) \right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 5208

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right) \right)$$

$$\frac{2bc \left(\frac{5}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx - bc \int \frac{1}{1-c^2x^2} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3} bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) \right) - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx)) \right) \right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 219

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right) \right)$$

$$\frac{2bc \left(\frac{5}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{x\sqrt{1-c^2x^2}} dx + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3} bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) - b \operatorname{arctanh}(cx) \right) - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)^{3/2}} + \frac{1}{2} bc \left(\frac{3}{2} (\operatorname{carctanh}(cx)) \right) \right)}{3d^3}$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3 x^3 (1-c^2x^2)^2}$$

↓ 5218

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right) \right)$$

$$2bc \left(\frac{5}{2} c^2 \left(\int \frac{a+b \arcsin(cx)}{cx} d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3} bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) - \operatorname{barctanh}(cx) \right) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 3042

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2b(i \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{2c} \right) \right) \right)$$

$$2bc \left(\frac{5}{2} c^2 \left(\int (a+b \arcsin(cx)) \operatorname{csc}(\arcsin(cx)) d \arcsin(cx) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \frac{1}{3} bc \left(\frac{\operatorname{arctanh}(cx)}{2c} + \frac{x}{2(1-c^2x^2)} \right) - \operatorname{barctanh}(cx) \right) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 4671

$$7 \left(5 \left(\frac{x(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - \frac{1}{2} bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{x}{2(1-c^2x^2)} + \frac{\operatorname{arctanh}(cx)}{2c} \right)}{3c} \right) \right) + \frac{3}{4} \left(\frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{a+b \arcsin(cx)}{c^2\sqrt{1-c^2x^2}} - \operatorname{barctanh}(cx) \right) \right) \right)$$

$$2b \left(\frac{5}{2} \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - \operatorname{barctanh}(cx) - \frac{1}{3} bc \left(\frac{x}{2(1-c^2x^2)} + \frac{\operatorname{arctanh}(cx)}{2c} \right) - \operatorname{barctanh}(cx) \right) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

↓ 2715

$$7 \left(5 \left(\frac{x(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - \frac{1}{2}bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{x}{2(1-c^2x^2)} + \frac{\operatorname{arctanh}(cx)}{2c} \right)}{3c} \right) \right) + \frac{3}{4} \left(\frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{a+b \arcsin(cx)}{c^2\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$2b \left(\frac{5}{2} \left(-2\operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - b\operatorname{arctanh}(cx) - \frac{1}{3}bc \left(\frac{x}{2(1-c^2x^2)} + \right) \right) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{3d^3x^3(1 - c^2x^2)^2}$$

↓ 2838

$$7 \left(5 \left(\frac{x(a+b \arcsin(cx))^2}{4(1-c^2x^2)^2} - \frac{1}{2}bc \left(\frac{a+b \arcsin(cx)}{3c^2(1-c^2x^2)^{3/2}} - \frac{b \left(\frac{x}{2(1-c^2x^2)} + \frac{\operatorname{arctanh}(cx)}{2c} \right)}{3c} \right) \right) + \frac{3}{4} \left(\frac{x(a+b \arcsin(cx))^2}{2(1-c^2x^2)} - bc \left(\frac{a+b \arcsin(cx)}{c^2\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$2b \left(\frac{5}{2} \left(-2\operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} - b\operatorname{arctanh}(cx) - \frac{1}{3}bc \left(\frac{x}{2(1-c^2x^2)} + \right) \right) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{3d^3x^3(1 - c^2x^2)^2}$$

↓ 7143

$$7c^2 \left(5c^2 \left(\frac{3}{4} \left(\frac{-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \operatorname{PolyLog}(3, -ie^{i \arcsin(cx)}) - 2b \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{2c}} \right) \right) \right)$$

$$2bc \left(\frac{5}{2}c^2 \left(-2\operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{3(1-c^2x^2)^{3/2}} + ib \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \right) \right)$$

$$\frac{(a + b \arcsin(cx))^2}{3d^3x^3(1 - c^2x^2)^2}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^3), x]`

output

```

-1/3*(a + b*ArcSin[c*x])^2/(d^3*x^3*(1 - c^2*x^2)^2) + (2*b*c*(-1/2*(a + b
*ArcSin[c*x])/(x^2*(1 - c^2*x^2)^(3/2)) + (b*c*(1/(2*x*(1 - c^2*x^2)) + (3
*(-x^(-1) + c*ArcTanh[c*x]))/2))/2 + (5*c^2*((a + b*ArcSin[c*x])/(3*(1 - c
^2*x^2)^(3/2)) + (a + b*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - 2*(a + b*ArcSin[c
*x])*ArcTanh[E^(I*ArcSin[c*x])] - b*ArcTanh[c*x] - (b*c*(x/(2*(1 - c^2*x^2
)) + ArcTanh[c*x]/(2*c))))/3 + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*Pol
yLog[2, E^(I*ArcSin[c*x])])/(2))/3 + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*Pol
yLog[2, E^(I*ArcSin[c*x])])/(3*d^3) + (7*c^2*(-((a + b*ArcSin[c*x])^2
/(x*(1 - c^2*x^2)^2)) + 2*b*c*((a + b*ArcSin[c*x])/(3*(1 - c^2*x^2)^(3/2))
+ (a + b*ArcSin[c*x])/Sqrt[1 - c^2*x^2] - 2*(a + b*ArcSin[c*x])*ArcTanh[E
^(I*ArcSin[c*x])] - b*ArcTanh[c*x] - (b*c*(x/(2*(1 - c^2*x^2)) + ArcTanh[c
*x]/(2*c))))/3 + I*b*PolyLog[2, -E^(I*ArcSin[c*x])] - I*b*PolyLog[2, E^(I*A
rcSin[c*x])]) + 5*c^2*((x*(a + b*ArcSin[c*x])^2)/(4*(1 - c^2*x^2)^2) - (b*
c*((a + b*ArcSin[c*x])/(3*c^2*(1 - c^2*x^2)^(3/2)) - (b*(x/(2*(1 - c^2*x^2
)) + ArcTanh[c*x]/(2*c))))/(3*c)))/2 + (3*((x*(a + b*ArcSin[c*x])^2)/(2*(1
- c^2*x^2)) - b*c*((a + b*ArcSin[c*x])/(c^2*Sqrt[1 - c^2*x^2]) - (b*ArcTan
h[c*x])/c^2) + ((-2*I)*(a + b*ArcSin[c*x])^2*ArcTan[E^(I*ArcSin[c*x])]) + 2
*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - b*PolyLog[3
, (-I)*E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, I*E^(I*
ArcSin[c*x])] - b*PolyLog[3, I*E^(I*ArcSin[c*x])])/(2*c))/4))/(3*d^3)

```

Defintions of rubi rules used

rule 27

```

Int[(a_)*(F_x), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x)] /; FreeQ[b, x]

```

rule 215

```

Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1)) Int[(a + b*x^2)^(p + 1)
, x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])

```

rule 219

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

rule 253 $\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^2)^p, x_Symbol] \rightarrow \text{Simp}[-(c \cdot x)^{m+1} \cdot (a + b \cdot x^2)^{p+1} / (2 \cdot a \cdot c \cdot (p+1)), x] + \text{Simp}[(m+2 \cdot p+3) / (2 \cdot a \cdot (p+1)) \text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^2)^{p+1}, x], x] /;$ FreeQ[{a, b, c, m}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]

rule 264 $\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^2)^p, x_Symbol] \rightarrow \text{Simp}[(c \cdot x)^{m+1} \cdot (a + b \cdot x^2)^{p+1} / (a \cdot c \cdot (m+1)), x] - \text{Simp}[b \cdot (m+2 \cdot p+3) / (a \cdot c^2 \cdot (m+1)) \text{Int}[(c \cdot x)^{m+2} \cdot (a + b \cdot x^2)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]

rule 2715 $\text{Int}[\text{Log}[a + b \cdot x] \cdot (F^{(e \cdot x + d \cdot x)})^n, x_Symbol] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

rule 2720 $\text{Int}[u, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v / D[v, x] \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x] / x, x], x, v], x] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w) \cdot (a) \cdot (v)^n] /; FreeQ[{a, m, n}, x] && IntegerQ[m \cdot n] && !MatchQ[u, E^{(a + b \cdot x)} \cdot (F)[v] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

rule 2838 $\text{Int}[\text{Log}[(c \cdot x)^d + (e \cdot x)^n] / (x), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n / n, x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c \cdot d, 1]

rule 3011 $\text{Int}[\text{Log}[1 + (e \cdot x)^n] \cdot (F^{(a + b \cdot x)})^m \cdot (f + g \cdot x)^m, x_Symbol] \rightarrow \text{Simp}[-(f + g \cdot x)^m \cdot (\text{PolyLog}[2, (-e) \cdot (F^{(a + b \cdot x)})^n] / (b \cdot c \cdot n \cdot \text{Log}[F])), x] + \text{Simp}[g \cdot m / (b \cdot c \cdot n \cdot \text{Log}[F]) \text{Int}[(f + g \cdot x)^{m-1} \cdot \text{PolyLog}[2, (-e) \cdot (F^{(a + b \cdot x)})^n], x], x] /;$ FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ FunctionOfTrigOfLinearQ[u, x]

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
  := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x]
  && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x]
  && IGtQ[m, 0]
```

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
  := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))
  Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x]
  && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x]
  && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
  := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```


rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.51

method	result
derivativedivides	$c^3 \left(-\frac{a^2 \left(\frac{1}{16(cx+1)^2} + \frac{11}{16(cx+1)} - \frac{35 \ln(cx+1)}{16} + \frac{1}{3c^3 x^3} + \frac{3}{cx} - \frac{1}{16(cx-1)^2} + \frac{11}{16(cx-1)} + \frac{35 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(\frac{105 \arcsin(cx)}{16} \right)}{d^3} \right)$
default	$c^3 \left(-\frac{a^2 \left(\frac{1}{16(cx+1)^2} + \frac{11}{16(cx+1)} - \frac{35 \ln(cx+1)}{16} + \frac{1}{3c^3 x^3} + \frac{3}{cx} - \frac{1}{16(cx-1)^2} + \frac{11}{16(cx-1)} + \frac{35 \ln(cx-1)}{16} \right)}{d^3} - \frac{b^2 \left(\frac{105 \arcsin(cx)}{16} \right)}{d^3} \right)$
parts	$-\frac{a^2 \left(-\frac{c^3}{16(cx-1)^2} + \frac{11c^3}{16(cx-1)} + \frac{35c^3 \ln(cx-1)}{16} + \frac{c^3}{16(cx+1)^2} + \frac{11c^3}{16(cx+1)} - \frac{35c^3 \ln(cx+1)}{16} + \frac{1}{3x^3} + \frac{3c^2}{x} \right)}{d^3} - \frac{b^2 c^3 \left(\frac{105 \arcsin(cx)}{16} \right)}{d^3}$

input `int((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```

c^3*(-a^2/d^3*(1/16/(c*x+1)^2+11/16/(c*x+1)-35/16*ln(c*x+1)+1/3/c^3/x^3+3/
c/x-1/16/(c*x-1)^2+11/16/(c*x-1)+35/16*ln(c*x-1))-b^2/d^3*(1/24*(105*arcsi
n(c*x)^2*c^6*x^6-58*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^5*c^5-175*arcsin(c*x)
^2*x^4*c^4+10*c^6*x^6+54*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^3*c^3+56*arcsin(
c*x)^2*x^2*c^2-18*c^4*x^4+8*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+8*arcsin(c*
x)^2+8*c^2*x^2)/(c^4*x^4-2*c^2*x^2+1)/c^3/x^3-35/8*arcsin(c*x)^2*ln(1-I*(I
*c*x+(-c^2*x^2+1)^(1/2)))+35/4*I*arcsin(c*x)*polylog(2,I*(I*c*x+(-c^2*x^2+
1)^(1/2)))-35/4*polylog(3,I*(I*c*x+(-c^2*x^2+1)^(1/2)))+19/3*arcsin(c*x)*l
n(1+I*c*x+(-c^2*x^2+1)^(1/2))-19/3*I*dilog(I*c*x+(-c^2*x^2+1)^(1/2))-19/3*
I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))+17/3*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2)
)+35/8*arcsin(c*x)^2*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-35/4*I*arcsin(c*x)
*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+35/4*polylog(3,-I*(I*c*x+(-c^2*x
^2+1)^(1/2))))-2*a*b/d^3*(1/24*(105*arcsin(c*x)*c^6*x^6-29*c^5*x^5*(-c^2*x
^2+1)^(1/2)-175*c^4*x^4*arcsin(c*x)+27*c^3*x^3*(-c^2*x^2+1)^(1/2)+56*c^2*x
^2*arcsin(c*x)+4*c*x*(-c^2*x^2+1)^(1/2)+8*arcsin(c*x))/(c^4*x^4-2*c^2*x^2+
1)/c^3/x^3-19/6*ln(I*c*x+(-c^2*x^2+1)^(1/2))-1)+19/6*ln(1+I*c*x+(-c^2*x^2+1)
^(1/2))+35/8*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-35/8*arcsin(c
*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-35/8*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)
^(1/2)))+35/8*I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))
    
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx$$

$$= -\frac{\int \frac{a^2}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx + \int \frac{b^2 \arcsin^2(cx)}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx + \int \frac{2ab \arcsin(cx)}{c^6 x^{10} - 3c^4 x^8 + 3c^2 x^6 - x^4} dx}{d^3}$$

input `integrate((a+b*asin(c*x))**2/x**4/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Integral(b**2*asin(c*x)**2/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Integral(2*a*b*asin(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x))/d**3`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2}{(c^2 dx^2 - d)^3 x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output

```
1/48*a^2*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6
*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^
3)) + 1/48*(105*(b^2*c^7*x^7 - 2*b^2*c^5*x^5 + b^2*c^3*x^3)*arctan2(c*x, s
qrt(c*x + 1)*sqrt(-c*x + 1))^2*log(c*x + 1) - 105*(b^2*c^7*x^7 - 2*b^2*c^5
*x^5 + b^2*c^3*x^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2*log(-c*x
+ 1) - 2*(105*b^2*c^6*x^6 - 175*b^2*c^4*x^4 + 56*b^2*c^2*x^2 + 8*b^2)*arct
an2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 48*(c^4*d^3*x^7 - 2*c^2*d^3*x^5
+ d^3*x^3)*integrate(-1/24*(48*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x +
1)) - (105*(b^2*c^8*x^8 - 2*b^2*c^6*x^6 + b^2*c^4*x^4)*arctan2(c*x, sqrt(
c*x + 1)*sqrt(-c*x + 1))*log(c*x + 1) - 105*(b^2*c^8*x^8 - 2*b^2*c^6*x^6 +
b^2*c^4*x^4)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*log(-c*x + 1) - 2
*(105*b^2*c^7*x^7 - 175*b^2*c^5*x^5 + 56*b^2*c^3*x^3 + 8*b^2*c*x)*arctan2(
c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^6*d^3
*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x))/(c^4*d^3*x^7 - 2*c^2
*d^3*x^5 + d^3*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx = \text{Timed out}$$

input

```
integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output

Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^3} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^4 (d - c^2 dx^2)^3} dx$$

input

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^3),x)
```

output

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^3), x)
```


3.207 $\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$

Optimal result	1993
Mathematica [A] (verified)	1994
Rubi [A] (verified)	1994
Maple [A] (verified)	1999
Fricas [A] (verification not implemented)	2000
Sympy [F]	2001
Maxima [A] (verification not implemented)	2001
Giac [F(-2)]	2002
Mupad [F(-1)]	2002
Reduce [F]	2003

Optimal result

Integrand size = 29, antiderivative size = 321

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \frac{52b^2 \sqrt{d - c^2 dx^2}}{225c^4} + \frac{26b^2 (d - c^2 dx^2)^{3/2}}{675c^4 d}$$

$$- \frac{2b^2 (d - c^2 dx^2)^{5/2}}{125c^4 d^2}$$

$$+ \frac{4bx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{15c^3 \sqrt{1 - c^2 x^2}}$$

$$+ \frac{2bx^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{45c \sqrt{1 - c^2 x^2}}$$

$$- \frac{2bcx^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{25 \sqrt{1 - c^2 x^2}}$$

$$- \frac{2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{15c^4}$$

$$- \frac{x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{15c^2}$$

$$+ \frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2$$

output

```
52/225*b^2*(-c^2*d*x^2+d)^(1/2)/c^4+26/675*b^2*(-c^2*d*x^2+d)^(3/2)/c^4/d-
2/125*b^2*(-c^2*d*x^2+d)^(5/2)/c^4/d^2+4/15*b*x*(-c^2*d*x^2+d)^(1/2)*(a+b*
arcsin(c*x))/c^3/(-c^2*x^2+1)^(1/2)+2/45*b*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-2/25*b*c*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
csin(c*x))/(-c^2*x^2+1)^(1/2)-2/15*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^
2/c^4-1/15*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^2+1/5*x^4*(-c^2*
d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.75

$$\int x^3 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d - c^2 x^2} (225 a^2 \sqrt{1 - c^2 x^2} (-2 - c^2 x^2 + 3 c^4 x^4) - 30 a b c x (-30 - 5 c^2 x^2 + 9 c^4 x^4) - 2 b^2 \sqrt{1 - c^2 x^2} (-42$$

input

```
Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(225*a^2*Sqrt[1 - c^2*x^2]*(-2 - c^2*x^2 + 3*c^4*x^4)
- 30*a*b*c*x*(-30 - 5*c^2*x^2 + 9*c^4*x^4) - 2*b^2*Sqrt[1 - c^2*x^2]*(-42
8 + 11*c^2*x^2 + 27*c^4*x^4) - 30*b*(15*a*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2 -
3*c^4*x^4) + b*c*x*(-30 - 5*c^2*x^2 + 9*c^4*x^4))*ArcSin[c*x] + 225*b^2*S
qrt[1 - c^2*x^2]*(-2 - c^2*x^2 + 3*c^4*x^4)*ArcSin[c*x]^2))/(3375*c^4*Sqrt
[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5198, 5138, 243, 53, 2009, 5210, 5138, 243, 53, 2009, 5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx \\
& \quad \downarrow \text{5198} \\
& - \frac{2bc\sqrt{d - c^2 dx^2} \int x^4 (a + b \arcsin(cx)) dx}{5\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{5138} \\
& - \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{5} bc \int \frac{x^5}{\sqrt{1 - c^2 x^2}} dx \right)}{5\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} + \frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{243} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} - \\
& \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{10} bc \int \frac{x^4}{\sqrt{1 - c^2 x^2}} dx^2 \right)}{5\sqrt{1 - c^2 x^2}} + \frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + \\
& \quad b \arcsin(cx))^2 \\
& \quad \downarrow \text{53} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} - \\
& \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{10} bc \int \left(\frac{(1 - c^2 x^2)^{3/2}}{c^4} - \frac{2\sqrt{1 - c^2 x^2}}{c^4} + \frac{1}{c^4 \sqrt{1 - c^2 x^2}} \right) dx^2 \right)}{5\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{2009} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} + \frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \\
& \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{10} bc \left(-\frac{2(1 - c^2 x^2)^{5/2}}{5c^6} + \frac{4(1 - c^2 x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1 - c^2 x^2}}{c^6} \right) \right)}{5\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5210}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{3c^2} + \frac{2b \int x^2(a+b \arcsin(cx)) dx}{3c} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \\
& \frac{\frac{1}{5}x^4 \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - 2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5\sqrt{1-c^2x^2}}}{5\sqrt{1-c^2x^2}} \quad \downarrow \text{5138} \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{3}bc \int \frac{x^3}{\sqrt{1-c^2x^2}} dx \right)}{3c} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \\
& \frac{\frac{1}{5}x^4 \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - 2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5\sqrt{1-c^2x^2}}}{5\sqrt{1-c^2x^2}} \quad \downarrow \text{243} \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx^2 \right)}{3c} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \\
& \frac{\frac{1}{5}x^4 \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - 2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5\sqrt{1-c^2x^2}}}{5\sqrt{1-c^2x^2}} \quad \downarrow \text{53} \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \left(\frac{1}{c^2 \sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{c^2} \right) dx^2 \right)}{3c} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \\
& \frac{\frac{1}{5}x^4 \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - 2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5\sqrt{1-c^2x^2}}}{5\sqrt{1-c^2x^2}}
\end{aligned}$$

↓ 2009

$$\sqrt{d - c^2 dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2 x^2}}}{3c^2} - \frac{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3c^2} + \frac{2b \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \left(\frac{2(1-c^2 x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2 x^2}}{c^4} \right) \right)}{3c} \right)$$

$$\frac{\frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - 2bc \sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{10} bc \left(-\frac{2(1-c^2 x^2)^{5/2}}{5c^6} + \frac{4(1-c^2 x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2 x^2}}{c^6} \right) \right)}{5\sqrt{1 - c^2 x^2}}$$

↓ 5182

$$\sqrt{d - c^2 dx^2} \left(\frac{2 \left(\frac{2b \int (a+b \arcsin(cx)) dx}{c} - \frac{\sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3c^2} + \frac{2b \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \left(\frac{2(1-c^2 x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2 x^2}}{c^4} \right) \right)}{3c} \right)$$

$$\frac{\frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - 2bc \sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{10} bc \left(-\frac{2(1-c^2 x^2)^{5/2}}{5c^6} + \frac{4(1-c^2 x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2 x^2}}{c^6} \right) \right)}{5\sqrt{1 - c^2 x^2}}$$

↓ 2009

$$\frac{\frac{1}{5} x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - 2bc \sqrt{d - c^2 dx^2} \left(\frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{10} bc \left(-\frac{2(1-c^2 x^2)^{5/2}}{5c^6} + \frac{4(1-c^2 x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2 x^2}}{c^6} \right) \right)}{5\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} \left(-\frac{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3c^2} + \frac{2 \left(\frac{2b \left(\frac{ax + bx \arcsin(cx) + \frac{b\sqrt{1-c^2 x^2}}{c}}{c} - \frac{\sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{c^2} \right)}{3c^2} \right)}{3c^2} + \frac{2b \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \left(\frac{2(1-c^2 x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2 x^2}}{c^4} \right) \right)}{3c} \right)$$

$$5\sqrt{1 - c^2 x^2}$$

input

Int [x^3*sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]

output

$$\begin{aligned} & (x^4 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2) / 5 - (2 b c \sqrt{d - c^2 d x^2} \\ & (-1/10 (b c ((-2 \sqrt{1 - c^2 x^2}) / c^6 + (4 (1 - c^2 x^2)^{3/2}) / (3 c^6) \\ & - (2 (1 - c^2 x^2)^{5/2}) / (5 c^6))) + (x^5 (a + b \operatorname{ArcSin}[c x]) / 5)) / (\\ & 5 \sqrt{1 - c^2 x^2}) + (\sqrt{d - c^2 d x^2} (-1/3 (x^2 \sqrt{1 - c^2 x^2} (\\ & a + b \operatorname{ArcSin}[c x])^2) / c^2 + (2 b (-1/6 (b c ((-2 \sqrt{1 - c^2 x^2}) / c^4 + \\ & (2 (1 - c^2 x^2)^{3/2}) / (3 c^4))) + (x^3 (a + b \operatorname{ArcSin}[c x]) / 3)) / (3 c) + \\ & (2 (-((\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2) / c^2) + (2 b (a x + (b \sqrt{1 - c^2 x^2}) / c + b x \operatorname{ArcSin}[c x])) / c)) / (3 c^2))) / (5 \sqrt{1 - c^2 x^2}) \end{aligned}$$

Defintions of rubi rules used

rule 53

$$\text{Int}[\{(a_.) + (b_.)(x_.)^{(m_.)}\} \{(c_.) + (d_.)(x_.)^{(n_.)}\}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \mid\mid (\text{EqQ}[c, 0] \&\& \text{LeQ}[7 m + 4 n + 4, 0]) \mid\mid \text{LtQ}[9 m + 5(n + 1), 0] \mid\mid \text{GtQ}[m + n + 2, 0])$$

rule 243

$$\text{Int}[(x_.)^{(m_.)} \{(a_.) + (b_.)(x_.)^2\}^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{Subst}[\text{Int}[x^{((m - 1)/2)} (a + b x)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \&\& \text{IntegerQ}[(m - 1)/2]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

rule 5138

$$\text{Int}[\{(a_.) + \operatorname{ArcSin}[c_.)(x_.)](b_.)^{(n_.)} \{(d_.)(x_.)^{(m_.)}\}, x_Symbol] \rightarrow \text{Simp}[(d x)^{m+1} \{(a + b \operatorname{ArcSin}[c x])^n / (d(m+1))\}, x] - \text{Simp}[b c (n / (d(m+1))) \text{Int}[(d x)^{m+1} \{(a + b \operatorname{ArcSin}[c x])^{n-1} / \sqrt{1 - c^2 x^2}\}], x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$

rule 5182

$$\text{Int}[\{(a_.) + \operatorname{ArcSin}[c_.)(x_.)](b_.)^{(n_.)}(x_.) \{(d_.) + (e_.)(x_.)^2\}^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e x^2)^{p+1} \{(a + b \operatorname{ArcSin}[c x])^n / (2 e (p + 1))\}, x] + \text{Simp}[b (n / (2 c (p + 1))) \text{Simp}[(d + e x^2)^p / (1 - c^2 x^2)^p] \text{Int}[(1 - c^2 x^2)^{(p+1/2)} (a + b \operatorname{ArcSin}[c x])^{n-1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{EqQ}[c^2 d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[p, -1]$$

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.57

method	result
orering	$\frac{(1647c^8x^8 - 2131c^6x^6 - 8610c^4x^4 + 13060c^2x^2 - 5136)\sqrt{-c^2dx^2 + d}(a + b\arcsin(cx))^2}{3375(c^2x^2 - 1)c^6x^2} - \frac{4(81c^6x^6 - 40c^4x^4 - 878c^2x^2 + 642)(3x^2 + 2)}{3375(c^2x^2 - 1)c^6x^2}$
default	Expression too large to display
parts	Expression too large to display

input

```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/3375*(1647*c^8*x^8-2131*c^6*x^6-8610*c^4*x^4+13060*c^2*x^2-5136)/(c^2*x^2-1)/c^6/x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-4/3375*(81*c^6*x^6-40*c^4*x^4-878*c^2*x^2+642)/c^6/x^4*(3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*c^2*d+2*b*c*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2))+1/3375*(27*c^4*x^4+11*c^2*x^2-428)/c^6*(c*x-1)/x^3*(c*x+1)*(6*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-7*x^3/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*c^2*d+12*b*c*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-x^5/(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2*c^4*d^2-4*x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*c^3*d*b/(-c^2*x^2+1)^(1/2)+2*b^2*c^2*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)+2*b*c^3*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.86

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{30(9abc^5x^5 - 5abc^3x^3 - 30abcx + (9b^2c^5x^5 - 5b^2c^3x^3 - 30b^2cx) \arcsin(cx)) \sqrt{-c^2dx^2 + d} \sqrt{-c^2x^2 + 1}}{1}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
1/3375*(30*(9*a*b*c^5*x^5 - 5*a*b*c^3*x^3 - 30*a*b*c*x + (9*b^2*c^5*x^5 - 5*b^2*c^3*x^3 - 30*b^2*c*x)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + (27*(25*a^2 - 2*b^2)*c^6*x^6 - 4*(225*a^2 - 8*b^2)*c^4*x^4 - (225*a^2 - 878*b^2)*c^2*x^2 + 225*(3*b^2*c^6*x^6 - 4*b^2*c^4*x^4 - b^2*c^2*x^2 + 2*b^2)*arcsin(c*x)^2 + 450*a^2 - 856*b^2 + 450*(3*a*b*c^6*x^6 - 4*a*b*c^4*x^4 - a*b*c^2*x^2 + 2*a*b)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int x^3 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2 dx$$

input `integrate(x**3*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2, x)`

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\begin{aligned} & \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx \\ &= -\frac{1}{15} b^2 \left(\frac{3(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^4 d} \right) \arcsin(cx)^2 \\ & \quad - \frac{2}{15} ab \left(\frac{3(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^4 d} \right) \arcsin(cx) \\ & \quad - \frac{1}{15} a^2 \left(\frac{3(-c^2 dx^2 + d)^{\frac{3}{2}} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{\frac{3}{2}}}{c^4 d} \right) \\ & \quad - \frac{2}{3375} b^2 \left(\frac{27 \sqrt{-c^2 x^2 + 1} c^2 \sqrt{dx^4} + 11 \sqrt{-c^2 x^2 + 1} \sqrt{dx^2} - \frac{428 \sqrt{-c^2 x^2 + 1} \sqrt{d}}{c^2}}{c^2} + \frac{15 (9 c^4 \sqrt{dx^5} - 5 c^2 \sqrt{dx^3} - 30 \sqrt{dx})}{c^3} \right) \\ & \quad - \frac{2 (9 c^4 \sqrt{dx^5} - 5 c^2 \sqrt{dx^3} - 30 \sqrt{dx}) ab}{225 c^3} \end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
-1/15*b^2*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)
/(c^4*d))*arcsin(c*x)^2 - 2/15*a*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) +
2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d))*arcsin(c*x) - 1/15*a^2*(3*(-c^2*d*x^2 +
d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) - 2/3375*b^2*(2
7*sqrt(-c^2*x^2 + 1)*c^2*sqrt(d)*x^4 + 11*sqrt(-c^2*x^2 + 1)*sqrt(d)*x^2 -
428*sqrt(-c^2*x^2 + 1)*sqrt(d)/c^2)/c^2 + 15*(9*c^4*sqrt(d)*x^5 - 5*c^2*s
qrt(d)*x^3 - 30*sqrt(d)*x)*arcsin(c*x)/c^3) - 2/225*(9*c^4*sqrt(d)*x^5 - 5
*c^2*sqrt(d)*x^3 - 30*sqrt(d)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac"
)
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^2 dx = \int x^3 (a + b \arcsin(cx))^2 \sqrt{d - c^2 x^2} dx$$

input

```
int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output

```
int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d} (3\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 - \sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a^2 + 30 \int \sqrt{-c^2 x^2 + 1} a \sin(cx) x^3 dx) a}{15c^4}$$

input

```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2,x)
```

output

```
(sqrt(d)*(3*sqrt(-c**2*x**2+1)*a**2*c**4*x**4 - sqrt(-c**2*x**2+1)
*a**2*c**2*x**2 - 2*sqrt(-c**2*x**2+1)*a**2 + 30*int(sqrt(-c**2*x**2
+1)*asin(c*x)*x**3,x)*a*b*c**4 + 15*int(sqrt(-c**2*x**2+1)*asin(c*x)
**2*x**3,x)*b**2*c**4))/(15*c**4)
```


3.208 $\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$

Optimal result	2004
Mathematica [A] (verified)	2005
Rubi [A] (verified)	2005
Maple [C] (verified)	2010
Fricas [F]	2011
Sympy [F]	2011
Maxima [F]	2012
Giac [A] (verification not implemented)	2012
Mupad [F(-1)]	2013
Reduce [F]	2013

Optimal result

Integrand size = 29, antiderivative size = 303

$$\begin{aligned}
 \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = & \frac{b^2 x \sqrt{d - c^2 dx^2}}{64c^2} - \frac{1}{32} b^2 x^3 \sqrt{d - c^2 dx^2} \\
 & - \frac{b^2 \sqrt{d - c^2 dx^2} \arcsin(cx)}{64c^3 \sqrt{1 - c^2 x^2}} \\
 & + \frac{bx^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{8c \sqrt{1 - c^2 x^2}} \\
 & - \frac{bcx^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{8 \sqrt{1 - c^2 x^2}} \\
 & - \frac{x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{8c^2} \\
 & + \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
 & + \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^3}{24bc^3 \sqrt{1 - c^2 x^2}}
 \end{aligned}$$

output

```
1/64*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2-1/32*b^2*x^3*(-c^2*d*x^2+d)^(1/2)-1/64
*b^2*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/c^3/(-c^2*x^2+1)^(1/2)+1/8*b*x^2*(-c
^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-1/8*b*c*x^4*(-c^2
*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/8*x*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))^2/c^2+1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x
))^2+1/24*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.81

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d - c^2 dx^2} (8a^3 + 3b^3 cx (1 - 2c^2 x^2) \sqrt{1 - c^2 x^2} - 24ab^2 c^2 x^2 (-1 + c^2 x^2) + 24a^2 bcx \sqrt{1 - c^2 x^2} (-1 + 2c^2 x^2))}{192 b^3 c^3 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(8*a^3 + 3*b^3*c*x*(1 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2]
- 24*a*b^2*c^2*x^2*(-1 + c^2*x^2) + 24*a^2*b*c*x*Sqrt[1 - c^2*x^2]*(-1 + 2
*c^2*x^2) - 3*b*(-8*a^2 + 16*a*b*c*x*(1 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + b
^2*(1 - 8*c^2*x^2 + 8*c^4*x^4))*ArcSin[c*x] + 24*b^2*(a + b*c*x*Sqrt[1 - c
^2*x^2]*(-1 + 2*c^2*x^2))*ArcSin[c*x]^2 + 8*b^3*ArcSin[c*x]^3)/(192*b*c^3
*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5198, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx \\
& \quad \downarrow \text{5198} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^3 (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{5138} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \\
& \frac{bc\sqrt{d - c^2 dx^2} \left(\frac{1}{4} x^4 (a + b \arcsin(cx)) - \frac{1}{4} bc \int \frac{x^4}{\sqrt{1 - c^2 x^2}} dx \right)}{2\sqrt{1 - c^2 x^2}} + \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{262} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \\
& \frac{bc\sqrt{d - c^2 dx^2} \left(\frac{1}{4} x^4 (a + b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{3 \int \frac{x^2}{\sqrt{1 - c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1 - c^2 x^2}}{4c^2} \right) \right)}{2\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{262} \\
& \frac{\sqrt{d - c^2 dx^2} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \\
& \frac{bc\sqrt{d - c^2 dx^2} \left(\frac{1}{4} x^4 (a + b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1 - c^2 x^2}} dx}{2c^2} - \frac{x \sqrt{1 - c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1 - c^2 x^2}}{4c^2} \right) \right)}{2\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{4} x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{223}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 - \\
& bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \\
& \hline
& 2\sqrt{1-c^2x^2} \\
& \downarrow \text{5210} \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x(a+b \arcsin(cx)) dx}{c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \\
& \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 - \\
& bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \\
& \hline
& 2\sqrt{1-c^2x^2} \\
& \downarrow \text{5138} \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \\
& \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 - \\
& bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \\
& \hline
& 2\sqrt{1-c^2x^2} \\
& \downarrow \text{262} \\
& \frac{\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \\
& \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 - \\
& bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \\
& \hline
& 2\sqrt{1-c^2x^2}
\end{aligned}$$

223

$$\frac{\sqrt{d-c^2x^2} \left(\frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right)}{4\sqrt{1-c^2x^2}} + \frac{\frac{1}{4}x^3\sqrt{d-c^2x^2}(a+b \arcsin(cx))^2 - bc\sqrt{d-c^2x^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2\sqrt{1-c^2x^2}}}{2\sqrt{1-c^2x^2}}$$

5152

$$\frac{\frac{1}{4}x^3\sqrt{d-c^2x^2}(a+b \arcsin(cx))^2 + \sqrt{d-c^2x^2} \left(\frac{(a+b \arcsin(cx))^3}{6bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right)}{4\sqrt{1-c^2x^2}}}{2\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2x^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2\sqrt{1-c^2x^2}}$$

input `Int [x^2*sqrt [d - c^2*d*x^2]*(a + b*ArcSin [c*x])^2,x]`

output `(x^3*sqrt [d - c^2*d*x^2]*(a + b*ArcSin [c*x])^2)/4 - (b*c*sqrt [d - c^2*d*x^2]*((x^4*(a + b*ArcSin [c*x]))/4 - (b*c*(-1/4*(x^3*sqrt [1 - c^2*x^2])/c^2 + (3*(-1/2*(x*sqrt [1 - c^2*x^2])/c^2 + ArcSin [c*x]/(2*c^3)))/(4*c^2)))/4))/ (2*sqrt [1 - c^2*x^2]) + (sqrt [d - c^2*d*x^2]*(-1/2*(x*sqrt [1 - c^2*x^2]*(a + b*ArcSin [c*x])^2)/c^2 + (a + b*ArcSin [c*x])^3/(6*b*c^3) + (b*((x^2*(a + b*ArcSin [c*x]))/2 - (b*c*(-1/2*(x*sqrt [1 - c^2*x^2])/c^2 + ArcSin [c*x]/(2*c^3)))/2))/c)/(4*sqrt [1 - c^2*x^2])`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5198 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+2))), x] + (\text{Simp}[(1/(m+2))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^m*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m+2)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])$

rule 5210

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]

```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.24

method	result
default	$-\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d} + \frac{a^2x\sqrt{-c^2dx^2+d}}{8c^2} + \frac{a^2d \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^3}{24c^3(c^2x^2-1)} + \dots \right)$
parts	$-\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4c^2d} + \frac{a^2x\sqrt{-c^2dx^2+d}}{8c^2} + \frac{a^2d \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{8c^2\sqrt{c^2d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^3}{24c^3(c^2x^2-1)} + \dots \right)$

input

```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a^2/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/24*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^3+1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(4*I*arcsin(c*x)+8*arcsin(c*x)^2-1)/c^3/(c^2*x^2-1)+1/512*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-4*I*arcsin(c*x)+8*arcsin(c*x)^2-1)/c^3/(c^2*x^2-1))+2*a*b*(-1/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^2+1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-I+4*arcsin(c*x))/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2 x^2 dx$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
integral((b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int x^2 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2 dx$$

input

```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2,x)
```


output `Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2 x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `1/8*a^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*arcsin(c*x)/c^3) + sqrt(d)*integrate((b^2*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)`

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.27

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{1}{4} \sqrt{-c^2 dx^2 + d} a^2 x^3 - \frac{\sqrt{-c^2 dx^2 + d} a^2 x}{8 c^2} - \frac{a^2 d \log(|-c\sqrt{-dx} + \sqrt{c^2 x^2 - 1}\sqrt{-d}|)}{8 c^3 \sqrt{-d}}$$

$$- \frac{48(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 \sqrt{dx} \arcsin(cx)^2 + 96(-c^2 x^2 + 1)^{\frac{3}{2}} ab \sqrt{dx} \arcsin(cx) - 24 \sqrt{-c^2 x^2 + 1} b^2 \sqrt{dx} \arcsin(cx)}{8 c^3 \sqrt{-d}}$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
1/4*sqrt(-c^2*d*x^2 + d)*a^2*x^3 - 1/8*sqrt(-c^2*d*x^2 + d)*a^2*x/c^2 - 1/
8*a^2*d*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)
) - 1/192*(48*(-c^2*x^2 + 1)^(3/2)*b^2*sqrt(d)*x*arcsin(c*x)^2 + 96*(-c^2*
x^2 + 1)^(3/2)*a*b*sqrt(d)*x*arcsin(c*x) - 24*sqrt(-c^2*x^2 + 1)*b^2*sqrt(
d)*x*arcsin(c*x)^2 - 6*(-c^2*x^2 + 1)^(3/2)*b^2*sqrt(d)*x - 48*sqrt(-c^2*x
^2 + 1)*a*b*sqrt(d)*x*arcsin(c*x) + 24*(c^2*x^2 - 1)^2*b^2*sqrt(d)*arcsin(
c*x)/c - 8*b^2*sqrt(d)*arcsin(c*x)^3/c + 3*sqrt(-c^2*x^2 + 1)*b^2*sqrt(d)*
x + 24*(c^2*x^2 - 1)^2*a*b*sqrt(d)/c + 24*(c^2*x^2 - 1)*b^2*sqrt(d)*arcsin
(c*x)/c - 24*a*b*sqrt(d)*arcsin(c*x)^2/c + 24*(c^2*x^2 - 1)*a*b*sqrt(d)/c
+ 3*b^2*sqrt(d)*arcsin(c*x)/c + 3*a*b*sqrt(d)/c/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^2 dx = \int x^2 (a + b \arcsin(cx))^2 \sqrt{d - c^2 x^2} dx$$

input

```
int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output

```
int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d} (a \sin(cx))^2 + 2\sqrt{-c^2 x^2 + 1} a^2 c^3 x^3 - \sqrt{-c^2 x^2 + 1} a^2 c x + 16 \left(\int \sqrt{-c^2 x^2 + 1} a \sin(cx) x^2 dx \right) a b c^3 + 8 b^2 c^3}{8c^3}$$

input

```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2,x)
```

output

```
(sqrt(d)*(asin(c*x))*a**2 + 2*sqrt(-c**2*x**2 + 1)*a**2*c**3*x**3 - sqrt(
-c**2*x**2 + 1)*a**2*c*x + 16*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,
x)*a*b*c**3 + 8*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**3
)/(8*c**3)
```

3.209 $\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2 dx$

Optimal result	2014
Mathematica [A] (verified)	2015
Rubi [A] (verified)	2015
Maple [B] (verified)	2018
Fricas [A] (verification not implemented)	2018
Sympy [F]	2019
Maxima [A] (verification not implemented)	2019
Giac [F(-2)]	2020
Mupad [F(-1)]	2020
Reduce [F]	2021

Optimal result

Integrand size = 27, antiderivative size = 181

$$\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2 dx = \frac{4b^2\sqrt{d - c^2dx^2}}{9c^2} + \frac{2b^2(d - c^2dx^2)^{3/2}}{27c^2d} + \frac{2bx\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{3c\sqrt{1 - c^2x^2}} - \frac{2bcx^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{9\sqrt{1 - c^2x^2}} - \frac{(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))^2}{3c^2d}$$

output

```
4/9*b^2*(-c^2*d*x^2+d)^(1/2)/c^2+2/27*b^2*(-c^2*d*x^2+d)^(3/2)/c^2/d+2/3*b
*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-2/9*b*c*x^3
*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2
+d)^(3/2)*(a+b*arcsin(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.66

$$\int x\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d - c^2 dx^2} \left((-1 + c^2 x^2) (a + b \arcsin(cx))^2 - \frac{2b(b\sqrt{1-c^2 x^2}(-7+c^2 x^2) + 3acx(-3+c^2 x^2) + 3bcx(-3+c^2 x^2) \arcsin(cx))}{9\sqrt{1-c^2 x^2}} \right)}{3c^2}$$

input `Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]`

output `(Sqrt[d - c^2*d*x^2]*((-1 + c^2*x^2)*(a + b*ArcSin[c*x])^2 - (2*b*(b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + 3*a*c*x*(-3 + c^2*x^2) + 3*b*c*x*(-3 + c^2*x^2)*ArcSin[c*x]))/(9*Sqrt[1 - c^2*x^2])))/(3*c^2)`

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5182, 5154, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2 dx$$

$$\downarrow 5182$$

$$\frac{2b\sqrt{d - c^2 dx^2} \int (1 - c^2 x^2) (a + b \arcsin(cx)) dx}{3c\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3c^2 d}$$

$$\downarrow 5154$$

$$\frac{2b\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x(3-c^2 x^2)}{3\sqrt{1-c^2 x^2}} dx - \frac{1}{3}c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{3c\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3c^2 d}$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\int\frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{\frac{3c\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}} \quad \downarrow 27$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{6}bc\int\frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{\frac{3c\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}} \quad \downarrow 353$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{\frac{3c\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}} \quad \downarrow 53$$

$$\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))-\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{\frac{3c\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}} \quad \downarrow 2009$$

input `Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]`

output `-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*Sqrt[d - c^2*d*x^2]*(-1/6*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSin[c*x]) - (c^2*x^3*(a + b*ArcSin[c*x]))/3)/(3*c*Sqrt[1 - c^2*x^2])`

Definitions of rubi rules used

- rule 27 `Int[(a_)*(Fx), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx) /; FreeQ[b, x]]`
- rule 53 `Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`
- rule 353 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 5154 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`
- rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(157) = 314.

Time = 0.51 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.43

method	result
orering	$\frac{(19c^6x^6-71c^4x^4+48c^2x^2-14)\sqrt{-c^2dx^2+d}(a+b\arcsin(cx))^2}{27(c^2x^2-1)c^4x^2} - \frac{2(3c^4x^4-16c^2x^2+7)\left(\sqrt{-c^2dx^2+d}(a+b\arcsin(cx))^2 - \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{-c^2dx^2+d}}\right)}{27c^4x^2}$
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d} + b^2 \left(\frac{\sqrt{-d(c^2x^2-1)}(4c^4x^4-5c^2x^2-4i\sqrt{-c^2x^2+1}x^3c^3+3i\sqrt{-c^2x^2+1}cx+1)(6i\arcsin(cx)+9\arcsin(cx))}{216c^2(c^2x^2-1)} \right)$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d} + b^2 \left(\frac{\sqrt{-d(c^2x^2-1)}(4c^4x^4-5c^2x^2-4i\sqrt{-c^2x^2+1}x^3c^3+3i\sqrt{-c^2x^2+1}cx+1)(6i\arcsin(cx)+9\arcsin(cx))}{216c^2(c^2x^2-1)} \right)$

```
input int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

```
output 1/27*(19*c^6*x^6-71*c^4*x^4+48*c^2*x^2-14)/(c^2*x^2-1)/c^4/x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-2/27*(3*c^4*x^4-16*c^2*x^2+7)/c^4/x^2*((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-x^2/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*c^2*d+2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/27*(c^2*x^2-7)/c^4*(c*x-1)/x*(c*x+1)*(-3/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*c^2*d*x+4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2)-x^3/(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2*c^4*d^2-4*x^2/(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*c^3*d*b/(-c^2*x^2+1)^(1/2)+2*x*(-c^2*d*x^2+d)^(1/2)*b^2*c^2/(-c^2*x^2+1)+2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.15

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 dx$$

$$= \frac{6(abc^3x^3-3abcx+(b^2c^3x^3-3b^2cx)\arcsin(cx))\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}+((9a^2-2b^2)c^4x^4-2(9a^2b-2b^3c)x^3+(9a^2c^2-2b^2c)x^2-2abcx+b^2)\arcsin(cx)}{27c^4x^2}$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output
$$\frac{1}{27} * (6 * (a * b * c^3 * x^3 - 3 * a * b * c * x + (b^2 * c^3 * x^3 - 3 * b^2 * c * x) * \arcsin(c * x)) * \sqrt{-c^2 * d * x^2 + d} * \sqrt{-c^2 * x^2 + 1} + ((9 * a^2 - 2 * b^2) * c^4 * x^4 - 2 * (9 * a^2 - 8 * b^2) * c^2 * x^2 + 9 * (b^2 * c^4 * x^4 - 2 * b^2 * c^2 * x^2 + b^2) * \arcsin(c * x)^2 + 9 * a^2 - 14 * b^2 + 18 * (a * b * c^4 * x^4 - 2 * a * b * c^2 * x^2 + a * b) * \arcsin(c * x)) * \sqrt{-c^2 * d * x^2 + d}) / (c^4 * x^2 - c^2)$$

Sympy [F]

$$\int x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int x \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2 dx$$

input `integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2, x)`

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.04

$$\begin{aligned} & \int x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx \\ &= -\frac{2}{27} b^2 \left(\frac{\sqrt{-c^2 x^2 + 1} d^{\frac{3}{2}} x^2 - \frac{7 \sqrt{-c^2 x^2 + 1} d^{\frac{3}{2}}}{c^2}}{d} + \frac{3 \left(c^2 d^{\frac{3}{2}} x^3 - 3 d^{\frac{3}{2}} x \right) \arcsin(cx)}{cd} \right) \\ & \quad - \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} b^2 \arcsin(cx)^2}{3 c^2 d} - \frac{2 (-c^2 dx^2 + d)^{\frac{3}{2}} ab \arcsin(cx)}{3 c^2 d} \\ & \quad - \frac{2 \left(c^2 d^{\frac{3}{2}} x^3 - 3 d^{\frac{3}{2}} x \right) ab}{9 cd} - \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} a^2}{3 c^2 d} \end{aligned}$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
-2/27*b^2*((sqrt(-c^2*x^2 + 1)*d^(3/2)*x^2 - 7*sqrt(-c^2*x^2 + 1)*d^(3/2)/
c^2)/d + 3*(c^2*d^(3/2)*x^3 - 3*d^(3/2)*x)*arcsin(c*x)/(c*d)) - 1/3*(-c^2*
d*x^2 + d)^(3/2)*b^2*arcsin(c*x)^2/(c^2*d) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*
b*arcsin(c*x)/(c^2*d) - 2/9*(c^2*d^(3/2)*x^3 - 3*d^(3/2)*x)*a*b/(c*d) - 1/
3*(-c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^2 dx = \int x(a+b\arcsin(cx))^2\sqrt{d-c^2x^2} dx$$

input

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d}(\sqrt{-c^2x^2+1}a^2c^2x^2 - \sqrt{-c^2x^2+1}a^2 + 6(\int \sqrt{-c^2x^2+1} \arcsin(cx) x dx) ab c^2 + 3(\int \sqrt{-c^2x^2+1} \arcsin(cx) x dx)^2)}{3c^2}$$

input `int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(d)*(sqrt(-c**2*x**2+1)*a**2*c**2*x**2 - sqrt(-c**2*x**2+1)*a**2 + 6*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*a*b*c**2 + 3*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x,x)*b**2*c**2))/(3*c**2)`

3.210 $\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$

Optimal result	2022
Mathematica [A] (verified)	2023
Rubi [A] (verified)	2023
Maple [C] (verified)	2026
Fricas [F]	2026
Sympy [F]	2027
Maxima [F]	2027
Giac [F(-2)]	2027
Mupad [F(-1)]	2028
Reduce [F]	2028

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = -\frac{1}{4} b^2 x \sqrt{d - c^2 dx^2} + \frac{b^2 \sqrt{d - c^2 dx^2} \arcsin(cx)}{4c\sqrt{1 - c^2 x^2}} - \frac{bcx^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 + \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^3}{6bc\sqrt{1 - c^2 x^2}}$$

output

```
-1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)+1/4*b^2*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/c
/(-c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c
^2*x^2+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2+1/6*(-c^2*d
*x^2+d)^(1/2)*(a+b*arcsin(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.67

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{1}{6} \sqrt{d - c^2 dx^2} \left(3x(a + b \arcsin(cx))^2 + \frac{(a + b \arcsin(cx))^3}{bc\sqrt{1 - c^2 x^2}} - \frac{3b(cx(2acx + b\sqrt{1 - c^2 x^2}) + b(-1 + 2c^2 x^2) \arcsin(cx))}{2c\sqrt{1 - c^2 x^2}} \right)$$

input `Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]`

output `(Sqrt[d - c^2*d*x^2]*(3*x*(a + b*ArcSin[c*x])^2 + (a + b*ArcSin[c*x])^3/(b*c*Sqrt[1 - c^2*x^2]) - (3*b*(c*x*(2*a*c*x + b*Sqrt[1 - c^2*x^2]) + b*(-1 + 2*c^2*x^2)*ArcSin[c*x]))/(2*c*Sqrt[1 - c^2*x^2]))/6`

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5156, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5156}$$

$$- \frac{bc\sqrt{d - c^2 dx^2} \int x(a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{(a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2$$

$$\downarrow \text{5138}$$

$$\begin{aligned}
& - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{\sqrt{1-c^2x^2}} + \\
& \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 \\
& \quad \downarrow 262 \\
& - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{\sqrt{1-c^2x^2}} + \\
& \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 \\
& \quad \downarrow 223 \\
& \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \\
& \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{\sqrt{1-c^2x^2}} \\
& \quad \downarrow 5152 \\
& \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \\
& \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{\sqrt{1-c^2x^2}}
\end{aligned}$$

input `Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2,x]`

output `(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^3)/(6*b*c*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*(x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)/Sqrt[1 - c^2*x^2]`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] \text{ ; FreeQ}\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1})/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] \text{ ; FreeQ}\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_)^{(m_)}], x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]), x], x] \text{ ; FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}]/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] \text{ ; FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5156 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^{n/2}), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.77

method	result
default	$\frac{x\sqrt{-c^2dx^2+da^2}}{2} + \frac{a^2d \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^3}{6(c^2x^2-1)c} + \frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2+1})}{6(c^2x^2-1)c} \right)$
parts	$\frac{x\sqrt{-c^2dx^2+da^2}}{2} + \frac{a^2d \arctan\left(\frac{\sqrt{c^2d}x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^3}{6(c^2x^2-1)c} + \frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2+1})}{6(c^2x^2-1)c} \right)$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & \frac{1}{2}x(-c^2dx^2+d)^{1/2}a^2 + \frac{1}{2}a^2d/(c^2d)^{1/2} \arctan((c^2d)^{1/2}x/(-c^2dx^2+d)^{1/2}) \\ & + b^2(-1/6(-d(c^2x^2-1))^{1/2}(-c^2x^2+1)^{1/2}/(c^2x^2-1)/c \arcsin(cx)^3 \\ & + 1/16(-d(c^2x^2-1))^{1/2}(-2I(-c^2x^2+1)^{1/2}x^2c^2+2c^3x^3+I(-c^2x^2+1)^{1/2}-2cx)(2I \arcsin(cx)+2 \arcsin(cx)^2-1)/(c^2x^2-1)/c \\ & + 1/16(-d(c^2x^2-1))^{1/2}(2I(-c^2x^2+1)^{1/2}x^2c^2+2c^3x^3-I(-c^2x^2+1)^{1/2}-2cx)(2 \arcsin(cx)^2-1-2I \arcsin(cx))/(c^2x^2-1)/c \\ & + 2ab(-1/4(-d(c^2x^2-1))^{1/2}(-c^2x^2+1)^{1/2}/(c^2x^2-1)/c \arcsin(cx)^2 \\ & + 1/16(-d(c^2x^2-1))^{1/2}(-2I(-c^2x^2+1)^{1/2}x^2c^2+2c^3x^3+I(-c^2x^2+1)^{1/2}-2cx)(I+2 \arcsin(cx))/(c^2x^2-1)/c \\ & + 1/16(-d(c^2x^2-1))^{1/2}(2I(-c^2x^2+1)^{1/2}x^2c^2+2c^3x^3-I(-c^2x^2+1)^{1/2}-2cx)(-I+2 \arcsin(cx))/(c^2x^2-1)/c \end{aligned}$$

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 dx = \int \sqrt{-c^2dx^2+d}(b \arcsin(cx)+a)^2 dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2 dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2 dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a^2 + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)`

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 \sqrt{d - c^2 dx^2} dx$$

input

```
int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output

```
int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d} (a \arcsin(cx) a^2 + \sqrt{-c^2 x^2 + 1} a^2 cx + 4 \int \sqrt{-c^2 x^2 + 1} a \arcsin(cx) dx) abc + 2 \left(\int \sqrt{-c^2 x^2 + 1} a \arcsin(cx)^2 dx \right)}{2c}$$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2,x)
```

output

```
(sqrt(d)*(asin(c*x)*a**2 + sqrt(-c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt(-
c**2*x**2 + 1)*asin(c*x),x)*a*b*c + 2*int(sqrt(-c**2*x**2 + 1)*asin(c*x
)**2,x)*b**2*c))/(2*c)
```

$$3.211 \quad \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x} dx$$

Optimal result	2029
Mathematica [A] (verified)	2030
Rubi [A] (verified)	2031
Maple [A] (verified)	2034
Fricas [F]	2035
Sympy [F]	2035
Maxima [F]	2035
Giac [F(-2)]	2036
Mupad [F(-1)]	2036
Reduce [F]	2036

Optimal result

Integrand size = 29, antiderivative size = 378

$$\begin{aligned}
& \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x} dx \\
&= -2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2b^2cx\sqrt{d-c^2dx^2} \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 \\
&\quad - \frac{2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{2ib\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2ib\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2b^2\sqrt{d-c^2dx^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}} + \frac{2b^2\sqrt{d-c^2dx^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}
\end{aligned}$$

output

```

-2*b^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-2*b^2*c*x*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.03

$$\begin{aligned}
& \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx \\
&= a^2 \sqrt{d - c^2 dx^2} + a^2 \sqrt{d} \log(cx) - a^2 \sqrt{d} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) \\
&+ \frac{2ab \sqrt{d - c^2 dx^2} (-cx + \sqrt{1 - c^2 x^2} \arcsin(cx) + \arcsin(cx) \log(1 - e^{i \arcsin(cx)}) - \arcsin(cx) \log(1 + e^{i \arcsin(cx)}))}{\sqrt{1 - c^2 x^2}} \\
&+ \frac{b^2 \sqrt{d - c^2 dx^2} (-2\sqrt{1 - c^2 x^2} - 2cx \arcsin(cx) + \sqrt{1 - c^2 x^2} \arcsin(cx)^2 + \arcsin(cx)^2 \log(1 - e^{i \arcsin(cx)}))}{\sqrt{1 - c^2 x^2}}
\end{aligned}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x,x]
```

output

```

a^2*Sqrt[d - c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2] + (b^2*Sqrt[d - c^2*d*x^2]*(-2*Sqrt[1 - c^2*x^2] - 2*c*x*ArcSin[c*x] + Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2 + ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] - ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x])]) + (2*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] - (2*I)*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])]) - 2*PolyLog[3, -E^(I*ArcSin[c*x])] + 2*PolyLog[3, E^(I*ArcSin[c*x])])/Sqrt[1 - c^2*x^2]

```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.62, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx \\
 & \quad \downarrow \text{5198} \\
 & -\frac{2bc\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2 x^2}} dx}{b \arcsin(cx)^2} + \sqrt{d - c^2 dx^2} (a + \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{d - c^2 dx^2} \int \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2 x^2}} dx}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \\
 & \quad \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{5218} \\
 & \frac{\sqrt{d - c^2 dx^2} \int \frac{(a + b \arcsin(cx))^2}{cx} d \arcsin(cx)}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \\
 & \quad \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{1 - c^2 x^2}} + \sqrt{d - c^2 dx^2} (a + \\
 & \quad b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{4671}
 \end{aligned}$$

$$\frac{\sqrt{d - c^2 dx^2} (-2b \int (a + b \arcsin(cx)) \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a + b \arcsin(cx)) \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}}}$$

↓ 3011

$$\frac{\sqrt{d - c^2 dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - 2b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}}}$$

↓ 2720

$$\frac{\sqrt{d - c^2 dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d e^{i \arcsin(cx)}) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d e^{i \arcsin(cx)})}{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}}}$$

↓ 7143

$$\frac{\sqrt{d - c^2 dx^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d e^{i \arcsin(cx)}) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d e^{i \arcsin(cx)})}{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{\sqrt{1 - c^2 x^2}}}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x,x]`

output `Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x]) + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x]])] - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x]])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/Sqrt[1 - c^2*x^2]`

Definitions of rubi rules used

- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`
- rule 3011 `Int[Log[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`
- rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`
- rule 5198 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.74

method	result
default	$-\sqrt{d} \ln \left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x} \right) a^2 + \sqrt{-c^2dx^2+d} a^2 + b^2 \left(\frac{\sqrt{-d(c^2x^2-1)} (c^2x^2-i\sqrt{-c^2x^2+1}cx-1)}{2c^2x^2-2} (\arcsin(c$
parts	$-\sqrt{d} \ln \left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x} \right) a^2 + \sqrt{-c^2dx^2+d} a^2 + b^2 \left(\frac{\sqrt{-d(c^2x^2-1)} (c^2x^2-i\sqrt{-c^2x^2+1}cx-1)}{2c^2x^2-2} (\arcsin(c$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output

```
-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a^2+(-c^2*d*x^2+d)^(1/
2)*a^2+b^2*(1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1
)*(arcsin(c*x))^2-2+2*I*arcsin(c*x))/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)
*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x))^2-2-2*I*arcsin(c*x))/(c
^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(arcsin(c*x))^2*ln(1+I*
c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x))^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*a
rcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,
I*c*x+(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(
3,-I*c*x-(-c^2*x^2+1)^(1/2)))/(c^2*x^2-1)+2*a*b*(1/2*(-d*(c^2*x^2-1))^(1/
2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)/(c^2*x^2-1)+1/2*(-
d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)/
(c^2*x^2-1)-I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(I*arc
sin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2
+1)^(1/2))+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+
1)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/x, x)`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2}{x} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2/x,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2/x, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output `-(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d))*a^2 + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(a + b \arcsin(cx))^2 \sqrt{d - c^2 dx^2}}{x} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx = \sqrt{d} \left(\sqrt{-c^2 x^2 + 1} a^2 \right. \\ \left. + 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} a \sin(cx)}{x} dx \right) ab \right. \\ \left. + \left(\int \frac{\sqrt{-c^2 x^2 + 1} a \sin^2(cx)}{x} dx \right) b^2 \right. \\ \left. + \log \left(\tan \left(\frac{a \sin(cx)}{2} \right) \right) a^2 - a^2 \right)$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2/x,x)`

output `sqrt(d)*(sqrt(-c**2*x**2+1)*a**2+2*int((sqrt(-c**2*x**2+1)*asin(c*x))/x,x)*a*b+int((sqrt(-c**2*x**2+1)*asin(c*x)**2)/x,x)*b**2+log(tan(asin(c*x)/2))*a**2-a**2)`

3.212 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx$

Optimal result	2038
Mathematica [A] (verified)	2039
Rubi [A] (verified)	2039
Maple [B] (verified)	2043
Fricas [F]	2044
Sympy [F]	2044
Maxima [F]	2045
Giac [F(-2)]	2045
Mupad [F(-1)]	2045
Reduce [F]	2046

Optimal result

Integrand size = 29, antiderivative size = 227

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx$$

$$= -\frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x} - \frac{ic\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}}$$

$$- \frac{c\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^3}{3b\sqrt{1-c^2x^2}}$$

$$+ \frac{2bc\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{ib^2c\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

output

```

-(c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x-I*c*(-c^2*d*x^2+d)^(1/2)*(a+b*
arcsin(c*x))^2/(-c^2*x^2+1)^(1/2)-1/3*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c
*x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*l
n(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*(-c^2*d*x^2+d
)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
    
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx$$

$$= -\frac{a^2 \sqrt{d - c^2 dx^2}}{x} + a^2 c \sqrt{d} \arctan\left(\frac{cx \sqrt{d - c^2 dx^2}}{\sqrt{d}(-1 + c^2 x^2)}\right)$$

$$- \frac{ab \sqrt{d - c^2 dx^2} (2\sqrt{1 - c^2 x^2} \arcsin(cx) + cx \arcsin(cx)^2 - 2cx \log(cx))}{x \sqrt{1 - c^2 x^2}}$$

$$- \frac{b^2 c \sqrt{d - c^2 dx^2} \left(\arcsin(cx) \left(\left(3i + \frac{3\sqrt{1 - c^2 x^2}}{cx} \right) \arcsin(cx) + \arcsin(cx)^2 - 6 \log(1 - e^{2i \arcsin(cx)}) \right) + 3i \right)}{3\sqrt{1 - c^2 x^2}}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```
-((a^2*Sqrt[d - c^2*d*x^2])/x) + a^2*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (a*b*Sqrt[d - c^2*d*x^2]*(2*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + c*x*ArcSin[c*x]^2 - 2*c*x*Log[c*x]))/(x*Sqrt[1 - c^2*x^2]) - (b^2*c*Sqrt[d - c^2*d*x^2]*(ArcSin[c*x]*((3*I + (3*Sqrt[1 - c^2*x^2]))/(c*x))*ArcSin[c*x] + ArcSin[c*x]^2 - 6*Log[1 - E^((2*I)*ArcSin[c*x])]) + (3*I)*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.80, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5196, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx$$

↓ 5196

$$\begin{aligned}
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x} dx}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} - \\
 & \qquad \qquad \qquad \downarrow \text{5136} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx} d\arcsin(cx)}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} - \\
 & \qquad \qquad \qquad \downarrow \text{3042} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \\
 & \frac{2bc\sqrt{d-c^2dx^2} \int -\left((a+b\arcsin(cx)) \tan\left(\arcsin(cx) + \frac{\pi}{2}\right)\right) d\arcsin(cx)}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} - \\
 & \qquad \qquad \qquad \downarrow \text{25} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} - \\
 & \frac{2bc\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d\arcsin(cx)}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} - \\
 & \qquad \qquad \qquad \downarrow \text{4200} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \\
 & \frac{2bc\sqrt{d-c^2dx^2} \left(2i \int -\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))^2}{2b}\right)}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} - \\
 & \qquad \qquad \qquad \downarrow \text{25}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \\
 & \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))^2}{2b} \right)}{\sqrt{1-c^2x^2}} - \\
 & \frac{\sqrt{1-c^2x^2}}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad x \\
 & \qquad \qquad \qquad \downarrow \text{2620} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \\
 & \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \frac{1}{2}ib \int \log(1-e^{2i\arcsin(cx)}) d\arcsin(cx) \right) - \frac{i(a+b\arcsin(cx))^2}{2b} \right)}{\sqrt{1-c^2x^2}} - \\
 & \frac{\sqrt{1-c^2x^2}}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad x \\
 & \qquad \qquad \qquad \downarrow \text{2715} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \\
 & \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \frac{1}{4}b \int e^{-2i\arcsin(cx)} \log(1-e^{2i\arcsin(cx)}) de^{2i\arcsin(cx)} \right) - \frac{i(a+b\arcsin(cx))^2}{2b} \right)}{\sqrt{1-c^2x^2}} - \\
 & \frac{\sqrt{1-c^2x^2}}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad x \\
 & \qquad \qquad \qquad \downarrow \text{2838} \\
 & -\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \\
 & \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \text{PolyLog}(2, e^{2i\arcsin(cx)}) \right) - \frac{i(a+b\arcsin(cx))^2}{2b} \right)}{\sqrt{1-c^2x^2}} - \\
 & \frac{\sqrt{1-c^2x^2}}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad x \\
 & \qquad \qquad \qquad \downarrow \text{5152} \\
 & \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \text{PolyLog}(2, e^{2i\arcsin(cx)}) \right) - \frac{i(a+b\arcsin(cx))^2}{2b} \right)}{\sqrt{1-c^2x^2}} - \\
 & \frac{c\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{3b\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad x
 \end{aligned}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^2,x]`

output `-((Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x) - (c*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^3)/(3*b*Sqrt[1 - c^2*x^2]) + (2*b*c*Sqrt[d - c^2*d*x^2]*(((1/2)*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4)))/Sqrt[1 - c^2*x^2]`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5196 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(225) = 450$.

Time = 0.65 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.49

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{dx} - a^2c^2x\sqrt{-c^2dx^2+d} - \frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}} + b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\arcsin(c)}{3c^2x^2-3}\right)$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{dx} - a^2c^2x\sqrt{-c^2dx^2+d} - \frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}} + b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\arcsin(c)}{3c^2x^2-3}\right)$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)`

output

```
-a^2/d/x*(-c^2*d*x^2+d)^(3/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(1/2)-a^2*c^2*d/(c^
2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/3*(-d*(c^2*
x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arcsin(c*x)^3*c-(-d*(c^2*x^2-
1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)^2/(c^2*x^2-1)/x
+2*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(I*arcsin(c*x)*
ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2)
)+arcsin(c*x)^2+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,I*c*x+(-c^2
*x^2+1)^(1/2)))*c)+2*a*b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c
^2*x^2-1)*arcsin(c*x)^2*c+2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c
^2*x^2-1)*arcsin(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c
^2*x^2-1)*arcsin(c*x)/(c^2*x^2-1)/x-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c)
```

Fricas [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x^2} dx$$

input

```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2
)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{\sqrt{-d (cx - 1) (cx + 1)} (a + b \arcsin(cx))^2}{x^2} dx$$

input

```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2/x**2,x)
```

output

```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="maxima")`

output `-(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a^2 + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 \sqrt{d - c^2 dx^2}}{x^2} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx$$

$$= \frac{\sqrt{d} \left(-a \sin(cx)^3 b^2 cx - 3a \sin(cx)^2 abcx - 3a \sin(cx) a^2 cx - 3\sqrt{-c^2 x^2 + 1} a^2 + 6 \left(\int \frac{a \sin(cx)}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) abx \right)}{3x}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2/x^2,x)`

output `(sqrt(d)*(-asin(c*x)**3*b**2*c*x - 3*asin(c*x)**2*a*b*c*x - 3*asin(c*x)*a**2*c*x - 3*sqrt(-c**2*x**2 + 1)*a**2 + 6*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**2),x)*a*b*x + 3*int(asin(c*x)**2/(sqrt(-c**2*x**2 + 1)*x**2),x)*b**2*x))/(3*x)`

3.213 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^3} dx$

Optimal result	2047
Mathematica [A] (verified)	2048
Rubi [A] (verified)	2049
Maple [A] (verified)	2054
Fricas [F]	2054
Sympy [F]	2055
Maxima [F]	2055
Giac [F(-2)]	2056
Mupad [F(-1)]	2056
Reduce [F]	2056

Optimal result

Integrand size = 29, antiderivative size = 398

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^3} dx$$

$$= -\frac{bc\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{x\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2x^2}$$

$$+ \frac{c^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{b^2c^2\sqrt{d-c^2dx^2} \operatorname{arctanh}(\sqrt{1-c^2x^2})}{\sqrt{1-c^2x^2}}$$

$$- \frac{ibc^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$+ \frac{ibc^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$+ \frac{b^2c^2\sqrt{d-c^2dx^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{b^2c^2\sqrt{d-c^2dx^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

output

```

-b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x/(-c^2*x^2+1)^(1/2)-1/2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^2+c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcs
in(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-b^2*c^2*(-
c^2*d*x^2+d)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-I*b*c^2*
(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2)
)/(-c^2*x^2+1)^(1/2)+I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylo
g(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+b^2*c^2*(-c^2*d*x^2+d)^(1
/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-b^2*c^2*(-c^2*
d*x^2+d)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.21

$$\begin{aligned}
& \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx \\
&= \frac{1}{8} \left(-\frac{4a^2 \sqrt{d - c^2 dx^2}}{x^2} - 4a^2 c^2 \sqrt{d} \log(x) + 4a^2 c^2 \sqrt{d} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) \right. \\
&\quad + \frac{2abc^2 d \sqrt{1 - c^2 x^2} \left(-2 \cot\left(\frac{1}{2} \arcsin(cx)\right) - \arcsin(cx) \csc^2\left(\frac{1}{2} \arcsin(cx)\right) - 4 \arcsin(cx) \log\left(1 - e^{i \arcsin(cx)}\right)\right)}{x^3} \\
&\quad \left. + \frac{b^2 c^2 d \sqrt{1 - c^2 x^2} \left(-4 \arcsin(cx) \cot\left(\frac{1}{2} \arcsin(cx)\right) - \arcsin(cx)^2 \csc^2\left(\frac{1}{2} \arcsin(cx)\right) - 4 \arcsin(cx)^2 \log\left(1 - e^{i \arcsin(cx)}\right)\right)}{x^3} \right)
\end{aligned}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^3,x]
```

output

```

((-4*a^2*Sqrt[d - c^2*d*x^2])/x^2 - 4*a^2*c^2*Sqrt[d]*Log[x] + 4*a^2*c^2*S
qrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*c^2*d*Sqrt[1 - c^2*x^
2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*
x]*Log[1 - E^(I*ArcSin[c*x])]) + 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] -
(4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])
] + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2]))/Sqrt[d - c^2
*d*x^2] + (b^2*c^2*d*Sqrt[1 - c^2*x^2]*(-4*ArcSin[c*x]*Cot[ArcSin[c*x]/2]
- ArcSin[c*x]^2*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*x]^2*Log[1 - E^(I*ArcSin
[c*x])]) + 4*ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x])]) + 8*Log[Tan[ArcSin[c*
x]/2]] - (8*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] + (8*I)*ArcSin[c
*x]*PolyLog[2, E^(I*ArcSin[c*x])] + 8*PolyLog[3, -E^(I*ArcSin[c*x])] - 8*P
olyLog[3, E^(I*ArcSin[c*x])] + ArcSin[c*x]^2*Sec[ArcSin[c*x]/2]^2 - 4*ArcS
in[c*x]*Tan[ArcSin[c*x]/2]))/Sqrt[d - c^2*d*x^2])/8

```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5196, 5138, 243, 73, 221, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx \\
 & \quad \downarrow \text{5196} \\
 & -\frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{bc \sqrt{d - c^2 dx^2} \int \frac{a + b \arcsin(cx)}{x^2} dx}{\sqrt{1 - c^2 x^2}} - \\
 & \quad \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{5138} \\
 & -\frac{c^2 \sqrt{d - c^2 dx^2} \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{1 - c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{bc \sqrt{d - c^2 dx^2} \left(bc \int \frac{1}{x \sqrt{1 - c^2 x^2}} dx - \frac{a + b \arcsin(cx)}{x} \right)}{\sqrt{1 - c^2 x^2}} - \\
 & \quad \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{243}
 \end{aligned}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int\frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arcsin(cx)}{x}\right)}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}$$

↓ 73

$$-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{b\int\frac{1}{c^2-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c}-\frac{a+b\arcsin(cx)}{x}\right)}{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}$$

↓ 221

$$-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{a+b\arcsin(cx)}{x}-b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2x^2}$$

↓ 5218

$$-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{cx}d\arcsin(cx)}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{a+b\arcsin(cx)}{x}-b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2x^2}$$

↓ 3042

$$-\frac{c^2\sqrt{d-c^2dx^2}\int(a+b\arcsin(cx))^2\csc(\arcsin(cx))d\arcsin(cx)}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{a+b\arcsin(cx)}{x}-b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2x^2}$$

↓ 4671

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(-2b\int(a+b\arcsin(cx))\log(1-e^{i\arcsin(cx)})d\arcsin(cx)+2b\int(a+b\arcsin(cx))\log(1+e^{i\arcsin(cx)})d\arcsin(cx)\right)}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{a+b\arcsin(cx)}{x}-b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2x^2}$$

↓ 3011

$$\frac{c^2\sqrt{d-c^2dx^2}(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - bc\sqrt{d-c^2dx^2}\left(-\frac{a+b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1-c^2x^2})\right)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a + b \arcsin(cx))^2}{2x^2}$$

↓ 2720

$$\frac{c^2\sqrt{d-c^2dx^2}(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - bc\sqrt{d-c^2dx^2}\left(-\frac{a+b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1-c^2x^2})\right)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a + b \arcsin(cx))^2}{2x^2}$$

↓ 7143

$$\frac{c^2\sqrt{d-c^2dx^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - bc\sqrt{d-c^2dx^2}\left(-\frac{a+b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1-c^2x^2})\right)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a + b \arcsin(cx))^2}{2x^2}$$

input

```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^3,x]
```

output

```
-1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^2 + (b*c*Sqrt[d - c^2*d*x^2]*(-(a + b*ArcSin[c*x])/x) - b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 - c^2*x^2] - (c^2*Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])]) - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/(2*Sqrt[1 - c^2*x^2])
```


Defintions of rubi rules used

- rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
 {p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^(n), x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
 Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
 inearQ[a, b, c, d, m, n, x]`
- rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
 /Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`
- rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int
 t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
 ntegerQ[(m - 1)/2]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
 Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
 ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
 [{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
 *(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`
- rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
 *(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
 b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
 m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
 , f, g, n}, x] && GtQ[m, 0]`
- rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
 Q[u, x]`

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5138

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5196

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x],
x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int
[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.48

method	result
default	$a^2 \left(-\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{2 d x^2} - \frac{c^2 \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right)}{2} \right) + b^2 \left(-\frac{(c^2 x^2 \arcsin(c x) - 2 c x \sqrt{-c^2 x^2 + 1} - a)}{2(c^2 x^2} \right)$
parts	$a^2 \left(-\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{2 d x^2} - \frac{c^2 \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right)}{2} \right) + b^2 \left(-\frac{(c^2 x^2 \arcsin(c x) - 2 c x \sqrt{-c^2 x^2 + 1} - a)}{2(c^2 x^2} \right)$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^3,x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & a^2 * (-1/2/d/x^2 * (-c^2*d*x^2+d)^{(3/2)} - 1/2*c^2 * ((-c^2*d*x^2+d)^{(1/2)} - d^{(1/2)} \\ & * \ln((2*d+2*d^{(1/2)} * (-c^2*d*x^2+d)^{(1/2)})/x)) + b^2 * (-1/2*(c^2*x^2*arcsin(c*x) \\ & - 2*c*x*(-c^2*x^2+1)^{(1/2)} - arcsin(c*x)) * arcsin(c*x) * (-d*(c^2*x^2-1))^{(1/2)} \\ & / (c^2*x^2-1)/x^2 - 1/2*(-d*(c^2*x^2-1))^{(1/2)} * (-c^2*x^2+1)^{(1/2)} * (arcsin(c*x) \\ & ^2 * \ln(1+I*c*x+(-c^2*x^2+1)^{(1/2)}) - arcsin(c*x)^2 * \ln(1-I*c*x-(-c^2*x^2+1)^{(1/2)}) \\ & - 2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^{(1/2)}) + 2*I*arcsin(c*x) \\ & *polylog(2,I*c*x+(-c^2*x^2+1)^{(1/2)}) + 2*polylog(3,-I*c*x-(-c^2*x^2+1)^{(1/2)} \\ &) - 2*polylog(3,I*c*x+(-c^2*x^2+1)^{(1/2)}) - 4*arctanh(I*c*x+(-c^2*x^2+1)^{(1/2)} \\ &)) * c^2 / (c^2*x^2-1) + 2*a*b * (-1/2*(c^2*x^2*arcsin(c*x) - c*x*(-c^2*x^2+1)^{(1/2)} \\ & - arcsin(c*x)) * (-d*(c^2*x^2-1))^{(1/2)} / (c^2*x^2-1)/x^2 + I * (-d*(c^2*x^2-1))^{(1/2)} \\ & * (-c^2*x^2+1)^{(1/2)} * (I*arcsin(c*x) * \ln(1+I*c*x+(-c^2*x^2+1)^{(1/2)}) - I*a \\ & rcsin(c*x) * \ln(1-I*c*x-(-c^2*x^2+1)^{(1/2)}) + polylog(2,-I*c*x-(-c^2*x^2+1)^{(1/2)} \\ &) - polylog(2,I*c*x+(-c^2*x^2+1)^{(1/2)})) * c^2 / (2*c^2*x^2-2) \end{aligned}$$

Fricas [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/x^3, x)`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2}{x^3} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2/x**3,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2/x**3, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="maxima")`

output `1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a^2 + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^3, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(a + b \arcsin(cx))^2 \sqrt{d - c^2 dx^2}}{x^3} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^3} dx$$

$$= \frac{\sqrt{d} \left(-\sqrt{-c^2 x^2 + 1} a^2 + 4 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^3} dx \right) ab x^2 + 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{x^3} dx \right) b^2 x^2 - \log \left(\tan \left(\frac{\arcsin(cx)}{2} \right) \right)}{2x^2}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2/x^3,x)`

output

```
(sqrt(d)*( - sqrt( - c**2*x**2 + 1)*a**2 + 4*int((sqrt( - c**2*x**2 + 1)*a
sin(c*x))/x**3,x)*a*b*x**2 + 2*int((sqrt( - c**2*x**2 + 1)*asin(c*x)**2)/x
**3,x)*b**2*x**2 - log(tan(asin(c*x)/2))*a**2*c**2*x**2))/(2*x**2)
```

3.214 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^4} dx$

Optimal result	2058
Mathematica [A] (verified)	2059
Rubi [A] (verified)	2059
Maple [B] (verified)	2063
Fricas [F]	2064
Sympy [F]	2065
Maxima [F]	2065
Giac [F(-2)]	2065
Mupad [F(-1)]	2066
Reduce [F]	2066

Optimal result

Integrand size = 29, antiderivative size = 314

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^4} dx$$

$$= -\frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2} \arcsin(cx)}{3\sqrt{1-c^2x^2}}$$

$$- \frac{bc\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{3x^2}$$

$$+ \frac{ic^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2}{3dx^3}$$

$$- \frac{2bc^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)})}{3\sqrt{1-c^2x^2}}$$

$$+ \frac{ib^2c^3\sqrt{d-c^2dx^2} \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{3\sqrt{1-c^2x^2}}$$

output

```
-1/3*b^2*c^2*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*(-c^2*d*x^2+d)^(1/2)*arcsi
n(c*x)/(-c^2*x^2+1)^(1/2)-1/3*b*c*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*
(a+b*arcsin(c*x))/x^2+1/3*I*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/(
-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/d/x^3-2/3*b
*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2)
)^2)/(-c^2*x^2+1)^(1/2)+1/3*I*b^2*c^3*(-c^2*d*x^2+d)^(1/2)*polylog(2,(I*c*
x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx$$

$$= \frac{\sqrt{d - c^2 dx^2} \left(2b^2 (ic^3 x^3 - \sqrt{1 - c^2 x^2} + c^2 x^2 \sqrt{1 - c^2 x^2}) \arcsin(cx)^2 - b \arcsin(cx) (2bcx + 3a\sqrt{1 - c^2 x^2}) \right)}{6x^3 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(2*b^2*(I*c^3*x^3 - Sqrt[1 - c^2*x^2] + c^2*x^2*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 - b*ArcSin[c*x]*(2*b*c*x + 3*a*Sqrt[1 - c^2*x^2] + a*Cos[3*ArcSin[c*x]] + 4*b*c^3*x^3*Log[1 - E^((2*I)*ArcSin[c*x])]) - 2*(a*b*c*x + b^2*c^2*x^2*Sqrt[1 - c^2*x^2] + a^2*(1 - c^2*x^2)^(3/2) + 2*a*b*c^3*x^3*Log[c*x]) + (2*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(6*x^3*Sqrt[1 - c^2*x^2]))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5186, 5190, 247, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx$$

$$\downarrow \text{5186}$$

$$\frac{2bc\sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{x^3} dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3dx^3}$$

$$\downarrow \text{5190}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int\frac{a+b\arcsin(cx)}{x}dx\right)+\frac{1}{2}bc\int\frac{\sqrt{1-c^2x^2}}{x^2}dx-\frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}} \\ \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3dx^3}$$

↓ 247

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int\frac{a+b\arcsin(cx)}{x}dx\right)+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}}{x}\right)-\frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}} \\ \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3dx^3}$$

↓ 223

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int\frac{a+b\arcsin(cx)}{x}dx\right)-\frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2}+\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)\right)}{3\sqrt{1-c^2x^2}} \\ \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3dx^3}$$

↓ 5136

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx}d\arcsin(cx)\right)-\frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2}+\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)\right)}{3\sqrt{1-c^2x^2}} \\ \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3dx^3}$$

↓ 3042

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int-\left((a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)\right)d\arcsin(cx)\right)-\frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2}+\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)\right)}{3\sqrt{1-c^2x^2}} \\ \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3dx^3}$$

↓ 25

$$\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\int(a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)d\arcsin(cx)-\frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2}+\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)\right)}{3\sqrt{1-c^2x^2}} \\ \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3dx^3}$$

$$\frac{\begin{aligned} & \downarrow 4200 \\ & -\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3dx^3} + \\ & 2bc\sqrt{d - c^2 dx^2} \left(-\left(c^2 \left(2i \int -\frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) \right) - \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{2x^2} \right) + \frac{1}{2} \end{aligned}}{3\sqrt{1 - c^2 x^2}}$$

$$\frac{\begin{aligned} & \downarrow 25 \\ & -\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3dx^3} + \\ & 2bc\sqrt{d - c^2 dx^2} \left(-\left(c^2 \left(-2i \int \frac{e^{2i \arcsin(cx)} (a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a + b \arcsin(cx))^2}{2b} \right) \right) - \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{2x^2} \right) + \frac{1}{2} \end{aligned}}{3\sqrt{1 - c^2 x^2}}$$

$$\frac{\begin{aligned} & \downarrow 2620 \\ & -\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3dx^3} + \\ & 2bc\sqrt{d - c^2 dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2} i b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) \right) \right) \right) \end{aligned}}{3\sqrt{1 - c^2 x^2}}$$

$$\frac{\begin{aligned} & \downarrow 2715 \\ & -\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3dx^3} + \\ & 2bc\sqrt{d - c^2 dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) \right) \right) \right) \end{aligned}}{3\sqrt{1 - c^2 x^2}}$$

$$\frac{\begin{aligned} & \downarrow 2838 \\ & -\frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3dx^3} + \\ & 2bc\sqrt{d - c^2 dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{4} b \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right) - \frac{i(a + b \arcsin(cx))}{2b} \right) \right) \end{aligned}}{3\sqrt{1 - c^2 x^2}}$$

input

```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

$$-1/3*((d - c^2*d*x^2)^{(3/2)}*(a + b*ArcSin[c*x])^2)/(d*x^3) + (2*b*c*Sqrt[d - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcSin[c*x])))/x^2 + (b*c*(-(Sqrt[1 - c^2*x^2]/x) - c*ArcSin[c*x]))/2 - c^2*(((-1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])]) + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4)))/(3*Sqrt[1 - c^2*x^2])$$

Defintions of rubi rules used

rule 25

$$\text{Int}[-(F x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F x, x], x]$$

rule 223

$$\text{Int}[1/\text{Sqrt}[(a) + (b) * (x)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2] * (x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$$

rule 247

$$\text{Int}[(c) * (x)^{(m)} * ((a) + (b) * (x)^2)^{(p)}, x_Symbol] \rightarrow \text{Simp}[(c*x)^{(m+1)} * ((a + b*x^2)^p / (c*(m+1))), x] - \text{Simp}[2*b*(p/(c^2*(m+1))) \quad \text{Int}[(c*x)^{(m+2)} * (a + b*x^2)^{(p-1)}, x], x] \text{ ; FreeQ}\{a, b, c\}, x \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{LtQ}[m + 2*p + 3)/2, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$$

rule 2620

$$\text{Int}[(F)^{(g) * ((e) + (f) * (x))})^{(n)} * ((c) + (d) * (x))^{(m)} / ((a) + (b) * ((F)^{(g) * ((e) + (f) * (x))})^{(n)}), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m / (b*f*g*n*Log[F]) * Log[1 + b*((F)^{(g*(e + f*x))})^n/a], x] - \text{Simp}[d*(m/(b*f*g*n*Log[F])) \quad \text{Int}[(c + d*x)^{(m-1)} * Log[1 + b*((F)^{(g*(e + f*x))})^n/a], x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$$

rule 2715

$$\text{Int}[\text{Log}[(a) + (b) * ((F)^{(e) * ((c) + (d) * (x))})^{(n)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*Log[F]) \quad \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F)^{(e*(c + d*x))})^n], x] \text{ ; FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838

$$\text{Int}[\text{Log}[(c) * ((d) + (e) * (x)^n)] / (x), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] \text{ ; FreeQ}\{c, d, e, n\}, x \ \&\& \ \text{EqQ}[c*d, 1]$$

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))]], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n * Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b * ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b * ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

rule 5190 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b * ArcSin[c*x])/(f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1))) Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b * ArcSin[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2040 vs. $2(294) = 588$.

Time = 0.76 (sec) , antiderivative size = 2041, normalized size of antiderivative = 6.50

method	result	size
default	Expression too large to display	2041
parts	Expression too large to display	2041

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^4,x,method=_RETURNVERBOSE)`

output `I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2*c^7-1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c^2*x^2-1)*(-c^2*x^2+1)*arcsin(c*x)*c^6-I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2*c^5+1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c^2*x^2-1)*(-c^2*x^2+1)*arcsin(c*x)*c^4-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(3/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c^2*x^2-1)*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c+1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2*c^3-1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c^2*x^2-1)*arcsin(c*x)*c^8-I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^7+2/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c^2*x^2-1)*arcsin(c*x)*c^6-1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c^2*x^2-1)*arcsin(c*x)*c^4+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^5+I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^5-3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c^2*x^2-1)*arcsin(c*x)^2*c^6+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c^2*x^2-1)*arcsin(c*x)^2*c^4-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c^2*x^2-1)*arcsin(c*x)^2*c^2-b^2*(...`

Fricas [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/x^4, x)`

Sympy [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{\sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2}{x^4} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2/x**4,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2/x**4, x)`

Maxima [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="maxima")`

output `1/3*((-1)^(-2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + c^2*d^(3/2)*log(x^2 - 1/c^2) - sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*d/x^2)*a*b*c/d - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*b*arcsin(c*x)/(d*x^3) + 1/3*((c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 - sqrt(d)*x^3*integrate(2*(c^3*x^2 - c)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)/x^3, x))*b^2/x^3 - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(a + b \arcsin(cx))^2 \sqrt{d - c^2 dx^2}}{x^4} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4, x)`

Reduce [F]

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^4} dx$$

$$= \frac{\sqrt{d} \left(\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} a^2 + 6 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{x^4} dx \right) ab x^3 + 3 \left(\int \frac{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{x^4} dx \right) b^2 \right)}{3x^3}$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2/x^4,x)`

output `(sqrt(d)*(sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(-c**2*x**2 + 1)*a
2 + 6*int((sqrt(-c2*x**2 + 1)*asin(c*x))/x**4,x)*a*b*x**3 + 3*int((s
qrt(-c**2*x**2 + 1)*asin(c*x)**2)/x**4,x)*b**2*x**3))/(3*x**3)`

3.215 $\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2067
Mathematica [A] (verified)	2068
Rubi [A] (verified)	2068
Maple [A] (verified)	2077
Fricas [A] (verification not implemented)	2077
Sympy [F]	2078
Maxima [A] (verification not implemented)	2079
Giac [F(-2)]	2080
Mupad [F(-1)]	2080
Reduce [F]	2080

Optimal result

Integrand size = 29, antiderivative size = 434

$$\begin{aligned}
 \int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = & \frac{304b^2 d \sqrt{d - c^2 dx^2}}{3675c^4} \\
 & + \frac{152b^2(d - c^2 dx^2)^{3/2}}{11025c^4} + \frac{38b^2(d - c^2 dx^2)^{5/2}}{6125c^4 d} - \frac{2b^2(d - c^2 dx^2)^{7/2}}{343c^4 d^2} \\
 & + \frac{4bdx\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{35c^3\sqrt{1 - c^2 x^2}} + \frac{2bdx^3\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{105c\sqrt{1 - c^2 x^2}} \\
 & - \frac{16bcdx^5\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{175\sqrt{1 - c^2 x^2}} \\
 & + \frac{2bc^3 dx^7 \sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{49\sqrt{1 - c^2 x^2}} \\
 & - \frac{2d\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2}{35c^4} - \frac{dx^2\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2}{35c^2} \\
 & + \frac{3}{35} dx^4 \sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2 + \frac{1}{7} x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2
 \end{aligned}$$

output

$$\begin{aligned} & 304/3675*b^2*d*(-c^2*d*x^2+d)^{(1/2)}/c^4+152/11025*b^2*(-c^2*d*x^2+d)^{(3/2)} \\ & /c^4+38/6125*b^2*(-c^2*d*x^2+d)^{(5/2)}/c^4/d-2/343*b^2*(-c^2*d*x^2+d)^{(7/2)} \\ & /c^4/d^2+4/35*b*d*x*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/c^3/(-c^2*x^2+1) \\ &)^{(1/2)}+2/105*b*d*x^3*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/c/(-c^2*x^2+1) \\ &)^{(1/2)}-16/175*b*c*d*x^5*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/(-c^2*x^2+1) \\ &)^{(1/2)}+2/49*b*c^3*d*x^7*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/(-c^2*x^2+1) \\ &)^{(1/2)}-2/35*d*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))^2/c^4-1/35*d*x^2*(-c^2*d*x^2+d) \\ &)^{(1/2)}*(a+b*\arcsin(c*x))^2/c^2+3/35*d*x^4*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))^2 \\ & +1/7*x^4*(-c^2*d*x^2+d)^{(3/2)}*(a+b*\arcsin(c*x))^2 \end{aligned}$$
Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.56

$$\int x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{d\sqrt{d - c^2 dx^2} \left(-11025a^2(1 - c^2 x^2)^{5/2} (2 + 5c^2 x^2) + 210abcx(210 + 35c^2 x^2 - 168c^4 x^4) \right)}{385875c^4 \sqrt{1 - c^2 x^2}}$$

input

Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]

output

$$\begin{aligned} & (d*\text{Sqrt}[d - c^2*d*x^2]*(-11025*a^2*(1 - c^2*x^2)^{(5/2)}*(2 + 5*c^2*x^2) + 2 \\ & 10*a*b*c*x*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6) + 2*b^2*\text{Sqrt}[1 - \\ & c^2*x^2]*(18692 - 1679*c^2*x^2 - 2178*c^4*x^4 + 1125*c^6*x^6) + 210*b*(-10 \\ & 5*a*(1 - c^2*x^2)^{(5/2)}*(2 + 5*c^2*x^2) + b*c*x*(210 + 35*c^2*x^2 - 168*c^4 \\ & 4*x^4 + 75*c^6*x^6))*\text{ArcSin}[c*x] - 11025*b^2*(1 - c^2*x^2)^{(5/2)}*(2 + 5*c^2 \\ & 2*x^2)*\text{ArcSin}[c*x]^2))/(385875*c^4*\text{Sqrt}[1 - c^2*x^2]) \end{aligned}$$
Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.29, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {5202, 5192, 27, 354, 86, 2009, 5198, 5138, 243, 53, 2009, 5210, 5138, 243, 53, 2009, 5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\begin{aligned}
 & \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \int x^4 (1 - c^2 x^2) (a + b \arcsin(cx)) dx}{7\sqrt{1 - c^2 x^2}} + \frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + \\
 & \quad b \arcsin(cx))^2 dx + \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5192} \\
 & \frac{\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx -}{7\sqrt{1 - c^2 x^2}} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^5 (7 - 5c^2 x^2)}{35\sqrt{1 - c^2 x^2}} dx - \frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{27} \\
 & \frac{\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx -}{7\sqrt{1 - c^2 x^2}} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{35}bc \int \frac{x^5 (7 - 5c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{354} \\
 & \frac{\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx -}{7\sqrt{1 - c^2 x^2}} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{70}bc \int \frac{x^4 (7 - 5c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx^2 - \frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{86} \\
 & \frac{\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx -}{7\sqrt{1 - c^2 x^2}} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{70}bc \int \left(\frac{5(1 - c^2 x^2)^{5/2}}{c^4} - \frac{8(1 - c^2 x^2)^{3/2}}{c^4} + \frac{\sqrt{1 - c^2 x^2}}{c^4} + \frac{2}{c^4 \sqrt{1 - c^2 x^2}} \right) dx^2 - \frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\frac{\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2 x^2)^{7/2}}{7c^6} + \frac{16(1-c^2 x^2)^{5/2}}{5c^6} - 2(1-c^2 x^2) \right) \right)}{7\sqrt{1 - c^2 x^2}}$$

↓ 5198

$$\frac{\frac{3}{7}d \left(-\frac{2bc\sqrt{d - c^2 dx^2} \int x^4 (a + b \arcsin(cx)) dx}{5\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} + \frac{1}{5}x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2 x^2)^{7/2}}{7c^6} + \frac{16(1-c^2 x^2)^{5/2}}{5c^6} - 2(1-c^2 x^2) \right) \right) \right)}{7\sqrt{1 - c^2 x^2}}$$

↓ 5138

$$\frac{\frac{3}{7}d \left(-\frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5}x^5 (a + b \arcsin(cx)) - \frac{1}{5}bc \int \frac{x^5}{\sqrt{1 - c^2 x^2}} dx \right)}{5\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} + \frac{1}{5}x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2 x^2)^{7/2}}{7c^6} + \frac{16(1-c^2 x^2)^{5/2}}{5c^6} - 2(1-c^2 x^2) \right) \right) \right)}{7\sqrt{1 - c^2 x^2}}$$

↓ 243

$$\frac{\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^3 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx}{5\sqrt{1 - c^2 x^2}} - \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5}x^5 (a + b \arcsin(cx)) - \frac{1}{10}bc \int \frac{x^4}{\sqrt{1 - c^2 x^2}} dx \right)}{5\sqrt{1 - c^2 x^2}} + \frac{1}{5}x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{1}{7}x^4 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5}x^5 (a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2 x^2)^{7/2}}{7c^6} + \frac{16(1-c^2 x^2)^{5/2}}{5c^6} - 2(1-c^2 x^2) \right) \right) \right)}{7\sqrt{1 - c^2 x^2}}$$

↓ 53

$$\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^3(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{5\sqrt{1 - c^2x^2}} - \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{10}bc \int \left(\frac{(1-c^2x^2)^{3/2}}{c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) dx \right)}{5\sqrt{1 - c^2x^2}} \right) - \frac{\frac{1}{7}x^4(d - c^2 dx^2)^{3/2}(a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - 2(1-c^2x^2) \right) \right)}{7\sqrt{1 - c^2x^2}}$$

↓ 2009

$$\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^3(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{5\sqrt{1 - c^2x^2}} + \frac{1}{5}x^4\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{10}bc \int \left(\frac{(1-c^2x^2)^{3/2}}{c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) dx \right)}{5\sqrt{1 - c^2x^2}} \right) - \frac{\frac{1}{7}x^4(d - c^2 dx^2)^{3/2}(a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - 2(1-c^2x^2) \right) \right)}{7\sqrt{1 - c^2x^2}}$$

↓ 5210

$$\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{2b \int x^2(a+b \arcsin(cx)) dx}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1 - c^2x^2}} + \frac{1}{5}x^4\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{10}bc \int \left(\frac{(1-c^2x^2)^{3/2}}{c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) dx \right)}{5\sqrt{1 - c^2x^2}} \right) - \frac{\frac{1}{7}x^4(d - c^2 dx^2)^{3/2}(a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - 2(1-c^2x^2) \right) \right)}{7\sqrt{1 - c^2x^2}}$$

↓ 5138

$$\frac{\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{3}bc \int \frac{x^3}{\sqrt{1-c^2x^2}} dx \right)}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x \right) + \frac{1}{7}x^4(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - 2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{7}c^2x^7(a+b \arcsin(cx)) + \frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{3c^2} \right) \right)}{7\sqrt{1-c^2x^2}}$$

↓ 243

$$\frac{\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx^2 \right)}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x \right) + \frac{1}{7}x^4(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - 2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{7}c^2x^7(a+b \arcsin(cx)) + \frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{3c^2} \right) \right)}{7\sqrt{1-c^2x^2}}$$

↓ 53

$$\frac{\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \left(\frac{1}{c^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{c^2} \right) dx^2 \right)}{3c} - \frac{x^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x \right) + \frac{1}{7}x^4(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - 2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{7}c^2x^7(a+b \arcsin(cx)) + \frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{3c^2} \right) \right)}{7\sqrt{1-c^2x^2}}$$

↓ 2009

$$\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) \right)}{3c} \right)}{5\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{\frac{1}{7}x^4(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - 2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{7}c^2x^7(a+b \arcsin(cx)) + \frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{c^4} \right) \right)}{7\sqrt{1-c^2x^2}} \right)$$

↓ 5182

$$\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{2 \left(\frac{2b \int (a+b \arcsin(cx)) dx}{c} - \frac{\sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{c^2} \right)}{3c^2} - \frac{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) \right)}{3c} \right)}{5\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{\frac{1}{7}x^4(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - 2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{7}c^2x^7(a+b \arcsin(cx)) + \frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{70}bc \left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{c^4} \right) \right)}{7\sqrt{1-c^2x^2}} \right)$$

↓ 2009

$$\frac{\frac{1}{7}x^4(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2dx^2}\left(-\frac{1}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6} + \frac{16(1-c^2x^2)^{5/2}}{5c^6} - \frac{2(1-c^2x^2)^{3/2}}{3c^6}\right)\right)}{7\sqrt{1 - c^2x^2}}$$

$$\left(\frac{3}{7}d \left(\frac{1}{5}x^4\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2 - \frac{2bc\sqrt{d - c^2dx^2}\left(\frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6}\right)\right)}{5\sqrt{1 - c^2x^2}} \right) \right)$$

input `Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]`

output `(x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/7 - (2*b*c*d*Sqrt[d - c^2*d*x^2]*(-1/70*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^6 - (2*(1 - c^2*x^2)^(3/2))/(3*c^6) + (16*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1 - c^2*x^2)^(7/2))/(7*c^6))) + (x^5*(a + b*ArcSin[c*x]))/5 - (c^2*x^7*(a + b*ArcSin[c*x]))/7)/(7*Sqrt[1 - c^2*x^2]) + (3*d*((x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*Sqrt[d - c^2*d*x^2]*(-1/10*(b*c*((-2*Sqrt[1 - c^2*x^2])/c^6 + (4*(1 - c^2*x^2)^(3/2))/(3*c^6) - (2*(1 - c^2*x^2)^(5/2))/(5*c^6))) + (x^5*(a + b*ArcSin[c*x]))/5))/(5*Sqrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2 + (2*b*(-1/6*(b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4))) + (x^3*(a + b*ArcSin[c*x]))/3))/(3*c) + (2*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2) + (2*b*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/c))/(3*c^2)))/(5*Sqrt[1 - c^2*x^2])))/7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] :> Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

- rule 53 `Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`
- rule 86 `Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`
- rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`
- rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 5138 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`
- rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.22

method	result
orering	$\frac{(47625c^{10}x^{10}-130566c^8x^8+68553c^6x^6+279840c^4x^4-260420c^2x^2+74768)(-c^2dx^2+d)^{\frac{3}{2}}(a+b\arcsin(cx))^2}{128625c^6x^2(c^2x^2-1)^2} - \frac{2(10125c^8x^8-24174c^6x^6-863c^4x^4+118868c^2x^2-56076)}{c^6/x^4/(c^2x^2-1)^2}$
default	Expression too large to display
parts	Expression too large to display

input `int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output
$$\frac{1}{128625} \cdot \frac{(47625c^{10}x^{10} - 130566c^8x^8 + 68553c^6x^6 + 279840c^4x^4 - 260420c^2x^2 + 74768)}{c^6/x^2/(c^2x^2-1)^2} \cdot (-c^2dx^2+d)^{3/2} \cdot (a+b\arcsin(cx))^2 - \frac{2(10125c^8x^8 - 24174c^6x^6 - 863c^4x^4 + 118868c^2x^2 - 56076)}{c^6/x^4/(c^2x^2-1)^2} \cdot (3x^2(-c^2dx^2+d)^{3/2}(a+b\arcsin(cx))^2 - 3x^4(-c^2dx^2+d)^{1/2}(a+b\arcsin(cx))^2c^2d+2x^3(-c^2dx^2+d)^{3/2}(a+b\arcsin(cx)) \cdot b \cdot c / (-c^2x^2+1)^{1/2} + 1/385875 \cdot (1125c^6x^6 - 2178c^4x^4 - 1679c^2x^2 + 18692) / c^6/x^3 \cdot (6x \cdot (-c^2dx^2+d)^{3/2}(a+b\arcsin(cx))^2 - 21x^3(-c^2dx^2+d)^{1/2}(a+b\arcsin(cx))^2c^2d + 12x^2(-c^2dx^2+d)^{3/2}(a+b\arcsin(cx)) \cdot b \cdot c / (-c^2x^2+1)^{1/2} + 3x^5/(-c^2dx^2+d)^{1/2}(a+b\arcsin(cx))^2c^4d^2 - 12x^4(-c^2dx^2+d)^{1/2}(a+b\arcsin(cx)) \cdot c^3 \cdot d \cdot b / (-c^2x^2+1)^{1/2} + 2x^3(-c^2dx^2+d)^{3/2} \cdot b^2 \cdot c^2 / (-c^2x^2+1) + 2x^4(-c^2dx^2+d)^{3/2}(a+b\arcsin(cx)) \cdot b \cdot c^3 / (-c^2x^2+1)^{3/2})$$

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.83

$$\int x^3(d - c^2dx^2)^{3/2}(a + b\arcsin(cx))^2 dx = \frac{210(75abc^7dx^7 - 168abc^5dx^5 + 35abc^3dx^3 + 210abcdx + (75b^2c^7dx^7 - 168b^2c^5dx^5 + 35b^2c^3dx^3 + 210b^2cdx + 210b^2c^2d^2x))}{(c^2dx^2+d)^{3/2}}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output

```
-1/385875*(210*(75*a*b*c^7*d*x^7 - 168*a*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 +
210*a*b*c*d*x + (75*b^2*c^7*d*x^7 - 168*b^2*c^5*d*x^5 + 35*b^2*c^3*d*x^3 +
210*b^2*c*d*x)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + (11
25*(49*a^2 - 2*b^2)*c^8*d*x^8 - 9*(15925*a^2 - 734*b^2)*c^6*d*x^6 + (99225
*a^2 - 998*b^2)*c^4*d*x^4 + (11025*a^2 - 40742*b^2)*c^2*d*x^2 + 11025*(5*b
^2*c^8*d*x^8 - 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 + b^2*c^2*d*x^2 - 2*b^2*
d)*arcsin(c*x)^2 - 2*(11025*a^2 - 18692*b^2)*d + 22050*(5*a*b*c^8*d*x^8 -
13*a*b*c^6*d*x^6 + 9*a*b*c^4*d*x^4 + a*b*c^2*d*x^2 - 2*a*b*d)*arcsin(c*x))
*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x^3 (-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))^2 dx$$

input

```
integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2,x)
```

output

```
Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.82

$$\begin{aligned}
& \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \\
& -\frac{1}{35} \left(\frac{5(-c^2 dx^2 + d)^{5/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{5/2}}{c^4 d} \right) b^2 \arcsin(cx)^2 \\
& -\frac{2}{35} \left(\frac{5(-c^2 dx^2 + d)^{5/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{5/2}}{c^4 d} \right) ab \arcsin(cx) \\
& -\frac{1}{35} \left(\frac{5(-c^2 dx^2 + d)^{5/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{5/2}}{c^4 d} \right) a^2 \\
& + \frac{2}{385875} b^2 \left(\frac{1125 \sqrt{-c^2 x^2 + 1} c^4 d^{3/2} x^6 - 2178 \sqrt{-c^2 x^2 + 1} c^2 d^{3/2} x^4 - 1679 \sqrt{-c^2 x^2 + 1} d^{3/2} x^2 + \frac{18692 \sqrt{-c^2 x^2 + 1}}{c^2}}{c^2} \right) \\
& + \frac{2 \left(75 c^6 d^{3/2} x^7 - 168 c^4 d^{3/2} x^5 + 35 c^2 d^{3/2} x^3 + 210 d^{3/2} x \right) ab}{3675 c^3}
\end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*b^2*arcsin(c*x)^2 - 2/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a*b*arcsin(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 + 2/385875*b^2*(1125*sqrt(-c^2*x^2 + 1)*c^4*d^(3/2)*x^6 - 2178*sqrt(-c^2*x^2 + 1)*c^2*d^(3/2)*x^4 - 1679*sqrt(-c^2*x^2 + 1)*d^(3/2)*x^2 + 18692*sqrt(-c^2*x^2 + 1)*d^(3/2)/c^2)/c^2 + 105*(75*c^6*d^(3/2)*x^7 - 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 + 210*d^(3/2)*x)*arcsin(c*x)/c^3 + 2/3675*(75*c^6*d^(3/2)*x^7 - 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 + 210*d^(3/2)*x)*a*b/c^3`

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x^3 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2} dx$$

input `int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d (-5\sqrt{-c^2 x^2 + 1} a^2 c^6 x^6 + 8\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 - \sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2)}{...}$$

input `int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2,x)`

output

```
(sqrt(d)*d*(- 5*sqrt(- c**2*x**2 + 1)*a**2*c**6*x**6 + 8*sqrt(- c**2*x*  
*2 + 1)*a**2*c**4*x**4 - sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt(-  
c**2*x**2 + 1)*a**2 - 70*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**5,x)*a*b  
*c**6 + 70*int(sqrt(- c**2*x**2 + 1)*asin(c*x)*x**3,x)*a*b*c**4 - 35*int(  
sqrt(- c**2*x**2 + 1)*asin(c*x)**2*x**5,x)*b**2*c**6 + 35*int(sqrt(- c**  
2*x**2 + 1)*asin(c*x)**2*x**3,x)*b**2*c**4))/(35*c**4)
```

3.216 $\int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2082
Mathematica [A] (verified)	2083
Rubi [A] (verified)	2084
Maple [C] (verified)	2091
Fricas [F]	2092
Sympy [F(-1)]	2093
Maxima [F]	2093
Giac [A] (verification not implemented)	2093
Mupad [F(-1)]	2094
Reduce [F]	2095

Optimal result

Integrand size = 29, antiderivative size = 421

$$\begin{aligned} \int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = & -\frac{7b^2dx\sqrt{d - c^2dx^2}}{1152c^2} \\ & -\frac{43b^2dx^3\sqrt{d - c^2dx^2}}{1728} + \frac{1}{108}b^2c^2dx^5\sqrt{d - c^2dx^2} \\ & + \frac{7b^2d\sqrt{d - c^2dx^2} \arcsin(cx)}{1152c^3\sqrt{1 - c^2x^2}} + \frac{bdx^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{16c\sqrt{1 - c^2x^2}} \\ & - \frac{7bcdx^4\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{48\sqrt{1 - c^2x^2}} + \frac{bc^3dx^6\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{18\sqrt{1 - c^2x^2}} \\ & - \frac{dx\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{16c^2} + \frac{1}{8}dx^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2 \\ & + \frac{1}{6}x^3(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 + \frac{d\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^3}{48bc^3\sqrt{1 - c^2x^2}} \end{aligned}$$

output

$$\begin{aligned}
& -7/1152*b^2*d*x*(-c^2*d*x^2+d)^{(1/2)}/c^2-43/1728*b^2*d*x^3*(-c^2*d*x^2+d)^{(1/2)} \\
& +1/108*b^2*c^2*d*x^5*(-c^2*d*x^2+d)^{(1/2)}+7/1152*b^2*d*(-c^2*d*x^2+d)^{(1/2)} \\
& *arcsin(c*x)/c^3/(-c^2*x^2+1)^{(1/2)}+1/16*b*d*x^2*(-c^2*d*x^2+d)^{(1/2)} \\
& *(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^{(1/2)}-7/48*b*c*d*x^4*(-c^2*d*x^2+d)^{(1/2)} \\
& *(a+b*arcsin(c*x))/(-c^2*x^2+1)^{(1/2)}+1/18*b*c^3*d*x^6*(-c^2*d*x^2+d)^{(1/2)} \\
& *(a+b*arcsin(c*x))/(-c^2*x^2+1)^{(1/2)}-1/16*d*x*(-c^2*d*x^2+d)^{(1/2)}*(a+b*arcsin(c*x))^2 \\
& /c^2+1/8*d*x^3*(-c^2*d*x^2+d)^{(1/2)}*(a+b*arcsin(c*x))^2+1/6*x^3*(-c^2*d*x^2+d)^{(3/2)} \\
& *(a+b*arcsin(c*x))^2+1/48*d*(-c^2*d*x^2+d)^{(1/2)}*(a+b*arcsin(c*x))^3/b/c^3/(-c^2*x^2+1)^{(1/2)}
\end{aligned}$$
Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.71

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{d\sqrt{d - c^2 dx^2}(72a^3 + 24ab^2c^2x^2(9 - 21c^2x^2 + 8c^4x^4) - 72a^2bcx\sqrt{1 - c^2x^2}(3 - 14c^2x^2))}{(3456b^3c^3\sqrt{1 - c^2x^2})}$$

input

`Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]`

output

$$\begin{aligned}
& (d*\text{Sqrt}[d - c^2*d*x^2]*(72*a^3 + 24*a*b^2*c^2*x^2*(9 - 21*c^2*x^2 + 8*c^4*x^4) \\
& - 72*a^2*b*c*x*\text{Sqrt}[1 - c^2*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) + b^3*c \\
& *x*\text{Sqrt}[1 - c^2*x^2]*(-21 - 86*c^2*x^2 + 32*c^4*x^4) + 3*b*(72*a^2 - 48*a* \\
& b*c*x*\text{Sqrt}[1 - c^2*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) + b^2*(7 + 72*c^2*x^2 \\
& - 168*c^4*x^4 + 64*c^6*x^6))*\text{ArcSin}[c*x] + 72*b^2*(3*a + b*c*x*\text{Sqrt}[1 - c \\
& ^2*x^2]*(-3 + 14*c^2*x^2 - 8*c^4*x^4))*\text{ArcSin}[c*x]^2 + 72*b^3*\text{ArcSin}[c*x]^3) \\
& / (3456*b*c^3*\text{Sqrt}[1 - c^2*x^2])
\end{aligned}$$

Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5202, 5192, 27, 363, 262, 262, 223, 5198, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - bcd\sqrt{d - c^2 dx^2} \int x^3 (1 - c^2 x^2) (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5192} \\
 & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - bcd\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^4 (3 - 2c^2 x^2)}{12\sqrt{1 - c^2 x^2}} dx - \frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{27} \\
 & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \int \frac{x^4 (3 - 2c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{363} \\
 & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \left(\frac{4}{3} \int \frac{x^4}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{3}x^5 \sqrt{1 - c^2 x^2} \right) - \frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2
 \end{aligned}$$

$$\begin{aligned} & \downarrow 262 \\ & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx -}{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) + \frac{1}{3}x^5 \sqrt{1 - c^2 x^2} \right) - \frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} \\ & \frac{\frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{6} \end{aligned}$$

$$\begin{aligned} & \downarrow 262 \\ & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx -}{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2 x^2}} dx}{2c^2} - \frac{x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) + \frac{1}{3}x^5 \sqrt{1 - c^2 x^2} \right) - \frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} \\ & \frac{\frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{6} \end{aligned}$$

$$\begin{aligned} & \downarrow 223 \\ & \frac{\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 -}{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \end{aligned}$$

$$\begin{aligned} & \downarrow 5198 \\ & \frac{\frac{1}{2}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^2 (a + b \arcsin(cx))^2 dx}{\sqrt{1-c^2 x^2}}}{4\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^3 (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{1}{6}x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 -}{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \end{aligned}$$

$$\downarrow 5138$$

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \int \frac{x^4}{\sqrt{1-c^2x^2}} dx \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2} - \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{3\sqrt{1-c^2x^2}} \right)$$

↓ 262

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2} - \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{3\sqrt{1-c^2x^2}} \right)$$

↓ 262

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} \right) \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2} - \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{3\sqrt{1-c^2x^2}} \right)$$

↓ 223

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx))^2 - \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{3\sqrt{1-c^2x^2}} \right)$$

↓ 5210

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x(a+b\arcsin(cx)) dx}{c} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{3\sqrt{1-c^2x^2}} \right)$$

↓ 5138

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c} + \frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{3\sqrt{1-c^2x^2}} \right)$$

↓ 262

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2}}{4\sqrt{1-c^2x^2}} \right)}{\frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b \arcsin(cx)) + \frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{3\sqrt{1-c^2x^2}} \right)}$$

↓ 223

$$\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right)}{4\sqrt{1-c^2x^2}} \right)}{\frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 - bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b \arcsin(cx)) + \frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{3\sqrt{1-c^2x^2}} \right)}$$

↓ 5152

$$\frac{1}{2}d \left(\frac{\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2 + \frac{\sqrt{d-c^2dx^2} \left(\frac{(a+b \arcsin(cx))^3}{6bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right)}{4\sqrt{1-c^2x^2}}}{\frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b \arcsin(cx))^2 + bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}c^2x^6(a+b \arcsin(cx)) + \frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{3\sqrt{1-c^2x^2}} \right)}$$

input `Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]`

output `(x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/6 - (b*c*d*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcSin[c*x]))/4 - (c^2*x^6*(a + b*ArcSin[c*x]))/6 - (b*c*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/3))/12)/((3*Sqrt[1 - c^2*x^2]) + (d*((x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/4 - (b*c*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcSin[c*x]))/4 - (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4))/2*Sqrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2 + (a + b*ArcSin[c*x])^3/(6*b*c^3) + (b*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c))/((4*Sqrt[1 - c^2*x^2])))/2`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 262 `Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 363 `Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 3, 0]`

rule 5138 $\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n \cdot (d \cdot x)^m, x_Symbol] \rightarrow \text{Simp}[(d \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n / (d \cdot (m+1)), x] - \text{Simp}[b \cdot c \cdot n / (d \cdot (m+1)) \cdot \text{Int}[(d \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n-1} / \text{Sqrt}[1 - c^2 \cdot x^2]], x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

rule 5152 $\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n / \text{Sqrt}[d + e \cdot x^2], x_Symbol] \rightarrow \text{Simp}[(1 / (b \cdot c \cdot (n+1))) \cdot \text{Simp}[\text{Sqrt}[1 - c^2 \cdot x^2] / \text{Sqrt}[d + e \cdot x^2]] \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n+1}, x] /;$ FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2 \cdot d + e, 0] && NeQ[n, -1]

rule 5192 $\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b) \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f \cdot x)^m \cdot (d + e \cdot x^2)^p, x]\}, \text{Simp}[a + b \cdot \text{ArcSin}[c \cdot x], u, x] - \text{Simp}[b \cdot c \cdot \text{Int}[\text{SimplifyIntegrand}[u / \text{Sqrt}[1 - c^2 \cdot x^2]], x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2 \cdot d + e, 0] && IGtQ[p, 0]

rule 5198 $\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n \cdot (f \cdot x)^m \cdot \text{Sqrt}[d + e \cdot x^2], x_Symbol] \rightarrow \text{Simp}[(f \cdot x)^{m+1} \cdot \text{Sqrt}[d + e \cdot x^2] \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n / (f \cdot (m+2)), x] + (\text{Simp}[(1 / (m+2)) \cdot \text{Simp}[\text{Sqrt}[d + e \cdot x^2] / \text{Sqrt}[1 - c^2 \cdot x^2]] \cdot \text{Int}[(f \cdot x)^m \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n / \text{Sqrt}[1 - c^2 \cdot x^2]], x], x] - \text{Simp}[b \cdot c \cdot n / (f \cdot (m+2)) \cdot \text{Simp}[\text{Sqrt}[d + e \cdot x^2] / \text{Sqrt}[1 - c^2 \cdot x^2]] \cdot \text{Int}[(f \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n-1}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2 \cdot d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])

rule 5202 $\text{Int}[(a + \text{ArcSin}[c \cdot x] \cdot b)^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x_Symbol] \rightarrow \text{Simp}[(f \cdot x)^{m+1} \cdot (d + e \cdot x^2)^p \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n / (f \cdot (m+2 \cdot p+1)), x] + (\text{Simp}[2 \cdot d \cdot (p / (m+2 \cdot p+1)) \cdot \text{Int}[(f \cdot x)^m \cdot (d + e \cdot x^2)^{p-1} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^n, x], x] - \text{Simp}[b \cdot c \cdot n / (f \cdot (m+2 \cdot p+1)) \cdot \text{Simp}[(d + e \cdot x^2)^p / (1 - c^2 \cdot x^2)^p] \cdot \text{Int}[(f \cdot x)^{m+1} \cdot (1 - c^2 \cdot x^2)^{p-1/2} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^{n-1}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2 \cdot d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]

rule 5210

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]

```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 1320, normalized size of antiderivative = 3.14

method	result	size
default	Expression too large to display	1320
parts	Expression too large to display	1320

input

```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```


output

```

-1/6*a^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+
1/16*a^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a^2/c^2*d^2/(c^2*d)^(1/2)*arctan
n((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/48*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^3*d-1/6912*(-d*(c^2*x^2-1))
^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)
)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^
2+1)^(1/2)-6*c*x)*(6*I*arcsin(c*x)+18*arcsin(c*x)^2-1)*d/c^3/(c^2*x^2-1)+1
/256*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-
c^2*x^2+1)^(1/2)-2*c*x)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))*d/c^3/(c^2*x^2
-1)+1/27648*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(1
32*I*arcsin(c*x)+144*arcsin(c*x)^2-23)*cos(5*arcsin(c*x))*d/c^3/(c^2*x^2-1
)-1/27648*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(84*
I*arcsin(c*x)+288*arcsin(c*x)^2-31)*sin(5*arcsin(c*x))*d/c^3/(c^2*x^2-1)-1
/1024*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(4*I*arc
sin(c*x)+16*arcsin(c*x)^2-5)*cos(3*arcsin(c*x))*d/c^3/(c^2*x^2-1)+3/1024*(
-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(I+4*arcsin(c*x
))*sin(3*arcsin(c*x))*d/c^3/(c^2*x^2-1))+2*a*b*(-1/32*(-d*(c^2*x^2-1))^(1/
2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^2*d-1/2304*(-d*(c^2*x^2-
1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(
1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-...

```

Fricas [F]

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 x^2 dx$$

input

```

integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")

```

output

```

integral(-(a^2*c^2*d*x^4 - a^2*d*x^2 + (b^2*c^2*d*x^4 - b^2*d*x^2)*arcsin(
c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d),
x)

```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2 x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `1/48*a^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + sqrt(d)*integrate(-((b^2*c^2*d*x^4 - b^2*d*x^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)`

Giac [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.34

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = -\frac{1}{6} \sqrt{-c^2 dx^2 + d} a^2 c^2 dx^5 + \frac{7}{24} \sqrt{-c^2 dx^2 + d} a^2 dx^3 - \frac{\sqrt{-c^2 dx^2 + d} a^2 dx}{16 c^2} - \frac{a^2 d^2 \log(|-c\sqrt{-d}x + \sqrt{c^2 x^2 - 1}\sqrt{-d}|)}{16 c^3 \sqrt{-d}} - \frac{576 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} b^2 d^{\frac{3}{2}} x \arcsin(cx)^2 + 1152 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} a b d^{\frac{3}{2}} x \arcsin(cx) - 144 (-$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `-1/6*sqrt(-c^2*d*x^2 + d)*a^2*c^2*d*x^5 + 7/24*sqrt(-c^2*d*x^2 + d)*a^2*d*x^3 - 1/16*sqrt(-c^2*d*x^2 + d)*a^2*d*x/c^2 - 1/16*a^2*d^2*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)) - 1/3456*(576*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^(3/2)*x*arcsin(c*x)^2 + 1152*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^(3/2)*x*arcsin(c*x) - 144*(-c^2*x^2 + 1)^(3/2)*b^2*d^(3/2)*x*arcsin(c*x)^2 - 32*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b^2*d^(3/2)*x - 288*(-c^2*x^2 + 1)^(3/2)*a*b*d^(3/2)*x*arcsin(c*x) - 216*sqrt(-c^2*x^2 + 1)*b^2*d^(3/2)*x*arcsin(c*x)^2 - 192*(c^2*x^2 - 1)^3*b^2*d^(3/2)*arcsin(c*x)/c - 22*(-c^2*x^2 + 1)^(3/2)*b^2*d^(3/2)*x - 432*sqrt(-c^2*x^2 + 1)*a*b*d^(3/2)*x*arcsin(c*x) - 192*(c^2*x^2 - 1)^3*a*b*d^(3/2)/c - 72*(c^2*x^2 - 1)^2*b^2*d^(3/2)*arcsin(c*x)/c - 72*b^2*d^(3/2)*arcsin(c*x)^3/c + 75*sqrt(-c^2*x^2 + 1)*b^2*d^(3/2)*x - 72*(c^2*x^2 - 1)^2*a*b*d^(3/2)/c + 216*(c^2*x^2 - 1)*b^2*d^(3/2)*arcsin(c*x)/c - 216*a*b*d^(3/2)*arcsin(c*x)^2/c + 216*(c^2*x^2 - 1)*a*b*d^(3/2)/c + 75*b^2*d^(3/2)*arcsin(c*x)/c + 75*a*b*d^(3/2)/c)/c^2`

Mupad [F(-1)]

Timed out.

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x^2 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2} dx$$

input `int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d (3 a \sin(cx) a^2 - 8 \sqrt{-c^2 x^2 + 1} a^2 c^5 x^5 + 14 \sqrt{-c^2 x^2 + 1} a^2 c^3 x^3 - 3 \sqrt{-c^2 x^2 + 1} a^2 c x - 96 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4, x) a b c^5 + 96 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) a b c^3 - 48 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4, x) b^2 c^5 + 48 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c^3)}{48 c^3}$$

input

```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2,x)
```

output

```
(sqrt(d)*d*(3*asin(c*x)*a**2 - 8*sqrt(-c**2*x**2 + 1)*a**2*c**5*x**5 + 14*sqrt(-c**2*x**2 + 1)*a**2*c**3*x**3 - 3*sqrt(-c**2*x**2 + 1)*a**2*c*x - 96*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*a*b*c**5 + 96*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*a*b*c**3 - 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**4,x)*b**2*c**5 + 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**3))/(48*c**3)
```

3.217 $\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2096
Mathematica [A] (verified)	2097
Rubi [A] (verified)	2097
Maple [B] (verified)	2100
Fricas [A] (verification not implemented)	2100
Sympy [F]	2101
Maxima [A] (verification not implemented)	2101
Giac [F(-2)]	2102
Mupad [F(-1)]	2102
Reduce [F]	2103

Optimal result

Integrand size = 27, antiderivative size = 258

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{16b^2 d \sqrt{d - c^2 dx^2}}{75c^2} + \frac{8b^2 (d - c^2 dx^2)^{3/2}}{225c^2} + \frac{2b^2 (d - c^2 dx^2)^{5/2}}{125c^2 d} + \frac{2bdx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{5c \sqrt{1 - c^2 x^2}} - \frac{4bcdx^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{15 \sqrt{1 - c^2 x^2}} + \frac{2bc^3 dx^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{25 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{5c^2 d}$$

output

```
16/75*b^2*d*(-c^2*d*x^2+d)^(1/2)/c^2+8/225*b^2*(-c^2*d*x^2+d)^(3/2)/c^2+2/125*b^2*(-c^2*d*x^2+d)^(5/2)/c^2/d+2/5*b*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-4/15*b*c*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+2/25*b*c^3*d*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = -\frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{5c^2 d} + \frac{2bd\sqrt{d - c^2 dx^2}(15acx(15 - 10c^2 x^2 + 3c^4 x^4) + b\sqrt{1 - c^2 x^2}(149 - 38c^2 x^2 + 9c^4 x^4) + 15bcx(15 - 10c^2 x^2 + 3c^4 x^4))}{1125c^2 \sqrt{1 - c^2 x^2}}$$

input

```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*d*Sqrt[d - c^2*d*x^2]*(15*a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*(149 - 38*c^2*x^2 + 9*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)*ArcSin[c*x]))/(1125*c^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5182, 5154, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5182$$

$$\frac{2bd\sqrt{d - c^2 dx^2} \int (1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{5c\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{5c^2 d}$$

$$\downarrow 5154$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-bc\int\frac{x(3c^4x^4-10c^2x^2+15)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arcsin(cx))-\frac{2}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{5c\sqrt{1-c^2x^2}\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{5c^2d}}$$

↓ 27

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{1}{15}bc\int\frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arcsin(cx))-\frac{2}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{5c\sqrt{1-c^2x^2}\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{5c^2d}}$$

↓ 1576

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{1}{30}bc\int\frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4x^5(a+b\arcsin(cx))-\frac{2}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{5c\sqrt{1-c^2x^2}\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{5c^2d}}$$

↓ 1140

$$\frac{2bd\sqrt{d-c^2dx^2}\left(-\frac{1}{30}bc\int\left(3(1-c^2x^2)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\arcsin(cx))-\frac{2}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{5c\sqrt{1-c^2x^2}\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{5c^2d}}$$

↓ 2009

$$\frac{2bd\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arcsin(cx))-\frac{2}{3}c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))-\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2}\right)\right)}{5c\sqrt{1-c^2x^2}\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{5c^2d}}$$

input

```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*d*Sqrt[d
- c^2*d*x^2]*(-1/30*(b*c*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(
3/2))/(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2))) + x*(a + b*ArcSin[c*x])
- (2*c^2*x^3*(a + b*ArcSin[c*x]))/3 + (c^4*x^5*(a + b*ArcSin[c*x]))/5)/(5
*c*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 1140

```
Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x
_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Simp[1/2 Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5154

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbo
l] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x
] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5182

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_))^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p
_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```


Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(224) = 448$.

Time = 0.77 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.80

method	result
ordering	$\frac{(549c^8x^8 - 1982c^6x^6 + 4355c^4x^4 - 1420c^2x^2 + 298)(-c^2dx^2 + d)^{\frac{3}{2}}(a + b\arcsin(cx))^2}{1125c^4x^2(c^2x^2 - 1)^2} - \frac{2(54c^6x^6 - 217c^4x^4 + 672c^2x^2 - 149)}{(-c^2dx^2 + d)^{\frac{3}{2}}}$
default	Expression too large to display
parts	Expression too large to display

input `int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output
$$\frac{1}{1125} \frac{(549c^8x^8 - 1982c^6x^6 + 4355c^4x^4 - 1420c^2x^2 + 298)}{c^4/x^2/(c^2x^2 - 1)^2} (-c^2dx^2 + d)^{3/2} (a + b\arcsin(cx))^2 - \frac{2(54c^6x^6 - 217c^4x^4 + 672c^2x^2 - 149)}{c^4/x^2/(c^2x^2 - 1)} ((-c^2dx^2 + d)^{3/2} (a + b\arcsin(cx))^2 - 3x^2(-c^2dx^2 + d)^{1/2} (a + b\arcsin(cx))^2 c^2d + 2x(-c^2dx^2 + d)^{3/2} (a + b\arcsin(cx)) * b c / (-c^2x^2 + 1)^{1/2}) + \frac{1}{1125} (9c^4x^4 - 38c^2x^2 + 149) / c^4/x^2 (-9c^2dx^2 * (-c^2dx^2 + d)^{1/2} (a + b\arcsin(cx))^2 + 4(-c^2dx^2 + d)^{3/2} (a + b\arcsin(cx)) * b c / (-c^2x^2 + 1)^{1/2} + 3x^3 / (-c^2dx^2 + d)^{1/2} (a + b\arcsin(cx))^2 c^4d^2 - 12b^2c^3dx^2 * (-c^2dx^2 + d)^{1/2} (a + b\arcsin(cx)) / (-c^2x^2 + 1)^{1/2} + 2x * (-c^2dx^2 + d)^{3/2} * b^2c^2 / (-c^2x^2 + 1) + 2x^2 * (-c^2dx^2 + d)^{3/2} (a + b\arcsin(cx)) * b^2c^3 / (-c^2x^2 + 1)^{3/2})$$

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.14

$$\int x(d - c^2dx^2)^{3/2} (a + b\arcsin(cx))^2 dx = \frac{30(3abc^5dx^5 - 10abc^3dx^3 + 15abcdx + (3b^2c^5dx^5 - 10b^2c^3dx^3 + 15b^2cdx)\arcsin(cx))\sqrt{-c^2dx^2 + d}}{\sqrt{-c^2dx^2 + d}}$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output

```
-1/1125*(30*(3*a*b*c^5*d*x^5 - 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x + (3*b^2*c^5*d*x^5 - 10*b^2*c^3*d*x^3 + 15*b^2*c*d*x)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + (9*(25*a^2 - 2*b^2)*c^6*d*x^6 - (675*a^2 - 94*b^2)*c^4*d*x^4 + (675*a^2 - 374*b^2)*c^2*d*x^2 + 225*(b^2*c^6*d*x^6 - 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 - (225*a^2 - 298*b^2)*d + 450*(a*b*c^6*d*x^6 - 3*a*b*c^4*d*x^4 + 3*a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))^2 dx$$

input

```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2,x)
```

output

```
Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.91

$$\begin{aligned} \int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx &= -\frac{(-c^2 dx^2 + d)^{5/2} b^2 \arcsin(cx)^2}{5 c^2 d} \\ &+ \frac{2}{1125} b^2 \left(\frac{9 \sqrt{-c^2 x^2 + 1} c^2 d^{5/2} x^4 - 38 \sqrt{-c^2 x^2 + 1} d^{5/2} x^2 + \frac{149 \sqrt{-c^2 x^2 + 1} d^{5/2}}{c^2}}{d} + \frac{15 (3 c^4 d^{5/2} x^5 - 10 c^2 d^{5/2} x^3 + 15 d^{5/2})}{cd} \right) \\ &- \frac{2(-c^2 dx^2 + d)^{5/2} ab \arcsin(cx)}{5 c^2 d} - \frac{(-c^2 dx^2 + d)^{5/2} a^2}{5 c^2 d} \\ &+ \frac{2(3 c^4 d^{5/2} x^5 - 10 c^2 d^{5/2} x^3 + 15 d^{5/2} x) ab}{75 cd} \end{aligned}$$

input

```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```
-1/5*(-c^2*d*x^2 + d)^(5/2)*b^2*arcsin(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqrt(-c^2*x^2 + 1)*c^2*d^(5/2)*x^4 - 38*sqrt(-c^2*x^2 + 1)*d^(5/2)*x^2 + 149*sqrt(-c^2*x^2 + 1)*d^(5/2)/c^2)/d + 15*(3*c^4*d^(5/2)*x^5 - 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*arcsin(c*x)/(c*d)) - 2/5*(-c^2*d*x^2 + d)^(5/2)*a*b*arcsin(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) + 2/75*(3*c^4*d^(5/2)*x^5 - 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2} dx$$

input

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d (-\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 + 2\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} a^2 - 10(\int \sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 dx - 10 \int \sqrt{-c^2 x^2 + 1} a^2 dx))}{5c^2}$$

input `int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(d)*d*(-sqrt(-c**2*x**2+1)*a**2*c**4*x**4+2*sqrt(-c**2*x**2+1)*a**2*c**2*x**2-sqrt(-c**2*x**2+1)*a**2-10*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*a*b*c**4+10*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*a*b*c**2-5*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x**3,x)*b**2*c**4+5*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x,x)*b**2*c**2))/(5*c**2)`

3.218 $\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2104
Mathematica [A] (verified)	2105
Rubi [A] (verified)	2105
Maple [C] (verified)	2109
Fricas [F]	2110
Sympy [F]	2111
Maxima [F]	2111
Giac [F(-2)]	2111
Mupad [F(-1)]	2112
Reduce [F]	2112

Optimal result

Integrand size = 26, antiderivative size = 296

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = -\frac{15}{64} b^2 dx \sqrt{d - c^2 dx^2} - \frac{1}{32} b^2 x (d - c^2 dx^2)^{3/2} + \frac{9b^2 d \sqrt{d - c^2 dx^2} \arcsin(cx)}{64c \sqrt{1 - c^2 x^2}} - \frac{3bcdx^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{8 \sqrt{1 - c^2 x^2}} + \frac{bd(1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{8c} + \frac{3}{8} dx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 + \frac{1}{4} x (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 + \frac{d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^3}{8bc \sqrt{1 - c^2 x^2}}$$

output

```
-15/64*b^2*d*x*(-c^2*d*x^2+d)^(1/2)-1/32*b^2*x*(-c^2*d*x^2+d)^(3/2)+9/64*b^2*d*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(1/2)-3/8*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+1/8*b*d*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c+3/8*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2+1/8*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.83

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{d\sqrt{d - c^2 dx^2}(8a^3 + 8ab^2c^2x^2(-5 + c^2x^2) + b^3cx\sqrt{1 - c^2x^2}(-17 + 2c^2x^2) - 8a^2bcx\sqrt{1 - c^2x^2})}{64b^3c\sqrt{1 - c^2x^2}} + \dots$$

input

```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d*Sqrt[d - c^2*d*x^2]*(8*a^3 + 8*a*b^2*c^2*x^2*(-5 + c^2*x^2) + b^3*c*x*Sqrt[1 - c^2*x^2]*(-17 + 2*c^2*x^2) - 8*a^2*b*c*x*Sqrt[1 - c^2*x^2]*(-5 + 2*c^2*x^2) + b*(24*a^2 + 16*a*b*c*x*(5 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + b^2*(17 - 40*c^2*x^2 + 8*c^4*x^4))*ArcSin[c*x] + 8*b^2*(3*a + b*c*x*(5 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 + 8*b^3*ArcSin[c*x]^3)/(64*b*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5158, 5156, 5138, 262, 223, 5152, 5182, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5158}$$

$$-\frac{bcd\sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{3}{4}d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{4}x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2$$

$$\downarrow \text{5156}$$

$$\begin{aligned}
& -\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \\
\frac{3}{4}d & \left(-\frac{bc\sqrt{d-c^2dx^2} \int x(a+b\arcsin(cx))dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right. \\
& \quad \left. + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 \right) \\
& \quad \downarrow \text{5138} \\
& -\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \\
\frac{3}{4}d & \left(-\frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right. \\
& \quad \left. + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 \right) \\
& \quad \downarrow \text{262} \\
& -\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \\
\frac{3}{4}d & \left(-\frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} \right. \\
& \quad \left. + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 \right) \\
& \quad \downarrow \text{223} \\
& -\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \\
\frac{3}{4}d & \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right. \\
& \quad \left. + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 \right) \\
& \quad \downarrow \text{5152} \\
& -\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 + \\
\frac{3}{4}d & \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow \text{5182}
\end{aligned}$$

$$\begin{aligned}
& \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \int (1-c^2x^2)^{3/2} dx}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2} (a + \\
& \quad b \arcsin(cx))^2 + \\
& \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow 211 \\
& \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \\
& \quad \frac{1}{4}x(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2 + \\
& \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow 211 \\
& \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \\
& \quad \frac{1}{4}x(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2 + \\
& \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow 223 \\
& \frac{1}{4}x(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2 - \\
& \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \\
& \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right)
\end{aligned}$$

input

```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```


output

$$\begin{aligned} & (x*(d - c^2*d*x^2)^{(3/2)}*(a + b*\text{ArcSin}[c*x])^2)/4 + (3*d*((x*\text{Sqrt}[d - c^2*d*x^2])*(a + b*\text{ArcSin}[c*x]) \\ & ^3)/(6*b*c*\text{Sqrt}[1 - c^2*x^2]) - (b*c*\text{Sqrt}[d - c^2*d*x^2]*((x^2*(a + b*\text{ArcSin}[c*x]))/2 - (b*c*(-1/2*(x*\text{Sqrt}[1 - c^2*x^2])/c^2 + \text{ArcSin}[c*x]/(2*c^3)))/2))/\text{Sqrt}[1 - c^2*x^2])/4 - (b*c*d*\text{Sqrt}[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*(a + b*\text{ArcSin}[c*x]))/c^2 + (b*((x*(1 - c^2*x^2)^{(3/2)})/4 + (3*((x*\text{Sqrt}[1 - c^2*x^2])/2 + \text{ArcSin}[c*x]/(2*c)))/4))/(4*c)))/(2*\text{Sqrt}[1 - c^2*x^2]) \end{aligned}$$
Defintions of rubi rules used

rule 211

$$\text{Int}[(a + (b*x)^2)^p, x_Symbol] \rightarrow \text{Simp}[x*(a + b*x^2)^p/(2*p + 1), x] + \text{Simp}[2*a*(p/(2*p + 1)) \text{Int}[(a + b*x^2)^{p-1}, x], x] /; \text{FreeQ}[a, b], x \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ (\text{IntegerQ}[4*p] \ || \ \text{IntegerQ}[6*p])$$

rule 223

$$\text{Int}[1/\text{Sqrt}[a + (b*x)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[a, b], x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$$

rule 262

$$\text{Int}[(c*x)^m*(a + (b*x)^2)^p, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{m-1}*((a + b*x^2)^{p+1}/(b*(m + 2*p + 1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m + 2*p + 1))) \text{Int}[(c*x)^{m-2}*(a + b*x^2)^p, x], x] /; \text{FreeQ}[a, b, c, p], x \ \&\& \ \text{GtQ}[m, 2 - 1] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$$

rule 5138

$$\text{Int}[(a + \text{ArcSin}[c*x]*b)^n*(d*x)^m, x_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcSin}[c*x])^n/(d*(m+1)), x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{Int}[(d*x)^{m+1}*(a + b*\text{ArcSin}[c*x])^{n-1}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[a, b, c, d, m], x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$$

rule 5152

$$\text{Int}[(a + \text{ArcSin}[c*x]*b)^n/\text{Sqrt}[d + e*x^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{n+1}, x] /; \text{FreeQ}[a, b, c, d, e, n], x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$$

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.14

method	result
default	$\frac{x(-c^2dx^2+d)^{\frac{3}{2}}a^2}{4} + \frac{3a^2dx\sqrt{-c^2dx^2+d}}{8} + \frac{3a^2d^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^3d}{8(c^2x^2-1)c} \right)$
parts	$\frac{x(-c^2dx^2+d)^{\frac{3}{2}}a^2}{4} + \frac{3a^2dx\sqrt{-c^2dx^2+d}}{8} + \frac{3a^2d^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1} \arcsin(cx)^3d}{8(c^2x^2-1)c} \right)$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```

1/4*x*(-c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^
2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/8*(-d
*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arcsin(c*x)^3*d-1/512
*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c
^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(4*I*arcsin
(c*x)+8*arcsin(c*x)^2-1)*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*I*
(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arcsin
(c*x)^2-1-2*I*arcsin(c*x))*d/(c^2*x^2-1)/c-1/512*(-d*(c^2*x^2-1))^(1/2)*(I
*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(68*I*arcsin(c*x)+56*arcsin(c*x)^2-31)*
cos(3*arcsin(c*x))*d/(c^2*x^2-1)/c+3/512*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x
^2+1)^(1/2)*c*x+c^2*x^2-1)*(20*I*arcsin(c*x)+24*arcsin(c*x)^2-11)*sin(3*ar
csin(c*x))*d/(c^2*x^2-1)/c)+2*a*b*(-3/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+
1)^(1/2)/(c^2*x^2-1)/c*arcsin(c*x)^2*d-1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*I*
(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3
*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))*d/(c^2*x^2-1)/c+1/16*(-
d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2
+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))*d/(c^2*x^2-1)/c-1/256*(-d*(c^2*x^2-1))
^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(17*I+28*arcsin(c*x))*cos(3*ar
csin(c*x))*d/(c^2*x^2-1)/c+3/256*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1
/2)*c*x+c^2*x^2-1)*(5*I+12*arcsin(c*x))*sin(3*arcsin(c*x))*d/(c^2*x^2-1...

```

Fricas [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 dx$$

input

```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```

integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 +
2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)

```

Sympy [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \operatorname{asin}(cx))^2 dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*arcsin(c*x)/c)*a^2 + sqrt(d)*integrate(-((b^2*c^2*d*x^2 - b^2*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)`

Giac [F(-2)]

Exception generated.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2} dx$$

input `int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2), x)`

output `int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d (3 \arcsin(cx) a^2 - 2 \sqrt{-c^2 x^2 + 1} a^2 c^3 x^3 + 5 \sqrt{-c^2 x^2 + 1} a^2 c x - 16 \int \sqrt{-c^2 x^2 + 1} a$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(d)*d*(3*asin(c*x)*a**2 - 2*sqrt(-c**2*x**2 + 1)*a**2*c**3*x**3 + 5*sqrt(-c**2*x**2 + 1)*a**2*c*x - 16*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*a*b*c**3 + 16*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*a*b*c - 8*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2,x)*b**2*c))/(8*c)`

3.219
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx$$

Optimal result	2113
Mathematica [A] (verified)	2114
Rubi [A] (verified)	2115
Maple [B] (verified)	2121
Fricas [F]	2122
Sympy [F]	2123
Maxima [F]	2123
Giac [F(-2)]	2123
Mupad [F(-1)]	2124
Reduce [F]	2124

Optimal result

Integrand size = 29, antiderivative size = 534

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = & -\frac{22}{9} b^2 d \sqrt{d - c^2 dx^2} \\ & - \frac{2abcdx\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} - \frac{2}{27} b^2 (d - c^2 dx^2)^{3/2} \\ & - \frac{2b^2 c dx \sqrt{d - c^2 dx^2} \arcsin(cx)}{\sqrt{1 - c^2 x^2}} - \frac{2bcdx\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))}{3\sqrt{1 - c^2 x^2}} \\ & + \frac{2bc^3 dx^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{9\sqrt{1 - c^2 x^2}} + d\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\ & + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - \frac{2d\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} + \frac{2ibd\sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```

-22/9*b^2*d*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^
2+1)^(1/2)-2/27*b^2*(-c^2*d*x^2+d)^(3/2)-2*b^2*c*d*x*(-c^2*d*x^2+d)^(1/2)*
arcsin(c*x)/(-c^2*x^2+1)^(1/2)-2/3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsi
n(c*x))/(-c^2*x^2+1)^(1/2)+2/9*b*c^3*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsi
n(c*x))/(-c^2*x^2+1)^(1/2)+d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2+1/3*
(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2-2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
sin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*d*(
-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))
/(-c^2*x^2+1)^(1/2)-2*I*b*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog
(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*d*(-c^2*d*x^2+d)^(1/
2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*d*(-c^2*d
*x^2+d)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.08

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = -\frac{1}{3} a^2 d (-4 + c^2 x^2) \sqrt{d - c^2 dx^2} + a^2 d^{3/2} \log(cx) - a^2 d^{3/2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) + \frac{2abd\sqrt{d - c^2 dx^2}(-cx + \sqrt{1 - c^2 x^2} \arcsin(cx) + \arcsin(cx) \log(1 - e^{ia}))}{x}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x,x]
```

output

```

-1/3*(a^2*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) + a^2*d^(3/2)*Log[c*x] - a
^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d*Sqrt[d - c^2*d*
x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]*Log[1 - E^(I*Ar
cSin[c*x])]) - ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])]) + I*PolyLog[2, -E^(I*
ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])])/Sqrt[1 - c^2*x^2] - (b^2
*d*Sqrt[d - c^2*d*x^2]*(2*Sqrt[1 - c^2*x^2] + 2*c*x*ArcSin[c*x] - Sqrt[1 -
c^2*x^2]*ArcSin[c*x]^2 - ArcSin[c*x]^2*(Log[1 - E^(I*ArcSin[c*x])]) - Log[
1 + E^(I*ArcSin[c*x])]) - (2*I)*ArcSin[c*x]*(PolyLog[2, -E^(I*ArcSin[c*x])
]) - PolyLog[2, E^(I*ArcSin[c*x])]) + 2*(PolyLog[3, -E^(I*ArcSin[c*x])]) - P
olyLog[3, E^(I*ArcSin[c*x])])/Sqrt[1 - c^2*x^2] - (a*b*d*Sqrt[d - c^2*d*
x^2]*(9*c*x - 3*ArcSin[c*x]*(3*Sqrt[1 - c^2*x^2] + Cos[3*ArcSin[c*x]]) + S
in[3*ArcSin[c*x]]))/(18*Sqrt[1 - c^2*x^2]) + (b^2*d*Sqrt[d - c^2*d*x^2]*(2
7*Sqrt[1 - c^2*x^2]*(-2 + ArcSin[c*x]^2) + (-2 + 9*ArcSin[c*x]^2)*Cos[3*Ar
cSin[c*x]] - 6*ArcSin[c*x]*(9*c*x + Sin[3*ArcSin[c*x]])))/(108*Sqrt[1 - c^
2*x^2])

```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.71, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5202, 5154, 27, 353, 53, 2009, 5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx$$

↓ 5202

$$-\frac{2bcd\sqrt{d - c^2 dx^2} \int (1 - c^2 x^2) (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx +$$

$$\frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2$$

↓ 5154

$$\begin{aligned}
& \frac{d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx - 2bcd\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x(3 - c^2 x^2)}{3\sqrt{1 - c^2 x^2}} dx - \frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{27} \\
& \frac{d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{3} bc \int \frac{x(3 - c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{353} \\
& \frac{d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{6} bc \int \frac{3 - c^2 x^2}{\sqrt{1 - c^2 x^2}} dx^2 - \frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{53} \\
& \frac{d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{6} bc \int \left(\sqrt{1 - c^2 x^2} + \frac{2}{\sqrt{1 - c^2 x^2}} \right) dx^2 - \frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \\
& \quad \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
& \quad \downarrow \text{2009} \\
& \frac{d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx + \frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{6} bc \left(-\frac{2(1 - c^2 x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1 - c^2 x^2}}{c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \\
& \quad \downarrow \text{5198}
\end{aligned}$$

$$d \left(-\frac{2bc\sqrt{d-c^2dx^2} \int (a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 \right) \\ - \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 - \\ \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) - \frac{1}{6}bc \left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1-c^2x^2}}{c^2} \right) \right)}{3\sqrt{1-c^2x^2}}$$

↓ 2009

$$d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax+bx \arcsin(cx) + b\sqrt{1-c^2x^2})}{\sqrt{1-c^2x^2}} \right) \\ - \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 - \\ \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) - \frac{1}{6}bc \left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1-c^2x^2}}{c^2} \right) \right)}{3\sqrt{1-c^2x^2}}$$

↓ 5218

$$d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b \arcsin(cx))^2}{cx} d \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax+bx \arcsin(cx) + b\sqrt{1-c^2x^2})}{\sqrt{1-c^2x^2}} \right) \\ - \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 - \\ \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) - \frac{1}{6}bc \left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1-c^2x^2}}{c^2} \right) \right)}{3\sqrt{1-c^2x^2}}$$

↓ 3042

$$d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax+bx \arcsin(cx) + b\sqrt{1-c^2x^2})}{\sqrt{1-c^2x^2}} \right) \\ - \frac{1}{3} (d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2 - \\ \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) - \frac{1}{6}bc \left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1-c^2x^2}}{c^2} \right) \right)}{3\sqrt{1-c^2x^2}}$$

↓ 4671

$$d \left(\frac{\sqrt{d - c^2 dx^2} (-2b \int (a + b \arcsin(cx)) \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a + b \arcsin(cx)) \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{1 - c^2 x^2}} \right. \\ \left. \frac{\frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{6} bc \left(-\frac{2(1 - c^2 x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1 - c^2 x^2}}{c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \right) \\ \downarrow \text{3011}$$

$$d \left(\frac{\sqrt{d - c^2 dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{1 - c^2 x^2}} \right. \\ \left. \frac{\frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{6} bc \left(-\frac{2(1 - c^2 x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1 - c^2 x^2}}{c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \right) \\ \downarrow \text{2720}$$

$$d \left(\frac{\sqrt{d - c^2 dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d e^{i \arcsin(cx)}}}{\sqrt{1 - c^2 x^2}} \right. \\ \left. \frac{\frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{6} bc \left(-\frac{2(1 - c^2 x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1 - c^2 x^2}}{c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \right) \\ \downarrow \text{7143}$$

$$d \left(\frac{\sqrt{d - c^2 dx^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{1 - c^2 x^2}} \right. \\ \left. \frac{\frac{1}{3} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - 2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{6} bc \left(-\frac{2(1 - c^2 x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1 - c^2 x^2}}{c^2} \right) \right)}{3\sqrt{1 - c^2 x^2}} \right)$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x,x]
```

output

```
((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*d*Sqrt[d - c^2*d*x^2]*(-1/6*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSin[c*x]) - (c^2*x^3*(a + b*ArcSin[c*x]))/3))/(3*Sqrt[1 - c^2*x^2]) + d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/Sqrt[1 - c^2*x^2] + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])]) - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5154 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5202 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]`

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(518) = 1036$.

Time = 0.77 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.01

method	result	size
default	Expression too large to display	1072
parts	Expression too large to display	1072

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output

```

1/3*(-c^2*d*x^2+d)^(3/2)*a^2-a^2*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a^2*d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*(6*I*arcsin(c*x)+9*arcsin(c*x)^2-2)*d/(c^2*x^2-1)+5/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))*d/(c^2*x^2-1)+5/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))*d/(c^2*x^2-1)-1/216*(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*(-6*I*arcsin(c*x)+9*arcsin(c*x)^2-2)*d/(c^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2)))*d)+2*a*b*(-1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*(I+3*arcsin(c*x))*d/(c^2*x^2-1)+5/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)*d/(c^2*x^2-1)+5/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)*d/(c^2*x^2-1)-1/72*(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*(-I+3*arcsin(c*x))*d/(c^2*x^2-1)-I...

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x} dx$$

input

```

integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="fricas"
)

```

output

```

integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x))^2 +
2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)

```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))^2}{x} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2/x,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2/x, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output `-1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a^2 - sqrt(d)*integrate(((b^2*c^2*d*x^2 - b^2*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
 PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
 index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2}}{x} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx = \frac{\sqrt{d} d \left(-\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 + 4\sqrt{-c^2 x^2 + 1} a^2 + 6 \left(\int \frac{\sqrt{-c^2 x^2 + 1} a}{x} dx \right) \right)}{3}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2/x,x)`

output `(sqrt(d)*d*(-sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 + 4*sqrt(-c**2*x**2 + 1)*a**2 + 6*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x,x)*a*b + 3*int((sqrt(-c**2*x**2 + 1)*asin(c*x)**2)/x,x)*b**2 - 6*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x,x)*a*b*c**2 - 3*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x,x)*b**2*c**2 + 3*log(tan(asin(c*x)/2))*a**2 - 4*a**2))/3`

3.220
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx$$

Optimal result	2125
Mathematica [A] (verified)	2126
Rubi [A] (verified)	2127
Maple [A] (verified)	2134
Fricas [F]	2135
Sympy [F]	2135
Maxima [F]	2135
Giac [F(-2)]	2136
Mupad [F(-1)]	2136
Reduce [F]	2137

Optimal result

Integrand size = 29, antiderivative size = 424

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx &= \frac{1}{4} b^2 c^2 dx \sqrt{d - c^2 dx^2} \\ &- \frac{5b^2 cd \sqrt{d - c^2 dx^2} \arcsin(cx)}{4\sqrt{1 - c^2 x^2}} + \frac{3bc^3 dx^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2\sqrt{1 - c^2 x^2}} \\ &+ bcd \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \\ &- \frac{3}{2} c^2 dx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{icd \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} \\ &- \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} - \frac{cd \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^3}{2b\sqrt{1 - c^2 x^2}} \\ &+ \frac{2bcd \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ &- \frac{ib^2 cd \sqrt{d - c^2 dx^2} \text{PolyLog}(2, e^{2i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```

1/4*b^2*c^2*d*x*(-c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*(-c^2*d*x^2+d)^(1/2)*arcs
in(c*x)/(-c^2*x^2+1)^(1/2)+3/2*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsi
n(c*x))/(-c^2*x^2+1)^(1/2)+b*c*d*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(
a+b*arcsin(c*x))-3/2*c^2*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-I*c*
d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(1/2)-(-c^2*d*x^2+
d)^(3/2)*(a+b*arcsin(c*x))^2/x-1/2*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*
x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*
ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*d*(-c^2*d*x^
2+d)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.93

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \frac{-12a^2 d \sqrt{1 - c^2 x^2} (2 + c^2 x^2) \sqrt{d - c^2 dx^2} + 36a^2 cd^{3/2} x \sqrt{1 - c^2 x^2}}{x^2}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```

(-12*a^2*d*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2] + 36*a^2*c*
d^(3/2)*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1
+ c^2*x^2))] - 24*a*b*d*Sqrt[d - c^2*d*x^2]*(2*Sqrt[1 - c^2*x^2]*ArcSin[c*
x] + c*x*ArcSin[c*x]^2 - 2*c*x*Log[c*x]) - 8*b^2*d*Sqrt[d - c^2*d*x^2]*(Ar
cSin[c*x]*(3*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + c*x*ArcSin[c*x]*(3*I + ArcSin
[c*x]) - 6*c*x*Log[1 - E^((2*I)*ArcSin[c*x])]) + (3*I)*c*x*PolyLog[2, E^((
2*I)*ArcSin[c*x])]) - b^2*c*d*x*Sqrt[d - c^2*d*x^2]*(4*ArcSin[c*x]^3 + 6*A
rcSin[c*x]*Cos[2*ArcSin[c*x]] + (-3 + 6*ArcSin[c*x]^2)*Sin[2*ArcSin[c*x]])
- 6*a*b*c*d*x*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcSin[c*x]] + 2*ArcSin[c*x]*(Ar
cSin[c*x] + Sin[2*ArcSin[c*x]])))/(24*x*Sqrt[1 - c^2*x^2])

```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.87, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5200, 5156, 5138, 262, 223, 5152, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx$$

$$\downarrow \text{5200}$$

$$-3c^2 d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{2bcd\sqrt{d - c^2 dx^2} \int \frac{(1-c^2 x^2)(a+b \arcsin(cx))}{x} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5156}$$

$$-3c^2 d \left(-\frac{bc\sqrt{d - c^2 dx^2} \int x(a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) - \frac{2bcd\sqrt{d - c^2 dx^2} \int \frac{(1-c^2 x^2)(a+b \arcsin(cx))}{x} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5138}$$

$$-3c^2 d \left(-\frac{bc\sqrt{d - c^2 dx^2} \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \int \frac{x^2}{\sqrt{1-c^2 x^2}} dx \right)}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{2} x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \right) - \frac{2bcd\sqrt{d - c^2 dx^2} \int \frac{(1-c^2 x^2)(a+b \arcsin(cx))}{x} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{262}$$

$$\begin{aligned}
& -3c^2d \left(\frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}}}{2\sqrt{1-c^2x^2}} \right. \\
& \quad \left. \frac{2bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{x} \right) \\
& \quad \downarrow \text{223} \\
& \quad \frac{2bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
& 3c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right. \\
& \quad \left. \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{x} \right) \\
& \quad \downarrow \text{5152} \\
& \quad \frac{2bcd\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{x} - \\
& 3c^2d \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow \text{5188} \\
& \quad \frac{2bcd\sqrt{d-c^2dx^2} \left(\int \frac{a+b\arcsin(cx)}{x} dx - \frac{1}{2}bc \int \sqrt{1-c^2x^2} dx + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} - \\
& \quad \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{x} - \\
& 3c^2d \left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) \\
& \quad \downarrow \text{211}
\end{aligned}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} - 3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 223

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arcsin(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} - 3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 5136

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} - 3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 3042

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\int-\left((a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)\right)d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} - 3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 25

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\int(a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\right)}{\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x}-$$

$$3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 4200

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\int-\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{i(a+b\arcsin(cx))^2}{2b}\right)}{\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x}-$$

$$3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 25

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-2i\int\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{i(a+b\arcsin(cx))^2}{2b}\right)}{\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x}-$$

$$3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 2620

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})(a+b\arcsin(cx))-\frac{1}{2}ib\int\log(1-e^{2i\arcsin(cx)})d\arcsin(cx)\right)+\frac{1}{2}\right)}{\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x}-$$

$$3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 2715

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))-\frac{1}{4}b\int e^{-2i\arcsin(cx)}\log(1-e^{2i\arcsin(cx)})de^{2i\arcsin(cx)}\right)}{\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} -$$

$$3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

↓ 2838

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{PolyLog}(2,\right)}{\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} -$$

$$3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}}\right)$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^2,x]`

output

```
-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x) - 3*c^2*d*((x*Sqrt[d -
c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c
*x])^3)/(6*b*c*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*
ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^
3)))/2))/Sqrt[1 - c^2*x^2]) + (2*b*c*d*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)
*(a + b*ArcSin[c*x]))/2 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*Sqrt[
1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 - (2*I)*((I/2)*(a + b*ArcSin[c*x])
*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/4
))/Sqrt[1 - c^2*x^2]
```


Definitions of rubi rules used

- rule 25 $\text{Int}[-(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[\text{Fx}, \text{x}], \text{x}]$
- rule 211 $\text{Int}[(\text{a}_) + (\text{b}_) * (\text{x}_)^2)^{\text{p}_}, \text{x_Symbol}] \rightarrow \text{Simp}[\text{x} * (\text{a} + \text{b} * \text{x}^2)^{\text{p}} / (2 * \text{p} + 1), \text{x}] + \text{Simp}[2 * \text{a} * (\text{p} / (2 * \text{p} + 1)) \quad \text{Int}[(\text{a} + \text{b} * \text{x}^2)^{\text{p} - 1}, \text{x}], \text{x}] /;$ $\text{FreeQ}[\{\text{a}, \text{b}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{p}, 0] \ \&\& \ (\text{IntegerQ}[4 * \text{p}] \ || \ \text{IntegerQ}[6 * \text{p}])$
- rule 223 $\text{Int}[1 / \text{Sqrt}[(\text{a}_) + (\text{b}_) * (\text{x}_)^2], \text{x_Symbol}] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-\text{b}, 2] * (\text{x} / \text{Sqrt}[\text{a}])] / \text{Rt}[-\text{b}, 2], \text{x}] /;$ $\text{FreeQ}[\{\text{a}, \text{b}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{a}, 0] \ \&\& \ \text{NegQ}[\text{b}]$
- rule 262 $\text{Int}[(\text{c}_) * (\text{x}_)]^{\text{m}_} * ((\text{a}_) + (\text{b}_) * (\text{x}_)^2)^{\text{p}_}, \text{x_Symbol}] \rightarrow \text{Simp}[\text{c} * (\text{c} * \text{x})^{\text{m} - 1} * ((\text{a} + \text{b} * \text{x}^2)^{\text{p} + 1} / (\text{b} * (\text{m} + 2 * \text{p} + 1))), \text{x}] - \text{Simp}[\text{a} * \text{c}^2 * ((\text{m} - 1) / (\text{b} * (\text{m} + 2 * \text{p} + 1))) \quad \text{Int}[(\text{c} * \text{x})^{\text{m} - 2} * (\text{a} + \text{b} * \text{x}^2)^{\text{p}}, \text{x}], \text{x}] /;$ $\text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{p}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{m}, 2 - 1] \ \&\& \ \text{NeQ}[\text{m} + 2 * \text{p} + 1, 0] \ \&\& \ \text{IntBinomialQ}[\text{a}, \text{b}, \text{c}, 2, \text{m}, \text{p}, \text{x}]$
- rule 2620 $\text{Int}[(\text{F}_)^{((\text{g}_) * ((\text{e}_) + (\text{f}_) * (\text{x}_)))^{\text{n}_}) * ((\text{c}_) + (\text{d}_) * (\text{x}_)]^{\text{m}_}) / ((\text{a}_) + (\text{b}_) * ((\text{F}_)^{((\text{g}_) * ((\text{e}_) + (\text{f}_) * (\text{x}_)))^{\text{n}_})}), \text{x_Symbol}] \rightarrow \text{Simp}[(\text{c} + \text{d} * \text{x})^{\text{m}} / (\text{b} * \text{f} * \text{g} * \text{n} * \text{Log}[\text{F}]) * \text{Log}[1 + \text{b} * ((\text{F}^{\text{g}} * (\text{e} + \text{f} * \text{x}))^{\text{n}} / \text{a})], \text{x}] - \text{Simp}[\text{d} * (\text{m} / (\text{b} * \text{f} * \text{g} * \text{n} * \text{Log}[\text{F}])) \quad \text{Int}[(\text{c} + \text{d} * \text{x})^{\text{m} - 1} * \text{Log}[1 + \text{b} * ((\text{F}^{\text{g}} * (\text{e} + \text{f} * \text{x}))^{\text{n}} / \text{a})], \text{x}], \text{x}] /;$ $\text{FreeQ}[\{\text{F}, \text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{g}, \text{n}\}, \text{x}] \ \&\& \ \text{IGtQ}[\text{m}, 0]$
- rule 2715 $\text{Int}[\text{Log}[(\text{a}_) + (\text{b}_) * ((\text{F}_)^{((\text{e}_) * ((\text{c}_) + (\text{d}_) * (\text{x}_)))^{\text{n}_})}], \text{x_Symbol}] \rightarrow \text{Simp}[1 / (\text{d} * \text{e} * \text{n} * \text{Log}[\text{F}]) \quad \text{Subst}[\text{Int}[\text{Log}[\text{a} + \text{b} * \text{x}] / \text{x}, \text{x}], \text{x}, (\text{F}^{\text{e}} * (\text{c} + \text{d} * \text{x}))^{\text{n}}], \text{x}] /;$ $\text{FreeQ}[\{\text{F}, \text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{n}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{a}, 0]$
- rule 2838 $\text{Int}[\text{Log}[(\text{c}_) * ((\text{d}_) + (\text{e}_) * (\text{x}_)^{\text{n}_})] / (\text{x}_), \text{x_Symbol}] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-\text{c}) * \text{e} * \text{x}^{\text{n}}] / \text{n}, \text{x}] /;$ $\text{FreeQ}[\{\text{c}, \text{d}, \text{e}, \text{n}\}, \text{x}] \ \&\& \ \text{EqQ}[\text{c} * \text{d}, 1]$
- rule 3042 $\text{Int}[\text{u}_, \text{x_Symbol}] \rightarrow \text{Int}[\text{DeactivateTrig}[\text{u}, \text{x}], \text{x}] /;$ $\text{FunctionOfTrigOfLinearQ}[\text{u}, \text{x}]$

rule 4200 $\text{Int}[\{(c_.) + (d_.)*(x_)\}^{(m_)}*\tan[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[I*((c + d*x)^{(m + 1)}/(d*(m + 1))), x] - \text{Simp}[2*I \text{ Int}[(c + d*x)^m * E^{(2*I*k*Pi)}*(E^{(2*I*(e + f*x)})/(1 + E^{(2*I*k*Pi)}*E^{(2*I*(e + f*x)}))], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

rule 5136 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)} / (x_), x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n * \text{Cot}[x], x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0]$

rule 5138 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)}*((d_.)*(x_)\}^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)}*((a + b*\text{ArcSin}[c*x])^n / (d*(m + 1))), x] - \text{Simp}[b*c*(n / (d*(m + 1))) \text{ Int}[(d*x)^{(m + 1)}*((a + b*\text{ArcSin}[c*x])^{(n - 1)} / \text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)} / \text{Sqrt}[(d_.) + (e_.)*(x_)\}^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n + 1))) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] * (a + b*\text{ArcSin}[c*x])^{(n + 1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[n, -1]$

rule 5156 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)} * \text{Sqrt}[(d_.) + (e_.)*(x_)\}^2], x_Symbol] \rightarrow \text{Simp}[x * \text{Sqrt}[d + e*x^2] * ((a + b*\text{ArcSin}[c*x])^{n/2}), x] + (\text{Simp}[(1/2) * \text{Simp}[\text{Sqrt}[d + e*x^2] / \text{Sqrt}[1 - c^2*x^2]] \text{ Int}[(a + b*\text{ArcSin}[c*x])^n / \text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2) * \text{Simp}[\text{Sqrt}[d + e*x^2] / \text{Sqrt}[1 - c^2*x^2]] \text{ Int}[x*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$

rule 5188 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}*((d_.) + (e_.)*(x_)\}^{(p_)} / (x_), x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^p * ((a + b*\text{ArcSin}[c*x]) / (2*p)), x] + (\text{Simp}[d \text{ Int}[(d + e*x^2)^{(p - 1)} * ((a + b*\text{ArcSin}[c*x]) / x), x], x] - \text{Simp}[b*c*(d^p / (2*p)) \text{ Int}[(1 - c^2*x^2)^{(p - 1/2)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[p, 0]$

rule 5200

```

Int[((a._) + ArcSin[(c._)*(x_)])*(b._)^(n._)*((f._)*(x_)^(m._))*((d_) + (e_.
)*(x_)^2)^(p._), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n/(f*(m + 1)), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]

```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.67

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{5}{2}}}{dx} - a^2c^2x(-c^2dx^2+d)^{\frac{3}{2}} - \frac{3\sqrt{-c^2dx^2+d}a^2c^2dx}{2} - \frac{3a^2c^2d^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + b^2\left(\sqrt{-d}\right)$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{5}{2}}}{dx} - a^2c^2x(-c^2dx^2+d)^{\frac{3}{2}} - \frac{3\sqrt{-c^2dx^2+d}a^2c^2dx}{2} - \frac{3a^2c^2d^2 \arctan\left(\frac{\sqrt{c^2dx}}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}} + b^2\left(\sqrt{-d}\right)$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output

```

-a^2/d/x*(-c^2*d*x^2+d)^(5/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*(-c^2*d*x
^2+d)^(1/2)*a^2*c^2*d*x-3/2*a^2*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)
)/(c^2*x^2-1)*arcsin(c*x)^3*c*d-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^
2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*I*arcsin(c*x)+
2*arcsin(c*x)^2-1)*c*d/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*
x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arcsin(c*x)^
2-1-2*I*arcsin(c*x))*c*d/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)
)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)^2*d/(c^2*x^2-1)/x+2*I*(-c^2*x^2+1)^(1/2)
)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)
)^(1/2))+I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2+polylo
g(2,-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c*d)+1
/4*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x*(4*arcsin(c
*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-2*c^3*x^3+6*arcsin(c*x)^2*c*x+8*I*arcsin(c*
x)*x*c-8*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*x*c+8*arcsin(c*x)*(-c^2*x^2+1)
^(1/2)+c*x)*d

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")`

output `integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))^2}{x^2} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2/x**2,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2/x**2, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="maxima")`

output

```
-1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)/x)*a^2 - sqrt(d)*integrate(((b^2*c^2*d*x^2 - b^2*d)*arctan
2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2
(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="giac"
)
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2}}{x^2} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx = \frac{\sqrt{d} d \left(-2 \arcsin(cx)^3 b^2 cx - 6 \arcsin(cx)^2 abcx - 9 \arcsin(cx) a^2 cx - 3 \right)}{6cx}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2/x^2,x)`

output `(sqrt(d)*d*(- 2*asin(c*x)**3*b**2*c*x - 6*asin(c*x)**2*a*b*c*x - 9*asin(c*x)*a**2*c*x - 3*sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - 6*sqrt(- c**2*x**2 + 1)*a**2 + 12*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*x**2),x)*a*b*x + 6*int(asin(c*x)**2/(sqrt(- c**2*x**2 + 1)*x**2),x)*b**2*x - 12*int(sqrt(- c**2*x**2 + 1)*asin(c*x),x)*a*b*c**2*x - 6*int(sqrt(- c**2*x**2 + 1)*asin(c*x)**2,x)*b**2*c**2*x))/(6*x)`

3.221 $\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx$

Optimal result	2138
Mathematica [A] (verified)	2139
Rubi [A] (verified)	2140
Maple [A] (verified)	2147
Fricas [F]	2148
Sympy [F]	2149
Maxima [F]	2149
Giac [F(-2)]	2149
Mupad [F(-1)]	2150
Reduce [F]	2150

Optimal result

Integrand size = 29, antiderivative size = 590

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx &= 2b^2 c^2 d \sqrt{d - c^2 dx^2} \\ &+ \frac{3abc^3 dx \sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} + \frac{3b^2 c^3 dx \sqrt{d - c^2 dx^2} \arcsin(cx)}{\sqrt{1 - c^2 x^2}} \\ &- \frac{bcd \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x \sqrt{1 - c^2 x^2}} - \frac{bc^3 dx \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}} \\ &- \frac{3}{2} c^2 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{2x^2} \\ &+ \frac{3c^2 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ &- \frac{b^2 c^2 d \sqrt{d - c^2 dx^2} \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{\sqrt{1 - c^2 x^2}} \\ &- \frac{3ibc^2 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ &+ \frac{3ibc^2 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ &+ \frac{3b^2 c^2 d \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \\ &- \frac{3b^2 c^2 d \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```

2*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)+3*a*b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*
x^2+1)^(1/2)+3*b^2*c^3*d*x*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(
1/2)-b*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/x/(-c^2*x^2+1)^(1/2)-b*c
^3*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-3/2*c^2*d
*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*ar
csin(c*x))^2/x^2+3*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(
I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-b^2*c^2*d*(-c^2*d*x^2+d)^(1/2
)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3*I*b*c^2*d*(-c^2*d*x^2+d
)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1
)^(1/2)+3*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x
+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+3*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)*p
olylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3*b^2*c^2*d*(-c^2*d
*x^2+d)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 7.00 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.45

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \text{Too large to display}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^3,x]
```


output

```

(-a^2*c^2*d) - (a^2*d)/(2*x^2))*Sqrt[-(d*(-1 + c^2*x^2))] - (3*a^2*c^2*d^(
(3/2)*Log[x])/2 + (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^
2))]])/2 - 2*a*b*c^2*d*Sqrt[d*(1 - c^2*x^2)]*(-((c*x)/Sqrt[1 - c^2*x^2]) +
ArcSin[c*x] + (ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x]]) - Log[1 + E^(I*Arc
Sin[c*x]])))/Sqrt[1 - c^2*x^2] + (I*(PolyLog[2, -E^(I*ArcSin[c*x]]) - Poly
Log[2, E^(I*ArcSin[c*x]])))/Sqrt[1 - c^2*x^2]) - b^2*c^2*d*Sqrt[d*(1 - c^2
*x^2)]*(-2 - (2*c*x*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + ArcSin[c*x]^2 + (ArcS
in[c*x]^2*(Log[1 - E^(I*ArcSin[c*x]]) - Log[1 + E^(I*ArcSin[c*x]])))/Sqrt[
1 - c^2*x^2] + ((2*I)*ArcSin[c*x]*(PolyLog[2, -E^(I*ArcSin[c*x]]) - PolyLo
g[2, E^(I*ArcSin[c*x]])))/Sqrt[1 - c^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcSin
[c*x]]) + PolyLog[3, E^(I*ArcSin[c*x]])))/Sqrt[1 - c^2*x^2]) + (a*b*c^2*d^
2*Sqrt[1 - c^2*x^2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2
]^2 - 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 4*ArcSin[c*x]*Log[1 + E^(
I*ArcSin[c*x])] - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2,
E^(I*ArcSin[c*x])] + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/
2]))/(4*Sqrt[d*(1 - c^2*x^2)]) + (b^2*c^2*d^2*Sqrt[1 - c^2*x^2]*(-4*ArcSin
[c*x]*Cot[ArcSin[c*x]/2] - ArcSin[c*x]^2*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c
*x]^2*Log[1 - E^(I*ArcSin[c*x])] + 4*ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x
]]) + 8*Log[Tan[ArcSin[c*x]/2]] - (8*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSi
n[c*x]]) + (8*I)*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] + 8*PolyLog[...

```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.64, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {5200, 5192, 25, 354, 90, 73, 221, 5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx$$

$$\downarrow 5200$$

$$-\frac{3}{2}c^2d \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} dx + \frac{bcd\sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{x^2} dx}{\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{2x^2}$$

$$\begin{array}{c}
\downarrow 5192 \\
\frac{-\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + bcd\sqrt{d-c^2dx^2} \left(-bc \int -\frac{c^2x^2+1}{x\sqrt{1-c^2x^2}} dx + c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}} (a+b\arcsin(cx))^2} \cdot \frac{1}{2x^2} \\
\downarrow 25 \\
\frac{-\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + bcd\sqrt{d-c^2dx^2} \left(bc \int \frac{c^2x^2+1}{x\sqrt{1-c^2x^2}} dx + c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}} (a+b\arcsin(cx))^2} \cdot \frac{1}{2x^2} \\
\downarrow 354 \\
\frac{-\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + bcd\sqrt{d-c^2dx^2} \left(\frac{1}{2}bc \int \frac{c^2x^2+1}{x^2\sqrt{1-c^2x^2}} dx^2 + c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}} (a+b\arcsin(cx))^2} \cdot \frac{1}{2x^2} \\
\downarrow 90 \\
\frac{-\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + bcd\sqrt{d-c^2dx^2} \left(\frac{1}{2}bc \left(\int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 - 2\sqrt{1-c^2x^2} \right) + c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{3/2}} (a+b\arcsin(cx))^2} \cdot \frac{1}{2x^2} \\
\downarrow 73
\end{array}$$

$$\begin{aligned}
& \frac{-\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + bcd\sqrt{d-c^2dx^2} \left(\frac{1}{2}bc \left(-\frac{2 \int \frac{1-x^4}{c^2-c^2} d\sqrt{1-c^2x^2}}{c^2} - 2\sqrt{1-c^2x^2} \right) + c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{(d-c^2dx^2)^{3/2} \frac{\sqrt{1-c^2x^2}}{2x^2} (a+b\arcsin(cx))^2} \\
& \quad \downarrow \text{221} \\
& \frac{-\frac{3}{2}c^2d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{(d-c^2dx^2)^{3/2} \frac{\sqrt{1-c^2x^2}}{2x^2} (a+b\arcsin(cx))^2} \\
& \quad \downarrow \text{5198} \\
& \frac{-\frac{3}{2}c^2d \left(-\frac{2bc\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right) + bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{(d-c^2dx^2)^{3/2} \frac{\sqrt{1-c^2x^2}}{2x^2} (a+b\arcsin(cx))^2} \\
& \quad \downarrow \text{2009} \\
& \frac{-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax+bx\arcsin(cx))}{\sqrt{1-c^2x^2}} \right) + bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{(d-c^2dx^2)^{3/2} \frac{\sqrt{1-c^2x^2}}{2x^2} (a+b\arcsin(cx))^2} \\
& \quad \downarrow \text{5218}
\end{aligned}$$

$$\begin{aligned}
 & -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2}(ax+bx)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{3042}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx))^2 \csc(\arcsin(cx)) d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{4671}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \left(-2b \int (a+b\arcsin(cx)) \log(1-e^{i\arcsin(cx)}) d\arcsin(cx) + 2b \int (a+b\arcsin(cx)) \log(1+e^{i\arcsin(cx)}) d\arcsin(cx) \right)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{3011}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} \left(2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) d\arcsin(cx) \right)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{bcd\sqrt{d-c^2dx^2} \left(c^2(-x)(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{1}{2}bc \left(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{2x^2} \\
 & \quad \downarrow \text{2720}
 \end{aligned}$$

$$-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) dx) - bcd\sqrt{d-c^2dx^2} (c^2(-x)(a + b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{1}{2}bc(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2}))}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2} (a + b \arcsin(cx))^2}{2x^2} \right)$$

↓ 7143

$$-\frac{3}{2}c^2d \left(\frac{\sqrt{d-c^2dx^2} (-2\operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - bcd\sqrt{d-c^2dx^2} (c^2(-x)(a + b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{1}{2}bc(-2\operatorname{arctanh}(\sqrt{1-c^2x^2}) - 2\sqrt{1-c^2x^2}))}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2} (a + b \arcsin(cx))^2}{2x^2} \right)$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^3,x]`

output `-1/2*((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^2 + (b*c*d*Sqrt[d - c^2*d*x^2]*(-(a + b*ArcSin[c*x])/x) - c^2*x*(a + b*ArcSin[c*x]) + (b*c*(-2*Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2))/Sqrt[1 - c^2*x^2] - (3*c^2*d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x])))/Sqrt[1 - c^2*x^2] + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x]])] - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x]])] - b*PolyLog[3, E^(I*ArcSin[c*x]])])))/Sqrt[1 - c^2*x^2))/2`

Definitions of rubi rules used

- rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`
- rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`
- rule 90 `Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)) Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]`
- rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`
- rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 $\text{Int}[\text{Log}[1 + (e_)*((F_)^((c_)*((a_)\ + (b_)*(x_)))^n)]*((f_)\ + (g_)*(x_))^m, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*\text{Log}[F])), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^(m - 1)*\text{PolyLog}[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_)\ + (f_)*(x_)]*((c_)\ + (d_)*(x_))^m, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^(I*(e + f*x))]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 - E^(I*(e + f*x))], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 + E^(I*(e + f*x))], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5192 $\text{Int}[(a_)\ + \text{ArcSin}[(c_)*(x_)]*(b_)]*((f_)*(x_))^m*((d_)\ + (e_)*(x_))^p, x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[p, 0]$

rule 5198 $\text{Int}[(a_)\ + \text{ArcSin}[(c_)*(x_)]*(b_)]^n*((f_)*(x_))^m*\text{Sqrt}[(d_)\ + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^(m + 1)*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m + 2))), x] + (\text{Simp}[(1/(m + 2))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^m*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m + 2)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^(m + 1)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])$

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n/(f*(m + 1)), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5218

```
Int((((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.50

method	result
default	$a^2 \left(-\frac{(-c^2 d x^2 + d)^{\frac{5}{2}}}{2 d x^2} - \frac{3 c^2 \left(\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{3} + d \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right) \right)}{2} \right) + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 + d)}}{2} \right)$
parts	$a^2 \left(-\frac{(-c^2 d x^2 + d)^{\frac{5}{2}}}{2 d x^2} - \frac{3 c^2 \left(\frac{(-c^2 d x^2 + d)^{\frac{3}{2}}}{3} + d \left(\sqrt{-c^2 d x^2 + d} - \sqrt{d} \ln \left(\frac{2 d + 2 \sqrt{d} \sqrt{-c^2 d x^2 + d}}{x} \right) \right) \right)}{2} \right) + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 + d)}}{2} \right)$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```


output

```

a^2*(-1/2/d/x^2*(-c^2*d*x^2+d)^(5/2)-3/2*c^2*(1/3*(-c^2*d*x^2+d)^(3/2)+d*(
(-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))))
+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(ar
csin(c*x)^2-2+2*I*arcsin(c*x))*c^2*d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2
)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))*c
^2*d/(c^2*x^2-1)-1/2*d*(c^2*x^2*arcsin(c*x)-2*c*x*(-c^2*x^2+1)^(1/2)-arcsi
n(c*x))*arcsin(c*x)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2-1/2*(-d*(c^2*x^
2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(3*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1
/2))-3*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-6*I*arcsin(c*x)*polylo
g(2,-I*c*x-(-c^2*x^2+1)^(1/2))+6*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1
)^(1/2))-4*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))+6*polylog(3,-I*c*x-(-c^2*x^2+
1)^(1/2))-6*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2)))*c^2*d/(c^2*x^2-1))+2*a*b*
(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(
c*x)+I)*c^2*d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2
)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)*c^2*d/(c^2*x^2-1)-1/2*d*(c^2*x^2*arcsin(c*
x)-c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/
x^2+3*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcsin(c*x)*ln(1+I*c*
x+(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog
(2,-I*c*x-(-c^2*x^2+1)^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c^2*d/(
2*c^2*x^2-2))

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x^3} dx$$

input

```

integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="fricas")

```

output

```

integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x))^2 +
2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)

```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(-d(cx - 1)(cx + 1))^{3/2} (a + b \arcsin(cx))^2}{x^3} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2/x**3,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2/x**3, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="maxima")`

output `1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 + d)^(5/2)/(d*x^2))*a^2 - sqrt(d)*integrate(((b^2*c^2*d*x^2 - b^2*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^3, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2}}{x^3} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^3} dx = \frac{\sqrt{d} d \left(-8\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 4\sqrt{-c^2 x^2 + 1} a^2 + 16 \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{x} dx \right) \right)}{8x^3}$$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2/x^3,x)
```

output

```
(sqrt(d)*d*(- 8*sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - 4*sqrt(- c**2*x*
*2 + 1)*a**2 + 16*int((sqrt(- c**2*x**2 + 1)*asin(c*x))/x**3,x)*a*b*x**2
- 16*int((sqrt(- c**2*x**2 + 1)*asin(c*x))/x,x)*a*b*c**2*x**2 + 8*int((sq
rt(- c**2*x**2 + 1)*asin(c*x)**2)/x**3,x)*b**2*x**2 - 8*int((sqrt(- c**2
*x**2 + 1)*asin(c*x)**2)/x,x)*b**2*c**2*x**2 - 12*log(tan(asin(c*x)/2))*a*
*2*c**2*x**2 + 9*a**2*c**2*x**2))/(8*x**2)
```

$$3.222 \quad \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx$$

Optimal result	2151
Mathematica [A] (verified)	2152
Rubi [A] (verified)	2153
Maple [B] (verified)	2160
Fricas [F]	2161
Sympy [F]	2162
Maxima [F]	2162
Giac [F(-2)]	2162
Mupad [F(-1)]	2163
Reduce [F]	2163

Optimal result

Integrand size = 29, antiderivative size = 400

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = & -\frac{b^2 c^2 d \sqrt{d - c^2 dx^2}}{3x} \\ & - \frac{b^2 c^3 d \sqrt{d - c^2 dx^2} \arcsin(cx)}{3\sqrt{1 - c^2 x^2}} - \frac{bcd\sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{3x^2} \\ & + \frac{c^2 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x} + \frac{4ic^3 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{3\sqrt{1 - c^2 x^2}} \\ & - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3x^3} + \frac{c^3 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^3}{3b\sqrt{1 - c^2 x^2}} \\ & - \frac{8bc^3 d \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{3\sqrt{1 - c^2 x^2}} \\ & + \frac{4ib^2 c^3 d \sqrt{d - c^2 dx^2} \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{3\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```
-1/3*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*d*(-c^2*d*x^2+d)^(1/2)*
rcsin(c*x)/(-c^2*x^2+1)^(1/2)-1/3*b*c*d*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))/x^2+c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2
/x+4/3*I*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(1/2)
-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^3+1/3*c^3*d*(-c^2*d*x^2+d)
^(1/2)*(a+b*arcsin(c*x))^3/b/(-c^2*x^2+1)^(1/2)-8/3*b*c^3*d*(-c^2*d*x^2+d)
^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(
1/2)+4/3*I*b^2*c^3*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1
/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.23

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \frac{-abcdx\sqrt{d - c^2 dx^2} - a^2 d \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} + 4a^2 c^2 dx^2 \sqrt{1 - c^2 x^2}}{x^4}$$

input

```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x]))^2/x^4,x]
```

output

```
(-(a*b*c*d*x*Sqrt[d - c^2*d*x^2]) - a^2*d*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d
*x^2] + 4*a^2*c^2*d*x^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] - b^2*c^2*d*
x^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] + b*d*Sqrt[d - c^2*d*x^2]*(3*a*c
^3*x^3 + b*((4*I)*c^3*x^3 - Sqrt[1 - c^2*x^2] + 4*c^2*x^2*Sqrt[1 - c^2*x^2
]))*ArcSin[c*x]^2 + b^2*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^3 - 3*a^
2*c^3*d^(3/2)*x^3*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt
[d]*(-1 + c^2*x^2))] - b*d*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]*(b*c*x + 2*a*(1
- 4*c^2*x^2)*Sqrt[1 - c^2*x^2] + 8*b*c^3*x^3*Log[1 - E^((2*I)*ArcSin[c*x]
)]) - 8*a*b*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*Log[c*x] + (4*I)*b^2*c^3*d*x^3*S
qrt[d - c^2*d*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(3*x^3*Sqrt[1 - c^2*
x^2])
```

Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.98, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5200, 5190, 247, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838, 5196, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx$$

$$\downarrow \text{5200}$$

$$c^2(-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx + \frac{2bcd\sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{x^3} dx}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{5190}$$

$$\frac{c^2(-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d - c^2 dx^2} \left(c^2 \left(- \int \frac{a + b \arcsin(cx)}{x} dx \right) + \frac{1}{2} bc \int \frac{\sqrt{1 - c^2 x^2}}{x^2} dx - \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{2x^2} \right)}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{247}$$

$$\frac{c^2(-d) \int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d - c^2 dx^2} \left(c^2 \left(- \int \frac{a + b \arcsin(cx)}{x} dx \right) + \frac{1}{2} bc \left(c^2 \left(- \int \frac{1}{\sqrt{1 - c^2 x^2}} dx \right) - \frac{\sqrt{1 - c^2 x^2}}{x} \right) - \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{2x^2} \right)}{3\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{223}$$

$$\begin{aligned}
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(c^2 \left(-\int \frac{a+b \arcsin(cx)}{x} dx \right) - \frac{(1-c^2x^2)(a+b \arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(-c \arcsin(cx) - \frac{\sqrt{1-c^2x^2}}{x} \right) \right)}{3\sqrt{1-c^2x^2} \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{3x^3}} \\
 & \quad \downarrow \text{5136} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(c^2 \left(-\int \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{cx} d \arcsin(cx) \right) - \frac{(1-c^2x^2)(a+b \arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(-c \arcsin(cx) - \frac{\sqrt{1-c^2x^2}}{x} \right) \right)}{3\sqrt{1-c^2x^2} \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{3x^3}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(c^2 \left(-\int -((a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2})) d \arcsin(cx) \right) - \frac{(1-c^2x^2)(a+b \arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(-c \arcsin(cx) - \frac{\sqrt{1-c^2x^2}}{x} \right) \right)}{3\sqrt{1-c^2x^2} \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{3x^3}} \\
 & \quad \downarrow \text{25} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(c^2 \int (a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) - \frac{(1-c^2x^2)(a+b \arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(-c \arcsin(cx) - \frac{\sqrt{1-c^2x^2}}{x} \right) \right)}{3\sqrt{1-c^2x^2} \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{3x^3}} \\
 & \quad \downarrow \text{4200} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(2i \int -\frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) \right) - \frac{(1-c^2x^2)(a+b \arcsin(cx))}{2x^2} + \frac{1}{2}bc \left(-c \arcsin(cx) - \frac{\sqrt{1-c^2x^2}}{x} \right) \right)}{3\sqrt{1-c^2x^2} \frac{(d-c^2dx^2)^{3/2} (a+b \arcsin(cx))^2}{3x^3}} \\
 & \quad \downarrow \text{25}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))^2}{2b} \right) \right) - \frac{(1-c^2x^2)(a+b\arcsin(cx))}{2x^2} \right)}{3\sqrt{1-c^2x^2}} \\
 & \qquad \qquad \qquad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3x^3} \\
 & \qquad \qquad \qquad \downarrow \text{2620} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \frac{1}{2}ib \int \log(1-e^{2i\arcsin(cx)}) d\arcsin(cx) \right) \right) \right)}{3\sqrt{1-c^2x^2}} \\
 & \qquad \qquad \qquad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3x^3} \\
 & \qquad \qquad \qquad \downarrow \text{2715} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \frac{1}{4}b \int e^{-2i\arcsin(cx)} \log(1-e^{2i\arcsin(cx)}) \right) \right) \right)}{3\sqrt{1-c^2x^2}} \\
 & \qquad \qquad \qquad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3x^3} \\
 & \qquad \qquad \qquad \downarrow \text{2838} \\
 & \frac{c^2(-d) \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x^2} dx + 2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{2} \right) \right)}{3\sqrt{1-c^2x^2}} \\
 & \qquad \qquad \qquad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3x^3} \\
 & \qquad \qquad \qquad \downarrow \text{5196} \\
 & \frac{c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \int \frac{a+b\arcsin(cx)}{x} dx}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{x} \right) + 2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{2} \right) \right)}{3\sqrt{1-c^2x^2}} \\
 & \qquad \qquad \qquad \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{3x^3}
 \end{aligned}$$

↓ 5136

$$\frac{c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}} \right) - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{2} \right)}{3\sqrt{1-c^2x^2}}}{\frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3}}$$

↓ 3042

$$\frac{c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \int -\left((a+b\arcsin(cx)) \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) \right) d\arcsin(cx)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}} \right) - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{2} \right)}{3\sqrt{1-c^2x^2}}}{\frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3}}$$

↓ 25

$$\frac{c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} - \frac{2bc\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) \tan\left(\arcsin(cx) + \frac{\pi}{2}\right) d\arcsin(cx)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}} \right) - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{2} \right)}{3\sqrt{1-c^2x^2}}}{\frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3}}$$

↓ 4200

$$\frac{c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \left(2i \int -\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))}{2} \right)}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}} \right) - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{2} \right)}{3\sqrt{1-c^2x^2}}}{\frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3}}$$

↓ 25

$$\begin{aligned}
 & c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{3\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3} \\
 & \quad \downarrow \text{2620}
 \end{aligned}$$

$$\begin{aligned}
 & c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{3\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3} \\
 & \quad \downarrow \text{2715}
 \end{aligned}$$

$$\begin{aligned}
 & c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{3\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3} \\
 & \quad \downarrow \text{2838}
 \end{aligned}$$

$$\begin{aligned}
 & c^2(-d) \left(-\frac{c^2\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{\sqrt{1-c^2x^2}} \right) \\
 & \frac{2bcd\sqrt{d-c^2dx^2} \left(-\left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) \right) - \frac{i(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} \right)}{3\sqrt{1-c^2x^2}} \\
 & \frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3} \\
 & \quad \downarrow \text{5152}
 \end{aligned}$$

$$c^2(-d) \left(\frac{2bc\sqrt{d-c^2dx^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) - \frac{i(a+b\arcsin(cx))}{2} \right)}{\sqrt{1-c^2x^2}} \right) - \frac{2bcd\sqrt{d-c^2dx^2} \left(- \left(c^2 \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \operatorname{PolyLog}(2, e^{2i\arcsin(cx)}) \right) - \frac{i(a+b\arcsin(cx))}{2} \right) \right)}{3\sqrt{1-c^2x^2}} \right)}{\frac{(d-c^2dx^2)^{3/2} (a+b\arcsin(cx))^2}{3x^3}}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^4,x]
```

output

```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/x^3 + (2*b*c*d*Sqrt[d - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcSin[c*x]))/x^2 + (b*c*(-(Sqrt[1 - c^2*x^2]/x) - c*ArcSin[c*x]))/2 - c^2*(((1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4))))/(3*Sqrt[1 - c^2*x^2]) - c^2*d*(-(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/x - (c*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^3)/(3*b*Sqrt[1 - c^2*x^2]) + (2*b*c*Sqrt[d - c^2*d*x^2]*(((1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4))))/Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1))) Int[(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2620 $\text{Int}[\frac{((F_.)^{((g_.) * (e_.) + (f_.) * (x_)))^{(n_.) * ((c_.) + (d_.) * (x_))^{(m_.)}}}{((a_.) + (b_.) * (F_.)^{((g_.) * (e_.) + (f_.) * (x_)))^{(n_.)}}), x_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m}{(b*f*g*n*\text{Log}[F])} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_.) + (b_.) * (F_.)^{((e_.) * ((c_.) + (d_.) * (x_)))^{(n_.)}}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_.) * ((d_.) + (e_.) * (x_.)^{(n_.)})]/(x_.), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4200 $\text{Int}[\frac{((c_.) + (d_.) * (x_.)^{(m_.)}) * \tan[(e_.) + \text{Pi} * (k_.) + (f_.) * (x_.)], x_Symbol]}{]} \rightarrow \text{Simp}[I * ((c + d*x)^{m+1} / (d*(m+1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * E^{(2*I*k*Pi)} * (E^{(2*I*(e + f*x))} / (1 + E^{(2*I*k*Pi)} * E^{(2*I*(e + f*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

rule 5136 $\text{Int}[\frac{((a_.) + \text{ArcSin}[(c_.) * (x_.)] * (b_.))^{(n_.)}}{(x_.)}, x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n * \text{Cot}[x], x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{IGtQ}[n, 0]$

rule 5152 $\text{Int}[\frac{((a_.) + \text{ArcSin}[(c_.) * (x_.)] * (b_.))^{(n_.)}}{\text{Sqrt}[(d_.) + (e_.) * (x_.)^2]}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1))) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]] * (a + b*\text{ArcSin}[c*x])^{n+1}, x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[n, -1]$

rule 5190

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x
])/ (f*(m + 1))), x] + (-Simp[b*c*(d^p/(f*(m + 1))) Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

rule 5196

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2]), x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x],
x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int
[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2249 vs. $2(372) = 744$.

Time = 0.83 (sec) , antiderivative size = 2250, normalized size of antiderivative = 5.62

method	result	size
default	Expression too large to display	2250
parts	Expression too large to display	2250

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output

```

-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arcsin(c*x)
^3*d*c^3-20/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c
^2*x^2-1)*c^8+29/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x
^3/(c^2*x^2-1)*c^6-10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2
+1)*x/(c^2*x^2-1)*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x
^2+1)/x/(c^2*x^2-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)/x^3/(c^2*x^2-1)*arcsin(c*x)^2+8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c
^4*x^4-9*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^5+1/3
*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(c^2*x^2-1)*arc
sin(c*x)*(-c^2*x^2+1)^(1/2)*c+4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x
^4-9*c^2*x^2+1)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2*c^3-16/3*I*b^
2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c^2*x^2-1)*arcsin
(c*x)*c^8-8*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c
^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^7+20/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c
^4*x^4-9*c^2*x^2+1)*x^3/(c^2*x^2-1)*arcsin(c*x)*c^6+3*I*b^2*(-d*(c^2*x^2-1
))^^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^5
-4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c^2*x^2-1)
*arcsin(c*x)*c^4-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a^2*c^4*x*(-c^2*d*x
^2+d)^(3/2)+a^2*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a^2*c^4*d^2/(c^2*d)^(1/2)*ar
ctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+2/3*a^2*c^2/d/x*(-c^2*d*x^2+...

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^2}{x^4} dx$$

input

```

integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="fricas")

```

output

```

integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x))^2 +
2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)

```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2}{x^4} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2/x**4,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2/x**4, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="maxima")`

output `1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a^2 - sqrt(d)*integrate(((b^2*c^2*d*x^2 - b^2*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^4, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2}}{x^4} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^4} dx = \frac{\sqrt{d} d (a \arcsin(cx)^3 b^2 c^3 x^3 + 3 a \arcsin(cx)^2 a b c^3 x^3 + 3 a \arcsin(cx) a^2 c^3 x^3 - \dots)}{x^4}$$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2/x^4,x)
```

output

```
(sqrt(d)*d*(asin(c*x)**3*b**2*c**3*x**3 + 3*asin(c*x)**2*a*b*c**3*x**3 + 3
*asin(c*x)*a**2*c**3*x**3 + 4*sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt
(-c**2*x**2 + 1)*a**2 - 6*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**2),x)
*a*b*c**2*x**3 - 3*int(asin(c*x)**2/(sqrt(-c**2*x**2 + 1)*x**2),x)*b**2*
c**2*x**3 + 6*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**4,x)*a*b*x**3 + 3*
int((sqrt(-c**2*x**2 + 1)*asin(c*x)**2)/x**4,x)*b**2*x**3))/(3*x**3)
```


3.223 $\int x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2164
Mathematica [A] (verified)	2165
Rubi [A] (verified)	2166
Maple [A] (verified)	2176
Fricas [A] (verification not implemented)	2176
Sympy [F(-1)]	2177
Maxima [A] (verification not implemented)	2178
Giac [F(-2)]	2179
Mupad [F(-1)]	2179
Reduce [F]	2179

Optimal result

Integrand size = 29, antiderivative size = 560

$$\begin{aligned}
 \int x^3(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = & \frac{160b^2d^2\sqrt{d - c^2dx^2}}{3969c^4} \\
 + \frac{80b^2d(d - c^2dx^2)^{3/2}}{11907c^4} + \frac{4b^2(d - c^2dx^2)^{5/2}}{1323c^4} + \frac{50b^2(d - c^2dx^2)^{7/2}}{27783c^4d} \\
 - \frac{2b^2(d - c^2dx^2)^{9/2}}{729c^4d^2} + \frac{4bd^2x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{63c^3\sqrt{1 - c^2x^2}} \\
 + \frac{2bd^2x^3\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{189c\sqrt{1 - c^2x^2}} - \frac{2bcd^2x^5\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{21\sqrt{1 - c^2x^2}} \\
 + \frac{38bc^3d^2x^7\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{441\sqrt{1 - c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d - c^2dx^2}(a + b \arcsin(cx))}{81\sqrt{1 - c^2x^2}} \\
 - \frac{2d^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{63c^4} - \frac{d^2x^2\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{63c^2} \\
 + \frac{1}{21}d^2x^4\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2 + \frac{5}{63}dx^4(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))^2 + \frac{1}{9}x^4(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))^2
 \end{aligned}$$

output

```
160/3969*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^4+80/11907*b^2*d*(-c^2*d*x^2+d)^(3/2)/c^4+4/1323*b^2*(-c^2*d*x^2+d)^(5/2)/c^4+50/27783*b^2*(-c^2*d*x^2+d)^(7/2)/c^4/d-2/729*b^2*(-c^2*d*x^2+d)^(9/2)/c^4/d^2+4/63*b*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c^3/(-c^2*x^2+1)^(1/2)+2/189*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-2/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+38/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-2/81*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-2/63*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^4-1/63*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^2+1/21*d^2*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2+5/63*d*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2+1/9*x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.48

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx =$$

$$\frac{d^2 \sqrt{d - c^2 dx^2} \left(3969 a^2 (1 - c^2 x^2)^{7/2} (2 + 7c^2 x^2) + 126 abc x (-126 - 21c^2 x^2 + 189c^4 x^4 - 171c^6 x^6 + 49c^8 x^8) \right)}{\dots}$$

input

```
Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/250047*(d^2*Sqrt[d - c^2*d*x^2]*(3969*a^2*(1 - c^2*x^2)^(7/2)*(2 + 7*c^2*x^2) + 126*a*b*c*x*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 171*c^6*x^6 + 49*c^8*x^8) + 2*b^2*Sqrt[1 - c^2*x^2]*(-6140 + 899*c^2*x^2 + 1005*c^4*x^4 - 1147*c^6*x^6 + 343*c^8*x^8) + 126*b*(63*a*(1 - c^2*x^2)^(7/2)*(2 + 7*c^2*x^2) + b*c*x*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 171*c^6*x^6 + 49*c^8*x^8))*ArcSin[c*x] + 3969*b^2*(1 - c^2*x^2)^(7/2)*(2 + 7*c^2*x^2)*ArcSin[c*x]^2))/(c^4*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 3.99 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.43, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.828$, Rules used = {5202, 5192, 27, 1578, 1195, 2009, 5202, 5192, 27, 354, 86, 2009, 5198, 5138, 243, 53, 2009, 5210, 5138, 243, 53, 2009, 5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & -\frac{2bcd^2 \sqrt{d - c^2 dx^2} \int x^4 (1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{9\sqrt{1 - c^2 x^2}} + \frac{5}{9} d \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx + \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5192} \\
 & \frac{\frac{5}{9} d \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^5 (35c^4 x^4 - 90c^2 x^2 + 63)}{315\sqrt{1 - c^2 x^2}} dx + \frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) \right)}{9\sqrt{1 - c^2 x^2}} + \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{27} \\
 & \frac{\frac{5}{9} d \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{315} bc \int \frac{x^5 (35c^4 x^4 - 90c^2 x^2 + 63)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) \right)}{9\sqrt{1 - c^2 x^2}} + \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{1578} \\
 & \frac{\frac{5}{9} d \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{630} bc \int \frac{x^4 (35c^4 x^4 - 90c^2 x^2 + 63)}{\sqrt{1 - c^2 x^2}} dx^2 + \frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) \right)}{9\sqrt{1 - c^2 x^2}} + \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2
 \end{aligned}$$

$$\begin{aligned} & \downarrow 1195 \\ & \frac{\frac{5}{9}d \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{630} bc \int \left(\frac{35(1-c^2 x^2)^{7/2}}{c^4} - \frac{50(1-c^2 x^2)^{5/2}}{c^4} + \frac{3(1-c^2 x^2)^{3/2}}{c^4} + \frac{4\sqrt{1-c^2 x^2}}{c^4} + \frac{8}{c^4 \sqrt{1-c^2 x^2}} \right) dx^2 + \frac{1}{9} c^4 x^9 \right)}{9\sqrt{1-c^2 x^2}} \\ & \frac{\frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{9\sqrt{1-c^2 x^2}} \end{aligned}$$

$$\begin{aligned} & \downarrow 2009 \\ & \frac{\frac{5}{9}d \int x^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx + \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{630} bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right)}{9\sqrt{1-c^2 x^2}} \end{aligned}$$

$$\begin{aligned} & \downarrow 5202 \\ & \frac{\frac{5}{9}d \left(-\frac{2bcd\sqrt{d-c^2 dx^2} \int x^4 (1-c^2 x^2) (a + b \arcsin(cx)) dx}{7\sqrt{1-c^2 x^2}} + \frac{3}{7}d \int x^3 \sqrt{d-c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{7} x^4 (a + b \arcsin(cx))^2 - \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{630} bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right) \right)}{9\sqrt{1-c^2 x^2}} \end{aligned}$$

$$\begin{aligned} & \downarrow 5192 \\ & \frac{\frac{5}{9}d \left(\frac{3}{7}d \int x^3 \sqrt{d-c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{2bcd\sqrt{d-c^2 dx^2} \left(-bc \int \frac{x^5 (7-5c^2 x^2)}{35\sqrt{1-c^2 x^2}} dx - \frac{1}{7} c^2 x^7 (a + b \arcsin(cx)) \right)}{7\sqrt{1-c^2 x^2}} - \frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{630} bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right) \right)}{9\sqrt{1-c^2 x^2}} \end{aligned}$$

$$\downarrow 27$$

$$\frac{5}{9}d \left(\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{35}bc \int \frac{x^5(7-5c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{1}{7}c^2x^7(a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right) \right) \\ \hline 9\sqrt{1 - c^2x^2}$$

↓ 354

$$\frac{5}{9}d \left(\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{70}bc \int \frac{x^4(7-5c^2x^2)}{\sqrt{1-c^2x^2}} dx^2 - \frac{1}{7}c^2x^7(a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right) \right) \\ \hline 9\sqrt{1 - c^2x^2}$$

↓ 86

$$\frac{5}{9}d \left(\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{70}bc \int \left(\frac{5(1-c^2x^2)^{5/2}}{c^4} - \frac{8(1-c^2x^2)^{3/2}}{c^4} + \frac{\sqrt{1-c^2x^2}}{c^4} \right) dx \right)}{7\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right) \right) \\ \hline 9\sqrt{1 - c^2x^2}$$

↓ 2009

$$\frac{5}{9}d \left(\frac{3}{7}d \int x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{7}x^4(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7}c^2x^7(a + b \arcsin(cx)) \right)}{7\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right) \right) \\ \hline 9\sqrt{1 - c^2x^2}$$

↓ 5198

$$\frac{5}{9}d \left(\frac{3}{7}d \left(-\frac{2bc\sqrt{d-c^2dx^2} \int x^4(a+b\arcsin(cx))dx}{5\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x^4\sqrt{d-c^2dx^2}(a + \right. \right.$$

$$\left. \left. \frac{1}{9}x^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \right.$$

$$\left. \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{9}c^4x^9(a+b\arcsin(cx)) - \frac{2}{7}c^2x^7(a+b\arcsin(cx)) + \frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right. \right. \right.$$

$$\left. \left. \left. \right. \right) \right) \right) \frac{1}{9\sqrt{1-c^2x^2}}$$

↓ 5138

$$\frac{5}{9}d \left(\frac{3}{7}d \left(-\frac{2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{5}bc \int \frac{x^5}{\sqrt{1-c^2x^2}}dx \right)}{5\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x^4\sqrt{d-c^2dx^2}(a + \right. \right.$$

$$\left. \left. \frac{1}{9}x^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \right.$$

$$\left. \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{9}c^4x^9(a+b\arcsin(cx)) - \frac{2}{7}c^2x^7(a+b\arcsin(cx)) + \frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right. \right. \right.$$

$$\left. \left. \left. \right. \right) \right) \right) \frac{1}{9\sqrt{1-c^2x^2}}$$

↓ 243

$$\frac{5}{9}d \left(\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}} - \frac{2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{10}bc \int \frac{x^4}{\sqrt{1-c^2x^2}}dx^2 \right)}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x^4\sqrt{d-c^2dx^2}(a + \right. \right.$$

$$\left. \left. \frac{1}{9}x^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \right.$$

$$\left. \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{9}c^4x^9(a+b\arcsin(cx)) - \frac{2}{7}c^2x^7(a+b\arcsin(cx)) + \frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right. \right. \right.$$

$$\left. \left. \left. \right. \right) \right) \right) \frac{1}{9\sqrt{1-c^2x^2}}$$

↓ 53

$$\frac{5}{9}d \left(\frac{3}{7}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}} - \frac{2bc\sqrt{d-c^2dx^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{10}bc \int \left(\frac{(1-c^2x^2)^{3/2}}{c^4} - 2\sqrt{1-c^2x^2} \right) dx \right)}{5\sqrt{1-c^2x^2}} + \frac{1}{5}x^4\sqrt{d-c^2dx^2}(a + \right. \right.$$

$$\left. \left. \frac{1}{9}x^4(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \right.$$

$$\left. \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{9}c^4x^9(a+b\arcsin(cx)) - \frac{2}{7}c^2x^7(a+b\arcsin(cx)) + \frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right. \right. \right.$$

$$\left. \left. \left. \right. \right) \right) \right) \frac{1}{9\sqrt{1-c^2x^2}}$$

$$\frac{5}{9}d \left(\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} dx}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \frac{x^2}{\sqrt{1-c^2 x^2}} dx^2 \right)}{3c} - \frac{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1 - c^2 x^2}} \right) \right. \\ \left. \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4 x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2 x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right)}{9\sqrt{1 - c^2 x^2}} \right)$$

↓ 53

$$\frac{5}{9}d \left(\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} dx}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \left(\frac{1}{c^2 \sqrt{1-c^2 x^2}} - \frac{\sqrt{1-c^2 x^2}}{c^2} \right) dx^2 \right)}{3c} - \frac{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3c^2} \right)}{5\sqrt{1 - c^2 x^2}} \right) \right. \\ \left. \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4 x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2 x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right)}{9\sqrt{1 - c^2 x^2}} \right)$$

↓ 2009

$$\frac{5}{9}d \left(\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} dx}{3c^2} - \frac{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3c^2} + \frac{2b \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2 x^2)^{3/2}}{3c^4} - 2 \int \frac{x^2}{\sqrt{1-c^2 x^2}} dx \right) \right)}{3c} \right)}{5\sqrt{1 - c^2 x^2}} \right) \right. \\ \left. \frac{1}{9}x^4(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9}c^4 x^9(a + b \arcsin(cx)) - \frac{2}{7}c^2 x^7(a + b \arcsin(cx)) + \frac{1}{5}x^5(a + b \arcsin(cx)) - \frac{1}{630}bc \left(-\frac{70(1-c^2)}{9c^6} \right) \right)}{9\sqrt{1 - c^2 x^2}} \right)$$

↓ 5182

$$\begin{aligned}
 & \frac{5}{9}d \left(\frac{3}{7}d \left(\frac{\sqrt{d - c^2 dx^2} \left(2 \left(\frac{2b \int (a + b \arcsin(cx)) dx}{c} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2}{c^2} \right) - \frac{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2}{3c^2} + \frac{2b}{c} \left(\frac{1}{3} x^3 (a + b \arcsin(cx)) \right)}{5\sqrt{1 - c^2 x^2}} \right. \right. \\
 & \left. \left. - \frac{\frac{1}{9} x^4 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9} c^4 x^9 (a + b \arcsin(cx)) - \frac{2}{7} c^2 x^7 (a + b \arcsin(cx)) + \frac{1}{5} x^5 (a + b \arcsin(cx)) - \frac{1}{630} bc \left(-\frac{70(1 - c^2)}{9c^6} \right)}{9\sqrt{1 - c^2 x^2}} \right)}{\right.} \\
 & \quad \downarrow \text{2009} \\
 & \left. \frac{\frac{1}{9} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 x^4 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{9} c^4 (a + b \arcsin(cx)) x^9 - \frac{2}{7} c^2 (a + b \arcsin(cx)) x^7 + \frac{1}{5} (a + b \arcsin(cx)) x^5 - \frac{1}{630} bc \left(-\frac{70(1 - c^2)}{9c^6} \right)}{9\sqrt{1 - c^2 x^2}} \right)}{\right.} \\
 & \left. \frac{5}{9}d \left(\frac{1}{7} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 x^4 - \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{7} c^2 (a + b \arcsin(cx)) x^7 + \frac{1}{5} (a + b \arcsin(cx)) x^5 - \frac{1}{630} bc \left(-\frac{70(1 - c^2)}{9c^6} \right)}{7\sqrt{1 - c^2 x^2}} \right)}{\right.} \right.
 \end{aligned}$$

input

`Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]`

output

```
(x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/9 - (2*b*c*d^2*Sqrt[d -
c^2*d*x^2]*(-1/630*(b*c*((-16*Sqrt[1 - c^2*x^2])/c^6 - (8*(1 - c^2*x^2)^(3
/2))/(3*c^6) - (6*(1 - c^2*x^2)^(5/2))/(5*c^6) + (100*(1 - c^2*x^2)^(7/2))
/(7*c^6) - (70*(1 - c^2*x^2)^(9/2))/(9*c^6))) + (x^5*(a + b*ArcSin[c*x]))/
5 - (2*c^2*x^7*(a + b*ArcSin[c*x]))/7 + (c^4*x^9*(a + b*ArcSin[c*x]))/9)/
(9*Sqrt[1 - c^2*x^2]) + (5*d*((x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x
])^2)/7 - (2*b*c*d*Sqrt[d - c^2*d*x^2]*(-1/70*(b*c*((-4*Sqrt[1 - c^2*x^2])
/c^6 - (2*(1 - c^2*x^2)^(3/2))/(3*c^6) + (16*(1 - c^2*x^2)^(5/2))/(5*c^6)
- (10*(1 - c^2*x^2)^(7/2))/(7*c^6))) + (x^5*(a + b*ArcSin[c*x]))/5 - (c^2*
x^7*(a + b*ArcSin[c*x]))/7))/(7*Sqrt[1 - c^2*x^2]) + (3*d*((x^4*Sqrt[d - c
^2*d*x^2]*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*Sqrt[d - c^2*d*x^2]*(-1/10*(b*
c*((-2*Sqrt[1 - c^2*x^2])/c^6 + (4*(1 - c^2*x^2)^(3/2))/(3*c^6) - (2*(1 -
c^2*x^2)^(5/2))/(5*c^6))) + (x^5*(a + b*ArcSin[c*x]))/5))/(5*Sqrt[1 - c^2*
x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*
x])^2)/c^2 + (2*b*(-1/6*(b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)
^(3/2))/(3*c^4))) + (x^3*(a + b*ArcSin[c*x]))/3))/(3*c) + (2*(-((Sqrt[1 -
c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2) + (2*b*(a*x + (b*Sqrt[1 - c^2*x^2])/
c + b*x*ArcSin[c*x]))/c))/(3*c^2)))/(5*Sqrt[1 - c^2*x^2])))/7)/9
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 243 $\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

rule 354 $\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^2)^{(p_.)}*((c_) + (d_.)*(x_)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m - 1)/2)*(a + b*x)^p*(c + d*x)^q}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

rule 1195 $\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}*((f_.) + (g_.)*(x_)^{(n_.)}*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

rule 1578 $\text{Int}[(x_)^{(m_.)}*((d_) + (e_.)*(x_)^2)^{(q_.)}*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

rule 5138 $\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_)]*(b_.))^{(n_.)}*((d_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{Int}[(d*x)^{m+1}*((a + b*\text{ArcSin}[c*x])^{n-1}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[c_.*(x_)]*(b_.))^{(n_.)}*(x_)*((d_) + (e_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{p+1}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{p+1/2}*(a + b*\text{ArcSin}[c*x])^{n-1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```


input `integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `1/250047*(126*(49*a*b*c^9*d^2*x^9 - 171*a*b*c^7*d^2*x^7 + 189*a*b*c^5*d^2*x^5 - 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x + (49*b^2*c^9*d^2*x^9 - 171*b^2*c^7*d^2*x^7 + 189*b^2*c^5*d^2*x^5 - 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + (343*(81*a^2 - 2*b^2)*c^10*d^2*x^10 - 2*(51597*a^2 - 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 - 2152*b^2)*c^6*d^2*x^6 - 4*(15876*a^2 - 53*b^2)*c^4*d^2*x^4 - (3969*a^2 - 14078*b^2)*c^2*d^2*x^2 + 2*(3969*a^2 - 6140*b^2)*d^2 + 3969*(7*b^2*c^10*d^2*x^10 - 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x^6 - 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 + 2*b^2*d^2)*arcsin(c*x)^2 + 7938*(7*a*b*c^10*d^2*x^10 - 26*a*b*c^8*d^2*x^8 + 34*a*b*c^6*d^2*x^6 - 16*a*b*c^4*d^2*x^4 - a*b*c^2*d^2*x^2 + 2*a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)`

Sympy [F(-1)]

Timed out.

$$\int x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.72

$$\begin{aligned}
& \int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \\
& -\frac{1}{63} \left(\frac{7(-c^2 dx^2 + d)^{7/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{7/2}}{c^4 d} \right) b^2 \arcsin(cx)^2 \\
& -\frac{2}{63} \left(\frac{7(-c^2 dx^2 + d)^{7/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{7/2}}{c^4 d} \right) ab \arcsin(cx) \\
& -\frac{1}{63} \left(\frac{7(-c^2 dx^2 + d)^{7/2} x^2}{c^2 d} + \frac{2(-c^2 dx^2 + d)^{7/2}}{c^4 d} \right) a^2 \\
& -\frac{2}{250047} b^2 \left(\frac{343 \sqrt{-c^2 x^2 + 1} c^6 d^{5/2} x^8 - 1147 \sqrt{-c^2 x^2 + 1} c^4 d^{5/2} x^6 + 1005 \sqrt{-c^2 x^2 + 1} c^2 d^{5/2} x^4 + 899 \sqrt{-c^2 x^2 + 1} d^{5/2}}{c^2} \right) \\
& -\frac{2 \left(49 c^8 d^{5/2} x^9 - 171 c^6 d^{5/2} x^7 + 189 c^4 d^{5/2} x^5 - 21 c^2 d^{5/2} x^3 - 126 d^{5/2} x \right) ab}{3969 c^3}
\end{aligned}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*b^2*arcsin(c*x)^2 - 2/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a*b*arcsin(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 - 2/250047*b^2*(343*sqrt(-c^2*x^2 + 1)*c^6*d^(5/2)*x^8 - 1147*sqrt(-c^2*x^2 + 1)*c^4*d^(5/2)*x^6 + 1005*sqrt(-c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 899*sqrt(-c^2*x^2 + 1)*d^(5/2)*x^2 - 6140*sqrt(-c^2*x^2 + 1)*d^(5/2)/c^2)/c^2 + 63*(49*c^8*d^(5/2)*x^9 - 171*c^6*d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 - 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*arcsin(c*x)/c^3 - 2/3969*(49*c^8*d^(5/2)*x^9 - 171*c^6*d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 - 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*a*b/c^3`

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int x^3 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2} dx$$

input `int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^3*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d^2 (7\sqrt{-c^2 x^2 + 1} a^2 c^8 x^8 - 19\sqrt{-c^2 x^2 + 1} a^2 c^6 x^6 + 15\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 - \sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 + a^2)}{24 c^2 d^2}$$

input `int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2,x)`

output

```
(sqrt(d)*d**2*(7*sqrt(-c**2*x**2+1)*a**2*c**8*x**8-19*sqrt(-c**2*x**2+1)*a**2*c**6*x**6+15*sqrt(-c**2*x**2+1)*a**2*c**4*x**4-sqrt(-c**2*x**2+1)*a**2*c**2*x**2-2*sqrt(-c**2*x**2+1)*a**2+126*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**7,x)*a*b*c**8-252*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**5,x)*a*b*c**6+126*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*a*b*c**4+63*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x**7,x)*b**2*c**8-126*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x**5,x)*b**2*c**6+63*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x**3,x)*b**2*c**4))/(63*c**4)
```

3.224 $\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2181
Mathematica [A] (verified)	2182
Rubi [A] (verified)	2183
Maple [C] (verified)	2194
Fricas [F]	2195
Sympy [F(-1)]	2196
Maxima [F]	2196
Giac [A] (verification not implemented)	2196
Mupad [F(-1)]	2197
Reduce [F]	2198

Optimal result

Integrand size = 29, antiderivative size = 556

$$\begin{aligned}
 & \int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \\
 & - \frac{359b^2 d^2 x \sqrt{d - c^2 dx^2}}{36864c^2} - \frac{1079b^2 d^2 x^3 \sqrt{d - c^2 dx^2}}{55296} \\
 & + \frac{209b^2 c^2 d^2 x^5 \sqrt{d - c^2 dx^2}}{13824} - \frac{1}{256} b^2 c^4 d^2 x^7 \sqrt{d - c^2 dx^2} \\
 & + \frac{359b^2 d^2 \sqrt{d - c^2 dx^2} \arcsin(cx)}{36864c^3 \sqrt{1 - c^2 x^2}} + \frac{5bd^2 x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{128c \sqrt{1 - c^2 x^2}} \\
 & - \frac{59bcd^2 x^4 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{384 \sqrt{1 - c^2 x^2}} \\
 & + \frac{17bc^3 d^2 x^6 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{144 \sqrt{1 - c^2 x^2}} \\
 & - \frac{bc^5 d^2 x^8 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{32 \sqrt{1 - c^2 x^2}} \\
 & - \frac{5d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{128c^2} \\
 & + \frac{5}{64} d^2 x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \\
 & + \frac{5}{48} dx^3 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \\
 & + \frac{1}{8} x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 + \frac{5d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^3}{384bc^3 \sqrt{1 - c^2 x^2}}
 \end{aligned}$$

output

```
-359/36864*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^2-1079/55296*b^2*d^2*x^3*(-c^2
*d*x^2+d)^(1/2)+209/13824*b^2*c^2*d^2*x^5*(-c^2*d*x^2+d)^(1/2)-1/256*b^2*c
^4*d^2*x^7*(-c^2*d*x^2+d)^(1/2)+359/36864*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*arc
sin(c*x)/c^3/(-c^2*x^2+1)^(1/2)+5/128*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*
arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-59/384*b*c*d^2*x^4*(-c^2*d*x^2+d)^(1/2)*
(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+17/144*b*c^3*d^2*x^6*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/32*b*c^5*d^2*x^8*(-c^2*d*x^2+d
)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-5/128*d^2*x*(-c^2*d*x^2+d)^(1
/2)*(a+b*arcsin(c*x))^2/c^2+5/64*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(
c*x))^2+5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2+1/8*x^3*(-c^2*
d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2+5/384*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
sin(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.63

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{d^2 \sqrt{d - c^2 dx^2} (1440a^3 - 96ab^2 c^2 x^2 (-45 + 177c^2 x^2 - 136c^4 x^4 + 36c^6 x^6) + 288a^2 bcx^2)}{\dots}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*Sqrt[d - c^2*d*x^2]*(1440*a^3 - 96*a*b^2*c^2*x^2*(-45 + 177*c^2*x^2 -
136*c^4*x^4 + 36*c^6*x^6) + 288*a^2*b*c*x*Sqrt[1 - c^2*x^2]*(-15 + 118*c^
2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) - b^3*c*x*Sqrt[1 - c^2*x^2]*(1077 + 2158
*c^2*x^2 - 1672*c^4*x^4 + 432*c^6*x^6) + 3*b*(1440*a^2 + 192*a*b*c*x*Sqrt[
1 - c^2*x^2]*(-15 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) + b^2*(359 + 1
440*c^2*x^2 - 5664*c^4*x^4 + 4352*c^6*x^6 - 1152*c^8*x^8))*ArcSin[c*x] + 2
88*b^2*(15*a + b*c*x*Sqrt[1 - c^2*x^2]*(-15 + 118*c^2*x^2 - 136*c^4*x^4 +
48*c^6*x^6))*ArcSin[c*x]^2 + 1440*b^3*ArcSin[c*x]^3))/(110592*b*c^3*Sqrt[1
- c^2*x^2])
```

Rubi [A] (verified)

Time = 3.73 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.37, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.931$, Rules used = {5202, 5192, 27, 1590, 25, 27, 363, 262, 262, 223, 5202, 5192, 27, 363, 262, 262, 223, 5198, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$$

↓ 5202

$$\frac{bcd^2\sqrt{d - c^2 dx^2} \int x^3(1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{4\sqrt{1 - c^2 x^2}} + \frac{5}{8}d \int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx + \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

↓ 5192

$$\frac{\frac{5}{8}d \int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^4(3c^4 x^4 - 8c^2 x^2 + 6)}{24\sqrt{1 - c^2 x^2}} dx + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} + \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{1}$$

↓ 27

$$\frac{\frac{5}{8}d \int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2\sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \int \frac{x^4(3c^4 x^4 - 8c^2 x^2 + 6)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} + \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{1}$$

↓ 1590

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \left(-\frac{\int -\frac{c^2 x^4 (48 - 43c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx}{8c^2} - \frac{3}{8}c^2 x^7 \sqrt{1 - c^2 x^2} \right) + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

↓ 25

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \left(\frac{\int \frac{c^2 x^4 (48 - 43c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx}{8c^2} - \frac{3}{8}c^2 x^7 \sqrt{1 - c^2 x^2} \right) + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

↓ 27

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \left(\frac{1}{8} \int \frac{x^4 (48 - 43c^2 x^2)}{\sqrt{1 - c^2 x^2}} dx - \frac{3}{8}c^2 x^7 \sqrt{1 - c^2 x^2} \right) + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

↓ 363

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \int \frac{x^4}{\sqrt{1 - c^2 x^2}} dx + \frac{43}{6}x^5 \sqrt{1 - c^2 x^2} \right) - \frac{3}{8}c^2 x^7 \sqrt{1 - c^2 x^2} \right) + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

↓ 262

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \int \frac{x^2}{\sqrt{1 - c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1 - c^2 x^2}}{4c^2} \right) + \frac{43}{6}x^5 \sqrt{1 - c^2 x^2} \right) - \frac{3}{8}c^2 x^7 \sqrt{1 - c^2 x^2} \right) + \frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) \right)}{4\sqrt{1 - c^2 x^2}} - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

↓ 262

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2 x^2}} dx - x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) + \frac{43}{6}x^5 \sqrt{1-c^2 x^2} \right) - \frac{3}{8}c^2 x^7 \sqrt{1-c^2 x^2} \right)}{4\sqrt{1-c^2 x^2}} \right)}{\frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}$$

223

$$\frac{\frac{5}{8}d \int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx + \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2 x^2}} dx - x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) + \frac{43}{6}x^5 \sqrt{1-c^2 x^2} \right) - \frac{3}{8}c^2 x^7 \sqrt{1-c^2 x^2} \right) \right)}{4\sqrt{1-c^2 x^2}}}{\frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}$$

5202

$$\frac{\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{bcd\sqrt{d - c^2 dx^2} \int x^3 (1 - c^2 x^2) (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2 x^2}} dx - x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) + \frac{43}{6}x^5 \sqrt{1-c^2 x^2} \right) - \frac{3}{8}c^2 x^7 \sqrt{1-c^2 x^2} \right) \right) \right)}{4\sqrt{1 - c^2 x^2}}}{\frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}$$

5192

$$\frac{\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{bcd\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^4 (3 - 2c^2 x^2)}{12\sqrt{1 - c^2 x^2}} dx - \frac{1}{6}c^2 x^6 (a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2 x^2}} + \frac{1}{6}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - \frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2 x^2}} dx - x \sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) + \frac{43}{6}x^5 \sqrt{1-c^2 x^2} \right) - \frac{3}{8}c^2 x^7 \sqrt{1-c^2 x^2} \right) \right) \right)}{4\sqrt{1 - c^2 x^2}}}{\frac{1}{8}x^3 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}$$

27

$$\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \int \frac{x^4(3-2c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{1}{6}c^2x^6(a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - \right. \\ \left. bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4x^8(a + b \arcsin(cx)) - \frac{1}{3}c^2x^6(a + b \arcsin(cx)) + \frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right) \right) \\ \hline 4\sqrt{1 - c^2x^2}$$

↓ 363

$$\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \left(\frac{4}{3} \int \frac{x^4}{\sqrt{1-c^2x^2}} dx + \frac{1}{3}x^5\sqrt{1 - c^2x^2} \right) - \frac{1}{6}c^2x^6(a + b \arcsin(cx)) \right)}{3\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - \right. \\ \left. bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4x^8(a + b \arcsin(cx)) - \frac{1}{3}c^2x^6(a + b \arcsin(cx)) + \frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right) \right) \\ \hline 4\sqrt{1 - c^2x^2}$$

↓ 262

$$\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) + \frac{1}{3}x^5\sqrt{1 - c^2x^2} \right) \right)}{3\sqrt{1 - c^2x^2}} \right. \\ \left. - \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - \right. \\ \left. bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4x^8(a + b \arcsin(cx)) - \frac{1}{3}c^2x^6(a + b \arcsin(cx)) + \frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right) \right) \\ \hline 4\sqrt{1 - c^2x^2}$$

↓ 262

$$\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx - \frac{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{12}bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2 x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2 x^2}}{4c^2} \right) \right)}{\right)}{\right)}{\right)} \\ \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\right)} \right) \right) \right) \right) \\ \hline 4\sqrt{1 - c^2 x^2}$$

↓ 223

$$\frac{5}{8}d \left(\frac{1}{2}d \int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{6}x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - \frac{bcd\sqrt{d - c^2 dx^2} \left(-\frac{1}{6}c^2 \right)}{\right)} \\ \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\right)} \right) \right) \right) \right) \\ \hline 4\sqrt{1 - c^2 x^2}$$

↓ 5198

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d - c^2 dx^2} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} dx}{4\sqrt{1 - c^2 x^2}} - \frac{bc\sqrt{d - c^2 dx^2} \int x^3(a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 \right) \right) \\ \frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2\sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8 (a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6 (a + b \arcsin(cx)) + \frac{1}{4}x^4 (a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\right)} \right) \right) \right) \right) \\ \hline 4\sqrt{1 - c^2 x^2}$$

↓ 5138

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \int \frac{x^4}{\sqrt{1-c^2x^2}} dx \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2} \right) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right) \right) \right) \right. \\ \left. \right) \frac{1}{4\sqrt{1-c^2x^2}}$$

↓ 262

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2\sqrt{1-c^2x^2}} \right) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right) \right) \right) \right. \\ \left. \right) \frac{1}{4\sqrt{1-c^2x^2}}$$

↓ 262

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} \right)}{2\sqrt{1-c^2x^2}} \right)}{4\sqrt{1-c^2x^2}} \right) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right) \right)}{4\sqrt{1-c^2x^2}}$$

↓ 223

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} \right)}{2\sqrt{1-c^2x^2}} \right)}{4\sqrt{1-c^2x^2}} \right) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right) \right)}{4\sqrt{1-c^2x^2}}$$

↓ 5210

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x(a+b\arcsin(cx)) dx}{c} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} \right)}{2\sqrt{1-c^2x^2}} \right)}{4\sqrt{1-c^2x^2}} \right) \right. \\ \left. - \frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b\arcsin(cx)) - \frac{1}{3}c^2x^6(a+b\arcsin(cx)) + \frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right) \right)}{4\sqrt{1-c^2x^2}}$$

↓ 5138

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} + \frac{1}{4} \right) \right.$$

$$\left. \frac{\frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b \arcsin(cx)) - \frac{1}{3}c^2x^6(a+b \arcsin(cx)) + \frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right)}{4\sqrt{1-c^2x^2}} \right)}{4\sqrt{1-c^2x^2}} \right.$$

↓ 262

$$\frac{5}{8}d \left(\frac{1}{2}d \left(\frac{\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{4\sqrt{1-c^2x^2}} \right) \right.$$

$$\left. \frac{\frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b \arcsin(cx))^2 - bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{8}c^4x^8(a+b \arcsin(cx)) - \frac{1}{3}c^2x^6(a+b \arcsin(cx)) + \frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right)}{4\sqrt{1-c^2x^2}} \right)}{4\sqrt{1-c^2x^2}} \right.$$

↓ 223

$$\frac{5}{8}d \left(\frac{1}{2}d \frac{\sqrt{d - c^2 dx^2} \left(\frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2 x^2}} - \frac{x\sqrt{1-c^2 x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2 x^2}}{2c^2} \right) \right)}{c}} \right)}{4\sqrt{1 - c^2 x^2}} \right.$$

$$\left. \frac{\frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8(a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6(a + b \arcsin(cx)) + \frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right)}{4\sqrt{1 - c^2 x^2}} \right)}{4\sqrt{1 - c^2 x^2}} \right.$$

↓ 5152

$$\frac{1}{8}x^3(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 +$$

$$\frac{5}{8}d \left(\frac{1}{6}x^3(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 + \frac{1}{2}d \left(\frac{1}{4}x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 + \frac{\sqrt{d - c^2 dx^2} \left(\frac{a+b \arcsin(cx)}{6bc^3} \right)}{\dots} \right) \right.$$

$$\left. \frac{bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{8}c^4 x^8(a + b \arcsin(cx)) - \frac{1}{3}c^2 x^6(a + b \arcsin(cx)) + \frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{24}bc \left(\frac{1}{8} \left(\frac{73}{6} \left(\frac{3}{\dots} \right) \right) \right) \right)}{4\sqrt{1 - c^2 x^2}} \right.$$

input Int [x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]

output

```
(x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/8 - (b*c*d^2*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcSin[c*x]))/4 - (c^2*x^6*(a + b*ArcSin[c*x]))/3 + (c^4*x^8*(a + b*ArcSin[c*x]))/8 - (b*c*((-3*c^2*x^7*Sqrt[1 - c^2*x^2])/8 + ((43*x^5*Sqrt[1 - c^2*x^2])/6 + (73*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/6)/8))/24)/(4*Sqrt[1 - c^2*x^2]) + (5*d*((x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/6 - (b*c*d*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcSin[c*x]))/4 - (c^2*x^6*(a + b*ArcSin[c*x]))/6 - (b*c*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/3))/12))/(3*Sqrt[1 - c^2*x^2]) + (d*((x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/4 - (b*c*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcSin[c*x]))/4 - (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4))/(2*Sqrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2 + (a + b*ArcSin[c*x])^3/(6*b*c^3) + (b*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c))/(4*Sqrt[1 - c^2*x^2]))/2)/8
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol]
:> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol]
:> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q + 1)) Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5138

```
Int[((a_) + ArcSin[(c_)*(x)]*(b_))^(n_)*((d_)*(x_))^(m_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5152

```
Int[((a_) + ArcSin[(c_)*(x)]*(b_))^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 5192

```
Int[((a_) + ArcSin[(c_)*(x)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol]
:> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 1939, normalized size of antiderivative = 3.49

method	result	size
default	Expression too large to display	1939
parts	Expression too large to display	1939

input

```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```

-1/8*a^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+
5/192*a^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a^2/c^2*d^2*x*(-c^2*d*x^2+d)^(
1/2)+5/128*a^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d
)^(1/2))+b^2*(-5/384*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^
2-1)*arcsin(c*x)^3*d^2+1/65536*(-d*(c^2*x^2-1))^(1/2)*(-128*I*(-c^2*x^2+1)
^(1/2)*x^8*c^8+128*c^9*x^9+256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7-16
0*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2
-88*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+8*c*x)*(8*I*arcsin(c*x)+32*arcsin(c*x)^2-
1)*d^2/c^3/(c^2*x^2-1)-1/6912*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(
1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-
c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*(6*I*arcsi
n(c*x)+18*arcsin(c*x)^2-1)*d^2/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2
)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2
*arcsin(c*x)^2-1-2*I*arcsin(c*x))*d^2/c^3/(c^2*x^2-1)+1/65536*(-d*(c^2*x^2
-1))^(1/2)*(128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9-256*I*(-c^2*x^2+1)
^(1/2)*x^6*c^6-320*c^7*x^7+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5-3
2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+8*c*x)*(-8*
I*arcsin(c*x)+32*arcsin(c*x)^2-1)*d^2/c^3/(c^2*x^2-1)+1/55296*(-d*(c^2*x^2
-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(156*I*arcsin(c*x)+72*arcs
in(c*x)^2-19)*cos(5*arcsin(c*x))*d^2/c^3/(c^2*x^2-1)-5/55296*(-d*(c^2*x...

```

Fricas [F]

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2 x^2 dx$$

input

```

integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")

```

output

```

integral((a^2*c^4*d^2*x^6 - 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2
*x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^6
- 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)

```


Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2,x)`

output Timed out

Maxima [F]

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2 x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin(c*x)/c^3)*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^6 - 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)`

Giac [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.35

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```

1/8*sqrt(-c^2*d*x^2 + d)*a^2*c^4*d^2*x^7 - 17/48*sqrt(-c^2*d*x^2 + d)*a^2*
c^2*d^2*x^5 + 59/192*sqrt(-c^2*d*x^2 + d)*a^2*d^2*x^3 - 5/128*sqrt(-c^2*d*
x^2 + d)*a^2*d^2*x/c^2 - 5/128*a^2*d^3*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^
2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)) + 1/110592*(13824*(c^2*x^2 - 1)^3*sqrt(-c
^2*x^2 + 1)*b^2*d^(5/2)*x*arcsin(c*x)^2 + 27648*(c^2*x^2 - 1)^3*sqrt(-c^2*
x^2 + 1)*a*b*d^(5/2)*x*arcsin(c*x) + 2304*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 +
1)*b^2*d^(5/2)*x*arcsin(c*x)^2 - 432*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b
^2*d^(5/2)*x + 4608*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*a*b*d^(5/2)*x*arcsin
(c*x) + 2880*(-c^2*x^2 + 1)^(3/2)*b^2*d^(5/2)*x*arcsin(c*x)^2 - 3456*(c^2*
x^2 - 1)^4*b^2*d^(5/2)*arcsin(c*x)/c + 376*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 +
1)*b^2*d^(5/2)*x + 5760*(-c^2*x^2 + 1)^(3/2)*a*b*d^(5/2)*x*arcsin(c*x) +
4320*sqrt(-c^2*x^2 + 1)*b^2*d^(5/2)*x*arcsin(c*x)^2 - 3456*(c^2*x^2 - 1)^4
*a*b*d^(5/2)/c - 768*(c^2*x^2 - 1)^3*b^2*d^(5/2)*arcsin(c*x)/c + 110*(-c^2
*x^2 + 1)^(3/2)*b^2*d^(5/2)*x + 8640*sqrt(-c^2*x^2 + 1)*a*b*d^(5/2)*x*arcs
in(c*x) - 768*(c^2*x^2 - 1)^3*a*b*d^(5/2)/c + 1440*(c^2*x^2 - 1)^2*b^2*d^(
5/2)*arcsin(c*x)/c + 1440*b^2*d^(5/2)*arcsin(c*x)^3/c - 1995*sqrt(-c^2*x^2
+ 1)*b^2*d^(5/2)*x + 1440*(c^2*x^2 - 1)^2*a*b*d^(5/2)/c - 4320*(c^2*x^2 -
1)*b^2*d^(5/2)*arcsin(c*x)/c + 4320*a*b*d^(5/2)*arcsin(c*x)^2/c - 4320*(c
^2*x^2 - 1)*a*b*d^(5/2)/c - 1995*b^2*d^(5/2)*arcsin(c*x)/c - 1995*a*b*d^(5
/2)/c)/c^2

```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int x^2 (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2} dx$$

input

```
int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output

```
int(x^2*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d^2 (15 a \sin(cx) a^2 + 48 \sqrt{-c^2 x^2 + 1} a^2 c^7 x^7 - 136 \sqrt{-c^2 x^2 + 1} a^2 c^5 x^5 + 118 \sqrt{-c^2 x^2 + 1} a^2 c^3 x^3 - 15 \sqrt{-c^2 x^2 + 1} a^2 c x + 768 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^6, x) a b c^7 - 1536 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4, x) a b c^5 + 768 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) a b c^3 + 384 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c^7 - 768 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c^5 + 384 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c^3) / (384 c^3)$$

input

```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2,x)
```

output

```
(sqrt(d)*d**2*(15*asin(c*x)*a**2 + 48*sqrt(-c**2*x**2 + 1)*a**2*c**7*x**7 - 136*sqrt(-c**2*x**2 + 1)*a**2*c**5*x**5 + 118*sqrt(-c**2*x**2 + 1)*a**2*c**3*x**3 - 15*sqrt(-c**2*x**2 + 1)*a**2*c*x + 768*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**6,x)*a*b*c**7 - 1536*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*a*b*c**5 + 768*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*a*b*c**3 + 384*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**6,x)*b**2*c**7 - 768*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**4,x)*b**2*c**5 + 384*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**3)/(384*c**3)
```

3.225 $\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2199
Mathematica [A] (verified)	2200
Rubi [A] (verified)	2200
Maple [A] (verified)	2203
Fricas [A] (verification not implemented)	2203
Sympy [F(-1)]	2204
Maxima [A] (verification not implemented)	2204
Giac [F(-2)]	2205
Mupad [F(-1)]	2205
Reduce [F]	2206

Optimal result

Integrand size = 27, antiderivative size = 343

$$\begin{aligned} \int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx &= \frac{32b^2 d^2 \sqrt{d - c^2 dx^2}}{245c^2} \\ &+ \frac{16b^2 d(d - c^2 dx^2)^{3/2}}{735c^2} + \frac{12b^2 (d - c^2 dx^2)^{5/2}}{1225c^2} + \frac{2b^2 (d - c^2 dx^2)^{7/2}}{343c^2 d} \\ &+ \frac{2bd^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{7c\sqrt{1 - c^2 x^2}} - \frac{2bcd^2 x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{7\sqrt{1 - c^2 x^2}} \\ &+ \frac{6bc^3 d^2 x^5 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{35\sqrt{1 - c^2 x^2}} \\ &- \frac{2bc^5 d^2 x^7 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{49\sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))^2}{7c^2 d} \end{aligned}$$

output

```
32/245*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^2+16/735*b^2*d*(-c^2*d*x^2+d)^(3/2)/
c^2+12/1225*b^2*(-c^2*d*x^2+d)^(5/2)/c^2+2/343*b^2*(-c^2*d*x^2+d)^(7/2)/c^
2/d+2/7*b*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2
)-2/7*b*c*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2
)+6/35*b*c^3*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(
1/2)-2/49*b*c^5*d^2*x^7*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1
)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arcsin(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.51

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{d^2 \sqrt{d - c^2 dx^2} \left((-1 + c^2 x^2)^3 (a + b \arcsin(cx))^2 - \frac{2b(105acx(-35 + 35c^2 x^2 - 21c^4 x^4 + 5c^6 x^6) + b\sqrt{1 - c^2 x^2}(-2161 + 757c^2 x^2 - 351c^4 x^4 + 75c^6 x^6) + 105b^2 c x^2(-35 + 35c^2 x^2 - 21c^4 x^4 + 5c^6 x^6) \arcsin(cx))}{3675 \sqrt{1 - c^2 x^2}} \right)}{7c^2}$$

input

```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*Sqrt[d - c^2*d*x^2]*((-1 + c^2*x^2)^3*(a + b*ArcSin[c*x])^2 - (2*b*(105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6) + 105*b*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcSin[c*x]))/(3675*Sqrt[1 - c^2*x^2]))/(7*c^2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5182, 5154, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5182$$

$$\frac{2bd^2 \sqrt{d - c^2 dx^2} \int (1 - c^2 x^2)^3 (a + b \arcsin(cx)) dx}{7c \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 dx^2)^{7/2} (a + b \arcsin(cx))^2}{7c^2 d}$$

$$\downarrow 5154$$

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arcsin(cx))+\frac{3}{5}c^4x^5(a+b\arcsin(cx))-c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{7c\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))^2}{7c^2d}$$

↓ 27

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arcsin(cx))+\frac{3}{5}c^4x^5(a+b\arcsin(cx))-c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{7c\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))^2}{7c^2d}$$

↓ 2331

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{70}bc\int\frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^6x^7(a+b\arcsin(cx))+\frac{3}{5}c^4x^5(a+b\arcsin(cx))-c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{7c\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))^2}{7c^2d}$$

↓ 2389

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{70}bc\int\left(5(1-c^2x^2)^{5/2}+6(1-c^2x^2)^{3/2}+8\sqrt{1-c^2x^2}+\frac{16}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{7}c^6x^7(a+b\arcsin(cx))+\frac{3}{5}c^4x^5(a+b\arcsin(cx))-c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{7c\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))^2}{7c^2d}$$

↓ 2009

$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^6x^7(a+b\arcsin(cx))+\frac{3}{5}c^4x^5(a+b\arcsin(cx))-c^2x^3(a+b\arcsin(cx))+x(a+b\arcsin(cx))\right)}{7c\sqrt{1-c^2x^2}}$$

$$\frac{(d-c^2dx^2)^{7/2}(a+b\arcsin(cx))^2}{7c^2d}$$

input `Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]`

output

```
-1/7*((d - c^2*d*x^2)^(7/2)*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*d^2*Sqrt
[d - c^2*d*x^2]*(-1/70*(b*c*((-32*Sqrt[1 - c^2*x^2])/c^2 - (16*(1 - c^2*x^
2)^(3/2))/(3*c^2) - (12*(1 - c^2*x^2)^(5/2))/(5*c^2) - (10*(1 - c^2*x^2)^(
7/2))/(7*c^2))) + x*(a + b*ArcSin[c*x]) - c^2*x^3*(a + b*ArcSin[c*x]) + (3
*c^4*x^5*(a + b*ArcSin[c*x]))/5 - (c^6*x^7*(a + b*ArcSin[c*x]))/7)/(7*c*S
qrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331

```
Int[(P_q)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, P_q, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[P_q, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389

```
Int[(P_q)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand
[P_q*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[P_q, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5154

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbo
l] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x
] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5182

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.55

method	result
orering	$\frac{(9525c^{10}x^{10}-41691c^8x^8+76515c^6x^6-124979c^4x^4+26152c^2x^2-4322)(-c^2dx^2+d)^{\frac{5}{2}}(a+b\arcsin(cx))^2}{25725c^4(cx-1)x^2(cx+1)(c^2x^2-1)^2} - \frac{2(675c^8x^8-3108c^6x^6}{$
default	Expression too large to display
parts	Expression too large to display

input `int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output
$$\frac{1}{25725} \cdot \frac{(9525c^{10}x^{10} - 41691c^8x^8 + 76515c^6x^6 - 124979c^4x^4 + 26152c^2x^2 - 4322)}{c^4} \cdot \frac{(cx-1)}{x^2} \cdot \frac{1}{(cx+1)} \cdot \frac{1}{(c^2x^2-1)^2} \cdot (-c^2dx^2+d)^{\frac{5}{2}} \cdot (a+b\arcsin(cx))^2 - \frac{2(675c^8x^8 - 3108c^6x^6 + 6352c^4x^4 - 14480c^2x^2 + 2161)}{c^4} \cdot \frac{(cx-1)}{x^2} \cdot \frac{1}{(cx+1)} \cdot \frac{1}{(c^2x^2-1)} \cdot (-c^2dx^2+d)^{\frac{5}{2}} \cdot (a+b\arcsin(cx))^2 - 5x^2 \cdot (-c^2dx^2+d)^{\frac{3}{2}} \cdot (a+b\arcsin(cx))^2 \cdot c^2 \cdot d \cdot 2x \cdot (-c^2dx^2+d)^{\frac{5}{2}} \cdot (a+b\arcsin(cx)) \cdot bc / (-c^2x^2+1)^{\frac{1}{2}} + \frac{1}{25725} \cdot (75c^6x^6 - 351c^4x^4 + 757c^2x^2 - 2161) / c^4 \cdot \frac{(cx-1)}{x} \cdot \frac{1}{(cx+1)} \cdot (-15c^2d \cdot x \cdot (-c^2dx^2+d)^{\frac{3}{2}} \cdot (a+b\arcsin(cx))^2 + 4 \cdot (-c^2dx^2+d)^{\frac{5}{2}} \cdot (a+b\arcsin(cx)) \cdot bc / (-c^2x^2+1)^{\frac{1}{2}} + 15x^3 \cdot (-c^2dx^2+d)^{\frac{1}{2}} \cdot (a+b\arcsin(cx))^2 \cdot c^4 \cdot d^2 - 20x^2 \cdot (-c^2dx^2+d)^{\frac{3}{2}} \cdot (a+b\arcsin(cx)) \cdot c^3 \cdot d \cdot b / (-c^2x^2+1)^{\frac{1}{2}} + 2x \cdot (-c^2dx^2+d)^{\frac{5}{2}} \cdot b^2 \cdot c^2 / (-c^2x^2+1) + 2x^2 \cdot (-c^2dx^2+d)^{\frac{5}{2}} \cdot (a+b\arcsin(cx)) \cdot bc \cdot c^3 / (-c^2x^2+1)^{\frac{3}{2}})$$

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.18

$$\int x(d - c^2dx^2)^{5/2} (a + b\arcsin(cx))^2 dx = \frac{210(5abc^7d^2x^7 - 21abc^5d^2x^5 + 35abc^3d^2x^3 - 35abcd^2x + (5b^2c^7d^2x^7 - 21b^2c^5d^2x^5$$

input `integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output

```
1/25725*(210*(5*a*b*c^7*d^2*x^7 - 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3
- 35*a*b*c*d^2*x + (5*b^2*c^7*d^2*x^7 - 21*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^
2*x^3 - 35*b^2*c*d^2*x)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 +
1) + (75*(49*a^2 - 2*b^2)*c^8*d^2*x^8 - 12*(1225*a^2 - 71*b^2)*c^6*d^2*x^6
+ 2*(11025*a^2 - 1108*b^2)*c^4*d^2*x^4 - 4*(3675*a^2 - 1459*b^2)*c^2*d^2*
x^2 + (3675*a^2 - 4322*b^2)*d^2 + 3675*(b^2*c^8*d^2*x^8 - 4*b^2*c^6*d^2*x^
6 + 6*b^2*c^4*d^2*x^4 - 4*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 7350*
(a*b*c^8*d^2*x^8 - 4*a*b*c^6*d^2*x^6 + 6*a*b*c^4*d^2*x^4 - 4*a*b*c^2*d^2*x
^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input

```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2,x)
```

output

Timed out

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.82

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx =$$

$$-\frac{(-c^2 dx^2 + d)^{7/2} b^2 \arcsin(cx)^2}{7 c^2 d} - \frac{2(-c^2 dx^2 + d)^{7/2} ab \arcsin(cx)}{7 c^2 d}$$

$$-\frac{2}{25725} b^2 \left(\frac{75 \sqrt{-c^2 x^2 + 1} c^4 d^{7/2} x^6 - 351 \sqrt{-c^2 x^2 + 1} c^2 d^{7/2} x^4 + 757 \sqrt{-c^2 x^2 + 1} d^{7/2} x^2 - \frac{2161 \sqrt{-c^2 x^2 + 1} d^{7/2}}{c^2}}{d} \right) +$$

$$-\frac{(-c^2 dx^2 + d)^{7/2} a^2}{7 c^2 d} - \frac{2(5 c^6 d^{7/2} x^7 - 21 c^4 d^{7/2} x^5 + 35 c^2 d^{7/2} x^3 - 35 d^{7/2} x) ab}{245 cd}$$

input

```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima"
)
```

output

```
-1/7*(-c^2*d*x^2 + d)^(7/2)*b^2*arcsin(c*x)^2/(c^2*d) - 2/7*(-c^2*d*x^2 +
d)^(7/2)*a*b*arcsin(c*x)/(c^2*d) - 2/25725*b^2*((75*sqrt(-c^2*x^2 + 1)*c^4
*d^(7/2)*x^6 - 351*sqrt(-c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(-c^2*x^2
+ 1)*d^(7/2)*x^2 - 2161*sqrt(-c^2*x^2 + 1)*d^(7/2)/c^2)/d + 105*(5*c^6*d^(
7/2)*x^7 - 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 - 35*d^(7/2)*x)*arcsin(
c*x)/(c*d) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) - 2/245*(5*c^6*d^(7/2
)*x^7 - 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 - 35*d^(7/2)*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int x(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2} dx$$

input

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output

```
int(x*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```


3.226 $\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2207
Mathematica [A] (verified)	2208
Rubi [A] (verified)	2209
Maple [C] (verified)	2214
Fricas [F]	2215
Sympy [F(-1)]	2215
Maxima [F]	2215
Giac [F(-2)]	2216
Mupad [F(-1)]	2216
Reduce [F]	2217

Optimal result

Integrand size = 26, antiderivative size = 411

$$\begin{aligned}
 \int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = & -\frac{245b^2 d^2 x \sqrt{d - c^2 dx^2}}{1152} \\
 & - \frac{65b^2 dx (d - c^2 dx^2)^{3/2}}{1728} - \frac{1}{108} b^2 x (d - c^2 dx^2)^{5/2} \\
 & + \frac{115b^2 d^2 \sqrt{d - c^2 dx^2} \arcsin(cx)}{1152c \sqrt{1 - c^2 x^2}} - \frac{5bcd^2 x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{16 \sqrt{1 - c^2 x^2}} \\
 & + \frac{5bd^2 (1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{48c} \\
 & + \frac{bd^2 (1 - c^2 x^2)^{5/2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{18c} \\
 & + \frac{5}{16} d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 + \frac{5}{24} dx (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 + \frac{1}{6} x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2
 \end{aligned}$$

output

```
-245/1152*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-65/1728*b^2*d*x*(-c^2*d*x^2+d)^(3/2)-1/108*b^2*x*(-c^2*d*x^2+d)^(5/2)+115/1152*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/c/(-c^2*x^2+1)^(1/2)-5/16*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+5/48*b*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c+1/18*b*d^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/c+5/16*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2+5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2+1/6*x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2+5/48*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.73

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{d^2 \sqrt{d - c^2 dx^2} (360a^3 + b^3 cx \sqrt{1 - c^2 x^2} (-897 + 194c^2 x^2 - 32c^4 x^4) - 24ab^2 c^2 x^2 (99 - 39c^2 x^2 + 8c^4 x^4) + 72a^2 b^2 c x \sqrt{1 - c^2 x^2} (33 - 26c^2 x^2 + 8c^4 x^4) + 3b^2 (360a^2 + 48ab^2 c x \sqrt{1 - c^2 x^2} (33 - 26c^2 x^2 + 8c^4 x^4) + b^2 (299 - 792c^2 x^2 + 312c^4 x^4 - 64c^6 x^6)) \arcsin[cx] + 72b^2 (15a + bc x \sqrt{1 - c^2 x^2} (33 - 26c^2 x^2 + 8c^4 x^4)) \arcsin[cx]^2 + 360b^3 \arcsin[cx]^3)}{(3456b^3 c \sqrt{1 - c^2 x^2})}$$

input

```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d^2*Sqrt[d - c^2*d*x^2]*(360*a^3 + b^3*c*x*Sqrt[1 - c^2*x^2]*(-897 + 194*c^2*x^2 - 32*c^4*x^4) - 24*a*b^2*c^2*x^2*(99 - 39*c^2*x^2 + 8*c^4*x^4) + 72*a^2*b^2*c*x*Sqrt[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4) + 3*b^2*(360*a^2 + 48*a*b*c*x*Sqrt[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4) + b^2*(299 - 792*c^2*x^2 + 312*c^4*x^4 - 64*c^6*x^6))*ArcSin[c*x] + 72*b^2*(15*a + b*c*x*Sqrt[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4))*ArcSin[c*x]^2 + 360*b^3*ArcSin[c*x]^3))/(3456*b^3*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5158, 5158, 5156, 5138, 262, 223, 5152, 5182, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5158} \\
 & -\frac{bcd^2 \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + \frac{5}{6} d \int (d - c^2 dx^2)^{3/2} (a + \\
 & \quad b \arcsin(cx))^2 dx + \frac{1}{6} x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5158} \\
 & -\frac{bcd^2 \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + \\
 & \frac{5}{6} d \left(-\frac{bcd \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{3}{4} d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{4} x (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 \right) + \frac{1}{6} x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5156} \\
 & -\frac{bcd^2 \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + \\
 & \frac{5}{6} d \left(-\frac{bcd \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{3}{4} d \left(-\frac{bc \sqrt{d - c^2 dx^2} \int x (a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2}}{2} (a + b \arcsin(cx))^2 \right) \right) + \frac{1}{6} x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \\
 & \quad \downarrow \text{5138} \\
 & -\frac{bcd^2 \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2 x^2}} + \\
 & \frac{5}{6} d \left(-\frac{bcd \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{3}{4} d \left(-\frac{bc \sqrt{d - c^2 dx^2} \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \int \sqrt{d - c^2 dx^2} dx \right)}{\sqrt{1 - c^2 x^2}} \right) \right) + \frac{1}{6} x (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2
 \end{aligned}$$

$$\begin{aligned}
& \downarrow 262 \\
& -\frac{bcd^2\sqrt{d-c^2dx^2} \int x(1-c^2x^2)^2(a+b\arcsin(cx))dx}{3\sqrt{1-c^2x^2}} + \\
\frac{5}{6}d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{3}{4}d \left(-\frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \right)}{\sqrt{1-c^2x^2}} \right. \right. \\
& \left. \left. + \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 \right) \right) \\
& \downarrow 223 \\
& -\frac{bcd^2\sqrt{d-c^2dx^2} \int x(1-c^2x^2)^2(a+b\arcsin(cx))dx}{3\sqrt{1-c^2x^2}} + \\
\frac{5}{6}d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2d} \right) \right. \\
& \left. + \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 \right) \\
& \downarrow 5152 \\
& -\frac{bcd^2\sqrt{d-c^2dx^2} \int x(1-c^2x^2)^2(a+b\arcsin(cx))dx}{3\sqrt{1-c^2x^2}} + \\
\frac{5}{6}d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 + \frac{3}{4}d \left(\frac{\sqrt{d-c^2d}}{\sqrt{1-c^2x^2}} \right) \right. \\
& \left. + \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 \right) \\
& \downarrow 5182 \\
& -\frac{bcd^2\sqrt{d-c^2dx^2} \left(\frac{b \int (1-c^2x^2)^{5/2} dx}{6c} - \frac{(1-c^2x^2)^3(a+b\arcsin(cx))}{6c^2} \right)}{3\sqrt{1-c^2x^2}} + \\
\frac{5}{6}d \left(-\frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \int (1-c^2x^2)^{3/2} dx}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 + \frac{3}{4}d \left(\frac{\sqrt{d-c^2d}}{\sqrt{1-c^2x^2}} \right) \right. \\
& \left. + \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 \right) \\
& \downarrow 211
\end{aligned}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{5}{6}\int(1-c^2x^2)^{3/2}dx+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\arcsin(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}} +$$

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))\right)$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2$$

211

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\arcsin(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}} +$$

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))\right)$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2$$

211

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\arcsin(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}} +$$

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))\right)$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2$$

223

$$\begin{aligned}
& \frac{bcd^2\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{5}{6} \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right) + \frac{1}{6}x(1-c^2x^2)^{5/2} \right)}{6c} - \frac{(1-c^2x^2)^3(a+b\arcsin(cx))}{6c^2} \right)}{3\sqrt{1-c^2x^2}} + \\
& \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 + \\
& \frac{5}{6}d \left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)}{2\sqrt{1-c^2x^2}} \right)}{2\sqrt{1-c^2x^2}} \right)
\end{aligned}$$

input

```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/6 - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-1/6*((1 - c^2*x^2)^3*(a + b*ArcSin[c*x]))/c^2 + (b*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/6))/(6*c)))/(3*Sqrt[1 - c^2*x^2]) + (5*d*((x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^3)/(6*b*c*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/Sqrt[1 - c^2*x^2]))/4 - (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/c^2 + (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 - c^2*x^2])))/6
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1))) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5138

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] := Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x],
x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 1349, normalized size of antiderivative = 3.28

method	result	size
default	Expression too large to display	1349
parts	Expression too large to display	1349

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned} & 1/6*x*(-c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3/(c^2*d)^(1/2)*\arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-5/48*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*\arcsin(c*x)^3*d^2+1/6912*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*(6*I*\arcsin(c*x)+18*\arcsin(c*x)^2-1)*d^2/(c^2*x^2-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*\arcsin(c*x)^2-1-2*I*\arcsin(c*x))*d^2/(c^2*x^2-1)/c-1/27648*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(348*I*\arcsin(c*x)+576*\arcsin(c*x)^2-77)*\cos(5*\arcsin(c*x))*d^2/(c^2*x^2-1)/c+5/27648*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(60*I*\arcsin(c*x)+144*\arcsin(c*x)^2-17)*\sin(5*\arcsin(c*x))*d^2/(c^2*x^2-1)/c-3/1024*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(44*I*\arcsin(c*x)+32*\arcsin(c*x)^2-19)*\cos(3*\arcsin(c*x))*d^2/(c^2*x^2-1)/c+9/1024*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(12*I*\arcsin(c*x)+16*\arcsin(c*x)^2-7)*\sin(3*\arcsin(c*x))*d^2/(c^2*x^2-1)/c+2*a*b*(-5/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*\arcsin(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*... \end{aligned}$$

Fricas [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2 dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d), x)`

Sympy [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2 dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a^2 + sqrt(d)*integrate
(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x +
1))*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*a
rctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1), x
)
```

Giac [F(-2)]

Exception generated.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2} dx$$

input

```
int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d} d^2 (15 a \sin(cx) a^2 + 8 \sqrt{-c^2 x^2 + 1} a^2 c^5 x^5 - 26 \sqrt{-c^2 x^2 + 1} a^2 c^3 x^3 + 33 \sqrt{-c^2 x^2 + 1} a^2 c x + 96 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4, x) a b c^5 - 192 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) a b c^3 + 96 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx), x) a b c + 48 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c^5 - 96 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c^3 + 48 \int (\sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2, x) b^2 c)}{48 c}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(d)*d**2*(15*asin(c*x)*a**2 + 8*sqrt(-c**2*x**2 + 1)*a**2*c**5*x**5 - 26*sqrt(-c**2*x**2 + 1)*a**2*c**3*x**3 + 33*sqrt(-c**2*x**2 + 1)*a**2*c*x + 96*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**4,x)*a*b*c**5 - 192*int(sqrt(-c**2*x**2 + 1)*asin(c*x)*x**2,x)*a*b*c**3 + 96*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*a*b*c + 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**4,x)*b**2*c**5 - 96*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**3 + 48*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2,x)*b**2*c))/(48*c)`

3.227
$$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2}{x} dx$$

Optimal result	2218
Mathematica [A] (verified)	2219
Rubi [A] (verified)	2220
Maple [B] (verified)	2228
Fricas [F]	2229
Sympy [F(-1)]	2230
Maxima [F]	2230
Giac [F(-2)]	2230
Mupad [F(-1)]	2231
Reduce [F]	2231

Optimal result

Integrand size = 29, antiderivative size = 660

$$\begin{aligned} \int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2}{x} dx = & -\frac{598}{225} b^2 d^2 \sqrt{d-c^2 dx^2} \\ & - \frac{2abcd^2 x \sqrt{d-c^2 dx^2}}{\sqrt{1-c^2 x^2}} - \frac{74}{675} b^2 d (d-c^2 dx^2)^{3/2} - \frac{2}{125} b^2 (d-c^2 dx^2)^{5/2} \\ & - \frac{2b^2 cd^2 x \sqrt{d-c^2 dx^2} \arcsin(cx)}{\sqrt{1-c^2 x^2}} - \frac{16bcd^2 x \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{15\sqrt{1-c^2 x^2}} \\ & + \frac{22bc^3 d^2 x^3 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{45\sqrt{1-c^2 x^2}} - \frac{2bc^5 d^2 x^5 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))}{25\sqrt{1-c^2 x^2}} \\ & + d^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2 + \frac{1}{3} d (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^2 + \frac{1}{5} (d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^2 \end{aligned}$$

output

```

-598/225*b^2*d^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-74/675*b^2*d*(-c^2*d*x^2+d)^(3/2)-2/125*b^2*(-c^2*d*x^2+d)^(5/2)-2*b^2*c*d^2*x*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(1/2)-16/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+22/45*b*c^3*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-2/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2+1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2-2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*I*b*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.17

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \frac{d^2 \left(3600a^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (23 - 11c^2 x^2 + 3c^4 x^4) + 54000 \right)}{x}$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x,x]
```


output

```
(d^2*(3600*a^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*
c^4*x^4) + 54000*a^2*Sqrt[d]*Sqrt[1 - c^2*x^2]*Log[c*x] - 54000*a^2*Sqrt[d
]*Sqrt[1 - c^2*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - 108000*a*b*Sqrt
[d - c^2*d*x^2]*(c*x - Sqrt[1 - c^2*x^2]*ArcSin[c*x] - ArcSin[c*x]*(Log[1
- E^(I*ArcSin[c*x])]) - Log[1 + E^(I*ArcSin[c*x])]) - I*(PolyLog[2, -E^(I*A
rcSin[c*x])] - PolyLog[2, E^(I*ArcSin[c*x])])) - 54000*b^2*Sqrt[d - c^2*d*
x^2]*(2*Sqrt[1 - c^2*x^2] + 2*c*x*ArcSin[c*x] - Sqrt[1 - c^2*x^2]*ArcSin[c
*x]^2 - ArcSin[c*x]^2*(Log[1 - E^(I*ArcSin[c*x])]) - Log[1 + E^(I*ArcSin[c*
x])]) - (2*I)*ArcSin[c*x]*(PolyLog[2, -E^(I*ArcSin[c*x])] - PolyLog[2, E^(
I*ArcSin[c*x])]) + 2*(PolyLog[3, -E^(I*ArcSin[c*x])] - PolyLog[3, E^(I*Arc
Sin[c*x])])) - 6000*a*b*Sqrt[d - c^2*d*x^2]*(9*c*x - 3*ArcSin[c*x]*(3*Sqrt
[1 - c^2*x^2] + Cos[3*ArcSin[c*x]]) + Sin[3*ArcSin[c*x]]) + 1000*b^2*Sqrt[
d - c^2*d*x^2]*(27*Sqrt[1 - c^2*x^2]*(-2 + ArcSin[c*x]^2) + (-2 + 9*ArcSin
[c*x]^2)*Cos[3*ArcSin[c*x]] - 6*ArcSin[c*x]*(9*c*x + Sin[3*ArcSin[c*x]]))
+ 30*a*b*Sqrt[d - c^2*d*x^2]*(450*c*x - 15*ArcSin[c*x]*(30*Sqrt[1 - c^2*x^
2] + 5*Cos[3*ArcSin[c*x]] - 3*Cos[5*ArcSin[c*x]]) + 25*Sin[3*ArcSin[c*x]]
- 9*Sin[5*ArcSin[c*x]]) - b^2*Sqrt[d - c^2*d*x^2]*(6750*Sqrt[1 - c^2*x^2]*
(-2 + ArcSin[c*x]^2) + 125*(-2 + 9*ArcSin[c*x]^2)*Cos[3*ArcSin[c*x]] - 27*
(-2 + 25*ArcSin[c*x]^2)*Cos[5*ArcSin[c*x]] + 30*ArcSin[c*x]*(-25*Sin[3*Arc
Sin[c*x]] + 9*(-50*c*x + Sin[5*ArcSin[c*x]])))))/(54000*Sqrt[1 - c^2*x^...
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.85, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.690$, Rules used = {5202, 5154, 27, 1576, 1140, 2009, 5202, 5154, 27, 353, 53, 2009, 5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx$$

$$\downarrow 5202$$

$$-\frac{2bcd^2 \sqrt{d - c^2 dx^2} \int (1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{5\sqrt{1 - c^2 x^2}} +$$

$$d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx + \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2$$

$$\begin{aligned} & \downarrow 5154 \\ & \frac{d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \int \frac{x(3c^4 x^4 - 10c^2 x^2 + 15)}{15\sqrt{1 - c^2 x^2}} dx + \frac{1}{5} c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5\sqrt{1 - c^2 x^2}} \\ & \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \end{aligned}$$

$$\begin{aligned} & \downarrow 27 \\ & \frac{d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{15} bc \int \frac{x(3c^4 x^4 - 10c^2 x^2 + 15)}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{5} c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5\sqrt{1 - c^2 x^2}} \\ & \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \end{aligned}$$

$$\begin{aligned} & \downarrow 1576 \\ & \frac{d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{30} bc \int \frac{3c^4 x^4 - 10c^2 x^2 + 15}{\sqrt{1 - c^2 x^2}} dx^2 + \frac{1}{5} c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5\sqrt{1 - c^2 x^2}} \\ & \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \end{aligned}$$

$$\begin{aligned} & \downarrow 1140 \\ & \frac{d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx - 2bcd^2 \sqrt{d - c^2 dx^2} \left(-\frac{1}{30} bc \int \left(3(1 - c^2 x^2)^{3/2} + 4\sqrt{1 - c^2 x^2} + \frac{8}{\sqrt{1 - c^2 x^2}} \right) dx^2 + \frac{1}{5} c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5\sqrt{1 - c^2 x^2}} \\ & \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 \end{aligned}$$

$$\begin{aligned} & \downarrow 2009 \\ & \frac{d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx + \frac{1}{5} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{5} c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3} c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{30} bc \left(-\frac{6(1 - c^2 x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1 - c^2 x^2}} \end{aligned}$$

$$\downarrow 5202$$

$$d \left(-\frac{2bcd\sqrt{d-c^2dx^2} \int (1-c^2x^2)(a+b\arcsin(cx))dx}{3\sqrt{1-c^2x^2}} + d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + \frac{1}{3}(d-c^2dx^2) \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \\ \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right) \right) \\ \hline 5\sqrt{1-c^2x^2}$$

↓ 5154

$$d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-bc \int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \\ \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right) \right) \\ \hline 5\sqrt{1-c^2x^2}$$

↓ 27

$$d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}bc \int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \\ \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right) \right) \\ \hline 5\sqrt{1-c^2x^2}$$

↓ 353

$$d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}bc \int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}} dx^2 - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - \right. \\ \left. 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right) \right) \\ \hline 5\sqrt{1-c^2x^2}$$

↓ 53

$$d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}bc \int \left(\sqrt{1-c^2x^2} + \frac{2}{\sqrt{1-c^2x^2}} \right) dx^2 - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 2009

$$d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + \frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}c^2x^3(a+b\arcsin(cx)) \right)}{5\sqrt{1-c^2x^2}} \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 5198

$$d \left(d \left(-\frac{2bc\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right) \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 2009

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax + bx\arcsin(cx))}{\sqrt{1-c^2x^2}} \right) \right. \\ \left. - \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 5218

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax+bx\arcsin(cx))}{\sqrt{1-c^2x^2}} \right. \right. \\ \left. \left. \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 3042

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx))^2 \csc(\arcsin(cx)) d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2} (a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2} (ax+bx\arcsin(cx))}{\sqrt{1-c^2x^2}} \right. \right. \\ \left. \left. \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 4671

$$d \left(d \left(\frac{\sqrt{d-c^2dx^2} (-2b \int (a+b\arcsin(cx)) \log(1-e^{i\arcsin(cx)}) d\arcsin(cx) + 2b \int (a+b\arcsin(cx)) \log(1+e^{i\arcsin(cx)}) d\arcsin(cx))}{\sqrt{1-c^2x^2}} \right. \right. \\ \left. \left. \frac{1}{5} (d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2 - 2bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{5}c^4x^5(a+b\arcsin(cx)) - \frac{2}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2} \right) \right)}{5\sqrt{1-c^2x^2}} \right)$$

↓ 3011

$$d \left(d \left(\frac{\sqrt{d - c^2 dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\frac{1}{5}(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{5}c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3}c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2 x^2)^{5/2}}{5c^2} \right)}{5\sqrt{1 - c^2 x^2}} \right. \right.$$

↓ 2720

$$d \left(d \left(\frac{\sqrt{d - c^2 dx^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\frac{1}{5}(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{5}c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3}c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2 x^2)^{5/2}}{5c^2} \right)}{5\sqrt{1 - c^2 x^2}} \right. \right.$$

↓ 7143

$$d \left(d \left(\frac{\sqrt{d - c^2 dx^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)))}{\frac{1}{5}(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{1}{5}c^4 x^5 (a + b \arcsin(cx)) - \frac{2}{3}c^2 x^3 (a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1-c^2 x^2)^{5/2}}{5c^2} \right)}{5\sqrt{1 - c^2 x^2}} \right. \right.$$

input

Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x,x]

output

```

((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/5 - (2*b*c*d^2*Sqrt[d - c^2*
d*x^2]*(-1/30*(b*c*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/
(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2))) + x*(a + b*ArcSin[c*x]) - (2*c
^2*x^3*(a + b*ArcSin[c*x]))/3 + (c^4*x^5*(a + b*ArcSin[c*x]))/5)/(5*Sqrt[
1 - c^2*x^2]) + d*(((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*
c*d*Sqrt[d - c^2*d*x^2]*(-1/6*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c
^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSin[c*x]) - (c^2*x^3*(a + b*ArcSin[
c*x]))/3))/(3*Sqrt[1 - c^2*x^2]) + d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*
x])^2 - (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*Ar
cSin[c*x]))/Sqrt[1 - c^2*x^2] + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x
])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E
^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcS
in[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])
)/Sqrt[1 - c^2*x^2]))

```

Defintions of rubi rules used

rule 27

```

Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]

```

rule 53

```

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

```

rule 353

```

Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
:= Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]

```

rule 1140

```

Int[((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]

```

rule 1576 `Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5154 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5218

```
Int((((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(630) = 1260$.

Time = 0.92 (sec) , antiderivative size = 1490, normalized size of antiderivative = 2.26

method	result	size
default	Expression too large to display	1490
parts	Expression too large to display	1490

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)`

output

```

1/5*(-c^2*d*x^2+d)^(5/2)*a^2+1/3*a^2*d*(-c^2*d*x^2+d)^(3/2)-a^2*d^(5/2)*ln
((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a^2*d^2*(-c^2*d*x^2+d)^(1/2)+b^2*
(1/4000*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4-16*I*(-c^2*x^2+1)^(1
/2)*x^5*c^5+13*c^2*x^2+20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-5*I*(-c^2*x^2+1)^(1
/2)*x*c-1)*(10*I*arcsin(c*x)+25*arcsin(c*x)^2-2)*d^2/(c^2*x^2-1)-7/864*(-d
*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*
I*(-c^2*x^2+1)^(1/2)*x*c+1)*(6*I*arcsin(c*x)+9*arcsin(c*x)^2-2)*d^2/(c^2*x
^2-1)+11/16*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(a
rcsin(c*x)^2-2+2*I*arcsin(c*x))*d^2/(c^2*x^2-1)+11/16*(-d*(c^2*x^2-1))^(1/
2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))*
d^2/(c^2*x^2-1)-7/864*(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/
2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*(-6*I*arcsin(c*x)+9*a
rcsin(c*x)^2-2)*d^2/(c^2*x^2-1)+1/4000*(-d*(c^2*x^2-1))^(1/2)*(16*I*c^5*x^
5*(-c^2*x^2+1)^(1/2)+16*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4
+5*I*(-c^2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*(-10*I*arcsin(c*x)+25*arcsin(c*x
)^2-2)*d^2/(c^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-
1)*(arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-
(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2
*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*polylog(3,I*c*x+(-c^2
*x^2+1)^(1/2))+2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2)))*d^2)-46/15*a*b*(...

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="fricas")`

output

```

integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2/x,x)`

output `Timed out`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output `-1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2 + d)*d^2)*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2}}{x} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x} dx = \frac{\sqrt{d} d^2 \left(3\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 - 11\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 + 23\sqrt{-c^2 x^2 + 1} a^2 \right)}{15}$$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2/x,x)
```

output

```
(sqrt(d)*d**2*(3*sqrt(-c**2*x**2+1)*a**2*c**4*x**4-11*sqrt(-c**2*x
**2+1)*a**2*c**2*x**2+23*sqrt(-c**2*x**2+1)*a**2+30*int((sqrt(-
c**2*x**2+1)*asin(c*x))/x,x)*a*b+15*int((sqrt(-c**2*x**2+1)*asin(
c*x)**2)/x,x)*b**2+30*int(sqrt(-c**2*x**2+1)*asin(c*x)*x**3,x)*a*b*c
**4-60*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*a*b*c**2+15*int(sqrt(
-c**2*x**2+1)*asin(c*x)**2*x**3,x)*b**2*c**4-30*int(sqrt(-c**2*x**
2+1)*asin(c*x)**2*x,x)*b**2*c**2+15*log(tan(asin(c*x)/2))*a**2-23*a*
*2))/15
```

3.228
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx$$

Optimal result	2232
Mathematica [A] (verified)	2233
Rubi [A] (verified)	2234
Maple [A] (verified)	2245
Fricas [F]	2246
Sympy [F]	2247
Maxima [F]	2247
Giac [F(-2)]	2247
Mupad [F(-1)]	2248
Reduce [F]	2248

Optimal result

Integrand size = 29, antiderivative size = 549

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx &= \frac{31}{64} b^2 c^2 d^2 x \sqrt{d - c^2 dx^2} \\ &+ \frac{1}{32} b^2 c^2 dx (d - c^2 dx^2)^{3/2} - \frac{89 b^2 c d^2 \sqrt{d - c^2 dx^2} \arcsin(cx)}{64 \sqrt{1 - c^2 x^2}} \\ &+ \frac{15 b c^3 d^2 x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{8 \sqrt{1 - c^2 x^2}} + b c d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \\ &- \frac{1}{8} b c d^2 (1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) - \frac{15}{8} c^2 d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{i c d^2 \sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} \end{aligned}$$

output

```

31/64*b^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2*d*x*(-c^2*d*x^2+d)^(
3/2)-89/64*b^2*c*d^2*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(1/2)+1
5/8*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2
)+b*c*d^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-1/8*b*
c*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))-15/8*c^2*d
^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2-I*c*d^2*(-c^2*d*x^2+d)^(1/2)
*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(1/2)-5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(
a+b*arcsin(c*x))^2-(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x-5/8*c*d^2*(-
c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*d^2*(-c^
2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^
2*x^2+1)^(1/2)-I*b^2*c*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2
+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \frac{d^2 \left(-256a^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} - 288a^2 c^2 x^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} \right)}{x^2}$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```

(d^2*(-256*a^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] - 288*a^2*c^2*x^2*Sqr
t[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 - c^2*x^2]*Sqrt
[d - c^2*d*x^2] - 160*b^2*c*x*Sqrt[d - c^2*d*x^2]*ArcSin[c*x]^3 + 480*a^2*
c*Sqrt[d]*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-
1 + c^2*x^2))] - 128*a*b*c*x*Sqrt[d - c^2*d*x^2]*Cos[2*ArcSin[c*x]] - 4*a*
b*c*x*Sqrt[d - c^2*d*x^2]*Cos[4*ArcSin[c*x]] + 512*a*b*c*x*Sqrt[d - c^2*d*
x^2]*Log[c*x] - (256*I)*b^2*c*x*Sqrt[d - c^2*d*x^2]*PolyLog[2, E^((2*I)*Ar
cSin[c*x])] + 64*b^2*c*x*Sqrt[d - c^2*d*x^2]*Sin[2*ArcSin[c*x]] + b^2*c*x*
Sqrt[d - c^2*d*x^2]*Sin[4*ArcSin[c*x]] - 4*b*Sqrt[d - c^2*d*x^2]*ArcSin[c*
x]*(128*a*Sqrt[1 - c^2*x^2] + 32*b*c*x*Cos[2*ArcSin[c*x]] + b*c*x*Cos[4*Ar
cSin[c*x]] - 128*b*c*x*Log[1 - E^((2*I)*ArcSin[c*x])] + 64*a*c*x*Sin[2*Arc
Sin[c*x]] + 4*a*c*x*Sin[4*ArcSin[c*x]]) - 8*b*Sqrt[d - c^2*d*x^2]*ArcSin[c
*x]^2*(60*a*c*x + (32*I)*b*c*x + 32*b*Sqrt[1 - c^2*x^2] + 16*b*c*x*Sin[2*A
rcSin[c*x]] + b*c*x*Sin[4*ArcSin[c*x]]))/(256*x*Sqrt[1 - c^2*x^2])

```

Rubi [A] (verified)

Time = 3.16 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.13, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.897$, Rules used = {5200, 5158, 5156, 5138, 262, 223, 5152, 5182, 211, 211, 223, 5188, 211, 211, 223, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx$$

$$\downarrow \text{5200}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx}{\sqrt{1 - c^2 x^2}} - 5c^2 d \int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5158}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx}{\sqrt{1 - c^2 x^2}} - 5c^2 d \left(-\frac{bcd \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{3}{4} d \int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx + \frac{1}{4} x(d - c^2 dx^2) \right) - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5156}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x} dx}{\sqrt{1 - c^2 x^2}} - 5c^2 d \left(-\frac{bcd \sqrt{d - c^2 dx^2} \int x(1 - c^2 x^2) (a + b \arcsin(cx)) dx}{2\sqrt{1 - c^2 x^2}} + \frac{3}{4} d \left(-\frac{bc \sqrt{d - c^2 dx^2} \int x(a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} + \frac{\sqrt{d - c^2 dx^2}}{4} \right) \right) - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x}$$

$$\downarrow \text{5138}$$

$$\begin{aligned}
& \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
5c^2d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{3}{4}d \left(-\frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \right)}{\sqrt{1-c^2x^2}} \right. \right. \\
& \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \right) \\
& \quad \downarrow \text{262} \\
& \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
5c^2d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{3}{4}d \left(-\frac{bc\sqrt{d-c^2dx^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \right)}{\sqrt{1-c^2x^2}} \right. \right. \\
& \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \right) \\
& \quad \downarrow \text{223} \\
& \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
5c^2d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2} \right. \right. \\
& \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \right) \\
& \quad \downarrow \text{5152} \\
& \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
5c^2d \left(-\frac{bcd\sqrt{d-c^2dx^2} \int x(1-c^2x^2)(a+b\arcsin(cx))dx}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 + \frac{3}{4}d \left(\frac{\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \right. \right. \\
& \left. \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \right) \\
& \quad \downarrow \text{5182}
\end{aligned}$$

$$\begin{aligned}
 & \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
 5c^2d \left(\frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \int (1-c^2x^2)^{3/2} dx}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 + \frac{3}{4}d \right. \\
 & \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \\
 & \quad \downarrow \text{211} \\
 & \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
 5c^2d \left(\frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 \right. \\
 & \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \\
 & \quad \downarrow \text{211} \\
 & \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \\
 5c^2d \left(\frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 \right. \\
 & \left. \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} \right) \\
 & \quad \downarrow \text{223} \\
 & \frac{2bcd^2\sqrt{d-c^2dx^2} \int \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{x} dx}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} - \\
 5c^2d \left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{bcd\sqrt{d-c^2dx^2} \left(\frac{b \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{4c^2} \right)}{2\sqrt{1-c^2x^2}} \right) \\
 & \quad \downarrow \text{5188}
 \end{aligned}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx-\frac{1}{4}bc\int(1-c^2x^2)^{3/2}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} - 5c^2d\left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c} - \frac{(1-c^2x^2)^2}{2\sqrt{1-c^2x^2}}\right)}{2\sqrt{1-c^2x^2}}\right)$$

↓ 211

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx-\frac{1}{4}bc\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} - 5c^2d\left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c} - \frac{(1-c^2x^2)^2}{2\sqrt{1-c^2x^2}}\right)}{2\sqrt{1-c^2x^2}}\right)$$

↓ 211

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))\right)}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x} - 5c^2d\left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c} - \frac{(1-c^2x^2)^2}{2\sqrt{1-c^2x^2}}\right)}{2\sqrt{1-c^2x^2}}\right)$$

↓ 223

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))-\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)\right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)}{2\sqrt{1-c^2x^2}}}$$

↓ 5188

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)}{2\sqrt{1-c^2x^2}}}$$

↓ 211

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)}{2\sqrt{1-c^2x^2}}}$$

↓ 223

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arcsin(cx)}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^2}{4c}\right)}{2\sqrt{1-c^2x^2}}-\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2\right)$$

↓ 5136

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx}d\arcsin(cx)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^2}{4c}\right)}{2\sqrt{1-c^2x^2}}-\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2\right)$$

↓ 3042

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int-\left((a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)\right)d\arcsin(cx)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^2}{4c}\right)}{2\sqrt{1-c^2x^2}}-\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2\right)$$

↓ 25

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\int(a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)d\arcsin(cx)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}\right)}{x}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^2}{4c}\right)}{2\sqrt{1-c^2x^2}}-\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2\right)$$

↓ 4200

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(2i\int-\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}}d\arcsin(cx)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)}{x}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^2}{4c}\right)}{2\sqrt{1-c^2x^2}}-\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2\right)$$

↓ 25

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2i\int\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}}d\arcsin(cx)+\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)}{x}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)-\frac{(1-c^2x^2)^2}{4c}\right)}{2\sqrt{1-c^2x^2}}-\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2\right)$$

↓ 2620

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))-\frac{1}{2}ib\int\log(1-e^{2i\arcsin(cx)})d\arcsin(cx)\right)+\frac{1}{4}}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)}{\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)$$

↓ 2715

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))-\frac{1}{4}b\int e^{-2i\arcsin(cx)}\log(1-e^{2i\arcsin(cx)})de^{2i\arcsin(cx)}\right)+\frac{1}{4}}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)}{\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)$$

↓ 2838

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{4}(1-c^2x^2)^2(a+b\arcsin(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)\right)+\frac{1}{4}}$$

$$5c^2d\left(\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)}{\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}-\frac{(1-c^2x^2)^{3/2}}{2\sqrt{1-c^2x^2}}\right)$$

input

`Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^2,x]`

output

```

-(((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x) - 5*c^2*d*((x*(d - c^2*
d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*d*x^2]*(a +
b*ArcSin[c*x])^2)/2 + (Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^3)/(6*b*c*S
qrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcSin[c*x]))/2
- (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/Sqrt[1
- c^2*x^2]))/4 - (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*(a + b*
ArcSin[c*x]))/c^2 + (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^
2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 - c^2*x^2])) + (2*b*c*d^
2*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)*(a + b*ArcSin[c*x]))/2 + ((1 - c^2*x
^2)^2*(a + b*ArcSin[c*x]))/4 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*
Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 - (b*c*((x*(1 - c^2*x^2)^(3/2
))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4 - (2*I)*((I
/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])]) + (b*PolyLog[2, E^
(2*I)*ArcSin[c*x]]))/4))/Sqrt[1 - c^2*x^2]

```

Definitions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620 $\text{Int}[\frac{((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_))}}{((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_))}, x_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m}{(b*f*g*n*\text{Log}[F])}*\text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_))})^{(n_)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n)], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4200 $\text{Int}[\frac{((c_) + (d_)*(x_))^{(m_)*\tan[(e_) + \text{Pi}*(k_) + (f_)*(x_)]}{(a_ + b*x)^{m+1}}, x_Symbol] \rightarrow \text{Simp}[I*(c + d*x)^{m+1}/(d*(m+1)), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * E^{(2*I*k*Pi)}*(E^{(2*I*(e + f*x))}/(1 + E^{(2*I*k*Pi)}*E^{(2*I*(e + f*x))})], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

rule 5136 $\text{Int}[\frac{((a_) + \text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)}}{(a_ + b*x)^n * \text{Cot}[x]}, x], x, \text{ArcSin}[c*x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{IGtQ}[n, 0]$

rule 5138 $\text{Int}[\frac{((a_) + \text{ArcSin}[(c_)*(x_)]*(b_))^{(n_)*((d_)*(x_))^{(m_)}}{(d*x)^{m+1}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{Int}[(d*x)^{m+1}*((a + b*\text{ArcSin}[c*x])^{n-1}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5158

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[x*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(2*p + 1)), x] + (Simp[2*d*(p/(2*p + 1)) Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5188

```
Int((((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_), x_Symbol]
:> Simp[(d + e*x^2)^p*((a + b*ArcSin[c*x])/(2*p)), x] + (Simp[d Int[(d + e*x^2)^(p - 1)*((a + b*ArcSin[c*x])/x), x], x] - Simp[b*c*(d^p/(2*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n/(f*(m + 1)), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 972, normalized size of antiderivative = 1.77

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{7}{2}}}{dx} - a^2c^2x(-c^2dx^2+d)^{\frac{5}{2}} - \frac{5(-c^2dx^2+d)^{\frac{3}{2}}a^2c^2dx}{4} - \frac{15a^2d^2\sqrt{-c^2dx^2+d}c^2x}{8} - \frac{15a^2c^2d^3 \arctan\left(\frac{cx}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{-c^2dx^2+d}}$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{7}{2}}}{dx} - a^2c^2x(-c^2dx^2+d)^{\frac{5}{2}} - \frac{5(-c^2dx^2+d)^{\frac{3}{2}}a^2c^2dx}{4} - \frac{15a^2d^2\sqrt{-c^2dx^2+d}c^2x}{8} - \frac{15a^2c^2d^3 \arctan\left(\frac{cx}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{-c^2dx^2+d}}$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output

```

-a^2/d/x*(-c^2*d*x^2+d)^(7/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*(-c^2*d*x
^2+d)^(3/2)*a^2*c^2*d*x-15/8*a^2*d^2*(-c^2*d*x^2+d)^(1/2)*c^2*x-15/8*a^2*c
^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(5/8
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arcsin(c*x)^3*d^2*c
+1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8
*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(4*I*
arcsin(c*x)+8*arcsin(c*x)^2-1)*d^2*c/(c^2*x^2-1)-1/8*(-d*(c^2*x^2-1))^(1/2
)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2
*arcsin(c*x)^2-1-2*I*arcsin(c*x))*d^2*c/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)
*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)^2*d^2/(c^2*x^2-1)/x+2*I*
(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1+
I*c*x+(-c^2*x^2+1)^(1/2))+I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+arc
sin(c*x)^2+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,I*c*x+(-c^2*x^2+
1)^(1/2)))d^2*c+3/512*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(
1/2)-I)*(44*I*arcsin(c*x)+40*arcsin(c*x)^2-21)*cos(3*arcsin(c*x))*d^2*c/(c
^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-
1)*(124*I*arcsin(c*x)+136*arcsin(c*x)^2-65)*sin(3*arcsin(c*x))*d^2*c/(c^2*
x^2-1))+1/64*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x*(
-32*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^5+144*arcsin(c*x)*(-c^2
*x^2+1)^(1/2)*c^2*x^2-72*c^3*x^3+120*arcsin(c*x)^2*c*x+128*I*arcsin(c*x)...

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x^2} dx$$

input

```

integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")

```

output

```

integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)

```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(-d(cx - 1)(cx + 1))^{5/2} (a + b \arcsin(cx))^2}{x^2} dx$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2/x**2,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))**2/x**2, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="maxima")`

output `-1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2}}{x^2} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^2} dx = \frac{\sqrt{d} d^2 (-8 \arcsin(cx)^3 b^2 cx - 24 \arcsin(cx)^2 abcx - 45 \arcsin(cx) a^2 cx}{x^2}$$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2/x^2,x)
```

output

```
(sqrt(d)*d**2*(- 8*asin(c*x)**3*b**2*c*x - 24*asin(c*x)**2*a*b*c*x - 45*a
sin(c*x)*a**2*c*x + 6*sqrt(- c**2*x**2 + 1)*a**2*c**4*x**4 - 27*sqrt(- c
**2*x**2 + 1)*a**2*c**2*x**2 - 24*sqrt(- c**2*x**2 + 1)*a**2 + 48*int(asi
n(c*x)/(sqrt(- c**2*x**2 + 1)*x**2),x)*a*b*x + 24*int(asin(c*x)**2/(sqrt(
- c**2*x**2 + 1)*x**2),x)*b**2*x + 48*int(sqrt(- c**2*x**2 + 1)*asin(c*x
)*x**2,x)*a*b*c**4*x - 96*int(sqrt(- c**2*x**2 + 1)*asin(c*x),x)*a*b*c**2
*x + 24*int(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**4*x - 48*i
nt(sqrt(- c**2*x**2 + 1)*asin(c*x)**2,x)*b**2*c**2*x))/(24*x)
```

3.229
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx$$

Optimal result	2249
Mathematica [A] (verified)	2250
Rubi [A] (warning: unable to verify)	2251
Maple [A] (verified)	2261
Fricas [F]	2262
Sympy [F]	2263
Maxima [F]	2263
Giac [F(-2)]	2264
Mupad [F(-1)]	2264
Reduce [F]	2264

Optimal result

Integrand size = 29, antiderivative size = 728

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \frac{40}{9} b^2 c^2 d^2 \sqrt{d - c^2 dx^2} + \frac{5abc^3 d^2 x \sqrt{d - c^2 dx^2}}{\sqrt{1 - c^2 x^2}} + \frac{2}{27} b^2 c^2 d (d - c^2 dx^2)^{3/2} + \frac{5b^2 c^3 d^2 x \sqrt{d - c^2 dx^2} \arcsin(cx)}{\sqrt{1 - c^2 x^2}} - \frac{bcd^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{x \sqrt{1 - c^2 x^2}} - \frac{bc^3 d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{3 \sqrt{1 - c^2 x^2}} - \frac{2bc^5 d^2 x^3 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{9 \sqrt{1 - c^2 x^2}} - \frac{5}{2} c^2 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 - \frac{5}{6} c^2 d (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{2x^2} + \frac{5c^2 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{\sqrt{1 - c^2 x^2}}$$

output

```

40/9*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)+5*a*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)
/(-c^2*x^2+1)^(1/2)+2/27*b^2*c^2*d*(-c^2*d*x^2+d)^(3/2)+5*b^2*c^3*d^2*x*(-
c^2*d*x^2+d)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(1/2)-b*c*d^2*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))/x/(-c^2*x^2+1)^(1/2)-1/3*b*c^3*d^2*x*(-c^2*d*x^2+d)
^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-2/9*b*c^5*d^2*x^3*(-c^2*d*x^2+
d)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-5/2*c^2*d^2*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))^2-5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^
2-1/2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^2+5*c^2*d^2*(-c^2*d*x^2+d)
^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)
^(1/2)-b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*
x^2+1)^(1/2)-5*I*b*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))*polylog(
2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+5*I*b*c^2*d^2*(-c^2*d*x^2+
d)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)
^(1/2)+5*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(
1/2))/(-c^2*x^2+1)^(1/2)-5*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,I*c
x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 7.16 (sec) , antiderivative size = 1073, normalized size of antiderivative = 1.47

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \text{Too large to display}$$

input

```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^3,x]
```

output

```

Sqrt[-(d*(-1 + c^2*x^2))]*((-7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^4*d^2*x^2)/3) - (5*a^2*c^2*d^(5/2)*Log[x])/2 + (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/2 - 4*a*b*c^2*d^2*Sqrt[d*(1 - c^2*x^2)]*(-((c*x)/Sqrt[1 - c^2*x^2]) + ArcSin[c*x] + (ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x]])] - Log[1 + E^(I*ArcSin[c*x]])])/Sqrt[1 - c^2*x^2] + (I*(PolyLog[2, -E^(I*ArcSin[c*x]])] - PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2]) - 2*b^2*c^2*d^2*Sqrt[d*(1 - c^2*x^2)]*(-2 - (2*c*x*ArcSin[c*x])/Sqrt[1 - c^2*x^2] + ArcSin[c*x]^2 + (ArcSin[c*x]^2*(Log[1 - E^(I*ArcSin[c*x]])] - Log[1 + E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2] + ((2*I)*ArcSin[c*x]*(PolyLog[2, -E^(I*ArcSin[c*x]])] - PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcSin[c*x])] + PolyLog[3, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2]) - (a*b*c^2*d^2*Sqrt[d*(1 - c^2*x^2)]*(-9*c*x + 9*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + 3*ArcSin[c*x]*Cos[3*ArcSin[c*x]] - Sin[3*ArcSin[c*x]]))/(18*Sqrt[1 - c^2*x^2]) - (b^2*c^2*d^2*Sqrt[d*(1 - c^2*x^2)]*(2*7*Sqrt[1 - c^2*x^2]*(-2 + ArcSin[c*x]^2) + (-2 + 9*ArcSin[c*x]^2)*Cos[3*ArcSin[c*x]] - 6*ArcSin[c*x]*(9*c*x + Sin[3*ArcSin[c*x]])))/(108*Sqrt[1 - c^2*x^2]) + (a*b*c^2*d^3*Sqrt[1 - c^2*x^2]*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 - 4*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] + (4*I)*PolyLog[2, E^(I*ArcSin[c*x])] + ArcSin[c*x]*Sec[ArcSin[c*x]/2...

```

Rubi [A] (warning: unable to verify)

Time = 3.59 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.77, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5200, 5192, 27, 1578, 1192, 25, 1467, 2009, 5202, 5154, 27, 353, 53, 2009, 5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx$$

$$\downarrow 5200$$

$$\frac{bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x^2} dx}{\sqrt{1 - c^2 x^2}} - \frac{5}{2} c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{2x^2}$$

$$\begin{array}{c}
\downarrow 5192 \\
\frac{-\frac{5}{2}c^2d \int \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} dx + bcd^2\sqrt{d-c^2dx^2} \left(-bc \int -\frac{-c^4x^4+6c^2x^2+3}{3x\sqrt{1-c^2x^2}} dx + \frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2}} \\
\downarrow 27 \\
\frac{-\frac{5}{2}c^2d \int \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} dx + bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}bc \int \frac{-c^4x^4+6c^2x^2+3}{x\sqrt{1-c^2x^2}} dx + \frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2}} \\
\downarrow 1578 \\
\frac{-\frac{5}{2}c^2d \int \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} dx + bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{6}bc \int \frac{-c^4x^4+6c^2x^2+3}{x^2\sqrt{1-c^2x^2}} dx^2 + \frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2}} \\
\downarrow 1192 \\
\frac{-\frac{5}{2}c^2d \int \frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x} dx + bcd^2\sqrt{d-c^2dx^2} \left(\frac{b \int -\frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4} d\sqrt{1-c^2x^2}}{3c^3} + \frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} \right)}{\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2}} \\
\downarrow 25
\end{array}$$

$$\begin{aligned}
 & -\frac{5}{2}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx + \\
 & bcd^2\sqrt{d - c^2dx^2} \left(-\frac{b \int \frac{-c^4x^8 - 4c^4x^4 + 8c^4}{1-x^4} d\sqrt{1-c^2x^2}}{3c^3} + \frac{1}{3}c^4x^3(a + b \arcsin(cx)) - 2c^2x(a + b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} \right) \\
 & \frac{\sqrt{1 - c^2x^2}}{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2} \\
 & \frac{2x^2}{}
 \end{aligned}$$

↓ 1467

$$\begin{aligned}
 & -\frac{5}{2}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx + \\
 & bcd^2\sqrt{d - c^2dx^2} \left(-\frac{b \int (x^4c^4 + \frac{3c^4}{1-x^4} + 5c^4) d\sqrt{1-c^2x^2}}{3c^3} + \frac{1}{3}c^4x^3(a + b \arcsin(cx)) - 2c^2x(a + b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} \right) \\
 & \frac{\sqrt{1 - c^2x^2}}{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2} \\
 & \frac{2x^2}{}
 \end{aligned}$$

↓ 2009

$$\begin{aligned}
 & -\frac{5}{2}c^2d \int \frac{(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2}{x} dx + \\
 & bcd^2\sqrt{d - c^2dx^2} \left(\frac{1}{3}c^4x^3(a + b \arcsin(cx)) - 2c^2x(a + b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{b(-3c^4 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{1}{3}c^4)}{3c^3} \right) \\
 & \frac{\sqrt{1 - c^2x^2}}{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2} \\
 & \frac{2x^2}{}
 \end{aligned}$$

↓ 5202

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(-\frac{2bcd\sqrt{d - c^2dx^2} \int (1 - c^2x^2) (a + b \arcsin(cx)) dx}{3\sqrt{1 - c^2x^2}} + d \int \frac{\sqrt{d - c^2dx^2} (a + b \arcsin(cx))^2}{x} dx + \frac{1}{3}(d - c^2x^2) \right) \\
 & bcd^2\sqrt{d - c^2dx^2} \left(\frac{1}{3}c^4x^3(a + b \arcsin(cx)) - 2c^2x(a + b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{b(-3c^4 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{1}{3}c^4)}{3c^3} \right) \\
 & \frac{\sqrt{1 - c^2x^2}}{(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2} \\
 & \frac{2x^2}{}
 \end{aligned}$$

↓ 5154

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-bc \int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4}{3c^3} \right) \right) \\
 & \qquad \qquad \qquad \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad \frac{2x^2}{2x^2}
 \end{aligned}$$

↓ 27

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{3}bc \int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4}{3c^3} \right) \right) \\
 & \qquad \qquad \qquad \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad \frac{2x^2}{2x^2}
 \end{aligned}$$

↓ 353

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}bc \int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}} dx^2 - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) \right)}{3\sqrt{1-c^2x^2}} \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4}{3c^3} \right) \right) \\
 & \qquad \qquad \qquad \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad \frac{2x^2}{2x^2}
 \end{aligned}$$

↓ 53

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{1}{6}bc \int \left(\sqrt{1-c^2x^2} + \frac{2}{\sqrt{1-c^2x^2}} \right) dx^2 - \frac{1}{3}c^2x^3 \right)}{3\sqrt{1-c^2x^2}} \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4}{3c^3} \right) \right) \\
 & \qquad \qquad \qquad \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2} \\
 & \qquad \qquad \qquad \frac{2x^2}{2x^2}
 \end{aligned}$$

↓ 2009

$$-\frac{5}{2}c^2d \left(d \int \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{x} dx + \frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{2bcd\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \left(-\frac{1}{3}c^2x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4)}{3c^3} \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2\sqrt{1-c^2x^2}}$$

↓ 5198

$$-\frac{5}{2}c^2d \left(d \left(-\frac{2bc\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx)) dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx)) \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2\sqrt{1-c^2x^2}}$$

↓ 2009

$$-\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2}(ax+bx\arcsin(cx))}{\sqrt{1-c^2x^2}} \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{2x^2\sqrt{1-c^2x^2}}$$

↓ 5218

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int \frac{(a+b\arcsin(cx))^2}{cx} d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc\sqrt{d-c^2dx^2}(ax+b)}{\sqrt{1-c^2x^2}} \right) \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4)}{3c^3} \right) \right) \\
 & \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2} \\
 & \frac{2x^2}{\phantom{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}} \\
 & \phantom{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}} \downarrow \text{3042}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2} \int (a+b\arcsin(cx))^2 \csc(\arcsin(cx)) d\arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2 - \frac{2bc}{\sqrt{1-c^2x^2}} \right) \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4)}{3c^3} \right) \right) \\
 & \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2} \\
 & \frac{2x^2}{\phantom{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}} \\
 & \phantom{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}} \downarrow \text{4671}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(-2b \int (a+b\arcsin(cx)) \log(1-e^i\arcsin(cx)) d\arcsin(cx) + 2b \int (a+b\arcsin(cx)) \log(1-e^{-i}\arcsin(cx)) d\arcsin(cx))}{\sqrt{1-c^2x^2}} \right) \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b\arcsin(cx)) - 2c^2x(a+b\arcsin(cx)) - \frac{a+b\arcsin(cx)}{x} + \frac{b(-3c^4\operatorname{arctanh}(\sqrt{1-c^2x^2})-\frac{1}{3}c^4)}{3c^3} \right) \right) \\
 & \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2} \\
 & \frac{2x^2}{\phantom{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}} \\
 & \phantom{\frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}} \downarrow \text{3011}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{d-c^2dx^2}} \right) \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b \arcsin(cx)) - 2c^2x(a+b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{b(-3c^4 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{1}{3}c^4)}{3c^3} \right) \right. \\
 & \left. \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}} \frac{(a+b \arcsin(cx))^2}{2x^2} \right) \\
 & \qquad \qquad \qquad \downarrow \text{2720}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{d-c^2dx^2}} \right) \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b \arcsin(cx)) - 2c^2x(a+b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{b(-3c^4 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{1}{3}c^4)}{3c^3} \right) \right. \\
 & \left. \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}} \frac{(a+b \arcsin(cx))^2}{2x^2} \right) \\
 & \qquad \qquad \qquad \downarrow \text{7143}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{5}{2}c^2d \left(d \left(\frac{\sqrt{d-c^2dx^2}(-2 \operatorname{arctanh}(e^{i \arcsin(cx)})(a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})(a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{d-c^2dx^2}} \right) \right. \\
 & \left. bcd^2\sqrt{d-c^2dx^2} \left(\frac{1}{3}c^4x^3(a+b \arcsin(cx)) - 2c^2x(a+b \arcsin(cx)) - \frac{a+b \arcsin(cx)}{x} + \frac{b(-3c^4 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{1}{3}c^4)}{3c^3} \right) \right. \\
 & \left. \frac{\sqrt{1-c^2x^2}}{(d-c^2dx^2)^{5/2}} \frac{(a+b \arcsin(cx))^2}{2x^2} \right)
 \end{aligned}$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^3,x]`

output

```
-1/2*((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^2 + (b*c*d^2*Sqrt[d -
c^2*d*x^2]*(-(a + b*ArcSin[c*x])/x) - 2*c^2*x*(a + b*ArcSin[c*x]) + (c^4
*x^3*(a + b*ArcSin[c*x]))/3 + (b*(-1/3*(c^4*x^6) - 5*c^4*Sqrt[1 - c^2*x^2]
- 3*c^4*ArcTanh[Sqrt[1 - c^2*x^2]]))/(3*c^3))/Sqrt[1 - c^2*x^2] - (5*c^2
*d*(((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/3 - (2*b*c*d*Sqrt[d - c^
2*d*x^2]*(-1/6*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/
(3*c^2))) + x*(a + b*ArcSin[c*x]) - (c^2*x^3*(a + b*ArcSin[c*x]))/3))/(3*S
qrt[1 - c^2*x^2]) + d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2 - (2*b*c*
Sqrt[d - c^2*d*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/Sqr
t[1 - c^2*x^2] + (Sqrt[d - c^2*d*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^
(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x]
)] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLo
g[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/Sqrt[1 - c^2*
x^2])))/2
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
:= Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1192

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[2/e^(n + 2*p + 1) Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```


rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5154

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5192

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_)*((f_.)*(x_))^(m_)*Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_)*((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5218

```
Int((((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 1326, normalized size of antiderivative = 1.82

method	result	size
default	Expression too large to display	1326
parts	Expression too large to display	1326

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output

```

a^2*(-1/2/d/x^2*(-c^2*d*x^2+d)^(7/2)-5/2*c^2*(1/5*(-c^2*d*x^2+d)^(5/2)+d*(
1/3*(-c^2*d*x^2+d)^(3/2)+d*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)
*(-c^2*d*x^2+d)^(1/2))/x)))))+b^2*(1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4
-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*(6
*I*arcsin(c*x)+9*arcsin(c*x)^2-2)*c^2*d^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))
^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*
x))*c^2*d^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c
*x+c^2*x^2-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))*c^2*d^2/(c^2*x^2-1)+1/216*
(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2
*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*(-6*I*arcsin(c*x)+9*arcsin(c*x)^2-2)*c^2*d^
2/(c^2*x^2-1)-1/2*d^2*(c^2*x^2*arcsin(c*x)-2*c*x*(-c^2*x^2+1)^(1/2)-arcsin
(c*x))*arcsin(c*x)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2-1/2*(-d*(c^2*x^2
-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(5*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/
2))-5*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-10*I*arcsin(c*x)*polylo
g(2,-I*c*x-(-c^2*x^2+1)^(1/2))+10*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+
1)^(1/2))-4*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))+10*polylog(3,-I*c*x-(-c^2*x^
2+1)^(1/2))-10*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2)))*c^2*d^2/(c^2*x^2-1))+2
*a*b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1
/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*(I+3*arcsin(c*x))*c^2*d^2/(c^2*x
^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(...

```

Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x^3} dx$$

input

```

integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="fricas")

```

output

```

integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)

```

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(-d(cx - 1)(cx + 1))^{5/2} (a + b \arcsin(cx))^2}{x^3} dx$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2/x**3,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))**2/x**3, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x^3} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="maxima")`

output `1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^3, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^3,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2}}{x^3} dx$$

input `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3,x)`

output `int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3, x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^3} dx = \frac{\sqrt{d} d^2 (8\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 - 56\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 12\sqrt{-c^2 x^2 + 1} a^2)}{x^3}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2/x^3,x)`

output

```
(sqrt(d)*d**2*(8*sqrt(-c**2*x**2+1)*a**2*c**4*x**4-56*sqrt(-c**2*x**2+1)*a**2*c**2*x**2-12*sqrt(-c**2*x**2+1)*a**2+48*int((sqrt(-c**2*x**2+1)*asin(c*x))/x**3,x)*a*b*x**2-96*int((sqrt(-c**2*x**2+1)*asin(c*x))/x,x)*a*b*c**2*x**2+24*int((sqrt(-c**2*x**2+1)*asin(c*x)**2)/x**3,x)*b**2*x**2-48*int((sqrt(-c**2*x**2+1)*asin(c*x)**2)/x,x)*b**2*c**2*x**2+48*int(sqrt(-c**2*x**2+1)*asin(c*x)*x,x)*a*b*c**4*x**2+24*int(sqrt(-c**2*x**2+1)*asin(c*x)**2*x,x)*b**2*c**4*x**2-60*log(tan(asin(c*x)/2))*a**2*c**2*x**2+65*a**2*c**2*x**2))/(24*x**2)
```

3.230
$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx$$

Optimal result	2266
Mathematica [A] (verified)	2267
Rubi [F]	2268
Maple [B] (verified)	2275
Fricas [F]	2276
Sympy [F]	2276
Maxima [F]	2276
Giac [F(-2)]	2277
Mupad [F(-1)]	2277
Reduce [F]	2278

Optimal result

Integrand size = 29, antiderivative size = 579

$$\begin{aligned} \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = & -\frac{7}{12} b^2 c^4 d^2 x \sqrt{d - c^2 dx^2} - \frac{b^2 c^2 d (d - c^2 dx^2)^{3/2}}{3x} \\ & + \frac{23b^2 c^3 d^2 \sqrt{d - c^2 dx^2} \arcsin(cx)}{12\sqrt{1 - c^2 x^2}} - \frac{5bc^5 d^2 x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{2\sqrt{1 - c^2 x^2}} \\ & - \frac{7}{3} bc^3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx)) \\ & - \frac{bcd^2 (1 - c^2 x^2)^{3/2} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))}{3x^2} \\ & + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 + \frac{7ic^3 d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{3\sqrt{1 - c^2 x^2}} + \frac{5c^2 d (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))}{3x} \end{aligned}$$

output

$$\begin{aligned}
& -7/12*b^2*c^4*d^2*x*(-c^2*d*x^2+d)^{(1/2)}-1/3*b^2*c^2*d*(-c^2*d*x^2+d)^{(3/2)} \\
&)/x+23/12*b^2*c^3*d^2*(-c^2*d*x^2+d)^{(1/2)}*\arcsin(c*x)/(-c^2*x^2+1)^{(1/2)}- \\
& 5/2*b*c^5*d^2*x^2*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/(-c^2*x^2+1)^{(1/2)} \\
&)-7/3*b*c^3*d^2*(-c^2*x^2+1)^{(1/2)}*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))- \\
& 1/3*b*c*d^2*(-c^2*x^2+1)^{(3/2)}*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))/x^2+ \\
& 5/2*c^4*d^2*x*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))^2+7/3*I*c^3*d^2*(-c^2 \\
& *d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))^2/(-c^2*x^2+1)^{(1/2)}+5/3*c^2*d*(-c^2*d*x \\
& ^2+d)^{(3/2)}*(a+b*\arcsin(c*x))^2/x-1/3*(-c^2*d*x^2+d)^{(5/2)}*(a+b*\arcsin(c*x \\
&))^2/x^3+5/6*c^3*d^2*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))^3/b/(-c^2*x^2+ \\
& 1)^{(1/2)}-14/3*b*c^3*d^2*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin(c*x))*\ln(1-(I*c*x \\
& +(-c^2*x^2+1)^{(1/2}))^2)/(-c^2*x^2+1)^{(1/2)}+7/3*I*b^2*c^3*d^2*(-c^2*d*x^2+d \\
&)^{(1/2)}*\operatorname{polylog}(2,(I*c*x+(-c^2*x^2+1)^{(1/2}))^2)/(-c^2*x^2+1)^{(1/2)}
\end{aligned}$$
Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.19

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \frac{d^2 \left(-4abcx\sqrt{d - c^2 dx^2} + 3abc^3 x^3 \sqrt{d - c^2 dx^2} - 6abc^5 x^5 \sqrt{d - c^2 dx^2} \right)}{x^4}$$

input

`Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^4,x]`

output

$$\begin{aligned}
& (d^2*(-4*a*b*c*x*\operatorname{Sqrt}[d - c^2*d*x^2] + 3*a*b*c^3*x^3*\operatorname{Sqrt}[d - c^2*d*x^2] - \\
& 6*a*b*c^5*x^5*\operatorname{Sqrt}[d - c^2*d*x^2] - 4*a^2*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d - c^2*d \\
& *x^2] + 28*a^2*c^2*x^2*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d - c^2*d*x^2] - 4*b^2*c^2* \\
& x^2*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d - c^2*d*x^2] + 6*a^2*c^4*x^4*\operatorname{Sqrt}[1 - c^2*x^2 \\
&]*\operatorname{Sqrt}[d - c^2*d*x^2] - 3*b^2*c^4*x^4*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d - c^2*d*x^2 \\
&] + 10*b^2*c^3*x^3*\operatorname{Sqrt}[d - c^2*d*x^2]*\operatorname{ArcSin}[c*x]^3 - 30*a^2*c^3*\operatorname{Sqrt}[d] * \\
& x^3*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{ArcTan}[(c*x*\operatorname{Sqrt}[d - c^2*d*x^2])/(\operatorname{Sqrt}[d]*(-1 + c^2* \\
& x^2))] - 56*a*b*c^3*x^3*\operatorname{Sqrt}[d - c^2*d*x^2]*\operatorname{Log}[c*x] + (28*I)*b^2*c^3*x^3* \\
& \operatorname{Sqrt}[d - c^2*d*x^2]*\operatorname{PolyLog}[2, E^{((2*I)*\operatorname{ArcSin}[c*x])}] + b*\operatorname{Sqrt}[d - c^2*d*x \\
& ^2]*\operatorname{ArcSin}[c*x]*(-4*b*c*x - 6*a*\operatorname{Sqrt}[1 - c^2*x^2] + 48*a*c^2*x^2*\operatorname{Sqrt}[1 - \\
& c^2*x^2] + 3*b*c^3*x^3*\operatorname{Cos}[2*\operatorname{ArcSin}[c*x]] - 2*a*\operatorname{Cos}[3*\operatorname{ArcSin}[c*x]] - 56*b* \\
& c^3*x^3*\operatorname{Log}[1 - E^{((2*I)*\operatorname{ArcSin}[c*x])}] + 6*a*c^3*x^3*\operatorname{Sin}[2*\operatorname{ArcSin}[c*x]]) + \\
& b*\operatorname{Sqrt}[d - c^2*d*x^2]*\operatorname{ArcSin}[c*x]^2*(30*a*c^3*x^3 + 4*b*((7*I)*c^3*x^3 - \\
& \operatorname{Sqrt}[1 - c^2*x^2] + 7*c^2*x^2*\operatorname{Sqrt}[1 - c^2*x^2]) + 3*b*c^3*x^3*\operatorname{Sin}[2*\operatorname{ArcSi} \\
& n[c*x]])))/(12*x^3*\operatorname{Sqrt}[1 - c^2*x^2])
\end{aligned}$$

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx$$

$$\downarrow \text{5200}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \int \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{x^3} dx}{3\sqrt{1 - c^2 x^2}} - \frac{5}{3} c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{5190}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \left(-2c^2 \int \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{x} dx + \frac{1}{2} bc \int \frac{(1 - c^2 x^2)^{3/2}}{x^2} dx - \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2} \right)}{3\sqrt{1 - c^2 x^2}} - \frac{5}{3} c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{247}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \left(-2c^2 \int \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{x} dx + \frac{1}{2} bc \left(-3c^2 \int \sqrt{1 - c^2 x^2} dx - \frac{(1 - c^2 x^2)^{3/2}}{x} \right) - \frac{(1 - c^2 x^2)^2 (a + b \arcsin(cx))}{2x^2} \right)}{3\sqrt{1 - c^2 x^2}} - \frac{5}{3} c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{211}$$

$$\frac{2bcd^2 \sqrt{d - c^2 dx^2} \left(-2c^2 \int \frac{(1 - c^2 x^2)(a + b \arcsin(cx))}{x} dx + \frac{1}{2} bc \left(-3c^2 \left(\frac{1}{2} \int \frac{1}{\sqrt{1 - c^2 x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2 x^2} \right) - \frac{(1 - c^2 x^2)^{3/2}}{x} \right) \right)}{3\sqrt{1 - c^2 x^2}} - \frac{5}{3} c^2 d \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{x^2} dx - \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{3x^3}$$

$$\downarrow \text{223}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx-\frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{2x^2}+\frac{1}{2}bc\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)\right)}{\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{3\sqrt{1-c^2x^2}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}}$$

↓ 5188

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)-\frac{(1-c^2x^2)^2(a+b\arcsin(cx))}{2x}\right)}{\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{3\sqrt{1-c^2x^2}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}}$$

↓ 211

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)}{\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{3\sqrt{1-c^2x^2}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}}$$

↓ 223

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\arcsin(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)\right)}{\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{3\sqrt{1-c^2x^2}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}}$$

↓ 5136

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)\right)}{\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{3\sqrt{1-c^2x^2}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}}$$

↓ 3042

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int -((a+b\arcsin(cx))\tan(\arcsin(cx)+\frac{\pi}{2}))d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)\right)$$

$$\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 25

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-\int(a+b\arcsin(cx))\tan(\arcsin(cx)+\frac{\pi}{2})d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)\right)$$

$$\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 4200

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(2i\int-\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{i(a+b\arcsin(cx))}{2b}\right)\right)$$

$$\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 25

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-2i\int\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{i(a+b\arcsin(cx))}{2b}\right)\right)$$

$$\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 2620

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})(a+b\arcsin(cx))-\frac{1}{2}ib\int\log(1-e^{2i\arcsin(cx)})d\arcsin(cx)\right)\right)\right)$$

$$\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 2715

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))-\frac{1}{4}b\int e^{-2i\arcsin(cx)}\log(1-e^{2i\arcsin(cx)})\right)\right)$$

$$\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 2838

$$-\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x^2}dx+$$

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5200

$$-\frac{5}{3}c^2d\left(-3c^2d\int\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}}{3x^3}\right)$$

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5156

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arcsin(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))\right)\right)$$

$$2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5138

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}\right)\right. \\ \left.2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 262

$$-\frac{5}{3}c^2d\left(-3c^2d\left(-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))-\frac{1}{2}bc\left(\int\frac{1}{\sqrt{1-c^2x^2}}dx-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}\right)\right. \\ \left.2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 223

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx}{\sqrt{1-c^2x^2}}-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))\right)\right. \\ \left.2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5152

$$-\frac{5}{3}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arcsin(cx))^2}{x}-3c^2d\left(\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))}{6bc\sqrt{1-c^2x^2}}\right)\right. \\ \left.2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{Pol}\right)\right)\right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5188

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(\int \frac{a+b\arcsin(cx)}{x} dx - \frac{1}{2}bc \int \sqrt{1-c^2x^2} dx + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)})(a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 211

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(\int \frac{a+b\arcsin(cx)}{x} dx - \frac{1}{2}bc \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)})(a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 223

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(\int \frac{a+b\arcsin(cx)}{x} dx + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)})(a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 5136

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx} d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}} \right) - \frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)})(a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{3x^3}$$

↓ 3042

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(\int -((a+b\arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2})) d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right)$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2}{3x^3}$$

↓ 25

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(-\int (a+b\arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}} \right)$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2}{3x^3}$$

↓ 4200

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(2i \int -\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - \frac{i(a+b\arcsin(cx))}{2b} \right)}{\sqrt{1-c^2x^2}} \right)$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2}{3x^3}$$

↓ 25

$$-\frac{5}{3}c^2d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(-2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) + \frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - \frac{i(a+b\arcsin(cx))}{2b} \right)}{\sqrt{1-c^2x^2}} \right)$$

$$2bcd^2\sqrt{d-c^2dx^2} \left(-2c^2 \left(\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) \right) (a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \right) \right)$$

$$\frac{(d-c^2dx^2)^{5/2} (a+b\arcsin(cx))^2}{3x^3}$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/x^4,x]`

output `$Aborted`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2649 vs. $2(529) = 1058$.

Time = 0.92 (sec) , antiderivative size = 2650, normalized size of antiderivative = 4.58

method	result	size
default	Expression too large to display	2650
parts	Expression too large to display	2650

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^4,x,method=_RETURNVERBOSE)`

output

```

-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c^2*x^2-1)*x^3+1/4*b^2*(-d*(c^2*x
^2-1))^(1/2)*d^2*c^4/(c^2*x^2-1)*x-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*
a^2*c^4*x*(-c^2*d*x^2+d)^(5/2)+5*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*
x^4-15*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^5-7/3*I*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c^2*x^2-1)*arcsin(c*x)*c^
4-49/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c^2
*x^2-1)*arcsin(c*x)*c^8+4/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(7/2)-5*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c^2*x^2-1)*arcsin(c*x)*(-c^
2*x^2+1)^(1/2)*c^3+14*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*d^2*c^
3/(3*c^2*x^2-3)*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-14*I*b^2*(-c^2*
x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(3*c^2*x^2-3)*arcsin(c*x)^2-14
*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(3*c^2*x^2-3)*pol
ylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-14*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2
-1))^(1/2)*d^2*c^3/(3*c^2*x^2-3)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-1/3*I
*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c^2*x^2-1)*(-c^
2*x^2+1)^(1/2)*c^3+14*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*d^2*c^
3/(3*c^2*x^2-3)*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+147*b^2*(-d*(c^
2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c^2*x^2-1)*arcsin(c*x)^
2*c^8-203*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c^
2*x^2-1)*arcsin(c*x)^2*c^6+190/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4...

```


Fricas [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="fricas")`

output `integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(-d(cx - 1)(cx + 1))^{5/2} (a + b \arcsin(cx))^2}{x^4} dx$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2/x**4,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))**2/x**4, x)`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2}{x^4} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="maxima")`

output

```
1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
+ 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d
*x^2 + d)^(7/2)/(d*x^3))*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2
*c^2*d^2*x^2 + b^2*d^2)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*(
a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(c*x, sqrt(c*x + 1))*
sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \text{Exception raised: TypeError}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2/x^4,x, algorithm="giac"
)
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \int \frac{(a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2}}{x^4} dx$$

input

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

output

```
int(((a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{x^4} dx = \frac{\sqrt{d} d^2 (4 \arcsin(cx)^3 b^2 c^3 x^3 + 12 \arcsin(cx)^2 ab c^3 x^3 + 15 \arcsin(cx) a^2 c^3 x^3 + 3 \sqrt{-c^2 x^2 + 1} a^2 c^3 x^3 + 14 \sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 2 \sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 24 \int(\arcsin(cx)/(\sqrt{-c^2 x^2 + 1}) x^2, x) a b c^2 x^3 - 12 \int(\arcsin(cx)^2/(\sqrt{-c^2 x^2 + 1}) x^2, x) b^2 c^2 x^3 + 12 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx))/x^4, x) a b x^3 + 6 \int((\sqrt{-c^2 x^2 + 1} \arcsin(cx))^2/x^4, x) b^2 x^3 + 12 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx), x) a b c^4 x^3 + 6 \int(\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2, x) b^2 c^4 x^3)}{6 x^3}$$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2/x^4,x)
```

output

```
(sqrt(d)*d**2*(4*asin(c*x)**3*b**2*c**3*x**3 + 12*asin(c*x)**2*a*b*c**3*x**3 + 15*asin(c*x)*a**2*c**3*x**3 + 3*sqrt(-c**2*x**2 + 1)*a**2*c**4*x**4 + 14*sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)*a**2 - 24*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x**2),x)*a*b*c**2*x**3 - 12*int(asin(c*x)**2/(sqrt(-c**2*x**2 + 1)*x**2),x)*b**2*c**2*x**3 + 12*int((sqrt(-c**2*x**2 + 1)*asin(c*x))/x**4,x)*a*b*x**3 + 6*int((sqrt(-c**2*x**2 + 1)*asin(c*x)**2)/x**4,x)*b**2*x**3 + 12*int(sqrt(-c**2*x**2 + 1)*asin(c*x),x)*a*b*c**4*x**3 + 6*int(sqrt(-c**2*x**2 + 1)*asin(c*x)**2,x)*b**2*c**4*x**3))/(6*x**3)
```

3.231 $\int \frac{x^5(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$

Optimal result	2279
Mathematica [A] (verified)	2280
Rubi [A] (verified)	2280
Maple [A] (verified)	2285
Fricas [A] (verification not implemented)	2286
Sympy [F]	2286
Maxima [A] (verification not implemented)	2287
Giac [F(-2)]	2288
Mupad [F(-1)]	2288
Reduce [F]	2288

Optimal result

Integrand size = 29, antiderivative size = 338

$$\int \frac{x^5(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx = \frac{298b^2\sqrt{d-c^2dx^2}}{225c^6d} - \frac{76b^2(d-c^2dx^2)^{3/2}}{675c^6d^2} + \frac{2b^2(d-c^2dx^2)^{5/2}}{125c^6d^3} + \frac{16bx\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{15c^5\sqrt{d-c^2dx^2}} + \frac{8bx^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{45c^3\sqrt{d-c^2dx^2}} + \frac{2bx^5\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{25c\sqrt{d-c^2dx^2}} - \frac{8\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{15c^6d} - \frac{4x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{15c^4d} - \frac{x^4\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{5c^2d}$$

output

$$\frac{298}{225}b^2(-c^2dx^2+d)^{1/2}/c^6/d-76/675b^2(-c^2dx^2+d)^{3/2}/c^6/d^2+2/125b^2(-c^2dx^2+d)^{5/2}/c^6/d^3+16/15b^2(-c^2x^2+1)^{1/2}(a+b\arcsin(cx))/c^5/(-c^2dx^2+d)^{1/2}+8/45b^2(-c^2x^2+1)^{1/2}(a+b\arcsin(cx))/c^3/(-c^2dx^2+d)^{1/2}+2/25b^2(-c^2x^2+1)^{1/2}(a+b\arcsin(cx))/c/(-c^2dx^2+d)^{1/2}-8/15(-c^2dx^2+d)^{1/2}(a+b\arcsin(cx))^2/c^4/d-1/5x^4(-c^2dx^2+d)^{1/2}(a+b\arcsin(cx))^2/c^2/d$$
Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.68

$$\int \frac{x^5(a+b\arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx$$

$$= \frac{30abcx\sqrt{1-c^2x^2}(120+20c^2x^2+9c^4x^4)+225a^2(-8+4c^2x^2+c^4x^4+3c^6x^6)-2b^2(-2072+1936c^2x^2+109c^4x^4+27c^6x^6)+30b(bcx\sqrt{1-c^2x^2}(120+20c^2x^2+9c^4x^4)+15a(-8+4c^2x^2+c^4x^4+3c^6x^6))\arcsin(cx)+225b^2(-8+4c^2x^2+c^4x^4+3c^6x^6)\arcsin(cx)^2}{(3375c^6\sqrt{d-c^2dx^2})}$$

input

`Integrate[(x^5*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2], x]`

output

$$(30*a*b*c*x*\sqrt{1-c^2*x^2}*(120+20*c^2*x^2+9*c^4*x^4)+225*a^2*(-8+4*c^2*x^2+c^4*x^4+3*c^6*x^6)-2*b^2*(-2072+1936*c^2*x^2+109*c^4*x^4+27*c^6*x^6)+30*b*(b*c*x*\sqrt{1-c^2*x^2}*(120+20*c^2*x^2+9*c^4*x^4)+15*a*(-8+4*c^2*x^2+c^4*x^4+3*c^6*x^6))*\text{ArcSin}[c*x]+225*b^2*(-8+4*c^2*x^2+c^4*x^4+3*c^6*x^6)*\text{ArcSin}[c*x]^2)/(3375*c^6*\sqrt{d-c^2*d*x^2})$$
Rubi [A] (verified)Time = 1.79 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5210, 5138, 243, 53, 2009, 5210, 5138, 243, 53, 2009, 5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^5(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx \\
& \quad \downarrow \text{5210} \\
& \frac{2b\sqrt{1-c^2x^2} \int x^4(a+b\arcsin(cx)) dx}{5c\sqrt{d-c^2dx^2}} + \frac{4 \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{5c^2} - \\
& \quad \frac{x^4\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{5c^2d} \\
& \quad \downarrow \text{5138} \\
& \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{5}bc \int \frac{x^5}{\sqrt{1-c^2x^2}} dx \right)}{5c\sqrt{d-c^2dx^2}} + \frac{4 \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{5c^2} - \\
& \quad \frac{x^4\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{5c^2d} \\
& \quad \downarrow \text{243} \\
& \frac{4 \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{5c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{10}bc \int \frac{x^4}{\sqrt{1-c^2x^2}} dx^2 \right)}{5c\sqrt{d-c^2dx^2}} - \\
& \quad \frac{x^4\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{5c^2d} \\
& \quad \downarrow \text{53} \\
& \frac{4 \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{5c^2} + \\
& \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{10}bc \int \left(\frac{(1-c^2x^2)^{3/2}}{c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{1}{c^4\sqrt{1-c^2x^2}} \right) dx^2 \right)}{5c\sqrt{d-c^2dx^2}} - \\
& \quad \frac{x^4\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{5c^2d} \\
& \quad \downarrow \text{2009} \\
& \frac{4 \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{5c^2} - \frac{x^4\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{5c^2d} + \\
& \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b\arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5c\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{5210}
\end{aligned}$$

$$\begin{aligned}
& 4 \left(\frac{2b\sqrt{1-c^2x^2} \int x^2(a+b \arcsin(cx))dx}{3c\sqrt{d-c^2dx^2}} + \frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{x^4\sqrt{d-c^2dx^2} \frac{5c^2}{(a+b \arcsin(cx))^2} + 5c^2d}{5c\sqrt{d-c^2dx^2}} + \\
& 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right) \\
& \downarrow \mathbf{5138} \\
& 4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{3}bc \int \frac{x^3}{\sqrt{1-c^2x^2}} dx \right)}{3c\sqrt{d-c^2dx^2}} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{x^4\sqrt{d-c^2dx^2} \frac{5c^2}{(a+b \arcsin(cx))^2} + 5c^2d}{5c\sqrt{d-c^2dx^2}} + \\
& 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right) \\
& \downarrow \mathbf{243} \\
& 4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx^2 \right)}{3c\sqrt{d-c^2dx^2}} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{x^4\sqrt{d-c^2dx^2} \frac{5c^2}{(a+b \arcsin(cx))^2} + 5c^2d}{5c\sqrt{d-c^2dx^2}} + \\
& 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right) \\
& \downarrow \mathbf{53} \\
& 4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \left(\frac{1}{c^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{c^2} \right) dx^2 \right)}{3c\sqrt{d-c^2dx^2}} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{x^4\sqrt{d-c^2dx^2} \frac{5c^2}{(a+b \arcsin(cx))^2} + 5c^2d}{5c\sqrt{d-c^2dx^2}} + \\
& 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)
\end{aligned}$$

↓ 2009

$$4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} - \frac{x^2 \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) \right)}{3c\sqrt{d-c^2dx^2}} \right)$$

$$\frac{\frac{x^4 \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{5c^2d} + 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5c\sqrt{d-c^2dx^2}}$$

↓ 5182

$$4 \left(\frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} - \frac{x^2 \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) \right)}{3c\sqrt{d-c^2dx^2}} \right)$$

$$\frac{\frac{x^4 \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{5c^2d} + 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5c\sqrt{d-c^2dx^2}}$$

↓ 2009

$$\frac{-\frac{x^4 \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{5c^2d} + 2b\sqrt{1-c^2x^2} \left(\frac{1}{5}x^5(a+b \arcsin(cx)) - \frac{1}{10}bc \left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6} + \frac{4(1-c^2x^2)^{3/2}}{3c^6} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right) \right)}{5c\sqrt{d-c^2dx^2}} + 4 \left(-\frac{x^2 \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) \right)}{3c\sqrt{d-c^2dx^2}} \right)$$

5c²

input `Int[(x^5*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]`

output

```
-1/5*(x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*Sqrt[1 - c^2*x^2]*(-1/10*(b*c*((-2*Sqrt[1 - c^2*x^2])/c^6 + (4*(1 - c^2*x^2)^(3/2))/(3*c^6) - (2*(1 - c^2*x^2)^(5/2))/(5*c^6))) + (x^5*(a + b*ArcSin[c*x])/5)/(5*c*Sqrt[d - c^2*d*x^2]) + (4*(-1/3*(x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*Sqrt[1 - c^2*x^2]*(-1/6*(b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))) + (x^3*(a + b*ArcSin[c*x])/3))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*Sqrt[1 - c^2*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(3*c^2)))/(5*c^2)
```

Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5138

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5210

```

Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]

```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.50

method	result
orering	$\frac{(1647c^8x^8+1684c^6x^6+34306c^4x^4-102032c^2x^2+62160)(a+b\arcsin(cx))^2}{3375c^8x^2\sqrt{-c^2dx^2+d}} - \frac{2(cx-1)(cx+1)(162c^6x^6+491c^4x^4+7472c^2x^2-10360)}{x^6/c^8(5x^4(a+b\arcsin(cx))^2/(-c^2dx^2+d)^{1/2}+2x^5(a+b\arcsin(cx))/(-c^2dx^2+d)^{1/2})+1/3375(27c^4x^4+136c^2x^2+2072)/c^8(cx-1)^2/x^5(cx+1)^2(20x^3(a+b\arcsin(cx))^2/(-c^2dx^2+d)^{1/2}+20x^4(a+b\arcsin(cx))/(-c^2dx^2+d)^{1/2})+11x^5(a+b\arcsin(cx))^2/(-c^2dx^2+d)^{3/2})+4x^6(a+b\arcsin(cx))/(-c^2dx^2+d)^{3/2})+3x^7(a+b\arcsin(cx))^2/(-c^2dx^2+d)^{5/2})+c^4d^2}$
default	Expression too large to display
parts	Expression too large to display

input

```
int(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```

1/3375*(1647*c^8*x^8+1684*c^6*x^6+34306*c^4*x^4-102032*c^2*x^2+62160)/c^8/
x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2)-2/3375*(c*x-1)*(c*x+1)*(162*c
^6*x^6+491*c^4*x^4+7472*c^2*x^2-10360)/x^6/c^8*(5*x^4*(a+b*arcsin(c*x))^2/
(-c^2*d*x^2+d)^(1/2)+2*x^5*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c/(-c^
2*x^2+1)^(1/2)+x^6*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2)*c^2*d)+1/3375*
(27*c^4*x^4+136*c^2*x^2+2072)/c^8*(c*x-1)^2/x^5*(c*x+1)^2*(20*x^3*(a+b*arc
sin(c*x))^2/(-c^2*d*x^2+d)^(1/2)+20*x^4*(a+b*arcsin(c*x))/(-c^2*d*x^2+d)^(
1/2)*b*c/(-c^2*x^2+1)^(1/2)+11*x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2
)*c^2*d+2*x^5*b^2*c^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+4*x^6*(a+b*arcsin(
c*x))/(-c^2*d*x^2+d)^(3/2)*b*c^3/(-c^2*x^2+1)^(1/2)*d+2*x^6*(a+b*arcsin(c*
x))/(-c^2*d*x^2+d)^(1/2)*b*c^3/(-c^2*x^2+1)^(3/2)+3*x^7*(a+b*arcsin(c*x))^
2/(-c^2*d*x^2+d)^(5/2)*c^4*d^2)

```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.82

$$\int \frac{x^5(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \frac{30(9abc^5x^5 + 20abc^3x^3 + 120abcx + (9b^2c^5x^5 + 20b^2c^3x^3 + 120b^2cx) \arcsin(cx))\sqrt{-c^2dx^2 + d}\sqrt{-d}}{\dots}$$

input

```
integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output

```
-1/3375*(30*(9*a*b*c^5*x^5 + 20*a*b*c^3*x^3 + 120*a*b*c*x + (9*b^2*c^5*x^5 + 20*b^2*c^3*x^3 + 120*b^2*c*x)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + (27*(25*a^2 - 2*b^2)*c^6*x^6 + (225*a^2 - 218*b^2)*c^4*x^4 + 4*(225*a^2 - 968*b^2)*c^2*x^2 + 225*(3*b^2*c^6*x^6 + b^2*c^4*x^4 + 4*b^2*c^2*x^2 - 8*b^2)*arcsin(c*x)^2 - 1800*a^2 + 4144*b^2 + 450*(3*a*b*c^6*x^6 + a*b*c^4*x^4 + 4*a*b*c^2*x^2 - 8*a*b)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))^2}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input

```
integrate(x**5*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output

```
Integral(x**5*(a + b*asin(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.08

$$\begin{aligned}
& \int \frac{x^5(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx \\
&= -\frac{1}{15} \left(\frac{3\sqrt{-c^2 dx^2 + dx^4}}{c^2 d} + \frac{4\sqrt{-c^2 dx^2 + dx^2}}{c^4 d} + \frac{8\sqrt{-c^2 dx^2 + d}}{c^6 d} \right) b^2 \arcsin(cx)^2 \\
&\quad - \frac{2}{15} \left(\frac{3\sqrt{-c^2 dx^2 + dx^4}}{c^2 d} + \frac{4\sqrt{-c^2 dx^2 + dx^2}}{c^4 d} + \frac{8\sqrt{-c^2 dx^2 + d}}{c^6 d} \right) ab \arcsin(cx) \\
&\quad - \frac{1}{15} \left(\frac{3\sqrt{-c^2 dx^2 + dx^4}}{c^2 d} + \frac{4\sqrt{-c^2 dx^2 + dx^2}}{c^4 d} + \frac{8\sqrt{-c^2 dx^2 + d}}{c^6 d} \right) a^2 \\
&\quad + \frac{2}{3375} b^2 \left(\frac{27\sqrt{-c^2 x^2 + 1} c^2 x^4 + 136\sqrt{-c^2 x^2 + 1} x^2 + \frac{2072\sqrt{-c^2 x^2 + 1}}{c^2}}{c^4 \sqrt{d}} + \frac{15(9c^4 x^5 + 20c^2 x^3 + 120x) \arcsin(cx)}{c^5 \sqrt{d}} \right) \\
&\quad + \frac{2(9c^4 x^5 + 20c^2 x^3 + 120x) ab}{225 c^5 \sqrt{d}}
\end{aligned}$$

input `integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b^2*arcsin(c*x)^2 - 2/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*a*b*arcsin(c*x) - 1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*a^2 + 2/3375*b^2*((27*sqrt(-c^2*x^2 + 1)*c^2*x^4 + 136*sqrt(-c^2*x^2 + 1)*x^2 + 2072*sqrt(-c^2*x^2 + 1)/c^2)/(c^4*sqrt(d)) + 15*(9*c^4*x^5 + 20*c^2*x^3 + 120*x)*arcsin(c*x)/(c^5*sqrt(d))) + 2/225*(9*c^4*x^5 + 20*c^2*x^3 + 120*x)*a*b/(c^5*sqrt(d))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^5*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^5*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{-3\sqrt{-c^2 x^2 + 1} a^2 c^4 x^4 - 4\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - 8\sqrt{-c^2 x^2 + 1} a^2 + 30 \left(\int \frac{\operatorname{asin}(cx) x^5}{\sqrt{-c^2 x^2 + 1}} dx \right) a b c^6 + 15 \left(\int \frac{\operatorname{asin}(cx) x^5}{\sqrt{-c^2 x^2 + 1}} dx \right) a b c^6}{15\sqrt{d} c^6}$$

input `int(x^5*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output

```
( - 3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a**  
2*c**2*x**2 - 8*sqrt( - c**2*x**2 + 1)*a**2 + 30*int((asin(c*x)*x**5)/sqrt  
( - c**2*x**2 + 1),x)*a*b*c**6 + 15*int((asin(c*x)**2*x**5)/sqrt( - c**2*x  
**2 + 1),x)*b**2*c**6)/(15*sqrt(d)*c**6)
```

3.232 $\int \frac{x^4(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$

Optimal result	2290
Mathematica [A] (verified)	2291
Rubi [A] (verified)	2291
Maple [B] (verified)	2296
Fricas [F]	2297
Sympy [F]	2297
Maxima [F]	2297
Giac [A] (verification not implemented)	2298
Mupad [F(-1)]	2298
Reduce [F]	2299

Optimal result

Integrand size = 29, antiderivative size = 323

$$\int \frac{x^4(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx = \frac{15b^2x\sqrt{d-c^2dx^2}}{64c^4d} + \frac{b^2x^3\sqrt{d-c^2dx^2}}{32c^2d} - \frac{15b^2\sqrt{1-c^2x^2} \arcsin(cx)}{64c^5\sqrt{d-c^2dx^2}} + \frac{3bx^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{8c^3\sqrt{d-c^2dx^2}} + \frac{bx^4\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{8c\sqrt{d-c^2dx^2}} - \frac{3x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{8c^4d} - \frac{x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{4c^2d} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{8bc^5\sqrt{d-c^2dx^2}}$$

output

$$\frac{15/64*b^2*x*(-c^2*d*x^2+d)^{(1/2)}/c^4/d+1/32*b^2*x^3*(-c^2*d*x^2+d)^{(1/2)}/c^2/d-15/64*b^2*(-c^2*x^2+1)^{(1/2)}*arcsin(c*x)/c^5/(-c^2*d*x^2+d)^{(1/2)}+3/8*b*x^2*(-c^2*x^2+1)^{(1/2)}*(a+b*arcsin(c*x))/c^3/(-c^2*d*x^2+d)^{(1/2)}+1/8*b*x^4*(-c^2*x^2+1)^{(1/2)}*(a+b*arcsin(c*x))/c/(-c^2*d*x^2+d)^{(1/2)}-3/8*x*(-c^2*d*x^2+d)^{(1/2)}*(a+b*arcsin(c*x))^2/c^4/d-1/4*x^3*(-c^2*d*x^2+d)^{(1/2)}*(a+b*arcsin(c*x))^2/c^2/d+1/8*(-c^2*x^2+1)^{(1/2)}*(a+b*arcsin(c*x))^3/b/c^5/(-c^2*d*x^2+d)^{(1/2)}}{1}$$
Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.88

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{32a^2c\sqrt{d}x(-1 + c^2x^2)(3 + 2c^2x^2) - 96a^2\sqrt{d - c^2dx^2} \arctan\left(\frac{cx\sqrt{d - c^2dx^2}}{\sqrt{d(-1 + c^2x^2)}}\right) + b^2\sqrt{d}\sqrt{1 - c^2x^2}(32 \arcsin(cx))^2}{1}$$

input

`Integrate[(x^4*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2], x]`

output

$$\frac{(32*a^2*c*\text{Sqrt}[d]*x*(-1 + c^2*x^2)*(3 + 2*c^2*x^2) - 96*a^2*\text{Sqrt}[d - c^2*d*x^2]*\text{ArcTan}[(c*x*\text{Sqrt}[d - c^2*d*x^2])/(\text{Sqrt}[d]*(-1 + c^2*x^2))] + b^2*\text{Sqrt}[d]*\text{Sqrt}[1 - c^2*x^2]*(32*\text{ArcSin}[c*x]^3 + 4*\text{ArcSin}[c*x]*(-16*\text{Cos}[2*\text{ArcSin}[c*x]] + \text{Cos}[4*\text{ArcSin}[c*x]]) + 32*\text{Sin}[2*\text{ArcSin}[c*x]] - \text{Sin}[4*\text{ArcSin}[c*x]] + 8*\text{ArcSin}[c*x]^2*(-8*\text{Sin}[2*\text{ArcSin}[c*x]] + \text{Sin}[4*\text{ArcSin}[c*x]])) - 4*a*b*\text{Sqrt}[d]*\text{Sqrt}[1 - c^2*x^2]*(16*\text{Cos}[2*\text{ArcSin}[c*x]] - \text{Cos}[4*\text{ArcSin}[c*x]] - 4*\text{ArcSin}[c*x]*(6*\text{ArcSin}[c*x] - 8*\text{Sin}[2*\text{ArcSin}[c*x]] + \text{Sin}[4*\text{ArcSin}[c*x]])))/256*c^5*\text{Sqrt}[d]*\text{Sqrt}[d - c^2*d*x^2])}{1}$$
Rubi [A] (verified)Time = 1.66 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5210, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^4(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx \\
& \quad \downarrow \text{5210} \\
& \frac{3 \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{4c^2} + \frac{b\sqrt{1-c^2x^2} \int x^3(a+b\arcsin(cx)) dx}{2c\sqrt{d-c^2dx^2}} - \\
& \quad \frac{x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{4c^2d} \\
& \quad \downarrow \text{5138} \\
& \frac{3 \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{4c^2} + \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \int \frac{x^4}{\sqrt{1-c^2x^2}} dx \right)}{2c\sqrt{d-c^2dx^2}} - \\
& \quad \frac{x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{4c^2d} \\
& \quad \downarrow \text{262} \\
& \frac{3 \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{4c^2} + \\
& \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2dx^2}} - \\
& \quad \frac{x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{4c^2d} \\
& \quad \downarrow \text{262} \\
& \frac{3 \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{4c^2} + \\
& \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2dx^2}} - \\
& \quad \frac{x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{4c^2d} \\
& \quad \downarrow \text{223} \\
& \frac{3 \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{4c^2} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{4c^2d} + \\
& \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2dx^2}}
\end{aligned}$$

$$\begin{array}{c}
\downarrow 5210 \\
\frac{3 \left(\frac{b\sqrt{1-c^2x^2} \int x(a+b \arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} \right)}{4c^2} \\
\frac{x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{4c^2d} + \\
\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2dx^2}} \\
\downarrow 5138 \\
\frac{3 \left(\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} \right)}{4c^2} \\
\frac{x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{4c^2d} + \\
\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2dx^2}} \\
\downarrow 262 \\
\frac{3 \left(\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} \right)}{4c^2} \\
\frac{x^3\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{4c^2d} + \\
\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{4}x^4(a+b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2dx^2}} \\
\downarrow 223
\end{array}$$

$$\begin{aligned}
 & 3 \left(\frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2 dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))^2}{2c^2 d} + \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{2} x^2 (a+b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2 x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2 dx^2}} \right) \\
 & \frac{4c^2 x^3 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{4c^2 d} + \\
 & \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{4} x^4 (a+b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2 dx^2}} \\
 & \quad \downarrow \text{5152} \\
 & \frac{-x^3 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{4c^2 d} + \\
 & 3 \left(-\frac{x\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{2c^2 d} + \frac{\sqrt{1-c^2 x^2} (a+b \arcsin(cx))^3}{6bc^3 \sqrt{d-c^2 dx^2}} + \frac{b\sqrt{1-c^2 x^2} \left(\frac{1}{2} x^2 (a+b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2 x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2 dx^2}} \right) \\
 & \frac{4c^2 b\sqrt{1-c^2 x^2} \left(\frac{1}{4} x^4 (a+b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2 x^2}}{2c^2} \right)}{4c^2} - \frac{x^3 \sqrt{1-c^2 x^2}}{4c^2} \right) \right)}{2c\sqrt{d-c^2 dx^2}}
 \end{aligned}$$

input `Int[(x^4*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]`

output `-1/4*(x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (b*Sqrt[1 - c^2*x^2]*((x^4*(a + b*ArcSin[c*x]))/4 - (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2))/4))/(2*c*Sqrt[d - c^2*d*x^2]) + (3*(-1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/(c*Sqrt[d - c^2*d*x^2])))/(4*c^2)`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /;$ $\text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5210 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\text{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \ \text{Int}[(f*x)^{(m-2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /;$ $\text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m+2*p+1, 0]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(283) = 566$.

Time = 0.54 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.24

method	result
default	$-\frac{a^2 x^3 \sqrt{-c^2 d x^2 + d}}{4c^2 d} - \frac{3a^2 x \sqrt{-c^2 d x^2 + d}}{8c^4 d} + \frac{3a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{8c^4 \sqrt{c^2 d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{8c^5 d (c^2 x^2 - 1)} + \dots \right)$
parts	$-\frac{a^2 x^3 \sqrt{-c^2 d x^2 + d}}{4c^2 d} - \frac{3a^2 x \sqrt{-c^2 d x^2 + d}}{8c^4 d} + \frac{3a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{8c^4 \sqrt{c^2 d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{8c^5 d (c^2 x^2 - 1)} + \dots \right)$

input `int(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output

$$\begin{aligned} & -1/4*a^2*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(-c^2*d*x^2+d)^(1/2) \\ & +3/8*a^2/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+ \\ & b^2*(-1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arcs \\ & in(c*x)^3+1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)* \\ & arcsin(c*x)+1/16*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*(2*arcsin(c*x)^2 \\ & -1)*x-1/128*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)*cos(5*arcs \\ & in(c*x))-1/512*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*(8*arcsin(c*x)^2- \\ & 1)*sin(5*arcsin(c*x))+15/128*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arcs \\ & in(c*x)*cos(3*arcsin(c*x))+1/512*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)* \\ & (56*arcsin(c*x)^2-31)*sin(3*arcsin(c*x))+2*a*b*(-3/16*(-d*(c^2*x^2-1))^(1 \\ & /2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)^2-1/16/c^5/(-d*(c^2*x \\ & ^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)+1/8*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2- \\ & 1)*arcsin(c*x)*x-1/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*cos(5*arcs \\ & in(c*x))-1/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)*sin(5*a \\ & rcsin(c*x))+15/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*cos(3*arcsin(c \\ & *x))+7/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arcsin(c*x)*sin(3*arcsi \\ & n(c*x))) \end{aligned}$$

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-(b^2*x^4*arcsin(c*x)^2 + 2*a*b*x^4*arcsin(c*x) + a^2*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate(x**4*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**4*(a + b*asin(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/8*a^2*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^4*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) - sqrt(d)*integrate((b^2*x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^2 - d), x)`

Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.95

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{16(-c^2 x^2 + 1)^{\frac{3}{2}} b^2 x \arcsin(cx)^2 + 32(-c^2 x^2 + 1)^{\frac{3}{2}} abx \arcsin(cx) - 40\sqrt{-c^2 x^2 + 1} b^2 x \arcsin(cx)^2 + 16}{}$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `1/64*(16*(-c^2*x^2 + 1)^(3/2)*b^2*x*arcsin(c*x)^2 + 32*(-c^2*x^2 + 1)^(3/2)*a*b*x*arcsin(c*x) - 40*sqrt(-c^2*x^2 + 1)*b^2*x*arcsin(c*x)^2 + 16*(-c^2*x^2 + 1)^(3/2)*a^2*x - 2*(-c^2*x^2 + 1)^(3/2)*b^2*x - 80*sqrt(-c^2*x^2 + 1)*a*b*x*arcsin(c*x) + 8*(c^2*x^2 - 1)^2*b^2*arcsin(c*x)/c + 8*b^2*arcsin(c*x)^3/c - 40*sqrt(-c^2*x^2 + 1)*a^2*x + 17*sqrt(-c^2*x^2 + 1)*b^2*x + 8*(c^2*x^2 - 1)^2*a*b/c + 40*(c^2*x^2 - 1)*b^2*arcsin(c*x)/c + 24*a*b*arcsin(c*x)^2/c + 40*(c^2*x^2 - 1)*a*b/c + 24*a^2*arcsin(c*x)/c + 17*b^2*arcsin(c*x)/c + 17*a*b/c)/(c^4*sqrt(d))`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{\sqrt{d - c^2x^2}} dx$$

$$= \frac{3a \sin(cx) a^2 - 2\sqrt{-c^2x^2 + 1} a^2 c^3 x^3 - 3\sqrt{-c^2x^2 + 1} a^2 cx + 16 \left(\int \frac{a \sin(cx) x^4}{\sqrt{-c^2x^2 + 1}} dx \right) ab c^5 + 8 \left(\int \frac{a \sin(cx)^2 x^4}{\sqrt{-c^2x^2 + 1}} dx \right)}{8\sqrt{d} c^5}$$

input `int(x^4*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(3*asin(c*x)*a**2 - 2*sqrt(-c**2*x**2 + 1)*a**2*c**3*x**3 - 3*sqrt(-c**2*x**2 + 1)*a**2*c*x + 16*int((asin(c*x)*x**4)/sqrt(-c**2*x**2 + 1),x)*a*b*c**5 + 8*int((asin(c*x)**2*x**4)/sqrt(-c**2*x**2 + 1),x)*b**2*c**5)/(8*sqrt(d)*c**5)`

3.233 $\int \frac{x^3(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$

Optimal result	2300
Mathematica [A] (verified)	2301
Rubi [A] (verified)	2301
Maple [B] (verified)	2304
Fricas [A] (verification not implemented)	2305
Sympy [F]	2306
Maxima [A] (verification not implemented)	2306
Giac [F(-2)]	2307
Mupad [F(-1)]	2307
Reduce [F]	2308

Optimal result

Integrand size = 29, antiderivative size = 224

$$\int \frac{x^3(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx = \frac{14b^2\sqrt{d-c^2dx^2}}{9c^4d} - \frac{2b^2(d-c^2dx^2)^{3/2}}{27c^4d^2} + \frac{4bx\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^3\sqrt{d-c^2dx^2}} + \frac{2bx^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{9c\sqrt{d-c^2dx^2}} - \frac{2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^4d} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d}$$

output

```
14/9*b^2*(-c^2*d*x^2+d)^(1/2)/c^4/d-2/27*b^2*(-c^2*d*x^2+d)^(3/2)/c^4/d^2+
4/3*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/(-c^2*d*x^2+d)^(1/2)+2/9*
b*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*d*x^2+d)^(1/2)-2/3*(-c^
2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^4/d-1/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a
+b*arcsin(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.79

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{6abcx\sqrt{1 - c^2 x^2}(6 + c^2 x^2) + 9a^2(-2 + c^2 x^2 + c^4 x^4) - 2b^2(-20 + 19c^2 x^2 + c^4 x^4) + 6b(bcx\sqrt{1 - c^2 x^2}(6 + c^2 x^2) + 3a(-2 + c^2 x^2 + c^4 x^4)) \operatorname{ArcSin}[cx] + 9b^2(-2 + c^2 x^2 + c^4 x^4) \operatorname{ArcSin}[cx]^2}{27c^4 \sqrt{d - c^2 dx^2}}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

output

```
(6*a*b*c*x*Sqrt[1 - c^2*x^2]*(6 + c^2*x^2) + 9*a^2*(-2 + c^2*x^2 + c^4*x^4) - 2*b^2*(-20 + 19*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*Sqrt[1 - c^2*x^2]*(6 + c^2*x^2) + 3*a*(-2 + c^2*x^2 + c^4*x^4))*ArcSin[c*x] + 9*b^2*(-2 + c^2*x^2 + c^4*x^4)*ArcSin[c*x]^2)/(27*c^4*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5210, 5138, 243, 53, 2009, 5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow \text{5210}$$

$$\frac{2b\sqrt{1 - c^2 x^2} \int x^2(a + b \arcsin(cx)) dx}{3c\sqrt{d - c^2 dx^2}} + \frac{2 \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx}{3c^2} -$$

$$\frac{x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{3c^2 d}$$

$$\downarrow \text{5138}$$

$$\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{3} bc \int \frac{x^3}{\sqrt{1-c^2 x^2}} dx \right)}{3c\sqrt{d-c^2 dx^2} x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2} - \frac{3c^2 d}{3c^2 d}$$

↓ 243

$$\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \int \frac{x^2}{\sqrt{1-c^2 x^2}} dx^2 \right)}{3c\sqrt{d-c^2 dx^2} x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2} - \frac{3c^2 d}{3c^2 d}$$

↓ 53

$$\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \int \left(\frac{1}{c^2 \sqrt{1-c^2 x^2}} - \frac{\sqrt{1-c^2 x^2}}{c^2} \right) dx^2 \right)}{3c\sqrt{d-c^2 dx^2} x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2} - \frac{3c^2 d}{3c^2 d}$$

↓ 2009

$$\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{3c^2} - \frac{x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3c^2 d} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \left(\frac{2(1-c^2 x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2 x^2}}{c^4} \right) \right)}{3c\sqrt{d-c^2 dx^2}}$$

↓ 5182

$$2 \left(\frac{2b\sqrt{1-c^2 x^2} \int (a+b \arcsin(cx)) dx}{c\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{c^2 d} \right) - \frac{x^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3c^2 d} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{1}{3} x^3 (a+b \arcsin(cx)) - \frac{1}{6} bc \left(\frac{2(1-c^2 x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2 x^2}}{c^4} \right) \right)}{3c\sqrt{d-c^2 dx^2}}$$

↓ 2009

$$\begin{aligned}
& -\frac{x^2\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3c^2d} + \\
& 2\left(\frac{2b\sqrt{1-c^2x^2}\left(ax+b\arcsin(cx)+\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{c^2d}\right) \\
& + \\
& \frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arcsin(cx)) - \frac{1}{6}bc\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c\sqrt{d-c^2dx^2}}
\end{aligned}$$

input `Int[(x^3*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]`

output `-1/3*(x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (2*b*Sqrt[1 - c^2*x^2]*(-1/6*(b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4))) + (x^3*(a + b*ArcSin[c*x]))/3))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d)) + (2*b*Sqrt[1 - c^2*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(3*c^2)`

Defintions of rubi rules used

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

```
rule 5138 Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

```
rule 5182 Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

```
rule 5210 Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x
)^m*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(196) = 392.
 Time = 0.75 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.14

method	result
orering	$\frac{(19c^6x^6+100c^4x^4-380c^2x^2+240)(a+b\arcsin(cx))^2}{27c^6x^2\sqrt{-c^2dx^2+d}} - \frac{2(cx-1)(cx+1)(c^4x^4+12c^2x^2-20)\left(\frac{3x^2(a+b\arcsin(cx))^2}{\sqrt{-c^2dx^2+d}} + \frac{2x^3(a+b\arcsin(cx))}{\sqrt{-c^2dx^2+d}}\right)}{9c^6x^4}$
default	$a^2\left(-\frac{x^2\sqrt{-c^2dx^2+d}}{3c^2d} - \frac{2\sqrt{-c^2dx^2+d}}{3dc^4}\right) + b^2\left(\frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1)}{432c^4d(c^2x^2-1)}(6i\arcsin(cx)+9\arcsin(cx))^2\right)$
parts	$a^2\left(-\frac{x^2\sqrt{-c^2dx^2+d}}{3c^2d} - \frac{2\sqrt{-c^2dx^2+d}}{3dc^4}\right) + b^2\left(\frac{\sqrt{-d(c^2x^2-1)}(-2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1)}{432c^4d(c^2x^2-1)}(6i\arcsin(cx)+9\arcsin(cx))^2\right)$

```
input int(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/27*(19*c^6*x^6+100*c^4*x^4-380*c^2*x^2+240)/c^6/x^2*(a+b*arcsin(c*x))^2/
(-c^2*d*x^2+d)^(1/2)-2/9*(c*x-1)*(c*x+1)*(c^4*x^4+12*c^2*x^2-20)/c^6/x^4*(
3*x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2)+2*x^3*(a+b*arcsin(c*x))/(-c
^2*d*x^2+d)^(1/2)*b*c/(-c^2*x^2+1)^(1/2)+x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x
^2+d)^(3/2)*c^2*d)+1/27*(c^2*x^2+20)/c^6*(c*x-1)^2/x^3*(c*x+1)^2*(6*x*(a+b
*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2)+12*x^2*(a+b*arcsin(c*x))/(-c^2*d*x^2+
d)^(1/2)*b*c/(-c^2*x^2+1)^(1/2)+7*x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(
3/2)*c^2*d+2*x^3*b^2*c^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+4*x^4*(a+b*arcs
in(c*x))/(-c^2*d*x^2+d)^(3/2)*b*c^3/(-c^2*x^2+1)^(1/2)*d+2*x^4*(a+b*arcsin
(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c^3/(-c^2*x^2+1)^(3/2)+3*x^5*(a+b*arcsin(c*x
))^2/(-c^2*d*x^2+d)^(5/2)*c^4*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx =$$

$$\frac{6(abc^3x^3 + 6abcx + (b^2c^3x^3 + 6b^2cx) \arcsin(cx))\sqrt{-c^2 dx^2 + d}\sqrt{-c^2 x^2 + 1} + ((9a^2 - 2b^2)c^4 x^4 + (9a^2 - 38b^2)c^2 x^2 + 9(b^2c^4 x^4 + b^2c^2 x^2 - 2b^2) \arcsin(cx))^2 - 18a^2 + 40b^2 + 18(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*\arcsin(c*x)}{(c^6*d*x^2 - c^4*d)}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output

```
-1/27*(6*(a*b*c^3*x^3 + 6*a*b*c*x + (b^2*c^3*x^3 + 6*b^2*c*x)*arcsin(c*x))
*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + ((9*a^2 - 2*b^2)*c^4*x^4 + (9*a
^2 - 38*b^2)*c^2*x^2 + 9*(b^2*c^4*x^4 + b^2*c^2*x^2 - 2*b^2)*arcsin(c*x)^2
- 18*a^2 + 40*b^2 + 18*(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*arcsin(c*x))*s
qrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate(x**3*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**3*(a + b*asin(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.12

$$\begin{aligned} & \int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx \\ &= -\frac{1}{3} b^2 \left(\frac{\sqrt{-c^2 dx^2 + dx^2}}{c^2 d} + \frac{2\sqrt{-c^2 dx^2 + d}}{c^4 d} \right) \arcsin(cx)^2 \\ & \quad - \frac{2}{3} ab \left(\frac{\sqrt{-c^2 dx^2 + dx^2}}{c^2 d} + \frac{2\sqrt{-c^2 dx^2 + d}}{c^4 d} \right) \arcsin(cx) \\ & \quad - \frac{1}{3} a^2 \left(\frac{\sqrt{-c^2 dx^2 + dx^2}}{c^2 d} + \frac{2\sqrt{-c^2 dx^2 + d}}{c^4 d} \right) \\ & \quad + \frac{2}{27} b^2 \left(\frac{\sqrt{-c^2 x^2 + 1} x^2 + \frac{20\sqrt{-c^2 x^2 + 1}}{c^2}}{c^2 \sqrt{d}} + \frac{3(c^2 x^3 + 6x) \arcsin(cx)}{c^3 \sqrt{d}} \right) \\ & \quad + \frac{2(c^2 x^3 + 6x) ab}{9 c^3 \sqrt{d}} \end{aligned}$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output

```
-1/3*b^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d
))*arcsin(c*x)^2 - 2/3*a*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2
*d*x^2 + d)/(c^4*d))*arcsin(c*x) - 1/3*a^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*
d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) + 2/27*b^2*((sqrt(-c^2*x^2 + 1)*x^2 +
20*sqrt(-c^2*x^2 + 1)/c^2)/(c^2*sqrt(d)) + 3*(c^2*x^3 + 6*x)*arcsin(c*x)/
(c^3*sqrt(d))) + 2/9*(c^2*x^3 + 6*x)*a*b/(c^3*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input

```
int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output

```
int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```


Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{\sqrt{d - c^2x^2}} dx$$

$$= \frac{-\sqrt{-c^2x^2 + 1} a^2 c^2 x^2 - 2\sqrt{-c^2x^2 + 1} a^2 + 6 \left(\int \frac{\arcsin(cx)x^3}{\sqrt{-c^2x^2 + 1}} dx \right) ab c^4 + 3 \left(\int \frac{\arcsin(cx)^2 x^3}{\sqrt{-c^2x^2 + 1}} dx \right) b^2 c^4}{3\sqrt{d} c^4}$$

input `int(x^3*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(-sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)*a**2 + 6*int((asin(c*x)*x**3)/sqrt(-c**2*x**2 + 1),x)*a*b*c**4 + 3*int((asin(c*x)**2*x**3)/sqrt(-c**2*x**2 + 1),x)*b**2*c**4)/(3*sqrt(d)*c**4)`

3.234 $\int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$

Optimal result	2309
Mathematica [A] (verified)	2310
Rubi [A] (verified)	2310
Maple [B] (verified)	2313
Fricas [F]	2313
Sympy [F]	2314
Maxima [F]	2314
Giac [A] (verification not implemented)	2315
Mupad [F(-1)]	2315
Reduce [F]	2316

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx = \frac{b^2x\sqrt{d-c^2dx^2}}{4c^2d} - \frac{b^2\sqrt{1-c^2x^2} \arcsin(cx)}{4c^3\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c\sqrt{d-c^2dx^2}} - \frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}$$

output

```
1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2/d-1/4*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)/c^3/(-c^2*d*x^2+d)^(1/2)+1/2*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*d*x^2+d)^(1/2)-1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^2/d+1/6*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.02

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{12a^2 cdx(-1 + c^2 x^2) - 12a^2 \sqrt{d} \sqrt{d - c^2 dx^2} \arctan\left(\frac{cx\sqrt{d-c^2 dx^2}}{\sqrt{d(-1+c^2 x^2)}}\right) - 6abd\sqrt{1 - c^2 x^2}(-2 \arcsin(cx))^2 + \cos$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

output

```
(12*a^2*c*d*x*(-1 + c^2*x^2) - 12*a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 6*a*b*d*Sqrt[1 - c^2*x^2]*(-2*ArcSin[c*x]^2 + Cos[2*ArcSin[c*x]] + 2*ArcSin[c*x]*Sin[2*ArcSin[c*x]]) + b^2*d*Sqrt[1 - c^2*x^2]*(4*ArcSin[c*x]^3 - 6*ArcSin[c*x]*Cos[2*ArcSin[c*x]] + (3 - 6*ArcSin[c*x]^2)*Sin[2*ArcSin[c*x]]))/(24*c^3*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow \text{5210}$$

$$\frac{b\sqrt{1 - c^2 x^2} \int x(a + b \arcsin(cx)) dx}{c\sqrt{d - c^2 dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{2c^2} - \frac{x\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2}{2c^2 d}$$

$$\downarrow \text{5138}$$

$$\begin{aligned}
& \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))-\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx\right)}{c\sqrt{d-c^2dx^2}\frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d}} + \frac{\int\frac{(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2} - \\
& \quad \downarrow 262 \\
& \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}\frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d}} + \frac{\int\frac{(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2} - \\
& \quad \downarrow 223 \\
& \frac{\int\frac{(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} + \\
& \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}} \\
& \quad \downarrow 5152 \\
& -\frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \\
& \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}
\end{aligned}$$

input `Int[(x^2*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]`

output `-1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/(c*Sqrt[d - c^2*d*x^2])`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)} / \text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5210 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\text{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \ \text{Int}[(f*x)^{(m-2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m+2*p+1, 0]$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(180) = 360$.

Time = 0.45 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.51

method	result
default	$-\frac{a^2 x \sqrt{-c^2 d x^2 + d}}{2c^2 d} + \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{2c^2 \sqrt{c^2 d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{6c^3 d(c^2 x^2 - 1)} + \frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1}}{8c^3 d(c^2 x^2 - 1)} \right)$
parts	$-\frac{a^2 x \sqrt{-c^2 d x^2 + d}}{2c^2 d} + \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{2c^2 \sqrt{c^2 d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{6c^3 d(c^2 x^2 - 1)} + \frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1}}{8c^3 d(c^2 x^2 - 1)} \right)$

input `int(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & -1/2*a^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a^2/c^2/(c^2*d)^(1/2)*\arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*\arcsin(c*x)^3+1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*\arcsin(c*x)+1/16*(-d*(c^2*x^2-1))^(1/2)/c^2/d/(c^2*x^2-1)*(2*\arcsin(c*x)^2-1)*x+1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*\arcsin(c*x)*\cos(3*\arcsin(c*x))+1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*(2*\arcsin(c*x)^2-1)*\sin(3*\arcsin(c*x)))+2*a*b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*\arcsin(c*x)^2-1/16/c^3/(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)+1/8*(-d*(c^2*x^2-1))^(1/2)/c^2/d/(c^2*x^2-1)*\arcsin(c*x)*x+1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*\cos(3*\arcsin(c*x))+1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*\arcsin(c*x)*\sin(3*\arcsin(c*x))) \end{aligned}$$

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))^2}{\sqrt{-d}(cx - 1)(cx + 1)} dx$$

input `integrate(x**2*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2), x)`

output `Integral(x**2*(a + b*asin(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2), x, algorithm="maxima")`

output `-1/2*a^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) - sqrt(d)*integrate((b^2*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^2 - d), x)`

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \frac{6\sqrt{-c^2 x^2 + 1} b^2 x \arcsin(cx)^2 + 12\sqrt{-c^2 x^2 + 1} abx \arcsin(cx) - \frac{2b^2 \arcsin(cx)^3}{c} + 6\sqrt{-c^2 x^2 + 1} a^2 x - 3}{12c^2}$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `-1/12*(6*sqrt(-c^2*x^2 + 1)*b^2*x*arcsin(c*x)^2 + 12*sqrt(-c^2*x^2 + 1)*a*b*x*arcsin(c*x) - 2*b^2*arcsin(c*x)^3/c + 6*sqrt(-c^2*x^2 + 1)*a^2*x - 3*sqrt(-c^2*x^2 + 1)*b^2*x - 6*(c^2*x^2 - 1)*b^2*arcsin(c*x)/c - 6*a*b*arcsin(c*x)^2/c - 6*(c^2*x^2 - 1)*a*b/c - 6*a^2*arcsin(c*x)/c - 3*b^2*arcsin(c*x)/c - 3*a*b/c)/(c^2*sqrt(d))`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d - c^2x^2}} dx$$

$$= \frac{\arcsin(cx) a^2 - \sqrt{-c^2x^2 + 1} a^2 cx + 4 \left(\int \frac{\arcsin(cx) x^2}{\sqrt{-c^2x^2 + 1}} dx \right) ab c^3 + 2 \left(\int \frac{\arcsin(cx)^2 x^2}{\sqrt{-c^2x^2 + 1}} dx \right) b^2 c^3}{2\sqrt{d} c^3}$$

input `int(x^2*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(asin(c*x)*a**2 - sqrt(-c**2*x**2 + 1)*a**2*c*x + 4*int((asin(c*x)*x**2)/sqrt(-c**2*x**2 + 1),x)*a*b*c**3 + 2*int((asin(c*x)**2*x**2)/sqrt(-c**2*x**2 + 1),x)*b**2*c**3)/(2*sqrt(d)*c**3)`

3.235 $\int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx$

Optimal result	2317
Mathematica [A] (verified)	2317
Rubi [A] (verified)	2318
Maple [C] (verified)	2319
Fricas [A] (verification not implemented)	2320
Sympy [F(-2)]	2320
Maxima [A] (verification not implemented)	2321
Giac [F(-2)]	2321
Mupad [F(-1)]	2322
Reduce [B] (verification not implemented)	2322

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2dx^2}} dx = \frac{2b^2\sqrt{d - c^2dx^2}}{c^2d} + \frac{2bx\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{c\sqrt{d - c^2dx^2}} - \frac{\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^2}{c^2d}$$

output

```
2*b^2*(-c^2*d*x^2+d)^(1/2)/c^2/d+2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2dx^2}} dx = \frac{(-1 + c^2x^2)(a + b \arcsin(cx))^2 + 2b\sqrt{1 - c^2x^2}(acx + b\sqrt{1 - c^2x^2} + bcx \arcsin(cx))}{c^2\sqrt{d - c^2dx^2}}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output $((-1 + c^2x^2)(a + b\text{ArcSin}[cx])^2 + 2b\sqrt{1 - c^2x^2}(acx + b\sqrt{1 - c^2x^2} + bcx\text{ArcSin}[cx]))/(c^2\sqrt{d - c^2dx^2})$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5182$$

$$\frac{2b\sqrt{1 - c^2 x^2} \int (a + b \arcsin(cx)) dx}{c\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{c^2 d}$$

$$\downarrow 2009$$

$$\frac{2b\sqrt{1 - c^2 x^2} \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2 x^2}}{c} \right)}{c\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{c^2 d}$$

input $\text{Int}[(x*(a + b\text{ArcSin}[c*x])^2)/\text{Sqrt}[d - c^2*d*x^2], x]$

output $-((\text{Sqrt}[d - c^2*d*x^2]*(a + b\text{ArcSin}[c*x])^2)/(c^2*d)) + (2*b*\text{Sqrt}[1 - c^2*x^2]*(a*x + (b*\text{Sqrt}[1 - c^2*x^2])/c + b*x*\text{ArcSin}[c*x]))/(c*\text{Sqrt}[d - c^2*d*x^2])$

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5182 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.04

method	result
default	$-\frac{a^2\sqrt{-c^2dx^2+d}}{c^2d} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}(c^2x^2 - i\sqrt{-c^2x^2+1}cx - 1)(\arcsin(cx)^2 - 2 + 2i\arcsin(cx))}{2c^2d(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}(i\sqrt{-c^2x^2+1}cx - 1)(\arcsin(cx)^2 - 2 + 2i\arcsin(cx))}{2c^2d(c^2x^2-1)} \right)$
parts	$-\frac{a^2\sqrt{-c^2dx^2+d}}{c^2d} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}(c^2x^2 - i\sqrt{-c^2x^2+1}cx - 1)(\arcsin(cx)^2 - 2 + 2i\arcsin(cx))}{2c^2d(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}(i\sqrt{-c^2x^2+1}cx - 1)(\arcsin(cx)^2 - 2 + 2i\arcsin(cx))}{2c^2d(c^2x^2-1)} \right)$
oring	$\frac{(c^4x^4 - 4c^2x^2 + 2)(a + b\arcsin(cx))^2}{c^4x^2\sqrt{-c^2dx^2+d}} + \frac{2(cx-1)(cx+1) \left(\frac{(a+b\arcsin(cx))^2}{\sqrt{-c^2dx^2+d}} + \frac{2x(a+b\arcsin(cx))bc}{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}} + \frac{x^2(a+b\arcsin(cx))^2c^2d}{(-c^2dx^2+d)^{\frac{3}{2}}} \right)}{c^4x^2} + \dots$

```
input int(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
output -a^2/c^2/d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))/c^2/d/(c^2*x^2-1))+2*a*b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)/c^2/d/(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.41

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \frac{2 \sqrt{-c^2 dx^2 + d}(b^2 cx \arcsin(cx) + abcx) \sqrt{-c^2 x^2 + 1} + ((a^2 - 2b^2)c^2 x^2 + (b^2 c^2 x^2 - b^2) \arcsin(cx)^2 - c^4 dx^2 - c^2 d)}{c^4 dx^2 - c^2 d}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `-(2*sqrt(-c^2*d*x^2 + d)*(b^2*c*x*arcsin(c*x) + a*b*c*x)*sqrt(-c^2*x^2 + 1) + ((a^2 - 2*b^2)*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*b^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*d*x^2 - c^2*d)`

Sympy [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)`

output `Exception raised: TypeError >> Invalid comparison of non-real zoo`

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.25

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = 2b^2 \left(\frac{x \arcsin(cx)}{c\sqrt{d}} + \frac{\sqrt{-c^2 x^2 + 1}}{c^2 \sqrt{d}} \right) + \frac{2abx}{c\sqrt{d}} - \frac{\sqrt{-c^2 dx^2 + db^2} \arcsin(cx)^2}{c^2 d} - \frac{2\sqrt{-c^2 dx^2 + d} ab \arcsin(cx)}{c^2 d} - \frac{\sqrt{-c^2 dx^2 + da^2}}{c^2 d}$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output

```
2*b^2*(x*arcsin(c*x)/(c*sqrt(d)) + sqrt(-c^2*x^2 + 1)/(c^2*sqrt(d))) + 2*a*b*x/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*b^2*arcsin(c*x)^2/(c^2*d) - 2*sqrt(-c^2*d*x^2 + d)*a*b*arcsin(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a^2/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x(a + b \operatorname{asin}(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)`

output `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

$$= \frac{\sqrt{d} (-\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx))^2 b^2 - 2\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) ab + 2\operatorname{asin}(cx) b^2 cx - \sqrt{-c^2 x^2 + 1} a^2 + 2\sqrt{-c^2 x^2 + 1} a^2}{c^2 d}$$

input `int(x*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(sqrt(d)*(-sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2 - 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b + 2*asin(c*x)*b**2*c*x - sqrt(-c**2*x**2 + 1)*a**2 + 2*sqrt(-c**2*x**2 + 1)*b**2 + 2*a*b*c*x))/(c**2*d)`

$$3.236 \quad \int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx$$

Optimal result	2323
Mathematica [A] (verified)	2323
Rubi [A] (verified)	2324
Maple [B] (verified)	2324
Fricas [F]	2325
Sympy [F]	2325
Maxima [A] (verification not implemented)	2326
Giac [A] (verification not implemented)	2326
Mupad [F(-1)]	2326
Reduce [B] (verification not implemented)	2327

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx = \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc\sqrt{d-c^2x^2}}$$

output $1/3*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^3/b/c/(-c^2*d*x^2+d)^{(1/2)}$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx = \frac{\sqrt{1-c^2x^2} \arcsin(cx) (3a^2 + 3ab \arcsin(cx) + b^2 \arcsin(cx)^2)}{3c\sqrt{d-c^2x^2}}$$

input `Integrate[(a + b*ArcSin[c*x])^2/Sqrt[d - c^2*d*x^2],x]`

output $(\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c*x]*(3*a^2 + 3*a*b*\text{ArcSin}[c*x] + b^2*\text{ArcSin}[c*x]^2))/(3*c*\text{Sqrt}[d - c^2*d*x^2])$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

↓ 5152

$$\frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^3}{3bc\sqrt{d - c^2 dx^2}}$$

input `Int[(a + b*ArcSin[c*x])^2/Sqrt[d - c^2*d*x^2],x]`

output `(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^3)/(3*b*c*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^n_/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(43) = 86$.

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.92

method	result	size
default	$\frac{a^2 \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{\sqrt{c^2 d}} - \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{3d(c^2 x^2 - 1)c} - \frac{ab \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{d(c^2 x^2 - 1)c}$	143
parts	$\frac{a^2 \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{\sqrt{c^2 d}} - \frac{b^2 \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{3d(c^2 x^2 - 1)c} - \frac{ab \sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2}{d(c^2 x^2 - 1)c}$	143

input `int((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$a^2/(c^2*d)^{(1/2)}*\arctan((c^2*d)^{(1/2)}*x/(-c^2*d*x^2+d)^{(1/2)})-1/3*b^2*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/c*\arcsin(c*x)^3-a*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/d/(c^2*x^2-1)/c*\arcsin(c*x)^2$$

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^2 - d), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \arcsin(cx))^2}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral((a + b*asin(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \frac{b^2 \arcsin(cx)^3}{3c\sqrt{d}} + \frac{ab \arcsin(cx)^2}{c\sqrt{d}} + \frac{a^2 \arcsin(cx)}{c\sqrt{d}}$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`output `1/3*b^2*arcsin(c*x)^3/(c*sqrt(d)) + a*b*arcsin(c*x)^2/(c*sqrt(d)) + a^2*arcsin(c*x)/(c*sqrt(d))`**Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \frac{b^2 \arcsin(cx)^3 + 3ab \arcsin(cx)^2 + 3a^2 \arcsin(cx)}{3c\sqrt{d}}$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`output `1/3*(b^2*arcsin(c*x)^3 + 3*a*b*arcsin(c*x)^2 + 3*a^2*arcsin(c*x))/(c*sqrt(d))`**Mupad [F(-1)]**

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^(1/2),x)`output `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^(1/2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 x^2}} dx = \frac{\sqrt{d} \operatorname{asin}(cx) (\operatorname{asin}(cx)^2 b^2 + 3 \operatorname{asin}(cx) ab + 3a^2)}{3cd}$$

input `int((a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(sqrt(d)*asin(c*x)*(asin(c*x)**2*b**2 + 3*asin(c*x)*a*b + 3*a**2))/(3*c*d)`

3.237 $\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx$

Optimal result	2328
Mathematica [A] (verified)	2329
Rubi [A] (verified)	2329
Maple [A] (verified)	2332
Fricas [F]	2332
Sympy [F]	2333
Maxima [F]	2333
Giac [F(-2)]	2333
Mupad [F(-1)]	2334
Reduce [F]	2334

Optimal result

Integrand size = 29, antiderivative size = 257

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2dx^2}} dx = -\frac{2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}} + \frac{2ib\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}} - \frac{2ib\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}} - \frac{2b^2\sqrt{1 - c^2x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}} + \frac{2b^2\sqrt{1 - c^2x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{d - c^2dx^2}}$$

output

```
-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2)
)/(-c^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(
2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)
*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)
)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*d*x^
2+d)^(1/2)+2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-
c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.17

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2 dx^2}} dx = \frac{a^2 \log(cx)}{\sqrt{d}} - \frac{a^2 \log(d + \sqrt{d}\sqrt{d - c^2 dx^2})}{\sqrt{d}} + \frac{2ab\sqrt{1 - c^2 x^2}(\arcsin(cx) (\log(1 - e^{i \arcsin(cx)}) - \log(1 + e^{i \arcsin(cx)})) + i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - \log(1 + e^{i \arcsin(cx)}))}{\sqrt{d - c^2 dx^2}} + \frac{b^2\sqrt{1 - c^2 x^2}(\arcsin(cx)^2 \log(1 - e^{i \arcsin(cx)}) - \arcsin(cx)^2 \log(1 + e^{i \arcsin(cx)}) + 2i \arcsin(cx) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - 2i \arcsin(cx) \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{\sqrt{d - c^2 dx^2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

output

```
(a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] + (2*a*b*Sqrt[1 - c^2*x^2]*(ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c*x]])] + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])]))/Sqrt[d - c^2*d*x^2] + (b^2*Sqrt[1 - c^2*x^2]*(ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] - ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x])] + (2*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] - (2*I)*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] - 2*PolyLog[3, -E^(I*ArcSin[c*x])] + 2*PolyLog[3, E^(I*ArcSin[c*x])]))/Sqrt[d - c^2*d*x^2]
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.56, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2 dx^2}} dx$$

↓ 5218

$$\frac{\sqrt{1 - c^2 x^2} \int \frac{(a + b \arcsin(cx))^2}{cx} d \arcsin(cx)}{\sqrt{d - c^2 dx^2}}$$

↓ 3042

$$\frac{\sqrt{1-c^2x^2} f(a+b\arcsin(cx))^2 \csc(\arcsin(cx)) d\arcsin(cx)}{\sqrt{d-c^2dx^2}}$$

↓ 4671

$$\frac{\sqrt{1-c^2x^2} (-2b f(a+b\arcsin(cx)) \log(1-e^{i\arcsin(cx)}) d\arcsin(cx) + 2b f(a+b\arcsin(cx)) \log(1+e^{i\arcsin(cx)}) d\arcsin(cx))}{\sqrt{d-c^2dx^2}}$$

↓ 3011

$$\frac{\sqrt{1-c^2x^2} (2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) d\arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - ib \int \operatorname{PolyLog}(2, e^{i\arcsin(cx)}) d\arcsin(cx))}{\sqrt{d-c^2dx^2}}$$

↓ 2720

$$\frac{\sqrt{1-c^2x^2} (2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - b \int e^{-i\arcsin(cx)} \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) de^{i\arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - b \int e^{i\arcsin(cx)} \operatorname{PolyLog}(2, e^{i\arcsin(cx)}) de^{i\arcsin(cx)})}{\sqrt{d-c^2dx^2}}$$

↓ 7143

$$\frac{\sqrt{1-c^2x^2} (-2\operatorname{arctanh}(e^{i\arcsin(cx)}) (a+b\arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - b \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx))) - 2b(i \operatorname{PolyLog}(2, e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - b \operatorname{PolyLog}(2, e^{i\arcsin(cx)}) (a+b\arcsin(cx)))}{\sqrt{d-c^2dx^2}}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x*sqrt[d - c^2*d*x^2]),x]
```

output

```
(sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] +
2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -
E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*
x])] - b*PolyLog[3, E^(I*ArcSin[c*x])]]))/sqrt[d - c^2*d*x^2]
```

Definitions of rubi rules used

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))] * ((f_) + (g_) * (x_))^(m_), x_Symbol] := Simp[(- (f + g*x)^m * (PolyLog[2, (-e)*(F^(c*(a + b*x)))^n] / (b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1) * PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)] * ((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m * (ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1) * Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1) * Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5218 `Int[(((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*(x_)^(m_))/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n * Sin[x]^m, x], x, ArcSin[c*x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]`

rule 7143 `Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)] / ((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p] / (e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.51

method	result
default	$-\frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{\sqrt{d}} + \frac{b^2\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(\arcsin(cx)^2 \ln(1+icx+\sqrt{-c^2x^2+1}) - \arcsin(cx)^2 \ln(1-icx-\sqrt{-c^2x^2+1})\right)}{\sqrt{d}}$
parts	$-\frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{\sqrt{d}} + \frac{b^2\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(\arcsin(cx)^2 \ln(1+icx+\sqrt{-c^2x^2+1}) - \arcsin(cx)^2 \ln(1-icx-\sqrt{-c^2x^2+1})\right)}{\sqrt{d}}$

input `int((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output
$$-a^2/d^{(1/2)}*\ln((2*d+2*d^{(1/2)}*(-c^2*d*x^2+d)^{(1/2)})/x)+b^2*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}*(\arcsin(c*x)^2*\ln(1+I*c*x+(-c^2*x^2+1)^{(1/2)})-\arcsin(c*x)^2*\ln(1-I*c*x-(-c^2*x^2+1)^{(1/2)})-2*I*\arcsin(c*x)*\text{polylog}(2,-I*c*x-(-c^2*x^2+1)^{(1/2)})+2*I*\arcsin(c*x)*\text{polylog}(2,I*c*x+(-c^2*x^2+1)^{(1/2)})-2*\text{polylog}(3,I*c*x+(-c^2*x^2+1)^{(1/2)})+2*\text{polylog}(3,-I*c*x-(-c^2*x^2+1)^{(1/2)}))/d/(c^2*x^2-1)-2*I*a*b*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}*(I*\arcsin(c*x)*\ln(1+I*c*x+(-c^2*x^2+1)^{(1/2)})-I*\arcsin(c*x)*\ln(1-I*c*x-(-c^2*x^2+1)^{(1/2)})+\text{polylog}(2,-I*c*x-(-c^2*x^2+1)^{(1/2)})-\text{polylog}(2,I*c*x+(-c^2*x^2+1)^{(1/2)}))/d/(c^2*x^2-1)$$

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2dx^2 + dx}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^3 - d*x), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2x^2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x\sqrt{-d}(cx - 1)(cx + 1)} dx$$

input `integrate((a+b*asin(c*x))**2/x/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral((a + b*asin(c*x))**2/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2x^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2x^2 + d}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-a^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) - sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^3 - d*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x\sqrt{d - c^2 dx^2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned} & \int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d - c^2 dx^2}} dx \\ &= \frac{2 \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1}} dx \right) ab + \left(\int \frac{\operatorname{asin}(cx)^2}{\sqrt{-c^2 x^2 + 1}} dx \right) b^2 + \log \left(\tan \left(\frac{\operatorname{asin}(cx)}{2} \right) \right) a^2}{\sqrt{d}} \end{aligned}$$

input

```
int((a+b*asin(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

output

```
(2*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*x),x)*a*b + int(asin(c*x)**2/(sqr
t(-c**2*x**2 + 1)*x),x)*b**2 + log(tan(asin(c*x)/2))*a**2)/sqrt(d)
```

3.238 $\int \frac{(a+b \arcsin(cx))^2}{x^2 \sqrt{d-c^2 dx^2}} dx$

Optimal result	2335
Mathematica [A] (verified)	2336
Rubi [A] (verified)	2336
Maple [B] (verified)	2339
Fricas [F]	2340
Sympy [F]	2340
Maxima [F]	2340
Giac [F(-2)]	2341
Mupad [F(-1)]	2341
Reduce [F]	2341

Optimal result

Integrand size = 29, antiderivative size = 183

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = -\frac{ic\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^2}{dx} + \frac{2bc\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{\sqrt{d - c^2 dx^2}} - \frac{ib^2 c \sqrt{1 - c^2 x^2} \text{PolyLog}(2, e^{2i \arcsin(cx)})}{\sqrt{d - c^2 dx^2}}$$

output

```
-I*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/d/x+2*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.87

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{1 - c^2 x^2} (b^2 (icx + \sqrt{1 - c^2 x^2}) \arcsin(cx)^2 + 2b \arcsin(cx) (a\sqrt{1 - c^2 x^2} - bcx \log(1 - e^{2i \arcsin(cx)}))}{x \sqrt{d - c^2 dx^2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output

```
-((Sqrt[1 - c^2*x^2]*(b^2*(I*c*x + Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 + 2*b*ArcSin[c*x]*(a*Sqrt[1 - c^2*x^2] - b*c*x*Log[1 - E^((2*I)*ArcSin[c*x])]) + a*(a*Sqrt[1 - c^2*x^2] - 2*b*c*x*Log[c*x]) + I*b^2*c*x*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(x*Sqrt[d - c^2*d*x^2]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {5186, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx \\ & \quad \downarrow \text{5186} \\ & \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x} dx}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{dx} \\ & \quad \downarrow \text{5136} \\ & \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{cx} d \arcsin(cx)}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{dx} \\ & \quad \downarrow \text{3042} \end{aligned}$$

$$\begin{aligned}
 & \frac{2bc\sqrt{1-c^2x^2} \int -((a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2})) d \arcsin(cx)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} \quad \text{---} \\
 & \qquad \qquad \qquad \downarrow \text{25} \\
 & \frac{2bc\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} \quad \text{---} \\
 & \qquad \qquad \qquad \downarrow \text{4200} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(2i \int -\frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} + \\
 & \qquad \qquad \qquad \downarrow \text{25} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(-2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} + \\
 & \qquad \qquad \qquad \downarrow \text{2620} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(-2i(\frac{1}{2}i \log(1-e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{2}ib \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx)) - \frac{i(a+b \arcsin(cx))^2}{2b} \right)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} + \\
 & \qquad \qquad \qquad \downarrow \text{2715} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(-2i(\frac{1}{2}i \log(1-e^{2i \arcsin(cx)})(a+b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \log(1-e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)}) - \frac{i(a+b \arcsin(cx))^2}{2b} \right)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} + \\
 & \qquad \qquad \qquad \downarrow \text{2838} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(-2i(\frac{1}{2}i \log(1-e^{2i \arcsin(cx)})(a+b \arcsin(cx)) + \frac{1}{4}b \text{PolyLog}(2, e^{2i \arcsin(cx)})) - \frac{i(a+b \arcsin(cx))^2}{2b} \right)}{\frac{\sqrt{d-c^2dx^2}}{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2} dx} \quad \text{---}
 \end{aligned}$$

input $\text{Int}[(a + b \cdot \text{ArcSin}[c \cdot x])^2 / (x^2 \cdot \text{Sqrt}[d - c^2 \cdot d \cdot x^2]), x]$

output $-\left(\frac{\text{Sqrt}[d - c^2 \cdot d \cdot x^2] \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^2}{d \cdot x}\right) + (2 \cdot b \cdot c \cdot \text{Sqrt}[1 - c^2 \cdot x^2] \cdot \left(\frac{-1/2 \cdot I}{b} \cdot (a + b \cdot \text{ArcSin}[c \cdot x])^2 - (2 \cdot I) \cdot \left(\frac{I}{2}\right) \cdot (a + b \cdot \text{ArcSin}[c \cdot x]) \cdot \text{Log}[1 - E^{(2 \cdot I) \cdot \text{ArcSin}[c \cdot x]}]\right) + (b \cdot \text{PolyLog}[2, E^{(2 \cdot I) \cdot \text{ArcSin}[c \cdot x]}]) / 4) / \text{Sqrt}[d - c^2 \cdot d \cdot x^2]$

Defintions of rubi rules used

rule 25 $\text{Int}[-(F x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F x, x], x]$

rule 2620 $\text{Int}[\left(\frac{(F) \cdot ((g) \cdot (e) + (f) \cdot (x))\right)^{(n)} \cdot ((c) + (d) \cdot (x))^{(m)}}{((a) + (b) \cdot (F) \cdot ((g) \cdot (e) + (f) \cdot (x)))^{(n)}}$, $x_Symbol] \rightarrow \text{Simp}[\left(\frac{(c + d \cdot x)^m}{(b \cdot f \cdot g \cdot n \cdot \text{Log}[F])}\right) \cdot \text{Log}[1 + b \cdot ((F \cdot (g \cdot (e + f \cdot x)))^n / a)], x] - \text{Simp}[d \cdot (m / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F])) \quad \text{Int}[(c + d \cdot x)^{(m-1)} \cdot \text{Log}[1 + b \cdot ((F \cdot (g \cdot (e + f \cdot x)))^n / a)], x], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \} \ \&\& \ \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a) + (b) \cdot (F) \cdot ((e) \cdot (c) + (d) \cdot (x))], x_Symbol] \rightarrow \text{Simp}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]) \quad \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F \cdot (e \cdot (c + d \cdot x)))^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x \} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c) \cdot ((d) + (e) \cdot (x))^{(n)}] / (x), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) \cdot e \cdot x^n] / n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x \} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4200 $\text{Int}[\left(\frac{(c) + (d) \cdot (x)}{(c) + (d) \cdot (x)}\right)^{(m)} \cdot \tan[(e) + \text{Pi} \cdot (k) + (f) \cdot (x)], x_Symbol] \rightarrow \text{Simp}[I \cdot ((c + d \cdot x)^{(m+1}) / (d \cdot (m+1))), x] - \text{Simp}[2 \cdot I \quad \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot (E^{(2 \cdot I \cdot (e + f \cdot x))} / (1 + E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot E^{(2 \cdot I \cdot (e + f \cdot x))}))], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x \} \ \&\& \ \text{IntegerQ}[4 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b *ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b *ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(187) = 374$.

Time = 0.64 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.33

method	result
default	$-\frac{a^2\sqrt{-c^2dx^2+d}}{dx} + b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}cx+c^2x^2-1\right)\arcsin(cx)^2}{(c^2x^2-1)xd} + \frac{2i\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(i\arcsin(cx)\right)}{(c^2x^2-1)xd}\right)$
parts	$-\frac{a^2\sqrt{-c^2dx^2+d}}{dx} + b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}cx+c^2x^2-1\right)\arcsin(cx)^2}{(c^2x^2-1)xd} + \frac{2i\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(i\arcsin(cx)\right)}{(c^2x^2-1)xd}\right)$

input `int((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output `-a^2/d/x*(-c^2*d*x^2+d)^(1/2)+b^2*(-(d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)^2/(c^2*x^2-1)/x/d+2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c)+2*a*b*(2*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*arcsin(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)/d/x/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c)`

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + dx^2}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^4 - d*x^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 \sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))**2/x**2/(-c**2*d*x**2+d)**(1/2), x)`

output `Integral((a + b*asin(c*x))**2/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + dx^2}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-((-1)^(-2*c^2*d*x^2 + 2*d)*sqrt(d)*log(-2*c^2*d + 2*d/x^2) + sqrt(d)*log(x^2 - 1/c^2))*a*b*c/d + b^2*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2), x)/sqrt(d) - 2*sqrt(-c^2*d*x^2 + d)*a*b*arcsin(c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a^2/(d*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)),x)`

output `int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} a^2 + 2 \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) abx + \left(\int \frac{\operatorname{asin}(cx)^2}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) b^2 x}{\sqrt{d} x}$$

input `int((a+b*asin(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x)`

output

```
( - sqrt( - c**2*x**2 + 1)*a**2 + 2*int(asin(c*x)/(sqrt( - c**2*x**2 + 1)*  
x**2),x)*a*b*x + int(asin(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*x)  
/(sqrt(d)*x)
```

3.239 $\int \frac{(a+b \arcsin(cx))^2}{x^3 \sqrt{d-c^2 dx^2}} dx$

Optimal result	2343
Mathematica [A] (verified)	2344
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Maple [A] (verified)	2349
Fricas [F]	2350
Sympy [F]	2350
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Optimal result

Integrand size = 29, antiderivative size = 402

$$\int \frac{(a+b \arcsin(cx))^2}{x^3 \sqrt{d-c^2 dx^2}} dx = -\frac{bc\sqrt{1-c^2 x^2}(a+b \arcsin(cx))}{x\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2}(a+b \arcsin(cx))^2}{2dx^2} - \frac{c^2\sqrt{1-c^2 x^2}(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{d-c^2 dx^2}} - \frac{b^2 c^2 \sqrt{1-c^2 x^2} \operatorname{arctanh}(\sqrt{1-c^2 x^2})}{\sqrt{d-c^2 dx^2}} + \frac{ibc^2 \sqrt{1-c^2 x^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{d-c^2 dx^2}} - \frac{ibc^2 \sqrt{1-c^2 x^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{d-c^2 dx^2}} - \frac{b^2 c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{d-c^2 dx^2}} + \frac{b^2 c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{d-c^2 dx^2}}$$

output

```
-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/x/(-c^2*d*x^2+d)^(1/2)-1/2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/d/x^2-c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcs
in(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*
(-c^2*x^2+1)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)+I*b*c^
2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2)
)/(-c^2*d*x^2+d)^(1/2)-I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylo
g(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*(-c^2*x^2+1)^(1
/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)+b^2*c^2*(-c^
2*x^2+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.21

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx$$

$$= \frac{-\frac{4a^2 \sqrt{d - c^2 dx^2}}{x^2} + 4a^2 c^2 \sqrt{d} \log(x) - 4a^2 c^2 \sqrt{d} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right) + \frac{2abc^2 d^2 (1 - c^2 x^2)^{3/2} (-2 \cot(\frac{1}{2} \arcsin(cx)) - \arcsin(cx))}{(d - c^2 dx^2)^{3/2}}}{(d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output

```
((-4*a^2*Sqrt[d - c^2*d*x^2])/x^2 + 4*a^2*c^2*Sqrt[d]*Log[x] - 4*a^2*c^2*S
qrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*c^2*d^2*(1 - c^2*x^2)
^(3/2)*(-2*Cot[ArcSin[c*x]/2] - ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 + 4*ArcSi
n[c*x]*Log[1 - E^(I*ArcSin[c*x])] - 4*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x]
)]) + (4*I)*PolyLog[2, -E^(I*ArcSin[c*x])] - (4*I)*PolyLog[2, E^(I*ArcSin[c
*x])] + ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 - 2*Tan[ArcSin[c*x]/2]))/(d - c^2
*d*x^2)^(3/2) + (b^2*c^2*d^2*(1 - c^2*x^2)^(3/2)*(-4*ArcSin[c*x]*Cot[ArcSi
n[c*x]/2] - ArcSin[c*x]^2*Csc[ArcSin[c*x]/2]^2 + 4*ArcSin[c*x]^2*Log[1 - E
^(I*ArcSin[c*x])] - 4*ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x])] + 8*Log[Tan
[ArcSin[c*x]/2]] + (8*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] - (8*I
)*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] - 8*PolyLog[3, -E^(I*ArcSin[c*
x])] + 8*PolyLog[3, E^(I*ArcSin[c*x])] + ArcSin[c*x]^2*Sec[ArcSin[c*x]/2]^
2 - 4*ArcSin[c*x]*Tan[ArcSin[c*x]/2]))/(d - c^2*d*x^2)^(3/2))/(8*d)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5204, 5138, 243, 73, 221, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx \\
 & \quad \downarrow \text{5204} \\
 & \frac{1}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d - c^2 dx^2}} dx + \frac{bc \sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x^2} dx}{\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2 dx^2} \\
 & \quad \downarrow \text{5138} \\
 & \frac{1}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d - c^2 dx^2}} dx + \frac{bc \sqrt{1 - c^2 x^2} \left(bc \int \frac{1}{x \sqrt{1 - c^2 x^2}} dx - \frac{a + b \arcsin(cx)}{x} \right)}{\sqrt{d - c^2 dx^2}} - \\
 & \quad \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2 dx^2} \\
 & \quad \downarrow \text{243} \\
 & \frac{1}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d - c^2 dx^2}} dx + \frac{bc \sqrt{1 - c^2 x^2} \left(\frac{1}{2} bc \int \frac{1}{x^2 \sqrt{1 - c^2 x^2}} dx^2 - \frac{a + b \arcsin(cx)}{x} \right)}{\sqrt{d - c^2 dx^2}} - \\
 & \quad \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2 dx^2} \\
 & \quad \downarrow \text{73} \\
 & \frac{1}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d - c^2 dx^2}} dx + \frac{bc \sqrt{1 - c^2 x^2} \left(-\frac{b \int \frac{1}{\frac{1}{c^2} - \frac{x^4}{c^2}} d \sqrt{1 - c^2 x^2}}{c} - \frac{a + b \arcsin(cx)}{x} \right)}{\sqrt{d - c^2 dx^2}} - \\
 & \quad \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2 dx^2} \\
 & \quad \downarrow \text{221} \\
 & \frac{1}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d - c^2 dx^2}} dx + \frac{bc \sqrt{1 - c^2 x^2} \left(-\frac{a + b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1 - c^2 x^2}) \right)}{\sqrt{d - c^2 dx^2}} - \\
 & \quad \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2 dx^2}
 \end{aligned}$$

$$\begin{aligned}
& \downarrow 5218 \\
& \frac{c^2\sqrt{1-c^2x^2} \int \frac{(a+b\arcsin(cx))^2}{cx} d\arcsin(cx) +}{2\sqrt{d-c^2dx^2}} + \\
& \frac{bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{x} - b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right) \right)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2dx^2} \\
& \downarrow 3042 \\
& \frac{c^2\sqrt{1-c^2x^2} \int (a+b\arcsin(cx))^2 \csc(\arcsin(cx)) d\arcsin(cx) +}{2\sqrt{d-c^2dx^2}} + \\
& \frac{bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{x} - b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right) \right)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2dx^2} \\
& \downarrow 4671 \\
& \frac{c^2\sqrt{1-c^2x^2} \left(-2b \int (a+b\arcsin(cx)) \log(1-e^{i\arcsin(cx)}) d\arcsin(cx) + 2b \int (a+b\arcsin(cx)) \log(1+e^{i\arcsin(cx)}) d\arcsin(cx) \right)}{2\sqrt{d-c^2dx^2}} + \\
& \frac{bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{x} - b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right) \right)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2dx^2} \\
& \downarrow 3011 \\
& \frac{c^2\sqrt{1-c^2x^2} \left(2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) d\arcsin(cx)) - 2b \right)}{2\sqrt{d-c^2dx^2}} + \\
& \frac{bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{x} - b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right) \right)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2dx^2} \\
& \downarrow 2720 \\
& \frac{c^2\sqrt{1-c^2x^2} \left(2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - b \int e^{-i\arcsin(cx)} \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) de^{i\arcsin(cx)}) \right)}{2\sqrt{d-c^2dx^2}} + \\
& \frac{bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{x} - b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right) \right)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2dx^2} \\
& \downarrow 7143 \\
& \frac{c^2\sqrt{1-c^2x^2} \left(-2\operatorname{arctanh}(e^{i\arcsin(cx)}) (a+b\arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) (a+b\arcsin(cx)) - b \int \operatorname{PolyLog}(2, -e^{i\arcsin(cx)}) d\arcsin(cx)) \right)}{2\sqrt{d-c^2dx^2}} + \\
& \frac{bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{x} - b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right) \right)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2dx^2}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]`

output `-1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(d*x^2) + (b*c*Sqrt[1 - c^2*x^2]*(-(a + b*ArcSin[c*x])/x) - b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[d - c^2*d*x^2] + (c^2*Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])]))/(2*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 $\text{Int}[\text{Log}[1 + (e_.) * ((F_.)^{((c_.) * (a_.) + (b_.) * (x_)))})^{(n_.)}] * ((f_.) + (g_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n]) / (b*c*n*\text{Log}[F]), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^{m-1} * \text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.) * (x_)] * ((c_.) + (d_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m * (\text{ArcTanh}[E^{(I*(e + f*x))}] / f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 5138 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)^{(n_.)} * ((d_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1} * ((a + b*\text{ArcSin}[c*x])^n / (d*(m+1))), x] - \text{Simp}[b*c*(n / (d*(m+1))) \text{Int}[(d*x)^{m+1} * ((a + b*\text{ArcSin}[c*x])^{n-1} / \text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)^{(n_.)} * ((f_.) * (x_))^{(m_.)} * ((d_.) + (e_.) * (x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{m+1} * (d + e*x^2)^{p+1} * ((a + b*\text{ArcSin}[c*x])^n / (d*f*(m+1))), x] + (\text{Simp}[c^2 * ((m + 2*p + 3) / (f^2*(m+1))) \text{Int}[(f*x)^{m+2} * (d + e*x^2)^p * (a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n / (f*(m+1))) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[(f*x)^{m+1} * (1 - c^2*x^2)^{p+1/2} * (a + b*\text{ArcSin}[c*x])^{n-1}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{ILtQ}[m, -1]$

rule 5218 $\text{Int}[(a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.)^{(n_.)} * (x_)^{(m_.)} / \text{Sqrt}[(d_.) + (e_.) * (x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/c^{m+1}) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] \text{Subst}[\text{Int}[(a + b*x)^n * \text{Sin}[x]^m, x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a^2\sqrt{-c^2dx^2+d}}{2dx^2} - \frac{a^2c^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{2\sqrt{d}} + b^2 \left(-\frac{(c^2x^2 \arcsin(cx) - 2cx\sqrt{-c^2x^2+1} - \arcsin(cx)) \arcsin(cx) \sqrt{-d}}{2x^2d(c^2x^2-1)} \right)$
parts	$-\frac{a^2\sqrt{-c^2dx^2+d}}{2dx^2} - \frac{a^2c^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{2\sqrt{d}} + b^2 \left(-\frac{(c^2x^2 \arcsin(cx) - 2cx\sqrt{-c^2x^2+1} - \arcsin(cx)) \arcsin(cx) \sqrt{-d}}{2x^2d(c^2x^2-1)} \right)$

input

```
int((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*a^2/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*
(-c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/2*(c^2*x^2*arcsin(c*x)-2*c*x*(-c^2*x^2+1)
^(1/2)-arcsin(c*x))*arcsin(c*x)*(-d*(c^2*x^2-1))^(1/2)/x^2/d/(c^2*x^2-1)+1
/2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*(arcsin(c*x))^2*
ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+
1)^(1/2))-arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*pol
ylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-2*
polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))+4*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))*c
^2)+2*a*b*(-1/2*(c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-arcsin(c*x))*
(-d*(c^2*x^2-1))^(1/2)/x^2/d/(c^2*x^2-1)-1/2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*
x^2-1))^(1/2)/d/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-
I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+polylog(2,-I*c*x-(-c^2*x^2+1)
^(1/2))-polylog(2,I*c*x+(-c^2*x^2+1)^(1/2)))*c^2)
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + dx^3}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^5 - d*x^3), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^3 \sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))**2/x**3/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + dx^3}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `-1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) + sqrt(-c^2*d*x^2 + d)/(d*x^2))*a^2 - sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^5 - d*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx$$

input `int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)),x)`

output `int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 \sqrt{d - c^2 dx^2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} a^2 + 4 \left(\int \frac{a \sin(cx)}{\sqrt{-c^2 x^2 + 1} x^3} dx \right) a b x^2 + 2 \left(\int \frac{a \sin^2(cx)}{\sqrt{-c^2 x^2 + 1} x^3} dx \right) b^2 x^2 + \log \left(\tan \left(\frac{a \sin(cx)}{2} \right) \right) a^2 c^2 x^2}{2 \sqrt{d} x^2}$$

input `int((a+b*asin(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)`

output

```
( - sqrt( - c**2*x**2 + 1)*a**2 + 4*int(asin(c*x)/(sqrt( - c**2*x**2 + 1)*  
x**3),x)*a*b*x**2 + 2*int(asin(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**3),x)*b*  
*2*x**2 + log(tan(asin(c*x)/2))*a**2*c**2*x**2)/(2*sqrt(d)*x**2)
```

3.240 $\int \frac{(a+b \arcsin(cx))^2}{x^4 \sqrt{d-c^2 dx^2}} dx$

Optimal result	2353
Mathematica [A] (verified)	2354
Rubi [A] (verified)	2354
Maple [B] (verified)	2359
Fricas [F]	2360
Sympy [F]	2360
Maxima [F]	2360
Giac [F(-2)]	2361
Mupad [F(-1)]	2361
Reduce [F]	2362

Optimal result

Integrand size = 29, antiderivative size = 312

$$\int \frac{(a+b \arcsin(cx))^2}{x^4 \sqrt{d-c^2 dx^2}} dx = -\frac{b^2 c^2 \sqrt{d-c^2 dx^2}}{3dx} - \frac{bc \sqrt{1-c^2 x^2} (a+b \arcsin(cx))}{3x^2 \sqrt{d-c^2 dx^2}} - \frac{2ic^3 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2}{3\sqrt{d-c^2 dx^2}} - \frac{\sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3dx^3} - \frac{2c^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3dx} + \frac{4bc^3 \sqrt{1-c^2 x^2} (a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)})}{3\sqrt{d-c^2 dx^2}} - \frac{2ib^2 c^3 \sqrt{1-c^2 x^2} \text{PolyLog}(2, e^{2i \arcsin(cx)})}{3\sqrt{d-c^2 dx^2}}$$

output

```
-1/3*b^2*c^2*(-c^2*d*x^2+d)^(1/2)/d/x-1/3*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/x^2/(-c^2*d*x^2+d)^(1/2)-2/3*I*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/d/x^3-2/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/d/x+4/3*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)-2/3*I*b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.86

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{1 - c^2 x^2} (abcx + a^2 \sqrt{1 - c^2 x^2} + 2a^2 c^2 x^2 \sqrt{1 - c^2 x^2} + b^2 c^2 x^2 \sqrt{1 - c^2 x^2} + b^2 (2ic^3 x^3 + \sqrt{1 - c^2 x^2} + 2$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output

```
-1/3*(Sqrt[1 - c^2*x^2]*(a*b*c*x + a^2*Sqrt[1 - c^2*x^2] + 2*a^2*c^2*x^2*Sqrt[1 - c^2*x^2] + b^2*c^2*x^2*Sqrt[1 - c^2*x^2] + b^2*((2*I)*c^3*x^3 + Sqrt[1 - c^2*x^2] + 2*c^2*x^2*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 - b*ArcSin[c*x]*(-(b*c*x) - 2*a*Sqrt[1 - c^2*x^2]*(1 + 2*c^2*x^2) + 4*b*c^3*x^3*Log[1 - E^((2*I)*ArcSin[c*x])]) - 4*a*b*c^3*x^3*Log[c*x] + (2*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5204, 5138, 242, 5186, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx$$

$$\downarrow 5204$$

$$\frac{2}{3}c^2 \int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d - c^2 dx^2}} dx + \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x^3} dx}{3\sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{3dx^3}$$

$$\downarrow 5138$$

$$\frac{2}{3}c^2 \left(-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2} \left(2i \int -\frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))}{2b} \right)}{\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3dx^3} \right)$$

↓ 25

$$\frac{2}{3}c^2 \left(-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2} \left(-2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1-e^{2i\arcsin(cx)}} d\arcsin(cx) - \frac{i(a+b\arcsin(cx))}{2b} \right)}{\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3dx^3} \right)$$

↓ 2620

$$\frac{2}{3}c^2 \left(-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \frac{1}{2}ib \int \dots \right) \right)}{\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3dx^3} \right)$$

↓ 2715

$$\frac{2}{3}c^2 \left(-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) - \frac{1}{4}b \int \dots \right) \right)}{\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3dx^3} \right)$$

↓ 2838

$$\frac{2}{3}c^2 \left(-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2} \left(-2i \left(\frac{1}{2}i \log(1-e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) + \frac{1}{4}b \text{Pol} \dots \right) \right)}{\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3dx^3} \right)$$

input `Int[(a + b*ArcSin[c*x])^2/(x^4*sqrt[d - c^2*d*x^2]),x]`

output

$$-1/3*(\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(d*x^3) + (2*b*c*\text{Sqrt}[1 - c^2*x^2]*(-1/2*(b*c*\text{Sqrt}[1 - c^2*x^2])/x - (a + b*\text{ArcSin}[c*x])/(2*x^2)))/(3*\text{Sqrt}[d - c^2*d*x^2]) + (2*c^2*(-(\text{Sqrt}[d - c^2*d*x^2]*(a + b*\text{ArcSin}[c*x])^2)/(d*x)) + (2*b*c*\text{Sqrt}[1 - c^2*x^2]*(((1/2*I)*(a + b*\text{ArcSin}[c*x])^2)/b - (2*I)*((I/2)*(a + b*\text{ArcSin}[c*x])*\text{Log}[1 - E^((2*I)*\text{ArcSin}[c*x])]) + (b*\text{PolyLog}[2, E^((2*I)*\text{ArcSin}[c*x])])]/4)))/\text{Sqrt}[d - c^2*d*x^2])/3$$

Defintions of rubi rules used

rule 25

$$\text{Int}[-(F_x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F_x, x], x]$$

rule 242

$$\text{Int}[((c_)*(x_))^m*((a_) + (b_)*(x_)^2)^p, x_Symbol] \rightarrow \text{Simp}[(c*x)^{m+1}*((a + b*x^2)^{p+1}/(a*c*(m+1))), x] /; \text{FreeQ}\{a, b, c, m, p\}, x] \&\& \text{EqQ}[m + 2*p + 3, 0] \&\& \text{NeQ}[m, -1]$$

rule 2620

$$\text{Int}[(((F_)^((g_)*((e_)+(f_)*(x_))))^n*((c_) + (d_)*(x_))^m)/((a_) + (b_)*((F_)^((g_)*((e_)+(f_)*(x_))))^n), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m/(b*f*g*n*\text{Log}[F])]*\text{Log}[1 + b*((F^(g*(e + f*x)))^n/a)], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \quad \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$$

rule 2715

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^n], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \quad \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$$

rule 2838

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^n)]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$$

rule 4200 $\text{Int}[(c_.) + (d_.)(x_)^{(m_.)} \tan[(e_.) + \text{Pi}(k_.) + (f_.)(x_)], x_Symbol] \rightarrow \text{Simp}[I*(c + d*x)^{(m + 1)}/(d*(m + 1)), x] - \text{Simp}[2*I \text{ Int}[(c + d*x)^m * E^{(2*I*k*Pi)} * (E^{(2*I*(e + f*x)})/(1 + E^{(2*I*k*Pi)} * E^{(2*I*(e + f*x)}))], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

rule 5136 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)}/(x_), x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n * \text{Cot}[x], x], x, \text{ArcSin}[c*x]] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0]$

rule 5138 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)} * ((d_.)(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)} * ((a + b*\text{ArcSin}[c*x])^n / (d*(m + 1))), x] - \text{Simp}[b*c*(n / (d*(m + 1))) \text{ Int}[(d*x)^{(m + 1)} * ((a + b*\text{ArcSin}[c*x])^{(n - 1)}) / \text{Sqrt}[1 - c^2 * x^2]], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5186 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)} * ((f_.)(x_)^{(m_.)} * ((d_) + (e_.)(x_)^2)^{(p_.)}), x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)} * (d + e*x^2)^{(p + 1)} * ((a + b*\text{ArcSin}[c*x])^n / (d*f*(m + 1))), x] - \text{Simp}[b*c*(n / (f*(m + 1))) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m + 1)} * (1 - c^2*x^2)^{(p + 1/2)} * (a + b*\text{ArcSin}[c*x])^{(n - 1)}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{EqQ}[m + 2*p + 3, 0] \&\& \text{NeQ}[m, -1]$

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.)]^{(n_.)} * ((f_.)(x_)^{(m_.)} * ((d_) + (e_.)(x_)^2)^{(p_.)}), x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)} * (d + e*x^2)^{(p + 1)} * ((a + b*\text{ArcSin}[c*x])^n / (d*f*(m + 1))), x] + (\text{Simp}[c^2 * ((m + 2*p + 3) / (f^2*(m + 1))) \text{ Int}[(f*x)^{(m + 2)} * (d + e*x^2)^p * (a + b*\text{ArcSin}[c*x])^n], x] - \text{Simp}[b*c*(n / (f*(m + 1))) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m + 1)} * (1 - c^2*x^2)^{(p + 1/2)} * (a + b*\text{ArcSin}[c*x])^{(n - 1)}], x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{ILtQ}[m, -1]$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(294) = 588$.

Time = 0.78 (sec) , antiderivative size = 2319, normalized size of antiderivative = 7.43

method	result	size
default	Expression too large to display	2319
parts	Expression too large to display	2319

input `int((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)`

output

$$\begin{aligned}
 & -4*I*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^2*(-c^2*x^2+1) \\
 & ^{(1/2)*arcsin(c*x)*c^5+a^2*(-1/3/d/x^3*(-c^2*d*x^2+d)^{(1/2)}-2/3*c^2/d/x*(-c^2*d*x^2+d)^{(1/2)})+4/3*I*b^2*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}/d/ \\
 & (c^2*x^2-1)*c^3*polylog(2,I*c*x+(-c^2*x^2+1)^{(1/2)})-2/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*(-c^2*x^2+1)^{(1/2)*arcsin(c*x)^2*c^3- \\
 & 4/3*I*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^5*c^8-4*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*arcsin(c*x)*c^6+2/3*I* \\
 & a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*c^6+2/3*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x*arcsin(c*x)*c^4+2/3*I*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x*c^4+8/3*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d/x*arcsin(c*x)*c^2+1/3*a*b*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d/x^2*(-c^2*x^2+1)^{(1/2)*c-4/3*a*b*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}/d/(c^2*x^2-1)*ln((I*c*x+(-c^2*x^2+1))^{(1/2)})^2-1)*c^3-4/3*b^2*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}/d/(c^2*x^2-1)*c^3*arcsin(c*x)*ln(1-I*c*x+(-c^2*x^2+1)^{(1/2)})+1/3*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d/x^2*(-c^2*x^2+1)^{(1/2)*arcsin(c*x)*c-4/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^5*arcsin(c*x)*c^8+2/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*arcsin(c*x)*c^6-I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2-1)/d*x^2*(-c^2*x^2+1)^{(1/2)*c^5+2/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(3*c^4*x^4-2*c^2*x^2...
 \end{aligned}$$

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + dx^4}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^2*d*x^6 - d*x^4), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate((a+b*asin(c*x))**2/x**4/(-c**2*d*x**2+d)**(1/2), x)`

output `Integral((a + b*asin(c*x))**2/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{-c^2 dx^2 + dx^4}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output

```
1/3*(4*c^2*log(x)/sqrt(d) - 1/(sqrt(d)*x^2))*a*b*c - 2/3*a*b*(2*sqrt(-c^2*
d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arcsin(c*x) - 1/3*a^2
*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3)) + b^2*i
ntegrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/(sqrt(c*x + 1)*sqrt(
-c*x + 1)*x^4), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^4 \sqrt{d - c^2 dx^2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 \sqrt{d - c^2 x^2}} dx$$

$$= \frac{-2\sqrt{-c^2 x^2 + 1} a^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} a^2 + 6 \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} x^4} dx \right) ab x^3 + 3 \left(\int \frac{\arcsin(cx)^2}{\sqrt{-c^2 x^2 + 1} x^4} dx \right) b^2 x^3}{3\sqrt{d} x^3}$$

input `int((a+b*asin(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x)`

output `(- 2*sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a**2 + 6*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*x**4),x)*a*b*x**3 + 3*int(asin(c*x)**2/(sqrt(- c**2*x**2 + 1)*x**4),x)*b**2*x**3)/(3*sqrt(d)*x**3)`

$$3.241 \quad \int \frac{x^5 (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

Optimal result	2363
Mathematica [A] (verified)	2364
Rubi [A] (verified)	2365
Maple [B] (verified)	2374
Fricas [F]	2375
Sympy [F]	2376
Maxima [F]	2376
Giac [F(-2)]	2377
Mupad [F(-1)]	2377
Reduce [F]	2377

Optimal result

Integrand size = 29, antiderivative size = 440

$$\begin{aligned} \int \frac{x^5 (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = & -\frac{32b^2 \sqrt{d - c^2 dx^2}}{9c^6 d^2} \\ & + \frac{2b^2 (d - c^2 dx^2)^{3/2}}{27c^6 d^3} - \frac{10bx \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{3c^5 d \sqrt{d - c^2 dx^2}} \\ & - \frac{2bx^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{9c^3 d \sqrt{d - c^2 dx^2}} + \frac{x^4 (a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} \\ & + \frac{8\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{3c^6 d^2} + \frac{4x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{3c^4 d^2} \\ & + \frac{4ib \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^6 d \sqrt{d - c^2 dx^2}} \\ & - \frac{2ib^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^6 d \sqrt{d - c^2 dx^2}} \\ & + \frac{2ib^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^6 d \sqrt{d - c^2 dx^2}} \end{aligned}$$

output

```
-32/9*b^2*(-c^2*d*x^2+d)^(1/2)/c^6/d^2+2/27*b^2*(-c^2*d*x^2+d)^(3/2)/c^6/d
^3-10/3*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^5/d/(-c^2*d*x^2+d)^(1/2)
)-2/9*b*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)
)+x^4*(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+8/3*(-c^2*d*x^2+d)^(1
/2)*(a+b*arcsin(c*x))^2/c^6/d^2+4/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c
*x))^2/c^4/d^2+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c
^2*x^2+1)^(1/2))/c^6/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*pol
ylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^6/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(
-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^6/d/(-c^2*d*x^
2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.03

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{576a^2 - 378b^2 - 288a^2c^2x^2 - 72a^2c^4x^4 + 810ab \arcsin(cx) + 405b^2 \arcsin(cx)^2}{(d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(x^5*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output

```
(576*a^2 - 378*b^2 - 288*a^2*c^2*x^2 - 72*a^2*c^4*x^4 + 810*a*b*ArcSin[c*x
] + 405*b^2*ArcSin[c*x]^2 - 376*b^2*Cos[2*ArcSin[c*x]] + 360*a*b*ArcSin[c*
x]*Cos[2*ArcSin[c*x]] + 180*b^2*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] + 2*b^2*Cos
[4*ArcSin[c*x]] - 18*a*b*ArcSin[c*x]*Cos[4*ArcSin[c*x]] - 9*b^2*ArcSin[c
*x]^2*Cos[4*ArcSin[c*x]] - 432*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I
*E^(I*ArcSin[c*x])] + 432*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I
*ArcSin[c*x])] + 432*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[Ar
cSin[c*x]/2]] - 432*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[Ar
cSin[c*x]/2]] - (432*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c
*x])] + (432*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])] - 37
2*a*b*Sin[2*ArcSin[c*x]] - 372*b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]] + 6*a*b*
Sin[4*ArcSin[c*x]] + 6*b^2*ArcSin[c*x]*Sin[4*ArcSin[c*x]])/(216*c^6*d*Sqrt
[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 3.05 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.15, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {5206, 5210, 243, 53, 2009, 5138, 243, 53, 2009, 5182, 2009, 5210, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^5(a+b\arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx \\
 & \quad \downarrow \text{5206} \\
 & \frac{2b\sqrt{1-c^2x^2} \int \frac{x^4(a+b\arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{4 \int \frac{x^3(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x^4(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{5210} \\
 & \frac{4 \left(\frac{2b\sqrt{1-c^2x^2} \int x^2(a+b\arcsin(cx)) dx}{3c\sqrt{d-c^2dx^2}} + \frac{2 \int \frac{x(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} - \frac{x^2\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3c^2d} \right)}{cd\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{243} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x^3}{\sqrt{1-c^2x^2}} dx}{3c} - \frac{x^3(a+b\arcsin(cx))}{3c^2} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^4(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{53} \\
 & \frac{4 \left(\frac{2b\sqrt{1-c^2x^2} \int x^2(a+b\arcsin(cx)) dx}{3c\sqrt{d-c^2dx^2}} + \frac{2 \int \frac{x(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{3c^2} - \frac{x^2\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{3c^2d} \right)}{cd\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{53} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{6c} - \frac{x^3(a+b\arcsin(cx))}{3c^2} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^4(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}
 \end{aligned}$$

$$\begin{aligned}
& 4 \left(\frac{2b\sqrt{1-c^2x^2} \int x^2(a+b \arcsin(cx)) dx}{3c\sqrt{d-c^2dx^2}} + \frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{3c^2} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{c^2d}{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx)) dx}{1-c^2x^2}}{c^2} + \frac{b \int \left(\frac{1}{c^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{c^2} \right) dx^2}{6c} - \frac{x^3(a+b \arcsin(cx))}{3c^2} \right)} \\
& \frac{cd\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} \\
& \frac{c^2d\sqrt{d-c^2dx^2}}{c^2d\sqrt{d-c^2dx^2}} \\
& \downarrow \text{2009} \\
& 4 \left(\frac{2b\sqrt{1-c^2x^2} \int x^2(a+b \arcsin(cx)) dx}{3c\sqrt{d-c^2dx^2}} + \frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{3c^2} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{c^2d}{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)} \\
& \frac{cd\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} \\
& \frac{c^2d\sqrt{d-c^2dx^2}}{c^2d\sqrt{d-c^2dx^2}} \\
& \downarrow \text{5138} \\
& 4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{3}bc \int \frac{x^3}{\sqrt{1-c^2x^2}} dx \right)}{3c\sqrt{d-c^2dx^2}} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} \right) \\
& \frac{c^2d}{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)} \\
& \frac{cd\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} \\
& \frac{c^2d\sqrt{d-c^2dx^2}}{c^2d\sqrt{d-c^2dx^2}} \\
& \downarrow \text{243}
\end{aligned}$$

$$\begin{aligned}
 & \frac{4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx^2 \right)}{3c\sqrt{d-c^2x^2}} - \frac{x^2\sqrt{d-c^2x^2}(a+b \arcsin(cx))^2}{3c^2d} \right)}{c^2d} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)}{c^2d\sqrt{d-c^2x^2}} + \\
 & \frac{cd\sqrt{d-c^2x^2}}{x^4(a+b \arcsin(cx))^2} \\
 & \quad \downarrow \text{53} \\
 & \frac{4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \int \left(\frac{1}{c^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}}{c^2} \right) dx^2 \right)}{3c\sqrt{d-c^2x^2}} - \frac{x^2\sqrt{d-c^2x^2}(a+b \arcsin(cx))^2}{3c^2d} \right)}{c^2d} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)}{c^2d\sqrt{d-c^2x^2}} + \\
 & \frac{cd\sqrt{d-c^2x^2}}{x^4(a+b \arcsin(cx))^2} \\
 & \quad \downarrow \text{2009} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)}{cd\sqrt{d-c^2x^2}} \\
 & \frac{4 \left(\frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2x^2}} dx}{3c^2} - \frac{x^2\sqrt{d-c^2x^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right) \right)}{3c\sqrt{d-c^2x^2}} \right)}{c^2d} \\
 & \frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2x^2}} \\
 & \quad \downarrow \text{5182}
 \end{aligned}$$

$$\begin{aligned}
 & 4 \left(\frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} - \frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{3c\sqrt{d-c^2dx^2}} \right) \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)}{cd\sqrt{d-c^2dx^2} \frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}} + \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x^2(a+b \arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)}{cd\sqrt{d-c^2dx^2} \frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}} + \\
 & 4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{3c^2d} \right) \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right)}{cd\sqrt{d-c^2dx^2} \frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}} + \\
 & 4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{3c^2d} \right)
 \end{aligned}$$

2009

5210

↓ 241

$$\begin{aligned}
 & 2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right) \\
 & \hline
 & \frac{cd\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} - \frac{c^2d\sqrt{d-c^2dx^2}}{c^2d} \\
 & 4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} \right) + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{c^2d} \\
 & \hline
 & c^2d
 \end{aligned}$$

↓ 5164

$$\begin{aligned}
 & 2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1-c^2x^2}}{c^4} \right)}{6c} \right) \\
 & \hline
 & \frac{cd\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} - \frac{c^2d\sqrt{d-c^2dx^2}}{c^2d} \\
 & 4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{3c^2} \right) + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{c^2d} \\
 & \hline
 & c^2d
 \end{aligned}$$

↓ 3042

$$2b\sqrt{1-c^2x^2} \left(\frac{\int (a+b \arcsin(cx)) \csc\left(\arcsin(cx)+\frac{\pi}{2}\right) d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} - \frac{x^3(a+b \arcsin(cx))}{3c^2} + \frac{b \left(\frac{2(1-c^2x^2)^{3/2}}{3c^4} - \dots \right)}{6c} \right)$$

$$4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{\frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{cd\sqrt{d-c^2dx^2}}{c^2d}}{3c^2} \right) + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{c^2d}$$

↓ 4669

$$2b\sqrt{1-c^2x^2} \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} \right)$$

$$4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{\frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{cd\sqrt{d-c^2dx^2}}{c^2d}}{3c^2} \right) + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{c^2d}$$

↓ 2715

$$2b\sqrt{1-c^2x^2} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})}{c^3} \right) \frac{cd\sqrt{d-c^2dx^2}}{c^2}$$

$$4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right) - \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c\sqrt{d-c^2dx^2}} \right)}{3c^2} \right) + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{c^2d}$$

↓ 2838

$$2b\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right) \frac{cd\sqrt{d-c^2dx^2}}{c^2}$$

$$4 \left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^2d} + \frac{x^4(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right) - \sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c\sqrt{d-c^2dx^2}} \right)}{3c^2} \right) + \frac{2b\sqrt{1-c^2x^2} \left(\frac{1}{3}x^3(a+b \arcsin(cx)) \right)}{c^2d}$$

input Int[(x^5*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]

output

$$\begin{aligned} & (x^4(a + b\text{ArcSin}[c*x])^2)/(c^2*d*\text{Sqrt}[d - c^2*d*x^2]) - (4*(-1/3*(x^2*\text{Sqrt}[d - c^2*d*x^2]*(a + b\text{ArcSin}[c*x])^2)/(c^2*d) + (2*b*\text{Sqrt}[1 - c^2*x^2]*(-1/6*(b*c*((-2*\text{Sqrt}[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4))) + (x^3*(a + b\text{ArcSin}[c*x]))/3))/(3*c*\text{Sqrt}[d - c^2*d*x^2]) + (2*(-((\text{Sqrt}[d - c^2*d*x^2]*(a + b\text{ArcSin}[c*x])^2)/(c^2*d) + (2*b*\text{Sqrt}[1 - c^2*x^2]*(a*x + (b*\text{Sqrt}[1 - c^2*x^2])/c + b*x*\text{ArcSin}[c*x]))/(c*\text{Sqrt}[d - c^2*d*x^2])))/(3*c^2)))/(c^2*d) - (2*b*\text{Sqrt}[1 - c^2*x^2]*((b*((-2*\text{Sqrt}[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/(6*c) - (x^3*(a + b\text{ArcSin}[c*x]))/(3*c^2) + (-((b*\text{Sqrt}[1 - c^2*x^2])/c^3) - (x*(a + b\text{ArcSin}[c*x]))/c^2 + ((-2*I)*(a + b\text{ArcSin}[c*x])*\text{ArcTan}[E^(I*\text{ArcSin}[c*x])] + I*b*\text{PolyLog}[2, (-I)*E^(I*\text{ArcSin}[c*x])] - I*b*\text{PolyLog}[2, I*E^(I*\text{ArcSin}[c*x])])/c^3)/c^2))/(c*d*\text{Sqrt}[d - c^2*d*x^2]) \end{aligned}$$

Defintions of rubi rules used

rule 53

$$\text{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$$

rule 241

$$\text{Int}[(x)^m*(a + b*x^2)^p, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$$

rule 243

$$\text{Int}[(x)^m*(a + b*x^2)^p, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$$

rule 2009

$$\text{Int}[u, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$$

rule 2715

$$\text{Int}[\text{Log}[(a + b*x)^m*(F)^n], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

rule 2838 $\text{Int}[\text{Log}[(c_)*(d_)+(e_)*(x_)^{(n_)}]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_)+\text{Pi}*(k_)+(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c+d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e+f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \ \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1-E^{(I*k*Pi)}*E^{(I*(e+f*x))}], x], x] + \text{Simp}[d*(m/f) \ \text{Int}[(c+d*x)^{(m-1)}*\text{Log}[1+E^{(I*k*Pi)}*E^{(I*(e+f*x))}], x], x) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5138 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*((d_)+(e_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a+b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a+b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1-c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5164 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}((d_)+(e_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[1/(c*d) \ \text{Subst}[\text{Int}[(a+b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a_)+\text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*(x_)*((d_)+(e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d+e*x^2)^{(p+1)}*((a+b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d+e*x^2)^p/(1-c^2*x^2)^p] \ \text{Int}[(1-c^2*x^2)^{(p+1/2)}*(a+b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d+e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5206

```

Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]

```

rule 5210

```

Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]

```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1086 vs. $2(417) = 834$.

Time = 0.86 (sec) , antiderivative size = 1087, normalized size of antiderivative = 2.47

method	result	size
default	Expression too large to display	1087
parts	Expression too large to display	1087

input

```
int(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```

a^2*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))-94/27*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*x^2-2*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+31/9*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x-65/24*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)^2-1/108*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*cos(4*arcsin(c*x))+1/24*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*cos(4*arcsin(c*x))*arcsin(c*x)^2+377/108*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcsin(c*x)^2*x^2-1/36*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)*sin(4*arcsin(c*x))-1/36*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*sin(4*arcsin(c*x))+10/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcsin(c*x)*x^2+31/9*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x-65/12*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)+1/12*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arcsin(c*x)*cos(...

```

Fricas [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^5}{(-c^2 dx^2 + d)^{3/2}} dx$$

input

```

integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

```

output

```

integral((b^2*x^5*arcsin(c*x)^2 + 2*a*b*x^5*arcsin(c*x) + a^2*x^5)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)

```

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^5(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate(x**5*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2), x)`

output `Integral(x**5*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^5}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2), x, algorithm="maxima")`

output `-1/3*a^2*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^4*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/3*((b^2*c^4*x^4 + 4*b^2*c^2*x^2 - 8*b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 3*(c^8*d^2*x^2 - c^6*d^2)*integrate(2/3*(3*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c^5*sqrt(d)*x^5*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (b^2*c^6*x^6 + 3*b^2*c^4*x^4 - 12*b^2*c^2*x^2 + 8*b^2)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/(c^9*d^2*x^4 - 2*c^7*d^2*x^2 + c^5*d^2), x)/(c^8*d^2*x^2 - c^6*d^2)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^5*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^5*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x^5}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) ab c^6 - 3\sqrt{-c^2 x^2 + 1} \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1}} dx \right) c^6 d}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} c^6 d}$$

input `int(x^5*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output

```
( - 6*sqrt( - c**2*x**2 + 1)*int((asin(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)*
c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c**6 - 3*sqrt( - c**2*x**2 + 1)
*int((asin(c*x)**2*x**5)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*
x**2 + 1)),x)*b**2*c**6 - a**2*c**4*x**4 - 4*a**2*c**2*x**2 + 8*a**2)/(3*s
qrt(d)*sqrt( - c**2*x**2 + 1)*c**6*d)
```

$$3.242 \quad \int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$$

Optimal result	2379
Mathematica [A] (verified)	2380
Rubi [A] (verified)	2380
Maple [B] (verified)	2387
Fricas [F]	2388
Sympy [F]	2389
Maxima [F]	2389
Giac [F(-2)]	2389
Mupad [F(-1)]	2390
Reduce [F]	2390

Optimal result

Integrand size = 29, antiderivative size = 414

$$\begin{aligned} \int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx = & -\frac{b^2x\sqrt{d-c^2dx^2}}{4c^4d^2} \\ & + \frac{b^2\sqrt{1-c^2x^2} \arcsin(cx)}{4c^5d\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^3d\sqrt{d-c^2dx^2}} \\ & + \frac{x^3(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{c^5d\sqrt{d-c^2dx^2}} \\ & + \frac{3x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^4d^2} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{2bc^5d\sqrt{d-c^2dx^2}} \\ & + \frac{2b\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{c^5d\sqrt{d-c^2dx^2}} \\ & - \frac{ib^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{c^5d\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
-1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^4/d^2+1/4*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c^5/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^4/d^2-1/2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c^5/d/(-c^2*d*x^2+d)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^5/d/(-c^2*d*x^2+d)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^5/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.75

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-4a^2 c \sqrt{d} x (-3 + c^2 x^2) + 12a^2 \sqrt{d - c^2 dx^2} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d(-1 + c^2 x^2)}}\right) + 2ab\sqrt{d}(8c^2 x^2 - 3)}{(d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output

```
(-4*a^2*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*b*Sqrt[d]*(8*c*x*ArcSin[c*x] + Sqrt[1 - c^2*x^2]*(-6*ArcSin[c*x]^2 + Cos[2*ArcSin[c*x]] + 4*Log[1 - c^2*x^2] + 2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])) + b^2*Sqrt[d]*(8*c*x*ArcSin[c*x]^2 - (8*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] + Sqrt[1 - c^2*x^2]*(-4*ArcSin[c*x]^3 + 2*ArcSin[c*x]*(Cos[2*ArcSin[c*x]] + 8*Log[1 + E^((2*I)*ArcSin[c*x])]) - Sin[2*ArcSin[c*x]] + 2*ArcSin[c*x]^2*(-4*I + Sin[2*ArcSin[c*x]])))))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)Time = 2.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5206, 5210, 262, 223, 5138, 262, 223, 5152, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^4(a+b\arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx \\
& \quad \downarrow \text{5206} \\
& \frac{3 \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} - \frac{2b\sqrt{1-c^2x^2} \int \frac{x^3(a+b\arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{5210} \\
& \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{2c} - \frac{x^2(a+b\arcsin(cx))}{2c^2} \right)}{cd\sqrt{d-c^2dx^2}} - \\
& \frac{3 \left(\frac{b\sqrt{1-c^2x^2} \int x(a+b\arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} \right)}{c^2d} + \\
& \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{262} \\
& \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{b \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} - \frac{x^2(a+b\arcsin(cx))}{2c^2} \right)}{cd\sqrt{d-c^2dx^2}} - \\
& \frac{3 \left(\frac{b\sqrt{1-c^2x^2} \int x(a+b\arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} \right)}{c^2d} + \\
& \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{223} \\
& \frac{3 \left(\frac{b\sqrt{1-c^2x^2} \int x(a+b\arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} \right)}{c^2d} - \\
& \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{x^2(a+b\arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{cd\sqrt{d-c^2dx^2}} + \\
& \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{5138}
\end{aligned}$$

$$\begin{aligned}
 & \frac{3 \left(\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b\arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} \right)}{c^2d} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b\arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^2(a+b\arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{c^2d\sqrt{d-c^2dx^2}} + \\
 & \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{262} \\
 & \frac{3 \left(\frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b\arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} \right)}{c^2d} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b\arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^2(a+b\arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{c^2d\sqrt{d-c^2dx^2}} + \\
 & \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{223} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b\arcsin(cx)) dx}{1-c^2x^2}}{c^2} - \frac{x^2(a+b\arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{c^2d\sqrt{d-c^2dx^2}} \\
 & \frac{3 \left(\frac{\int \frac{(a+b\arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{2c^2} - \frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} + \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2dx^2}} \right)}{c^2d} \\
 & \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{5152}
 \end{aligned}$$

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{x^2(a+b \arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{c^2d} + \frac{cd\sqrt{d-c^2dx^2}}{x^3(a+b \arcsin(cx))^2} - \frac{cd\sqrt{d-c^2dx^2}}{c^2d\sqrt{d-c^2dx^2}} - 3 \left(-\frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2dx^2}} \right) \right)}{c^2d}$$

5180

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^4} - \frac{x^2(a+b \arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{c^2d} + \frac{cd\sqrt{d-c^2dx^2}}{x^3(a+b \arcsin(cx))^2} - \frac{cd\sqrt{d-c^2dx^2}}{c^2d\sqrt{d-c^2dx^2}} - 3 \left(-\frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2dx^2}} \right) \right)}{c^2d}$$

3042

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^4} - \frac{x^2(a+b \arcsin(cx))}{2c^2} + \frac{b \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{2c} \right)}{c^2d} + \frac{cd\sqrt{d-c^2dx^2}}{x^3(a+b \arcsin(cx))^2} - \frac{cd\sqrt{d-c^2dx^2}}{c^2d\sqrt{d-c^2dx^2}} - 3 \left(-\frac{x\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2} \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c\sqrt{d-c^2dx^2}} \right) \right)}{c^2d}$$

4202

$$\begin{aligned}
 & 2b\sqrt{1-c^2x^2} \left(\frac{\frac{i(a+b\arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1+e^{2i\arcsin(cx)}} d\arcsin(cx)}{c^4} - \frac{x^2(a+b\arcsin(cx))}{2c^2} + \frac{b\left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c} \right) \\
 & \frac{cd\sqrt{d-c^2dx^2}}{x^3(a+b\arcsin(cx))^2} - \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & 3 \left(-\frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}} \right) \\
 & \qquad \qquad \qquad c^2d \\
 & \qquad \qquad \qquad \downarrow \text{2620} \\
 & 2b\sqrt{1-c^2x^2} \left(\frac{\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(\frac{1}{2}ib \int \log(1+e^{2i\arcsin(cx)}) d\arcsin(cx) - \frac{1}{2}i \log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx))\right)}{c^4} - \frac{x^2(a+b\arcsin(cx))}{2c^2} \right) \\
 & \frac{cd\sqrt{d-c^2dx^2}}{x^3(a+b\arcsin(cx))^2} - \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & 3 \left(-\frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}} \right) \\
 & \qquad \qquad \qquad c^2d \\
 & \qquad \qquad \qquad \downarrow \text{2715} \\
 & 2b\sqrt{1-c^2x^2} \left(\frac{\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(\frac{1}{4}b \int e^{-2i\arcsin(cx)} \log(1+e^{2i\arcsin(cx)}) de^{2i\arcsin(cx)} - \frac{1}{2}i \log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx))\right)}{c^4} - \frac{x^2(a+b\arcsin(cx))}{2c^2} \right) \\
 & \frac{cd\sqrt{d-c^2dx^2}}{x^3(a+b\arcsin(cx))^2} - \frac{x^3(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & 3 \left(-\frac{x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{6bc^3\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}} \right) \\
 & \qquad \qquad \qquad c^2d \\
 & \qquad \qquad \qquad \downarrow \text{2838}
 \end{aligned}$$

$$\frac{x^3(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{3 \left(-\frac{x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{2c^2 d} + \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^3}{6bc^3 \sqrt{d - c^2 dx^2}} + \frac{b \sqrt{1 - c^2 x^2} \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x \sqrt{1 - c^2 x^2}}{2c^2} \right) \right)}{c \sqrt{d - c^2 dx^2}} \right)}{c^2 d} - \frac{2b \sqrt{1 - c^2 x^2} \left(\frac{i(a + b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{c^4} - \frac{x^2 (a + b \arcsin(cx))}{2c^2} + \frac{b}{cd \sqrt{d - c^2 dx^2}}$$

input `Int[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]`

output `(x^3*(a + b*ArcSin[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (3*(-1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d) + (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))))/(c*Sqrt[d - c^2*d*x^2])))/(c^2*d) - (2*b*Sqrt[1 - c^2*x^2]*(-1/2*(x^2*(a + b*ArcSin[c*x]))/c^2 + (b*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(2*c) + (((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])]/4))/c^4)/(c*d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 223 `Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 262 `Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*(m - 1)/(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 2620 $\text{Int}[\left(\frac{(F_.)^{((g_.) * (e_.) + (f_.) * (x_))})^{(n_.)} * ((c_.) + (d_.) * (x_))^{(m_.)}}{((a_.) + (b_.) * (F_.)^{((g_.) * (e_.) + (f_.) * (x_))})^{(n_.)}}\right), x_Symbol] \rightarrow \text{Simp}[\left(\frac{(c + d*x)^m}{(b*f*g*n*\text{Log}[F])}\right) * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_.) + (b_.) * (F_.)^{((e_.) * (c_.) + (d_.) * (x_))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_.) * ((d_.) + (e_.) * (x_.)^{(n_.)})]/(x_.), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[\left(\frac{(c_.) + (d_.) * (x_.)^{(m_.)} * \tan[(e_.) + (f_.) * (x_.)]}{(c + d*x)^{m+1} / (d*(m+1))}\right), x] \rightarrow \text{Simp}[I * ((c + d*x)^{m+1} / (d*(m+1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * (E^{(2*I*(e + f*x))} / (1 + E^{(2*I*(e + f*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 5138 $\text{Int}[\left(\frac{(a_.) + \text{ArcSin}[(c_.) * (x_.)] * (b_.)}{(d_.) * (x_.)^{(m_.)}}\right)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1} * ((a + b*\text{ArcSin}[c*x])^n / (d*(m+1))), x] - \text{Simp}[b*c*(n / (d*(m+1))) \text{Int}[(d*x)^{m+1} * ((a + b*\text{ArcSin}[c*x])^{n-1} / \text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[\left(\frac{(a_.) + \text{ArcSin}[(c_.) * (x_.)] * (b_.)}{\text{Sqrt}[(d_.) + (e_.) * (x_.)^2]}\right)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1))) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] * (a + b*\text{ArcSin}[c*x])^{n+1}, x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[n, -1]$

rule 5180

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x
)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(392) = 784.

Time = 0.74 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.95

method	result
default	$-\frac{a^2 x^3}{2c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{3a^2 x}{2c^4 d \sqrt{-c^2 d x^2 + d}} - \frac{3a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{2c^4 d \sqrt{c^2 d}} + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{2d^2 c^5 (c^2 x^2 - 1)} + \dots \right)$
parts	$-\frac{a^2 x^3}{2c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{3a^2 x}{2c^4 d \sqrt{-c^2 d x^2 + d}} - \frac{3a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{2c^4 d \sqrt{c^2 d}} + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{2d^2 c^5 (c^2 x^2 - 1)} + \dots \right)$

input `int(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output

$$\begin{aligned}
 & -1/2*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(-c^2*d*x^2+d)^(1/2) \\
 & -3/2*a^2/c^4/d/(c^2*d)^(1/2)*\arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)) \\
 & +b^2*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*\arcsin(c*x) \\
 & +3*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)* \\
 & (2*I*\arcsin(c*x)*\ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*\arcsin(c*x)^2+\text{polylog}(2, \\
 & -(I*c*x+(-c^2*x^2+1)^(1/2))^2))-1/8*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^5/(c^2*x^2-1)*\arcsin(c*x)*(-I+8*\arcsin(c*x))-1/16*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^4/(c^2*x^2-1)*(18*\arcsin(c*x)^2-1)*x-1/8*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^5/(c^2*x^2-1)*\arcsin(c*x)*\cos(3*\arcsin(c*x))-1/16*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^5/(c^2*x^2-1)*(2*\arcsin(c*x)^2-1)*\sin(3*\arcsin(c*x)))+3/2*a*b*(-d*(c^2*x^2-1))^(1/2) \\
 & *(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*\arcsin(c*x)^2+2*I*a*b*(-c^2*x^2+1)^(1/2) \\
 & *(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*\arcsin(c*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2) \\
 & *(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*\ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1/8*a*b*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-9/4*a*b*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^4/(c^2*x^2-1)*\arcsin(c*x)*x-1/8*a*b*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^5/(c^2*x^2-1)*\cos(3*\arcsin(c*x))-1/4*a*b*(-d*(c^2*x^2-1))^(1/2) \\
 & /d^2/c^5/(c^2*x^2-1)*\arcsin(c*x)*\sin(3*\arcsin(c*x))
 \end{aligned}$$

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral((b^2*x^4*arcsin(c*x)^2 + 2*a*b*x^4*arcsin(c*x) + a^2*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^4(a + b \sin(cx))^2}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate(x**4*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2), x)`

output `Integral(x**4*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2), x, algorithm="maxima")`

output `-1/2*a^2*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*d) + 3*arcsin(c*x)/(c^5*d^(3/2))) + sqrt(d)*integrate((b^2*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2), x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^4(a + b \sin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input

```
int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output

```
int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-2\sqrt{-c^2 x^2 + 1} \arcsin(cx)^3 b^2 - 6\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 ab - 2\arcsin(cx)^2 b^2 c^3 x^2}{(d - c^2 dx^2)^{3/2}}$$

input

```
int(x^4*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
( - 2*sqrt( - c**2*x**2 + 1)*asin(c*x)**3*b**2 - 6*sqrt( - c**2*x**2 + 1)*
asin(c*x)**2*a*b - 2*asin(c*x)**2*b**2*c**3*x**3 + 2*asin(c*x)**2*b**2*c*x
- 6*sqrt( - c**2*x**2 + 1)*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*asin
(c*x)*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*asin(c*x)*b**2 - 4*asin(c*x)
*a*b*c**3*x**3 + 4*asin(c*x)*a*b*c*x - 8*sqrt( - c**2*x**2 + 1)*int(asin(c
*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c -
4*sqrt( - c**2*x**2 + 1)*int(asin(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**
*2 - sqrt( - c**2*x**2 + 1)),x)*b**2*c - 2*sqrt( - c**2*x**2 + 1)*a*b*c**2
*x**2 + 2*sqrt( - c**2*x**2 + 1)*a*b - 2*a**2*c**3*x**3 + 6*a**2*c*x + b**
2*c**3*x**3 - b**2*c*x)/(4*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**5*d)
```

3.243 $\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$

Optimal result	2391
Mathematica [A] (verified)	2392
Rubi [A] (verified)	2392
Maple [B] (verified)	2397
Fricas [F]	2398
Sympy [F]	2398
Maxima [F]	2399
Giac [F(-2)]	2399
Mupad [F(-1)]	2400
Reduce [F]	2400

Optimal result

Integrand size = 29, antiderivative size = 317

$$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx = -\frac{2b^2\sqrt{d-c^2dx^2}}{c^4d^2}$$

$$-\frac{2bx\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{c^3d\sqrt{d-c^2dx^2}}$$

$$+\frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^4d^2}$$

$$+\frac{4ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^4d\sqrt{d-c^2dx^2}}$$

$$-\frac{2ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^4d\sqrt{d-c^2dx^2}}$$

$$+\frac{2ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, ie^{i \arcsin(cx)})}{c^4d\sqrt{d-c^2dx^2}}$$

output

```
-2*b^2*(-c^2*d*x^2+d)^(1/2)/c^4/d^2-2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^2*(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2/c^4/d^2+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^4/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^4/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^4/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.16

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{4a^2 - 2b^2 - 2a^2 c^2 x^2 + 6ab \arcsin(cx) + 3b^2 \arcsin(cx)^2 - 2b^2 \cos(2 \arcsin(cx))}{(d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output

```
(4*a^2 - 2*b^2 - 2*a^2*c^2*x^2 + 6*a*b*ArcSin[c*x] + 3*b^2*ArcSin[c*x]^2 - 2*b^2*Cos[2*ArcSin[c*x]] + 2*a*b*ArcSin[c*x]*Cos[2*ArcSin[c*x]] + b^2*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] - 4*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 4*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 4*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - 4*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] - (4*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])] - 2*a*b*Sin[2*ArcSin[c*x]] - 2*b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])/(2*c^4*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)Time = 1.39 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5206, 5182, 2009, 5210, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx \\
 & \quad \downarrow \text{5206} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{5182} \\
 & \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{c^2d} \\
 & \quad - \frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{2009} \\
 & \frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{c^2d} \\
 & \quad \downarrow \text{5210} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{c^2d} \\
 & \quad \downarrow \text{241} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
 & \quad - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{c^2d} \\
 & \quad \downarrow \text{5164}
 \end{aligned}$$

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{cd\sqrt{d-c^2dx^2}}{c\sqrt{d-c^2dx^2}} \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right) \right)}{c^2d} - \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

↓ 3042

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{cd\sqrt{d-c^2dx^2}}{c\sqrt{d-c^2dx^2}} \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right) \right)}{c^2d} - \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

↓ 4669

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{cd\sqrt{d-c^2dx^2}}{c\sqrt{d-c^2dx^2}} \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right) \right)}{c^2d} - \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{2b\sqrt{1-c^2x^2} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{cd\sqrt{d-c^2dx^2}}{c\sqrt{d-c^2dx^2}} \left(\frac{2b\sqrt{1-c^2x^2} \left(ax+bx \arcsin(cx) + \frac{b\sqrt{1-c^2x^2}}{c} \right)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right) \right)}{c^2d} - \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

↓ 2838

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)})}{c^3}-\frac{x(a+b\arcsin(cx))}{c^2}\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{2\left(\frac{cd\sqrt{d-c^2dx^2}}{c\sqrt{d-c^2dx^2}}\left(\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arcsin(cx)+\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{c^2d}\right)\right)}{c^2d}$$

input `Int[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]`

output `(x^2*(a + b*ArcSin[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d)) + (2*b*Sqrt[1 - c^2*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(c^2*d) - (2*b*Sqrt[1 - c^2*x^2]*(-(b*Sqrt[1 - c^2*x^2])/c^3) - (x*(a + b*ArcSin[c*x]))/c^2 + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])]/c^3))/(c*d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 241 `Int[(x_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5164 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^((n_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^((n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

rule 5206 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^((n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]`

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(314) = 628.

Time = 0.67 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.11

method	result
default	$a^2 \left(-\frac{x^2}{c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{2}{d c^4 \sqrt{-c^2 d x^2 + d}} \right) + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} (c^2 x^2 - i \sqrt{-c^2 x^2 + 1} c x - 1) (\arcsin(cx)^2 - 2 + 2i \arcsin(cx))}{2d^2 c^4 (c^2 x^2 - 1)} \right)$
parts	$a^2 \left(-\frac{x^2}{c^2 d \sqrt{-c^2 d x^2 + d}} + \frac{2}{d c^4 \sqrt{-c^2 d x^2 + d}} \right) + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} (c^2 x^2 - i \sqrt{-c^2 x^2 + 1} c x - 1) (\arcsin(cx)^2 - 2 + 2i \arcsin(cx))}{2d^2 c^4 (c^2 x^2 - 1)} \right)$

input

```
int(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```

a^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b^2*(1/
2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)
^2-2+2*I*arcsin(c*x))/d^2/c^4/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-
c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))/d^2/c^4/
(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcsin(c*x)^2-2*(-c
^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^
2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(I
*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))/d^2/c^4
/(c^2*x^2-1)+2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1
)^(1/2)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arcsin(c*x)*x^2
-4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcsin(c*x)-2*a*b*(-c^2*
x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2
+1)^(1/2)-I)+2*a*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*
x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)

```

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^3}{(-c^2 dx^2 + d)^{3/2}} dx$$

input

```

integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

```

output

```

integral((b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3)*sqrt(-c
^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)

```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^3(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input

```

integrate(x**3*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)

```

output `Integral(x**3*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^3}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `-a*b*c*(2*x/(c^4*d^(3/2)) + log(c*x + 1)/(c^5*d^(3/2)) - log(c*x - 1)/(c^5*d^(3/2))) - 2*a*b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d))*arcsin(c*x) - a^2*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d)) + ((c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 - (c^6*d^2*x^2 - c^4*d^2)*sqrt(d)*integrate(2*(c^2*x^4 - 2*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^3*d^2*x^2 - c*d^2), x))*b^2/(c^6*d^2*x^2 - c^4*d^2)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-2\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x^3}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) ab c^4 - \sqrt{-c^2 x^2 + 1} \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1}} dx \right)}{\sqrt{d} \sqrt{-c^2 x^2 + 1} c^4 d}$$

input `int(x^3*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(-2*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**3)/(sqrt(-c**2*x**2 + 1)*c**2*x**2 - sqrt(-c**2*x**2 + 1)),x)*a*b*c**4 - sqrt(-c**2*x**2 + 1)*int((asin(c*x)**2*x**3)/(sqrt(-c**2*x**2 + 1)*c**2*x**2 - sqrt(-c**2*x**2 + 1)),x)*b**2*c**4 - a**2*c**2*x**2 + 2*a**2)/(sqrt(d)*sqrt(-c**2*x**2 + 1)*c**4*d)`

3.244
$$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$$

Optimal result	2401
Mathematica [A] (verified)	2402
Rubi [A] (verified)	2402
Maple [B] (verified)	2405
Fricas [F]	2406
Sympy [F]	2406
Maxima [F]	2407
Giac [F(-2)]	2407
Mupad [F(-1)]	2408
Reduce [F]	2408

Optimal result

Integrand size = 29, antiderivative size = 250

$$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx = \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} + \frac{2b\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{c^3d\sqrt{d-c^2dx^2}} - \frac{ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{c^3d\sqrt{d-c^2dx^2}}$$

output

```
x*(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c^3/d/(-c^2*d*x^2+d)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.18

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = -\frac{a^2 x \sqrt{-d(-1 + c^2 x^2)}}{c^2 d^2 (-1 + c^2 x^2)} + \frac{a^2 \arctan\left(\frac{cx \sqrt{-d(-1 + c^2 x^2)}}{\sqrt{d(-1 + c^2 x^2)}}\right)}{c^3 d^{3/2}}$$

$$+ \frac{ab(2cx \arcsin(cx) - \sqrt{1 - c^2 x^2}(\arcsin(cx))^2 - 2 \log(\sqrt{1 - c^2 x^2}))}{c^3 d \sqrt{d(1 - c^2 x^2)}}$$

$$+ \frac{b^2(\arcsin(cx)(3cx \arcsin(cx) - \sqrt{1 - c^2 x^2} \arcsin(cx)(3i + \arcsin(cx)) + 6\sqrt{1 - c^2 x^2} \log(1 + e^{2i \arcsin(cx)}))}{3c^3 d \sqrt{d(1 - c^2 x^2)}}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output

```
-((a^2*x*Sqrt[-(d*(-1 + c^2*x^2))])/(c^2*d^2*(-1 + c^2*x^2))) + (a^2*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(c^3*d^(3/2)) + (a*b*(2*c*x*ArcSin[c*x] - Sqrt[1 - c^2*x^2]*(ArcSin[c*x]^2 - 2*Log[Sqrt[1 - c^2*x^2]])))/(c^3*d*Sqrt[d*(1 - c^2*x^2)]) + (b^2*(ArcSin[c*x]*(3*c*x*ArcSin[c*x] - Sqrt[1 - c^2*x^2]*ArcSin[c*x]*(3*I + ArcSin[c*x]) + 6*Sqrt[1 - c^2*x^2]*Log[1 + E^((2*I)*ArcSin[c*x])]) - (3*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^((2*I)*ArcSin[c*x])]))/(3*c^3*d*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5206, 5152, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow \text{5206}$$

$$-\frac{2b\sqrt{1 - c^2 x^2} \int \frac{x(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{cd\sqrt{d - c^2 dx^2}} - \frac{\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx}{c^2 d} + \frac{x(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}}$$

$$\begin{aligned}
& \downarrow 5152 \\
& -\frac{2b\sqrt{1-c^2x^2} \int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \downarrow 5180 \\
& -\frac{2b\sqrt{1-c^2x^2} \int \frac{cx(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} d\arcsin(cx)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \\
& \quad \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \downarrow 3042 \\
& -\frac{2b\sqrt{1-c^2x^2} \int (a+b\arcsin(cx)) \tan(\arcsin(cx)) d\arcsin(cx)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \\
& \quad \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \downarrow 4202 \\
& -\frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1+e^{2i\arcsin(cx)}} d\arcsin(cx) \right)}{c^3d\sqrt{d-c^2dx^2}} + \\
& \quad \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \downarrow 2620 \\
& -\frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i\arcsin(cx)}) d\arcsin(cx) - \frac{1}{2} i \log(1+e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) \right) \right)}{c^3d\sqrt{d-c^2dx^2}} \\
& \quad \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \downarrow 2715 \\
& -\frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i\arcsin(cx)} \log(1+e^{2i\arcsin(cx)}) de^{2i\arcsin(cx)} - \frac{1}{2} i \log(1+e^{2i\arcsin(cx)}) (a+b\arcsin(cx)) \right) \right)}{c^3d\sqrt{d-c^2dx^2}} \\
& \quad \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \downarrow 2838
\end{aligned}$$

$$\frac{x(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{i(a + b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2}i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{c^3 d \sqrt{d - c^2 dx^2}} - \frac{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^3}{3bc^3 d \sqrt{d - c^2 dx^2}}$$

input `Int[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]`

output `(x*(a + b*ArcSin[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)))/(c^3*d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4202 $\text{Int}[\left((c_{.}) + (d_{.}) \cdot (x_{.})\right)^{(m_{.})} \cdot \tan\left[\left(e_{.}\right) + \left(f_{.}\right) \cdot (x_{.})\right], x_{\text{Symbol}}] \rightarrow \text{Simp}\left[\text{I} \cdot \left(\left(c + d \cdot x\right)^{(m + 1)} / (d \cdot (m + 1))\right), x\right] - \text{Simp}\left[2 \cdot \text{I} \cdot \text{Int}\left[\left(c + d \cdot x\right)^m \cdot \left(E^{(2 \cdot \text{I} \cdot (e + f \cdot x))} / (1 + E^{(2 \cdot \text{I} \cdot (e + f \cdot x))})\right)\right], x\right] /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

rule 5152 $\text{Int}\left[\left(\left(a_{.}\right) + \text{ArcSin}\left[\left(c_{.}\right) \cdot \left(x_{.}\right)\right] \cdot \left(b_{.}\right)\right)^{(n_{.})} / \text{Sqrt}\left[\left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)^2\right], x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\left(1 / (b \cdot c \cdot (n + 1))\right) \cdot \text{Simp}\left[\text{Sqrt}\left[1 - c^2 \cdot x^2\right] / \text{Sqrt}\left[d + e \cdot x^2\right]\right] \cdot \left(a + b \cdot \text{ArcSin}\left[c \cdot x\right]\right)^{(n + 1)}, x\right] /;$ FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]

rule 5180 $\text{Int}\left[\left(\left(a_{.}\right) + \text{ArcSin}\left[\left(c_{.}\right) \cdot \left(x_{.}\right)\right] \cdot \left(b_{.}\right)\right)^{(n_{.})} \cdot \left(x_{.}\right) / \left(\left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)^2\right), x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[-e^{(-1)} \cdot \text{Subst}\left[\text{Int}\left[\left(a + b \cdot x\right)^n \cdot \text{Tan}\left[x\right], x\right], x, \text{ArcSin}\left[c \cdot x\right]\right], x\right] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]

rule 5206 $\text{Int}\left[\left(\left(a_{.}\right) + \text{ArcSin}\left[\left(c_{.}\right) \cdot \left(x_{.}\right)\right] \cdot \left(b_{.}\right)\right)^{(n_{.})} \cdot \left(\left(f_{.}\right) \cdot \left(x_{.}\right)\right)^{(m_{.})} \cdot \left(\left(d_{.}\right) + \left(e_{.}\right) \cdot \left(x_{.}\right)^2\right)^{(p_{.})}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[f \cdot \left(f \cdot x\right)^{(m - 1)} \cdot \left(d + e \cdot x^2\right)^{(p + 1)} \cdot \left(a + b \cdot \text{ArcSin}\left[c \cdot x\right]\right)^n / (2 \cdot e \cdot (p + 1)), x\right] + \left(-\text{Simp}\left[f^2 \cdot (m - 1) / (2 \cdot e \cdot (p + 1))\right] \cdot \text{Int}\left[\left(f \cdot x\right)^{(m - 2)} \cdot \left(d + e \cdot x^2\right)^{(p + 1)} \cdot \left(a + b \cdot \text{ArcSin}\left[c \cdot x\right]\right)^n, x\right] + \text{Simp}\left[b \cdot f \cdot (n / (2 \cdot c \cdot (p + 1))) \cdot \text{Simp}\left[\left(d + e \cdot x^2\right)^p / (1 - c^2 \cdot x^2)^p\right] \cdot \text{Int}\left[\left(f \cdot x\right)^{(m - 1)} \cdot (1 - c^2 \cdot x^2)^{(p + 1/2)} \cdot \left(a + b \cdot \text{ArcSin}\left[c \cdot x\right]\right)^{(n - 1)}, x\right], x\right) /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(248) = 496$.

Time = 0.64 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.02

method	result
default	$\frac{a^2 x}{c^2 d \sqrt{-c^2 d x^2 + d}} - \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{c^2 d \sqrt{c^2 d}} + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{3d^2 c^3 (c^2 x^2 - 1)} - \frac{\sqrt{-d(c^2 x^2 - 1)} (i\sqrt{-c^2 x^2 + 1})}{d^2 c^3 (c^2 x^2 - 1)} \right)$
parts	$\frac{a^2 x}{c^2 d \sqrt{-c^2 d x^2 + d}} - \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d x}}{\sqrt{-c^2 d x^2 + d}}\right)}{c^2 d \sqrt{c^2 d}} + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{3d^2 c^3 (c^2 x^2 - 1)} - \frac{\sqrt{-d(c^2 x^2 - 1)} (i\sqrt{-c^2 x^2 + 1})}{d^2 c^3 (c^2 x^2 - 1)} \right)$

input `int(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output `a^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arcsin(c*x)^3-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)+c*x)*arcsin(c*x)^2/d^2/c^3/(c^2*x^2-1)+I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(2*I*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*arcsin(c*x)^2+polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))+a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arcsin(c*x)^2+2*I*a*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*arcsin(c*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)`

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral((b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^2(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate(x**2*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral(x**2*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `a^2*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + sqrt(d)*integrate((b^2*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^2(a + b \sin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\sqrt{-c^2 x^2 + 1} \arcsin(cx)^3 b^2 - 3\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 ab - 3\sqrt{-c^2 x^2 + 1} \arcsin(cx) a^2}{(d - c^2 dx^2)^{3/2}}$$

input `int(x^2*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(-sqrt(-c**2*x**2 + 1)*asin(c*x)**3*b**2 - 3*sqrt(-c**2*x**2 + 1)*asin(c*x)**2*a*b - 3*sqrt(-c**2*x**2 + 1)*asin(c*x)*a**2 - 6*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**2*x**2 - sqrt(-c**2*x**2 + 1)),x)*a*b*c - 3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)**2/(sqrt(-c**2*x**2 + 1)*c**2*x**2 - sqrt(-c**2*x**2 + 1)),x)*b**2*c + 3*a**2*c*x)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*c**3*d)`

3.245
$$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$$

Optimal result	2409
Mathematica [A] (verified)	2410
Rubi [A] (verified)	2410
Maple [A] (verified)	2412
Fricas [F]	2413
Sympy [F]	2413
Maxima [F]	2414
Giac [F(-2)]	2414
Mupad [F(-1)]	2414
Reduce [F]	2415

Optimal result

Integrand size = 27, antiderivative size = 208

$$\begin{aligned} \int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx &= \frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\ &+ \frac{4ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^2d\sqrt{d-c^2dx^2}} \\ &- \frac{2ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^2d\sqrt{d-c^2dx^2}} \\ &+ \frac{2ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, ie^{i \arcsin(cx)})}{c^2d\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+4*I*b*(-c^2*x^2+1)^(1/2)*(a
+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^2/d/(-c^2*d*x^2+d)^(1/2
)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^2/
d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2
*x^2+1)^(1/2)))/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.33

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{a^2 + 2ab \arcsin(cx) + b^2 \arcsin(cx)^2 - 2b^2 \sqrt{1 - c^2 x^2} \arcsin(cx) \log(1 - ie^{i \arcsin(cx)})}{(d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output

```
(a^2 + 2*a*b*ArcSin[c*x] + b^2*ArcSin[c*x]^2 - 2*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] - (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5182, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow 5182$$

$$\frac{(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b \sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{1 - c^2 x^2} dx}{cd \sqrt{d - c^2 dx^2}}$$

$$\downarrow 5164$$

$$\frac{(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b \sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} d \arcsin(cx)}{c^2 d \sqrt{d - c^2 dx^2}}$$

$$\begin{aligned}
& \downarrow 3042 \\
& \frac{(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b \sqrt{1 - c^2 x^2} \int (a + b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^2 d \sqrt{d - c^2 dx^2}} \\
& \downarrow 4669 \\
& \frac{(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b \sqrt{1 - c^2 x^2} (-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}))}{c^2 d \sqrt{d - c^2 dx^2}} \\
& \downarrow 2715 \\
& \frac{(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b \sqrt{1 - c^2 x^2} (ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{c^2 d \sqrt{d - c^2 dx^2}} \\
& \downarrow 2838 \\
& \frac{(a + b \arcsin(cx))^2}{c^2 d \sqrt{d - c^2 dx^2}} - \frac{2b \sqrt{1 - c^2 x^2} (-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{c^2 d \sqrt{d - c^2 dx^2}}
\end{aligned}$$

input `Int[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]`

output `(a + b*ArcSin[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/(c^2*d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))]^(n_)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 5164 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.93

method	result
default	$\frac{a^2}{c^2 d \sqrt{-c^2 d x^2 + d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \arcsin(cx)^2}{d^2 c^2 (c^2 x^2 - 1)} - \frac{2\sqrt{-c^2 x^2 + 1} \sqrt{-d(c^2 x^2 - 1)} (\arcsin(cx) \ln(1 + i(icx + \sqrt{-c^2 x^2 + 1})))}{d^2 c^2 (c^2 x^2 - 1)} \right)$
parts	$\frac{a^2}{c^2 d \sqrt{-c^2 d x^2 + d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \arcsin(cx)^2}{d^2 c^2 (c^2 x^2 - 1)} - \frac{2\sqrt{-c^2 x^2 + 1} \sqrt{-d(c^2 x^2 - 1)} (\arcsin(cx) \ln(1 + i(icx + \sqrt{-c^2 x^2 + 1})))}{d^2 c^2 (c^2 x^2 - 1)} \right)$

input `int(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output

```
a^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+b^2*(-(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arcsin(c*x)^2-2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))/d^2/c^2/(c^2*x^2-1))+2*a*b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arcsin(c*x)+(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)-(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I))
```

Fricas [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(-c^2 dx^2 + d)^{3/2}} dx$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*x*arcsin(c*x)^2 + 2*a*b*x*arcsin(c*x) + a^2*x)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input

```
integrate(x*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output

```
Integral(x*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**3/2, x)
```

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `sqrt(d)*integrate((b^2*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x) + a^2/(sqrt(-c^2*d*x^2 + d)*c^2*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)`

output `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-2\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)x}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) ab c^2 - \sqrt{-c^2 x^2 + 1} \left(\int \frac{c^2}{\sqrt{-c^2 x^2 + 1}} dx \right)}{\sqrt{d} \sqrt{-c^2 x^2 + 1} c^2 d}$$

input `int(x*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 2*sqrt(- c**2*x**2 + 1)*int((asin(c*x)*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*a*b*c**2 - sqrt(- c**2*x**2 + 1)*int((asin(c*x)**2*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b**2*c**2 + a**2)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*c**2*d)`

3.246 $\int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx$

Optimal result	2416
Mathematica [A] (verified)	2417
Rubi [A] (verified)	2417
Maple [A] (verified)	2420
Fricas [F]	2420
Sympy [F]	2421
Maxima [F]	2421
Giac [F(-2)]	2421
Mupad [F(-1)]	2422
Reduce [F]	2422

Optimal result

Integrand size = 26, antiderivative size = 195

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2dx^2)^{3/2}} dx = \frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2dx^2}} - \frac{i\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2}{cd\sqrt{d - c^2dx^2}} + \frac{2b\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)})}{cd\sqrt{d - c^2dx^2}} - \frac{ib^2\sqrt{1 - c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{cd\sqrt{d - c^2dx^2}}$$

output

```
x*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c/d/(-c^2*d*x^2+d)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c/d/(-c^2*d*x^2+d)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{b^2(cx - i\sqrt{1 - c^2x^2}) \arcsin(cx)^2 + 2b \arcsin(cx) (acx + b\sqrt{1 - c^2x^2} \log(1 + e^{2i \arcsin(cx)})) + a^2 \arcsin(cx)^2 + a(a + b \arcsin(cx)) \sqrt{1 - c^2x^2} \log(1 + e^{2i \arcsin(cx)})}{(d - c^2 dx^2)^{3/2}}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^(3/2),x]`

output `(b^2*(c*x - I*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 + 2*b*ArcSin[c*x]*(a*c*x + b*Sqrt[1 - c^2*x^2]*Log[1 + E^((2*I)*ArcSin[c*x])]) + a*(a*c*x + b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2]) - I*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])`

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5160, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx \\ & \quad \downarrow \text{5160} \\ & \frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2bc\sqrt{1 - c^2x^2} \int \frac{x(a + b \arcsin(cx))}{1 - c^2x^2} dx}{d\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{5180} \\ & \frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2x^2} \int \frac{cx(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} d \arcsin(cx)}{cd\sqrt{d - c^2 dx^2}} \\ & \quad \downarrow \text{3042} \\ & \frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2x^2} \int (a + b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{cd\sqrt{d - c^2 dx^2}} \end{aligned}$$

$$\begin{array}{c}
\downarrow 4202 \\
\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{cd\sqrt{d - c^2 dx^2}} \\
\downarrow 2620 \\
\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} i b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) \right) \right)}{cd\sqrt{d - c^2 dx^2}} \\
\downarrow 2715 \\
\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{2} i \log(1 + e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d - c^2 dx^2}} \\
\downarrow 2838 \\
\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1 + e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4} b \text{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d - c^2 dx^2}}
\end{array}$$

input `Int[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^(3/2),x]`

output `(x*(a + b*ArcSin[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])]/4)))/(c*d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 2620 $\text{Int}[\frac{((F_)^{((g_.) * (e_.) + (f_.) * (x_.))})^{(n_.)} * ((c_.) + (d_.) * (x_.))^{(m_.)}}{((a_.) + (b_.) * ((F_)^{((g_.) * (e_.) + (f_.) * (x_.))})^{(n_.)})}, x_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m}{(b*f*g*n*\text{Log}[F])} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_) * ((F_)^{((e_.) * (c_.) + (d_.) * (x_.))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n}], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_.) * ((d_) + (e_.) * (x_.)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[\frac{((c_.) + (d_.) * (x_.))^{(m_.)} * \tan[(e_.) + (f_.) * (x_.)]}{(c + d*x)^{(m+1)}/(d*(m+1))}, x_Symbol] \rightarrow \text{Simp}[I * \frac{(c + d*x)^{(m+1)}}{d*(m+1)}, x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * (E^{(2*I*(e + f*x))}/(1 + E^{(2*I*(e + f*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 5160 $\text{Int}[\frac{((a_.) + \text{ArcSin}[(c_.) * (x_.)] * (b_.))^{(n_.)}}{((d_) + (e_.) * (x_.)^2)^{(3/2)}}, x_Symbol] \rightarrow \text{Simp}[x * \frac{(a + b*\text{ArcSin}[c*x])^n}{d*\text{Sqrt}[d + e*x^2]}, x] - \text{Simp}[b * c * \frac{(n/d) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]] \text{Int}[x * ((a + b*\text{ArcSin}[c*x])^{(n-1)})/(1 - c^2*x^2)], x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$

rule 5180 $\text{Int}[\frac{((a_.) + \text{ArcSin}[(c_.) * (x_.)] * (b_.))^{(n_.)} * (x_.)}{((d_) + (e_.) * (x_.)^2)}, x_Symbol] \rightarrow \text{Simp}[-e^{(-1)} \text{Subst}[\text{Int}[(a + b*x)^n * \text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.82

method	result
default	$\frac{a^2x}{d\sqrt{-c^2dx^2+d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} (i\sqrt{-c^2x^2+1}+cx) \arcsin(cx)^2}{cd^2(c^2x^2-1)} + \frac{i\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)}}{cd^2(c^2x^2-1)} \left(2i \arcsin(cx) \ln \left(1 + \left(i\sqrt{-c^2x^2+1} + cx \right) \right) \right) \right)$
parts	$\frac{a^2x}{d\sqrt{-c^2dx^2+d}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} (i\sqrt{-c^2x^2+1}+cx) \arcsin(cx)^2}{cd^2(c^2x^2-1)} + \frac{i\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)}}{cd^2(c^2x^2-1)} \left(2i \arcsin(cx) \ln \left(1 + \left(i\sqrt{-c^2x^2+1} + cx \right) \right) \right) \right)$

input `int((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)`

output
$$a^2*x/d/(-c^2*d*x^2+d)^{(1/2)}+b^2*(-(-d*(c^2*x^2-1))^{(1/2)}*(I*(-c^2*x^2+1)^{(1/2)}+c*x)*\arcsin(c*x)^2/c/d^2/(c^2*x^2-1)+I*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}/c/d^2/(c^2*x^2-1)*(2*I*\arcsin(c*x)*\ln(1+(I*c*x+(-c^2*x^2+1)^{(1/2)})^2)+2*\arcsin(c*x)^2+\text{polylog}(2,-(I*c*x+(-c^2*x^2+1)^{(1/2)})^2)))+2*I*a*b*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1))^{(1/2)}/c/d^2/(c^2*x^2-1)*\arcsin(c*x)-2*a*b*(-d*(c^2*x^2-1))^{(1/2)}*\arcsin(c*x)/d^2/(c^2*x^2-1)*x-2*a*b*(-d*(c^2*x^2-1))^{(1/2)}*(-c^2*x^2+1)^{(1/2)}/c/d^2/(c^2*x^2-1)*\ln(1+(I*c*x+(-c^2*x^2+1)^{(1/2)})^2)$$

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `2*a*b*x*arcsin(c*x)/(sqrt(-c^2*d*x^2 + d)*d) - b^2*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d) - a*b*log(x^2 - 1/c^2)/(c*d^(3/2))`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \sin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^(3/2),x)`

output `int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-2\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx \right) ab - \sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x} dx \right)}{\sqrt{d} \sqrt{-c^2 x^2 + 1} d}$$

input `int((a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 2*sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x*
*2 - sqrt(- c**2*x**2 + 1)),x)*a*b - sqrt(- c**2*x**2 + 1)*int(asin(c*x)
2/(sqrt(- c2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b**2 +
a**2*x)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*d)`

$$3.247 \quad \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx$$

Optimal result	2423
Mathematica [A] (verified)	2424
Rubi [A] (verified)	2425
Maple [A] (verified)	2429
Fricas [F]	2430
Sympy [F]	2430
Maxima [F]	2431
Giac [F(-2)]	2431
Mupad [F(-1)]	2431
Reduce [F]	2432

Optimal result

Integrand size = 29, antiderivative size = 467

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx &= \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{4ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &- \frac{2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{2ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &- \frac{2ib^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{2ib^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &- \frac{2ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &- \frac{2b^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{2b^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.43

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \frac{a^2 d + a^2 \sqrt{d} \sqrt{d - c^2 dx^2} \log(cx) - a^2 \sqrt{d} \sqrt{d - c^2 dx^2} \log\left(d + \sqrt{d} \sqrt{d - c^2 dx^2}\right)}{x(d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output

```
(a^2*d + a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*Log[c*x] - a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + 2*a*b*d*(ArcSin[c*x] + Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] - Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] + Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] + I*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcSin[c*x])] - I*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcSin[c*x])]) + b^2*d*(ArcSin[c*x]^2 + Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] - 2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] + 2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x])] + (2*I)*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] - (2*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])] - (2*I)*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] - 2*Sqrt[1 - c^2*x^2]*PolyLog[3, -E^(I*ArcSin[c*x])] + 2*Sqrt[1 - c^2*x^2]*PolyLog[3, E^(I*ArcSin[c*x])]))/(d^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.60, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5208, 5164, 3042, 4669, 2715, 2838, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx \\
 & \quad \downarrow \text{5208} \\
 & -\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{d\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a + b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{5164} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a + b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} - \frac{2b\sqrt{1-c^2x^2} \int (a + b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{(a + b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \text{4669} \\
 & -\frac{2b\sqrt{1-c^2x^2}(-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}}}{d} \\
 & \quad \downarrow \text{2715} \\
 & -\frac{2b\sqrt{1-c^2x^2}(ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\
 & \quad \downarrow \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a + b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}
 \end{aligned}$$

$$\begin{aligned}
 & \int \frac{(a+b \arcsin(cx))^2 dx}{x\sqrt{d-c^2dx^2}} \quad \downarrow \text{2838} \\
 & \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d} \\
 & \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\
 & \downarrow \text{5218} \\
 & \frac{\sqrt{1-c^2x^2} \int \frac{(a+b \arcsin(cx))^2}{cx} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} \quad \downarrow \text{3042} \\
 & \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \\
 & \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\
 & \downarrow \text{4671} \\
 & \frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} \\
 & \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \\
 & \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\
 & \downarrow \text{3011} \\
 & \frac{\sqrt{1-c^2x^2}(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, e^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) d \arcsin(cx))}{d\sqrt{d-c^2dx^2}} \\
 & \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}}
 \end{aligned}$$

↓ 2720

$$\frac{\sqrt{1 - c^2 x^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx))) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{2b\sqrt{1 - c^2 x^2} (-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))} \frac{d\sqrt{d - c^2 dx^2}}{(a + b \arcsin(cx))^2} \frac{1}{d\sqrt{d - c^2 dx^2}}$$

↓ 7143

$$\frac{2b\sqrt{1 - c^2 x^2} (-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{\sqrt{1 - c^2 x^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))} \frac{d\sqrt{d - c^2 dx^2}}{(a + b \arcsin(cx))^2} \frac{1}{d\sqrt{d - c^2 dx^2}}$$

```
input Int[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
output (a + b*ArcSin[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/(d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

```
rule 2715 Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```


rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.27

method	result
default	$\frac{a^2}{d\sqrt{-c^2dx^2+d}} - \frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} \arcsin(cx)^2}{d^2(c^2x^2-1)} + \frac{i\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)} (i \arcsin(c$
parts	$\frac{a^2}{d\sqrt{-c^2dx^2+d}} - \frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} \arcsin(cx)^2}{d^2(c^2x^2-1)} + \frac{i\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)} (i \arcsin(c$

input

```
int((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
a^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+b^2*(-(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arcsin(c*x)^2+I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*(I*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2)))+2*I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))+2*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-2*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))-2*I*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))+2*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))+2*a*b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arcsin(c*x)+(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*arctan(I*c*x+(-c^2*x^2+1)^(1/2))-I*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-I*dilog(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{3/2} x} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input

```
integrate((a+b*asin(c*x))**2/x/(-c**2*d*x**2+d)**(3/2),x)
```

output

```
Integral((a + b*asin(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{\frac{3}{2}} x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `-a^2*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(sqrt(-c^2*d*x^2 + d)*d)) + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx$$

input `int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)),x)`

output `int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx = \frac{-2\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^3 - \sqrt{-c^2 x^2 + 1} x} dx \right) ab - \sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^3 - \sqrt{-c^2 x^2 + 1} x} dx \right)}{\sqrt{d} \sqrt{-c^2 x^2 + 1}}$$

input `int((a+b*asin(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 2*sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**3 - sqrt(- c**2*x**2 + 1)*x),x)*a*b - sqrt(- c**2*x**2 + 1)*int(asin(c*x)**2/(sqrt(- c**2*x**2 + 1)*c**2*x**3 - sqrt(- c**2*x**2 + 1)*x),x)*b**2 + sqrt(- c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**2 - sqrt(- c**2*x**2 + 1)*a**2 + a**2)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*d)`

3.248 $\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^{3/2}} dx$

Optimal result	2433
Mathematica [A] (verified)	2434
Rubi [A] (verified)	2434
Maple [A] (verified)	2440
Fricas [F]	2441
Sympy [F]	2441
Maxima [F]	2441
Giac [F(-2)]	2442
Mupad [F(-1)]	2442
Reduce [F]	2443

Optimal result

Integrand size = 29, antiderivative size = 333

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^{3/2}} dx &= -\frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}} \\ &+ \frac{2c^2x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2ic\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} \\ &- \frac{4bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &+ \frac{4bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))\log(1+e^{2i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &- \frac{ib^2c\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,-e^{2i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ &- \frac{ib^2c\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,e^{2i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \end{aligned}$$

output

$$\begin{aligned}
& -(a+b\arcsin(cx))^2/d/x/(-c^2dx^2+d)^{(1/2)}+2c^2x(a+b\arcsin(cx))^2/ \\
& d/(-c^2dx^2+d)^{(1/2)}-2Ic(-c^2x^2+1)^{(1/2)}(a+b\arcsin(cx))^2/d/(-c^ \\
& 2dx^2+d)^{(1/2)}-4b*c*(-c^2x^2+1)^{(1/2)}(a+b\arcsin(cx))*\operatorname{arctanh}((Icx \\
& +(-c^2x^2+1)^{(1/2)})^2)/d/(-c^2dx^2+d)^{(1/2)}+4b*c*(-c^2x^2+1)^{(1/2)}(a \\
& +b\arcsin(cx))*\ln(1+(Icx+(-c^2x^2+1)^{(1/2)})^2)/d/(-c^2dx^2+d)^{(1/2)}- \\
& I*b^2*c*(-c^2x^2+1)^{(1/2)}\operatorname{polylog}(2,-(Icx+(-c^2x^2+1)^{(1/2)})^2)/d/(-c^ \\
& 2dx^2+d)^{(1/2)}-I*b^2*c*(-c^2x^2+1)^{(1/2)}\operatorname{polylog}(2,(Icx+(-c^2x^2+1) \\
& (1/2))^2)/d/(-c^2dx^2+d)^{(1/2)}
\end{aligned}$$
Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.97

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \frac{-a^2 + 2a^2 c^2 x^2 - 2ab \arcsin(cx) + 4abc^2 x^2 \arcsin(cx) - b^2 \arcsin(cx)^2 + 2b^2 c^2 x^2}{x^2 (d - c^2 dx^2)^{3/2}}$$

input

`Integrate[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]`

output

$$\begin{aligned}
& (-a^2 + 2a^2c^2x^2 - 2a*b*ArcSin[c*x] + 4a*b*c^2x^2*ArcSin[c*x] - b^ \\
& 2*ArcSin[c*x]^2 + 2*b^2*c^2x^2*ArcSin[c*x]^2 - (2*I)*b^2*c*x*sqrt[1 - c^2 \\
& *x^2]*ArcSin[c*x]^2 + 2*b^2*c*x*sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((\\
& 2*I)*ArcSin[c*x])] + 2*b^2*c*x*sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + E^((\\
& 2*I)*ArcSin[c*x])] + 2*a*b*c*x*sqrt[1 - c^2*x^2]*Log[c*x] + a*b*c*x*sqrt[1 \\
& - c^2*x^2]*Log[1 - c^2*x^2] - I*b^2*c*x*sqrt[1 - c^2*x^2]*PolyLog[2, -E^((\\
& 2*I)*ArcSin[c*x])] - I*b^2*c*x*sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSi \\
& n[c*x])])]/(d*x*sqrt[d - c^2*d*x^2])
\end{aligned}$$
Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5204, 5160, 5180, 3042, 4202, 2620, 2715, 2838, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx \\
& \quad \downarrow \text{5204} \\
& 2c^2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx + \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5160} \\
& 2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}} \right) + \\
& \quad \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5180} \\
& 2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} d \arcsin(cx)}{cd\sqrt{d - c^2 dx^2}} \right) + \\
& \quad \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{3042} \\
& \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} + \\
& 2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \int (a + b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{cd\sqrt{d - c^2 dx^2}} \right) - \\
& \quad \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{4202} \\
& 2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)} (a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{cd\sqrt{d - c^2 dx^2}} \right) + \\
& \quad \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{2620}
\end{aligned}$$

$$\begin{aligned}
 & \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + \\
 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} \right) b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right)}{cd\sqrt{d-c^2dx^2}} \right. \\
 & \left. - \frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}} \right) \\
 & \quad \downarrow \text{2715} \\
 & \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + \\
 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} \right) b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right)}{cd\sqrt{d-c^2dx^2}} \right. \\
 & \left. - \frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}} \right) \\
 & \quad \downarrow \text{2838} \\
 & \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + \\
 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} \right) i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right)}{cd\sqrt{d-c^2dx^2}} \right. \\
 & \left. - \frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}} \right) \\
 & \quad \downarrow \text{5184} \\
 & \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2x^2}} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \\
 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} \right) i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right)}{cd\sqrt{d-c^2dx^2}} \right. \\
 & \left. - \frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}} \right) \\
 & \quad \downarrow \text{4919}
 \end{aligned}$$

$$\frac{4bc\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} +$$

$$2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})) (a+b \arcsin(cx)) - \frac{1}{4}b \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

↓ 3042

$$\frac{4bc\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} +$$

$$2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})) (a+b \arcsin(cx)) - \frac{1}{4}b \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

↓ 4671

$$\frac{4bc\sqrt{1-c^2x^2} \left(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})) \right)}{d\sqrt{d-c^2dx^2}}$$

$$2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})) (a+b \arcsin(cx)) - \frac{1}{4}b \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{4bc\sqrt{1-c^2x^2} \left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1-e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} \right)}{d\sqrt{d-c^2dx^2}}$$

$$2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})) (a+b \arcsin(cx)) - \frac{1}{4}b \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

↓ 2838

$$4bc\sqrt{1-c^2x^2}\left(-\operatorname{arctanh}(e^{2i\arcsin(cx)})(a+b\arcsin(cx))\right) + \frac{1}{4}ib\operatorname{PolyLog}(2, -e^{2i\arcsin(cx)}) - \frac{1}{4}ib\operatorname{PolyLog}(2, e^{2i\arcsin(cx)})$$

$$2c^2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx)) - \frac{1}{4}b\right)\right)}{cd\sqrt{d-c^2dx^2}}\right)$$

$$\frac{(a+b\arcsin(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]`

output `-((a + b*ArcSin[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + 2*c^2*((x*(a + b*ArcSin[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])) + (4*b*c*Sqrt[1 - c^2*x^2]*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])))/(d*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\tan[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[I*((c + d*x)^{(m + 1)}/(d*(m + 1))), x] - \text{Simp}[2*I \text{ Int}[(c + d*x)^m*(E^{(2*I*(e + f*x))}/(1 + E^{(2*I*(e + f*x))}))], x], x] \text{ ; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{ Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) \text{ ; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sec}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{ Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^n], x] \text{ ; FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{RationalQ}[m]$

rule 5160 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}/((d_.) + (e_.)*(x_.)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[x*((a + b*\text{ArcSin}[c*x])^n/(d*\text{Sqrt}[d + e*x^2])), x] - \text{Simp}[b*c*(n/d)*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]] \text{ Int}[x*((a + b*\text{ArcSin}[c*x])^{(n - 1)})/(1 - c^2*x^2)], x], x] \text{ ; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

rule 5180 $\text{Int}[(((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}*(x_.))/((d_.) + (e_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[-e^{(-1)} \text{ Subst}[\text{Int}[(a + b*x)^n*\text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] \text{ ; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5184 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.))^{(n_.)}/((x_.)*((d_.) + (e_.)*(x_.)^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{ Subst}[\text{Int}[(a + b*x)^n/(\text{Cos}[x]*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] \text{ ; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.54

method	result
default	$a^2 \left(-\frac{1}{dx\sqrt{-c^2dx^2+d}} + \frac{2c^2x}{d\sqrt{-c^2dx^2+d}} \right) + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} (2i\sqrt{-c^2x^2+1} cx+2c^2x^2-1) \arcsin(cx)^2}{(c^2x^2-1)d^2x} + \frac{i\sqrt{-c^2x^2+1}}{\dots} \right)$
parts	$a^2 \left(-\frac{1}{dx\sqrt{-c^2dx^2+d}} + \frac{2c^2x}{d\sqrt{-c^2dx^2+d}} \right) + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} (2i\sqrt{-c^2x^2+1} cx+2c^2x^2-1) \arcsin(cx)^2}{(c^2x^2-1)d^2x} + \frac{i\sqrt{-c^2x^2+1}}{\dots} \right)$

input

```
int((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
a^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-(-d
*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*arcsin(c*x)^2
/(c^2*x^2-1)/d^2/x+I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)
/d^2*(2*I*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*ln(1+
(I*c*x+(-c^2*x^2+1)^(1/2))^2)+2*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2)
))+4*arcsin(c*x)^2+2*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+polylog(2,-(I*c*x
+(-c^2*x^2+1)^(1/2))^2)+2*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2)))*c)+2*a*b*(
4*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/d^2*arcsin(c*x)*
c-(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*arcsin(c
*x)/(c^2*x^2-1)/d^2/x-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)
)/d^2*ln((I*c*x+(-c^2*x^2+1)^(1/2))^4-1)*c)
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^2 (-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))**2/x**2/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output

```
a*b*c*(log(c*x + 1)/d^(3/2) + log(c*x - 1)/d^(3/2) + 2*log(x)/d^(3/2)) + 2
*(2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d))*a*b*arc
sin(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*x
))*a^2 - b^2*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/((c^2*
d*x^4 - d*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx = \frac{-2\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^4 - \sqrt{-c^2 x^2 + 1} x^2} dx \right) abx - \sqrt{-c^2 x^2 + 1} \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1}} dx \right)}{\sqrt{d} \sqrt{-c^2 x^2 + 1}}$$

input `int((a+b*asin(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(- 2*sqrt(- c**2*x**2 + 1)*int(asin(c*x)/(sqrt(- c**2*x**2 + 1)*c**2*x**4 - sqrt(- c**2*x**2 + 1)*x**2),x)*a*b*x - sqrt(- c**2*x**2 + 1)*int(asin(c*x)**2/(sqrt(- c**2*x**2 + 1)*c**2*x**4 - sqrt(- c**2*x**2 + 1)*x**2),x)*b**2*x + 2*a**2*c**2*x**2 - a**2)/(sqrt(d)*sqrt(- c**2*x**2 + 1)*d*x)`

$$3.249 \quad \int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^{3/2}} dx$$

Optimal result	2445
Mathematica [A] (warning: unable to verify)	2446
Rubi [A] (verified)	2447
Maple [A] (verified)	2455
Fricas [F]	2456
Sympy [F]	2457
Maxima [F]	2457
Giac [F(-2)]	2457
Mupad [F(-1)]	2458
Reduce [F]	2458

Optimal result

Integrand size = 29, antiderivative size = 634

$$\begin{aligned}
& \int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = -\frac{bc\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))}{dx\sqrt{d - c^2 dx^2}} \\
& + \frac{3c^2(a + b \arcsin(cx))^2}{2d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{2dx^2\sqrt{d - c^2 dx^2}} \\
& + \frac{4ibc^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& - \frac{3c^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& - \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{d\sqrt{d - c^2 dx^2}} \\
& + \frac{3ibc^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& - \frac{2ib^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& + \frac{2ib^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& - \frac{3ibc^2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& - \frac{3b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}} \\
& + \frac{3b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{d\sqrt{d - c^2 dx^2}}
\end{aligned}$$

output

```

-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)+3/2*c^2
*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arcsin(c*x))^2/d/x^2/
(-c^2*d*x^2+d)^(1/2)+4*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan
(I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-3*c^2*(-c^2*x^2+1)^(1/2)
*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1
/2)-b^2*c^2*(-c^2*x^2+1)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d
)^(1/2)+3*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-
c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*c^2*(-c^2*x^2+1)^(1/2)*po
lylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*c^2*
(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d
)^(1/2)-3*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c
^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-3*b^2*c^2*(-c^2*x^2+1)^(1/2)*polyl
og(3,-I*c*x-(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+3*b^2*c^2*(-c^2*x^2
+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)

```

Mathematica [A] (warning: unable to verify)

Time = 7.75 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.33

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output

```

Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a^2/(d^2*x^2) - (a^2*c^2)/(d^2*(-1 + c^2*x
^2))) + (3*a^2*c^2*Log[x])/(2*d^(3/2)) - (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-
(d*(-1 + c^2*x^2))])]/(2*d^(3/2)) + (a*b*c*((6*I)*PolyLog[2, -E^(I*ArcSin[
c*x])]*Sin[2*ArcSin[c*x]] - (6*I)*PolyLog[2, E^(I*ArcSin[c*x])]*Sin[2*ArcSi
in[c*x]] - (-2*ArcSin[c*x] + 6*ArcSin[c*x]*Cos[2*ArcSin[c*x]] + 3*ArcSin[c
*x]*Cos[3*ArcSin[c*x]]*Log[1 - E^(I*ArcSin[c*x])] - 3*ArcSin[c*x]*Cos[3*Ar
cSin[c*x]]*Log[1 + E^(I*ArcSin[c*x])] + 2*Cos[3*ArcSin[c*x]]*Log[Cos[ArcSi
n[c*x]/2] - Sin[ArcSin[c*x]/2]] - 2*Cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]
/2] + Sin[ArcSin[c*x]/2]] + Sqrt[1 - c^2*x^2]*(-3*ArcSin[c*x]*Log[1 - E^(I
*ArcSin[c*x])] + 3*ArcSin[c*x]*Log[1 + E^(I*ArcSin[c*x])] - 2*Log[Cos[ArcS
in[c*x]/2] - Sin[ArcSin[c*x]/2]] + 2*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c
*x]/2])) + 2*Sin[2*ArcSin[c*x]]/(c*x))/(4*d*x*Sqrt[d*(1 - c^2*x^2)]) + (
b^2*c^2*Sqrt[1 - c^2*x^2]*(8*ArcSin[c*x]^2 - 4*ArcSin[c*x]*Cot[ArcSin[c*x]
/2] - ArcSin[c*x]^2*Csc[ArcSin[c*x]/2]^2 + 8*Log[Tan[ArcSin[c*x]/2]] - 16*
(ArcSin[c*x]*(Log[1 - I*E^(I*ArcSin[c*x])] - Log[1 + I*E^(I*ArcSin[c*x])])
+ I*(PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - PolyLog[2, I*E^(I*ArcSin[c*x])])
)) + 12*(ArcSin[c*x]^2*(Log[1 - E^(I*ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c
*x])])) + (2*I)*ArcSin[c*x]*(PolyLog[2, -E^(I*ArcSin[c*x])] - PolyLog[2, E^
(I*ArcSin[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcSin[c*x])] + PolyLog[3, E^(I*A
rcSin[c*x])])) + ArcSin[c*x]^2*Sec[ArcSin[c*x]/2]^2 + (8*ArcSin[c*x]^2*...

```

Rubi [A] (verified)

Time = 4.57 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.72, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5204, 5204, 243, 73, 221, 5164, 3042, 4669, 2715, 2838, 5208, 5164, 3042, 4669, 2715, 2838, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx \\
 & \quad \downarrow 5204 \\
 & \frac{3}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x (d - c^2 dx^2)^{3/2}} dx + \frac{bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x^2(1 - c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{2dx^2\sqrt{d - c^2 dx^2}} \\
 & \quad \downarrow 5204
 \end{aligned}$$

$$\begin{aligned}
& \frac{\frac{3}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx + bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + bc \int \frac{1}{x\sqrt{1-c^2x^2}} dx - \frac{a+b \arcsin(cx)}{x} \right)}{d\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{243} \\
& \frac{\frac{3}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx + bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{1}{2}bc \int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{a+b \arcsin(cx)}{x} \right)}{d\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{73} \\
& \frac{bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{b \int \frac{1}{\frac{1}{2}-x^2} d\sqrt{1-c^2x^2}}{c} - \frac{a+b \arcsin(cx)}{x} \right)}{d\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{221} \\
& \frac{bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{a+b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{d\sqrt{d-c^2dx^2}} + \frac{\frac{3}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{5164} \\
& \frac{bc\sqrt{1-c^2x^2} \left(c \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx) - \frac{a+b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{d\sqrt{d-c^2dx^2}} + \frac{\frac{3}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{3042} \\
& \frac{bc\sqrt{1-c^2x^2} \left(c \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) - \frac{a+b \arcsin(cx)}{x} - b \operatorname{arctanh}(\sqrt{1-c^2x^2}) \right)}{d\sqrt{d-c^2dx^2}} + \frac{\frac{3}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{4669}
\end{aligned}$$

$$\frac{bc\sqrt{1-c^2x^2}\left(c(-b\int\log(1-ie^{i\arcsin(cx)})d\arcsin(cx)+b\int\log(1+ie^{i\arcsin(cx)})d\arcsin(cx)-2i\arctan(e^{i\arcsin(cx)})\right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{3}{2}c^2\int\frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}}dx-\frac{(a+b\arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{bc\sqrt{1-c^2x^2}\left(c(ib\int e^{-i\arcsin(cx)}\log(1-ie^{i\arcsin(cx)})de^{i\arcsin(cx)}-ib\int e^{-i\arcsin(cx)}\log(1+ie^{i\arcsin(cx)})de^{i\arcsin(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{3}{2}c^2\int\frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}}dx-\frac{(a+b\arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}}$$

↓ 2838

$$\frac{\frac{3}{2}c^2\int\frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}}dx+bc\sqrt{1-c^2x^2}\left(c(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)})\right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{(a+b\arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}}$$

↓ 5208

$$\frac{\frac{3}{2}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arcsin(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arcsin(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}\right)+bc\sqrt{1-c^2x^2}\left(c(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)})\right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{(a+b\arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}}$$

↓ 5164

$$\frac{\frac{3}{2}c^2\left(-\frac{2b\sqrt{1-c^2x^2}\int\frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}}d\arcsin(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arcsin(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}\right)+bc\sqrt{1-c^2x^2}\left(c(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)})\right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{(a+b\arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}}{d\sqrt{d-c^2dx^2}}$$

↓ 3042

$$\frac{3}{2}c^2 \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{(a+b \arcsin(cx))}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(c(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})) \right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 4669

$$\frac{3}{2}c^2 \left(-\frac{2b\sqrt{1-c^2x^2} (-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(c(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})) \right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{3}{2}c^2 \left(-\frac{2b\sqrt{1-c^2x^2} (ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(c(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})) \right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 2838

$$\frac{3}{2}c^2 \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} - \frac{2b\sqrt{1-c^2x^2} (-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(c(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})) \right)}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 5218

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} \int \frac{(a+b \arcsin(cx))^2}{cx} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 3042

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 4671

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2}(-2b \int (a+b \arcsin(cx)) \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+e^{i \arcsin(cx)}) d \arcsin(cx))}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 3011

$$\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2}(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})(a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - 2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

↓ 2720

$$\frac{\frac{3}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) dx) + bc\sqrt{1-c^2x^2} (c(-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)}{(a + b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}} \downarrow 7143$$

$$\frac{\frac{3}{2}c^2 \left(-\frac{2b\sqrt{1-c^2x^2} (-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} + bc\sqrt{1-c^2x^2} (c(-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)}{(a + b \arcsin(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

```
input Int[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
output -1/2*(a + b*ArcSin[c*x])^2/(d*x^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*(-(a + b*ArcSin[c*x])/x) - b*c*ArcTanh[Sqrt[1 - c^2*x^2]] + c*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2]) + (3*c^2*((a + b*ArcSin[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/(d*Sqrt[d - c^2*d*x^2])))/2
```

Definitions of rubi rules used

- rule 73 $\text{Int}[(a_.) + (b_.)(x_)^{(m_)} * ((c_.) + (d_.)(x_))^{(n_)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Simp}[p/b \text{ Subst}[\text{Int}[x^{(p*(m+1)-1)} * (c - a*(d/b) + d*(x^p/b))^{(n)}, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$
- rule 221 $\text{Int}[(a_) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) * \text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$
- rule 243 $\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/2 \text{ Subst}[\text{Int}[x^{((m-1)/2) * (a + b*x)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \&\& \text{IntegerQ}[(m-1)/2]$
- rule 2715 $\text{Int}[\text{Log}[(a_) + (b_.)((F_)^{((e_.)((c_.) + (d_.)(x_)))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \text{ Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_)*((a_.)(v_)^{(n_)})^{(m_)} /; \text{FreeQ}[\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n]] \&\& \text{!MatchQ}[u, E^{((c_.)((a_.) + (b_.)*x))} * (F_) [v_] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$
- rule 2838 $\text{Int}[\text{Log}[(c_.)((d_) + (e_.)(x_)^{(n_.)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_.)((F_)^{((c_.)((a_.) + (b_.)(x_)))})^{(n_.)}] * ((f_.) + (g_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n]) / (b*c*n*\text{Log}[F]), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{ Int}[(f + g*x)^{(m-1)} * \text{PolyLog}[2, (-e)*(F^{(c*(a + b*x))})^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5164 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5204 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]`

rule 5208 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1)) Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])`

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_)]/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.37

method	result
default	$a^2 \left(-\frac{1}{2dx^2\sqrt{-c^2dx^2+d}} + \frac{3c^2 \left(\frac{1}{d\sqrt{-c^2dx^2+d}} - \frac{\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2} \right) + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} \arcsin(cx) (3c^2x^2 a}{2d^2(c}$
parts	$a^2 \left(-\frac{1}{2dx^2\sqrt{-c^2dx^2+d}} + \frac{3c^2 \left(\frac{1}{d\sqrt{-c^2dx^2+d}} - \frac{\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2} \right) + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)} \arcsin(cx) (3c^2x^2 a}{2d^2(c}$

input

```
int((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```

a^2*(-1/2/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(1/d/(-c^2*d*x^2+d)^(1/2)-1/d
^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))+b^2*(-1/2*(-d*(c^2*x^2
-1))^(1/2)/d^2/(c^2*x^2-1)/x^2*arcsin(c*x)*(3*c^2*x^2*arcsin(c*x)-2*c*x*(-
c^2*x^2+1)^(1/2)-arcsin(c*x))+1/2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2
)/(c^2*x^2-1)/d^2*(3*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-3*arcsin
(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-6*I*arcsin(c*x)*polylog(2,-I*c*x-(-
c^2*x^2+1)^(1/2))+6*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-4*ar
csin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+4*arcsin(c*x)*ln(1-I*(I*c*x+(
-c^2*x^2+1)^(1/2)))+4*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-4*I*dilog(1-
I*(I*c*x+(-c^2*x^2+1)^(1/2)))+2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+6*polylog(3
,-I*c*x-(-c^2*x^2+1)^(1/2))-6*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))-2*ln(I*c
*x+(-c^2*x^2+1)^(1/2)-1))*c^2-I*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(
1/2)*(3*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^4*c^4+4*arctan(I*c*
x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+
3*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*I*(-c^2*x^2+1)^(1/2)*arcsin(c*
x)*c^2*x^2-3*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^2*c^2+I*x^3*c^
3-4*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-3*dilog(1+I*c*x+(-c^2*x^2+1)^(
1/2))*c^2*x^2-3*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-I*(-c^2*x^2+1)^(1
/2)*arcsin(c*x)-I*c*x)/d^2/(c^4*x^4-2*c^2*x^2+1)/x^2

```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{3/2} x^3} dx$$

input

```

integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

```

output

```

integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2
)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)

```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^3 (-d(cx - 1)(cx + 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))**2/x**3/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{\frac{3}{2}} x^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `-1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a^2 + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{3/2}} dx = \frac{-16\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^5 - \sqrt{-c^2 x^2 + 1} x^3} dx \right) ab x^2 - 8\sqrt{-c^2 x^2 + 1} \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} x^3} dx \right)}$$

input

```
int((a+b*asin(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
( - 16*sqrt( - c**2*x**2 + 1)*int(asin(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**5 - sqrt( - c**2*x**2 + 1)*x**3),x)*a*b*x**2 - 8*sqrt( - c**2*x**2 + 1)*
int(asin(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**5 - sqrt( - c**2*x**2 + 1
)*x**3),x)*b**2*x**2 + 12*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**
2*c**2*x**2 - 9*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 12*a**2*c**2*x**2
- 4*a**2)/(8*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**2)
```

$$3.250 \quad \int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^{3/2}} dx$$

Optimal result	2459
Mathematica [A] (verified)	2460
Rubi [A] (verified)	2461
Maple [B] (verified)	2467
Fricas [F]	2468
Sympy [F]	2469
Maxima [F]	2469
Giac [F(-2)]	2469
Mupad [F(-1)]	2470
Reduce [F]	2470

Optimal result

Integrand size = 29, antiderivative size = 473

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^{3/2}} dx = & -\frac{b^2c^2\sqrt{d-c^2dx^2}}{3d^2x} - \frac{bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3dx^2\sqrt{d-c^2dx^2}} \\ & - \frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2(a+b \arcsin(cx))^2}{3dx\sqrt{d-c^2dx^2}} \\ & + \frac{8c^4x(a+b \arcsin(cx))^2}{3d\sqrt{d-c^2dx^2}} - \frac{8ic^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3d\sqrt{d-c^2dx^2}} \\ & - \frac{20bc^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{3d\sqrt{d-c^2dx^2}} \\ & + \frac{16bc^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))\log(1+e^{2i \arcsin(cx)})}{3d\sqrt{d-c^2dx^2}} \\ & - \frac{ib^2c^3\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,-e^{2i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \\ & - \frac{5ib^2c^3\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,e^{2i \arcsin(cx)})}{3d\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
-1/3*b^2*c^2*(-c^2*d*x^2+d)^(1/2)/d^2/x-1/3*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/d/x^2/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arcsin(c*x))^2/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arcsin(c*x))^2/d/x/(-c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-8/3*I*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-20/3*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)+16/3*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-I*b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-5/3*I*b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.98

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \frac{-a^2 - 4a^2 c^2 x^2 - b^2 c^2 x^2 + 8a^2 c^4 x^4 + b^2 c^4 x^4 - abcx\sqrt{1 - c^2 x^2} - 2ab \arcsin(cx)}{x^4 (d - c^2 dx^2)^{3/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output

```
(-a^2 - 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 + b^2*c^4*x^4 - a*b*c*x*Sqrt[1 - c^2*x^2] - 2*a*b*ArcSin[c*x] - 8*a*b*c^2*x^2*ArcSin[c*x] + 16*a*b*c^4*x^4*ArcSin[c*x] - b^2*c*x*Sqrt[1 - c^2*x^2]*ArcSin[c*x] - b^2*ArcSin[c*x]^2 - 4*b^2*c^2*x^2*ArcSin[c*x]^2 + 8*b^2*c^4*x^4*ArcSin[c*x]^2 - (8*I)*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2 + 10*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 6*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + E^((2*I)*ArcSin[c*x])] + 10*a*b*c^3*x^3*Sqrt[1 - c^2*x^2]*Log[c*x] + 3*a*b*c^3*x^3*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] - (3*I)*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (5*I)*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(3*d*x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 3.39 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {5204, 5204, 242, 5160, 5180, 3042, 4202, 2620, 2715, 2838, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx$$

$$\downarrow 5204$$

$$\frac{4}{3} c^2 \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{3/2}} dx + \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x^3(1-c^2 x^2)} dx}{3d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{3dx^3\sqrt{d - c^2 dx^2}}$$

$$\downarrow 5204$$

$$\frac{4}{3} c^2 \left(2c^2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx + \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \right) +$$

$$\frac{2bc\sqrt{1 - c^2 x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx + \frac{1}{2} bc \int \frac{1}{x^2\sqrt{1-c^2 x^2}} dx - \frac{a+b \arcsin(cx)}{2x^2} \right)}{3d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{3dx^3\sqrt{d - c^2 dx^2}}$$

$$\downarrow 242$$

$$\frac{4}{3} c^2 \left(2c^2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx + \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \right) +$$

$$\frac{2bc\sqrt{1 - c^2 x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2 x^2}}{2x} \right)}{3d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{3dx^3\sqrt{d - c^2 dx^2}}$$

$$\downarrow 5160$$

$$\frac{4}{3} c^2 \left(2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}} \right) + \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx}{d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx\sqrt{d - c^2 dx^2}} \right) +$$

$$\frac{2bc\sqrt{1 - c^2 x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2 x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2 x^2}}{2x} \right)}{3d\sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{3dx^3\sqrt{d - c^2 dx^2}}$$

$$\downarrow 5180$$

$$\frac{4}{3}c^2 \left(2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{cd\sqrt{d-c^2dx^2}} \right) + \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}} \right)$$

↓ 3042

$$\frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} + \\ \frac{4}{3}c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \tan(\arcsin(cx))}{cd\sqrt{d-c^2dx^2}} \right) \right. \\ \left. \frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}} \right)$$

↓ 4202

$$\frac{4}{3}c^2 \left(2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{cd\sqrt{d-c^2dx^2}} \right) \right) + \\ \frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 2620

$$\frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} + \\ \frac{4}{3}c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log \right) \right)}{cd\sqrt{d-c^2dx^2}} \right) \right. \\ \left. \frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}} \right)$$

↓ 2715

$$\frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{a+b \arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} + \\ \frac{4}{3}c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \right) \right)}{cd\sqrt{d-c^2dx^2}} \right) \right. \\ \left. \frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}} \right)$$

↓ 2838

$$\frac{4}{3}c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)} dx}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i(-\frac{1}{2}i \log \dots) \right)}{d\sqrt{d-c^2dx^2}} \right) \right)$$

$$\frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)} dx - \frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} - \frac{(a+b\arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 5184

$$\frac{4}{3}c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b\arcsin(cx)}{cx\sqrt{1-c^2x^2}} d\arcsin(cx)}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i \dots \right)}{d\sqrt{d-c^2dx^2}} \right) \right)$$

$$\frac{2bc\sqrt{1-c^2x^2} \left(c^2 \int \frac{a+b\arcsin(cx)}{cx\sqrt{1-c^2x^2}} d\arcsin(cx) - \frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} - \frac{(a+b\arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 4919

$$\frac{4}{3}c^2 \left(\frac{4bc\sqrt{1-c^2x^2} \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx)}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \dots}{d\sqrt{d-c^2dx^2}} \right) \right)$$

$$\frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx) - \frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} - \frac{(a+b\arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 3042

$$\frac{4}{3}c^2 \left(\frac{4bc\sqrt{1-c^2x^2} \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx)}{d\sqrt{d-c^2dx^2}} + 2c^2 \left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \dots}{d\sqrt{d-c^2dx^2}} \right) \right)$$

$$\frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx) - \frac{a+b\arcsin(cx)}{2x^2} - \frac{bc\sqrt{1-c^2x^2}}{2x} \right)}{3d\sqrt{d-c^2dx^2}} - \frac{(a+b\arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 4671

$$\frac{\frac{4}{3}c^2 \left(\frac{4bc\sqrt{1-c^2x^2} \left(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - (\arctanh(e^{2i \arcsin(cx)})(a+b \arcsin(cx))) \right)}{d\sqrt{d-c^2dx^2}} \right)}{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - (\arctanh(e^{2i \arcsin(cx)})(a+b \arcsin(cx))) \right) \right)}{3d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 2715

$$\frac{\frac{4}{3}c^2 \left(\frac{4bc\sqrt{1-c^2x^2} \left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1-e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - (\text{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx))) \right)}{d\sqrt{d-c^2dx^2}} \right)}{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1-e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - (\text{PolyLog}(2, -e^{2i \arcsin(cx)})(a+b \arcsin(cx))) \right) \right)}{3d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

↓ 2838

$$\frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(-(\arctanh(e^{2i \arcsin(cx)})(a+b \arcsin(cx))) + \frac{1}{4}ib \text{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \text{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right)}{3d\sqrt{d-c^2dx^2}}$$

$$\frac{\frac{4}{3}c^2 \left(\frac{4bc\sqrt{1-c^2x^2} \left(-(\arctanh(e^{2i \arcsin(cx)})(a+b \arcsin(cx))) + \frac{1}{4}ib \text{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \text{PolyLog}(2, e^{2i \arcsin(cx)}) \right)}{d\sqrt{d-c^2dx^2}} \right)}{(a+b \arcsin(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]`

output

```
-1/3*(a + b*ArcSin[c*x])^2/(d*x^3*Sqrt[d - c^2*d*x^2]) + (2*b*c*Sqrt[1 - c^2*x^2]*(-1/2*(b*c*Sqrt[1 - c^2*x^2])/x - (a + b*ArcSin[c*x])/(2*x^2) + 2*c^2*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x]])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x]])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(3*d*Sqrt[d - c^2*d*x^2]) + (4*c^2*(-((a + b*ArcSin[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + 2*c^2*((x*(a + b*ArcSin[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x]]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x]])]/4)))/(c*d*Sqrt[d - c^2*d*x^2])) + (4*b*c*Sqrt[1 - c^2*x^2]*(-((a + b*ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x]])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x]])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2]))/3
```

Defintions of rubi rules used

rule 242

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 4202 $\text{Int}[(c + d*x)^m * \tan(e + f*x), x_Symbol] \rightarrow \text{Simp}[I * ((c + d*x)^{m+1} / (d*(m+1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * (E^{2*I*(e + f*x)} / (1 + E^{2*I*(e + f*x)}))], x], x] /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

rule 4671 $\text{Int}[\text{csc}(e + f*x) * (c + d*x)^m, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m * (\text{ArcTanh}[E^{I*(e + f*x)}] / f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 - E^{I*(e + f*x)}]], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + E^{I*(e + f*x)}]], x], x] /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

rule 4919 $\text{Int}[\text{Csc}(a + b*x)^n * (c + d*x)^m * \text{Sec}(a + b*x)^m, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m * \text{Csc}[2*a + 2*b*x]^n, x], x] /;$ FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]

rule 5160 $\text{Int}[(a + \text{ArcSin}(c*x) * b)^n / ((d + e*x^2)^{3/2}), x_Symbol] \rightarrow \text{Simp}[x * (a + b * \text{ArcSin}[c*x])^n / (d * \text{Sqrt}[d + e*x^2]), x] - \text{Simp}[b * c * (n/d) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] \text{Int}[x * (a + b * \text{ArcSin}[c*x])^{n-1} / (1 - c^2*x^2)], x], x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]

rule 5180 $\text{Int}[(a + \text{ArcSin}(c*x) * b)^n * x / (d + e*x^2), x_Symbol] \rightarrow \text{Simp}[-e^{-1} \text{Subst}[\text{Int}[(a + b*x)^n * \text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]

rule 5184 $\text{Int}[(a + \text{ArcSin}(c*x) * b)^n / (x * (d + e*x^2)), x_Symbol] \rightarrow \text{Simp}[1/d \text{Subst}[\text{Int}[(a + b*x)^n / (\text{Cos}[x] * \text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]

rule 5204

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]

```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2843 vs. $2(460) = 920$.

Time = 0.89 (sec) , antiderivative size = 2844, normalized size of antiderivative = 6.01

method	result	size
default	Expression too large to display	2844
parts	Expression too large to display	2844

input

```
int((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```


output

```
-128/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^5+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^7*c^10-40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^5*c^8+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^3*arcsin(c*x)^2+a^2*(-1/3/d/x^3/(-c^2*d*x^2+d)^(1/2)+4/3*c^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2))+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*(-c^2*x^2+1)^(1/2)*c^3+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^3*arcsin(c*x)+64/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^5*(-c^2*x^2+1)*c^8-32/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*(-c^2*x^2+1)*c^6+32/3*I*a*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arcsin(c*x)*c^3-8/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*(-c^2*x^2+1)*c^4-16/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*c^3+64/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^5*(-c^2*x^2+1)*arcsin(c*x)*c^8-32/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*(-c^2*x^2+1)*arcsin(c*x)*c^6-64/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2*c^5-8/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7...
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{3/2} x^4} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^4 (-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate((a+b*asin(c*x))**2/x**4/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{3/2} x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) - 1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a^2 + sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{3/2}} dx = \frac{-6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^6 - \sqrt{-c^2 x^2 + 1} x^4} dx \right) ab x^3 - 3\sqrt{-c^2 x^2 + 1} \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1}} dx \right)}{3\sqrt{d} \sqrt{-c^2 x^2 + 1} dx^3}$$

input

```
int((a+b*asin(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
( - 6*sqrt( - c**2*x**2 + 1)*int(asin(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*6 - sqrt( - c**2*x**2 + 1)*x**4),x)*a*b*x**3 - 3*sqrt( - c**2*x**2 + 1)*i
nt(asin(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**6 - sqrt( - c**2*x**2 + 1)
*x**4),x)*b**2*x**3 + 8*a**2*c**4*x**4 - 4*a**2*c**2*x**2 - a**2)/(3*sqrt(
d)*sqrt( - c**2*x**2 + 1)*d*x**3)
```

3.251
$$\int \frac{x^5(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2471
Mathematica [A] (verified)	2472
Rubi [A] (verified)	2473
Maple [A] (verified)	2480
Fricas [F]	2481
Sympy [F]	2482
Maxima [F]	2482
Giac [F(-2)]	2483
Mupad [F(-1)]	2483
Reduce [F]	2483

Optimal result

Integrand size = 29, antiderivative size = 447

$$\begin{aligned} \int \frac{x^5(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx &= \frac{b^2}{3c^6d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{2b^2\sqrt{d-c^2dx^2}}{c^6d^3} - \frac{bx^3(a+b \arcsin(cx))}{3c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} \\ &+ \frac{5bx\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\ &- \frac{4x^2(a+b \arcsin(cx))^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{8\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{3c^6d^3} \\ &- \frac{22ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{3c^6d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{11ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{3c^6d^2\sqrt{d-c^2dx^2}} \\ &- \frac{11ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, ie^{i \arcsin(cx)})}{3c^6d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

$$\begin{aligned} & 1/3*b^2/c^6/d^2/(-c^2*d*x^2+d)^{(1/2)}+2*b^2*(-c^2*d*x^2+d)^{(1/2)}/c^6/d^3-1/ \\ & 3*b*x^3*(a+b*\arcsin(c*x))/c^3/d^2/(-c^2*x^2+1)^{(1/2)}/(-c^2*d*x^2+d)^{(1/2)}+ \\ & 5/3*b*x*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))/c^5/d^2/(-c^2*d*x^2+d)^{(1/2)}+ \\ & 1/3*x^4*(a+b*\arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^{(3/2)}-4/3*x^2*(a+b*\arcsin \\ & (c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^{(1/2)}-8/3*(-c^2*d*x^2+d)^{(1/2)}*(a+b*\arcsin \\ & (c*x))^2/c^6/d^3-22/3*I*b*(-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))*\arctan(I*c* \\ & x+(-c^2*x^2+1)^{(1/2)})/c^6/d^2/(-c^2*d*x^2+d)^{(1/2)}+11/3*I*b^2*(-c^2*x^2+1) \\ & ^{(1/2)}*\operatorname{polylog}(2,-I*(I*c*x+(-c^2*x^2+1)^{(1/2)}))/c^6/d^2/(-c^2*d*x^2+d)^{(1/2)} \\ & -11/3*I*b^2*(-c^2*x^2+1)^{(1/2)}*\operatorname{polylog}(2,I*(I*c*x+(-c^2*x^2+1)^{(1/2)}))/c \\ & ^6/d^2/(-c^2*d*x^2+d)^{(1/2)} \end{aligned}$$
Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.33

$$\int \frac{x^5(a+b\arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx = \frac{\sqrt{d-c^2dx^2}(-64a^2+22b^2+96a^2c^2x^2-24a^2c^4x^4-50ab\arcsin(cx)-25b^2)}{(d-c^2dx^2)^{5/2}}$$

input

$$\text{Integrate}[(x^5*(a + b*\text{ArcSin}[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]$$

output

$$\begin{aligned} & (\text{Sqrt}[d - c^2*d*x^2]*(-64*a^2 + 22*b^2 + 96*a^2*c^2*x^2 - 24*a^2*c^4*x^4 - \\ & 50*a*b*\text{ArcSin}[c*x] - 25*b^2*\text{ArcSin}[c*x]^2 + 28*b^2*\text{Cos}[2*\text{ArcSin}[c*x]] - 7 \\ & 2*a*b*\text{ArcSin}[c*x]*\text{Cos}[2*\text{ArcSin}[c*x]] - 36*b^2*\text{ArcSin}[c*x]^2*\text{Cos}[2*\text{ArcSin}[c \\ & *x]] + 6*b^2*\text{Cos}[4*\text{ArcSin}[c*x]] - 6*a*b*\text{ArcSin}[c*x]*\text{Cos}[4*\text{ArcSin}[c*x]] - 3 \\ & *b^2*\text{ArcSin}[c*x]^2*\text{Cos}[4*\text{ArcSin}[c*x]] + 66*b^2*\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c* \\ & x]*\text{Log}[1 - I*E^(I*\text{ArcSin}[c*x])] + 22*b^2*\text{ArcSin}[c*x]*\text{Cos}[3*\text{ArcSin}[c*x]]* \\ & \text{Log}[1 - I*E^(I*\text{ArcSin}[c*x])] - 66*b^2*\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c*x]*\text{Log}[1 + \\ & I*E^(I*\text{ArcSin}[c*x])] - 22*b^2*\text{ArcSin}[c*x]*\text{Cos}[3*\text{ArcSin}[c*x]]*\text{Log}[1 + I*E^(\\ & I*\text{ArcSin}[c*x])] - 66*a*b*\text{Sqrt}[1 - c^2*x^2]*\text{Log}[\text{Cos}[\text{ArcSin}[c*x]/2] - \text{Sin}[\text{Arc} \\ & \text{Sin}[c*x]/2]] - 22*a*b*\text{Cos}[3*\text{ArcSin}[c*x]]*\text{Log}[\text{Cos}[\text{ArcSin}[c*x]/2] - \text{Sin}[\text{Arc} \\ & \text{Sin}[c*x]/2]] + 66*a*b*\text{Sqrt}[1 - c^2*x^2]*\text{Log}[\text{Cos}[\text{ArcSin}[c*x]/2] + \text{Sin}[\text{ArcSi} \\ & n[c*x]/2]] + 22*a*b*\text{Cos}[3*\text{ArcSin}[c*x]]*\text{Log}[\text{Cos}[\text{ArcSin}[c*x]/2] + \text{Sin}[\text{ArcSi} \\ & n[c*x]/2]] + (88*I)*b^2*(1 - c^2*x^2)^(3/2)*\text{PolyLog}[2, (-I)*E^(I*\text{ArcSin}[c*x \\ &])] - (88*I)*b^2*(1 - c^2*x^2)^(3/2)*\text{PolyLog}[2, I*E^(I*\text{ArcSin}[c*x])] + 8*a \\ & *b*\text{Sin}[2*\text{ArcSin}[c*x]] + 8*b^2*\text{ArcSin}[c*x]*\text{Sin}[2*\text{ArcSin}[c*x]] + 6*a*b*\text{Sin}[4 \\ & *\text{ArcSin}[c*x]] + 6*b^2*\text{ArcSin}[c*x]*\text{Sin}[4*\text{ArcSin}[c*x]]))/(24*c^6*d^3*(-1 + c \\ & ^2*x^2)^2) \end{aligned}$$

Rubi [A] (verified)

Time = 3.05 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5206, 5206, 243, 53, 2009, 5182, 2009, 5210, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx \\
 & \quad \downarrow \text{5206} \\
 & \frac{2b\sqrt{1 - c^2 x^2} \int \frac{x^4(a + b \arcsin(cx))}{(1 - c^2 x^2)^2} dx}{3cd^2\sqrt{d - c^2 dx^2}} - \frac{4 \int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx}{3c^2 d} + \frac{x^4(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{5206} \\
 & \frac{2b\sqrt{1 - c^2 x^2} \left(-\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{2c^2} - \frac{b \int \frac{x^3}{(1 - c^2 x^2)^{3/2}} dx}{2c} + \frac{x^3(a + b \arcsin(cx))}{2c^2(1 - c^2 x^2)} \right)}{3cd^2\sqrt{d - c^2 dx^2}} \\
 & \frac{4 \left(-\frac{2b\sqrt{1 - c^2 x^2} \int \frac{x^2(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{cd\sqrt{d - c^2 dx^2}} - \frac{2 \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx}{c^2 d} + \frac{x^2(a + b \arcsin(cx))^2}{c^2 d\sqrt{d - c^2 dx^2}} \right)}{3c^2 d} + \\
 & \frac{x^4(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{243} \\
 & \frac{2b\sqrt{1 - c^2 x^2} \left(-\frac{3 \int \frac{x^2(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{2c^2} - \frac{b \int \frac{x^2}{(1 - c^2 x^2)^{3/2}} dx}{4c} + \frac{x^3(a + b \arcsin(cx))}{2c^2(1 - c^2 x^2)} \right)}{3cd^2\sqrt{d - c^2 dx^2}} \\
 & \frac{4 \left(-\frac{2b\sqrt{1 - c^2 x^2} \int \frac{x^2(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{cd\sqrt{d - c^2 dx^2}} - \frac{2 \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx}{c^2 d} + \frac{x^2(a + b \arcsin(cx))^2}{c^2 d\sqrt{d - c^2 dx^2}} \right)}{3c^2 d} + \\
 & \frac{x^4(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{53}
 \end{aligned}$$

$$\begin{aligned}
 & 2b\sqrt{1-c^2x^2} \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2} - \frac{b \int \left(\frac{1}{c^2(1-c^2x^2)^{3/2}} - \frac{1}{c^2\sqrt{1-c^2x^2}} \right) dx^2}{4c} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right) \\
 & \frac{3cd^2\sqrt{d-c^2dx^2}}{4 \left(-\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \right)} \\
 & \frac{3c^2d}{x^4(a+b \arcsin(cx))^2} \\
 & \frac{3c^2d}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \downarrow \text{2009} \\
 & 4 \left(-\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{2 \int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \right) \\
 & \frac{3c^2d}{2b\sqrt{1-c^2x^2} \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right)} \\
 & \frac{3cd^2\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} \\
 & \frac{3c^2d}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \downarrow \text{5182} \\
 & 4 \left(-\frac{2 \left(\frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) dx}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^2}{c^2d} \right)}{c^2d} - \frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \right) \\
 & \frac{3c^2d}{2b\sqrt{1-c^2x^2} \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right)} \\
 & \frac{3cd^2\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} \\
 & \frac{3c^2d}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \downarrow \text{2009}
 \end{aligned}$$

$$4 \left(-\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} (ax+b \arcsin(cx)) + \frac{b\sqrt{1-c^2x^2}}{c}}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))}{c^2d} \right)}{c^2d} \right)$$

$$\frac{2b\sqrt{1-c^2x^2} \left(-\frac{3 \int \frac{x^2(a+b \arcsin(cx))}{1-c^2x^2} dx}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right)}{3c^2d} + \frac{3cd^2\sqrt{d-c^2dx^2} x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

↓ 5210

$$4 \left(-\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} (ax+b \arcsin(cx)) + \frac{b\sqrt{1-c^2x^2}}{c}}{c\sqrt{d-c^2dx^2}} \right)}{c^2d} \right)$$

$$2b\sqrt{1-c^2x^2} \left(-\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} + \frac{b \int \frac{x}{\sqrt{1-c^2x^2}} dx}{c} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right) + \frac{3cd^2\sqrt{d-c^2dx^2} x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

↓ 241

$$4 \left(-\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} (ax+b \arcsin(cx)) + \frac{b\sqrt{1-c^2x^2}}{c}}{c\sqrt{d-c^2dx^2}} \right)}{c^2d} \right)$$

$$2b\sqrt{1-c^2x^2} \left(-\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c^2} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right) + \frac{3cd^2\sqrt{d-c^2dx^2} x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

↓ 5164

$$4 \left(-\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} (ax+b \arcsin(cx)) + \frac{b^2}{c^2}}{c\sqrt{d-c^2dx^2}} \right)}{c^2d\sqrt{d-c^2dx^2}} \right)$$

$$2b\sqrt{1-c^2x^2} \left(-\frac{3 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right)$$

$$\frac{3cd^2\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} - \frac{3c^2d}{2c^2(1-c^2x^2)}$$

↓ 3042

$$4 \left(-\frac{2b\sqrt{1-c^2x^2} \left(\frac{\int (a+b \arcsin(cx)) \csc\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{cd\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{2b\sqrt{1-c^2x^2} (ax+b \arcsin(cx)) + \frac{b^2}{c^2}}{c\sqrt{d-c^2dx^2}} \right)}{c^2d\sqrt{d-c^2dx^2}} \right)$$

$$2b\sqrt{1-c^2x^2} \left(-\frac{3 \left(\frac{\int (a+b \arcsin(cx)) \csc\left(\arcsin(cx) + \frac{\pi}{2}\right) d \arcsin(cx)}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{2c^2} + \frac{x^3(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{2\sqrt{1-c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1-c^2x^2}} \right)}{4c} \right)$$

$$\frac{3cd^2\sqrt{d-c^2dx^2}}{x^4(a+b \arcsin(cx))^2} - \frac{3c^2d}{2c^2(1-c^2x^2)}$$

↓ 4669

$$4 \left(\frac{2b\sqrt{1-c^2x^2} \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx)) (a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$2b\sqrt{1-c^2x^2} \left(\frac{3 \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx)) (a+b \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} \right)}{2c^2} \right)$$

$$\frac{x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \quad \frac{3cd^2\sqrt{d-c^2dx^2}}$$

↓ 2715

$$4 \left(\frac{2b\sqrt{1-c^2x^2} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{c^3} \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$2b\sqrt{1-c^2x^2} \left(\frac{3 \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{c^3} \right)}{2c^2} \right)$$

$$\frac{x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \quad \frac{3cd^2\sqrt{d-c^2dx^2}}$$

↓ 2838

$$4 \left(\frac{2b\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx)) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{cd\sqrt{d-c^2dx^2}} \right)$$

$$2b\sqrt{1-c^2x^2} \left(\frac{3 \left(\frac{-2i \arctan(e^i \arcsin(cx)) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{c^3} - \frac{x(a+b \arcsin(cx))}{c^2} - \frac{b\sqrt{1-c^2x^2}}{c^3} \right)}{2c^2} \right)$$

$$\frac{x^4(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \quad \frac{3cd^2\sqrt{d-c^2dx^2}}$$

input `Int[(x^5*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]`

output `(x^4*(a + b*ArcSin[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (2*b*Sqrt[1 - c^2*x^2]*(-1/4*(b*(2/(c^4*Sqrt[1 - c^2*x^2]) + (2*Sqrt[1 - c^2*x^2])/c^4)))/c + (x^3*(a + b*ArcSin[c*x]))/(2*c^2*(1 - c^2*x^2)) - (3*(-((b*Sqrt[1 - c^2*x^2])/c^3) - (x*(a + b*ArcSin[c*x]))/c^2 + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/c^3))/(2*c^2))/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (4*((x^2*(a + b*ArcSin[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^2)/(c^2*d)) + (2*b*Sqrt[1 - c^2*x^2]*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(c^2*d) - (2*b*Sqrt[1 - c^2*x^2]*(-((b*Sqrt[1 - c^2*x^2])/c^3) - (x*(a + b*ArcSin[c*x]))/c^2 + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/c^3))/(c*d*Sqrt[d - c^2*d*x^2])))/(3*c^2*d)`

Defintions of rubi rules used

rule 53 `Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 243 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^((n_))], x_Symbol]$
 $\text{:> Simp}[1/(d*e*n*\text{Log}[F]) \text{ Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] \text{:> Simp}[-\text{PolyLog}[2,$
 $(-c)*e*x^n/n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \text{:> Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinear}$
 $\text{Q}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol]$
 $\text{:> Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f)$
 $\text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f)$
 $\text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x) /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)/((d_) + (e_)*(x_)^2), x_Symbol]$
 $\text{:> Simp}[1/(c*d) \text{ Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x]$
 $/;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_)$
 $, x_Symbol] \text{:> Simp}[(d + e*x^2)^(p + 1)*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x]$
 $+ \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(1 - c^2*x^2)^(p + 1/2)*$
 $(a + b*\text{ArcSin}[c*x])^(n - 1), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, p\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5206 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^(n_)*((f_)*(x_))^(m_)*((d_) + (e_)$
 $)*(x_)^2)^(p_), x_Symbol] \text{:> Simp}[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +$
 $b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + (-\text{Simp}[f^2*((m - 1)/(2*e*(p + 1)))$
 $\text{Int}[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}$
 $[b*f*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^(m -$
 $1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x] /;$ $\text{FreeQ}\{a,$
 $b, c, d, e, f\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IGtQ}[m, 1]$

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.82

method	result
default	$a^2 \left(-\frac{x^4}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} + \frac{\frac{4x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{8}{3d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}}}{c^2} \right) + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} (c^2 x^2 - i\sqrt{-c^2 x^2 + 1} cx - 1)}{2d^3 c^6 (c^2 x^2 - 1)} \right)$
parts	$a^2 \left(-\frac{x^4}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} + \frac{\frac{4x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{8}{3d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}}}{c^2} \right) + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} (c^2 x^2 - i\sqrt{-c^2 x^2 + 1} cx - 1)}{2d^3 c^6 (c^2 x^2 - 1)} \right)$

input

```
int(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```

a^2*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)
-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2)))+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^
2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))/d^3/c^6/(c
^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*
(arcsin(c*x)^2-2-2*I*arcsin(c*x))/d^3/c^6/(c^2*x^2-1)+1/3*(-d*(c^2*x^2-1))
^(1/2)*(6*arcsin(c*x)^2*x^2*c^2-arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x-c^2*x^2
-5*arcsin(c*x)^2+1)/(c^2*x^2-1)^2/d^3/c^6+11/3*(-c^2*x^2+1)^(1/2)*(-d*(c^2
*x^2-1))^(1/2)*(arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)
*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))
)+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))/d^3/c^6/(c^2*x^2-1))+2*a*b*(-1/
2*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)
+I)/d^3/c^6/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c
*x+c^2*x^2-1)*(arcsin(c*x)-I)/d^3/c^6/(c^2*x^2-1)+1/6*(-d*(c^2*x^2-1))^(1/
2)*(12*c^2*x^2*arcsin(c*x)-c*x*(-c^2*x^2+1)^(1/2)-10*arcsin(c*x))/(c^2*x^2
-1)^2/d^3/c^6-11/6*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^6/(c^2*
x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)+11/6*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2
-1))^(1/2)/d^3/c^6/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I))

```

Fricas [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^5}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```

integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

```

output

```

integral(-(b^2*x^5*arcsin(c*x)^2 + 2*a*b*x^5*arcsin(c*x) + a^2*x^5)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)

```

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^5(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate(x**5*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral(x**5*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**5/2, x)`

Maxima [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^5}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `-1/3*a^2*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2 + d)^(3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d)) - 1/3*((3*b^2*c^4*x^4 - 12*b^2*c^2*x^2 + 8*b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 3*(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3)*integrate(2/3*(3*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c^5*sqrt(d)*x^5*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) - (3*b^2*c^6*x^6 - 15*b^2*c^4*x^4 + 20*b^2*c^2*x^2 - 8*b^2)*sqrt(d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/(c^11*d^3*x^6 - 3*c^9*d^3*x^4 + 3*c^7*d^3*x^2 - c^5*d^3), x))/(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x^5*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)`

output `int((x^5*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x^5(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x^5}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) ab c^8 x^2 - 6\sqrt{-c^2 x^2 + 1}}{6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)x^5}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) ab c^8 x^2 - 6\sqrt{-c^2 x^2 + 1}}$$

input `int(x^5*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)`

output

```
(6*sqrt(-c**2*x**2+1)*int((asin(c*x)*x**5)/(sqrt(-c**2*x**2+1)*c**
4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*a
*b*c**8*x**2-6*sqrt(-c**2*x**2+1)*int((asin(c*x)*x**5)/(sqrt(-c**2
*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x
**2+1)),x)*a*b*c**6+3*sqrt(-c**2*x**2+1)*int((asin(c*x)**2*x**5)/(
sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sq
rt(-c**2*x**2+1)),x)*b**2*c**8*x**2-3*sqrt(-c**2*x**2+1)*int((as
in(c*x)**2*x**5)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+
1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b**2*c**6+3*a**2*c**4*x**4-
12*a**2*c**2*x**2+8*a**2)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*c**6*d**2*(c
**2*x**2-1))
```

3.252
$$\int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2485
Mathematica [A] (verified)	2486
Rubi [A] (verified)	2487
Maple [B] (verified)	2492
Fricas [F]	2493
Sympy [F]	2494
Maxima [F]	2494
Giac [F(-2)]	2494
Mupad [F(-1)]	2495
Reduce [F]	2495

Optimal result

Integrand size = 29, antiderivative size = 421

$$\begin{aligned} \int \frac{x^4(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx = & \frac{b^2x}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{1-c^2x^2} \arcsin(cx)}{3c^5d^2\sqrt{d-c^2dx^2}} \\ & - \frac{bx^2(a+b \arcsin(cx))}{3c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} - \frac{x(a+b \arcsin(cx))^2}{c^4d^2\sqrt{d-c^2dx^2}} \\ & + \frac{4i\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^5d^2\sqrt{d-c^2dx^2}} \\ & - \frac{8b\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{3c^5d^2\sqrt{d-c^2dx^2}} \\ & + \frac{4ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{3c^5d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```

1/3*b^2*x/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c
*x)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x^2*(a+b*arcsin(c*x))/c^3/d^2/(-c^2
*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arcsin(c*x))^2/c^2/d/(-c^2
*d*x^2+d)^(3/2)-x*(a+b*arcsin(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*I*(
-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*(-c
^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*b*(
-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^5
/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(
-c^2*x^2+1)^(1/2))^2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)

```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.89

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{a^2 c \sqrt{d} x (-3 + 4c^2 x^2) + 3a^2 (-1 + c^2 x^2) \sqrt{d - c^2 dx^2} \arctan\left(\frac{cx\sqrt{d - c^2 dx^2}}{\sqrt{d(-1 + c^2 x^2)}}\right) + b^2 \sqrt{d} \left[(c^3 x^3 - \sqrt{1 - c^2 x^2}) \arcsin(cx) - 3cx \arcsin^2(cx) + 4 \arcsin^3(cx) \right] + (4I)(1 - c^2 x^2)^{3/2} \arcsin^2(cx) + (1 - c^2 x^2)^{3/2} \arcsin^3(cx) - 8(1 - c^2 x^2)^{3/2} \arcsin(cx) \log[1 + E^{(2I) \arcsin(cx)}] + (4I)(1 - c^2 x^2)^{3/2} \text{PolyLog}[2, -E^{(2I) \arcsin(cx)}] - ab \sqrt{d} (\sqrt{1 - c^2 x^2} + (1 - c^2 x^2)^{3/2} (-3 \arcsin(cx) + 4 \log[1 - c^2 x^2]) + 2 \arcsin(cx) \sin[3 \arcsin(cx)])}{(3c^5 d^{5/2} (1 - c^2 x^2) \sqrt{d - c^2 dx^2})}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output

```

(a^2*c*Sqrt[d]*x*(-3 + 4*c^2*x^2) + 3*a^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^
2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b^2*Sqrt[d
]*(c*x - c^3*x^3 - Sqrt[1 - c^2*x^2]*ArcSin[c*x] - 3*c*x*ArcSin[c*x]^2 + 4
*c^3*x^3*ArcSin[c*x]^2 + (4*I)*(1 - c^2*x^2)^(3/2)*ArcSin[c*x]^2 + (1 - c^
2*x^2)^(3/2)*ArcSin[c*x]^3 - 8*(1 - c^2*x^2)^(3/2)*ArcSin[c*x]*Log[1 + E^(
(2*I)*ArcSin[c*x])] + (4*I)*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^((2*I)*ArcSi
n[c*x])]) - a*b*Sqrt[d]*(Sqrt[1 - c^2*x^2] + (1 - c^2*x^2)^(3/2)*(-3*ArcSi
n[c*x]^2 + 4*Log[1 - c^2*x^2]) + 2*ArcSin[c*x]*Sin[3*ArcSin[c*x]]))/(3*c^5
*d^(5/2)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])

```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5206, 5206, 252, 223, 5152, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx \\
 & \quad \downarrow \text{5206} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x^3(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} - \frac{\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx}{c^2d} + \frac{x^3(a + b \arcsin(cx))^2}{3c^2d(d - c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5206} \\
 & -\frac{2b\sqrt{1-c^2x^2} \left(-\frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{b \int \frac{x^2}{(1-c^2x^2)^{3/2}} dx}{2c} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3cd^2\sqrt{d-c^2dx^2}} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{x^3(a + b \arcsin(cx))^2}{3c^2d(d - c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{252} \\
 & -\frac{2b\sqrt{1-c^2x^2} \left(-\frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} - b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{c^2} \right) + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3cd^2\sqrt{d-c^2dx^2}} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{x^3(a + b \arcsin(cx))^2}{3c^2d(d - c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{223}
 \end{aligned}$$

$$\begin{aligned}
& - \frac{2b\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} - \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d-c^2dx^2}} dx}{c^2d} + \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
& \frac{2b\sqrt{1-c^2x^2} \left(- \frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{c^2d} + \\
& \frac{3cd^2\sqrt{d-c^2dx^2}}{x^3(a+b \arcsin(cx))^2} \\
& \frac{3c^2d(d-c^2dx^2)^{3/2}}{3c^2d(d-c^2dx^2)^{3/2}} \\
& \downarrow 5152 \\
& \frac{2b\sqrt{1-c^2x^2} \left(- \frac{\int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{c^2d} - \\
& - \frac{2b\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} + \frac{x^3(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
& \downarrow 5180 \\
& - \frac{2b\sqrt{1-c^2x^2} \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \frac{2b\sqrt{1-c^2x^2} \left(- \frac{\int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^4} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{c^2d} + \\
& \frac{3cd^2\sqrt{d-c^2dx^2}}{x^3(a+b \arcsin(cx))^2} \\
& \frac{3c^2d(d-c^2dx^2)^{3/2}}{3c^2d(d-c^2dx^2)^{3/2}} \\
& \downarrow 3042 \\
& - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} \\
& \frac{2b\sqrt{1-c^2x^2} \left(- \frac{\int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^4} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{c^2d} + \\
& \frac{3cd^2\sqrt{d-c^2dx^2}}{x^3(a+b \arcsin(cx))^2} \\
& \frac{3c^2d(d-c^2dx^2)^{3/2}}{3c^2d(d-c^2dx^2)^{3/2}} \\
& \downarrow 4202
\end{aligned}$$

$$\begin{aligned}
 & - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{c^3 d \sqrt{d-c^2 dx^2}} + \frac{x(a+b \arcsin(cx))^2}{c^2 d \sqrt{d-c^2 dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^3 d \sqrt{d-c^2 dx^2}} \\
 & \frac{c^2 d}{2b\sqrt{1-c^2x^2} \left(- \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)}{c^4} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2 \sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)} \\
 & \frac{3cd^2 \sqrt{d-c^2 dx^2}}{x^3(a+b \arcsin(cx))^2} \\
 & \frac{3c^2 d (d-c^2 dx^2)^{3/2}}{3c^2 d (d-c^2 dx^2)^{3/2}} \\
 & \downarrow \text{2620} \\
 & - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)})(a+b \arcsin(cx)) \right) \right)}{c^3 d \sqrt{d-c^2 dx^2}} + \frac{x(a+b \arcsin(cx))}{c^2 d \sqrt{d-c^2 dx^2}} \\
 & \frac{c^2 d}{2b\sqrt{1-c^2x^2} \left(- \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)})(a+b \arcsin(cx)) \right)}{c^4} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right)} \\
 & \frac{3cd^2 \sqrt{d-c^2 dx^2}}{x^3(a+b \arcsin(cx))^2} \\
 & \frac{3c^2 d (d-c^2 dx^2)^{3/2}}{3c^2 d (d-c^2 dx^2)^{3/2}} \\
 & \downarrow \text{2715} \\
 & - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)})(a+b \arcsin(cx)) \right) \right)}{c^3 d \sqrt{d-c^2 dx^2}} + x \\
 & \frac{c^2 d}{2b\sqrt{1-c^2x^2} \left(- \frac{\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)})(a+b \arcsin(cx)) \right)}{c^4} + \frac{x^2(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right)} \\
 & \frac{3cd^2 \sqrt{d-c^2 dx^2}}{x^3(a+b \arcsin(cx))^2} \\
 & \frac{3c^2 d (d-c^2 dx^2)^{3/2}}{3c^2 d (d-c^2 dx^2)^{3/2}} \\
 & \downarrow \text{2838}
 \end{aligned}$$

$$\frac{x^3(a+b\arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} - \frac{x(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx)) - \frac{1}{4}b\text{PolyLog}(2, -e^{2i\arcsin(cx)})\right)\right)}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}}{c^2d}$$

$$2b\sqrt{1-c^2x^2}\left(-\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx)) - \frac{1}{4}b\text{PolyLog}(2, -e^{2i\arcsin(cx)})\right)\right) + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{3cd^2\sqrt{d-c^2dx^2}}{c^4}$$

input `Int[(x^4*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]`

output `(x^3*(a + b*ArcSin[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (2*b*sqrt[1 - c^2*x^2]*((x^2*(a + b*ArcSin[c*x]))/(2*c^2*(1 - c^2*x^2)) - (b*(x/(c^2*sqrt[1 - c^2*x^2]) - ArcSin[c*x]/c^3))/(2*c) - (((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4))/c^4)/(3*c*d^2*sqrt[d - c^2*d*x^2]) - ((x*(a + b*ArcSin[c*x])^2)/(c^2*d*sqrt[d - c^2*d*x^2]) - (sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^3)/(3*b*c^3*d*sqrt[d - c^2*d*x^2]) - (2*b*sqrt[1 - c^2*x^2]*(((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)))/c^3*d*sqrt[d - c^2*d*x^2]))/(c^2*d)`

Defintions of rubi rules used

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 252 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m-1)*((a + b*x^2)^(p+1)/(2*b*(p+1))), x] - Simp[c^2*((m-1)/(2*b*(p+1)) Int[(c*x)^(m-2)*(a + b*x^2)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && LtQ[p, -1] && GtQ[m, 1] && !ILtQ[(m+2*p+3)/2, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 2620 $\text{Int}[\left(\frac{(F_{-})^{((g_{-}) * (e_{-}) + (f_{-}) * (x_{-}))}}{(a_{-}) + (b_{-}) * (F_{-})^{((g_{-}) * (e_{-}) + (f_{-}) * (x_{-}))}}\right)^{(n_{-})} * ((c_{-}) + (d_{-}) * (x_{-}))^{(m_{-})} / \left(\frac{(c + d * x)^m}{(b * f * g * n * \text{Log}[F])} * \text{Log}[1 + b * (F^{(g * (e + f * x)))^n / a}]\right), x] - \text{Simp}[d * (m / (b * f * g * n * \text{Log}[F])) \text{Int}[(c + d * x)^{(m - 1)} * \text{Log}[1 + b * (F^{(g * (e + f * x)))^n / a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_{-}) + (b_{-}) * (F_{-})^{((e_{-}) * (c_{-}) + (d_{-}) * (x_{-}))}]^{(n_{-})}], x_Symbol] \rightarrow \text{Simp}[1 / (d * e * n * \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b * x] / x, x], x, (F^{(e * (c + d * x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_{-}) * ((d_{-}) + (e_{-}) * (x_{-})^{(n_{-})})] / (x_{-}), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) * e * x^n] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c * d, 1]$

rule 3042 $\text{Int}[u_{-}, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[\left(\frac{(c_{-}) + (d_{-}) * (x_{-})^{(m_{-})}}{(c + d * x)^{(m + 1)} / (d * (m + 1))}\right) * \tan[(e_{-}) + (f_{-}) * (x_{-})], x_Symbol] \rightarrow \text{Simp}[I * ((c + d * x)^{(m + 1)} / (d * (m + 1))), x] - \text{Simp}[2 * I \text{Int}[(c + d * x)^m * (E^{(2 * I * (e + f * x))} / (1 + E^{(2 * I * (e + f * x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 5152 $\text{Int}[\left(\frac{(a_{-}) + \text{ArcSin}[(c_{-}) * (x_{-})] * (b_{-})}{\sqrt{(d_{-}) + (e_{-}) * (x_{-})^2}}\right)^{(n_{-})}, x_Symbol] \rightarrow \text{Simp}[(1 / (b * c * (n + 1))) * \text{Simp}[\sqrt{1 - c^2 * x^2} / \sqrt{d + e * x^2}] * (a + b * \text{ArcSin}[c * x])^{(n + 1)}, x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{EqQ}[c^2 * d + e, 0] \&\& \text{NeQ}[n, -1]$

rule 5180 $\text{Int}[\left(\frac{(a_{-}) + \text{ArcSin}[(c_{-}) * (x_{-})] * (b_{-})}{(d_{-}) + (e_{-}) * (x_{-})^2}\right)^{(n_{-})} * (x_{-}), x_Symbol] \rightarrow \text{Simp}[-e^{(-1)} \text{Subst}[\text{Int}[(a + b * x)^n * \text{Tan}[x], x], x, \text{ArcSin}[c * x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2 * d + e, 0] \&\& \text{IGtQ}[n, 0]$

rule 5206

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]

```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(393) = 786$.

Time = 0.84 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.90

method	result
default	$\frac{a^2 x^3}{3c^2 d(-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{a^2 x}{c^4 d^2 \sqrt{-c^2 d x^2 + d}} + \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{c^4 d^2 \sqrt{c^2 d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{3d^3 c^5 (c^2 x^2 - 1)} + \dots \right)$
parts	$\frac{a^2 x^3}{3c^2 d(-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{a^2 x}{c^4 d^2 \sqrt{-c^2 d x^2 + d}} + \frac{a^2 \arctan\left(\frac{\sqrt{c^2 d} x}{\sqrt{-c^2 d x^2 + d}}\right)}{c^4 d^2 \sqrt{c^2 d}} + b^2 \left(-\frac{\sqrt{-d(c^2 x^2 - 1)} \sqrt{-c^2 x^2 + 1} \arcsin(cx)^3}{3d^3 c^5 (c^2 x^2 - 1)} + \dots \right)$

input

```
int(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```

1/3*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)-a^2/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+
a^2/c^4/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2
*(-1/3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arcsi
n(c*x)^3+1/12*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x)*(-1+6*arc
sin(c*x)^2-2*I*arcsin(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/c^5-4/3*I*(-c^2*x^2+
1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(2*I*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(
1/2))^2)+2*arcsin(c*x)^2+polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2))/d^3/c^
5/(c^2*x^2-1)+1/12*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+
2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(10*arcsin(c*x)^2-2*I*arcsin(c*x)-3)
/d^3/(c^4*x^4-2*c^2*x^2+1)/c^5-1/12*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+
1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))*x/c^4/(c^4*x
^4-2*c^2*x^2+1)/d^3)-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(3*
arcsin(c*x)^2*x^4*c^4+8*I*arcsin(c*x)*x^4*c^4-8*ln(1+(I*c*x+(-c^2*x^2+1)^(
1/2))^2)*x^4*c^4+8*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^3*c^3-6*arcsin(c*x)^2*
x^2*c^2-16*I*arcsin(c*x)*x^2*c^2+16*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)*x^2
*c^2-6*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2+3*arcsin(c*x)^2+8*I*arcs
in(c*x)-8*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)-1)/d^3/(c^6*x^6-3*c^4*x^4+3*c
^2*x^2-1)/c^5

```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```

integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

```

output

```

integral(-(b^2*x^4*arcsin(c*x)^2 + 2*a*b*x^4*arcsin(c*x) + a^2*x^4)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)

```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^4(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{\frac{5}{2}}} dx$$

input `integrate(x**4*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2), x)`

output `Integral(x**4*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)`

Maxima [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^4}{(-c^2 dx^2 + d)^{\frac{5}{2}}} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2), x, algorithm="maxima")`

output `1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2))*a^2 - sqrt(d)*integrate((b^2*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^4*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2), x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^4(a + b \sin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((x^4*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) a^2 c^2 x^2 - 3\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) a^2 + 6\sqrt{-c^2 x^2 + 1}}{(d - c^2 dx^2)^{5/2}}$$

input

```
int(x^4*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(3*sqrt(-c**2*x**2 + 1)*asin(c*x)*a**2*c**2*x**2 - 3*sqrt(-c**2*x**2 +
1)*asin(c*x)*a**2 + 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**4)/(sqrt(
-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-
c**2*x**2 + 1)),x)*a*b*c**7*x**2 - 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)
*x**4)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x
**2 + sqrt(-c**2*x**2 + 1)),x)*a*b*c**5 + 3*sqrt(-c**2*x**2 + 1)*int((
asin(c*x)**2*x**4)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2
+ 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2*c**7*x**2 - 3*sqrt(-c*
**2*x**2 + 1)*int((asin(c*x)**2*x**4)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2
*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2*c**5 -
4*a**2*c**3*x**3 + 3*a**2*c*x)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*c**5*d**
2*(c**2*x**2 - 1))
```

3.253
$$\int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2496
Mathematica [A] (verified)	2497
Rubi [A] (verified)	2497
Maple [A] (verified)	2502
Fricas [F]	2503
Sympy [F]	2503
Maxima [F]	2504
Giac [F(-2)]	2504
Mupad [F(-1)]	2505
Reduce [F]	2505

Optimal result

Integrand size = 29, antiderivative size = 332

$$\begin{aligned} \int \frac{x^3(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx &= \frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} \\ &- \frac{bx(a+b \arcsin(cx))}{3c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\ &- \frac{2(a+b \arcsin(cx))^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{10ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{3c^4d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{5ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{3c^4d^2\sqrt{d-c^2dx^2}} \\ &- \frac{5ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, ie^{i \arcsin(cx)})}{3c^4d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
1/3*b^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x*(a+b*arcsin(c*x))/c^3/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^2*(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3*(a+b*arcsin(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-10/3*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+5/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.54

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{8a^2 - 2b^2 - 12a^2 c^2 x^2 + 4ab \arcsin(cx) + 2b^2 \arcsin(cx)^2 - 2b^2 \cos(2 \arcsin(c$$

input `Integrate[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]`

output

```
(8*a^2 - 2*b^2 - 12*a^2*c^2*x^2 + 4*a*b*ArcSin[c*x] + 2*b^2*ArcSin[c*x]^2
- 2*b^2*Cos[2*ArcSin[c*x]] + 12*a*b*ArcSin[c*x]*Cos[2*ArcSin[c*x]] + 6*b^2
*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] - 15*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*L
og[1 - I*E^(I*ArcSin[c*x])] - 5*b^2*ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 -
I*E^(I*ArcSin[c*x])] + 15*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(
I*ArcSin[c*x])] + 5*b^2*ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 + I*E^(I*ArcS
in[c*x])] + 15*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c
*x]/2]] + 5*a*b*Cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x
]/2]] - 15*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/
2]] - 5*a*b*Cos[3*ArcSin[c*x]]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2
]] - (20*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2
0*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, I*E^(I*ArcSin[c*x])] + 2*a*b*Sin[2
*ArcSin[c*x]] + 2*b^2*ArcSin[c*x]*Sin[2*ArcSin[c*x]]/(12*c^4*d^2*(-1 + c^
2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5206, 5182, 5164, 3042, 4669, 2715, 2838, 5206, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

↓ 5206

$$\begin{aligned}
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2 \int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx}{3c^2d} + \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5182} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{cd\sqrt{d-c^2dx^2}} \right)}{3c^2d} + \\
 & \quad \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5164} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2 \left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^2d\sqrt{d-c^2dx^2}} \right)}{3c^2d} + \\
 & \quad \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} - \\
 & \quad 2 \left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^2d\sqrt{d-c^2dx^2}} \right) + \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{4669} \\
 & -\frac{2 \left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} (-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx)) (a+b \arcsin(cx))}{c^2d\sqrt{d-c^2dx^2}} \right)}{3c^2d} \\
 & \quad \frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2715} \\
 & -\frac{2 \left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} (ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx)}{c^2d\sqrt{d-c^2dx^2}} \right)}{3c^2d} \\
 & \quad \frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} + \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2838}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2b\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3cd^2\sqrt{d-c^2dx^2}} \\
 & \frac{2\left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d} \\
 & \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5206} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(-\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{2c^2} - \frac{b \int \frac{x}{(1-c^2x^2)^{3/2}} dx}{2c} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3c^2d\sqrt{d-c^2dx^2}} \\
 & \frac{2\left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d} \\
 & \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{241} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(-\frac{\int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{2c^2} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b}{2c^3\sqrt{1-c^2x^2}} \right)}{3c^2d\sqrt{d-c^2dx^2}} \\
 & \frac{2\left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d} \\
 & \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5164} \\
 & \frac{2b\sqrt{1-c^2x^2} \left(-\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c^3} + \frac{x(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b}{2c^3\sqrt{1-c^2x^2}} \right)}{3c^2d\sqrt{d-c^2dx^2}} \\
 & \frac{2\left(\frac{(a+b \arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d} \\
 & \frac{x^2(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{3042}
 \end{aligned}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int(a+b\arcsin(cx))\csc(\arcsin(cx)+\frac{\pi}{2})d\arcsin(cx)}{2c^3}+\frac{x(a+b\arcsin(cx))}{2c^2(1-c^2x^2)}-\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2\left(\frac{(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{x^2(a+b\arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

↓ 4669

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{b\int\log(1-ie^{i\arcsin(cx)})d\arcsin(cx)+b\int\log(1+ie^{i\arcsin(cx)})d\arcsin(cx)-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))}{2c^3}}{3cd^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{x^2(a+b\arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

↓ 2715

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{ib\int e^{-i\arcsin(cx)}\log(1-ie^{i\arcsin(cx)})de^{i\arcsin(cx)}-ib\int e^{-i\arcsin(cx)}\log(1+ie^{i\arcsin(cx)})de^{i\arcsin(cx)}-2i\arctan(e^{i\arcsin(cx)})}{2c^3}}{3cd^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{x^2(a+b\arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

↓ 2838

$$\frac{2\left(\frac{(a+b\arcsin(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}(-2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)}))}{c^2d\sqrt{d-c^2dx^2}}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2b\sqrt{1-c^2x^2}\left(-\frac{2i\arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))+ib\text{PolyLog}(2,-ie^{i\arcsin(cx)})-ib\text{PolyLog}(2,ie^{i\arcsin(cx)})}{2c^3}+\frac{x(a+b\arcsin(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}} + \frac{x^2(a+b\arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

input Int[(x^3*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]

output

$$\begin{aligned} & (x^2(a + b\text{ArcSin}[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^{(3/2)}) - (2*b*\text{Sqrt}[1 \\ & - c^2*x^2]*(-1/2*b/(c^3*\text{Sqrt}[1 - c^2*x^2]) + (x*(a + b*\text{ArcSin}[c*x]))/(2*c^ \\ & 2*(1 - c^2*x^2)) - ((-2*I)*(a + b*\text{ArcSin}[c*x])*\text{ArcTan}[E^(I*\text{ArcSin}[c*x])] + \\ & I*b*\text{PolyLog}[2, (-I)*E^(I*\text{ArcSin}[c*x])] - I*b*\text{PolyLog}[2, I*E^(I*\text{ArcSin}[c*x \\ &])])/(2*c^3)))/(3*c*d^2*\text{Sqrt}[d - c^2*d*x^2]) - (2*((a + b*\text{ArcSin}[c*x])^2/(\\ & c^2*d*\text{Sqrt}[d - c^2*d*x^2]) - (2*b*\text{Sqrt}[1 - c^2*x^2]*((-2*I)*(a + b*\text{ArcSin}[\\ & c*x])*\text{ArcTan}[E^(I*\text{ArcSin}[c*x])] + I*b*\text{PolyLog}[2, (-I)*E^(I*\text{ArcSin}[c*x])] - \\ & I*b*\text{PolyLog}[2, I*E^(I*\text{ArcSin}[c*x])])))/(c^2*d*\text{Sqrt}[d - c^2*d*x^2])))/(3*c^ \\ & 2*d) \end{aligned}$$

Defintions of rubi rules used

rule 241

$$\text{Int}[(x_*)*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p + 1)}/(2*b*(p + 1)), x] \text{ ; FreeQ}\{a, b, p, x\} \ \&\& \ \text{NeQ}\{p, -1\}$$

rule 2715

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^{n}], x] \text{ ; FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \text{GtQ}\{a, 0\}$$

rule 2838

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)}]/(x_)), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] \text{ ; FreeQ}\{c, d, e, n, x\} \ \&\& \ \text{EqQ}\{c*d, 1\}$$

rule 3042

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$$

rule 4669

$$\begin{aligned} & \text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^{(m_)}, x_Symbol \\ &] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-\text{Si} \\ & \text{mp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], \\ & x] + \text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^(I*k*Pi)*E^(I*(e + f*x \\ &))], x], x]) \text{ ; FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0] \end{aligned}$$

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:= Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))),
x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)
*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.52

method	result
default	$a^2 \left(\frac{x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{2}{3 d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}} \right) + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} \left(3 \arcsin(cx)^2 x^2 c^2 - \arcsin(cx) \sqrt{-c^2 x^2 + 1} c x - c^2 x^2 - 2 \right)}{3 (c^2 x^2 - 1)^2 d^3 c^4} \right)$
parts	$a^2 \left(\frac{x^2}{c^2 d (-c^2 d x^2 + d)^{\frac{3}{2}}} - \frac{2}{3 d c^4 (-c^2 d x^2 + d)^{\frac{3}{2}}} \right) + b^2 \left(\frac{\sqrt{-d(c^2 x^2 - 1)} \left(3 \arcsin(cx)^2 x^2 c^2 - \arcsin(cx) \sqrt{-c^2 x^2 + 1} c x - c^2 x^2 - 2 \right)}{3 (c^2 x^2 - 1)^2 d^3 c^4} \right)$

input

```
int(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```

a^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b^2*(1
/3*(-d*(c^2*x^2-1))^(1/2)*(3*arcsin(c*x)^2*x^2*c^2-arcsin(c*x)*(-c^2*x^2+1
)^(1/2)*c*x-c^2*x^2-2*arcsin(c*x)^2+1)/(c^2*x^2-1)^2/d^3/c^4+5/3*(-c^2*x^2
+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(
1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(I*c*x+(
-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))/d^3/(c^2*x^2-
1)/c^4)+2*a*b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(6*c^2*x^2*arcsin(c*x)-c*x*(-c^2
*x^2+1)^(1/2)-4*arcsin(c*x))/(c^2*x^2-1)^2/d^3/c^4-5/6*(-c^2*x^2+1)^(1/2)*
(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c^4*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)+
5/6*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c^4*ln(I*c*x
+(-c^2*x^2+1)^(1/2)-I))

```

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^3}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```

integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

```

output

```

integral(-(b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)

```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input

```

integrate(x**3*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)

```

output

```

Integral(x**3*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**5/2, x)

```

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^3}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/6*a*b*c*(2*x/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) + 5*log(c*x + 1)/(c^5*d^(5/2)) - 5*log(c*x - 1)/(c^5*d^(5/2))) + 2/3*a*b*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arcsin(c*x) + 1/3*a^2*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2*integrate(x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)`

output `int((x^3*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1}}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} \left(\int \frac{\operatorname{asin}(cx) x^3}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) ab c^6 x^2 - 6\sqrt{-c^2 x^2 + 1}$$

input `int(x^3*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)`

output `(6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**3)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b*c**6*x**2 - 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**3)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*a*b*c**4 + 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)**2*x**3)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2*c**6*x**2 - 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)**2*x**3)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2*c**4 - 3*a**2*c**2*x**2 + 2*a**2)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*c**4*d**2*(c**2*x**2 - 1))`

3.254
$$\int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2506
Mathematica [A] (verified)	2507
Rubi [A] (verified)	2507
Maple [B] (verified)	2511
Fricas [F]	2512
Sympy [F]	2513
Maxima [F]	2513
Giac [F(-2)]	2513
Mupad [F(-1)]	2514
Reduce [F]	2514

Optimal result

Integrand size = 29, antiderivative size = 332

$$\begin{aligned} \int \frac{x^2(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx &= \frac{b^2x}{3c^2d^2\sqrt{d-c^2dx^2}} \\ &- \frac{b^2\sqrt{1-c^2x^2} \arcsin(cx)}{3c^3d^2\sqrt{d-c^2dx^2}} - \frac{bx^2(a+b \arcsin(cx))}{3cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} \\ &+ \frac{x^3(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} + \frac{i\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3c^3d^2\sqrt{d-c^2dx^2}} \\ &- \frac{2b\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{3c^3d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{3c^3d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```
1/3*b^2*x/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x^2*(a+b*arcsin(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)+1/3*I*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c^3/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{-b^2 cx - a^2 c^3 x^3 + b^2 c^3 x^3 + ab\sqrt{1 - c^2 x^2} + ib^2(ic^3 x^3 - \sqrt{1 - c^2 x^2} + c^2 x^2 \sqrt{1 - c^2 x^2})}{(d - c^2 dx^2)^{5/2}}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]
```

output

```
(-(b^2*c*x) - a^2*c^3*x^3 + b^2*c^3*x^3 + a*b*Sqrt[1 - c^2*x^2] + I*b^2*(I*c^3*x^3 - Sqrt[1 - c^2*x^2] + c^2*x^2*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 + b*ArcSin[c*x]*(-2*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] + 2*b*(1 - c^2*x^2)^(3/2))*Log[1 + E^((2*I)*ArcSin[c*x])]) + a*b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] - a*b*c^2*x^2*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] - I*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^((2*I)*ArcSin[c*x])]/(3*c^3*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.66, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5186, 5206, 252, 223, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow \text{5186}$$

$$\frac{x^3(a + b \arcsin(cx))^2}{3d(d - c^2 dx^2)^{3/2}} - \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x^3(a + b \arcsin(cx))}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}}$$

$$\downarrow \text{5206}$$

$$\begin{aligned}
& \frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \\
& \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{b \int \frac{x^2}{(1-c^2x^2)^{3/2}} dx}{2c} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{252} \\
& \frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \\
& \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{c^2} \right)}{2c} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{223} \\
& \frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \\
& \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\int \frac{x(a+b\arcsin(cx))}{1-c^2x^2} dx}{c^2} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{5180} \\
& \frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \\
& \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\int \frac{cx(a+b\arcsin(cx))}{\sqrt{1-c^2x^2}} d\arcsin(cx)}{c^4} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{3042} \\
& \frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \\
& \frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\int (a+b\arcsin(cx)) \tan(\arcsin(cx)) d\arcsin(cx)}{c^4} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b \left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3} \right)}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \\
& \quad \downarrow \text{4202}
\end{aligned}$$

$$\frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \frac{i(a+b\arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i\arcsin(cx)}(a+b\arcsin(cx))}{1+e^{2i\arcsin(cx)}} d\arcsin(cx)}{c^4} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}} - \frac{\arcsin(cx)}{c^3}\right)}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

2620

$$\frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(\frac{1}{2}ib \int \log(1+e^{2i\arcsin(cx)}) d\arcsin(cx) - \frac{1}{2}i \log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx))\right)}{c^4} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

2715

$$\frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(\frac{1}{4}b \int e^{-2i\arcsin(cx)} \log(1+e^{2i\arcsin(cx)}) de^{2i\arcsin(cx)} - \frac{1}{2}i \log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx))\right)}{c^4} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

2838

$$\frac{2bc\sqrt{1-c^2x^2} \left(-\frac{\frac{x^3(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1+e^{2i\arcsin(cx)})(a+b\arcsin(cx)) - \frac{1}{4}b \text{PolyLog}(2, -e^{2i\arcsin(cx)})\right)}{c^4} + \frac{x^2(a+b\arcsin(cx))}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}}$$

input `Int[(x^2*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]`

output `(x^3*(a + b*ArcSin[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 - c^2*x^2]*((x^2*(a + b*ArcSin[c*x]))/(2*c^2*(1 - c^2*x^2)) - (b*(x/(c^2*Sqrt[1 - c^2*x^2]) - ArcSin[c*x]/c^3))/(2*c) - (((1/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)/c^4))/(3*d^2*Sqrt[d - c^2*d*x^2])`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 252 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)/(2*b*(p+1))}), x] - \text{Simp}[c^2*(m-1)/(2*b*(p+1)) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^{(p+1)}, x], x] \text{ ; FreeQ}\{a, b, c, x\} \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ !\text{ILtQ}[(m + 2*p + 3)/2, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 2620 $\text{Int}[(((F_)^{((g_)*((e_) + (f_)*(x_)))})^{(n_)}*((c_) + (d_)*(x_))^{(m_)} / ((a_) + (b_)*((F_)^{((g_)*((e_) + (f_)*(x_)))})^{(n_)})), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m/(b*f*g*n*\text{Log}[F])*\text{Log}[1 + b*((F^{(g*(e + f*x)))})^n/a], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \ \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + b*((F^{(g*(e + f*x)))})^n/a], x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \ \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)))})^n], x] \text{ ; FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] \text{ ; FreeQ}\{c, d, e, n, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[((c_)+(d_)*(x_))^{(m_)}*\text{tan}[(e_)+(f_)*(x_)], x_Symbol] \rightarrow \text{Simp}[I*((c + d*x)^{(m+1)}/(d*(m+1))), x] - \text{Simp}[2*I \ \text{Int}[(c + d*x)^m*(E^{(2*I*(e + f*x))}/(1 + E^{(2*I*(e + f*x))})), x], x] \text{ ; FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{IGtQ}[m, 0]$

rule 5186

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5186

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^ (m_))*((d_) + (e_.
)*(x_)^2)^ (p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5206

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_)^ (m_))*((d_) + (e_.
)*(x_)^2)^ (p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m -
1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3297 vs. $2(312) = 624$.

Time = 0.79 (sec) , antiderivative size = 3298, normalized size of antiderivative = 9.93

method	result	size
default	Expression too large to display	3298
parts	Expression too large to display	3298

input

```
int(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```

b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)
)*c^4*arcsin(c*x)^2*x^7-1/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*
c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4*x^7+1/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d
^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*(-c^2*x^2+1)*x^3+2*a*b*(-d
*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4*a
rcsin(c*x)*x^7-a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*
x^4-5*c^2*x^2+1)*c*(-c^2*x^2+1)^(1/2)*x^4-2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3
/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2*arcsin(c*x)*x^5+2/3*I*a*
b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*
c^2*x^5+a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c
^2*x^2+1)/c*(-c^2*x^2+1)^(1/2)*x^2+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/d^3/(c^2*x^2-1)/c^3*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)+a^2*(1/2
*x/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/2/c^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^
2*x/(-c^2*d*x^2+d)^(1/2)))-I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3*(-c^2*x^2+1)^(1/2)*x^6-1/3*I*b^2*(-d*(c
^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4*arcs
in(c*x)*x^7+2*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4
*x^4-5*c^2*x^2+1)*c*(-c^2*x^2+1)^(1/2)*x^4-4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c*x^2*(-c^2*x^2+1)^(1/2
)-1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-...

```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```

integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

```

output

```

integral(-(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)

```

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^2(a + b \sin(cx))^2}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate(x**2*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral(x**2*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)`

Maxima [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) - log(c*x + 1)/(c^4*d^(5/2)) - log(c*x - 1)/(c^4*d^(5/2))) - 2/3*a*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arcsin(c*x) - 1/3*a^2*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^2(a + b \sin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((x^2*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1}}{ab c^2 x^2 - 6\sqrt{-c^2 x^2 + 1}} \left(\int \frac{\arcsin(cx) x^2}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right)$$

input

```
int(x^2*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**2)/(sqrt(-c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*a
*b*c**2*x**2 - 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x**2)/(sqrt(-c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x
**2 + 1)),x)*a*b + 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)**2*x**2)/(sqrt(
-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-
c**2*x**2 + 1)),x)*b**2*c**2*x**2 - 3*sqrt(-c**2*x**2 + 1)*int((asin(c*
x)**2*x**2)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c
**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2 - a**2*x**3)/(3*sqrt(d)*sqrt(-
c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.255
$$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2515
Mathematica [A] (verified)	2516
Rubi [A] (verified)	2516
Maple [A] (verified)	2519
Fricas [F]	2520
Sympy [F]	2520
Maxima [F]	2521
Giac [F(-2)]	2521
Mupad [F(-1)]	2521
Reduce [F]	2522

Optimal result

Integrand size = 27, antiderivative size = 294

$$\int \frac{x(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx = \frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} - \frac{bx(a+b \arcsin(cx))}{3cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{(a+b \arcsin(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}} + \frac{2ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{3c^2d^2\sqrt{d-c^2dx^2}} - \frac{ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, ie^{i \arcsin(cx)})}{3c^2d^2\sqrt{d-c^2dx^2}}$$

output

```
1/3*b^2/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x*(a+b*arcsin(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+2/3*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```


Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.57

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{a^2 \sqrt{-d(-1 + c^2 x^2)}}{3c^2 d^3 (-1 + c^2 x^2)^2} + \frac{ab(8 \arcsin(cx) + 3\sqrt{1 - c^2 x^2}(\log(\cos(\frac{1}{2} \arcsin(cx))) - \sin(\frac{1}{2} \arcsin(cx)))) - \log(\cos(\frac{1}{2} \arcsin(cx))) + \sin(\frac{1}{2} \arcsin(cx))}{3c^2 d^3 (-1 + c^2 x^2)^2} + \frac{b^2(2 + 4 \arcsin(cx)^2 + 2 \cos(2 \arcsin(cx)) - 3\sqrt{1 - c^2 x^2} \arcsin(cx) \log(1 - ie^{i \arcsin(cx)}) - \arcsin(cx) \cos(\frac{1}{2} \arcsin(cx)))}{3c^2 d^3 (-1 + c^2 x^2)^2}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]
```

output

```
(a^2*Sqrt[-(d*(-1 + c^2*x^2))])/(3*c^2*d^3*(-1 + c^2*x^2)^2) + (a*b*(8*ArcSin[c*x] + 3*Sqrt[1 - c^2*x^2]*(Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]])) + Cos[3*ArcSin[c*x]]*(Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]]) - 2*Sin[2*ArcSin[c*x]])/(12*c^2*d*(d*(1 - c^2*x^2))^(3/2)) + (b^2*(2 + 4*ArcSin[c*x]^2 + 2*Cos[2*ArcSin[c*x]] - 3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 - I*E^(I*ArcSin[c*x])] + 3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 + I*E^(I*ArcSin[c*x])]) - (4*I)*(1 - c^2*x^2)^(3/2)*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (4*I)*(1 - c^2*x^2)^(3/2)*PolyLog[2, I*E^(I*ArcSin[c*x])] - 2*ArcSin[c*x]*Sin[2*ArcSin[c*x]])/(12*c^2*d*(d*(1 - c^2*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5182, 5162, 241, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx \\
& \quad \downarrow \text{5182} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{(1 - c^2 x^2)^2} dx}{3cd^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5162} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{1 - c^2 x^2} dx - \frac{1}{2} bc \int \frac{x}{(1 - c^2 x^2)^{3/2}} dx + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} \right)}{3cd^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{241} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{1}{2} \int \frac{a + b \arcsin(cx)}{1 - c^2 x^2} dx + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} - \frac{b}{2c\sqrt{1 - c^2 x^2}} \right)}{3cd^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{5164} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{\int \frac{a + b \arcsin(cx)}{\sqrt{1 - c^2 x^2}} d \arcsin(cx)}{2c} + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} - \frac{b}{2c\sqrt{1 - c^2 x^2}} \right)}{3cd^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{3042} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{\int (a + b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} - \frac{b}{2c\sqrt{1 - c^2 x^2}} \right)}{3cd^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{4669} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx))}{2c} + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} \right)}{3cd^2 \sqrt{d - c^2 dx^2}} \\
& \quad \downarrow \text{2715} \\
& \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - 2i \arctan(e^{i \arcsin(cx)})}{2c} + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)} \right)}{3cd^2 \sqrt{d - c^2 dx^2}}
\end{aligned}$$

$$\begin{array}{c}
 \downarrow 2838 \\
 \frac{(a + b \arcsin(cx))^2}{3c^2 d (d - c^2 dx^2)^{3/2}} - \\
 \frac{2b\sqrt{1 - c^2 x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)}) (a + b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a + b \arcsin(cx))}{2(1 - c^2 x^2)}}{3cd^2 \sqrt{d - c^2 dx^2}}
 \end{array}$$

input `Int[(x*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]`

output `(a + b*ArcSin[c*x])^2/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) - (2*b*Sqrt[1 - c^2*x^2]*(-1/2*b/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x])))/(2*(1 - c^2*x^2)) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c)))/(3*c*d^2*Sqrt[d - c^2*d*x^2])`

Defintions of rubi rules used

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f)
  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x]
  && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol]
  := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))
  Int[(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
  && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)^2), x_Symbol]
  := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x]
  && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol]
  := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]
  Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x]
  && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.59

method	result
default	$\frac{a^2}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}} + b^2 \left(\frac{\sqrt{-d(c^2x^2-1)} \left(-\arcsin(cx)\sqrt{-c^2x^2+1}cx - c^2x^2 + \arcsin(cx)^2 + 1 \right)}{3d^3(c^4x^4 - 2c^2x^2 + 1)c^2} - \frac{\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)}}{3d^3(c^4x^4 - 2c^2x^2 + 1)c^2} \right)$
parts	$\frac{a^2}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}} + b^2 \left(\frac{\sqrt{-d(c^2x^2-1)} \left(-\arcsin(cx)\sqrt{-c^2x^2+1}cx - c^2x^2 + \arcsin(cx)^2 + 1 \right)}{3d^3(c^4x^4 - 2c^2x^2 + 1)c^2} - \frac{\sqrt{-c^2x^2+1} \sqrt{-d(c^2x^2-1)}}{3d^3(c^4x^4 - 2c^2x^2 + 1)c^2} \right)$

input

```
int(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*a^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+b^2*(1/3*(-d*(c^2*x^2-1))^(1/2)*(-arcsi
n(c*x)*(-c^2*x^2+1)^(1/2)*c*x-c^2*x^2+arcsin(c*x)^2+1)/d^3/(c^4*x^4-2*c^2*
x^2+1)/c^2-1/3*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arcsin(c*x)*ln(1
+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2
))))-I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(I*c*x+(-c^2*x^2+1
)^(1/2))))/d^3/(c^2*x^2-1)/c^2)+2*a*b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(-c*x*(-
c^2*x^2+1)^(1/2)+2*arcsin(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/c^2+1/6*(-d*(c^2
*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(I*c*x+(-c^2*x^2+1
)^(1/2)+I)-1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^2/(c^2*x^2-
1)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I))
```

Fricas [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(-c^2 dx^2 + d)^{5/2}} dx$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output

```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*x*arcsin(c*x)^2 + 2*a*b*x*arcsin(c*x)
+ a^2*x)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x(a + b \operatorname{asin}(cx))^2}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input

```
integrate(x*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output

```
Integral(x*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**5/2, x)
```

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `-sqrt(d)*integrate((b^2*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 1/3*a^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)`

output `int((x*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)x}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) ab c^4 x^2 - 6\sqrt{-c^2 x^2}}$$

input `int(x*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2), x)`

output `(6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*a*b*c**4*x**2 - 6*sqrt(-c**2*x**2 + 1)*int((asin(c*x)*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*a*b*c**2 + 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)**2*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2*c**4*x**2 - 3*sqrt(-c**2*x**2 + 1)*int((asin(c*x)**2*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**4 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**2 + sqrt(-c**2*x**2 + 1)),x)*b**2*c**2 - a**2)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*c**2*d**2*(c**2*x**2 - 1))`

3.256 $\int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx$

Optimal result	2523
Mathematica [A] (verified)	2524
Rubi [A] (verified)	2524
Maple [B] (verified)	2529
Fricas [F]	2530
Sympy [F]	2530
Maxima [F]	2530
Giac [F(-2)]	2531
Mupad [F(-1)]	2531
Reduce [F]	2532

Optimal result

Integrand size = 26, antiderivative size = 311

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2dx^2)^{5/2}} dx = \frac{b^2x}{3d^2\sqrt{d - c^2dx^2}} - \frac{b(a + b \arcsin(cx))}{3cd^2\sqrt{1 - c^2x^2}\sqrt{d - c^2dx^2}}$$

$$+ \frac{x(a + b \arcsin(cx))^2}{3d(d - c^2dx^2)^{3/2}} + \frac{2x(a + b \arcsin(cx))^2}{3d^2\sqrt{d - c^2dx^2}} - \frac{2i\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2}{3cd^2\sqrt{d - c^2dx^2}}$$

$$+ \frac{4b\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)})}{3cd^2\sqrt{d - c^2dx^2}}$$

$$- \frac{2ib^2\sqrt{1 - c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{3cd^2\sqrt{d - c^2dx^2}}$$

output

```
1/3*b^2*x/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*(a+b*arcsin(c*x))/c/d^2/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(
3/2)+2/3*x*(a+b*arcsin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*I*(-c^2*x^2+1
)^(1/2)*(a+b*arcsin(c*x))^2/c/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*b*(-c^2*x^2+1)^(
1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c/d^2/(-c^2*d*x
^2+d)^(1/2)-2/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1
/2))^2)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```


Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.03

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{-3a^2 cx - b^2 cx + 2a^2 c^3 x^3 + b^2 c^3 x^3 + ab\sqrt{1 - c^2 x^2} + b^2(-3cx + 2c^3 x^3 + 2i\sqrt{1 - c^2 x^2})}{(d - c^2 dx^2)^{5/2}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output

```
(-3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 + b^2*c^3*x^3 + a*b*Sqrt[1 - c^2*x^2] + b^2*(-3*c*x + 2*c^3*x^3 + (2*I)*Sqrt[1 - c^2*x^2] - (2*I)*c^2*x^2*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 + b*ArcSin[c*x]*(-6*a*c*x + 4*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] - 4*b*(1 - c^2*x^2)^(3/2)*Log[1 + E^((2*I)*ArcSin[c*x])]) - 2*a*b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] + 2*a*b*c^2*x^2*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] + (2*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/(3*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)Time = 1.41 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5162, 5160, 5180, 3042, 4202, 2620, 2715, 2838, 5182, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow 5162$$

$$-\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x(a + b \arcsin(cx))}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx}{3d} + \frac{x(a + b \arcsin(cx))^2}{3d(d - c^2 dx^2)^{3/2}}$$

$$\downarrow 5160$$

$$\begin{aligned}
 & -\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{d\sqrt{d-c^2dx^2}}\right)}{3d} + \\
 & \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5180} \\
 & -\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{2\left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d} + \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{2\left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d} + \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{4202} \\
 & -\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{2\left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx)\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d} + \\
 & \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2620} \\
 & -\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{2\left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(\frac{1}{2}ib \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2}i \log(1+e^{2i \arcsin(cx)})(a+b \arcsin(cx))\right)\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d} + \\
 & \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2715}
 \end{aligned}$$

$$2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2 dx^2}} - \frac{2bc\sqrt{1-c^2 x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^2} dx}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2 dx^2}} \right)$$

$$\frac{x(a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}} \quad 3d$$

↓ 2838

$$2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2 dx^2}} - \frac{2bc\sqrt{1-c^2 x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2 x^2)^2} dx}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2 dx^2}} \right)$$

$$\frac{x(a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}} \quad 3d$$

↓ 5182

$$2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2 dx^2}} - \frac{2bc\sqrt{1-c^2 x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2 x^2)} - \frac{b \int \frac{1}{(1-c^2 x^2)^{3/2}} dx}{2c} \right)}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2 dx^2}} \right)$$

$$\frac{x(a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}} \quad 3d$$

↓ 208

$$2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2 dx^2}} - \frac{2bc\sqrt{1-c^2 x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2 x^2)} - \frac{bx}{2c\sqrt{1-c^2 x^2}} \right)}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{2b\sqrt{1-c^2 x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2 dx^2}} \right)$$

$$\frac{x(a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}} \quad 3d$$

input `Int[(a + b*ArcSin[c*x])^2/(d - c^2*d*x^2)^(5/2),x]`

output `(x*(a + b*ArcSin[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 - c^2*x^2]*(-1/2*(b*x)/(c*Sqrt[1 - c^2*x^2]) + (a + b*ArcSin[c*x])/(2*c^2*(1 - c^2*x^2))))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*(a + b*ArcSin[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2]))/(3*d)`

Defintions of rubi rules used

rule 208 `Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[x/(a*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b}, x]`

rule 2620 `Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4202

```
Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[
I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x))))], x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5160

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*((a + b*ArcSin[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]
```

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p +
1))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

rule 5180

```
Int((((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2894 vs. $2(293) = 586$.

Time = 0.64 (sec) , antiderivative size = 2895, normalized size of antiderivative = 9.31

method	result	size
default	Expression too large to display	2895
parts	Expression too large to display	2895

input `int((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```

-2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*x-4*
I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3*(
-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^4+28/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3
*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^2+8/3
*I*a*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c*arcsin(
c*x)-10/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)*c^2*(-c^2*x^2+1)*x^3-10/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6
-10*c^4*x^4+11*c^2*x^2-4)*c^2*(-c^2*x^2+1)*arcsin(c*x)*x^3+14/3*I*b^2*(-d*
(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c*(-c^2*x^2+1)^(
1/2)*arcsin(c*x)^2*x^2+4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10
*c^4*x^4+11*c^2*x^2-4)*c^4*(-c^2*x^2+1)*arcsin(c*x)*x^5-2*I*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3*(-c^2*x^2+1)^(1/2
)*arcsin(c*x)^2*x^4+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x
^4+11*c^2*x^2-4)*(-c^2*x^2+1)*x+a^2*(1/3*x/d/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*
x/(-c^2*d*x^2+d)^(1/2))-14/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-1
0*c^4*x^4+11*c^2*x^2-4)*c^4*x^5-4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^
6-10*c^4*x^4+11*c^2*x^2-4)*c^4*arcsin(c*x)*x^5+2*I*a*b*(-d*(c^2*x^2-1))^(1
/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*(-c^2*x^2+1)*x+16/3*I*a*b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*x^3+4/3*I*a
*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6*x...

```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**5/2, x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output

```
1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 - c^2*d^(5/2)) + 2*log(c*x + 1)/(c^2*d^(5/2)
) + 2*log(c*x - 1)/(c^2*d^(5/2))) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2 + d)*d^2
) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arcsin(c*x) + 1/3*a^2*(2*x/(sqrt(-c^2*d*
x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(arctan2(c*x,
sqrt(c*x + 1)*sqrt(-c*x + 1))^2/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt
(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx);OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^(5/2),x)
```

output

```
int((a + b*asin(c*x))^2/(d - c^2*d*x^2)^(5/2), x)
```


Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) ab c^2 x^2 - 6\sqrt{-c^2 x^2 + 1}}$$

input `int((a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)`

output `(6*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*a*b*c**2*x**2-6*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*a*b+3*sqrt(-c**2*x**2+1)*int(asin(c*x)**2/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b**2*c**2*x**2-3*sqrt(-c**2*x**2+1)*int(asin(c*x)**2/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b**2+2*a**2*c**2*x**3-3*a**2*x)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*d**2*(c**2*x**2-1))`

$$3.257 \quad \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2534
Mathematica [A] (warning: unable to verify)	2535
Rubi [A] (verified)	2536
Maple [A] (verified)	2544
Fricas [F]	2545
Sympy [F]	2545
Maxima [F]	2545
Giac [F(-2)]	2546
Mupad [F(-1)]	2546
Reduce [F]	2547

Optimal result

Integrand size = 29, antiderivative size = 577

$$\begin{aligned}
& \int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \frac{b^2}{3d^2 \sqrt{d - c^2 dx^2}} \\
& - \frac{bcx(a + b \arcsin(cx))}{3d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2}} + \frac{(a + b \arcsin(cx))^2}{3d(d - c^2 dx^2)^{3/2}} + \frac{(a + b \arcsin(cx))^2}{d^2 \sqrt{d - c^2 dx^2}} \\
& + \frac{14ib\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{3d^2 \sqrt{d - c^2 dx^2}} \\
& - \frac{2\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
& + \frac{2ib\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
& - \frac{7ib^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{3d^2 \sqrt{d - c^2 dx^2}} \\
& + \frac{7ib^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{3d^2 \sqrt{d - c^2 dx^2}} \\
& - \frac{2ib\sqrt{1 - c^2 x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
& - \frac{2b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
& + \frac{2b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}}
\end{aligned}$$

output

```

1/3*b^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arcsin(c*x))/d^2/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(3
/2)+(a+b*arcsin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+14/3*I*b*(-c^2*x^2+1)^(1/
2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(
1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(
1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*
polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-7/3*I*b^2*(-
c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2/(-c^2*d*x^2+
d)^(1/2)+7/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2
)))/d^2/(-c^2*d*x^2+d)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*po
lylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-c^2*x^2
+1)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+2*
b^2*(-c^2*x^2+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2
+d)^(1/2)

```

Mathematica [A] (warning: unable to verify)

Time = 8.24 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.62

$$\int \frac{(a + b \arcsin(cx))^2}{x (d - c^2 dx^2)^{5/2}} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output

```

Sqrt[-(d*(-1 + c^2*x^2))]*(a^2/(3*d^3*(-1 + c^2*x^2)^2) - a^2/(d^3*(-1 + c
^2*x^2))) + (a^2*Log[c*x])/d^(5/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c
^2*x^2))]])/d^(5/2) + (b^2*(1 - c^2*x^2)^(3/2)*(4 - ((-2 + ArcSin[c*x])*Ar
cSin[c*x])/(-1 + c*x) + 14*ArcSin[c*x]^2 + 12*ArcSin[c*x]^2*(Log[1 - E^(I*
ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c*x])]) - 28*(ArcSin[c*x]*(Log[1 - I*E
^(I*ArcSin[c*x])] - Log[1 + I*E^(I*ArcSin[c*x])])) + I*(PolyLog[2, (-I)*E^(
I*ArcSin[c*x])] - PolyLog[2, I*E^(I*ArcSin[c*x])])) + (24*I)*ArcSin[c*x]*(
PolyLog[2, -E^(I*ArcSin[c*x])] - PolyLog[2, E^(I*ArcSin[c*x])]) + 24*(-Pol
yLog[3, -E^(I*ArcSin[c*x])] + PolyLog[3, E^(I*ArcSin[c*x])]) + (2*ArcSin[c
*x]^2*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2])^3 + (2
*(2 + 7*ArcSin[c*x]^2)*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] - Sin[ArcSi
n[c*x]/2]) - (2*ArcSin[c*x]^2*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] + Si
n[ArcSin[c*x]/2])^3 + (ArcSin[c*x]*(2 + ArcSin[c*x]))/(Cos[ArcSin[c*x]/2]
+ Sin[ArcSin[c*x]/2])^2 - (2*(2 + 7*ArcSin[c*x]^2)*Sin[ArcSin[c*x]/2])/(Co
s[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2])/(12*d*(d*(1 - c^2*x^2)^(3/2)) +
(a*b*(20*ArcSin[c*x] + 12*ArcSin[c*x]*Cos[2*ArcSin[c*x]] + 18*Sqrt[1 - c^2
*x^2]*ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])] + 6*ArcSin[c*x]*Cos[3*ArcSin[
c*x]]*Log[1 - E^(I*ArcSin[c*x])] - 18*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1
+ E^(I*ArcSin[c*x])] - 6*ArcSin[c*x]*Cos[3*ArcSin[c*x]]*Log[1 + E^(I*ArcSi
n[c*x])]) + 21*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]...

```

Rubi [A] (verified)

Time = 5.01 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.82, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.690$, Rules used = {5208, 5162, 241, 5164, 3042, 4669, 2715, 2838, 5208, 5164, 3042, 4669, 2715, 2838, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow \text{5208}$$

$$-\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{3/2}} dx}{d} + \frac{(a + b \arcsin(cx))^2}{3d(d - c^2 dx^2)^{3/2}}$$

$$\downarrow \text{5162}$$

$$\begin{aligned}
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b\arcsin(cx))}{2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{\int \frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{241} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{1}{2} \int \frac{a+b\arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b\arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \\
 & \frac{(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5164} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b\arcsin(cx)}{\sqrt{1-c^2x^2}} d\arcsin(cx)}{2c} + \frac{x(a+b\arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{\int \frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{\int (a+b\arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d\arcsin(cx)}{2c} + \frac{x(a+b\arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{\int \frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{4669} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-b \int \log(1-ie^{i\arcsin(cx)}) d\arcsin(cx) + b \int \log(1+ie^{i\arcsin(cx)}) d\arcsin(cx) - 2i \arctan(e^{i\arcsin(cx)})(a+b\arcsin(cx))}{2c} + \frac{x(a+b\arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \\
 & \frac{\int \frac{(a+b\arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b\arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2715}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2838} \\
 & \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} - \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{3d^2\sqrt{d-c^2dx^2}} \\
 & \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5208} \\
 & \frac{-\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{d\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}}{d} - \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{3d^2\sqrt{d-c^2dx^2}} \\
 & \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5164} \\
 & \frac{-\frac{2b\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}}{d} - \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{3d^2\sqrt{d-c^2dx^2}} \\
 & \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{3042}
 \end{aligned}$$

$$\frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2 dx^2}} dx - \frac{2b\sqrt{1-c^2 x^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx)+\frac{\pi}{2}) d \arcsin(cx)}{d\sqrt{d-c^2 dx^2}} + \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2 dx^2}}}{2bc\sqrt{1-c^2 x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2 x^2)} \right)}$$

$$\frac{3d^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}}$$

↓ 4669

$$\frac{-\frac{2b\sqrt{1-c^2 x^2} (-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx)+b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx)-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)))}{d\sqrt{d-c^2 dx^2}} + \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2 dx^2}} dx}{2bc\sqrt{1-c^2 x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2 x^2)} \right)}$$

$$\frac{3d^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}}$$

↓ 2715

$$\frac{-\frac{2b\sqrt{1-c^2 x^2} (ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}-ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)))}{d\sqrt{d-c^2 dx^2}} + \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2 dx^2}} dx}{2bc\sqrt{1-c^2 x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2 x^2)} \right)}$$

$$\frac{3d^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}}$$

↓ 2838

$$\frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2 dx^2}} dx - \frac{2b\sqrt{1-c^2 x^2} (-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2 dx^2}} + \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2 dx^2}} dx}{2bc\sqrt{1-c^2 x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2 x^2)} \right)}$$

$$\frac{3d^2 \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{3d(d-c^2 dx^2)^{3/2}}$$

↓ 5218

$$\frac{\sqrt{1-c^2x^2} \int \frac{(a+b \arcsin(cx))^2}{cx} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} (-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx)))}{d\sqrt{d-c^2dx^2}}$$

$$2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} \frac{d}{d\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) -$$

$$\frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

↓ 3042

$$\frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} (-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))+ib \operatorname{PolyLog}(2,ie^i \arcsin(cx)))}{d\sqrt{d-c^2dx^2}}$$

$$2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} \frac{d}{d\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) -$$

$$\frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

↓ 4671

$$\frac{\sqrt{1-c^2x^2} (-2b \int (a+b \arcsin(cx)) \log(1-e^i \arcsin(cx)) d \arcsin(cx)+2b \int (a+b \arcsin(cx)) \log(1+e^i \arcsin(cx)) d \arcsin(cx)-2a \operatorname{arctanh}(e^i \arcsin(cx)))}{d\sqrt{d-c^2dx^2}}$$

$$2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} \frac{d}{d\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) -$$

$$\frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

↓ 3011

$$\frac{\sqrt{1-c^2x^2} (2b(i \operatorname{PolyLog}(2,-e^i \arcsin(cx))(a+b \arcsin(cx))-ib \int \operatorname{PolyLog}(2,-e^i \arcsin(cx)) d \arcsin(cx))-2b(i \operatorname{PolyLog}(2,e^i \arcsin(cx))(a+b \arcsin(cx))+ib \int \operatorname{PolyLog}(2,e^i \arcsin(cx)) d \arcsin(cx))}{d\sqrt{d-c^2dx^2}}$$

$$2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} \frac{d}{d\sqrt{d-c^2dx^2}} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) -$$

$$\frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

↓ 2720

$$\frac{\sqrt{1-c^2x^2} \left(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) \right)}{d\sqrt{d-c^2dx^2}}$$

$$2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)$$

$$\frac{3d^2\sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

↓ 7143

$$-\frac{2b\sqrt{1-c^2x^2} (-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{1-c^2x^2} (-2a \arctan(\dots))}{2(1-c^2x^2)}$$

$$2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)$$

$$\frac{3d^2\sqrt{d-c^2dx^2} (a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output

```
(a + b*ArcSin[c*x])^2/(3*d*(d - c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 - c^2*x^2]*(-1/2*b/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcSin[c*x]))/(2*(1 - c^2*x^2))) + ((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])])/(2*c))/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((a + b*ArcSin[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/(d*Sqrt[d - c^2*d*x^2]))/d
```

Definitions of rubi rules used

- rule 241 $\text{Int}[(x_*)*((a_*) + (b_*)*(x_*)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p + 1)}/(2*b*(p + 1)), x] /; \text{FreeQ}\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 2715 $\text{Int}[\text{Log}[(a_*) + (b_*)*((F_*)^{((e_*)*((c_*) + (d_*)*(x_*)}))})^{(n_*)}], x_Symbol] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \ \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \ \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_*)*(v_*)^{(n_*)})^{(m_*)} /; \text{FreeQ}\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_*)*((a_*) + (b_*)*x))*}(F_)[v_]] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$
- rule 2838 $\text{Int}[\text{Log}[(c_*)*((d_*) + (e_*)*(x_*)^{(n_*)})]/(x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_*)*((F_*)^{((c_*)*((a_*) + (b_*)*(x_*)}))})^{(n_*)}]*((f_*) + (g_*)*(x_*)^{(m_*)}), x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m*(\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x)})^n])^m)/(b*c*n*\text{Log}[F]), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \ \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, (-e)*(F^{(c*(a + b*x)})^n]), x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4669 $\text{Int}[\text{csc}[(e_*) + \text{Pi}*(k_*) + (f_*)*(x_*)]*((c_*) + (d_*)*(x_*)^{(m_*)}), x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \ \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x) /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*((d_.) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

rule 5164

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_/((d_.) + (e_.)*(x_)^2), x_Symbo
l] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x]
/; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*((f_.)*(x_))^(m_)*((d_.) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*(x_)^(m_))/Sqrt[(d_.) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.60

method	result
default	$\frac{a^2}{3d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{a^2}{d^2\sqrt{-c^2dx^2+d}} - \frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}(3\arcsin(cx)^2x^2c^2+\arcsin(cx))}{3(c^2x^2-1)} \right)$
parts	$\frac{a^2}{3d(-c^2dx^2+d)^{\frac{3}{2}}} + \frac{a^2}{d^2\sqrt{-c^2dx^2+d}} - \frac{a^2 \ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}} + b^2 \left(-\frac{\sqrt{-d(c^2x^2-1)}(3\arcsin(cx)^2x^2c^2+\arcsin(cx))}{3(c^2x^2-1)} \right)$

input `int((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

```

1/3*a^2/d/(-c^2*d*x^2+d)^(3/2)+a^2/d^2/(-c^2*d*x^2+d)^(1/2)-a^2/d^(5/2)*ln
((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/3*(-d*(c^2*x^2-1))^(1/2)*
(3*arcsin(c*x)^2*x^2*c^2+arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-4*arcs
in(c*x)^2-1)/(c^2*x^2-1)^2/d^3+1/3*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(
1/2)*(3*I*arcsin(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-3*I*arcsin(c*x)^2*ln
(1+I*c*x+(-c^2*x^2+1)^(1/2))+7*I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(
1/2))))-7*I*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+6*arcsin(c*x)*po
lylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-6*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2
+1)^(1/2))+6*I*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))-6*I*polylog(3,-I*c*x-(-
c^2*x^2+1)^(1/2))+7*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-7*dilog(1-I*(I*c
*x+(-c^2*x^2+1)^(1/2)))/d^3/(c^2*x^2-1))-1/3*I*a*b*(-c^2*x^2+1)^(1/2)*(-d
*(c^2*x^2-1))^(1/2)*(-8*I*arcsin(c*x)*(-c^2*x^2+1)^(1/2)+6*dilog(1+I*c*x+(
-c^2*x^2+1)^(1/2))*c^4*x^4+6*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+14*ar
ctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+6*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^
2+1)^(1/2))-I*x^3*c^3-12*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*x^2*
c^2-12*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2-12*dilog(I*c*x+(-c^2*x^2+
1)^(1/2))*c^2*x^2-28*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^2*x^2+6*I*arcsin(c
*x)*(-c^2*x^2+1)^(1/2)*x^2*c^2+6*I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/
2))*x^4*c^4+I*c*x+6*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))+6*dilog(I*c*x+(-c^2*
x^2+1)^(1/2))+14*arctan(I*c*x+(-c^2*x^2+1)^(1/2)))/(c^6*x^6-3*c^4*x^4+3...

```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x(-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))**2/x/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output

```
-1/3*a^2*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2)
) - 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d) - sqrt(d)
*integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arcta
n2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d
^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((a+b*arcsin(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^5 - 2\sqrt{-c^2 x^2 + 1} c^2 x^3 + \sqrt{-c^2 x^2 + 1} x} dx \right) ab c^2 x^2 - 6\sqrt{-c^2 x^2}}$$

input `int((a+b*asin(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)`

output `(6*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**5-2*sqrt(-c**2*x**2+1)*c**2*x**3+sqrt(-c**2*x**2+1)*x),x)*a*b*c**2*x**2-6*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**5-2*sqrt(-c**2*x**2+1)*c**2*x**3+sqrt(-c**2*x**2+1)*x),x)*a*b+3*sqrt(-c**2*x**2+1)*int(asin(c*x)**2/(sqrt(-c**2*x**2+1)*c**4*x**5-2*sqrt(-c**2*x**2+1)*c**2*x**3+sqrt(-c**2*x**2+1)*x),x)*b**2*c**2*x**2-3*sqrt(-c**2*x**2+1)*int(asin(c*x)**2/(sqrt(-c**2*x**2+1)*c**4*x**5-2*sqrt(-c**2*x**2+1)*c**2*x**3+sqrt(-c**2*x**2+1)*x),x)*b**2+3*sqrt(-c**2*x**2+1)*log(tan(asin(c*x)/2))*a**2*c**2*x**2-3*sqrt(-c**2*x**2+1)*log(tan(asin(c*x)/2))*a**2-4*sqrt(-c**2*x**2+1)*a**2*c**2*x**2+4*sqrt(-c**2*x**2+1)*a**2+3*a**2*c**2*x**2-4*a**2)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*d**2*(c**2*x**2-1))`

3.258
$$\int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2548
Mathematica [A] (verified)	2549
Rubi [A] (verified)	2550
Maple [B] (verified)	2558
Fricas [F]	2559
Sympy [F]	2560
Maxima [F]	2560
Giac [F(-2)]	2560
Mupad [F(-1)]	2561
Reduce [F]	2561

Optimal result

Integrand size = 29, antiderivative size = 452

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x^2(d-c^2dx^2)^{5/2}} dx &= \frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} - \frac{bc(a+b \arcsin(cx))}{3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} \\ &- \frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}} + \frac{4c^2x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\ &+ \frac{8c^2x(a+b \arcsin(cx))^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{8ic\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3d^2\sqrt{d-c^2dx^2}} \\ &- \frac{4bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{16bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))\log(1+e^{2i \arcsin(cx)})}{3d^2\sqrt{d-c^2dx^2}} \\ &- \frac{5ib^2c\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,-e^{2i \arcsin(cx)})}{3d^2\sqrt{d-c^2dx^2}} \\ &- \frac{ib^2c\sqrt{1-c^2x^2}\operatorname{PolyLog}(2,e^{2i \arcsin(cx)})}{d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```

1/3*b^2*c^2*x/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arcsin(c*x))/d^2/(-c^2
*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b*arcsin(c*x))^2/d/x/(-c^2*d*x^2+d)^(
3/2)+4/3*c^2*x*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)+8/3*c^2*x*(a+b*
arcsin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*I*c*(-c^2*x^2+1)^(1/2)*(a+b*ar
csin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-4*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin
(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)+16/3
*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^
2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c
*x+(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1
/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)

```

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.78

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx =$$

$$c \left(\frac{a^2(3-12c^2x^2+8c^4x^4)}{cx} + \frac{2ab(3-12c^2x^2+8c^4x^4) \arcsin(cx)}{cx} + ab\sqrt{1-c^2x^2}(1+6(-1+c^2x^2)\log(cx)) + 5(-1+c^2x^2) \right)$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output

```

-1/3*(c*((a^2*(3 - 12*c^2*x^2 + 8*c^4*x^4))/(c*x) + (2*a*b*(3 - 12*c^2*x^2
+ 8*c^4*x^4)*ArcSin[c*x]))/(c*x) + a*b*Sqrt[1 - c^2*x^2]*(1 + 6*(-1 + c^2*
x^2)*Log[c*x] + 5*(-1 + c^2*x^2)*Log[1 - c^2*x^2]) - b^2*(1 - c^2*x^2)^(3/
2)*((c*x)/Sqrt[1 - c^2*x^2] + ArcSin[c*x])/(-1 + c^2*x^2) - (8*I)*ArcSin[c*
x]^2 + (c*x*ArcSin[c*x]^2)/(1 - c^2*x^2)^(3/2) + (5*c*x*ArcSin[c*x]^2)/Sqr
t[1 - c^2*x^2] - (3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(c*x) + 6*ArcSin[c*x]
*Log[1 - E^((2*I)*ArcSin[c*x])] + 10*ArcSin[c*x]*Log[1 + E^((2*I)*ArcSin[c
*x])] - (5*I)*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (3*I)*PolyLog[2, E^((2*
I)*ArcSin[c*x])])]/(d*(d - c^2*d*x^2)^(3/2))

```

Rubi [A] (verified)

Time = 3.42 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.655$, Rules used = {5204, 5162, 5160, 5180, 3042, 4202, 2620, 2715, 2838, 5182, 208, 5208, 208, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx \\
 & \quad \downarrow \text{5204} \\
 & \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2 \sqrt{d - c^2 dx^2}} + 4c^2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx - \frac{(a + b \arcsin(cx))^2}{dx (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{5162} \\
 & 4c^2 \left(-\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x(a + b \arcsin(cx))}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx}{3d} + \frac{x(a + b \arcsin(cx))^2}{3d (d - c^2 dx^2)^{3/2}} \right) + \\
 & \quad \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2 \sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{5160} \\
 & 4c^2 \left(-\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x(a + b \arcsin(cx))}{(1 - c^2 x^2)^2} dx}{3d^2 \sqrt{d - c^2 dx^2}} + \frac{2 \left(\frac{x(a + b \arcsin(cx))^2}{d\sqrt{d - c^2 dx^2}} - \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{x(a + b \arcsin(cx))}{1 - c^2 x^2} dx}{d\sqrt{d - c^2 dx^2}} \right)}{3d} + \frac{x(a + b \arcsin(cx))^2}{3d (d - c^2 dx^2)^{3/2}} \right) + \\
 & \quad \frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2 \sqrt{d - c^2 dx^2}} - \frac{(a + b \arcsin(cx))^2}{dx (d - c^2 dx^2)^{3/2}} \\
 & \quad \downarrow \text{5180}
 \end{aligned}$$

$$\begin{aligned}
 & 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} + \frac{x(a+b \arcsin(cx))}{3d(d-c^2dx^2)} \right) \\
 & \quad - \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{3042} \\
 & 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right) \\
 & \quad - \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{4202} \\
 & 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)} (a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} \right)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right) \\
 & \quad - \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2620} \\
 & 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right) \\
 & \quad - \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{2715}
 \end{aligned}$$

$$4c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} + \frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) dx \right) \right)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 2838

$$4c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right)$$

$$\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 5182

$$4c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{b \int \frac{1}{(1-c^2x^2)^{3/2}} dx}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) \right)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right)$$

$$\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 208

$$\begin{aligned}
 & \frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} + \\
 4c^2 & \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) \right)}{3d} \right)}{3d} \right) \\
 & \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5208} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx - \frac{1}{2} bc \int \frac{1}{(1-c^2x^2)^{3/2}} dx + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} + \\
 4c^2 & \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) \right)}{3d} \right)}{3d} \right) \\
 & \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{208} \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(\int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2\sqrt{d-c^2dx^2}} + \\
 4c^2 & \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) \right)}{3d} \right)}{3d} \right) \\
 & \frac{(a+b \arcsin(cx))^2}{dx (d-c^2dx^2)^{3/2}} \\
 & \quad \downarrow \text{5184}
 \end{aligned}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\int \frac{a+b\arcsin(cx)}{cx\sqrt{1-c^2x^2}}d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}} +$$

$$4c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})\right)\right)}{3d}\right)}{3d}\right)$$

$$\frac{(a+b\arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 4919

$$\frac{2bc\sqrt{1-c^2x^2}\left(2\int(a+b\arcsin(cx))\csc(2\arcsin(cx))d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}} +$$

$$4c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})\right)\right)}{3d}\right)}{3d}\right)$$

$$\frac{(a+b\arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 3042

$$\frac{2bc\sqrt{1-c^2x^2}\left(2\int(a+b\arcsin(cx))\csc(2\arcsin(cx))d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}} +$$

$$4c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})\right)\right)}{3d}\right)}{3d}\right)$$

$$\frac{(a+b\arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 4671

$$\frac{2bc\sqrt{1-c^2x^2} \left(2\left(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) dx + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) dx - (\operatorname{arctanh}(e^{2i \arcsin(cx)})) \right) \right)}{4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{d^2\sqrt{d-c^2dx^2}}{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})\right) \right)}{3d} \right)}{3d} \right)}$$

$$\frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 2715

$$\frac{2bc\sqrt{1-c^2x^2} \left(2\left(\frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1-e^{2i \arcsin(cx)}) dx - \frac{1}{4}ib \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) dx \right) \right)}{4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{d^2\sqrt{d-c^2dx^2}}{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})\right) \right)}{3d} \right)}{3d} \right)}$$

$$\frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

↓ 2838

$$\frac{2bc\sqrt{1-c^2x^2} \left(2\left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)})) (a+b \arcsin(cx))\right) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right)}{4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{d^2\sqrt{d-c^2dx^2}}{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i\left(-\frac{1}{2}i \log(1+e^{2i \arcsin(cx)})\right) \right)}{3d} \right)}{3d} \right)}$$

$$\frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]`

output

$$\begin{aligned}
& -((a + b \operatorname{ArcSin}[c x])^2 / (d x (d - c^2 d x^2)^{3/2})) + 4 c^2 ((x (a + b \operatorname{ArcSin}[c x])^2) / (3 d (d - c^2 d x^2)^{3/2}) - (2 b c \operatorname{Sqrt}[1 - c^2 x^2] (-1/2 * (b x) / (c \operatorname{Sqrt}[1 - c^2 x^2]) + (a + b \operatorname{ArcSin}[c x]) / (2 c^2 (1 - c^2 x^2)))) / (3 d^2 \operatorname{Sqrt}[d - c^2 d x^2]) + (2 ((x (a + b \operatorname{ArcSin}[c x])^2) / (d \operatorname{Sqrt}[d - c^2 d x^2]) - (2 b \operatorname{Sqrt}[1 - c^2 x^2] (((I/2) (a + b \operatorname{ArcSin}[c x])^2) / b - (2 I) ((-1/2 I) (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}[1 + E^{((2 I) \operatorname{ArcSin}[c x])}] - (b \operatorname{PolyLog}[2, -E^{((2 I) \operatorname{ArcSin}[c x])}]) / 4))) / (c d \operatorname{Sqrt}[d - c^2 d x^2])) / (3 d)) + (2 b c \operatorname{Sqrt}[1 - c^2 x^2] (-1/2 (b c x) / \operatorname{Sqrt}[1 - c^2 x^2] + (a + b \operatorname{ArcSin}[c x]) / (2 (1 - c^2 x^2)) + 2 (-((a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}[E^{((2 I) \operatorname{ArcSin}[c x])}]) + (I/4) b \operatorname{PolyLog}[2, -E^{((2 I) \operatorname{ArcSin}[c x])}] - (I/4) b \operatorname{PolyLog}[2, E^{((2 I) \operatorname{ArcSin}[c x])}])])) / (d^2 \operatorname{Sqrt}[d - c^2 d x^2])
\end{aligned}$$

Defintions of rubi rules used

rule 208

$$\operatorname{Int}[(a + (b x)^2)^{-3/2}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[x / (a \operatorname{Sqrt}[a + b x^2]), x] \text{ ; FreeQ}\{a, b\}, x]$$

rule 2620

$$\begin{aligned}
& \operatorname{Int}[(F^{(g(x) + f(x))})^{(c + d x)^m} / ((a + b x) (F^{(g(x) + f(x))})^{(c + d x)^m}), x_{\text{Symbol}}] \rightarrow \operatorname{Simp} \\
& [((c + d x)^m / (b f g n \operatorname{Log}[F])) \operatorname{Log}[1 + b (F^{(g(x) + f(x))})^n / a], x] - \operatorname{Simp} \\
& [d (m / (b f g n \operatorname{Log}[F])) \operatorname{Int}[(c + d x)^{m-1} \operatorname{Log}[1 + b (F^{(g(x) + f(x))})^n / a], x], x] \text{ ; FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \operatorname{IGtQ}[m, 0]
\end{aligned}$$

rule 2715

$$\begin{aligned}
& \operatorname{Int}[\operatorname{Log}[(a + b x) (F^{(e + d x)})^{(c + d x)^n}], x_{\text{Symbol}}] \\
& \rightarrow \operatorname{Simp}[1 / (d e n \operatorname{Log}[F]) \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b x] / x, x], x, (F^{(e + d x)})^n], x] \text{ ; FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \operatorname{GtQ}[a, 0]
\end{aligned}$$

rule 2838

$$\begin{aligned}
& \operatorname{Int}[\operatorname{Log}[(c + d x) (e + f x)^n] / (x), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[-\operatorname{PolyLog}[2, \\
& (-c) e x^n / n, x] \text{ ; FreeQ}\{c, d, e, n\}, x] \ \&\& \operatorname{EqQ}[c d, 1]
\end{aligned}$$

rule 3042

$$\operatorname{Int}[u, x_{\text{Symbol}}] \rightarrow \operatorname{Int}[\operatorname{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$$

rule 4202 $\text{Int}[\{(c_.) + (d_.)*(x_)\}^{(m_)}*\tan[(e_.) + (f_.)*(x_)], x_Symbol] \rightarrow \text{Simp}[I*((c + d*x)^{(m + 1)}/(d*(m + 1))), x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m*(E^{(2*I*(e + f*x))}/(1 + E^{(2*I*(e + f*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]*\{(c_.) + (d_.)*(x_)\}^{(m_)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.)*(x_)]^{(n_)}*\{(c_.) + (d_.)*(x_)\}^{(m_)}*\text{Sec}[(a_.) + (b_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m*\text{Csc}[2*a + 2*b*x]^{2n}, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{RationalQ}[m]$

rule 5160 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)}/\{(d_.) + (e_.)*(x_)^2\}^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[x*((a + b*\text{ArcSin}[c*x])^n/(d*\text{Sqrt}[d + e*x^2])), x] - \text{Simp}[b*c*(n/d)*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]] \text{Int}[x*((a + b*\text{ArcSin}[c*x])^{(n - 1)}/(1 - c^2*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

rule 5162 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)}*\{(d_.) + (e_.)*(x_)^2\}^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(-x)*(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*d*(p + 1))), x] + (\text{Simp}[(2*p + 3)/(2*d*(p + 1)) \text{Int}[(d + e*x^2)^{(p + 1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[x*(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

rule 5180 $\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_)}*(x_)/\{(d_.) + (e_.)*(x_)^2\}, x_Symbol] \rightarrow \text{Simp}[-e^{(-1)} \text{Subst}[\text{Int}[(a + b*x)^n*\text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[c_.*x_]*b_.)^{n_.*x_*(d_ + (e_.*x_)^2)^{p_} .), x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{p+1}*(a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1)), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{p+1/2}*(a + b*\text{ArcSin}[c*x])^{n-1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5184 $\text{Int}[(a_.) + \text{ArcSin}[c_.*x_]*b_.)^{n_./((x_)*(d_ + (e_.*x_)^2))}, x_Symbol] \rightarrow \text{Simp}[1/d \ \text{Subst}[\text{Int}[(a + b*x)^n/(\text{Cos}[x]*\text{Sin}[x]), x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5204 $\text{Int}[(a_.) + \text{ArcSin}[c_.*x_]*b_.)^{n_.*((f_.*x_)^m)*(d_ + (e_.*x_)^2)^{p_} .), x_Symbol] \rightarrow \text{Simp}[(f*x)^{m+1}*(d + e*x^2)^{p+1}*((a + b*\text{ArcSin}[c*x])^n/(d*f*(m+1))), x] + (\text{Simp}[c^2*((m+2*p+3)/(f^2*(m+1))) \ \text{Int}[(f*x)^{m+2}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{m+1}*(1 - c^2*x^2)^{p+1/2}*(a + b*\text{ArcSin}[c*x])^{n-1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{ILtQ}[m, -1]$

rule 5208 $\text{Int}[(a_.) + \text{ArcSin}[c_.*x_]*b_.)^{n_.*((f_.*x_)^m)*(d_ + (e_.*x_)^2)^{p_} .), x_Symbol] \rightarrow \text{Simp}[(-f*x)^{m+1}*(d + e*x^2)^{p+1}*((a + b*\text{ArcSin}[c*x])^n/(2*d*f*(p+1))), x] + (\text{Simp}[(m+2*p+3)/(2*d*(p+1)) \ \text{Int}[(f*x)^m*(d + e*x^2)^{p+1}*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*c*(n/(2*f*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{m+1}*(1 - c^2*x^2)^{p+1/2}*(a + b*\text{ArcSin}[c*x])^{n-1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !\text{GtQ}[m, 1] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegerQ}[p] \ || \ \text{EqQ}[n, 1])$

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3772 vs. $2(443) = 886$.

Time = 0.93 (sec) , antiderivative size = 3773, normalized size of antiderivative = 8.35

method	result	size
default	Expression too large to display	3773
parts	Expression too large to display	3773

input `int((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)`

output

$$\begin{aligned}
 & -8*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*(- \\
 & c^2*x^2+1)*c^2+3*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x \\
 & ^2-9)/d^3*arcsin(c*x)*(-c^2*x^2+1)^{(1/2)}*c-64/3*b^2*(-d*(c^2*x^2-1))^{(1/2)} \\
 & /((8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*arcsin(c*x)^2*c^6+56*b^2*(-d* \\
 & (c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^3*arcsin(c*x) \\
 & ^2*c^4-44*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d \\
 & ^3*x*arcsin(c*x)^2*c^2-3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^ \\
 & 4+26*c^2*x^2-9)/d^3*(-c^2*x^2+1)^{(1/2)}*c-224/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)} \\
 &)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^7*arcsin(c*x)*c^8+280/3*I*b^2* \\
 & (-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*arcsin(\\
 & c*x)*c^6-8/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2 \\
 & -9)/d^3*x^4*(-c^2*x^2+1)^{(1/2)}*c^5-2*b^2*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2*x^2-1 \\
 &))^{(1/2)}/d^3/(c^2*x^2-1)*c*arcsin(c*x)*ln(1-I*c*x+(-c^2*x^2+1)^{(1/2)})-48*I \\
 & *b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^3*ar \\
 & csin(c*x)*c^4+17/3*I*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c \\
 & ^2*x^2-9)/d^3*x^2*(-c^2*x^2+1)^{(1/2)}*c^3-2*b^2*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2 \\
 & *x^2-1))^{(1/2)}/d^3/(c^2*x^2-1)*c*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^{(1/2)} \\
 &)-8/3*b^2*(-d*(c^2*x^2-1))^{(1/2)}/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x \\
 & ^2*arcsin(c*x)*(-c^2*x^2+1)^{(1/2)}*c^3-10/3*b^2*(-c^2*x^2+1)^{(1/2)}*(-d*(c^2 \\
 & *x^2-1))^{(1/2)}/d^3/(c^2*x^2-1)*c*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^{...}
 \end{aligned}$$

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^2 (-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))**2/x**2/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/3*a^2*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/2)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) - sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2 x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^6 - 2\sqrt{-c^2 x^2 + 1} c^2 x^4 + \sqrt{-c^2 x^2 + 1} x^2} dx \right) ab c^2 x^3 - 6\sqrt{-c^2 x^2 + 1}}$$

input

```
int((a+b*asin(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(6*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**6
- 2*sqrt(-c**2*x**2 + 1)*c**2*x**4 + sqrt(-c**2*x**2 + 1)*x**2),x)*a*b
*c**2*x**3 - 6*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)
)*c**4*x**6 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**4 + sqrt(-c**2*x**2 + 1)*
x**2),x)*a*b*x + 3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)**2/(sqrt(-c**2*x
**2 + 1)*c**4*x**6 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**4 + sqrt(-c**2*x**
2 + 1)*x**2),x)*b**2*c**2*x**3 - 3*sqrt(-c**2*x**2 + 1)*int(asin(c*x)**2
/(sqrt(-c**2*x**2 + 1)*c**4*x**6 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**4 +
sqrt(-c**2*x**2 + 1)*x**2),x)*b**2*x + 8*a**2*c**4*x**4 - 12*a**2*c**2*x
**2 + 3*a**2)/(3*sqrt(d)*sqrt(-c**2*x**2 + 1)*d**2*x*(c**2*x**2 - 1))
```

$$3.259 \quad \int \frac{(a+b \arcsin(cx))^2}{x^3(d-c^2dx^2)^{5/2}} dx$$

Optimal result	2563
Mathematica [A] (verified)	2564
Rubi [F]	2565
Maple [A] (warning: unable to verify)	2572
Fricas [F]	2573
Sympy [F]	2574
Maxima [F]	2574
Giac [F(-2)]	2574
Mupad [F(-1)]	2575
Reduce [F]	2575

Optimal result

Integrand size = 29, antiderivative size = 752

$$\begin{aligned}
\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx &= \frac{b^2 c^2}{3d^2 \sqrt{d - c^2 dx^2}} - \frac{bc^3 x (a + b \arcsin(cx))}{3d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 dx^2}} \\
&- \frac{bc \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{d^2 x \sqrt{d - c^2 dx^2}} + \frac{5c^2 (a + b \arcsin(cx))^2}{6d (d - c^2 dx^2)^{3/2}} \\
&- \frac{(a + b \arcsin(cx))^2}{2dx^2 (d - c^2 dx^2)^{3/2}} + \frac{5c^2 (a + b \arcsin(cx))^2}{2d^2 \sqrt{d - c^2 dx^2}} \\
&+ \frac{26ibc^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{3d^2 \sqrt{d - c^2 dx^2}} \\
&- \frac{5c^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
&- \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{d^2 \sqrt{d - c^2 dx^2}} \\
&+ \frac{5ibc^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
&- \frac{13ib^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{3d^2 \sqrt{d - c^2 dx^2}} \\
&+ \frac{13ib^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{3d^2 \sqrt{d - c^2 dx^2}} \\
&- \frac{5ibc^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
&- \frac{5b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}} \\
&+ \frac{5b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{d^2 \sqrt{d - c^2 dx^2}}
\end{aligned}$$

output

```

1/3*b^2*c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c^3*x*(a+b*arcsin(c*x))/d^2/(-c
^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*
x))/d^2/x/(-c^2*d*x^2+d)^(1/2)+5/6*c^2*(a+b*arcsin(c*x))^2/d/(-c^2*d*x^2+d
)^(3/2)-1/2*(a+b*arcsin(c*x))^2/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/2*c^2*(a+b*ar
csin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+26/3*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b
*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-5*
c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2
))/d^2/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*(-c^2*x^2+1)^(1/2)*arctanh((-c^2*x^2+1
)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+5*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin
(c*x))*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-13/3*
I*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d^2/
(-c^2*d*x^2+d)^(1/2)+13/3*I*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+
(-c^2*x^2+1)^(1/2)))/d^2/(-c^2*d*x^2+d)^(1/2)-5*I*b*c^2*(-c^2*x^2+1)^(1/2)
*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(
1/2)-5*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(3,-I*c*x+(-c^2*x^2+1)^(1/2))/d^
2/(-c^2*d*x^2+d)^(1/2)+5*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(3,I*c*x+(-c^2*
x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)

```

Mathematica [A] (verified)

Time = 9.73 (sec) , antiderivative size = 1090, normalized size of antiderivative = 1.45

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output

```

Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a^2/(d^3*x^2) + (a^2*c^2)/(3*d^3*(-1 + c^2
*x^2)^2) - (2*a^2*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a^2*c^2*Log[x])/(2*d^(5/
2)) - (5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) +
(a*b*c^2*Sqrt[1 - c^2*x^2]*((-2*(-1 + ArcSin[c*x])))/(-1 + c*x) + 52*ArcSi
n[c*x] - 6*Cot[ArcSin[c*x]/2] - 3*ArcSin[c*x]*Csc[ArcSin[c*x]/2]^2 + 60*Ar
cSin[c*x]*(Log[1 - E^(I*ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c*x])]) + 52*L
og[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] - 52*Log[Cos[ArcSin[c*x]/2] +
Sin[ArcSin[c*x]/2]] + (60*I)*(PolyLog[2, -E^(I*ArcSin[c*x])] - PolyLog[2,
E^(I*ArcSin[c*x])]) + 3*ArcSin[c*x]*Sec[ArcSin[c*x]/2]^2 + (4*ArcSin[c*x]*
Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2])^3 + (52*ArcS
in[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]) - (4
*ArcSin[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2])
^3 + (2*(1 + ArcSin[c*x]))/(Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2])^2 - (
52*ArcSin[c*x]*Sin[ArcSin[c*x]/2])/(Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2
]) - 6*Tan[ArcSin[c*x]/2]))/(12*d^2*Sqrt[d*(1 - c^2*x^2)]) + (b^2*c^2*Sqrt
[1 - c^2*x^2]*(8 - (2*(-2 + ArcSin[c*x])*ArcSin[c*x]))/(-1 + c*x) + 52*ArcS
in[c*x]^2 - 12*ArcSin[c*x]*Cot[ArcSin[c*x]/2] - 3*ArcSin[c*x]^2*Csc[ArcSin
[c*x]/2]^2 + 24*Log[Tan[ArcSin[c*x]/2]] - 104*(ArcSin[c*x]*(Log[1 - I*E^(I
*ArcSin[c*x])] - Log[1 + I*E^(I*ArcSin[c*x])]) + I*(PolyLog[2, (-I)*E^(I*A
rcSin[c*x])] - PolyLog[2, I*E^(I*ArcSin[c*x])])) + 60*(ArcSin[c*x]^2*(L...

```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow 5204$$

$$\frac{bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x^2(1 - c^2 x^2)^2} dx}{d^2 \sqrt{d - c^2 dx^2}} + \frac{5}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x (d - c^2 dx^2)^{5/2}} dx - \frac{(a + b \arcsin(cx))^2}{2 dx^2 (d - c^2 dx^2)^{3/2}}$$

$$\downarrow 5204$$

$$\frac{bc\sqrt{1 - c^2 x^2} \left(3c^2 \int \frac{a + b \arcsin(cx)}{(1 - c^2 x^2)^2} dx + bc \int \frac{1}{x(1 - c^2 x^2)^{3/2}} dx - \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)} \right)}{d^2 \sqrt{d - c^2 dx^2}} + \frac{5}{2} c^2 \int \frac{(a + b \arcsin(cx))^2}{x (d - c^2 dx^2)^{5/2}} dx - \frac{(a + b \arcsin(cx))^2}{2 dx^2 (d - c^2 dx^2)^{3/2}}$$

$$\begin{aligned}
& \downarrow 243 \\
& \frac{bc\sqrt{1-c^2x^2}\left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{1}{2}bc \int \frac{1}{x^2(1-c^2x^2)^{3/2}} dx^2 - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}} + \\
& \frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}} \\
& \downarrow 61 \\
& \frac{bc\sqrt{1-c^2x^2}\left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{1}{2}bc\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}} dx^2 + \frac{2}{\sqrt{1-c^2x^2}}\right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}} + \\
& \frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}} \\
& \downarrow 73 \\
& \frac{bc\sqrt{1-c^2x^2}\left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx + \frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}} - \frac{2 \int \frac{1-x^4}{c^2-x^2} d\sqrt{1-c^2x^2}}{c^2}\right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}} + \\
& \frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}} \\
& \downarrow 221 \\
& \frac{bc\sqrt{1-c^2x^2}\left(3c^2 \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)\right)}{d^2\sqrt{d-c^2dx^2}} + \\
& \frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}} \\
& \downarrow 5162 \\
& \frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}\right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)\right)}{d^2\sqrt{d-c^2dx^2}} + \\
& \frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}} \\
& \downarrow 241 \\
& \frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}}\right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}} - 2\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)\right)}{d^2\sqrt{d-c^2dx^2}} + \\
& \frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}
\end{aligned}$$

↓ 5164

$$bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \arctan \left(\frac{cx}{\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$\frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{d^2\sqrt{d-c^2dx^2}}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 3042

$$bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx)+\frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \arctan \left(\frac{cx}{\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$\frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{d^2\sqrt{d-c^2dx^2}}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 4669

$$bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx) + b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \arctan \left(\frac{cx}{\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$\frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{d^2\sqrt{d-c^2dx^2}}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 2715

$$bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx) - ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx) - 2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \arctan \left(\frac{cx}{\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$\frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx - \frac{d^2\sqrt{d-c^2dx^2}}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 2838

$$\frac{5}{2}c^2 \int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{5/2}} dx +$$

$$bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^i \arcsin(cx)) - ib \operatorname{PolyLog}(2, ie^i \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right) - \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2}{\sqrt{1-c^2x^2}} - 2 \arctan \left(\frac{cx}{\sqrt{1-c^2x^2}} \right) \right) \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 5208

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) +$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

5162

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx - \frac{1}{2}bc \int \frac{x}{(1-c^2x^2)^{3/2}} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) +$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

241

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{1}{2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) +$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

5164

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{\int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) +$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

3042

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{\int (a+b \arcsin(cx)) \csc(\arcsin(cx)+\frac{\pi}{2}) d \arcsin(cx)}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} - \frac{b}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}}}{d} \right)$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 4669

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{-b \int \log(1-ie^i \arcsin(cx)) d \arcsin(cx)+b \int \log(1+ie^i \arcsin(cx)) d \arcsin(cx)-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \right)$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 2715

$$\frac{5}{2}c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{ib \int e^{-i \arcsin(cx)} \log(1-ie^i \arcsin(cx)) de^i \arcsin(cx)-ib \int e^{-i \arcsin(cx)} \log(1+ie^i \arcsin(cx)) de^i \arcsin(cx)-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \right)$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 2838

$$\frac{5}{2}c^2 \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{x(d-c^2dx^2)^{3/2}} dx}{d} - \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} \right)}{3d^2\sqrt{d-c^2dx^2}} \right)$$

$$\frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^i \arcsin(cx))(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^i \arcsin(cx))-ib \operatorname{PolyLog}(2,ie^i \arcsin(cx))}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 5208

$$\frac{5}{2}c^2 \left(\frac{-\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{d\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}}{d} - \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))}{2c} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{d^2\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 5164

$$\frac{5}{2}c^2 \left(\frac{-\frac{2b\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} + \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}}{d} - \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})}{2c} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{d^2\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 3042

$$\frac{5}{2}c^2 \left(\frac{\frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}} dx}{d} - \frac{2b\sqrt{1-c^2x^2} \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} + \frac{(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}}{d} - \frac{2bc\sqrt{1-c^2x^2} \left(\frac{-2i \arctan(e^{i \arcsin(cx)})}{2c} + \frac{ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)}}{d^2\sqrt{d-c^2dx^2}} \right)$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2 (d-c^2dx^2)^{3/2}}$$

↓ 4669

$$\frac{5}{2}c^2 \left(\frac{-2b\sqrt{1-c^2x^2}(-b \int \log(1-ie^{i \arcsin(cx)})d \arcsin(cx)+b \int \log(1+ie^{i \arcsin(cx)})d \arcsin(cx)-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)))}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 2715

$$\frac{5}{2}c^2 \left(\frac{-2b\sqrt{1-c^2x^2}(ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)})de^{i \arcsin(cx)}-ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)})de^{i \arcsin(cx)}-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx)))}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 2838

$$\frac{5}{2}c^2 \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d-c^2dx^2}}dx - 2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 5218

$$\frac{5}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} \int \frac{(a+b \arcsin(cx))^2}{cx}d \arcsin(cx) - 2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right) + \frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} \right)}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 3042

$$\frac{\frac{5}{2}c^2 \left(\frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)}))}{d\sqrt{d-c^2dx^2}} \right)}{d} + \frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

↓ 4671

$$\frac{\frac{5}{2}c^2 \left(\frac{\sqrt{1-c^2x^2}(-2b \int (a+b \arcsin(cx)) \log(1-e^{i \arcsin(cx)}) d \arcsin(cx)+2b \int (a+b \arcsin(cx)) \log(1+e^{i \arcsin(cx)}) d \arcsin(cx)-2\operatorname{arctanh}(e^{i \arcsin(cx)})}{d\sqrt{d-c^2dx^2}} \right)}{d} + \frac{bc\sqrt{1-c^2x^2} \left(3c^2 \left(\frac{-2i \arctan(e^{i \arcsin(cx)})(a+b \arcsin(cx))+ib \operatorname{PolyLog}(2,-ie^{i \arcsin(cx)})-ib \operatorname{PolyLog}(2,ie^{i \arcsin(cx)})}{2c} \right) + \frac{x(a+b \arcsin(cx))}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b \arcsin(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output

```
$Aborted
```

Maple [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 1121, normalized size of antiderivative = 1.49

method	result	size
default	Expression too large to display	1121
parts	Expression too large to display	1121

input

```
int((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```

-1/2*a^2/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a^2*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2
*a^2*c^2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a^2*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-
c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(15*arcsin(c*x)^2*
x^4*c^4-4*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*x^3*c^3+2*c^4*x^4-20*arcsin(c*x)^
2*x^2*c^2+6*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x-2*c^2*x^2+3*arcsin(c*x)^2)/
d^3/(c^4*x^4-2*c^2*x^2+1)/x^2+1/6*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2
)/d^3/(c^2*x^2-1)*(15*arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-15*arcs
in(c*x)^2*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-30*I*arcsin(c*x)*polylog(2,-I*c*x
-(-c^2*x^2+1)^(1/2))+30*I*arcsin(c*x)*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-
26*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+26*arcsin(c*x)*ln(1-I*(I
*c*x+(-c^2*x^2+1)^(1/2)))+26*I*dilog(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-26*I*
dilog(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+6*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+30*
polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))-30*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2)
)-6*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)*c^2)-1/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)*(-30*I*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*arcsin(c*x)*x^4*c
^4+15*dilog(I*c*x+(-c^2*x^2+1)^(1/2))*c^6*x^6+15*dilog(1+I*c*x+(-c^2*x^2+1
)^(1/2))*c^6*x^6+26*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^6*x^6-5*I*x^3*c^3+2
*I*x^5*c^5+15*I*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*arcsin(c*x)*x^2*c^2-30*dilo
g(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4-30*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))*c
^4*x^4-52*arctan(I*c*x+(-c^2*x^2+1)^(1/2))*c^4*x^4+3*I*(-c^2*x^2+1)^(1/2)...

```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x^3} dx$$

input

```

integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")

```

output

```

integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^
2)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)

```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^3 (-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))**2/x**3/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x^3} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `-1/6*a^2*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2) - sqrt(d)*integrate((b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2 + 2*a*b*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \sin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^3 (d - c^2 dx^2)^{5/2}} dx = \frac{48\sqrt{-c^2x^2 + 1} \left(\int \frac{\arcsin(cx)}{\sqrt{-c^2x^2 + 1} c^4 x^7 - 2\sqrt{-c^2x^2 + 1} c^2 x^5 + \sqrt{-c^2x^2 + 1} x^3} dx \right) ab c^2 x^4 - 48\sqrt{-c^2x^2 + 1}}{(-c^2 dx^2 + d)^{5/2}}$$

input

```
int((a+b*asin(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(48*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 + 1)*c**4*x**7
- 2*sqrt(-c**2*x**2 + 1)*c**2*x**5 + sqrt(-c**2*x**2 + 1)*x**3),x)*a*
b*c**2*x**4 - 48*sqrt(-c**2*x**2 + 1)*int(asin(c*x)/(sqrt(-c**2*x**2 +
1)*c**4*x**7 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**5 + sqrt(-c**2*x**2 + 1)
*x**3),x)*a*b*x**2 + 24*sqrt(-c**2*x**2 + 1)*int(asin(c*x)**2/(sqrt(-
c**2*x**2 + 1)*c**4*x**7 - 2*sqrt(-c**2*x**2 + 1)*c**2*x**5 + sqrt(-c
**2*x**2 + 1)*x**3),x)*b**2*c**2*x**4 - 24*sqrt(-c**2*x**2 + 1)*int(asin(
c*x)**2/(sqrt(-c**2*x**2 + 1)*c**4*x**7 - 2*sqrt(-c**2*x**2 + 1)*c**2*
x**5 + sqrt(-c**2*x**2 + 1)*x**3),x)*b**2*x**2 + 60*sqrt(-c**2*x**2 +
1)*log(tan(asin(c*x)/2))*a**2*c**4*x**4 - 60*sqrt(-c**2*x**2 + 1)*log(ta
n(asin(c*x)/2))*a**2*c**2*x**2 - 65*sqrt(-c**2*x**2 + 1)*a**2*c**4*x**4
+ 65*sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 + 60*a**2*c**4*x**4 - 80*a**2*c
**2*x**2 + 12*a**2)/(24*sqrt(d)*sqrt(-c**2*x**2 + 1)*d**2*x**2*(c**2*x**
2 - 1))
```

3.260 $\int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^{5/2}} dx$

Optimal result	2577
Mathematica [A] (verified)	2578
Rubi [A] (verified)	2579
Maple [B] (verified)	2589
Fricas [F]	2589
Sympy [F]	2590
Maxima [F]	2590
Giac [F(-2)]	2591
Mupad [F(-1)]	2591
Reduce [F]	2591

Optimal result

Integrand size = 29, antiderivative size = 538

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x^4(d-c^2dx^2)^{5/2}} dx = & -\frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} + \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} \\ & - \frac{bc(a+b \arcsin(cx))}{3d^2x^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \\ & - \frac{2c^2(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}} + \frac{8c^4x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\ & + \frac{16c^4x(a+b \arcsin(cx))^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{16ic^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{3d^2\sqrt{d-c^2dx^2}} \\ & - \frac{32bc^3\sqrt{1-c^2x^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{3d^2\sqrt{d-c^2dx^2}} \\ & + \frac{32bc^3\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{3d^2\sqrt{d-c^2dx^2}} \\ & - \frac{8ib^2c^3\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{3d^2\sqrt{d-c^2dx^2}} \\ & - \frac{8ib^2c^3\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{3d^2\sqrt{d-c^2dx^2}} \end{aligned}$$

output

```

-1/3*b^2*c^2/d^2/x/(-c^2*d*x^2+d)^(1/2)+2/3*b^2*c^4*x/d^2/(-c^2*d*x^2+d)^(
1/2)-1/3*b*c*(a+b*arcsin(c*x))/d^2/x^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(
1/2)-1/3*(a+b*arcsin(c*x))^2/d/x^3/(-c^2*d*x^2+d)^(3/2)-2*c^2*(a+b*arcsin(
c*x))^2/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arcsin(c*x))^2/d/(-c^2*d*x
^2+d)^(3/2)+16/3*c^4*x*(a+b*arcsin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*I
*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-32/3*
b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/
2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)+32/3*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(
c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*I*b
^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2/(-c
^2*d*x^2+d)^(1/2)-8/3*I*b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x
^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)

```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.82

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \frac{-\frac{a^2(1+6c^2x^2-24c^4x^4+16c^6x^6)}{x^3} - \frac{ab(2(1+6c^2x^2-24c^4x^4+16c^6x^6) \arcsin(cx) + cx\sqrt{1-c^2x^2}(1+16c^2x^2))}{x^3}}{x^3}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output

```

(-((a^2*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6))/x^3) - (a*b*(2*(1 + 6*c
^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6)*ArcSin[c*x] + c*x*Sqrt[1 - c^2*x^2]*(1 +
16*c^2*x^2*(-1 + c^2*x^2)*Log[c*x] + 8*c^2*x^2*(-1 + c^2*x^2)*Log[1 - c^2
*x^2]))) / x^3 + b^2*c^3*(1 - c^2*x^2)^(3/2)*((c*x)/Sqrt[1 - c^2*x^2] - Sqrt
[1 - c^2*x^2]/(c*x) - ArcSin[c*x]/(c^2*x^2) + ArcSin[c*x]/(-1 + c^2*x^2) -
(16*I)*ArcSin[c*x]^2 + (c*x*ArcSin[c*x]^2)/(1 - c^2*x^2)^(3/2) + (8*c*x*A
rcSin[c*x]^2)/Sqrt[1 - c^2*x^2] - (Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(c^3*x
^3) - (8*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(c*x) + 16*ArcSin[c*x]*Log[1 - E
^((2*I)*ArcSin[c*x])] + 16*ArcSin[c*x]*Log[1 + E^((2*I)*ArcSin[c*x])] - (8
*I)*PolyLog[2, -E^((2*I)*ArcSin[c*x])] - (8*I)*PolyLog[2, E^((2*I)*ArcSin[
c*x])])]) / (3*d*(d - c^2*d*x^2)^(3/2))

```

Rubi [A] (verified)

Time = 5.05 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.34, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5204, 5204, 245, 208, 5162, 5160, 5180, 3042, 4202, 2620, 2715, 2838, 5182, 208, 5208, 208, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx$$

$$\downarrow 5204$$

$$\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x^3(1 - c^2 x^2)^2} dx}{3d^2\sqrt{d - c^2 dx^2}} + 2c^2 \int \frac{(a + b \arcsin(cx))^2}{x^2 (d - c^2 dx^2)^{5/2}} dx - \frac{(a + b \arcsin(cx))^2}{3dx^3 (d - c^2 dx^2)^{3/2}}$$

$$\downarrow 5204$$

$$2c^2 \left(\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2\sqrt{d - c^2 dx^2}} + 4c^2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx - \frac{(a + b \arcsin(cx))^2}{dx (d - c^2 dx^2)^{3/2}} \right) +$$

$$\frac{2bc\sqrt{1 - c^2 x^2} \left(2c^2 \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx + \frac{1}{2} bc \int \frac{1}{x^2(1 - c^2 x^2)^{3/2}} dx - \frac{a + b \arcsin(cx)}{2x^2(1 - c^2 x^2)} \right)}{3d^2\sqrt{d - c^2 dx^2} \frac{(a + b \arcsin(cx))^2}{3dx^3 (d - c^2 dx^2)^{3/2}}}$$

$$\downarrow 245$$

$$2c^2 \left(\frac{2bc\sqrt{1 - c^2 x^2} \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx}{d^2\sqrt{d - c^2 dx^2}} + 4c^2 \int \frac{(a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx - \frac{(a + b \arcsin(cx))^2}{dx (d - c^2 dx^2)^{3/2}} \right) +$$

$$\frac{2bc\sqrt{1 - c^2 x^2} \left(2c^2 \int \frac{a + b \arcsin(cx)}{x(1 - c^2 x^2)^2} dx + \frac{1}{2} bc \left(2c^2 \int \frac{1}{(1 - c^2 x^2)^{3/2}} dx - \frac{1}{x\sqrt{1 - c^2 x^2}} \right) - \frac{a + b \arcsin(cx)}{2x^2(1 - c^2 x^2)} \right)}{3d^2\sqrt{d - c^2 dx^2} \frac{(a + b \arcsin(cx))^2}{3dx^3 (d - c^2 dx^2)^{3/2}}}$$

$$\downarrow 208$$

$$2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{5/2}} dx - \frac{(a+b \arcsin(cx))^2}{dx(d-c^2dx^2)^{3/2}} \right) + \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2} \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}}$$

↓ 5162

$$2c^2 \left(4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \int \frac{(a+b \arcsin(cx))^2}{(d-c^2dx^2)^{3/2}} dx}{3d} + \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) + \frac{2bc\sqrt{1-c^2x^2}}{d^2\sqrt{d-c^2dx^2}} \right) - \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2} \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}}$$

↓ 5160

$$2c^2 \left(4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{d\sqrt{d-c^2dx^2}} \right)}{3d} + \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) \right) - \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2} \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}}$$

↓ 5180

$$2c^2 \left(4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} + \frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} \right) \right) - \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2} \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}}$$

$$\begin{aligned}
 & \downarrow 3042 \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} + \\
 2c^2 & \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right) \right. \\
 & \left. \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 4202 \\
 2c^2 & \left(4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} \right)}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right) \right. \\
 & \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 2620 \\
 & \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b \arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} + \\
 2c^2 & \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \int \frac{x(a+b \arcsin(cx))}{(1-c^2x^2)^2} dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}}{cd\sqrt{d-c^2dx^2}} \right)}{3d} \right) \right. \\
 & \left. \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right)
 \end{aligned}$$

$$\downarrow 2715$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arcsin(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)}+\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}} +$$

$$2c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arcsin(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arcsin(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}}{3d}\right)}{(a+b\arcsin(cx))^2}\right)\right)$$

$$\frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}$$

↓ 2838

$$2c^2\left(4c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arcsin(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})\right)\right)}{3d}\right)}{(a+b\arcsin(cx))^2}\right)\right)$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arcsin(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)}+\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}$$

↓ 5182

$$2c^2\left(4c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)}-\frac{b\int\frac{1}{(1-c^2x^2)^{3/2}}dx}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{i(a+b\arcsin(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log(1+e^{2i\arcsin(cx)})\right)\right)}{3d}\right)}{(a+b\arcsin(cx))^2}\right)\right)$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arcsin(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)}+\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}$$

↓ 208

$$\begin{aligned}
 & 2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)^2} dx}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b\arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \right. \\
 & \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)^2} dx - \frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} - \right. \\
 & \left. \frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right)
 \end{aligned}$$

↓ 5208

$$\begin{aligned}
 & 2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \left(\int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)} dx - \frac{1}{2}bc \int \frac{1}{(1-c^2x^2)^{3/2}} dx + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} \right)}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \right. \\
 & \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(\int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)} dx - \frac{1}{2}bc \int \frac{1}{(1-c^2x^2)^{3/2}} dx + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} \right) - \frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} - \right. \\
 & \left. \frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right)
 \end{aligned}$$

↓ 208

$$\begin{aligned}
 & 2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \left(\int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)} dx + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} \right) \right. \\
 & \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(\int \frac{a+b\arcsin(cx)}{x(1-c^2x^2)} dx + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right) - \frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} - \right. \\
 & \left. \frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right)
 \end{aligned}$$

↓ 5184

$$\begin{aligned}
& 2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \left(\int \frac{a+b\arcsin(cx)}{cx\sqrt{1-c^2x^2}} d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \right. \\
& \left. \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(\int \frac{a+b\arcsin(cx)}{cx\sqrt{1-c^2x^2}} d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right) - \frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)} + \frac{1}{2}bc \left(\frac{2c^2x}{\sqrt{1-c^2x^2}} - \frac{1}{x\sqrt{1-c^2x^2}} \right) \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \right. \\
& \left. \left. \frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right) \right.
\end{aligned}$$

↓ 4919

$$\begin{aligned}
& 2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \left(2 \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \right. \\
& \left. \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(2 \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right) - \frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \right. \\
& \left. \left. \frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right) \right.
\end{aligned}$$

↓ 3042

$$\begin{aligned}
& 2c^2 \left(\frac{2bc\sqrt{1-c^2x^2} \left(2 \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right)}{d^2\sqrt{d-c^2dx^2}} + 4c^2 \left(-\frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b\arcsin(cx)}{2c^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \right. \\
& \left. \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(2 \int (a+b\arcsin(cx)) \csc(2\arcsin(cx)) d\arcsin(cx) + \frac{a+b\arcsin(cx)}{2(1-c^2x^2)} - \frac{bcx}{2\sqrt{1-c^2x^2}} \right) - \frac{a+b\arcsin(cx)}{2x^2(1-c^2x^2)} \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \right. \\
& \left. \left. \frac{(a+b\arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right) \right.
\end{aligned}$$

↓ 4671

$$2c^2 \frac{2bc\sqrt{1-c^2x^2} \left(2\left(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - (\arctan\right)}{d^2\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(2\left(-\frac{1}{2}b \int \log(1-e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2}b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - (\arctan\right)}{3d^2\sqrt{d-c^2dx^2}} \right) \right.}{\frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}}$$

↓ 2715

$$2 \left(4 \left(\frac{x(a+b \arcsin(cx))^2}{3d(d-c^2dx^2)^{3/2}} - \frac{2bc\sqrt{1-c^2x^2} \left(\frac{a+b \arcsin(cx)}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{2 \left(\frac{x(a+b \arcsin(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2} \left(\frac{i(a+b \arcsin(cx))}{2c^2(1-c^2x^2)} - \frac{bx}{2c\sqrt{1-c^2x^2}} \right)}{3d^2\sqrt{d-c^2dx^2}} \right)}{2b\sqrt{1-c^2x^2} \left(2 \left(-\frac{bcx}{2\sqrt{1-c^2x^2}} + \frac{a+b \arcsin(cx)}{2(1-c^2x^2)} + 2 \left(-((a+b \arcsin(cx)) \operatorname{arctanh}(e^{2i \arcsin(cx)}) \right) + \frac{1}{4}ib \int e^{-2i \arcsin(cx)} dx \right)} \right. \right. \\ \left. \left. \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right) \right.$$

↓ 2838

$$\frac{2bc\sqrt{1-c^2x^2} \left(2c^2 \left(2 \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a+b \arcsin(cx))) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right)}{3d^2\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{2bc\sqrt{1-c^2x^2} \left(2 \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)}) (a+b \arcsin(cx))) + \frac{1}{4}ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4}ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right)}{d^2\sqrt{d-c^2dx^2}} \right. \\ \left. \frac{(a+b \arcsin(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \right.$$

input Int[(a + b*ArcSin[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]

output

```

-1/3*(a + b*ArcSin[c*x])^2/(d*x^3*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 -
c^2*x^2]*((b*c*(-1/(x*Sqrt[1 - c^2*x^2])) + (2*c^2*x)/Sqrt[1 - c^2*x^2])
)/2 - (a + b*ArcSin[c*x])/(2*x^2*(1 - c^2*x^2)) + 2*c^2*(-1/2*(b*c*x)/Sqrt
[1 - c^2*x^2] + (a + b*ArcSin[c*x])/(2*(1 - c^2*x^2)) + 2*(-((a + b*ArcSin
[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcS
in[c*x])]) - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])))/(3*d^2*Sqrt[d -
c^2*d*x^2]) + 2*c^2*(-((a + b*ArcSin[c*x])^2/(d*x*(d - c^2*d*x^2)^(3/2)))
+ 4*c^2*((x*(a + b*ArcSin[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) - (2*b*c*Sq
rt[1 - c^2*x^2]*(-1/2*(b*x)/(c*Sqrt[1 - c^2*x^2]) + (a + b*ArcSin[c*x])/(2
*c^2*(1 - c^2*x^2)))))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*(a + b*ArcSin[c
*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((I/2)*(a + b*Ar
cSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*Arc
Sin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)))/(c*d*Sqrt[d - c^2
*d*x^2])))/(3*d) + (2*b*c*Sqrt[1 - c^2*x^2]*(-1/2*(b*c*x)/Sqrt[1 - c^2*x^
2] + (a + b*ArcSin[c*x])/(2*(1 - c^2*x^2)) + 2*(-((a + b*ArcSin[c*x])*ArcT
anh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcSin[c*x])]) -
(I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])))/(d^2*Sqrt[d - c^2*d*x^2])

```

Defintions of rubi rules used

rule 208

```

Int[((a_) + (b_)*(x_)^2)^(-3/2), x_Symbol] := Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]

```

rule 245

```

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[x^(m + 1)*((a +
b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]

```

rule 2620

```

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4202 `Int[((c_) + (d_)*(x_)^(m_))*tan[(e_) + (f_)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(
e + f*x)))/(1 + E^(2*I*(e + f*x)))], x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_)^(m_)), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)
^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]`

rule 4919 `Int[Csc[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_)^(m_))*Sec[(a_) + (b
)*(x)]^(n_), x_Symbol] :> Simp[2^n Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]`

rule 5160 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/((d_) + (e_)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*(a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2]), x] - Simp[b
c(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*(a + b*ArcSin[c*x
])^(n - 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]`

rule 5162

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1)) Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cSin[c*x])^n, x], x] + Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p] Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
-1] && NeQ[p, -3/2]
```

rule 5180

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5184

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSi
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5208

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
  Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])

```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5224 vs. $2(517) = 1034$.

Time = 0.94 (sec) , antiderivative size = 5225, normalized size of antiderivative = 9.71

method	result	size
default	Expression too large to display	5225
parts	Expression too large to display	5225

input

```
int((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
result too large to display
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x^4} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^
2)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^4 (-d(cx - 1)(cx + 1))^{5/2}} dx$$

input `integrate((a+b*asin(c*x))**2/x**4/(-c**2*d*x**2+d)**(5/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(-c^2 dx^2 + d)^{5/2} x^4} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `1/3*a*b*c*(8*c^2*log(c*x + 1)/d^(5/2) + 8*c^2*log(c*x - 1)/d^(5/2) + 16*c^2*log(x)/d^(5/2) + 1/(c^2*d^(5/2)*x^4 - d^(5/2)*x^2)) + 2/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*arcsin(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*a^2 + b^2*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))^2/((c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx$$

input `int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)),x)`

output `int((a + b*asin(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^4 (d - c^2 dx^2)^{5/2}} dx = \frac{6\sqrt{-c^2x^2 + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{-c^2x^2 + 1} c^4 x^8 - 2\sqrt{-c^2x^2 + 1} c^2 x^6 + \sqrt{-c^2x^2 + 1} x^4} dx \right) ab c^2 x^5 - 6\sqrt{-c^2x^2 + 1}}$$

input `int((a+b*asin(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x)`

output

```
(6*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)*c**4*x**8
-2*sqrt(-c**2*x**2+1)*c**2*x**6+sqrt(-c**2*x**2+1)*x**4),x)*a*b
*c**2*x**5-6*sqrt(-c**2*x**2+1)*int(asin(c*x)/(sqrt(-c**2*x**2+1)
)*c**4*x**8-2*sqrt(-c**2*x**2+1)*c**2*x**6+sqrt(-c**2*x**2+1)*
x**4),x)*a*b*x**3+3*sqrt(-c**2*x**2+1)*int(asin(c*x)**2/(sqrt(-c**
2*x**2+1)*c**4*x**8-2*sqrt(-c**2*x**2+1)*c**2*x**6+sqrt(-c**2*x
**2+1)*x**4),x)*b**2*c**2*x**5-3*sqrt(-c**2*x**2+1)*int(asin(c*x)
**2/(sqrt(-c**2*x**2+1)*c**4*x**8-2*sqrt(-c**2*x**2+1)*c**2*x**6
+sqrt(-c**2*x**2+1)*x**4),x)*b**2*x**3+16*a**2*c**6*x**6-24*a**2
*c**4*x**4+6*a**2*c**2*x**2+a**2)/(3*sqrt(d)*sqrt(-c**2*x**2+1)*d*
*2*x**3*(c**2*x**2-1))
```

3.261 $\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal result	2593
Mathematica [A] (verified)	2594
Rubi [A] (verified)	2594
Maple [A] (verified)	2598
Fricas [A] (verification not implemented)	2598
Sympy [A] (verification not implemented)	2599
Maxima [F]	2599
Giac [A] (verification not implemented)	2600
Mupad [F(-1)]	2600
Reduce [F]	2601

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{15x\sqrt{1-a^2x^2}}{64a^4} + \frac{x^3\sqrt{1-a^2x^2}}{32a^2} - \frac{15 \arcsin(ax)}{64a^5} + \frac{3x^2 \arcsin(ax)}{8a^3} + \frac{x^4 \arcsin(ax)}{8a} - \frac{3x\sqrt{1-a^2x^2} \arcsin(ax)^2}{8a^4} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \frac{\arcsin(ax)^3}{8a^5}$$

output

```
15/64*x*(-a^2*x^2+1)^(1/2)/a^4+1/32*x^3*(-a^2*x^2+1)^(1/2)/a^2-15/64*arcsi
n(a*x)/a^5+3/8*x^2*arcsin(a*x)/a^3+1/8*x^4*arcsin(a*x)/a-3/8*x*(-a^2*x^2+1
)^(1/2)*arcsin(a*x)^2/a^4-1/4*x^3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2/a^2+1/8
*arcsin(a*x)^3/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{ax\sqrt{1-a^2x^2}(15+2a^2x^2) + (-15+24a^2x^2+8a^4x^4)\arcsin(ax) - 8ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arcsin(ax)}{64a^5}$$

input

```
Integrate[(x^4*ArcSin[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output

```
(a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2) + (-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcSin[a*x] - 8*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcSin[a*x]^2 + 8*ArcSin[a*x]^3)/(64*a^5)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.43, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5210, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

$$\downarrow \text{5210}$$

$$\frac{3 \int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{\int x^3 \arcsin(ax) dx}{2a} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2}$$

$$\downarrow \text{5138}$$

$$\frac{3 \int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \int \frac{x^4}{\sqrt{1-a^2x^2}} dx}{2a} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2}$$

$$\downarrow \text{262}$$

$$\begin{aligned}
& \frac{3 \int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \int \frac{x^2}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
& \qquad \qquad \qquad \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} \\
& \qquad \qquad \qquad \downarrow \text{262} \\
& \frac{3 \int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x \sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
& \qquad \qquad \qquad \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} \\
& \qquad \qquad \qquad \downarrow \text{223} \\
& \frac{3 \int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \\
& \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x \sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
& \qquad \qquad \qquad \downarrow \text{5210} \\
& \frac{3 \left(\frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x \arcsin(ax) dx}{a} - \frac{x \sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \\
& \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x \sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
& \qquad \qquad \qquad \downarrow \text{5138} \\
& \frac{3 \left(\frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \int \frac{x^2}{\sqrt{1-a^2x^2}} dx}{a} + \frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x \sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} \right)}{4a^2} - \\
& \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x \sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3 \sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
& \qquad \qquad \qquad \downarrow \text{262}
\end{aligned}$$

$$\begin{aligned}
 & \frac{3 \left(\frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{a} + \frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} \right)}{4a^2} \\
 & + \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
 & \quad \downarrow \text{223} \\
 & \frac{3 \left(\frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} + \frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{a} \right)}{4a^2} \\
 & + \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2} \right)}{2a} \\
 & \quad \downarrow \text{5152} \\
 & - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^2}{4a^2} + \\
 & \frac{3 \left(\frac{\arcsin(ax)^3}{6a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} + \frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{a} \right)}{4a^2} + \\
 & \frac{\frac{1}{4}x^4 \arcsin(ax) - \frac{1}{4}a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2} \right)}{2a}
 \end{aligned}$$

input

`Int[(x^4*ArcSin[a*x]^2)/Sqrt[1 - a^2*x^2],x]`

output

`-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/a^2 + ((x^4*ArcSin[a*x])/4 - (a*(-1/4*(x^3*Sqrt[1 - a^2*x^2]))/a^2 + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]))/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2))/4)/(2*a) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2])*ArcSin[a*x]^2)/a^2 + ArcSin[a*x]^3/(6*a^3) + ((x^2*ArcSin[a*x])/2 - (a*(-1/2*(x*Sqrt[1 - a^2*x^2]))/a^2 + ArcSin[a*x]/(2*a^3)))/2)/a)/(4*a^2)`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /;$ $\text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5210 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\text{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \ \text{Int}[(f*x)^{(m-2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /;$ $\text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m+2*p+1, 0]$

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

method	result
default	$\frac{-16 \arcsin(ax)^2 \sqrt{-a^2x^2+1} a^3x^3 + 8 \arcsin(ax) a^4x^4 + 2\sqrt{-a^2x^2+1} a^3x^3 - 24\sqrt{-a^2x^2+1} \arcsin(ax)^2 ax + 24 \arcsin(ax) a^2x^2 + 8 \arcsin(ax)}{64a^5}$

input `int(x^4*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `1/64*(-16*arcsin(a*x)^2*(-a^2*x^2+1)^(1/2)*a^3*x^3+8*arcsin(a*x)*a^4*x^4+2*(-a^2*x^2+1)^(1/2)*a^3*x^3-24*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2*a*x+24*arcsin(a*x)*a^2*x^2+8*arcsin(a*x)^3+15*(-a^2*x^2+1)^(1/2)*x*a-15*arcsin(a*x))/a^5`

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{8 \arcsin(ax)^3 + (8a^4x^4 + 24a^2x^2 - 15) \arcsin(ax) + (2a^3x^3 - 8(2a^3x^3 + 3ax) \arcsin(ax)^2 + 15ax) \sqrt{1-a^2x^2}}{64a^5}$$

input `integrate(x^4*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `1/64*(8*arcsin(a*x)^3 + (8*a^4*x^4 + 24*a^2*x^2 - 15)*arcsin(a*x) + (2*a^3*x^3 - 8*(2*a^3*x^3 + 3*a*x)*arcsin(a*x)^2 + 15*a*x)*sqrt(-a^2*x^2 + 1))/a^5`

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

$$= \begin{cases} \frac{x^4 \arcsin(ax)}{8a} - \frac{x^3 \sqrt{-a^2x^2+1} \arcsin^2(ax)}{4a^2} + \frac{x^3 \sqrt{-a^2x^2+1}}{32a^2} + \frac{3x^2 \arcsin(ax)}{8a^3} - \frac{3x \sqrt{-a^2x^2+1} \arcsin^2(ax)}{8a^4} + \frac{15x \sqrt{-a^2x^2+1}}{64a^4} + \frac{\arcsin^3(ax)}{8a^5} \\ 0 \end{cases}$$

input `integrate(x**4*asin(a*x)**2/(-a**2*x**2+1)**(1/2),x)`output `Piecewise((x**4*asin(a*x)/(8*a) - x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(4*a**2) + x**3*sqrt(-a**2*x**2 + 1)/(32*a**2) + 3*x**2*asin(a*x)/(8*a**3) - 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(8*a**4) + 15*x*sqrt(-a**2*x**2 + 1)/(64*a**4) + asin(a*x)**3/(8*a**5) - 15*asin(a*x)/(64*a**5), Ne(a, 0)), (0, True))`**Maxima [F]**

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{x^4 \arcsin(ax)^2}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^4*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `integrate(x^4*arcsin(a*x)^2/sqrt(-a^2*x^2 + 1), x)`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.91

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{(-a^2x^2+1)^{\frac{3}{2}}x \arcsin(ax)^2}{4a^4} - \frac{5\sqrt{-a^2x^2+1}x \arcsin(ax)^2}{8a^4} - \frac{(-a^2x^2+1)^{\frac{3}{2}}x}{32a^4} + \frac{(a^2x^2-1)^2 \arcsin(ax)}{8a^5} + \frac{\arcsin(ax)^3}{8a^5} + \frac{17\sqrt{-a^2x^2+1}x}{64a^4} + \frac{5(a^2x^2-1) \arcsin(ax)}{8a^5} + \frac{17 \arcsin(ax)}{64a^5}$$

input `integrate(x^4*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `1/4*(-a^2*x^2 + 1)^(3/2)*x*arcsin(a*x)^2/a^4 - 5/8*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^2/a^4 - 1/32*(-a^2*x^2 + 1)^(3/2)*x/a^4 + 1/8*(a^2*x^2 - 1)^2*arcsin(a*x)/a^5 + 1/8*arcsin(a*x)^3/a^5 + 17/64*sqrt(-a^2*x^2 + 1)*x/a^4 + 5/8*(a^2*x^2 - 1)*arcsin(a*x)/a^5 + 17/64*arcsin(a*x)/a^5`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{x^4 \operatorname{asin}(ax)^2}{\sqrt{1-a^2x^2}} dx$$

input `int((x^4*asin(a*x)^2)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^4*asin(a*x)^2)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^4 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{a \sin(ax)^2 x^4}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^4*asin(a*x)^2/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**2*x**4)/sqrt(-a**2*x**2+1),x)`

3.262 $\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal result	2602
Mathematica [A] (verified)	2602
Rubi [A] (verified)	2603
Maple [A] (verified)	2606
Fricas [A] (verification not implemented)	2606
Sympy [A] (verification not implemented)	2607
Maxima [A] (verification not implemented)	2607
Giac [F(-2)]	2608
Mupad [F(-1)]	2608
Reduce [F]	2608

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{14\sqrt{1-a^2x^2}}{9a^4} - \frac{2(1-a^2x^2)^{3/2}}{27a^4} + \frac{4x \arcsin(ax)}{3a^3} + \frac{2x^3 \arcsin(ax)}{9a} - \frac{2\sqrt{1-a^2x^2} \arcsin(ax)^2}{3a^4} - \frac{x^2\sqrt{1-a^2x^2} \arcsin(ax)^2}{3a^2}$$

output

```
14/9*(-a^2*x^2+1)^(1/2)/a^4-2/27*(-a^2*x^2+1)^(3/2)/a^4+4/3*x*arcsin(a*x)/a^3+2/9*x^3*arcsin(a*x)/a-2/3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{2\sqrt{1-a^2x^2}(20+a^2x^2) + 6ax(6+a^2x^2) \arcsin(ax) - 9\sqrt{1-a^2x^2}(2+a^2x^2) \arcsin(ax)^2}{27a^4}$$

input

```
Integrate[(x^3*ArcSin[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output

$$(2\sqrt{1 - a^2x^2}*(20 + a^2x^2) + 6ax*(6 + a^2x^2)*\text{ArcSin}[ax] - 9*\sqrt{1 - a^2x^2}*(2 + a^2x^2)*\text{ArcSin}[ax]^2)/(27a^4)$$

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5210, 5138, 243, 53, 2009, 5182, 5130, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^3 \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx \\ & \quad \downarrow \text{5210} \\ & \frac{2 \int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx}{3a^2} + \frac{2 \int x^2 \arcsin(ax) dx}{3a} - \frac{x^2 \sqrt{1 - a^2x^2} \arcsin(ax)^2}{3a^2} \\ & \quad \downarrow \text{5138} \\ & \frac{2 \int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx}{3a^2} + \frac{2 \left(\frac{1}{3} x^3 \arcsin(ax) - \frac{1}{3} a \int \frac{x^3}{\sqrt{1 - a^2x^2}} dx \right)}{3a} - \frac{x^2 \sqrt{1 - a^2x^2} \arcsin(ax)^2}{3a^2} \\ & \quad \downarrow \text{243} \\ & \frac{2 \int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx}{3a^2} + \frac{2 \left(\frac{1}{3} x^3 \arcsin(ax) - \frac{1}{6} a \int \frac{x^2}{\sqrt{1 - a^2x^2}} dx^2 \right)}{3a} - \frac{x^2 \sqrt{1 - a^2x^2} \arcsin(ax)^2}{3a^2} \\ & \quad \downarrow \text{53} \\ & \frac{2 \int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx}{3a^2} + \frac{2 \left(\frac{1}{3} x^3 \arcsin(ax) - \frac{1}{6} a \int \left(\frac{1}{a^2 \sqrt{1 - a^2x^2}} - \frac{\sqrt{1 - a^2x^2}}{a^2} \right) dx^2 \right)}{3a} - \frac{x^2 \sqrt{1 - a^2x^2} \arcsin(ax)^2}{3a^2} \\ & \quad \downarrow \text{2009} \\ & \frac{2 \int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx}{3a^2} - \frac{x^2 \sqrt{1 - a^2x^2} \arcsin(ax)^2}{3a^2} + \\ & \frac{2 \left(\frac{1}{3} x^3 \arcsin(ax) - \frac{1}{6} a \left(\frac{2(1 - a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1 - a^2x^2}}{a^4} \right) \right)}{3a} \end{aligned}$$

$$\begin{aligned}
& \downarrow 5182 \\
& \frac{2\left(\frac{2\int \arcsin(ax)dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}\right)}{3a^2} - \frac{x^2\sqrt{1-a^2x^2} \arcsin(ax)^2}{3a^2} + \\
& \frac{2\left(\frac{1}{3}x^3 \arcsin(ax) - \frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right)\right)}{3a} \\
& \downarrow 5130 \\
& \frac{2\left(\frac{2\left(x \arcsin(ax) - a \int \frac{x}{\sqrt{1-a^2x^2}} dx\right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}\right)}{3a^2} - \frac{x^2\sqrt{1-a^2x^2} \arcsin(ax)^2}{3a^2} + \\
& \frac{2\left(\frac{1}{3}x^3 \arcsin(ax) - \frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right)\right)}{3a} \\
& \downarrow 241 \\
& -\frac{x^2\sqrt{1-a^2x^2} \arcsin(ax)^2}{3a^2} + \frac{2\left(\frac{2\left(\frac{\sqrt{1-a^2x^2}}{a} + x \arcsin(ax)\right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}\right)}{3a^2} + \\
& \frac{2\left(\frac{1}{3}x^3 \arcsin(ax) - \frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4} - \frac{2\sqrt{1-a^2x^2}}{a^4}\right)\right)}{3a}
\end{aligned}$$

input `Int[(x^3*ArcSin[a*x]^2)/Sqrt[1 - a^2*x^2], x]`

output `-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/a^2 + (2*(-1/6*(a*((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4))) + (x^3*ArcSin[a*x])/3)/(3*a) + (2*(-((Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/a^2) + (2*(Sqrt[1 - a^2*x^2])/a + x*ArcSin[a*x])/a))/(3*a^2)`

Definitions of rubi rules used

- rule 53 $\text{Int}[(a_.) + (b_.)(x_)^{(m_.)}((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$
- rule 241 $\text{Int}[(x_)*((a_) + (b_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x^2)^{(p + 1)}/(2*b*(p + 1)), x] /; \text{FreeQ}\{a, b, p\}, x] \ \&\& \ \text{NeQ}[p, -1]$
- rule 243 $\text{Int}[(x_)^{(m_.)}((a_) + (b_.)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m - 1)/2}*(a + b*x)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 5130 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Simp}[b*c*n \ \text{Int}[x*((a + b*\text{ArcSin}[c*x])^{(n - 1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$
- rule 5138 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}((d_.)(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m + 1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m + 1))), x] - \text{Simp}[b*c*(n/(d*(m + 1))) \ \text{Int}[(d*x)^{(m + 1)}*((a + b*\text{ArcSin}[c*x])^{(n - 1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5182 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}(x_)*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p + 1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(1 - c^2*x^2)^{(p + 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

method	result
default	$-\frac{(9 \arcsin(ax)^2 a^4 x^4 + 9x^2 \arcsin(ax)^2 a^2 + 6 \arcsin(ax) \sqrt{-a^2 x^2 + 1} a^3 x^3 - 2a^4 x^4 - 38a^2 x^2 - 18 \arcsin(ax)^2 + 36 \arcsin(ax) \sqrt{-a^2 x^2 + 1})}{27a^4(a^2 x^2 - 1)}$
orering	$\frac{(19a^6 x^6 + 100a^4 x^4 - 380a^2 x^2 + 240) \arcsin(ax)^2}{27a^6 x^2 \sqrt{-a^2 x^2 + 1}} - \frac{2(ax-1)(ax+1)(a^4 x^4 + 12a^2 x^2 - 20)}{9x^4 a^6} \left(\frac{3x^2 \arcsin(ax)^2}{\sqrt{-a^2 x^2 + 1}} + \frac{2x^3 \arcsin(ax)a}{-a^2 x^2 + 1} + \frac{x^4 \arcsin(ax)}{-a^2} \right)$

input

```
int(x^3*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/27/a^4*(9*arcsin(a*x)^2*a^4*x^4+9*x^2*arcsin(a*x)^2*a^2+6*arcsin(a*x)*
(-a^2*x^2+1)^(1/2)*a^3*x^3-2*a^4*x^4-38*a^2*x^2-18*arcsin(a*x)^2+36*arcsin(
a*x)*(-a^2*x^2+1)^(1/2)*a*x+40)*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1 - a^2 x^2}} dx$$

$$= \frac{6(a^3 x^3 + 6ax) \arcsin(ax) + (2a^2 x^2 - 9(a^2 x^2 + 2) \arcsin(ax)^2 + 40) \sqrt{-a^2 x^2 + 1}}{27a^4}$$

input

```
integrate(x^3*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output $\frac{1}{27}*(6*(a^3*x^3 + 6*a*x)*\arcsin(ax) + (2*a^2*x^2 - 9*(a^2*x^2 + 2)*\arcsin(ax)^2 + 40)*\sqrt{-a^2*x^2 + 1})/a^4$

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{2x^3 \arcsin(ax)}{9a} - \frac{x^2 \sqrt{-a^2x^2+1} \arcsin^2(ax)}{3a^2} + \frac{2x^2 \sqrt{-a^2x^2+1}}{27a^2} + \frac{4x \arcsin(ax)}{3a^3} - \frac{2\sqrt{-a^2x^2+1} \arcsin^2(ax)}{3a^4} + \frac{40\sqrt{-a^2x^2+1}}{27a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x**3*asin(a*x)**2/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((2*x**3*asin(a*x)/(9*a) - x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(3*a**2) + 2*x**2*sqrt(-a**2*x**2 + 1)/(27*a**2) + 4*x*asin(a*x)/(3*a**3) - 2*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(3*a**4) + 40*sqrt(-a**2*x**2 + 1)/(27*a**4), Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = -\frac{1}{3} \left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arcsin(ax)^2 + \frac{2 \left(\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2} \right)}{27a^2} + \frac{2(a^2x^3 + 6x) \arcsin(ax)}{9a^3}$$

input `integrate(x^3*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output $-1/3*(\sqrt{-a^2*x^2 + 1}*x^2/a^2 + 2*\sqrt{-a^2*x^2 + 1}/a^4)*\arcsin(a*x)^2 + 2/27*(\sqrt{-a^2*x^2 + 1}*x^2 + 20*\sqrt{-a^2*x^2 + 1}/a^2)/a^2 + 2/9*(a^2*x^3 + 6*x)*\arcsin(a*x)/a^3$

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{x^3 \operatorname{asin}(ax)^2}{\sqrt{1-a^2x^2}} dx$$

input `int((x^3*asin(a*x)^2)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^3*asin(a*x)^2)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^3 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^2 x^3}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^3*asin(a*x)^2/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**2*x**3)/sqrt(- a**2*x**2 + 1),x)`

3.263 $\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal result	2609
Mathematica [A] (verified)	2609
Rubi [A] (verified)	2610
Maple [A] (verified)	2612
Fricas [A] (verification not implemented)	2612
Sympy [A] (verification not implemented)	2612
Maxima [F]	2613
Giac [A] (verification not implemented)	2613
Mupad [F(-1)]	2614
Reduce [F]	2614

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{x\sqrt{1-a^2x^2}}{4a^2} - \frac{\arcsin(ax)}{4a^3} + \frac{x^2 \arcsin(ax)}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} + \frac{\arcsin(ax)^3}{6a^3}$$

output

```
1/4*x*(-a^2*x^2+1)^(1/2)/a^2-1/4*arcsin(a*x)/a^3+1/2*x^2*arcsin(a*x)/a-1/2
*x*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2/a^2+1/6*arcsin(a*x)^3/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{3ax\sqrt{1-a^2x^2} + (-3 + 6a^2x^2) \arcsin(ax) - 6ax\sqrt{1-a^2x^2} \arcsin(ax)^2 + 2 \arcsin(ax)^3}{12a^3}$$

input

```
Integrate[(x^2*ArcSin[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output

$$(3*a*x*\text{Sqrt}[1 - a^2*x^2] + (-3 + 6*a^2*x^2)*\text{ArcSin}[a*x] - 6*a*x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^2 + 2*\text{ArcSin}[a*x]^3)/(12*a^3)$$
Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

↓ 5210

$$\frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x \arcsin(ax) dx}{a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2}$$

↓ 5138

$$\frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \int \frac{x^2}{\sqrt{1-a^2x^2}} dx}{a} + \frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2}$$

↓ 262

$$\frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{a} + \frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2}$$

↓ 223

$$\frac{\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} + \frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{a}$$

↓ 5152

$$\frac{\arcsin(ax)^3}{6a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^2}{2a^2} + \frac{\frac{1}{2}x^2 \arcsin(ax) - \frac{1}{2}a \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{a}$$

input

$$\text{Int}[(x^2*\text{ArcSin}[a*x]^2)/\text{Sqrt}[1 - a^2*x^2], x]$$

output

$$-1/2*(x*\sqrt{1 - a^2*x^2}*\text{ArcSin}[a*x]^2)/a^2 + \text{ArcSin}[a*x]^3/(6*a^3) + ((x^2*\text{ArcSin}[a*x])/2 - (a*(-1/2*(x*\sqrt{1 - a^2*x^2}))/a^2 + \text{ArcSin}[a*x]/(2*a^3)))/2)/a$$
Defintions of rubi rules used

rule 223

$$\text{Int}[1/\sqrt{(a_) + (b_)*(x_)^2}, x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\sqrt{a})]/\text{Rt}[-b, 2], x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \text{GtQ}\{a, 0\} \ \&\& \text{NegQ}\{b\}$$

rule 262

$$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \text{ Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] \text{ ; FreeQ}\{a, b, c, p\}, x \ \&\& \text{GtQ}\{m, 2-1\} \ \&\& \text{NeQ}\{m+2*p+1, 0\} \ \&\& \text{IntBinomialQ}\{a, b, c, 2, m, p, x\}$$

rule 5138

$$\text{Int}[((a_) + \text{ArcSin}[c_)*(x_)]*(b_))^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{ Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\sqrt{1 - c^2*x^2}], x], x] \text{ ; FreeQ}\{a, b, c, d, m\}, x \ \&\& \text{IGtQ}\{n, 0\} \ \&\& \text{NeQ}\{m, -1\}$$

rule 5152

$$\text{Int}[((a_) + \text{ArcSin}[c_)*(x_)]*(b_))^{(n_)}/\sqrt{(d_) + (e_)*(x_)^2}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\sqrt{1 - c^2*x^2}/\sqrt{d + e*x^2}]*((a + b*\text{ArcSin}[c*x])^{(n+1)}), x] \text{ ; FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \text{EqQ}\{c^2*d + e, 0\} \ \&\& \text{NeQ}\{n, -1\}$$

rule 5210

$$\text{Int}[((a_) + \text{ArcSin}[c_)*(x_)]*(b_))^{(n_)}*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\text{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \text{ Int}[(f*x)^{(m-2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}\{a, b, c, d, e, f, p\}, x \ \&\& \text{EqQ}\{c^2*d + e, 0\} \ \&\& \text{GtQ}\{n, 0\} \ \&\& \text{IGtQ}\{m, 1\} \ \&\& \text{NeQ}\{m+2*p+1, 0\}$$

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{-6\sqrt{-a^2x^2+1} \arcsin(ax)^2 ax + 6 \arcsin(ax) a^2 x^2 + 2 \arcsin(ax)^3 + 3\sqrt{-a^2x^2+1} xa - 3 \arcsin(ax)}{12a^3}$	71

input `int(x^2*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `1/12*(-6*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2*a*x+6*arcsin(a*x)*a^2*x^2+2*arcsin(a*x)^3+3*(-a^2*x^2+1)^(1/2)*x*a-3*arcsin(a*x))/a^3`

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{2 \arcsin(ax)^3 + 3(2a^2x^2 - 1) \arcsin(ax) - 3\sqrt{-a^2x^2+1}(2ax \arcsin(ax)^2 - ax)}{12a^3}$$

input `integrate(x^2*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `1/12*(2*arcsin(a*x)^3 + 3*(2*a^2*x^2 - 1)*arcsin(a*x) - 3*sqrt(-a^2*x^2 + 1)*(2*a*x*arcsin(a*x)^2 - a*x))/a^3`

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

$$= \begin{cases} \frac{x^2 \arcsin(ax)}{2a} - \frac{x\sqrt{-a^2x^2+1} \arcsin^2(ax)}{2a^2} + \frac{x\sqrt{-a^2x^2+1}}{4a^2} + \frac{\arcsin^3(ax)}{6a^3} - \frac{\arcsin(ax)}{4a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x**2*asin(a*x)**2/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((x**2*asin(a*x)/(2*a) - x*sqrt(-a**2*x**2 + 1)*asin(a*x)**2/(2*a**2) + x*sqrt(-a**2*x**2 + 1)/(4*a**2) + asin(a*x)**3/(6*a**3) - asin(a*x)/(4*a**3), Ne(a, 0)), (0, True))`

Maxima [F]

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \arcsin(ax)^2}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^2*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(x^2*arcsin(a*x)^2/sqrt(-a^2*x^2 + 1), x)`

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{-a^2x^2+1}x \arcsin(ax)^2}{2a^2} + \frac{\arcsin(ax)^3}{6a^3} + \frac{\sqrt{-a^2x^2+1}x}{4a^2} + \frac{(a^2x^2-1) \arcsin(ax)}{2a^3} + \frac{\arcsin(ax)}{4a^3}$$

input `integrate(x^2*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `-1/2*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^2/a^2 + 1/6*arcsin(a*x)^3/a^3 + 1/4*sqrt(-a^2*x^2 + 1)*x/a^2 + 1/2*(a^2*x^2 - 1)*arcsin(a*x)/a^3 + 1/4*arcsin(a*x)/a^3`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \operatorname{asin}(ax)^2}{\sqrt{1-a^2x^2}} dx$$

input `int((x^2*asin(a*x)^2)/(1 - a^2*x^2)^(1/2),x)`output `int((x^2*asin(a*x)^2)/(1 - a^2*x^2)^(1/2), x)`**Reduce [F]**

$$\int \frac{x^2 \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^2 x^2}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^2*asin(a*x)^2/(-a^2*x^2+1)^(1/2),x)`output `int((asin(a*x)**2*x**2)/sqrt(- a**2*x**2 + 1),x)`

3.264 $\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal result	2615
Mathematica [A] (verified)	2615
Rubi [A] (verified)	2616
Maple [A] (verified)	2617
Fricas [A] (verification not implemented)	2617
Sympy [A] (verification not implemented)	2618
Maxima [A] (verification not implemented)	2618
Giac [A] (verification not implemented)	2619
Mupad [F(-1)]	2619
Reduce [B] (verification not implemented)	2619

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{2\sqrt{1-a^2x^2}}{a^2} + \frac{2x \arcsin(ax)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}$$

output

$$2*(-a^2*x^2+1)^{(1/2)}/a^2+2*x*\arcsin(a*x)/a-(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^2/a^2$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{2\sqrt{1-a^2x^2} + 2ax \arcsin(ax) - \sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}$$

input

$$\text{Integrate}[(x*\text{ArcSin}[a*x]^2)/\text{Sqrt}[1 - a^2*x^2], x]$$

output

$$(2*\text{Sqrt}[1 - a^2*x^2] + 2*a*x*\text{ArcSin}[a*x] - \text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^2)/a^2$$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5182, 5130, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

$$\downarrow 5182$$

$$\frac{2 \int \arcsin(ax) dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}$$

$$\downarrow 5130$$

$$\frac{2 \left(x \arcsin(ax) - a \int \frac{x}{\sqrt{1-a^2x^2}} dx \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}$$

$$\downarrow 241$$

$$\frac{2 \left(\frac{\sqrt{1-a^2x^2}}{a} + x \arcsin(ax) \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{a^2}$$

input `Int[(x*ArcSin[a*x]^2)/Sqrt[1 - a^2*x^2],x]`

output `-((Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/a^2) + (2*(Sqrt[1 - a^2*x^2]/a + x*ArcSin[a*x]))/a`

Defintions of rubi rules used

rule 241 `Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]`

rule 5130

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^(n/(2*e*(p + 1))))], x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\sqrt{-a^2x^2+1} \left(x^2 \arcsin(ax)^2 a^2 - \arcsin(ax)^2 + 2 \arcsin(ax) \sqrt{-a^2x^2+1} ax - 2a^2x^2 + 2 \right)}{a^2(a^2x^2-1)}$
ordering	$\frac{(a^4x^4-4a^2x^2+2) \arcsin(ax)^2}{a^4x^2\sqrt{-a^2x^2+1}} + \frac{2(ax-1)(ax+1) \left(\frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}} + \frac{2x \arcsin(ax)a}{-a^2x^2+1} + \frac{x^2 \arcsin(ax)^2 a^2}{(-a^2x^2+1)^{\frac{3}{2}}} \right)}{x^2 a^4} + \frac{(ax+1)^2 (ax-1)^2 \left(\frac{4 \arcsin(ax)}{-a^2x^2+1} \right)}{x^2 a^4}$

input

```
int(x*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/a^2*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(x^2*arcsin(a*x)^2*a^2-arcsin(a*x)^2+2*arcsin(a*x)*(-a^2*x^2+1)^(1/2)*a*x-2*a^2*x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{2ax \arcsin(ax) - \sqrt{-a^2x^2+1} (\arcsin(ax)^2 - 2)}{a^2}$$

input

```
integrate(x*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output $(2ax \arcsin(ax) - \sqrt{-a^2x^2 + 1}(\arcsin(ax)^2 - 2))/a^2$

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx = \begin{cases} \frac{2x \arcsin(ax)}{a} - \frac{\sqrt{-a^2x^2+1} \arcsin^2(ax)}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x*asin(a*x)**2/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((2*x*asin(a*x)/a - sqrt(-a**2*x**2 + 1)*asin(a*x)**2/a**2 + 2*sqrt(-a**2*x**2 + 1)/a**2, Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1 - a^2x^2}} dx = -\frac{\sqrt{-a^2x^2 + 1} \arcsin(ax)^2}{a^2} + \frac{2(ax \arcsin(ax) + \sqrt{-a^2x^2 + 1})}{a^2}$$

input `integrate(x*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^2/a^2 + 2*(a*x*arcsin(a*x) + sqrt(-a^2*x^2 + 1))/a^2`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{-a^2x^2+1} \arcsin(ax)^2}{a^2} + \frac{2(ax \arcsin(ax) + \sqrt{-a^2x^2+1})}{a^2}$$

input `integrate(x*arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^2/a^2 + 2*(a*x*arcsin(a*x) + sqrt(-a^2*x^2 + 1))/a^2`**Mupad [F(-1)]**

Timed out.

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \int \frac{x \operatorname{asin}(ax)^2}{\sqrt{1-a^2x^2}} dx$$

input `int((x*asin(a*x)^2)/(1 - a^2*x^2)^(1/2),x)`output `int((x*asin(a*x)^2)/(1 - a^2*x^2)^(1/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x \arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{-\sqrt{-a^2x^2+1} \operatorname{asin}(ax)^2 + 2\operatorname{asin}(ax) ax + 2\sqrt{-a^2x^2+1}}{a^2}$$

input `int(x*asin(a*x)^2/(-a^2*x^2+1)^(1/2),x)`output `(- sqrt(- a**2*x**2 + 1)*asin(a*x)**2 + 2*asin(a*x)*a*x + 2*sqrt(- a**2*x**2 + 1))/a**2`

3.265 $\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal result	2620
Mathematica [A] (verified)	2620
Rubi [A] (verified)	2621
Maple [A] (verified)	2621
Fricas [A] (verification not implemented)	2622
Sympy [A] (verification not implemented)	2622
Maxima [A] (verification not implemented)	2623
Giac [A] (verification not implemented)	2623
Mupad [B] (verification not implemented)	2623
Reduce [B] (verification not implemented)	2624

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^3}{3a}$$

output `1/3*arcsin(a*x)^3/a`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^3}{3a}$$

input `Integrate[ArcSin[a*x]^2/Sqrt[1 - a^2*x^2],x]`

output `ArcSin[a*x]^3/(3*a)`

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx$$

↓ 5152

$$\frac{\arcsin(ax)^3}{3a}$$

input `Int[ArcSin[a*x]^2/Sqrt[1 - a^2*x^2], x]`

output `ArcSin[a*x]^3/(3*a)`

Defintions of rubi rules used

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
;/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\arcsin(ax)^3}{3a}$	12
default	$\frac{\arcsin(ax)^3}{3a}$	12

input `int(arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `1/3*arcsin(a*x)^3/a`

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^3}{3a}$$

input `integrate(arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `1/3*arcsin(a*x)^3/a`

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{\arcsin^3(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(asin(a*x)**2/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((asin(a*x)**3/(3*a), Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^3}{3a}$$

input `integrate(arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `1/3*arcsin(a*x)^3/a`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^3}{3a}$$

input `integrate(arcsin(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `1/3*arcsin(a*x)^3/a`**Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^3}{3a}$$

input `int(asin(a*x)^2/(1 - a^2*x^2)^(1/2),x)`output `asin(a*x)^3/(3*a)`

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{a \sin(ax)^3}{3a}$$

input `int(asin(a*x)^2/(-a^2*x^2+1)^(1/2),x)`

output `asin(a*x)**3/(3*a)`

3.266 $\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx$

Optimal result	2625
Mathematica [A] (verified)	2625
Rubi [A] (verified)	2626
Maple [A] (verified)	2628
Fricas [F]	2629
Sympy [F]	2629
Maxima [F]	2629
Giac [F]	2630
Mupad [F(-1)]	2630
Reduce [F]	2630

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = -2 \arcsin(ax)^2 \operatorname{arctanh}(e^{i \arcsin(ax)})$$

$$+ 2i \arcsin(ax) \operatorname{PolyLog}(2, -e^{i \arcsin(ax)})$$

$$- 2i \arcsin(ax) \operatorname{PolyLog}(2, e^{i \arcsin(ax)})$$

$$- 2 \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) + 2 \operatorname{PolyLog}(3, e^{i \arcsin(ax)})$$

output

```
-2*arcsin(a*x)^2*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))+2*I*arcsin(a*x)*polylog
(2,-I*a*x-(-a^2*x^2+1)^(1/2))-2*I*arcsin(a*x)*polylog(2,I*a*x+(-a^2*x^2+1)
^(1/2))-2*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))+2*polylog(3,I*a*x+(-a^2*x^2
+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \arcsin(ax)^2 \log(1 - e^{i \arcsin(ax)}) - \arcsin(ax)^2 \log(1 + e^{i \arcsin(ax)})$$

$$+ 2i \arcsin(ax) \operatorname{PolyLog}(2, -e^{i \arcsin(ax)})$$

$$- 2i \arcsin(ax) \operatorname{PolyLog}(2, e^{i \arcsin(ax)})$$

$$- 2 \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) + 2 \operatorname{PolyLog}(3, e^{i \arcsin(ax)})$$

input `Integrate[ArcSin[a*x]^2/(x*sqrt[1 - a^2*x^2]),x]`

output `ArcSin[a*x]^2*Log[1 - E^(I*ArcSin[a*x])] - ArcSin[a*x]^2*Log[1 + E^(I*ArcSin[a*x])] + (2*I)*ArcSin[a*x]*PolyLog[2, -E^(I*ArcSin[a*x])] - (2*I)*ArcSin[a*x]*PolyLog[2, E^(I*ArcSin[a*x])] - 2*PolyLog[3, -E^(I*ArcSin[a*x])] + 2*PolyLog[3, E^(I*ArcSin[a*x])]`

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx \\
 & \quad \downarrow \text{5218} \\
 & \int \frac{\arcsin(ax)^2}{ax} d \arcsin(ax) \\
 & \quad \downarrow \text{3042} \\
 & \int \arcsin(ax)^2 \csc(\arcsin(ax)) d \arcsin(ax) \\
 & \quad \downarrow \text{4671} \\
 & -2 \int \arcsin(ax) \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + \\
 & 2 \int \arcsin(ax) \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax)^2 \operatorname{arctanh}(e^{i \arcsin(ax)}) \\
 & \quad \downarrow \text{3011} \\
 & 2 \left(i \arcsin(ax) \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - i \int \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) d \arcsin(ax) \right) - \\
 & 2 \left(i \arcsin(ax) \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) - i \int \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) d \arcsin(ax) \right) - \\
 & 2 \arcsin(ax)^2 \operatorname{arctanh}(e^{i \arcsin(ax)})
 \end{aligned}$$

↓ 2720

$$2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - \int e^{-i \arcsin(ax)} \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) de^{i \arcsin(ax)} \right) -$$

$$2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, e^{i \arcsin(ax)} \right) - \int e^{-i \arcsin(ax)} \operatorname{PolyLog} \left(2, e^{i \arcsin(ax)} \right) de^{i \arcsin(ax)} \right) -$$

$$2 \arcsin(ax)^2 \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right)$$

↓ 7143

$$-2 \arcsin(ax)^2 \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right) +$$

$$2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - \operatorname{PolyLog} \left(3, -e^{i \arcsin(ax)} \right) \right) -$$

$$2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, e^{i \arcsin(ax)} \right) - \operatorname{PolyLog} \left(3, e^{i \arcsin(ax)} \right) \right)$$

input `Int[ArcSin[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]`

output `-2*ArcSin[a*x]^2*ArcTanh[E^(I*ArcSin[a*x])] + 2*(I*ArcSin[a*x]*PolyLog[2, -E^(I*ArcSin[a*x])] - PolyLog[3, -E^(I*ArcSin[a*x])]) - 2*(I*ArcSin[a*x]*PolyLog[2, E^(I*ArcSin[a*x])] - PolyLog[3, E^(I*ArcSin[a*x])])`

Defintions of rubi rules used

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a + b*x))))^n]/(b*c*n*Log[F]), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x))))^n], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5218 `Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]`

rule 7143 `Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75

method	result
default	$\arcsin(ax)^2 \ln(1 - iax - \sqrt{-a^2x^2 + 1}) - 2i \arcsin(ax) \operatorname{polylog}(2, iax + \sqrt{-a^2x^2 + 1}) + 2 \operatorname{polylog}(3, -Iax - \sqrt{-a^2x^2 + 1})$

input `int(arcsin(a*x)^2/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `arcsin(a*x)^2*ln(1-I*a*x-(-a^2*x^2+1)^(1/2))-2*I*arcsin(a*x)*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))+2*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))-arcsin(a*x)^2*ln(1+I*a*x+(-a^2*x^2+1)^(1/2))+2*I*arcsin(a*x)*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-2*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))`

Fricas [F]

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1x}} dx$$

input `integrate(arcsin(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^2/(a^2*x^3 - x), x)`

Sympy [F]

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F]

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1x}} dx$$

input `integrate(arcsin(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(arcsin(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)`

Giac [F]

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}x} dx$$

input `integrate(arcsin(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}x} dx$$

input `int(asin(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)**2/(sqrt(- a**2*x**2 + 1)*x),x)`

3.267 $\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx$

Optimal result	2631
Mathematica [A] (verified)	2631
Rubi [A] (verified)	2632
Maple [A] (verified)	2634
Fricas [F]	2635
Sympy [F]	2635
Maxima [F]	2636
Giac [F]	2636
Mupad [F(-1)]	2636
Reduce [F]	2637

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = -ia \arcsin(ax)^2 - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x} + 2a \arcsin(ax) \log(1 - e^{2i \arcsin(ax)}) - ia \operatorname{PolyLog}(2, e^{2i \arcsin(ax)})$$

output

```
-I*a*arcsin(a*x)^2-(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2/x+2*a*arcsin(a*x)*ln(1-(I*a*x+(-a^2*x^2+1)^(1/2))^2)-I*a*polylog(2,(I*a*x+(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \arcsin(ax) \left(-\frac{(iax + \sqrt{1-a^2x^2}) \arcsin(ax)}{x} + 2a \log(1 - e^{2i \arcsin(ax)}) \right) - ia \operatorname{PolyLog}(2, e^{2i \arcsin(ax)})$$

input

```
Integrate[ArcSin[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output

```
ArcSin[a*x]*(-(((I*a*x + Sqrt[1 - a^2*x^2])*ArcSin[a*x])/x) + 2*a*Log[1 -
E^((2*I)*ArcSin[a*x])]) - I*a*PolyLog[2, E^((2*I)*ArcSin[a*x])]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5186, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^2}{x^2 \sqrt{1-a^2x^2}} dx$$

$$\downarrow 5186$$

$$2a \int \frac{\arcsin(ax)}{x} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x}$$

$$\downarrow 5136$$

$$2a \int \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{ax} d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x}$$

$$\downarrow 3042$$

$$2a \int -\arcsin(ax) \tan\left(\arcsin(ax) + \frac{\pi}{2}\right) d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x}$$

$$\downarrow 25$$

$$-2a \int \arcsin(ax) \tan\left(\arcsin(ax) + \frac{\pi}{2}\right) d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x}$$

$$\downarrow 4200$$

$$-\frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x} + 2a \left(2i \int -\frac{e^{2i \arcsin(ax)} \arcsin(ax)}{1 - e^{2i \arcsin(ax)}} d \arcsin(ax) - \frac{1}{2} i \arcsin(ax)^2 \right)$$

$$\downarrow 25$$

$$-\frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x} + 2a \left(-2i \int \frac{e^{2i \arcsin(ax)} \arcsin(ax)}{1 - e^{2i \arcsin(ax)}} d \arcsin(ax) - \frac{1}{2} i \arcsin(ax)^2 \right)$$

$$\begin{aligned}
 & \downarrow 2620 \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x} + \\
 & 2a \left(-2i \left(\frac{1}{2} i \arcsin(ax) \log \left(1 - e^{2i \arcsin(ax)} \right) - \frac{1}{2} i \int \log \left(1 - e^{2i \arcsin(ax)} \right) d \arcsin(ax) \right) - \frac{1}{2} i \arcsin(ax)^2 \right) \\
 & \downarrow 2715 \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x} + \\
 & 2a \left(-2i \left(\frac{1}{2} i \arcsin(ax) \log \left(1 - e^{2i \arcsin(ax)} \right) - \frac{1}{4} \int e^{-2i \arcsin(ax)} \log \left(1 - e^{2i \arcsin(ax)} \right) de^{2i \arcsin(ax)} \right) - \frac{1}{2} i \arcsin(ax)^2 \right) \\
 & \downarrow 2838 \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{x} + \\
 & 2a \left(-2i \left(\frac{1}{4} \text{PolyLog} \left(2, e^{2i \arcsin(ax)} \right) + \frac{1}{2} i \arcsin(ax) \log \left(1 - e^{2i \arcsin(ax)} \right) \right) - \frac{1}{2} i \arcsin(ax)^2 \right)
 \end{aligned}$$

input `Int[ArcSin[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]`

output `-((Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/x) + 2*a*((-1/2*I)*ArcSin[a*x]^2 - (2*I)*((I/2)*ArcSin[a*x]*Log[1 - E^((2*I)*ArcSin[a*x])] + PolyLog[2, E^((2*I)*ArcSin[a*x])])/4))`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp [((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.86

method	result
default	$\frac{(iax - \sqrt{-a^2x^2 + 1}) \arcsin(ax)^2}{x} - 2ia(i \arcsin(ax) \ln(1 - iax - \sqrt{-a^2x^2 + 1}) + i \arcsin(ax) \ln(1 + iax - \sqrt{-a^2x^2 + 1}))$

input `int(arcsin(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `(I*a*x-(-a^2*x^2+1)^(1/2))*arcsin(a*x)^2/x-2*I*a*(I*arcsin(a*x)*ln(1-I*a*x-(-a^2*x^2+1)^(1/2))+I*arcsin(a*x)*ln(1+I*a*x+(-a^2*x^2+1)^(1/2))+arcsin(a*x)^2+polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))+polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2)))`

Fricas [F]

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1x^2}} dx$$

input `integrate(arcsin(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^2/(a^2*x^4 - x^2), x)`

Sympy [F]

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F]

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1x^2}} dx$$

input `integrate(arcsin(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `-(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^2 - 2*a*x*integrate(arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))/x, x))/x`

Giac [F]

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1x^2}} dx$$

input `integrate(arcsin(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)^2}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^2}{\sqrt{-a^2x^2+1}x^2} dx$$

input `int(asin(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)**2/(sqrt(-a**2*x**2+1)*x**2),x)`

$$3.268 \quad \int \frac{\arcsin(ax)^2}{x^3 \sqrt{1-a^2x^2}} dx$$

Optimal result	2638
Mathematica [A] (verified)	2639
Rubi [A] (verified)	2639
Maple [A] (verified)	2643
Fricas [F]	2644
Sympy [F]	2644
Maxima [F]	2644
Giac [F]	2645
Mupad [F(-1)]	2645
Reduce [F]	2645

Optimal result

Integrand size = 24, antiderivative size = 163

$$\begin{aligned} \int \frac{\arcsin(ax)^2}{x^3 \sqrt{1-a^2x^2}} dx = & -\frac{a \arcsin(ax)}{x} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\ & - a^2 \arcsin(ax)^2 \operatorname{arctanh}(e^{i \arcsin(ax)}) - a^2 \operatorname{arctanh}(\sqrt{1-a^2x^2}) \\ & + ia^2 \arcsin(ax) \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) \\ & - ia^2 \arcsin(ax) \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) \\ & - a^2 \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) + a^2 \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) \end{aligned}$$

output

```
-a*arcsin(a*x)/x-1/2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2/x^2-a^2*arcsin(a*x)^2*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))-a^2*arctanh((-a^2*x^2+1)^(1/2))+I*a^2*arcsin(a*x)*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-I*a^2*arcsin(a*x)*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))+a^2*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.19

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \frac{1}{8}a^2 \left(-4 \arcsin(ax) \cot\left(\frac{1}{2} \arcsin(ax)\right) \right. \\ \left. - \arcsin(ax)^2 \csc^2\left(\frac{1}{2} \arcsin(ax)\right) \right. \\ \left. + 4 \arcsin(ax)^2 (\log(1 - e^{i \arcsin(ax)}) - \log(1 + e^{i \arcsin(ax)})) \right. \\ \left. + 8 \log\left(\tan\left(\frac{1}{2} \arcsin(ax)\right)\right) \right. \\ \left. + 8i \arcsin(ax) (\text{PolyLog}(2, -e^{i \arcsin(ax)}) \right. \\ \left. - \text{PolyLog}(2, e^{i \arcsin(ax)})) \right. \\ \left. + 8(-\text{PolyLog}(3, -e^{i \arcsin(ax)}) + \text{PolyLog}(3, e^{i \arcsin(ax)})) \right. \\ \left. + \arcsin(ax)^2 \sec^2\left(\frac{1}{2} \arcsin(ax)\right) \right. \\ \left. - 4 \arcsin(ax) \tan\left(\frac{1}{2} \arcsin(ax)\right) \right)$$

input `Integrate[ArcSin[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]`

output `(a^2*(-4*ArcSin[a*x]*Cot[ArcSin[a*x]/2] - ArcSin[a*x]^2*Csc[ArcSin[a*x]/2]^2 + 4*ArcSin[a*x]^2*(Log[1 - E^(I*ArcSin[a*x])] - Log[1 + E^(I*ArcSin[a*x])])) + 8*Log[Tan[ArcSin[a*x]/2]] + (8*I)*ArcSin[a*x]*(PolyLog[2, -E^(I*ArcSin[a*x])] - PolyLog[2, E^(I*ArcSin[a*x])]) + 8*(-PolyLog[3, -E^(I*ArcSin[a*x])] + PolyLog[3, E^(I*ArcSin[a*x])]) + ArcSin[a*x]^2*Sec[ArcSin[a*x]/2]^2 - 4*ArcSin[a*x]*Tan[ArcSin[a*x]/2]))/8`

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5204, 5138, 243, 73, 221, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx \\
& \quad \downarrow \text{5204} \\
& \frac{1}{2}a^2 \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx + a \int \frac{\arcsin(ax)}{x^2} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{5138} \\
& \frac{1}{2}a^2 \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx + a \left(a \int \frac{1}{x\sqrt{1-a^2x^2}} dx - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{243} \\
& \frac{1}{2}a^2 \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx + a \left(\frac{1}{2}a \int \frac{1}{x^2\sqrt{1-a^2x^2}} dx^2 - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{73} \\
& \frac{1}{2}a^2 \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx + a \left(-\frac{\int \frac{1}{\frac{1}{a^2}-x^4} d\sqrt{1-a^2x^2}}{a} - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{221} \\
& \frac{1}{2}a^2 \int \frac{\arcsin(ax)^2}{x\sqrt{1-a^2x^2}} dx + a \left(-a \operatorname{arctanh}(\sqrt{1-a^2x^2}) - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{5218} \\
& \frac{1}{2}a^2 \int \frac{\arcsin(ax)^2}{ax} d \arcsin(ax) + a \left(-a \operatorname{arctanh}(\sqrt{1-a^2x^2}) - \frac{\arcsin(ax)}{x} \right) - \\
& \quad \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{3042} \\
& \frac{1}{2}a^2 \int \arcsin(ax)^2 \csc(\arcsin(ax)) d \arcsin(ax) + a \left(-a \operatorname{arctanh}(\sqrt{1-a^2x^2}) - \frac{\arcsin(ax)}{x} \right) - \\
& \quad \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \\
& \quad \downarrow \text{4671} \\
& \frac{1}{2}a^2 \left(-2 \int \arcsin(ax) \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + 2 \int \arcsin(ax) \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2a \right. \\
& \quad \left. a \left(-a \operatorname{arctanh}(\sqrt{1-a^2x^2}) - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{2x^2} \right)
\end{aligned}$$

↓ 3011

$$\frac{1}{2}a^2 \left(2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - i \int \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) d \arcsin(ax) \right) - 2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) \right) \right. \\ \left. a \left(-a \operatorname{arctanh} \left(\sqrt{1 - a^2 x^2} \right) - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)^2}{2x^2} \right)$$

↓ 2720

$$\frac{1}{2}a^2 \left(2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - \int e^{-i \arcsin(ax)} \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) de^{i \arcsin(ax)} \right) - 2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) \right) \right. \\ \left. a \left(-a \operatorname{arctanh} \left(\sqrt{1 - a^2 x^2} \right) - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)^2}{2x^2} \right)$$

↓ 7143

$$\frac{1}{2}a^2 \left(-2 \arcsin(ax)^2 \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right) + 2 \left(i \arcsin(ax) \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - \operatorname{PolyLog} \left(3, -e^{i \arcsin(ax)} \right) \right) \right. \\ \left. a \left(-a \operatorname{arctanh} \left(\sqrt{1 - a^2 x^2} \right) - \frac{\arcsin(ax)}{x} \right) - \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)^2}{2x^2} \right)$$

input `Int[ArcSin[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]`

output `-1/2*(Sqrt[1 - a^2*x^2]*ArcSin[a*x]^2)/x^2 + a*(-(ArcSin[a*x]/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]) + (a^2*(-2*ArcSin[a*x]^2*ArcTanh[E^(I*ArcSin[a*x])]] + 2*(I*ArcSin[a*x]*PolyLog[2, -E^(I*ArcSin[a*x])]] - PolyLog[3, -E^(I*ArcSin[a*x])])) - 2*(I*ArcSin[a*x]*PolyLog[2, E^(I*ArcSin[a*x])]] - PolyLog[3, E^(I*ArcSin[a*x])])))/2`

Defintions of rubi rules used

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && Lt Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

- rule 221 $\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a) \cdot \text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$
- rule 243 $\text{Int}[(x_)^{(m_)} \cdot ((a_) + (b_ \cdot)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{((m - 1)/2) \cdot (a + b \cdot x)^p}, x], x, x^2], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$
- rule 2720 $\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Simp}[v/D[v, x] \ \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ \text{!MatchQ}[u, (w_) \cdot ((a_) \cdot (v_)^{(n_)})^{(m_)} /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m \cdot n] \ \&\& \ \text{!MatchQ}[u, E^{((c_) \cdot ((a_) + (b_ \cdot)x))} \cdot (F_)^{[v_]} /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$
- rule 3011 $\text{Int}[\text{Log}[1 + (e_) \cdot ((F_)^{((c_) \cdot ((a_) + (b_ \cdot)(x_))))^{(n_)}}] \cdot ((f_) + (g_) \cdot (x_)^{(m_)}), x_Symbol] \rightarrow \text{Simp}[(- (f + g \cdot x)^m \cdot (\text{PolyLog}[2, (-e) \cdot (F^{(c \cdot (a + b \cdot x))})^n] / (b \cdot c \cdot n \cdot \text{Log}[F]))], x] + \text{Simp}[g \cdot (m / (b \cdot c \cdot n \cdot \text{Log}[F])) \ \text{Int}[(f + g \cdot x)^{(m - 1)} \cdot \text{PolyLog}[2, (-e) \cdot (F^{(c \cdot (a + b \cdot x))})^n], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 4671 $\text{Int}[\text{csc}[(e_) + (f_) \cdot (x_)] \cdot ((c_) + (d_) \cdot (x_)^{(m_)}), x_Symbol] \rightarrow \text{Simp}[-2 \cdot (c + d \cdot x)^m \cdot (\text{ArcTanh}[E^{(I \cdot (e + f \cdot x))}]/f), x] + (-\text{Simp}[d \cdot (m/f) \ \text{Int}[(c + d \cdot x)^{(m - 1)} \cdot \text{Log}[1 - E^{(I \cdot (e + f \cdot x))}], x], x] + \text{Simp}[d \cdot (m/f) \ \text{Int}[(c + d \cdot x)^{(m - 1)} \cdot \text{Log}[1 + E^{(I \cdot (e + f \cdot x))}], x], x]) /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0]$
- rule 5138 $\text{Int}[(a_) + \text{ArcSin}[(c_) \cdot (x_)] \cdot (b_)^{(n_)} \cdot ((d_) \cdot (x_)^{(m_)}), x_Symbol] \rightarrow \text{Simp}[(d \cdot x)^{(m + 1)} \cdot ((a + b \cdot \text{ArcSin}[c \cdot x])^n / (d \cdot (m + 1))), x] - \text{Simp}[b \cdot c \cdot (n / (d \cdot (m + 1))) \ \text{Int}[(d \cdot x)^{(m + 1)} \cdot ((a + b \cdot \text{ArcSin}[c \cdot x])^{(n - 1)} / \text{Sqrt}[1 - c^2 \cdot x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{-a^2x^2+1} \arcsin(ax) \left(\arcsin(ax) a^2 x^2 - 2\sqrt{-a^2x^2+1} x a - \arcsin(ax) \right)}{2x^2(a^2x^2-1)} + \frac{a^2 \left(\arcsin(ax)^2 \ln(1-iax-\sqrt{-a^2x^2+1}) - \arcsin(ax) \right)}{2x^2(a^2x^2-1)}$

input

```
int(arcsin(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(-a^2*x^2+1)^(1/2)/x^2/(a^2*x^2-1)*arcsin(a*x)*(arcsin(a*x)*a^2*x^2-2
*(-a^2*x^2+1)^(1/2)*x*a-arcsin(a*x))+1/2*a^2*(arcsin(a*x)^2*ln(1-I*a*x-(-a
^2*x^2+1)^(1/2))-arcsin(a*x)^2*ln(1+I*a*x+(-a^2*x^2+1)^(1/2))-2*I*arcsin(a
*x)*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))+2*I*arcsin(a*x)*polylog(2,-I*a*x-(
-a^2*x^2+1)^(1/2))+2*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))-2*polylog(3,-I*a*
x-(-a^2*x^2+1)^(1/2))-4*arctanh(I*a*x+(-a^2*x^2+1)^(1/2)))
```


Fricas [F]

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}x^3} dx$$

input `integrate(arcsin(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^2/(a^2*x^5 - x^3), x)`

Sympy [F]

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F]

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}x^3} dx$$

input `integrate(arcsin(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(arcsin(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)`

Giac [F]

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}x^3} dx$$

input `integrate(arcsin(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)^2}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^2}{\sqrt{-a^2x^2+1}x^3} dx$$

input `int(asin(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)**2/(sqrt(- a**2*x**2 + 1)*x**3),x)`

3.269 $\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx$

Optimal result	2646
Mathematica [A] (warning: unable to verify)	2647
Rubi [A] (verified)	2648
Maple [F]	2657
Fricas [F]	2657
Sympy [F]	2657
Maxima [F]	2658
Giac [F]	2659
Mupad [F(-1)]	2660
Reduce [F]	2660

Optimal result

Integrand size = 27, antiderivative size = 1312

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

output

```

96*b^2*c^2*d^3*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],c^2*x^2)/(3+m)^2/(5+m)/(7+m)/(m^2+3*m+2)-30*b*c*d^3*x^(2+m)*(a+b*arcsin(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(5+m)/(7+m)^2/(m^2+5*m+6)-36*b*c*d^3*x^(2+m)*(a+b*arcsin(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(5+m)^2/(7+m)/(m^2+5*m+6)-96*b*c*d^3*x^(2+m)*(a+b*arcsin(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(5+m)/(7+m)/(m^3+6*m^2+11*m+6)-30*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/(7+m)^2/(m^2+8*m+15)+2*b^2*c^6*d^3*x^(7+m)/(7+m)^3+d^3*x^(1+m)*(-c^2*x^2+1)^3*(a+b*arcsin(c*x))^2/(7+m)+30*b^2*c^2*d^3*x^(3+m)/(3+m)^2/(5+m)/(7+m)^2+36*b^2*c^2*d^3*x^(3+m)/(3+m)^2/(5+m)^2/(7+m)+12*b^2*c^2*d^3*x^(3+m)/(3+m)/(5+m)^2/(7+m)+48*b^2*c^2*d^3*x^(3+m)/(3+m)^3/(5+m)/(7+m)-2*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))/(7+m)^2+2*b^2*c^2*d^3*x^(3+m)/(3+m)/(7+m)^2+10*b^2*c^2*d^3*x^(3+m)/(7+m)^2/(m^2+8*m+15)-10*b^2*c^4*d^3*x^(5+m)/(5+m)^2/(7+m)^2-4*b^2*c^4*d^3*x^(5+m)/(5+m)/(7+m)^2-12*b^2*c^4*d^3*x^(5+m)/(5+m)^3/(7+m)+48*d^3*x^(1+m)*(a+b*arcsin(c*x))^2/(5+m)/(7+m)/(m^2+4*m+3)+24*d^3*x^(1+m)*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/(7+m)/(m^2+8*m+15)+6*d^3*x^(1+m)*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/(5+m)/(7+m)-36*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/(3+m)/(5+m)^2/(7+m)-48*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/(3+m)^2/(5+m)/(7+m)-48*b*c*d^3*x^(2+m)*(a+b*arcsin(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(2+m)/(3+m)^2/(5+m)...
    
```

Mathematica [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.41

$$\int x^m (d - c^2 x^2)^3 (a + b \arcsin(cx))^2 dx = d^3 x^{1+m} \left(\frac{(a + b \arcsin(cx))^2}{1 + m} \right.$$

$$- \frac{3c^2 x^2 (a + b \arcsin(cx))^2}{3 + m} + \frac{3c^4 x^4 (a + b \arcsin(cx))^2}{5 + m} - \frac{c^6 x^6 (a + b \arcsin(cx))^2}{7 + m}$$

$$+ \frac{2bcx \left(-((3 + m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)\right) + bcx {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right)}{(1 + m)(2 + m)(3 + m)}$$

$$- \frac{6bc^3 x^3 \left(-((5 + m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, c^2 x^2\right)\right) + bcx {}_3F_2\left(1, \frac{5}{2} + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right)}{(3 + m)(4 + m)(5 + m)}$$

$$+ \frac{6bc^5 x^5 \left(-((7 + m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{6+m}{2}, \frac{8+m}{2}, c^2 x^2\right)\right) + bcx {}_3F_2\left(1, \frac{7}{2} + \frac{m}{2}, \frac{7}{2} + \frac{m}{2}\right)}{(5 + m)(6 + m)(7 + m)}$$

$$- \frac{2bc^7 x^7 \left(-((9 + m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 4 + \frac{m}{2}, 5 + \frac{m}{2}, c^2 x^2\right)\right) + bcx {}_3F_2\left(1, \frac{9}{2} + \frac{m}{2}, \frac{9}{2} + \frac{m}{2}\right)}{(7 + m)(8 + m)(9 + m)}$$

input `Integrate[x^m*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]`

output
$$\begin{aligned} & d^3 x^{(1+m)} ((a + b \operatorname{ArcSin}[c x])^2 / (1+m) - (3 c^2 x^2 (a + b \operatorname{ArcSin}[c x])^2) / (3+m) + (3 c^4 x^4 (a + b \operatorname{ArcSin}[c x])^2) / (5+m) - (c^6 x^6 (a + b \operatorname{ArcSin}[c x])^2) / (7+m) + (2 b c x (-((3+m)(a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, c^2 x^2]) + b c x \operatorname{HypergeometricPFQ}[\{1, 3/2 + m/2, 3/2 + m/2\}, \{2 + m/2, 5/2 + m/2\}, c^2 x^2])) / ((1+m)(2+m)(3+m)) - (6 b c^3 x^3 (-((5+m)(a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}[1/2, (4+m)/2, (6+m)/2, c^2 x^2]) + b c x \operatorname{HypergeometricPFQ}[\{1, 5/2 + m/2, 5/2 + m/2\}, \{3 + m/2, 7/2 + m/2\}, c^2 x^2])) / ((3+m)(4+m)(5+m)) + (6 b c^5 x^5 (-((7+m)(a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}[1/2, (6+m)/2, (8+m)/2, c^2 x^2]) + b c x \operatorname{HypergeometricPFQ}[\{1, 7/2 + m/2, 7/2 + m/2\}, \{4 + m/2, 9/2 + m/2\}, c^2 x^2])) / ((5+m)(6+m)(7+m)) - (2 b c^7 x^7 (-((9+m)(a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}[1/2, 4 + m/2, 5 + m/2, c^2 x^2]) + b c x \operatorname{HypergeometricPFQ}[\{1, 9/2 + m/2, 9/2 + m/2\}, \{5 + m/2, 11/2 + m/2\}, c^2 x^2])) / ((7+m)(8+m)(9+m)) \end{aligned}$$

Rubi [A] (verified)

Time = 3.38 (sec) , antiderivative size = 1031, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5202, 27, 5202, 244, 2009, 5202, 244, 2009, 5138, 5198, 15, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx \\ & \quad \downarrow 5202 \\ & \frac{2bcd^3 \int x^{m+1} (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx)) dx}{m+7} + \\ & \frac{6d \int d^2 x^m (1 - c^2 x^2)^2 (a + b \arcsin(cx))^2 dx}{m+7} + \frac{d^3 (1 - c^2 x^2)^3 x^{m+1} (a + b \arcsin(cx))^2}{m+7} \\ & \quad \downarrow 27 \end{aligned}$$

$$\frac{2bcd^3 \int x^{m+1}(1-c^2x^2)^{5/2}(a+b\arcsin(cx))dx}{m+7} + \frac{6d^3 \int x^m(1-c^2x^2)^2(a+b\arcsin(cx))^2dx}{m+7} + \frac{d^3(1-c^2x^2)^3 x^{m+1}(a+b\arcsin(cx))^2}{m+7}$$

↓ 5202

$$\frac{2bcd^3 \left(\frac{5 \int x^{m+1}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx}{m+7} - \frac{bc \int x^{m+2}(1-c^2x^2)^2dx}{m+7} + \frac{(1-c^2x^2)^{5/2}x^{m+2}(a+b\arcsin(cx))}{m+7} \right)}{m+7} + \frac{6d^3 \left(-\frac{2bc \int x^{m+1}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx}{m+5} + \frac{4 \int x^m(1-c^2x^2)(a+b\arcsin(cx))^2dx}{m+5} + \frac{(1-c^2x^2)^2x^{m+1}(a+b\arcsin(cx))^2}{m+5} \right)}{m+7} + \frac{d^3(1-c^2x^2)^3 x^{m+1}(a+b\arcsin(cx))^2}{m+7}$$

↓ 244

$$\frac{6d^3 \left(-\frac{2bc \int x^{m+1}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx}{m+5} + \frac{4 \int x^m(1-c^2x^2)(a+b\arcsin(cx))^2dx}{m+5} + \frac{(1-c^2x^2)^2x^{m+1}(a+b\arcsin(cx))^2}{m+5} \right)}{m+7} - \frac{2bcd^3 \left(\frac{5 \int x^{m+1}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx}{m+7} - \frac{bc \int (x^{m+2}-2c^2x^{m+4}+c^4x^{m+6})dx}{m+7} + \frac{(1-c^2x^2)^{5/2}x^{m+2}(a+b\arcsin(cx))}{m+7} \right)}{m+7} + \frac{d^3(1-c^2x^2)^3 x^{m+1}(a+b\arcsin(cx))^2}{m+7}$$

↓ 2009

$$\frac{6d^3 \left(-\frac{2bc \int x^{m+1}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx}{m+5} + \frac{4 \int x^m(1-c^2x^2)(a+b\arcsin(cx))^2dx}{m+5} + \frac{(1-c^2x^2)^2x^{m+1}(a+b\arcsin(cx))^2}{m+5} \right)}{m+7} - \frac{2bcd^3 \left(\frac{5 \int x^{m+1}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))dx}{m+7} + \frac{(1-c^2x^2)^{5/2}x^{m+2}(a+b\arcsin(cx))}{m+7} - \frac{bc \left(\frac{c^4x^{m+7}}{m+7} - \frac{2c^2x^{m+5}}{m+5} + \frac{x^{m+3}}{m+3} \right)}{m+7} \right)}{m+7} + \frac{d^3(1-c^2x^2)^3 x^{m+1}(a+b\arcsin(cx))^2}{m+7}$$

↓ 5202

$$6d^3 \left(-\frac{2bc \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} - \frac{bc \int x^{m+2} (1-c^2x^2) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} \right)}{m+5} \right) + \frac{4 \left(-\frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} \right)}{m+7}$$

$$2bcd^3 \left(\frac{5 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} - \frac{bc \int x^{m+2} (1-c^2x^2) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} \right)}{m+7} \right) + \frac{(1-c^2x^2)^{5/2} x^{m+2} (a+b \arcsin(cx))}{m+7}$$

$$\frac{d^3(1-c^2x^2)^3 x^{m+1} (a+b \arcsin(cx))^2}{m+7}$$

↓ 244

$$6d^3 \left(-\frac{2bc \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} - \frac{bc \int (x^{m+2} - c^2x^{m+4}) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} \right)}{m+5} \right) + \frac{4 \left(-\frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} \right)}{m+7}$$

$$2bcd^3 \left(\frac{5 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} - \frac{bc \int (x^{m+2} - c^2x^{m+4}) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} \right)}{m+7} \right) + \frac{(1-c^2x^2)^{5/2} x^{m+2} (a+b \arcsin(cx))}{m+7}$$

$$\frac{d^3(1-c^2x^2)^3 x^{m+1} (a+b \arcsin(cx))^2}{m+7}$$

↓ 2009

$$6d^3 \left(-\frac{2bc \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2x^{m+5}}{m+5} \right)}{m+5} \right)}{m+5} \right) + \frac{4 \left(-\frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} \right)}{m+7}$$

$$2bcd^3 \left(\frac{5 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2x^{m+5}}{m+5} \right)}{m+5} \right)}{m+7} \right) + \frac{(1-c^2x^2)^{5/2} x^{m+2} (a+b \arcsin(cx))}{m+7}$$

$$\frac{d^3(1-c^2x^2)^3 x^{m+1} (a+b \arcsin(cx))^2}{m+7}$$

↓ 5138

$$6d^3 \left(\frac{4 \left(\frac{2 \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right)}{m+3} \right)}{m+5} - \frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} + \frac{(1-c^2x^2)x^{m+1}(a+b \arcsin(cx))^2}{m+3} \right)$$

$$2bcd^3 \left(\frac{5 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2 x^{m+5}}{m+5} \right)}{m+5} \right)}{m+7} + \frac{(1-c^2x^2)^{5/2} x^{m+2} (a+b \arcsin(cx))}{m+7} \right)$$

$$\frac{d^3 (1-c^2x^2)^3 x^{m+1} (a+b \arcsin(cx))^2}{m+7}$$

↓ 5198

$$6d^3 \left(\frac{4 \left(\frac{2 \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right)}{m+3} \right) - 2bc \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} - \frac{bc \int x^{m+2} dx}{m+3} + \frac{\sqrt{1-c^2x^2} x^{m+2}(a+b \arcsin(cx))}{m+3} \right)}{m+5} \right)$$

$$2bcd^3 \left(\frac{5 \left(\frac{3 \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} - \frac{bc \int x^{m+2} dx}{m+3} + \frac{\sqrt{1-c^2x^2} x^{m+2}(a+b \arcsin(cx))}{m+3} \right)}{m+5} \right) + \frac{(1-c^2x^2)^{3/2} x^{m+2}(a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2 x^{m+5}}{m+5} \right)}{m+5}}{m+7} \right)$$

$$\frac{d^3(1-c^2x^2)^3 x^{m+1}(a+b \arcsin(cx))^2}{m+7} \qquad m+7$$

↓ 15

$$6d^3 \left(\frac{2 \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right) - 2bc \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} + \frac{\sqrt{1-c^2x^2}x^{m+2}(a+b \arcsin(cx))}{m+3} - \frac{bcx^{m+3}}{(m+3)^2} \right)}{m+3} \right) \frac{1}{m+5}$$

$$2bcd^3 \left(\frac{3 \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} + \frac{\sqrt{1-c^2x^2}x^{m+2}(a+b \arcsin(cx))}{m+3} - \frac{bcx^{m+3}}{(m+3)^2} \right) + \frac{(1-c^2x^2)^{3/2}x^{m+2}(a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2x^{m+5}}{m+5} \right)}{m+5}}{m+5} \right) \frac{1}{m+7}$$

$$\frac{d^3(1-c^2x^2)^3 x^{m+1}(a+b \arcsin(cx))^2}{m+7} \qquad m+7$$

↓ 5220

$$6d^3 \left(\frac{(1-c^2x^2)^2 (a+b \arcsin(cx))^2 x^{m+1}}{m+5} + \left(\frac{(1-c^2x^2) (a+b \arcsin(cx))^2 x^{m+1}}{m+3} + \left(\frac{x^{m+1} (a+b \arcsin(cx))^2}{m+1} - 2bc \frac{x^{m+2} (a+b \arcsin(cx)) \text{Hypergeometric}}{m+1} \right) \right) \right)$$

$$2bcd^3 \left(\frac{(1-c^2x^2)^{5/2} (a+b \arcsin(cx)) x^{m+2}}{m+7} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{2c^2 x^{m+5}}{m+5} + \frac{c^4 x^{m+7}}{m+7} \right)}{m+7} + \left(\frac{(1-c^2x^2)^{3/2} (a+b \arcsin(cx)) x^{m+2}}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2 x^{m+5}}{m+5} \right)}{m+5} \right) \right)$$

input `Int [x^m*(d - c^2*d*x^2)^3*(a + b*ArcSin[c*x])^2,x]`

output

```
(d^3*x^(1+m)*(1-c^2*x^2)^3*(a+b*ArcSin[c*x])^2)/(7+m) + (6*d^3*((x
^(1+m)*(1-c^2*x^2)^2*(a+b*ArcSin[c*x])^2)/(5+m) + (4*((x^(1+m)*
(1-c^2*x^2)*(a+b*ArcSin[c*x])^2)/(3+m) + (2*((x^(1+m)*(a+b*ArcSin
[c*x])^2)/(1+m) - (2*b*c*((x^(2+m)*(a+b*ArcSin[c*x])*Hypergeometric2
F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(2+m) - (b*c*x^(3+m)*Hypergeom
etricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, c^2*x^2])/(6+5
*m+m^2)))/(1+m)))/(3+m) - (2*b*c*(-((b*c*x^(3+m))/(3+m)^2) + (x^
(2+m)*Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])))/(3+m) + ((x^(2+m)*(a+
b*ArcSin[c*x])*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(2+
m) - (b*c*x^(3+m)*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2
, 5/2+m/2}, c^2*x^2])/(6+5*m+m^2)))/(3+m)))/(3+m)))/(5+m) - (2*
b*c*(-((b*c*(x^(3+m))/(3+m) - (c^2*x^(5+m))/(5+m)))/(5+m) + (x^
(2+m)*(1-c^2*x^2)^(3/2)*(a+b*ArcSin[c*x])))/(5+m) + (3*(-((b*c*x^(3
+m))/(3+m)^2) + (x^(2+m)*Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])))/(3+
m) + ((x^(2+m)*(a+b*ArcSin[c*x])*Hypergeometric2F1[1/2, (2+m)/2, (4
+m)/2, c^2*x^2])/(2+m) - (b*c*x^(3+m)*HypergeometricPFQ[{1, 3/2+m/2
, 3/2+m/2}, {2+m/2, 5/2+m/2}, c^2*x^2])/(6+5*m+m^2)))/(3+m)))/(
5+m)))/(5+m)))/(7+m) - (2*b*c*d^3*(-((b*c*(x^(3+m))/(3+m) - (2*c^
2*x^(5+m))/(5+m) + (c^4*x^(7+m))/(7+m)))/(7+m) + (x^(2+m)*(1
-c^2*x^2)^(5/2)*(a+b*ArcSin[c*x])))/(7+m) + (5*(-((b*c*(x^(3+m)/(...
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m+1)/(m+1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 244

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand
Integrand[(c*x)^m*(a+b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5138

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5198

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5220

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int x^m (-c^2 d x^2 + d)^3 (a + b \arcsin(cx))^2 dx$$

input `int(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x)`

Fricas [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arcsin(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arcsin(c*x))*x^m, x)`

Sympy [F]

$$\begin{aligned} & \int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx \\ &= -d^3 \left(\int (-a^2 x^m) dx + \int (-b^2 x^m \operatorname{asin}^2(cx)) dx + \int (-2abx^m \operatorname{asin}(cx)) dx \right. \\ & \quad + \int 3a^2 c^2 x^2 x^m dx + \int (-3a^2 c^4 x^4 x^m) dx + \int a^2 c^6 x^6 x^m dx \\ & \quad + \int 3b^2 c^2 x^2 x^m \operatorname{asin}^2(cx) dx + \int (-3b^2 c^4 x^4 x^m \operatorname{asin}^2(cx)) dx \\ & \quad + \int b^2 c^6 x^6 x^m \operatorname{asin}^2(cx) dx + \int 6abc^2 x^2 x^m \operatorname{asin}(cx) dx \\ & \quad \left. + \int (-6abc^4 x^4 x^m \operatorname{asin}(cx)) dx + \int 2abc^6 x^6 x^m \operatorname{asin}(cx) dx \right) \end{aligned}$$

input `integrate(x**m*(-c**2*d*x**2+d)**3*(a+b*asin(c*x))**2,x)`

output `-d**3*(Integral(-a**2*x**m, x) + Integral(-b**2*x**m*asin(c*x)**2, x) + Integral(-2*a*b*x**m*asin(c*x), x) + Integral(3*a**2*c**2*x**2*x**m, x) + Integral(-3*a**2*c**4*x**4*x**m, x) + Integral(a**2*c**6*x**6*x**m, x) + Integral(3*b**2*c**2*x**2*x**m*asin(c*x)**2, x) + Integral(-3*b**2*c**4*x**4*x**m*asin(c*x)**2, x) + Integral(b**2*c**6*x**6*x**m*asin(c*x)**2, x) + Integral(6*a*b*c**2*x**2*x**m*asin(c*x), x) + Integral(-6*a*b*c**4*x**4*x**m*asin(c*x), x) + Integral(2*a*b*c**6*x**6*x**m*asin(c*x), x))`

Maxima [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

-a^2*c^6*d^3*x^(m + 7)/(m + 7) + 3*a^2*c^4*d^3*x^(m + 5)/(m + 5) - 3*a^2*c
^2*d^3*x^(m + 3)/(m + 3) + a^2*d^3*x^(m + 1)/(m + 1) - (((b^2*c^6*d^3*m^3
+ 9*b^2*c^6*d^3*m^2 + 23*b^2*c^6*d^3*m + 15*b^2*c^6*d^3)*x^7 - 3*(b^2*c^4*
d^3*m^3 + 11*b^2*c^4*d^3*m^2 + 31*b^2*c^4*d^3*m + 21*b^2*c^4*d^3)*x^5 + 3*
(b^2*c^2*d^3*m^3 + 13*b^2*c^2*d^3*m^2 + 47*b^2*c^2*d^3*m + 35*b^2*c^2*d^3)
*x^3 - (b^2*d^3*m^3 + 15*b^2*d^3*m^2 + 71*b^2*d^3*m + 105*b^2*d^3)*x)*x^m*
arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + (m^4 + 16*m^3 + 86*m^2 + 17
6*m + 105)*integrate(-2*(((b^2*c^7*d^3*m^3 + 9*b^2*c^7*d^3*m^2 + 23*b^2*c^
7*d^3*m + 15*b^2*c^7*d^3)*x^7 - 3*(b^2*c^5*d^3*m^3 + 11*b^2*c^5*d^3*m^2 +
31*b^2*c^5*d^3*m + 21*b^2*c^5*d^3)*x^5 + 3*(b^2*c^3*d^3*m^3 + 13*b^2*c^3*d
^3*m^2 + 47*b^2*c^3*d^3*m + 35*b^2*c^3*d^3)*x^3 - (b^2*c*d^3*m^3 + 15*b^2*
c*d^3*m^2 + 71*b^2*c*d^3*m + 105*b^2*c*d^3)*x)*sqrt(c*x + 1))*sqrt(-c*x + 1
)*x^m*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + (a*b*d^3*m^4 + (a*b*c^8
*d^3*m^4 + 16*a*b*c^8*d^3*m^3 + 86*a*b*c^8*d^3*m^2 + 176*a*b*c^8*d^3*m + 1
05*a*b*c^8*d^3)*x^8 + 16*a*b*d^3*m^3 + 86*a*b*d^3*m^2 - 4*(a*b*c^6*d^3*m^4
+ 16*a*b*c^6*d^3*m^3 + 86*a*b*c^6*d^3*m^2 + 176*a*b*c^6*d^3*m + 105*a*b*c
^6*d^3)*x^6 + 176*a*b*d^3*m + 105*a*b*d^3 + 6*(a*b*c^4*d^3*m^4 + 16*a*b*c^
4*d^3*m^3 + 86*a*b*c^4*d^3*m^2 + 176*a*b*c^4*d^3*m + 105*a*b*c^4*d^3)*x^4
- 4*(a*b*c^2*d^3*m^4 + 16*a*b*c^2*d^3*m^3 + 86*a*b*c^2*d^3*m^2 + 176*a*b*c
^2*d^3*m + 105*a*b*c^2*d^3)*x^2)*x^m*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c...

```

Giac [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d)^3 (b \arcsin(cx) + a)^2 x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
integrate(-(c^2*d*x^2 - d)^3*(b*arcsin(c*x) + a)^2*x^m, x)
```


Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \int x^m (a + b \operatorname{asin}(cx))^2 (d - c^2 dx^2)^3 dx$$

input `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3,x)`

output `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^3, x)`

Reduce [F]

$$\int x^m (d - c^2 dx^2)^3 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `int(x^m*(-c^2*d*x^2+d)^3*(a+b*asin(c*x))^2,x)`

output

```
(d**3*( - x**m*a**2*c**6*m**3*x**7 - 9*x**m*a**2*c**6*m**2*x**7 - 23*x**m*
a**2*c**6*m*x**7 - 15*x**m*a**2*c**6*x**7 + 3*x**m*a**2*c**4*m**3*x**5 + 3
3*x**m*a**2*c**4*m**2*x**5 + 93*x**m*a**2*c**4*m*x**5 + 63*x**m*a**2*c**4*
x**5 - 3*x**m*a**2*c**2*m**3*x**3 - 39*x**m*a**2*c**2*m**2*x**3 - 141*x**m
*a**2*c**2*m*x**3 - 105*x**m*a**2*c**2*x**3 + x**m*a**2*m**3*x + 15*x**m*a
**2*m**2*x + 71*x**m*a**2*m*x + 105*x**m*a**2*x - 2*int(x**m*asin(c*x)*x**
6,x)*a*b*c**6*m**4 - 32*int(x**m*asin(c*x)*x**6,x)*a*b*c**6*m**3 - 172*int
(x**m*asin(c*x)*x**6,x)*a*b*c**6*m**2 - 352*int(x**m*asin(c*x)*x**6,x)*a*b
*c**6*m - 210*int(x**m*asin(c*x)*x**6,x)*a*b*c**6 + 6*int(x**m*asin(c*x)*x
**4,x)*a*b*c**4*m**4 + 96*int(x**m*asin(c*x)*x**4,x)*a*b*c**4*m**3 + 516*i
nt(x**m*asin(c*x)*x**4,x)*a*b*c**4*m**2 + 1056*int(x**m*asin(c*x)*x**4,x)*
a*b*c**4*m + 630*int(x**m*asin(c*x)*x**4,x)*a*b*c**4 - 6*int(x**m*asin(c*x
)*x**2,x)*a*b*c**2*m**4 - 96*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m**3 - 51
6*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m**2 - 1056*int(x**m*asin(c*x)*x**2,
x)*a*b*c**2*m - 630*int(x**m*asin(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m*asin(
c*x),x)*a*b*m**4 + 32*int(x**m*asin(c*x),x)*a*b*m**3 + 172*int(x**m*asin(c
*x),x)*a*b*m**2 + 352*int(x**m*asin(c*x),x)*a*b*m + 210*int(x**m*asin(c*x)
,x)*a*b - int(x**m*asin(c*x)**2*x**6,x)*b**2*c**6*m**4 - 16*int(x**m*asin(
c*x)**2*x**6,x)*b**2*c**6*m**3 - 86*int(x**m*asin(c*x)**2*x**6,x)*b**2*c**
6*m**2 - 176*int(x**m*asin(c*x)**2*x**6,x)*b**2*c**6*m - 105*int(x**m*a...
```

3.270 $\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx$

Optimal result	2663
Mathematica [A] (warning: unable to verify)	2664
Rubi [A] (verified)	2665
Maple [F]	2670
Fricas [F]	2670
Sympy [F]	2671
Maxima [F]	2671
Giac [F]	2672
Mupad [F(-1)]	2672
Reduce [F]	2673

Optimal result

Integrand size = 27, antiderivative size = 756

$$\begin{aligned}
& \int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
&= \frac{6b^2 c^2 d^2 x^{3+m}}{(3+m)^2 (5+m)^2} + \frac{2b^2 c^2 d^2 x^{3+m}}{(3+m)(5+m)^2} + \frac{8b^2 c^2 d^2 x^{3+m}}{(3+m)^3 (5+m)} - \frac{2b^2 c^4 d^2 x^{5+m}}{(5+m)^3} \\
&\quad - \frac{6bcd^2 x^{2+m} \sqrt{1-c^2 x^2} (a + b \arcsin(cx))}{(3+m)(5+m)^2} - \frac{8bcd^2 x^{2+m} \sqrt{1-c^2 x^2} (a + b \arcsin(cx))}{(3+m)^2 (5+m)} \\
&\quad - \frac{2bcd^2 x^{2+m} (1-c^2 x^2)^{3/2} (a + b \arcsin(cx))}{(5+m)^2} + \frac{8d^2 x^{1+m} (a + b \arcsin(cx))^2}{(5+m)(3+4m+m^2)} \\
&\quad + \frac{4d^2 x^{1+m} (1-c^2 x^2) (a + b \arcsin(cx))^2}{15+8m+m^2} + \frac{d^2 x^{1+m} (1-c^2 x^2)^2 (a + b \arcsin(cx))^2}{5+m} \\
&\quad - \frac{8bcd^2 x^{2+m} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{(2+m)(3+m)^2 (5+m)} \\
&\quad - \frac{6bcd^2 x^{2+m} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{(5+m)^2 (6+5m+m^2)} \\
&\quad - \frac{16bcd^2 x^{2+m} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{(5+m)(6+11m+6m^2+m^3)} \\
&\quad + \frac{6b^2 c^2 d^2 x^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; c^2 x^2\right)}{(2+m)(3+m)^2 (5+m)^2} \\
&\quad + \frac{8b^2 c^2 d^2 x^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; c^2 x^2\right)}{(2+m)(3+m)^3 (5+m)} \\
&\quad + \frac{16b^2 c^2 d^2 x^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; c^2 x^2\right)}{(3+m)^2 (5+m)(2+3m+m^2)}
\end{aligned}$$

output

```

6*b^2*c^2*d^2*x^(3+m)/(3+m)^2/(5+m)^2+2*b^2*c^2*d^2*x^(3+m)/(3+m)/(5+m)^2+
8*b^2*c^2*d^2*x^(3+m)/(3+m)^3/(5+m)-2*b^2*c^4*d^2*x^(5+m)/(5+m)^3-6*b*c*d^
2*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/(3+m)/(5+m)^2-8*b*c*d^2*x^(
2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/(3+m)^2/(5+m)-2*b*c*d^2*x^(2+m)*
(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))/(5+m)^2+8*d^2*x^(1+m)*(a+b*arcsin(c*x
))^2/(5+m)/(m^2+4*m+3)+4*d^2*x^(1+m)*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/(m^2
+8*m+15)+d^2*x^(1+m)*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/(5+m)-8*b*c*d^2*x^
(2+m)*(a+b*arcsin(c*x))*hypergeom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)/(2+m)/
(3+m)^2/(5+m)-6*b*c*d^2*x^(2+m)*(a+b*arcsin(c*x))*hypergeom([1/2, 1+1/2*m]
, [2+1/2*m], c^2*x^2)/(5+m)^2/(m^2+5*m+6)-16*b*c*d^2*x^(2+m)*(a+b*arcsin(c*x
))*hypergeom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)/(5+m)/(m^3+6*m^2+11*m+6)+6*
b^2*c^2*d^2*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m], [2+1/2*m, 5/2+1/2*
m], c^2*x^2)/(2+m)/(3+m)^2/(5+m)^2+8*b^2*c^2*d^2*x^(3+m)*hypergeom([1, 3/2+
1/2*m, 3/2+1/2*m], [2+1/2*m, 5/2+1/2*m], c^2*x^2)/(2+m)/(3+m)^3/(5+m)+16*b^2
*c^2*d^2*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m], [2+1/2*m, 5/2+1/2*m]
, c^2*x^2)/(3+m)^2/(5+m)/(m^2+3*m+2)

```

Mathematica [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.53

$$\begin{aligned}
& \int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
&= d^2 x^{1+m} \left(\frac{(a + b \arcsin(cx))^2}{1+m} - \frac{2c^2 x^2 (a + b \arcsin(cx))^2}{3+m} + \frac{c^4 x^4 (a + b \arcsin(cx))^2}{5+m} \right. \\
&+ \frac{2bcx \left(-((3+m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right) + bcx {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}, c^2 x^2\right) \right)}{(1+m)(2+m)(3+m)} \\
&- \frac{4bc^3 x^3 \left(-((5+m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, c^2 x^2\right) + bcx {}_3F_2\left(1, \frac{5}{2} + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}, \frac{7}{2} + \frac{m}{2}, c^2 x^2\right) \right)}{(3+m)(4+m)(5+m)} \\
&+ \left. \frac{2bc^5 x^5 \left(-((7+m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{6+m}{2}, \frac{8+m}{2}, c^2 x^2\right) + bcx {}_3F_2\left(1, \frac{7}{2} + \frac{m}{2}, \frac{7}{2} + \frac{m}{2}, \frac{9}{2} + \frac{m}{2}, c^2 x^2\right) \right)}{(5+m)(6+m)(7+m)} \right)
\end{aligned}$$

input

```
Integrate[x^m*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]
```

output

```

d^2*x^(1 + m)*((a + b*ArcSin[c*x])^2/(1 + m) - (2*c^2*x^2*(a + b*ArcSin[c*
x])^2)/(3 + m) + (c^4*x^4*(a + b*ArcSin[c*x])^2)/(5 + m) + (2*b*c*x*(-((3
+ m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*
x^2]) + b*c*x*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 +
m/2}, c^2*x^2]))/((1 + m)*(2 + m)*(3 + m)) - (4*b*c^3*x^3*(-((5 + m)*(a +
b*ArcSin[c*x])*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2]) + b
*c*x*HypergeometricPFQ[{1, 5/2 + m/2, 5/2 + m/2}, {3 + m/2, 7/2 + m/2}, c^
2*x^2]))/((3 + m)*(4 + m)*(5 + m)) + (2*b*c^5*x^5*(-((7 + m)*(a + b*ArcSin
[c*x])*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2]) + b*c*x*Hype
rgeometricPFQ[{1, 7/2 + m/2, 7/2 + m/2}, {4 + m/2, 9/2 + m/2}, c^2*x^2]))/
((5 + m)*(6 + m)*(7 + m))

```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5202, 27, 5202, 244, 2009, 5138, 5198, 15, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow 5202 \\
 & -\frac{2bcd^2 \int x^{m+1} (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx}{m+5} + \\
 & \frac{4d \int dx^m (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx}{m+5} + \frac{d^2 (1 - c^2 x^2)^2 x^{m+1} (a + b \arcsin(cx))^2}{m+5} \\
 & \quad \downarrow 27 \\
 & -\frac{2bcd^2 \int x^{m+1} (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx)) dx}{m+5} + \\
 & \frac{4d^2 \int x^m (1 - c^2 x^2) (a + b \arcsin(cx))^2 dx}{m+5} + \frac{d^2 (1 - c^2 x^2)^2 x^{m+1} (a + b \arcsin(cx))^2}{m+5} \\
 & \quad \downarrow 5202
 \end{aligned}$$

$$\frac{2bcd^2 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} - \frac{bc \int x^{m+2} (1-c^2x^2) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} \right)}{+}$$

$$\frac{4d^2 \left(-\frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} + \frac{2 \int x^m (a+b \arcsin(cx))^2 dx}{m+3} + \frac{(1-c^2x^2) x^{m+1} (a+b \arcsin(cx))^2}{m+3} \right)}{+}$$

$$\frac{d^2 (1-c^2x^2)^2 x^{m+1} (a+b \arcsin(cx))^2}{m+5}$$

↓ 244

$$\frac{2bcd^2 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} - \frac{bc \int (x^{m+2} - c^2 x^{m+4}) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} \right)}{+}$$

$$\frac{4d^2 \left(-\frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} + \frac{2 \int x^m (a+b \arcsin(cx))^2 dx}{m+3} + \frac{(1-c^2x^2) x^{m+1} (a+b \arcsin(cx))^2}{m+3} \right)}{+}$$

$$\frac{d^2 (1-c^2x^2)^2 x^{m+1} (a+b \arcsin(cx))^2}{m+5}$$

↓ 2009

$$\frac{2bcd^2 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2 x^{m+5}}{m+5} \right)}{m+5} \right)}{+}$$

$$\frac{4d^2 \left(-\frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} + \frac{2 \int x^m (a+b \arcsin(cx))^2 dx}{m+3} + \frac{(1-c^2x^2) x^{m+1} (a+b \arcsin(cx))^2}{m+3} \right)}{+}$$

$$\frac{d^2 (1-c^2x^2)^2 x^{m+1} (a+b \arcsin(cx))^2}{m+5}$$

↓ 5138

$$\frac{4d^2 \left(\frac{2 \left(\frac{x^{m+1} (a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1} (a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right)}{m+3} - \frac{2bc \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+3} + \frac{(1-c^2x^2) x^{m+1} (a+b \arcsin(cx))^2}{m+3} \right)}{+}$$

$$\frac{2bcd^2 \left(\frac{3 \int x^{m+1} \sqrt{1-c^2x^2} (a+b \arcsin(cx)) dx}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2} (a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2 x^{m+5}}{m+5} \right)}{m+5} \right)}{+}$$

$$\frac{d^2 (1-c^2x^2)^2 x^{m+1} (a+b \arcsin(cx))^2}{m+5}$$

↓ 5198

$$4d^2 \left(\frac{2 \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right)}{m+3} - \frac{2bc \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} - \frac{bc \int x^{m+2} dx}{m+3} + \frac{\sqrt{1-c^2x^2} x^{m+2}(a+b \arcsin(cx))}{m+3} \right)}{m+3} \right)$$

$$2bcd^2 \left(\frac{3 \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} - \frac{bc \int x^{m+2} dx}{m+3} + \frac{\sqrt{1-c^2x^2} x^{m+2}(a+b \arcsin(cx))}{m+3} \right)}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2}(a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2}{m+5} \right)}{m+5} \right)$$

$$\frac{d^2(1-c^2x^2)^2 x^{m+1}(a+b \arcsin(cx))^2}{m+5}$$

↓ 15

$$4d^2 \left(\frac{2 \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right)}{m+3} - \frac{2bc \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} + \frac{\sqrt{1-c^2x^2} x^{m+2}(a+b \arcsin(cx))}{m+3} - \frac{bcx^{m+3}}{(m+3)^2} \right)}{m+3} \right)$$

$$2bcd^2 \left(\frac{3 \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} + \frac{\sqrt{1-c^2x^2} x^{m+2}(a+b \arcsin(cx))}{m+3} - \frac{bcx^{m+3}}{(m+3)^2} \right)}{m+5} + \frac{(1-c^2x^2)^{3/2} x^{m+2}(a+b \arcsin(cx))}{m+5} - \frac{bc \left(\frac{x^{m+3}}{m+3} - \frac{c^2 x^{m+5}}{m+5} \right)}{m+5} \right)$$

$$\frac{d^2(1-c^2x^2)^2 x^{m+1}(a+b \arcsin(cx))^2}{m+5}$$

↓ 5220

$$\begin{aligned}
 & \left(\frac{2 \left(\frac{x^{m+1} (a+b \arcsin(cx))^2}{m+1} - \frac{2bc \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right) (a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2 x^2\right)}{m^2 + 5m + 6} \right)}{m+1}}{m+3} \right. \\
 & \left. + \frac{2bcd^2 \left(\frac{3 \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right) (a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2 x^2\right)}{m^2 + 5m + 6} \right)}{m+3} + \frac{\sqrt{1-c^2 x^2} x^{m+2} (a+b \arcsin(cx))}{m+3} \right)}{m+5} \right) \\
 & \frac{d^2 (1-c^2 x^2)^2 x^{m+1} (a+b \arcsin(cx))^2}{m+5}
 \end{aligned}$$

input `Int [x^m*(d - c^2*d*x^2)^2*(a + b*ArcSin[c*x])^2,x]`

output

```

(d^2*x^(1+m)*(1-c^2*x^2)^2*(a+b*ArcSin[c*x])^2)/(5+m) + (4*d^2*((x
^(1+m)*(1-c^2*x^2)*(a+b*ArcSin[c*x])^2)/(3+m) + (2*((x^(1+m)*(a
+b*ArcSin[c*x])^2)/(1+m) - (2*b*c*((x^(2+m)*(a+b*ArcSin[c*x])*Hyper
geometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(2+m) - (b*c*x^(3+m)
*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, c^2*x^
2])/(6+5*m+m^2)))/(1+m)))/(3+m) - (2*b*c*(-((b*c*x^(3+m))/(3+m)
)^2) + (x^(2+m)*Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])))/(3+m) + ((x^(2
+m)*(a+b*ArcSin[c*x])*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*
x^2])/(2+m) - (b*c*x^(3+m)*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}
, {2+m/2, 5/2+m/2}, c^2*x^2])/(6+5*m+m^2))/(3+m))/(3+m))/(5
+m) - (2*b*c*d^2*(-((b*c*(x^(3+m))/(3+m) - (c^2*x^(5+m))/(5+m)))/(
5+m) + (x^(2+m)*(1-c^2*x^2)^(3/2)*(a+b*ArcSin[c*x])))/(5+m) + (3
*(-((b*c*x^(3+m))/(3+m)^2) + (x^(2+m)*Sqrt[1-c^2*x^2]*(a+b*ArcSi
n[c*x])))/(3+m) + ((x^(2+m)*(a+b*ArcSin[c*x])*Hypergeometric2F1[1/2,
(2+m)/2, (4+m)/2, c^2*x^2])/(2+m) - (b*c*x^(3+m)*HypergeometricPFQ
[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, c^2*x^2])/(6+5*m+m^2)
))/(3+m))/(5+m))/(5+m)
    
```

Defintions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 27 $\text{Int}[(a_)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \ \text{Int}[Fx, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ \text{!MatchQ}[Fx, (b_)(Gx_)] \text{ ; FreeQ}[b, x]$
- rule 244 $\text{Int}[((c_.)(x_))^{(m_.)}*((a_) + (b_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, m\}, x] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$
- rule 5138 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]), x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5198 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((f_.)(x_))^{(m_.)}*\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+2))), x] + (\text{Simp}[(1/(m+2))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^m*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m+2)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])$
- rule 5202 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((f_.)(x_))^{(m_.)}*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^p*((a + b*\text{ArcSin}[c*x])^n/(f*(m+2*p+1))), x] + (\text{Simp}[2*d*(p/(m+2*p+1)) \ \text{Int}[(f*x)^m*(d + e*x^2)^{(p-1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p-1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{!LtQ}[m, -1]$

rule 5220

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_)]/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] := Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 - c^2*
x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, c^2*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] && !IntegerQ[m]
```

Maple [F]

$$\int x^m (-c^2 d x^2 + d)^2 (a + b \arcsin(cx))^2 dx$$

input

```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x)
```

output

```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x)
```

Fricas [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2 x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x))^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*x^m, x)
```

Sympy [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = d^2 \left(\int a^2 x^m dx + \int b^2 x^m \arcsin^2(cx) dx \right. \\ \left. + \int 2abx^m \arcsin(cx) dx \right. \\ \left. + \int (-2a^2c^2x^2x^m) dx + \int a^2c^4x^4x^m dx \right. \\ \left. + \int (-2b^2c^2x^2x^m \arcsin^2(cx)) dx \right. \\ \left. + \int b^2c^4x^4x^m \arcsin^2(cx) dx \right. \\ \left. + \int (-4abc^2x^2x^m \arcsin(cx)) dx \right. \\ \left. + \int 2abc^4x^4x^m \arcsin(cx) dx \right)$$

input `integrate(x**m*(-c**2*d*x**2+d)**2*(a+b*asin(c*x))**2,x)`

output `d**2*(Integral(a**2*x**m, x) + Integral(b**2*x**m*asin(c*x)**2, x) + Integral(2*a*b*x**m*asin(c*x), x) + Integral(-2*a**2*c**2*x**2*x**m, x) + Integral(a**2*c**4*x**4*x**m, x) + Integral(-2*b**2*c**2*x**2*x**m*asin(c*x)**2, x) + Integral(b**2*c**4*x**4*x**m*asin(c*x)**2, x) + Integral(-4*a*b*c**2*x**2*x**m*asin(c*x), x) + Integral(2*a*b*c**4*x**4*x**m*asin(c*x), x))`

Maxima [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```

a^2*c^4*d^2*x^(m + 5)/(m + 5) - 2*a^2*c^2*d^2*x^(m + 3)/(m + 3) + a^2*d^2*
x^(m + 1)/(m + 1) + (((b^2*c^4*d^2*m^2 + 4*b^2*c^4*d^2*m + 3*b^2*c^4*d^2)*
x^5 - 2*(b^2*c^2*d^2*m^2 + 6*b^2*c^2*d^2*m + 5*b^2*c^2*d^2)*x^3 + (b^2*d^2
*m^2 + 8*b^2*d^2*m + 15*b^2*d^2)*x)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c
*x + 1))^2 + (m^3 + 9*m^2 + 23*m + 15)*integrate(-2*(((b^2*c^5*d^2*m^2 + 4
*b^2*c^5*d^2*m + 3*b^2*c^5*d^2)*x^5 - 2*(b^2*c^3*d^2*m^2 + 6*b^2*c^3*d^2*m
+ 5*b^2*c^3*d^2)*x^3 + (b^2*c*d^2*m^2 + 8*b^2*c*d^2*m + 15*b^2*c*d^2)*x)*
sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)
) - (a*b*d^2*m^3 - (a*b*c^6*d^2*m^3 + 9*a*b*c^6*d^2*m^2 + 23*a*b*c^6*d^2*m
+ 15*a*b*c^6*d^2)*x^6 + 9*a*b*d^2*m^2 + 23*a*b*d^2*m + 3*(a*b*c^4*d^2*m^3
+ 9*a*b*c^4*d^2*m^2 + 23*a*b*c^4*d^2*m + 15*a*b*c^4*d^2)*x^4 + 15*a*b*d^2
- 3*(a*b*c^2*d^2*m^3 + 9*a*b*c^2*d^2*m^2 + 23*a*b*c^2*d^2*m + 15*a*b*c^2*
d^2)*x^2)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))/(m^3 - (c^2*m^3
+ 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x)/(m^3 + 9*m^
2 + 23*m + 15)

```

Giac [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int (c^2 dx^2 - d)^2 (b \arcsin(cx) + a)^2 x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
integrate((c^2*d*x^2 - d)^2*(b*arcsin(c*x) + a)^2*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \int x^m (a + b \arcsin(cx))^2 (d - c^2 dx^2)^2 dx$$

input

```
int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output

```
int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^m (d - c^2 dx^2)^2 (a + b \arcsin(cx))^2 dx = \text{Too large to display}$$

input `int(x^m*(-c^2*d*x^2+d)^2*(a+b*asin(c*x))^2,x)`

output

```
(d**2*(x**m*a**2*c**4*m**2*x**5 + 4*x**m*a**2*c**4*m*x**5 + 3*x**m*a**2*c**4*x**5 - 2*x**m*a**2*c**2*m**2*x**3 - 12*x**m*a**2*c**2*m*x**3 - 10*x**m*a**2*c**2*x**3 + x**m*a**2*m**2*x + 8*x**m*a**2*m*x + 15*x**m*a**2*x + 2*int(x**m*asin(c*x)*x**4,x)*a*b*c**4*m**3 + 18*int(x**m*asin(c*x)*x**4,x)*a*b*c**4*m**2 + 46*int(x**m*asin(c*x)*x**4,x)*a*b*c**4*m + 30*int(x**m*asin(c*x)*x**4,x)*a*b*c**4 - 4*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m**3 - 36*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m**2 - 92*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m - 60*int(x**m*asin(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m*asin(c*x),x)*a*b*m**3 + 18*int(x**m*asin(c*x),x)*a*b*m**2 + 46*int(x**m*asin(c*x),x)*a*b*m + 30*int(x**m*asin(c*x),x)*a*b + int(x**m*asin(c*x)**2*x**4,x)*b**2*c**4*m**3 + 9*int(x**m*asin(c*x)**2*x**4,x)*b**2*c**4*m**2 + 23*int(x**m*asin(c*x)**2*x**4,x)*b**2*c**4*m + 15*int(x**m*asin(c*x)**2*x**4,x)*b**2*c**4 - 2*int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2*m**3 - 18*int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2*m**2 - 46*int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2*m - 30*int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*asin(c*x)**2,x)*b**2*m**3 + 9*int(x**m*asin(c*x)**2,x)*b**2*m**2 + 23*int(x**m*asin(c*x)**2,x)*b**2*m + 15*int(x**m*asin(c*x)**2,x)*b**2))/ (m**3 + 9*m**2 + 23*m + 15)
```

3.271 $\int x^m(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$

Optimal result	2674
Mathematica [A] (warning: unable to verify)	2675
Rubi [A] (verified)	2676
Maple [F]	2679
Fricas [F]	2679
Sympy [F]	2679
Maxima [F]	2680
Giac [F]	2680
Mupad [F(-1)]	2681
Reduce [F]	2681

Optimal result

Integrand size = 25, antiderivative size = 371

$$\begin{aligned}
 & \int x^m(d - c^2 dx^2) (a + b \arcsin(cx))^2 dx \\
 &= \frac{2b^2 c^2 dx^{3+m}}{(3+m)^3} - \frac{2bcdx^{2+m} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))}{(3+m)^2} \\
 &+ \frac{2dx^{1+m} (a + b \arcsin(cx))^2}{3 + 4m + m^2} + \frac{dx^{1+m} (1 - c^2 x^2) (a + b \arcsin(cx))^2}{3 + m} \\
 &- \frac{2bcdx^{2+m} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{(2+m)(3+m)^2} \\
 &- \frac{4bcdx^{2+m} (a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right)}{6 + 11m + 6m^2 + m^3} \\
 &+ \frac{2b^2 c^2 dx^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; c^2 x^2\right)}{(2+m)(3+m)^3} \\
 &+ \frac{4b^2 c^2 dx^{3+m} {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}; 2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}; c^2 x^2\right)}{(3+m)^2 (2 + 3m + m^2)}
 \end{aligned}$$

output

```

2*b^2*c^2*d*x^(3+m)/(3+m)^3-2*b*c*d*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin
(c*x))/(3+m)^2+2*d*x^(1+m)*(a+b*arcsin(c*x))^2/(m^2+4*m+3)+d*x^(1+m)*(-c^2
*x^2+1)*(a+b*arcsin(c*x))^2/(3+m)-2*b*c*d*x^(2+m)*(a+b*arcsin(c*x))*hyperg
eom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)/(2+m)/(3+m)^2-4*b*c*d*x^(2+m)*(a+b*a
rcsin(c*x))*hypergeom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)/(m^3+6*m^2+11*m+6)
+2*b^2*c^2*d*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m], [2+1/2*m, 5/2+1/2
*m], c^2*x^2)/(2+m)/(3+m)^3+4*b^2*c^2*d*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/
2+1/2*m], [2+1/2*m, 5/2+1/2*m], c^2*x^2)/(3+m)^2/(m^2+3*m+2)

```

Mathematica [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

$$\begin{aligned}
& \int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx \\
&= dx^{1+m} \left(\frac{(a + b \arcsin(cx))^2}{1+m} - \frac{c^2 x^2 (a + b \arcsin(cx))^2}{3+m} \right. \\
&\quad + \frac{2bcx \left(-((3+m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right) + bcx {}_3F_2\left(1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}, c^2 x^2\right) \right)}{(1+m)(2+m)(3+m)} \\
&\quad \left. - \frac{2bc^3 x^3 \left(-((5+m)(a + b \arcsin(cx)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, c^2 x^2\right) + bcx {}_3F_2\left(1, \frac{5}{2} + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}, \frac{7}{2} + \frac{m}{2}, \frac{7}{2} + \frac{m}{2}, c^2 x^2\right) \right)}{(3+m)(4+m)(5+m)} \right)
\end{aligned}$$

input

```
Integrate[x^m*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

```

d*x^(1+m)*((a + b*ArcSin[c*x])^2/(1+m) - (c^2*x^2*(a + b*ArcSin[c*x])^2)/(3+m) +
(2*b*c*x*(-((3+m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2]) +
b*c*x*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2]))/((1+m)*(2+m)*(3+m)) -
(2*b*c^3*x^3*(-((5+m)*(a + b*ArcSin[c*x])*Hypergeometric2F1[1/2, (4+m)/2, (6+m)/2, c^2*x^2]) +
b*c*x*HypergeometricPFQ[{1, 5/2 + m/2, 5/2 + m/2}, {3 + m/2, 7/2 + m/2}, c^2*x^2]))/((3+m)*(4+m)*(5+m))

```


Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5202, 5138, 5198, 15, 5220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & - \frac{2bcd \int x^{m+1} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx}{\frac{m+3}{d(1 - c^2 x^2)} x^{m+1} (a + b \arcsin(cx))^2} + \frac{2d \int x^m (a + b \arcsin(cx))^2 dx}{m+3} + \\
 & \quad \downarrow \text{5138} \\
 & \frac{2d \left(\frac{x^{m+1} (a + b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1} (a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} \right)}{m+3} - \\
 & \frac{2bcd \int x^{m+1} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) dx}{m+3} + \frac{d(1 - c^2 x^2) x^{m+1} (a + b \arcsin(cx))^2}{m+3} \\
 & \quad \downarrow \text{5198} \\
 & \frac{2d \left(\frac{x^{m+1} (a + b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1} (a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}} \right)}{m+3} - \\
 & \frac{2bcd \left(\frac{\int \frac{x^{m+1} (a + b \arcsin(cx)) dx}{\sqrt{1 - c^2 x^2}}}{m+3} - \frac{bc \int x^{m+2} dx}{m+3} + \frac{\sqrt{1 - c^2 x^2} x^{m+2} (a + b \arcsin(cx))}{m+3} \right)}{m+3} + \\
 & \frac{d(1 - c^2 x^2) x^{m+1} (a + b \arcsin(cx))^2}{m+3} \\
 & \quad \downarrow \text{15}
 \end{aligned}$$

$$\frac{2d \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+1} \right)}{m+3} - \frac{2bcd \left(\frac{\int \frac{x^{m+1}(a+b \arcsin(cx)) dx}{\sqrt{1-c^2x^2}}}{m+3} + \frac{\sqrt{1-c^2x^2}x^{m+2}(a+b \arcsin(cx))}{m+3} - \frac{bcx^{m+3}}{(m+3)^2} \right)}{m+3} + \frac{d(1-c^2x^2)x^{m+1}(a+b \arcsin(cx))^2}{m+3}$$

↓ 5220

$$\frac{2d \left(\frac{x^{m+1}(a+b \arcsin(cx))^2}{m+1} - \frac{2bc \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)(a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2x^2\right)}{m^2+5m+6} \right)}{m+1} \right)}{m+3} - \frac{2bcd \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)(a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2x^2\right)}{m^2+5m+6} + \frac{\sqrt{1-c^2x^2}x^{m+2}(a+b \arcsin(cx))}{m+3} \right)}{m+3} + \frac{d(1-c^2x^2)x^{m+1}(a+b \arcsin(cx))^2}{m+3}$$

input

```
Int[x^m*(d - c^2*d*x^2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d*x^(1+m)*(1-c^2*x^2)*(a+b*ArcSin[c*x])^2)/(3+m) + (2*d*((x^(1+m)*(a+b*ArcSin[c*x])^2)/(1+m) - (2*b*c*((x^(2+m)*(a+b*ArcSin[c*x]) *Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(2+m) - (b*c*x^(3+m)*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, c^2*x^2])/(6+5*m+m^2)))/(1+m)))/(3+m) - (2*b*c*d*(-((b*c*x^(3+m))/(3+m)^2) + (x^(2+m)*Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x]))/(3+m) + ((x^(2+m)*(a+b*ArcSin[c*x])*Hypergeometric2F1[1/2, (2+m)/2, (4+m)/2, c^2*x^2])/(2+m) - (b*c*x^(3+m)*HypergeometricPFQ[{1, 3/2+m/2, 3/2+m/2}, {2+m/2, 5/2+m/2}, c^2*x^2])/(6+5*m+m^2)))/(3+m)))/(3+m)
```

Definitions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] /; \text{FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5138 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((d_.)(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]], x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5198 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((f_.)(x_))^{(m_)}*\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+2))), x] + (\text{Simp}[(1/(m+2))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^m*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m+2)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])$
- rule 5202 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))^{(n_.)}*((f_.)(x_))^{(m_)}*((d_) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(d + e*x^2)^p*((a + b*\text{ArcSin}[c*x])^n/(f*(m+2*p+1))), x] + (\text{Simp}[2*d*(p/(m+2*p+1)) \ \text{Int}[(f*x)^m*(d + e*x^2)^{(p-1)}*(a + b*\text{ArcSin}[c*x])^n, x], x] - \text{Simp}[b*c*(n/(f*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m+1)}*(1 - c^2*x^2)^{(p-1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ !\text{LtQ}[m, -1]$
- rule 5220 $\text{Int}[(((a_.) + \text{ArcSin}[(c_.)(x_)]*(b_.))*((f_.)(x_))^{(m_)})/\text{Sqrt}[(d_) + (e_.)(x_)^2], x_Symbol] \rightarrow \text{Simp}[((f*x)^{(m+1)})/(f*(m+1))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2], x] - \text{Simp}[b*c*((f*x)^{(m+2)})/(f^2*(m+1)*(m+2))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*HypergeometricPFQ[\{1, 1+m/2, 1+m/2\}, \{3/2+m/2, 2+m/2\}, c^2*x^2], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ !\text{IntegerQ}[m]$

Maple [F]

$$\int x^m (-c^2 d x^2 + d) (a + b \arcsin(cx))^2 dx$$

input `int(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x)`

Fricas [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d) (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*x^m, x)`

Sympy [F]

$$\begin{aligned} \int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = & -d \left(\int (-a^2 x^m) dx \right. \\ & + \int (-b^2 x^m \operatorname{asin}^2(cx)) dx \\ & + \int (-2abx^m \operatorname{asin}(cx)) dx \\ & + \int a^2 c^2 x^2 x^m dx + \int b^2 c^2 x^2 x^m \operatorname{asin}^2(cx) dx \\ & \left. + \int 2abc^2 x^2 x^m \operatorname{asin}(cx) dx \right) \end{aligned}$$

input `integrate(x**m*(-c**2*d*x**2+d)*(a+b*asin(c*x))**2,x)`

output

```
-d*(Integral(-a**2*x**m, x) + Integral(-b**2*x**m*asin(c*x)**2, x) + Integral(-2*a*b*x**m*asin(c*x), x) + Integral(a**2*c**2*x**2*x**m, x) + Integral(b**2*c**2*x**2*x**m*asin(c*x)**2, x) + Integral(2*a*b*c**2*x**2*x**m*asin(c*x), x))
```

Maxima [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d) (b \arcsin(cx) + a)^2 x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```
-a^2*c^2*d*x^(m + 3)/(m + 3) + a^2*d*x^(m + 1)/(m + 1) - (((b^2*c^2*d*m + b^2*c^2*d)*x^3 - (b^2*d*m + 3*b^2*d)*x)*x^m*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + (m^2 + 4*m + 3)*integrate(2*(((b^2*c^3*d*m + b^2*c^3*d)*x^3 - (b^2*c*d*m + 3*b^2*c*d)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + (a*b*d*m^2 + (a*b*c^4*d*m^2 + 4*a*b*c^4*d*m + 3*a*b*c^4*d)*x^4 + 4*a*b*d*m + 3*a*b*d - 2*(a*b*c^2*d*m^2 + 4*a*b*c^2*d*m + 3*a*b*c^2*d)*x^2)*x^m*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x)/(m^2 + 4*m + 3)
```

Giac [F]

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int -(c^2 dx^2 - d) (b \arcsin(cx) + a)^2 x^m dx$$

input

```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
integrate(-(c^2*d*x^2 - d)*(b*arcsin(c*x) + a)^2*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx = \int x^m (a + b \arcsin(cx))^2 (d - c^2 dx^2) dx$$

input `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`output `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2), x)`**Reduce [F]**

$$\int x^m (d - c^2 dx^2) (a + b \arcsin(cx))^2 dx$$

$$= \frac{d(-x^m a^2 c^2 m x^3 - x^m a^2 c^2 x^3 + x^m a^2 m x + 3x^m a^2 x - 2(\int x^m \arcsin(cx) x^2 dx) a b c^2 m^2 - 8(\int x^m \arcsin(cx) a$$

input `int(x^m*(-c^2*d*x^2+d)*(a+b*asin(c*x))^2,x)`output `(d*(-x**m*a**2*c**2*m*x**3 - x**m*a**2*c**2*x**3 + x**m*a**2*m*x + 3*x**m*a**2*x - 2*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m**2 - 8*int(x**m*asin(c*x)*x**2,x)*a*b*c**2*m - 6*int(x**m*asin(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m*asin(c*x),x)*a*b*m**2 + 8*int(x**m*asin(c*x),x)*a*b*m + 6*int(x**m*asin(c*x),x)*a*b - int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2*m**2 - 4*int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2*m - 3*int(x**m*asin(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*asin(c*x)**2,x)*b**2*m**2 + 4*int(x**m*asin(c*x)**2,x)*b**2*m + 3*int(x**m*asin(c*x)**2,x)*b**2))/ (m**2 + 4*m + 3)`

3.272 $\int \frac{x^m(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$

Optimal result	2682
Mathematica [N/A]	2682
Rubi [N/A]	2683
Maple [N/A]	2683
Fricas [N/A]	2684
Sympy [N/A]	2684
Maxima [N/A]	2685
Giac [F(-2)]	2685
Mupad [N/A]	2685
Reduce [N/A]	2686

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(a+b \arcsin(cx))^2}{d-c^2dx^2} dx = \text{Int}\left(\frac{x^m(a+b \arcsin(cx))^2}{d-c^2dx^2}, x\right)$$

output

```
Defer(Int)(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b \arcsin(cx))^2}{d-c^2dx^2} dx = \int \frac{x^m(a+b \arcsin(cx))^2}{d-c^2dx^2} dx$$

input

```
Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2),x]
```

output

```
Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

↓ 5234

$$\int \frac{x^m(a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

input `Int[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2), x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a + b \arcsin(cx))^2}{-c^2 dx^2 + d} dx$$

input `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d), x)`

output `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d), x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^m (a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^m}{c^2 dx^2 - d} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m/(c^2*d*x^2 - d), x)`

Sympy [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \frac{x^m (a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = -\frac{\int \frac{a^2 x^m}{c^2 x^2 - 1} dx + \int \frac{b^2 x^m \arcsin^2(cx)}{c^2 x^2 - 1} dx + \int \frac{2abx^m \arcsin(cx)}{c^2 x^2 - 1} dx}{d}$$

input `integrate(x**m*(a+b*asin(c*x))**2/(-c**2*d*x**2+d),x)`

output `-(Integral(a**2*x**m/(c**2*x**2 - 1), x) + Integral(b**2*x**m*asin(c*x)**2/(c**2*x**2 - 1), x) + Integral(2*a*b*x**m*asin(c*x)/(c**2*x**2 - 1), x))/d`

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^m (a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^m}{c^2 dx^2 - d} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")`

output `-integrate((b*arcsin(c*x) + a)^2*x^m/(c^2*d*x^2 - d), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{d - c^2 dx^2} dx = \int \frac{x^m (a + b \operatorname{asin}(cx))^2}{d - c^2 dx^2} dx$$

input `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2),x)`

output `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \frac{x^m (a + b \arcsin(cx))^2}{d - c^2 dx^2} dx$$

$$= \frac{-\left(\int \frac{x^m}{c^2 x^2 - 1} dx\right) a^2 - 2\left(\int \frac{x^m \arcsin(cx)}{c^2 x^2 - 1} dx\right) ab - \left(\int \frac{x^m \arcsin(cx)^2}{c^2 x^2 - 1} dx\right) b^2}{d}$$

input `int(x^m*(a+b*asin(c*x))^2/(-c^2*d*x^2+d), x)`

output `(- int(x**m/(c**2*x**2 - 1),x)*a**2 - 2*int((x**m*asin(c*x))/(c**2*x**2 - 1),x)*a*b - int((x**m*asin(c*x)**2)/(c**2*x**2 - 1),x)*b**2)/d`

$$3.273 \quad \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

Optimal result	2687
Mathematica [N/A]	2687
Rubi [N/A]	2688
Maple [N/A]	2689
Fricas [N/A]	2690
Sympy [N/A]	2690
Maxima [N/A]	2691
Giac [F(-2)]	2691
Mupad [N/A]	2691
Reduce [N/A]	2692

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \text{Int}\left(\frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2}, x\right)$$

output `Defer(Int)(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

Mathematica [N/A]

Not integrable

Time = 6.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

input `Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]`

output `Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2, x]`

Rubi [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx \\
 & \quad \downarrow \text{5208} \\
 & -\frac{bc \int \frac{x^{m+1}(a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx}{d^2} + \frac{(1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{d(1-c^2 x^2)} dx}{2d} + \frac{x^{m+1}(a+b \arcsin(cx))^2}{2d^2(1-c^2 x^2)} \\
 & \quad \downarrow \text{27} \\
 & -\frac{bc \int \frac{x^{m+1}(a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx}{d^2} + \frac{(1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx}{2d^2} + \frac{x^{m+1}(a+b \arcsin(cx))^2}{2d^2(1-c^2 x^2)} \\
 & \quad \downarrow \text{5208} \\
 & -\frac{bc \left(-(m+1) \int \frac{x^{m+1}(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx - bc \int \frac{x^{m+2}}{1-c^2 x^2} dx + \frac{x^{m+2}(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} \right)}{d^2} + \\
 & \quad \frac{(1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx}{2d^2} + \frac{x^{m+1}(a+b \arcsin(cx))^2}{2d^2(1-c^2 x^2)} \\
 & \quad \downarrow \text{278} \\
 & -\frac{bc \left(-(m+1) \int \frac{x^{m+1}(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} dx + \frac{x^{m+2}(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{m+3} \right)}{d^2} + \\
 & \quad \frac{(1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx}{2d^2} + \frac{x^{m+1}(a+b \arcsin(cx))^2}{2d^2(1-c^2 x^2)} \\
 & \quad \downarrow \text{5220}
 \end{aligned}$$

$$\frac{(1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx}{2d^2} - \frac{bc \left(-(m+1) \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right) (a+b \arcsin(cx))}{m+2} - \frac{bc x^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2 x^2\right)}{m^2 + 5m + 6} \right) + x^m}{d^2}}{d^2}$$

$$\frac{x^{m+1} (a+b \arcsin(cx))^2}{2d^2 (1-c^2 x^2)}$$

↓ 5234

$$\frac{(1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx}{2d^2} - \frac{bc \left(-(m+1) \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2\right) (a+b \arcsin(cx))}{m+2} - \frac{bc x^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}; c^2 x^2\right)}{m^2 + 5m + 6} \right) + x^m}{d^2}}{d^2}$$

$$\frac{x^{m+1} (a+b \arcsin(cx))^2}{2d^2 (1-c^2 x^2)}$$

input `Int[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(-c^2 d x^2 + d)^2} dx$$

input `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

output `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [N/A]

Not integrable

Time = 14.95 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.41

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{a^2 x^m}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{b^2 x^m \arcsin^2(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx + \int \frac{2abx^m \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx$$

input `integrate(x**m*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**2,x)`

output `(Integral(a**2*x**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**m*asin(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**m*asin(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2`

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{(c^2 dx^2 - d)^2} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)^2*x^m/(c^2*d*x^2 - d)^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx = \int \frac{x^m (a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^2} dx$$

input `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2,x)`

output `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^2, x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^2} dx$$

$$= \frac{\left(\int \frac{x^m}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) a^2 + 2 \left(\int \frac{x^m \arcsin(cx)}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) ab + \left(\int \frac{x^m \arcsin(cx)^2}{c^4 x^4 - 2c^2 x^2 + 1} dx \right) b^2}{d^2}$$

input `int(x^m*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^2,x)`

output `(int(x**m/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a**2 + 2*int((x**m*asin(c*x))/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b + int((x**m*asin(c*x)**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2)/d**2`

3.274
$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

Optimal result	2693
Mathematica [N/A]	2693
Rubi [N/A]	2694
Maple [N/A]	2696
Fricas [N/A]	2696
Sympy [N/A]	2697
Maxima [N/A]	2697
Giac [F(-2)]	2698
Mupad [N/A]	2698
Reduce [N/A]	2698

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \text{Int}\left(\frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3}, x\right)$$

output

```
Defer(Int)(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 7.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

input

```
Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output

```
Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx \\
 & \quad \downarrow \text{5208} \\
 & -\frac{bc \int \frac{x^{m+1} (a+b \arcsin(cx))}{(1-c^2 x^2)^{5/2}} dx}{2d^3} + \frac{(3-m) \int \frac{x^m (a+b \arcsin(cx))^2}{d^2 (1-c^2 x^2)^2} dx}{4d} + \frac{x^{m+1} (a + b \arcsin(cx))^2}{4d^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{27} \\
 & -\frac{bc \int \frac{x^{m+1} (a+b \arcsin(cx))}{(1-c^2 x^2)^{5/2}} dx}{2d^3} + \frac{(3-m) \int \frac{x^m (a+b \arcsin(cx))^2}{(1-c^2 x^2)^2} dx}{4d^3} + \frac{x^{m+1} (a + b \arcsin(cx))^2}{4d^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{5208} \\
 & -\frac{bc \left(\frac{1}{3} (1-m) \int \frac{x^{m+1} (a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx - \frac{1}{3} bc \int \frac{x^{m+2}}{(1-c^2 x^2)^2} dx + \frac{x^{m+2} (a+b \arcsin(cx))}{3(1-c^2 x^2)^{3/2}} \right)}{2d^3} + \\
 & \frac{(3-m) \left(-bc \int \frac{x^{m+1} (a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx + \frac{1}{2} (1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx + \frac{x^{m+1} (a+b \arcsin(cx))^2}{2(1-c^2 x^2)} \right)}{4d^3} + \\
 & \frac{x^{m+1} (a + b \arcsin(cx))^2}{4d^3 (1 - c^2 x^2)^2} \\
 & \quad \downarrow \text{278} \\
 & -\frac{bc \left(\frac{1}{3} (1-m) \int \frac{x^{m+1} (a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx + \frac{x^{m+2} (a+b \arcsin(cx))}{3(1-c^2 x^2)^{3/2}} - \frac{bc x^{m+3} \text{Hypergeometric2F1} \left(2, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2 \right)}{3(m+3)} \right)}{2d^3} + \\
 & \frac{(3-m) \left(-bc \int \frac{x^{m+1} (a+b \arcsin(cx))}{(1-c^2 x^2)^{3/2}} dx + \frac{1}{2} (1-m) \int \frac{x^m (a+b \arcsin(cx))^2}{1-c^2 x^2} dx + \frac{x^{m+1} (a+b \arcsin(cx))^2}{2(1-c^2 x^2)} \right)}{4d^3} + \\
 & \frac{x^{m+1} (a + b \arcsin(cx))^2}{4d^3 (1 - c^2 x^2)^2}
 \end{aligned}$$

↓ 5208

$$\frac{bc \left(\frac{1}{3}(1-m) \left(-(m+1) \int \frac{x^{m+1}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx - bc \int \frac{x^{m+2}}{1-c^2x^2} dx + \frac{x^{m+2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} \right) + \frac{x^{m+2}(a+b \arcsin(cx))}{3(1-c^2x^2)^{3/2}} \right)}{(3-m) \left(-bc \left(-(m+1) \int \frac{x^{m+1}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx - bc \int \frac{x^{m+2}}{1-c^2x^2} dx + \frac{x^{m+2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} \right) + \frac{1}{2}(1-m) \int \frac{x^m(a+b \arcsin(cx))}{1-c^2x^2} dx \right)}$$

$$\frac{2d^3}{4d^3}$$

$$\frac{x^{m+1}(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 278

$$\frac{bc \left(\frac{1}{3}(1-m) \left(-(m+1) \int \frac{x^{m+1}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx + \frac{x^{m+2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, c^2x^2\right)}{m+3} \right) \right)}{(3-m) \left(-bc \left(-(m+1) \int \frac{x^{m+1}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} dx + \frac{x^{m+2}(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, c^2x^2\right)}{m+3} \right) \right)}$$

$$\frac{2d^3}{4d^3}$$

$$\frac{x^{m+1}(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 5220

$$\frac{(3-m) \left(\frac{1}{2}(1-m) \int \frac{x^m(a+b \arcsin(cx))^2}{1-c^2x^2} dx - bc \left(-(m+1) \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)(a+b \arcsin(cx))}{m+2} \right) \right) \right)}{bc \left(\frac{1}{3}(1-m) \left(-(m+1) \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)(a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}\right)}{m^2+5m+6} \right) \right) \right)}$$

$$\frac{x^{m+1}(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

↓ 5234

$$\frac{(3-m) \left(\frac{1}{2}(1-m) \int \frac{x^m(a+b \arcsin(cx))^2}{1-c^2x^2} dx - bc \left(-(m+1) \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)(a+b \arcsin(cx))}{m+2} \right) \right) \right)}{bc \left(\frac{1}{3}(1-m) \left(-(m+1) \left(\frac{x^{m+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)(a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} {}_3F_2\left(1, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + 2, \frac{m}{2} + \frac{5}{2}\right)}{m^2+5m+6} \right) \right) \right)}$$

$$\frac{x^{m+1}(a+b \arcsin(cx))^2}{4d^3(1-c^2x^2)^2}$$

input `Int[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^3,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(-c^2 d x^2 + d)^3} dx$$

input `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x)`

output `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^m}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [N/A]

Not integrable

Time = 100.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.41

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= -\frac{\int \frac{a^2 x^m}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{b^2 x^m \arcsin^2(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx + \int \frac{2abx^m \arcsin(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx}{d^3}$$

input `integrate(x**m*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**3,x)`

output `-(Integral(a**2*x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b**2*x**m*asin(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(2*a*b*x**m*asin(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3`

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int -\frac{(b \arcsin(cx) + a)^2 x^m}{(c^2 dx^2 - d)^3} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")`

output `-integrate((b*arcsin(c*x) + a)^2*x^m/(c^2*d*x^2 - d)^3, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx = \int \frac{x^m (a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^3} dx$$

input `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3,x)`

output `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^3, x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.74

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^3} dx$$

$$= \frac{-\left(\int \frac{x^m}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx\right) a^2 - 2\left(\int \frac{x^m \operatorname{asin}(cx)}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx\right) ab - \left(\int \frac{x^m \operatorname{asin}(cx)^2}{c^6 x^6 - 3c^4 x^4 + 3c^2 x^2 - 1} dx\right) b^2}{d^3}$$

input `int(x^m*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^3,x)`

output `(- int(x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a**2 - 2*int((
x**m*asin(c*x))/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b - int((
x**m*asin(c*x)**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2)/d*
*3`

3.275 $\int x^m(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2700
Mathematica [N/A]	2700
Rubi [N/A]	2701
Maple [N/A]	2706
Fricas [N/A]	2707
Sympy [F(-1)]	2707
Maxima [N/A]	2707
Giac [F(-2)]	2708
Mupad [N/A]	2708
Reduce [N/A]	2709

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int x^m(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \frac{d^2 \sqrt{d - c^2dx^2} \text{Int}\left(x^m(1 - c^2x^2)^{5/2} (a + b \arcsin(cx))^2, x\right)}{\sqrt{1 - c^2x^2}}$$

output

```
d^2*(-c^2*d*x^2+d)^(1/2)*Defer(Int)(x^m*(-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))^2,x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int x^m(d - c^2dx^2)^{5/2} (a + b \arcsin(cx))^2 dx$$

input

```
Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]
```

```
output Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & - \frac{2bcd^2 \sqrt{d - c^2 dx^2} \int x^{m+1} (1 - c^2 x^2)^2 (a + b \arcsin(cx)) dx}{(m + 6) \sqrt{1 - c^2 x^2}} + \\
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{m + 6} + \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m + 6} \\
 & \quad \downarrow \text{5192} \\
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{m + 6} - \\
 & 2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^{m+2} \left(\frac{c^4 x^4}{m+6} - \frac{2c^2 x^2}{m+4} + \frac{1}{m+2} \right)}{\sqrt{1 - c^2 x^2}} dx + \frac{c^4 x^{m+6} (a + b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a + b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a + b \arcsin(cx))}{m+2} \right) \\
 & \quad \downarrow \text{1590} \\
 & \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m + 6}
 \end{aligned}$$

$$\frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \left(\frac{\int \frac{c^2 x^{m+2} \left(\frac{m+6}{m+2} - \frac{c^2 (m^2+15m+52)x^2}{(m+4)(m+6)} \right) dx}{c^2(m+6)} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+5}}{(m+6)^2} \right) + \frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+6} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6}$$

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$$\frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \left(\frac{\int \frac{c^2 x^{m+2} \left(\frac{m+6}{m+2} - \frac{c^2 (m^2+15m+52)x^2}{(m+4)(m+6)} \right) dx}{c^2(m+6)} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+5}}{(m+6)^2} \right) + \frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+6} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6}$$

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$$\frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \left(\frac{\int \frac{x^{m+2} \left(\frac{m+6}{m+2} - \frac{c^2 (m^2+15m+52)x^2}{(m+4)(m+6)} \right) dx}{m+6} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+5}}{(m+6)^2} \right) + \frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+6} \right)}{(m+6)\sqrt{1-c^2 x^2}}$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6}$$

363

$$\begin{aligned}
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{2bcd^2 \sqrt{d - c^2 dx^2} \left(-bc \left(\frac{\frac{m+6}{(m+2)(m+4)^2(m+6)} \int \frac{x^{m+2}}{\sqrt{1-c^2 x^2}} dx + \frac{(m^2+15m+52)\sqrt{1-c^2 x^2} x^{m+3}}{(m+4)^2(m+6)}}{m+6} - \frac{c^2 \sqrt{1-c^2 x^2} x^{m+5}}{(m+6)^2} \right) + \frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} \right)} \\
 & \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6} \\
 & \quad \downarrow \text{278} \\
 & \frac{5d \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx}{m+6} + \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6} \\
 & \frac{2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264) x^{m+3} \text{Hyp}}{(m+2)(m+4)^2(m+6)} \right) \right)}{(m+6)\sqrt{1-c^2 x^2}} \\
 & \quad \downarrow \text{5202} \\
 & 5d \left(-\frac{2bcd\sqrt{d-c^2 dx^2} \int x^{m+1} (1-c^2 x^2) (a+b \arcsin(cx)) dx}{(m+4)\sqrt{1-c^2 x^2}} + \frac{3d \int x^m \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2 dx}{m+4} + \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))}{m+4} \right) \\
 & \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6} \\
 & \frac{2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264) x^{m+3} \text{Hyp}}{(m+2)(m+4)^2(m+6)} \right) \right)}{(m+6)\sqrt{1-c^2 x^2}} \\
 & \quad \downarrow \text{5192} \\
 & 5d \left(-\frac{2bcd\sqrt{d-c^2 dx^2} \left(-bc \int \frac{x^{m+2} \left(\frac{1}{m+2} - \frac{c^2 x^2}{m+4} \right) dx - \frac{c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} \right)}{(m+4)\sqrt{1-c^2 x^2}} + \frac{3d \int x^m \sqrt{d-c^2 dx^2} (a+b \arcsin(cx)) dx}{m+4} \right) \\
 & \frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m+6} \\
 & \frac{2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264) x^{m+3} \text{Hyp}}{(m+2)(m+4)^2(m+6)} \right) \right)}{(m+6)\sqrt{1-c^2 x^2}}
 \end{aligned}$$

↓ 363

$$5d \left(\frac{2bcd\sqrt{d-c^2dx^2} \left(-bc \left(\frac{(3m+10) \int \frac{x^{m+2}}{\sqrt{1-c^2x^2}} dx + \frac{\sqrt{1-c^2x^2} x^{m+3}}{(m+4)^2} \right) - \frac{c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} \right)}{(m+4)\sqrt{1-c^2x^2}} + \frac{3d \int x^m \sqrt{d-c^2dx^2}}{m+6} \right)$$

$$\frac{x^{m+1}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{m+6} - \frac{2bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^{m+6}(a+b\arcsin(cx))}{m+6} - \frac{2c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264)x^{m+3} \text{Hyp}}{(m+2)(m+6)\sqrt{1-c^2x^2}} \right) \right)}{(m+6)\sqrt{1-c^2x^2}}$$

↓ 278

$$5d \left(\frac{3d \int x^m \sqrt{d-c^2dx^2} (a+b\arcsin(cx))^2 dx}{m+4} - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(3m+10)x^{m+3} \text{Hypergeom}}{(m+2)(m+6)\sqrt{1-c^2x^2}} \right) \right)}{(m+4)\sqrt{1-c^2x^2}} \right)$$

$$\frac{x^{m+1}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{m+6} - \frac{2bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^{m+6}(a+b\arcsin(cx))}{m+6} - \frac{2c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264)x^{m+3} \text{Hyp}}{(m+2)(m+6)\sqrt{1-c^2x^2}} \right) \right)}{(m+6)\sqrt{1-c^2x^2}}$$

↓ 5202

$$5d \left(\frac{3d \left(-\frac{2bc\sqrt{d-c^2dx^2} \int x^{m+1}(a+b\arcsin(cx)) dx}{(m+2)\sqrt{1-c^2x^2}} + \frac{d \int \frac{x^m(a+b\arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{m+2} + \frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{m+2} \right)}{m+4} - \frac{2bcd\sqrt{d-c^2dx^2} \left(-\frac{c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264)x^{m+3} \text{Hyp}}{(m+2)(m+6)\sqrt{1-c^2x^2}} \right) \right)}{(m+4)\sqrt{1-c^2x^2}} \right)$$

$$\frac{x^{m+1}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{m+6} - \frac{2bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^{m+6}(a+b\arcsin(cx))}{m+6} - \frac{2c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264)x^{m+3} \text{Hyp}}{(m+2)(m+6)\sqrt{1-c^2x^2}} \right) \right)}{(m+6)\sqrt{1-c^2x^2}}$$

↓ 5138

$$5d \left(\frac{3d \left(\frac{2bc\sqrt{d-c^2dx^2} \left(\frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - \frac{bc \int \frac{x^{m+2}}{\sqrt{1-c^2x^2}} dx}{m+2} \right)}{(m+2)\sqrt{1-c^2x^2}} + \frac{d \int \frac{x^m(a+b\arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{m+2} + \frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{m+2} \right)}{m+4} - \frac{2bcd\sqrt{d-c^2dx^2}}{m+4} \right)$$

$$\frac{x^{m+1}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{m+6} - \frac{2bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^{m+6}(a+b\arcsin(cx))}{m+6} - \frac{2c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264)x^{m+3} \text{Hyp}}{(m+2)(m+3)} \right) \right)}{(m+6)\sqrt{1-c^2x^2}}$$

↓ 278

$$5d \left(\frac{3d \left(\frac{d \int \frac{x^m(a+b\arcsin(cx))^2 dx}{\sqrt{d-c^2dx^2}}}{m+2} - \frac{2bc\sqrt{d-c^2dx^2} \left(\frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - \frac{bcx^{m+3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2x^2\right)}{(m+2)(m+3)} \right)}{(m+2)\sqrt{1-c^2x^2}} \right)}{m+4} + \frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^2}{m+2} \right)$$

$$\frac{x^{m+1}(d-c^2dx^2)^{5/2}(a+b\arcsin(cx))^2}{m+6} - \frac{2bcd^2\sqrt{d-c^2dx^2} \left(\frac{c^4x^{m+6}(a+b\arcsin(cx))}{m+6} - \frac{2c^2x^{m+4}(a+b\arcsin(cx))}{m+4} + \frac{x^{m+2}(a+b\arcsin(cx))}{m+2} - bc \left(\frac{(15m^2+130m+264)x^{m+3} \text{Hyp}}{(m+2)(m+3)} \right) \right)}{(m+6)\sqrt{1-c^2x^2}}$$

↓ 5234

$$5d \left(\frac{d \int \frac{x^m (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx - 2bc \sqrt{d-c^2 dx^2} \left(\frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)} \right) + \frac{x^{m+1} \sqrt{d-c^2 dx^2} (a+b \arcsin(cx))^2}{m+2}}{m+4} \right)$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2}{m + 6} - 2bcd^2 \sqrt{d - c^2 dx^2} \left(\frac{c^4 x^{m+6} (a+b \arcsin(cx))}{m+6} - \frac{2c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - bc \left(\frac{(15m^2 + 130m + 264) x^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)} \right) \right)$$

$$(m + 6) \sqrt{1 - c^2 x^2}$$

input `Int[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^m (-c^2 d x^2 + d)^{5/2} (a + b \arcsin(cx))^2 dx$$

input `int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arcsin(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)`

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x**m*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^2*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^2 dx = \int x^m (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{5/2} dx$$

input `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(5/2), x)`

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 9.10

$$\begin{aligned}
& \int x^m (d - c^2 dx^2)^{5/2} (a \\
& + b \arcsin(cx))^2 dx = \sqrt{d} d^2 \left(2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) x^4 dx \right) ab c^4 \right. \\
& - 4 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2 dx \right) ab c^2 \\
& + 2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx \right) ab \\
& + \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 x^4 dx \right) b^2 c^4 \\
& - 2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 x^2 dx \right) b^2 c^2 \\
& + \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 dx \right) b^2 + \left(\int x^m \sqrt{-c^2 x^2 + 1} x^4 dx \right) a^2 c^4 \\
& \left. - 2 \left(\int x^m \sqrt{-c^2 x^2 + 1} x^2 dx \right) a^2 c^2 + \left(\int x^m \sqrt{-c^2 x^2 + 1} dx \right) a^2 \right)
\end{aligned}$$

input

```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^2,x)
```

output

```
sqrt(d)*d**2*(2*int(x**m*sqrt(-c**2*x**2+1)*asin(c*x)*x**4,x)*a*b*c**4
- 4*int(x**m*sqrt(-c**2*x**2+1)*asin(c*x)*x**2,x)*a*b*c**2+ 2*int(x*
**m*sqrt(-c**2*x**2+1)*asin(c*x),x)*a*b+ int(x**m*sqrt(-c**2*x**2+
1)*asin(c*x)**2*x**4,x)*b**2*c**4- 2*int(x**m*sqrt(-c**2*x**2+1)*asin
(c*x)**2*x**2,x)*b**2*c**2+ int(x**m*sqrt(-c**2*x**2+1)*asin(c*x)**2,
x)*b**2+ int(x**m*sqrt(-c**2*x**2+1)*x**4,x)*a**2*c**4- 2*int(x**m*s
qrt(-c**2*x**2+1)*x**2,x)*a**2*c**2+ int(x**m*sqrt(-c**2*x**2+1),
x)*a**2)
```

3.276 $\int x^m(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$

Optimal result	2710
Mathematica [N/A]	2710
Rubi [N/A]	2711
Maple [N/A]	2713
Fricas [N/A]	2714
Sympy [F(-1)]	2714
Maxima [N/A]	2715
Giac [F(-2)]	2715
Mupad [N/A]	2715
Reduce [N/A]	2716

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int x^m(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{d\sqrt{d - c^2dx^2} \text{Int}\left(x^m(1 - c^2x^2)^{3/2} (a + b \arcsin(cx))^2, x\right)}{\sqrt{1 - c^2x^2}}$$

output

```
d*(-c^2*d*x^2+d)^(1/2)*Defer(Int)(x^m*(-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))^2,x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x^m(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^2 dx$$

input

```
Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \int x^{m+1} (1 - c^2 x^2) (a + b \arcsin(cx)) dx}{(m + 4)\sqrt{1 - c^2 x^2}} + \\
 & \frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx}{m + 4} + \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{m + 4} \\
 & \quad \downarrow \text{5192} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \left(-bc \int \frac{x^{m+2} \left(\frac{1}{m+2} - \frac{c^2 x^2}{m+4} \right) dx}{\sqrt{1 - c^2 x^2}} - \frac{c^2 x^{m+4} (a + b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a + b \arcsin(cx))}{m+2} \right)}{(m + 4)\sqrt{1 - c^2 x^2}} + \\
 & \frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx}{m + 4} + \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{m + 4} \\
 & \quad \downarrow \text{363} \\
 & \frac{2bcd\sqrt{d - c^2 dx^2} \left(-bc \left(\frac{(3m+10) \int \frac{x^{m+2}}{\sqrt{1 - c^2 x^2}} dx}{(m+2)(m+4)^2} + \frac{\sqrt{1 - c^2 x^2} x^{m+3}}{(m+4)^2} \right) - \frac{c^2 x^{m+4} (a + b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a + b \arcsin(cx))}{m+2} \right)}{(m + 4)\sqrt{1 - c^2 x^2}} + \\
 & \frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx}{m + 4} + \frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{m + 4} \\
 & \quad \downarrow \text{278}
 \end{aligned}$$

$$\frac{3d \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx}{m + 4} - \frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{c^2 x^{m+4} (a + b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - bc \left(\frac{(3m+10)x^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)(m+4)^2} \right) \right)}{(m+4)\sqrt{1 - c^2 x^2}}$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{m + 4}$$

↓ 5202

$$3d \left(-\frac{2bc\sqrt{d - c^2 dx^2} \int x^{m+1} (a + b \arcsin(cx)) dx}{(m+2)\sqrt{1 - c^2 x^2}} + \frac{d \int \frac{x^m (a + b \arcsin(cx))^2 dx}{\sqrt{d - c^2 dx^2}}}{m+2} + \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{m+2} \right)$$

$$\frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{c^2 x^{m+4} (a + b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - bc \left(\frac{(3m+10)x^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)(m+4)^2} \right) \right)}{(m+4)\sqrt{1 - c^2 x^2}}$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{m + 4}$$

↓ 5138

$$3d \left(-\frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - \frac{bc \int \frac{x^{m+2} dx}{\sqrt{1 - c^2 x^2}}}{m+2} \right)}{(m+2)\sqrt{1 - c^2 x^2}} + \frac{d \int \frac{x^m (a + b \arcsin(cx))^2 dx}{\sqrt{d - c^2 dx^2}}}{m+2} + \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{m+2} \right)$$

$$\frac{2bcd\sqrt{d - c^2 dx^2} \left(-\frac{c^2 x^{m+4} (a + b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - bc \left(\frac{(3m+10)x^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)(m+4)^2} \right) \right)}{(m+4)\sqrt{1 - c^2 x^2}}$$

$$\frac{x^{m+1} (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2}{m + 4}$$

↓ 278

$$\begin{aligned}
 & 3d \left(\frac{d \int \frac{x^m (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{m+2} - \frac{2bc\sqrt{d-c^2 dx^2} \left(\frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)} \right)}{(m+2)\sqrt{1-c^2 x^2}} \right) + \frac{x^{m+1} \sqrt{d-c^2 dx^2}}{m} \\
 & \frac{2bcd\sqrt{d-c^2 dx^2} \left(-\frac{c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - bc \left(\frac{(3m+10)x^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)(m+4)^2} \right) \right)}{(m+4)\sqrt{1-c^2 x^2}} \\
 & \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^2}{m+4} \\
 & \quad \downarrow \text{5234} \\
 & 3d \left(\frac{d \int \frac{x^m (a+b \arcsin(cx))^2}{\sqrt{d-c^2 dx^2}} dx}{m+2} - \frac{2bc\sqrt{d-c^2 dx^2} \left(\frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)} \right)}{(m+2)\sqrt{1-c^2 x^2}} \right) + \frac{x^{m+1} \sqrt{d-c^2 dx^2}}{m} \\
 & \frac{2bcd\sqrt{d-c^2 dx^2} \left(-\frac{c^2 x^{m+4} (a+b \arcsin(cx))}{m+4} + \frac{x^{m+2} (a+b \arcsin(cx))}{m+2} - bc \left(\frac{(3m+10)x^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)(m+4)^2} \right) \right)}{(m+4)\sqrt{1-c^2 x^2}} \\
 & \frac{x^{m+1} (d-c^2 dx^2)^{3/2} (a+b \arcsin(cx))^2}{m+4}
 \end{aligned}$$

input `Int[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^m (-c^2 dx^2 + d)^{3/2} (a + b \arcsin(cx))^2 dx$$

input `int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arcsin(c*x))^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcsin(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)`

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x**m*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^2*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^2 dx = \int x^m (a + b \arcsin(cx))^2 (d - c^2 dx^2)^{3/2} dx$$

input `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(3/2), x)`

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 5.90

$$\begin{aligned} & \int x^m (d - c^2 dx^2)^{3/2} (a \\ & + b \arcsin(cx))^2 dx = \sqrt{d} d \left(-2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) x^2 dx \right) ab c^2 \right. \\ & + 2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx \right) ab \\ & - \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 x^2 dx \right) b^2 c^2 \\ & + \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 dx \right) b^2 \\ & \left. - \left(\int x^m \sqrt{-c^2 x^2 + 1} x^2 dx \right) a^2 c^2 + \left(\int x^m \sqrt{-c^2 x^2 + 1} dx \right) a^2 \right) \end{aligned}$$

input `int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^2,x)`

output `sqrt(d)*d*(- 2*int(x**m*sqrt(- c**2*x**2 + 1)*asin(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m*sqrt(- c**2*x**2 + 1)*asin(c*x),x)*a*b - int(x**m*sqrt(- c**2*x**2 + 1)*asin(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*sqrt(- c**2*x**2 + 1)*asin(c*x)**2,x)*b**2 - int(x**m*sqrt(- c**2*x**2 + 1)*x**2,x)*a**2*c**2 + int(x**m*sqrt(- c**2*x**2 + 1),x)*a**2)`

3.277 $\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$

Optimal result	2717
Mathematica [N/A]	2717
Rubi [N/A]	2718
Maple [N/A]	2719
Fricas [N/A]	2719
Sympy [N/A]	2720
Maxima [N/A]	2720
Giac [F(-2)]	2721
Mupad [N/A]	2721
Reduce [N/A]	2721

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{d - c^2 dx^2} \text{Int}(x^m \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2, x)}{\sqrt{1 - c^2 x^2}}$$

output $(-c^2 d x^2 + d)^{(1/2)} \text{Defer}(\text{Int}(x^m (-c^2 x^2 + 1)^{(1/2)} (a + b \arcsin(c x))^2, x)) / (-c^2 x^2 + 1)^{(1/2)}$

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx$$

input $\text{Integrate}[x^m \text{Sqrt}[d - c^2 d x^2] (a + b \text{ArcSin}[c x])^2, x]$

output $\text{Integrate}[x^m \text{Sqrt}[d - c^2 d x^2] (a + b \text{ArcSin}[c x])^2, x]$

Rubi [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx \\
 & \quad \downarrow \text{5202} \\
 & -\frac{2bc\sqrt{d - c^2 dx^2} \int x^{m+1} (a + b \arcsin(cx)) dx}{(m+2)\sqrt{1 - c^2 x^2}} + \frac{d \int \frac{x^m (a + b \arcsin(cx))^2 dx}{\sqrt{d - c^2 dx^2}}}{m+2} + \\
 & \quad \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{m+2} \\
 & \quad \downarrow \text{5138} \\
 & -\frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - \frac{bc \int \frac{x^{m+2}}{\sqrt{1 - c^2 x^2}} dx}{m+2} \right)}{(m+2)\sqrt{1 - c^2 x^2}} + \frac{d \int \frac{x^m (a + b \arcsin(cx))^2 dx}{\sqrt{d - c^2 dx^2}}}{m+2} + \\
 & \quad \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{m+2} \\
 & \quad \downarrow \text{278} \\
 & \quad \frac{d \int \frac{x^m (a + b \arcsin(cx))^2 dx}{\sqrt{d - c^2 dx^2}}}{m+2} - \\
 & \quad \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)} \right)}{(m+2)\sqrt{1 - c^2 x^2}} + \\
 & \quad \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{m+2} \\
 & \quad \downarrow \text{5234}
 \end{aligned}$$

$$\frac{d \int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx}{m + 2} - \frac{2bc\sqrt{d - c^2 dx^2} \left(\frac{x^{m+2} (a + b \arcsin(cx))}{m+2} - \frac{bcx^{m+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}, c^2 x^2\right)}{(m+2)(m+3)} \right)}{(m+2)\sqrt{1 - c^2 x^2}} + \frac{x^{m+1} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2}{m + 2}$$

input `Int [x^m*sqrt [d - c^2*d*x^2]*(a + b*ArcSin [c*x])^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^m \sqrt{-c^2 d x^2 + d} (a + b \arcsin (cx))^2 dx$$

input `int (x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x)`

output `int (x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin (cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m, x)`

Sympy [N/A]

Not integrable

Time = 18.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int x^m \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2 dx$$

input `integrate(x**m*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(x**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^2 x^m dx$$

input `integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^2*x^m, x)`

Giac [F(-2)]

Exception generated.

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \int x^m (a + b \arcsin(cx))^2 \sqrt{d - c^2 dx^2} dx$$

input `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2),x)`

output `int(x^m*(a + b*asin(c*x))^2*(d - c^2*d*x^2)^(1/2), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.66

$$\int x^m \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^2 dx = \sqrt{d} \left(2 \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx) dx \right) ab + \left(\int x^m \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 dx \right) b^2 + \left(\int x^m \sqrt{-c^2 x^2 + 1} dx \right) a^2 \right)$$

input `int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^2,x)`

output `sqrt(d)*(2*int(x**m*sqrt(-c**2*x**2+1)*asin(c*x),x)*a*b + int(x**m*sqrt(-c**2*x**2+1)*asin(c*x)**2,x)*b**2 + int(x**m*sqrt(-c**2*x**2+1),x)*a**2)`

3.278
$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

Optimal result	2723
Mathematica [N/A]	2723
Rubi [N/A]	2724
Maple [N/A]	2724
Fricas [N/A]	2725
Sympy [N/A]	2725
Maxima [N/A]	2726
Giac [F(-2)]	2726
Mupad [N/A]	2727
Reduce [N/A]	2727

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \frac{\sqrt{1 - c^2 x^2} \operatorname{Int}\left(\frac{x^m (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}}, x\right)}{\sqrt{d - c^2 dx^2}}$$

output

```
(-c^2*x^2+1)^(1/2)*Defer(Int)(x^m*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(1/2),x)
)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input

```
Integrate[(x^m*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output

```
Integrate[(x^m*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```


Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

↓ 5234

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `Int[(x^m*(a + b*ArcSin[c*x])^2)/Sqrt[d - c^2*d*x^2],x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{-c^2 dx^2 + d}} dx$$

input `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m/(c^2*d*x^2 - d), x)`

Sympy [N/A]

Not integrable

Time = 7.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{-d(cx - 1)(cx + 1)}} dx$$

input `integrate(x**m*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)`

output `Integral(x**m*(a + b*asin(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)`

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{\sqrt{-c^2 dx^2 + d}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)^2*x^m/sqrt(-c^2*d*x^2 + d), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx = \int \frac{x^m (a + b \operatorname{asin}(cx))^2}{\sqrt{d - c^2 dx^2}} dx$$

input `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)`

output `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\begin{aligned} & \int \frac{x^m (a + b \arcsin(cx))^2}{\sqrt{d - c^2 dx^2}} dx \\ &= \frac{\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1}} dx \right) a^2 + 2 \left(\int \frac{x^m \operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1}} dx \right) ab + \left(\int \frac{x^m \operatorname{asin}(cx)^2}{\sqrt{-c^2 x^2 + 1}} dx \right) b^2}{\sqrt{d}} \end{aligned}$$

input `int(x^m*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)`

output `(int(x**m/sqrt(-c**2*x**2 + 1),x)*a**2 + 2*int((x**m*asin(c*x))/sqrt(-c**2*x**2 + 1),x)*a*b + int((x**m*asin(c*x)**2)/sqrt(-c**2*x**2 + 1),x)*b**2)/sqrt(d)`

3.279
$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

Optimal result	2728
Mathematica [N/A]	2728
Rubi [N/A]	2729
Maple [N/A]	2729
Fricas [N/A]	2730
Sympy [N/A]	2730
Maxima [N/A]	2731
Giac [F(-2)]	2731
Mupad [N/A]	2732
Reduce [N/A]	2732

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{\sqrt{1 - c^2 x^2} \operatorname{Int}\left(\frac{x^m (a + b \arcsin(cx))^2}{(1 - c^2 x^2)^{3/2}}, x\right)}{d \sqrt{d - c^2 dx^2}}$$

output `(-c^2*x^2+1)^(1/2)*Defer(Int)(x^m*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(3/2),x)/d/(-c^2*d*x^2+d)^(1/2)`

Mathematica [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]`

output `Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]`

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

↓ 5234

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `Int[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(-c^2 dx^2 + d)^{\frac{3}{2}}} dx$$

input `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)`

Sympy [N/A]

Not integrable

Time = 10.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{(-d(cx - 1)(cx + 1))^{3/2}} dx$$

input `integrate(x**m*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)`

output `Integral(x**m*(a + b*asin(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)`

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{(-c^2 dx^2 + d)^{3/2}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)^2*x^m/(-c^2*d*x^2 + d)^(3/2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx)::OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \int \frac{x^m (a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{3/2}} dx$$

input `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)`

output `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.28

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{3/2}} dx = \frac{-\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx\right) a^2 - 2\left(\int \frac{x^m \operatorname{asin}(cx)}{\sqrt{-c^2 x^2 + 1} c^2 x^2 - \sqrt{-c^2 x^2 + 1}} dx\right) ab - \left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1}} dx\right) b^2}{\sqrt{d} d}$$

input `int(x^m*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)`

output `(- int(x**m/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*a**2 - 2*int((x**m*asin(c*x))/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*a*b - int((x**m*asin(c*x)**2)/(sqrt(- c**2*x**2 + 1)*c**2*x**2 - sqrt(- c**2*x**2 + 1)),x)*b**2)/(sqrt(d)*d)`

3.280
$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

Optimal result	2733
Mathematica [N/A]	2733
Rubi [N/A]	2734
Maple [N/A]	2734
Fricas [N/A]	2735
Sympy [F(-1)]	2735
Maxima [N/A]	2735
Giac [F(-2)]	2736
Mupad [N/A]	2736
Reduce [N/A]	2737

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \frac{\sqrt{1 - c^2 x^2} \operatorname{Int}\left(\frac{x^m (a + b \arcsin(cx))^2}{(1 - c^2 x^2)^{5/2}}, x\right)}{d^2 \sqrt{d - c^2 dx^2}}$$

output `(-c^2*x^2+1)^(1/2)*Defer(Int)(x^m*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(5/2),x)/d^2/(-c^2*d*x^2+d)^(1/2)`

Mathematica [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input `Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]`

output `Integrate[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]`

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

↓ 5234

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input `Int[(x^m*(a + b*ArcSin[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)`

output `int(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)`

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.83

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*x^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Timed out}$$

input `integrate(x**m*(a+b*asin(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^m}{(-c^2 dx^2 + d)^{5/2}} dx$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)^2*x^m/(-c^2*d*x^2 + d)^(5/2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(a+b*arcsin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \int \frac{x^m (a + b \operatorname{asin}(cx))^2}{(d - c^2 dx^2)^{5/2}} dx$$

input `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)`

output `int((x^m*(a + b*asin(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 6.97

$$\int \frac{x^m (a + b \arcsin(cx))^2}{(d - c^2 dx^2)^{5/2}} dx = \left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) a^2 + 2 \left(\int \frac{x^m \arcsin(cx)}{\sqrt{-c^2 x^2 + 1} c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} c^2 x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) \sqrt{d}$$

input

```
int(x^m*(a+b*asin(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(int(x**m/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*a**2+2*int((x**m*asin(c*x))/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*a*b+int((x**m*asin(c*x)**2)/(sqrt(-c**2*x**2+1)*c**4*x**4-2*sqrt(-c**2*x**2+1)*c**2*x**2+sqrt(-c**2*x**2+1)),x)*b**2)/(sqrt(d)*d**2)
```

$$3.281 \quad \int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

Optimal result	2738
Mathematica [N/A]	2738
Rubi [N/A]	2739
Maple [N/A]	2739
Fricas [N/A]	2740
Sympy [N/A]	2740
Maxima [N/A]	2740
Giac [N/A]	2741
Mupad [N/A]	2741
Reduce [N/A]	2742

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \text{Int}\left(\frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}}, x\right)$$

output `Defer(Int)(x^m*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x)`

Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `Integrate[(x^m*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2],x]`

output `Integrate[(x^m*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2], x]`

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

↓ 5234

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `Int[(x^m*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2], x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^m*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2), x)`

output `int(x^m*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2), x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^m*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^m*arcsin(a*x)^3/(a^2*x^2 - 1), x)`

Sympy [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}^3(ax)}{\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(x**m*asin(a*x)**3/(-a**2*x**2+1)**(1/2),x)`

output `Integral(x**m*asin(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)`

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^m*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(x^m*arcsin(a*x)^3/sqrt(-a^2*x^2 + 1), x)`

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^m*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(x^m*arcsin(a*x)^3/sqrt(-a^2*x^2 + 1), x)`

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `int((x^m*asin(a*x)^3)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^m*asin(a*x)^3)/(1 - a^2*x^2)^(1/2), x)`

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^m*asin(a*x)^3/(-a^2*x^2+1)^(1/2),x)`output `int((x**m*asin(a*x)**3)/sqrt(-a**2*x**2+1),x)`

3.282 $\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result	2743
Mathematica [A] (verified)	2744
Rubi [A] (verified)	2744
Maple [A] (verified)	2748
Fricas [A] (verification not implemented)	2748
Sympy [A] (verification not implemented)	2749
Maxima [F]	2749
Giac [A] (verification not implemented)	2750
Mupad [F(-1)]	2750
Reduce [F]	2751

Optimal result

Integrand size = 24, antiderivative size = 191

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{45x^2}{128a^3} - \frac{3x^4}{128a} + \frac{45x\sqrt{1-a^2x^2} \arcsin(ax)}{64a^4} + \frac{3x^3\sqrt{1-a^2x^2} \arcsin(ax)}{32a^2} - \frac{45 \arcsin(ax)^2}{128a^5} + \frac{9x^2 \arcsin(ax)^2}{16a^3} + \frac{3x^4 \arcsin(ax)^2}{16a} - \frac{3x\sqrt{1-a^2x^2} \arcsin(ax)^3}{8a^4} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2} + \frac{3 \arcsin(ax)^4}{32a^5}$$

output

```
-45/128*x^2/a^3-3/128*x^4/a+45/64*x*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^4+3/32*x^3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^2-45/128*arcsin(a*x)^2/a^5+9/16*x^2*arcsin(a*x)^2/a^3+3/16*x^4*arcsin(a*x)^2/a-3/8*x*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/a^4-1/4*x^3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/a^2+3/32*arcsin(a*x)^4/a^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{-3a^2x^2(15+a^2x^2) + 6ax\sqrt{1-a^2x^2}(15+2a^2x^2)\arcsin(ax) + 3(-15+24a^2x^2+8a^4x^4)\arcsin(ax)^2 - 16a^3x^3\arcsin(ax)^3 + 12a^2x^2\arcsin(ax)^4}{128a^5}$$

input

```
Integrate[(x^4*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output

```
(-3*a^2*x^2*(15 + a^2*x^2) + 6*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2)*ArcSin[a*x] + 3*(-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcSin[a*x]^2 - 16*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcSin[a*x]^3 + 12*ArcSin[a*x]^4)/(128*a^5)
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5210, 5138, 5210, 15, 5138, 5152, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$\downarrow \text{5210}$$

$$\frac{3 \int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{3 \int x^3 \arcsin(ax)^2 dx}{4a} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2}$$

$$\downarrow \text{5138}$$

$$\frac{3 \int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{3 \left(\frac{1}{4} x^4 \arcsin(ax)^2 - \frac{1}{2} a \int \frac{x^4 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx \right)}{4a} - \frac{x^3 \sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2}$$

$$\downarrow \text{5210}$$

$$\begin{aligned}
& \frac{3 \left(\frac{\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{3 \int x \arcsin(ax)^2 dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} \right)}{4a^2} + \\
& \frac{3 \left(\frac{1}{4}x^4 \arcsin(ax)^2 - \frac{1}{2}a \left(\frac{3 \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{4a^2} + \frac{\int x^3 dx}{4a} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} \right) \right)}{4a^2} \\
& \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2} \\
& \downarrow \text{15} \\
& \frac{3 \left(\frac{\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{3 \int x \arcsin(ax)^2 dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} \right)}{4a^2} + \\
& \frac{3 \left(\frac{1}{4}x^4 \arcsin(ax)^2 - \frac{1}{2}a \left(\frac{3 \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \right) \right)}{4a^2} \\
& \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2} \\
& \downarrow \text{5138} \\
& \frac{3 \left(\frac{3 \left(\frac{1}{2}x^2 \arcsin(ax)^2 - a \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx \right)}{2a} + \frac{\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} \right)}{4a^2} + \\
& \frac{3 \left(\frac{1}{4}x^4 \arcsin(ax)^2 - \frac{1}{2}a \left(\frac{3 \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \right) \right)}{4a^2} \\
& \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2} \\
& \downarrow \text{5152} \\
& \frac{3 \left(\frac{1}{4}x^4 \arcsin(ax)^2 - \frac{1}{2}a \left(\frac{3 \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \right) \right)}{4a^2} + \\
& \frac{3 \left(\frac{3 \left(\frac{1}{2}x^2 \arcsin(ax)^2 - a \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx \right)}{2a} + \frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} \right)}{4a^2} \\
& \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2} \\
& \downarrow \text{5210}
\end{aligned}$$

$$\begin{aligned}
 & 3 \left(\frac{1}{4} x^4 \arcsin(ax)^2 - \frac{1}{2} a \left(\frac{3 \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \right) \right) \\
 & \frac{4a}{3 \left(\frac{1}{2} x^2 \arcsin(ax)^2 - a \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} \right) \right)} + \frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} \\
 & \frac{4a^2}{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3} \\
 & \quad \downarrow 15 \\
 & 3 \left(\frac{1}{4} x^4 \arcsin(ax)^2 - \frac{1}{2} a \left(\frac{3 \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{x^4}{16a} \right) \right) \\
 & \frac{4a}{3 \left(\frac{1}{2} x^2 \arcsin(ax)^2 - a \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right) \right)} + \frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} \\
 & \frac{4a^2}{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3} \\
 & \quad \downarrow 5152 \\
 & - \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)^3}{4a^2} + \\
 & 3 \left(\frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} + \frac{3 \left(\frac{1}{2} x^2 \arcsin(ax)^2 - a \left(\frac{\arcsin(ax)^2}{4a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right) \right)}{2a} \right) \\
 & \frac{4a^2}{3 \left(\frac{1}{4} x^4 \arcsin(ax)^2 - \frac{1}{2} a \left(- \frac{x^3\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} + \frac{3 \left(\frac{\arcsin(ax)^2}{4a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right)}{4a^2} + \frac{x^4}{16a} \right) \right)} \\
 & \quad 4a
 \end{aligned}$$

input `Int[(x^4*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2], x]`

output

$$\begin{aligned}
& -1/4*(x^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/a^2 + (3*((x^4*\text{ArcSin}[a*x]^2)/4 \\
& - (a*(x^4/(16*a) - (x^3*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(4*a^2) + (3*(x^2/ \\
& (4*a) - (x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(2*a^2) + \text{ArcSin}[a*x]^2/(4*a^3)) \\
&))/(4*a^2)))/2)/(4*a) + (3*(-1/2*(x*\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/a^2 + \\
& \text{ArcSin}[a*x]^4/(8*a^3) + (3*((x^2*\text{ArcSin}[a*x]^2)/2 - a*(x^2/(4*a) - (x*\text{Sqr} \\
& \text{t}[1 - a^2*x^2]*\text{ArcSin}[a*x])/(2*a^2) + \text{ArcSin}[a*x]^2/(4*a^3)))))/(2*a)))/(4* \\
& a^2)
\end{aligned}$$

Defintions of rubi rules used

rule 15

$$\text{Int}[(a_.)*(x_)^(m_.), x_Symbol] \rightarrow \text{Simp}[a*(x^(m + 1)/(m + 1)), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$$

rule 5138

$$\begin{aligned}
& \text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] \\
& \rightarrow \text{Simp}[(d*x)^(m + 1)*((a + b*\text{ArcSin}[c*x])^n/(d*(m + 1))), x] - \text{Simp}[b*c*(n \\
& /((d*(m + 1))) \ \text{Int}[(d*x)^(m + 1)*((a + b*\text{ArcSin}[c*x])^(n - 1)/\text{Sqrt}[1 - c^2 \\
& *x^2]), x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]
\end{aligned}$$

rule 5152

$$\begin{aligned}
& \text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^(n_.)/\text{Sqrt}[(d_) + (e_.)*(x_)^2], x_S \\
& \text{ymbol}] \rightarrow \text{Simp}[(1/(b*c*(n + 1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a \\
& + b*\text{ArcSin}[c*x])^(n + 1), x] \text{ ; FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d \\
& + e, 0] \ \&\& \ \text{NeQ}[n, -1]
\end{aligned}$$

rule 5210

$$\begin{aligned}
& \text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_. \\
&)*(x_)^2)^(p_), x_Symbol] \rightarrow \text{Simp}[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + \\
& b*\text{ArcSin}[c*x])^n/(e*(m + 2*p + 1))), x] + (\text{Simp}[f^2*((m - 1)/(c^2*(m + 2*p \\
& + 1)) \ \text{Int}[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{S} \\
& \text{imp}[b*f*(n/(c*(m + 2*p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x \\
& x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*\text{ArcSin}[c*x])^(n - 1), x], x]) \text{ ;} \\
& \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m \\
& , 1] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0]
\end{aligned}$$

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.84

method	result
default	$\frac{-128 \arcsin(ax)^3 \sqrt{-a^2x^2+1} a^3 x^3 + 96 \arcsin(ax)^2 a^4 x^4 + 48 \arcsin(ax) \sqrt{-a^2x^2+1} a^3 x^3 - 12a^4 x^4 - 192 \arcsin(ax)^3 \sqrt{-a^2x^2+1} ax + \dots}{512a^5}$

input `int(x^4*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{512} * (-128 * \arcsin(ax)^3 * (-a^2 * x^2 + 1)^{(1/2)} * a^3 * x^3 + 96 * \arcsin(ax)^2 * a^4 * x^4 + 48 * \arcsin(ax) * (-a^2 * x^2 + 1)^{(1/2)} * a^3 * x^3 - 12 * a^4 * x^4 - 192 * \arcsin(ax)^3 * (-a^2 * x^2 + 1)^{(1/2)} * a * x + 288 * x^2 * \arcsin(ax)^2 * a^2 + 48 * \arcsin(ax)^4 + 360 * \arcsin(ax) * (-a^2 * x^2 + 1)^{(1/2)} * a * x - 180 * a^2 * x^2 - 180 * \arcsin(ax)^2 - 27) / a^5$$

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{3a^4x^4 + 45a^2x^2 - 12\arcsin(ax)^4 - 3(8a^4x^4 + 24a^2x^2 - 15)\arcsin(ax)^2 + 2\sqrt{-a^2x^2+1}(8(2a^3x^3 + 3ax)\arcsin(ax)^3 - 3(2a^3x^3 + 15ax)\arcsin(ax))}{128a^5}$$

input `integrate(x^4*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output
$$-1/128 * (3 * a^4 * x^4 + 45 * a^2 * x^2 - 12 * \arcsin(ax)^4 - 3 * (8 * a^4 * x^4 + 24 * a^2 * x^2 - 15) * \arcsin(ax)^2 + 2 * \sqrt{-a^2 * x^2 + 1} * (8 * (2 * a^3 * x^3 + 3 * a * x) * \arcsin(ax)^3 - 3 * (2 * a^3 * x^3 + 15 * a * x) * \arcsin(ax))) / a^5$$

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \begin{cases} \frac{3x^4 \operatorname{asin}^2(ax)}{16a} - \frac{3x^4}{128a} - \frac{x^3 \sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{4a^2} + \frac{3x^3 \sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{32a^2} + \frac{9x^2 \operatorname{asin}^2(ax)}{16a^3} - \frac{45x^2}{128a^3} - \frac{3x \sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{8a^4} \\ 0 \end{cases}$$

input `integrate(x**4*asin(a*x)**3/(-a**2*x**2+1)**(1/2),x)`output `Piecewise(((3*x**4*asin(a*x)**2/(16*a) - 3*x**4/(128*a) - x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(4*a**2) + 3*x**3*sqrt(-a**2*x**2 + 1)*asin(a*x)/(32*a**2) + 9*x**2*asin(a*x)**2/(16*a**3) - 45*x**2/(128*a**3) - 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(8*a**4) + 45*x*sqrt(-a**2*x**2 + 1)*asin(a*x)/(64*a**4) + 3*asin(a*x)**4/(32*a**5) - 45*asin(a*x)**2/(128*a**5), Ne(a, 0)), (0, True))`**Maxima [F]**

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^4 \arcsin(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^4*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `integrate(x^4*arcsin(a*x)^3/sqrt(-a^2*x^2 + 1), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{(-a^2x^2+1)^{\frac{3}{2}}x \arcsin(ax)^3}{4a^4} - \frac{5\sqrt{-a^2x^2+1}x \arcsin(ax)^3}{8a^4}$$

$$- \frac{3(-a^2x^2+1)^{\frac{3}{2}}x \arcsin(ax)}{32a^4} + \frac{3(a^2x^2-1)^2 \arcsin(ax)^2}{16a^5}$$

$$+ \frac{3 \arcsin(ax)^4}{32a^5} + \frac{51\sqrt{-a^2x^2+1}x \arcsin(ax)}{64a^4}$$

$$+ \frac{15(a^2x^2-1) \arcsin(ax)^2}{16a^5} - \frac{3(a^2x^2-1)^2}{128a^5}$$

$$+ \frac{51 \arcsin(ax)^2}{128a^5} - \frac{51(a^2x^2-1)}{128a^5} - \frac{195}{1024a^5}$$

input `integrate(x^4*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `1/4*(-a^2*x^2 + 1)^(3/2)*x*arcsin(a*x)^3/a^4 - 5/8*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^3/a^4 - 3/32*(-a^2*x^2 + 1)^(3/2)*x*arcsin(a*x)/a^4 + 3/16*(a^2*x^2 - 1)^2*arcsin(a*x)^2/a^5 + 3/32*arcsin(a*x)^4/a^5 + 51/64*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a^4 + 15/16*(a^2*x^2 - 1)*arcsin(a*x)^2/a^5 - 3/128*(a^2*x^2 - 1)^2/a^5 + 51/128*arcsin(a*x)^2/a^5 - 51/128*(a^2*x^2 - 1)/a^5 - 195/1024/a^5`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^4 \operatorname{asin}(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `int((x^4*asin(a*x)^3)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^4*asin(a*x)^3)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^4 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{a \sin(ax)^3 x^4}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^4*asin(a*x)^3/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**3*x**4)/sqrt(-a**2*x**2+1),x)`

3.283 $\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result	2752
Mathematica [A] (verified)	2753
Rubi [A] (verified)	2753
Maple [A] (verified)	2757
Fricas [A] (verification not implemented)	2757
Sympy [A] (verification not implemented)	2758
Maxima [A] (verification not implemented)	2758
Giac [F(-2)]	2759
Mupad [F(-1)]	2759
Reduce [F]	2759

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{40x}{9a^3} - \frac{2x^3}{27a} + \frac{40\sqrt{1-a^2x^2} \arcsin(ax)}{9a^4} + \frac{2x^2\sqrt{1-a^2x^2} \arcsin(ax)}{9a^2} + \frac{2x \arcsin(ax)^2}{a^3} + \frac{x^3 \arcsin(ax)^2}{3a} - \frac{2\sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^4} - \frac{x^2\sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^2}$$

output

```
-40/9*x/a^3-2/27*x^3/a+40/9*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^4+2/9*x^2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^2+2*x*arcsin(a*x)^2/a^3+1/3*x^3*arcsin(a*x)^2/a-2/3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{-2ax(60 + a^2x^2) + 6\sqrt{1-a^2x^2}(20 + a^2x^2) \arcsin(ax) + 9ax(6 + a^2x^2) \arcsin(ax)^2 - 9\sqrt{1-a^2x^2}(2 + a^2x^2) \arcsin(ax)^3}{27a^4}$$

input

```
Integrate[(x^3*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output

```
(-2*a*x*(60 + a^2*x^2) + 6*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2)*ArcSin[a*x] +
9*a*x*(6 + a^2*x^2)*ArcSin[a*x]^2 - 9*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcSin[a*x]^3)/(27*a^4)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5210, 5138, 5182, 5130, 5182, 24, 5210, 15, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$\downarrow \text{5210}$$

$$\frac{2 \int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{3a^2} + \frac{\int x^2 \arcsin(ax)^2 dx}{a} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^2}$$

$$\downarrow \text{5138}$$

$$\frac{2 \int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{3a^2} + \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{a} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^2}$$

$$\downarrow \text{5182}$$

$$\begin{aligned}
 & \frac{2\left(\frac{3\int \arcsin(ax)^2 dx}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)^3}{a^2}\right)}{3a^2} + \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{a} \\
 & \qquad \qquad \qquad \frac{x^2\sqrt{1-a^2x^2}\arcsin(ax)^3}{3a^2} \\
 & \qquad \qquad \qquad \downarrow \text{5130} \\
 & \frac{2\left(\frac{3\left(x \arcsin(ax)^2 - 2a \int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx\right)}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)^3}{a^2}\right)}{3a^2} + \\
 & \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{a} - \frac{x^2\sqrt{1-a^2x^2}\arcsin(ax)^3}{3a^2} \\
 & \qquad \qquad \qquad \downarrow \text{5182} \\
 & \frac{2\left(\frac{3\left(x \arcsin(ax)^2 - 2a\left(\frac{\int 1 dx}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)}{a^2}\right)\right)}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)^3}{a^2}\right)}{3a^2} + \\
 & \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{a} - \frac{x^2\sqrt{1-a^2x^2}\arcsin(ax)^3}{3a^2} \\
 & \qquad \qquad \qquad \downarrow \text{24} \\
 & \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \int \frac{x^3 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{a} - \frac{x^2\sqrt{1-a^2x^2}\arcsin(ax)^3}{3a^2} + \\
 & \frac{2\left(\frac{3\left(x \arcsin(ax)^2 - 2a\left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)}{a^2}\right)\right)}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)^3}{a^2}\right)}{3a^2} \\
 & \qquad \qquad \qquad \downarrow \text{5210} \\
 & \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a\left(\frac{2\int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{3a^2} + \frac{\int x^2 dx}{3a} - \frac{x^2\sqrt{1-a^2x^2}\arcsin(ax)}{3a^2}\right)}{a} \\
 & \frac{x^2\sqrt{1-a^2x^2}\arcsin(ax)^3}{3a^2} + \frac{2\left(\frac{3\left(x \arcsin(ax)^2 - 2a\left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)}{a^2}\right)\right)}{a} - \frac{\sqrt{1-a^2x^2}\arcsin(ax)^3}{a^2}\right)}{3a^2} \\
 & \qquad \qquad \qquad \downarrow \text{15}
 \end{aligned}$$

$$\begin{aligned}
& \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \left(\frac{2 \int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{3a^2} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} + \frac{x^3}{9a} \right)}{a} - \\
& \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^2} + \frac{2 \left(\frac{3 \left(x \arcsin(ax)^2 - 2a \left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right) \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \right)}{3a^2} \\
& \quad \downarrow 5182 \\
& \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \left(\frac{2 \left(\frac{\int 1 dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right)}{3a^2} - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} + \frac{x^3}{9a} \right)}{a} - \\
& \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^2} + \frac{2 \left(\frac{3 \left(x \arcsin(ax)^2 - 2a \left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right) \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \right)}{3a^2} \\
& \quad \downarrow 24 \\
& - \frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)^3}{3a^2} + \frac{2 \left(\frac{3 \left(x \arcsin(ax)^2 - 2a \left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right) \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \right)}{3a^2} + \\
& \frac{\frac{1}{3}x^3 \arcsin(ax)^2 - \frac{2}{3}a \left(-\frac{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}{3a^2} + \frac{2 \left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right)}{3a^2} + \frac{x^3}{9a} \right)}{a}
\end{aligned}$$

input `Int[(x^3*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2], x]`

output `-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/a^2 + ((x^3*ArcSin[a*x]^2)/3 - (2*a*(x^3/(9*a) - (x^2*Sqrt[1 - a^2*x^2]*ArcSin[a*x]))/(3*a^2) + (2*(x/a - (Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a^2))/(3*a^2)))/3/a + (2*(-((Sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/a^2) + (3*(x*ArcSin[a*x]^2 - 2*a*(x/a - (Sqrt[1 - a^2*x^2]*ArcSin[a*x])/a^2)))/a))/(3*a^2)`

Definitions of rubi rules used

- rule 15 $\text{Int}[(a_.)(x_)^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[a*(x^{(m+1)})/(m+1), x] \text{ ; FreeQ}[\{a, m\}, x] \ \&\& \ \text{NeQ}[m, -1]$
- rule 24 $\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] \text{ ; FreeQ}[a, x]$
- rule 5130 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSin}[c*x])^n, x] - \text{Simp}[b*c*n \text{ Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] \text{ ; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$
- rule 5138 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((d_.)(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{ Int}[(d*x)^{(m+1)*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] \text{ ; FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$
- rule 5182 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x^2)^{(p+1)*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p+1))), x] + \text{Simp}[b*(n/(2*c*(p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(1 - c^2*x^2)^{(p+1/2)*((a + b*\text{ArcSin}[c*x])^{(n-1)}), x], x] \text{ ; FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$
- rule 5210 $\text{Int}[(a_. + \text{ArcSin}[c_.)(x_)]*(b_.)^{(n_.)*((f_.)(x_)^{(m_.)*((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)*(d + e*x^2)^{(p+1)*((a + b*\text{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\text{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \text{ Int}[(f*x)^{(m-2)*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m-1)*(1 - c^2*x^2)^{(p+1/2)*((a + b*\text{ArcSin}[c*x])^{(n-1)}), x], x]) \text{ ; FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0]$

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15

method	result
default	$-\frac{(9 \arcsin(ax)^3 a^4 x^4 + 9 \arcsin(ax)^3 a^2 x^2 + 9 \arcsin(ax)^2 \sqrt{-a^2 x^2 + 1} a^3 x^3 - 6 \arcsin(ax) a^4 x^4 - 114 \arcsin(ax) a^2 x^2 - 2 \sqrt{-a^2 x^2 + 1} a^3 x^3 - 27 a^4 (a^2 x^2 - 1))}{27 a^4 (a^2 x^2 - 1)}$
orering	$\frac{5(13 a^6 x^6 + 144 a^4 x^4 - 936 a^2 x^2 + 864) \arcsin(ax)^3}{81 a^6 x^2 \sqrt{-a^2 x^2 + 1}} - \frac{(25 a^6 x^6 + 578 a^4 x^4 - 2940 a^2 x^2 + 2520) \left(\frac{3 x^2 \arcsin(ax)^3}{\sqrt{-a^2 x^2 + 1}} + \frac{3 x^3 \arcsin(ax)^2 a}{-a^2 x^2 + 1} + \frac{x^4}{\sqrt{-a^2 x^2 + 1}} \right)}{81 a^6 x^4}$

input `int(x^3*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & -1/27/a^4*(9*\arcsin(a*x)^3*a^4*x^4+9*\arcsin(a*x)^3*a^2*x^2+9*\arcsin(a*x)^2 \\ & *(-a^2*x^2+1)^(1/2)*a^3*x^3-6*\arcsin(a*x)*a^4*x^4-114*\arcsin(a*x)*a^2*x^2- \\ & 2*(-a^2*x^2+1)^(1/2)*a^3*x^3-18*\arcsin(a*x)^3+54*(-a^2*x^2+1)^(1/2)*\arcsin \\ & (a*x)^2*a*x+120*\arcsin(a*x)-120*(-a^2*x^2+1)^(1/2)*x*a*(-a^2*x^2+1)^(1/2) \\ & /(a^2*x^2-1) \end{aligned}$$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{2 a^3 x^3 - 9 (a^3 x^3 + 6 a x) \arcsin(ax)^2 + 120 a x + 3 \sqrt{-a^2 x^2 + 1} (3 (a^2 x^2 + 2) \arcsin(ax)^3 - 2 (a^2 x^2 + 2) \arcsin(ax) \sqrt{-a^2 x^2 + 1})}{27 a^4}$$

input `integrate(x^3*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output
$$\begin{aligned} & -1/27*(2*a^3*x^3 - 9*(a^3*x^3 + 6*a*x)*\arcsin(a*x)^2 + 120*a*x + 3*\sqrt{-a \\ & ^2*x^2 + 1}*(3*(a^2*x^2 + 2)*\arcsin(a*x)^3 - 2*(a^2*x^2 + 20)*\arcsin(a*x) \\ &)/a^4 \end{aligned}$$

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \begin{cases} \frac{x^3 \arcsin^2(ax)}{3a} - \frac{2x^3}{27a} - \frac{x^2 \sqrt{-a^2x^2+1} \arcsin^3(ax)}{3a^2} + \frac{2x^2 \sqrt{-a^2x^2+1} \arcsin(ax)}{9a^2} + \frac{2x \arcsin^2(ax)}{a^3} - \frac{40x}{9a^3} - \frac{2\sqrt{-a^2x^2+1} \arcsin^3(ax)}{3a^4} + 4 \\ 0 \end{cases}$$

input `integrate(x**3*asin(a*x)**3/(-a**2*x**2+1)**(1/2),x)`output `Piecewise((x**3*asin(a*x)**2/(3*a) - 2*x**3/(27*a) - x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(3*a**2) + 2*x**2*sqrt(-a**2*x**2 + 1)*asin(a*x)/(9*a**2) + 2*x*asin(a*x)**2/a**3 - 40*x/(9*a**3) - 2*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(3*a**4) + 40*sqrt(-a**2*x**2 + 1)*asin(a*x)/(9*a**4), Ne(a, 0)), (0, True))`**Maxima [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= -\frac{1}{3} \left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arcsin(ax)^3$$

$$+ \frac{2}{27} a \left(\frac{3 \left(\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2} \right) \arcsin(ax)}{a^3} - \frac{a^2x^3 + 60x}{a^4} \right)$$

$$+ \frac{(a^2x^3 + 6x) \arcsin(ax)^2}{3a^3}$$

input `integrate(x^3*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `-1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arcsin(a*x)^3 + 2/27*a*(3*(sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-a^2*x^2 + 1)/a^2)*arcsin(a*x)/a^3 - (a^2*x^3 + 60*x)/a^4) + 1/3*(a^2*x^3 + 6*x)*arcsin(a*x)^2/a^3`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^3 \operatorname{asin}(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `int((x^3*asin(a*x)^3)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^3*asin(a*x)^3)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^3 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^3 x^3}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^3*asin(a*x)^3/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**3*x**3)/sqrt(- a**2*x**2 + 1),x)`

3.284 $\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result	2760
Mathematica [A] (verified)	2760
Rubi [A] (verified)	2761
Maple [A] (verified)	2763
Fricas [A] (verification not implemented)	2763
Sympy [A] (verification not implemented)	2764
Maxima [F]	2764
Giac [A] (verification not implemented)	2764
Mupad [F(-1)]	2765
Reduce [F]	2765

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{3x^2}{8a} + \frac{3x\sqrt{1-a^2x^2} \arcsin(ax)}{4a^2} - \frac{3 \arcsin(ax)^2}{8a^3} + \frac{3x^2 \arcsin(ax)^2}{4a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2} + \frac{\arcsin(ax)^4}{8a^3}$$

output

```
-3/8*x^2/a+3/4*x*(-a^2*x^2+1)^(1/2)*arcsin(a*x)/a^2-3/8*arcsin(a*x)^2/a^3+
3/4*x^2*arcsin(a*x)^2/a-1/2*x*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/a^2+1/8*arc
sin(a*x)^4/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{-3a^2x^2 + 6ax\sqrt{1-a^2x^2} \arcsin(ax) + (-3 + 6a^2x^2) \arcsin(ax)^2 - 4ax\sqrt{1-a^2x^2} \arcsin(ax)^3 + \arcsin(ax)^4}{8a^3}$$

input

```
Integrate[(x^2*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output

$$(-3a^2x^2 + 6ax\sqrt{1-a^2x^2})\text{ArcSin}[ax] + (-3 + 6a^2x^2)\text{ArcSin}[ax]^2 - 4ax\sqrt{1-a^2x^2}\text{ArcSin}[ax]^3 + \text{ArcSin}[ax]^4 / (8a^3)$$

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5210, 5138, 5152, 5210, 15, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$\downarrow 5210$$

$$\frac{\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{3 \int x \arcsin(ax)^2 dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2}$$

$$\downarrow 5138$$

$$\frac{3\left(\frac{1}{2}x^2 \arcsin(ax)^2 - a \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx\right)}{2a} + \frac{\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2}$$

$$\downarrow 5152$$

$$\frac{3\left(\frac{1}{2}x^2 \arcsin(ax)^2 - a \int \frac{x^2 \arcsin(ax)}{\sqrt{1-a^2x^2}} dx\right)}{2a} + \frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2}$$

$$\downarrow 5210$$

$$\frac{3\left(\frac{1}{2}x^2 \arcsin(ax)^2 - a\left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} + \frac{\int x dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2}\right)\right)}{2a} + \frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2}$$

$$\downarrow 15$$

$$\begin{aligned}
& \frac{3 \left(\frac{1}{2} x^2 \arcsin(ax)^2 - a \left(\frac{\int \frac{\arcsin(ax)}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right) \right)}{\frac{2a}{x\sqrt{1-a^2x^2} \arcsin(ax)^3} + \frac{\arcsin(ax)^4}{8a^3}} \\
& \quad \downarrow \text{5152} \\
& \frac{3 \left(\frac{1}{2} x^2 \arcsin(ax)^2 - a \left(\frac{\arcsin(ax)^2}{4a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)}{2a^2} + \frac{x^2}{4a} \right) \right)}{2a} + \frac{\arcsin(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arcsin(ax)^3}{2a^2}
\end{aligned}$$

input `Int[(x^2*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2], x]`

output `-1/2*(x*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/a^2 + ArcSin[a*x]^4/(8*a^3) + (3*((x^2*ArcSin[a*x]^2)/2 - a*(x^2/(4*a) - (x*Sqrt[1 - a^2*x^2]*ArcSin[a*x]))/(2*a^2) + ArcSin[a*x]^2/(4*a^3)))/(2*a)`

Defintions of rubi rules used

rule 15 `Int[(a_.)*(x_)^(m_.), x_Symbol] := Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[{a, m}, x] && NeQ[m, -1]`

rule 5138 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5210

```

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]

```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{-4 \arcsin(ax)^3 \sqrt{-a^2x^2+1} ax + 6x^2 \arcsin(ax)^2 a^2 + \arcsin(ax)^4 + 6 \arcsin(ax) \sqrt{-a^2x^2+1} ax - 3a^2x^2 - 3 \arcsin(ax)^2}{8a^3}$	85

input

```
int(x^2*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```

1/8*(-4*arcsin(a*x)^3*(-a^2*x^2+1)^(1/2)*a*x+6*x^2*arcsin(a*x)^2*a^2+arcsi
n(a*x)^4+6*arcsin(a*x)*(-a^2*x^2+1)^(1/2)*a*x-3*a^2*x^2-3*arcsin(a*x)^2)/a
^3

```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{3a^2x^2 - \arcsin(ax)^4 - 3(2a^2x^2 - 1)\arcsin(ax)^2 + 2(2ax \arcsin(ax))^3 - 3ax \arcsin(ax)\sqrt{-a^2x^2 + 1}}{8a^3}$$

input

```
integrate(x^2*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```

-1/8*(3*a^2*x^2 - arcsin(a*x)^4 - 3*(2*a^2*x^2 - 1)*arcsin(a*x)^2 + 2*(2*a
*x*arcsin(a*x)^3 - 3*a*x*arcsin(a*x))*sqrt(-a^2*x^2 + 1))/a^3

```


Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{3x^2 \operatorname{asin}^2(ax)}{4a} - \frac{3x^2}{8a} - \frac{x\sqrt{-a^2x^2+1} \operatorname{asin}^3(ax)}{2a^2} + \frac{3x\sqrt{-a^2x^2+1} \operatorname{asin}(ax)}{4a^2} + \frac{\operatorname{asin}^4(ax)}{8a^3} - \frac{3 \operatorname{asin}^2(ax)}{8a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x**2*asin(a*x)**3/(-a**2*x**2+1)**(1/2),x)`output `Piecewise((3*x**2*asin(a*x)**2/(4*a) - 3*x**2/(8*a) - x*sqrt(-a**2*x**2 + 1)*asin(a*x)**3/(2*a**2) + 3*x*sqrt(-a**2*x**2 + 1)*asin(a*x)/(4*a**2) + asin(a*x)**4/(8*a**3) - 3*asin(a*x)**2/(8*a**3), Ne(a, 0)), (0, True))`**Maxima [F]**

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \arcsin(ax)^3}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^2*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `integrate(x^2*arcsin(a*x)^3/sqrt(-a^2*x^2 + 1), x)`**Giac [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{-a^2x^2+1} x \arcsin(ax)^3}{2a^2} + \frac{\arcsin(ax)^4}{8a^3} + \frac{3\sqrt{-a^2x^2+1} x \arcsin(ax)}{4a^2} + \frac{3(a^2x^2-1) \arcsin(ax)^2}{4a^3} + \frac{3 \arcsin(ax)^2}{8a^3} - \frac{3(a^2x^2-1)}{8a^3} - \frac{3}{16a^3}$$

input `integrate(x^2*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `-1/2*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^3/a^2 + 1/8*arcsin(a*x)^4/a^3 + 3/4*sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)/a^2 + 3/4*(a^2*x^2 - 1)*arcsin(a*x)^2/a^3 + 3/8*arcsin(a*x)^2/a^3 - 3/8*(a^2*x^2 - 1)/a^3 - 3/16/a^3`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \operatorname{asin}(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `int((x^2*asin(a*x)^3)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^2*asin(a*x)^3)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^2 \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^3 x^2}{\sqrt{-a^2x^2 + 1}} dx$$

input `int(x^2*asin(a*x)^3/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**3*x**2)/sqrt(- a**2*x**2 + 1),x)`

3.285 $\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result	2766
Mathematica [A] (verified)	2766
Rubi [A] (verified)	2767
Maple [A] (verified)	2768
Fricas [A] (verification not implemented)	2769
Sympy [A] (verification not implemented)	2769
Maxima [A] (verification not implemented)	2770
Giac [A] (verification not implemented)	2770
Mupad [F(-1)]	2771
Reduce [B] (verification not implemented)	2771

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{6x}{a} + \frac{6\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} + \frac{3x \arcsin(ax)^2}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2}$$

output

$-6*x/a+6*(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)/a^2+3*x*\arcsin(a*x)^2/a-(-a^2*x^2+1)^{(1/2)}*\arcsin(a*x)^3/a^2$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{-6ax + 6\sqrt{1-a^2x^2} \arcsin(ax) + 3ax \arcsin(ax)^2 - \sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2}$$

input

`Integrate[(x*ArcSin[a*x]^3)/Sqrt[1 - a^2*x^2],x]`

output

$$(-6*a*x + 6*sqrt[1 - a^2*x^2]*ArcSin[a*x] + 3*a*x*ArcSin[a*x]^2 - sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/a^2$$

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5182, 5130, 5182, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx \\ & \quad \downarrow \text{5182} \\ & \frac{3 \int \arcsin(ax)^2 dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \\ & \quad \downarrow \text{5130} \\ & \frac{3 \left(x \arcsin(ax)^2 - 2a \int \frac{x \arcsin(ax)}{\sqrt{1-a^2x^2}} dx \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \\ & \quad \downarrow \text{5182} \\ & \frac{3 \left(x \arcsin(ax)^2 - 2a \left(\frac{\int 1 dx}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right) \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \\ & \quad \downarrow \text{24} \\ & \frac{3 \left(x \arcsin(ax)^2 - 2a \left(\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)}{a^2} \right) \right)}{a} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{a^2} \end{aligned}$$

input

$$\text{Int}[(x*\text{ArcSin}[a*x]^3)/\text{Sqrt}[1 - a^2*x^2], x]$$

output

$$-((\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x]^3)/a^2) + (3*(x*\text{ArcSin}[a*x]^2 - 2*a*(x/a - (\text{Sqrt}[1 - a^2*x^2]*\text{ArcSin}[a*x])/a^2)))/a$$

Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 5130 `Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^n_.], x_Symbol] := Simp[x*(a + b*ArcSin[c*x])^n, x] - Simp[b*c*n Int[x*(a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]`

rule 5182 `Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^n_.)*(x_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]`

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{-a^2x^2+1} \left(\arcsin(ax)^3 a^2 x^2 - \arcsin(ax)^3 + 3\sqrt{-a^2x^2+1} \arcsin(ax)^2 ax - 6 \arcsin(ax) a^2 x^2 + 6 \arcsin(ax) - 6\sqrt{-a^2x^2+1} xa \right)}{a^2(a^2x^2-1)}$
orering	$\frac{(a^4x^4-8a^2x^2+8) \arcsin(ax)^3}{a^4x^2\sqrt{-a^2x^2+1}} - \frac{(a^4x^4-6a^2x^2+8) \left(\frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1}} + \frac{3x \arcsin(ax)^2 a}{-a^2x^2+1} + \frac{x^2 \arcsin(ax)^3 a^2}{(-a^2x^2+1)^{\frac{3}{2}}} \right)}{x^2 a^4} - \frac{2(ax-1)(ax+1)(a^2x^2+1)}{a^2(a^2x^2-1)}$

input `int(x*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `-1/a^2*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(arcsin(a*x)^3*a^2*x^2-arcsin(a*x)^3+3*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^2*a*x-6*arcsin(a*x)*a^2*x^2+6*arcsin(a*x)-6*(-a^2*x^2+1)^(1/2)*x*a)`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{3ax \arcsin(ax)^2 - 6ax - \sqrt{-a^2x^2+1}(\arcsin(ax)^3 - 6 \arcsin(ax))}{a^2}$$

input `integrate(x*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`output `(3*a*x*arcsin(a*x)^2 - 6*a*x - sqrt(-a^2*x^2 + 1)*(arcsin(a*x)^3 - 6*arcsin(a*x)))/a^2`**Sympy [A] (verification not implemented)**

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \begin{cases} \frac{3x \arcsin^2(ax)}{a} - \frac{6x}{a} - \frac{\sqrt{-a^2x^2+1} \arcsin^3(ax)}{a^2} + \frac{6\sqrt{-a^2x^2+1} \arcsin(ax)}{a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(x*asin(a*x)**3/(-a**2*x**2+1)**(1/2),x)`output `Piecewise((3*x*asin(a*x)**2/a - 6*x/a - sqrt(-a**2*x**2 + 1)*asin(a*x)**3/a**2 + 6*sqrt(-a**2*x**2 + 1)*asin(a*x)/a**2, Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{3x \arcsin(ax)^2}{a} - \frac{\sqrt{-a^2x^2+1} \arcsin(ax)^3}{a^2} - \frac{6 \left(x - \frac{\sqrt{-a^2x^2+1} \arcsin(ax)}{a} \right)}{a}$$

input `integrate(x*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `3*x*arcsin(a*x)^2/a - sqrt(-a^2*x^2 + 1)*arcsin(a*x)^3/a^2 - 6*(x - sqrt(-a^2*x^2 + 1)*arcsin(a*x)/a)/a`**Giac [A] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{\sqrt{-a^2x^2+1} \arcsin(ax)^3}{a^2} + \frac{3 \left(x \arcsin(ax)^2 - 2x + \frac{2\sqrt{-a^2x^2+1} \arcsin(ax)}{a} \right)}{a}$$

input `integrate(x*arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^3/a^2 + 3*(x*arcsin(a*x)^2 - 2*x + 2*sqrt(-a^2*x^2 + 1)*arcsin(a*x)/a)/a`

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \int \frac{x \operatorname{asin}(ax)^3}{\sqrt{1-a^2x^2}} dx$$

input `int((x*asin(a*x)^3)/(1 - a^2*x^2)^(1/2), x)`output `int((x*asin(a*x)^3)/(1 - a^2*x^2)^(1/2), x)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{x \arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$= \frac{-\sqrt{-a^2x^2+1} \operatorname{asin}(ax)^3 + 3 \operatorname{asin}(ax)^2 ax + 6\sqrt{-a^2x^2+1} \operatorname{asin}(ax) - 6ax}{a^2}$$

input `int(x*asin(a*x)^3/(-a^2*x^2+1)^(1/2), x)`output `(- sqrt(- a**2*x**2 + 1)*asin(a*x)**3 + 3*asin(a*x)**2*a*x + 6*sqrt(- a**2*x**2 + 1)*asin(a*x) - 6*a*x)/a**2`

3.286 $\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result	2772
Mathematica [A] (verified)	2772
Rubi [A] (verified)	2773
Maple [A] (verified)	2773
Fricas [A] (verification not implemented)	2774
Sympy [A] (verification not implemented)	2774
Maxima [A] (verification not implemented)	2775
Giac [A] (verification not implemented)	2775
Mupad [B] (verification not implemented)	2775
Reduce [B] (verification not implemented)	2776

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

output `1/4*arcsin(a*x)^4/a`

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

input `Integrate[ArcSin[a*x]^3/Sqrt[1 - a^2*x^2],x]`

output `ArcSin[a*x]^4/(4*a)`

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx$$

↓ 5152

$$\frac{\arcsin(ax)^4}{4a}$$

input `Int[ArcSin[a*x]^3/Sqrt[1 - a^2*x^2], x]`

output `ArcSin[a*x]^4/(4*a)`

Defintions of rubi rules used

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
;/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\arcsin(ax)^4}{4a}$	12
default	$\frac{\arcsin(ax)^4}{4a}$	12

input `int(arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `1/4*arcsin(a*x)^4/a`

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

input `integrate(arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `1/4*arcsin(a*x)^4/a`

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{\arcsin^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input `integrate(asin(a*x)**3/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((asin(a*x)**4/(4*a), Ne(a, 0)), (0, True))`

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

input `integrate(arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `1/4*arcsin(a*x)^4/a`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

input `integrate(arcsin(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `1/4*arcsin(a*x)^4/a`**Mupad [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

input `int(asin(a*x)^3/(1 - a^2*x^2)^(1/2),x)`output `asin(a*x)^4/(4*a)`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arcsin(ax)^3}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^4}{4a}$$

input `int(asin(a*x)^3/(-a^2*x^2+1)^(1/2),x)`

output `asin(a*x)**4/(4*a)`

3.287 $\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx$

Optimal result	2777
Mathematica [A] (verified)	2778
Rubi [A] (verified)	2778
Maple [A] (verified)	2781
Fricas [F]	2782
Sympy [F]	2782
Maxima [F]	2782
Giac [F]	2783
Mupad [F(-1)]	2783
Reduce [F]	2783

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = -2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)})$$

$$+ 3i \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)})$$

$$- 3i \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)})$$

$$- 6 \arcsin(ax) \operatorname{PolyLog}(3, -e^{i \arcsin(ax)})$$

$$+ 6 \arcsin(ax) \operatorname{PolyLog}(3, e^{i \arcsin(ax)})$$

$$- 6i \operatorname{PolyLog}(4, -e^{i \arcsin(ax)}) + 6i \operatorname{PolyLog}(4, e^{i \arcsin(ax)})$$

output

```
-2*arcsin(a*x)^3*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))+3*I*arcsin(a*x)^2*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-3*I*arcsin(a*x)^2*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))-6*arcsin(a*x)*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))+6*arcsin(a*x)*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))-6*I*polylog(4,-I*a*x-(-a^2*x^2+1)^(1/2))+6*I*polylog(4,I*a*x+(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = -\frac{1}{8}i(\pi^4 - 2\arcsin(ax)^4 + 8i\arcsin(ax)^3 \log(1 - e^{-i\arcsin(ax)}) - 8i\arcsin(ax)^3 \log(1 + e^{i\arcsin(ax)}) - 24\arcsin(ax)^2 \text{PolyLog}(2, e^{-i\arcsin(ax)}) - 24\arcsin(ax)^2 \text{PolyLog}(2, -e^{i\arcsin(ax)}) + 48i\arcsin(ax) \text{PolyLog}(3, e^{-i\arcsin(ax)}) - 48i\arcsin(ax) \text{PolyLog}(3, -e^{i\arcsin(ax)}) + 48 \text{PolyLog}(4, e^{-i\arcsin(ax)}) + 48 \text{PolyLog}(4, -e^{i\arcsin(ax)}))$$

input

```
Integrate[ArcSin[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output

```
(-1/8*I)*(Pi^4 - 2*ArcSin[a*x]^4 + (8*I)*ArcSin[a*x]^3*Log[1 - E^((-I)*ArcSin[a*x])] - (8*I)*ArcSin[a*x]^3*Log[1 + E^(I*ArcSin[a*x])] - 24*ArcSin[a*x]^2*PolyLog[2, E^((-I)*ArcSin[a*x])] - 24*ArcSin[a*x]^2*PolyLog[2, -E^(I*ArcSin[a*x])] + (48*I)*ArcSin[a*x]*PolyLog[3, E^((-I)*ArcSin[a*x])] - (48*I)*ArcSin[a*x]*PolyLog[3, -E^(I*ArcSin[a*x])] + 48*PolyLog[4, E^((-I)*ArcSin[a*x])] + 48*PolyLog[4, -E^(I*ArcSin[a*x])])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5218, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx \quad \downarrow \quad 5218$$

$$\int \frac{\arcsin(ax)^3}{ax} d\arcsin(ax)$$

$$\begin{aligned}
& \downarrow 3042 \\
& \int \arcsin(ax)^3 \csc(\arcsin(ax)) d \arcsin(ax) \\
& \downarrow 4671 \\
& -3 \int \arcsin(ax)^2 \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + \\
& 3 \int \arcsin(ax)^2 \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)}) \\
& \downarrow 3011 \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - 2i \int \arcsin(ax) \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) d \arcsin(ax) \right) - \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) - 2i \int \arcsin(ax) \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) d \arcsin(ax) \right) - \\
& 2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)}) \\
& \downarrow 7163 \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - 2i \left(i \int \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) d \arcsin(ax) - i \arcsin(ax) \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) \right) \right) - \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) - 2i \left(i \int \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) d \arcsin(ax) - i \arcsin(ax) \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) \right) \right) - \\
& 2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)}) \\
& \downarrow 2720 \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - 2i \left(\int e^{-i \arcsin(ax)} \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) de^{i \arcsin(ax)} - i \arcsin(ax) \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) \right) \right) - \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) - 2i \left(\int e^{-i \arcsin(ax)} \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) de^{i \arcsin(ax)} - i \arcsin(ax) \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) \right) \right) - \\
& 2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)}) \\
& \downarrow 7143 \\
& -2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)}) + \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - 2i \left(\operatorname{PolyLog}(4, -e^{i \arcsin(ax)}) - i \arcsin(ax) \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) \right) \right) - \\
& 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) - 2i \left(\operatorname{PolyLog}(4, e^{i \arcsin(ax)}) - i \arcsin(ax) \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) \right) \right)
\end{aligned}$$

input `Int[ArcSin[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]`

output `-2*ArcSin[a*x]^3*ArcTanh[E^(I*ArcSin[a*x])] + 3*(I*ArcSin[a*x]^2*PolyLog[2, -E^(I*ArcSin[a*x])] - (2*I)*((-I)*ArcSin[a*x]*PolyLog[3, -E^(I*ArcSin[a*x])]) + PolyLog[4, -E^(I*ArcSin[a*x])]) - 3*(I*ArcSin[a*x]^2*PolyLog[2, E^(I*ArcSin[a*x])] - (2*I)*((-I)*ArcSin[a*x]*PolyLog[3, E^(I*ArcSin[a*x])]) + PolyLog[4, E^(I*ArcSin[a*x])])`

Defintions of rubi rules used

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_)^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F])) Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.63

method	result
default	$i(i \arcsin(ax))^3 \ln(1 + iax + \sqrt{-a^2x^2 + 1}) + 3 \arcsin(ax)^2 \operatorname{polylog}(2, -iax - \sqrt{-a^2x^2 + 1}) +$

input

```
int(arcsin(a*x)^3/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
I*(I*arcsin(a*x)^3*ln(1+I*a*x+(-a^2*x^2+1)^(1/2))+3*arcsin(a*x)^2*polylog(
2,-I*a*x-(-a^2*x^2+1)^(1/2))+6*I*arcsin(a*x)*polylog(3,-I*a*x-(-a^2*x^2+1)
^(1/2))-6*polylog(4,-I*a*x-(-a^2*x^2+1)^(1/2))-I*arcsin(a*x)^3*ln(1-I*a*x-
(-a^2*x^2+1)^(1/2))-3*arcsin(a*x)^2*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))-6*
I*arcsin(a*x)*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))+6*polylog(4,I*a*x+(-a^2*
x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1x}} dx$$

input `integrate(arcsin(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^3/(a^2*x^3 - x), x)`

Sympy [F]

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F]

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1x}} dx$$

input `integrate(arcsin(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(arcsin(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)`

Giac [F]

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1}x} dx$$

input `integrate(arcsin(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^3}{x\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^3}{\sqrt{-a^2x^2+1}x} dx$$

input `int(asin(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)**3/(sqrt(- a**2*x**2 + 1)*x),x)`

3.288 $\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx$

Optimal result	2784
Mathematica [A] (verified)	2785
Rubi [A] (verified)	2785
Maple [A] (verified)	2788
Fricas [F]	2789
Sympy [F]	2789
Maxima [F]	2789
Giac [F]	2790
Mupad [F(-1)]	2790
Reduce [F]	2790

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = -ia \arcsin(ax)^3 - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + 3a \arcsin(ax)^2 \log(1 - e^{2i \arcsin(ax)}) - 3ia \arcsin(ax) \text{PolyLog}(2, e^{2i \arcsin(ax)}) + \frac{3}{2}a \text{PolyLog}(3, e^{2i \arcsin(ax)})$$

output

```
-I*a*arcsin(a*x)^3-(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/x+3*a*arcsin(a*x)^2*ln
(1-(I*a*x+(-a^2*x^2+1)^(1/2))^2)-3*I*a*arcsin(a*x)*polylog(2,(I*a*x+(-a^2*
x^2+1)^(1/2))^2)+3/2*a*polylog(3,(I*a*x+(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \frac{1}{8}a \left(-i\pi^3 + 8i \arcsin(ax)^3 - \frac{8\sqrt{1-a^2x^2} \arcsin(ax)^3}{ax} \right. \\ \left. + 24 \arcsin(ax)^2 \log(1 - e^{-2i \arcsin(ax)}) \right. \\ \left. + 24i \arcsin(ax) \operatorname{PolyLog}(2, e^{-2i \arcsin(ax)}) \right. \\ \left. + 12 \operatorname{PolyLog}(3, e^{-2i \arcsin(ax)}) \right)$$

input `Integrate[ArcSin[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]`

output `(a*((-I)*Pi^3 + (8*I)*ArcSin[a*x]^3 - (8*Sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/
(a*x) + 24*ArcSin[a*x]^2*Log[1 - E^((-2*I)*ArcSin[a*x])] + (24*I)*ArcSin[a
*x]*PolyLog[2, E^((-2*I)*ArcSin[a*x])] + 12*PolyLog[3, E^((-2*I)*ArcSin[a*
x])]))/8`

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5186, 5136, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx \\ \downarrow \text{5186} \\ 3a \int \frac{\arcsin(ax)^2}{x} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} \\ \downarrow \text{5136}$$

$$\begin{aligned}
 & 3a \int \frac{\sqrt{1-a^2x^2} \arcsin(ax)^2}{ax} d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} \\
 & \quad \downarrow \text{3042} \\
 & 3a \int -\arcsin(ax)^2 \tan\left(\arcsin(ax) + \frac{\pi}{2}\right) d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} \\
 & \quad \downarrow \text{25} \\
 & -3a \int \arcsin(ax)^2 \tan\left(\arcsin(ax) + \frac{\pi}{2}\right) d \arcsin(ax) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} \\
 & \quad \downarrow \text{4200} \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + 3a \left(2i \int -\frac{e^{2i \arcsin(ax)} \arcsin(ax)^2}{1-e^{2i \arcsin(ax)}} d \arcsin(ax) - \frac{1}{3} i \arcsin(ax)^3 \right) \\
 & \quad \downarrow \text{25} \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + 3a \left(-2i \int \frac{e^{2i \arcsin(ax)} \arcsin(ax)^2}{1-e^{2i \arcsin(ax)}} d \arcsin(ax) - \frac{1}{3} i \arcsin(ax)^3 \right) \\
 & \quad \downarrow \text{2620} \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + \\
 & 3a \left(-2i \left(\frac{1}{2} i \arcsin(ax)^2 \log\left(1-e^{2i \arcsin(ax)}\right) - i \int \arcsin(ax) \log\left(1-e^{2i \arcsin(ax)}\right) d \arcsin(ax) \right) - \frac{1}{3} i \arcsin(ax)^3 \right) \\
 & \quad \downarrow \text{3011} \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + \\
 & 3a \left(-2i \left(\frac{1}{2} i \arcsin(ax)^2 \log\left(1-e^{2i \arcsin(ax)}\right) - i \left(\frac{1}{2} i \arcsin(ax) \operatorname{PolyLog}\left(2, e^{2i \arcsin(ax)}\right) - \frac{1}{2} i \int \operatorname{PolyLog}\left(2, e^{2i \arcsin(ax)}\right) d \arcsin(ax) \right) - \frac{1}{3} i \arcsin(ax)^3 \right) \\
 & \quad \downarrow \text{2720} \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + \\
 & 3a \left(-2i \left(\frac{1}{2} i \arcsin(ax)^2 \log\left(1-e^{2i \arcsin(ax)}\right) - i \left(\frac{1}{2} i \arcsin(ax) \operatorname{PolyLog}\left(2, e^{2i \arcsin(ax)}\right) - \frac{1}{4} \int e^{-2i \arcsin(ax)} \operatorname{PolyLog}\left(2, e^{2i \arcsin(ax)}\right) d \arcsin(ax) \right) - \frac{1}{3} i \arcsin(ax)^3 \right) \\
 & \quad \downarrow \text{7143} \\
 & -\frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{x} + \\
 & 3a \left(-2i \left(\frac{1}{2} i \arcsin(ax)^2 \log\left(1-e^{2i \arcsin(ax)}\right) - i \left(\frac{1}{2} i \arcsin(ax) \operatorname{PolyLog}\left(2, e^{2i \arcsin(ax)}\right) - \frac{1}{4} \operatorname{PolyLog}\left(3, e^{2i \arcsin(ax)}\right) \right) - \frac{1}{3} i \arcsin(ax)^3 \right)
 \end{aligned}$$

input `Int[ArcSin[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]`

output `-((Sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/x) + 3*a*((-1/3*I)*ArcSin[a*x]^3 - (2*I)*((I/2)*ArcSin[a*x]^2*Log[1 - E^((2*I)*ArcSin[a*x])] - I*((I/2)*ArcSin[a*x]*PolyLog[2, E^((2*I)*ArcSin[a*x])] - PolyLog[3, E^((2*I)*ArcSin[a*x])]/4)))`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m * E^(2*I*k*Pi) * (E^(2*I*(e + f*x)) / (1 + E^(2*I*k*Pi) * E^(2*I*(e + f*x))))], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n * Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b * ArcSin[c*x])^n / (d*f*(m + 1))), x] - Simp[b*c*(n / (f*(m + 1))) * Simp[(d + e*x^2)^p / (1 - c^2*x^2)^p] Int[(f*x)^(m + 1) * (1 - c^2*x^2)^(p + 1/2) * (a + b * ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

rule 7143 `Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p] / (e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]`

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.07

method	result
default	$\frac{(iax - \sqrt{-a^2x^2 + 1}) \arcsin(ax)^3}{x} - a(2i \arcsin(ax)^3 - 3 \arcsin(ax)^2 \ln(1 + iax + \sqrt{-a^2x^2 + 1}) - 3 \arcsin(ax) \ln^2(1 + iax + \sqrt{-a^2x^2 + 1}) - \ln^3(1 + iax + \sqrt{-a^2x^2 + 1}))$

input `int(arcsin(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `(I*a*x - (-a^2*x^2+1)^(1/2))*arcsin(a*x)^3/x - a*(2*I*arcsin(a*x)^3 - 3*arcsin(a*x)^2*ln(1+I*a*x+(-a^2*x^2+1)^(1/2)) - 3*arcsin(a*x)*ln(1-I*a*x-(-a^2*x^2+1)^(1/2)) + 6*I*arcsin(a*x)*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2)) + 6*I*arcsin(a*x)*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2)) - 6*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2)) - 6*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2)))`

Fricas [F]

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1x^2}} dx$$

input `integrate(arcsin(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^3/(a^2*x^4 - x^2), x)`

Sympy [F]

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^3(ax)}{x^2\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F]

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1x^2}} dx$$

input `integrate(arcsin(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `(3*a^3*x*integrate(x*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^2, x) - sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(a*x, sqrt(a*x + 1)*sqrt(-a*x + 1))^3)/x`

Giac [F]

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1}x^2} dx$$

input `integrate(arcsin(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)^3}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1}x^2} dx$$

input `int(asin(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)**3/(sqrt(- a**2*x**2 + 1)*x**2),x)`

$$3.289 \quad \int \frac{\arcsin(ax)^3}{x^3 \sqrt{1-a^2x^2}} dx$$

Optimal result	2791
Mathematica [A] (verified)	2792
Rubi [A] (verified)	2793
Maple [A] (verified)	2797
Fricas [F]	2798
Sympy [F]	2798
Maxima [F]	2799
Giac [F]	2799
Mupad [F(-1)]	2799
Reduce [F]	2800

Optimal result

Integrand size = 24, antiderivative size = 264

$$\begin{aligned} \int \frac{\arcsin(ax)^3}{x^3 \sqrt{1-a^2x^2}} dx = & -\frac{3a \arcsin(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2} \\ & - 6a^2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) \\ & - a^2 \arcsin(ax)^3 \operatorname{arctanh}(e^{i \arcsin(ax)}) \\ & + 3ia^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) \\ & + \frac{3}{2} ia^2 \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) \\ & - 3ia^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) \\ & - \frac{3}{2} ia^2 \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) \\ & - 3a^2 \arcsin(ax) \operatorname{PolyLog}(3, -e^{i \arcsin(ax)}) \\ & + 3a^2 \arcsin(ax) \operatorname{PolyLog}(3, e^{i \arcsin(ax)}) \\ & - 3ia^2 \operatorname{PolyLog}(4, -e^{i \arcsin(ax)}) + 3ia^2 \operatorname{PolyLog}(4, e^{i \arcsin(ax)}) \end{aligned}$$

output

```

-3/2*a*arcsin(a*x)^2/x-1/2*(-a^2*x^2+1)^(1/2)*arcsin(a*x)^3/x^2-6*a^2*arcs
in(a*x)*arctanh(I*a*x+(-a^2*x^2+1)^(1/2))-a^2*arcsin(a*x)^3*arctanh(I*a*x+
(-a^2*x^2+1)^(1/2))+3*I*a^2*polylog(2,-I*a*x+(-a^2*x^2+1)^(1/2))+3/2*I*a^2
*arcsin(a*x)^2*polylog(2,-I*a*x+(-a^2*x^2+1)^(1/2))-3*I*a^2*polylog(2,I*a*
x+(-a^2*x^2+1)^(1/2))-3/2*I*a^2*arcsin(a*x)^2*polylog(2,I*a*x+(-a^2*x^2+1)
^(1/2))-3*a^2*arcsin(a*x)*polylog(3,-I*a*x+(-a^2*x^2+1)^(1/2))+3*a^2*arcsi
n(a*x)*polylog(3,I*a*x+(-a^2*x^2+1)^(1/2))-3*I*a^2*polylog(4,-I*a*x+(-a^2*
x^2+1)^(1/2))+3*I*a^2*polylog(4,I*a*x+(-a^2*x^2+1)^(1/2))

```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.20

$$\begin{aligned}
\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = & \frac{1}{16}a^2 \left(-i\pi^4 + 2i \arcsin(ax)^4 - 12 \arcsin(ax)^2 \cot \left(\frac{1}{2} \arcsin(ax) \right) \right. \\
& - 2 \arcsin(ax)^3 \csc^2 \left(\frac{1}{2} \arcsin(ax) \right) \\
& + 8 \arcsin(ax)^3 \log(1 - e^{-i \arcsin(ax)}) \\
& + 48 \arcsin(ax) \log(1 - e^{i \arcsin(ax)}) \\
& - 48 \arcsin(ax) \log(1 + e^{i \arcsin(ax)}) \\
& - 8 \arcsin(ax)^3 \log(1 + e^{i \arcsin(ax)}) \\
& + 24i \arcsin(ax)^2 \text{PolyLog}(2, e^{-i \arcsin(ax)}) \\
& + 24i(2 + \arcsin(ax)^2) \text{PolyLog}(2, -e^{i \arcsin(ax)}) \\
& - 48i \text{PolyLog}(2, e^{i \arcsin(ax)}) \\
& + 48 \arcsin(ax) \text{PolyLog}(3, e^{-i \arcsin(ax)}) \\
& - 48 \arcsin(ax) \text{PolyLog}(3, -e^{i \arcsin(ax)}) \\
& - 48i \text{PolyLog}(4, e^{-i \arcsin(ax)}) - 48i \text{PolyLog}(4, -e^{i \arcsin(ax)}) \\
& + 2 \arcsin(ax)^3 \sec^2 \left(\frac{1}{2} \arcsin(ax) \right) \\
& \left. - 12 \arcsin(ax)^2 \tan \left(\frac{1}{2} \arcsin(ax) \right) \right)
\end{aligned}$$

input

```
Integrate[ArcSin[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output

```
(a^2*((-I)*Pi^4 + (2*I)*ArcSin[a*x]^4 - 12*ArcSin[a*x]^2*Cot[ArcSin[a*x]/2] - 2*ArcSin[a*x]^3*Csc[ArcSin[a*x]/2]^2 + 8*ArcSin[a*x]^3*Log[1 - E^((-I)*ArcSin[a*x])]) + 48*ArcSin[a*x]*Log[1 - E^(I*ArcSin[a*x])] - 48*ArcSin[a*x]*Log[1 + E^(I*ArcSin[a*x])] - 8*ArcSin[a*x]^3*Log[1 + E^(I*ArcSin[a*x])] + (24*I)*ArcSin[a*x]^2*PolyLog[2, E^((-I)*ArcSin[a*x])] + (24*I)*(2 + ArcSin[a*x]^2)*PolyLog[2, -E^(I*ArcSin[a*x])] - (48*I)*PolyLog[2, E^(I*ArcSin[a*x])] + 48*ArcSin[a*x]*PolyLog[3, E^((-I)*ArcSin[a*x])] - 48*ArcSin[a*x]*PolyLog[3, -E^(I*ArcSin[a*x])] - (48*I)*PolyLog[4, E^((-I)*ArcSin[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcSin[a*x])] + 2*ArcSin[a*x]^3*Sec[ArcSin[a*x]/2]^2 - 12*ArcSin[a*x]^2*Tan[ArcSin[a*x]/2]))/16
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5204, 5138, 5218, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx$$

$$\downarrow 5204$$

$$\frac{1}{2}a^2 \int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx + \frac{3}{2}a \int \frac{\arcsin(ax)^2}{x^2} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2}$$

$$\downarrow 5138$$

$$\frac{3}{2}a \left(2a \int \frac{\arcsin(ax)}{x\sqrt{1-a^2x^2}} dx - \frac{\arcsin(ax)^2}{x} \right) + \frac{1}{2}a^2 \int \frac{\arcsin(ax)^3}{x\sqrt{1-a^2x^2}} dx - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2}$$

$$\downarrow 5218$$

$$\frac{1}{2}a^2 \int \frac{\arcsin(ax)^3}{ax} d \arcsin(ax) + \frac{3}{2}a \left(2a \int \frac{\arcsin(ax)}{ax} d \arcsin(ax) - \frac{\arcsin(ax)^2}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2}$$

$$\downarrow 3042$$

$$\frac{1}{2}a^2 \int \arcsin(ax)^3 \csc(\arcsin(ax)) d \arcsin(ax) + \frac{3}{2}a \left(2a \int \arcsin(ax) \csc(\arcsin(ax)) d \arcsin(ax) - \frac{\arcsin(ax)^2}{x} \right) - \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2}$$

↓ 4671

$$\frac{1}{2}a^2 \left(-3 \int \arcsin(ax)^2 \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + 3 \int \arcsin(ax)^2 \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax) \right) - \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(-\int \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + \int \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax) \right) \right) + \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2}$$

↓ 2715

$$\frac{1}{2}a^2 \left(-3 \int \arcsin(ax)^2 \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + 3 \int \arcsin(ax)^2 \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax) \right) - \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(i \int e^{-i \arcsin(ax)} \log(1 - e^{i \arcsin(ax)}) de^{i \arcsin(ax)} - i \int e^{-i \arcsin(ax)} \log(1 + e^{i \arcsin(ax)}) de^{i \arcsin(ax)} \right) \right) + \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2}$$

↓ 2838

$$\frac{1}{2}a^2 \left(-3 \int \arcsin(ax)^2 \log(1 - e^{i \arcsin(ax)}) d \arcsin(ax) + 3 \int \arcsin(ax)^2 \log(1 + e^{i \arcsin(ax)}) d \arcsin(ax) - 2 \arcsin(ax) \right) + \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2} + \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(-2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) + i \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) \right) \right)$$

↓ 3011

$$\frac{1}{2}a^2 \left(3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - 2i \int \arcsin(ax) \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) d \arcsin(ax) \right) - 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) - 2i \int \arcsin(ax) \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) d \arcsin(ax) \right) \right) + \frac{\sqrt{1-a^2x^2} \arcsin(ax)^3}{2x^2} + \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(-2 \arcsin(ax) \operatorname{arctanh}(e^{i \arcsin(ax)}) + i \operatorname{PolyLog}(2, -e^{i \arcsin(ax)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(ax)}) \right) \right)$$

↓ 7163

$$\frac{1}{2}a^2 \left(3 \left(i \arcsin(ax)^2 \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - 2i \left(i \int \operatorname{PolyLog} \left(3, -e^{i \arcsin(ax)} \right) d \arcsin(ax) - i \arcsin(ax) \right) \right) \right. \\ \left. \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)^3}{2x^2} + \right.$$

$$\left. \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(-2 \arcsin(ax) \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right) + i \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - i \operatorname{PolyLog} \left(2, e^{i \arcsin(ax)} \right) \right) \right)$$

↓ 2720

$$\frac{1}{2}a^2 \left(3 \left(i \arcsin(ax)^2 \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - 2i \left(\int e^{-i \arcsin(ax)} \operatorname{PolyLog} \left(3, -e^{i \arcsin(ax)} \right) d e^{i \arcsin(ax)} - i a \right) \right) \right. \\ \left. \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)^3}{2x^2} + \right.$$

$$\left. \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(-2 \arcsin(ax) \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right) + i \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - i \operatorname{PolyLog} \left(2, e^{i \arcsin(ax)} \right) \right) \right)$$

↓ 7143

$$\frac{1}{2}a^2 \left(-2 \arcsin(ax)^3 \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right) + 3 \left(i \arcsin(ax)^2 \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - 2i \left(\operatorname{PolyLog} \left(4, -e^{i \arcsin(ax)} \right) \right) \right) \right. \\ \left. \frac{\sqrt{1 - a^2 x^2} \arcsin(ax)^3}{2x^2} + \right.$$

$$\left. \frac{3}{2}a \left(-\frac{\arcsin(ax)^2}{x} + 2a \left(-2 \arcsin(ax) \operatorname{arctanh} \left(e^{i \arcsin(ax)} \right) + i \operatorname{PolyLog} \left(2, -e^{i \arcsin(ax)} \right) - i \operatorname{PolyLog} \left(2, e^{i \arcsin(ax)} \right) \right) \right)$$

input `Int[ArcSin[a*x]^3/(x^3*sqrt[1 - a^2*x^2]),x]`

output `-1/2*(sqrt[1 - a^2*x^2]*ArcSin[a*x]^3)/x^2 + (3*a*(-(ArcSin[a*x]^2/x) + 2*a*(-2*ArcSin[a*x]*ArcTanh[E^(I*ArcSin[a*x])] + I*PolyLog[2, -E^(I*ArcSin[a*x]])] - I*PolyLog[2, E^(I*ArcSin[a*x])])))/2 + (a^2*(-2*ArcSin[a*x]^3*ArcTanh[E^(I*ArcSin[a*x])] + 3*(I*ArcSin[a*x]^2*PolyLog[2, -E^(I*ArcSin[a*x])]) - (2*I)*((-I)*ArcSin[a*x]*PolyLog[3, -E^(I*ArcSin[a*x])] + PolyLog[4, -E^(I*ArcSin[a*x])])) - 3*(I*ArcSin[a*x]^2*PolyLog[2, E^(I*ArcSin[a*x])]) - (2*I)*((-I)*ArcSin[a*x]*PolyLog[3, E^(I*ArcSin[a*x])] + PolyLog[4, E^(I*ArcSin[a*x])])))/2`

Defintions of rubi rules used

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2720 `Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] :> Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)
^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]`

rule 5138 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] := Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F])) Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.51

method	result
default	$-\frac{\sqrt{-a^2x^2+1} \arcsin(ax)^2 \left(\arcsin(ax)a^2x^2 - 3\sqrt{-a^2x^2+1}xa - \arcsin(ax) \right)}{2x^2(a^2x^2-1)} + \frac{ia^2 \left(i \arcsin(ax)^3 \ln(1+iax+\sqrt{-a^2x^2+1}) - i \arcsin(ax) \right)}{2x^2(a^2x^2-1)}$

input

```
int(arcsin(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(-a^2*x^2+1)^(1/2)/x^2/(a^2*x^2-1)*arcsin(a*x)^2*(arcsin(a*x)*a^2*x^2
-3*(-a^2*x^2+1)^(1/2)*x*a-arcsin(a*x))+1/2*I*a^2*(I*arcsin(a*x)^3*ln(1+I*a
*x+(-a^2*x^2+1)^(1/2))-I*arcsin(a*x)^3*ln(1-I*a*x-(-a^2*x^2+1)^(1/2))+3*ar
csin(a*x)^2*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-3*arcsin(a*x)^2*polylog(2
,I*a*x+(-a^2*x^2+1)^(1/2))+6*I*arcsin(a*x)*ln(1+I*a*x+(-a^2*x^2+1)^(1/2))+
6*I*arcsin(a*x)*polylog(3,-I*a*x-(-a^2*x^2+1)^(1/2))-6*I*arcsin(a*x)*ln(1-
I*a*x-(-a^2*x^2+1)^(1/2))-6*I*arcsin(a*x)*polylog(3,I*a*x+(-a^2*x^2+1)^(1/
2))+6*polylog(2,-I*a*x-(-a^2*x^2+1)^(1/2))-6*polylog(4,-I*a*x-(-a^2*x^2+1)
^(1/2))-6*polylog(2,I*a*x+(-a^2*x^2+1)^(1/2))+6*polylog(4,I*a*x+(-a^2*x^2+
1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1x^3}} dx$$

input

```
integrate(arcsin(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output

```
integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}} dx$$

input

```
integrate(asin(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

output

```
Integral(asin(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1}x^3} dx$$

input `integrate(arcsin(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `integrate(arcsin(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)`

Giac [F]

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^3}{\sqrt{-a^2x^2+1}x^3} dx$$

input `integrate(arcsin(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^3}{x^3\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{\arcsin(ax)^3}{x^3\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^3}{\sqrt{-a^2x^2+1}x^3} dx$$

input `int(asin(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)`

output `int(asin(a*x)**3/(sqrt(-a**2*x**2+1)*x**3),x)`

3.290 $\int \frac{x^4 \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$

Optimal result	2801
Mathematica [A] (verified)	2802
Rubi [A] (verified)	2802
Maple [A] (verified)	2804
Fricas [F]	2804
Sympy [F]	2804
Maxima [F]	2805
Giac [B] (verification not implemented)	2805
Mupad [F(-1)]	2806
Reduce [F]	2806

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{x^4 \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = -\frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc^5} - \frac{\cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{16bc^5} + \frac{\cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc^5} + \frac{\log(a+b \arcsin(cx))}{16bc^5} - \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc^5} - \frac{\sin\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{16bc^5} + \frac{\sin\left(\frac{6a}{b}\right) \text{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc^5}$$

output

```
-1/32*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c^5-1/16*cos(4*a/b)*Ci(4*(a+b*arcsin(c*x))/b)/b/c^5+1/32*cos(6*a/b)*Ci(6*(a+b*arcsin(c*x))/b)/b/c^5+1/16*ln(a+b*arcsin(c*x))/b/c^5-1/32*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c^5-1/16*sin(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b/c^5+1/32*sin(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx =$$

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 2 \cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) - \cos\left(\frac{6a}{b}\right) \operatorname{CosIntegral}\left(6\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{b^5}$$

input `Integrate[(x^4*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]`

output `-1/32*(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] + 2*Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])] - Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcSin[c*x])] - 2*Log[a + b*ArcSin[c*x]] + Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 2*Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] - Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])])/(b*c^5)`

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx$$

$$\downarrow 5224$$

$$\int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))$$

$$\frac{\quad}{bc^5}$$

$$\downarrow 4906$$

$$\int \left(\frac{\cos\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} - \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{1}{16(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

$$bc^5$$

$$\downarrow \text{2009}$$

$$-\frac{1}{32} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{16} \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{32} \cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]
```

output

```
(-1/32*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcSin[c*x]))/b]) - (Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcSin[c*x]))/b])/16 + (Cos[(6*a)/b]*CosIntegral[(6*(a + b*ArcSin[c*x]))/b])/32 + Log[a + b*ArcSin[c*x]]/16 - (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 - (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/16 + (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x]))/b])/32)/(b*c^5)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```


Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) + 2 \operatorname{Si}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) + \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) - \operatorname{Si}(6 \arcsin(cx) + \frac{6a}{b}) \sin(\frac{6a}{b}) - \operatorname{Ci}(6 \arcsin(cx) + \frac{6a}{b}) \cos(\frac{6a}{b}) + 2 \operatorname{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) - 2 \ln(a + b \arcsin(cx))}{32c^5 b}$

input `int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$-1/32/c^5*(\operatorname{Si}(2*\arcsin(c*x)+2*a/b)*\sin(2*a/b)+2*\operatorname{Si}(4*\arcsin(c*x)+4*a/b)*\sin(4*a/b)+\operatorname{Ci}(2*\arcsin(c*x)+2*a/b)*\cos(2*a/b)-\operatorname{Si}(6*\arcsin(c*x)+6*a/b)*\sin(6*a/b)-\operatorname{Ci}(6*\arcsin(c*x)+6*a/b)*\cos(6*a/b)+2*\operatorname{Ci}(4*\arcsin(c*x)+4*a/b)*\cos(4*a/b)-2*\ln(a+b*\arcsin(c*x)))/b$$

Fricas [F]

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^4}{b \arcsin(cx) + a} dx$$

input `integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^4/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^4 \sqrt{-(cx - 1)(cx + 1)}}{a + b \arcsin(cx)} dx$$

input `integrate(x**4*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**4*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^4}{b \arcsin(cx) + a} dx$$

input `integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(192) = 384.

Time = 0.15 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.29

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `cos(a/b)^6*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) + cos(a/b)^5*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) - 3/2*cos(a/b)^4*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) - 1/2*cos(a/b)^4*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) - cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) - 1/2*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) + 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) + 1/2*cos(a/b)^2*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) - 1/16*cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^5) + 3/16*cos(a/b)*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) + 1/4*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) - 1/16*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c^5) - 1/32*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^5) - 1/16*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) + 1/32*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^5) + 1/16*log(b*arcsin(c*x) + a)/(b*c^5)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)),x)`

output `int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)), x)`

Reduce [F]

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^4}{a \operatorname{sin}(cx) b + a} dx$$

input `int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2 + 1)*x**4)/(asin(c*x)*b + a),x)`

3.291 $\int \frac{x^3 \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$

Optimal result	2807
Mathematica [A] (verified)	2808
Rubi [A] (verified)	2808
Maple [A] (verified)	2810
Fricas [F]	2810
Sympy [F]	2811
Maxima [F]	2811
Giac [F(-2)]	2811
Mupad [F(-1)]	2812
Reduce [F]	2812

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{x^3 \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = -\frac{\text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{8bc^4} - \frac{\text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{16bc^4} + \frac{\text{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{5a}{b}\right)}{16bc^4} + \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8bc^4} + \frac{\cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16bc^4} - \frac{\cos\left(\frac{5a}{b}\right) \text{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16bc^4}$$

output

```
-1/8*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^4-1/16*Ci(3*(a+b*arcsin(c*x))/b)*sin(3*a/b)/b/c^4+1/16*Ci(5*(a+b*arcsin(c*x))/b)*sin(5*a/b)/b/c^4+1/8*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^4+1/16*cos(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b/c^4-1/16*cos(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx$$

$$= \frac{-2 \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{3a}{b}\right) + \operatorname{CosIntegral}\left(5\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{5a}{b}\right)}{16b^4 c^4}$$

input

```
Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]
```

output

```
(-2*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - CosIntegral[3*(a/b + ArcSin[c*x]])*Sin[(3*a)/b] + CosIntegral[5*(a/b + ArcSin[c*x]])*Sin[(5*a)/b] + 2*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] - Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])])/(16*b*c^4)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx$$

$$\downarrow \text{5224}$$

$$\int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^4}$$

$$\downarrow \text{25}$$

$$\int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^4}$$

$$\frac{\int \left(-\frac{\sin\left(\frac{5a}{b} - \frac{5(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{\sin\left(\frac{3a}{b} - \frac{3(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{8(a+b\arcsin(cx))} \right) d(a+b\arcsin(cx))}{bc^4}$$

↓ 4906

$$\frac{-\frac{1}{8} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b\arcsin(cx)}{b}\right) - \frac{1}{16} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b\arcsin(cx))}{b}\right) + \frac{1}{16} \sin\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b\arcsin(cx))}{b}\right)}{bc^4}$$

input `Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]`

output `(-1/8*(CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b]) - (CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/16 + (CosIntegral[(5*(a + b*ArcSin[c*x])/b]*Sin[(5*a)/b])/16 + (Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/16 - (Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/16)/(b*c^4)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2 \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b}) - 2 \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) + \operatorname{Si}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b}) - \operatorname{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b}) - \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) + \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b})}{16c^4b}$

input

```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-1/16/c^4*(2*Ci(arcsin(c*x)+a/b)*sin(a/b)-2*Si(arcsin(c*x)+a/b)*cos(a/b)+S
i(5*arcsin(c*x)+5*a/b)*cos(5*a/b)-Ci(5*arcsin(c*x)+5*a/b)*sin(5*a/b)-Si(3*
arcsin(c*x)+3*a/b)*cos(3*a/b)+Ci(3*arcsin(c*x)+3*a/b)*sin(3*a/b))/b
```

Fricas [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^3}{b \arcsin(cx) + a} dx$$

input

```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(sqrt(-c^2*x^2 + 1)*x^3/(b*arcsin(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^3 \sqrt{-(cx - 1)(cx + 1)}}{a + b \arcsin(cx)} dx$$

input `integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^3}{b \arcsin(cx) + a} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arcsin(c*x) + a), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)),x)`

output `int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)), x)`

Reduce [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^3}{a \operatorname{sin}(cx) b + a} dx$$

input `int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)*b + a),x)`

3.292 $\int \frac{x^2\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$

Optimal result	2813
Mathematica [A] (verified)	2813
Rubi [A] (verified)	2814
Maple [A] (verified)	2815
Fricas [F]	2816
Sympy [F]	2816
Maxima [F]	2816
Giac [B] (verification not implemented)	2817
Mupad [F(-1)]	2817
Reduce [F]	2818

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = -\frac{\cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8bc^3} + \frac{\log(a+b \arcsin(cx))}{8bc^3} - \frac{\sin\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8bc^3}$$

output

```
-1/8*cos(4*a/b)*Ci(4*(a+b*arcsin(c*x))/b)/b/c^3+1/8*ln(a+b*arcsin(c*x))/b/c^3-1/8*sin(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = \frac{\cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) - \log(8(a+b \arcsin(cx))) + \sin\left(\frac{4a}{b}\right) \text{Si}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{8bc^3}$$

input

```
Integrate[(x^2*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]
```

output

```
-1/8*(Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])] - Log[8*(a + b*ArcSin[c*x])] + Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])])/(b*c^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx$$

$$\downarrow \text{5224}$$

$$\frac{\int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^3}$$

$$\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8(a+b \arcsin(cx))} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))}{bc^3}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{1}{8} \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) - \frac{1}{8} \sin\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \log(a + b \arcsin(cx))}{bc^3}$$

input

```
Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]
```

output

```
(-1/8*(Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcSin[c*x]))/b]) + Log[a + b*ArcSin[c*x]]/8 - (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8)/(b*c^3)
```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\operatorname{Si}\left(4\arcsin\left(\frac{cx}{b}\right)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)+\operatorname{Ci}\left(4\arcsin\left(\frac{cx}{b}\right)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)-\ln\left(4b\arcsin\left(\frac{cx}{b}\right)+4a\right)}{8c^3b}$	68

input `int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `-1/8/c^3*(Si(4*arcsin(c*x)+4*a/b)*sin(4*a/b)+Ci(4*arcsin(c*x)+4*a/b)*cos(4*a/b)-ln(4*b*arcsin(c*x)+4*a))/b`

Fricas [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^2}{b \arcsin(cx) + a} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^2/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^2 \sqrt{-(cx - 1)(cx + 1)}}{a + b \arcsin(cx)} dx$$

input `integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^2}{b \arcsin(cx) + a} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(76) = 152$.

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.06

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = -\frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{bc^3} - \frac{\cos\left(\frac{a}{b}\right)^3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{bc^3} + \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{bc^3} + \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{2bc^3} - \frac{\operatorname{Ci}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{8bc^3} + \frac{\log(b \arcsin(cx) + a)}{8bc^3}$$

input `integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-cos(a/b)^4*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) - cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + cos(a/b)^2*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + 1/2*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) - 1/8*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + 1/8*log(b*arcsin(c*x) + a)/(b*c^3)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)),x)`

output `int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)), x)`

Reduce [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^2}{\arcsin(cx) b + a} dx$$

input `int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2+1)*x**2)/(asin(c*x)*b+a),x)`

3.293 $\int \frac{x\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$

Optimal result	2819
Mathematica [A] (verified)	2820
Rubi [A] (verified)	2820
Maple [A] (verified)	2822
Fricas [F]	2822
Sympy [F]	2822
Maxima [F]	2823
Giac [A] (verification not implemented)	2823
Mupad [F(-1)]	2824
Reduce [F]	2824

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = -\frac{\text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{4bc^2} - \frac{\text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{4bc^2} + \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{4bc^2} + \frac{\cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{4bc^2}$$

output

```
-1/4*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^2-1/4*Ci(3*(a+b*arcsin(c*x))/b)*
sin(3*a/b)/b/c^2+1/4*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^2+1/4*cos(3*a/b)
*Si(3*(a+b*arcsin(c*x))/b)/b/c^2
```


Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$$

$$= \frac{-\operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{3a}{b}\right) + \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{4bc^2}$$

input

```
Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]
```

output

```
(-(CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + ArcSin[c*x]])*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])])/(4*b*c^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$$

$$\downarrow 5224$$

$$\frac{\int -\frac{\cos^2\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2}$$

$$\downarrow 4906$$

$$\frac{\int \left(\frac{\sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{bc^2}$$

↓ 2009

$$\frac{-\frac{1}{4} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{4} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{bc^2}$$

input `Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]`

output `(-1/4*(CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b]) - (CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/4 + (Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/4 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/4)/(b*c^2)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\text{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) - \text{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) + \text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{4c^2b}$	92

input `int(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `1/4/c^2*(Si(3*arcsin(c*x)+3*a/b)*cos(3*a/b)-Ci(3*arcsin(c*x)+3*a/b)*sin(3*a/b)+Si(arcsin(c*x)+a/b)*cos(a/b)-Ci(arcsin(c*x)+a/b)*sin(a/b))/b`

Fricas [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\arcsin(cx)+a} dx$$

input `integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{x\sqrt{-(cx-1)(cx+1)}}{a+b\arcsin(cx)} dx$$

input `integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\arcsin(cx)+a} dx$$

input `integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)*x/(b*arcsin(c*x) + a), x)`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\begin{aligned} \int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = & -\frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{3a}{b} + 3\arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{bc^2} \\ & + \frac{\cos\left(\frac{a}{b}\right)^3 \operatorname{Si}\left(\frac{3a}{b} + 3\arcsin(cx)\right)}{bc^2} \\ & + \frac{\operatorname{Ci}\left(\frac{3a}{b} + 3\arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{4bc^2} - \frac{\operatorname{Ci}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{4bc^2} \\ & - \frac{3\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3\arcsin(cx)\right)}{4bc^2} \\ & + \frac{\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{4bc^2} \end{aligned}$$

input `integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-cos(a/b)^2*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^2) + cos(a/b)^3*sin_integral(3*a/b + 3*arcsin(c*x))/(b*c^2) + 1/4*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^2) - 1/4*cos_integral(a/b + arcsin(c*x))*sin(a/b)/(b*c^2) - 3/4*cos(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b*c^2) + 1/4*cos(a/b)*sin_integral(a/b + arcsin(c*x))/(b*c^2)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{x\sqrt{1-c^2x^2}}{a+b\sin(cx)} dx$$

input `int((x*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)),x)`

output `int((x*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)), x)`

Reduce [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-c^2x^2+1}x}{a\sin(cx)b+a} dx$$

input `int(x*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2+1)*x)/(asin(c*x)*b+a),x)`

3.294 $\int \frac{\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$

Optimal result	2825
Mathematica [A] (verified)	2825
Rubi [A] (verified)	2826
Maple [A] (verified)	2827
Fricas [F]	2828
Sympy [F]	2828
Maxima [F]	2828
Giac [A] (verification not implemented)	2829
Mupad [F(-1)]	2829
Reduce [F]	2830

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = \frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc} + \frac{\log(a+b \arcsin(cx))}{2bc} + \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc}$$

output

$1/2*\cos(2*a/b)*Ci(2*(a+b*\arcsin(c*x))/b)/b/c+1/2*\ln(a+b*\arcsin(c*x))/b/c+1/2*\sin(2*a/b)*Si(2*(a+b*\arcsin(c*x))/b)/b/c$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = \frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \log(a+b \arcsin(cx)) + \sin\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{2bc}$$

input

`Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcSin[c*x]),x]`

output

```
(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] + Log[a + b*ArcSin[c*x]]
+ Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])])/(2*b*c)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5168, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$$

$$\downarrow \text{5168}$$

$$\int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))$$

$$\frac{bc}{bc}$$

$$\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b} + \frac{\pi}{2}\right)^2}{a+b\arcsin(cx)} d(a+b\arcsin(cx))$$

$$\frac{bc}{bc}$$

$$\downarrow \text{3793}$$

$$\int \left(\frac{\cos\left(\frac{2a}{b} - \frac{2(a+b\arcsin(cx))}{b}\right)}{2(a+b\arcsin(cx))} + \frac{1}{2(a+b\arcsin(cx))} \right) d(a+b\arcsin(cx))$$

$$\frac{bc}{bc}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{1}{2} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b\arcsin(cx))}{b}\right) + \frac{1}{2} \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b\arcsin(cx))}{b}\right) + \frac{1}{2} \log(a+b\arcsin(cx))}{bc}$$

input

```
Int[Sqrt[1 - c^2*x^2]/(a + b*ArcSin[c*x]), x]
```

output
$$\left(\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left[\frac{2(a+b\text{ArcSin}[c*x])}{b}\right]}{2} + \frac{\log[a+b\text{ArcSin}[c*x]]}{2} + \frac{\sin\left(\frac{2a}{b}\right)\text{SinIntegral}\left[\frac{2(a+b\text{ArcSin}[c*x])}{b}\right]}{2}\right)/(b*c)$$

Defintions of rubi rules used

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 3793 $\text{Int}[\left((c_.) + (d_.)*(x_)\right)^{(m_)}*\sin[(e_.) + (f_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] \text{ ; FreeQ}\{c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (!\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$

rule 5168 $\text{Int}[\left((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\right)^{(n_)}*((d_.) + (e_.)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Subst}[\text{Int}[x^n*\text{Cos}[-a/b + x/b]^{(2*p + 1)}, x], x, a + b*\text{ArcSin}[c*x]], x] \text{ ; FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[2*p, 0]$

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\text{Si}\left(2\arcsin\left(\frac{cx}{b}\right) + \frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right) + \text{Ci}\left(2\arcsin\left(\frac{cx}{b}\right) + \frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right) + \ln\left(a + b\arcsin\left(\frac{cx}{b}\right)\right)}{2cb}$	63

input $\text{int}\left(\left(-c^2*x^2+1\right)^{(1/2)}/(a+b*\arcsin(c*x)),x,\text{method}=_RETURNVERBOSE\right)$

output
$$1/2/c*(\text{Si}(2*\arcsin(c*x)+2*a/b)*\sin(2*a/b)+\text{Ci}(2*\arcsin(c*x)+2*a/b)*\cos(2*a/b)+\ln(a+b*\arcsin(c*x)))/b$$

Fricas [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-c^2x^2+1}}{b\arcsin(cx)+a} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{a+b\arcsin(cx)} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-c^2x^2+1}}{b\arcsin(cx)+a} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{bc} + \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{bc} - \frac{\operatorname{Ci}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{2bc} + \frac{\log(b\arcsin(cx) + a)}{2bc}$$

input `integrate((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c) + cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c) - 1/2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c) + 1/2*log(b*arcsin(c*x) + a)/(b*c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx$$

input `int((1 - c^2*x^2)^(1/2)/(a + b*asin(c*x)),x)`

output `int((1 - c^2*x^2)^(1/2)/(a + b*asin(c*x)), x)`

Reduce [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arcsin(cx)} dx = \int \frac{\sqrt{-c^2x^2+1}}{\arcsin(cx)b+a} dx$$

input `int((-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int(sqrt(-c**2*x**2+1)/(asin(c*x)*b+a),x)`

3.295 $\int \frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))} dx$

Optimal result	2831
Mathematica [N/A]	2831
Rubi [N/A]	2832
Maple [N/A]	2832
Fricas [N/A]	2833
Sympy [N/A]	2833
Maxima [N/A]	2834
Giac [F(-2)]	2834
Mupad [N/A]	2834
Reduce [N/A]	2835

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))} dx = \int \frac{\sqrt{1-c^2x^2}}{x(a+b \arcsin(cx))} dx$$

input

```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcSin[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx$$

↓ 5226

$$\int \left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} - \frac{c^2x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} \right) dx$$

↓ 2009

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx + \frac{\sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b\arcsin(cx)}{b}\right)}{b} - \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b\arcsin(cx)}{b}\right)}{b}$$

input `Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x(a+b\arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)x} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b*x*arcsin(c*x) + a*x), x)`

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x(a+b\arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/x/(a+b*asin(c*x)),x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)x} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)/((b*arcsin(c*x) + a)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx = \int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))} dx$$

input `int((1 - c^2*x^2)^(1/2)/(x*(a + b*asin(c*x))),x)`

output `int((1 - c^2*x^2)^(1/2)/(x*(a + b*asin(c*x))), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1 - c^2 x^2}}{x(a + b \arcsin(cx))} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\arcsin(cx) bx + ax} dx$$

input `int((-c^2*x^2+1)^(1/2)/x/(a+b*asin(c*x)),x)`

output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)*b*x + a*x),x)`

3.296 $\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))} dx$

Optimal result	2836
Mathematica [N/A]	2836
Rubi [N/A]	2837
Maple [N/A]	2837
Fricas [N/A]	2838
Sympy [N/A]	2838
Maxima [N/A]	2838
Giac [N/A]	2839
Mupad [N/A]	2839
Reduce [N/A]	2840

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))} dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))} dx$$

input

```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcSin[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))} dx$$

↓ 5226

$$\int \left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))} - \frac{c^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} \right) dx$$

↓ 2009

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx - \frac{c \log(a+b\arcsin(cx))}{b}$$

input `Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2(a+b\arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b*x^2*arcsin(c*x) + a*x^2), x)`

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2(a+b\arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*asin(c*x)),x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)/((b*arcsin(c*x) + a)*x^2), x)`

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^2 (a + b \arcsin(cx))} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{(b \arcsin(cx) + a) x^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(sqrt(-c^2*x^2 + 1)/((b*arcsin(c*x) + a)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^2 (a + b \arcsin(cx))} dx = \int \frac{\sqrt{1 - c^2 x^2}}{x^2 (a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*asin(c*x))),x)`

output `int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*asin(c*x))), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))} dx$$

$$= \frac{\left(\int \frac{1}{\sqrt{-c^2x^2+1} \arcsin(cx) b x^2 + \sqrt{-c^2x^2+1} a x^2} dx \right) b - \log(\arcsin(cx) b + a) c}{b}$$

input `int((-c^2*x^2+1)^(1/2)/x^2/(a+b*asin(c*x)),x)`output `(int(1/(sqrt(-c**2*x**2+1)*asin(c*x)*b*x**2+sqrt(-c**2*x**2+1)*a*x**2),x)*b-log(asin(c*x)*b+a)*c)/b`

$$3.297 \quad \int \frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))} dx$$

Optimal result	2841
Mathematica [N/A]	2841
Rubi [N/A]	2842
Maple [N/A]	2842
Fricas [N/A]	2843
Sympy [N/A]	2843
Maxima [N/A]	2843
Giac [F(-2)]	2844
Mupad [N/A]	2844
Reduce [N/A]	2845

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))} dx = \int \frac{\sqrt{1-c^2x^2}}{x^3(a+b \arcsin(cx))} dx$$

input `Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcSin[c*x])),x]`

output `Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))} dx$$

input `Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 x^2 + 1}}{x^3 (a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)x^3} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b*x^3*arcsin(c*x) + a*x^3), x)`

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3(a+b\arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*asin(c*x)),x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)x^3} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)/((b*arcsin(c*x) + a)*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*asin(c*x))),x)`

output `int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*asin(c*x))), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))} dx = \int \frac{\sqrt{-c^2x^2+1}}{\operatorname{asin}(cx)bx^3+ax^3} dx$$

input `int((-c^2*x^2+1)^(1/2)/x^3/(a+b*asin(c*x)),x)`output `int(sqrt(-c**2*x**2+1)/(asin(c*x)*b*x**3+a*x**3),x)`

3.298 $\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$

Optimal result	2846
Mathematica [A] (verified)	2847
Rubi [A] (verified)	2847
Maple [A] (verified)	2849
Fricas [F]	2849
Sympy [F]	2850
Maxima [F]	2850
Giac [B] (verification not implemented)	2850
Mupad [F(-1)]	2851
Reduce [F]	2851

Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx = -\frac{3 \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{64bc^4}$$

$$-\frac{3 \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{64bc^4}$$

$$+\frac{\operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{5a}{b}\right)}{64bc^4} + \frac{\operatorname{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{7a}{b}\right)}{64bc^4}$$

$$+\frac{3 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{64bc^4} + \frac{3 \cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{64bc^4}$$

$$-\frac{\cos\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{64bc^4} - \frac{\cos\left(\frac{7a}{b}\right) \operatorname{Si}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{64bc^4}$$

output

```
-3/64*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^4-3/64*Ci(3*(a+b*arcsin(c*x))/b)*sin(3*a/b)/b/c^4+1/64*Ci(5*(a+b*arcsin(c*x))/b)*sin(5*a/b)/b/c^4+1/64*Ci(7*(a+b*arcsin(c*x))/b)*sin(7*a/b)/b/c^4+3/64*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^4+3/64*cos(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b/c^4-1/64*cos(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b/c^4-1/64*cos(7*a/b)*Si(7*(a+b*arcsin(c*x))/b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = \frac{-3\operatorname{CosIntegral}\left(\frac{a}{b}+\arcsin(cx)\right)\sin\left(\frac{a}{b}\right)-3\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arcsin(cx)\right)\right)\sin\left(3\left(\frac{a}{b}+\arcsin(cx)\right)\right)}{64b^4c^4}$$

input `Integrate[(x^3*(1-c^2*x^2)^(3/2))/(a+b*ArcSin[c*x]),x]`

output `(-3*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - 3*CosIntegral[3*(a/b + ArcSin[c*x]])*Sin[(3*a)/b] + CosIntegral[5*(a/b + ArcSin[c*x]])*Sin[(5*a)/b] + CosIntegral[7*(a/b + ArcSin[c*x]])*Sin[(7*a)/b] + 3*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 3*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] - Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])] - Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcSin[c*x])])/(64*b*c^4)`

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx$$

$$\downarrow 5224$$

$$\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^4}$$

$$\downarrow 25$$

$$\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^4}$$

$$\downarrow 4906$$

$$\int \left(-\frac{\sin\left(\frac{7a}{b} - \frac{7(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} - \frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{3 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{3 \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{64(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

$$bc^4$$

↓ 2009

$$-\frac{3}{64} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{3}{64} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{64} \sin\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]
```

output

```
((-3*CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b])/64 - (3*CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/64 + (CosIntegral[(5*(a + b*ArcSin[c*x])/b]*Sin[(5*a)/b])/64 + (CosIntegral[(7*(a + b*ArcSin[c*x])/b]*Sin[(7*a)/b])/64 + (3*Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/64 + (3*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/64 - (Cos[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x])/b])/64)/(b*c^4)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.75

method	result
default	$\frac{3 \operatorname{Si}\left(3 \arcsin(cx) + \frac{3a}{b}\right) \cos\left(\frac{3a}{b}\right) - 3 \operatorname{Ci}\left(3 \arcsin(cx) + \frac{3a}{b}\right) \sin\left(\frac{3a}{b}\right) + 3 \operatorname{Si}\left(\arcsin(cx) + \frac{a}{b}\right) \cos\left(\frac{a}{b}\right) - 3 \operatorname{Ci}\left(\arcsin(cx) + \frac{a}{b}\right) \sin\left(\frac{a}{b}\right) - \operatorname{Si}\left(5 \arcsin(cx) + \frac{5a}{b}\right) \cos\left(\frac{5a}{b}\right) + \operatorname{Ci}\left(5 \arcsin(cx) + \frac{5a}{b}\right) \sin\left(\frac{5a}{b}\right) - \operatorname{Si}\left(7 \arcsin(cx) + \frac{7a}{b}\right) \cos\left(\frac{7a}{b}\right) + \operatorname{Ci}\left(7 \arcsin(cx) + \frac{7a}{b}\right) \sin\left(\frac{7a}{b}\right)}{64c^4b}$

input

```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/64/c^4*(3*Si(3*arcsin(c*x)+3*a/b)*cos(3*a/b)-3*Ci(3*arcsin(c*x)+3*a/b)*s
in(3*a/b)+3*Si(arcsin(c*x)+a/b)*cos(a/b)-3*Ci(arcsin(c*x)+a/b)*sin(a/b)-Si
(5*arcsin(c*x)+5*a/b)*cos(5*a/b)+Ci(5*arcsin(c*x)+5*a/b)*sin(5*a/b)-Si(7*a
rcsin(c*x)+7*a/b)*cos(7*a/b)+Ci(7*arcsin(c*x)+7*a/b)*sin(7*a/b))/b
```

Fricas [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^3}{b \arcsin(cx) + a} dx$$

input

```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{x^3(-(cx - 1)(cx + 1))^{3/2}}{a + b \arcsin(cx)} dx$$

input `integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{3/2} x^3}{b \arcsin(cx) + a} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(229) = 458$.

Time = 0.18 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.51

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

cos(a/b)^6*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^4) - cos(a/b)
^7*sin_integral(7*a/b + 7*arcsin(c*x))/(b*c^4) - 5/4*cos(a/b)^4*cos_integr
al(7*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^4) + 1/4*cos(a/b)^4*cos_integral(5
*a/b + 5*arcsin(c*x))*sin(a/b)/(b*c^4) + 7/4*cos(a/b)^5*sin_integral(7*a/b
+ 7*arcsin(c*x))/(b*c^4) - 1/4*cos(a/b)^5*sin_integral(5*a/b + 5*arcsin(c
*x))/(b*c^4) + 3/8*cos(a/b)^2*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)
/(b*c^4) - 3/16*cos(a/b)^2*cos_integral(5*a/b + 5*arcsin(c*x))*sin(a/b)/(b
*c^4) - 3/16*cos(a/b)^2*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^
4) - 7/8*cos(a/b)^3*sin_integral(7*a/b + 7*arcsin(c*x))/(b*c^4) + 5/16*cos
(a/b)^3*sin_integral(5*a/b + 5*arcsin(c*x))/(b*c^4) + 3/16*cos(a/b)^3*sin_
integral(3*a/b + 3*arcsin(c*x))/(b*c^4) - 1/64*cos_integral(7*a/b + 7*arcs
in(c*x))*sin(a/b)/(b*c^4) + 1/64*cos_integral(5*a/b + 5*arcsin(c*x))*sin(a
/b)/(b*c^4) + 3/64*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^4) -
3/64*cos_integral(a/b + arcsin(c*x))*sin(a/b)/(b*c^4) + 7/64*cos(a/b)*sin_
integral(7*a/b + 7*arcsin(c*x))/(b*c^4) - 5/64*cos(a/b)*sin_integral(5*a/b
+ 5*arcsin(c*x))/(b*c^4) - 9/64*cos(a/b)*sin_integral(3*a/b + 3*arcsin(c*
x))/(b*c^4) + 3/64*cos(a/b)*sin_integral(a/b + arcsin(c*x))/(b*c^4)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \operatorname{asin}(cx)} dx$$

input

```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)),x)
```

output

```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = - \left(\int \frac{\sqrt{-c^2x^2 + 1} x^5}{\operatorname{asin}(cx) b + a} dx \right) c^2 + \int \frac{\sqrt{-c^2x^2 + 1} x^3}{\operatorname{asin}(cx) b + a} dx$$

input

```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)
```


output

```
- int((sqrt(- c**2*x**2 + 1)*x**5)/(asin(c*x)*b + a),x)*c**2 + int((sqrt  
(- c**2*x**2 + 1)*x**3)/(asin(c*x)*b + a),x)
```

3.299
$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$$

Optimal result	2853
Mathematica [A] (verified)	2854
Rubi [A] (verified)	2854
Maple [A] (verified)	2856
Fricas [F]	2856
Sympy [F]	2856
Maxima [F]	2857
Giac [B] (verification not implemented)	2857
Mupad [F(-1)]	2858
Reduce [F]	2858

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx = \frac{\cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{16bc^3} - \frac{\cos\left(\frac{6a}{b}\right) \operatorname{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc^3} + \frac{\log(a+b \arcsin(cx))}{16bc^3} + \frac{\sin\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{4a}{b}\right) \operatorname{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{16bc^3} - \frac{\sin\left(\frac{6a}{b}\right) \operatorname{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc^3}$$

output

```
1/32*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c^3-1/16*cos(4*a/b)*Ci(4*(a+b*arcsin(c*x))/b)/b/c^3-1/32*cos(6*a/b)*Ci(6*(a+b*arcsin(c*x))/b)/b/c^3+1/16*ln(a+b*arcsin(c*x))/b/c^3+1/32*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c^3-1/16*sin(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b/c^3-1/32*sin(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx =$$

$$-\cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 2\cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \cos\left(\frac{6a}{b}\right) \operatorname{CosIntegral}\left(6\left(\frac{a}{b} + \arcsin(cx)\right)\right) - \sin\left(\frac{2a}{b}\right) \operatorname{SinIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 2\sin\left(\frac{4a}{b}\right) \operatorname{SinIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \sin\left(\frac{6a}{b}\right) \operatorname{SinIntegral}\left(6\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \frac{2a^2 - b^2}{b^2 c^3}$$

input `Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]`

output `-1/32*(-(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])]) + 2*Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])]) + Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcSin[c*x])] + 2*Log[a + b*ArcSin[c*x]] - 4*Log[8*(a + b*ArcSin[c*x])] - Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 2*Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])])/(b*c^3)`

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx$$

$$\downarrow 5224$$

$$\int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))$$

$$\frac{bc^3}{\downarrow 4906}$$

$$\int \left(-\frac{\cos\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{1}{16(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

$$bc^3$$

$$\downarrow \text{2009}$$

$$\frac{1}{32} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{16} \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) - \frac{1}{32} \cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]
```

output

```
((Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 - (Cos[(4*a)/b]*
CosIntegral[(4*(a + b*ArcSin[c*x]))/b])/16 - (Cos[(6*a)/b]*CosIntegral[(6*
(a + b*ArcSin[c*x]))/b])/32 + Log[a + b*ArcSin[c*x]]/16 + (Sin[(2*a)/b]*Si
nIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 - (Sin[(4*a)/b]*SinIntegral[(4*(a
+ b*ArcSin[c*x]))/b])/16 - (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x
]))/b])/32)/(b*c^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*sin[-a/b + x/b]^m*cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

method	result
default	$-\frac{2 \operatorname{Si}\left(4 \arcsin\left(\frac{cx}{b}\right)+\frac{4a}{b}\right) \sin\left(\frac{4a}{b}\right)+2 \operatorname{Ci}\left(4 \arcsin\left(\frac{cx}{b}\right)+\frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right)-\operatorname{Si}\left(2 \arcsin\left(\frac{cx}{b}\right)+\frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right)-\operatorname{Ci}\left(2 \arcsin\left(\frac{cx}{b}\right)+\frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right)+\operatorname{Si}\left(6 \arcsin\left(\frac{cx}{b}\right)+\frac{6a}{b}\right) \sin\left(\frac{6a}{b}\right)+\operatorname{Ci}\left(6 \arcsin\left(\frac{cx}{b}\right)+\frac{6a}{b}\right) \cos\left(\frac{6a}{b}\right)-2 \ln\left(a+b \arcsin\left(\frac{cx}{b}\right)\right)}{32c^3b}$

input `int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$-\frac{1}{32c^3} \left(2 \operatorname{Si}\left(4 \arcsin\left(\frac{cx}{b}\right)+\frac{4a}{b}\right) \sin\left(\frac{4a}{b}\right) + 2 \operatorname{Ci}\left(4 \arcsin\left(\frac{cx}{b}\right)+\frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right) - \operatorname{Si}\left(2 \arcsin\left(\frac{cx}{b}\right)+\frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) - \operatorname{Ci}\left(2 \arcsin\left(\frac{cx}{b}\right)+\frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) + \operatorname{Si}\left(6 \arcsin\left(\frac{cx}{b}\right)+\frac{6a}{b}\right) \sin\left(\frac{6a}{b}\right) + \operatorname{Ci}\left(6 \arcsin\left(\frac{cx}{b}\right)+\frac{6a}{b}\right) \cos\left(\frac{6a}{b}\right) - 2 \ln\left(a+b \arcsin\left(\frac{cx}{b}\right)\right) \right) / b$$

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{b \arcsin(cx)+a} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx = \int \frac{x^2(-\left(\frac{cx-1}{cx+1}\right)^{\frac{3}{2}})}{a+b \arcsin(cx)} dx$$

input `integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^2}{b \arcsin(cx) + a} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(192) = 384.

Time = 0.16 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.30

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-cos(a/b)^6*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) - cos(a/b)^5*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) + 3/2*cos(a/b)^4*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) - 1/2*cos(a/b)^4*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) - 1/2*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) - 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) + 1/2*cos(a/b)^2*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + 1/16*cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) - 3/16*cos(a/b)*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) + 1/4*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + 1/16*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) + 1/32*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) - 1/16*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) - 1/32*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) + 1/16*log(b*arcsin(c*x) + a)/(b*c^3)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx$$

input `int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)),x)`output `int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)), x)`**Reduce [F]**

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = - \left(\int \frac{\sqrt{-c^2x^2 + 1} x^4}{\arcsin(cx) b + a} dx \right) c^2 + \int \frac{\sqrt{-c^2x^2 + 1} x^2}{\arcsin(cx) b + a} dx$$

input `int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`output `- int((sqrt(-c**2*x**2 + 1)*x**4)/(asin(c*x)*b + a),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)*b + a),x)`

3.300 $\int \frac{x(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$

Optimal result	2859
Mathematica [A] (verified)	2860
Rubi [A] (verified)	2860
Maple [A] (verified)	2862
Fricas [F]	2862
Sympy [F]	2862
Maxima [F]	2863
Giac [B] (verification not implemented)	2863
Mupad [F(-1)]	2864
Reduce [F]	2864

Optimal result

Integrand size = 26, antiderivative size = 183

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx = -\frac{\text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{8bc^2} - \frac{3 \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{16bc^2} - \frac{\text{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{5a}{b}\right)}{16bc^2} + \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8bc^2} + \frac{3 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16bc^2} + \frac{\cos\left(\frac{5a}{b}\right) \text{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16bc^2}$$

output

```
-1/8*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^2-3/16*Ci(3*(a+b*arcsin(c*x))/b)*sin(3*a/b)/b/c^2-1/16*Ci(5*(a+b*arcsin(c*x))/b)*sin(5*a/b)/b/c^2+1/8*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^2+3/16*cos(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b/c^2+1/16*cos(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b/c^2
```


Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \frac{-2 \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - 3 \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{a}{b}\right)}{16 b c^2}$$

input `Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]`

output `(-2*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - 3*CosIntegral[3*(a/b + ArcSin[c*x]))*Sin[(3*a)/b] - CosIntegral[5*(a/b + ArcSin[c*x]]*Sin[(5*a)/b] + 2*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 3*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] + Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])])/(16*b*c^2)`

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx \\ & \quad \downarrow \text{5224} \\ & \frac{\int -\frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^2} \\ & \quad \downarrow \text{25} \\ & \frac{\int \cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^2} \\ & \quad \downarrow \text{4906} \end{aligned}$$

$$\frac{\int \left(\frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{3 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{bc^2}$$

↓ 2009

$$\frac{-\frac{1}{8} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{3}{16} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{1}{16} \sin\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{bc^2}$$

input

```
Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]
```

output

```
(-1/8*(CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b]) - (3*CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/16 - (CosIntegral[(5*(a + b*ArcSin[c*x])/b]*Sin[(5*a)/b])/16 + (Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 + (3*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/16 + (Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/16)/(b*c^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

method	result
default	$\frac{\text{Si}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b}) - \text{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b}) + 3 \text{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) - 3 \text{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) + 2 \text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - 2 \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{16c^2b}$

input `int(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{16/c^2} * (\text{Si}(5*\arcsin(c*x)+5*a/b)*\cos(5*a/b) - \text{Ci}(5*\arcsin(c*x)+5*a/b)*\sin(5*a/b) + 3*\text{Si}(3*\arcsin(c*x)+3*a/b)*\cos(3*a/b) - 3*\text{Ci}(3*\arcsin(c*x)+3*a/b)*\sin(3*a/b) + 2*\text{Si}(\arcsin(c*x)+a/b)*\cos(a/b) - 2*\text{Ci}(\arcsin(c*x)+a/b)*\sin(a/b)) / b$$

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{b\arcsin(cx)+a} dx$$

input `integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = \int \frac{x(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\arcsin(cx)} dx$$

input `integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2 x^2 + 1)^{3/2} x}{b \arcsin(cx) + a} dx$$

input `integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(171) = 342.

Time = 0.16 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.97

$$\begin{aligned} \int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = & -\frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{5a}{b} + 5 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{bc^2} \\ & + \frac{\cos\left(\frac{a}{b}\right)^5 \operatorname{Si}\left(\frac{5a}{b} + 5 \arcsin(cx)\right)}{bc^2} + \frac{3 \cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{5a}{b} + 5 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{4bc^2} \\ & - \frac{3 \cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{4bc^2} \\ & - \frac{5 \cos\left(\frac{a}{b}\right)^3 \operatorname{Si}\left(\frac{5a}{b} + 5 \arcsin(cx)\right)}{4bc^2} + \frac{3 \cos\left(\frac{a}{b}\right)^3 \operatorname{Si}\left(\frac{3a}{b} + 3 \arcsin(cx)\right)}{4bc^2} \\ & - \frac{\operatorname{Ci}\left(\frac{5a}{b} + 5 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{16bc^2} + \frac{3 \operatorname{Ci}\left(\frac{3a}{b} + 3 \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{16bc^2} \\ & - \frac{\operatorname{Ci}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{8bc^2} + \frac{5 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{5a}{b} + 5 \arcsin(cx)\right)}{16bc^2} \\ & - \frac{9 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3 \arcsin(cx)\right)}{16bc^2} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{8bc^2} \end{aligned}$$

input `integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```
-cos(a/b)^4*cos_integral(5*a/b + 5*arcsin(c*x))*sin(a/b)/(b*c^2) + cos(a/b)
)^5*sin_integral(5*a/b + 5*arcsin(c*x))/(b*c^2) + 3/4*cos(a/b)^2*cos_integ
ral(5*a/b + 5*arcsin(c*x))*sin(a/b)/(b*c^2) - 3/4*cos(a/b)^2*cos_integral(
3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^2) - 5/4*cos(a/b)^3*sin_integral(5*a/
b + 5*arcsin(c*x))/(b*c^2) + 3/4*cos(a/b)^3*sin_integral(3*a/b + 3*arcsin(
c*x))/(b*c^2) - 1/16*cos_integral(5*a/b + 5*arcsin(c*x))*sin(a/b)/(b*c^2)
+ 3/16*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^2) - 1/8*cos_inte
gral(a/b + arcsin(c*x))*sin(a/b)/(b*c^2) + 5/16*cos(a/b)*sin_integral(5*a/
b + 5*arcsin(c*x))/(b*c^2) - 9/16*cos(a/b)*sin_integral(3*a/b + 3*arcsin(c
*x))/(b*c^2) + 1/8*cos(a/b)*sin_integral(a/b + arcsin(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx$$

input

```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)),x)
```

output

```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)), x)
```

Reduce [F]

$$\int \frac{x(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = - \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^3}{a \arcsin(cx) b + a} dx \right) c^2 + \int \frac{\sqrt{-c^2 x^2 + 1} x}{a \arcsin(cx) b + a} dx$$

input

```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)
```

output

```
- int((sqrt(- c**2*x**2 + 1)*x**3)/(asin(c*x)*b + a),x)*c**2 + int((sqrt
(- c**2*x**2 + 1)*x)/(asin(c*x)*b + a),x)
```

3.301 $\int \frac{(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx$

Optimal result	2865
Mathematica [A] (verified)	2866
Rubi [A] (verified)	2866
Maple [A] (verified)	2868
Fricas [F]	2868
Sympy [F]	2868
Maxima [F]	2869
Giac [A] (verification not implemented)	2869
Mupad [F(-1)]	2870
Reduce [F]	2870

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b \arcsin(cx)} dx = \frac{\cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc} + \frac{\cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8bc} + \frac{3 \log(a+b \arcsin(cx))}{8bc} + \frac{\sin\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc} + \frac{\sin\left(\frac{4a}{b}\right) \operatorname{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8bc}$$

output

```
1/2*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c+1/8*cos(4*a/b)*Ci(4*(a+b*arcsin(c*x))/b)/b/c+3/8*ln(a+b*arcsin(c*x))/b/c+1/2*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c+1/8*sin(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \frac{4 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{8bc}$$

input `Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcSin[c*x]),x]`

output `(4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] + Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])] + 4*Log[a + b*ArcSin[c*x]] - Log[8*(a + b*ArcSin[c*x])] + 4*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])])/(8*b*c)`

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5168, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx \\ & \quad \downarrow \text{5168} \\ & \int \frac{\cos^4\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right)}{a + b \arcsin(cx)} d(a + b \arcsin(cx)) \\ & \quad \quad \quad \frac{bc}{bc} \\ & \quad \quad \quad \downarrow \text{3042} \\ & \int \frac{\sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^4}{a + b \arcsin(cx)} d(a + b \arcsin(cx)) \\ & \quad \quad \quad \frac{bc}{bc} \\ & \quad \quad \quad \downarrow \text{3793} \end{aligned}$$

$$\int \left(\frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{2(a+b \arcsin(cx))} + \frac{3}{8(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

bc
↓ 2009

$$\frac{1}{2} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{2} \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcSin[c*x]),x]
```

output

```
((Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcSin[c*x]))/b])/2 + (Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcSin[c*x]))/b])/8 + (3*Log[a + b*ArcSin[c*x]])/8 + (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/2 + (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8)/(b*c)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5168

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```


Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

method	result
default	$\frac{\text{Si}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) + \text{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) + 4 \text{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) + 4 \text{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) + 3 \ln(a + b \arcsin(cx))}{8cb}$

input `int((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\frac{1}{8c} \left(\text{Si}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) + \text{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) + 4 \text{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) + 4 \text{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) + 3 \ln(a + b \arcsin(cx)) \right) / b$$

Fricas [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{b \arcsin(cx) + a} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{a + b \arcsin(cx)} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{b \arcsin(cx) + a} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)/(b*arcsin(c*x) + a), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.75

$$\begin{aligned} \int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx &= \frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{bc} \\ &+ \frac{\cos\left(\frac{a}{b}\right)^3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{bc} \\ &+ \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{2bc} \\ &+ \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{bc} + \frac{\operatorname{Ci}\left(\frac{4a}{b} + 4 \arcsin(cx)\right)}{8bc} \\ &- \frac{\operatorname{Ci}\left(\frac{2a}{b} + 2 \arcsin(cx)\right)}{2bc} + \frac{3 \log(b \arcsin(cx) + a)}{8bc} \end{aligned}$$

input `integrate((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `cos(a/b)^4*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c) + cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c) - cos(a/b)^2*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c) + cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c) - 1/2*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c) + cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c) + 1/8*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c) - 1/2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c) + 3/8*log(b*arcsin(c*x) + a)/(b*c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((1 - c^2*x^2)^(3/2)/(a + b*asin(c*x)),x)`output `int((1 - c^2*x^2)^(3/2)/(a + b*asin(c*x)), x)`**Reduce [F]**

$$\int \frac{(1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) b + a} dx - \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^2}{\operatorname{asin}(cx) b + a} dx \right) c^2$$

input `int((-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)*b + a),x) - int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)*b + a),x)*c**2`

3.302 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))} dx$

Optimal result	2871
Mathematica [N/A]	2871
Rubi [N/A]	2872
Maple [N/A]	2873
Fricas [N/A]	2873
Sympy [N/A]	2873
Maxima [N/A]	2874
Giac [F(-2)]	2874
Mupad [N/A]	2875
Reduce [N/A]	2875

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1 - c^2x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \text{Int} \left(\frac{(1 - c^2x^2)^{3/2}}{x(a + b \arcsin(cx))}, x \right)$$

output `Defer(Int)((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1 - c^2x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2x^2)^{3/2}}{x(a + b \arcsin(cx))} dx$$

input `Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcSin[c*x])),x]`

output `Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))} dx$$

↓ 5226

$$\int \left(-\frac{2c^2 x}{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} + \frac{1}{x\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} + \frac{c^4 x^3}{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} \right) dx$$

↓ 2009

$$\int \frac{1}{x\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} dx + \frac{5 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a + b \arcsin(cx)}{b}\right)}{4b} +$$

$$\frac{\sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a + b \arcsin(cx))}{b}\right)}{4b} - \frac{5 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a + b \arcsin(cx)}{b}\right)}{4b} -$$

$$\frac{\cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a + b \arcsin(cx))}{b}\right)}{4b}$$

input `Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}}{x(a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x)),x)`output `int((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x)),x)`**Fricas [N/A]**

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)x} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x)),x, algorithm="fricas")`output `integral((-c^2*x^2 + 1)^(3/2)/(b*x*arcsin(c*x) + a*x), x)`**Sympy [N/A]**

Not integrable

Time = 2.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(1 - c^2x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{x(a + b \arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/x/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(x*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{3/2}}{(b \arcsin(cx) + a)x} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)/((b*arcsin(c*x) + a)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(3/2)/(x*(a + b*asin(c*x))),x)`output `int((1 - c^2*x^2)^(3/2)/(x*(a + b*asin(c*x))), x)`**Reduce [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) bx + ax} dx - \left(\int \frac{\sqrt{-c^2 x^2 + 1} x}{\operatorname{asin}(cx) b + a} dx \right) c^2$$

input `int((-c^2*x^2+1)^(3/2)/x/(a+b*asin(c*x)),x)`output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)*b*x + a*x),x) - int((sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)*b + a),x)*c**2`

3.303 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))} dx$

Optimal result	2876
Mathematica [N/A]	2876
Rubi [N/A]	2877
Maple [N/A]	2877
Fricas [N/A]	2878
Sympy [N/A]	2878
Maxima [N/A]	2879
Giac [N/A]	2879
Mupad [N/A]	2879
Reduce [N/A]	2880

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1 - c^2x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx = \text{Int}\left(\frac{(1 - c^2x^2)^{3/2}}{x^2(a + b \arcsin(cx))}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1 - c^2x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx$$

input

```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcSin[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))} dx$$

↓ 5226

$$\int \left(-\frac{2c^2}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} + \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} + \frac{c^4 x^2}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} \right) dx$$

↓ 2009

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx - \frac{c \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a + b \arcsin(cx))}{b}\right)}{2b} - \frac{c \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a + b \arcsin(cx))}{b}\right)}{2b} - \frac{3c \log(a + b \arcsin(cx))}{2b}$$

input

```
Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcSin[c*x])),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{3/2}}{x^2 (a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a) x^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b*x^2*arcsin(c*x) + a*x^2), x)`

Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))} dx = \int \frac{-(cx - 1)(cx + 1)^{\frac{3}{2}}}{x^2 (a + b \arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(x**2*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)/((b*arcsin(c*x) + a)*x^2), x)`

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate((-c^2*x^2 + 1)^(3/2)/((b*arcsin(c*x) + a)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{x^2(a + b \arcsin(cx))} dx$$

input `int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*asin(c*x))),x)`

output `int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*asin(c*x))), x)`

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.21

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))} dx = \frac{-\left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\arcsin(cx) b + a} dx\right) b c^2 + \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b x^2 + \sqrt{-c^2 x^2 + 1} a x^2} dx\right) b - \log(\arcsin(cx))}{b}$$

input `int((-c^2*x^2+1)^(3/2)/x^2/(a+b*asin(c*x)),x)`

output `(- int(sqrt(- c**2*x**2 + 1)/(asin(c*x)*b + a),x)*b*c**2 + int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)*b*x**2 + sqrt(- c**2*x**2 + 1)*a*x**2),x)*b - log(asin(c*x)*b + a)*c)/b`

3.304 $\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))} dx$

Optimal result	2881
Mathematica [N/A]	2881
Rubi [N/A]	2882
Maple [N/A]	2882
Fricas [N/A]	2883
Sympy [N/A]	2883
Maxima [N/A]	2883
Giac [F(-2)]	2884
Mupad [N/A]	2884
Reduce [N/A]	2885

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))} dx = \int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))} dx$$

input

```
Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcSin[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))} dx$$

input `Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{3/2}}{x^3 (a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)x^3} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b*x^3*arcsin(c*x) + a*x^3), x)`

Sympy [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3(a + b \arcsin(cx))} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{x^3(a + b \arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(x**3*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)x^3} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)/((b*arcsin(c*x) + a)*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*asin(c*x))),x)`

output `int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*asin(c*x))), x)`

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.29

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{a \sin(cx) b x^3 + a x^3} dx - \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{a \sin(cx) b x + a x} dx \right) c^2$$

input `int((-c^2*x^2+1)^(3/2)/x^3/(a+b*asin(c*x)),x)`

output `int(sqrt(-c**2*x**2+1)/(asin(c*x)*b*x**3+a*x**3),x)-int(sqrt(-c**2*x**2+1)/(asin(c*x)*b*x+a*x),x)*c**2`

3.305 $\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$

Optimal result	2886
Mathematica [A] (verified)	2887
Rubi [A] (verified)	2887
Maple [A] (verified)	2889
Fricas [F]	2889
Sympy [F]	2890
Maxima [F]	2890
Giac [B] (verification not implemented)	2890
Mupad [F(-1)]	2891
Reduce [F]	2892

Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx = -\frac{3 \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{128bc^4}$$

$$- \frac{\operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{32bc^4}$$

$$+ \frac{3 \operatorname{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{7a}{b}\right)}{256bc^4} + \frac{\operatorname{CosIntegral}\left(\frac{9(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{9a}{b}\right)}{256bc^4}$$

$$+ \frac{3 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{128bc^4} + \frac{\cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{32bc^4}$$

$$- \frac{3 \cos\left(\frac{7a}{b}\right) \operatorname{Si}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{256bc^4} - \frac{\cos\left(\frac{9a}{b}\right) \operatorname{Si}\left(\frac{9(a+b \arcsin(cx))}{b}\right)}{256bc^4}$$

output

```
-3/128*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^4-1/32*Ci(3*(a+b*arcsin(c*x))/b)*sin(3*a/b)/b/c^4+3/256*Ci(7*(a+b*arcsin(c*x))/b)*sin(7*a/b)/b/c^4+1/256*Ci(9*(a+b*arcsin(c*x))/b)*sin(9*a/b)/b/c^4+3/128*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^4+1/32*cos(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b/c^4-3/256*cos(7*a/b)*Si(7*(a+b*arcsin(c*x))/b)/b/c^4-1/256*cos(9*a/b)*Si(9*(a+b*arcsin(c*x))/b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \frac{-6 \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - 8 \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{a}{b}\right)}{256bc^4}$$

input `Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]`

output `(-6*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - 8*CosIntegral[3*(a/b + ArcSin[c*x]))*Sin[(3*a)/b] + 3*CosIntegral[7*(a/b + ArcSin[c*x]))*Sin[(7*a)/b] + CosIntegral[9*(a/b + ArcSin[c*x]))*Sin[(9*a)/b] + 6*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 8*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] - 3*Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcSin[c*x])] - Cos[(9*a)/b]*SinIntegral[9*(a/b + ArcSin[c*x])])/(256*b*c^4)`

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

$$\downarrow 5224$$

$$\int \frac{\cos^6\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^4}$$

$$\downarrow 25$$

$$\int \frac{\cos^6\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^4}$$

$$\downarrow 4906$$

$$\int \left(-\frac{\sin\left(\frac{9a}{b} - \frac{9(a+b \arcsin(cx))}{b}\right)}{256(a+b \arcsin(cx))} - \frac{3 \sin\left(\frac{7a}{b} - \frac{7(a+b \arcsin(cx))}{b}\right)}{256(a+b \arcsin(cx))} + \frac{\sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{3 \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{128(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

 bc^4

↓ 2009

$$-\frac{3}{128} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{32} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{3}{256} \sin\left(\frac{7a}{b}\right) \operatorname{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right) - \frac{3}{256} \sin\left(\frac{9a}{b}\right) \operatorname{CosIntegral}\left(\frac{9(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]
```

output

```
((-3*CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b])/128 - (CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/32 + (3*CosIntegral[(7*(a + b*ArcSin[c*x])/b]*Sin[(7*a)/b])/256 + (CosIntegral[(9*(a + b*ArcSin[c*x])/b]*Sin[(9*a)/b])/256 + (3*Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/128 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/32 - (3*Cos[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x])/b])/256 - (Cos[(9*a)/b]*SinIntegral[(9*(a + b*ArcSin[c*x])/b])/256)/(b*c^4)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

method	result
default	$\frac{8 \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) - 8 \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) + 6 \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - 6 \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b}) - 3 \operatorname{Si}(\frac{a}{b}) - 3 \operatorname{Ci}(\frac{a}{b})}{256c^4b}$

input

```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/256/c^4*(8*Si(3*arcsin(c*x)+3*a/b)*cos(3*a/b)-8*Ci(3*arcsin(c*x)+3*a/b)*
sin(3*a/b)+6*Si(arcsin(c*x)+a/b)*cos(a/b)-6*Ci(arcsin(c*x)+a/b)*sin(a/b)-3
*Si(7*arcsin(c*x)+7*a/b)*cos(7*a/b)+3*Ci(7*arcsin(c*x)+7*a/b)*sin(7*a/b)-S
i(9*arcsin(c*x)+9*a/b)*cos(9*a/b)+Ci(9*arcsin(c*x)+9*a/b)*sin(9*a/b))/b
```

Fricas [F]

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x^3}{b \arcsin(cx) + a} dx$$

input

```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a
), x)
```

Sympy [F]

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x^3(-(cx - 1)(cx + 1))^{5/2}}{a + b \arcsin(cx)} dx$$

input `integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(x**3*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x^3}{b \arcsin(cx) + a} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(229) = 458$.

Time = 0.17 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.04

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

cos(a/b)^8*cos_integral(9*a/b + 9*arcsin(c*x))*sin(a/b)/(b*c^4) - cos(a/b)
^9*sin_integral(9*a/b + 9*arcsin(c*x))/(b*c^4) - 7/4*cos(a/b)^6*cos_integr
al(9*a/b + 9*arcsin(c*x))*sin(a/b)/(b*c^4) + 3/4*cos(a/b)^6*cos_integral(7
*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^4) + 9/4*cos(a/b)^7*sin_integral(9*a/b
+ 9*arcsin(c*x))/(b*c^4) - 3/4*cos(a/b)^7*sin_integral(7*a/b + 7*arcsin(c
*x))/(b*c^4) + 15/16*cos(a/b)^4*cos_integral(9*a/b + 9*arcsin(c*x))*sin(a/
b)/(b*c^4) - 15/16*cos(a/b)^4*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)
/(b*c^4) - 27/16*cos(a/b)^5*sin_integral(9*a/b + 9*arcsin(c*x))/(b*c^4) +
21/16*cos(a/b)^5*sin_integral(7*a/b + 7*arcsin(c*x))/(b*c^4) - 5/32*cos(a/
b)^2*cos_integral(9*a/b + 9*arcsin(c*x))*sin(a/b)/(b*c^4) + 9/32*cos(a/b)^
2*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^4) - 1/8*cos(a/b)^2*co
s_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^4) + 15/32*cos(a/b)^3*sin_
integral(9*a/b + 9*arcsin(c*x))/(b*c^4) - 21/32*cos(a/b)^3*sin_integral(7*
a/b + 7*arcsin(c*x))/(b*c^4) + 1/8*cos(a/b)^3*sin_integral(3*a/b + 3*arcsi
n(c*x))/(b*c^4) + 1/256*cos_integral(9*a/b + 9*arcsin(c*x))*sin(a/b)/(b*c^
4) - 3/256*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^4) + 1/32*cos
_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^4) - 3/128*cos_integral(a/b
+ arcsin(c*x))*sin(a/b)/(b*c^4) - 9/256*cos(a/b)*sin_integral(9*a/b + 9*a
rcsin(c*x))/(b*c^4) + 21/256*cos(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/
(b*c^4) - 3/32*cos(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b*c^4) + 3...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x^3(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

input

```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)),x)
```

output

```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)), x)
```


Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arcsin(cx)} dx = \left(\int \frac{\sqrt{-c^2x^2+1}x^7}{\arcsin(cx)b+a} dx \right) c^4$$

$$- 2 \left(\int \frac{\sqrt{-c^2x^2+1}x^5}{\arcsin(cx)b+a} dx \right) c^2 + \int \frac{\sqrt{-c^2x^2+1}x^3}{\arcsin(cx)b+a} dx$$

input `int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2+1)*x**7)/(asin(c*x)*b+a),x)*c**4 - 2*int((sqrt(-c**2*x**2+1)*x**5)/(asin(c*x)*b+a),x)*c**2 + int((sqrt(-c**2*x**2+1)*x**3)/(asin(c*x)*b+a),x)`

3.306 $\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$

Optimal result	2893
Mathematica [A] (verified)	2894
Rubi [A] (verified)	2894
Maple [A] (verified)	2896
Fricas [F]	2896
Sympy [F]	2896
Maxima [F]	2897
Giac [B] (verification not implemented)	2897
Mupad [F(-1)]	2898
Reduce [F]	2899

Optimal result

Integrand size = 28, antiderivative size = 268

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx = \frac{\cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{6a}{b}\right) \operatorname{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{8a}{b}\right) \operatorname{CosIntegral}\left(\frac{8(a+b \arcsin(cx))}{b}\right)}{128bc^3} + \frac{5 \log(a+b \arcsin(cx))}{128bc^3} + \frac{\sin\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{4a}{b}\right) \operatorname{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{6a}{b}\right) \operatorname{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{8a}{b}\right) \operatorname{Si}\left(\frac{8(a+b \arcsin(cx))}{b}\right)}{128bc^3}$$

output

```
1/32*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c^3-1/32*cos(4*a/b)*Ci(4*(a+b*
arcsin(c*x))/b)/b/c^3-1/32*cos(6*a/b)*Ci(6*(a+b*arcsin(c*x))/b)/b/c^3-1/12
8*cos(8*a/b)*Ci(8*(a+b*arcsin(c*x))/b)/b/c^3+5/128*ln(a+b*arcsin(c*x))/b/c
^3+1/32*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c^3-1/32*sin(4*a/b)*Si(4*(a
+b*arcsin(c*x))/b)/b/c^3-1/32*sin(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b/c^3-1
/128*sin(8*a/b)*Si(8*(a+b*arcsin(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.78

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx =$$

$$-4 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 4 \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 4 \cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(6\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \dots$$

input `Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]`

output `-1/128*(-4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] + 4*Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])] + 4*Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcSin[c*x])] + Cos[(8*a)/b]*CosIntegral[8*(a/b + ArcSin[c*x])] + 11*Log[a + b*ArcSin[c*x]] - 16*Log[8*(a + b*ArcSin[c*x])] - 4*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 4*Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + 4*Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])] + Sin[(8*a)/b]*SinIntegral[8*(a/b + ArcSin[c*x])])/(b*c^3)`

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

$$\downarrow \text{5224}$$

$$\int \frac{\cos^6\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))$$

$$\downarrow \text{4906}$$

$$\frac{\dots}{bc^3}$$

$$\int \left(-\frac{\cos\left(\frac{8a}{b} - \frac{8(a+b \arcsin(cx))}{b}\right)}{128(a+b \arcsin(cx))} - \frac{\cos\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{5}{128(a+b \arcsin(cx))} \right) dx$$

 bc^3

↓ 2009

$$\frac{1}{32} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{32} \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) - \frac{1}{32} \cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) + \frac{1}{32} \cos\left(\frac{8a}{b}\right) \text{CosIntegral}\left(\frac{8(a+b \arcsin(cx))}{b}\right) + \frac{5}{128} \text{Log}\left(a + b \arcsin(cx)\right)$$

input

```
Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]
```

output

```
((Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 - (Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcSin[c*x]))/b])/32 - (Cos[(6*a)/b]*CosIntegral[(6*(a + b*ArcSin[c*x]))/b])/32 - (Cos[(8*a)/b]*CosIntegral[(8*(a + b*ArcSin[c*x]))/b])/128 + (5*Log[a + b*ArcSin[c*x]])/128 + (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 - (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/32 - (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x]))/b])/32 - (Sin[(8*a)/b]*SinIntegral[(8*(a + b*ArcSin[c*x]))/b])/128)/(b*c^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^(2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\text{Ci}(8 \arcsin(cx) + \frac{8a}{b}) \cos(\frac{8a}{b}) - 4 \text{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) - 4 \text{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) + 4 \text{Si}(6 \arcsin(cx) + \frac{6a}{b}) \sin(\frac{6a}{b})}{b}$

input `int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `-1/128/c^3*(Ci(8*arcsin(c*x)+8*a/b)*cos(8*a/b)-4*Si(2*arcsin(c*x)+2*a/b)*sin(2*a/b)-4*Ci(2*arcsin(c*x)+2*a/b)*cos(2*a/b)+4*Si(6*arcsin(c*x)+6*a/b)*sin(6*a/b)+4*Ci(6*arcsin(c*x)+6*a/b)*cos(6*a/b)+4*Si(4*arcsin(c*x)+4*a/b)*sin(4*a/b)+4*Ci(4*arcsin(c*x)+4*a/b)*cos(4*a/b)+Si(8*arcsin(c*x)+8*a/b)*sin(8*a/b)-5*ln(a+b*arcsin(c*x)))/b`

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx = \int \frac{(-c^2x^2+1)^{5/2}x^2}{b \arcsin(cx)+a} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx = \int \frac{x^2(- (cx-1)(cx+1))^{5/2}}{a+b \arcsin(cx)} dx$$

input `integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x^2}{b \arcsin(cx) + a} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(250) = 500$.

Time = 0.16 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.82

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

-cos(a/b)^8*cos_integral(8*a/b + 8*arcsin(c*x))/(b*c^3) - cos(a/b)^7*sin(a
/b)*sin_integral(8*a/b + 8*arcsin(c*x))/(b*c^3) + 2*cos(a/b)^6*cos_integra
l(8*a/b + 8*arcsin(c*x))/(b*c^3) - cos(a/b)^6*cos_integral(6*a/b + 6*arcsi
n(c*x))/(b*c^3) + 3/2*cos(a/b)^5*sin(a/b)*sin_integral(8*a/b + 8*arcsin(c*
x))/(b*c^3) - cos(a/b)^5*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c
^3) - 5/4*cos(a/b)^4*cos_integral(8*a/b + 8*arcsin(c*x))/(b*c^3) + 3/2*cos
(a/b)^4*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) - 1/4*cos(a/b)^4*cos_i
ntegral(4*a/b + 4*arcsin(c*x))/(b*c^3) - 5/8*cos(a/b)^3*sin(a/b)*sin_integ
ral(8*a/b + 8*arcsin(c*x))/(b*c^3) + cos(a/b)^3*sin(a/b)*sin_integral(6*a/
b + 6*arcsin(c*x))/(b*c^3) - 1/4*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b +
4*arcsin(c*x))/(b*c^3) + 1/4*cos(a/b)^2*cos_integral(8*a/b + 8*arcsin(c*x)
)/(b*c^3) - 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) +
1/4*cos(a/b)^2*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^3) + 1/16*cos(a/b)
^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) + 1/16*cos(a/b)*sin(a/b)*si
n_integral(8*a/b + 8*arcsin(c*x))/(b*c^3) - 3/16*cos(a/b)*sin(a/b)*sin_int
egral(6*a/b + 6*arcsin(c*x))/(b*c^3) + 1/8*cos(a/b)*sin(a/b)*sin_integral(
4*a/b + 4*arcsin(c*x))/(b*c^3) + 1/16*cos(a/b)*sin(a/b)*sin_integral(2*a/b
+ 2*arcsin(c*x))/(b*c^3) - 1/128*cos_integral(8*a/b + 8*arcsin(c*x))/(b*c
^3) + 1/32*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c^3) - 1/32*cos_integral
(4*a/b + 4*arcsin(c*x))/(b*c^3) - 1/32*cos_integral(2*a/b + 2*arcsin(c*...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x^2(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

input

```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)),x)
```

output

```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arcsin(cx)} dx = \left(\int \frac{\sqrt{-c^2x^2+1}x^6}{\arcsin(cx)b+a} dx \right) c^4$$

$$- 2 \left(\int \frac{\sqrt{-c^2x^2+1}x^4}{\arcsin(cx)b+a} dx \right) c^2 + \int \frac{\sqrt{-c^2x^2+1}x^2}{\arcsin(cx)b+a} dx$$

input `int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2+1)*x**6)/(asin(c*x)*b+a),x)*c**4 - 2*int((sqrt(-c**2*x**2+1)*x**4)/(asin(c*x)*b+a),x)*c**2 + int((sqrt(-c**2*x**2+1)*x**2)/(asin(c*x)*b+a),x)`

3.307 $\int \frac{x(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$

Optimal result	2900
Mathematica [A] (verified)	2901
Rubi [A] (verified)	2901
Maple [A] (verified)	2903
Fricas [F]	2903
Sympy [F]	2904
Maxima [F]	2904
Giac [B] (verification not implemented)	2904
Mupad [F(-1)]	2905
Reduce [F]	2905

Optimal result

Integrand size = 26, antiderivative size = 245

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx = -\frac{5 \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{64bc^2}$$

$$- \frac{9 \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{64bc^2}$$

$$- \frac{5 \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{5a}{b}\right)}{64bc^2} - \frac{\operatorname{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{7a}{b}\right)}{64bc^2}$$

$$+ \frac{5 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{64bc^2} + \frac{9 \cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{64bc^2}$$

$$+ \frac{5 \cos\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{64bc^2} + \frac{\cos\left(\frac{7a}{b}\right) \operatorname{Si}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{64bc^2}$$

output

```
-5/64*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^2-9/64*Ci(3*(a+b*arcsin(c*x))/b)*sin(3*a/b)/b/c^2-5/64*Ci(5*(a+b*arcsin(c*x))/b)*sin(5*a/b)/b/c^2-1/64*Ci(7*(a+b*arcsin(c*x))/b)*sin(7*a/b)/b/c^2+5/64*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^2+9/64*cos(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b/c^2+5/64*cos(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b/c^2+1/64*cos(7*a/b)*Si(7*(a+b*arcsin(c*x))/b)/b/c^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.73

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \frac{-5 \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - 9 \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{a}{b}\right)}{b^2}$$

input `Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]`

output `(-5*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - 9*CosIntegral[3*(a/b + ArcSin[c*x]])*Sin[(3*a)/b] - 5*CosIntegral[5*(a/b + ArcSin[c*x]])*Sin[(5*a)/b] - CosIntegral[7*(a/b + ArcSin[c*x]])*Sin[(7*a)/b] + 5*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 9*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] + 5*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])] + Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcSin[c*x])])/(64*b*c^2)`

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

↓ 5224

$$\frac{\int -\frac{\cos^6\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^2}$$

↓ 25

$$-\frac{\int \frac{\cos^6\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{bc^2}$$

↓ 4906

$$\int \left(\frac{\sin\left(\frac{7a}{b} - \frac{7(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{5 \sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{9 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{5 \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{64(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

$$bc^2$$

↓ 2009

$$-\frac{5}{64} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{9}{64} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{5}{64} \sin\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]
```

output

```
((-5*CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b])/64 - (9*CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/64 - (5*CosIntegral[(5*(a + b*ArcSin[c*x])/b]*Sin[(5*a)/b])/64 - (CosIntegral[(7*(a + b*ArcSin[c*x])/b]*Sin[(7*a)/b])/64 + (5*Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/64 + (9*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/64 + (5*Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/64 + (Cos[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x])/b])/64)/(b*c^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

method	result
default	$\frac{\text{Si}(7 \arcsin(cx) + \frac{7a}{b}) \cos(\frac{7a}{b}) - \text{Ci}(7 \arcsin(cx) + \frac{7a}{b}) \sin(\frac{7a}{b}) + 5 \text{Si}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b}) - 5 \text{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b}) + 9 \text{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) - 9 \text{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) + 5 \text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - 5 \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{64c^2b}$

input

```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/64/c^2*(Si(7*arcsin(c*x)+7*a/b)*cos(7*a/b)-Ci(7*arcsin(c*x)+7*a/b)*sin(7
*a/b)+5*Si(5*arcsin(c*x)+5*a/b)*cos(5*a/b)-5*Ci(5*arcsin(c*x)+5*a/b)*sin(5
*a/b)+9*Si(3*arcsin(c*x)+3*a/b)*cos(3*a/b)-9*Ci(3*arcsin(c*x)+3*a/b)*sin(3
*a/b)+5*Si(arcsin(c*x)+a/b)*cos(a/b)-5*Ci(arcsin(c*x)+a/b)*sin(a/b))/b
```

Fricas [F]

$$\int \frac{x(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x}{b \arcsin(cx) + a} dx$$

input

```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a),
x)
```

Sympy [F]

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x(-cx - 1)(cx + 1)^{5/2}}{a + b \arcsin(cx)} dx$$

input `integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(x*(-(c*x - 1)*(c*x + 1))**5/2/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2 x^2 + 1)^{5/2} x}{b \arcsin(cx) + a} dx$$

input `integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(229) = 458$.

Time = 0.17 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.51

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

-cos(a/b)^6*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^2) + cos(a/b)
)^7*sin_integral(7*a/b + 7*arcsin(c*x))/(b*c^2) + 5/4*cos(a/b)^4*cos_integ
ral(7*a/b + 7*arcsin(c*x))*sin(a/b)/(b*c^2) - 5/4*cos(a/b)^4*cos_integral(
5*a/b + 5*arcsin(c*x))*sin(a/b)/(b*c^2) - 7/4*cos(a/b)^5*sin_integral(7*a/
b + 7*arcsin(c*x))/(b*c^2) + 5/4*cos(a/b)^5*sin_integral(5*a/b + 5*arcsin(
c*x))/(b*c^2) - 3/8*cos(a/b)^2*cos_integral(7*a/b + 7*arcsin(c*x))*sin(a/b)
)/(b*c^2) + 15/16*cos(a/b)^2*cos_integral(5*a/b + 5*arcsin(c*x))*sin(a/b)/
(b*c^2) - 9/16*cos(a/b)^2*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*
c^2) + 7/8*cos(a/b)^3*sin_integral(7*a/b + 7*arcsin(c*x))/(b*c^2) - 25/16*
cos(a/b)^3*sin_integral(5*a/b + 5*arcsin(c*x))/(b*c^2) + 9/16*cos(a/b)^3*s
in_integral(3*a/b + 3*arcsin(c*x))/(b*c^2) + 1/64*cos_integral(7*a/b + 7*a
rcsin(c*x))*sin(a/b)/(b*c^2) - 5/64*cos_integral(5*a/b + 5*arcsin(c*x))*si
n(a/b)/(b*c^2) + 9/64*cos_integral(3*a/b + 3*arcsin(c*x))*sin(a/b)/(b*c^2)
- 5/64*cos_integral(a/b + arcsin(c*x))*sin(a/b)/(b*c^2) - 7/64*cos(a/b)*s
in_integral(7*a/b + 7*arcsin(c*x))/(b*c^2) + 25/64*cos(a/b)*sin_integral(5
*a/b + 5*arcsin(c*x))/(b*c^2) - 27/64*cos(a/b)*sin_integral(3*a/b + 3*arcs
in(c*x))/(b*c^2) + 5/64*cos(a/b)*sin_integral(a/b + arcsin(c*x))/(b*c^2)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \operatorname{asin}(cx)} dx$$

input

```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)),x)
```

output

```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)), x)
```

Reduce [F]

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^5}{\operatorname{asin}(cx) b + a} dx \right) c^4 - 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^3}{\operatorname{asin}(cx) b + a} dx \right) c^2 + \int \frac{\sqrt{-c^2 x^2 + 1} x}{\operatorname{asin}(cx) b + a} dx$$

input `int(x*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`

output `int((sqrt(-c**2*x**2+1)*x**5)/(asin(c*x)*b+a),x)*c**4 - 2*int((sqrt(-c**2*x**2+1)*x**3)/(asin(c*x)*b+a),x)*c**2 + int((sqrt(-c**2*x**2+1)*x)/(asin(c*x)*b+a),x)`

3.308 $\int \frac{(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx$

Optimal result	2907
Mathematica [A] (verified)	2908
Rubi [A] (verified)	2908
Maple [A] (verified)	2910
Fricas [F]	2910
Sympy [F]	2910
Maxima [F]	2911
Giac [B] (verification not implemented)	2911
Mupad [F(-1)]	2912
Reduce [F]	2912

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b \arcsin(cx)} dx = \frac{15 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc} + \frac{3 \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{16bc} + \frac{\cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc} + \frac{5 \log(a+b \arcsin(cx))}{16bc} + \frac{15 \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{32bc} + \frac{3 \sin\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{16bc} + \frac{\sin\left(\frac{6a}{b}\right) \text{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{32bc}$$

output

```
15/32*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c+3/16*cos(4*a/b)*Ci(4*(a+b*arcsin(c*x))/b)/b/c+1/32*cos(6*a/b)*Ci(6*(a+b*arcsin(c*x))/b)/b/c+5/16*ln(a+b*arcsin(c*x))/b/c+15/32*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c+3/16*sin(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b/c+1/32*sin(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b/c
```


Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \frac{15 \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 6 \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{32bc}$$

input `Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcSin[c*x]),x]`

output `(15*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] + 6*Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])] + Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcSin[c*x])] + 18*Log[a + b*ArcSin[c*x]] - 8*Log[8*(a + b*ArcSin[c*x])] + 15*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 6*Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])])/(32*b*c)`

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5168, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

$$\downarrow \text{5168}$$

$$\int \frac{\cos^6\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))$$

$$\frac{bc}{bc}$$

$$\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^6}{a+b \arcsin(cx)} d(a + b \arcsin(cx))$$

$$\frac{bc}{bc}$$

$$\downarrow \text{3793}$$

$$\int \left(\frac{\cos\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{3 \cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{15 \cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{5}{16(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

bc

↓ 2009

$$\frac{15}{32} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{3}{16} \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{32} \cos\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right)$$

input

```
Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcSin[c*x]),x]
```

output

```
((15*Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 + (3*Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcSin[c*x]))/b])/16 + (Cos[(6*a)/b]*CosIntegral[(6*(a + b*ArcSin[c*x]))/b])/32 + (5*Log[a + b*ArcSin[c*x]])/16 + (15*Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 + (3*Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/16 + (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x]))/b])/32)/(b*c)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 3793

```
Int[((c._) + (d._)*(x._))^(m._)*sin[(e._) + (f._)*(x._)]^(n._), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5168

```
Int[((a._) + ArcSin[(c._)*(x._)]*(b._))^(n._)*((d._) + (e._)*(x._)^2)^(p._), x_Symbol] := Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

method	result
default	$\frac{\text{Si}(6 \arcsin(cx) + \frac{6a}{b}) \sin(\frac{6a}{b}) + \text{Ci}(6 \arcsin(cx) + \frac{6a}{b}) \cos(\frac{6a}{b}) + 6 \text{Si}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) + 6 \text{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) + 15 \text{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) + 15 \text{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) + 10 \ln(a + b \arcsin(cx))}{32cb}$

input `int((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\frac{1/32/c*(\text{Si}(6*\arcsin(c*x)+6*a/b)*\sin(6*a/b)+\text{Ci}(6*\arcsin(c*x)+6*a/b)*\cos(6*a/b)+6*\text{Si}(4*\arcsin(c*x)+4*a/b)*\sin(4*a/b)+6*\text{Ci}(4*\arcsin(c*x)+4*a/b)*\cos(4*a/b)+15*\text{Si}(2*\arcsin(c*x)+2*a/b)*\sin(2*a/b)+15*\text{Ci}(2*\arcsin(c*x)+2*a/b)*\cos(2*a/b)+10*\ln(a+b*\arcsin(c*x)))}{b}$$

Fricas [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{b \arcsin(cx) + a} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*arcsin(c*x) + a), x)`

Sympy [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-(cx - 1)(cx + 1))^{5/2}}{a + b \arcsin(cx)} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(a + b*asin(c*x)), x)`

Maxima [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{b \arcsin(cx) + a} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)/(b*arcsin(c*x) + a), x)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(192) = 384.

Time = 0.16 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.29

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Too large to display}$$

input `integrate((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `cos(a/b)^6*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c) + cos(a/b)^5*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c) - 3/2*cos(a/b)^4*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c) + 3/2*cos(a/b)^4*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c) - cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c) + 3/2*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c) + 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c) - 3/2*cos(a/b)^2*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c) + 15/16*cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c) + 3/16*cos(a/b)*sin(a/b)*sin_integral(6*a/b + 6*arcsin(c*x))/(b*c) - 3/4*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c) + 15/16*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c) - 1/32*cos_integral(6*a/b + 6*arcsin(c*x))/(b*c) + 3/16*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c) - 15/32*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c) + 5/16*log(b*arcsin(c*x) + a)/(b*c)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((1 - c^2*x^2)^(5/2)/(a + b*asin(c*x)),x)`output `int((1 - c^2*x^2)^(5/2)/(a + b*asin(c*x)), x)`**Reduce [F]**

$$\int \frac{(1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) b + a} dx$$

$$+ \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^4}{\operatorname{asin}(cx) b + a} dx \right) c^4 - 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^2}{\operatorname{asin}(cx) b + a} dx \right) c^2$$

input `int((-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)*b + a),x) + int((sqrt(-c**2*x**2 + 1)*x**4)/(asin(c*x)*b + a),x)*c**4 - 2*int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)*b + a),x)*c**2`

3.309 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))} dx$

Optimal result	2913
Mathematica [N/A]	2913
Rubi [N/A]	2914
Maple [N/A]	2915
Fricas [N/A]	2915
Sympy [N/A]	2915
Maxima [N/A]	2916
Giac [F(-2)]	2916
Mupad [N/A]	2917
Reduce [N/A]	2917

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1 - c^2x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \text{Int} \left(\frac{(1 - c^2x^2)^{5/2}}{x(a + b \arcsin(cx))}, x \right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1 - c^2x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2x^2)^{5/2}}{x(a + b \arcsin(cx))} dx$$

input

```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcSin[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))} dx$$

↓ 5226

$$\int \left(-\frac{3c^2 x}{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} + \frac{1}{x\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} - \frac{c^6 x^5}{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} + \frac{1}{\sqrt{1 - c^2 x^2}} \right) dx$$

↓ 2009

$$\int \frac{1}{x\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} dx + \frac{11 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a + b \arcsin(cx)}{b}\right)}{8b} +$$

$$\frac{7 \sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a + b \arcsin(cx))}{b}\right)}{16b} + \frac{\sin\left(\frac{5a}{b}\right) \text{CosIntegral}\left(\frac{5(a + b \arcsin(cx))}{b}\right)}{16b} -$$

$$\frac{11 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a + b \arcsin(cx)}{b}\right)}{8b} - \frac{7 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a + b \arcsin(cx))}{b}\right)}{16b} - \frac{\cos\left(\frac{5a}{b}\right) \text{Si}\left(\frac{5(a + b \arcsin(cx))}{b}\right)}{16b}$$

input

```
Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcSin[c*x])),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}}{x(a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x)),x)`output `int((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x)),x)`**Fricas [N/A]**

Not integrable

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{(1 - c^2x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}}{(b \arcsin(cx) + a)x} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x)),x, algorithm="fricas")`output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x*arcsin(c*x) + a*x), x)`**Sympy [N/A]**

Not integrable

Time = 4.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(1 - c^2x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{5}{2}}}{x(a + b \arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/x/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(x*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)x} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)/((b*arcsin(c*x) + a)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(5/2)/(x*(a + b*asin(c*x))),x)`output `int((1 - c^2*x^2)^(5/2)/(x*(a + b*asin(c*x))), x)`**Reduce [N/A]**

Not integrable

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) bx + ax} dx$$

$$+ \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^3}{\operatorname{asin}(cx) b + a} dx \right) c^4 - 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} x}{\operatorname{asin}(cx) b + a} dx \right) c^2$$

input `int((-c^2*x^2+1)^(5/2)/x/(a+b*asin(c*x)),x)`output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)*b*x + a*x),x) + int((sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)*b + a),x)*c**4 - 2*int((sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)*b + a),x)*c**2`

3.310
$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b \arcsin(cx))} dx$$

Optimal result	2918
Mathematica [N/A]	2918
Rubi [N/A]	2919
Maple [N/A]	2920
Fricas [N/A]	2920
Sympy [N/A]	2920
Maxima [N/A]	2921
Giac [N/A]	2921
Mupad [N/A]	2922
Reduce [N/A]	2922

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx = \text{Int}\left(\frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx$$

input

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcSin[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx$$

↓ 5226

$$\int \left(-\frac{3c^2}{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} + \frac{1}{x^2 \sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} - \frac{c^6 x^4}{\sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} + \frac{1}{\sqrt{1 - c^2 x^2}} \right) dx$$

↓ 2009

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2}(a + b \arcsin(cx))} dx - \frac{c \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a + b \arcsin(cx))}{b}\right)}{b} - \frac{c \cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a + b \arcsin(cx))}{b}\right)}{8b} - \frac{c \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a + b \arcsin(cx))}{b}\right)}{b} - \frac{c \sin\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a + b \arcsin(cx))}{b}\right)}{8b} - \frac{15c \log(a + b \arcsin(cx))}{8b}$$

input `Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}}{x^2 (a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}}{(b \arcsin(cx) + a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^2*arcsin(c*x) + a*x^2), x)`

Sympy [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{-(cx - 1)(cx + 1)^{\frac{5}{2}}}{x^2(a + b \arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(x**2*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)/((b*arcsin(c*x) + a)*x^2), x)`

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2(a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)x^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate((-c^2*x^2 + 1)^(5/2)/((b*arcsin(c*x) + a)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*asin(c*x))),x)`output `int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*asin(c*x))), x)`**Reduce [N/A]**

Not integrable

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.36

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))} dx = \frac{-2 \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) b + a} dx \right) b c^2 + \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^2}{\operatorname{asin}(cx) b + a} dx \right) b c^4 + \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) b x^2 + \sqrt{-c^2 x^2 + 1}} dx \right) b}{b}$$

input `int((-c^2*x^2+1)^(5/2)/x^2/(a+b*asin(c*x)),x)`output `(- 2*int(sqrt(- c**2*x**2 + 1)/(asin(c*x)*b + a),x)*b*c**2 + int((sqrt(- c**2*x**2 + 1)*x**2)/(asin(c*x)*b + a),x)*b*c**4 + int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)*b*x**2 + sqrt(- c**2*x**2 + 1)*a*x**2),x)*b - log(asin(c*x)*b + a)*c)/b`

3.311 $\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))} dx$

Optimal result	2923
Mathematica [N/A]	2923
Rubi [N/A]	2924
Maple [N/A]	2924
Fricas [N/A]	2925
Sympy [N/A]	2925
Maxima [N/A]	2925
Giac [F(-2)]	2926
Mupad [N/A]	2926
Reduce [N/A]	2927

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^3(a + b \arcsin(cx))} dx = \text{Int}\left(\frac{(1 - c^2x^2)^{5/2}}{x^3(a + b \arcsin(cx))}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^3(a + b \arcsin(cx))} dx = \int \frac{(1 - c^2x^2)^{5/2}}{x^3(a + b \arcsin(cx))} dx$$

input

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcSin[c*x])),x]
```

output

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcSin[c*x])), x]
```


Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx$$

input `Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{5/2}}{x^3 (a + b \arcsin(cx))} dx$$

input `int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x)),x)`

output `int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)x^3} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^3*arcsin(c*x) + a*x^3), x)`

Sympy [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{(-(cx - 1)(cx + 1))^{5/2}}{x^3 (a + b \arcsin(cx))} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*asin(c*x)),x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(x**3*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)x^3} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)/((b*arcsin(c*x) + a)*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \operatorname{asin}(cx))} dx$$

input `int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*asin(c*x))),x)`

output `int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*asin(c*x))), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) b x^3 + a x^3} dx$$

$$- 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx) b x + a x} dx \right) c^2 + \left(\int \frac{\sqrt{-c^2 x^2 + 1} x}{\operatorname{asin}(cx) b + a} dx \right) c^4$$

input `int((-c^2*x^2+1)^(5/2)/x^3/(a+b*asin(c*x)),x)`output `int(sqrt(-c**2*x**2+1)/(asin(c*x)*b*x**3+a*x**3),x)-2*int(sqrt(-c**2*x**2+1)/(asin(c*x)*b*x+a*x),x)*c**2+int((sqrt(-c**2*x**2+1)*x)/(asin(c*x)*b+a),x)*c**4`

3.312 $\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2928
Mathematica [A] (verified)	2928
Rubi [A] (verified)	2929
Maple [A] (verified)	2930
Fricas [F]	2931
Sympy [F]	2931
Maxima [F]	2931
Giac [A] (verification not implemented)	2932
Mupad [F(-1)]	2932
Reduce [F]	2932

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{CosIntegral}(2 \arcsin(ax))}{2a^5} + \frac{\text{CosIntegral}(4 \arcsin(ax))}{8a^5} + \frac{3 \log(\arcsin(ax))}{8a^5}$$

output `-1/2*Ci(2*arcsin(a*x))/a^5+1/8*Ci(4*arcsin(a*x))/a^5+3/8*ln(arcsin(a*x))/a^5`

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{4 \text{CosIntegral}(2 \arcsin(ax)) - \text{CosIntegral}(4 \arcsin(ax)) - 3 \log(\arcsin(ax))}{8a^5}$$

input `Integrate[x^4/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output

```
-1/8*(4*CosIntegral[2*ArcSin[a*x]] - CosIntegral[4*ArcSin[a*x]] - 3*Log[ArcSin[a*x]])/a^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx \\
 & \quad \downarrow \text{5224} \\
 & \frac{\int \frac{a^4 x^4}{\arcsin(ax)} d \arcsin(ax)}{a^5} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\int \frac{\sin(\arcsin(ax))^4}{\arcsin(ax)} d \arcsin(ax)}{a^5} \\
 & \quad \downarrow \text{3793} \\
 & \frac{\int \left(-\frac{\cos(2 \arcsin(ax))}{2 \arcsin(ax)} + \frac{\cos(4 \arcsin(ax))}{8 \arcsin(ax)} + \frac{3}{8 \arcsin(ax)} \right) d \arcsin(ax)}{a^5} \\
 & \quad \downarrow \text{2009} \\
 & \frac{-\frac{1}{2} \text{CosIntegral}(2 \arcsin(ax)) + \frac{1}{8} \text{CosIntegral}(4 \arcsin(ax)) + \frac{3}{8} \log(\arcsin(ax))}{a^5}
 \end{aligned}$$

input

```
Int[x^4/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]
```

output

```
(-1/2*CosIntegral[2*ArcSin[a*x]] + CosIntegral[4*ArcSin[a*x]]/8 + (3*Log[ArcSin[a*x]])/8)/a^5
```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_)^(m_)*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{3 \ln(\arcsin(ax)) - 4 \operatorname{Ci}(2 \arcsin(ax)) + \operatorname{Ci}(4 \arcsin(ax))}{8a^5}$	30

input `int(x^4/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x,method=_RETURNVERBOSE)`

output `1/8*(3*ln(arcsin(a*x))-4*Ci(2*arcsin(a*x))+Ci(4*arcsin(a*x)))/a^5`

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^4}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^4/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^4/((a^2*x^2 - 1)*arcsin(a*x)), x)`

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^4}{\sqrt{-(ax-1)(ax+1)} \operatorname{asin}(ax)} dx$$

input `integrate(x**4/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(x**4/(sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^4}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^4/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(x^4/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\text{Ci}(4 \arcsin(ax))}{8a^5} - \frac{\text{Ci}(2 \arcsin(ax))}{2a^5} + \frac{3 \log(\arcsin(ax))}{8a^5}$$

input `integrate(x^4/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `1/8*cos_integral(4*arcsin(a*x))/a^5 - 1/2*cos_integral(2*arcsin(a*x))/a^5 + 3/8*log(arcsin(a*x))/a^5`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^4}{\arcsin(ax) \sqrt{1-a^2x^2}} dx$$

input `int(x^4/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(x^4/(asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^4}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `int(x^4/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`

output `int(x**4/(sqrt(-a**2*x**2+1)*asin(a*x)),x)`

3.313 $\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2933
Mathematica [A] (verified)	2933
Rubi [A] (verified)	2934
Maple [A] (verified)	2935
Fricas [F]	2936
Sympy [F]	2936
Maxima [F]	2936
Giac [F(-2)]	2937
Mupad [F(-1)]	2937
Reduce [F]	2937

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{3\text{Si}(\arcsin(ax))}{4a^4} - \frac{\text{Si}(3 \arcsin(ax))}{4a^4}$$

output `3/4*Si(arcsin(a*x))/a^4-1/4*Si(3*arcsin(a*x))/a^4`

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{-3\text{Si}(\arcsin(ax)) + \text{Si}(3 \arcsin(ax))}{4a^4}$$

input `Integrate[x^3/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `-1/4*(-3*SinIntegral[ArcSin[a*x]] + SinIntegral[3*ArcSin[a*x]])/a^4`

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx \\
 & \quad \downarrow \text{5224} \\
 & \int \frac{a^3x^3}{\arcsin(ax)} d \arcsin(ax) \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sin(\arcsin(ax))^3}{\arcsin(ax)} d \arcsin(ax) \\
 & \quad \downarrow \text{3793} \\
 & \int \left(\frac{3ax}{4 \arcsin(ax)} - \frac{\sin(3 \arcsin(ax))}{4 \arcsin(ax)} \right) d \arcsin(ax) \\
 & \quad \downarrow \text{2009} \\
 & \frac{\frac{3}{4} \text{Si}(\arcsin(ax)) - \frac{1}{4} \text{Si}(3 \arcsin(ax))}{a^4}
 \end{aligned}$$

input `Int[x^3/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `((3*SinIntegral[ArcSin[a*x]])/4 - SinIntegral[3*ArcSin[a*x]]/4)/a^4`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{3 \operatorname{Si}(\arcsin(ax)) - \operatorname{Si}(3 \arcsin(ax))}{4a^4}$	23

input `int(x^3/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x,method=_RETURNVERBOSE)`

output `1/4*(3*Si(arcsin(a*x))-Si(3*arcsin(a*x)))/a^4`

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^3}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^3/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^3/((a^2*x^2 - 1)*arcsin(a*x)), x)`

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^3}{\sqrt{-(ax-1)(ax+1)} \operatorname{asin}(ax)} dx$$

input `integrate(x**3/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^3}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^3/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(x^3/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^3}{\arcsin(ax) \sqrt{1-a^2x^2}} dx$$

input `int(x^3/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(x^3/(asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^3}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `int(x^3/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`

output `int(x**3/(sqrt(-a**2*x**2+1)*asin(a*x)),x)`

3.314 $\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2938
Mathematica [A] (verified)	2938
Rubi [A] (verified)	2939
Maple [A] (verified)	2940
Fricas [F]	2941
Sympy [F]	2941
Maxima [F]	2941
Giac [A] (verification not implemented)	2942
Mupad [F(-1)]	2942
Reduce [F]	2942

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{CosIntegral}(2 \arcsin(ax))}{2a^3} + \frac{\log(\arcsin(ax))}{2a^3}$$

output `-1/2*Ci(2*arcsin(a*x))/a^3+1/2*ln(arcsin(a*x))/a^3`

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{CosIntegral}(2 \arcsin(ax)) - \log(\arcsin(ax))}{2a^3}$$

input `Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `-1/2*(CosIntegral[2*ArcSin[a*x]] - Log[ArcSin[a*x]])/a^3`

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx \\
 & \quad \downarrow \text{5224} \\
 & \int \frac{a^2x^2}{\arcsin(ax)} d \arcsin(ax) \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sin(\arcsin(ax))^2}{\arcsin(ax)} d \arcsin(ax) \\
 & \quad \downarrow \text{3793} \\
 & \int \left(\frac{1}{2 \arcsin(ax)} - \frac{\cos(2 \arcsin(ax))}{2 \arcsin(ax)} \right) d \arcsin(ax) \\
 & \quad \downarrow \text{2009} \\
 & \frac{\frac{1}{2} \log(\arcsin(ax)) - \frac{1}{2} \text{CosIntegral}(2 \arcsin(ax))}{a^3}
 \end{aligned}$$

input `Int[x^2/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `(-1/2*CosIntegral[2*ArcSin[a*x]] + Log[ArcSin[a*x]]/2)/a^3`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_)^(m_)*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln(\arcsin(ax)) - \text{Ci}(2 \arcsin(ax))}{2a^3}$	21

input `int(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x,method=_RETURNVERBOSE)`

output `1/2*(ln(arcsin(a*x))-Ci(2*arcsin(a*x)))/a^3`

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arcsin(a*x)), x)`

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)} \operatorname{asin}(ax)} dx$$

input `integrate(x**2/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(x^2/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{Ci}(2 \arcsin(ax))}{2a^3} + \frac{\log(\arcsin(ax))}{2a^3}$$

input `integrate(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `-1/2*cos_integral(2*arcsin(a*x))/a^3 + 1/2*log(arcsin(a*x))/a^3`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\arcsin(ax) \sqrt{1-a^2x^2}} dx$$

input `int(x^2/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(x^2/(asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `int(x^2/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`

output `int(x**2/(sqrt(-a**2*x**2+1)*asin(a*x)),x)`

3.315 $\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2943
Mathematica [A] (verified)	2943
Rubi [A] (verified)	2944
Maple [A] (verified)	2945
Fricas [F]	2946
Sympy [F]	2946
Maxima [F]	2946
Giac [A] (verification not implemented)	2947
Mupad [F(-1)]	2947
Reduce [F]	2947

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{CosIntegral}(2 \arcsin(ax))}{2a^3} + \frac{\log(\arcsin(ax))}{2a^3}$$

output -1/2*Ci(2*arcsin(a*x))/a^3+1/2*ln(arcsin(a*x))/a^3

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{CosIntegral}(2 \arcsin(ax)) - \log(\arcsin(ax))}{2a^3}$$

input Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]

output -1/2*(CosIntegral[2*ArcSin[a*x]] - Log[ArcSin[a*x]])/a^3

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx \\
 & \quad \downarrow \text{5224} \\
 & \int \frac{a^2x^2}{\arcsin(ax)} d \arcsin(ax) \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sin(\arcsin(ax))^2}{\arcsin(ax)} d \arcsin(ax) \\
 & \quad \downarrow \text{3793} \\
 & \int \left(\frac{1}{2 \arcsin(ax)} - \frac{\cos(2 \arcsin(ax))}{2 \arcsin(ax)} \right) d \arcsin(ax) \\
 & \quad \downarrow \text{2009} \\
 & \frac{\frac{1}{2} \log(\arcsin(ax)) - \frac{1}{2} \text{CosIntegral}(2 \arcsin(ax))}{a^3}
 \end{aligned}$$

input `Int[x^2/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `(-1/2*CosIntegral[2*ArcSin[a*x]] + Log[ArcSin[a*x]]/2)/a^3`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln(\arcsin(ax)) - \text{Ci}(2 \arcsin(ax))}{2a^3}$	21

input `int(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x,method=_RETURNVERBOSE)`

output `1/2*(ln(arcsin(a*x))-Ci(2*arcsin(a*x)))/a^3`

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arcsin(a*x)), x)`

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)} \operatorname{asin}(ax)} dx$$

input `integrate(x**2/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(x^2/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = -\frac{\text{Ci}(2 \arcsin(ax))}{2a^3} + \frac{\log(\arcsin(ax))}{2a^3}$$

input `integrate(x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `-1/2*cos_integral(2*arcsin(a*x))/a^3 + 1/2*log(arcsin(a*x))/a^3`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\arcsin(ax) \sqrt{1-a^2x^2}} dx$$

input `int(x^2/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(x^2/(asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^2}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `int(x^2/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`

output `int(x**2/(sqrt(-a**2*x**2+1)*asin(a*x)),x)`

3.316 $\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2948
Mathematica [A] (verified)	2948
Rubi [A] (verified)	2949
Maple [A] (verified)	2950
Fricas [F]	2950
Sympy [F]	2951
Maxima [F]	2951
Giac [A] (verification not implemented)	2951
Mupad [F(-1)]	2952
Reduce [F]	2952

Optimal result

Integrand size = 22, antiderivative size = 9

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\text{Si}(\arcsin(ax))}{a^2}$$

output `Si(arcsin(a*x))/a^2`

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\text{Si}(\arcsin(ax))}{a^2}$$

input `Integrate[x/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `SinIntegral[ArcSin[a*x]]/a^2`

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5224, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$$

↓ 5224

$$\frac{\int \frac{ax}{\arcsin(ax)} d \arcsin(ax)}{a^2}$$

↓ 3042

$$\frac{\int \frac{\sin(\arcsin(ax))}{\arcsin(ax)} d \arcsin(ax)}{a^2}$$

↓ 3780

$$\frac{\text{Si}(\arcsin(ax))}{a^2}$$

input `Int[x/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `SinIntegral[ArcSin[a*x]]/a^2`

Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\text{Si}(\arcsin(ax))}{a^2}$	10

input

```
int(x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x,method=_RETURNVERBOSE)
```

output

```
Si(arcsin(a*x))/a^2
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input

```
integrate(x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")
```

output

```
integral(-sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*arcsin(a*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x}{\sqrt{-(ax-1)(ax+1)} \operatorname{asin}(ax)} dx$$

input `integrate(x/(-a**2*x**2+1)**(1/2)/asin(a*x), x)`

output `Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [F]

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x/(-a^2*x^2+1)^(1/2)/arcsin(a*x), x, algorithm="maxima")`

output `integrate(x/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\operatorname{Si}(\arcsin(ax))}{a^2}$$

input `integrate(x/(-a^2*x^2+1)^(1/2)/arcsin(a*x), x, algorithm="giac")`

output `sin_integral(arcsin(a*x))/a^2`

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x}{\arcsin(ax) \sqrt{1-a^2x^2}} dx$$

input `int(x/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`output `int(x/(asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`**Reduce [F]**

$$\int \frac{x}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `int(x/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`output `int(x/(sqrt(-a**2*x**2 + 1)*asin(a*x)),x)`

3.317 $\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2953
Mathematica [A] (verified)	2953
Rubi [A] (verified)	2954
Maple [A] (verified)	2954
Fricas [A] (verification not implemented)	2955
Sympy [A] (verification not implemented)	2955
Maxima [A] (verification not implemented)	2955
Giac [A] (verification not implemented)	2956
Mupad [B] (verification not implemented)	2956
Reduce [B] (verification not implemented)	2956

Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(\arcsin(ax))}{a}$$

output ln(arcsin(a*x))/a

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(\arcsin(ax))}{a}$$

input Integrate[1/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]

output Log[ArcSin[a*x]]/a

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$$

↓ 5150

$$\frac{\log(\arcsin(ax))}{a}$$

input `Int[1/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `Log[ArcSin[a*x]]/a`

Defintions of rubi rules used

rule 5150 `Int[1/(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Symbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]`

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\ln(\arcsin(ax))}{a}$	10
default	$\frac{\ln(\arcsin(ax))}{a}$	10

input `int(1/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x,method=_RETURNVERBOSE)`

output `ln(arcsin(a*x))/a`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(-\arcsin(ax))}{a}$$

input `integrate(1/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `log(-arcsin(a*x))/a`

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(\arcsin(ax))}{a}$$

input `integrate(1/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `log(asin(a*x))/a`

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(\arcsin(ax))}{a}$$

input `integrate(1/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `log(arcsin(a*x))/a`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(|\arcsin(ax)|)}{a}$$

input `integrate(1/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `log(abs(arcsin(a*x)))/a`

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\ln(\arcsin(ax))}{a}$$

input `int(1/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `log(asin(a*x))/a`

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \frac{\log(\arcsin(ax))}{a}$$

input `int(1/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`

output `log(asin(a*x))/a`

3.318 $\int \frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2957
Mathematica [N/A]	2957
Rubi [N/A]	2958
Maple [N/A]	2958
Fricas [N/A]	2959
Sympy [N/A]	2959
Maxima [N/A]	2959
Giac [N/A]	2960
Mupad [N/A]	2960
Reduce [N/A]	2961

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)} dx = \text{Int}\left(\frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)}, x\right)$$

output `Defer(Int)(1/x/(-a^2*x^2+1)^(1/2)/arcsin(a*x), x)`

Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{1}{x\sqrt{1-a^2x^2} \arcsin(ax)} dx$$

input `Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcSin[a*x]), x]`

output `Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcSin[a*x]), x]`

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx$$

↓ 5234

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx$$

input `Int[1/(x*Sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{-a^2x^2+1}\arcsin(ax)} dx$$

input `int(1/x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x)`

output `int(1/x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2x^2+1x}\arcsin(ax)} dx$$

input `integrate(1/x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^3 - x)*arcsin(a*x)), x)`

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx = \int \frac{1}{x\sqrt{-(ax-1)(ax+1)}\arcsin(ax)} dx$$

input `integrate(1/x/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(1/(x*sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2x^2+1x}\arcsin(ax)} dx$$

input `integrate(1/x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(1/(sqrt(-a^2*x^2 + 1))*x*arcsin(a*x)), x)`

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2x^2+1}x\arcsin(ax)} dx$$

input `integrate(1/x/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `integrate(1/(sqrt(-a^2*x^2 + 1))*x*arcsin(a*x)), x)`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx = \int \frac{1}{x\arcsin(ax)\sqrt{1-a^2x^2}} dx$$

input `int(1/(x*asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(1/(x*asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2x^2+1}\arcsin(ax)x} dx$$

input `int(1/x/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`output `int(1/(sqrt(-a**2*x**2+1)*asin(a*x)*x),x)`

3.319 $\int \frac{1}{x^2 \sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	2962
Mathematica [N/A]	2962
Rubi [N/A]	2963
Maple [N/A]	2963
Fricas [N/A]	2964
Sympy [N/A]	2964
Maxima [N/A]	2964
Giac [N/A]	2965
Mupad [N/A]	2965
Reduce [N/A]	2966

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2 \sqrt{1-a^2x^2} \arcsin(ax)} dx = \text{Int}\left(\frac{1}{x^2 \sqrt{1-a^2x^2} \arcsin(ax)}, x\right)$$

output `Defer(Int)(1/x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x)`

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{1}{x^2 \sqrt{1-a^2x^2} \arcsin(ax)} dx$$

input `Integrate[1/(x^2*sqrt[1 - a^2*x^2]*ArcSin[a*x]),x]`

output `Integrate[1/(x^2*sqrt[1 - a^2*x^2]*ArcSin[a*x]), x]`

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx$$

↓ 5234

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx$$

input `Int [1/(x^2*sqrt [1 - a^2*x^2]*ArcSin[a*x]), x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \sqrt{-a^2 x^2 + 1} \arcsin(ax)} dx$$

input `int (1/x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x) , x)`

output `int (1/x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x) , x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2 x^2 + 1} x^2 \arcsin(ax)} dx$$

input `integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^4 - x^2)*arcsin(a*x)), x)`

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{1}{x^2 \sqrt{-(ax - 1)(ax + 1)} \operatorname{asin}(ax)} dx$$

input `integrate(1/x**2/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(1/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*asin(a*x)), x)`

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2 x^2 + 1} x^2 \arcsin(ax)} dx$$

input `integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(1/(sqrt(-a^2*x^2 + 1))*x^2*arcsin(a*x)), x)`

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2 x^2 + 1} x^2 \arcsin(ax)} dx$$

input `integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `integrate(1/(sqrt(-a^2*x^2 + 1))*x^2*arcsin(a*x)), x)`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{1}{x^2 \arcsin(ax) \sqrt{1 - a^2 x^2}} dx$$

input `int(1/(x^2*asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(1/(x^2*asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 \sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{1}{\sqrt{-a^2 x^2 + 1} \arcsin(ax) x^2} dx$$

input `int(1/x^2/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`output `int(1/(sqrt(-a**2*x**2+1)*asin(a*x)*x**2),x)`

3.320 $\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	2967
Mathematica [A] (verified)	2968
Rubi [A] (verified)	2968
Maple [A] (verified)	2970
Fricas [F]	2970
Sympy [F]	2971
Maxima [F]	2971
Giac [F(-2)]	2971
Mupad [F(-1)]	2972
Reduce [F]	2972

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = -\frac{5 \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{8bc^6} + \frac{5 \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{16bc^6} - \frac{\operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{5a}{b}\right)}{16bc^6} + \frac{5 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8bc^6} - \frac{5 \cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16bc^6} + \frac{\cos\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16bc^6}$$

output

```
-5/8*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^6+5/16*Ci(3*(a+b*arcsin(c*x))/b)*sin(3*a/b)/b/c^6-1/16*Ci(5*(a+b*arcsin(c*x))/b)*sin(5*a/b)/b/c^6+5/8*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^6-5/16*cos(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b/c^6+1/16*cos(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b/c^6
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{10 \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - 5 \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{3a}{b}\right) + \operatorname{CosIntegral}\left(5\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{5a}{b}\right)}{bc^6}$$

input

```
Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]
```

output

```
-1/16*(10*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - 5*CosIntegral[3*(a/b + ArcSin[c*x]])*Sin[(3*a)/b] + CosIntegral[5*(a/b + ArcSin[c*x]])*Sin[(5*a)/b] - 10*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 5*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] - Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])])/(b*c^6)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5224, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx \\ & \quad \downarrow \text{5224} \\ & \int \frac{\sin^5\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) \\ & \quad \downarrow \text{25} \\ & \int \frac{\sin^5\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) \\ & \quad \text{---} \frac{\hspace{10em}}{bc^6} \end{aligned}$$

$$\begin{array}{c}
 \downarrow 3042 \\
 \int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)^5}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) \\
 \hline
 bc^6 \\
 \downarrow 3793 \\
 \int \left(\frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} - \frac{5 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{5 \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx)) \\
 \hline
 bc^6 \\
 \downarrow 2009 \\
 \frac{-\frac{5}{8} \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \sin\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{1}{16} \sin\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{bc^6}
 \end{array}$$

input `Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `((-5*CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b])/8 + (5*CosIntegral[(3*(a + b*ArcSin[c*x])/b]*Sin[(3*a)/b])/16 - (CosIntegral[(5*(a + b*ArcSin[c*x])/b]*Sin[(5*a)/b])/16 + (5*Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 - (5*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/16 + (Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/16)/(b*c^6)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793

```
Int[((c_.) + (d_.)*(x_)^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

method	result
default	$\frac{\text{Si}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b}) - \text{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b}) - 5 \text{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) + 5 \text{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) + 10 \text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - 10 \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{16c^6b}$

input

```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/16/c^6*(Si(5*arcsin(c*x)+5*a/b)*cos(5*a/b)-Ci(5*arcsin(c*x)+5*a/b)*sin(5*a/b)-5*Si(3*arcsin(c*x)+3*a/b)*cos(3*a/b)+5*Ci(3*arcsin(c*x)+3*a/b)*sin(3*a/b)+10*Si(arcsin(c*x)+a/b)*cos(a/b)-10*Ci(arcsin(c*x)+a/b)*sin(a/b))/b
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{x^5}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a)} dx$$

input

```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsin(c*x) - a), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(x**5/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^5/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^5}{(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input `int(x^5/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^5/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^5}{\sqrt{-c^2x^2+1} \arcsin(cx) b + \sqrt{-c^2x^2+1} a} dx$$

input `int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int(x**5/(sqrt(-c**2*x**2+1)*asin(c*x)*b + sqrt(-c**2*x**2+1)*a),x)`

3.321 $\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	2973
Mathematica [A] (verified)	2974
Rubi [A] (verified)	2974
Maple [A] (verified)	2976
Fricas [F]	2976
Sympy [F]	2976
Maxima [F]	2977
Giac [A] (verification not implemented)	2977
Mupad [F(-1)]	2978
Reduce [F]	2978

Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = -\frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc^5} + \frac{\cos\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8bc^5} + \frac{3 \log(a+b \arcsin(cx))}{8bc^5} - \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc^5} + \frac{\sin\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8bc^5}$$

output

```
-1/2*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c^5+1/8*cos(4*a/b)*Ci(4*(a+b*arcsin(c*x))/b)/b/c^5+3/8*ln(a+b*arcsin(c*x))/b/c^5-1/2*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c^5+1/8*sin(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

$$= \frac{-4 \cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) + \cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) + 3 \log(a + b \arcsin(cx))}{8bc^5}$$

input

```
Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]
```

output

```
(-4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] + Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcSin[c*x])] + 3*Log[a + b*ArcSin[c*x]] - 4*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])])/(8*b*c^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

$$\downarrow \text{5224}$$

$$\frac{\int \frac{\sin^4\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^5}$$

$$\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)^4}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^5}$$

$$\downarrow \text{3793}$$

$$\int \left(\frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} - \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{2(a+b \arcsin(cx))} + \frac{3}{8(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))$$

$$bc^5$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{2} \cos\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \cos\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) - \frac{1}{2} \sin\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{bc^5}$$

input

```
Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]
```

output

```
(-1/2*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcSin[c*x]))/b]) + (Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcSin[c*x]))/b])/8 + (3*Log[a + b*ArcSin[c*x]])/8 - (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/2 + (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8)/(b*c^5)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

method	result
default	$\frac{\text{Si}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) + \text{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) - 4 \text{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) - 4 \text{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) + 3 \ln(a + b \arcsin(cx))}{8c^5b}$

input `int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output
$$\frac{1/8/c^5*(\text{Si}(4*\arcsin(c*x)+4*a/b)*\sin(4*a/b)+\text{Ci}(4*\arcsin(c*x)+4*a/b)*\cos(4*a/b)-4*\text{Si}(2*\arcsin(c*x)+2*a/b)*\sin(2*a/b)-4*\text{Ci}(2*\arcsin(c*x)+2*a/b)*\cos(2*a/b)+3*\ln(a+b*\arcsin(c*x)))}{b}$$

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^4}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(x**4/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^4/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.76

$$\begin{aligned} \int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = & \frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{4a}{b} + 4\arcsin(cx)\right)}{bc^5} \\ & + \frac{\cos\left(\frac{a}{b}\right)^3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b} + 4\arcsin(cx)\right)}{bc^5} \\ & - \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{4a}{b} + 4\arcsin(cx)\right)}{bc^5} \\ & - \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{bc^5} \\ & - \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b} + 4\arcsin(cx)\right)}{2bc^5} \\ & - \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{bc^5} \\ & + \frac{\operatorname{Ci}\left(\frac{4a}{b} + 4\arcsin(cx)\right)}{8bc^5} \\ & + \frac{\operatorname{Ci}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{2bc^5} \\ & + \frac{3 \log(b\arcsin(cx) + a)}{8bc^5} \end{aligned}$$

input `integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```
cos(a/b)^4*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) + cos(a/b)^3*sin(a/
b)*sin_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) - cos(a/b)^2*cos_integral(4
*a/b + 4*arcsin(c*x))/(b*c^5) - cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c
*x))/(b*c^5) - 1/2*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcsin(c*x))/(
b*c^5) - cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c^5) + 1
/8*cos_integral(4*a/b + 4*arcsin(c*x))/(b*c^5) + 1/2*cos_integral(2*a/b +
2*arcsin(c*x))/(b*c^5) + 3/8*log(b*arcsin(c*x) + a)/(b*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^4}{(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input

```
int(x^4/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output

```
int(x^4/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^4}{\sqrt{-c^2x^2+1} \arcsin(cx) b + \sqrt{-c^2x^2+1} a} dx$$

input

```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)
```

output

```
int(x**4/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b + sqrt(-c**2*x**2 + 1)*a),x
)
```

3.322 $\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	2979
Mathematica [A] (verified)	2980
Rubi [A] (verified)	2980
Maple [A] (verified)	2982
Fricas [F]	2982
Sympy [F]	2983
Maxima [F]	2983
Giac [F(-2)]	2983
Mupad [F(-1)]	2984
Reduce [F]	2984

Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = -\frac{3 \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{4bc^4} + \frac{\operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{3a}{b}\right)}{4bc^4} + \frac{3 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{4bc^4} - \frac{\cos\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{4bc^4}$$

output

```
-3/4*Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^4+1/4*Ci(3*(a+b*arcsin(c*x))/b)*
sin(3*a/b)/b/c^4+3/4*cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^4-1/4*cos(3*a/b)
*Si(3*(a+b*arcsin(c*x))/b)/b/c^4
```


Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{3 \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) - \operatorname{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{3a}{b}\right) - 3 \cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{4bc^4}$$

input `Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `-1/4*(3*CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b] - CosIntegral[3*(a/b + ArcSin[c*x]])*Sin[(3*a)/b] - 3*Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]] + Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])])/(b*c^4)`

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5224, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx \\ & \quad \downarrow 5224 \\ & \int \frac{\sin^3\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) \\ & \quad \downarrow 25 \\ & \int \frac{\sin^3\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) \\ & \quad \downarrow 3042 \end{aligned}$$

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\text{Si}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) - \text{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) - 3 \text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) + 3 \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{4c^4b}$	93

input

```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
-1/4/c^4*(Si(3*arcsin(c*x)+3*a/b)*cos(3*a/b)-Ci(3*arcsin(c*x)+3*a/b)*sin(3
*a/b)-3*Si(arcsin(c*x)+a/b)*cos(a/b)+3*Ci(arcsin(c*x)+a/b)*sin(a/b))/b
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input

```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsin(c*x)
- a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(x**3/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^3}{(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input `int(x^3/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`output `int(x^3/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`**Reduce [F]**

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^3}{\sqrt{-c^2x^2+1} \arcsin(cx) b + \sqrt{-c^2x^2+1} a} dx$$

input `int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`output `int(x**3/(sqrt(-c**2*x**2+1)*asin(c*x)*b + sqrt(-c**2*x**2+1)*a),x)`

3.323 $\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	2985
Mathematica [A] (verified)	2985
Rubi [A] (verified)	2986
Maple [A] (verified)	2987
Fricas [F]	2988
Sympy [F]	2988
Maxima [F]	2988
Giac [A] (verification not implemented)	2989
Mupad [F(-1)]	2989
Reduce [F]	2990

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = -\frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc^3} + \frac{\log(a+b \arcsin(cx))}{2bc^3} - \frac{\sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{2bc^3}$$

output

```
-1/2*cos(2*a/b)*Ci(2*(a+b*arcsin(c*x))/b)/b/c^3+1/2*ln(a+b*arcsin(c*x))/b/c^3-1/2*sin(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \frac{\cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) - \log(a+b \arcsin(cx)) + \sin\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{2bc^3}$$

input `Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `-1/2*(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcSin[c*x])] - Log[a + b*ArcSin[c*x]] + Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])])/(b*c^3)`

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx \\
 & \quad \downarrow \text{5224} \\
 & \int \frac{\sin^2\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right)}{a + b \arcsin(cx)} d(a + b \arcsin(cx)) \\
 & \quad \quad \quad \frac{bc^3}{bc^3} \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right)^2}{a + b \arcsin(cx)} d(a + b \arcsin(cx)) \\
 & \quad \quad \quad \frac{bc^3}{bc^3} \\
 & \quad \downarrow \text{3793} \\
 & \int \left(\frac{1}{2(a + b \arcsin(cx))} - \frac{\cos\left(\frac{2a}{b} - \frac{2(a + b \arcsin(cx))}{b}\right)}{2(a + b \arcsin(cx))} \right) d(a + b \arcsin(cx)) \\
 & \quad \quad \quad \frac{bc^3}{bc^3} \\
 & \quad \downarrow \text{2009} \\
 & \frac{-\frac{1}{2} \cos\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a + b \arcsin(cx))}{b}\right) - \frac{1}{2} \sin\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a + b \arcsin(cx))}{b}\right) + \frac{1}{2} \log(a + b \arcsin(cx))}{bc^3}
 \end{aligned}$$

input `Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output

$$\frac{(-1/2*(\text{Cos}[(2*a)/b]*\text{CosIntegral}[(2*(a + b*\text{ArcSin}[c*x]))/b]) + \text{Log}[a + b*\text{ArcSin}[c*x]])/2 - (\text{Sin}[(2*a)/b]*\text{SinIntegral}[(2*(a + b*\text{ArcSin}[c*x]))/b])/2)/(b*c^3)}$$
Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Simp}[\text{IntSum}[u, x], x] \text{ /; } \text{SumQ}[u]$$

rule 3042

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ /; } \text{FunctionOfTrigOfLinearQ}[u, x]$$

rule 3793

$$\text{Int}[(c_. + (d_.)*(x_)^(m_)*\sin[(e_. + (f_.)*(x_)^(n_)]), x_Symbol] \text{ :> } \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] \text{ /; } \text{FreeQ}\{c, d, e, f, m\}, x \text{ \&\& } \text{IGtQ}[n, 1] \text{ \&\& } (!\text{RationalQ}[m] \text{ || } (\text{GeQ}[m, -1] \text{ \&\& } \text{LtQ}[m, 1]))$$

rule 5224

$$\text{Int}[(a_. + \text{ArcSin}[c_.*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(2)^(p_.), x_Symbol] \text{ :> } \text{Simp}[(1/(b*c^(m + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p \text{ Subst}[\text{Int}[x^n*\text{Sin}[-a/b + x/b]^m*\text{Cos}[-a/b + x/b]^(2*p + 1), x], x, a + b*\text{ArcSin}[c*x]], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, n\}, x \text{ \&\& } \text{EqQ}[c^2*d + e, 0] \text{ \&\& } \text{IGtQ}[2*p + 2, 0] \text{ \&\& } \text{IGtQ}[m, 0]$$
Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{-\text{Si}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) - \text{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) + \ln(a + b \arcsin(cx))}{2c^3b}$	65

input

$$\text{int}(x^2/(-c^2*x^2+1)^(1/2)/(a+b*\arcsin(c*x)), x, \text{method}=_RETURNVERBOSE)$$

output

$$1/2/c^3*(-\text{Si}(2*\arcsin(c*x)+2*a/b)*\sin(2*a/b)-\text{Ci}(2*\arcsin(c*x)+2*a/b)*\cos(2*a/b)+\ln(a+b*\arcsin(c*x)))/b$$

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(x**2/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = -\frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{bc^3} - \frac{\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{bc^3} + \frac{\operatorname{Ci}\left(\frac{2a}{b} + 2\arcsin(cx)\right)}{2bc^3} + \frac{\log(b\arcsin(cx) + a)}{2bc^3}$$

input `integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-cos(a/b)^2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) - cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) + 1/2*cos_integral(2*a/b + 2*arcsin(c*x))/(b*c^3) + 1/2*log(b*arcsin(c*x) + a)/(b*c^3)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^2}{(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input `int(x^2/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^2/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^2}{\sqrt{-c^2x^2+1} \operatorname{asin}(cx)b + \sqrt{-c^2x^2+1} a} dx$$

input

```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)
```

output

```
int(x**2/(sqrt(-c**2*x**2+1)*asin(c*x)*b + sqrt(-c**2*x**2+1)*a),x)
```

3.324 $\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	2991
Mathematica [A] (verified)	2991
Rubi [A] (verified)	2992
Maple [A] (verified)	2994
Fricas [F]	2994
Sympy [F]	2995
Maxima [F]	2995
Giac [A] (verification not implemented)	2995
Mupad [F(-1)]	2996
Reduce [F]	2996

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = -\frac{\text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b}\right)}{bc^2} + \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{bc^2}$$

output

```
-Ci((a+b*arcsin(c*x))/b)*sin(a/b)/b/c^2+cos(a/b)*Si((a+b*arcsin(c*x))/b)/b/c^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \frac{-\text{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right) + \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{bc^2}$$

input

```
Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]
```

output

```
(-(CosIntegral[a/b + ArcSin[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b + ArcSin[c*x]])/(b*c^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5224, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx \\
 & \quad \downarrow \text{5224} \\
 & \frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2} \\
 & \quad \downarrow \text{25} \\
 & \frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2} \\
 & \quad \downarrow \text{3784} \\
 & \frac{-\sin\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) - \cos\left(\frac{a}{b}\right) \int -\frac{\sin\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2} \\
 & \quad \downarrow \text{25} \\
 & \frac{\cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) - \sin\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{bc^2} \\
 & \quad \downarrow \text{3042}
 \end{aligned}$$

$$\frac{\cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{bc^2}$$

↓ 3780

$$\frac{\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) - \sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{bc^2}$$

↓ 3783

$$\frac{\cos\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) - \sin\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{bc^2}$$

input `Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `(-(CosIntegral[(a + b*ArcSin[c*x])/b]*Sin[a/b]) + Cos[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/(b*c^2)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

rule 3783 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

rule 3784

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[Cos[(d*
e - c*f)/d] Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d] Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\text{Si}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) - \text{Ci}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})}{c^2 b}$	46

input

```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/c^2*(Si(arcsin(c*x)+a/b)*cos(a/b)-Ci(arcsin(c*x)+a/b)*sin(a/b))/b
```

Fricas [F]

$$\int \frac{x}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{x}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a)} dx$$

input

```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsin(c*x) -
a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(x/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = -\frac{\text{Ci}\left(\frac{a}{b} + \arcsin(cx)\right) \sin\left(\frac{a}{b}\right)}{bc^2} + \frac{\cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{bc^2}$$

input `integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `-cos_integral(a/b + arcsin(c*x))*sin(a/b)/(b*c^2) + cos(a/b)*sin_integral(a/b + arcsin(c*x))/(b*c^2)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x}{(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input `int(x/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`output `int(x/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`**Reduce [F]**

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x}{\sqrt{-c^2x^2+1} \arcsin(cx) b + \sqrt{-c^2x^2+1} a} dx$$

input `int(x/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`output `int(x/(sqrt(-c**2*x**2+1)*asin(c*x)*b + sqrt(-c**2*x**2+1)*a),x)`

3.325 $\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	2997
Mathematica [A] (verified)	2997
Rubi [A] (verified)	2998
Maple [A] (verified)	2998
Fricas [A] (verification not implemented)	2999
Sympy [C] (verification not implemented)	2999
Maxima [A] (verification not implemented)	3000
Giac [A] (verification not implemented)	3000
Mupad [B] (verification not implemented)	3000
Reduce [B] (verification not implemented)	3001

Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \frac{\log(a+b \arcsin(cx))}{bc}$$

output `ln(a+b*arcsin(c*x))/b/c`

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \frac{\log(a+b \arcsin(cx))}{bc}$$

input `Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `Log[a + b*ArcSin[c*x]]/(b*c)`

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

↓ 5150

$$\frac{\log(a+b\arcsin(cx))}{bc}$$

input

```
Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]
```

output

```
Log[a + b*ArcSin[c*x]]/(b*c)
```

Defintions of rubi rules used

rule 5150

```
Int[1/(((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> Simp[(1/(b*c))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*Ar
cSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\arcsin(cx))}{bc}$	17
default	$\frac{\ln(a+b\arcsin(cx))}{bc}$	17

input

```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output $\ln(a+b\arcsin(cx))/b/c$

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{\log(-b\arcsin(cx) - a)}{bc}$$

input `integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output $\log(-b\arcsin(cx) - a)/(b*c)$

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge c = 0 \\ \begin{cases} -\frac{i \operatorname{acosh}(cx)}{c} & \text{for } |c^2x^2| > 1 \\ \frac{\operatorname{asin}(cx)}{c} & \text{otherwise} \end{cases} & \text{for } b = 0 \\ \frac{x}{a} & \text{for } c = 0 \\ \frac{\log\left(\frac{a}{b} + \operatorname{asin}(cx)\right)}{bc} & \text{otherwise} \end{cases}$$

input `integrate(1/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), (Piecewise((-I*acosh(c*x)/c, Abs(c**2*x**2) > 1), (asin(c*x)/c, True))/a, Eq(b, 0)), (x/a, Eq(c, 0)), (log(a/b + asin(c*x))/(b*c), True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{\log(b\arcsin(cx) + a)}{bc}$$

input `integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`output `log(b*arcsin(c*x) + a)/(b*c)`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{\log(|b\arcsin(cx) + a|)}{bc}$$

input `integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`output `log(abs(b*arcsin(c*x) + a))/(b*c)`**Mupad [B] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{\ln(a + b\arcsin(cx))}{bc}$$

input `int(1/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`output `log(a + b*asin(c*x))/(b*c)`

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \frac{\log(\operatorname{asin}(cx)b+a)}{bc}$$

input `int(1/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `log(asin(c*x)*b + a)/(b*c)`

3.326 $\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$

Optimal result	3002
Mathematica [N/A]	3002
Rubi [N/A]	3003
Maple [N/A]	3003
Fricas [N/A]	3004
Sympy [N/A]	3004
Maxima [N/A]	3004
Giac [F(-2)]	3005
Mupad [N/A]	3005
Reduce [N/A]	3006

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))}, x\right)$$

output `Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

input `Integrate[1/(x*Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])),x]`

output `Integrate[1/(x*Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])),x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

↓ 5234

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

input `Int[1/(x*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{-c^2x^2+1}(a+b\arcsin(cx))} dx$$

input `int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

output `int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^3 - a*x + (b*c^2*x^3 - b*x)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{1}{x\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(1/x/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input `int(1/(x*(a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`

output `int(1/(x*(a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2x^2+1} \arcsin(cx) bx + \sqrt{-c^2x^2+1} ax} dx$$

input `int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`output `int(1/(sqrt(-c**2*x**2+1)*asin(c*x)*b*x+sqrt(-c**2*x**2+1)*a*x),x)`

$$3.327 \quad \int \frac{1}{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$$

Optimal result	3007
Mathematica [N/A]	3007
Rubi [N/A]	3008
Maple [N/A]	3008
Fricas [N/A]	3009
Sympy [N/A]	3009
Maxima [N/A]	3009
Giac [N/A]	3010
Mupad [N/A]	3010
Reduce [N/A]	3011

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{1}{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))}, x\right)$$

output `Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \int \frac{1}{x^2 \sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$$

input `Integrate[1/(x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/(x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx$$

input `Int[1/(x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 \sqrt{-c^2 x^2 + 1} (a + b \arcsin(cx))} dx$$

input `int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

output `int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^4 - a*x^2 + (b*c^2*x^4 - b*x^2)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{1}{x^2 \sqrt{-(cx - 1)(cx + 1)} (a + b \arcsin(cx))} dx$$

input `integrate(1/x**2/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)*x^2), x)`

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx = \int \frac{1}{x^2 (a + b \arcsin(cx)) \sqrt{1 - c^2 x^2}} dx$$

input `int(1/(x^2*(a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`

output `int(1/(x^2*(a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))} dx$$

$$= \int \frac{1}{\sqrt{-c^2 x^2 + 1} a \sin(cx) b x^2 + \sqrt{-c^2 x^2 + 1} a x^2} dx$$

input `int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`

output `int(1/(sqrt(-c**2*x**2+1)*asin(c*x)*b*x**2+sqrt(-c**2*x**2+1)*a*x**2),x)`

$$3.328 \quad \int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

Optimal result	3012
Mathematica [N/A]	3012
Rubi [N/A]	3013
Maple [N/A]	3013
Fricas [N/A]	3014
Sympy [N/A]	3014
Maxima [N/A]	3014
Giac [N/A]	3015
Mupad [N/A]	3015
Reduce [N/A]	3016

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

input `Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

input `Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))} dx$$

input `int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

output `int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^2/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a), x)`

Sympy [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x**2/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{3/2}} dx$$

input `int(x^2/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)),x)`

output `int(x^2/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.64

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = - \left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx) b + \sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1}} \right) \frac{1}{b c^3}$$

input `int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `(- (int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)*b*c**2*x**2 - sqrt(- c**2*x**2 + 1)*asin(c*x)*b + sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a),x)*b*c + log(asin(c*x)*b + a)))/(b*c**3)`

$$3.329 \quad \int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

Optimal result	3017
Mathematica [N/A]	3017
Rubi [N/A]	3018
Maple [N/A]	3018
Fricas [N/A]	3019
Sympy [N/A]	3019
Maxima [N/A]	3019
Giac [F(-2)]	3020
Mupad [N/A]	3020
Reduce [N/A]	3021

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \int \frac{x}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

input `Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

input `Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))} dx$$

input `int(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

output `int(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a), x)`

Sympy [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{3/2}} dx$$

input `int(x/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)),x)`

output `int(x/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx =$$

$$-\left(\int \frac{x}{\sqrt{-c^2 x^2 + 1} a \sin(cx) b c^2 x^2 - \sqrt{-c^2 x^2 + 1} a \sin(cx) b + \sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a} dx \right)$$

input `int(x/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `- int(x/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**2*x**2 - sqrt(-c**2*x**2+1)*asin(c*x)*b + sqrt(-c**2*x**2+1)*a*c**2*x**2 - sqrt(-c**2*x**2+1)*a),x)`

$$3.330 \quad \int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

Optimal result	3022
Mathematica [N/A]	3022
Rubi [N/A]	3023
Maple [N/A]	3023
Fricas [N/A]	3024
Sympy [N/A]	3024
Maxima [N/A]	3024
Giac [N/A]	3025
Mupad [N/A]	3025
Reduce [N/A]	3026

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

input `Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

↓ 5174

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

input `Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))} dx$$

input `int(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

output `int(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a), x)`

Sympy [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(1/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{3/2}} dx$$

input `int(1/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)),x)`

output `int(1/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx) b + \sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a} dx \right)$$

input `int(1/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `- int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)*b*c**2*x**2 - sqrt(- c**2*x**2 + 1)*asin(c*x)*b + sqrt(- c**2*x**2 + 1)*a*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a),x)`

$$3.331 \quad \int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$$

Optimal result	3027
Mathematica [N/A]	3027
Rubi [N/A]	3028
Maple [N/A]	3028
Fricas [N/A]	3029
Sympy [N/A]	3029
Maxima [N/A]	3029
Giac [F(-2)]	3030
Mupad [N/A]	3030
Reduce [N/A]	3031

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \text{Int}\left(\frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}, x\right)$$

output `Defer(Int)(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 6.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$$

input `Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$$

↓ 5234

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$$

input `Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(-c^2x^2+1)^{3/2}(a+b\arcsin(cx))} dx$$

input `int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

output `int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.29

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arcsin(cx)+a)x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^5 - 2*a*c^2*x^3 + a*x + (b*c^4*x^5 - 2*b*c^2*x^3 + b*x)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\arcsin(cx))} dx$$

input `integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arcsin(cx)+a)x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(a+b\arcsin(cx))(1-c^2x^2)^{3/2}} dx$$

input `int(1/(x*(a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)),x)`

output `int(1/(x*(a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.96

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\arcsin(cx)bc^2x^3 - \sqrt{-c^2x^2+1}\arcsin(cx)bx + \sqrt{-c^2x^2+1}ac^2x^3 - \sqrt{-c^2x^2+1}ax} dx\right)$$

input `int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `- int(1/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**2*x**3 - sqrt(-c**2*x**2+1)*asin(c*x)*b*x + sqrt(-c**2*x**2+1)*a*c**2*x**3 - sqrt(-c**2*x**2+1)*a*x),x)`

$$3.332 \quad \int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$$

Optimal result	3032
Mathematica [N/A]	3032
Rubi [N/A]	3033
Maple [N/A]	3033
Fricas [N/A]	3034
Sympy [N/A]	3034
Maxima [N/A]	3035
Giac [N/A]	3035
Mupad [N/A]	3035
Reduce [N/A]	3036

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \text{Int}\left(\frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arcsin(cx))}, x\right)$$

output `Defer(Int)(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arcsin(cx))} dx$$

input `Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

input

```
Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))} dx$$

input

```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)
```

output

```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^6 - 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 - 2*b*c^2*x^4 + b*x^2)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{x^2 (-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)*x^2), x)`

Giac [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{1}{x^2 (a + b \arcsin(cx)) (1 - c^2 x^2)^{3/2}} dx$$

input `int(1/(x^2*(a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)),x)`

output `int(1/(x^2*(a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^4 - \sqrt{-c^2 x^2 + 1} \arcsin(cx) b x^2 + \sqrt{-c^2 x^2 + 1} a c^2 x^4 - \sqrt{-c^2 x^2 + 1} a x^2} dx \right)$$

input `int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `- int(1/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b*c**2*x**4 - sqrt(-c**2*x**2 + 1)*asin(c*x)*b*x**2 + sqrt(-c**2*x**2 + 1)*a*c**2*x**4 - sqrt(-c**2*x**2 + 1)*a*x**2),x)`

$$3.333 \quad \int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

Optimal result	3037
Mathematica [N/A]	3037
Rubi [N/A]	3038
Maple [N/A]	3038
Fricas [N/A]	3039
Sympy [N/A]	3039
Maxima [N/A]	3039
Giac [N/A]	3040
Mupad [N/A]	3040
Reduce [N/A]	3041

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{x^2}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

input `Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

input `Int[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))} dx$$

input `int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(x**2/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(x**2/((-c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^2}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{5/2}} dx$$

input `int(x^2/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)),x)`

output `int(x^2/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.46

$$\int \frac{x^2}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^2 + \sqrt{-c^2 x^2 + 1} a} dx$$

input `int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`output `int(x**2/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**4*x**4-2*sqrt(-c**2*x**2+1)*asin(c*x)*b+c*sqrt(-c**2*x**2+1)*a*c**4*x**4-2*sqrt(-c**2*x**2+1)*a*c**2*x**2+sqrt(-c**2*x**2+1)*a),x)`

$$3.334 \quad \int \frac{x}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

Optimal result	3042
Mathematica [N/A]	3042
Rubi [N/A]	3043
Maple [N/A]	3043
Fricas [N/A]	3044
Sympy [N/A]	3044
Maxima [N/A]	3044
Giac [F(-2)]	3045
Mupad [N/A]	3045
Reduce [N/A]	3046

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{x}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)(x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \int \frac{x}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

input `Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

input `Int[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))} dx$$

input `int(x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(-(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(x/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(x/((-c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{5/2}} dx$$

input `int(x/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)),x)`

output `int(x/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.73

$$\int \frac{x}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x}{\sqrt{-c^2 x^2 + 1} a \sin(cx) b c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} a \sin(cx) b c^2 x^2 + \dots}$$

input `int(x/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`output `int(x/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**4*x**4-2*sqrt(-c**2*x**2+1)*asin(c*x)*b+sqrt(-c**2*x**2+1)*a*c**4*x**4-2*sqrt(-c**2*x**2+1)*a*c**2*x**2+sqrt(-c**2*x**2+1)*a),x)`

$$3.335 \quad \int \frac{1}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

Optimal result	3047
Mathematica [N/A]	3047
Rubi [N/A]	3048
Maple [N/A]	3048
Fricas [N/A]	3049
Sympy [N/A]	3049
Maxima [N/A]	3049
Giac [N/A]	3050
Mupad [N/A]	3050
Reduce [N/A]	3051

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \text{Int} \left(\frac{1}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))}, x \right)$$

output `Defer(Int)(1/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \int \frac{1}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

input `Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

↓ 5174

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

input `Int[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))} dx$$

input `int(1/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(1/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(1/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(1/((-c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{5/2}} dx$$

input `int(1/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)),x)`

output `int(1/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.84

$$\int \frac{1}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^2 + \sqrt{-c^2 x^2 + 1} a} dx$$

input `int(1/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`output `int(1/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**4*x**4-2*sqrt(-c**2*x**2+1)*asin(c*x)*b+sqrt(-c**2*x**2+1)*a*c**4*x**4-2*sqrt(-c**2*x**2+1)*a*c**2*x**2+sqrt(-c**2*x**2+1)*a),x)`

$$3.336 \quad \int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$$

Optimal result	3052
Mathematica [N/A]	3052
Rubi [N/A]	3053
Maple [N/A]	3053
Fricas [N/A]	3054
Sympy [N/A]	3054
Maxima [N/A]	3054
Giac [F(-2)]	3055
Mupad [N/A]	3055
Reduce [N/A]	3056

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \text{Int}\left(\frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}, x\right)$$

output `Defer(Int)(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 5.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$$

input `Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$$

↓ 5234

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$$

input `Int[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 3.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(-c^2x^2+1)^{5/2}(a+b\arcsin(cx))} dx$$

input `int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arcsin(cx)+a)x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2+1)/(a*c^6*x^7-3*a*c^4*x^5+3*a*c^2*x^3-a*x+(b*c^6*x^7-3*b*c^4*x^5+3*b*c^2*x^3-b*x)*arcsin(c*x)),x)`

Sympy [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\arcsin(cx))} dx$$

input `integrate(1/x/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(1/(x*(-(c*x-1)*(c*x+1))**(5/2)*(a+b*asin(c*x))),x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arcsin(cx)+a)x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \text{Exception raised: TypeError}$$

input `integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x(a+b\arcsin(cx))(1-c^2x^2)^{5/2}} dx$$

input `int(1/(x*(a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)),x)`

output `int(1/(x*(a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.39

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2x^2+1} \arcsin(cx) b c^4x^5 - 2\sqrt{-c^2x^2+1} \arcsin(cx) b c^2x^3 + \dots}$$

input `int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`

output `int(1/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**4*x**5-2*sqrt(-c**2*x**2+1)*asin(c*x)*b*c**2*x**3+sqrt(-c**2*x**2+1)*a*c**4*x**5-2*sqrt(-c**2*x**2+1)*a*c**2*x**3+sqrt(-c**2*x**2+1)*a*x),x)`

$$3.337 \quad \int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$$

Optimal result	3057
Mathematica [N/A]	3057
Rubi [N/A]	3058
Maple [N/A]	3058
Fricas [N/A]	3059
Sympy [N/A]	3059
Maxima [N/A]	3060
Giac [N/A]	3060
Mupad [N/A]	3060
Reduce [N/A]	3061

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \text{Int}\left(\frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))}, x\right)$$

output `Defer(Int)(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 9.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx = \int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arcsin(cx))} dx$$

input `Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

input `Int[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))} dx$$

input `int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)/(a*c^6*x^8 - 3*a*c^4*x^6 + 3*a*c^2*x^4 - a*x^2 + (b*c^6*x^8 - 3*b*c^4*x^6 + 3*b*c^2*x^4 - b*x^2)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 3.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{x^2 (-(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(1/x**2/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)*x^2), x)`

Giac [N/A]

Not integrable

Time = 7.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a) x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{x^2 (a + b \arcsin(cx)) (1 - c^2 x^2)^{5/2}} dx$$

input `int(1/(x^2*(a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)),x)`

output `int(1/(x^2*(a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^4 x^6 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^4 - \dots}$$

input `int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`

output `int(1/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b*c**4*x**6 - 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*b*x**2 + sqrt(-c**2*x**2 + 1)*a*c**4*x**6 - 2*sqrt(-c**2*x**2 + 1)*a*c**2*x**4 + sqrt(-c**2*x**2 + 1)*a*x**2),x)`

$$3.338 \quad \int \frac{x^m (1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

Optimal result	3062
Mathematica [N/A]	3062
Rubi [N/A]	3063
Maple [N/A]	3063
Fricas [N/A]	3064
Sympy [F(-1)]	3064
Maxima [N/A]	3064
Giac [F(-2)]	3065
Mupad [N/A]	3065
Reduce [N/A]	3066

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m (1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Int} \left(\frac{x^m (1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)}, x \right)$$

output `Defer(Int)(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m (1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x^m (1 - c^2 x^2)^{5/2}}{a + b \arcsin(cx)} dx$$

input `Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]`

output `Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]), x]`

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2} x^m}{a + b \arcsin(cx)} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{5/2} x^m}{a + b \arcsin(cx)} dx$$

input `Int[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x]),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m (-c^2 x^2 + 1)^{5/2}}{a + b \arcsin(cx)} dx$$

input `int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}} x^m}{b \arcsin(cx) + a} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arcsin(c*x) + a), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Timed out}$$

input `integrate(x**m*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}} x^m}{b \arcsin(cx) + a} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(5/2)*x^m/(b*arcsin(c*x) + a), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{a + b \arcsin(cx)} dx = \int \frac{x^m(1 - c^2x^2)^{5/2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)),x)`

output `int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x)), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.46

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arcsin(cx)} dx = \left(\int \frac{x^m\sqrt{-c^2x^2+1}x^4}{\arcsin(cx)b+a} dx \right) c^4 - 2 \left(\int \frac{x^m\sqrt{-c^2x^2+1}x^2}{\arcsin(cx)b+a} dx \right) c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{\arcsin(cx)b+a} dx$$

input `int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`

output `int((x**m*sqrt(-c**2*x**2+1)*x**4)/(asin(c*x)*b+a),x)*c**4 - 2*int((x**m*sqrt(-c**2*x**2+1)*x**2)/(asin(c*x)*b+a),x)*c**2 + int((x**m*sqrt(-c**2*x**2+1))/(asin(c*x)*b+a),x)`

$$3.339 \quad \int \frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx$$

Optimal result	3067
Mathematica [N/A]	3067
Rubi [N/A]	3068
Maple [N/A]	3068
Fricas [N/A]	3069
Sympy [N/A]	3069
Maxima [N/A]	3069
Giac [F(-2)]	3070
Mupad [N/A]	3070
Reduce [N/A]	3071

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \text{Int} \left(\frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)}, x \right)$$

output `Defer(Int)(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \arcsin(cx)} dx$$

input `Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]`

output `Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]), x]`

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2} x^m}{a + b \arcsin(cx)} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{3/2} x^m}{a + b \arcsin(cx)} dx$$

input `Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x]),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m (-c^2 x^2 + 1)^{3/2}}{a + b \arcsin(cx)} dx$$

input `int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

output `int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{b\arcsin(cx)+a} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arcsin(c*x) + a), x)`

Sympy [N/A]

Not integrable

Time = 13.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = \int \frac{x^m(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\arcsin(cx)} dx$$

input `integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x)), x)`

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{b\arcsin(cx)+a} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arcsin(c*x) + a), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1 - c^2x^2)^{3/2}}{a + b \arcsin(cx)} dx = \int \frac{x^m(1 - c^2x^2)^{3/2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)),x)`

output `int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arcsin(cx)} dx = -\left(\int \frac{x^m\sqrt{-c^2x^2+1}x^2}{\arcsin(cx)b+a} dx\right) c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{\arcsin(cx)b+a} dx$$

input `int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `- int((x**m*sqrt(-c**2*x**2+1)*x**2)/(asin(c*x)*b+a),x)*c**2 + int((x**m*sqrt(-c**2*x**2+1))/(asin(c*x)*b+a),x)`

3.340 $\int \frac{x^m \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$

Optimal result	3072
Mathematica [N/A]	3072
Rubi [N/A]	3073
Maple [N/A]	3073
Fricas [N/A]	3074
Sympy [N/A]	3074
Maxima [N/A]	3074
Giac [F(-2)]	3075
Mupad [N/A]	3075
Reduce [N/A]	3076

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = \text{Int} \left(\frac{x^m \sqrt{1-c^2x^2}}{a+b \arcsin(cx)}, x \right)$$

output `Defer(Int)(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx = \int \frac{x^m \sqrt{1-c^2x^2}}{a+b \arcsin(cx)} dx$$

input `Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]`

output `Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]), x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1 - c^2 x^2} x^m}{a + b \arcsin(cx)} dx$$

↓ 5234

$$\int \frac{\sqrt{1 - c^2 x^2} x^m}{a + b \arcsin(cx)} dx$$

input `Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x]),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m \sqrt{-c^2 x^2 + 1}}{a + b \arcsin(cx)} dx$$

input `int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

output `int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^m}{b \arcsin(cx) + a} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^m/(b*arcsin(c*x) + a), x)`

Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^m \sqrt{-(cx - 1)(cx + 1)}}{a + b \arcsin(cx)} dx$$

input `integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x)), x)`

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^m}{b \arcsin(cx) + a} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arcsin(c*x) + a), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \operatorname{asin}(cx)} dx$$

input `int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)),x)`

output `int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x)), x)`

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \arcsin(cx)} dx = \int \frac{x^m \sqrt{-c^2 x^2 + 1}}{a \sin(cx) b + a} dx$$

input `int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`output `int((x**m*sqrt(-c**2*x**2+1))/(asin(c*x)*b+a),x)`

3.341 $\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$

Optimal result	3077
Mathematica [N/A]	3077
Rubi [N/A]	3078
Maple [N/A]	3078
Fricas [N/A]	3079
Sympy [N/A]	3079
Maxima [N/A]	3079
Giac [N/A]	3080
Mupad [N/A]	3080
Reduce [N/A]	3081

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}, x\right)$$

output `Defer(Int)(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx = \int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))} dx$$

input `Integrate[x^m/(Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])),x]`

output `Integrate[x^m/(Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])),x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx$$

input `Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}(a+b\arcsin(cx))} dx$$

input `int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

output `int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^m}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))} dx$$

input `integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x)),x)`

output `Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^m}{(a+b\arcsin(cx))\sqrt{1-c^2x^2}} dx$$

input `int(x^m/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^m/((a + b*asin(c*x))*(1 - c^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1} \operatorname{asin}(cx) b + \sqrt{-c^2x^2+1} a} dx$$

input `int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x)),x)`output `int(x**m/(sqrt(-c**2*x**2+1)*asin(c*x)*b+sqrt(-c**2*x**2+1)*a),x)`

$$3.342 \quad \int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

Optimal result	3082
Mathematica [N/A]	3082
Rubi [N/A]	3083
Maple [N/A]	3083
Fricas [N/A]	3084
Sympy [N/A]	3084
Maxima [N/A]	3084
Giac [N/A]	3085
Mupad [N/A]	3085
Reduce [N/A]	3086

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))}, x\right)$$

output `Defer(Int)(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx = \int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))} dx$$

input `Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx$$

input `Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))} dx$$

input `int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

output `int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^m/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arcsin(c*x) + a), x)`

Sympy [N/A]

Not integrable

Time = 9.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x)),x)`

output `Integral(x**m/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{3/2}} dx$$

input `int(x^m/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)),x)`

output `int(x^m/((a + b*asin(c*x))*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))} dx =$$

$$-\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1} a \sin(cx) b c^2 x^2 - \sqrt{-c^2 x^2 + 1} a \sin(cx) b + \sqrt{-c^2 x^2 + 1} a c^2 x^2 - \sqrt{-c^2 x^2 + 1} a} dx \right)$$

input `int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x)),x)`

output `- int(x**m/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**2*x**2 - sqrt(-c**2*x**2+1)*asin(c*x)*b + sqrt(-c**2*x**2+1)*a*c**2*x**2 - sqrt(-c**2*x**2+1)*a),x)`

$$3.343 \quad \int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

Optimal result	3087
Mathematica [N/A]	3087
Rubi [N/A]	3088
Maple [N/A]	3088
Fricas [N/A]	3089
Sympy [N/A]	3089
Maxima [N/A]	3089
Giac [N/A]	3090
Mupad [N/A]	3090
Reduce [N/A]	3091

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))}, x\right)$$

output `Defer(Int)(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx = \int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))} dx$$

input `Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])), x]`

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

↓ 5234

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx$$

input `Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))} dx$$

input `int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

output `int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arcsin(c*x) - a), x)`

Sympy [N/A]

Not integrable

Time = 16.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b \arcsin(cx))} dx$$

input `integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x)),x)`

output `Integral(x**m/((-c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))), x)`

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Giac [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)), x)`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{(a + b \arcsin(cx)) (1 - c^2 x^2)^{5/2}} dx$$

input `int(x^m/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)),x)`

output `int(x^m/((a + b*asin(c*x))*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.46

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))} dx = \int \frac{x^m}{\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) b c^2 x^2 + \sqrt{-c^2 x^2 + 1} a} dx$$

input `int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x)),x)`output `int(x**m/(sqrt(-c**2*x**2+1)*asin(c*x)*b*c**4*x**4-2*sqrt(-c**2*x**2+1)*asin(c*x)*b+sqrt(-c**2*x**2+1)*a*c**4*x**4-2*sqrt(-c**2*x**2+1)*a*c**2*x**2+sqrt(-c**2*x**2+1)*a),x)`

3.344 $\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$

Optimal result	3092
Mathematica [N/A]	3092
Rubi [N/A]	3093
Maple [N/A]	3093
Fricas [N/A]	3094
Sympy [N/A]	3094
Maxima [N/A]	3094
Giac [N/A]	3095
Mupad [N/A]	3095
Reduce [N/A]	3096

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \text{Int}\left(\frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)}, x\right)$$

output `Defer(Int)(x^m/(-a^2*x^2+1)^(1/2)/arcsin(a*x), x)`

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$$

input `Integrate[x^m/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]), x]`

output `Integrate[x^m/(Sqrt[1 - a^2*x^2]*ArcSin[a*x]), x]`

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$$

↓ 5234

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx$$

input `Int [x^m/(Sqrt [1 - a^2*x^2]*ArcSin [a*x]), x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `int (x^m/(-a^2*x^2+1)^(1/2)/arcsin(a*x) , x)`

output `int (x^m/(-a^2*x^2+1)^(1/2)/arcsin(a*x) , x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^m}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^m/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2+1)*x^m/((a^2*x^2-1)*arcsin(a*x)),x)`

Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^m}{\sqrt{-(ax-1)(ax+1)} \operatorname{asin}(ax)} dx$$

input `integrate(x**m/(-a**2*x**2+1)**(1/2)/asin(a*x),x)`

output `Integral(x**m/(sqrt(-(a*x-1)*(a*x+1))*asin(a*x)),x)`

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-a^2x^2} \arcsin(ax)} dx = \int \frac{x^m}{\sqrt{-a^2x^2+1} \arcsin(ax)} dx$$

input `integrate(x^m/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="maxima")`

output `integrate(x^m/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{x^m}{\sqrt{-a^2 x^2 + 1} \arcsin(ax)} dx$$

input `integrate(x^m/(-a^2*x^2+1)^(1/2)/arcsin(a*x),x, algorithm="giac")`

output `integrate(x^m/(sqrt(-a^2*x^2 + 1)*arcsin(a*x)), x)`

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{x^m}{\arcsin(ax) \sqrt{1 - a^2 x^2}} dx$$

input `int(x^m/(asin(a*x)*(1 - a^2*x^2)^(1/2)),x)`

output `int(x^m/(asin(a*x)*(1 - a^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{\sqrt{1 - a^2 x^2} \arcsin(ax)} dx = \int \frac{x^m}{\sqrt{-a^2 x^2 + 1} \operatorname{asin}(ax)} dx$$

input `int(x^m/(-a^2*x^2+1)^(1/2)/asin(a*x),x)`output `int(x**m/(sqrt(- a**2*x**2 + 1)*asin(a*x)),x)`

3.345
$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$$

Optimal result	3097
Mathematica [N/A]	3097
Rubi [N/A]	3098
Maple [N/A]	3098
Fricas [N/A]	3099
Sympy [F(-1)]	3099
Maxima [N/A]	3099
Giac [F(-2)]	3100
Mupad [N/A]	3100
Reduce [N/A]	3101

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{x^m(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2}, x\right)$$

output

```
Defer(Int)(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx = \int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$$

input

```
Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2} x^m}{(a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{5/2} x^m}{(a + b \arcsin(cx))^2} dx$$

input `Int[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m (-c^2 x^2 + 1)^{5/2}}{(a + b \arcsin(cx))^2} dx$$

input `int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}x^m}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*x^m/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \text{Timed out}$$

input `integrate(x**m*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 186, normalized size of antiderivative = 6.64

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}x^m}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*x^m - (b^2*c*arctan2(c*x, sqrt(c*x
+ 1)*sqrt(-c*x + 1)) + a*b*c)*integrate(((c^6*m + 6*c^6)*x^6 - 3*(c^4*m +
4*c^4)*x^4 + 3*(c^2*m + 2*c^2)*x^2 - m)*x^m/(b^2*c*x*arctan2(c*x, sqrt(c*x
+ 1)*sqrt(-c*x + 1)) + a*b*c*x), x))/(b^2*c*arctan2(c*x, sqrt(c*x + 1)*sq
rt(-c*x + 1)) + a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^m(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx$$

input

```
int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2,x)
```

output

```
int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.96

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx = \left(\int \frac{x^m\sqrt{-c^2x^2+1}x^4}{\operatorname{asin}(cx)^2b^2+2\operatorname{asin}(cx)ab+a^2} dx \right) c^4$$

$$- 2 \left(\int \frac{x^m\sqrt{-c^2x^2+1}x^2}{\operatorname{asin}(cx)^2b^2+2\operatorname{asin}(cx)ab+a^2} dx \right) c^2$$

$$+ \int \frac{x^m\sqrt{-c^2x^2+1}}{\operatorname{asin}(cx)^2b^2+2\operatorname{asin}(cx)ab+a^2} dx$$

input `int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x))^2,x)`

output `int((x**m*sqrt(-c**2*x**2+1)*x**4)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)*c**4-2*int((x**m*sqrt(-c**2*x**2+1)*x**2)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)*c**2+int((x**m*sqrt(-c**2*x**2+1))/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)`

3.346 $\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3102
Mathematica [N/A]	3102
Rubi [N/A]	3103
Maple [N/A]	3103
Fricas [N/A]	3104
Sympy [N/A]	3104
Maxima [N/A]	3105
Giac [F(-2)]	3105
Mupad [N/A]	3106
Reduce [N/A]	3106

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{x^m(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2}, x\right)$$

output

```
Defer(Int)(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx = \int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$$

input

```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2} x^m}{(a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{3/2} x^m}{(a + b \arcsin(cx))^2} dx$$

input `Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m (-c^2 x^2 + 1)^{3/2}}{(a + b \arcsin(cx))^2} dx$$

input `int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{x^m(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^m}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [N/A]

Not integrable

Time = 31.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^m(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x))**2, x)`

Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.75

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-((c^4*x^4 - 2*c^2*x^2 + 1)*x^m - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(((c^4*m + 4*c^4)*x^4 - 2*(c^2*m + 2*c^2)*x^2 + m)*x^m/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x, x))/((b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^m(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx$$

input `int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2,x)`

output `int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2, x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \frac{x^m(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = - \left(\int \frac{x^m \sqrt{-c^2 x^2 + 1} x^2}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^2$$

$$+ \int \frac{x^m \sqrt{-c^2 x^2 + 1}}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx$$

input `int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int((x**m*sqrt(- c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((x**m*sqrt(- c**2*x**2 + 1))/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

$$3.347 \quad \int \frac{x^m \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$$

Optimal result	3107
Mathematica [N/A]	3107
Rubi [N/A]	3108
Maple [N/A]	3108
Fricas [N/A]	3109
Sympy [N/A]	3109
Maxima [N/A]	3110
Giac [F(-2)]	3110
Mupad [N/A]	3111
Reduce [N/A]	3111

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = \text{Int} \left(\frac{x^m \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2}, x \right)$$

output `Defer(Int)(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = \int \frac{x^m \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$$

input `Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]`

output `Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2, x]`

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1 - c^2 x^2} x^m}{(a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{\sqrt{1 - c^2 x^2} x^m}{(a + b \arcsin(cx))^2} dx$$

input `Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m \sqrt{-c^2 x^2 + 1}}{(a + b \arcsin(cx))^2} dx$$

input `int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^m}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^m/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^m \sqrt{-(cx - 1)(cx + 1)}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x))**2, x)`

Maxima [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.93

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^m}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `((c^2*x^2 - 1)*x^m - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(((c^2*m + 2*c^2)*x^2 - m)*x^m/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x), x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \sin(cx))^2} dx$$

input `int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2,x)`output `int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2, x)`**Reduce [N/A]**

Not integrable

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^m \sqrt{-c^2 x^2 + 1}}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx$$

input `int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`output `int((x**m*sqrt(- c**2*x**2 + 1))/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.348 $\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3112
Mathematica [N/A]	3112
Rubi [N/A]	3113
Maple [N/A]	3113
Fricas [N/A]	3114
Sympy [N/A]	3114
Maxima [N/A]	3115
Giac [N/A]	3115
Mupad [N/A]	3116
Reduce [N/A]	3116

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = \int \frac{x^m}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$$

input `Integrate[x^m/(Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])^2),x]`

output `Integrate[x^m/(Sqrt[1-c^2*x^2]*(a+b*ArcSin[c*x])^2),x]`

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$\downarrow 5222$$

$$\frac{m \int \frac{x^{m-1}}{a+b\arcsin(cx)} dx}{bc} - \frac{x^m}{bc(a+b\arcsin(cx))}$$

$$\downarrow 5148$$

$$\frac{m \int \frac{x^{m-1}}{a+b\arcsin(cx)} dx}{bc} - \frac{x^m}{bc(a+b\arcsin(cx))}$$

input `Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}(a+b\arcsin(cx))^2} dx$$

input `int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^m}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input `integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `((b^2*c*m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*m)*integrate(x^m/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x), x) - x^m)/(b^2*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c)`

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^m}{(a+b\arcsin(cx))^2 \sqrt{1-c^2x^2}} dx$$

input `int(x^m/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^m/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$= \frac{a\sin(cx) \left(\int \frac{x^m}{a\sin(cx)bx+ax} dx \right) bm - x^m + \left(\int \frac{x^m}{a\sin(cx)bx+ax} dx \right) am}{bc(a\sin(cx)b+a)}$$

input `int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `(asin(c*x)*int(x**m/(asin(c*x)*b*x + a*x),x)*b*m - x**m + int(x**m/(asin(c*x)*b*x + a*x),x)*a*m)/(b*c*(asin(c*x)*b + a))`

$$3.349 \quad \int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$$

Optimal result	3117
Mathematica [N/A]	3117
Rubi [N/A]	3118
Maple [N/A]	3118
Fricas [N/A]	3119
Sympy [N/A]	3119
Maxima [N/A]	3120
Giac [N/A]	3120
Mupad [N/A]	3121
Reduce [N/A]	3121

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx = \int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$$

input `Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arcsin(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 73.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**m/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 7.79

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-((a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate(((c^2*m - 2*c^2)*x^2 - m)*x^m/(a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x) - x^m)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)^2), x)`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(a + b \arcsin(cx))^2 (1 - c^2 x^2)^{3/2}} dx$$

input `int(x^m/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`

output `int(x^m/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.04

$$\int \frac{x^m}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx =$$

$$-\left(\int \frac{x^m}{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab} dx \right)$$

input `int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int(x**m/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**2 - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a**2),x)`

$$3.350 \quad \int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))^2} dx$$

Optimal result	3122
Mathematica [N/A]	3122
Rubi [N/A]	3123
Maple [N/A]	3123
Fricas [N/A]	3124
Sympy [N/A]	3124
Maxima [N/A]	3125
Giac [N/A]	3125
Mupad [N/A]	3126
Reduce [N/A]	3126

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))^2} dx = \int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b \arcsin(cx))^2} dx$$

input `Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx$$

input `Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))^2} dx$$

input `int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.14

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 74.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**m/((-c*x - 1)*(c*x + 1))**(5/2)*(a + b*asin(c*x))**2, x)`

Maxima [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 9.93

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `((a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate(((c^2*m - 4*c^2)*x^2 - m)*x^m/(a*b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x + (b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 - b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x) - x^m)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arcsin(c*x) + a)^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{(a + b \arcsin(cx))^2 (1 - c^2 x^2)^{5/2}} dx$$

input `int(x^m/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(5/2)),x)`

output `int(x^m/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(5/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 7.57

$$\int \frac{x^m}{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^m}{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^4 x^4 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^2 x^2}$$

input `int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x))^2,x)`

output `int(x**m/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2*c**4*x**4-2*sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2*c**2*x**2+sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b*c**4*x**4-4*sqrt(-c**2*x**2+1)*asin(c*x)*a*b*c**2*x**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b+sqrt(-c**2*x**2+1)*a**2*c**4*x**4-2*sqrt(-c**2*x**2+1)*a**2*c**2*x**2+sqrt(-c**2*x**2+1)*a**2),x)`

3.351 $\int \frac{x^3 \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3127
Mathematica [A] (verified)	3128
Rubi [A] (verified)	3128
Maple [A] (verified)	3130
Fricas [F]	3131
Sympy [F]	3131
Maxima [F]	3132
Giac [F(-2)]	3132
Mupad [F(-1)]	3132
Reduce [F]	3133

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{x^3 \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = -\frac{x^3(1-c^2x^2)}{bc(a+b \arcsin(cx))} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8b^2c^4}$$

$$+ \frac{3 \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16b^2c^4}$$

$$- \frac{5 \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16b^2c^4}$$

$$+ \frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8b^2c^4} + \frac{3 \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16b^2c^4}$$

$$- \frac{5 \sin\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16b^2c^4}$$

output

```
-x^3*(-c^2*x^2+1)/b/c/(a+b*arcsin(c*x))+1/8*cos(a/b)*Ci((a+b*arcsin(c*x))/
b)/b^2/c^4+3/16*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^4-5/16*cos(5*a/
b)*Ci(5*(a+b*arcsin(c*x))/b)/b^2/c^4+1/8*sin(a/b)*Si((a+b*arcsin(c*x))/b)/
b^2/c^4+3/16*sin(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^4-5/16*sin(5*a/b)*
Si(5*(a+b*arcsin(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx$$

$$= -\frac{16bc^3 x^3}{a + b \arcsin(cx)} + \frac{16bc^5 x^5}{a + b \arcsin(cx)} + 2 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) + 3 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right)$$

input

```
Integrate[(x^3*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]
```

output

```
((-16*b*c^3*x^3)/(a + b*ArcSin[c*x]) + (16*b*c^5*x^5)/(a + b*ArcSin[c*x])
+ 2*Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] + 3*Cos[(3*a)/b]*CosIntegral[3
*(a/b + ArcSin[c*x])] - 5*Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcSin[c*x])]
+ 2*Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 3*Sin[(3*a)/b]*SinIntegral[3
*(a/b + ArcSin[c*x])] - 5*Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])])
/(16*b^2*c^4)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5214, 5146, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx$$

$$\downarrow \text{5214}$$

$$-\frac{5c \int \frac{x^4}{a + b \arcsin(cx)} dx}{b} + \frac{3 \int \frac{x^2}{a + b \arcsin(cx)} dx}{bc} - \frac{x^3 (1 - c^2 x^2)}{bc(a + b \arcsin(cx))}$$

$$\downarrow \text{5146}$$

$$\frac{5 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^4} + \frac{3 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^4} - \frac{x^3(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

↓ 4906

$$\frac{5 \int \left(\frac{\cos\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} - \frac{3 \cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^4} + \frac{3 \int \left(\frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4(a+b \arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^4} - \frac{x^3(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

↓ 2009

$$\frac{3 \left(\frac{1}{4} \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{4} \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) \right)}{b^2 c^4} - \frac{5 \left(\frac{1}{8} \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{3}{16} \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \text{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \right)}{b^2 c^4} - \frac{x^3(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

input

```
Int[(x^3*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]
```

output

```

-((x^3*(1 - c^2*x^2))/(b*c*(a + b*ArcSin[c*x]))) + (3*((Cos[a/b]*CosIntegral
al[(a + b*ArcSin[c*x])/b])/4 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[
c*x])/b])/4 + (Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/4 - (Sin[(3*a
)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/4))/(b^2*c^4) - (5*((Cos[a/b]
*CosIntegral[(a + b*ArcSin[c*x])/b])/8 - (3*Cos[(3*a)/b]*CosIntegral[(3*(a
+ b*ArcSin[c*x])/b])/16 + (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x
])/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 - (3*Sin[(3*a
)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/16 + (Sin[(5*a)/b]*SinIntegra
l[(5*(a + b*ArcSin[c*x])/b])/16))/(b^2*c^4)

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5146 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^m, x_Symbol] := Simp[1/(b*c^(m + 1)) Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]`

rule 5214 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1, 0] && IGtQ[m, -3]`

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.59

method	result
default	$\frac{3 \arcsin(cx) \operatorname{Si}\left(3 \arcsin(cx) + \frac{3a}{b}\right) \sin\left(\frac{3a}{b}\right) b + 3 \arcsin(cx) \operatorname{Ci}\left(3 \arcsin(cx) + \frac{3a}{b}\right) \cos\left(\frac{3a}{b}\right) b + 2 \arcsin(cx) \operatorname{Si}\left(\arcsin(cx) + \frac{a}{b}\right) \sin\left(\frac{a}{b}\right) b + 2 \arcsin(cx) \operatorname{Ci}\left(\arcsin(cx) + \frac{a}{b}\right) \cos\left(\frac{a}{b}\right) b}{(a + b \arcsin(cx))^2}$

input `int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output

```
1/16/c^4*(3*arcsin(c*x)*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*b+3*arcsin(c*x)
*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b+2*arcsin(c*x)*Si(arcsin(c*x)+a/b)*si
n(a/b)*b+2*arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b-5*arcsin(c*x)*Si(5*a
rcsin(c*x)+5*a/b)*sin(5*a/b)*b-5*arcsin(c*x)*Ci(5*arcsin(c*x)+5*a/b)*cos(5
*a/b)*b+3*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*a+3*Ci(3*arcsin(c*x)+3*a/b)*c
os(3*a/b)*a+2*Si(arcsin(c*x)+a/b)*sin(a/b)*a+2*Ci(arcsin(c*x)+a/b)*cos(a/b
)*a-5*Si(5*arcsin(c*x)+5*a/b)*sin(5*a/b)*a-5*Ci(5*arcsin(c*x)+5*a/b)*cos(5
*a/b)*a-2*x*b*c-sin(3*arcsin(c*x))*b+sin(5*arcsin(c*x))*b)/(a+b*arcsin(c*x
))/b^2
```

Fricas [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^3}{(b \arcsin(cx) + a)^2} dx$$

input

```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas"
)
```

output

```
integral(sqrt(-c^2*x^2 + 1)*x^3/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^3 \sqrt{-(cx - 1)(cx + 1)}}{(a + b \arcsin(cx))^2} dx$$

input

```
integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)
```

output

```
Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x))**2, x)
```


Maxima [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^3}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^5 - x^3 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate((5*c^2*x^4 - 3*x^2)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \operatorname{asin}(cx))^2} dx$$

input `int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2,x)`

output `int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{x^3 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^3}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx$$

input `int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int((sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.352 $\int \frac{x^2 \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3134
Mathematica [A] (verified)	3134
Rubi [B] (verified)	3135
Maple [A] (verified)	3140
Fricas [F]	3140
Sympy [F]	3140
Maxima [F]	3141
Giac [B] (verification not implemented)	3141
Mupad [F(-1)]	3142
Reduce [F]	3142

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{x^2 \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = -\frac{x^2(1-c^2x^2)}{bc(a+b \arcsin(cx))} - \frac{\text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cos\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{2b^2c^3}$$

output

```
-x^2*(-c^2*x^2+1)/b/c/(a+b*arcsin(c*x))-1/2*Ci(4*(a+b*arcsin(c*x))/b)*sin(4*a/b)/b^2/c^3+1/2*cos(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{x^2 \sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = \frac{\frac{2bc^2x^2(-1+c^2x^2)}{a+b \arcsin(cx)} - \text{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{4a}{b}\right) + \cos\left(\frac{4a}{b}\right) \text{Si}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{2b^2c^3}$$

input `Integrate[(x^2*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]`

output `((2*b*c^2*x^2*(-1 + c^2*x^2))/(a + b*ArcSin[c*x]) - CosIntegral[4*(a/b + ArcSin[c*x])]*Sin[(4*a)/b] + Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])])/(2*b^2*c^3)`

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. $2(94) = 188$.

Time = 1.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5214, 5146, 25, 4906, 27, 2009, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & -\frac{4c \int \frac{x^3}{a+b \arcsin(cx)} dx}{b} + \frac{2 \int \frac{x}{a+b \arcsin(cx)} dx}{bc} - \frac{x^2(1 - c^2 x^2)}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{5146} \\
 & -\frac{4 \int -\frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c^3} + \\
 & \frac{2 \int -\frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1 - c^2 x^2)}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{25} \\
 & \frac{4 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c^3} - \\
 & \frac{2 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1 - c^2 x^2)}{bc(a + b \arcsin(cx))}
 \end{aligned}$$

$$\begin{aligned} & \downarrow 4906 \\ & - \frac{2 \int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{2(a+b \arcsin(cx))} d(a+b \arcsin(cx))}{b^2 c^3} + \\ & \frac{4 \int \left(\frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} - \frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1-c^2x^2)}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\begin{aligned} & \downarrow 27 \\ & - \frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} + \\ & \frac{4 \int \left(\frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} - \frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1-c^2x^2)}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\begin{aligned} & \downarrow 2009 \\ & - \frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \\ & \frac{4 \left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \right)}{b^2 c^3} - \\ & \frac{x^2(1-c^2x^2)}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\begin{aligned} & \downarrow 3042 \\ & - \frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \\ & \frac{4 \left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \right)}{b^2 c^3} - \\ & \frac{x^2(1-c^2x^2)}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\downarrow 3784$$

$$\frac{-\sin\left(\frac{2a}{b}\right) \int \frac{\cos\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \cos\left(\frac{2a}{b}\right) \int -\frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3}$$

$$\frac{4\left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^3}$$

$$\frac{x^2(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

↓ 25

$$\frac{\cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{2a}{b}\right) \int \frac{\cos\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3}$$

$$\frac{4\left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^3}$$

$$\frac{x^2(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

↓ 3042

$$\frac{\cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3}$$

$$\frac{4\left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^3}$$

$$\frac{x^2(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

↓ 3780

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \sin\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3}$$

$$\frac{4\left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^3}$$

$$\frac{x^2(1-c^2 x^2)}{bc(a+b \arcsin(cx))}$$

↓ 3783

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b\arcsin(cx))}{b}\right) - \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b\arcsin(cx))}{b}\right)}{b^2 c^3} - \frac{4\left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b\arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b\arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b\arcsin(cx))}{b}\right)\right)}{b^2 c^3} - \frac{x^2(1-c^2x^2)}{bc(a+b\arcsin(cx))}$$

input `Int[(x^2*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]`

output `-((x^2*(1 - c^2*x^2))/(b*c*(a + b*ArcSin[c*x]))) + (-((CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/(b^2*c^3) - (4*(-1/4*(CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b]) + (CosIntegral[(4*(a + b*ArcSin[c*x]))/b]*Sin[(4*a)/b])/8 + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/4 - (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8))/(b^2*c^3)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

rule 3783 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

rule 3784 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[Cos[(d*e - c*f)/d] Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*f)/d] Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5146 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n*(x_)^m, x_Symbol] := Simp[1/(b*c^(m + 1)) Subst[Int[x^n*sin[-a/b + x/b]^m*cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]`

rule 5214 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1, 0] && IGtQ[m, -3]`

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

method	result
default	$\frac{4 \arcsin(cx) \operatorname{Si}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right) b - 4 \arcsin(cx) \sin\left(\frac{4a}{b}\right) \operatorname{Ci}\left(4 \arcsin(cx) + \frac{4a}{b}\right) b + 4 \operatorname{Si}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right) a - 4 \sin\left(\frac{4a}{b}\right) a}{8c^3 (a + b \arcsin(cx))^2}$

input `int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output
$$\frac{1/8/c^3*(4*\arcsin(c*x)*\operatorname{Si}(4*\arcsin(c*x)+4*a/b)*\cos(4*a/b)*b-4*\arcsin(c*x)*\sin(4*a/b)*\operatorname{Ci}(4*\arcsin(c*x)+4*a/b)*b+4*\operatorname{Si}(4*\arcsin(c*x)+4*a/b)*\cos(4*a/b)*a-4*\sin(4*a/b)*\operatorname{Ci}(4*\arcsin(c*x)+4*a/b)*a+\cos(4*\arcsin(c*x))*b-b}{(a+b*\arcsin(c*x))^2}$$

Fricas [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^2}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^2/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^2 \sqrt{-(cx - 1)(cx + 1)}}{(a + b \operatorname{asin}(cx))^2} dx$$

input `integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^2}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^4 - x^2 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(2*(2*c^2*x^3 - x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c, x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(88) = 176.

Time = 0.22 (sec) , antiderivative size = 563, normalized size of antiderivative = 5.99

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```

-4*b*arcsin(c*x)*cos(a/b)^3*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(
b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 4*b*arcsin(c*x)*cos(a/b)^4*sin_integral
(4*a/b + 4*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 4*a*cos(a/b)^3
*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2
*c^3) + 4*a*cos(a/b)^4*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c^3*arcsin
(c*x) + a*b^2*c^3) + 2*b*arcsin(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arcsi
n(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 4*b*arcsin(c*x)*cos(a
/b)^2*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3
) + 2*a*cos(a/b)*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c^3*arc
sin(c*x) + a*b^2*c^3) - 4*a*cos(a/b)^2*sin_integral(4*a/b + 4*arcsin(c*x))
/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + (c^2*x^2 - 1)^2*b/(b^3*c^3*arcsin(c*x
) + a*b^2*c^3) + 1/2*b*arcsin(c*x)*sin_integral(4*a/b + 4*arcsin(c*x))/(b^
3*c^3*arcsin(c*x) + a*b^2*c^3) + (c^2*x^2 - 1)*b/(b^3*c^3*arcsin(c*x) + a*
b^2*c^3) + 1/2*a*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c^3*arcsin(c*x)
+ a*b^2*c^3)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx$$

input

```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2,x)
```

output

```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1} x^2}{a \sin(cx)^2 b^2 + 2 a \sin(cx) a b + a^2} dx$$

input

```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)
```

output

```
int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)
```

3.353 $\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$

Optimal result	3144
Mathematica [A] (verified)	3145
Rubi [A] (verified)	3145
Maple [A] (verified)	3149
Fricas [F]	3150
Sympy [F]	3150
Maxima [F]	3150
Giac [B] (verification not implemented)	3151
Mupad [F(-1)]	3151
Reduce [F]	3152

Optimal result

Integrand size = 26, antiderivative size = 150

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = -\frac{x(1-c^2x^2)}{bc(a+b\arcsin(cx))} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b\arcsin(cx)}{b}\right)}{4b^2c^2}$$

$$+ \frac{3\cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b\arcsin(cx))}{b}\right)}{4b^2c^2}$$

$$+ \frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b\arcsin(cx)}{b}\right)}{4b^2c^2} + \frac{3\sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b\arcsin(cx))}{b}\right)}{4b^2c^2}$$

output

```
-x*(-c^2*x^2+1)/b/c/(a+b*arcsin(c*x))+1/4*cos(a/b)*Ci((a+b*arcsin(c*x))/b)
/b^2/c^2+3/4*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^2+1/4*sin(a/b)*Si(
(a+b*arcsin(c*x))/b)/b^2/c^2+3/4*sin(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/
c^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$$

$$= \frac{-\frac{4bcx}{a+b\arcsin(cx)} + \frac{4bc^3x^3}{a+b\arcsin(cx)} + \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) + 3\cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(3\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{4b^2c^2}$$

input

```
Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]
```

output

```
((-4*b*c*x)/(a + b*ArcSin[c*x]) + (4*b*c^3*x^3)/(a + b*ArcSin[c*x]) + Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] + 3*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcSin[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 3*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])])/(4*b^2*c^2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5214, 5134, 3042, 3784, 25, 3042, 3780, 3783, 5146, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$$

$$\downarrow \text{5214}$$

$$-\frac{3c \int \frac{x^2}{a+b\arcsin(cx)} dx}{b} + \frac{\int \frac{1}{a+b\arcsin(cx)} dx}{bc} - \frac{x(1-c^2x^2)}{bc(a+b\arcsin(cx))}$$

$$\downarrow \text{5134}$$

$$\frac{\int \frac{\cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^2} - \frac{3c \int \frac{x^2}{a+b\arcsin(cx)} dx}{b} - \frac{x(1-c^2x^2)}{bc(a+b\arcsin(cx))}$$

$$\downarrow \text{3042}$$

$$\begin{aligned}
& \frac{\int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{3c \int \frac{x^2}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{3784} \\
& \frac{\cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{a}{b}\right) \int -\frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} \\
& \quad - \frac{3c \int \frac{x^2}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{25} \\
& \frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} \\
& \quad - \frac{3c \int \frac{x^2}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{3042} \\
& \frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} \\
& \quad - \frac{3c \int \frac{x^2}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{3780} \\
& \frac{\cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} \\
& \quad - \frac{3c \int \frac{x^2}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{3783} \\
& -\frac{3c \int \frac{x^2}{a+b \arcsin(cx)} dx}{b} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} \\
& \quad - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{5146}
\end{aligned}$$

$$\begin{aligned}
 & \frac{3 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} + \\
 & \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
 & \quad \downarrow \text{4906} \\
 & \frac{3 \int \left(\frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4(a+b \arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^2} + \\
 & \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} - \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} - \\
 & \frac{3\left(\frac{1}{4} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{4} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) - \right)}{b^2 c^2} - \\
 & \frac{x(1-c^2 x^2)}{bc(a+b \arcsin(cx))}
 \end{aligned}$$

input

```
Int[(x*sqrt[1 - c^2*x^2])/(a + b*ArcSin[c*x])^2,x]
```

output

```

-((x*(1 - c^2*x^2))/(b*c*(a + b*ArcSin[c*x]))) + (Cos[a/b]*CosIntegral[(a
+ b*ArcSin[c*x])/b] + Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/(b^2*c^
2) - (3*((Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/4 - (Cos[(3*a)/b]*C
osIntegral[(3*(a + b*ArcSin[c*x]))/b])/4 + (Sin[a/b]*SinIntegral[(a + b*Ar
cSin[c*x])/b])/4 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x]))/b])/4
))/(b^2*c^2)

```


Definitions of rubi rules used

- rule 25 $\text{Int}[-(F_x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \text{ Int}[F_x, x], x]$
- rule 2009 $\text{Int}[u, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$
- rule 3042 $\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ /; FunctionOfTrigOfLinearQ}[u, x]$
- rule 3780 $\text{Int}[\sin[(e.) + (f.)(x)]/((c.) + (d.)(x)), x_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] \text{ /; FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{EqQ}[d*e - c*f, 0]$
- rule 3783 $\text{Int}[\sin[(e.) + (f.)(x)]/((c.) + (d.)(x)), x_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] \text{ /; FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$
- rule 3784 $\text{Int}[\sin[(e.) + (f.)(x)]/((c.) + (d.)(x)), x_Symbol] \rightarrow \text{Simp}[\text{Cos}[(d*e - c*f)/d] \text{ Int}[\text{Sin}[c*(f/d) + f*x]/(c + d*x), x], x] + \text{Simp}[\text{Sin}[(d*e - c*f)/d] \text{ Int}[\text{Cos}[c*(f/d) + f*x]/(c + d*x), x], x] \text{ /; FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{NeQ}[d*e - c*f, 0]$
- rule 4906 $\text{Int}[\text{Cos}[(a.) + (b.)(x)]^{(p.)} * ((c.) + (d.)(x))^{(m.)} * \text{Sin}[(a.) + (b.)(x)]^{(n.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^{n*} \text{Cos}[a + b*x]^p, x], x] \text{ /; FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 5134 $\text{Int}[(a.) + \text{ArcSin}[c.)(x)]*(b.)^{(n.)}, x_Symbol] \rightarrow \text{Simp}[1/(b*c) \text{ Subst}[\text{Int}[x^n * \text{Cos}[-a/b + x/b], x], x, a + b * \text{ArcSin}[c*x]], x] \text{ /; FreeQ}\{a, b, c, n\}, x]$

rule 5146

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*(x_)^m_., x_Symbol] := Simp[1
/(b*c^(m + 1)) Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b], x], x, a
+ b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*((f_.)*(x_)^m_)*((d_) + (e_.
)*(x_)^2)^p_., x_Symbol] := Simp[(f*x)^m*sqrt[1 - c^2*x^2]*(d + e*x^2)^p*
((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p
- 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*(m + 2*p + 1)/(b*f*(n
+ 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2
)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.49

method	result
default	$\frac{3 \arcsin(cx) \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b})b + 3 \arcsin(cx) \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b})b + \arcsin(cx) \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})b + \arcsin(cx) \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b})b}{(a + b \arcsin(cx))^2}$

input

```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/4/c^2*(3*arcsin(c*x)*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*b+3*arcsin(c*x)*
Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b+arcsin(c*x)*Si(arcsin(c*x)+a/b)*sin(a
/b)*b+arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+3*Si(3*arcsin(c*x)+3*a/b)
*sin(3*a/b)*a+3*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*a+Si(arcsin(c*x)+a/b)*s
in(a/b)*a+Ci(arcsin(c*x)+a/b)*cos(a/b)*a-x*b*c-sin(3*arcsin(c*x))*b)/(a+b*
arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}x}{(b\arcsin(cx)+a)^2} dx$$

input `integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{x\sqrt{-(cx-1)(cx+1)}}{(a+b\arcsin(cx))^2} dx$$

input `integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}x}{(b\arcsin(cx)+a)^2} dx$$

input `integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^3 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate((3*c^2*x^2 - 1)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c, x) - x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(140) = 280$.

Time = 0.22 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.05

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```

3*b*arcsin(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 3*b*arcsin(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 3*a*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 3*a*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + (c^2*x^2 - 1)*b*c*x/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 9/4*b*arcsin(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 1/4*b*arcsin(c*x)*cos(a/b)*cos_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 3/4*b*arcsin(c*x)*sin(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 1/4*b*arcsin(c*x)*sin(a/b)*sin_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 9/4*a*cos(a/b)*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 1/4*a*cos(a/b)*cos_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 3/4*a*sin(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 1/4*a*sin(a/b)*sin_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2)

```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx$$

input `int((x*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2,x)`

output `int((x*(1 - c^2*x^2)^(1/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}x}{\operatorname{asin}(cx)^2 b^2 + 2\operatorname{asin}(cx)ab + a^2} dx$$

input `int(x*(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int((sqrt(-c**2*x**2+1)*x)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)`

3.354 $\int \frac{\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3153
Mathematica [A] (verified)	3153
Rubi [A] (verified)	3154
Maple [A] (verified)	3157
Fricas [F]	3158
Sympy [F]	3158
Maxima [F]	3158
Giac [B] (verification not implemented)	3159
Mupad [F(-1)]	3160
Reduce [F]	3160

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = -\frac{1-c^2x^2}{bc(a+b \arcsin(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{b^2c} - \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{b^2c}$$

output

$$-(-c^2x^2+1)/b/c/(a+b*\arcsin(c*x))+Ci(2*(a+b*\arcsin(c*x))/b)*\sin(2*a/b)/b^2/c-\cos(2*a/b)*Si(2*(a+b*\arcsin(c*x))/b)/b^2/c$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b \arcsin(cx))^2} dx = \frac{\frac{b(-1+c^2x^2)}{a+b \arcsin(cx)} + \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{2a}{b}\right) - \cos\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{b^2c}$$

input `Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcSin[c*x])^2,x]`

output `((b*(-1 + c^2*x^2))/(a + b*ArcSin[c*x]) + CosIntegral[2*(a/b + ArcSin[c*x])]*Sin[(2*a)/b] - Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])])/(b^2*c)`

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5166, 5146, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{1 - c^2 x^2}}{(a + b \arcsin(cx))^2} dx \\
 & \quad \downarrow 5166 \\
 & -\frac{2c \int \frac{x}{a + b \arcsin(cx)} dx}{b} - \frac{1 - c^2 x^2}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow 5146 \\
 & -\frac{2 \int -\frac{\cos\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right)}{a + b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c} - \frac{1 - c^2 x^2}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow 25 \\
 & \frac{2 \int \frac{\cos\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right)}{a + b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c} - \frac{1 - c^2 x^2}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow 4906 \\
 & \frac{2 \int \frac{\sin\left(\frac{2a}{b} - \frac{2(a + b \arcsin(cx))}{b}\right)}{2(a + b \arcsin(cx))} d(a + b \arcsin(cx))}{b^2 c} - \frac{1 - c^2 x^2}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow 27
 \end{aligned}$$

$$\frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c} - \frac{1-c^2 x^2}{bc(a+b \arcsin(cx))}$$

↓ 3042

$$\frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c} - \frac{1-c^2 x^2}{bc(a+b \arcsin(cx))}$$

↓ 3784

$$\frac{-\sin\left(\frac{2a}{b}\right) \int \frac{\cos\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{1-c^2 x^2} \frac{b^2 c}{bc(a+b \arcsin(cx))}$$

↓ 25

$$\frac{\cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{2a}{b}\right) \int \frac{\cos\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{1-c^2 x^2} \frac{b^2 c}{bc(a+b \arcsin(cx))}$$

↓ 3042

$$\frac{\cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx)) + \pi}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{1-c^2 x^2} \frac{b^2 c}{bc(a+b \arcsin(cx))}$$

↓ 3780

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \sin\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx)) + \pi}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{1-c^2 x^2} \frac{b^2 c}{bc(a+b \arcsin(cx))}$$

↓ 3783

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{b^2 c} - \frac{1-c^2 x^2}{bc(a+b \arcsin(cx))}$$

input `Int[Sqrt[1 - c^2*x^2]/(a + b*ArcSin[c*x])^2,x]`

output `-((1 - c^2*x^2)/(b*c*(a + b*ArcSin[c*x]))) - (-(CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x])/b]))/(b^2*c)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

rule 3783 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

rule 3784 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[Cos[(d*e - c*f)/d] Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*f)/d] Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]`

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5146

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[1/(b*c^(m + 1)) Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5166

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

method	result
default	$-\frac{2 \arcsin(cx) \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) b - 2 \arcsin(cx) \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) b + 2 \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) a - 2 \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) a}{2c(a + b \arcsin(cx))b^2}$

input

```
int((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2/c*(2*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*b-2*arcsin(c*x)*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*b+2*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a-2*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*a+cos(2*arcsin(c*x))*b+b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{(a+b\arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^2 - 2*(b^2*c^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c^2)*integrate(x/(b^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b), x) - 1)/(b^2*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(84) = 168$.

Time = 0.21 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.37

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \frac{2b\arcsin(cx)\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\arcsin(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c\arcsin(cx)+ab^2c} - \frac{2b\arcsin(cx)\cos\left(\frac{a}{b}\right)^2\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c\arcsin(cx)+ab^2c} + \frac{2a\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\arcsin(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c\arcsin(cx)+ab^2c} - \frac{2a\cos\left(\frac{a}{b}\right)^2\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c\arcsin(cx)+ab^2c} + \frac{b\arcsin(cx)\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c\arcsin(cx)+ab^2c} + \frac{(c^2x^2-1)b}{b^3c\arcsin(cx)+ab^2c} + \frac{a\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c\arcsin(cx)+ab^2c}$$

input

```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
2*b*arcsin(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) - 2*b*arcsin(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) + 2*a*cos(a/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) - 2*a*cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) + b*arcsin(c*x)*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) + (c^2*x^2 - 1)*b/(b^3*c*arcsin(c*x) + a*b^2*c) + a*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{1-c^2x^2}}{(a+b\sin(cx))^2} dx$$

input `int((1 - c^2*x^2)^(1/2)/(a + b*asin(c*x))^2,x)`

output `int((1 - c^2*x^2)^(1/2)/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{\sin^2(cx)b^2+2\sin(cx)ab+a^2} dx$$

input `int((-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2+1)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)`

$$3.355 \quad \int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx$$

Optimal result	3161
Mathematica [N/A]	3161
Rubi [N/A]	3162
Maple [N/A]	3164
Fricas [N/A]	3164
Sympy [N/A]	3164
Maxima [N/A]	3165
Giac [F(-2)]	3165
Mupad [N/A]	3166
Reduce [N/A]	3166

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2}, x\right)$$

output `Defer(Int)((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 8.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx$$

input `Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & -\frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{c \int \frac{1}{a+b\arcsin(cx)} dx}{b} - \frac{1-c^2x^2}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{5134} \\
 & -\frac{\int \frac{\cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2} - \frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{1-c^2x^2}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{\int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2} - \frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{1-c^2x^2}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3784} \\
 & \frac{\cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) - \sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2} \\
 & \quad - \frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{1-c^2x^2}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{25} \\
 & \frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx)) + \cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2} \\
 & \quad - \frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{1-c^2x^2}{bcx(a+b\arcsin(cx))}
 \end{aligned}$$

↓ 3042

$$\frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{\int \frac{1}{x^2(a+b \arcsin(cx))} dx - \frac{b^2}{bc} - \frac{1-c^2x^2}{bcx(a+b \arcsin(cx))}}$$

↓ 3780

$$\frac{\cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{\int \frac{1}{x^2(a+b \arcsin(cx))} dx - \frac{b^2}{bc} - \frac{1-c^2x^2}{bcx(a+b \arcsin(cx))}}$$

↓ 3783

$$\frac{\int \frac{1}{x^2(a+b \arcsin(cx))} dx}{bc} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{\frac{1-c^2x^2}{bcx(a+b \arcsin(cx))} - \frac{b^2}{bc}}$$

↓ 5148

$$\frac{\int \frac{1}{x^2(a+b \arcsin(cx))} dx}{bc} - \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{\frac{1-c^2x^2}{bcx(a+b \arcsin(cx))} - \frac{b^2}{bc}}$$

input `Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2 + 1}}{x(a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x))^2,x)`output `int((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x))^2,x)`**Fricas [N/A]**

Not integrable

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{1 - c^2x^2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2 + 1}}{(b \arcsin(cx) + a)^2 x} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`output `integral(sqrt(-c^2*x^2 + 1)/(b^2*x*arcsin(c*x)^2 + 2*a*b*x*arcsin(c*x) + a^2*x), x)`**Sympy [N/A]**

Not integrable

Time = 1.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1 - c^2x^2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-(cx - 1)(cx + 1)}}{x(a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/x/(a+b*asin(c*x))**2,x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.57

$$\int \frac{\sqrt{1 - c^2 x^2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{(b \arcsin(cx) + a)^2 x} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^2 - (b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x)*
integrate((c^2*x^2 + 1)/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x +
1)) + a*b*c*x^2), x) - 1)/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x +
1)) + a*b*c*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x(a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{1-c^2x^2}}{x(a+b\sin(cx))^2} dx$$

input `int((1 - c^2*x^2)^(1/2)/(x*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(1/2)/(x*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{\sin(cx)^2 b^2x + 2\sin(cx) abx + a^2x} dx$$

input `int((-c^2*x^2+1)^(1/2)/x/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2*x + 2*asin(c*x)*a*b*x + a**2*x),x)`

3.356 $\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))^2} dx$

Optimal result	3167
Mathematica [N/A]	3167
Rubi [N/A]	3168
Maple [N/A]	3168
Fricas [N/A]	3169
Sympy [N/A]	3169
Maxima [N/A]	3170
Giac [N/A]	3170
Mupad [N/A]	3171
Reduce [N/A]	3171

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))^2} dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b \arcsin(cx))^2} dx$$

input `Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcSin[c*x])^2),x]`

output `Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^2 (a + b \arcsin(cx))^2} dx$$

$$\downarrow 5212$$

$$-\frac{2 \int \frac{1}{x^3 (a + b \arcsin(cx))} dx}{bc} - \frac{1 - c^2 x^2}{bcx^2 (a + b \arcsin(cx))}$$

$$\downarrow 5148$$

$$-\frac{2 \int \frac{1}{x^3 (a + b \arcsin(cx))} dx}{bc} - \frac{1 - c^2 x^2}{bcx^2 (a + b \arcsin(cx))}$$

input `Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 x^2 + 1}}{x^2 (a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2x^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2), x)`

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2(a+b\arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*asin(c*x))**2,x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2x^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^2 - 2*(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^2)*integrate(1/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^3), x) - 1)/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^2)`

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2x^2} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(sqrt(-c^2*x^2 + 1)/((b*arcsin(c*x) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.11

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arcsin(cx))^2} dx$$

$$= \frac{\operatorname{asin}(cx) \left(\int \frac{1}{\sqrt{-c^2x^2+1} \operatorname{asin}(cx)^2 b^2 x^2 + 2\sqrt{-c^2x^2+1} \operatorname{asin}(cx) a b x^2 + \sqrt{-c^2x^2+1} a^2 x^2} dx \right) a b - \operatorname{asin}(cx) c + \left(\int \frac{1}{\sqrt{-c^2x^2+1} \operatorname{asin}(cx)} dx \right) a}{a (\operatorname{asin}(cx) b + a)}$$

input `int((-c^2*x^2+1)^(1/2)/x^2/(a+b*asin(c*x))^2,x)`

output `(asin(c*x)*int(1/(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(-c**2*x**2 + 1)*a**2*x**2),x) *a*b - asin(c*x)*c + int(1/(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(-c**2*x**2 + 1)*a**2*x**2),x)*a**2)/(a*(asin(c*x)*b + a))`

$$3.357 \quad \int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx$$

Optimal result	3172
Mathematica [N/A]	3172
Rubi [N/A]	3173
Maple [N/A]	3173
Fricas [N/A]	3174
Sympy [N/A]	3174
Maxima [N/A]	3175
Giac [F(-2)]	3175
Mupad [N/A]	3176
Reduce [N/A]	3176

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2}, x\right)$$

output `Defer(Int)((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 10.74 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx$$

input `Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 x^2 + 1}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2x^3} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3), x)`

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3(a+b\arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*asin(c*x))**2,x)`

output `Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.82

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arcsin(cx)+a)^2x^3} dx$$

input `integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^2*x^2 + (b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^3)*integrate((c^2*x^2 - 3)/(b^2*c*x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^4), x) - 1)/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 x^3 + 2 \operatorname{asin}(cx) a b x^3 + a^2 x^3} dx$$

input `int((-c^2*x^2+1)^(1/2)/x^3/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2*x**3 + 2*asin(c*x)*a*b*x**3 + a**2*x**3),x)`

3.358 $\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3177
Mathematica [A] (verified)	3178
Rubi [A] (verified)	3179
Maple [A] (verified)	3181
Fricas [F]	3182
Sympy [F]	3182
Maxima [F]	3182
Giac [B] (verification not implemented)	3183
Mupad [F(-1)]	3184
Reduce [F]	3184

Optimal result

Integrand size = 28, antiderivative size = 278

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx = -\frac{x^3(1-c^2x^2)^2}{bc(a+b \arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{64b^2c^4} + \frac{9 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{64b^2c^4} - \frac{5 \cos\left(\frac{5a}{b}\right) \text{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{64b^2c^4} - \frac{7 \cos\left(\frac{7a}{b}\right) \text{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{64b^2c^4} + \frac{3 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{64b^2c^4} + \frac{9 \sin\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{64b^2c^4} - \frac{5 \sin\left(\frac{5a}{b}\right) \text{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{64b^2c^4} - \frac{7 \sin\left(\frac{7a}{b}\right) \text{Si}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{64b^2c^4}$$

output

```
-x^3*(-c^2*x^2+1)^2/b/c/(a+b*arcsin(c*x))+3/64*cos(a/b)*Ci((a+b*arcsin(c*x))
)/b)/b^2/c^4+9/64*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^4-5/64*cos(5
*a/b)*Ci(5*(a+b*arcsin(c*x))/b)/b^2/c^4-7/64*cos(7*a/b)*Ci(7*(a+b*arcsin(c
*x))/b)/b^2/c^4+3/64*sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^4+9/64*sin(3*a
/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^4-5/64*sin(5*a/b)*Si(5*(a+b*arcsin(c*x
))/b)/b^2/c^4-7/64*sin(7*a/b)*Si(7*(a+b*arcsin(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.44

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx = \frac{64bc^3x^3 - 128bc^5x^5 + 64bc^7x^7 - 3(a+b\arcsin(cx))\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) - 9(a+b\arcsin(cx))}{(a+b\arcsin(cx))^2}$$

input

```
Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/64*(64*b*c^3*x^3 - 128*b*c^5*x^5 + 64*b*c^7*x^7 - 3*(a + b*ArcSin[c*x])
)*Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] - 9*(a + b*ArcSin[c*x])*Cos[(3*a
)/b]*CosIntegral[3*(a/b + ArcSin[c*x])] + 5*a*Cos[(5*a)/b]*CosIntegral[5*(a
/b + ArcSin[c*x])] + 5*b*ArcSin[c*x]*Cos[(5*a)/b]*CosIntegral[5*(a/b + Arc
Sin[c*x])] + 7*a*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcSin[c*x])] + 7*b*Arc
Sin[c*x]*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcSin[c*x])] - 3*a*Sin[a/b]*Si
nIntegral[a/b + ArcSin[c*x]] - 3*b*ArcSin[c*x]*Sin[a/b]*SinIntegral[a/b +
ArcSin[c*x]] - 9*a*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] - 9*b*A
rcSin[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] + 5*a*Sin[(5*a
)/b]*SinIntegral[5*(a/b + ArcSin[c*x])] + 5*b*ArcSin[c*x]*Sin[(5*a)/b]*Si
nIntegral[5*(a/b + ArcSin[c*x])] + 7*a*Sin[(7*a)/b]*SinIntegral[7*(a/b + Arc
Sin[c*x])] + 7*b*ArcSin[c*x]*Sin[(7*a)/b]*SinIntegral[7*(a/b + ArcSin[c*x]
)])/b^2*c^4*(a + b*ArcSin[c*x])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5214, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & \frac{3 \int \frac{x^2(1-c^2x^2)}{a+b\arcsin(cx)} dx}{bc} - \frac{7c \int \frac{x^4(1-c^2x^2)}{a+b\arcsin(cx)} dx}{b} - \frac{x^3(1-c^2x^2)^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{5224} \\
 & - \frac{7 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^4} + \\
 & \frac{3 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^4} - \frac{x^3(1-c^2x^2)^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{4906} \\
 & \frac{7 \int \left(\frac{\cos\left(\frac{7a}{b} - \frac{7(a+b\arcsin(cx))}{b}\right)}{64(a+b\arcsin(cx))} - \frac{\cos\left(\frac{5a}{b} - \frac{5(a+b\arcsin(cx))}{b}\right)}{64(a+b\arcsin(cx))} - \frac{3 \cos\left(\frac{3a}{b} - \frac{3(a+b\arcsin(cx))}{b}\right)}{64(a+b\arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{64(a+b\arcsin(cx))} \right) d(a+b\arcsin(cx))}{b^2c^4} \\
 & \frac{3 \int \left(-\frac{\cos\left(\frac{5a}{b} - \frac{5(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{\cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{8(a+b\arcsin(cx))} \right) d(a+b\arcsin(cx))}{b^2c^4} \\
 & \frac{x^3(1-c^2x^2)^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\frac{3\left(\frac{1}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)\right)}{7\left(\frac{3}{64} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{3}{64} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{1}{64} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)\right)}$$

$$\frac{x^3(1-c^2x^2)^2}{bc(a+b \arcsin(cx))}$$

input `Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]`

output

```

-((x^3*(1 - c^2*x^2)^2)/(b*c*(a + b*ArcSin[c*x]))) + (3*((Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/8 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x])/b])/16 - (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x])/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/16 - (Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/16))/(b^2*c^4) - (7*((3*Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/64 - (3*Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x])/b])/64 + (Cos[(7*a)/b]*CosIntegral[(7*(a + b*ArcSin[c*x])/b])/64 + (3*Sine[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/64 - (3*Sine[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (Sine[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/64 + (Sine[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x])/b])/64))/(b^2*c^4)

```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*
((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p
- 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n
+ 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2
)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.64

method	result
default	$-\frac{7 \arcsin(cx) \cos(\frac{7a}{b}) \text{Ci}(7 \arcsin(cx) + \frac{7a}{b}) b + 5 \arcsin(cx) \text{Si}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b}) b + 5 \arcsin(cx) \text{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b})}{(a + b \arcsin(cx))^2}$

input

```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/64/c^4*(7*arcsin(c*x)*cos(7*a/b)*Ci(7*arcsin(c*x)+7*a/b)*b+5*arcsin(c*x
)*Si(5*arcsin(c*x)+5*a/b)*sin(5*a/b)*b+5*arcsin(c*x)*Ci(5*arcsin(c*x)+5*a/
b)*cos(5*a/b)*b-9*arcsin(c*x)*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*b-9*arcsi
n(c*x)*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b-3*arcsin(c*x)*Si(arcsin(c*x)+a
/b)*sin(a/b)*b-3*arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+7*arcsin(c*x)*
Si(7*arcsin(c*x)+7*a/b)*sin(7*a/b)*b+7*cos(7*a/b)*Ci(7*arcsin(c*x)+7*a/b)*
a+5*Si(5*arcsin(c*x)+5*a/b)*sin(5*a/b)*a+5*Ci(5*arcsin(c*x)+5*a/b)*cos(5*a
/b)*a-9*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*a-9*Ci(3*arcsin(c*x)+3*a/b)*cos
(3*a/b)*a-3*Si(arcsin(c*x)+a/b)*sin(a/b)*a-3*Ci(arcsin(c*x)+a/b)*cos(a/b)*
a+7*Si(7*arcsin(c*x)+7*a/b)*sin(7*a/b)*a+3*x*b*c-sin(5*arcsin(c*x))*b+3*si
n(3*arcsin(c*x))*b-sin(7*arcsin(c*x))*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^3}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^3(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^3}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
-(c^4*x^7 - 2*c^2*x^5 + x^3 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x
+ 1)) + a*b*c)*integrate((7*c^4*x^6 - 10*c^2*x^4 + 3*x^2)/(b^2*c*arctan2(c
*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x)/(b^2*c*arctan2(c*x, sqrt(c
*x + 1))*sqrt(-c*x + 1)) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2065 vs. 2(261) = 522.

Time = 0.25 (sec) , antiderivative size = 2065, normalized size of antiderivative = 7.43

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input

```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
-7*b*arcsin(c*x)*cos(a/b)^7*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*a
rcsin(c*x) + a*b^2*c^4) - 7*b*arcsin(c*x)*cos(a/b)^6*sin(a/b)*sin_integral
(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 7*a*cos(a/b)^7
*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 7
*a*cos(a/b)^6*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin
(c*x) + a*b^2*c^4) + 49/4*b*arcsin(c*x)*cos(a/b)^5*cos_integral(7*a/b + 7*
arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 5/4*b*arcsin(c*x)*cos(a/b
)^5*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4)
+ 35/4*b*arcsin(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x
))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 5/4*b*arcsin(c*x)*cos(a/b)^4*sin(a/
b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) +
49/4*a*cos(a/b)^5*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x
) + a*b^2*c^4) - 5/4*a*cos(a/b)^5*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3
*c^4*arcsin(c*x) + a*b^2*c^4) + 35/4*a*cos(a/b)^4*sin(a/b)*sin_integral(7*
a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 5/4*a*cos(a/b)^4*
sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*
c^4) - (c^2*x^2 - 1)^3*b*c*x/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 49/8*b*ar
csin(c*x)*cos(a/b)^3*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c
*x) + a*b^2*c^4) + 25/16*b*arcsin(c*x)*cos(a/b)^3*cos_integral(5*a/b + 5*a
rcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) + 9/16*b*arcsin(c*x)*cos(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \sin(cx))^2} dx$$

input `int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2,x)`

output `int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{x^3(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = - \left(\int \frac{\sqrt{-c^2x^2 + 1} x^5}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^2$$

$$+ \int \frac{\sqrt{-c^2x^2 + 1} x^3}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx$$

input `int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int((sqrt(- c**2*x**2 + 1)*x**5)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(- c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.359 $\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3185
Mathematica [A] (verified)	3186
Rubi [A] (verified)	3186
Maple [A] (verified)	3189
Fricas [F]	3189
Sympy [F]	3190
Maxima [F]	3190
Giac [B] (verification not implemented)	3191
Mupad [F(-1)]	3192
Reduce [F]	3192

Optimal result

Integrand size = 28, antiderivative size = 220

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx = -\frac{x^2(1-c^2x^2)^2}{bc(a+b \arcsin(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{4a}{b}\right)}{4b^2c^3} - \frac{3 \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{16b^2c^3} + \frac{\cos\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{4b^2c^3} + \frac{3 \cos\left(\frac{6a}{b}\right) \text{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{16b^2c^3}$$

output

```
-x^2*(-c^2*x^2+1)^2/b/c/(a+b*arcsin(c*x))+1/16*Ci(2*(a+b*arcsin(c*x))/b)*sin(2*a/b)/b^2/c^3-1/4*Ci(4*(a+b*arcsin(c*x))/b)*sin(4*a/b)/b^2/c^3-3/16*Ci(6*(a+b*arcsin(c*x))/b)*sin(6*a/b)/b^2/c^3-1/16*cos(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b^2/c^3+1/4*cos(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b^2/c^3+3/16*cos(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.39

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx = \frac{16bc^2x^2 - 32bc^4x^4 + 16bc^6x^6 - (a+b\arcsin(cx))\operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)\sin\left(\frac{2a}{b}\right) + 4(a+b\arcsin(cx))\operatorname{SinIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)\cos\left(\frac{2a}{b}\right)}{(b^2c^3(a+b\arcsin(cx)))}$$

input

```
Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/16*(16*b*c^2*x^2 - 32*b*c^4*x^4 + 16*b*c^6*x^6 - (a + b*ArcSin[c*x])*CosIntegral[2*(a/b + ArcSin[c*x])]*Sin[(2*a)/b] + 4*(a + b*ArcSin[c*x])*CosIntegral[4*(a/b + ArcSin[c*x])]*Sin[(4*a)/b] + 3*a*CosIntegral[6*(a/b + ArcSin[c*x])]*Sin[(6*a)/b] + 3*b*ArcSin[c*x]*CosIntegral[6*(a/b + ArcSin[c*x])]*Sin[(6*a)/b] + a*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + b*ArcSin[c*x]*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] - 4*a*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] - 4*b*ArcSin[c*x]*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] - 3*a*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])] - 3*b*ArcSin[c*x]*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])])/(b^2*c^3*(a + b*ArcSin[c*x]))
```

Rubi [A] (verified)Time = 0.95 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5214, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx$$

↓ 5214

$$\frac{2 \int \frac{x(1-c^2x^2)}{a+b\arcsin(cx)} dx}{bc} - \frac{6c \int \frac{x^3(1-c^2x^2)}{a+b\arcsin(cx)} dx}{b} - \frac{x^2(1-c^2x^2)^2}{bc(a+b\arcsin(cx))}$$

$$\begin{aligned} & \downarrow 5224 \\ & \frac{6 \int -\frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} + \\ & \frac{2 \int -\frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\begin{aligned} & \downarrow 25 \\ & \frac{6 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \\ & \frac{2 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\begin{aligned} & \downarrow 4906 \\ & \frac{6 \int \left(\frac{3 \sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} - \frac{\sin\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^3} - \\ & \frac{2 \int \left(\frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} + \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))} \end{aligned}$$

$$\begin{aligned} & \downarrow 2009 \\ & \frac{2\left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{8} \sin\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^3} \\ & \frac{6\left(-\frac{3}{32} \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{32} \sin\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) + \frac{3}{32} \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^3} \\ & \frac{x^2(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))} \end{aligned}$$

input

`Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]`

output

$$\begin{aligned}
& -((x^2(1 - c^2x^2)^2)/(b*c*(a + b*\text{ArcSin}[c*x])) + (2*(-1/4*(\text{CosIntegral} \\
& [(2*(a + b*\text{ArcSin}[c*x])/b]*\text{Sin}[(2*a)/b]) - (\text{CosIntegral}[(4*(a + b*\text{ArcSin}[\\
& c*x])/b]*\text{Sin}[(4*a)/b])/8 + (\text{Cos}[(2*a)/b]*\text{SinIntegral}[(2*(a + b*\text{ArcSin}[c*x] \\
&])/b])/4 + (\text{Cos}[(4*a)/b]*\text{SinIntegral}[(4*(a + b*\text{ArcSin}[c*x])/b])/8))/(b^2 \\
& *c^3) - (6*((-3*\text{CosIntegral}[(2*(a + b*\text{ArcSin}[c*x])/b]*\text{Sin}[(2*a)/b])/32 + \\
& (\text{CosIntegral}[(6*(a + b*\text{ArcSin}[c*x])/b]*\text{Sin}[(6*a)/b])/32 + (3*\text{Cos}[(2*a)/b] \\
& *\text{SinIntegral}[(2*(a + b*\text{ArcSin}[c*x])/b])/32 - (\text{Cos}[(6*a)/b]*\text{SinIntegral}[(6 \\
& *(a + b*\text{ArcSin}[c*x])/b])/32))/(b^2*c^3)
\end{aligned}$$

Defintions of rubi rules used

rule 25

$$\text{Int}[-(F_x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F_x, x], x]$$

rule 2009

$$\text{Int}[u, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$$

rule 4906

$$\begin{aligned}
& \text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(p_.)*((c_.) + (d_.)*(x_))^{(m_.)*\text{Sin}[(a_.) + (b \\
& _.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x \\
&]^n*\text{Cos}[a + b*x]^p, x], x] \text{ /; FreeQ}\{a, b, c, d, m\}, x \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IG} \\
& \text{tQ}[p, 0]
\end{aligned}$$

rule 5214

$$\begin{aligned}
& \text{Int}[(a_.) + \text{ArcSin}[c_.)*(x_)]*(b_.)^{(n_.)*((f_.)*(x_))^{(m_.)*((d_.) + (e_. \\
&)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^m*\text{Sqrt}[1 - c^2*x^2]*(d + e*x^2)^p* \\
& ((a + b*\text{ArcSin}[c*x])^{(n + 1)/(b*c*(n + 1))}, x] + (-\text{Simp}[f*(m/(b*c*(n + 1)) \\
&]*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \quad \text{Int}[(f*x)^{(m - 1)}*(1 - c^2*x^2)^{(p \\
& - 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x] + \text{Simp}[c*((m + 2*p + 1)/(b*f*(n \\
& + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \quad \text{Int}[(f*x)^{(m + 1)}*(1 - c^2*x^2 \\
&)^{(p - 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x]) \text{ /; FreeQ}\{a, b, c, d, e, f \\
& \}, x \ \&\& \text{EqQ}[c^2*d + e, 0] \ \&\& \text{LtQ}[n, -1] \ \&\& \text{IGtQ}[2*p, 0] \ \&\& \text{NeQ}[m + 2*p + 1 \\
& , 0] \ \&\& \text{IGtQ}[m, -3]
\end{aligned}$$

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.65

method	result
default	$-\frac{2 \arcsin(cx) \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) b - 2 \arcsin(cx) \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) b - 6 \arcsin(cx) \operatorname{Si}(6 \arcsin(cx) + \frac{6a}{b}) \cos(\frac{6a}{b}) b - 6 \arcsin(cx) \operatorname{Ci}(6 \arcsin(cx) + \frac{6a}{b}) \sin(\frac{6a}{b}) b - 8 \arcsin(cx) \operatorname{Si}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) b - 8 \arcsin(cx) \operatorname{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) b - 2 \arcsin(cx) \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) b - 2 \arcsin(cx) \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) b + a^2 \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) b - a^2 \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) b + a^2 \operatorname{Ci}(6 \arcsin(cx) + \frac{6a}{b}) \sin(\frac{6a}{b}) b - a^2 \operatorname{Si}(6 \arcsin(cx) + \frac{6a}{b}) \cos(\frac{6a}{b}) b + a^2 \operatorname{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) b - a^2 \operatorname{Si}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) b + a^2 \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) b - a^2 \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) b}{(a + b \arcsin(cx))^2}$

input

```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/32/c^3*(2*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*b-2*arcsin(c*x)
)*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*b-6*arcsin(c*x)*Si(6*arcsin(c*x)+6*a/
b)*cos(6*a/b)*b+6*arcsin(c*x)*Ci(6*arcsin(c*x)+6*a/b)*sin(6*a/b)*b-8*arcsi
n(c*x)*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*b+8*arcsin(c*x)*sin(4*a/b)*Ci(4*
arcsin(c*x)+4*a/b)*b+2*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a-2*Ci(2*arcsin(
c*x)+2*a/b)*sin(2*a/b)*a-6*Si(6*arcsin(c*x)+6*a/b)*cos(6*a/b)*a+6*Ci(6*arc
sin(c*x)+6*a/b)*sin(6*a/b)*a-8*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*a+8*sin(
4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*a+cos(2*arcsin(c*x))*b-cos(6*arcsin(c*x))*b
-2*cos(4*arcsin(c*x))*b+2*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^2}{(b \arcsin(cx) + a)^2} dx$$

input

```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas"
)
```

output `integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^2(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x^2}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(c^4*x^6 - 2*c^2*x^4 + x^2 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(2*(3*c^4*x^5 - 4*c^2*x^3 + x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1553 vs. $2(207) = 414$.

Time = 0.24 (sec) , antiderivative size = 1553, normalized size of antiderivative = 7.06

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
-6*b*arcsin(c*x)*cos(a/b)^5*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(
b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 6*b*arcsin(c*x)*cos(a/b)^6*sin_integral
(6*a/b + 6*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 6*a*cos(a/b)^5
*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2
*c^3) + 6*a*cos(a/b)^6*sin_integral(6*a/b + 6*arcsin(c*x))/(b^3*c^3*arcsin
(c*x) + a*b^2*c^3) + 6*b*arcsin(c*x)*cos(a/b)^3*cos_integral(6*a/b + 6*arc
sin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 2*b*arcsin(c*x)*cos
(a/b)^3*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x)
+ a*b^2*c^3) - 9*b*arcsin(c*x)*cos(a/b)^4*sin_integral(6*a/b + 6*arcsin(c*
x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 2*b*arcsin(c*x)*cos(a/b)^4*sin_int
egral(4*a/b + 4*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 6*a*cos(a
/b)^3*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) +
a*b^2*c^3) - 2*a*cos(a/b)^3*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(
b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 9*a*cos(a/b)^4*sin_integral(6*a/b + 6*a
rcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 2*a*cos(a/b)^4*sin_integra
l(4*a/b + 4*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 9/8*b*arcsin(
c*x)*cos(a/b)*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin
(c*x) + a*b^2*c^3) + b*arcsin(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arcsin(
c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 1/8*b*arcsin(c*x)*cos(a
/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^2(1 - c^2x^2)^{3/2}}{(a + b \sin(cx))^2} dx$$

input `int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2,x)`

output `int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{x^2(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = - \left(\int \frac{\sqrt{-c^2x^2 + 1} x^4}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^2$$

$$+ \int \frac{\sqrt{-c^2x^2 + 1} x^2}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx$$

input `int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int((sqrt(-c**2*x**2 + 1)*x**4)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.360
$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$$

Optimal result	3193
Mathematica [A] (verified)	3194
Rubi [A] (verified)	3194
Maple [A] (verified)	3198
Fricas [F]	3198
Sympy [F]	3199
Maxima [F]	3199
Giac [B] (verification not implemented)	3200
Mupad [F(-1)]	3201
Reduce [F]	3201

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx = -\frac{x(1-c^2x^2)^2}{bc(a+b \arcsin(cx))} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8b^2c^2} + \frac{9 \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16b^2c^2} + \frac{5 \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16b^2c^2} + \frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8b^2c^2} + \frac{9 \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16b^2c^2} + \frac{5 \sin\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16b^2c^2}$$

output

```
-x*(-c^2*x^2+1)^2/b/c/(a+b*arcsin(c*x))+1/8*cos(a/b)*Ci((a+b*arcsin(c*x))/b)/b^2/c^2+9/16*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^2+5/16*cos(5*a/b)*Ci(5*(a+b*arcsin(c*x))/b)/b^2/c^2+1/8*sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^2+9/16*sin(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^2+5/16*sin(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int \frac{x(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \frac{-16bcx + 32bc^3x^3 - 16bc^5x^5 + 2(a + b \arcsin(cx)) \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right)}{(a + b \arcsin(cx))^2}$$

input

```
Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
(-16*b*c*x + 32*b*c^3*x^3 - 16*b*c^5*x^5 + 2*(a + b*ArcSin[c*x])*Cos[a/b]*
CosIntegral[a/b + ArcSin[c*x]] + 9*(a + b*ArcSin[c*x])*Cos[(3*a)/b]*CosInt
egral[3*(a/b + ArcSin[c*x])] + 5*a*cos[(5*a)/b]*CosIntegral[5*(a/b + ArcSi
n[c*x])] + 5*b*ArcSin[c*x]*Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcSin[c*x])]
+ 2*a*Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 2*b*ArcSin[c*x]*Sin[a/b]*
SinIntegral[a/b + ArcSin[c*x]] + 9*a*Sin[(3*a)/b]*SinIntegral[3*(a/b + Arc
Sin[c*x])] + 9*b*ArcSin[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x]
)] + 5*a*Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])] + 5*b*ArcSin[c*x]
*Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])])/(16*b^2*c^2*(a + b*ArcSi
n[c*x]))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5214, 5168, 3042, 3793, 2009, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx$$

$$\downarrow \text{5214}$$

$$\frac{\int \frac{1-c^2x^2}{a+b \arcsin(cx)} dx}{bc} - \frac{5c \int \frac{x^2(1-c^2x^2)}{a+b \arcsin(cx)} dx}{b} - \frac{x(1 - c^2x^2)^2}{bc(a + b \arcsin(cx))}$$

$$\downarrow \text{5168}$$

$$\frac{\int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{5c \int \frac{x^2(1-c^2 x^2)}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))}$$

↓ 3042

$$\frac{\int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^3}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{5c \int \frac{x^2(1-c^2 x^2)}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))}$$

↓ 3793

$$\frac{\int \left(\frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^2} - \frac{5c \int \frac{x^2(1-c^2 x^2)}{a+b \arcsin(cx)} dx}{b} - \frac{x(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))}$$

↓ 2009

$$\frac{\frac{3}{4} \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{3}{4} \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \frac{5c \int \frac{x^2(1-c^2 x^2)}{a+b \arcsin(cx)} dx}{b}}{b^2 c^2} + \frac{x(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))}$$

↓ 5224

$$\frac{\frac{3}{4} \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{3}{4} \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \frac{5 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2}}{b^2 c^2} + \frac{x(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))}$$

↓ 4906

$$\begin{aligned}
 & \frac{5 \int \left(-\frac{\cos\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))}{b^2 c^2} + \\
 & \frac{\frac{3}{4} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{3}{4} \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4}}{b^2 c^2} \\
 & \frac{x(1 - c^2 x^2)^2}{bc(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{2009} \\
 & \frac{\frac{3}{4} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{3}{4} \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{4}}{b^2 c^2} \\
 & \frac{5\left(\frac{1}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{1}{8} \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)\right)}{b^2 c^2} \\
 & \frac{x(1 - c^2 x^2)^2}{bc(a + b \arcsin(cx))}
 \end{aligned}$$

input

```
Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```

-((x*(1 - c^2*x^2)^2)/(b*c*(a + b*ArcSin[c*x]))) + ((3*Cos[a/b]*CosIntegral
1[(a + b*ArcSin[c*x])/b])/4 + (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c
*x]))/b])/4 + (3*Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/4 + (Sin[(3*
a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x]))/b])/4)/(b^2*c^2) - (5*((Cos[a/b]
*CosIntegral[(a + b*ArcSin[c*x])/b])/8 - (Cos[(3*a)/b]*CosIntegral[(3*(a +
b*ArcSin[c*x]))/b])/16 - (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x])
)/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 - (Sin[(3*a)/b]
*SinIntegral[(3*(a + b*ArcSin[c*x]))/b])/16 - (Sin[(5*a)/b]*SinIntegral[(5
*(a + b*ArcSin[c*x])/b])/16))/(b^2*c^2)

```

Definitions of rubi rules used

- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$
- rule 3793 $\text{Int}[(c_.) + (d_.)(x_)^{(m_)} \sin[(e_.) + (f_.)(x_)^{(n_)}], x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (!\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$
- rule 4906 $\text{Int}[\text{Cos}[(a_.) + (b_.)(x_)]^{(p_.)}((c_.) + (d_.)(x_))^{(m_.)} \text{Sin}[(a_.) + (b_.)(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n \text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 5168 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)](b_.)^{(n_.)}((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Subst}[\text{Int}[x^n \text{Cos}[-a/b + x/b]^{(2*p + 1)}, x], x, a + b*\text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[2*p, 0]$
- rule 5214 $\text{Int}[(a_.) + \text{ArcSin}[c_.)(x_)](b_.)^{(n_.)}((f_.)(x_))^{(m_.)}((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^m \text{Sqrt}[1 - c^2*x^2] * (d + e*x^2)^p * ((a + b*\text{ArcSin}[c*x])^{(n + 1)} / (b*c*(n + 1))), x] + (-\text{Simp}[f*(m/(b*c*(n + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m - 1)} * (1 - c^2*x^2)^{(p - 1/2)} * (a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x] + \text{Simp}[c*((m + 2*p + 1)/(b*f*(n + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m + 1)} * (1 - c^2*x^2)^{(p - 1/2)} * (a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IGtQ}[2*p, 0] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0] \ \&\& \ \text{IGtQ}[m, -3]$

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.59

method	result
default	$\frac{9 \arcsin(cx) \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b})b + 9 \arcsin(cx) \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b})b + 2 \arcsin(cx) \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})b + 2 \arcsin(cx) \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b})b + 5 \arcsin(cx) \operatorname{Si}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b})b + 5 \arcsin(cx) \operatorname{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b})b + 9 \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b})a + 9 \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b})a + 2 \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b})a + 2 \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b})a + 5 \operatorname{Si}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b})a + 5 \operatorname{Ci}(5 \arcsin(cx) + \frac{5a}{b}) \cos(\frac{5a}{b})a - 2*x*b*c - 3*\sin(3*\arcsin(c*x))*b - \sin(5*\arcsin(c*x))*b}{(a+b*\arcsin(c*x))^2}$

input

```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/16/c^2*(9*arcsin(c*x)*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*b+9*arcsin(c*x)
*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b+2*arcsin(c*x)*Si(arcsin(c*x)+a/b)*si
n(a/b)*b+2*arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+5*arcsin(c*x)*Si(5*a
rcsin(c*x)+5*a/b)*sin(5*a/b)*b+5*arcsin(c*x)*Ci(5*arcsin(c*x)+5*a/b)*cos(5
*a/b)*b+9*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*a+9*Ci(3*arcsin(c*x)+3*a/b)*c
os(3*a/b)*a+2*Si(arcsin(c*x)+a/b)*sin(a/b)*a+2*Ci(arcsin(c*x)+a/b)*cos(a/b
)*a+5*Si(5*arcsin(c*x)+5*a/b)*sin(5*a/b)*a+5*Ci(5*arcsin(c*x)+5*a/b)*cos(5
*a/b)*a-2*x*b*c-3*sin(3*arcsin(c*x))*b-sin(5*arcsin(c*x))*b)/(a+b*arcsin(c
*x))/b^2
```

Fricas [F]

$$\int \frac{x(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}x}{(b \arcsin(cx) + a)^2} dx$$

input

```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output `integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x(-(cx - 1)(cx + 1))^{3/2}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{3/2}x}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(c^4*x^5 - 2*c^2*x^3 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate((5*c^4*x^4 - 6*c^2*x^2 + 1)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x) + x/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(201) = 402$.

Time = 0.24 (sec) , antiderivative size = 1215, normalized size of antiderivative = 5.68

$$\int \frac{x(1 - c^2x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
5*b*arcsin(c*x)*cos(a/b)^5*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 5*b*arcsin(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 5*a*cos(a/b)^5*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 5*a*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 25/4*b*arcsin(c*x)*cos(a/b)^3*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 9/4*b*arcsin(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 15/4*b*arcsin(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 9/4*b*arcsin(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - (c^2*x^2 - 1)^2*b*c*x/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 25/4*a*cos(a/b)^3*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 9/4*a*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 15/4*a*cos(a/b)^2*sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 9/4*a*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 25/16*b*arcsin(c*x)*cos(a/b)*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 27/16*b*arcsin(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 1/8*b*arcsin(c*x)*cos(a/b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x(1 - c^2 x^2)^{3/2}}{(a + b \sin(cx))^2} dx$$

input `int((x*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2,x)`

output `int((x*(1 - c^2*x^2)^(3/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{x(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = - \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^3}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^2$$

$$+ \int \frac{\sqrt{-c^2 x^2 + 1} x}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx$$

input `int(x*(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int((sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.361 $\int \frac{(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3202
Mathematica [A] (verified)	3203
Rubi [A] (verified)	3203
Maple [A] (verified)	3205
Fricas [F]	3206
Sympy [F]	3206
Maxima [F]	3206
Giac [B] (verification not implemented)	3207
Mupad [F(-1)]	3208
Reduce [F]	3208

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b \arcsin(cx))^2} dx = -\frac{(1-c^2x^2)^2}{bc(a+b \arcsin(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{b^2c} + \frac{\text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{4a}{b}\right)}{2b^2c} - \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{b^2c} - \frac{\cos\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{2b^2c}$$

output

```

-(-c^2*x^2+1)^2/b/c/(a+b*arcsin(c*x))+Ci(2*(a+b*arcsin(c*x))/b)*sin(2*a/b)
/b^2/c+1/2*Ci(4*(a+b*arcsin(c*x))/b)*sin(4*a/b)/b^2/c-cos(2*a/b)*Si(2*(a+b
*arcsin(c*x))/b)/b^2/c-1/2*cos(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b^2/c
    
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = -\frac{2b(-1+c^2x^2)^2}{a+b \arcsin(cx)} + 2 \operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{2a}{b}\right) + \operatorname{CosIntegral}\left(4\left(\frac{a}{b}\right)\right)$$

input `Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcSin[c*x])^2,x]`

output `((-2*b*(-1 + c^2*x^2)^2)/(a + b*ArcSin[c*x]) + 2*CosIntegral[2*(a/b + ArcSin[c*x]])*Sin[(2*a)/b] + CosIntegral[4*(a/b + ArcSin[c*x]])*Sin[(4*a)/b] - 2*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] - Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])])/(2*b^2*c)`

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5166, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx \\ & \quad \downarrow \text{5166} \\ & -\frac{4c \int \frac{x(1-c^2x^2)}{a+b \arcsin(cx)} dx}{b} - \frac{(1 - c^2 x^2)^2}{bc(a + b \arcsin(cx))} \\ & \quad \downarrow \text{5224} \\ & -\frac{4 \int -\frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2 c} - \frac{(1 - c^2 x^2)^2}{bc(a + b \arcsin(cx))} \\ & \quad \downarrow \text{25} \end{aligned}$$

$$\begin{aligned}
& \frac{4 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c} - \frac{(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{4906} \\
& \frac{4 \int \left(\frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} + \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c} - \frac{(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{2009} \\
& \frac{4 \left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{8} \sin\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \right)}{b^2 c} - \frac{(1-c^2 x^2)^2}{bc(a+b \arcsin(cx))}
\end{aligned}$$

input `Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcSin[c*x])^2,x]`

output `-((1 - c^2*x^2)^2/(b*c*(a + b*ArcSin[c*x]))) - (4*(-1/4*(CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b]) - (CosIntegral[(4*(a + b*ArcSin[c*x]))/b]*Sin[(4*a)/b])/8 + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/4 + (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8)/(b^2*c)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5166

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)
)/(b*c*(n + 1)), x] + Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p/(1
- c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.67

method	result
default	$-\frac{4 \arcsin(cx) \operatorname{Si}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) b - 4 \arcsin(cx) \sin(\frac{4a}{b}) \operatorname{Ci}(4 \arcsin(cx) + \frac{4a}{b}) b + 8 \arcsin(cx) \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b})}{(a + b \arcsin(cx))^2}$

input

```
int((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/8/c*(4*arcsin(c*x)*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*b-4*arcsin(c*x)*
in(4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*b+8*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b)*
cos(2*a/b)*b-8*arcsin(c*x)*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*b+4*Si(4*arc
sin(c*x)+4*a/b)*cos(4*a/b)*a-4*sin(4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*a+8*Si(2
*arcsin(c*x)+2*a/b)*cos(2*a/b)*a-8*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*a+co
s(4*arcsin(c*x))*b+4*cos(2*arcsin(c*x))*b+3*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{(a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(c^4*x^4 - 2*c^2*x^2 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(4*(c^3*x^3 - c*x)/(b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b), x) + 1)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(145) = 290$.

Time = 0.23 (sec) , antiderivative size = 747, normalized size of antiderivative = 4.98

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate((-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
4*b*arcsin(c*x)*cos(a/b)^3*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b
^3*c*arcsin(c*x) + a*b^2*c) - 4*b*arcsin(c*x)*cos(a/b)^4*sin_integral(4*a/
b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) + 4*a*cos(a/b)^3*cos_inte
gral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) - 4*a*c
os(a/b)^4*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c
) - 2*b*arcsin(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/
(b^3*c*arcsin(c*x) + a*b^2*c) + 2*b*arcsin(c*x)*cos(a/b)*cos_integral(2*a/
b + 2*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 4*b*arcsin(c*x
)*cos(a/b)^2*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^
2*c) - 2*b*arcsin(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3
*c*arcsin(c*x) + a*b^2*c) - 2*a*cos(a/b)*cos_integral(4*a/b + 4*arcsin(c*x
))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 2*a*cos(a/b)*cos_integral(2*a/
b + 2*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 4*a*cos(a/b)^2
*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) - 2*a*c
os(a/b)^2*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c
) - (c^2*x^2 - 1)^2*b/(b^3*c*arcsin(c*x) + a*b^2*c) - 1/2*b*arcsin(c*x)*si
n_integral(4*a/b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) + b*arcsin
(c*x)*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) -
1/2*a*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) +
a*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \sin(cx))^2} dx$$

input `int((1 - c^2*x^2)^(3/2)/(a + b*asin(c*x))^2,x)`

output `int((1 - c^2*x^2)^(3/2)/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx$$

$$- \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^2}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^2$$

input `int((-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x) - int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2`

3.362 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2} dx$

Optimal result	3209
Mathematica [N/A]	3209
Rubi [N/A]	3210
Maple [N/A]	3211
Fricas [N/A]	3212
Sympy [N/A]	3212
Maxima [N/A]	3212
Giac [F(-2)]	3213
Mupad [N/A]	3213
Reduce [N/A]	3214

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2} dx = \int \frac{(1-c^2x^2)^{3/2}}{x(a+b \arcsin(cx))^2} dx$$

input

```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcSin[c*x])^2),x]
```

output

```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcSin[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & -\frac{3c \int \frac{1-c^2 x^2}{a+b \arcsin(cx)} dx}{b} - \frac{\int \frac{1-c^2 x^2}{x^2(a+b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^2}{bcx(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{5168} \\
 & -\frac{3 \int \frac{\cos^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2} - \frac{\int \frac{1-c^2 x^2}{x^2(a+b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^2}{bcx(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{3 \int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^3}{a+b \arcsin(cx)} d(a + b \arcsin(cx))}{b^2} - \frac{\int \frac{1-c^2 x^2}{x^2(a+b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^2}{bcx(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{3793} \\
 & -\frac{3 \int \left(\frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a + b \arcsin(cx))}{b^2} - \\
 & \quad \frac{\int \frac{1-c^2 x^2}{x^2(a+b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^2}{bcx(a + b \arcsin(cx))} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\frac{\int \frac{1-c^2x^2}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{3\left(\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arcsin(cx)}{b}\right) + \frac{1}{4}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arcsin(cx))}{b}\right) + \frac{3}{4}\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arcsin(cx)}{b}\right) + \frac{3}{4}\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arcsin(cx))}{b}\right)\right)}{b^2}$$

$$\frac{(1-c^2x^2)^2}{bcx(a+b\arcsin(cx))}$$

↓ 5234

$$\frac{\int \frac{1-c^2x^2}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{3\left(\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arcsin(cx)}{b}\right) + \frac{1}{4}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arcsin(cx))}{b}\right) + \frac{3}{4}\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arcsin(cx)}{b}\right) + \frac{3}{4}\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arcsin(cx))}{b}\right)\right)}{b^2}$$

$$\frac{(1-c^2x^2)^2}{bcx(a+b\arcsin(cx))}$$

input `Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2 + 1)^{\frac{3}{2}}}{x(a + b\arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b^2*x*arcsin(c*x)^2 + 2*a*b*x*arcsin(c*x) + a^2*x), x)`

Sympy [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{-(cx - 1)(cx + 1)^{\frac{3}{2}}}{x(a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/x/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(x*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.21

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + a*b*c*x)*integrate((3*c^4*x^4 - 2*c^2*x^2 - 1)/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2), x) + 1)/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + a*b*c*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(3/2)/(x*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(3/2)/(x*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{a \sin(cx)^2 b^2 x + 2a \sin(cx) abx + a^2 x} dx$$

$$- \left(\int \frac{\sqrt{-c^2 x^2 + 1} x}{a \sin(cx)^2 b^2 + 2a \sin(cx) ab + a^2} dx \right) c^2$$

input

```
int((-c^2*x^2+1)^(3/2)/x/(a+b*asin(c*x))^2,x)
```

output

```
int(sqrt(-c**2*x**2+1)/(asin(c*x)**2*b**2*x+2*asin(c*x)*a*b*x+a**2*x),x) - int((sqrt(-c**2*x**2+1)*x)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)*c**2
```

$$3.363 \quad \int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2} dx$$

Optimal result	3215
Mathematica [N/A]	3215
Rubi [N/A]	3216
Maple [N/A]	3216
Fricas [N/A]	3217
Sympy [N/A]	3217
Maxima [N/A]	3218
Giac [N/A]	3218
Mupad [N/A]	3219
Reduce [N/A]	3219

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2} dx = \int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b \arcsin(cx))^2} dx$$

input `Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx$$

$$\downarrow 5214$$

$$-\frac{2c \int \frac{1-c^2 x^2}{x(a+b \arcsin(cx))} dx}{b} - \frac{2 \int \frac{1-c^2 x^2}{x^3(a+b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^2}{bcx^2(a + b \arcsin(cx))}$$

$$\downarrow 5234$$

$$-\frac{2c \int \frac{1-c^2 x^2}{x(a+b \arcsin(cx))} dx}{b} - \frac{2 \int \frac{1-c^2 x^2}{x^3(a+b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^2}{bcx^2(a + b \arcsin(cx))}$$

input `Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2), x)`

Sympy [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{x^2 (a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(x**2*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.21

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2)*integrate(2*(c^4*x^4 - 1)/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^3, x) + 1/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2`

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate((-c^2*x^2 + 1)^(3/2)/((b*arcsin(c*x) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 9.50

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \arcsin(cx))^2} dx = \frac{-\operatorname{asin}(cx) \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) ab c^2 + \operatorname{asin}(cx) \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx)} dx \right)}{x^2 (a + b \arcsin(cx))^2}$$

input `int((-c^2*x^2+1)^(3/2)/x^2/(a+b*asin(c*x))^2,x)`

output `(- asin(c*x)*int(sqrt(- c**2*x**2 + 1)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a*b*c**2 + asin(c*x)*int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(- c**2*x**2 + 1)*a**2*x**2),x)*a*b - asin(c*x)*c - int(sqrt(- c**2*x**2 + 1)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a**2*c**2 + int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(- c**2*x**2 + 1)*a**2*x**2),x)*a**2)/(a*(asin(c*x)*b + a))`

$$3.364 \quad \int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2} dx$$

Optimal result	3220
Mathematica [N/A]	3220
Rubi [N/A]	3221
Maple [N/A]	3221
Fricas [N/A]	3222
Sympy [N/A]	3222
Maxima [N/A]	3223
Giac [F(-2)]	3223
Mupad [N/A]	3224
Reduce [N/A]	3224

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2} dx = \text{Int} \left(\frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2}, x \right)$$

output `Defer(Int)((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 10.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2} dx = \int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b \arcsin(cx))^2} dx$$

input `Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x^3} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3), x)`

Sympy [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{3}{2}}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(3/2)/(x**3*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.46

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{3}{2}}}{(b \arcsin(cx) + a)^2 x^3} dx$$

input `integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^3)*integrate((c^4*x^4 + 2*c^2*x^2 - 3)/(b^2*c*x^4*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^4), x) + 1)/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c*x^3)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.43

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 x^3 + 2 \operatorname{asin}(cx) a b x^3 + a^2 x^3} dx - \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 x + 2 \operatorname{asin}(cx) a b x + a^2 x} dx \right) c^2$$

input `int((-c^2*x^2+1)^(3/2)/x^3/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2*x**3 + 2*asin(c*x)*a*b*x**3 + a**2*x**3),x) - int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2*x + 2*asin(c*x)*a*b*x + a**2*x),x)*c**2`

3.365 $\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3225
Mathematica [A] (verified)	3226
Rubi [A] (verified)	3227
Maple [A] (verified)	3229
Fricas [F]	3230
Sympy [F]	3230
Maxima [F]	3230
Giac [B] (verification not implemented)	3231
Mupad [F(-1)]	3232
Reduce [F]	3232

Optimal result

Integrand size = 28, antiderivative size = 278

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx = -\frac{x^3(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{128b^2c^4} + \frac{3 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{32b^2c^4} - \frac{21 \cos\left(\frac{7a}{b}\right) \text{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{256b^2c^4} - \frac{9 \cos\left(\frac{9a}{b}\right) \text{CosIntegral}\left(\frac{9(a+b \arcsin(cx))}{b}\right)}{256b^2c^4} + \frac{3 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{128b^2c^4} + \frac{3 \sin\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{32b^2c^4} - \frac{21 \sin\left(\frac{7a}{b}\right) \text{Si}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{256b^2c^4} - \frac{9 \sin\left(\frac{9a}{b}\right) \text{Si}\left(\frac{9(a+b \arcsin(cx))}{b}\right)}{256b^2c^4}$$

output

```
-x^3*(-c^2*x^2+1)^3/b/c/(a+b*arcsin(c*x))+3/128*cos(a/b)*Ci((a+b*arcsin(c*x))/b)/b^2/c^4+3/32*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^4-21/256*cos(7*a/b)*Ci(7*(a+b*arcsin(c*x))/b)/b^2/c^4-9/256*cos(9*a/b)*Ci(9*(a+b*arcsin(c*x))/b)/b^2/c^4+3/128*sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^4+3/32*sin(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^4-21/256*sin(7*a/b)*Si(7*(a+b*arcsin(c*x))/b)/b^2/c^4-9/256*sin(9*a/b)*Si(9*(a+b*arcsin(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.47

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx = \frac{256bc^3x^3 - 768bc^5x^5 + 768bc^7x^7 - 256bc^9x^9 - 6(a+b\arcsin(cx))\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) - \dots}{(a+b\arcsin(cx))^2}$$

input

```
Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/256*(256*b*c^3*x^3 - 768*b*c^5*x^5 + 768*b*c^7*x^7 - 256*b*c^9*x^9 - 6*(a + b*ArcSin[c*x])*Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] - 24*(a + b*ArcSin[c*x])*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcSin[c*x])] + 21*a*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcSin[c*x])] + 21*b*ArcSin[c*x]*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcSin[c*x])] + 9*a*Cos[(9*a)/b]*CosIntegral[9*(a/b + ArcSin[c*x])] + 9*b*ArcSin[c*x]*Cos[(9*a)/b]*CosIntegral[9*(a/b + ArcSin[c*x])] - 6*a*Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] - 6*b*ArcSin[c*x]*Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] - 24*a*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] - 24*b*ArcSin[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] + 21*a*Sin[(7*a)/b]*SinIntegral[7*(a/b + ArcSin[c*x])] + 21*b*ArcSin[c*x]*Sin[(7*a)/b]*SinIntegral[7*(a/b + ArcSin[c*x])] + 9*a*Sin[(9*a)/b]*SinIntegral[9*(a/b + ArcSin[c*x])] + 9*b*ArcSin[c*x]*Sin[(9*a)/b]*SinIntegral[9*(a/b + ArcSin[c*x])])/(b^2*c^4*(a + b*ArcSin[c*x]))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5214, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & \frac{3 \int \frac{x^2(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{bc} - \frac{9c \int \frac{x^4(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{b} - \frac{x^3(1-c^2x^2)^3}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{5224} \\
 & - \frac{9 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^4} + \\
 & \frac{3 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^4} - \frac{x^3(1-c^2x^2)^3}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{4906} \\
 & 9 \int \left(\frac{\cos\left(\frac{9a}{b} - \frac{9(a+b\arcsin(cx))}{b}\right)}{256(a+b\arcsin(cx))} + \frac{\cos\left(\frac{7a}{b} - \frac{7(a+b\arcsin(cx))}{b}\right)}{256(a+b\arcsin(cx))} - \frac{\cos\left(\frac{5a}{b} - \frac{5(a+b\arcsin(cx))}{b}\right)}{64(a+b\arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b\arcsin(cx))}{b}\right)}{64(a+b\arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{128(a+b\arcsin(cx))} \right) \\
 & \quad \downarrow \text{2009} \\
 & \frac{x^3(1-c^2x^2)^3}{bc(a+b\arcsin(cx))}
 \end{aligned}$$

$$\frac{3\left(\frac{5}{64} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{64} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{3}{64} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) - \frac{3}{64} \cos\left(\frac{7a}{b}\right) \operatorname{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right) - \frac{3}{64} \cos\left(\frac{9a}{b}\right) \operatorname{CosIntegral}\left(\frac{9(a+b \arcsin(cx))}{b}\right)\right)}{bc(a+b \arcsin(cx))} - \frac{x^3(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}$$

input `Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]`

output `-((x^3*(1 - c^2*x^2)^3)/(b*c*(a + b*ArcSin[c*x]))) + (3*((5*Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/64 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (3*Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x])/b])/64 - (Cos[(7*a)/b]*CosIntegral[(7*(a + b*ArcSin[c*x])/b])/64 + (5*Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/64 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (3*Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/64 - (Sin[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x])/b])/64))/b^2*c^4 - (9*((3*Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/128 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x])/b])/64 + (Cos[(7*a)/b]*CosIntegral[(7*(a + b*ArcSin[c*x])/b])/256 + (Cos[(9*a)/b]*CosIntegral[(9*(a + b*ArcSin[c*x])/b])/256 + (3*Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/128 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/64 - (Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/64 + (Sin[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x])/b])/256 + (Sin[(9*a)/b]*SinIntegral[(9*(a + b*ArcSin[c*x])/b])/256))/b^2*c^4)`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*
((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p
- 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n
+ 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2
)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.64

method	result
default	$-\frac{9 \arcsin(cx) \operatorname{Si}(9 \arcsin(cx) + \frac{9a}{b}) \sin(\frac{9a}{b}) b + 9 \arcsin(cx) \operatorname{Ci}(9 \arcsin(cx) + \frac{9a}{b}) \cos(\frac{9a}{b}) b - 24 \arcsin(cx) \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) b + 24 \arcsin(cx) \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) b + 21 \arcsin(cx) \operatorname{Si}(7 \arcsin(cx) + \frac{7a}{b}) \sin(\frac{7a}{b}) b + 21 \arcsin(cx) \operatorname{Ci}(7 \arcsin(cx) + \frac{7a}{b}) \cos(\frac{7a}{b}) b - 6 \arcsin(cx) \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b}) b - 6 \arcsin(cx) \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) b + 9 \operatorname{Si}(9 \arcsin(cx) + \frac{9a}{b}) \sin(\frac{9a}{b}) a + 9 \operatorname{Ci}(9 \arcsin(cx) + \frac{9a}{b}) \cos(\frac{9a}{b}) a - 24 \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) a - 24 \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) a + 21 \operatorname{Si}(7 \arcsin(cx) + \frac{7a}{b}) \sin(\frac{7a}{b}) a + 21 \operatorname{Ci}(7 \arcsin(cx) + \frac{7a}{b}) \cos(\frac{7a}{b}) a - 6 \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b}) a - 6 \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) a + 6 x b c - \sin(9 \arcsin(cx)) b + 8 \sin(3 \arcsin(cx)) b - 3 \sin(7 \arcsin(cx)) b}{(a + b \arcsin(cx)) b^2}$

input

```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/256/c^4*(9*arcsin(c*x)*Si(9*arcsin(c*x)+9*a/b)*sin(9*a/b)*b+9*arcsin(c*
x)*Ci(9*arcsin(c*x)+9*a/b)*cos(9*a/b)*b-24*arcsin(c*x)*Si(3*arcsin(c*x)+3*
a/b)*sin(3*a/b)*b-24*arcsin(c*x)*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b+21*a
rcsin(c*x)*Si(7*arcsin(c*x)+7*a/b)*sin(7*a/b)*b+21*arcsin(c*x)*cos(7*a/b)*
Ci(7*arcsin(c*x)+7*a/b)*b-6*arcsin(c*x)*Si(arcsin(c*x)+a/b)*sin(a/b)*b-6*a
rcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+9*Si(9*arcsin(c*x)+9*a/b)*sin(9*
a/b)*a+9*Ci(9*arcsin(c*x)+9*a/b)*cos(9*a/b)*a-24*Si(3*arcsin(c*x)+3*a/b)*s
in(3*a/b)*a-24*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*a+21*Si(7*arcsin(c*x)+7*
a/b)*sin(7*a/b)*a+21*cos(7*a/b)*Ci(7*arcsin(c*x)+7*a/b)*a-6*Si(arcsin(c*x)
+a/b)*sin(a/b)*a-6*Ci(arcsin(c*x)+a/b)*cos(a/b)*a+6*x*b*c-sin(9*arcsin(c*x
))*b+8*sin(3*arcsin(c*x))*b-3*sin(7*arcsin(c*x))*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}x^3}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^3(-(cx - 1)(cx + 1))^{\frac{5}{2}}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**3*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}x^3}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output

```
(c^6*x^9 - 3*c^4*x^7 + 3*c^2*x^5 - x^3 - (b^2*c*arctan2(c*x, sqrt(c*x + 1)
*sqrt(-c*x + 1)) + a*b*c)*integrate(3*(3*c^6*x^8 - 7*c^4*x^6 + 5*c^2*x^4 -
x^2)/(b^2*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c), x))/(b^2
*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2479 vs. $2(260) = 520$.

Time = 0.26 (sec) , antiderivative size = 2479, normalized size of antiderivative = 8.92

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input

```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
-9*b*arcsin(c*x)*cos(a/b)^9*cos_integral(9*a/b + 9*arcsin(c*x))/(b^3*c^4*a
rcsin(c*x) + a*b^2*c^4) - 9*b*arcsin(c*x)*cos(a/b)^8*sin(a/b)*sin_integral
(9*a/b + 9*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 9*a*cos(a/b)^9
*cos_integral(9*a/b + 9*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 9
*a*cos(a/b)^8*sin(a/b)*sin_integral(9*a/b + 9*arcsin(c*x))/(b^3*c^4*arcsin
(c*x) + a*b^2*c^4) + 81/4*b*arcsin(c*x)*cos(a/b)^7*cos_integral(9*a/b + 9*
arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 21/4*b*arcsin(c*x)*cos(a/
b)^7*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4)
+ 63/4*b*arcsin(c*x)*cos(a/b)^6*sin(a/b)*sin_integral(9*a/b + 9*arcsin(c*
x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 21/4*b*arcsin(c*x)*cos(a/b)^6*sin(
a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4)
+ 81/4*a*cos(a/b)^7*cos_integral(9*a/b + 9*arcsin(c*x))/(b^3*c^4*arcsin(c
*x) + a*b^2*c^4) - 21/4*a*cos(a/b)^7*cos_integral(7*a/b + 7*arcsin(c*x))/(
b^3*c^4*arcsin(c*x) + a*b^2*c^4) + 63/4*a*cos(a/b)^6*sin(a/b)*sin_integral
(9*a/b + 9*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 21/4*a*cos(a/b)
^6*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*
b^2*c^4) - 243/16*b*arcsin(c*x)*cos(a/b)^5*cos_integral(9*a/b + 9*arcsin(c
*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) + 147/16*b*arcsin(c*x)*cos(a/b)^5*c
os_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^4*arcsin(c*x) + a*b^2*c^4) - 135
/16*b*arcsin(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(9*a/b + 9*arcsin(c*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \sin(cx))^2} dx$$

input `int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2,x)`

output `int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\begin{aligned} \int \frac{x^3(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx &= \left(\int \frac{\sqrt{-c^2x^2 + 1} x^7}{\sin^2(cx) b^2 + 2\sin(cx) ab + a^2} dx \right) c^4 \\ &\quad - 2 \left(\int \frac{\sqrt{-c^2x^2 + 1} x^5}{\sin^2(cx) b^2 + 2\sin(cx) ab + a^2} dx \right) c^2 \\ &\quad + \int \frac{\sqrt{-c^2x^2 + 1} x^3}{\sin^2(cx) b^2 + 2\sin(cx) ab + a^2} dx \end{aligned}$$

input `int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x))^2,x)`

output `int((sqrt(-c**2*x**2 + 1)*x**7)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(-c**2*x**2 + 1)*x**5)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.366 $\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3233
Mathematica [A] (verified)	3234
Rubi [A] (verified)	3234
Maple [A] (verified)	3237
Fricas [F]	3237
Sympy [F]	3238
Maxima [F]	3238
Giac [B] (verification not implemented)	3239
Mupad [F(-1)]	3240
Reduce [F]	3240

Optimal result

Integrand size = 28, antiderivative size = 282

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx = -\frac{x^2(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{4a}{b}\right)}{8b^2c^3} - \frac{3 \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\text{CosIntegral}\left(\frac{8(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{8a}{b}\right)}{16b^2c^3} - \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{16b^2c^3} + \frac{\cos\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{8b^2c^3} + \frac{3 \cos\left(\frac{6a}{b}\right) \text{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{16b^2c^3} + \frac{\cos\left(\frac{8a}{b}\right) \text{Si}\left(\frac{8(a+b \arcsin(cx))}{b}\right)}{16b^2c^3}$$

output

```
-x^2*(-c^2*x^2+1)^3/b/c/(a+b*arcsin(c*x))+1/16*Ci(2*(a+b*arcsin(c*x))/b)*sin(2*a/b)/b^2/c^3-1/8*Ci(4*(a+b*arcsin(c*x))/b)*sin(4*a/b)/b^2/c^3-3/16*Ci(6*(a+b*arcsin(c*x))/b)*sin(6*a/b)/b^2/c^3-1/16*Ci(8*(a+b*arcsin(c*x))/b)*sin(8*a/b)/b^2/c^3-1/16*cos(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b^2/c^3+1/8*cos(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b^2/c^3+3/16*cos(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b^2/c^3+1/16*cos(8*a/b)*Si(8*(a+b*arcsin(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.47

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx = \frac{-16bc^2x^2 + 48bc^4x^4 - 48bc^6x^6 + 16bc^8x^8 + (a+b\arcsin(cx))\operatorname{CosIntegral}(2\arcsin(cx))}{(a+b\arcsin(cx))^2}$$

input `Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]`

output
$$\frac{(-16*b*c^2*x^2 + 48*b*c^4*x^4 - 48*b*c^6*x^6 + 16*b*c^8*x^8 + (a + b*ArcSin[c*x])*CosIntegral[2*(a/b + ArcSin[c*x])]*Sin[(2*a)/b] - 2*(a + b*ArcSin[c*x])*CosIntegral[4*(a/b + ArcSin[c*x])]*Sin[(4*a)/b] - 3*a*CosIntegral[6*(a/b + ArcSin[c*x])]*Sin[(6*a)/b] - 3*b*ArcSin[c*x]*CosIntegral[6*(a/b + ArcSin[c*x])]*Sin[(6*a)/b] - a*CosIntegral[8*(a/b + ArcSin[c*x])]*Sin[(8*a)/b] - b*ArcSin[c*x]*CosIntegral[8*(a/b + ArcSin[c*x])]*Sin[(8*a)/b] - a*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] - b*ArcSin[c*x]*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 2*a*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + 2*b*ArcSin[c*x]*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + 3*a*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])] + 3*b*ArcSin[c*x]*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])] + a*Cos[(8*a)/b]*SinIntegral[8*(a/b + ArcSin[c*x])] + b*ArcSin[c*x]*Cos[(8*a)/b]*SinIntegral[8*(a/b + ArcSin[c*x])])/(16*b^2*c^3*(a + b*ArcSin[c*x]))$$

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.43, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5214, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx$$

↓ 5214

$$\frac{2 \int \frac{x(1-c^2x^2)^2}{a+b \arcsin(cx)} dx}{bc} - \frac{8c \int \frac{x^3(1-c^2x^2)^2}{a+b \arcsin(cx)} dx}{b} - \frac{x^2(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}$$

↓ 5224

$$\frac{8 \int -\frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2c^3} +$$

$$\frac{2 \int -\frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2c^3} - \frac{x^2(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}$$

↓ 25

$$\frac{8 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2c^3} -$$

$$\frac{2 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2c^3} - \frac{x^2(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}$$

↓ 4906

$$8 \int \left(-\frac{\sin\left(\frac{8a}{b} - \frac{8(a+b \arcsin(cx))}{b}\right)}{128(a+b \arcsin(cx))} - \frac{\sin\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{3 \sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))$$

$$2 \int \left(\frac{\sin\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} + \frac{5 \sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))$$

$$\frac{x^2(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}$$

↓ 2009

$$2 \left(-\frac{5}{32} \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{8} \sin\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) - \frac{1}{32} \sin\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) \right)$$

$$8 \left(-\frac{3}{64} \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{64} \sin\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{64} \sin\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) \right)$$

$$\frac{x^2(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}$$

input

Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]

output

```

-((x^2*(1 - c^2*x^2)^3)/(b*c*(a + b*ArcSin[c*x]))) + (2*((-5*CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b])/32 - (CosIntegral[(4*(a + b*ArcSin[c*x]))/b]*Sin[(4*a)/b])/8 - (CosIntegral[(6*(a + b*ArcSin[c*x]))/b]*Sin[(6*a)/b])/32 + (5*Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 + (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8 + (Cos[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x]))/b])/32)/(b^2*c^3) - (8*((-3*CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b])/64 - (CosIntegral[(4*(a + b*ArcSin[c*x]))/b]*Sin[(4*a)/b])/64 + (CosIntegral[(6*(a + b*ArcSin[c*x]))/b]*Sin[(6*a)/b])/64 + (CosIntegral[(8*(a + b*ArcSin[c*x]))/b]*Sin[(8*a)/b])/128 + (3*Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/64 + (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/64 - (Cos[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x]))/b])/64 - (Cos[(8*a)/b]*SinIntegral[(8*(a + b*ArcSin[c*x]))/b])/128))/(b^2*c^3)

```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.70

method	result
default	$-\frac{16 \arcsin(cx) \sin\left(\frac{4a}{b}\right) \text{Ci}\left(4 \arcsin(cx) + \frac{4a}{b}\right) b - 8 \arcsin(cx) \text{Si}\left(8 \arcsin(cx) + \frac{8a}{b}\right) \cos\left(\frac{8a}{b}\right) b + 8 \arcsin(cx) \text{Ci}\left(8 \arcsin(cx) + \frac{8a}{b}\right) \sin\left(\frac{8a}{b}\right) b - 8 \arcsin(cx) \text{Si}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) b + 8 \arcsin(cx) \text{Ci}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) b - 24 \arcsin(cx) \text{Si}\left(6 \arcsin(cx) + \frac{6a}{b}\right) \cos\left(\frac{6a}{b}\right) b + 24 \arcsin(cx) \text{Ci}\left(6 \arcsin(cx) + \frac{6a}{b}\right) \sin\left(\frac{6a}{b}\right) b - 16 \arcsin(cx) \text{Si}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right) b + 16 \arcsin(cx) \text{Ci}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \sin\left(\frac{4a}{b}\right) b - 8 \arcsin(cx) \text{Si}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) a + 8 \arcsin(cx) \text{Ci}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) a - 24 \arcsin(cx) \text{Si}\left(6 \arcsin(cx) + \frac{6a}{b}\right) \cos\left(\frac{6a}{b}\right) a + 24 \arcsin(cx) \text{Ci}\left(6 \arcsin(cx) + \frac{6a}{b}\right) \sin\left(\frac{6a}{b}\right) a - 16 \arcsin(cx) \text{Si}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right) a + 16 \arcsin(cx) \text{Ci}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \sin\left(\frac{4a}{b}\right) a - 8 \arcsin(cx) \text{Si}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) b + 4 \arcsin(cx) \text{Ci}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) b - 4 \arcsin(cx) \text{Si}\left(6 \arcsin(cx) + \frac{6a}{b}\right) \cos\left(\frac{6a}{b}\right) b + 4 \arcsin(cx) \text{Ci}\left(6 \arcsin(cx) + \frac{6a}{b}\right) \sin\left(\frac{6a}{b}\right) b - 4 \arcsin(cx) \text{Si}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right) b + 4 \arcsin(cx) \text{Ci}\left(4 \arcsin(cx) + \frac{4a}{b}\right) \sin\left(\frac{4a}{b}\right) b - 4 \arcsin(cx) \text{Si}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) b + 4 \arcsin(cx) \text{Ci}\left(2 \arcsin(cx) + \frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) b}{(a + b \arcsin(cx))^2}$

input

```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/128/c^3*(16*arcsin(c*x)*sin(4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*b-8*arcsin(c
*x)*Si(8*arcsin(c*x)+8*a/b)*cos(8*a/b)*b+8*arcsin(c*x)*Ci(8*arcsin(c*x)+8*
a/b)*sin(8*a/b)*b+8*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*b-8*arc
sin(c*x)*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*b-24*arcsin(c*x)*Si(6*arcsin(c
*x)+6*a/b)*cos(6*a/b)*b+24*arcsin(c*x)*Ci(6*arcsin(c*x)+6*a/b)*sin(6*a/b)*
b-16*arcsin(c*x)*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*b+16*arcsin(c*x)*Ci(4*a
rcsin(c*x)+4*a/b)*sin(4*a/b)*b-8*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a+8*Ci(8*arcsin(c
*x)+8*a/b)*sin(8*a/b)*a+8*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a-8*Ci(2*arcs
in(c*x)+2*a/b)*sin(2*a/b)*a-24*Si(6*arcsin(c*x)+6*a/b)*cos(6*a/b)*a+24*Ci(
6*arcsin(c*x)+6*a/b)*sin(6*a/b)*a-16*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*a-
cos(8*arcsin(c*x))*b+4*cos(2*arcsin(c*x))*b-4*cos(6*arcsin(c*x))*b-4*cos(4
*arcsin(c*x))*b+5*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{5/2} x^2}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^2(-(cx - 1)(cx + 1))^{5/2}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**2*(-(c*x - 1)*(c*x + 1))**5/2/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x^2}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^6*x^8 - 3*c^4*x^6 + 3*c^2*x^4 - x^2 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(2*(4*c^6*x^7 - 9*c^4*x^5 + 6*c^2*x^3 - x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c, x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2461 vs. $2(264) = 528$.

Time = 0.25 (sec) , antiderivative size = 2461, normalized size of antiderivative = 8.73

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
-8*b*arcsin(c*x)*cos(a/b)^7*cos_integral(8*a/b + 8*arcsin(c*x))*sin(a/b)/(
b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 8*b*arcsin(c*x)*cos(a/b)^8*sin_integral
(8*a/b + 8*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 8*a*cos(a/b)^7
*cos_integral(8*a/b + 8*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2
*c^3) + 8*a*cos(a/b)^8*sin_integral(8*a/b + 8*arcsin(c*x))/(b^3*c^3*arcsin
(c*x) + a*b^2*c^3) + 12*b*arcsin(c*x)*cos(a/b)^5*cos_integral(8*a/b + 8*ar
csin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 6*b*arcsin(c*x)*co
s(a/b)^5*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x)
+ a*b^2*c^3) - 16*b*arcsin(c*x)*cos(a/b)^6*sin_integral(8*a/b + 8*arcsin(
c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 6*b*arcsin(c*x)*cos(a/b)^6*sin_i
ntegral(6*a/b + 6*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 12*a*co
s(a/b)^5*cos_integral(8*a/b + 8*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x)
+ a*b^2*c^3) - 6*a*cos(a/b)^5*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b
)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 16*a*cos(a/b)^6*sin_integral(8*a/b +
8*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 6*a*cos(a/b)^6*sin_int
egral(6*a/b + 6*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 5*b*arcsi
n(c*x)*cos(a/b)^3*cos_integral(8*a/b + 8*arcsin(c*x))*sin(a/b)/(b^3*c^3*ar
csin(c*x) + a*b^2*c^3) + 6*b*arcsin(c*x)*cos(a/b)^3*cos_integral(6*a/b + 6
*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - b*arcsin(c*x)*c
os(a/b)^3*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x^2(1 - c^2x^2)^{5/2}}{(a + b \sin(cx))^2} dx$$

input `int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2,x)`

output `int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\begin{aligned} \int \frac{x^2(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx &= \left(\int \frac{\sqrt{-c^2x^2 + 1} x^6}{\sin^2(cx) b^2 + 2\sin(cx) ab + a^2} dx \right) c^4 \\ &\quad - 2 \left(\int \frac{\sqrt{-c^2x^2 + 1} x^4}{\sin^2(cx) b^2 + 2\sin(cx) ab + a^2} dx \right) c^2 \\ &\quad + \int \frac{\sqrt{-c^2x^2 + 1} x^2}{\sin^2(cx) b^2 + 2\sin(cx) ab + a^2} dx \end{aligned}$$

input `int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x))^2,x)`

output `int((sqrt(-c**2*x**2 + 1)*x**6)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(-c**2*x**2 + 1)*x**4)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.367 $\int \frac{x(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3241
Mathematica [A] (verified)	3242
Rubi [A] (verified)	3243
Maple [A] (verified)	3246
Fricas [F]	3247
Sympy [F]	3247
Maxima [F]	3247
Giac [B] (verification not implemented)	3248
Mupad [F(-1)]	3249
Reduce [F]	3249

Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx = -\frac{x(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} + \frac{5 \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{64b^2c^2} + \frac{27 \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{64b^2c^2} + \frac{25 \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{64b^2c^2} + \frac{7 \cos\left(\frac{7a}{b}\right) \operatorname{CosIntegral}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{64b^2c^2} + \frac{5 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{64b^2c^2} + \frac{27 \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{64b^2c^2} + \frac{25 \sin\left(\frac{5a}{b}\right) \operatorname{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{64b^2c^2} + \frac{7 \sin\left(\frac{7a}{b}\right) \operatorname{Si}\left(\frac{7(a+b \arcsin(cx))}{b}\right)}{64b^2c^2}$$

output

```
-x*(-c^2*x^2+1)^3/b/c/(a+b*arcsin(c*x))+5/64*cos(a/b)*Ci((a+b*arcsin(c*x))
/b)/b^2/c^2+27/64*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^2+25/64*cos(5
*a/b)*Ci(5*(a+b*arcsin(c*x))/b)/b^2/c^2+7/64*cos(7*a/b)*Ci(7*(a+b*arcsin(c
*x))/b)/b^2/c^2+5/64*sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^2+27/64*sin(3*
a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^2+25/64*sin(5*a/b)*Si(5*(a+b*arcsin(c
*x))/b)/b^2/c^2+7/64*sin(7*a/b)*Si(7*(a+b*arcsin(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.46

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx = \frac{-64bcx + 192bc^3x^3 - 192bc^5x^5 + 64bc^7x^7 + 5(a+b\arcsin(cx))\cos\left(\frac{a}{b}\right)\text{CosInt}}{(a+b\arcsin(cx))^2}$$

input

```
Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```
(-64*b*c*x + 192*b*c^3*x^3 - 192*b*c^5*x^5 + 64*b*c^7*x^7 + 5*(a + b*ArcSi
n[c*x])*Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] + 27*(a + b*ArcSin[c*x])*C
os[(3*a)/b]*CosIntegral[3*(a/b + ArcSin[c*x])] + 25*a*Cos[(5*a)/b]*CosInte
gral[5*(a/b + ArcSin[c*x])] + 25*b*ArcSin[c*x]*Cos[(5*a)/b]*CosIntegral[5*
(a/b + ArcSin[c*x])] + 7*a*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcSin[c*x])]
+ 7*b*ArcSin[c*x]*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcSin[c*x])] + 5*a*S
in[a/b]*SinIntegral[a/b + ArcSin[c*x]] + 5*b*ArcSin[c*x]*Sin[a/b]*SinInteg
ral[a/b + ArcSin[c*x]] + 27*a*Ssin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x]
)] + 27*b*ArcSin[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] + 2
5*a*Ssin[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])] + 25*b*ArcSin[c*x]*Sin
[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])] + 7*a*Ssin[(7*a)/b]*SinIntegra
l[7*(a/b + ArcSin[c*x])] + 7*b*ArcSin[c*x]*Sin[(7*a)/b]*SinIntegral[7*(a/b
+ ArcSin[c*x])]/(64*b^2*c^2*(a + b*ArcSin[c*x]))
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5214, 5168, 3042, 3793, 2009, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & \frac{\int \frac{(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{bc} - \frac{7c \int \frac{x^2(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{b} - \frac{x(1-c^2x^2)^3}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{5168} \\
 & \frac{\int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^2} - \frac{7c \int \frac{x^2(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{b} - \frac{x(1-c^2x^2)^3}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b} + \frac{\pi}{2}\right)^5}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^2} - \frac{7c \int \frac{x^2(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{b} - \frac{x(1-c^2x^2)^3}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3793} \\
 & \frac{\int \left(\frac{\cos\left(\frac{5a}{b} - \frac{5(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{5 \cos\left(\frac{3a}{b} - \frac{3(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{5 \cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{8(a+b\arcsin(cx))} \right) d(a+b\arcsin(cx))}{b^2c^2} \\
 & \quad \downarrow \text{2009} \\
 & \frac{7c \int \frac{x^2(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{b} - \frac{x(1-c^2x^2)^3}{bc(a+b\arcsin(cx))}
 \end{aligned}$$

$$\begin{aligned}
& -\frac{7c \int \frac{x^2(1-c^2x^2)^2}{a+b \arcsin(cx)} dx}{b} + \\
& \frac{\frac{5}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{b^2c^2} \\
& \frac{x(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{5224} \\
& -\frac{7 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2c^2} + \\
& \frac{\frac{5}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{b^2c^2} \\
& \frac{x(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{4906} \\
& -\frac{7 \int \left(-\frac{\cos\left(\frac{7a}{b} - \frac{7(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} - \frac{3 \cos\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{64(a+b \arcsin(cx))} + \frac{5 \cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{64(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2c^2} \\
& \frac{\frac{5}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{b^2c^2} \\
& \frac{x(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{2009} \\
& \frac{\frac{5}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{b^2c^2} \\
& \frac{7\left(\frac{5}{64} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{64} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) - \frac{3}{64} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)\right)}{b^2c^2} \\
& \frac{x(1-c^2x^2)^3}{bc(a+b \arcsin(cx))}
\end{aligned}$$

input

```
Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcSin[c*x])^2,x]
```

output

```

-((x*(1 - c^2*x^2)^3)/(b*c*(a + b*ArcSin[c*x]))) + ((5*Cos[a/b]*CosIntegral[
(a + b*ArcSin[c*x])/b])/8 + (5*Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin
[c*x]))/b])/16 + (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x]))/b])/16
+ (5*Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 + (5*Sin[(3*a)/b]*SinI
ntegral[(3*(a + b*ArcSin[c*x]))/b])/16 + (Sin[(5*a)/b]*SinIntegral[(5*(a +
b*ArcSin[c*x]))/b])/16)/(b^2*c^2) - (7*((5*Cos[a/b]*CosIntegral[(a + b*Ar
cSin[c*x])/b])/64 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x]))/b])/
64 - (3*Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x]))/b])/64 - (Cos[(7*
a)/b]*CosIntegral[(7*(a + b*ArcSin[c*x]))/b])/64 + (5*Sin[a/b]*SinIntegral
[(a + b*ArcSin[c*x])/b])/64 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c
*x]))/b])/64 - (3*Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x]))/b])/64
- (Sin[(7*a)/b]*SinIntegral[(7*(a + b*ArcSin[c*x]))/b])/64))/(b^2*c^2)

```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_)^(m_))*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_)^(m_.))*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5168

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[
x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b
, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*
((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p
- 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n
+ 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2
)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.65

method	result
default	$\frac{7 \arcsin(cx) \operatorname{Si}(7 \arcsin(cx) + \frac{7a}{b}) \sin(\frac{7a}{b}) b + 7 \arcsin(cx) \cos(\frac{7a}{b}) \operatorname{Ci}(7 \arcsin(cx) + \frac{7a}{b}) b + 25 \arcsin(cx) \operatorname{Si}(5 \arcsin(cx) + \frac{5a}{b}) \sin(\frac{5a}{b})}{(a + b \arcsin(cx))^2}$

input

```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/64/c^2*(7*arcsin(c*x)*Si(7*arcsin(c*x)+7*a/b)*sin(7*a/b)*b+7*arcsin(c*x)
*cos(7*a/b)*Ci(7*arcsin(c*x)+7*a/b)*b+25*arcsin(c*x)*Si(5*arcsin(c*x)+5*a/
b)*sin(5*a/b)*b+25*arcsin(c*x)*Ci(5*arcsin(c*x)+5*a/b)*cos(5*a/b)*b+27*arc
sin(c*x)*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*b+27*arcsin(c*x)*Ci(3*arcsin(c
*x)+3*a/b)*cos(3*a/b)*b+5*arcsin(c*x)*Si(arcsin(c*x)+a/b)*sin(a/b)*b+5*arc
sin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+7*Si(7*arcsin(c*x)+7*a/b)*sin(7*a/
b)*a+7*cos(7*a/b)*Ci(7*arcsin(c*x)+7*a/b)*a+25*Si(5*arcsin(c*x)+5*a/b)*sin
(5*a/b)*a+25*Ci(5*arcsin(c*x)+5*a/b)*cos(5*a/b)*a+27*Si(3*arcsin(c*x)+3*a/
b)*sin(3*a/b)*a+27*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*a+5*Si(arcsin(c*x)+a
/b)*sin(a/b)*a+5*Ci(arcsin(c*x)+a/b)*cos(a/b)*a-5*x*b*c-sin(7*arcsin(c*x))
*b-5*sin(5*arcsin(c*x))*b-9*sin(3*arcsin(c*x))*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2), x)`

Sympy [F]

$$\int \frac{x(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x(-cx - 1)(cx + 1)^{5/2}}{(a + b \arcsin(cx))^2} dx$$

input `integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x*(-(c*x - 1)*(c*x + 1))**5/2/(a + b*asin(c*x))**2, x)`

Maxima [F]

$$\int \frac{x(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2x^2 + 1)^{5/2}x}{(b \arcsin(cx) + a)^2} dx$$

input `integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^6*x^7 - 3*c^4*x^5 + 3*c^2*x^3 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate((7*c^6*x^6 - 15*c^4*x^4 + 9*c^2*x^2 - 1)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x) - x/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. $2(258) = 516$.

Time = 0.26 (sec) , antiderivative size = 2026, normalized size of antiderivative = 7.34

$$\int \frac{x(1 - c^2x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
7*b*arcsin(c*x)*cos(a/b)^7*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 7*b*arcsin(c*x)*cos(a/b)^6*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 7*a*cos(a/b)^7*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 7*a*cos(a/b)^6*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 49/4*b*arcsin(c*x)*cos(a/b)^5*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 25/4*b*arcsin(c*x)*cos(a/b)^5*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 35/4*b*arcsin(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 25/4*b*arcsin(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 49/4*a*cos(a/b)^5*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 25/4*a*cos(a/b)^5*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 35/4*a*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 25/4*a*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + (c^2*x^2 - 1)^3*b*c*x/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 49/8*b*arcsin(c*x)*cos(a/b)^3*cos_integral(7*a/b + 7*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - 125/16*b*arcsin(c*x)*cos(a/b)^3*cos_integral(5*a/b + 5*arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + 27/16*b*arcsin(c*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{x(1 - c^2 x^2)^{5/2}}{(a + b \sin(cx))^2} dx$$

input `int((x*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2,x)`

output `int((x*(1 - c^2*x^2)^(5/2))/(a + b*asin(c*x))^2, x)`

Reduce [F]

$$\begin{aligned} \int \frac{x(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx &= \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^5}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^4 \\ &\quad - 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^3}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^2 \\ &\quad + \int \frac{\sqrt{-c^2 x^2 + 1} x}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \end{aligned}$$

input `int(x*(-c^2*x^2+1)^(5/2)/(a+b*asin(c*x))^2,x)`

output `int((sqrt(-c**2*x**2 + 1)*x**5)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)`

3.368 $\int \frac{(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx$

Optimal result	3250
Mathematica [A] (verified)	3251
Rubi [A] (verified)	3251
Maple [A] (verified)	3253
Fricas [F]	3254
Sympy [F]	3254
Maxima [F]	3255
Giac [B] (verification not implemented)	3255
Mupad [F(-1)]	3256
Reduce [F]	3257

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b \arcsin(cx))^2} dx = -\frac{(1-c^2x^2)^3}{bc(a+b \arcsin(cx))} + \frac{15 \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{16b^2c} + \frac{3 \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{4a}{b}\right)}{4b^2c} + \frac{3 \operatorname{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{6a}{b}\right)}{16b^2c} - \frac{15 \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{16b^2c} - \frac{3 \cos\left(\frac{4a}{b}\right) \operatorname{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{4b^2c} - \frac{3 \cos\left(\frac{6a}{b}\right) \operatorname{Si}\left(\frac{6(a+b \arcsin(cx))}{b}\right)}{16b^2c}$$

output

```
-(-c^2*x^2+1)^3/b/c/(a+b*arcsin(c*x))+15/16*Ci(2*(a+b*arcsin(c*x))/b)*sin(2*a/b)/b^2/c+3/4*Ci(4*(a+b*arcsin(c*x))/b)*sin(4*a/b)/b^2/c+3/16*Ci(6*(a+b*arcsin(c*x))/b)*sin(6*a/b)/b^2/c-15/16*cos(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b^2/c-3/4*cos(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b^2/c-3/16*cos(6*a/b)*Si(6*(a+b*arcsin(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.43

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \frac{16b - 48bc^2 x^2 + 48bc^4 x^4 - 16bc^6 x^6 - 15(a + b \arcsin(cx)) \operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{2a}{b}\right) - 12(a + b \arcsin(cx)) \operatorname{SinIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) \cos\left(\frac{2a}{b}\right)}{(b^2 c^2 (a + b \arcsin(cx)))^2}$$

input

```
Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/16*(16*b - 48*b*c^2*x^2 + 48*b*c^4*x^4 - 16*b*c^6*x^6 - 15*(a + b*ArcSin[c*x])*CosIntegral[2*(a/b + ArcSin[c*x])]*Sin[(2*a)/b] - 12*(a + b*ArcSin[c*x])*CosIntegral[4*(a/b + ArcSin[c*x])]*Sin[(4*a)/b] - 3*a*CosIntegral[6*(a/b + ArcSin[c*x])]*Sin[(6*a)/b] - 3*b*ArcSin[c*x]*CosIntegral[6*(a/b + ArcSin[c*x])]*Sin[(6*a)/b] + 15*a*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 15*b*ArcSin[c*x]*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] + 12*a*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + 12*b*ArcSin[c*x]*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcSin[c*x])] + 3*a*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])] + 3*b*ArcSin[c*x]*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcSin[c*x])])/(b^2*c*(a + b*ArcSin[c*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5166, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx$$

↓ 5166

$$-\frac{6c \int \frac{x(1 - c^2 x^2)^2}{a + b \arcsin(cx)} dx}{b} - \frac{(1 - c^2 x^2)^3}{bc(a + b \arcsin(cx))}$$

$$\begin{array}{c}
\downarrow 5224 \\
\frac{6 \int -\frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c} - \frac{(1-c^2 x^2)^3}{bc(a+b \arcsin(cx))} \\
\downarrow 25 \\
\frac{6 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c} - \frac{(1-c^2 x^2)^3}{bc(a+b \arcsin(cx))} \\
\downarrow 4906 \\
\frac{6 \int \left(\frac{\sin\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} + \frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} + \frac{5 \sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{\frac{b^2 c}{bc(a+b \arcsin(cx))} (1-c^2 x^2)^3} \\
\downarrow 2009 \\
\frac{6 \left(-\frac{5}{32} \sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \frac{1}{8} \sin\left(\frac{4a}{b}\right) \text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) - \frac{1}{32} \sin\left(\frac{6a}{b}\right) \text{CosIntegral}\left(\frac{6(a+b \arcsin(cx))}{b}\right) \right)}{(1-c^2 x^2)^3} \\
bc(a+b \arcsin(cx))
\end{array}$$

input

```
Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcSin[c*x])^2,x]
```

output

```
-((1 - c^2*x^2)^3/(b*c*(a + b*ArcSin[c*x]))) - (6*((-5*CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b])/32 - (CosIntegral[(4*(a + b*ArcSin[c*x]))/b]*Sin[(4*a)/b])/8 - (CosIntegral[(6*(a + b*ArcSin[c*x]))/b]*Sin[(6*a)/b])/32 + (5*Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/32 + (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8 + (Cos[(6*a)/b]*SinIntegral[(6*(a + b*ArcSin[c*x]))/b])/32))/(b^2*c)
```

Definitions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5166 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*(a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1)), x] + Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.68

method	result
default	$-\frac{6 \arcsin(cx) \operatorname{Si}(6 \arcsin(cx) + \frac{6a}{b}) \cos(\frac{6a}{b}) b - 6 \arcsin(cx) \operatorname{Ci}(6 \arcsin(cx) + \frac{6a}{b}) \sin(\frac{6a}{b}) b + 24 \arcsin(cx) \operatorname{Si}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) b - 24 \arcsin(cx) \operatorname{Ci}(4 \arcsin(cx) + \frac{4a}{b}) \sin(\frac{4a}{b}) b}{(c^2 x^2 + 1)^{5/2} (a + b \arcsin(cx))^2}$

input `int((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output

```
-1/32/c*(6*arcsin(c*x)*Si(6*arcsin(c*x)+6*a/b)*cos(6*a/b)*b-6*arcsin(c*x)*
Ci(6*arcsin(c*x)+6*a/b)*sin(6*a/b)*b+24*arcsin(c*x)*Si(4*arcsin(c*x)+4*a/b
)*cos(4*a/b)*b-24*arcsin(c*x)*sin(4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*b+30*arcs
in(c*x)*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*b-30*arcsin(c*x)*Ci(2*arcsin(c*
x)+2*a/b)*sin(2*a/b)*b+6*Si(6*arcsin(c*x)+6*a/b)*cos(6*a/b)*a-6*Ci(6*arcsi
n(c*x)+6*a/b)*sin(6*a/b)*a+24*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*a-24*sin(
4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*a+30*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a-3
0*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*a+cos(6*arcsin(c*x))*b+6*cos(4*arcsin
(c*x))*b+15*cos(2*arcsin(c*x))*b+10*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2} dx$$

input

```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*arcsin(c*x)^2 +
2*a*b*arcsin(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{5/2}}{(a + b \arcsin(cx))^2} dx$$

input

```
integrate((-c**2*x**2+1)**(5/2)/(a+b*asin(c*x))**2,x)
```

output

```
Integral((-c*x - 1)*(c*x + 1)**(5/2)/(a + b*asin(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(6*(c^5*x^5 - 2*c^3*x^3 + c*x)/(b^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b), x) - 1)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. 2(203) = 406.

Time = 0.23 (sec) , antiderivative size = 1394, normalized size of antiderivative = 6.42

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate((-c^2*x^2+1)^(5/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```

6*b*arcsin(c*x)*cos(a/b)^5*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(b
^3*c*arcsin(c*x) + a*b^2*c) - 6*b*arcsin(c*x)*cos(a/b)^6*sin_integral(6*a/
b + 6*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c) + 6*a*cos(a/b)^5*cos_inte
gral(6*a/b + 6*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) - 6*a*c
os(a/b)^6*sin_integral(6*a/b + 6*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c
) - 6*b*arcsin(c*x)*cos(a/b)^3*cos_integral(6*a/b + 6*arcsin(c*x))*sin(a/b
)/(b^3*c*arcsin(c*x) + a*b^2*c) + 6*b*arcsin(c*x)*cos(a/b)^3*cos_integral(
4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 9*b*arcsin
(c*x)*cos(a/b)^4*sin_integral(6*a/b + 6*arcsin(c*x))/(b^3*c*arcsin(c*x) +
a*b^2*c) - 6*b*arcsin(c*x)*cos(a/b)^4*sin_integral(4*a/b + 4*arcsin(c*x))/
(b^3*c*arcsin(c*x) + a*b^2*c) - 6*a*cos(a/b)^3*cos_integral(6*a/b + 6*arcs
in(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 6*a*cos(a/b)^3*cos_integ
ral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 9*a*co
s(a/b)^4*sin_integral(6*a/b + 6*arcsin(c*x))/(b^3*c*arcsin(c*x) + a*b^2*c)
- 6*a*cos(a/b)^4*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c*arcsin(c*x) +
a*b^2*c) + 9/8*b*arcsin(c*x)*cos(a/b)*cos_integral(6*a/b + 6*arcsin(c*x))
*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) - 3*b*arcsin(c*x)*cos(a/b)*cos_int
egral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c*arcsin(c*x) + a*b^2*c) + 15/8
*b*arcsin(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*
c*arcsin(c*x) + a*b^2*c) - 27/8*b*arcsin(c*x)*cos(a/b)^2*sin_integral(6...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx$$

input

```
int((1 - c^2*x^2)^(5/2)/(a + b*asin(c*x))^2,x)
```

output

```
int((1 - c^2*x^2)^(5/2)/(a + b*asin(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx$$

$$+ \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^4}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) c^4$$

$$- 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^2}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) c^2$$

input `int((-c^2*x^2+1)^(5/2)/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2+1)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)
+int((sqrt(-c**2*x**2+1)*x**4)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b
+a**2),x)*c**4-2*int((sqrt(-c**2*x**2+1)*x**2)/(asin(c*x)**2*b**2+
2*asin(c*x)*a*b+a**2),x)*c**2`

$$3.369 \quad \int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2} dx$$

Optimal result	3258
Mathematica [N/A]	3258
Rubi [N/A]	3259
Maple [N/A]	3260
Fricas [N/A]	3261
Sympy [N/A]	3261
Maxima [N/A]	3261
Giac [F(-2)]	3262
Mupad [N/A]	3262
Reduce [N/A]	3263

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 8.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2} dx = \int \frac{(1-c^2x^2)^{5/2}}{x(a+b \arcsin(cx))^2} dx$$

input `Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcSin[c*x]))^2,x]`

output `Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcSin[c*x]))^2, x]`

Rubi [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5214} \\
 & -\frac{5c \int \frac{(1-c^2x^2)^2}{a+b\arcsin(cx)} dx}{b} - \frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{(1-c^2x^2)^3}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{5168} \\
 & -\frac{5 \int \frac{\cos^5\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2} - \frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{(1-c^2x^2)^3}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3042} \\
 & -\frac{5 \int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b} + \frac{\pi}{2}\right)^5}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2} - \frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{(1-c^2x^2)^3}{bcx(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3793} \\
 & -\frac{5 \int \left(\frac{\cos\left(\frac{5a}{b} - \frac{5(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{5 \cos\left(\frac{3a}{b} - \frac{3(a+b\arcsin(cx))}{b}\right)}{16(a+b\arcsin(cx))} + \frac{5 \cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{8(a+b\arcsin(cx))} \right) d(a+b\arcsin(cx))}{b^2} \\
 & \quad \downarrow \text{2009} \\
 & -\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{(1-c^2x^2)^3}{bcx(a+b\arcsin(cx))}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b \arcsin(cx))} dx}{bc} - \\
 & \frac{5\left(\frac{5}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)\right)}{b^2} \\
 & \frac{(1-c^2x^2)^3}{bcx(a+b \arcsin(cx))} \\
 & \quad \downarrow \text{5234} \\
 & \frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b \arcsin(cx))} dx}{bc} - \\
 & \frac{5\left(\frac{5}{8} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \frac{5}{16} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \operatorname{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)\right)}{b^2} \\
 & \frac{(1-c^2x^2)^3}{bcx(a+b \arcsin(cx))}
 \end{aligned}$$

input `Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2 + 1)^{\frac{5}{2}}}{x(a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2 x} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x*arcsin(c*x)^2 + 2*a*b*x*arcsin(c*x) + a^2*x), x)`

Sympy [N/A]

Not integrable

Time = 7.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{5/2}}{x(a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/x/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(x*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.75

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2 x} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x)*integrate((5*c^6*x^6 - 9*c^4*x^4 + 3*c^2*x^2 + 1)/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2), x) - 1)/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(5/2)/(x*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(5/2)/(x*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.71

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x(a + b \arcsin(cx))^2} dx = \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 x + 2 \operatorname{asin}(cx) abx + a^2 x} dx$$

$$+ \left(\int \frac{\sqrt{-c^2 x^2 + 1} x^3}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) c^4$$

$$- 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1} x}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) c^2$$

input `int((-c^2*x^2+1)^(5/2)/x/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2+1)/(asin(c*x)**2*b**2*x+2*asin(c*x)*a*b*x+a**2*x),x)+int((sqrt(-c**2*x**2+1)*x**3)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)*c**4-2*int((sqrt(-c**2*x**2+1)*x)/(asin(c*x)**2*b**2+2*asin(c*x)*a*b+a**2),x)*c**2`

3.370 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b \arcsin(cx))^2} dx$

Optimal result	3264
Mathematica [N/A]	3264
Rubi [N/A]	3265
Maple [N/A]	3265
Fricas [N/A]	3266
Sympy [N/A]	3266
Maxima [N/A]	3267
Giac [N/A]	3267
Mupad [N/A]	3268
Reduce [N/A]	3268

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))^2} dx = \text{Int}\left(\frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))^2}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2x^2)^{5/2}}{x^2(a + b \arcsin(cx))^2} dx$$

input

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcSin[c*x])^2),x]
```

output

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcSin[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx$$

$$\downarrow 5214$$

$$-\frac{4c \int \frac{(1 - c^2 x^2)^2}{x(a + b \arcsin(cx))} dx}{b} - \frac{2 \int \frac{(1 - c^2 x^2)^2}{x^3(a + b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^3}{bcx^2(a + b \arcsin(cx))}$$

$$\downarrow 5234$$

$$-\frac{4c \int \frac{(1 - c^2 x^2)^2}{x(a + b \arcsin(cx))} dx}{b} - \frac{2 \int \frac{(1 - c^2 x^2)^2}{x^3(a + b \arcsin(cx))} dx}{bc} - \frac{(1 - c^2 x^2)^3}{bcx^2(a + b \arcsin(cx))}$$

input

```
Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcSin[c*x])^2),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx$$

input

```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x))^2,x)
```

output `int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{\frac{5}{2}}}{(b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2), x)`

Sympy [N/A]

Not integrable

Time = 6.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{\frac{5}{2}}}{x^2 (a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(x**2*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.79

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2)*integrate(2*(2*c^6*x^6 - 3*c^4*x^4 + 1)/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^3), x) - 1/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2)`

Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate((-c^2*x^2 + 1)^(5/2)/((b*arcsin(c*x) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 365, normalized size of antiderivative = 13.04

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \arcsin(cx))^2} dx = \frac{-2 \operatorname{asin}(cx) \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) ab c^2 + \operatorname{asin}(cx) \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right)}{}$$

input `int((-c^2*x^2+1)^(5/2)/x^2/(a+b*asin(c*x))^2,x)`

output `(- 2*asin(c*x)*int(sqrt(- c**2*x**2 + 1)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a*b*c**2 + asin(c*x)*int((sqrt(- c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a*b*c**4 + asin(c*x)*int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(- c**2*x**2 + 1)*a**2*x**2),x)*a*b - asin(c*x)*c - 2*int(sqrt(- c**2*x**2 + 1)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a**2*c**2 + int((sqrt(- c**2*x**2 + 1)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a**2*c**4 + int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(- c**2*x**2 + 1)*a**2*x**2),x)*a**2)/(a*(asin(c*x)*b + a))`

3.371
$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2} dx$$

Optimal result	3269
Mathematica [N/A]	3269
Rubi [N/A]	3270
Maple [N/A]	3270
Fricas [N/A]	3271
Sympy [N/A]	3271
Maxima [N/A]	3272
Giac [F(-2)]	3272
Mupad [N/A]	3273
Reduce [N/A]	3273

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2}, x\right)$$

output

```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.89 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2} dx = \int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b \arcsin(cx))^2} dx$$

input

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcSin[c*x])^2), x]
```

output

```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcSin[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 x^2 + 1)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x))^2,x)`

output `int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2 x^3} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^3*arcsin(c*x)^2 + 2*a*b*x^3*arcsin(c*x) + a^2*x^3), x)`

Sympy [N/A]

Not integrable

Time = 7.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(-(cx - 1)(cx + 1))^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx$$

input `integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*asin(c*x))**2,x)`

output `Integral((-c*x - 1)*(c*x + 1)**(5/2)/(x**3*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 169, normalized size of antiderivative = 6.04

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(-c^2 x^2 + 1)^{5/2}}{(b \arcsin(cx) + a)^2 x^3} dx$$

input `integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^3)*integrate(3*(c^6*x^6 - c^4*x^4 - c^2*x^2 + 1)/(b^2*c*x^4*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^4, x) - 1)/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^3`

Giac [F(-2)]

Exception generated.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx = \int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \operatorname{asin}(cx))^2} dx$$

input `int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*asin(c*x))^2),x)`

output `int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*asin(c*x))^2), x)`

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.96

$$\begin{aligned} \int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \arcsin(cx))^2} dx &= \int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 x^3 + 2 \operatorname{asin}(cx) a b x^3 + a^2 x^3} dx \\ &- 2 \left(\int \frac{\sqrt{-c^2 x^2 + 1}}{\operatorname{asin}(cx)^2 b^2 x + 2 \operatorname{asin}(cx) a b x + a^2 x} dx \right) c^2 \\ &+ \left(\int \frac{\sqrt{-c^2 x^2 + 1} x}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) a b + a^2} dx \right) c^4 \end{aligned}$$

input `int((-c^2*x^2+1)^(5/2)/x^3/(a+b*asin(c*x))^2,x)`

output `int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2*x**3 + 2*asin(c*x)*a*b*x**3 + a**2*x**3),x) - 2*int(sqrt(-c**2*x**2 + 1)/(asin(c*x)**2*b**2*x + 2*asin(c*x)*a*b*x + a**2*x),x)*c**2 + int((sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**4`

3.372 $\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3274
Mathematica [A] (verified)	3275
Rubi [A] (verified)	3275
Maple [A] (verified)	3277
Fricas [F]	3278
Sympy [F]	3278
Maxima [F]	3279
Giac [F(-2)]	3279
Mupad [F(-1)]	3279
Reduce [F]	3280

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{x^5}{bc(a+b \arcsin(cx))} + \frac{5 \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8b^2c^6} - \frac{15 \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16b^2c^6} + \frac{5 \cos\left(\frac{5a}{b}\right) \text{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16b^2c^6} + \frac{5 \sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{8b^2c^6} - \frac{15 \sin\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{16b^2c^6} + \frac{5 \sin\left(\frac{5a}{b}\right) \text{Si}\left(\frac{5(a+b \arcsin(cx))}{b}\right)}{16b^2c^6}$$

output

```
-x^5/b/c/(a+b*arcsin(c*x))+5/8*cos(a/b)*Ci((a+b*arcsin(c*x))/b)/b^2/c^6-15/16*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^6+5/16*cos(5*a/b)*Ci(5*(a+b*arcsin(c*x))/b)/b^2/c^6+5/8*sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^6-15/16*sin(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^6+5/16*sin(5*a/b)*Si(5*(a+b*arcsin(c*x))/b)/b^2/c^6
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = -\frac{x^5}{bc(a+b\arcsin(cx))} + \frac{5(2\cos(\frac{a}{b})\text{CosIntegral}(\frac{a}{b}+\arcsin(cx)) - 3\cos(\frac{3a}{b})\text{CosIntegral}(3(\frac{a}{b}+\arcsin(cx)))) + \cos(\frac{5a}{b})\text{CosIntegral}(5(\frac{a}{b}+\arcsin(cx)))}{16b^2c^6}$$

input

```
Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]
```

output

```
-(x^5/(b*c*(a + b*ArcSin[c*x]))) + (5*(2*Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] - 3*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcSin[c*x])] + Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcSin[c*x])] + 2*Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] - 3*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])] + Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcSin[c*x])]))/(16*b^2*c^6)
```

Rubi [A] (verified)Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5222, 5146, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

↓ 5222

$$\begin{aligned}
 & \frac{5 \int \frac{x^4}{a+b \arcsin(cx)} dx}{bc} - \frac{x^5}{bc(a+b \arcsin(cx))} \\
 & \quad \downarrow 5146 \\
 & \frac{5 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^6} - \frac{x^5}{bc(a+b \arcsin(cx))} \\
 & \quad \downarrow 4906 \\
 & \frac{5 \int \left(\frac{\cos\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} - \frac{3 \cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16(a+b \arcsin(cx))} + \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^6} - \frac{x^5}{bc(a+b \arcsin(cx))} \\
 & \quad \downarrow 2009 \\
 & \frac{5 \left(\frac{1}{8} \cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{3}{16} \cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{16} \cos\left(\frac{5a}{b}\right) \text{CosIntegral}\left(\frac{5(a+b \arcsin(cx))}{b}\right) \right)}{b^2 c^6} - \frac{x^5}{bc(a+b \arcsin(cx))}
 \end{aligned}$$

input `Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `-(x^5/(b*c*(a + b*ArcSin[c*x]))) + (5*((Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/8 - (3*Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x])/b])/16 + (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcSin[c*x])/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/8 - (3*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/16 + (Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcSin[c*x])/b])/16)))/(b^2*c^6)`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5146 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := Simp[1/(b*c^(m + 1)) Subst[Int[x^n*sin[-a/b + x/b]^m*cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]`

rule 5222 `Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] - Simp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]`

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.67

method	result
default	$-\frac{15 \arcsin(cx) \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) b - 10 \arcsin(cx) \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b}) b - 10 \arcsin(cx) \operatorname{Ci}(\arcsin(cx) + \frac{a}{b}) \cos(\frac{a}{b}) b}{b^2}$

input `int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output

```
-1/16/c^6*(15*arcsin(c*x)*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b-10*arcsin(c
*x)*Si(arcsin(c*x)+a/b)*sin(a/b)*b-10*arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(
a/b)*b-5*arcsin(c*x)*Si(5*arcsin(c*x)+5*a/b)*sin(5*a/b)*b-5*arcsin(c*x)*Ci
(5*arcsin(c*x)+5*a/b)*cos(5*a/b)*b+15*arcsin(c*x)*Si(3*arcsin(c*x)+3*a/b)*
sin(3*a/b)*b+15*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*a-10*Si(arcsin(c*x)+a/b
)*sin(a/b)*a-10*Ci(arcsin(c*x)+a/b)*cos(a/b)*a-5*Si(5*arcsin(c*x)+5*a/b)*s
in(5*a/b)*a-5*Ci(5*arcsin(c*x)+5*a/b)*cos(5*a/b)*a+15*Si(3*arcsin(c*x)+3*a
/b)*sin(3*a/b)*a+10*x*b*c+sin(5*arcsin(c*x))*b-5*sin(3*arcsin(c*x))*b)/(a+
b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input

```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas"
)
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input

```
integrate(x**5/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)
```

output

```
Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(x^5 - 5*(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(x^4/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^5}{(a+b\arcsin(cx))^2 \sqrt{1-c^2x^2}} dx$$

input `int(x^5/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^5/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^5}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx$$

$$= \int \frac{x^5}{\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx)^2 b^2 + 2\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) ab + \sqrt{-c^2 x^2 + 1} a^2} dx$$

input `int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(x**5/(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(-c**2*x**2 + 1)*a**2),x)`

3.373 $\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3281
Mathematica [A] (verified)	3282
Rubi [A] (verified)	3282
Maple [A] (verified)	3284
Fricas [F]	3285
Sympy [F]	3285
Maxima [F]	3285
Giac [B] (verification not implemented)	3286
Mupad [F(-1)]	3287
Reduce [F]	3287

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{x^4}{bc(a+b \arcsin(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{b^2c^5} + \frac{\text{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{4a}{b}\right)}{2b^2c^5} + \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{b^2c^5} - \frac{\cos\left(\frac{4a}{b}\right) \text{Si}\left(\frac{4(a+b \arcsin(cx))}{b}\right)}{2b^2c^5}$$

output

```
-x^4/b/c/(a+b*arcsin(c*x))-Ci(2*(a+b*arcsin(c*x))/b)*sin(2*a/b)/b^2/c^5+1/2*Ci(4*(a+b*arcsin(c*x))/b)*sin(4*a/b)/b^2/c^5+cos(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b^2/c^5-1/2*cos(4*a/b)*Si(4*(a+b*arcsin(c*x))/b)/b^2/c^5
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$= \frac{-\frac{2bc^4x^4}{a+b\arcsin(cx)} - 2\operatorname{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)\sin\left(\frac{2a}{b}\right) + \operatorname{CosIntegral}\left(4\left(\frac{a}{b} + \arcsin(cx)\right)\right)\sin\left(\frac{4a}{b}\right) + 2\operatorname{Cos}\left[2\left(\frac{a}{b} + \arcsin(cx)\right)\right]\operatorname{SinIntegral}\left[2\left(\frac{a}{b} + \arcsin(cx)\right)\right] - \operatorname{Cos}\left[\frac{4a}{b}\right]\operatorname{SinIntegral}\left[4\left(\frac{a}{b} + \arcsin(cx)\right)\right]}{2b^2c^5}$$

input

```
Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]
```

output

```
((-2*b*c^4*x^4)/(a + b*ArcSin[c*x]) - 2*CosIntegral[2*(a/b + ArcSin[c*x])]
*Sin[(2*a)/b] + CosIntegral[4*(a/b + ArcSin[c*x])]*Sin[(4*a)/b] + 2*Cos[(2
*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])] - Cos[(4*a)/b]*SinIntegral[4*(a/
b + ArcSin[c*x])])/(2*b^2*c^5)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5222, 5146, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$\downarrow \text{5222}$$

$$\frac{4 \int \frac{x^3}{a+b\arcsin(cx)} dx}{bc} - \frac{x^4}{bc(a+b\arcsin(cx))}$$

$$\downarrow \text{5146}$$

$$\frac{4 \int -\frac{\cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)\sin^3\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^5} - \frac{x^4}{bc(a+b\arcsin(cx))}$$

$$\downarrow \text{25}$$

$$\begin{aligned}
& \frac{4 \int \frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^5} - \frac{x^4}{bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{4906} \\
& \frac{4 \int \left(\frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} - \frac{\sin\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{\frac{b^2 c^5}{x^4} bc(a+b \arcsin(cx))} \\
& \quad \downarrow \text{2009} \\
& \frac{4 \left(-\frac{1}{4} \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) + \frac{1}{8} \sin\left(\frac{4a}{b}\right) \operatorname{CosIntegral}\left(\frac{4(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \right)}{x^4 bc(a+b \arcsin(cx))}
\end{aligned}$$

input `Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `-(x^4/(b*c*(a + b*ArcSin[c*x]))) + (4*(-1/4*(CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b]) + (CosIntegral[(4*(a + b*ArcSin[c*x]))/b]*Sin[(4*a)/b])/8 + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/4 - (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcSin[c*x]))/b])/8)/(b^2*c^5)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5146

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] := Simp[1
/(b*c^(m + 1)) Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b], x], x, a
+ b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5222

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_)^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^
2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] - Simp[f*(m/(b*c*(n
+ 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[(f*x)^(m - 1)*(a + b*
ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4 \arcsin(cx) \operatorname{Si}(4 \arcsin(cx) + \frac{4a}{b}) \cos(\frac{4a}{b}) b - 4 \arcsin(cx) \sin(\frac{4a}{b}) \operatorname{Ci}(4 \arcsin(cx) + \frac{4a}{b}) b - 8 \arcsin(cx) \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b})}{(a + b \arcsin(cx))^2}$

input

```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/8/c^5*(4*arcsin(c*x)*Si(4*arcsin(c*x)+4*a/b)*cos(4*a/b)*b-4*arcsin(c*x)
*sin(4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*b-8*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b
)*cos(2*a/b)*b+8*arcsin(c*x)*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*b+4*Si(4*a
rcsin(c*x)+4*a/b)*cos(4*a/b)*a-4*sin(4*a/b)*Ci(4*arcsin(c*x)+4*a/b)*a-8*Si
(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a+8*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*a+
cos(4*arcsin(c*x))*b-4*cos(2*arcsin(c*x))*b+3*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)`

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^4}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input `integrate(x**4/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)`

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(x^4 - 4*(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(x^3/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c, x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(137) = 274$.

Time = 0.23 (sec) , antiderivative size = 876, normalized size of antiderivative = 6.21

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \text{Too large to display}$$

input `integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
4*b*arcsin(c*x)*cos(a/b)^3*cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b
^3*c^5*arcsin(c*x) + a*b^2*c^5) - 4*b*arcsin(c*x)*cos(a/b)^4*sin_integral(
4*a/b + 4*arcsin(c*x))/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) + 4*a*cos(a/b)^3*
cos_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c^5*arcsin(c*x) + a*b^2*
c^5) - 4*a*cos(a/b)^4*sin_integral(4*a/b + 4*arcsin(c*x))/(b^3*c^5*arcsin(
c*x) + a*b^2*c^5) - 2*b*arcsin(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arcsin
(c*x))*sin(a/b)/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) - 2*b*arcsin(c*x)*cos(a/
b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c^5*arcsin(c*x) + a*b
^2*c^5) + 4*b*arcsin(c*x)*cos(a/b)^2*sin_integral(4*a/b + 4*arcsin(c*x))/(
b^3*c^5*arcsin(c*x) + a*b^2*c^5) + 2*b*arcsin(c*x)*cos(a/b)^2*sin_integral
(2*a/b + 2*arcsin(c*x))/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) - 2*a*cos(a/b)*c
os_integral(4*a/b + 4*arcsin(c*x))*sin(a/b)/(b^3*c^5*arcsin(c*x) + a*b^2*c
^5) - 2*a*cos(a/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c^5*a
rcsin(c*x) + a*b^2*c^5) + 4*a*cos(a/b)^2*sin_integral(4*a/b + 4*arcsin(c*x
))/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) + 2*a*cos(a/b)^2*sin_integral(2*a/b +
2*arcsin(c*x))/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) - (c^2*x^2 - 1)^2*b/(b^3
*c^5*arcsin(c*x) + a*b^2*c^5) - 1/2*b*arcsin(c*x)*sin_integral(4*a/b + 4*a
rcsin(c*x))/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) - b*arcsin(c*x)*sin_integral
(2*a/b + 2*arcsin(c*x))/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) - 2*(c^2*x^2 - 1
)*b/(b^3*c^5*arcsin(c*x) + a*b^2*c^5) - 1/2*a*sin_integral(4*a/b + 4*ar...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^4}{(a+b\arcsin(cx))^2 \sqrt{1-c^2x^2}} dx$$

input `int(x^4/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^4/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx \\ &= \int \frac{x^4}{\sqrt{-c^2x^2+1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2x^2+1} \arcsin(cx) ab + \sqrt{-c^2x^2+1} a^2} dx \end{aligned}$$

input `int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(x**4/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b+sqrt(-c**2*x**2+1)*a**2),x)`

3.374 $\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3288
Mathematica [A] (verified)	3289
Rubi [A] (verified)	3289
Maple [A] (verified)	3291
Fricas [F]	3291
Sympy [F]	3292
Maxima [F]	3292
Giac [F(-2)]	3292
Mupad [F(-1)]	3293
Reduce [F]	3293

Optimal result

Integrand size = 28, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{x^3}{bc(a+b \arcsin(cx))} + \frac{3 \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{4b^2c^4} - \frac{3 \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{4b^2c^4} + \frac{3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{4b^2c^4} - \frac{3 \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right)}{4b^2c^4}$$

output

```
-x^3/b/c/(a+b*arcsin(c*x))+3/4*cos(a/b)*Ci((a+b*arcsin(c*x))/b)/b^2/c^4-3/4*cos(3*a/b)*Ci(3*(a+b*arcsin(c*x))/b)/b^2/c^4+3/4*sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^4-3/4*sin(3*a/b)*Si(3*(a+b*arcsin(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = -\frac{x^3}{bc(a+b\arcsin(cx))} + \frac{3(\cos(\frac{a}{b})\text{CosIntegral}(\frac{a}{b}+\arcsin(cx)) - \cos(\frac{3a}{b})\text{CosIntegral}(3(\frac{a}{b}+\arcsin(cx)))) + \sin(\frac{a}{b})\text{Si}(\frac{a}{b}+a)}{4b^2c^4}$$

input `Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output
$$\frac{-(x^3/(b*c*(a + b*ArcSin[c*x]))) + (3*(Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] - Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcSin[c*x]]) + Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]] - Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcSin[c*x])]))}{(4*b^2*c^4)}$$

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5222, 5146, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx \\ & \quad \downarrow \text{5222} \\ & \frac{3 \int \frac{x^2}{a+b\arcsin(cx)} dx}{bc} - \frac{x^3}{bc(a+b\arcsin(cx))} \\ & \quad \downarrow \text{5146} \\ & \frac{3 \int \frac{\cos(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}) \sin^2(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b})}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^4} - \frac{x^3}{bc(a+b\arcsin(cx))} \\ & \quad \downarrow \text{4906} \end{aligned}$$

$$\frac{3 \int \left(\frac{\cos\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4(a+b \arcsin(cx))} - \frac{\cos\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4(a+b \arcsin(cx))} \right) d(a+b \arcsin(cx))}{b^2 c^4} - \frac{x^3}{bc(a+b \arcsin(cx))}$$

↓ 2009

$$\frac{3 \left(\frac{1}{4} \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{4} \cos\left(\frac{3a}{b}\right) \operatorname{CosIntegral}\left(\frac{3(a+b \arcsin(cx))}{b}\right) + \frac{1}{4} \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right) - \frac{1}{4} \sin\left(\frac{3a}{b}\right) \operatorname{Si}\left(\frac{3(a+b \arcsin(cx))}{b}\right) \right)}{b^2 c^4} - \frac{x^3}{bc(a+b \arcsin(cx))}$$

input `Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `-(x^3/(b*c*(a + b*ArcSin[c*x]))) + (3*((Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b])/4 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcSin[c*x])/b])/4 + (Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/4 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcSin[c*x])/b])/4))/(b^2*c^4)`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^(n)*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5146 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^m, x_Symbol] := Simp[1/(b*c^(m + 1)) Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]`

rule 5222

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_)*((f_.)*(x_))^(m_.)/Sqrt[(d_
+ (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^
2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] - Simp[f*(m/(b*c*(n
+ 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[(f*x)^(m - 1)*(a + b*
ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.60

method	result
default	$-\frac{3 \arcsin(cx) \operatorname{Si}(3 \arcsin(cx) + \frac{3a}{b}) \sin(\frac{3a}{b}) b + 3 \arcsin(cx) \operatorname{Ci}(3 \arcsin(cx) + \frac{3a}{b}) \cos(\frac{3a}{b}) b - 3 \arcsin(cx) \operatorname{Si}(\arcsin(cx) + \frac{a}{b}) \sin(\frac{a}{b}) b}{b^2}$

input

```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/4/c^4*(3*arcsin(c*x)*Si(3*arcsin(c*x)+3*a/b)*sin(3*a/b)*b+3*arcsin(c*x)
*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*b-3*arcsin(c*x)*Si(arcsin(c*x)+a/b)*si
n(a/b)*b-3*arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+3*Si(3*arcsin(c*x)+3
*a/b)*sin(3*a/b)*a+3*Ci(3*arcsin(c*x)+3*a/b)*cos(3*a/b)*a-3*Si(arcsin(c*x)
+a/b)*sin(a/b)*a-3*Ci(arcsin(c*x)+a/b)*cos(a/b)*a+3*x*b*c-sin(3*arcsin(c*x)
))*b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx = \int \frac{x^3}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a)^2} dx$$

input

```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas"
)
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)
```


Sympy [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input `integrate(x**3/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)`

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(x^3 - 3*(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(x^2/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + a*b*c)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^3}{(a+b\arcsin(cx))^2 \sqrt{1-c^2x^2}} dx$$

input `int(x^3/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^3/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx \\ &= \int \frac{x^3}{\sqrt{-c^2x^2+1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2x^2+1} \arcsin(cx) ab + \sqrt{-c^2x^2+1} a^2} dx \end{aligned}$$

input `int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(x**3/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b+sqrt(-c**2*x**2+1)*a**2),x)`

3.375 $\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3294
Mathematica [A] (verified)	3294
Rubi [A] (verified)	3295
Maple [A] (verified)	3298
Fricas [F]	3299
Sympy [F]	3299
Maxima [F]	3299
Giac [B] (verification not implemented)	3300
Mupad [F(-1)]	3301
Reduce [F]	3301

Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{x^2}{bc(a+b \arcsin(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{b^2c^3} + \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{b^2c^3}$$

output

```
-x^2/b/c/(a+b*arcsin(c*x))-Ci(2*(a+b*arcsin(c*x))/b)*sin(2*a/b)/b^2/c^3+cos(2*a/b)*Si(2*(a+b*arcsin(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = \frac{-\frac{bc^2x^2}{a+b \arcsin(cx)} - \text{CosIntegral}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right) \sin\left(\frac{2a}{b}\right) + \cos\left(\frac{2a}{b}\right) \text{Si}\left(2\left(\frac{a}{b} + \arcsin(cx)\right)\right)}{b^2c^3}$$

input `Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `((-(b*c^2*x^2)/(a + b*ArcSin[c*x])) - CosIntegral[2*(a/b + ArcSin[c*x]])*Sin[(2*a)/b] + Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcSin[c*x])])/(b^2*c^3)`

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5222, 5146, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow 5222 \\
 & \frac{2 \int \frac{x}{a+b\arcsin(cx)} dx}{bc} - \frac{x^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow 5146 \\
 & \frac{2 \int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^3} - \frac{x^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow 25 \\
 & -\frac{2 \int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arcsin(cx)}{b}\right)}{a+b\arcsin(cx)} d(a+b\arcsin(cx))}{b^2c^3} - \frac{x^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow 4906 \\
 & -\frac{2 \int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arcsin(cx))}{b}\right)}{2(a+b\arcsin(cx))} d(a+b\arcsin(cx))}{b^2c^3} - \frac{x^2}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow 27
 \end{aligned}$$

$$\frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2}{bc(a+b \arcsin(cx))}$$

↓ 3042

$$\frac{\int \frac{\sin\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{x^2}{bc(a+b \arcsin(cx))}$$

↓ 3784

$$\frac{-\sin\left(\frac{2a}{b}\right) \int \frac{\cos\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{\frac{b^2 c^3}{x^2} bc(a+b \arcsin(cx))}$$

↓ 25

$$\frac{\cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{2a}{b}\right) \int \frac{\cos\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{\frac{b^2 c^3}{x^2} bc(a+b \arcsin(cx))}$$

↓ 3042

$$\frac{\cos\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) - \sin\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{\frac{b^2 c^3}{x^2} bc(a+b \arcsin(cx))}$$

↓ 3780

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \sin\left(\frac{2a}{b}\right) \int \frac{\sin\left(\frac{2(a+b \arcsin(cx))}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{\frac{b^2 c^3}{x^2} bc(a+b \arcsin(cx))}$$

↓ 3783

$$\frac{\cos\left(\frac{2a}{b}\right) \operatorname{Si}\left(\frac{2(a+b \arcsin(cx))}{b}\right) - \sin\left(\frac{2a}{b}\right) \operatorname{CosIntegral}\left(\frac{2(a+b \arcsin(cx))}{b}\right)}{b^2 c^3} - \frac{x^2}{bc(a+b \arcsin(cx))}$$

input `Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `-(x^2/(b*c*(a + b*ArcSin[c*x]))) + (-(CosIntegral[(2*(a + b*ArcSin[c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcSin[c*x]))/b])/(b^2*c^3)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

rule 3783 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

rule 3784 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[Cos[(d*e - c*f)/d] Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*f)/d] Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]`

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5146

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^m, x_Symbol] := Simp[1/(b*c^(m + 1)) Subst[Int[x^n*sin[-a/b + x/b]^m*cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5222

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.))/Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] - Simp[f*(m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

method	result
default	$\frac{2 \arcsin(cx) \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) b - 2 \arcsin(cx) \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) b + 2 \operatorname{Si}(2 \arcsin(cx) + \frac{2a}{b}) \cos(\frac{2a}{b}) a - 2 \operatorname{Ci}(2 \arcsin(cx) + \frac{2a}{b}) \sin(\frac{2a}{b}) a}{2c^3(a + b \arcsin(cx))b^2}$

input

```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/c^3*(2*arcsin(c*x)*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*b-2*arcsin(c*x)*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*b+2*Si(2*arcsin(c*x)+2*a/b)*cos(2*a/b)*a-2*Ci(2*arcsin(c*x)+2*a/b)*sin(2*a/b)*a+cos(2*arcsin(c*x))*b-b)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)`

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input `integrate(x**2/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)`

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(x^2 - 2*(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c)*integrate(x/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c), x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1) + a*b*c)`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(79) = 158$.

Time = 0.23 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.38

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$= -\frac{2b\arcsin(cx)\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\arcsin(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c^3\arcsin(cx)+ab^2c^3}$$

$$+\frac{2b\arcsin(cx)\cos\left(\frac{a}{b}\right)^2\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c^3\arcsin(cx)+ab^2c^3}$$

$$-\frac{2a\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{2a}{b}+2\arcsin(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c^3\arcsin(cx)+ab^2c^3} + \frac{2a\cos\left(\frac{a}{b}\right)^2\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c^3\arcsin(cx)+ab^2c^3}$$

$$-\frac{b\arcsin(cx)\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c^3\arcsin(cx)+ab^2c^3} - \frac{(c^2x^2-1)b}{b^3c^3\arcsin(cx)+ab^2c^3}$$

$$-\frac{a\text{Si}\left(\frac{2a}{b}+2\arcsin(cx)\right)}{b^3c^3\arcsin(cx)+ab^2c^3} - \frac{b}{b^3c^3\arcsin(cx)+ab^2c^3}$$

input `integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `-2*b*arcsin(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 2*b*arcsin(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - 2*a*cos(a/b)*cos_integral(2*a/b + 2*arcsin(c*x))*sin(a/b)/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) + 2*a*cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - b*arcsin(c*x)*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - (c^2*x^2 - 1)*b/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - a*sin_integral(2*a/b + 2*arcsin(c*x))/(b^3*c^3*arcsin(c*x) + a*b^2*c^3) - b/(b^3*c^3*arcsin(c*x) + a*b^2*c^3)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x^2}{(a+b\arcsin(cx))^2 \sqrt{1-c^2x^2}} dx$$

input `int(x^2/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(x^2/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\begin{aligned} & \int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx \\ &= \int \frac{x^2}{\sqrt{-c^2x^2+1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2x^2+1} \arcsin(cx) ab + \sqrt{-c^2x^2+1} a^2} dx \end{aligned}$$

input `int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(x**2/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b+sqrt(-c**2*x**2+1)*a**2),x)`

3.376 $\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3302
Mathematica [A] (verified)	3302
Rubi [A] (verified)	3303
Maple [A] (verified)	3305
Fricas [F]	3306
Sympy [F]	3306
Maxima [F]	3306
Giac [B] (verification not implemented)	3307
Mupad [F(-1)]	3308
Reduce [F]	3308

Optimal result

Integrand size = 26, antiderivative size = 72

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{x}{bc(a+b \arcsin(cx))} + \frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2c^2} + \frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2c^2}$$

output

```
-x/b/c/(a+b*arcsin(c*x))+cos(a/b)*Ci((a+b*arcsin(c*x))/b)/b^2/c^2+sin(a/b)*Si((a+b*arcsin(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = \frac{-\frac{bcx}{a+b \arcsin(cx)} + \cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a}{b} + \arcsin(cx)\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b} + \arcsin(cx)\right)}{b^2c^2}$$

input `Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `((-(b*c*x)/(a + b*ArcSin[c*x])) + Cos[a/b]*CosIntegral[a/b + ArcSin[c*x]] + Sin[a/b]*SinIntegral[a/b + ArcSin[c*x]])/(b^2*c^2)`

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5222, 5134, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx \\
 & \quad \downarrow \text{5222} \\
 & \frac{\int \frac{1}{a+b\arcsin(cx)} dx}{bc} - \frac{x}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{5134} \\
 & \frac{\int \frac{\cos\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) d(a+b\arcsin(cx))}{b^2c^2}}{bc(a+b\arcsin(cx))} - \frac{x}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3042} \\
 & \frac{\int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b} + \frac{\pi}{2}\right) d(a+b\arcsin(cx))}{b^2c^2}}{bc(a+b\arcsin(cx))} - \frac{x}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{3784} \\
 & \frac{\cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b\arcsin(cx)}{b}\right) d(a+b\arcsin(cx))}{a+b\arcsin(cx)} - \sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b\arcsin(cx)}{b}\right) d(a+b\arcsin(cx))}{a+b\arcsin(cx)}}{b^2c^2x} - \frac{x}{bc(a+b\arcsin(cx))} \\
 & \quad \downarrow \text{25}
 \end{aligned}$$

$$\frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \cos\left(\frac{a}{b}\right) \int \frac{\cos\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{bc(a+b \arcsin(cx))}{b^2 c^2}$$

↓ 3042

$$\frac{\sin\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{bc(a+b \arcsin(cx))}{b^2 c^2}$$

↓ 3780

$$\frac{\cos\left(\frac{a}{b}\right) \int \frac{\sin\left(\frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)}{a+b \arcsin(cx)} d(a+b \arcsin(cx)) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} - \frac{bc(a+b \arcsin(cx))}{b^2 c^2}$$

↓ 3783

$$\frac{\cos\left(\frac{a}{b}\right) \operatorname{CosIntegral}\left(\frac{a+b \arcsin(cx)}{b}\right) + \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a+b \arcsin(cx)}{b}\right)}{b^2 c^2} - \frac{x}{bc(a+b \arcsin(cx))}$$

input `Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2), x]`

output `-(x/(b*c*(a + b*ArcSin[c*x]))) + (Cos[a/b]*CosIntegral[(a + b*ArcSin[c*x])/b] + Sin[a/b]*SinIntegral[(a + b*ArcSin[c*x])/b])/(b^2*c^2)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 3780 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

rule 3783 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]`

rule 3784 `Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[Cos[(d*
e - c*f)/d] Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d] Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]`

rule 5134 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n, x_Symbol] := Simp[1/(b*c) Su
bst[Int[x^n*cos[-a/b + x/b], x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b,
c, n}, x]`

rule 5222 `Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n)*((f_.)*(x_))^m/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] := Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c^
2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] - Simp[f*(m/(b*c*(n
+ 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[(f*x)^(m - 1)*(a + b*
ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]`

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.50

method	result
default	$\frac{\arcsin(cx) \operatorname{Si}\left(\arcsin(cx) + \frac{a}{b}\right) \sin\left(\frac{a}{b}\right) b + \arcsin(cx) \operatorname{Ci}\left(\arcsin(cx) + \frac{a}{b}\right) \cos\left(\frac{a}{b}\right) b + \operatorname{Si}\left(\arcsin(cx) + \frac{a}{b}\right) \sin\left(\frac{a}{b}\right) a + \operatorname{Ci}\left(\arcsin(cx) + \frac{a}{b}\right) \cos\left(\frac{a}{b}\right) a}{c^2 (a + b \arcsin(cx)) b^2}$

input `int(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output

```
1/c^2*(arcsin(c*x)*Si(arcsin(c*x)+a/b)*sin(a/b)*b+arcsin(c*x)*Ci(arcsin(c*x)+a/b)*cos(a/b)*b+Si(arcsin(c*x)+a/b)*sin(a/b)*a+Ci(arcsin(c*x)+a/b)*cos(a/b)*a-x*b*c)/(a+b*arcsin(c*x))/b^2
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input

```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")
```

output

```
integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x}{\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input

```
integrate(x/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)
```

output

```
Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2} dx$$

input

```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")
```

output

```
((b^2*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c)*integrate(1/(b^2*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c), x) - x)/(b^2*c*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(72) = 144$.

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.78

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \frac{b\arcsin(cx)\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\arcsin(cx)\right)}{b^3c^2\arcsin(cx)+ab^2c^2} + \frac{b\arcsin(cx)\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arcsin(cx)\right)}{b^3c^2\arcsin(cx)+ab^2c^2} - \frac{bcx}{b^3c^2\arcsin(cx)+ab^2c^2} + \frac{a\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\arcsin(cx)\right)}{b^3c^2\arcsin(cx)+ab^2c^2} + \frac{a\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arcsin(cx)\right)}{b^3c^2\arcsin(cx)+ab^2c^2}$$

input

```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")
```

output

```
b*arcsin(c*x)*cos(a/b)*cos_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + b*arcsin(c*x)*sin(a/b)*sin_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) - b*c*x/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + a*cos(a/b)*cos_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2) + a*sin(a/b)*sin_integral(a/b + arcsin(c*x))/(b^3*c^2*arcsin(c*x) + a*b^2*c^2)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{x}{(a+b\sin(cx))^2\sqrt{1-c^2x^2}} dx$$

input `int(x/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(x/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$= \int \frac{x}{\sqrt{-c^2x^2+1} \operatorname{asin}(cx)^2 b^2 + 2\sqrt{-c^2x^2+1} \operatorname{asin}(cx) ab + \sqrt{-c^2x^2+1} a^2} dx$$

input `int(x/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(x/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b+sqrt(-c**2*x**2+1)*a**2),x)`

3.377 $\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx$

Optimal result	3309
Mathematica [A] (verified)	3309
Rubi [A] (verified)	3310
Maple [A] (verified)	3310
Fricas [A] (verification not implemented)	3311
Sympy [C] (verification not implemented)	3311
Maxima [A] (verification not implemented)	3312
Giac [A] (verification not implemented)	3312
Mupad [B] (verification not implemented)	3312
Reduce [B] (verification not implemented)	3313

Optimal result

Integrand size = 25, antiderivative size = 18

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{1}{bc(a+b \arcsin(cx))}$$

output -1/b/c/(a+b*arcsin(c*x))

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2} dx = -\frac{1}{bc(a+b \arcsin(cx))}$$

input Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]

output -(1/(b*c*(a + b*ArcSin[c*x])))

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

↓ 5152

$$-\frac{1}{bc(a+b\arcsin(cx))}$$

input `Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `-(1/(b*c*(a + b*ArcSin[c*x])))`

Defintions of rubi rules used

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{1}{bc(a+b\arcsin(cx))}$	19
default	$-\frac{1}{bc(a+b\arcsin(cx))}$	19

input `int(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)`

output `-1/b/c/(a+b*arcsin(c*x))`

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = -\frac{1}{b^2c\arcsin(cx) + abc}$$

input `integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `-1/(b^2*c*arcsin(c*x) + a*b*c)`

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \begin{cases} \frac{x}{a^2} & \text{for } b = 0 \wedge c = 0 \\ \begin{cases} -\frac{i\operatorname{acosh}(cx)}{c} & \text{for } |c^2x^2| > 1 \\ \frac{\operatorname{asin}(cx)}{c} & \text{otherwise} \end{cases} & \text{for } b = 0 \\ \frac{x}{a^2} & \text{for } c = 0 \\ -\frac{1}{abc+b^2c\operatorname{asin}(cx)} & \text{otherwise} \end{cases}$$

input `integrate(1/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), (Piecewise((-I*acosh(c*x)/c, Abs(c**2*x**2) > 1), (asin(c*x)/c, True))/a**2, Eq(b, 0)), (x/a**2, Eq(c, 0)), (-1/(a*b*c + b**2*c*asin(c*x)), True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = -\frac{1}{(b\arcsin(cx)+a)bc}$$

input `integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`output `-1/((b*arcsin(c*x) + a)*b*c)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = -\frac{1}{(b\arcsin(cx)+a)bc}$$

input `integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`output `-1/((b*arcsin(c*x) + a)*b*c)`**Mupad [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = -\frac{1}{c\arcsin(cx)b^2+acb}$$

input `int(1/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`output `-1/(b^2*c*asin(c*x) + a*b*c)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \frac{\arcsin(cx)}{ac(\arcsin(cx)b+a)}$$

input `int(1/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `asin(c*x)/(a*c*(asin(c*x)*b + a)`

3.378 $\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$

Optimal result	3314
Mathematica [N/A]	3314
Rubi [N/A]	3315
Maple [N/A]	3315
Fricas [N/A]	3316
Sympy [N/A]	3316
Maxima [N/A]	3316
Giac [F(-2)]	3317
Mupad [N/A]	3317
Reduce [N/A]	3318

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}, x\right)$$

output `Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

input `Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

↓ 5222

$$-\frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{1}{bcx(a+b\arcsin(cx))}$$

↓ 5148

$$-\frac{\int \frac{1}{x^2(a+b\arcsin(cx))} dx}{bc} - \frac{1}{bcx(a+b\arcsin(cx))}$$

input `Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2), x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{-c^2x^2+1}(a+b\arcsin(cx))^2} dx$$

input `int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

output `int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2+1)/(a^2*c^2*x^3-a^2*x+(b^2*c^2*x^3-b^2*x)*arcsin(c*x)^2+2*(a*b*c^2*x^3-a*b*x)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{x\sqrt{-(cx-1)(cx+1)}(a+b\arcsin(cx))^2} dx$$

input `integrate(1/x/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(1/(x*sqrt(-(c*x-1)*(c*x+1))*(a+b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arcsin(cx)+a)^2x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-((b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x)*integrate(1/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2), x) + 1)/(b^2*c*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{x(a+b\arcsin(cx))^2\sqrt{1-c^2x^2}} dx$$

input `int(1/(x*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(1/(x*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2} dx$$

$$= \int \frac{1}{\sqrt{-c^2x^2+1} \operatorname{asin}(cx)^2 b^2x + 2\sqrt{-c^2x^2+1} \operatorname{asin}(cx) abx + \sqrt{-c^2x^2+1} a^2x} dx$$

input `int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)`

output `int(1/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2*x+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b*x+sqrt(-c**2*x**2+1)*a**2*x),x)`

3.379
$$\int \frac{1}{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2} dx$$

Optimal result	3319
Mathematica [N/A]	3319
Rubi [N/A]	3320
Maple [N/A]	3320
Fricas [N/A]	3321
Sympy [N/A]	3321
Maxima [N/A]	3321
Giac [N/A]	3322
Mupad [N/A]	3322
Reduce [N/A]	3323

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{1}{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2} dx = \int \frac{1}{x^2 \sqrt{1-c^2x^2} (a+b \arcsin(cx))^2} dx$$

input `Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2),x]`

output `Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx$$

$$\downarrow 5222$$

$$-\frac{2 \int \frac{1}{x^3 (a + b \arcsin(cx))} dx}{bc} - \frac{1}{bcx^2 (a + b \arcsin(cx))}$$

$$\downarrow 5148$$

$$-\frac{2 \int \frac{1}{x^3 (a + b \arcsin(cx))} dx}{bc} - \frac{1}{bcx^2 (a + b \arcsin(cx))}$$

input `Int [1/(x^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2), x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 \sqrt{-c^2 x^2 + 1} (a + b \arcsin(cx))^2} dx$$

input `int (1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

output `int (1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^4 - a^2*x^2 + (b^2*c^2*x^4 - b^2*x^2)*arcsin(c*x)^2 + 2*(a*b*c^2*x^4 - a*b*x^2)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{x^2 \sqrt{-(cx - 1)(cx + 1)} (a + b \arcsin(cx))^2} dx$$

input `integrate(1/x**2/(-c**2*x**2+1)**(1/2)/(a+b*asin(c*x))**2,x)`

output `Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.29

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `-(2*(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2)*integrate(1/(b^2*c*x^3*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^3), x) + 1/(b^2*c*x^2*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)) + a*b*c*x^2)`

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{\sqrt{-c^2 x^2 + 1} (b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arcsin(c*x) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{x^2 (a + b \arcsin(cx))^2 \sqrt{1 - c^2 x^2}} dx$$

input `int(1/(x^2*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)),x)`

output `int(1/(x^2*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^2 \sqrt{1-c^2 x^2} (a+b \arcsin(cx))^2} dx$$

$$= \int \frac{1}{\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx)^2 b^2 x^2 + 2\sqrt{-c^2 x^2 + 1} \operatorname{asin}(cx) ab x^2 + \sqrt{-c^2 x^2 + 1} a^2 x^2} dx$$

input

```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*asin(c*x))^2,x)
```

output

```
int(1/(sqrt(-c**2*x**2+1)*asin(c*x)**2*b**2*x**2+2*sqrt(-c**2*x**2+1)*asin(c*x)*a*b*x**2+sqrt(-c**2*x**2+1)*a**2*x**2),x)
```


$$3.380 \quad \int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

Optimal result	3324
Mathematica [N/A]	3324
Rubi [N/A]	3325
Maple [N/A]	3325
Fricas [N/A]	3326
Sympy [N/A]	3326
Maxima [N/A]	3327
Giac [F(-2)]	3327
Mupad [N/A]	3328
Reduce [N/A]	3328

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2}, x\right)$$

output `Defer(Int)(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 41.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

input `Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `Int[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^3}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arcsin(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^3}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(x**3/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**3/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 205, normalized size of antiderivative = 7.32

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^3}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(x^3 - (a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate((c^2*x^4 - 3*x^2)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^3}{(a + b \arcsin(cx))^2 (1 - c^2 x^2)^{3/2}} dx$$

input `int(x^3/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`

output `int(x^3/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.04

$$\int \frac{x^3}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx =$$

$$-\left(\int \frac{x^3}{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab} dx \right)$$

input `int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int(x**3/(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**2 - sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**2 - 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(-c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(-c**2*x**2 + 1)*a**2),x)`

$$3.381 \quad \int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

Optimal result	3329
Mathematica [N/A]	3329
Rubi [N/A]	3330
Maple [N/A]	3330
Fricas [N/A]	3331
Sympy [N/A]	3331
Maxima [N/A]	3332
Giac [N/A]	3332
Mupad [N/A]	3333
Reduce [N/A]	3333

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2}, x\right)$$

output `Defer(Int)(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

input `Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{3/2} (a+b\arcsin(cx))^2} dx$$

$$\downarrow \text{5212}$$

$$\frac{2 \int \frac{x}{(1-c^2x^2)^2 (a+b\arcsin(cx))} dx}{bc} - \frac{x^2}{bc(1-c^2x^2)(a+b\arcsin(cx))}$$

$$\downarrow \text{5234}$$

$$\frac{2 \int \frac{x}{(1-c^2x^2)^2 (a+b\arcsin(cx))} dx}{bc} - \frac{x^2}{bc(1-c^2x^2)(a+b\arcsin(cx))}$$

input

```
Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2x^2 + 1)^{\frac{3}{2}} (a+b\arcsin(cx))^2} dx$$

input

```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)
```

output `int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arcsin(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^2}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x**2/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2, x)`

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 193, normalized size of antiderivative = 6.89

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(x^2 + 2*(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate(x/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^2}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)^2), x)`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x^2}{(a + b \arcsin(cx))^2 (1 - c^2 x^2)^{3/2}} dx$$

input `int(x^2/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`

output `int(x^2/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 11.04

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \frac{-\arcsin(cx)}{\sqrt{-c^2 x^2 + 1}} \left(\int \frac{\arcsin(cx)^2 b^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx)}{\sqrt{-c^2 x^2 + 1}} dx \right)$$

input `int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `(- (asin(c*x)*int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**2 - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a**2),x)*a*b*c + asin(c*x) + int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**2 - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a**2),x)*a**2*c)/ (a*c**3*(asin(c*x)*b + a))`

$$3.382 \quad \int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

Optimal result	3334
Mathematica [N/A]	3334
Rubi [N/A]	3335
Maple [N/A]	3335
Fricas [N/A]	3336
Sympy [N/A]	3336
Maxima [N/A]	3337
Giac [F(-2)]	3337
Mupad [N/A]	3338
Reduce [N/A]	3338

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2}, x\right)$$

output

```
Defer(Int)(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 36.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

input

```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]
```

output

```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2 x^2 + 1)^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `int(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.00

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)*x/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arcsin(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(x/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 198, normalized size of antiderivative = 7.62

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `((a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*integrate((c^2*x^2 + 1)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1)), x) + x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))`

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{x}{(a + b \arcsin(cx))^2 (1 - c^2 x^2)^{3/2}} dx$$

input `int(x/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`

output `int(x/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.35

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = - \left(\int \frac{x}{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab} dx \right)$$

input `int(x/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int(x/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**2 - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a**2),x)`

$$3.383 \quad \int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$$

Optimal result	3339
Mathematica [N/A]	3339
Rubi [N/A]	3340
Maple [N/A]	3340
Fricas [N/A]	3341
Sympy [N/A]	3341
Maxima [N/A]	3342
Giac [N/A]	3342
Mupad [N/A]	3343
Reduce [N/A]	3343

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx = \text{Int} \left(\frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}, x \right)$$

output `Defer(Int)(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx = \int \frac{1}{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$$

input `Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

$$\downarrow \text{5166}$$

$$\frac{2c \int \frac{x}{(1 - c^2 x^2)^2 (a + b \arcsin(cx))} dx}{b} - \frac{1}{bc(1 - c^2 x^2) (a + b \arcsin(cx))}$$

$$\downarrow \text{5234}$$

$$\frac{2c \int \frac{x}{(1 - c^2 x^2)^2 (a + b \arcsin(cx))} dx}{b} - \frac{1}{bc(1 - c^2 x^2) (a + b \arcsin(cx))}$$

input `Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `int(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arcsin(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(1/((-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2),x)`

output `Integral(1/((-c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 7.68

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `(2*(a*b*c^4*x^2 - a*b*c^2 + (b^2*c^4*x^2 - b^2*c^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate(x/(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x) + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2} dx$$

input `integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)^2), x)`

Mupad [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(a + b \arcsin(cx))^2 (1 - c^2 x^2)^{3/2}} dx$$

input `int(1/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`

output `int(1/((a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^2 x^2 - \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab} dx \right)$$

input `int(1/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**2 - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b + sqrt(- c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt(- c**2*x**2 + 1)*a**2),x)`

$$3.384 \quad \int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

Optimal result	3344
Mathematica [N/A]	3344
Rubi [N/A]	3345
Maple [N/A]	3345
Fricas [N/A]	3346
Sympy [N/A]	3346
Maxima [N/A]	3347
Giac [F(-2)]	3347
Mupad [N/A]	3348
Reduce [N/A]	3348

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \text{Int}\left(\frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2}, x\right)$$

output `Defer(Int)(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 34.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

input `Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

output `Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx$$

input `Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(-c^2x^2+1)^{3/2}(a+b\arcsin(cx))^2} dx$$

input `int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.86

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arcsin(cx)+a)^2x} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^5 - 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4*x^5 - 2*b^2*c^2*x^3 + b^2*x)*arcsin(c*x)^2 + 2*(a*b*c^4*x^5 - 2*a*b*c^2*x^3 + a*b*x)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 6.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{x(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\arcsin(cx))^2} dx$$

input `integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 209, normalized size of antiderivative = 7.46

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arcsin(cx)+a)^2} dx$$

input `integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `((a*b*c^3*x^3 - a*b*c*x + (b^2*c^3*x^3 - b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate((3*c^2*x^2 - 1)/(a*b*c^5*x^6 - 2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x) + 1)/(a*b*c^3*x^3 - a*b*c*x + (b^2*c^3*x^3 - b^2*c*x)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \text{Exception raised: TypeError}$$

input `integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx = \int \frac{1}{x(a+b\arcsin(cx))^2(1-c^2x^2)^{3/2}} dx$$

input `int(1/(x*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`

output `int(1/(x*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.00

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2} dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1} \arcsin(cx)^2 b^2 c^2 x^3 - \sqrt{-c^2x^2+1} \arcsin(cx)^2 b^2 x + 2\sqrt{-c^2x^2+1} \arcsin(cx) ab c^2 x^3 - 2\sqrt{-c^2x^2+1} \arcsin(cx) ab c^2 x^2 + 2\sqrt{-c^2x^2+1} \arcsin(cx) ab c^2 x - 2\sqrt{-c^2x^2+1} \arcsin(cx) ab} \right)$$

input `int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`

output `- int(1/(sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**3 - sqrt(-c**2*x**2 + 1)*asin(c*x)**2*b**2*x + 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**3 - 2*sqrt(-c**2*x**2 + 1)*asin(c*x)*a*b*x + sqrt(-c**2*x**2 + 1)*a**2*c**2*x**3 - sqrt(-c**2*x**2 + 1)*a**2*x),x)`

3.385 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$

Optimal result	3349
Mathematica [N/A]	3349
Rubi [N/A]	3350
Maple [N/A]	3350
Fricas [N/A]	3351
Sympy [N/A]	3351
Maxima [N/A]	3352
Giac [N/A]	3352
Mupad [N/A]	3353
Reduce [N/A]	3353

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx = \text{Int}\left(\frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}, x\right)$$

output `Defer(Int)(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Mathematica [N/A]

Not integrable

Time = 22.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx = \int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2} dx$$

input `Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2), x]`

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

↓ 5234

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2 x^2 + 1)^{3/2} (a + b \arcsin(cx))^2} dx$$

input `int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

output `int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x)`

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^6 - 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c^4*x^6 - 2*b^2*c^2*x^4 + b^2*x^2)*arcsin(c*x)^2 + 2*(a*b*c^4*x^6 - 2*a*b*c^2*x^4 + a*b*x^2)*arcsin(c*x)), x)`

Sympy [N/A]

Not integrable

Time = 5.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{x^2 (-(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^2} dx$$

input `integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*asin(c*x))**2,x)`

output `Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**2), x)`

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 218, normalized size of antiderivative = 7.79

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `((a*b*c^3*x^4 - a*b*c*x^2 + (b^2*c^3*x^4 - b^2*c*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))*integrate(2*(2*c^2*x^2 - 1)/(a*b*c^5*x^7 - 2*a*b*c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 - 2*b^2*c^3*x^5 + b^2*c*x^3)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))), x) + 1)/(a*b*c^3*x^4 - a*b*c*x^2 + (b^2*c^3*x^4 - b^2*c*x^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)))`

Giac [N/A]

Not integrable

Time = 17.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 x^2} dx$$

input `integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arcsin(c*x) + a)^2*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx = \int \frac{1}{x^2 (a + b \arcsin(cx))^2 (1 - c^2 x^2)^{3/2}} dx$$

input `int(1/(x^2*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)),x)`output `int(1/(x^2*(a + b*asin(c*x))^2*(1 - c^2*x^2)^(3/2)), x)`**Reduce [N/A]**

Not integrable

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.21

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2} dx =$$

$$-\left(\int \frac{1}{\sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 c^2 x^4 - \sqrt{-c^2 x^2 + 1} \arcsin(cx)^2 b^2 x^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^4 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2 + 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2 - 2\sqrt{-c^2 x^2 + 1} \arcsin(cx) ab c^2 x^2} dx \right)$$

input `int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*asin(c*x))^2,x)`output `- int(1/(sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*c**2*x**4 - sqrt(- c**2*x**2 + 1)*asin(c*x)**2*b**2*x**2 + 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*c**2*x**4 - 2*sqrt(- c**2*x**2 + 1)*asin(c*x)*a*b*x**2 + sqrt(- c**2*x**2 + 1)*a**2*c**2*x**4 - sqrt(- c**2*x**2 + 1)*a**2*x**2),x)`

$$3.386 \quad \int \left(-\frac{1}{(1-x^2) \arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2} \arcsin(x)} \right) dx$$

Optimal result	3354
Mathematica [A] (verified)	3354
Rubi [A] (verified)	3355
Maple [F]	3355
Fricas [A] (verification not implemented)	3356
Sympy [F]	3356
Maxima [B] (verification not implemented)	3357
Giac [B] (verification not implemented)	3357
Mupad [F(-1)]	3358
Reduce [B] (verification not implemented)	3358

Optimal result

Integrand size = 33, antiderivative size = 16

$$\int \left(-\frac{1}{(1-x^2) \arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2} \arcsin(x)} \right) dx = \frac{1}{\sqrt{1-x^2} \arcsin(x)}$$

output `1/(-x^2+1)^(1/2)/arcsin(x)`

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{(1-x^2) \arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2} \arcsin(x)} \right) dx = \frac{1}{\sqrt{1-x^2} \arcsin(x)}$$

input `Integrate[-(1/((1-x^2)*ArcSin[x]^2)) + x/((1-x^2)^(3/2)*ArcSin[x]),x]`

output `1/(Sqrt[1-x^2]*ArcSin[x])`

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{(1-x^2)^{3/2} \arcsin(x)} - \frac{1}{(1-x^2) \arcsin(x)^2} \right) dx$$

↓ 2009

$$\frac{1}{\sqrt{1-x^2} \arcsin(x)}$$

input `Int[-1/((1 - x^2)*ArcSin[x]^2)) + x/((1 - x^2)^(3/2)*ArcSin[x]),x]`

output `1/(Sqrt[1 - x^2]*ArcSin[x])`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [F]

$$\int \left(-\frac{1}{(-x^2+1) \arcsin(x)^2} + \frac{x}{(-x^2+1)^{3/2} \arcsin(x)} \right) dx$$

input `int(-1/(-x^2+1)/arcsin(x)^2+x/(-x^2+1)^(3/2)/arcsin(x),x)`

output `int(-1/(-x^2+1)/arcsin(x)^2+x/(-x^2+1)^(3/2)/arcsin(x),x)`

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \left(-\frac{1}{(1-x^2)\arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2}\arcsin(x)} \right) dx = -\frac{\sqrt{-x^2+1}}{(x^2-1)\arcsin(x)}$$

input `integrate(-1/(-x^2+1)/arcsin(x)^2+x/(-x^2+1)^(3/2)/arcsin(x),x, algorithm="fricas")`

output `-sqrt(-x^2 + 1)/((x^2 - 1)*arcsin(x))`

Sympy [F]

$$\begin{aligned} & \int \left(-\frac{1}{(1-x^2)\arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2}\arcsin(x)} \right) dx = \\ & - \int \left(-\frac{x}{x^4\sqrt{1-x^2}\arcsin(x) - 2x^2\sqrt{1-x^2}\arcsin(x) + \sqrt{1-x^2}\arcsin(x)} \right) dx \\ & - \int \left(-\frac{x^2}{x^4\arcsin^2(x) - 2x^2\arcsin^2(x) + \arcsin^2(x)} \right) dx \\ & - \int \frac{x^3}{x^4\sqrt{1-x^2}\arcsin(x) - 2x^2\sqrt{1-x^2}\arcsin(x) + \sqrt{1-x^2}\arcsin(x)} dx \\ & - \int \frac{1}{x^4\arcsin^2(x) - 2x^2\arcsin^2(x) + \arcsin^2(x)} dx \end{aligned}$$

input `integrate(-1/(-x**2+1)/asin(x)**2+x/(-x**2+1)**(3/2)/asin(x),x)`

output `-Integral(-x/(x**4*sqrt(1-x**2)*asin(x) - 2*x**2*sqrt(1-x**2)*asin(x) + sqrt(1-x**2)*asin(x)), x) - Integral(-x**2/(x**4*asin(x)**2 - 2*x**2*asin(x)**2 + asin(x)**2), x) - Integral(x**3/(x**4*sqrt(1-x**2)*asin(x) - 2*x**2*sqrt(1-x**2)*asin(x) + sqrt(1-x**2)*asin(x)), x) - Integral(1/(x**4*asin(x)**2 - 2*x**2*asin(x)**2 + asin(x)**2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.

Time = 0.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \left(-\frac{1}{(1-x^2)\arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2}\arcsin(x)} \right) dx =$$

$$-\frac{\sqrt{x+1}\sqrt{-x+1}}{(x^2-1)\arctan(x, \sqrt{x+1}\sqrt{-x+1})}$$

input `integrate(-1/(-x^2+1)/arcsin(x)^2+x/(-x^2+1)^(3/2)/arcsin(x),x, algorithm="maxima")`

output `-sqrt(x + 1)*sqrt(-x + 1)/((x^2 - 1)*arctan2(x, sqrt(x + 1)*sqrt(-x + 1)))`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(14) = 28$.

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.56

$$\int \left(-\frac{1}{(1-x^2)\arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2}\arcsin(x)} \right) dx =$$

$$-\frac{1}{\frac{x^2\arcsin(x)}{(\sqrt{-x^2+1}+1)^2} - \arcsin(x)} - \frac{x^2}{\left(\frac{x^2\arcsin(x)}{(\sqrt{-x^2+1}+1)^2} - \arcsin(x) \right) (\sqrt{-x^2+1}+1)^2}$$

input `integrate(-1/(-x^2+1)/arcsin(x)^2+x/(-x^2+1)^(3/2)/arcsin(x),x, algorithm="giac")`

output `-1/(x^2*arcsin(x)/(sqrt(-x^2 + 1) + 1)^2 - arcsin(x)) - x^2/((x^2*arcsin(x))/(sqrt(-x^2 + 1) + 1)^2 - arcsin(x))*(sqrt(-x^2 + 1) + 1)^2)`

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{1}{(1-x^2)\arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2}\arcsin(x)} \right) dx = \int \frac{1}{\arcsin(x)^2(x^2-1)} + \frac{x}{\arcsin(x)(1-x^2)^{3/2}} dx$$

input `int(1/(asin(x)^2*(x^2 - 1)) + x/(asin(x)*(1 - x^2)^(3/2)),x)`

output `int(1/(asin(x)^2*(x^2 - 1)) + x/(asin(x)*(1 - x^2)^(3/2)), x)`

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \left(-\frac{1}{(1-x^2)\arcsin(x)^2} + \frac{x}{(1-x^2)^{3/2}\arcsin(x)} \right) dx = \frac{1}{\sqrt{-x^2+1}\arcsin(x)}$$

input `int(-1/(-x^2+1)/asin(x)^2+x/(-x^2+1)^(3/2)/asin(x),x)`

output `1/(sqrt(-x**2+1)*asin(x))`

3.387 $\int \frac{x^3(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$

Optimal result	3359
Mathematica [C] (verified)	3360
Rubi [B] (verified)	3361
Maple [A] (verified)	3363
Fricas [F(-2)]	3364
Sympy [F]	3364
Maxima [F]	3365
Giac [F]	3365
Mupad [F(-1)]	3365
Reduce [F]	3366

Optimal result

Integrand size = 27, antiderivative size = 251

$$\int \frac{x^3(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx = -\frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} - \frac{d\sqrt{3\pi} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^4} + \frac{3d\sqrt{\pi} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8b^{3/2}c^4} + \frac{3d\sqrt{\pi} \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \sin\left(\frac{2a}{b}\right)}{8b^{3/2}c^4} - \frac{d\sqrt{3\pi} \text{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{6a}{b}\right)}{8b^{3/2}c^4}$$

output

```
-2*d*x^3*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arcsin(c*x))^(1/2)-1/8*d*3^(1/2)*Pi^(1/2)*cos(6*a/b)*FresnelC(2*3^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4+3/8*d*Pi^(1/2)*cos(2*a/b)*FresnelC(2*(a+b*arcsin(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^4+3/8*d*Pi^(1/2)*FresnelS(2*(a+b*arcsin(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c^4-1/8*d*3^(1/2)*Pi^(1/2)*FresnelS(2*3^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(6*a/b)/b^(3/2)/c^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.14

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx =$$

$$ide^{-\frac{6ia}{b}} \left(3\sqrt{2}e^{\frac{4ia}{b}} \sqrt{-\frac{i(a+b \arcsin(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \arcsin(cx))}{b}\right) - 3\sqrt{2}e^{\frac{8ia}{b}} \sqrt{\frac{i(a+b \arcsin(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{2i(a+b \arcsin(cx))}{b}\right) \right)$$

input

```
Integrate[(x^3*(d - c^2*d*x^2))/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
((-1/32*I)*d*(3*Sqrt[2]*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-2*I)*(a + b*ArcSin[c*x]))/b] - 3*Sqrt[2]*E^(((8*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[6]*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-6*I)*(a + b*ArcSin[c*x]))/b] + Sqrt[6]*E^(((12*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((6*I)*(a + b*ArcSin[c*x]))/b] - (6*I)*E^(((6*I)*a)/b)*Sin[2*ArcSin[c*x]] + (2*I)*E^(((6*I)*a)/b)*Sin[6*ArcSin[c*x]]))/(b*c^4*E^(((6*I)*a)/b)*Sqrt[a + b*ArcSin[c*x]])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 507 vs. $2(251) = 502$.

Time = 1.51 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5214, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx \\
 & \quad \downarrow \text{5214} \\
 & \frac{6d \int \frac{x^2 \sqrt{1-c^2 x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{12cd \int \frac{x^4 \sqrt{1-c^2 x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2dx^3(1-c^2 x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{5224} \\
 & \frac{12d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^4} + \\
 & \frac{6d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^4} - \frac{2dx^3(1-c^2 x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{4906} \\
 & \frac{6d \int \left(\frac{1}{8\sqrt{a+b \arcsin(cx)}} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^4} - \\
 & \frac{12d \int \left(\frac{\cos\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32\sqrt{a+b \arcsin(cx)}} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{16\sqrt{a+b \arcsin(cx)}} - \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32\sqrt{a+b \arcsin(cx)}} + \frac{1}{16\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^4} \\
 & \frac{2dx^3(1-c^2 x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\begin{aligned}
& 6d \left(-\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{4a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{4} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{4} \sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{4a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \right) \\
& \frac{b^2 c^4}{12d} \left(-\frac{1}{16} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{32} \sqrt{\frac{\pi}{3}} \sqrt{b} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{32} \sqrt{\frac{\pi}{3}} \sqrt{b} \sin\left(\frac{6a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \right) \\
& \frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}}
\end{aligned}$$

input

```
Int[(x^3*(d - c^2*d*x^2))/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
(-2*d*x^3*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSin[c*x]]) + (6*d*(Sqrt[a + b*ArcSin[c*x]]/4 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/8)/(b^2*c^4) - (12*d*(Sqrt[a + b*ArcSin[c*x]]/8 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/16 + (Sqrt[b]*Sqrt[Pi/3]*Cos[(6*a)/b]*FresnelC[(2*Sqrt[3/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcSin[c*x]])/(Sqrt[b]*Sqrt[Pi])])/32 - (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcSin[c*x]])/(Sqrt[b]*Sqrt[Pi])])*Sin[(2*a)/b])/32 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/16 + (Sqrt[b]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[3/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(6*a)/b])/32)/(b^2*c^4)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*
((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p
- 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*(m + 2*p + 1)/(b*f*(n
+ 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2
)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.21

method	result
default	$\frac{d \left(-\sqrt{-\frac{6}{b}} \operatorname{FresnelC} \left(\frac{6\sqrt{2}\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{-\frac{6}{b}}b} \right) \sqrt{a+b\arcsin(cx)} \sqrt{2} \cos\left(\frac{6a}{b}\right) \sqrt{\pi} + \sqrt{-\frac{6}{b}} \operatorname{FresnelS} \left(\frac{6\sqrt{2}\sqrt{a+b\arcsin(cx)}}{\sqrt{\pi}\sqrt{-\frac{6}{b}}b} \right) \sqrt{a+b\arcsin(cx)}}{\dots}$

input

```
int(x^3*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/16*d/c^4/b*(-(-6/b)^(1/2)*FresnelC(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*
arcsin(c*x))^(1/2)/b)*(a+b*arcsin(c*x))^(1/2)*2^(1/2)*cos(6*a/b)*Pi^(1/2)+
(-6/b)^(1/2)*FresnelS(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arcsin(c*x))^(1
/2)/b)*(a+b*arcsin(c*x))^(1/2)*2^(1/2)*sin(6*a/b)*Pi^(1/2)+6*(-1/b)^(1/2)*
Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-
2/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)-6*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arcsi
n(c*x))^(1/2)*sin(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcsi
n(c*x))^(1/2)/b)+3*sin(-2*(a+b*arcsin(c*x))/b+2*a/b)-sin(-6*(a+b*arcsin(c*
x))/b+6*a/b))/(a+b*arcsin(c*x))^(1/2)
```


Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx =$$

$$-d \left(\int \left(-\frac{x^3}{a \sqrt{a + b \arcsin(cx)} + b \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right)$$

$$+ \int \frac{c^2 x^5}{a \sqrt{a + b \arcsin(cx)} + b \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx$$

input `integrate(x**3*(-c**2*d*x**2+d)/(a+b*asin(c*x))**(3/2),x)`

output `-d*(Integral(-x**3/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{(c^2 dx^2 - d)x^3}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x^3*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `-integrate((c^2*d*x^2 - d)*x^3/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{(c^2 dx^2 - d)x^3}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x^3*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)*x^3/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx$$

input `int((x^3*(d - c^2*d*x^2))/(a + b*asin(c*x))^(3/2),x)`

output `int((x^3*(d - c^2*d*x^2))/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\int \frac{x^3(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = d \left(- \left(\int \frac{\sqrt{\arcsin(cx) b + a} x^5}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^2 \right. \\ \left. + \int \frac{\sqrt{\arcsin(cx) b + a} x^3}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right)$$

input `int(x^3*(-c^2*d*x^2+d)/(a+b*asin(c*x))^(3/2),x)`

output `d*(- int((sqrt(asin(c*x)*b + a)*x**5)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(asin(c*x)*b + a)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x))`

$$3.388 \quad \int \frac{x^2(d-c^2dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$$

Optimal result	3368
Mathematica [C] (verified)	3369
Rubi [A] (verified)	3370
Maple [A] (verified)	3372
Fricas [F(-2)]	3373
Sympy [F]	3373
Maxima [F]	3374
Giac [F]	3374
Mupad [F(-1)]	3375
Reduce [F]	3375

Optimal result

Integrand size = 27, antiderivative size = 591

$$\begin{aligned}
& \int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = -\frac{2dx^2(1 - c^2 x^2)^{3/2}}{bc\sqrt{a + b \arcsin(cx)}} \\
& - \frac{5d\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^3} \\
& + \frac{d\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} \\
& - \frac{5d\sqrt{\frac{\pi}{6}} \cos\left(\frac{3a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} \\
& + \frac{d\sqrt{\frac{2\pi}{3}} \cos\left(\frac{3a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} \\
& + \frac{d\sqrt{\frac{5\pi}{2}} \cos\left(\frac{5a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} \\
& + \frac{5d\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{2b^{3/2}c^3} \\
& - \frac{d\sqrt{2\pi} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{b^{3/2}c^3} \\
& + \frac{5d\sqrt{\frac{\pi}{6}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{3a}{b}\right)}{4b^{3/2}c^3} \\
& - \frac{d\sqrt{\frac{2\pi}{3}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{3a}{b}\right)}{b^{3/2}c^3} \\
& - \frac{d\sqrt{\frac{5\pi}{2}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{5a}{b}\right)}{4b^{3/2}c^3}
\end{aligned}$$

output

```
-2*d*x^2*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arcsin(c*x))^(1/2)-1/4*d*2^(1/2)*Pi^(
1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c^3+1/8*d*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+
b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+1/8*d*10^(1/2)*Pi^(1/2)*cos(5*a/
b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3
+1/4*d*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/
b^(1/2))*sin(a/b)/b^(3/2)/c^3-1/8*d*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(
1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c^3-1/8*d*10^(1/2
)*Pi^(1/2)*FresnelC(10^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin
(5*a/b)/b^(3/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.87

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \frac{de^{-\frac{5i(a+b \arcsin(cx))}{b}} \left(e^{\frac{5ia}{b}} + e^{\frac{5ia}{b} + 2i \arcsin(cx)} - 2e^{\frac{5ia}{b} + 4i \arcsin(cx)} - 2e^{\frac{5ia}{b} + 6i \arcsin(cx)} \right)}{...}$$

input

```
Integrate[(x^2*(d - c^2*d*x^2))/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
(d*(E^(((5*I)*a)/b) + E^(((5*I)*a)/b + (2*I)*ArcSin[c*x]) - 2*E^(((5*I)*a)
/b + (4*I)*ArcSin[c*x]) - 2*E^(((5*I)*a)/b + (6*I)*ArcSin[c*x]) + E^(((5*I)
)*a)/b + (8*I)*ArcSin[c*x]) + E^(((5*I)*(a + 2*b*ArcSin[c*x]))/b) + 2*E^((
(4*I)*a)/b + (5*I)*ArcSin[c*x])*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1
/2, ((-I)*(a + b*ArcSin[c*x]))/b] + 2*E^(((6*I)*a)/b + (5*I)*ArcSin[c*x])*
Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcSin[c*x]))/b] - Sq
rt[3]*E^(((2*I)*a)/b + (5*I)*ArcSin[c*x])*Sqrt[((-I)*(a + b*ArcSin[c*x]))/
b]*Gamma[1/2, ((-3*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[3]*E^(((8*I)*a)/b + (
5*I)*ArcSin[c*x])*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b
*ArcSin[c*x]))/b] - Sqrt[5]*E^(((5*I)*ArcSin[c*x])*Sqrt[((-I)*(a + b*ArcSin
[c*x]))/b]*Gamma[1/2, ((-5*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[5]*E^(((5*I)*
(2*a + b*ArcSin[c*x]))/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((5*I)
)*(a + b*ArcSin[c*x]))/b]))/(16*b*c^3*E^(((5*I)*(a + b*ArcSin[c*x]))/b)*Sq
rt[a + b*ArcSin[c*x]])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5214, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx$$

$$\downarrow 5214$$

$$\frac{4d \int \frac{x\sqrt{1-c^2x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{10cd \int \frac{x^3\sqrt{1-c^2x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 5224$$

$$\frac{10d \int -\frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c^3} +$$

$$\frac{4d \int -\frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c^3} - \frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 25$$

$$\frac{10d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c^3} -$$

$$\frac{4d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c^3} - \frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 4906$$

$$\frac{10d \int \left(-\frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16\sqrt{a+b \arcsin(cx)}} + \frac{\sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16\sqrt{a+b \arcsin(cx)}} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2c^3} -$$

$$\frac{4d \int \left(\frac{\sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4\sqrt{a+b \arcsin(cx)}} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2c^3} - \frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.76

method	result
default	$-\frac{d \left(-2\sqrt{-\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b}} b}\right) - 2\sqrt{-\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b}} b}\right) \right)}{1}$

input

```
int(x^2*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/8*d/c^3/b*(-2*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*cos
(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)-2*
(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(a/b)*FresnelC(2^
(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)+Pi^(1/2)*2^(1/2)*(a
+b*arcsin(c*x))^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*
(a+b*arcsin(c*x))^(1/2)/b)*(-3/b)^(1/2)+Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))
^(1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arcsin(c*x)
))^(1/2)/b)*(-3/b)^(1/2)+cos(5*a/b)*FresnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/
2)*(a+b*arcsin(c*x))^(1/2)/b)*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-5
/b)^(1/2)+sin(5*a/b)*FresnelC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arcsin(
c*x))^(1/2)/b)*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-5/b)^(1/2)+2*cos
(-(a+b*arcsin(c*x))/b+a/b)-cos(-3*(a+b*arcsin(c*x))/b+3*a/b)-cos(-5*(a+b*a
rcsin(c*x))/b+5*a/b))/(a+b*arcsin(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas"
)
```

output

```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx =$$

$$-d \left(\int \left(\frac{x^2}{a \sqrt{a + b \arcsin(cx)} + b \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right.$$

$$\left. + \int \frac{c^2 x^4}{a \sqrt{a + b \arcsin(cx)} + b \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

input `integrate(x**2*(-c**2*d*x**2+d)/(a+b*asin(c*x))**(3/2),x)`

output `-d*(Integral(-x**2/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{(c^2 dx^2 - d)x^2}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x^2*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `-integrate((c^2*d*x^2 - d)*x^2/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{(c^2 dx^2 - d)x^2}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x^2*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)*x^2/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{x^2(d - c^2 dx^2)}{(a + b \sin(cx))^{3/2}} dx$$

input `int((x^2*(d - c^2*d*x^2))/(a + b*asin(c*x))^(3/2),x)`

output `int((x^2*(d - c^2*d*x^2))/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\int \frac{x^2(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = d \left(- \left(\int \frac{\sqrt{\sin(cx) b + a} x^4}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right) c^2 \right. \\ \left. + \int \frac{\sqrt{\sin(cx) b + a} x^2}{\sin^2(cx) b^2 + 2 \sin(cx) ab + a^2} dx \right)$$

input `int(x^2*(-c^2*d*x^2+d)/(a+b*asin(c*x))^(3/2),x)`

output `d*(- int((sqrt(asin(c*x)*b + a)*x**4)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(asin(c*x)*b + a)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x))`

3.389 $\int \frac{x(d-c^2 dx^2)}{(a+b \arcsin(cx))^{3/2}} dx$

Optimal result	3376
Mathematica [C] (verified)	3377
Rubi [A] (verified)	3377
Maple [A] (verified)	3381
Fricas [F(-2)]	3381
Sympy [F]	3382
Maxima [F]	3382
Giac [F]	3383
Mupad [F(-1)]	3383
Reduce [F]	3383

Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = -\frac{2dx(1 - c^2 x^2)^{3/2}}{bc\sqrt{a + b \arcsin(cx)}} + \frac{d\sqrt{\frac{\pi}{2}} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{d\sqrt{\pi} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} + \frac{d\sqrt{\pi} \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \sin\left(\frac{2a}{b}\right)}{b^{3/2}c^2} + \frac{d\sqrt{\frac{\pi}{2}} \text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{4a}{b}\right)}{b^{3/2}c^2}$$

output

```
-2*d*x*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arcsin(c*x))^(1/2)+1/2*d*2^(1/2)*Pi^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2+d*Pi^(1/2)*cos(2*a/b)*FresnelC(2*(a+b*arcsin(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^2+d*Pi^(1/2)*FresnelS(2*(a+b*arcsin(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c^2+1/2*d*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(4*a/b)/b^(3/2)/c^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.15

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \frac{ide^{-\frac{4ia}{b}} \left(-\sqrt{2}e^{\frac{2ia}{b}} \sqrt{-\frac{i(a+b \arcsin(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \arcsin(cx))}{b}\right) + \sqrt{2}e^{\frac{6ia}{b}} \sqrt{\frac{i(a+b \arcsin(cx))}{b}} \right)}{b^2}$$

input `Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcSin[c*x])^(3/2),x]`

output `((I/4)*d*(-(Sqrt[2]*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-2*I)*(a + b*ArcSin[c*x]))/b]) + Sqrt[2]*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-4*I)*(a + b*ArcSin[c*x]))/b] + E^(((8*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((4*I)*(a + b*ArcSin[c*x]))/b] + (2*I)*E^(((4*I)*a)/b)*Sin[2*ArcSin[c*x]] + I*E^(((4*I)*a)/b)*Sin[4*ArcSin[c*x]]))/(b*c^2*E^(((4*I)*a)/b)*Sqrt[a + b*ArcSin[c*x]])`

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5214, 5168, 3042, 3793, 2009, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx$$

$$\downarrow \text{5214}$$

$$\frac{2d \int \frac{\sqrt{1-c^2x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{8cd \int \frac{x^2 \sqrt{1-c^2x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2dx(1 - c^2x^2)^{3/2}}{bc\sqrt{a + b \arcsin(cx)}}$$

$$\downarrow \text{5168}$$

$$\frac{2d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{8cd \int \frac{x^2 \sqrt{1-c^2 x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2dx(1-c^2 x^2)^{3/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 3042

$$\frac{2d \int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^2}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^2} - \frac{8cd \int \frac{x^2 \sqrt{1-c^2 x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2dx(1-c^2 x^2)^{3/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 3793

$$\frac{2d \int \left(\frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{2\sqrt{a+b \arcsin(cx)}} + \frac{1}{2\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^2} - \frac{8cd \int \frac{x^2 \sqrt{1-c^2 x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} -$$

$$\frac{2dx(1-c^2 x^2)^{3/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 2009

$$\frac{2d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a+b \arcsin(cx)} \right)}{b^2 c^2} - \frac{8cd \int \frac{x^2 \sqrt{1-c^2 x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} +$$

$$\frac{2dx(1-c^2 x^2)^{3/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 5224

$$\frac{2d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a+b \arcsin(cx)} \right)}{b^2 c^2} - \frac{8d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^2} +$$

$$\frac{2dx(1-c^2 x^2)^{3/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 4906

$$\frac{8d \int \left(\frac{1}{8\sqrt{a+b \arcsin(cx)}} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8\sqrt{a+b \arcsin(cx)}} \right) d(a + b \arcsin(cx))}{b^2 c^2} + \frac{2d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \operatorname{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a + b \arcsin(cx)} \right)}{b^2 c^2} \\ \frac{2dx(1 - c^2 x^2)^{3/2}}{bc\sqrt{a + b \arcsin(cx)}} \quad \downarrow \text{2009} \\ \frac{2d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \operatorname{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a + b \arcsin(cx)} \right)}{b^2 c^2} \\ \frac{8d \left(-\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{4a}{b}\right) \operatorname{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{4} \sqrt{a + b \arcsin(cx)} \right)}{b^2 c^2} \\ \frac{2dx(1 - c^2 x^2)^{3/2}}{bc\sqrt{a + b \arcsin(cx)}}$$

input `Int[(x*(d - c^2*d*x^2))/(a + b*ArcSin[c*x])^(3/2),x]`

output `(-2*d*x*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSin[c*x]]) + (2*d*(Sqrt[a + b*ArcSin[c*x]] + (Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcSin[c*x]])/(Sqrt[b]*Sqrt[Pi])])/2 + (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcSin[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/2))/(b^2*c^2) - (8*d*(Sqrt[a + b*ArcSin[c*x]]/4 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/8))/(b^2*c^2)`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 $\text{Int}[(c_.) + (d_.)(x_)^{(m_.)} \sin[(e_.) + (f_.)(x_)^{(n_.)}], x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (!\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$

rule 4906 $\text{Int}[\text{Cos}[(a_.) + (b_.)(x_)]^{(p_.)} ((c_.) + (d_.)(x_))^{(m_.)} \text{Sin}[(a_.) + (b_.)(x_)]^{(n_.)}], x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n \text{Cos}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

rule 5168 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)} ((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c)) \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Subst}[\text{Int}[x^n \text{Cos}[-a/b + x/b]^{(2*p + 1)}, x], x, a + b \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[2*p, 0]$

rule 5214 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)} ((f_.)(x_))^{(m_.)} ((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(f*x)^m \text{Sqrt}[1 - c^2*x^2] (d + e*x^2)^p ((a + b \text{ArcSin}[c*x])^{(n + 1)} / (b*c*(n + 1))), x] + (-\text{Simp}[f*(m/(b*c*(n + 1))) \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m - 1)} (1 - c^2*x^2)^{(p - 1/2)} (a + b \text{ArcSin}[c*x])^{(n + 1)}, x], x] + \text{Simp}[c*((m + 2*p + 1)/(b*f*(n + 1))) \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[(f*x)^{(m + 1)} (1 - c^2*x^2)^{(p - 1/2)} (a + b \text{ArcSin}[c*x])^{(n + 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IGtQ}[2*p, 0] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0] \ \&\& \ \text{IGtQ}[m, -3]$

rule 5224 $\text{Int}[(a_.) + \text{ArcSin}[(c_.)(x_)](b_.)]^{(n_.)} (x_)^{(m_.)} ((d_.) + (e_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(1/(b*c^{(m + 1)})) \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Subst}[\text{Int}[x^n \text{Sin}[-a/b + x/b]^m \text{Cos}[-a/b + x/b]^{(2*p + 1)}, x], x, a + b \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[2*p + 2, 0] \ \&\& \ \text{IGtQ}[m, 0]$

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.26

method	result
default	$d \left(2\sqrt{-\frac{1}{b}} \sqrt{2} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b} b}}\right) - 2\sqrt{-\frac{1}{b}} \sqrt{2} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{4a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b} b}}\right) \right)$

input `int(x*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)`

output
$$\begin{aligned} & \frac{1}{4} \frac{d}{c^2 b} \left(2 \left(-\frac{1}{b}\right)^{\frac{1}{2}} 2^{\frac{1}{2}} \pi^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2 \cdot 2^{\frac{1}{2}} / \pi^{\frac{1}{2}}}{(-1/b)^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} / b} \right) - \right. \\ & 2 \left(-\frac{1}{b}\right)^{\frac{1}{2}} 2^{\frac{1}{2}} \pi^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} \sin\left(\frac{4a}{b}\right) \text{FresnelS}\left(\frac{2 \cdot 2^{\frac{1}{2}} / \pi^{\frac{1}{2}}}{(-1/b)^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} / b} \right) + 4 \left(-\frac{1}{b}\right)^{\frac{1}{2}} \\ & \left. \pi^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2 \cdot 2^{\frac{1}{2}} / \pi^{\frac{1}{2}}}{(-2/b)^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} / b} \right) - 4 \left(-\frac{1}{b}\right)^{\frac{1}{2}} \pi^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} \sin\left(\frac{2a}{b}\right) \right. \\ & \left. \text{FresnelS}\left(\frac{2 \cdot 2^{\frac{1}{2}} / \pi^{\frac{1}{2}}}{(-2/b)^{\frac{1}{2}} (a+b \arcsin(cx))^{\frac{1}{2}} / b} \right) + 2 \sin\left(-2 \frac{(a+b \arcsin(cx))}{b+2a/b} + \sin\left(-4 \frac{(a+b \arcsin(cx))}{b+4a/b}\right) \right) \right) / (a+b \arcsin(cx))^{\frac{1}{2}} \end{aligned}$$

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx =$$

$$-d \left(\int \left(-\frac{x}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right.$$

$$\left. + \int \frac{c^2 x^3}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

input `integrate(x*(-c**2*d*x**2+d)/(a+b*asin(c*x))**(3/2),x)`

output `-d*(Integral(-x/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{(c^2 dx^2 - d)x}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `-integrate((c^2*d*x^2 - d)*x/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{(c^2 dx^2 - d)x}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate(x*(-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)*x/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx$$

input `int((x*(d - c^2*d*x^2))/(a + b*asin(c*x))^(3/2),x)`

output `int((x*(d - c^2*d*x^2))/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\int \frac{x(d - c^2 dx^2)}{(a + b \arcsin(cx))^{3/2}} dx = d \left(- \left(\int \frac{\sqrt{\arcsin(cx) b + a} x^3}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^2 \right. \\ \left. + \int \frac{\sqrt{\arcsin(cx) b + a} x}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right)$$

input `int(x*(-c^2*d*x^2+d)/(a+b*asin(c*x))^(3/2),x)`

output `d*(- int((sqrt(asin(c*x)*b + a)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(asin(c*x)*b + a)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x))`

3.390 $\int \frac{d-c^2 dx^2}{(a+b \arcsin(cx))^{3/2}} dx$

Optimal result	3384
Mathematica [C] (verified)	3385
Rubi [A] (verified)	3386
Maple [A] (verified)	3388
Fricas [F(-2)]	3388
Sympy [F]	3389
Maxima [F]	3389
Giac [F]	3390
Mupad [F(-1)]	3390
Reduce [F]	3390

Optimal result

Integrand size = 24, antiderivative size = 253

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = -\frac{2d(1 - c^2 x^2)^{3/2}}{bc\sqrt{a + b \arcsin(cx)}} - \frac{3d\sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{d\sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{3d\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{b^{3/2}c} + \frac{d\sqrt{\frac{3\pi}{2}} \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{3a}{b}\right)}{b^{3/2}c}$$

output

```
-2*d*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arcsin(c*x))^(1/2)-3/2*d*2^(1/2)*Pi^(1/2)
*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)
/c-1/2*d*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arcs
in(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+3/2*d*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/
Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(3/2)/c+1/2*d*6^(1/2)
*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(3
*a/b)/b^(3/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.38

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = \frac{de^{-\frac{3i(a+b \arcsin(cx))}{b}} \left(-e^{\frac{3ia}{b}} - 3e^{\frac{3ia}{b} + 2i \arcsin(cx)} - 3e^{\frac{3ia}{b} + 4i \arcsin(cx)} - e^{\frac{3i(a+2b \arcsin(cx))}{b}} \right)}{...}$$

input

```
Integrate[(d - c^2*d*x^2)/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
(d*(-E^(((3*I)*a)/b) - 3*E^(((3*I)*a)/b + (2*I)*ArcSin[c*x]) - 3*E^(((3*I)
*a)/b + (4*I)*ArcSin[c*x]) - E^(((3*I)*(a + 2*b*ArcSin[c*x]))/b) + 3*E^(((
2*I)*a)/b + (3*I)*ArcSin[c*x])*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/
2, ((-I)*(a + b*ArcSin[c*x]))/b] + 3*E^(((4*I)*a)/b + (3*I)*ArcSin[c*x])*S
qrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcSin[c*x]))/b] + Sqr
t[3]*E^(((3*I)*ArcSin[c*x])*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, (
(-3*I)*(a + b*ArcSin[c*x]))/b] + Sqrt[3]*E^(((3*I)*((2*a)/b + ArcSin[c*x]))
*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcSin[c*x]))/b]
))/(4*b*c*E^(((3*I)*(a + b*ArcSin[c*x]))/b)*Sqrt[a + b*ArcSin[c*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5166, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx \\
 & \quad \downarrow \text{5166} \\
 & -\frac{6cd \int \frac{x\sqrt{1-c^2x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{5224} \\
 & -\frac{6d \int -\frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c} - \frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{25} \\
 & -\frac{6d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c} - \frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{4906} \\
 & -\frac{6d \int \left(\frac{\sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{4\sqrt{a+b \arcsin(cx)}} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{4\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2c} - \frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{2009} \\
 & -\frac{6d \left(-\frac{1}{2} \sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{2} \sqrt{\frac{\pi}{6}} \sqrt{b} \sin\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\frac{\pi}{2}} \right)}{b^2c} - \frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b \arcsin(cx)}}
 \end{aligned}$$

input `Int[(d - c^2*d*x^2)/(a + b*ArcSin[c*x])^(3/2),x]`

output `(-2*d*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcSin[c*x]]) - (6*d*((Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/2 - (Sqrt[b]*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2))/(b^2*c)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5166 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1))/(b*c*(n + 1)), x] + Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d \left(-3\sqrt{-\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b} b}}\right) - 3\sqrt{-\frac{1}{b}} \sqrt{\pi} \sqrt{2} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b} b}}\right)}{\right)}$

input `int((-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)`

output
$$-1/2/c*d/b/(a+b*arcsin(c*x))^{1/2}*(-3*(-1/b)^{1/2}*Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\cos(a/b)*\operatorname{FresnelS}(2^{1/2}/Pi^{1/2}/(-1/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)-3*(-1/b)^{1/2}*Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\sin(a/b)*\operatorname{FresnelC}(2^{1/2}/Pi^{1/2}/(-1/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)-Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\cos(3*a/b)*\operatorname{FresnelS}(3*2^{1/2}/Pi^{1/2}/(-3/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)*(-3/b)^{1/2}-Pi^{1/2}*2^{1/2}*(a+b*arcsin(c*x))^{1/2}*\sin(3*a/b)*\operatorname{FresnelC}(3*2^{1/2}/Pi^{1/2}/(-3/b)^{1/2}*(a+b*arcsin(c*x))^{1/2}/b)*(-3/b)^{1/2}+3*\cos(-(a+b*arcsin(c*x)))/b+a/b+\cos(-3*(a+b*arcsin(c*x))/b+3*a/b)$$

Fricas [F(-2)]

Exception generated.

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx =$$

$$-d \left(\int \frac{c^2 x^2}{a \sqrt{a + b \arcsin(cx)} + b \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

$$+ \int \left(-\frac{1}{a \sqrt{a + b \arcsin(cx)} + b \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx$$

input `integrate((-c**2*d*x**2+d)/(a+b*asin(c*x))**(3/2),x)`

output `-d*(Integral(c**2*x**2/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(-1/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{c^2 dx^2 - d}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate((-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `-integrate((c^2*d*x^2 - d)/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{c^2 dx^2 - d}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate((-c^2*d*x^2+d)/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx$$

input `int((d - c^2*d*x^2)/(a + b*asin(c*x))^(3/2),x)`

output `int((d - c^2*d*x^2)/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\int \frac{d - c^2 dx^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{Too large to display}$$

input `int((-c^2*d*x^2+d)/(a+b*asin(c*x))^(3/2),x)`

output

```
(d*(12*sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*asin(c*x)*b + 12*asin(c*x)*int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c - 12*asin(c*x)*int((sqrt(asin(c*x)*b + a)*x**4)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**5 + 24*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*asin(c*x)*x**3)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**4 - 12*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**3*c**2 + 24*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*b*c**4 + 12*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*b*c**2 + 3*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**3*c**2 - 8*sqrt(asin(c*x)...
```

3.391
$$\int \frac{d-c^2 dx^2}{x(a+b \arcsin(cx))^{3/2}} dx$$

Optimal result	3392
Mathematica [N/A]	3392
Rubi [N/A]	3393
Maple [N/A]	3394
Fricas [F(-2)]	3395
Sympy [N/A]	3395
Maxima [N/A]	3396
Giac [F(-2)]	3396
Mupad [N/A]	3396
Reduce [N/A]	3397

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = \text{Int}\left(\frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}}, x\right)$$

output `Defer(Int)((-c^2*d*x^2+d)/x/(a+b*arcsin(c*x))^(3/2),x)`

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = \int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx$$

input `Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcSin[c*x])^(3/2)),x]`

output `Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcSin[c*x])^(3/2)), x]`

Rubi [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx \\
 & \quad \downarrow \text{5214} \\
 & - \frac{4cd \int \frac{\sqrt{1-c^2x^2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d \int \frac{\sqrt{1-c^2x^2}}{x^2 \sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{5168} \\
 & - \frac{4d \int \frac{\cos^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2} - \frac{2d \int \frac{\sqrt{1-c^2x^2}}{x^2 \sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{3042} \\
 & - \frac{4d \int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^2}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2} - \frac{2d \int \frac{\sqrt{1-c^2x^2}}{x^2 \sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{3793} \\
 & - \frac{4d \int \left(\frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{2\sqrt{a+b \arcsin(cx)}} + \frac{1}{2\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2} - \frac{2d \int \frac{\sqrt{1-c^2x^2}}{x^2 \sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\frac{2d \int \frac{\sqrt{1-c^2x^2}}{x^2 \sqrt{a+b \arcsin(cx)}} dx}{4d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a+b \arcsin(cx)} \right) \frac{bc}{b^2}}$$

$$\frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}}$$

↓ 5226

$$\frac{2d \int \left(\frac{1}{x^2 \sqrt{1-c^2x^2} \sqrt{a+b \arcsin(cx)}} - \frac{c^2}{\sqrt{1-c^2x^2} \sqrt{a+b \arcsin(cx)}} \right) dx}{4d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a+b \arcsin(cx)} \right) \frac{bc}{b^2}}$$

$$\frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}}$$

↓ 2009

$$\frac{2d \left(\int \frac{1}{x^2 \sqrt{1-c^2x^2} \sqrt{a+b \arcsin(cx)}} dx - \frac{2c \sqrt{a+b \arcsin(cx)}}{b} \right)}{4d \left(\frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \sqrt{a+b \arcsin(cx)} \right) \frac{bc}{b^2}}$$

$$\frac{2d(1-c^2x^2)^{3/2}}{bcx \sqrt{a+b \arcsin(cx)}}$$

input

```
Int[(d - c^2*d*x^2)/(x*(a + b*ArcSin[c*x])^(3/2)),x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{-c^2 d x^2 + d}{x (a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

input `int((-c^2*d*x^2+d)/x/(a+b*arcsin(c*x))^(3/2),x)`

output `int((-c^2*d*x^2+d)/x/(a+b*arcsin(c*x))^(3/2),x)`

Fricas [F(-2)]

Exception generated.

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)/x/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx =$$

$$-d \left(\int \frac{c^2 x^2}{ax \sqrt{a + b \arcsin(cx)} + bx \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right.$$

$$\left. + \int \left(-\frac{1}{ax \sqrt{a + b \arcsin(cx)} + bx \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right)$$

input `integrate((-c**2*d*x**2+d)/x/(a+b*asin(c*x))**(3/2),x)`

output `-d*(Integral(c**2*x**2/(a*x*sqrt(a + b*asin(c*x)) + b*x*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(-1/(a*x*sqrt(a + b*asin(c*x)) + b*x*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{c^2 dx^2 - d}{(b \arcsin(cx) + a)^{\frac{3}{2}} x} dx$$

input `integrate((-c^2*d*x^2+d)/x/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `-integrate((c^2*d*x^2 - d)/((b*arcsin(c*x) + a)^(3/2)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)/x/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = \int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx$$

input `int((d - c^2*d*x^2)/(x*(a + b*asin(c*x))^(3/2)),x)`

output `int((d - c^2*d*x^2)/(x*(a + b*asin(c*x))^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{d - c^2 dx^2}{x(a + b \arcsin(cx))^{3/2}} dx = d \left(\int \frac{\sqrt{\arcsin(cx) b + a}}{\arcsin(cx)^2 b^2 x + 2 \arcsin(cx) abx + a^2 x} dx \right. \\ \left. - \left(\int \frac{\sqrt{\arcsin(cx) b + a} x}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^2 \right)$$

input `int((-c^2*d*x^2+d)/x/(a+b*asin(c*x))^(3/2),x)`

output `d*(int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*x + 2*asin(c*x)*a*b*x + a**2*x),x) - int((sqrt(asin(c*x)*b + a)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2)`

3.392 $\int \frac{d-c^2 dx^2}{x^2(a+b \arcsin(cx))^{3/2}} dx$

Optimal result	3398
Mathematica [N/A]	3398
Rubi [N/A]	3399
Maple [N/A]	3400
Fricas [F(-2)]	3400
Sympy [N/A]	3401
Maxima [N/A]	3401
Giac [N/A]	3402
Mupad [N/A]	3402
Reduce [N/A]	3402

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \text{Int}\left(\frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}}, x\right)$$

output Defer(Int)((-c^2*d*x^2+d)/x^2/(a+b*arcsin(c*x))^(3/2),x)

Mathematica [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx$$

input Integrate[(d - c^2*d*x^2)/(x^2*(a + b*ArcSin[c*x])^(3/2)),x]

output Integrate[(d - c^2*d*x^2)/(x^2*(a + b*ArcSin[c*x])^(3/2)), x]

Rubi [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{d - c^2 x^2}{x^2 (a + b \arcsin(cx))^{3/2}} dx \\
 & \quad \downarrow \text{5214} \\
 & - \frac{2cd \int \frac{\sqrt{1-c^2x^2}}{x\sqrt{a+b\arcsin(cx)}} dx}{b} - \frac{4d \int \frac{\sqrt{1-c^2x^2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d(1-c^2x^2)^{3/2}}{bcx^2\sqrt{a+b\arcsin(cx)}} \\
 & \quad \downarrow \text{5226} \\
 & - \frac{2cd \int \left(\frac{1}{x\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} - \frac{c^2x}{\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} \right) dx}{b} - \frac{4d \int \frac{\sqrt{1-c^2x^2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \\
 & \quad \frac{2d(1-c^2x^2)^{3/2}}{bcx^2\sqrt{a+b\arcsin(cx)}} \\
 & \quad \downarrow \text{2009} \\
 & - \frac{2cd \left(\int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} dx + \frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{\sqrt{b}} \right)}{b} \\
 & \quad - \frac{4d \int \frac{\sqrt{1-c^2x^2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d(1-c^2x^2)^{3/2}}{bcx^2\sqrt{a+b\arcsin(cx)}} \\
 & \quad \downarrow \text{5234}
 \end{aligned}$$

$$2cd \left(\int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} dx + \frac{\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2\pi} \cos\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{\sqrt{b}} \right) - \frac{4d \int \frac{\sqrt{1-c^2x^2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{b}{bcx^2\sqrt{a+b\arcsin(cx)}} - \frac{2d(1-c^2x^2)^{3/2}}{bcx^2\sqrt{a+b\arcsin(cx)}}$$

input `Int[(d - c^2*d*x^2)/(x^2*(a + b*ArcSin[c*x])^(3/2)),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{-c^2 d x^2 + d}{x^2 (a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

input `int((-c^2*d*x^2+d)/x^2/(a+b*arcsin(c*x))^(3/2),x)`

output `int((-c^2*d*x^2+d)/x^2/(a+b*arcsin(c*x))^(3/2),x)`

Fricas [F(-2)]

Exception generated.

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)/x^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [N/A]

Not integrable

Time = 5.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = -d \left(\int \frac{c^2 x^2}{ax^2 \sqrt{a + b \arcsin(cx)} + bx^2 \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx + \int \left(-\frac{1}{ax^2 \sqrt{a + b \arcsin(cx)} + bx^2 \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right)$$

input `integrate((-c**2*d*x**2+d)/x**2/(a+b*asin(c*x))**(3/2),x)`

output `-d*(Integral(c**2*x**2/(a*x**2*sqrt(a + b*asin(c*x)) + b*x**2*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(-1/(a*x**2*sqrt(a + b*asin(c*x)) + b*x**2*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{c^2 dx^2 - d}{(b \arcsin(cx) + a)^{\frac{3}{2}} x^2} dx$$

input `integrate((-c^2*d*x^2+d)/x^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `-integrate((c^2*d*x^2 - d)/((b*arcsin(c*x) + a)^(3/2)*x^2), x)`

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int -\frac{c^2 dx^2 - d}{(b \arcsin(cx) + a)^{\frac{3}{2}} x^2} dx$$

input `integrate((-c^2*d*x^2+d)/x^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate(-(c^2*d*x^2 - d)/((b*arcsin(c*x) + a)^(3/2)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx$$

input `int((d - c^2*d*x^2)/(x^2*(a + b*asin(c*x))^(3/2)),x)`

output `int((d - c^2*d*x^2)/(x^2*(a + b*asin(c*x))^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 9.26

$$\int \frac{d - c^2 dx^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \frac{d \left(\arcsin(cx) \left(\int \frac{\sqrt{\arcsin(cx)b+a}}{\arcsin(cx)^2 b^2 x^2 + 2 \arcsin(cx) a b x^2 + a^2 x^2} dx \right) b^2 + 2 \arcsin(cx) \left(\int \frac{\sqrt{\arcsin(cx)b+a}}{\sqrt{-c^2 x^2 + 1}} dx \right) \right)}{x^2(a + b \arcsin(cx))^{3/2}}$$

input `int((-c^2*d*x^2+d)/x^2/(a+b*asin(c*x))^(3/2),x)`

output

```
(d*(asin(c*x)*int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*x**2 + 2*asin(c*x)*a*b*x**2 + a**2*x**2),x)*b**2 + 2*asin(c*x)*int((sqrt(asin(c*x)*b + a)*x)/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b + sqrt(-c**2*x**2 + 1)*a),x)*b*c**3 + 2*sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*c + int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*x**2 + 2*asin(c*x)*a*b*x**2 + a**2*x**2),x)*a*b + 2*int((sqrt(asin(c*x)*b + a)*x)/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b + sqrt(-c**2*x**2 + 1)*a),x)*a*c**3))/(b*(asin(c*x)*b + a))
```


$$3.393 \quad \int \frac{x^3(d-c^2dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$$

Optimal result	3405
Mathematica [C] (verified)	3406
Rubi [A] (verified)	3407
Maple [A] (verified)	3410
Fricas [F(-2)]	3411
Sympy [F]	3411
Maxima [F]	3412
Giac [F]	3412
Mupad [F(-1)]	3412
Reduce [F]	3413

Optimal result

Integrand size = 29, antiderivative size = 485

$$\begin{aligned}
& \int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = -\frac{2d^2 x^3(1 - c^2 x^2)^{5/2}}{bc\sqrt{a + b \arcsin(cx)}} \\
& + \frac{d^2 \sqrt{\frac{\pi}{2}} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^4} \\
& - \frac{d^2 \sqrt{3\pi} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} \\
& + \frac{3d^2 \sqrt{\pi} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{16b^{3/2}c^4} \\
& - \frac{d^2 \sqrt{\pi} \cos\left(\frac{8a}{b}\right) \text{FresnelC}\left(\frac{4\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{16b^{3/2}c^4} \\
& + \frac{3d^2 \sqrt{\pi} \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \sin\left(\frac{2a}{b}\right)}{16b^{3/2}c^4} \\
& + \frac{d^2 \sqrt{\frac{\pi}{2}} \text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{4a}{b}\right)}{8b^{3/2}c^4} \\
& - \frac{d^2 \sqrt{3\pi} \text{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{6a}{b}\right)}{16b^{3/2}c^4} \\
& - \frac{d^2 \sqrt{\pi} \text{FresnelS}\left(\frac{4\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \sin\left(\frac{8a}{b}\right)}{16b^{3/2}c^4}
\end{aligned}$$

output

```

-2*d^2*x^3*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arcsin(c*x))^(1/2)+1/16*d^2*2^(1/2)
*Pi^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b
^(1/2))/b^(3/2)/c^4-1/16*d^2*3^(1/2)*Pi^(1/2)*cos(6*a/b)*FresnelC(2*3^(1/2)
)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4+3/16*d^2*Pi^(1/2)*
cos(2*a/b)*FresnelC(2*(a+b*arcsin(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^
4-1/16*d^2*Pi^(1/2)*cos(8*a/b)*FresnelC(4*(a+b*arcsin(c*x))^(1/2)/b^(1/2)/
Pi^(1/2))/b^(3/2)/c^4+3/16*d^2*Pi^(1/2)*FresnelS(2*(a+b*arcsin(c*x))^(1/2)
/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c^4+1/16*d^2*2^(1/2)*Pi^(1/2)*Fresne
lS(2*2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(4*a/b)/b^(3/2)/
c^4-1/16*d^2*3^(1/2)*Pi^(1/2)*FresnelS(2*3^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x)
)^(1/2)/b^(1/2))*sin(6*a/b)/b^(3/2)/c^4-1/16*d^2*Pi^(1/2)*FresnelS(4*(a+b*
arcsin(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(8*a/b)/b^(3/2)/c^4

```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.11

$$\int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx =$$

$$id^2 e^{-\frac{8ia}{b}} \left(3\sqrt{2} e^{\frac{6ia}{b}} \sqrt{-\frac{i(a+b \arcsin(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \arcsin(cx))}{b}\right) - 3\sqrt{2} e^{\frac{10ia}{b}} \sqrt{\frac{i(a+b \arcsin(cx))}{b}} \Gamma\left(\frac{1}{2}, \frac{2i(a+b \arcsin(cx))}{b}\right) \right)$$

input

```
Integrate[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
((-1/64*I)*d^2*(3*Sqrt[2]*E^(((6*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-2*I)*(a + b*ArcSin[c*x]))/b] - 3*Sqrt[2]*E^(((10*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcSin[c*x]))/b] + 2*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-4*I)*(a + b*ArcSin[c*x]))/b] - 2*E^(((12*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((4*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[6]*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-6*I)*(a + b*ArcSin[c*x]))/b] + Sqrt[6]*E^(((14*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((6*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[2]*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-8*I)*(a + b*ArcSin[c*x]))/b] + Sqrt[2]*E^(((16*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((8*I)*(a + b*ArcSin[c*x]))/b] - (6*I)*E^(((8*I)*a)/b)*Sin[2*ArcSin[c*x]] - (2*I)*E^(((8*I)*a)/b)*Sin[4*ArcSin[c*x]] + (2*I)*E^(((8*I)*a)/b)*Sin[6*ArcSin[c*x]] + I*E^(((8*I)*a)/b)*Sin[8*ArcSin[c*x]]))/(b*c^4*E^(((8*I)*a)/b)*Sqrt[a + b*ArcSin[c*x]])
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5214, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d - c^2x^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

$$\downarrow 5214$$

$$\frac{6d^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{16cd^2 \int \frac{x^4(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d^2x^3(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 5224$$

$$- \frac{16d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a + b \arcsin(cx))}{b^2c^4} +$$

$$\frac{6d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a + b \arcsin(cx))}{b^2c^4} - \frac{2d^2x^3(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

↓ 4906

$$\frac{16d^2 \int \left(\frac{\cos\left(\frac{8a}{b} - \frac{8(a+b \arcsin(cx))}{b}\right)}{128\sqrt{a+b \arcsin(cx)}} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{32\sqrt{a+b \arcsin(cx)}} + \frac{3}{128\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^4} +$$

$$\frac{6d^2 \int \left(-\frac{\cos\left(\frac{6a}{b} - \frac{6(a+b \arcsin(cx))}{b}\right)}{32\sqrt{a+b \arcsin(cx)}} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{16\sqrt{a+b \arcsin(cx)}} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{32\sqrt{a+b \arcsin(cx)}} + \frac{1}{16\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^4}$$

$$\frac{2d^2 x^3 (1 - c^2 x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

↓ 2009

$$\frac{6d^2 \left(-\frac{1}{16} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{32} \sqrt{\frac{\pi}{3}} \sqrt{b} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \dots \right)}{b^2 c^4}$$

$$\frac{16d^2 \left(-\frac{1}{32} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{256} \sqrt{\pi} \sqrt{b} \cos\left(\frac{8a}{b}\right) \text{FresnelC}\left(\frac{4\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) - \frac{1}{32} \dots \right)}{b^2 c^4}$$

$$\frac{2d^2 x^3 (1 - c^2 x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

input

```
Int[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcSin[c*x])^(3/2),x]
```

output

$$\begin{aligned} & (-2*d^2*x^3*(1 - c^2*x^2)^{(5/2)})/(b*c*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) + (6*d^2*(\text{Sqrt}[a + b*\text{ArcSin}[c*x]]/8 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[(4*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/16 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/3]*\text{Cos}[(6*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/32 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])])/32 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[(2*a)/b])/32 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(4*a)/b])/16 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/3]*\text{FresnelS}[(2*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(6*a)/b])/32))/(b^2*c^4) - (16*d^2*((3*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/64 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[(4*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/32 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{Cos}[(8*a)/b]*\text{FresnelC}[(4*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])])/256 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(4*a)/b])/32 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(4*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])]*\text{Sin}[(8*a)/b])/256))/(b^2*c^4) \end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> } \text{Simp}[\text{IntSum}[u, x], x] \text{ /; } \text{SumQ}[u]$$

rule 4906

$$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(p_.)}*((c_.) + (d_.)*(x_))^{(m_.)}*\text{Sin}[(a_.) + (b_.)*(x_)]^{(n_.)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x]^n*\text{Cos}[a + b*x]^p, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$$

rule 5214

$$\begin{aligned} & \text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))^{(n_.)}*((f_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x_Symbol] \text{ :> } \text{Simp}[(f*x)^m*\text{Sqrt}[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*\text{ArcSin}[c*x])^{(n + 1)})/(b*c*(n + 1)), x] + (-\text{Simp}[f*(m/(b*c*(n + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m - 1)}*(1 - c^2*x^2)^{(p - 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x] + \text{Simp}[c*((m + 2*p + 1)/(b*f*(n + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{ Int}[(f*x)^{(m + 1)}*(1 - c^2*x^2)^{(p - 1/2)}*(a + b*\text{ArcSin}[c*x])^{(n + 1)}, x], x]) \text{ /; } \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IGtQ}[2*p, 0] \ \&\& \ \text{NeQ}[m + 2*p + 1, 0] \ \&\& \ \text{IGtQ}[m, -3] \end{aligned}$$

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.22

method	result
default	$\frac{d^2 \left(4\sqrt{-\frac{1}{b}} \sqrt{2} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{4a}{b}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b}} b}\right) - 4\sqrt{-\frac{1}{b}} \sqrt{2} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{4a}{b}\right) \operatorname{FresnelS}\left(\frac{2\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b}} b}\right) \right)}{\dots}$

input

```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/64*d^2/c^4/b*(4*(-1/b)^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*co
s(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/
b)-4*(-1/b)^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(4*a/b)*Fres
nelS(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)-2*(-6/b)^(
1/2)*FresnelC(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*(
a+b*arcsin(c*x))^(1/2)*2^(1/2)*cos(6*a/b)*Pi^(1/2)+2*(-6/b)^(1/2)*FresnelS
(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*(a+b*arcsin(c*
x))^(1/2)*2^(1/2)*sin(6*a/b)*Pi^(1/2)+12*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arcsin
(c*x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcs
in(c*x))^(1/2)/b)-12*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(2*a
/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)-4*
cos(8*a/b)*FresnelC(4*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcsin(c*x))^(1/2
)/b)*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*(-1/b)^(1/2)+4*sin(8*a/b)*FresnelS(4
*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*(a+b*arcsin(c*x)
)^(1/2)*Pi^(1/2)*(-1/b)^(1/2)+6*sin(-2*(a+b*arcsin(c*x))/b+2*a/b)+2*sin(-4
*(a+b*arcsin(c*x))/b+4*a/b)-2*sin(-6*(a+b*arcsin(c*x))/b+6*a/b)-sin(-8*(a+
b*arcsin(c*x))/b+8*a/b))/(a+b*arcsin(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\begin{aligned} \int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx &= d^2 \left(\int \frac{x^3}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right. \\ &+ \int \left(-\frac{2c^2 x^5}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \\ &\left. + \int \frac{c^4 x^7}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right) \end{aligned}$$

input `integrate(x**3*(-c**2*d*x**2+d)**2/(a+b*asin(c*x))**(3/2),x)`

output `d**2*(Integral(x**3/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(-2*c**2*x**5/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2 x^3}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `integrate((c^2*d*x^2 - d)^2*x^3/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2 x^3}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate((c^2*d*x^2 - d)^2*x^3/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

input `int((x^3*(d - c^2*d*x^2)^2)/(a + b*asin(c*x))^(3/2),x)`

output `int((x^3*(d - c^2*d*x^2)^2)/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\int \frac{x^3(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\left(\int \frac{\sqrt{\arcsin(cx) b + a} x^7}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^4 \right. \\ \left. - 2 \left(\int \frac{\sqrt{\arcsin(cx) b + a} x^5}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^2 \right. \\ \left. + \int \frac{\sqrt{\arcsin(cx) b + a} x^3}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right)$$

input

```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*asin(c*x))^(3/2),x)
```

output

```
d**2*(int((sqrt(asin(c*x)*b + a)*x**7)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*
b + a**2),x)*c**4 - 2*int((sqrt(asin(c*x)*b + a)*x**5)/(asin(c*x)**2*b**2
+ 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(asin(c*x)*b + a)*x**3)/(asin
(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x))
```

$$3.394 \quad \int \frac{x^2(d-c^2dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$$

Optimal result	3415
Mathematica [C] (verified)	3416
Rubi [A] (verified)	3417
Maple [A] (verified)	3420
Fricas [F(-2)]	3421
Sympy [F]	3422
Maxima [F]	3422
Giac [F]	3423
Mupad [F(-1)]	3423
Reduce [F]	3423

Optimal result

Integrand size = 29, antiderivative size = 511

$$\begin{aligned}
& \int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = -\frac{2d^2 x^2(1 - c^2 x^2)^{5/2}}{bc\sqrt{a + b \arcsin(cx)}} \\
& - \frac{5d^2 \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} \\
& + \frac{d^2 \sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} \\
& + \frac{3d^2 \sqrt{\frac{5\pi}{2}} \cos\left(\frac{5a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} \\
& + \frac{d^2 \sqrt{\frac{7\pi}{2}} \cos\left(\frac{7a}{b}\right) \operatorname{FresnelS}\left(\frac{\sqrt{\frac{14}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} \\
& + \frac{5d^2 \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{16b^{3/2}c^3} \\
& - \frac{d^2 \sqrt{\frac{3\pi}{2}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{3a}{b}\right)}{16b^{3/2}c^3} \\
& - \frac{3d^2 \sqrt{\frac{5\pi}{2}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{5a}{b}\right)}{16b^{3/2}c^3} \\
& - \frac{d^2 \sqrt{\frac{7\pi}{2}} \operatorname{FresnelC}\left(\frac{\sqrt{\frac{14}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{7a}{b}\right)}{16b^{3/2}c^3}
\end{aligned}$$

output

```

-2*d^2*x^2*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arcsin(c*x))^(1/2)-5/32*d^2*2^(1/2)
*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2)
)/b^(3/2)/c^3+1/32*d^2*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(
1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+3/32*d^2*10^(1/2)*Pi^(1/2)
*cos(5*a/b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/
b^(3/2)/c^3+1/32*d^2*14^(1/2)*Pi^(1/2)*cos(7*a/b)*FresnelS(14^(1/2)/Pi^(1/2)
*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+5/32*d^2*2^(1/2)*Pi^(1/2)*
FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(3/2)
)/c^3-1/32*d^2*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x)
)^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c^3-3/32*d^2*10^(1/2)*Pi^(1/2)*Fresnel
C(10^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(5*a/b)/b^(3/2)/c^
3-1/32*d^2*14^(1/2)*Pi^(1/2)*FresnelC(14^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(
1/2)/b^(1/2))*sin(7*a/b)/b^(3/2)/c^3

```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.34

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \frac{d^2 e^{-\frac{7i(a+b \arcsin(cx))}{b}} \left(e^{\frac{7ia}{b}} + 3e^{\frac{7ia}{b} + 2i \arcsin(cx)} + e^{\frac{7ia}{b} + 4i \arcsin(cx)} - 5e^{\frac{7ia}{b} + 6i \arcsin(cx)} \right)}{...}$$

input

```
Integrate[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
(d^2*(E^(((7*I)*a)/b) + 3*E^(((7*I)*a)/b + (2*I)*ArcSin[c*x]) + E^(((7*I)*a)/b + (4*I)*ArcSin[c*x]) - 5*E^(((7*I)*a)/b + (6*I)*ArcSin[c*x]) - 5*E^(((7*I)*a)/b + (8*I)*ArcSin[c*x]) + E^(((7*I)*a)/b + (10*I)*ArcSin[c*x]) + 3*E^(((7*I)*a)/b + (12*I)*ArcSin[c*x]) + E^(((7*I)*a + 2*b*ArcSin[c*x])/b) + 5*E^(((6*I)*a)/b + (7*I)*ArcSin[c*x])*Sqrt[(-I)*(a + b*ArcSin[c*x])/b]*Gamma[1/2, ((-I)*(a + b*ArcSin[c*x])/b)] + 5*E^(((8*I)*a)/b + (7*I)*ArcSin[c*x])*Sqrt[(I*(a + b*ArcSin[c*x])/b)*Gamma[1/2, (I*(a + b*ArcSin[c*x])/b)] - Sqrt[3]*E^(((4*I)*a)/b + (7*I)*ArcSin[c*x])*Sqrt[(-I)*(a + b*ArcSin[c*x])/b]*Gamma[1/2, ((-3*I)*(a + b*ArcSin[c*x])/b)] - Sqrt[3]*E^(((10*I)*a)/b + (7*I)*ArcSin[c*x])*Sqrt[(I*(a + b*ArcSin[c*x])/b)*Gamma[1/2, ((3*I)*(a + b*ArcSin[c*x])/b)] - 3*Sqrt[5]*E^(((2*I)*a)/b + (7*I)*ArcSin[c*x])*Sqrt[(-I)*(a + b*ArcSin[c*x])/b]*Gamma[1/2, ((-5*I)*(a + b*ArcSin[c*x])/b)] - 3*Sqrt[5]*E^(((12*I)*a)/b + (7*I)*ArcSin[c*x])*Sqrt[(I*(a + b*ArcSin[c*x])/b)*Gamma[1/2, ((5*I)*(a + b*ArcSin[c*x])/b)] - Sqrt[7]*E^(((7*I)*ArcSin[c*x])*Sqrt[(-I)*(a + b*ArcSin[c*x])/b]*Gamma[1/2, ((-7*I)*(a + b*ArcSin[c*x])/b)] - Sqrt[7]*E^(((7*I)*(2*a + b*ArcSin[c*x])/b)*Sqrt[(I*(a + b*ArcSin[c*x])/b)*Gamma[1/2, ((7*I)*(a + b*ArcSin[c*x])/b)]))/(64*b*c^3*E^(((7*I)*(a + b*ArcSin[c*x])/b)*Sqrt[a + b*ArcSin[c*x]])
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5214, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

$$\downarrow 5214$$

$$\frac{4d^2 \int \frac{x(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{14cd^2 \int \frac{x^3(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d^2x^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 5224$$

$$\frac{14d^2 \int -\frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^3} + \frac{4d^2 \int -\frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{2d^2 x^2 (1 - c^2 x^2)^{5/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 25

$$\frac{14d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin^3\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{4d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2 c^3} - \frac{2d^2 x^2 (1 - c^2 x^2)^{5/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 4906

$$\frac{14d^2 \int \left(-\frac{\sin\left(\frac{7a}{b} - \frac{7(a+b \arcsin(cx))}{b}\right)}{64 \sqrt{a+b \arcsin(cx)}} - \frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{64 \sqrt{a+b \arcsin(cx)}} + \frac{3 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{64 \sqrt{a+b \arcsin(cx)}} + \frac{3 \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{64 \sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^3} - \frac{4d^2 \int \left(\frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16 \sqrt{a+b \arcsin(cx)}} + \frac{3 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16 \sqrt{a+b \arcsin(cx)}} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8 \sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2 c^3} - \frac{2d^2 x^2 (1 - c^2 x^2)^{5/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

↓ 2009

$$\frac{4d^2 \left(\frac{1}{4} \sqrt{b} \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{8} \sqrt{b} \sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{8} \sqrt{b} \sqrt{\frac{6}{\pi}} \cos\left(\frac{5a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{8} \sqrt{b} \sqrt{\frac{8}{\pi}} \cos\left(\frac{7a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{14}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \right)}{b^2 c^3} - \frac{14d^2 \left(\frac{3}{32} \sqrt{b} \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{32} \sqrt{b} \sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{32} \sqrt{b} \sqrt{\frac{6}{\pi}} \cos\left(\frac{5a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{32} \sqrt{b} \sqrt{\frac{8}{\pi}} \cos\left(\frac{7a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{14}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \right)}{b^2 c^3} - \frac{2d^2 x^2 (1 - c^2 x^2)^{5/2}}{bc \sqrt{a+b \arcsin(cx)}}$$

input

```
Int[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcSin[c*x])^(3/2),x]
```

output

$$\begin{aligned}
& (-2*d^2*x^2*(1 - c^2*x^2)^{(5/2)})/(b*c*Sqrt[a + b*ArcSin[c*x]]) + (4*d^2*((\\
& Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/ \\
& Sqrt[b]])/4 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[\\
& a + b*ArcSin[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]*Fre \\
& snelS[(Sqrt[10/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi \\
& /2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[a/b])/4 - (\\
& Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[\\
& b]]*Sin[(3*a)/b])/8 - (Sqrt[b]*Sqrt[Pi/10]*FresnelC[(Sqrt[10/Pi]*Sqrt[a + \\
& b*ArcSin[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/8)/(b^2*c^3) - (14*d^2*((3*Sqrt[b] \\
& *Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b] \\
&])/32 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + \\
& b*ArcSin[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]*FresnelS \\
& [(Sqrt[10/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi/14] \\
& *Cos[(7*a)/b]*FresnelS[(Sqrt[14/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/32 \\
& - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt \\
& [b]]*Sin[a/b])/32 - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + \\
& b*ArcSin[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/32 + (Sqrt[b]*Sqrt[Pi/10]*FresnelC[\\
& (Sqrt[10/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/32 + (Sqrt[b] \\
& *Sqrt[Pi/14]*FresnelC[(Sqrt[14/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(\\
& 7*a)/b])/32))/(b^2*c^3)
\end{aligned}$$

Defintions of rubi rules used

rule 25

$$\text{Int}[-(F_x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F_x, x], x]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$$

rule 4906

$$\begin{aligned}
& \text{Int}[\text{Cos}[(a_.) + (b_.)*(x_)]^{(p_.)*((c_.) + (d_.)*(x_))^{(m_.)*\text{Sin}[(a_.) + (b \\
& _.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x \\
&]^n*\text{Cos}[a + b*x]^p, x], x] \text{ /; FreeQ}\{a, b, c, d, m\}, x \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IG} \\
& \text{tQ}[p, 0]
\end{aligned}$$

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*
((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1))
)*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p
- 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*(m + 2*p + 1)/(b*f*(n
+ 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2
)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1
, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.16

method	result
default	$-\frac{d^2 \left(\cos\left(\frac{7a}{b}\right) \operatorname{FresnelS}\left(\frac{7\sqrt{2}\sqrt{a+b}\arcsin(cx)}{\sqrt{\pi}\sqrt{-\frac{7}{b}}}\right) \sqrt{a+b}\arcsin(cx) \sqrt{\pi} \sqrt{2} \sqrt{-\frac{7}{b}} + \sin\left(\frac{7a}{b}\right) \operatorname{FresnelC}\left(\frac{7\sqrt{2}\sqrt{a+b}\arcsin(cx)}{\sqrt{\pi}\sqrt{-\frac{7}{b}}}\right) \sqrt{a+b}\arcsin(cx)}{\dots}$

input

```
int(x^2*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```

-1/32*d^2/c^3/b/(a+b*arcsin(c*x))^(1/2)*(cos(7*a/b)*FresnelS(7*2^(1/2)/Pi^(1/2)/(-7/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-7/b)^(1/2)+sin(7*a/b)*FresnelC(7*2^(1/2)/Pi^(1/2)/(-7/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-7/b)^(1/2)+Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b*(-3/b)^(1/2)+Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b*(-3/b)^(1/2)-5*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)-5*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)+3*cos(5*a/b)*FresnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-5/b)^(1/2)+3*sin(5*a/b)*FresnelC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-5/b)^(1/2)+5*cos(-(a+b*arcsin(c*x))/b+a/b)-cos(-3*(a+b*arcsin(c*x))/b+3*a/b)-3*cos(-5*(a+b*arcsin(c*x))/b+5*a/b)-cos(-7*(a+b*arcsin(c*x))/b+7*a/b))

```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\int \frac{x^2}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right. \\ \left. + \int \left(-\frac{2c^2 x^4}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right. \\ \left. + \int \frac{c^4 x^6}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

input `integrate(x**2*(-c**2*d*x**2+d)**2/(a+b*asin(c*x))**(3/2),x)`

output `d**2*(Integral(x**2/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(-2*c**2*x**4/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2 x^2}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `integrate((c^2*d*x^2 - d)^2*x^2/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2 x^2}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate((c^2*d*x^2 - d)^2*x^2/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

input `int((x^2*(d - c^2*d*x^2)^2)/(a + b*asin(c*x))^(3/2),x)`

output `int((x^2*(d - c^2*d*x^2)^2)/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\begin{aligned} \int \frac{x^2(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx &= d^2 \left(\left(\int \frac{\sqrt{a \sin(cx) b + a x^6}}{a \sin^2(cx) b^2 + 2a \sin(cx) ab + a^2} dx \right) c^4 \right. \\ &\quad \left. - 2 \left(\int \frac{\sqrt{a \sin(cx) b + a x^4}}{a \sin^2(cx) b^2 + 2a \sin(cx) ab + a^2} dx \right) c^2 \right. \\ &\quad \left. + \int \frac{\sqrt{a \sin(cx) b + a x^2}}{a \sin^2(cx) b^2 + 2a \sin(cx) ab + a^2} dx \right) \end{aligned}$$

input `int(x^2*(-c^2*d*x^2+d)^2/(a+b*asin(c*x))^(3/2),x)`

output

```
d**2*(int((sqrt(asin(c*x)*b + a)*x**6)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*
b + a**2),x)*c**4 - 2*int((sqrt(asin(c*x)*b + a)*x**4)/(asin(c*x)**2*b**2
+ 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(asin(c*x)*b + a)*x**2)/(asin
(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x))
```

3.395 $\int \frac{x(d-c^2 dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx$

Optimal result	3425
Mathematica [C] (verified)	3426
Rubi [A] (verified)	3427
Maple [A] (verified)	3430
Fricas [F(-2)]	3431
Sympy [F]	3431
Maxima [F]	3432
Giac [F]	3432
Mupad [F(-1)]	3433
Reduce [F]	3433

Optimal result

Integrand size = 27, antiderivative size = 373

$$\int \frac{x(d-c^2 dx^2)^2}{(a+b \arcsin(cx))^{3/2}} dx = -\frac{2d^2 x(1-c^2 x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}} + \frac{d^2 \sqrt{\frac{\pi}{2}} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{d^2 \sqrt{3\pi} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^2} + \frac{5d^2 \sqrt{\pi} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8b^{3/2}c^2} + \frac{5d^2 \sqrt{\pi} \text{FresnelS}\left(\frac{2\sqrt{a+b \arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \sin\left(\frac{2a}{b}\right)}{8b^{3/2}c^2} + \frac{d^2 \sqrt{\frac{\pi}{2}} \text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{4a}{b}\right)}{b^{3/2}c^2} + \frac{d^2 \sqrt{3\pi} \text{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{6a}{b}\right)}{8b^{3/2}c^2}$$

output

$$\begin{aligned}
& -2d^2x(-c^2x^2+1)^{5/2}/b/c/(a+b\arcsin(cx))^{1/2}+1/2d^2x^{1/2}\pi^{1/2}\cos(4a/b)\operatorname{FresnelC}(2x^{1/2}/\pi^{1/2}(a+b\arcsin(cx))^{1/2}/b^{1/2})/b^{3/2}/c^2+1/8d^2x^{3/2}\pi^{1/2}\cos(6a/b)\operatorname{FresnelC}(2x^{3/2}/\pi^{1/2}(a+b\arcsin(cx))^{1/2}/b^{1/2})/b^{3/2}/c^2+5/8d^2\pi^{1/2}\cos(2a/b)\operatorname{FresnelC}(2(a+b\arcsin(cx))^{1/2}/b^{1/2}/\pi^{1/2})/b^{3/2}/c^2+5/8d^2\pi^{1/2}\operatorname{FresnelS}(2(a+b\arcsin(cx))^{1/2}/b^{1/2}/\pi^{1/2})\sin(2a/b)/b^{3/2}/c^2+1/2d^2x^{1/2}\pi^{1/2}\operatorname{FresnelS}(2x^{1/2}/\pi^{1/2}(a+b\arcsin(cx))^{1/2}/b^{1/2})\sin(4a/b)/b^{3/2}/c^2+1/8d^2x^{3/2}\pi^{1/2}\operatorname{FresnelS}(2x^{3/2}/\pi^{1/2}(a+b\arcsin(cx))^{1/2}/b^{1/2})\sin(6a/b)/b^{3/2}/c^2
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.10

$$\int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \frac{id^2 e^{-\frac{6ia}{b}} \left(-5\sqrt{2} e^{\frac{4ia}{b}} \sqrt{-\frac{i(a+b \arcsin(cx))}{b}} \Gamma\left(\frac{1}{2}, -\frac{2i(a+b \arcsin(cx))}{b}\right) \right) + 5\sqrt{2} e^{\frac{8ia}{b}} \sqrt{i(a+b \arcsin(cx))}}{b^2}$$

input

```
Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```

((I/32)*d^2*(-5*Sqrt[2]*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]
*Gamma[1/2, ((-2*I)*(a + b*ArcSin[c*x]))/b] + 5*Sqrt[2]*E^(((8*I)*a)/b)*Sqrt[
(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcSin[c*x]))/b] -
8*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((-4*I)*(a
+ b*ArcSin[c*x]))/b] + 8*E^(((10*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]
*Gamma[1/2, ((4*I)*(a + b*ArcSin[c*x]))/b] - Sqrt[6]*Sqrt[((-I)*(a + b*Arc
Sin[c*x]))/b]*Gamma[1/2, ((-6*I)*(a + b*ArcSin[c*x]))/b] + Sqrt[6]*E^(((12
*I)*a)/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, ((6*I)*(a + b*ArcSin[
c*x]))/b] + (10*I)*E^(((6*I)*a)/b)*Sin[2*ArcSin[c*x]] + (8*I)*E^(((6*I)*a)
/b)*Sin[4*ArcSin[c*x]] + (2*I)*E^(((6*I)*a)/b)*Sin[6*ArcSin[c*x]]))/(b*c^2
*E^(((6*I)*a)/b)*Sqrt[a + b*ArcSin[c*x]])

```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.62, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5214, 5168, 3042, 3793, 2009, 5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx \\
 & \quad \downarrow \text{5214} \\
 & \frac{2d^2 \int \frac{(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{bc} - \frac{12cd^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{5168} \\
 & \frac{2d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a + b \arcsin(cx))}{b^2c^2} - \frac{12cd^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{2d^2 \int \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^4}{\sqrt{a+b \arcsin(cx)}} d(a + b \arcsin(cx))}{b^2c^2} - \frac{12cd^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \\
 & \quad \frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{3793} \\
 & \frac{2d^2 \int \left(\frac{\cos\left(\frac{4a}{b} - \frac{4(a+b \arcsin(cx))}{b}\right)}{8\sqrt{a+b \arcsin(cx)}} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b}\right)}{2\sqrt{a+b \arcsin(cx)}} + \frac{3}{8\sqrt{a+b \arcsin(cx)}} \right) d(a + b \arcsin(cx))}{b^2c^2} - \\
 & \quad \frac{12cd^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\frac{12cd^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arcsin(cx)}} dx}{b} + 2d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \right)$$

b^2c^2

$$\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arcsin(cx)}}$$

5224

$$\frac{12d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{\sqrt{a+b\arcsin(cx)}} d(a+b\arcsin(cx))}{b^2c^2} + 2d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \right)$$

b^2c^2

$$\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arcsin(cx)}}$$

4906

$$\frac{12d^2 \int \left(-\frac{\cos\left(\frac{6a}{b} - \frac{6(a+b\arcsin(cx))}{b}\right)}{32\sqrt{a+b\arcsin(cx)}} - \frac{\cos\left(\frac{4a}{b} - \frac{4(a+b\arcsin(cx))}{b}\right)}{16\sqrt{a+b\arcsin(cx)}} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b\arcsin(cx))}{b}\right)}{32\sqrt{a+b\arcsin(cx)}} + \frac{1}{16\sqrt{a+b\arcsin(cx)}} \right) d(a+b\arcsin(cx))}{b^2c^2} + 2d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \right)$$

b^2c^2

$$\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arcsin(cx)}}$$

2009

$$2d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) \right) + 12d^2 \left(-\frac{1}{16} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{32} \sqrt{\frac{\pi}{3}} \sqrt{b} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{32} \sqrt{\frac{\pi}{3}} \sqrt{b} \cos\left(\frac{6a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) \right)$$

b^2c^2

$$\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arcsin(cx)}}$$

input $\text{Int}[(x*(d - c^2*d*x^2)^2)/(a + b*\text{ArcSin}[c*x])^{3/2}, x]$

output
$$\begin{aligned} & (-2*d^2*x*(1 - c^2*x^2)^{5/2})/(b*c*\text{Sqrt}[a + b*\text{ArcSin}[c*x]]) + (2*d^2*((3* \\ & \text{Sqrt}[a + b*\text{ArcSin}[c*x]])/4 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[(4*a)/b]*\text{FresnelC}[(2* \\ & \text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/8 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{Cos}[(2 \\ & *a)/b]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))/2 + (\text{Sqrt} \\ & [b]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))*\text{Sin} \\ & [(2*a)/b])/2 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin} \\ & [c*x]])/\text{Sqrt}[b]]*\text{Sin}[(4*a)/b])/8)/(b^2*c^2) - (12*d^2*(\text{Sqrt}[a + b*\text{ArcSin}[\\ & c*x]])/8 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{Cos}[(4*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + \\ & b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/16 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/3]*\text{Cos}[(6*a)/b]*\text{FresnelC} \\ & [(2*\text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]])/32 + (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{Co} \\ & s[(2*a)/b]*\text{FresnelC}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]))/32 + \\ & (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{FresnelS}[(2*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/(\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}])) \\ & *\text{Sin}[(2*a)/b])/32 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/2]*\text{FresnelS}[(2*\text{Sqrt}[2/\text{Pi}]*\text{Sqrt}[a + b* \\ & \text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(4*a)/b])/16 - (\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}/3]*\text{FresnelS}[(2* \\ & \text{Sqrt}[3/\text{Pi}]*\text{Sqrt}[a + b*\text{ArcSin}[c*x]])/\text{Sqrt}[b]]*\text{Sin}[(6*a)/b])/32)/(b^2*c^2) \end{aligned}$$

Defintions of rubi rules used

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinear} \\ \text{Q}[u, x]$

rule 3793 $\text{Int}(((c_.) + (d_.)*(x_.))^{(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol) \rightarrow \text{Int} \\ [\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] \text{ ; FreeQ}\{c, d, e, f \\ , m\}, x \ \&\& \text{IGtQ}[n, 1] \ \&\& (\text{!RationalQ}[m] \ || (\text{GeQ}[m, -1] \ \&\& \text{LtQ}[m, 1]))$

rule 4906 $\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sin}[(a_.) + (b \\ _.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[a + b*x \\]^n*\text{Cos}[a + b*x]^p, x], x] \text{ ; FreeQ}\{a, b, c, d, m\}, x \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IG} \\ \text{tQ}[p, 0]$

rule 5168

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5214

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(f*x)^m*Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Simp[f*(m/(b*c*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x] + Simp[c*((m + 2*p + 1)/(b*f*(n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1, 0] && IGtQ[m, -3]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.20

method	result
default	$\frac{d^2 \left(8\sqrt{-\frac{1}{b}} \sqrt{2} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{4a}{b}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b}} b}\right) - 8\sqrt{-\frac{1}{b}} \sqrt{2} \sqrt{\pi} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{4a}{b}\right) \operatorname{FresnelS}\left(\frac{2\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{1}{b}} b}\right) \right)}{d^2}$

input

```
int(x*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```

1/16*d^2/c^2/b*(8*(-1/b)^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*co
s(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/
b)-8*(-1/b)^(1/2)*2^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(4*a/b)*Fres
nelS(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)+(-6/b)^(1/
2)*FresnelC(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*(a+
b*arcsin(c*x))^(1/2)*2^(1/2)*cos(6*a/b)*Pi^(1/2)-(-6/b)^(1/2)*FresnelS(6*2
^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*(a+b*arcsin(c*x))^(
1/2)*2^(1/2)*sin(6*a/b)*Pi^(1/2)+10*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x
))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcsin(c
*x))^(1/2)/b)-10*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(2*a/b)*
FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)+5*sin(
-2*(a+b*arcsin(c*x))/b+2*a/b)+4*sin(-4*(a+b*arcsin(c*x))/b+4*a/b)+sin(-6*(
a+b*arcsin(c*x))/b+6*a/b))/(a+b*arcsin(c*x))^(1/2)

```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input

```

integrate(x*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas"
)

```

output

```

Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)

```

Sympy [F]

$$\begin{aligned}
& \int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\int \frac{x}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right. \\
& + \int \left(-\frac{2c^2 x^3}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \\
& \left. + \int \frac{c^4 x^5}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)
\end{aligned}$$

input `integrate(x*(-c**2*d*x**2+d)**2/(a+b*asin(c*x))**(3/2),x)`

output `d**2*(Integral(x/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(-2*c**2*x**3/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2 x}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `integrate((c^2*d*x^2 - d)^2*x/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2 x}{(b \arcsin(cx) + a)^{\frac{3}{2}}} dx$$

input `integrate(x*(-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate((c^2*d*x^2 - d)^2*x/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{x(d - c^2 dx^2)^2}{(a + b \sin(cx))^{3/2}} dx$$

input `int((x*(d - c^2*d*x^2)^2)/(a + b*asin(c*x))^(3/2),x)`

output `int((x*(d - c^2*d*x^2)^2)/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\begin{aligned} \int \frac{x(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx &= d^2 \left(\left(\int \frac{\sqrt{\arcsin(cx) b + a} x^5}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^4 \right. \\ &\quad \left. - 2 \left(\int \frac{\sqrt{\arcsin(cx) b + a} x^3}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) c^2 \right. \\ &\quad \left. + \int \frac{\sqrt{\arcsin(cx) b + a} x}{\arcsin(cx)^2 b^2 + 2 \arcsin(cx) ab + a^2} dx \right) \end{aligned}$$

input `int(x*(-c^2*d*x^2+d)^2/(a+b*asin(c*x))^(3/2),x)`

output `d**2*(int((sqrt(asin(c*x)*b + a)*x**5)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(asin(c*x)*b + a)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2 + int((sqrt(asin(c*x)*b + a)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x))`

3.396
$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

Optimal result	3434
Mathematica [C] (verified)	3435
Rubi [A] (verified)	3436
Maple [A] (verified)	3438
Fricas [F(-2)]	3439
Sympy [F]	3439
Maxima [F]	3440
Giac [F]	3440
Mupad [F(-1)]	3440
Reduce [F]	3441

Optimal result

Integrand size = 26, antiderivative size = 390

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = -\frac{2d^2(1 - c^2 x^2)^{5/2}}{bc\sqrt{a + b \arcsin(cx)}} - \frac{5d^2 \sqrt{\frac{\pi}{2}} \cos\left(\frac{a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c} - \frac{5d^2 \sqrt{\frac{3\pi}{2}} \cos\left(\frac{3a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{d^2 \sqrt{\frac{5\pi}{2}} \cos\left(\frac{5a}{b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{5d^2 \sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{a}{b}\right)}{2b^{3/2}c} + \frac{5d^2 \sqrt{\frac{3\pi}{2}} \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{3a}{b}\right)}{4b^{3/2}c} + \frac{d^2 \sqrt{\frac{5\pi}{2}} \text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a + b \arcsin(cx)}}{\sqrt{b}}\right) \sin\left(\frac{5a}{b}\right)}{4b^{3/2}c}$$

output

```
-2*d^2*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arcsin(c*x))^(1/2)-5/4*d^2*2^(1/2)*Pi^(
1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c-5/8*d^2*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+
b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-1/8*d^2*10^(1/2)*Pi^(1/2)*cos(5*a/
b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+5
/4*d^2*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/
b^(1/2))*sin(a/b)/b^(3/2)/c+5/8*d^2*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(
1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c+1/8*d^2*10^(1/2
)*Pi^(1/2)*FresnelC(10^(1/2)/Pi^(1/2)*(a+b*arcsin(c*x))^(1/2)/b^(1/2))*sin
(5*a/b)/b^(3/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.34

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \frac{d^2 e^{-\frac{5i(a+b \arcsin(cx))}{b}} \left(-e^{\frac{5ia}{b}} - 5e^{\frac{5ia}{b} + 2i \arcsin(cx)} - 10e^{\frac{5ia}{b} + 4i \arcsin(cx)} - 10e^{\frac{5ia}{b} + 6i \arcsin(cx)} \right)}{\dots}$$

input

```
Integrate[(d - c^2*d*x^2)^2/(a + b*ArcSin[c*x])^(3/2),x]
```

output

```
(d^2*(-E^(((5*I)*a)/b) - 5*E^(((5*I)*a)/b + (2*I)*ArcSin[c*x]) - 10*E^(((5
*I)*a)/b + (4*I)*ArcSin[c*x]) - 10*E^(((5*I)*a)/b + (6*I)*ArcSin[c*x]) - 5
*E^(((5*I)*a)/b + (8*I)*ArcSin[c*x]) - E^(((5*I)*(a + 2*b*ArcSin[c*x]))/b)
+ 10*E^(((4*I)*a)/b + (5*I)*ArcSin[c*x])*Sqrt[(-I)*(a + b*ArcSin[c*x]))/
b]*Gamma[1/2, (-I)*(a + b*ArcSin[c*x])/b] + 10*E^(((6*I)*a)/b + (5*I)*Ar
cSin[c*x])*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcSin[c*x
]))/b] + 5*Sqrt[3]*E^(((2*I)*a)/b + (5*I)*ArcSin[c*x])*Sqrt[(-I)*(a + b*A
rcSin[c*x]))/b]*Gamma[1/2, (-3*I)*(a + b*ArcSin[c*x])/b] + 5*Sqrt[3]*E^(
((8*I)*a)/b + (5*I)*ArcSin[c*x])*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*Gamma[1/2
, ((3*I)*(a + b*ArcSin[c*x]))/b] + Sqrt[5]*E^((5*I)*ArcSin[c*x])*Sqrt[(-I
)*(a + b*ArcSin[c*x]))/b]*Gamma[1/2, (-5*I)*(a + b*ArcSin[c*x])/b] + Sqr
t[5]*E^(((5*I)*(2*a + b*ArcSin[c*x]))/b)*Sqrt[(I*(a + b*ArcSin[c*x]))/b]*G
amma[1/2, ((5*I)*(a + b*ArcSin[c*x]))/b]))/(16*b*c*E^(((5*I)*(a + b*ArcSin
[c*x]))/b)*Sqrt[a + b*ArcSin[c*x]])
```


Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5166, 5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

$$\downarrow 5166$$

$$-\frac{10cd^2 \int \frac{x(1-c^2x^2)^{3/2}}{\sqrt{a+b \arcsin(cx)}} dx}{b} - \frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 5224$$

$$\frac{10d^2 \int -\frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c} - \frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 25$$

$$\frac{10d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{\sqrt{a+b \arcsin(cx)}} d(a+b \arcsin(cx))}{b^2c} - \frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 4906$$

$$\frac{10d^2 \int \left(\frac{\sin\left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b}\right)}{16\sqrt{a+b \arcsin(cx)}} + \frac{3 \sin\left(\frac{3a}{b} - \frac{3(a+b \arcsin(cx))}{b}\right)}{16\sqrt{a+b \arcsin(cx)}} + \frac{\sin\left(\frac{a}{b} - \frac{a+b \arcsin(cx)}{b}\right)}{8\sqrt{a+b \arcsin(cx)}} \right) d(a+b \arcsin(cx))}{b^2c} - \frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

$$\downarrow 2009$$

$$10d^2 \left(-\frac{1}{4} \sqrt{\frac{\pi}{2}} \sqrt{b} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{8} \sqrt{\frac{3\pi}{2}} \sqrt{b} \sin\left(\frac{3a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) - \frac{1}{8} \sqrt{\frac{10\pi}{2}} \sqrt{b} \sin\left(\frac{5a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}} \sqrt{a+b \arcsin(cx)}}{\sqrt{b}}\right) \right) - \frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b \arcsin(cx)}}$$

input `Int[(d - c^2*d*x^2)^2/(a + b*ArcSin[c*x])^(3/2),x]`

output `(-2*d^2*(1 - c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcSin[c*x]]) - (10*d^2*((Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/4 + (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]*FresnelS[(Sqrt[10/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[a/b])/4 - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/8 - (Sqrt[b]*Sqrt[Pi/10]*FresnelC[(Sqrt[10/Pi]*Sqrt[a + b*ArcSin[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/8)/(b^2*c)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5166

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] := Simp[Sqrt[1 - c^2*x^2]*(d + e*x^2)^p*((a + b*ArcSin[c*x])^(n + 1)
)/(b*c*(n + 1)), x] + Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p/(1
- c^2*x^2)^p] Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.16

method	result
default	$-\frac{d^2 \left(-5\sqrt{\pi} \sqrt{2} \sqrt{a+b \arcsin(cx)} \cos\left(\frac{3a}{b}\right) \operatorname{FresnelS}\left(\frac{3\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{3}{b}}}\right) \sqrt{-\frac{3}{b}} - 5\sqrt{\pi} \sqrt{2} \sqrt{a+b \arcsin(cx)} \sin\left(\frac{3a}{b}\right) \operatorname{FresnelC}\left(\frac{3\sqrt{2} \sqrt{a+b \arcsin(cx)}}{\sqrt{\pi} \sqrt{-\frac{3}{b}}}\right) \sqrt{-\frac{3}{b}} \right)}{\dots}$

input

```
int((-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/8/c*d^2/b/(a+b*arcsin(c*x))^(1/2)*(-5*Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x)
)^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arcsin(c*
x))^(1/2)/b)*(-3/b)^(1/2)-5*Pi^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*sin(3
*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)*
(-3/b)^(1/2)-cos(5*a/b)*FresnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arcs
in(c*x))^(1/2)/b*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-5/b)^(1/2)-si
n(5*a/b)*FresnelC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/
b*(a+b*arcsin(c*x))^(1/2)*Pi^(1/2)*2^(1/2)*(-5/b)^(1/2)-10*(-1/b)^(1/2)*P
i^(1/2)*2^(1/2)*(a+b*arcsin(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)
/(-1/b)^(1/2)*(a+b*arcsin(c*x))^(1/2)/b)-10*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*
(a+b*arcsin(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a
+b*arcsin(c*x))^(1/2)/b)+cos(-5*(a+b*arcsin(c*x))/b+5*a/b)+5*cos(-3*(a+b*a
rcsin(c*x))/b+3*a/b)+10*cos(-(a+b*arcsin(c*x))/b+a/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\int \left(-\frac{2c^2 x^2}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right. \\ \left. + \int \frac{c^4 x^4}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right. \\ \left. + \int \frac{1}{a\sqrt{a + b \arcsin(cx)} + b\sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

input `integrate((-c**2*d*x**2+d)**2/(a+b*asin(c*x))**(3/2),x)`

output `d**2*(Integral(-2*c**2*x**2/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(1/(a*sqrt(a + b*asin(c*x)) + b*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [F]

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate((-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `integrate((c^2*d*x^2 - d)^2/(b*arcsin(c*x) + a)^(3/2), x)`

Giac [F]

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2}{(b \arcsin(cx) + a)^{3/2}} dx$$

input `integrate((-c^2*d*x^2+d)^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate((c^2*d*x^2 - d)^2/(b*arcsin(c*x) + a)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx$$

input `int((d - c^2*d*x^2)^2/(a + b*asin(c*x))^(3/2),x)`

output `int((d - c^2*d*x^2)^2/(a + b*asin(c*x))^(3/2), x)`

Reduce [F]

$$\int \frac{(d - c^2 dx^2)^2}{(a + b \arcsin(cx))^{3/2}} dx = \text{too large to display}$$

input `int((-c^2*d*x^2+d)^2/(a+b*asin(c*x))^(3/2),x)`

output

```
(d**2*(20*sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*asin(c*x)*b + 20*asin(c*x)*int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c + 8*asin(c*x)*int((sqrt(asin(c*x)*b + a)*x**6)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**7 - 28*asin(c*x)*int((sqrt(asin(c*x)*b + a)*x**4)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**5 + 24*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*asin(c*x)*x**3)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**4 - 20*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*asin(c*x)**2*x)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**3*c**2 + 24*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*x**3)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*b*c**4 + 20*asin(c*x)*int((sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*x)/(asin(c*x)**2*b**2*c**2*x**2 - asin(c*x)**2*b**2 + 2*asin(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*b*c**2 + 5*asin(c*x)*int((sqrt(asin(c*x)...
```

$$3.397 \quad \int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx$$

Optimal result	3442
Mathematica [N/A]	3442
Rubi [N/A]	3443
Maple [N/A]	3445
Fricas [F(-2)]	3445
Sympy [N/A]	3445
Maxima [N/A]	3446
Giac [F(-2)]	3446
Mupad [N/A]	3447
Reduce [N/A]	3447

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = \text{Int} \left(\frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}}, x \right)$$

output `Defer(Int)((-c^2*d*x^2+d)^2/x/(a+b*arcsin(c*x))^(3/2),x)`

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx$$

input `Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcSin[c*x])^(3/2)),x]`

output `Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcSin[c*x])^(3/2)), x]`

Rubi [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d - c^2 x^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx \\
 & \quad \downarrow \text{5214} \\
 & \frac{8cd^2 \int \frac{(1-c^2x^2)^{3/2}}{\sqrt{a+b\arcsin(cx)}} dx}{b} - \frac{2d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^2 \sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d^2 (1-c^2x^2)^{5/2}}{bcx \sqrt{a+b\arcsin(cx)}} \\
 & \quad \downarrow \text{5168} \\
 & \frac{8d^2 \int \frac{\cos^4\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b}\right)}{\sqrt{a+b\arcsin(cx)}} d(a+b\arcsin(cx))}{b^2} - \frac{2d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^2 \sqrt{a+b\arcsin(cx)}} dx}{bc} - \\
 & \quad \frac{2d^2 (1-c^2x^2)^{5/2}}{bcx \sqrt{a+b\arcsin(cx)}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{8d^2 \int \frac{\sin\left(\frac{a}{b} - \frac{a+b\arcsin(cx)}{b} + \frac{\pi}{2}\right)^4}{\sqrt{a+b\arcsin(cx)}} d(a+b\arcsin(cx))}{b^2} - \frac{2d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^2 \sqrt{a+b\arcsin(cx)}} dx}{bc} - \\
 & \quad \frac{2d^2 (1-c^2x^2)^{5/2}}{bcx \sqrt{a+b\arcsin(cx)}} \\
 & \quad \downarrow \text{3793} \\
 & \frac{8d^2 \int \left(\frac{\cos\left(\frac{4a}{b} - \frac{4(a+b\arcsin(cx))}{b}\right)}{8\sqrt{a+b\arcsin(cx)}} + \frac{\cos\left(\frac{2a}{b} - \frac{2(a+b\arcsin(cx))}{b}\right)}{2\sqrt{a+b\arcsin(cx)}} + \frac{3}{8\sqrt{a+b\arcsin(cx)}} \right) d(a+b\arcsin(cx))}{b^2} \\
 & \quad \frac{2d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^2 \sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d^2 (1-c^2x^2)^{5/2}}{bcx \sqrt{a+b\arcsin(cx)}}
 \end{aligned}$$

$$\begin{aligned}
 & \downarrow 2009 \\
 & \frac{2d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^2\sqrt{a+b\arcsin(cx)}} dx}{bc} - \\
 & \frac{8d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \text{s} \right)}{b^2} \\
 & \frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arcsin(cx)}} \\
 & \downarrow 5226 \\
 & \frac{2d^2 \int \left(\frac{x^2c^4}{\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} - \frac{2c^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} + \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} \right) dx}{bc} - \\
 & \frac{8d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \text{s} \right)}{b^2} \\
 & \frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arcsin(cx)}} \\
 & \downarrow 2009 \\
 & \frac{2d^2 \left(\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} dx - \frac{\sqrt{\pi}c \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}} - \frac{\sqrt{\pi}c \sin\left(\frac{2a}{b}\right) \text{FresnelS}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}} - 3c \right)}{bc} - \\
 & \frac{8d^2 \left(\frac{1}{8} \sqrt{\frac{\pi}{2}} \sqrt{b} \cos\left(\frac{4a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \cos\left(\frac{2a}{b}\right) \text{FresnelC}\left(\frac{2\sqrt{a+b\arcsin(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + \frac{1}{2} \sqrt{\pi} \sqrt{b} \text{s} \right)}{b^2} \\
 & \frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arcsin(cx)}}
 \end{aligned}$$

input

`Int[(d - c^2*d*x^2)^2/(x*(a + b*ArcSin[c*x])^(3/2)),x]`

output

`$Aborted`

Maple [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d x^2 + d)^2}{x (a + b \arcsin(cx))^{\frac{3}{2}}} dx$$

input `int((-c^2*d*x^2+d)^2/x/(a+b*arcsin(c*x))^(3/2),x)`

output `int((-c^2*d*x^2+d)^2/x/(a+b*arcsin(c*x))^(3/2),x)`

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^2/x/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [N/A]

Not integrable

Time = 5.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\int \left(-\frac{2c^2 x^2}{ax \sqrt{a + b \arcsin(cx)} + bx \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right. \\ \left. + \int \frac{c^4 x^4}{ax \sqrt{a + b \arcsin(cx)} + bx \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right. \\ \left. + \int \frac{1}{ax \sqrt{a + b \arcsin(cx)} + bx \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

input `integrate((-c**2*d*x**2+d)**2/x/(a+b*asin(c*x))**(3/2),x)`

output `d**2*(Integral(-2*c**2*x**2/(a*x*sqrt(a + b*asin(c*x)) + b*x*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*asin(c*x)) + b*x*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(1/(a*x*sqrt(a + b*asin(c*x)) + b*x*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2}{(b \arcsin(cx) + a)^{3/2} x} dx$$

input `integrate((-c^2*d*x^2+d)^2/x/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `integrate((c^2*d*x^2 - d)^2/((b*arcsin(c*x) + a)^(3/2)*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate((-c^2*d*x^2+d)^2/x/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(d - c^2 dx^2)^2}{x(a + b \operatorname{asin}(cx))^{3/2}} dx$$

input `int((d - c^2*d*x^2)^2/(x*(a + b*asin(c*x))^(3/2)),x)`output `int((d - c^2*d*x^2)^2/(x*(a + b*asin(c*x))^(3/2)), x)`**Reduce [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.48

$$\int \frac{(d - c^2 dx^2)^2}{x(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\int \frac{\sqrt{\operatorname{asin}(cx) b + a}}{\operatorname{asin}(cx)^2 b^2 x + 2 \operatorname{asin}(cx) abx + a^2 x} dx \right. \\ \left. + \left(\int \frac{\sqrt{\operatorname{asin}(cx) b + a} x^3}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) c^4 \right. \\ \left. - 2 \left(\int \frac{\sqrt{\operatorname{asin}(cx) b + a} x}{\operatorname{asin}(cx)^2 b^2 + 2 \operatorname{asin}(cx) ab + a^2} dx \right) c^2 \right)$$

input `int((-c^2*d*x^2+d)^2/x/(a+b*asin(c*x))^(3/2),x)`output `d**2*(int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*x + 2*asin(c*x)*a*b*x + a**2*x),x) + int((sqrt(asin(c*x)*b + a)*x**3)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(asin(c*x)*b + a)*x)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*c**2)`

$$3.398 \quad \int \frac{(d - c^2 dx^2)^2}{x^2 (a + b \arcsin(cx))^{3/2}} dx$$

Optimal result	3448
Mathematica [N/A]	3448
Rubi [N/A]	3449
Maple [N/A]	3450
Fricas [F(-2)]	3450
Sympy [N/A]	3451
Maxima [N/A]	3451
Giac [N/A]	3452
Mupad [N/A]	3452
Reduce [N/A]	3453

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d - c^2 dx^2)^2}{x^2 (a + b \arcsin(cx))^{3/2}} dx = \text{Int} \left(\frac{(d - c^2 dx^2)^2}{x^2 (a + b \arcsin(cx))^{3/2}}, x \right)$$

output `Defer(Int)((-c^2*d*x^2+d)^2/x^2/(a+b*arcsin(c*x))^(3/2),x)`

Mathematica [N/A]

Not integrable

Time = 10.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^2}{x^2 (a + b \arcsin(cx))^{3/2}} dx = \int \frac{(d - c^2 dx^2)^2}{x^2 (a + b \arcsin(cx))^{3/2}} dx$$

input `Integrate[(d - c^2*d*x^2)^2/(x^2*(a + b*ArcSin[c*x])^(3/2)),x]`

output `Integrate[(d - c^2*d*x^2)^2/(x^2*(a + b*ArcSin[c*x])^(3/2)), x]`

Rubi [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx$$

$$\downarrow \text{5214}$$

$$\frac{6cd^2 \int \frac{(1-c^2x^2)^{3/2}}{x\sqrt{a+b\arcsin(cx)}} dx}{b} - \frac{4d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d^2(1-c^2x^2)^{5/2}}{bcx^2\sqrt{a+b\arcsin(cx)}}$$

$$\downarrow \text{5226}$$

$$\frac{4d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{6cd^2 \int \left(\frac{x^3c^4}{\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} - \frac{2xc^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} + \frac{1}{x\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} \right) dx}{b} - \frac{2d^2(1-c^2x^2)^{5/2}}{bcx^2\sqrt{a+b\arcsin(cx)}}$$

$$\downarrow \text{2009}$$

$$\frac{6cd^2 \left(\int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} dx + \frac{2\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{3\sqrt{\frac{\pi}{2}} \sin\left(\frac{a}{b}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}} \right)}{b} - \frac{4d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d^2(1-c^2x^2)^{5/2}}{bcx^2\sqrt{a+b\arcsin(cx)}}$$

$$\downarrow \text{5234}$$

$$6cd^2 \left(\int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{a+b\arcsin(cx)}} dx + \frac{2\sqrt{2\pi} \sin\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{3\sqrt{\frac{\pi}{2}} \sin\left(\frac{a}{b}\right) \operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arcsin(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}} \right) - \frac{4d^2 \int \frac{(1-c^2x^2)^{3/2}}{x^3\sqrt{a+b\arcsin(cx)}} dx}{bc} - \frac{2d^2(1-c^2x^2)^{5/2}}{bcx^2\sqrt{a+b\arcsin(cx)}}$$

input `Int[(d - c^2*d*x^2)^2/(x^2*(a + b*ArcSin[c*x])^(3/2)),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2dx^2 + d)^2}{x^2(a + b\arcsin(cx))^{\frac{3}{2}}} dx$$

input `int((-c^2*d*x^2+d)^2/x^2/(a+b*arcsin(c*x))^(3/2),x)`

output `int((-c^2*d*x^2+d)^2/x^2/(a+b*arcsin(c*x))^(3/2),x)`

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)^2}{x^2(a + b\arcsin(cx))^{3/2}} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^2/x^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="fricas")`

output Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [N/A]

Not integrable

Time = 5.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.93

$$\int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = d^2 \left(\int \left(-\frac{2c^2 x^2}{ax^2 \sqrt{a + b \arcsin(cx)} + bx^2 \sqrt{a + b \arcsin(cx)} \arcsin(cx)} \right) dx \right. \\ \left. + \int \frac{c^4 x^4}{ax^2 \sqrt{a + b \arcsin(cx)} + bx^2 \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right. \\ \left. + \int \frac{1}{ax^2 \sqrt{a + b \arcsin(cx)} + bx^2 \sqrt{a + b \arcsin(cx)} \arcsin(cx)} dx \right)$$

input `integrate((-c**2*d*x**2+d)**2/x**2/(a+b*asin(c*x))**(3/2),x)`

output `d**2*(Integral(-2*c**2*x**2/(a*x**2*sqrt(a + b*asin(c*x)) + b*x**2*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(c**4*x**4/(a*x**2*sqrt(a + b*asin(c*x)) + b*x**2*sqrt(a + b*asin(c*x))*asin(c*x)), x) + Integral(1/(a*x**2*sqrt(a + b*asin(c*x)) + b*x**2*sqrt(a + b*asin(c*x))*asin(c*x)), x))`

Maxima [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2}{(b \arcsin(cx) + a)^{\frac{3}{2}} x^2} dx$$

input `integrate((-c^2*d*x^2+d)^2/x^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="maxima")`

output `integrate((c^2*d*x^2 - d)^2/((b*arcsin(c*x) + a)^(3/2)*x^2), x)`

Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(c^2 dx^2 - d)^2}{(b \arcsin(cx) + a)^{3/2} x^2} dx$$

input `integrate((-c^2*d*x^2+d)^2/x^2/(a+b*arcsin(c*x))^(3/2),x, algorithm="giac")`

output `integrate((c^2*d*x^2 - d)^2/((b*arcsin(c*x) + a)^(3/2)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx$$

input `int((d - c^2*d*x^2)^2/(x^2*(a + b*asin(c*x))^(3/2)),x)`

output `int((d - c^2*d*x^2)^2/(x^2*(a + b*asin(c*x))^(3/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 347, normalized size of antiderivative = 11.97

$$\int \frac{(d - c^2 dx^2)^2}{x^2(a + b \arcsin(cx))^{3/2}} dx = \frac{d^2 \left(\arcsin(cx) \left(\int \frac{\sqrt{\arcsin(cx)b+a}}{\arcsin(cx)^2 b^2 x^2 + 2\arcsin(cx)abx^2 + a^2 x^2} dx \right) b^2 + \arcsin(cx) \left(\int \frac{\sqrt{\arcsin(cx)b+a}}{\arcsin(cx)^2 b^2} dx \right) \right)}{x^2(a + b \arcsin(cx))^{3/2}}$$

input `int((-c^2*d*x^2+d)^2/x^2/(a+b*asin(c*x))^(3/2),x)`

output `(d**2*(asin(c*x)*int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*x**2 + 2*asin(c*x)*a*b*x**2 + a**2*x**2),x)*b**2 + asin(c*x)*int((sqrt(asin(c*x)*b + a)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*b**2*c**4 + 4*asin(c*x)*int((sqrt(asin(c*x)*b + a)*x)/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b + sqrt(-c**2*x**2 + 1)*a),x)*b*c**3 + 4*sqrt(asin(c*x)*b + a)*sqrt(-c**2*x**2 + 1)*c + int(sqrt(asin(c*x)*b + a)/(asin(c*x)**2*b**2*x**2 + 2*asin(c*x)*a*b*x**2 + a**2*x**2),x)*a*b + int((sqrt(asin(c*x)*b + a)*x**2)/(asin(c*x)**2*b**2 + 2*asin(c*x)*a*b + a**2),x)*a*b*c**4 + 4*int((sqrt(asin(c*x)*b + a)*x)/(sqrt(-c**2*x**2 + 1)*asin(c*x)*b + sqrt(-c**2*x**2 + 1)*a),x)*a*c**3))/(b*(asin(c*x)*b + a))`

$$3.399 \quad \int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx$$

Optimal result	3454
Mathematica [F]	3454
Rubi [A] (verified)	3455
Maple [C] (verified)	3456
Fricas [F(-2)]	3456
Sympy [F]	3457
Maxima [F(-2)]	3457
Giac [A] (verification not implemented)	3458
Mupad [F(-1)]	3458
Reduce [B] (verification not implemented)	3459

Optimal result

Integrand size = 38, antiderivative size = 42

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = -\frac{3x\sqrt{\arcsin(x)}}{4\sqrt{1-x^2}} + \frac{\arcsin(x)^{3/2}}{2(1-x^2)}$$

output `-3/4*x*arcsin(x)^(1/2)/(-x^2+1)^(1/2)+arcsin(x)^(3/2)/(-2*x^2+2)`

Mathematica [F]

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = \int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx$$

input `Integrate[(-3*x)/(8*(1-x^2)*Sqrt[ArcSin[x]]) + (x*ArcSin[x]^(3/2))/(1-x^2)^2,x]`

output

```
Integrate[(-3*x)/(8*(1 - x^2)*Sqrt[ArcSin[x]]) + (x*ArcSin[x]^(3/2))/(1 - x^2)^2, x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} - \frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} \right) dx$$

↓ 2009

$$\frac{\arcsin(x)^{3/2}}{2(1-x^2)} - \frac{3x\sqrt{\arcsin(x)}}{4\sqrt{1-x^2}}$$

input

```
Int[(-3*x)/(8*(1 - x^2)*Sqrt[ArcSin[x]]) + (x*ArcSin[x]^(3/2))/(1 - x^2)^2, x]
```

output

```
(-3*x*Sqrt[ArcSin[x]])/(4*Sqrt[1 - x^2]) + ArcSin[x]^(3/2)/(2*(1 - x^2))
```

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{3i\sqrt{\arcsin(x)}}{4} - \frac{(3ix^2 - 3x\sqrt{-x^2+1} + 2\arcsin(x) - 3i)\sqrt{\arcsin(x)}}{4(x^2-1)}$	47
parts	$\frac{3i\sqrt{\arcsin(x)}}{4} - \frac{(3ix^2 - 3x\sqrt{-x^2+1} + 2\arcsin(x) - 3i)\sqrt{\arcsin(x)}}{4(x^2-1)}$	47

input `int(-3/8*x/(-x^2+1)/arcsin(x)^(1/2)+x*arcsin(x)^(3/2)/(-x^2+1)^2,x,method=_RETURNVERBOSE)`

output `3/4*I*arcsin(x)^(1/2)-1/4*(3*I*x^2-3*x*(-x^2+1)^(1/2)+2*arcsin(x)-3*I)*arcsin(x)^(1/2)/(x^2-1)`

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = \text{Exception raised: TypeError}$$

input `integrate(-3/8*x/(-x^2+1)/arcsin(x)^(1/2)+x*arcsin(x)^(3/2)/(-x^2+1)^2,x,algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = \frac{\int \left(-\frac{3x}{x^4\sqrt{\arcsin(x)} - 2x^2\sqrt{\arcsin(x)} + \sqrt{\arcsin(x)}} \right) dx + \int \frac{3x^3}{x^4\sqrt{\arcsin(x)} - 2x^2\sqrt{\arcsin(x)} + \sqrt{\arcsin(x)}} dx + \int \frac{1}{x} dx}{8}$$

input `integrate(-3/8*x/(-x**2+1)/asin(x)**(1/2)+x*asin(x)**(3/2)/(-x**2+1)**2,x)`

output `(Integral(-3*x/(x**4*sqrt(asin(x)) - 2*x**2*sqrt(asin(x)) + sqrt(asin(x))), x) + Integral(3*x**3/(x**4*sqrt(asin(x)) - 2*x**2*sqrt(asin(x)) + sqrt(asin(x))), x) + Integral(8*x*asin(x)**2/(x**4*sqrt(asin(x)) - 2*x**2*sqrt(asin(x)) + sqrt(asin(x))), x))/8`

Maxima [F(-2)]

Exception generated.

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = \text{Exception raised: RuntimeError}$$

input `integrate(-3/8*x/(-x^2+1)/arcsin(x)^(1/2)+x*arcsin(x)^(3/2)/(-x^2+1)^2,x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx =$$

$$-\frac{x^2 \arcsin(x)^{3/2}}{2(x^2-1)} + \frac{1}{2} \arcsin(x)^{3/2} + \frac{3\sqrt{-x^2+1}x\sqrt{\arcsin(x)}}{4(x^2-1)}$$

input `integrate(-3/8*x/(-x^2+1)/arcsin(x)^(1/2)+x*arcsin(x)^(3/2)/(-x^2+1)^2,x,
algorithm="giac")`

output `-1/2*x^2*arcsin(x)^(3/2)/(x^2 - 1) + 1/2*arcsin(x)^(3/2) + 3/4*sqrt(-x^2 +
1)*x*sqrt(arcsin(x))/(x^2 - 1)`

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = \int \frac{3x}{8\sqrt{\arcsin(x)}(x^2-1)} + \frac{x \arcsin(x)^{3/2}}{(x^2-1)^2} dx$$

input `int((3*x)/(8*asin(x)^(1/2)*(x^2 - 1)) + (x*asin(x)^(3/2))/(x^2 - 1)^2,x)`

output `int((3*x)/(8*asin(x)^(1/2)*(x^2 - 1)) + (x*asin(x)^(3/2))/(x^2 - 1)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arcsin(x)}} + \frac{x \arcsin(x)^{3/2}}{(1-x^2)^2} \right) dx = \frac{\sqrt{\arcsin(x)}(-2\arcsin(x) + 3\sqrt{-x^2+1}x)}{4x^2-4}$$

input `int(-3/8*x/(-x^2+1)/asin(x)^(1/2)+x*asin(x)^(3/2)/(-x^2+1)^2,x)`

output `(sqrt(asin(x))*(-2*asin(x) + 3*sqrt(-x**2 + 1)*x))/(4*(x**2 - 1))`

3.400 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx$

Optimal result	3460
Mathematica [C] (verified)	3460
Rubi [A] (verified)	3461
Maple [A] (verified)	3462
Fricas [F(-2)]	3463
Sympy [F]	3463
Maxima [F(-2)]	3463
Giac [C] (verification not implemented)	3464
Mupad [F(-1)]	3464
Reduce [F]	3465

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = \sqrt{2\pi} \operatorname{FresnelS} \left(\sqrt{\frac{2}{\pi}} \sqrt{\arcsin(x)} \right)$$

output `2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arcsin(x)^(1/2))`

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = -\frac{\sqrt{-i \arcsin(x)}\Gamma(\frac{1}{2}, -i \arcsin(x)) + \sqrt{i \arcsin(x)}\Gamma(\frac{1}{2}, i \arcsin(x))}{2\sqrt{\arcsin(x)}}$$

input `Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcSin[x]]), x]`

output

```
-1/2*(Sqrt[(-I)*ArcSin[x]]*Gamma[1/2, (-I)*ArcSin[x]] + Sqrt[I*ArcSin[x]]*
Gamma[1/2, I*ArcSin[x]])/Sqrt[ArcSin[x]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5224, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx \\
 & \quad \downarrow \text{5224} \\
 & \int \frac{x}{\sqrt{\arcsin(x)}} d\arcsin(x) \\
 & \quad \downarrow \text{3042} \\
 & \int \frac{\sin(\arcsin(x))}{\sqrt{\arcsin(x)}} d\arcsin(x) \\
 & \quad \downarrow \text{3786} \\
 & 2 \int x d\sqrt{\arcsin(x)} \\
 & \quad \downarrow \text{3832} \\
 & \sqrt{2\pi} \operatorname{FresnelS} \left(\sqrt{\frac{2}{\pi}} \sqrt{\arcsin(x)} \right)
 \end{aligned}$$

input

```
Int[x/(Sqrt[1 - x^2]*Sqrt[ArcSin[x]]), x]
```

output

```
Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcSin[x]]]
```

Definitions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3786 `Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[2/d Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]`

rule 3832 `Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] := Simp[(Sqrt[Pi/2]/(f*Rt[d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_*(x_)^m_*(d_ + (e_.)*(x_)^2)^p_], x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\sqrt{2} \sqrt{\pi} \operatorname{FresnelS}\left(\frac{\sqrt{2} \sqrt{\arcsin(x)}}{\sqrt{\pi}}\right)$	20

input `int(x/(-x^2+1)^(1/2)/arcsin(x)^(1/2),x,method=_RETURNVERBOSE)`

output `2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arcsin(x)^(1/2))`

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = \text{Exception raised: TypeError}$$

input `integrate(x/(-x^2+1)^(1/2)/arcsin(x)^(1/2),x, algorithm="fricas")`

output `Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)`

Sympy [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = \int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\arcsin(x)}} dx$$

input `integrate(x/(-x**2+1)**(1/2)/asin(x)**(1/2),x)`

output `Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(asin(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x/(-x^2+1)^(1/2)/arcsin(x)^(1/2),x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = \left(\frac{1}{4}i - \frac{1}{4}\right) \sqrt{2}\sqrt{\pi} \operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2}\sqrt{\arcsin(x)}\right) - \left(\frac{1}{4}i + \frac{1}{4}\right) \sqrt{2}\sqrt{\pi} \operatorname{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2}\sqrt{\arcsin(x)}\right)$$

input `integrate(x/(-x^2+1)^(1/2)/arcsin(x)^(1/2),x, algorithm="giac")`

output `((1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arcsin(x))) - (1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arcsin(x))))`

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = \int \frac{x}{\sqrt{\arcsin(x)}\sqrt{1-x^2}} dx$$

input `int(x/(asin(x)^(1/2)*(1-x^2)^(1/2)),x)`

output `int(x/(asin(x)^(1/2)*(1-x^2)^(1/2)), x)`

Reduce [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arcsin(x)}} dx = - \left(\int \frac{\sqrt{-x^2+1}\sqrt{\arcsin(x)}x}{\arcsin(x)x^2 - \arcsin(x)} dx \right)$$

input `int(x/(-x^2+1)^(1/2)/asin(x)^(1/2),x)`

output `- int((sqrt(-x**2 + 1)*sqrt(asin(x))*x)/(asin(x)*x**2 - asin(x)),x)`

3.401 $\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx$

Optimal result	3466
Mathematica [A] (verified)	3467
Rubi [A] (verified)	3467
Maple [F]	3469
Fricas [F]	3469
Sympy [F]	3469
Maxima [F]	3470
Giac [F]	3470
Mupad [F(-1)]	3470
Reduce [F]	3471

Optimal result

Integrand size = 29, antiderivative size = 259

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^{1+n}}{8bc^3(1+n)\sqrt{1 - c^2 x^2}} + \frac{i^{2-2(3+n)} e^{-\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{4i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} - \frac{i^{2-2(3+n)} e^{\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{4i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}}$$

output

```
1/8*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^(1+n)/b/c^3/(1+n)/(-c^2*x^2+1)^(1/2)+I*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-4*I*(a+b*arcsin(c*x))/b)/(2^(6+2*n))/c^3/exp(4*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-I*exp(4*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,4*I*(a+b*arcsin(c*x))/b)/(2^(6+2*n))/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$\int x^2 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^n dx$$

$$= \frac{d\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{8a + 8b \arcsin(cx)}{b + bn} + i4^{-n} e^{-\frac{4ia}{b}} \left(\frac{(a + b \arcsin(cx))^2}{b^2} \right)^{-n} \left(\frac{i(a + b \arcsin(cx))}{b} \right)^n \Gamma(1 + n) \right)}{64c^3 \sqrt{d(1 - c^2 x^2)}}$$

input

```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*((8*a + 8*b*ArcSin[c*x])/(b + b*n) + (I*(((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b] - E^(((8*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b]))/(4^n*E^(((4*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^n))/(64*c^3*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^n dx$$

$$\downarrow 5224$$

$$\frac{\sqrt{d - c^2 x^2} \int (a + b \arcsin(cx))^n \cos^2 \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} \right) \sin^2 \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} \right) d(a + b \arcsin(cx))}{bc^3 \sqrt{1 - c^2 x^2}}$$

$$\downarrow 4906$$

$$\frac{\sqrt{d-c^2x^2} \int \left(\frac{1}{8}(a+b\arcsin(cx))^n - \frac{1}{8}(a+b\arcsin(cx))^n \cos\left(\frac{4a}{b} - \frac{4(a+b\arcsin(cx))}{b}\right) \right) d(a+b\arcsin(cx))}{bc^3\sqrt{1-c^2x^2}}$$

↓ 2009

$$\frac{\sqrt{d-c^2x^2} \left(\frac{(a+b\arcsin(cx))^{n+1}}{8(n+1)} + ib2^{-2(n+3)}e^{-\frac{4ia}{b}}(a+b\arcsin(cx))^n \left(-\frac{i(a+b\arcsin(cx))}{b}\right)^{-n} \Gamma\left(n+1, -\frac{4i(a+b\arcsin(cx))}{b}\right) \right)}{bc^3\sqrt{1-c^2x^2}}$$

input

```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*((a + b*ArcSin[c*x])^(1 + n)/(8*(1 + n)) + (I*b*(a +
b*ArcSin[c*x])^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b])/(2^(2*(3 +
n))*E^(((4*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n - (I*b*E^(((4*I)*a)/
b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b])/(2^(
2*(3 + n))*((I*(a + b*ArcSin[c*x]))/b)^n))/(b*c^3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \sqrt{-c^2 d x^2 + d} (a + b \arcsin(cx))^n dx$$

input `int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x)`

output `int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n*x^2, x)`

Sympy [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int x^2 \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^n dx$$

input `integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**n,x)`

output `Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**n, x)`

Maxima [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n*x^2, x)`

Giac [F]

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n*x^2, x)`

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int x^2 (a + b \arcsin(cx))^n \sqrt{d - c^2 dx^2} dx$$

input `int(x^2*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2),x)`

output `int(x^2*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int x^2 \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^n dx = \sqrt{d} \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^2 dx \right)$$

input `int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^n,x)`

output `sqrt(d)*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**2,x)`

3.402 $\int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n dx$

Optimal result	3472
Mathematica [A] (verified)	3473
Rubi [A] (verified)	3474
Maple [F]	3475
Fricas [F]	3476
Sympy [F]	3476
Maxima [F]	3476
Giac [F(-2)]	3477
Mupad [F(-1)]	3477
Reduce [F]	3477

Optimal result

Integrand size = 27, antiderivative size = 391

$$\begin{aligned}
 & \int x\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n dx \\
 = & -\frac{e^{-\frac{ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{i(a+b \arcsin(cx))}{b}\right)}{8c^2\sqrt{1 - c^2x^2}} \\
 & -\frac{e^{\frac{ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{i(a+b \arcsin(cx))}{b}\right)}{8c^2\sqrt{1 - c^2x^2}} \\
 & -\frac{3^{-1-n}e^{-\frac{3ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{3i(a+b \arcsin(cx))}{b}\right)}{8c^2\sqrt{1 - c^2x^2}} \\
 & -\frac{3^{-1-n}e^{\frac{3ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{3i(a+b \arcsin(cx))}{b}\right)}{8c^2\sqrt{1 - c^2x^2}}
 \end{aligned}$$

output

```
-1/8*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-I*(a+b*arcsin(c*x)))/b)/c^2/exp(I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x)))/b)^n)-1/8*exp(I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,I*(a+b*arcsin(c*x)))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x)))/b)^n)-1/8*3^(-1-n)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-3*I*(a+b*arcsin(c*x)))/b)/c^2/exp(3*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x)))/b)^n)-1/8*3^(-1-n)*exp(3*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,3*I*(a+b*arcsin(c*x)))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x)))/b)^n)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.70

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^n dx$$

$$de^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arcsin(cx))^n \left(3e^{\frac{2ia}{b}} \left(-\left(-\frac{i(a+b\arcsin(cx))}{b} \right)^{-n} \Gamma\left(1+n, -\frac{i(a+b\arcsin(cx))}{b} \right) \right) - e^{\frac{2ia}{b}} \left(\frac{i(a+b\arcsin(cx))}{b} \right)^{-n} \Gamma\left(1+n, \frac{i(a+b\arcsin(cx))}{b} \right) \right)$$

input

```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*(3*E^(((2*I)*a)/b)*(-(Gamma[1 + n, ((-I)*(a + b*ArcSin[c*x]))/b])/(((I)*(a + b*ArcSin[c*x]))/b)^n) - (E^(((2*I)*a)/b)*Gamma[1 + n, (I*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n) - (((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-3*I)*(a + b*ArcSin[c*x]))/b] + E^(((6*I)*a)/b)*(((I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((3*I)*(a + b*ArcSin[c*x]))/b])/(3^n*((a + b*ArcSin[c*x])^2/b^2)^n)))/(24*c^2*E^(((3*I)*a)/b)*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx$$

$$\downarrow 5224$$

$$\frac{\sqrt{d - c^2 dx^2} \int -(a + b \arcsin(cx))^n \cos^2\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

$$\downarrow 25$$

$$\frac{\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^2\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

$$\downarrow 4906$$

$$\frac{\sqrt{d - c^2 dx^2} \int \left(\frac{1}{4} \sin\left(\frac{3a}{b} - \frac{3(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))^n + \frac{1}{4} \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) (a + b \arcsin(cx))^n\right)}{bc^2 \sqrt{1 - c^2 x^2}}$$

$$\downarrow 2009$$

$$\frac{\sqrt{d - c^2 dx^2} \left(-\frac{1}{8} b e^{-\frac{ia}{b}} (a + b \arcsin(cx))^n \left(-\frac{i(a + b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, -\frac{i(a + b \arcsin(cx))}{b}\right) - \frac{1}{8} b 3^{-n-1} e^{-\frac{3ia}{b}} (a + b \arcsin(cx))^n\right)}{bc^2 \sqrt{1 - c^2 x^2}}$$

input

```
Int[x*sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n,x]
```

output

```
(Sqrt[d - c^2*d*x^2]*(-1/8*(b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-I)*(a + b*ArcSin[c*x]))/b])/(E^((I*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n) - (b*E^((I*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, (I*(a + b*ArcSin[c*x]))/b])/(8*((I*(a + b*ArcSin[c*x]))/b)^n) - (3^(-1 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-3*I)*(a + b*ArcSin[c*x]))/b])/(8*E^(((3*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n) - (3^(-1 - n)*b*E^(((3*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((3*I)*(a + b*ArcSin[c*x]))/b])/(8*((I*(a + b*ArcSin[c*x]))/b)^n)))/(b*c^2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*sin[-a/b + x/b]^m*cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\sqrt{-c^2dx^2+d}(a+b\arcsin(cx))^n dx$$

input

```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x)
```


output `int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^n dx = \int \sqrt{-c^2dx^2+d}(b\arcsin(cx)+a)^n x dx$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n*x, x)`

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^n dx = \int x\sqrt{-d(cx-1)(cx+1)}(a+b\arcsin(cx))^n dx$$

input `integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**n,x)`

output `Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**n, x)`

Maxima [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arcsin(cx))^n dx = \int \sqrt{-c^2dx^2+d}(b\arcsin(cx)+a)^n x dx$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n*x, x)`

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^n dx = \text{Exception raised: TypeError}$$

input `integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^n dx = \int x(a+b\arcsin(cx))^n \sqrt{d-c^2x^2} dx$$

input `int(x*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2),x)`

output `int(x*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int x\sqrt{d-c^2x^2}(a+b\arcsin(cx))^n dx = \sqrt{d} \left(\int (a+b\arcsin(cx))^n \sqrt{-c^2x^2+1} dx \right)$$

input `int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^n,x)`

output `sqrt(d)*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x,x)`

3.403 $\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx$

Optimal result	3478
Mathematica [A] (verified)	3479
Rubi [A] (verified)	3479
Maple [F]	3481
Fricas [F]	3481
Sympy [F]	3481
Maxima [F]	3482
Giac [F(-2)]	3482
Mupad [F(-1)]	3482
Reduce [F]	3483

Optimal result

Integrand size = 26, antiderivative size = 259

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^{1+n}}{2bc(1+n)\sqrt{1 - c^2 x^2}} - \frac{i2^{-3-n} e^{-\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}} + \frac{i2^{-3-n} e^{\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{2i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

```
output 1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^(1+n)/b/c/(1+n)/(-c^2*x^2+1)^(1/2)-I*2^(-3-n)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-2*I*(a+b*arcsin(c*x))/b)/c/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)+I*2^(-3-n)*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,2*I*(a+b*arcsin(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.70

$$\int \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^n dx$$

$$= \frac{d\sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{4a + 4b \arcsin(cx)}{b + bn} - i 2^{-n} e^{-\frac{2ia}{b}} \left(-\frac{i(a + b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(1 + n, -\frac{2i(a + b \arcsin(cx))}{b}\right) \right)}{8c\sqrt{d(1 - c^2 x^2)}}$$

input

```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*((4*a + 4*b*ArcSin[c*x])/(b + b
*n) - (I*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x]))/b])/(2^n*E^(((2*I)*a)/b
))*((( -I)*(a + b*ArcSin[c*x]))/b)^n) + (I*E^(((2*I)*a)/b)*Gamma[1 + n, ((2*
I)*(a + b*ArcSin[c*x]))/b])/(2^n*((I*(a + b*ArcSin[c*x]))/b)^n))/(8*c*Sqr
t[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5168, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^n dx$$

$$\downarrow \text{5168}$$

$$\frac{\sqrt{d - c^2 x^2} \int (a + b \arcsin(cx))^n \cos^2 \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} \right) d(a + b \arcsin(cx))}{bc\sqrt{1 - c^2 x^2}}$$

$$\downarrow \text{3042}$$

$$\frac{\sqrt{d - c^2 x^2} \int (a + b \arcsin(cx))^n \sin \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} + \frac{\pi}{2} \right)^2 d(a + b \arcsin(cx))}{bc\sqrt{1 - c^2 x^2}}$$

↓ 3793

$$\frac{\sqrt{d - c^2 x^2} \int \left(\frac{1}{2} \cos \left(\frac{2a}{b} - \frac{2(a+b \arcsin(cx))}{b} \right) (a + b \arcsin(cx))^n + \frac{1}{2} (a + b \arcsin(cx))^n \right) d(a + b \arcsin(cx))}{bc\sqrt{1 - c^2 x^2}}$$

↓ 2009

$$\frac{\sqrt{d - c^2 x^2} \left(\frac{(a+b \arcsin(cx))^{n+1}}{2(n+1)} - ib2^{-n-3} e^{-\frac{2ia}{b}} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma \left(n + 1, -\frac{2i(a+b \arcsin(cx))}{b} \right) \right)}{bc\sqrt{1 - c^2 x^2}}$$

input `Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n,x]`

output `(Sqrt[d - c^2*d*x^2]*((a + b*ArcSin[c*x])^(1 + n)/(2*(1 + n)) - (I*2^(-3 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x]))/b])/ (E^(((2*I)*a)/b)*(((-I)*(a + b*ArcSin[c*x]))/b)^n) + (I*2^(-3 - n)*b*E^(((2*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n)/(b*c*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5168 `Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]`

Maple [F]

$$\int \sqrt{-c^2 d x^2 + d} (a + b \arcsin(cx))^n dx$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x)`

output `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n, x)`

Sympy [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int \sqrt{-d(cx - 1)(cx + 1)} (a + b \arcsin(cx))^n dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**n,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**n, x)`

Maxima [F]

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int \sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n, x)`

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n dx = \int (a + b \arcsin(cx))^n \sqrt{d - c^2 dx^2} dx$$

input `int((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2),x)`

output `int((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2), x)`

Reduce [F]

$$\int \sqrt{d - c^2 x^2} (a + b \arcsin(cx))^n dx = \sqrt{d} \left(\int (a \sin(cx) + b)^n \sqrt{-c^2 x^2 + 1} dx \right)$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^n,x)`

output `sqrt(d)*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1),x)`

3.404 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x} dx$

Optimal result	3484
Mathematica [N/A]	3484
Rubi [N/A]	3485
Maple [N/A]	3486
Fricas [N/A]	3486
Sympy [N/A]	3486
Maxima [N/A]	3487
Giac [F(-2)]	3487
Mupad [N/A]	3488
Reduce [N/A]	3488

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x} dx = \frac{\sqrt{d-c^2dx^2} \operatorname{Int}\left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^n}{x}, x\right)}{\sqrt{1-c^2x^2}}$$

output $(-c^2*d*x^2+d)^{(1/2)}*Defer(\operatorname{Int}((-c^2*x^2+1)^{(1/2)}*(a+b*\arcsin(c*x))^n/x,x)/(-c^2*x^2+1)^{(1/2)})$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x} dx = \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x} dx$$

input $\operatorname{Integrate}[(\operatorname{Sqrt}[d-c^2*d*x^2]*(a+b*\operatorname{ArcSin}[c*x]))^n/x,x]$

output $\operatorname{Integrate}[(\operatorname{Sqrt}[d-c^2*d*x^2]*(a+b*\operatorname{ArcSin}[c*x]))^n/x,x]$

Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx$$

↓ 5226

$$\int \left(\frac{d(a + b \arcsin(cx))^n}{x\sqrt{d - c^2 dx^2}} - \frac{c^2 dx (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} \right) dx$$

↓ 2009

$$\frac{d \int \frac{(a + b \arcsin(cx))^n}{x\sqrt{d - c^2 dx^2}} dx + de^{-\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{i(a + b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, -\frac{i(a + b \arcsin(cx))}{b}\right)}{2\sqrt{d - c^2 dx^2}} + \frac{de^{\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a + b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, \frac{i(a + b \arcsin(cx))}{b}\right)}{2\sqrt{d - c^2 dx^2}}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n)/x,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 d x^2 + d} (a + b \arcsin(cx))^n}{x} dx$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x,x)`

output `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x,x)`

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n/x, x)`

Sympy [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{\sqrt{-d (cx - 1) (cx + 1)} (a + b \arcsin(cx))^n}{x} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**n/x,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**n/x, x)`

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(a + b \arcsin(cx))^n \sqrt{d - c^2 dx^2}}{x} dx$$

input `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2))/x,x)`output `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2))/x, x)`**Reduce [N/A]**

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x} dx = \sqrt{d} \left(\int \frac{(a \arcsin(cx) + b)^n \sqrt{-c^2 x^2 + 1}}{x} dx \right)$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^n/x,x)`output `sqrt(d)*int(((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1))/x,x)`

3.405 $\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x^2} dx$

Optimal result	3489
Mathematica [N/A]	3489
Rubi [N/A]	3490
Maple [N/A]	3490
Fricas [N/A]	3491
Sympy [N/A]	3491
Maxima [N/A]	3491
Giac [F(-2)]	3492
Mupad [N/A]	3492
Reduce [N/A]	3493

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x^2} dx = \frac{\sqrt{d-c^2dx^2} \operatorname{Int}\left(\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^n}{x^2}, x\right)}{\sqrt{1-c^2x^2}}$$

output

```
(-c^2*d*x^2+d)^(1/2)*Defer(Int)((-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^n/x^2, x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x^2} dx = \int \frac{\sqrt{d-c^2dx^2}(a+b \arcsin(cx))^n}{x^2} dx$$

input

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n)/x^2, x]
```

output

```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx$$

↓ 5226

$$\int \left(\frac{d(a + b \arcsin(cx))^n}{x^2 \sqrt{d - c^2 dx^2}} - \frac{c^2 d (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} \right) dx$$

↓ 2009

$$d \int \frac{(a + b \arcsin(cx))^n}{x^2 \sqrt{d - c^2 dx^2}} dx - \frac{cd \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^{n+1}}{b(n+1) \sqrt{d - c^2 dx^2}}$$

input `Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcSin[c*x])^n)/x^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 d x^2 + d} (a + b \arcsin(cx))^n}{x^2} dx$$

input `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x^2,x)`

output `int((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x^2,x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="fricas")`

output `integral(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n/x^2, x)`

Sympy [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{\sqrt{-d (cx - 1) (cx + 1)} (a + b \arcsin(cx))^n}{x^2} dx$$

input `integrate((-c**2*d*x**2+d)**(1/2)*(a+b*asin(c*x))**n/x**2,x)`

output `Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*asin(c*x))**n/x**2, x)`

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{\sqrt{-c^2 dx^2 + d} (b \arcsin(cx) + a)^n}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="maxima")`

output `integrate(sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n/x^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(a + b \arcsin(cx))^n \sqrt{d - c^2 dx^2}}{x^2} dx$$

input `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2,x)`

output `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2, x)`

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.69

$$\int \frac{\sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n}{x^2} dx$$

$$= \frac{\sqrt{d} \left(-(a \sin(cx) b + a)^n \sin(cx) bc - (a \sin(cx) b + a)^n ac + \left(\int \frac{(a \sin(cx) b + a)^n}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) bn + \left(\int \frac{(a \sin(cx) b + a)^n}{\sqrt{-c^2 x^2 + 1} x^2} dx \right) a \right)}{b(n + 1)}$$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*asin(c*x))^n/x^2,x)
```

output

```
(sqrt(d)*(- (asin(c*x)*b + a)**n*asin(c*x)*b*c - (asin(c*x)*b + a)**n*a*c
+ int((asin(c*x)*b + a)**n/(sqrt(- c**2*x**2 + 1)*x**2),x)*b*n + int((as
in(c*x)*b + a)**n/(sqrt(- c**2*x**2 + 1)*x**2),x)*b))/(b*(n + 1))
```

3.406 $\int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^n dx$

Optimal result	3494
Mathematica [A] (verified)	3495
Rubi [A] (verified)	3496
Maple [F]	3498
Fricas [F]	3498
Sympy [F(-1)]	3498
Maxima [F]	3499
Giac [F]	3499
Mupad [F(-1)]	3499
Reduce [F]	3500

Optimal result

Integrand size = 29, antiderivative size = 684

$$\int x^2(d - c^2dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \frac{d\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^{1+n}}{16bc^3(1 + n)\sqrt{1 - c^2x^2}}$$

$$- \frac{i2^{-7-n}de^{-\frac{2ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{c^3\sqrt{1 - c^2x^2}}$$

$$+ \frac{i2^{-7-n}de^{\frac{2ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{2i(a+b \arcsin(cx))}{b}\right)}{c^3\sqrt{1 - c^2x^2}}$$

$$+ \frac{i2^{-7-2n}de^{-\frac{4ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{4i(a+b \arcsin(cx))}{b}\right)}{c^3\sqrt{1 - c^2x^2}}$$

$$- \frac{i2^{-7-2n}de^{\frac{4ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{4i(a+b \arcsin(cx))}{b}\right)}{c^3\sqrt{1 - c^2x^2}}$$

$$+ \frac{i2^{-7-n}3^{-1-n}de^{-\frac{6ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{6i(a+b \arcsin(cx))}{b}\right)}{c^3\sqrt{1 - c^2x^2}}$$

$$- \frac{i2^{-7-n}3^{-1-n}de^{\frac{6ia}{b}}\sqrt{d - c^2dx^2}(a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{6i(a+b \arcsin(cx))}{b}\right)}{c^3\sqrt{1 - c^2x^2}}$$

output

```

1/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^(1+n)/b/c^3/(1+n)/(-c^2*x^2+
1)^(1/2)-I*2^(-7-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-
2*I*(a+b*arcsin(c*x))/b)/c^3/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arc
sin(c*x))/b)^n)+I*2^(-7-n)*d*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin
(c*x))^n*GAMMA(1+n,2*I*(a+b*arcsin(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+
b*arcsin(c*x))/b)^n)+I*2^(-7-2*n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))
^n*GAMMA(1+n,-4*I*(a+b*arcsin(c*x))/b)/c^3/exp(4*I*a/b)/(-c^2*x^2+1)^(1/2)
/((-I*(a+b*arcsin(c*x))/b)^n)-I*2^(-7-2*n)*d*exp(4*I*a/b)*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,4*I*(a+b*arcsin(c*x))/b)/c^3/(-c^2*x^2+
1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)+I*2^(-7-n)*3^(-1-n)*d*(-c^2*d*x^2+d)^(
1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-6*I*(a+b*arcsin(c*x))/b)/c^3/exp(6*I*
a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-I*2^(-7-n)*3^(-1-n)*d
*exp(6*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,6*I*(a+b*
arcsin(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)

```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.74

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \frac{d^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{24a}{b+bn} + \frac{24 \arcsin(cx)}{1+n} - 3i2^{-n} e^{-\frac{2ia}{b}} \left(\frac{i(a+b \arcsin(cx))}{b} \right)^n \right)}{1}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*((24*a)/(b + b*n) + (24*ArcSi
n[c*x])/(1 + n) - ((3*I)*((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-2*I
)*(a + b*ArcSin[c*x]))/b])/(2^n*E^(((2*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2
)^n) + ((3*I)*E^(((2*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n
, ((2*I)*(a + b*ArcSin[c*x]))/b])/(2^n*((a + b*ArcSin[c*x])^2/b^2)^n) + ((
3*I)*((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]
))/b])/(4^n*E^(((4*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^n) - ((3*I)*E^(((4
*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((4*I)*(a + b*ArcS
in[c*x]))/b])/(4^n*((a + b*ArcSin[c*x])^2/b^2)^n) + (I*((I*(a + b*ArcSin[c
*x]))/b)^n*Gamma[1 + n, ((-6*I)*(a + b*ArcSin[c*x]))/b])/(6^n*E^(((6*I)*a
)/b)*((a + b*ArcSin[c*x])^2/b^2)^n) - (I*E^(((6*I)*a)/b)*((-I)*(a + b*ArcS
in[c*x]))/b)^n*Gamma[1 + n, ((6*I)*(a + b*ArcSin[c*x]))/b])/(6^n*((a + b*A
rcSin[c*x])^2/b^2)^n))/(384*c^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx$$

↓ 5224

$$\frac{d\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^4\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin^2\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^3\sqrt{1 - c^2 x^2}}$$

↓ 4906

$$\frac{d\sqrt{d - c^2 dx^2} \int \left(-\frac{1}{32} \cos\left(\frac{6a}{b} - \frac{6(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))^n - \frac{1}{16} \cos\left(\frac{4a}{b} - \frac{4(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))\right)}{bc^3\sqrt{1 - c^2 x^2}}$$

↓ 2009

$$d\sqrt{d-c^2x^2} \left(\frac{(a+b\arcsin(cx))^{n+1}}{16(n+1)} - ib2^{-n-7}e^{-\frac{2ia}{b}}(a+b\arcsin(cx))^n \left(-\frac{i(a+b\arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, -\frac{2i(a+b\arcsin(cx))}{b} \right) \right)$$

input `Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n,x]`

output `(d*Sqrt[d - c^2*d*x^2]*((a + b*ArcSin[c*x])^(1 + n)/(16*(1 + n)) - (I*2^(-7 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x]))/b])/((E^(((2*I)*a)/b)*(((-I)*(a + b*ArcSin[c*x]))/b)^n) + (I*2^(-7 - n)*b*E^(((2*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n + (I*2^(-7 - 2*n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b])/((E^(((4*I)*a)/b)*(((-I)*(a + b*ArcSin[c*x]))/b)^n) - (I*2^(-7 - 2*n)*b*E^(((4*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n + (I*2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-6*I)*(a + b*ArcSin[c*x]))/b])/((E^(((6*I)*a)/b)*(((-I)*(a + b*ArcSin[c*x]))/b)^n) - (I*2^(-7 - n)*3^(-1 - n)*b*E^(((6*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((6*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n))/(b*c^3*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [F]

$$\int x^2(-c^2dx^2 + d)^{\frac{3}{2}}(a + b \arcsin(cx))^n dx$$

input `int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x)`

output `int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int x^2(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))^n dx = \int (-c^2dx^2 + d)^{\frac{3}{2}}(b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral(-(c^2*d*x^4 - d*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n, x)`

Sympy [F(-1)]

Timed out.

$$\int x^2(d - c^2dx^2)^{3/2}(a + b \arcsin(cx))^n dx = \text{Timed out}$$

input `integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**n,x)`

output `Timed out`

Maxima [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n*x^2, x)`

Giac [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n*x^2, x)`

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int x^2 (a + b \arcsin(cx))^n (d - c^2 dx^2)^{3/2} dx$$

input `int(x^2*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2),x)`

output `int(x^2*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \sqrt{d} d \left(- \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^4 dx \right) c^2 + \int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^2 dx \right)$$

input

```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^n,x)
```

output

```
sqrt(d)*d*( - int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**4,x)*c**2 + int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**2,x))
```

3.407 $\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx$

Optimal result	3501
Mathematica [A] (verified)	3502
Rubi [A] (verified)	3503
Maple [F]	3505
Fricas [F]	3505
Sympy [F(-1)]	3505
Maxima [F]	3506
Giac [F(-2)]	3506
Mupad [F(-1)]	3506
Reduce [F]	3507

Optimal result

Integrand size = 27, antiderivative size = 595

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx =$$

$$\frac{de^{-\frac{ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{i(a+b \arcsin(cx))}{b}\right)}{16c^2 \sqrt{1 - c^2 x^2}}$$

$$\frac{de^{\frac{ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{i(a+b \arcsin(cx))}{b}\right)}{16c^2 \sqrt{1 - c^2 x^2}}$$

$$\frac{3^{-n} de^{-\frac{3ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{3i(a+b \arcsin(cx))}{b}\right)}{32c^2 \sqrt{1 - c^2 x^2}}$$

$$\frac{3^{-n} de^{\frac{3ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{3i(a+b \arcsin(cx))}{b}\right)}{32c^2 \sqrt{1 - c^2 x^2}}$$

$$\frac{5^{-1-n} de^{-\frac{5ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{5i(a+b \arcsin(cx))}{b}\right)}{32c^2 \sqrt{1 - c^2 x^2}}$$

$$\frac{5^{-1-n} de^{\frac{5ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{5i(a+b \arcsin(cx))}{b}\right)}{32c^2 \sqrt{1 - c^2 x^2}}$$

output

```

-1/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-I*(a+b*arcsin(
c*x))/b)/c^2/exp(I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-1/
16*d*exp(I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,I*(a+b*
arcsin(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)-1/32*d*
(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-3*I*(a+b*arcsin(c*x))/
b)/(3^n)/c^2/exp(3*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-
1/32*d*exp(3*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,3*I
*(a+b*arcsin(c*x))/b)/(3^n)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b
)^n)-1/32*5^(-1-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-5
*I*(a+b*arcsin(c*x))/b)/c^2/exp(5*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcs
in(c*x))/b)^n)-1/32*5^(-1-n)*d*exp(5*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcs
in(c*x))^n*GAMMA(1+n,5*I*(a+b*arcsin(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(
a+b*arcsin(c*x))/b)^n)

```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.78

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx =$$

$$15^{-1-n} d^2 e^{-\frac{5ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{(a + b \arcsin(cx))^2}{b^2} \right)^{-3n} \left(2 \cdot 15^{1+n} e^{\frac{4ia}{b}} \left(\frac{i(a + b \arcsin(cx))}{b} \right)^n \left(\frac{(a + b \arcsin(cx))}{b^2} \right)^n \right)$$

input

```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```

-1/32*(15^(-1 - n)*d^2*sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*(2*15^(1 +
n)*E^(((4*I)*a)/b)*((I*(a + b*ArcSin[c*x]))/b)^n*((a + b*ArcSin[c*x])^2/b^
2)^(2*n)*Gamma[1 + n, ((-I)*(a + b*ArcSin[c*x]))/b] + (((-I)*(a + b*ArcSin
[c*x]))/b)^n*(2*15^(1 + n)*E^(((6*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^(2*
n)*Gamma[1 + n, (I*(a + b*ArcSin[c*x]))/b] + 3*(5^(1 + n)*E^(((2*I)*a)/b)*
((I*(a + b*ArcSin[c*x]))/b)^(2*n)*((a + b*ArcSin[c*x])^2/b^2)^n*Gamma[1 +
n, ((-3*I)*(a + b*ArcSin[c*x]))/b] + 5^(1 + n)*E^(((8*I)*a)/b)*((a + b*Arc
Sin[c*x])^2/b^2)^(2*n)*Gamma[1 + n, ((3*I)*(a + b*ArcSin[c*x]))/b] + 3^n*(
(((I*(a + b*ArcSin[c*x]))/b)^n*((I*(a + b*ArcSin[c*x]))/b)^(3*n)*Gamma[1
+ n, ((-5*I)*(a + b*ArcSin[c*x]))/b] + E^(((10*I)*a)/b)*((a + b*ArcSin[c*
x])^2/b^2)^(2*n)*Gamma[1 + n, ((5*I)*(a + b*ArcSin[c*x]))/b]))))/(c^2*E^((
(5*I)*a)/b)*sqrt[d - c^2*d*x^2]*((a + b*ArcSin[c*x])^2/b^2)^(3*n))

```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx$$

↓ 5224

$$\frac{d\sqrt{d - c^2 dx^2} \int -(a + b \arcsin(cx))^n \cos^4\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

↓ 25

$$\frac{d\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^4\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

↓ 4906

$$\frac{d\sqrt{d - c^2 dx^2} \int \left(\frac{1}{16} \sin\left(\frac{5a}{b} - \frac{5(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))^n + \frac{3}{16} \sin\left(\frac{3a}{b} - \frac{3(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))^n\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

↓ 2009

$$\frac{d\sqrt{d - c^2 dx^2} \left(-\frac{1}{16} b e^{-\frac{ia}{b}} (a + b \arcsin(cx))^n \left(-\frac{i(a + b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, -\frac{i(a + b \arcsin(cx))}{b}\right) - \frac{1}{32} b^3 e^{-\frac{3ia}{b}} (a + b \arcsin(cx))^n\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

input

```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d*Sqrt[d - c^2*d*x^2]*(-1/16*(b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-I)*
(a + b*ArcSin[c*x]))/b])/(E^((I*a)/b)*(((-I)*(a + b*ArcSin[c*x]))/b)^n) -
(b*E^((I*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, (I*(a + b*ArcSin[c*x]))/
b])/(16*(((I*(a + b*ArcSin[c*x]))/b)^n) - (b*(a + b*ArcSin[c*x])^n*Gamma[1
+ n, ((-3*I)*(a + b*ArcSin[c*x]))/b])/(32*3^n*E^(((3*I)*a)/b)*(((-I)*(a +
b*ArcSin[c*x]))/b)^n) - (b*E^(((3*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 +
n, ((3*I)*(a + b*ArcSin[c*x]))/b])/(32*3^n*((I*(a + b*ArcSin[c*x]))/b)^n)
- (5^(-1 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-5*I)*(a + b*ArcSin[
c*x]))/b])/(32*E^(((5*I)*a)/b)*(((-I)*(a + b*ArcSin[c*x]))/b)^n) - (5^(-1
- n)*b*E^(((5*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((5*I)*(a + b*Ar
cSin[c*x]))/b])/(32*((I*(a + b*ArcSin[c*x]))/b)^n))/(b*c^2*Sqrt[1 - c^2*x
^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x(-c^2 d x^2 + d)^{\frac{3}{2}} (a + b \arcsin(cx))^n dx$$

input `int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x)`

output `int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n x dx$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral(-(c^2*d*x^3 - d*x)*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n, x)`

Sympy [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \text{Timed out}$$

input `integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**n,x)`

output `Timed out`

Maxima [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{3/2} (b \arcsin(cx) + a)^n x dx$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n*x, x)`

Giac [F(-2)]

Exception generated.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \text{Exception raised: TypeError}$$

input `integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int x(a + b \arcsin(cx))^n (d - c^2 dx^2)^{3/2} dx$$

input `int(x*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2),x)`

output `int(x*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int x(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \sqrt{d} d \left(- \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^3 dx \right) c^2 + \int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x dx \right)$$

input

```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^n,x)
```

output

```
sqrt(d)*d*( - int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**3,x)*c**2 + int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x,x))
```


3.408 $\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx$

Optimal result	3508
Mathematica [A] (verified)	3509
Rubi [A] (verified)	3510
Maple [F]	3511
Fricas [F]	3512
Sympy [F(-1)]	3512
Maxima [F]	3512
Giac [F(-2)]	3513
Mupad [F(-1)]	3513
Reduce [F]	3513

Optimal result

Integrand size = 26, antiderivative size = 466

$$\begin{aligned}
 \int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx &= \frac{3d\sqrt{d - c^2 dx^2}(a + b \arcsin(cx))^{1+n}}{8bc(1 + n)\sqrt{1 - c^2 x^2}} \\
 &- \frac{i2^{-3-n} de^{-\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}} \\
 &+ \frac{i2^{-3-n} de^{\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{2i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}} \\
 &- \frac{i4^{-3-n} de^{-\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{4i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}} \\
 &+ \frac{i4^{-3-n} de^{\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{4i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}
 \end{aligned}$$

output

```

3/8*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^(1+n)/b/c/(1+n)/(-c^2*x^2+1)^(
1/2)-I*2^(-3-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-2*I
*(a+b*arcsin(c*x))/b)/c/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c
*x))/b)^n)+I*2^(-3-n)*d*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x)
)^n*GAMMA(1+n,2*I*(a+b*arcsin(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsi
n(c*x))/b)^n)-I*4^(-3-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(
1+n,-4*I*(a+b*arcsin(c*x))/b)/c/exp(4*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*
arcsin(c*x))/b)^n)+I*4^(-3-n)*d*exp(4*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
sin(c*x))^n*GAMMA(1+n,4*I*(a+b*arcsin(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a
+b*arcsin(c*x))/b)^n)

```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.70

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \frac{d^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{8(a + b \arcsin(cx))}{b(1+n)} + 8 \left(\frac{4a + 4b \arcsin(cx)}{b + bn} - i 2^{-n} e^{-\frac{2ia}{b}} \left(-i \right) \right) \right)}{64 c \sqrt{d - c^2 dx^2}}$$

input

```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```

(d^2*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*((-8*(a + b*ArcSin[c*x]))/(b*
(1 + n)) + 8*((4*a + 4*b*ArcSin[c*x]))/(b + b*n) - (I*Gamma[1 + n, ((-2*I)*
(a + b*ArcSin[c*x]))/b])/(2^n*E^(((2*I)*a)/b)*(((I)*(a + b*ArcSin[c*x]))/
b)^n) + (I*E^(((2*I)*a)/b)*Gamma[1 + n, ((2*I)*(a + b*ArcSin[c*x]))/b])/(2
^n*((I*(a + b*ArcSin[c*x]))/b)^n) + (I*(-(((I*(a + b*ArcSin[c*x]))/b)^n*G
amma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b]) + E^(((8*I)*a)/b)*(((I)*(a +
b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b]))/(4^n*E
^(((4*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^n))/(64*c*Sqrt[d - c^2*d*x^2])

```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5168, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx$$

$$\downarrow 5168$$

$$\frac{d\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^4\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc\sqrt{1 - c^2 x^2}}$$

$$\downarrow 3042$$

$$\frac{d\sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} + \frac{\pi}{2}\right)^4 d(a + b \arcsin(cx))}{bc\sqrt{1 - c^2 x^2}}$$

$$\downarrow 3793$$

$$\frac{d\sqrt{d - c^2 dx^2} \int \left(\frac{1}{8} \cos\left(\frac{4a}{b} - \frac{4(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))^n + \frac{1}{2} \cos\left(\frac{2a}{b} - \frac{2(a + b \arcsin(cx))}{b}\right) (a + b \arcsin(cx))\right)}{bc\sqrt{1 - c^2 x^2}}$$

$$\downarrow 2009$$

$$\frac{d\sqrt{d - c^2 dx^2} \left(\frac{3(a + b \arcsin(cx))^{n+1}}{8(n+1)} - ib2^{-n-3} e^{-\frac{2ia}{b}} (a + b \arcsin(cx))^n \left(-\frac{i(a + b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, -\frac{2i(a + b \arcsin(cx))}{b}\right)\right)}{bc\sqrt{1 - c^2 x^2}}$$

input

```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d*Sqrt[d - c^2*d*x^2]*((3*(a + b*ArcSin[c*x])^(1 + n))/(8*(1 + n)) - (I*2
^(-3 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x])
)/b])/E^(((2*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b^n + (I*2^(-3 - n)*b
*E^(((2*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcSin[c
*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n - (I*b*(a + b*ArcSin[c*x])^n*Gamma
[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b])/2^(2*(3 + n))*E^(((4*I)*a)/b)*((
(-I)*(a + b*ArcSin[c*x]))/b)^n + (I*b*E^(((4*I)*a)/b)*(a + b*ArcSin[c*x])
^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b])/2^(2*(3 + n))*((I*(a + b*
ArcSin[c*x]))/b)^n))/(b*c*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_) + (d_)*(x_)^(m_)*sin[(e_) + (f_)*(x_)^(n_)], x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5168

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((d_) + (e_)*(x_)^2)^(p_), x
_Symbol] := Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[
x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b
, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int (-c^2 d x^2 + d)^{\frac{3}{2}} (a + b \arcsin(cx))^n dx$$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x)
```

output `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n, x)`

Sympy [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**n,x)`

output `Timed out`

Maxima [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n, x)`

Giac [F(-2)]

Exception generated.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \int (a + b \arcsin(cx))^n (d - c^2 dx^2)^{3/2} dx$$

input `int((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2),x)`

output `int((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2), x)`

Reduce [F]

$$\int (d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n dx = \sqrt{d} d \left(- \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^2 dx \right) c^2 + \int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} dx \right)$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^n,x)`

output

```
sqrt(d)*d*( - int((asin(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*c**2  
+ int((asin(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x))
```

3.409
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx$$

Optimal result	3515
Mathematica [N/A]	3515
Rubi [N/A]	3516
Maple [N/A]	3517
Fricas [N/A]	3517
Sympy [N/A]	3517
Maxima [N/A]	3518
Giac [F(-2)]	3518
Mupad [N/A]	3519
Reduce [N/A]	3519

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \frac{d\sqrt{d - c^2 dx^2} \operatorname{Int}\left(\frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^n}{x}, x\right)}{\sqrt{1 - c^2 x^2}}$$

output `d*(-c^2*d*x^2+d)^(1/2)*Defer(Int)((-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))^n/x, x)/(-c^2*x^2+1)^(1/2)`

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx$$

input `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n)/x, x]`

output `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n)/x, x]`

Rubi [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx$$

↓ 5226

$$\int \left(-\frac{2c^2 d^2 x (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} + \frac{d^2 (a + b \arcsin(cx))^n}{x \sqrt{d - c^2 dx^2}} + \frac{c^4 d^2 x^3 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} \right) dx$$

↓ 2009

$$\begin{aligned} & d^2 \int \frac{(a + b \arcsin(cx))^n}{x \sqrt{d - c^2 dx^2}} dx + \\ & \frac{5d^2 e^{-\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, -\frac{i(a+b \arcsin(cx))}{b}\right)}{8\sqrt{d - c^2 dx^2}} + \\ & \frac{d^2 3^{-n-1} e^{-\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, -\frac{3i(a+b \arcsin(cx))}{b}\right)}{8\sqrt{d - c^2 dx^2}} + \\ & \frac{5d^2 e^{\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, \frac{i(a+b \arcsin(cx))}{b}\right)}{8\sqrt{d - c^2 dx^2}} + \\ & \frac{d^2 3^{-n-1} e^{\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, \frac{3i(a+b \arcsin(cx))}{b}\right)}{8\sqrt{d - c^2 dx^2}} \end{aligned}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n)/x,x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d x^2 + d)^{\frac{3}{2}} (a + b \arcsin(cx))^n}{x} dx$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x,x)`output `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x,x)`**Fricas [N/A]**

Not integrable

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="fricas")`output `integral((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n/x, x)`**Sympy [N/A]**

Not integrable

Time = 104.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \arcsin(cx))^n}{x} dx$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**n/x,x)`

output `Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*asin(c*x))**n/x, x)`

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(a + b \arcsin(cx))^n (d - c^2 dx^2)^{3/2}}{x} dx$$

input `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2))/x,x)`

output `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2))/x, x)`

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x} dx = \sqrt{d} d \left(\int \frac{(a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1}}{x} dx \right. \\ \left. - \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x dx \right) c^2 \right)$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^n/x,x)`

output `sqrt(d)*d*(int(((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1))/x,x) - int((a sin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x,x)*c**2)`

3.410
$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx$$

Optimal result	3520
Mathematica [N/A]	3520
Rubi [N/A]	3521
Maple [N/A]	3522
Fricas [N/A]	3522
Sympy [F(-1)]	3522
Maxima [N/A]	3523
Giac [F(-2)]	3523
Mupad [N/A]	3524
Reduce [N/A]	3524

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \frac{d\sqrt{d - c^2 dx^2} \operatorname{Int}\left(\frac{(1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^n}{x^2}, x\right)}{\sqrt{1 - c^2 x^2}}$$

output `d*(-c^2*d*x^2+d)^(1/2)*Defer(Int)((-c^2*x^2+1)^(3/2)*(a+b*arcsin(c*x))^n/x^2,x)/(-c^2*x^2+1)^(1/2)`

Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx$$

input `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n)/x^2,x]`

output `Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n)/x^2, x]`

Rubi [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx$$

↓ 5226

$$\int \left(-\frac{2c^2 d^2 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} + \frac{d^2 (a + b \arcsin(cx))^n}{x^2 \sqrt{d - c^2 dx^2}} + \frac{c^4 d^2 x^2 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} \right) dx$$

↓ 2009

$$\frac{d^2 \int \frac{(a + b \arcsin(cx))^n}{x^2 \sqrt{d - c^2 dx^2}} dx - \frac{3cd^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^{n+1}}{2b(n+1) \sqrt{d - c^2 dx^2}} + \frac{icd^2 2^{-n-3} e^{-\frac{2ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{\sqrt{d - c^2 dx^2}}}{icd^2 2^{-n-3} e^{\frac{2ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, \frac{2i(a+b \arcsin(cx))}{b}\right)}{\sqrt{d - c^2 dx^2}}$$

input `Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcSin[c*x])^n)/x^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d x^2 + d)^{\frac{3}{2}} (a + b \arcsin(cx))^n}{x^2} dx$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x^2,x)`

output `int((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x^2,x)`

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="fricas")`

output `integral((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n/x^2, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(3/2)*(a+b*asin(c*x))**n/x**2,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{\frac{3}{2}} (b \arcsin(cx) + a)^n}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(3/2)*(b*arcsin(c*x) + a)^n/x^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(a + b \arcsin(cx))^n (d - c^2 dx^2)^{3/2}}{x^2} dx$$

input `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2,x)`

output `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2, x)`

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.83

$$\int \frac{(d - c^2 dx^2)^{3/2} (a + b \arcsin(cx))^n}{x^2} dx = \frac{\sqrt{d} d \left(-(\arcsin(cx) b + a)^n \arcsin(cx) bc - (\arcsin(cx) b + a)^n ac + \right)}{x^2}$$

input `int((-c^2*d*x^2+d)^(3/2)*(a+b*asin(c*x))^n/x^2,x)`

output `(sqrt(d)*d*(- (asin(c*x)*b + a)**n*asin(c*x)*b*c - (asin(c*x)*b + a)**n*a*c + int((asin(c*x)*b + a)**n/(sqrt(-c**2*x**2 + 1)*x**2),x)*b*n + int((asin(c*x)*b + a)**n/(sqrt(-c**2*x**2 + 1)*x**2),x)*b - int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1),x)*b*c**2*n - int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1),x)*b*c**2))/(b*(n + 1))`

3.411 $\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$

Optimal result	3526
Mathematica [A] (verified)	3527
Rubi [A] (verified)	3528
Maple [F]	3530
Fricas [F]	3530
Sympy [F(-1)]	3531
Maxima [F]	3531
Giac [F]	3531
Mupad [F(-1)]	3532
Reduce [F]	3532

Optimal result

Integrand size = 29, antiderivative size = 906

$$\begin{aligned}
& \int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \frac{5d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^{1+n}}{128bc^3(1+n)\sqrt{1 - c^2 x^2}} \\
& \frac{i2^{-7-n} d^2 e^{-\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& + \frac{i2^{-7-n} d^2 e^{\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{2i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& + \frac{i2^{-2(4+n)} d^2 e^{-\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{4i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& - \frac{i2^{-2(4+n)} d^2 e^{\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{4i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& + \frac{i2^{-7-n} 3^{-1-n} d^2 e^{-\frac{6ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{6i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& - \frac{i2^{-7-n} 3^{-1-n} d^2 e^{\frac{6ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{6i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& + \frac{i2^{-11-3n} d^2 e^{-\frac{8ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{8i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}} \\
& - \frac{i2^{-11-3n} d^2 e^{\frac{8ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{8i(a+b \arcsin(cx))}{b}\right)}{c^3 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

output

```

5/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^(1+n)/b/c^3/(1+n)/(-c^2*x
^2+1)^(1/2)-I*2^(-7-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(
1+n,-2*I*(a+b*arcsin(c*x))/b)/c^3/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+
b*arcsin(c*x))/b)^n+I*2^(-7-n)*d^2*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b
*arcsin(c*x))^n*GAMMA(1+n,2*I*(a+b*arcsin(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/
((I*(a+b*arcsin(c*x))/b)^n+I*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n
*GAMMA(1+n,-4*I*(a+b*arcsin(c*x))/b)/(2^(8+2*n))/c^3/exp(4*I*a/b)/(-c^2*x^
2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n-I*d^2*exp(4*I*a/b)*(-c^2*d*x^2+d)^
(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,4*I*(a+b*arcsin(c*x))/b)/(2^(8+2*n))/c
^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n+I*2^(-7-n)*3^(-1-n)*d^2*
(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-6*I*(a+b*arcsin(c*x))/
b)/c^3/exp(6*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n-I*2^(-
7-n)*3^(-1-n)*d^2*exp(6*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GA
MMA(1+n,6*I*(a+b*arcsin(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*
x))/b)^n+I*2^(-11-3*n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA
(1+n,-8*I*(a+b*arcsin(c*x))/b)/c^3/exp(8*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a
+b*arcsin(c*x))/b)^n-I*2^(-11-3*n)*d^2*exp(8*I*a/b)*(-c^2*d*x^2+d)^(1/2)*
(a+b*arcsin(c*x))^n*GAMMA(1+n,8*I*(a+b*arcsin(c*x))/b)/c^3/(-c^2*x^2+1)^(1
/2)/((I*(a+b*arcsin(c*x))/b)^n)

```

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.09

$$\int x^2(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \text{Too large to display}$$

input

```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*(5*2
^(4 + 3*n)*3^(1 + n)*a*E^(((8*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^n + 5*2
^(4 + 3*n)*3^(1 + n)*b*E^(((8*I)*a)/b)*ArcSin[c*x]*((a + b*ArcSin[c*x])^2/
b^2)^n - I*3^(1 + n)*4^(2 + n)*b*E^(((6*I)*a)/b)*(1 + n)*((I*(a + b*ArcSin
[c*x]))/b)^n*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x]))/b] + I*3^(1 + n)*4^
(2 + n)*b*E^(((10*I)*a)/b)*(1 + n)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[
1 + n, ((2*I)*(a + b*ArcSin[c*x]))/b] + I*2^(3 + n)*3^(1 + n)*b*E^(((4*I)*
a)/b)*((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x
]))/b] + I*2^(3 + n)*3^(1 + n)*b*E^(((4*I)*a)/b)*n*((I*(a + b*ArcSin[c*x])
)/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b] - I*2^(3 + n)*3^(1 + n
)*b*E^(((12*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((4*I)*
(a + b*ArcSin[c*x]))/b] - I*2^(3 + n)*3^(1 + n)*b*E^(((12*I)*a)/b)*n*((-I
)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b] +
I*4^(2 + n)*b*E^(((2*I)*a)/b)*((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, (
(-6*I)*(a + b*ArcSin[c*x]))/b] + I*4^(2 + n)*b*E^(((2*I)*a)/b)*n*((I*(a +
b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-6*I)*(a + b*ArcSin[c*x]))/b] - I*4^(2
+ n)*b*E^(((14*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((6
*I)*(a + b*ArcSin[c*x]))/b] - I*4^(2 + n)*b*E^(((14*I)*a)/b)*n*((-I)*(a +
b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((6*I)*(a + b*ArcSin[c*x]))/b] + I*3^(1
+ n)*b*((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-8*I)*(a + b*ArcSi...
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 635, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5224, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$$

$$\downarrow 5224$$

$$\frac{d^2 \sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^6 \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} \right) \sin^2 \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} \right) d(a + b \arcsin(cx))}{bc^3 \sqrt{1 - c^2 x^2}}$$

$$\downarrow 4906$$

$$d^2\sqrt{d-c^2x^2} \int \left(-\frac{1}{128} \cos\left(\frac{8a}{b} - \frac{8(a+b\arcsin(cx))}{b}\right) (a+b\arcsin(cx))^n - \frac{1}{32} \cos\left(\frac{6a}{b} - \frac{6(a+b\arcsin(cx))}{b}\right) (a+b\arcsin(cx))^n \right) dx$$

↓ 2009

$$d^2\sqrt{d-c^2x^2} \left(\frac{5(a+b\arcsin(cx))^{n+1}}{128(n+1)} - ib2^{-n-7}e^{-\frac{2ia}{b}}(a+b\arcsin(cx))^n \left(-\frac{i(a+b\arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, -\frac{2i(a+b\arcsin(cx))}{b}\right) \right)$$

input

```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d^2*sqrt[d - c^2*d*x^2]*((5*(a + b*ArcSin[c*x])^(1 + n))/(128*(1 + n)) -
(I*2^(-7 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c
*x]))/b]))/(E^(((2*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n + (I*2^(-7 -
n)*b*E^(((2*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcS
in[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n + (I*b*(a + b*ArcSin[c*x])^n*G
amma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b])/(2^(2*(4 + n))*E^(((4*I)*a)/b
)*((-I)*(a + b*ArcSin[c*x]))/b)^n - (I*b*E^(((4*I)*a)/b)*(a + b*ArcSin[c
*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b])/(2^(2*(4 + n))*((I*(a
+ b*ArcSin[c*x]))/b)^n + (I*2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcSin[c*x])^n
*Gamma[1 + n, ((-6*I)*(a + b*ArcSin[c*x]))/b])/(E^(((6*I)*a)/b)*((-I)*(a
+ b*ArcSin[c*x]))/b)^n - (I*2^(-7 - n)*3^(-1 - n)*b*E^(((6*I)*a)/b)*(a +
b*ArcSin[c*x])^n*Gamma[1 + n, ((6*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*A
rcSin[c*x]))/b)^n + (I*2^(-11 - 3*n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n,
((-8*I)*(a + b*ArcSin[c*x]))/b])/(E^(((8*I)*a)/b)*((-I)*(a + b*ArcSin[c*x
]))/b)^n - (I*2^(-11 - 3*n)*b*E^(((8*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma
[1 + n, ((8*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n)/(b
*c^3*sqrt[1 - c^2*x^2])
```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 4906 `Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [F]

$$\int x^2(-c^2dx^2 + d)^{\frac{5}{2}}(a + b \arcsin(cx))^n dx$$

input `int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x)`

output `int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x)`

Fricas [F]

$$\int x^2(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))^n dx = \int (-c^2dx^2 + d)^{\frac{5}{2}}(b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")`

output `integral((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n, x)`

Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \text{Timed out}$$

input `integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**n,x)`

output Timed out

Maxima [F]

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^n*x^2, x)`

Giac [F]

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^n x^2 dx$$

input `integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^n*x^2, x)`

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int x^2 (a + b \arcsin(cx))^n (d - c^2 dx^2)^{5/2} dx$$

input `int(x^2*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2),x)`

output `int(x^2*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\begin{aligned} \int x^2 (d - c^2 dx^2)^{5/2} (a \\ + b \arcsin(cx))^n dx = \sqrt{d} d^2 \left(\left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^6 dx \right) c^4 \right. \\ \left. - 2 \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^4 dx \right) c^2 \right. \\ \left. + \int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^2 dx \right) \end{aligned}$$

input `int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^n,x)`

output `sqrt(d)*d**2*(int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**6,x)*c**4 - 2*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**4,x)*c**2 + int((a sin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**2,x))`

3.412 $\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$

Optimal result	3534
Mathematica [A] (verified)	3535
Rubi [A] (verified)	3536
Maple [F]	3538
Fricas [F]	3538
Sympy [F(-1)]	3539
Maxima [F]	3539
Giac [F(-2)]	3539
Mupad [F(-1)]	3540
Reduce [F]	3540

Optimal result

Integrand size = 27, antiderivative size = 815

$$\begin{aligned}
& \int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \\
& \frac{5d^2 e^{-\frac{ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{5d^2 e^{\frac{ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{3^{1-n} d^2 e^{-\frac{3ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{3i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{3^{1-n} d^2 e^{\frac{3ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{3i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{5^{-n} d^2 e^{-\frac{5ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{5i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{5^{-n} d^2 e^{\frac{5ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{5i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{7^{-1-n} d^2 e^{-\frac{7ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, -\frac{7i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}} \\
& \frac{7^{-1-n} d^2 e^{\frac{7ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1 + n, \frac{7i(a+b \arcsin(cx))}{b}\right)}{128c^2 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

output

```

-5/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-I*(a+b*arcsin(c*x))/b)/c^2/exp(I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)
-5/128*d^2*exp(I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,I*(a+b*arcsin(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)-1/128*3^(1-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-3*I*(a+b*arcsin(c*x))/b)/c^2/exp(3*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-1/128*3^(1-n)*d^2*exp(3*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,3*I*(a+b*arcsin(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)-1/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-5*I*(a+b*arcsin(c*x))/b)/(5^n)/c^2/exp(5*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-1/128*d^2*exp(5*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,5*I*(a+b*arcsin(c*x))/b)/(5^n)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)-1/128*7^(-1-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-7*I*(a+b*arcsin(c*x))/b)/c^2/exp(7*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)-1/128*7^(-1-n)*d^2*exp(7*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,7*I*(a+b*arcsin(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)

```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 603, normalized size of antiderivative = 0.74

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx =$$

$$5^{-n} 21^{-1-n} d^3 e^{-\frac{7ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{(a + b \arcsin(cx))^2}{b^2} \right)^{-3n} \left(105^{1+n} e^{\frac{6ia}{b}} \left(\frac{i(a + b \arcsin(cx))}{b} \right)^n \left(\frac{(a + b \arcsin(cx))}{b} \right)^n \right)$$

input

```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```

-1/128*(21^(-1 - n)*d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*(105^(1 +
n)*E^(((6*I)*a)/b)*((I*(a + b*ArcSin[c*x]))/b)^n*((a + b*ArcSin[c*x])^2/b^
2)^(2*n)*Gamma[1 + n, ((-I)*(a + b*ArcSin[c*x]))/b] + (((-I)*(a + b*ArcSin
[c*x]))/b)^n*(105^(1 + n)*E^(((8*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^(2*n
)*Gamma[1 + n, (I*(a + b*ArcSin[c*x]))/b] + 9*5^n*7^(1 + n)*E^(((4*I)*a)/b
)*((I*(a + b*ArcSin[c*x]))/b)^(2*n)*((a + b*ArcSin[c*x])^2/b^2)^n*Gamma[1
+ n, ((-3*I)*(a + b*ArcSin[c*x]))/b] + 9*5^n*7^(1 + n)*E^(((10*I)*a)/b)*((
a + b*ArcSin[c*x])^2/b^2)^(2*n)*Gamma[1 + n, ((3*I)*(a + b*ArcSin[c*x]))/b
] + 3^(1 + n)*(7^(1 + n)*E^(((2*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*
((I*(a + b*ArcSin[c*x]))/b)^(3*n)*Gamma[1 + n, ((-5*I)*(a + b*ArcSin[c*x])
)/b] + 7^(1 + n)*E^(((12*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^(2*n)*Gamma[
1 + n, ((5*I)*(a + b*ArcSin[c*x]))/b] + 5^n*(((I)*(a + b*ArcSin[c*x]))/b
)^n*((I*(a + b*ArcSin[c*x]))/b)^(3*n)*Gamma[1 + n, ((-7*I)*(a + b*ArcSin[c
*x]))/b] + E^(((14*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^(2*n)*Gamma[1 + n,
((7*I)*(a + b*ArcSin[c*x]))/b])))/(5^n*c^2*E^(((7*I)*a)/b)*Sqrt[d - c^2
*d*x^2]*((a + b*ArcSin[c*x])^2/b^2)^(3*n))

```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5224, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$$

$$\downarrow 5224$$

$$\frac{d^2 \sqrt{d - c^2 dx^2} \int -(a + b \arcsin(cx))^n \cos^6\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

$$\downarrow 25$$

$$\frac{d^2 \sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^6\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) \sin\left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b}\right) d(a + b \arcsin(cx))}{bc^2 \sqrt{1 - c^2 x^2}}$$

4906

$$\frac{d^2 \sqrt{d - c^2 dx^2} \int \left(\frac{1}{64} \sin \left(\frac{7a}{b} - \frac{7(a+b \arcsin(cx))}{b} \right) (a + b \arcsin(cx))^n + \frac{5}{64} \sin \left(\frac{5a}{b} - \frac{5(a+b \arcsin(cx))}{b} \right) (a + b \arcsin(cx))^n \right)}{dx^2}$$

2009

$$\frac{d^2 \sqrt{d - c^2 dx^2} \left(-\frac{5}{128} b e^{-\frac{ia}{b}} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma \left(n + 1, -\frac{i(a+b \arcsin(cx))}{b} \right) - \frac{1}{128} b 3^{1-n} e^{-\frac{3ia}{b}} \right)}{dx^2}$$

input

```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d^2*Sqrt[d - c^2*d*x^2]*((-5*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-I)*(a + b*ArcSin[c*x]))/b])/(128*E^((I*a)/b)*(((I)*(a + b*ArcSin[c*x]))/b)^n) - (5*b*E^((I*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, (I*(a + b*ArcSin[c*x]))/b])/(128*((I*(a + b*ArcSin[c*x]))/b)^n) - (3^(1 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-3*I)*(a + b*ArcSin[c*x]))/b])/(128*E^(((3*I)*a)/b)*(((I)*(a + b*ArcSin[c*x]))/b)^n) - (3^(1 - n)*b*E^(((3*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((3*I)*(a + b*ArcSin[c*x]))/b])/(128*((I*(a + b*ArcSin[c*x]))/b)^n) - (b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-5*I)*(a + b*ArcSin[c*x]))/b])/(128*5^n*E^(((5*I)*a)/b)*(((I)*(a + b*ArcSin[c*x]))/b)^n) - (b*E^(((5*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((5*I)*(a + b*ArcSin[c*x]))/b])/(128*5^n*((I*(a + b*ArcSin[c*x]))/b)^n) - (7^(-1 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-7*I)*(a + b*ArcSin[c*x]))/b])/(128*E^(((7*I)*a)/b)*(((I)*(a + b*ArcSin[c*x]))/b)^n) - (7^(-1 - n)*b*E^(((7*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((7*I)*(a + b*ArcSin[c*x]))/b])/(128*((I*(a + b*ArcSin[c*x]))/b)^n))/(b*c^2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 5224

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Ssin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x(-c^2dx^2 + d)^{\frac{5}{2}}(a + b \arcsin(cx))^n dx$$

input

```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x)
```

output

```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x)
```

Fricas [F]

$$\int x(d - c^2dx^2)^{5/2}(a + b \arcsin(cx))^n dx = \int (-c^2dx^2 + d)^{\frac{5}{2}}(b \arcsin(cx) + a)^n x dx$$

input

```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")
```

output

```
integral((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \text{Timed out}$$

input `integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**n,x)`

output `Timed out`

Maxima [F]

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^n x dx$$

input `integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^n*x, x)`

Giac [F(-2)]

Exception generated.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \text{Exception raised: TypeError}$$

input `integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int x(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int x(a + b \arcsin(cx))^n (d - c^2 dx^2)^{5/2} dx$$

input `int(x*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2),x)`

output `int(x*(a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\begin{aligned} \int x(d - c^2 dx^2)^{5/2} (a \\ + b \arcsin(cx))^n dx = \sqrt{d} d^2 \left(\left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^5 dx \right) c^4 \right. \\ \left. - 2 \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^3 dx \right) c^2 \right. \\ \left. + \int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x dx \right) \end{aligned}$$

input `int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^n,x)`

output `sqrt(d)*d**2*(int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**5,x)*c**4 - 2*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**3,x)*c**2 + int((a sin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x,x))`

3.413 $\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx$

Optimal result	3541
Mathematica [A] (verified)	3542
Rubi [A] (verified)	3543
Maple [F]	3545
Fricas [F]	3545
Sympy [F(-1)]	3545
Maxima [F]	3546
Giac [F(-2)]	3546
Mupad [F(-1)]	3546
Reduce [F]	3547

Optimal result

Integrand size = 26, antiderivative size = 698

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \frac{5d^2 \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^{1+n}}{16bc(1+n)\sqrt{1 - c^2 x^2}}$$

$$- \frac{15i2^{-7-n} d^2 e^{-\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

$$+ \frac{15i2^{-7-n} d^2 e^{\frac{2ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{2i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

$$- \frac{3i2^{-7-2n} d^2 e^{-\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{4i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

$$+ \frac{3i2^{-7-2n} d^2 e^{\frac{4ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{4i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

$$- \frac{i2^{-7-n} 3^{-1-n} d^2 e^{-\frac{6ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, -\frac{6i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

$$+ \frac{i2^{-7-n} 3^{-1-n} d^2 e^{\frac{6ia}{b}} \sqrt{d - c^2 dx^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(1+n, \frac{6i(a+b \arcsin(cx))}{b}\right)}{c\sqrt{1 - c^2 x^2}}$$

output

```

5/16*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^(1+n)/b/c/(1+n)/(-c^2*x^2+
1)^(1/2)-15*I*2^(-7-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(
1+n,-2*I*(a+b*arcsin(c*x))/b)/c/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*
arcsin(c*x))/b)^n)+15*I*2^(-7-n)*d^2*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+
b*arcsin(c*x))^n*GAMMA(1+n,2*I*(a+b*arcsin(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((
I*(a+b*arcsin(c*x))/b)^n)-3*I*2^(-7-2*n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
csin(c*x))^n*GAMMA(1+n,-4*I*(a+b*arcsin(c*x))/b)/c/exp(4*I*a/b)/(-c^2*x^2+
1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)+3*I*2^(-7-2*n)*d^2*exp(4*I*a/b)*(-c^
2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,4*I*(a+b*arcsin(c*x))/b)/c/
(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^n)-I*2^(-7-n)*3^(-1-n)*d^2*(-c
^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1+n,-6*I*(a+b*arcsin(c*x))/b)/
c/exp(6*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcsin(c*x))/b)^n)+I*2^(-7-n)*
3^(-1-n)*d^2*exp(6*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcsin(c*x))^n*GAMMA(1
+n,6*I*(a+b*arcsin(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcsin(c*x))/b)^
n)

```

Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.68

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \frac{d^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{120a}{b+bn} + \frac{120 \arcsin(cx)}{1+n} - 45i2^{-n} e^{-\frac{2ia}{b}} \left(-\frac{i(a+b \arcsin(cx))}{b} \right) \right)}{1}$$

input

```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n,x]
```

output

```
(d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^n*((120*a)/(b + b*n) + (120*Arc
Sin[c*x])/(1 + n) - ((45*I)*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x]))/b])/
(2^n*E^(((2*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n + ((45*I)*E^(((2*I)
*a)/b)*Gamma[1 + n, ((2*I)*(a + b*ArcSin[c*x]))/b])/(2^n*((I*(a + b*ArcSin
[c*x]))/b)^n) - ((9*I)*((I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*
(a + b*ArcSin[c*x]))/b])/(4^n*E^(((4*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^
n) + ((9*I)*E^(((4*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n,
((4*I)*(a + b*ArcSin[c*x]))/b])/(4^n*((a + b*ArcSin[c*x])^2/b^2)^n) - (I*(
(I*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((-6*I)*(a + b*ArcSin[c*x]))/b])
/(6^n*E^(((6*I)*a)/b)*((a + b*ArcSin[c*x])^2/b^2)^n) + (I*E^(((6*I)*a)/b)*
((-I)*(a + b*ArcSin[c*x]))/b)^n*Gamma[1 + n, ((6*I)*(a + b*ArcSin[c*x]))/
b])/(6^n*((a + b*ArcSin[c*x])^2/b^2)^n))/(384*c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5168, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx \\
 & \quad \downarrow \text{5168} \\
 & \frac{d^2 \sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \cos^6 \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} \right) d(a + b \arcsin(cx))}{bc \sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{3042} \\
 & \frac{d^2 \sqrt{d - c^2 dx^2} \int (a + b \arcsin(cx))^n \sin \left(\frac{a}{b} - \frac{a + b \arcsin(cx)}{b} + \frac{\pi}{2} \right)^6 d(a + b \arcsin(cx))}{bc \sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{3793} \\
 & \frac{d^2 \sqrt{d - c^2 dx^2} \int \left(\frac{1}{32} \cos \left(\frac{6a}{b} - \frac{6(a + b \arcsin(cx))}{b} \right) (a + b \arcsin(cx))^n + \frac{3}{16} \cos \left(\frac{4a}{b} - \frac{4(a + b \arcsin(cx))}{b} \right) (a + b \arcsin(cx))^n \right)}{bc \sqrt{1 - c^2 x^2}} \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$d^2 \sqrt{d - c^2 x^2} \left(\frac{5(a + b \arcsin(cx))^{n+1}}{16(n+1)} - 15ib2^{-n-7} e^{-\frac{2ia}{b}} (a + b \arcsin(cx))^n \left(-\frac{i(a + b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n + 1, -\frac{2i(a + b \arcsin(cx))}{b}\right) \right)$$

input `Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n,x]`

output `(d^2*Sqrt[d - c^2*d*x^2]*((5*(a + b*ArcSin[c*x])^(1 + n))/(16*(1 + n)) - ((15*I)*2^(-7 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcSin[c*x]))/b])/(E^(((2*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n) + ((15*I)*2^(-7 - n)*b*E^(((2*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n - ((3*I)*2^(-7 - 2*n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-4*I)*(a + b*ArcSin[c*x]))/b])/(E^(((4*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n) + ((3*I)*2^(-7 - 2*n)*b*E^(((4*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n - (I*2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((-6*I)*(a + b*ArcSin[c*x]))/b])/(E^(((6*I)*a)/b)*((-I)*(a + b*ArcSin[c*x]))/b)^n) + (I*2^(-7 - n)*3^(-1 - n)*b*E^(((6*I)*a)/b)*(a + b*ArcSin[c*x])^n*Gamma[1 + n, ((6*I)*(a + b*ArcSin[c*x]))/b])/((I*(a + b*ArcSin[c*x]))/b)^n)/(b*c*Sqrt[1 - c^2*x^2])`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5168

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[(1/(b*c))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[
x^n*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b
, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int (-c^2 d x^2 + d)^{\frac{5}{2}} (a + b \arcsin(cx))^n dx$$

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x)
```

output

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x)
```

Fricas [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{5}{2}} (b \arcsin(cx) + a)^n dx$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="fricas")
```

output

```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcsi
n(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \text{Timed out}$$

input

```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**n,x)
```

output

```
Timed out
```

Maxima [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int (-c^2 dx^2 + d)^{\frac{5}{2}} (b \arcsin(cx) + a)^n dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^n, x)`

Giac [F(-2)]

Exception generated.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \int (a + b \arcsin(cx))^n (d - c^2 dx^2)^{5/2} dx$$

input `int((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2),x)`

output `int((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2), x)`

Reduce [F]

$$\int (d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n dx = \sqrt{d} d^2 \left(\left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^4 dx \right) c^4 - 2 \left(\int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^2 dx \right) c^2 + \int (a \sin(cx) b + a)^n \sqrt{-c^2 x^2 + 1} dx \right)$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^n,x)`

output `sqrt(d)*d**2*(int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**4,x)*c**4 - 2*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**2,x)*c**2 + int((a sin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1),x))`

3.414 $\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx$

Optimal result	3548
Mathematica [N/A]	3548
Rubi [N/A]	3549
Maple [N/A]	3550
Fricas [N/A]	3550
Sympy [F(-1)]	3551
Maxima [N/A]	3551
Giac [F(-2)]	3551
Mupad [N/A]	3552
Reduce [N/A]	3552

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \frac{d^2 \sqrt{d - c^2 dx^2} \operatorname{Int}\left(\frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^n}{x}, x\right)}{\sqrt{1 - c^2 x^2}}$$

output `d^2*(-c^2*d*x^2+d)^(1/2)*Defer(Int)((-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))^n/x,x)/(-c^2*x^2+1)^(1/2)`

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx$$

input `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n)/x,x]`

output `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n)/x, x]`

Rubi [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx$$

↓ 5226

$$\int \left(-\frac{3c^2 d^3 x (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} + \frac{d^3 (a + b \arcsin(cx))^n}{x \sqrt{d - c^2 dx^2}} - \frac{c^6 d^3 x^5 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} + \frac{3c^4 d^3 x^3 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} \right) dx$$

↓ 2009

$$\frac{11d^3 e^{-\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \Gamma\left(n + 1, -\frac{i(a+b \arcsin(cx))}{b}\right) \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n}}{16\sqrt{d - c^2 dx^2}} -$$

$$\frac{5 \cdot 3^{-n-1} d^3 e^{-\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \Gamma\left(n + 1, -\frac{3i(a+b \arcsin(cx))}{b}\right) \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n}}{32\sqrt{d - c^2 dx^2}} +$$

$$\frac{3^{-n} d^3 e^{-\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \Gamma\left(n + 1, -\frac{3i(a+b \arcsin(cx))}{b}\right) \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n}}{8\sqrt{d - c^2 dx^2}} +$$

$$\frac{5^{-n-1} d^3 e^{-\frac{5ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \Gamma\left(n + 1, -\frac{5i(a+b \arcsin(cx))}{b}\right) \left(-\frac{i(a+b \arcsin(cx))}{b}\right)^{-n}}{32\sqrt{d - c^2 dx^2}} +$$

$$\frac{11d^3 e^{\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, \frac{i(a+b \arcsin(cx))}{b}\right)}{16\sqrt{d - c^2 dx^2}} -$$

$$\frac{5 \cdot 3^{-n-1} d^3 e^{\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, \frac{3i(a+b \arcsin(cx))}{b}\right)}{32\sqrt{d - c^2 dx^2}} +$$

$$\frac{3^{-n} d^3 e^{\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, \frac{3i(a+b \arcsin(cx))}{b}\right)}{8\sqrt{d - c^2 dx^2}} +$$

$$\frac{5^{-n-1} d^3 e^{\frac{5ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b}\right)^{-n} \Gamma\left(n + 1, \frac{5i(a+b \arcsin(cx))}{b}\right)}{32\sqrt{d - c^2 dx^2}} +$$

$$d^3 \int \frac{(a + b \arcsin(cx))^n}{x \sqrt{d - c^2 dx^2}} dx$$

input `Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n)/x,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d x^2 + d)^{\frac{5}{2}} (a + b \arcsin(cx))^n}{x} dx$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x,x)`

output `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x,x)`

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(-c^2 d x^2 + d)^{5/2} (b \arcsin(cx) + a)^n}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="fricas")`

output `integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n/x, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**n/x,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^n}{x} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^n/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x,x, algorithm="giac")`

output Exception raised: TypeError >> an error occurred running a Giac command:IN
 PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
 index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \int \frac{(a + b \operatorname{asin}(cx))^n (d - c^2 dx^2)^{5/2}}{x} dx$$

input `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2))/x,x)`

output `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2))/x, x)`

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x} dx = \sqrt{d} d^2 \left(\int \frac{(a \operatorname{asin}(cx) b + a)^n \sqrt{-c^2 x^2 + 1}}{x} dx \right. \\
+ \left(\int (a \operatorname{asin}(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x^3 dx \right) c^4 \\
\left. - 2 \left(\int (a \operatorname{asin}(cx) b + a)^n \sqrt{-c^2 x^2 + 1} x dx \right) c^2 \right)$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^n/x,x)`

output `sqrt(d)*d**2*(int(((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1))/x,x) + int
 ((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**3,x)*c**4 - 2*int((asin(c*
 x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x,x)*c**2)`

3.415
$$\int \frac{(d-c^2 dx^2)^{5/2} (a+b \arcsin(cx))^n}{x^2} dx$$

Optimal result	3553
Mathematica [N/A]	3553
Rubi [N/A]	3554
Maple [N/A]	3555
Fricas [N/A]	3555
Sympy [F(-1)]	3555
Maxima [N/A]	3556
Giac [F(-2)]	3556
Mupad [N/A]	3557
Reduce [N/A]	3557

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \frac{d^2 \sqrt{d - c^2 dx^2} \operatorname{Int}\left(\frac{(1 - c^2 x^2)^{5/2} (a + b \arcsin(cx))^n}{x^2}, x\right)}{\sqrt{1 - c^2 x^2}}$$

output `d^2*(-c^2*d*x^2+d)^(1/2)*Defer(Int)((-c^2*x^2+1)^(5/2)*(a+b*arcsin(c*x))^n/x^2,x)/(-c^2*x^2+1)^(1/2)`

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx$$

input `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n)/x^2,x]`

output `Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n)/x^2, x]`

Rubi [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx$$

↓ 5226

$$\int \left(-\frac{3c^2 d^3 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} + \frac{d^3 (a + b \arcsin(cx))^n}{x^2 \sqrt{d - c^2 dx^2}} - \frac{c^6 d^3 x^4 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} + \frac{3c^4 d^3 x^2 (a + b \arcsin(cx))^n}{\sqrt{d - c^2 dx^2}} \right) dx$$

↓ 2009

$$\frac{d^3 \int \frac{(a + b \arcsin(cx))^n}{x^2 \sqrt{d - c^2 dx^2}} dx - \frac{15cd^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^{n+1}}{8b(n+1) \sqrt{d - c^2 dx^2}} + \frac{icd^3 2^{-n-2} e^{-\frac{2ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, -\frac{2i(a+b \arcsin(cx))}{b}\right)}{\sqrt{d - c^2 dx^2}} + \frac{icd^3 2^{-2(n+3)} e^{-\frac{4ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(-\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, -\frac{4i(a+b \arcsin(cx))}{b}\right)}{\sqrt{d - c^2 dx^2}} - \frac{icd^3 2^{-n-2} e^{\frac{2ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, \frac{2i(a+b \arcsin(cx))}{b}\right)}{\sqrt{d - c^2 dx^2}} - \frac{icd^3 2^{-2(n+3)} e^{\frac{4ia}{b}} \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^n \left(\frac{i(a+b \arcsin(cx))}{b} \right)^{-n} \Gamma\left(n+1, \frac{4i(a+b \arcsin(cx))}{b}\right)}{\sqrt{d - c^2 dx^2}}}{\sqrt{d - c^2 dx^2}}$$

input

```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcSin[c*x])^n)/x^2,x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2 d x^2 + d)^{\frac{5}{2}} (a + b \arcsin(cx))^n}{x^2} dx$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x^2,x)`

output `int((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x^2,x)`

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^n}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="fricas")`

output `integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcsin(c*x) + a)^n/x^2, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \text{Timed out}$$

input `integrate((-c**2*d*x**2+d)**(5/2)*(a+b*asin(c*x))**n/x**2,x)`

output Timed out

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(-c^2 dx^2 + d)^{5/2} (b \arcsin(cx) + a)^n}{x^2} dx$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="maxima")`

output `integrate((-c^2*d*x^2 + d)^(5/2)*(b*arcsin(c*x) + a)^n/x^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \text{Exception raised: TypeError}$$

input `integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arcsin(c*x))^n/x^2,x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \int \frac{(a + b \arcsin(cx))^n (d - c^2 dx^2)^{5/2}}{x^2} dx$$

input `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2,x)`

output `int(((a + b*asin(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2, x)`

Reduce [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 236, normalized size of antiderivative = 8.14

$$\int \frac{(d - c^2 dx^2)^{5/2} (a + b \arcsin(cx))^n}{x^2} dx = \frac{\sqrt{d} d^2 \left(-(a \sin(cx) b + a)^n \arcsin(cx) bc - (a \sin(cx) b + a)^n ac + \right)}{x^2}$$

input `int((-c^2*d*x^2+d)^(5/2)*(a+b*asin(c*x))^n/x^2,x)`

output `(sqrt(d)*d**2*(- (asin(c*x)*b + a)**n*asin(c*x)*b*c - (asin(c*x)*b + a)**n*a*c + int((asin(c*x)*b + a)**n/(sqrt(-c**2*x**2 + 1)*x**2),x)*b*n + int((asin(c*x)*b + a)**n/(sqrt(-c**2*x**2 + 1)*x**2),x)*b + int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**2,x)*b*c**4*n + int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1)*x**2,x)*b*c**4 - 2*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1),x)*b*c**2*n - 2*int((asin(c*x)*b + a)**n*sqrt(-c**2*x**2 + 1),x)*b*c**2))/(b*(n + 1))`

3.416 $\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$

Optimal result	3558
Mathematica [N/A]	3558
Rubi [N/A]	3559
Maple [N/A]	3559
Fricas [N/A]	3560
Sympy [N/A]	3560
Maxima [F(-2)]	3560
Giac [F(-1)]	3561
Mupad [N/A]	3561
Reduce [N/A]	3562

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Int}\left(\frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}}, x\right)$$

output

```
Defer(Int)(x^m*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$$

input

```
Integrate[(x^m*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output

```
Integrate[(x^m*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$$

↓ 5234

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$$

input `Int[(x^m*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2], x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^m*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2), x)`

output `int(x^m*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2), x)`

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^m*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^m*arcsin(a*x)^n/(a^2*x^2 - 1), x)`

Sympy [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}^n(ax)}{\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(x**m*asin(a*x)**n/(-a**2*x**2+1)**(1/2),x)`

output `Integral(x**m*asin(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^m*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [F(-1)]

Timed out.

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Timed out}$$

input `integrate(x^m*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output Timed out

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)^n}{\sqrt{1-a^2x^2}} dx$$

input `int((x^m*asin(a*x)^n)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^m*asin(a*x)^n)/(1 - a^2*x^2)^(1/2), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^m \operatorname{asin}(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^m*asin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`output `int((x**m*asin(a*x)**n)/sqrt(-a**2*x**2+1),x)`

3.417 $\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$

Optimal result	3563
Mathematica [A] (verified)	3564
Rubi [A] (verified)	3564
Maple [F]	3566
Fricas [F]	3566
Sympy [F]	3566
Maxima [F(-2)]	3567
Giac [F(-2)]	3567
Mupad [F(-1)]	3567
Reduce [F]	3568

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = -\frac{3(-i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, -i \arcsin(ax))}{8a^4} - \frac{3(i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, i \arcsin(ax))}{8a^4} + \frac{3^{-1-n}(-i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, -3i \arcsin(ax))}{8a^4} + \frac{3^{-1-n}(i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, 3i \arcsin(ax))}{8a^4}$$

output

```
-3/8*arcsin(a*x)^n*GAMMA(1+n,-I*arcsin(a*x))/a^4/((-I*arcsin(a*x))^n)-3/8*
arcsin(a*x)^n*GAMMA(1+n,I*arcsin(a*x))/a^4/((I*arcsin(a*x))^n)+1/8*3^(-1-n
)*arcsin(a*x)^n*GAMMA(1+n,-3*I*arcsin(a*x))/a^4/((-I*arcsin(a*x))^n)+1/8*3
^(-1-n)*arcsin(a*x)^n*GAMMA(1+n,3*I*arcsin(a*x))/a^4/((I*arcsin(a*x))^n)
```


Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{3^{-1-n} \arcsin(ax)^n (\arcsin(ax)^2)^{-2n} (3^{2+n} (i \arcsin(ax))^n (\arcsin(ax)^2)^n \Gamma(1+n, -i \arcsin(ax)) + (-i$$

input

```
Integrate[(x^3*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output

```
-1/8*(3^(-1 - n)*ArcSin[a*x]^n*(3^(2 + n)*(I*ArcSin[a*x])^n*(ArcSin[a*x]^2)^n*Gamma[1 + n, (-I)*ArcSin[a*x]] + ((-I)*ArcSin[a*x])^n*(3^(2 + n)*(ArcSin[a*x]^2)^n*Gamma[1 + n, I*ArcSin[a*x]] - (I*ArcSin[a*x])^(2*n)*Gamma[1 + n, (-3*I)*ArcSin[a*x]] - (ArcSin[a*x]^2)^n*Gamma[1 + n, (3*I)*ArcSin[a*x]])))/(a^4*(ArcSin[a*x]^2)^(2*n))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx \\ \downarrow \text{5224} \\ \frac{\int a^3 x^3 \arcsin(ax)^n d \arcsin(ax)}{a^4} \\ \downarrow \text{3042} \\ \frac{\int \arcsin(ax)^n \sin(\arcsin(ax))^3 d \arcsin(ax)}{a^4} \\ \downarrow \text{3793} \end{array}$$

$$\frac{\int \left(\frac{3}{4} ax \arcsin(ax)^n - \frac{1}{4} \arcsin(ax)^n \sin(3 \arcsin(ax)) \right) d \arcsin(ax)}{a^4}$$

↓ 2009

$$-\frac{3}{8} \arcsin(ax)^n (-i \arcsin(ax))^{-n} \Gamma(n+1, -i \arcsin(ax)) + \frac{1}{8} 3^{-n-1} \arcsin(ax)^n (-i \arcsin(ax))^{-n} \Gamma(n+1, -3i \arcsin(ax))$$

input `Int[(x^3*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2], x]`

output `((-3*ArcSin[a*x]^n*Gamma[1 + n, (-I)*ArcSin[a*x]])/(8*((-I)*ArcSin[a*x])^n) - (3*ArcSin[a*x]^n*Gamma[1 + n, I*ArcSin[a*x]])/(8*(I*ArcSin[a*x])^n) + (3^(-1 - n)*ArcSin[a*x]^n*Gamma[1 + n, (-3*I)*ArcSin[a*x]])/(8*((-I)*ArcSin[a*x])^n) + (3^(-1 - n)*ArcSin[a*x]^n*Gamma[1 + n, (3*I)*ArcSin[a*x]])/(8*(I*ArcSin[a*x])^n))/a^4`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3793 `Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

rule 5224 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [F]

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^3*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

output `int(x^3*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

Fricas [F]

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^3 \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^3*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^3*arcsin(a*x)^n/(a^2*x^2 - 1), x)`

Sympy [F]

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^3 \operatorname{asin}^n(ax)}{\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(x**3*asin(a*x)**n/(-a**2*x**2+1)**(1/2),x)`

output `Integral(x**3*asin(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: TypeError}$$

input `integrate(x^3*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^3 \operatorname{asin}(ax)^n}{\sqrt{1-a^2x^2}} dx$$

input `int((x^3*asin(a*x)^n)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^3*asin(a*x)^n)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^3 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{a \sin(ax)^n x^3}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^3*asin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**n*x**3)/sqrt(-a**2*x**2+1),x)`

3.418 $\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$

Optimal result	3569
Mathematica [A] (verified)	3569
Rubi [A] (verified)	3570
Maple [F]	3571
Fricas [F]	3572
Sympy [F]	3572
Maxima [F(-2)]	3572
Giac [F]	3573
Mupad [F(-1)]	3573
Reduce [F]	3573

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^{1+n}}{2a^3(1+n)} + \frac{i2^{-3-n}(-i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, -2i \arcsin(ax))}{a^3} - \frac{i2^{-3-n}(i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, 2i \arcsin(ax))}{a^3}$$

output

```
1/2*arcsin(a*x)^(1+n)/a^3/(1+n)+I*2^(-3-n)*arcsin(a*x)^n*GAMMA(1+n,-2*I*arcsin(a*x))/a^3/((-I*arcsin(a*x))^n)-I*2^(-3-n)*arcsin(a*x)^n*GAMMA(1+n,2*I*arcsin(a*x))/a^3/((I*arcsin(a*x))^n)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{2^{-3-n} \arcsin(ax)^n (\arcsin(ax)^2)^{-n} (2^{2+n} \arcsin(ax) (\arcsin(ax)^2)^n + i(1+n)(i \arcsin(ax))^n \Gamma(1+n, -2i \arcsin(ax)) - i(1+n)(-i \arcsin(ax))^n \Gamma(1+n, 2i \arcsin(ax))}{a^3(1+n)}$$

input `Integrate[(x^2*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2],x]`

output $(2^{-3-n} \text{ArcSin}[a*x]^n (2^{2+n} \text{ArcSin}[a*x] (\text{ArcSin}[a*x]^2)^n + I(1+n) (I \text{ArcSin}[a*x])^n \Gamma[1+n, (-2I) \text{ArcSin}[a*x]] - I(1+n) ((-I) \text{ArcSin}[a*x])^n \Gamma[1+n, (2I) \text{ArcSin}[a*x]])) / (a^3 (1+n) (\text{ArcSin}[a*x]^2)^n)$

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5224, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx \\ & \quad \downarrow \text{5224} \\ & \frac{\int a^2 x^2 \arcsin(ax)^n d \arcsin(ax)}{a^3} \\ & \quad \downarrow \text{3042} \\ & \frac{\int \arcsin(ax)^n \sin(\arcsin(ax))^2 d \arcsin(ax)}{a^3} \\ & \quad \downarrow \text{3793} \\ & \frac{\int \left(\frac{1}{2} \arcsin(ax)^n - \frac{1}{2} \arcsin(ax)^n \cos(2 \arcsin(ax)) \right) d \arcsin(ax)}{a^3} \\ & \quad \downarrow \text{2009} \\ & \frac{\frac{\arcsin(ax)^{n+1}}{2(n+1)} + i 2^{-n-3} \arcsin(ax)^n (-i \arcsin(ax))^{-n} \Gamma(n+1, -2i \arcsin(ax)) - i 2^{-n-3} (i \arcsin(ax))^{-n} \arcsin(ax)}{a^3} \end{aligned}$$

input `Int[(x^2*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2],x]`

output $(\text{ArcSin}[a*x]^{(1+n)/(2*(1+n))} + (I*2^{(-3-n)}*\text{ArcSin}[a*x]^n*\text{Gamma}[1+n, (-2*I)*\text{ArcSin}[a*x]])/((-I)*\text{ArcSin}[a*x]^n - (I*2^{(-3-n)}*\text{ArcSin}[a*x]^n*\text{Gamma}[1+n, (2*I)*\text{ArcSin}[a*x]])/(I*\text{ArcSin}[a*x])^n)/a^3$

Defintions of rubi rules used

rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 3793 $\text{Int}[(c_. + (d_.)*(x_)^(m_)*\sin[(e_. + (f_.)*(x_)^(n_)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] || (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

rule 5224 $\text{Int}[(a_. + \text{ArcSin}[c_.*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(2)^(p_.), x_Symbol] \rightarrow \text{Simp}[(1/(b*c^(m+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Subst}[\text{Int}[x^n*\text{Sin}[-a/b + x/b]^m*\text{Cos}[-a/b + x/b]^(2*p+1), x], x, a + b*\text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[2*p + 2, 0] \&\& \text{IGtQ}[m, 0]$

Maple [F]

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input $\text{int}(x^2*\arcsin(a*x)^n/(-a^2*x^2+1)^(1/2), x)$

output $\text{int}(x^2*\arcsin(a*x)^n/(-a^2*x^2+1)^(1/2), x)$

Fricas [F]

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^2*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x^2*arcsin(a*x)^n/(a^2*x^2 - 1), x)`

Sympy [F]

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \operatorname{asin}^n(ax)}{\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(x**2*asin(a*x)**n/(-a**2*x**2+1)**(1/2),x)`

output `Integral(x**2*asin(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^2*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [F]

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x^2*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(x^2*arcsin(a*x)^n/sqrt(-a^2*x^2 + 1), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x^2 \operatorname{asin}(ax)^n}{\sqrt{1-a^2x^2}} dx$$

input `int((x^2*asin(a*x)^n)/(1 - a^2*x^2)^(1/2),x)`

output `int((x^2*asin(a*x)^n)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x^2 \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^n x^2}{\sqrt{-a^2x^2+1}} dx$$

input `int(x^2*asin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**n*x**2)/sqrt(- a**2*x**2 + 1),x)`

3.419 $\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$

Optimal result	3574
Mathematica [A] (verified)	3574
Rubi [A] (verified)	3575
Maple [F]	3576
Fricas [F]	3577
Sympy [F]	3577
Maxima [F(-2)]	3577
Giac [F]	3578
Mupad [F(-1)]	3578
Reduce [F]	3578

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = -\frac{(-i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, -i \arcsin(ax))}{2a^2} - \frac{(i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(1+n, i \arcsin(ax))}{2a^2}$$

output

```
-1/2*arcsin(a*x)^n*GAMMA(1+n,-I*arcsin(a*x))/a^2/((-I*arcsin(a*x))^n)-1/2*arcsin(a*x)^n*GAMMA(1+n,I*arcsin(a*x))/a^2/((I*arcsin(a*x))^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = -\frac{\arcsin(ax)^n (\arcsin(ax)^2)^{-n} ((i \arcsin(ax))^n \Gamma(1+n, -i \arcsin(ax)) + (-i \arcsin(ax))^n \Gamma(1+n, i \arcsin(ax)))}{2a^2}$$

input

```
Integrate[(x*ArcSin[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output

$$-1/2*(\text{ArcSin}[a*x]^n*((I*\text{ArcSin}[a*x])^n*\text{Gamma}[1+n, (-I)*\text{ArcSin}[a*x]] + ((-I)*\text{ArcSin}[a*x])^n*\text{Gamma}[1+n, I*\text{ArcSin}[a*x]]))/((a^2*(\text{ArcSin}[a*x]^2)^n)$$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5224, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx \\ & \quad \downarrow \text{5224} \\ & \frac{\int ax \arcsin(ax)^n d \arcsin(ax)}{a^2} \\ & \quad \downarrow \text{3042} \\ & \frac{\int \arcsin(ax)^n \sin(\arcsin(ax)) d \arcsin(ax)}{a^2} \\ & \quad \downarrow \text{3789} \\ & \frac{\frac{1}{2}i \int e^{-i \arcsin(ax)} \arcsin(ax)^n d \arcsin(ax) - \frac{1}{2}i \int e^{i \arcsin(ax)} \arcsin(ax)^n d \arcsin(ax)}{a^2} \\ & \quad \downarrow \text{2612} \\ & \frac{-\frac{1}{2} \arcsin(ax)^n (-i \arcsin(ax))^{-n} \Gamma(n+1, -i \arcsin(ax)) - \frac{1}{2} (i \arcsin(ax))^{-n} \arcsin(ax)^n \Gamma(n+1, i \arcsin(ax))}{a^2} \end{aligned}$$

input

$$\text{Int}[(x*\text{ArcSin}[a*x]^n)/\text{Sqrt}[1 - a^2*x^2], x]$$

output

$$(-1/2*(\text{ArcSin}[a*x]^n*\text{Gamma}[1+n, (-I)*\text{ArcSin}[a*x]])/((-I)*\text{ArcSin}[a*x])^n - (\text{ArcSin}[a*x]^n*\text{Gamma}[1+n, I*\text{ArcSin}[a*x]])/(2*(I*\text{ArcSin}[a*x])^n))/a^2$$

Definitions of rubi rules used

rule 2612 `Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]])*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 3789 `Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] :> Simp[I
/2 Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2 Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]`

rule 5224 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_))^(n_)*(x_)^(m_)*((d_) + (e_)*(x_)^
2)^(p_), x_Symbol] :> Simp[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x
^2)^p] Subst[Int[x^n*Sin[-a/b + x/b]^m*Cos[-a/b + x/b]^(2*p + 1), x], x,
a + b*ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]`

Maple [F]

$$\int \frac{x \arcsin(ax)^n}{\sqrt{-a^2x^2 + 1}} dx$$

input `int(x*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

output `int(x*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

Fricas [F]

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*x*arcsin(a*x)^n/(a^2*x^2 - 1), x)`

Sympy [F]

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x \operatorname{asin}^n(ax)}{\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(x*asin(a*x)**n/(-a**2*x**2+1)**(1/2),x)`

output `Integral(x*asin(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output `Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.`

Giac [F]

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x \arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(x*arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(x*arcsin(a*x)^n/sqrt(-a^2*x^2 + 1), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{x \operatorname{asin}(ax)^n}{\sqrt{1-a^2x^2}} dx$$

input `int((x*asin(a*x)^n)/(1 - a^2*x^2)^(1/2),x)`

output `int((x*asin(a*x)^n)/(1 - a^2*x^2)^(1/2), x)`

Reduce [F]

$$\int \frac{x \arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^n x}{\sqrt{-a^2x^2+1}} dx$$

input `int(x*asin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

output `int((asin(a*x)**n*x)/sqrt(- a**2*x**2 + 1),x)`

3.420 $\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$

Optimal result	3579
Mathematica [A] (verified)	3579
Rubi [A] (verified)	3580
Maple [A] (verified)	3580
Fricas [A] (verification not implemented)	3581
Sympy [B] (verification not implemented)	3581
Maxima [A] (verification not implemented)	3582
Giac [A] (verification not implemented)	3582
Mupad [B] (verification not implemented)	3582
Reduce [B] (verification not implemented)	3583

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^{1+n}}{a(1+n)}$$

output

`arcsin(a*x)^(1+n)/a/(1+n)`

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^{1+n}}{a(1+n)}$$

input

`Integrate[ArcSin[a*x]^n/Sqrt[1 - a^2*x^2],x]`

output

`ArcSin[a*x]^(1 + n)/(a*(1 + n))`

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx$$

↓ 5152

$$\frac{\arcsin(ax)^{n+1}}{a(n+1)}$$

input `Int[ArcSin[a*x]^n/Sqrt[1 - a^2*x^2],x]`

output `ArcSin[a*x]^(1 + n)/(a*(1 + n))`

Defintions of rubi rules used

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_./Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\arcsin(ax)^{1+n}}{a(1+n)}$	18
default	$\frac{\arcsin(ax)^{1+n}}{a(1+n)}$	18

input `int(arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)`

output `arcsin(a*x)^(1+n)/a/(1+n)`

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^n \arcsin(ax)}{an+a}$$

input `integrate(arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `arcsin(a*x)^n*arcsin(a*x)/(a*n + a)`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(12) = 24.

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge n = -1 \\ 0^n x & \text{for } a = 0 \\ \frac{\log(\operatorname{asin}(ax))}{a} & \text{for } n = -1 \\ \frac{\operatorname{asin}(ax) \operatorname{asin}^n(ax)}{an+a} & \text{otherwise} \end{cases}$$

input `integrate(asin(a*x)**n/(-a**2*x**2+1)**(1/2),x)`

output `Piecewise((zoo*x, Eq(a, 0) & Eq(n, -1)), (0**n*x, Eq(a, 0)), (log(asin(a*x))/a, Eq(n, -1)), (asin(a*x)*asin(a*x)**n/(a*n + a), True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^{n+1}}{a(n+1)}$$

input `integrate(arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`output `arcsin(a*x)^(n + 1)/(a*(n + 1))`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^{n+1}}{a(n+1)}$$

input `integrate(arcsin(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`output `arcsin(a*x)^(n + 1)/(a*(n + 1))`**Mupad [B] (verification not implemented)**

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \begin{cases} \frac{\ln(\operatorname{asin}(ax))}{a} & \text{if } n = -1 \\ \frac{\operatorname{asin}(ax)^{n+1}}{a(n+1)} & \text{if } n \neq -1 \end{cases}$$

input `int(asin(a*x)^n/(1 - a^2*x^2)^(1/2),x)`output `piecewise(n == -1, log(asin(a*x))/a, n ~= -1, asin(a*x)^(n + 1)/(a*(n + 1))`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\arcsin(ax)^n}{\sqrt{1-a^2x^2}} dx = \frac{\arcsin(ax)^n \arcsin(ax)}{a(n+1)}$$

input `int(asin(a*x)^n/(-a^2*x^2+1)^(1/2),x)`

output `(asin(a*x)**n*asin(a*x))/(a*(n + 1))`

3.421 $\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx$

Optimal result	3584
Mathematica [N/A]	3584
Rubi [N/A]	3585
Maple [N/A]	3585
Fricas [N/A]	3586
Sympy [N/A]	3586
Maxima [F(-2)]	3586
Giac [N/A]	3587
Mupad [N/A]	3587
Reduce [N/A]	3588

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \text{Int}\left(\frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}}, x\right)$$

output

```
Defer(Int)(arcsin(a*x)^n/x/(-a^2*x^2+1)^(1/2), x)
```

Mathematica [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx$$

input

```
Integrate[ArcSin[a*x]^n/(x*Sqrt[1 - a^2*x^2]), x]
```

output

```
Integrate[ArcSin[a*x]^n/(x*Sqrt[1 - a^2*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx$$

↓ 5234

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx$$

input `Int[ArcSin[a*x]^n/(x*Sqrt[1 - a^2*x^2]),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arcsin(ax)^n}{x\sqrt{-a^2x^2+1}} dx$$

input `int(arcsin(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)`

output `int(arcsin(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)`

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{\sqrt{-a^2x^2+1}} dx$$

input `integrate(arcsin(a*x)^n/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^n/(a^2*x^3 - x), x)`

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^n(ax)}{x\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**n/x/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**n/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(arcsin(a*x)^n/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{\sqrt{-a^2x^2+1}x} dx$$

input `integrate(arcsin(a*x)^n/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^n/(sqrt(-a^2*x^2 + 1)*x), x)`

Mupad [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^n/(x*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^n/(x*(1 - a^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\arcsin(ax)^n}{x\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}(ax)^n}{\sqrt{-a^2x^2+1}x} dx$$

input `int(asin(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)`output `int(asin(a*x)**n/(sqrt(-a**2*x**2+1)*x),x)`

3.422 $\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx$

Optimal result	3589
Mathematica [N/A]	3589
Rubi [N/A]	3590
Maple [N/A]	3590
Fricas [N/A]	3591
Sympy [N/A]	3591
Maxima [F(-2)]	3591
Giac [N/A]	3592
Mupad [N/A]	3592
Reduce [N/A]	3593

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx = \text{Int}\left(\frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}}, x\right)$$

output

```
Defer(Int)(arcsin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2), x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx$$

input

```
Integrate[ArcSin[a*x]^n/(x^2*Sqrt[1 - a^2*x^2]), x]
```

output

```
Integrate[ArcSin[a*x]^n/(x^2*Sqrt[1 - a^2*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx$$

↓ 5234

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx$$

input `Int[ArcSin[a*x]^n/(x^2*Sqrt[1 - a^2*x^2]),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{-a^2x^2+1}} dx$$

input `int(arcsin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)`

output `int(arcsin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)`

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{\sqrt{-a^2x^2+1x^2}} dx$$

input `integrate(arcsin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")`

output `integral(-sqrt(-a^2*x^2 + 1)*arcsin(a*x)^n/(a^2*x^4 - x^2), x)`

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx = \int \frac{\operatorname{asin}^n(ax)}{x^2\sqrt{-(ax-1)(ax+1)}} dx$$

input `integrate(asin(a*x)**n/x**2/(-a**2*x**2+1)**(1/2),x)`

output `Integral(asin(a*x)**n/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arcsin(ax)^n}{x^2\sqrt{1-a^2x^2}} dx = \text{Exception raised: RuntimeError}$$

input `integrate(arcsin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")`

output Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negative exponent.

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{x^2 \sqrt{1-a^2x^2}} dx = \int \frac{\arcsin(ax)^n}{\sqrt{-a^2x^2+1}x^2} dx$$

input `integrate(arcsin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")`

output `integrate(arcsin(a*x)^n/(sqrt(-a^2*x^2 + 1)*x^2), x)`

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arcsin(ax)^n}{x^2 \sqrt{1-a^2x^2}} dx = \int \frac{\asin(ax)^n}{x^2 \sqrt{1-a^2x^2}} dx$$

input `int(asin(a*x)^n/(x^2*(1 - a^2*x^2)^(1/2)),x)`

output `int(asin(a*x)^n/(x^2*(1 - a^2*x^2)^(1/2)), x)`

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\arcsin(ax)^n}{x^2 \sqrt{1 - a^2 x^2}} dx = \int \frac{a \sin(ax)^n}{\sqrt{-a^2 x^2 + 1} x^2} dx$$

input `int(asin(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)`output `int(asin(a*x)**n/(sqrt(-a**2*x**2+1)*x**2),x)`

3.423 $\int x^4(d + ex^2) (a + b \arcsin(cx)) dx$

Optimal result	3594
Mathematica [A] (verified)	3595
Rubi [A] (verified)	3595
Maple [A] (verified)	3597
Fricas [A] (verification not implemented)	3598
Sympy [A] (verification not implemented)	3598
Maxima [A] (verification not implemented)	3599
Giac [B] (verification not implemented)	3599
Mupad [F(-1)]	3601
Reduce [B] (verification not implemented)	3601

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int x^4(d + ex^2) (a + b \arcsin(cx)) dx = \frac{b(7c^2d + 5e) \sqrt{1 - c^2x^2}}{35c^7} - \frac{b(14c^2d + 15e) (1 - c^2x^2)^{3/2}}{105c^7} + \frac{b(7c^2d + 15e) (1 - c^2x^2)^{5/2}}{175c^7} - \frac{be(1 - c^2x^2)^{7/2}}{49c^7} + \frac{1}{5}dx^5(a + b \arcsin(cx)) + \frac{1}{7}ex^7(a + b \arcsin(cx))$$

output

```
1/35*b*(7*c^2*d+5*e)*(-c^2*x^2+1)^(1/2)/c^7-1/105*b*(14*c^2*d+15*e)*(-c^2*x^2+1)^(3/2)/c^7+1/175*b*(7*c^2*d+15*e)*(-c^2*x^2+1)^(5/2)/c^7-1/49*b*e*(-c^2*x^2+1)^(7/2)/c^7+1/5*d*x^5*(a+b*arcsin(c*x))+1/7*e*x^7*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int x^4(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{105ax^5(7d + 5ex^2) + \frac{b\sqrt{1-c^2x^2}(240e+8c^2(49d+15ex^2)+2c^4(98dx^2+45ex^4)+3c^6(49dx^4+25ex^6))}{c^7} + 105bx^5(7d + 5ex^2) \arcsin(cx)}{3675}$$

input

```
Integrate[x^4*(d + e*x^2)*(a + b*ArcSin[c*x]),x]
```

output

```
(105*a*x^5*(7*d + 5*e*x^2) + (b*Sqrt[1 - c^2*x^2]*(240*e + 8*c^2*(49*d + 15*e*x^2) + 2*c^4*(98*d*x^2 + 45*e*x^4) + 3*c^6*(49*d*x^4 + 25*e*x^6)))/c^7 + 105*b*x^5*(7*d + 5*e*x^2)*ArcSin[c*x])/3675
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5230, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d + ex^2)(a + b \arcsin(cx)) dx$$

$$\downarrow 5230$$

$$-bc \int \frac{x^5(5ex^2 + 7d)}{35\sqrt{1-c^2x^2}} dx + \frac{1}{5} dx^5(a + b \arcsin(cx)) + \frac{1}{7} ex^7(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{35} bc \int \frac{x^5(5ex^2 + 7d)}{\sqrt{1-c^2x^2}} dx + \frac{1}{5} dx^5(a + b \arcsin(cx)) + \frac{1}{7} ex^7(a + b \arcsin(cx))$$

$$\downarrow 354$$

$$-\frac{1}{70} bc \int \frac{x^4(5ex^2 + 7d)}{\sqrt{1-c^2x^2}} dx^2 + \frac{1}{5} dx^5(a + b \arcsin(cx)) + \frac{1}{7} ex^7(a + b \arcsin(cx))$$

↓ 86

$$-\frac{1}{70}bc \int \left(-\frac{5e(1-c^2x^2)^{5/2}}{c^6} + \frac{(7dc^2+15e)(1-c^2x^2)^{3/2}}{c^6} + \frac{(-14dc^2-15e)\sqrt{1-c^2x^2}}{c^6} + \frac{7dc^2+5e}{c^6\sqrt{1-c^2x^2}} \right) dx^2$$

$$\frac{1}{5}dx^5(a+b\arcsin(cx)) + \frac{1}{7}ex^7(a+b\arcsin(cx))$$

↓ 2009

$$\frac{1}{70}bc \left(-\frac{2(1-c^2x^2)^{5/2}(7c^2d+15e)}{5c^8} + \frac{2(1-c^2x^2)^{3/2}(14c^2d+15e)}{3c^8} - \frac{2\sqrt{1-c^2x^2}(7c^2d+5e)}{c^8} + \frac{10e(1-c^2x^2)}{7c^8} \right) - \frac{1}{5}dx^5(a+b\arcsin(cx)) + \frac{1}{7}ex^7(a+b\arcsin(cx))$$

input `Int[x^4*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `-1/70*(b*c*((-2*(7*c^2*d + 5*e)*Sqrt[1 - c^2*x^2])/c^8 + (2*(14*c^2*d + 15*e)*(1 - c^2*x^2)^(3/2))/(3*c^8) - (2*(7*c^2*d + 15*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (10*e*(1 - c^2*x^2)^(7/2))/(7*c^8))) + (d*x^5*(a + b*ArcSin[c*x]))/5 + (e*x^7*(a + b*ArcSin[c*x]))/7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 86 `Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

rule 354 `Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5230 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{1}{7}e x^7 + \frac{1}{5}d x^5\right) + \frac{b\left(\frac{c^5 \arcsin(cx) e x^7}{7} + \frac{\arcsin(cx) c^5 x^5 d}{5} - 5e\left(-\frac{c^6 x^6 \sqrt{-c^2 x^2 + 1}}{7} - \frac{6c^4 x^4 \sqrt{-c^2 x^2 + 1}}{35} - \frac{8c^2 x^2 \sqrt{-c^2 x^2 + 1}}{35}\right)\right)}{c^5}$
derivativedivides	$\frac{a\left(\frac{1}{5}d c^7 x^5 + \frac{1}{7}e c^7 x^7\right)}{c^2} + \frac{b\left(\frac{\arcsin(cx) d c^7 x^5}{5} + \frac{\arcsin(cx) e c^7 x^7}{7} - e\left(-\frac{c^6 x^6 \sqrt{-c^2 x^2 + 1}}{7} - \frac{6c^4 x^4 \sqrt{-c^2 x^2 + 1}}{35} - \frac{8c^2 x^2 \sqrt{-c^2 x^2 + 1}}{35}\right)\right)}{c^2}$
default	$\frac{a\left(\frac{1}{5}d c^7 x^5 + \frac{1}{7}e c^7 x^7\right)}{c^2} + \frac{b\left(\frac{\arcsin(cx) d c^7 x^5}{5} + \frac{\arcsin(cx) e c^7 x^7}{7} - e\left(-\frac{c^6 x^6 \sqrt{-c^2 x^2 + 1}}{7} - \frac{6c^4 x^4 \sqrt{-c^2 x^2 + 1}}{35} - \frac{8c^2 x^2 \sqrt{-c^2 x^2 + 1}}{35}\right)\right)}{c^2}$
orering	$\frac{(975c^8e^2x^{10} + 2442c^8dex^8 + 1323c^8d^2x^6 + 90x^8e^2c^6 + 354x^6ec^6d + 196c^6d^2x^4 + 180e^2x^6c^4 + 1296c^4dex^4 + 784c^4d^2x^2 + 3675x(e x^2 + d)c^8)}{3675x(e x^2 + d)c^8}$

```
input int(x^4*(e*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arcsin(c*x)*e*x^7+1/5*arcsin(c*x)*c^5*x^5*d-1/35/c^2*(5*e*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))+7*d*c^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

$$\int x^4(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{525 ac^7 ex^7 + 735 ac^7 dx^5 + 105 (5 bc^7 ex^7 + 7 bc^7 dx^5) \arcsin(cx) + (75 bc^6 ex^6 + 3 (49 bc^6 d + 30 bc^4 e)x^4 + 2 bc^2 d + 4 (49 bc^4 d + 30 bc^2 e)x^2 + 240 b e) \sqrt{-c^2 x^2 + 1}}{3675 c^7}$$

input `integrate(x^4*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`output `1/3675*(525*a*c^7*e*x^7 + 735*a*c^7*d*x^5 + 105*(5*b*c^7*e*x^7 + 7*b*c^7*d*x^5)*arcsin(c*x) + (75*b*c^6*e*x^6 + 3*(49*b*c^6*d + 30*b*c^4*e)*x^4 + 392*b*c^2*d + 4*(49*b*c^4*d + 30*b*c^2*e)*x^2 + 240*b*e)*sqrt(-c^2*x^2 + 1))/c^7`**Sympy [A] (verification not implemented)**

Time = 0.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.47

$$\int x^4(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{adx^5}{5} + \frac{aex^7}{7} + \frac{bdx^5 \arcsin(cx)}{5} + \frac{bex^7 \arcsin(cx)}{7} + \frac{bdx^4 \sqrt{-c^2 x^2 + 1}}{25c} + \frac{bex^6 \sqrt{-c^2 x^2 + 1}}{49c} + \frac{4bdx^2 \sqrt{-c^2 x^2 + 1}}{75c^3} + \frac{6bex^4 \sqrt{-c^2 x^2 + 1}}{245c^3} + \\ a \left(\frac{dx^5}{5} + \frac{ex^7}{7} \right) \end{cases}$$

input `integrate(x**4*(e*x**2+d)*(a+b*asin(c*x)),x)`output `Piecewise((a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*asin(c*x)/5 + b*e*x**7*asin(c*x)/7 + b*d*x**4*sqrt(-c**2*x**2 + 1)/(25*c) + b*e*x**6*sqrt(-c**2*x**2 + 1)/(49*c) + 4*b*d*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) + 6*b*e*x**4*sqrt(-c**2*x**2 + 1)/(245*c**3) + 8*b*d*sqrt(-c**2*x**2 + 1)/(75*c**5) + 8*b*e*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) + 16*b*e*sqrt(-c**2*x**2 + 1)/(245*c**7), Ne(c, 0)), (a*(d*x**5/5 + e*x**7/7), True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.20

$$\int x^4(d + ex^2)(a + b \arcsin(cx)) dx = \frac{1}{7} aex^7 + \frac{1}{5} adx^5 + \frac{1}{75} \left(15x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6} \right) c \right) bd + \frac{1}{245} \left(35x^7 \arcsin(cx) + \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8} \right) c \right) bde$$

input `integrate(x^4*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d + 1/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(132) = 264.

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.08

$$\begin{aligned}
 \int x^4(d+ex^2)(a+b\arcsin(cx))dx &= \frac{1}{7}aex^7 + \frac{1}{5}adx^5 + \frac{(c^2x^2-1)^2bdx\arcsin(cx)}{5c^4} \\
 &+ \frac{2(c^2x^2-1)bdx\arcsin(cx)}{5c^4} \\
 &+ \frac{(c^2x^2-1)^3bex\arcsin(cx)}{7c^6} + \frac{bdx\arcsin(cx)}{5c^4} \\
 &+ \frac{3(c^2x^2-1)^2bex\arcsin(cx)}{7c^6} \\
 &+ \frac{(c^2x^2-1)^2\sqrt{-c^2x^2+1}bd}{25c^5} \\
 &+ \frac{3(c^2x^2-1)bex\arcsin(cx)}{7c^6} - \frac{2(-c^2x^2+1)^{\frac{3}{2}}bd}{15c^5} \\
 &+ \frac{(c^2x^2-1)^3\sqrt{-c^2x^2+1}be}{49c^7} + \frac{bex\arcsin(cx)}{7c^6} \\
 &+ \frac{\sqrt{-c^2x^2+1}bd}{5c^5} + \frac{3(c^2x^2-1)^2\sqrt{-c^2x^2+1}be}{35c^7} \\
 &- \frac{(-c^2x^2+1)^{\frac{3}{2}}be}{7c^7} + \frac{\sqrt{-c^2x^2+1}be}{7c^7}
 \end{aligned}$$

input `integrate(x^4*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/5*(c^2*x^2 - 1)^2*b*d*x*arcsin(c*x)/c^4 + 2/5*(c^2*x^2 - 1)*b*d*x*arcsin(c*x)/c^4 + 1/7*(c^2*x^2 - 1)^3*b*e*x*arcsin(c*x)/c^6 + 1/5*b*d*x*arcsin(c*x)/c^4 + 3/7*(c^2*x^2 - 1)^2*b*e*x*arcsin(c*x)/c^6 + 1/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d/c^5 + 3/7*(c^2*x^2 - 1)*b*e*x*arcsin(c*x)/c^6 - 2/15*(-c^2*x^2 + 1)^(3/2)*b*d/c^5 + 1/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*e/c^7 + 1/7*b*e*x*arcsin(c*x)/c^6 + 1/5*sqrt(-c^2*x^2 + 1)*b*d/c^5 + 3/35*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e/c^7 - 1/7*(-c^2*x^2 + 1)^(3/2)*b*e/c^7 + 1/7*sqrt(-c^2*x^2 + 1)*b*e/c^7`

Mupad [F(-1)]

Timed out.

$$\int x^4 (d + ex^2) (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (ex^2 + d) dx$$

input `int(x^4*(a + b*asin(c*x))*(d + e*x^2),x)`output `int(x^4*(a + b*asin(c*x))*(d + e*x^2), x)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.26

$$\int x^4 (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \frac{735 \operatorname{asin}(cx) b c^7 d x^5 + 525 \operatorname{asin}(cx) b c^7 e x^7 + 147 \sqrt{-c^2 x^2 + 1} b c^6 d x^4 + 75 \sqrt{-c^2 x^2 + 1} b c^6 e x^6 + 196 \sqrt{-c^2 x^2 + 1} b c^5 d x^3 + 120 \sqrt{-c^2 x^2 + 1} b c^5 e x^5 + 392 \sqrt{-c^2 x^2 + 1} b c^4 d x^2 + 90 \sqrt{-c^2 x^2 + 1} b c^4 e x^4 + 120 \sqrt{-c^2 x^2 + 1} b c^3 d x + 240 \sqrt{-c^2 x^2 + 1} b c^3 e x^3 + 735 a c^7 d x^5 + 525 a c^7 e x^7}{3675 c^7}$$

input `int(x^4*(e*x^2+d)*(a+b*asin(c*x)),x)`output `(735*asin(c*x)*b*c**7*d*x**5 + 525*asin(c*x)*b*c**7*e*x**7 + 147*sqrt(-c**2*x**2 + 1)*b*c**6*d*x**4 + 75*sqrt(-c**2*x**2 + 1)*b*c**6*e*x**6 + 196*sqrt(-c**2*x**2 + 1)*b*c**4*d*x**2 + 90*sqrt(-c**2*x**2 + 1)*b*c**4*e*x**4 + 392*sqrt(-c**2*x**2 + 1)*b*c**2*d + 120*sqrt(-c**2*x**2 + 1)*b*c**2*e*x**2 + 240*sqrt(-c**2*x**2 + 1)*b*e + 735*a*c**7*d*x**5 + 525*a*c**7*e*x**7)/(3675*c**7)`

3.424 $\int x^3(d + ex^2) (a + b \arcsin(cx)) dx$

Optimal result	3602
Mathematica [A] (verified)	3603
Rubi [A] (verified)	3603
Maple [A] (verified)	3606
Fricas [A] (verification not implemented)	3606
Sympy [A] (verification not implemented)	3607
Maxima [A] (verification not implemented)	3607
Giac [A] (verification not implemented)	3608
Mupad [F(-1)]	3609
Reduce [B] (verification not implemented)	3609

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int x^3(d + ex^2) (a + b \arcsin(cx)) dx = \frac{b(9c^2d + 5e) x \sqrt{1 - c^2x^2}}{96c^5} + \frac{b(9c^2d + 5e) x^3 \sqrt{1 - c^2x^2}}{144c^3} + \frac{bex^5 \sqrt{1 - c^2x^2}}{36c} - \frac{b(9c^2d + 5e) \arcsin(cx)}{96c^6} + \frac{1}{4}dx^4(a + b \arcsin(cx)) + \frac{1}{6}ex^6(a + b \arcsin(cx))$$

```
output 1/96*b*(9*c^2*d+5*e)*x*(-c^2*x^2+1)^(1/2)/c^5+1/144*b*(9*c^2*d+5*e)*x^3*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*e*x^5*(-c^2*x^2+1)^(1/2)/c-1/96*b*(9*c^2*d+5*e)*arcsin(c*x)/c^6+1/4*d*x^4*(a+b*arcsin(c*x))+1/6*e*x^6*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{24ac^6x^4(3d + 2ex^2) + bcx\sqrt{1 - c^2x^2}(15e + c^2(27d + 10ex^2) + 2c^4(9dx^2 + 4ex^4)) + 3b(-9c^2d - 5e + 8c^6(3d^2x^4 + 2ex^6)) \operatorname{ArcSin}[cx]}{288c^6}$$

input

```
Integrate[x^3*(d + e*x^2)*(a + b*ArcSin[c*x]),x]
```

output

```
(24*a*c^6*x^4*(3*d + 2*e*x^2) + b*c*x*Sqrt[1 - c^2*x^2]*(15*e + c^2*(27*d + 10*e*x^2) + 2*c^4*(9*d*x^2 + 4*e*x^4)) + 3*b*(-9*c^2*d - 5*e + 8*c^6*(3*d*x^4 + 2*e*x^6))*ArcSin[c*x])/(288*c^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5230, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx$$

$$\downarrow \text{5230}$$

$$-bc \int \frac{x^4(2ex^2 + 3d)}{12\sqrt{1 - c^2x^2}} dx + \frac{1}{4} dx^4(a + b \arcsin(cx)) + \frac{1}{6} ex^6(a + b \arcsin(cx))$$

$$\downarrow \text{27}$$

$$-\frac{1}{12} bc \int \frac{x^4(2ex^2 + 3d)}{\sqrt{1 - c^2x^2}} dx + \frac{1}{4} dx^4(a + b \arcsin(cx)) + \frac{1}{6} ex^6(a + b \arcsin(cx))$$

$$\downarrow \text{363}$$

$$\begin{aligned}
& -\frac{1}{12}bc \left(\frac{1}{3} \left(\frac{5e}{c^2} + 9d \right) \int \frac{x^4}{\sqrt{1-c^2x^2}} dx - \frac{ex^5\sqrt{1-c^2x^2}}{3c^2} \right) + \frac{1}{4}dx^4(a + b \arcsin(cx)) + \frac{1}{6}ex^6(a + \\
& \qquad \qquad \qquad b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow \text{262} \\
& -\frac{1}{12}bc \left(\frac{1}{3} \left(\frac{5e}{c^2} + 9d \right) \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) - \frac{ex^5\sqrt{1-c^2x^2}}{3c^2} \right) + \frac{1}{4}dx^4(a + \\
& \qquad \qquad \qquad b \arcsin(cx)) + \frac{1}{6}ex^6(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow \text{262} \\
& -\frac{1}{12}bc \left(\frac{1}{3} \left(\frac{5e}{c^2} + 9d \right) \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) - \frac{ex^5\sqrt{1-c^2x^2}}{3c^2} \right) + \\
& \qquad \qquad \qquad \frac{1}{4}dx^4(a + b \arcsin(cx)) + \frac{1}{6}ex^6(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow \text{223} \\
& \qquad \qquad \qquad \frac{1}{4}dx^4(a + b \arcsin(cx)) + \frac{1}{6}ex^6(a + b \arcsin(cx)) - \\
& \frac{1}{12}bc \left(\frac{1}{3} \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \left(\frac{5e}{c^2} + 9d \right) - \frac{ex^5\sqrt{1-c^2x^2}}{3c^2} \right)
\end{aligned}$$

input `Int[x^3*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `(d*x^4*(a + b*ArcSin[c*x]))/4 + (e*x^6*(a + b*ArcSin[c*x]))/6 - (b*c*(-1/3*(e*x^5*Sqrt[1 - c^2*x^2])/c^2 + ((9*d + (5*e)/c^2)*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3))))/(4*c^2)))/3)/12`

Defintions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 223 $\text{Int}[1/\text{Sqrt}[(a_*) + (b_*)(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$
- rule 262 $\text{Int}[((c_*)(x_))^{(m_*)}((a_*) + (b_*)(x_)^2)^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}((a + b*x^2)^{(p+1})/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \text{ Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$
- rule 363 $\text{Int}[((e_*)(x_))^{(m_*)}((a_*) + (b_*)(x_)^2)^{(p_*)}((c_*) + (d_*)(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*(e*x)^{(m+1)}((a + b*x^2)^{(p+1})/(b*e*(m+2*p+3))), x] - \text{Simp}[(a*d*(m+1) - b*c*(m+2*p+3))/(b*(m+2*p+3)) \text{ Int}[(e*x)^m*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m+2*p+3, 0]$
- rule 5230 $\text{Int}[(a_*) + \text{ArcSin}[(c_*)(x_)]*(b_*)*((f_*)(x_))^{(m_*)}((d_*) + (e_*)(x_)^2)^{(p_*)}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) u, x] - \text{Simp}[b*c \text{ Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (\text{GtQ}[p, 0] \ || \ (\text{IGtQ}[(m-1)/2, 0] \ \&\& \ \text{LeQ}[m+p, 0]))$

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

method	result
parts	$a\left(\frac{1}{6}e x^6 + \frac{1}{4}d x^4\right) + \frac{b\left(\frac{c^4 \arcsin(cx) e x^6}{6} + \frac{\arcsin(cx) c^4 x^4 d}{4} - 2e\left(-\frac{c^5 x^5 \sqrt{-c^2 x^2 + 1}}{6} - \frac{5c^3 x^3 \sqrt{-c^2 x^2 + 1}}{24} - \frac{5cx \sqrt{-c^2 x^2 + 1}}{16} + 5\right)\right)}{c^4}$
derivativdivides	$\frac{a\left(\frac{1}{4}x^4 d c^6 + \frac{1}{6}c^6 e x^6\right)}{c^2} + \frac{b\left(\frac{\arcsin(cx) d c^6 x^4}{4} + \frac{\arcsin(cx) e c^6 x^6}{6} - e\left(-\frac{c^5 x^5 \sqrt{-c^2 x^2 + 1}}{6} - \frac{5c^3 x^3 \sqrt{-c^2 x^2 + 1}}{24} - \frac{5cx \sqrt{-c^2 x^2 + 1}}{16} + 5\right)\right)}{c^2}$
default	$\frac{a\left(\frac{1}{4}x^4 d c^6 + \frac{1}{6}c^6 e x^6\right)}{c^2} + \frac{b\left(\frac{\arcsin(cx) d c^6 x^4}{4} + \frac{\arcsin(cx) e c^6 x^6}{6} - e\left(-\frac{c^5 x^5 \sqrt{-c^2 x^2 + 1}}{6} - \frac{5c^3 x^3 \sqrt{-c^2 x^2 + 1}}{24} - \frac{5cx \sqrt{-c^2 x^2 + 1}}{16} + 5\right)\right)}{c^2}$
oring	$\frac{(88x^8 e^2 c^6 + 234x^6 e c^6 d + 126c^6 d^2 x^4 + 10e^2 x^6 c^4 + 51c^4 d e x^4 + 27c^4 d^2 x^2 + 25c^2 e^2 x^4 - 147c^2 d e x^2 - 108c^2 d^2 - 90e^2 x^2 - 60a)}{288(e x^2 + d)c^6}$

input `int(x^3*(e*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arcsin(c*x)*e*x^6+1/4*arcsin(c*x)*c^4*x^4*d-1/12/c^2*(2*e*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))+3*d*c^2*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x)))`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int x^3 (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \frac{48 ac^6 ex^6 + 72 ac^6 dx^4 + 3(16 bc^6 ex^6 + 24 bc^6 dx^4 - 9 bc^2 d - 5 be) \arcsin(cx) + (8 bc^5 ex^5 + 2(9 bc^5 d + 5 b^2))}{288 c^6}$$

input `integrate(x^3*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output

$$\frac{1}{288}(48ac^6e^x^6 + 72ac^6d^2x^4 + 3(16b^2c^6e^x^6 + 24b^2c^6d^2x^4 - 9b^2c^2d - 5b^2e)\arcsin(cx) + (8b^2c^5e^x^5 + 2(9b^2c^5d + 5b^2c^3e)x^3 + 3(9b^2c^3d + 5b^2ce)x)\sqrt{-c^2x^2 + 1})/c^6$$

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.38

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4 \arcsin(cx)}{4} + \frac{bex^6 \arcsin(cx)}{6} + \frac{bdx^3 \sqrt{-c^2x^2+1}}{16c} + \frac{bex^5 \sqrt{-c^2x^2+1}}{36c} + \frac{3bdx \sqrt{-c^2x^2+1}}{32c^3} + \frac{5bex^3 \sqrt{-c^2x^2+1}}{144c^3} - \\ a\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) \end{cases}$$

input

```
integrate(x**3*(e*x**2+d)*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asin(c*x)/4 + b*e*x**6*asin(c*x)/6 + b*d*x**3*sqrt(-c**2*x**2 + 1)/(16*c) + b*e*x**5*sqrt(-c**2*x**2 + 1)/(36*c) + 3*b*d*x*sqrt(-c**2*x**2 + 1)/(32*c**3) + 5*b*e*x**3*sqrt(-c**2*x**2 + 1)/(144*c**3) - 3*b*d*asin(c*x)/(32*c**4) + 5*b*e*x*sqrt(-c**2*x**2 + 1)/(96*c**5) - 5*b*e*asin(c*x)/(96*c**6), Ne(c, 0)), (a*(d*x**4/4 + e*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx = \frac{1}{6} aex^6 + \frac{1}{4} adx^4$$

$$+ \frac{1}{32} \left(8x^4 \arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5} \right) c \right) bd$$

$$+ \frac{1}{288} \left(48x^6 \arcsin(cx) + \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7} \right) c \right) bd$$

input

```
integrate(x^3*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)
)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d + 1/288
*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 +
1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.70

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx = \frac{1}{6} aex^6 + \frac{1}{4} adx^4 - \frac{(-c^2x^2 + 1)^{\frac{3}{2}} bdx}{16c^3} + \frac{(c^2x^2 - 1)^2 bd \arcsin(cx)}{4c^4} + \frac{5\sqrt{-c^2x^2 + 1} bdx}{32c^3} + \frac{(c^2x^2 - 1)^2 \sqrt{-c^2x^2 + 1} bex}{36c^5} + \frac{(c^2x^2 - 1) bd \arcsin(cx)}{2c^4} + \frac{(c^2x^2 - 1)^3 be \arcsin(cx)}{6c^6} - \frac{13(-c^2x^2 + 1)^{\frac{3}{2}} bex}{144c^5} + \frac{5 bd \arcsin(cx)}{32c^4} + \frac{(c^2x^2 - 1)^2 be \arcsin(cx)}{2c^6} + \frac{11\sqrt{-c^2x^2 + 1} bex}{96c^5} + \frac{(c^2x^2 - 1) be \arcsin(cx)}{2c^6} + \frac{11 be \arcsin(cx)}{96c^6}$$

input

```
integrate(x^3*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
1/6*a*e*x^6 + 1/4*a*d*x^4 - 1/16*(-c^2*x^2 + 1)^(3/2)*b*d*x/c^3 + 1/4*(c^2
*x^2 - 1)^2*b*d*arcsin(c*x)/c^4 + 5/32*sqrt(-c^2*x^2 + 1)*b*d*x/c^3 + 1/36
*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e*x/c^5 + 1/2*(c^2*x^2 - 1)*b*d*arcs
in(c*x)/c^4 + 1/6*(c^2*x^2 - 1)^3*b*e*arcsin(c*x)/c^6 - 13/144*(-c^2*x^2 +
1)^(3/2)*b*e*x/c^5 + 5/32*b*d*arcsin(c*x)/c^4 + 1/2*(c^2*x^2 - 1)^2*b*e*a
rcsin(c*x)/c^6 + 11/96*sqrt(-c^2*x^2 + 1)*b*e*x/c^5 + 1/2*(c^2*x^2 - 1)*b*
e*arcsin(c*x)/c^6 + 11/96*b*e*arcsin(c*x)/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx = \int x^3(a + b \arcsin(cx))(ex^2 + d) dx$$

input `int(x^3*(a + b*asin(c*x))*(d + e*x^2),x)`output `int(x^3*(a + b*asin(c*x))*(d + e*x^2), x)`**Reduce [B] (verification not implemented)**

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15

$$\int x^3(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{72 \arcsin(cx) b c^6 d x^4 + 48 \arcsin(cx) b c^6 e x^6 - 27 \arcsin(cx) b c^2 d - 15 \arcsin(cx) b e + 18 \sqrt{-c^2 x^2 + 1} b c^5 d x^3 + \dots}{(288 c^6)}$$

input `int(x^3*(e*x^2+d)*(a+b*asin(c*x)),x)`output `(72*asin(c*x)*b*c**6*d*x**4 + 48*asin(c*x)*b*c**6*e*x**6 - 27*asin(c*x)*b*c**2*d - 15*asin(c*x)*b*e + 18*sqrt(-c**2*x**2 + 1)*b*c**5*d*x**3 + 8*sqrt(-c**2*x**2 + 1)*b*c**5*e*x**5 + 27*sqrt(-c**2*x**2 + 1)*b*c**3*d*x + 10*sqrt(-c**2*x**2 + 1)*b*c**3*e*x**3 + 15*sqrt(-c**2*x**2 + 1)*b*c*e*x + 72*a*c**6*d*x**4 + 48*a*c**6*e*x**6)/(288*c**6)`

3.425 $\int x^2(d + ex^2) (a + b \arcsin(cx)) dx$

Optimal result	3610
Mathematica [A] (verified)	3610
Rubi [A] (verified)	3611
Maple [A] (verified)	3613
Fricas [A] (verification not implemented)	3614
Sympy [A] (verification not implemented)	3614
Maxima [A] (verification not implemented)	3615
Giac [B] (verification not implemented)	3615
Mupad [F(-1)]	3616
Reduce [B] (verification not implemented)	3616

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int x^2(d + ex^2) (a + b \arcsin(cx)) dx = \frac{b(5c^2d + 3e) \sqrt{1 - c^2x^2}}{15c^5} - \frac{b(5c^2d + 6e) (1 - c^2x^2)^{3/2}}{45c^5} + \frac{be(1 - c^2x^2)^{5/2}}{25c^5} + \frac{1}{3}dx^3(a + b \arcsin(cx)) + \frac{1}{5}ex^5(a + b \arcsin(cx))$$

output

```
1/15*b*(5*c^2*d+3*e)*(-c^2*x^2+1)^(1/2)/c^5-1/45*b*(5*c^2*d+6*e)*(-c^2*x^2+1)^(3/2)/c^5+1/25*b*e*(-c^2*x^2+1)^(5/2)/c^5+1/3*d*x^3*(a+b*arcsin(c*x))+1/5*e*x^5*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int x^2(d + ex^2) (a + b \arcsin(cx)) dx = \frac{1}{225} \left(15ax^3(5d + 3ex^2) + \frac{b\sqrt{1 - c^2x^2}(24e + 2c^2(25d + 6ex^2) + c^4(25dx^2 + 9ex^4))}{c^5} + 15bx^3(5d + 3ex^2) \arcsin(cx) \right)$$

input `Integrate[x^2*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `(15*a*x^3*(5*d + 3*e*x^2) + (b*sqrt[1 - c^2*x^2]*(24*e + 2*c^2*(25*d + 6*e*x^2) + c^4*(25*d*x^2 + 9*e*x^4)))/c^5 + 15*b*x^3*(5*d + 3*e*x^2)*ArcSin[c*x])/225`

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5230, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2(d + ex^2)(a + b \arcsin(cx)) dx \\
 & \quad \downarrow \text{5230} \\
 & -bc \int \frac{x^3(3ex^2 + 5d)}{15\sqrt{1 - c^2x^2}} dx + \frac{1}{3} dx^3(a + b \arcsin(cx)) + \frac{1}{5} ex^5(a + b \arcsin(cx)) \\
 & \quad \downarrow \text{27} \\
 & -\frac{1}{15} bc \int \frac{x^3(3ex^2 + 5d)}{\sqrt{1 - c^2x^2}} dx + \frac{1}{3} dx^3(a + b \arcsin(cx)) + \frac{1}{5} ex^5(a + b \arcsin(cx)) \\
 & \quad \downarrow \text{354} \\
 & -\frac{1}{30} bc \int \frac{x^2(3ex^2 + 5d)}{\sqrt{1 - c^2x^2}} dx^2 + \frac{1}{3} dx^3(a + b \arcsin(cx)) + \frac{1}{5} ex^5(a + b \arcsin(cx)) \\
 & \quad \downarrow \text{86} \\
 & -\frac{1}{30} bc \int \left(\frac{3e(1 - c^2x^2)^{3/2}}{c^4} + \frac{(-5dc^2 - 6e)\sqrt{1 - c^2x^2}}{c^4} + \frac{5dc^2 + 3e}{c^4\sqrt{1 - c^2x^2}} \right) dx^2 + \frac{1}{3} dx^3(a + \\
 & \quad b \arcsin(cx)) + \frac{1}{5} ex^5(a + b \arcsin(cx)) \\
 & \quad \downarrow \text{2009}
 \end{aligned}$$

$$\frac{1}{3}dx^3(a + b \arcsin(cx)) + \frac{1}{5}ex^5(a + b \arcsin(cx)) - \frac{1}{30}bc \left(\frac{2(1 - c^2x^2)^{3/2}(5c^2d + 6e)}{3c^6} - \frac{2\sqrt{1 - c^2x^2}(5c^2d + 3e)}{c^6} - \frac{6e(1 - c^2x^2)^{5/2}}{5c^6} \right)$$

input `Int[x^2*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `-1/30*(b*c*((-2*(5*c^2*d + 3*e)*Sqrt[1 - c^2*x^2])/c^6 + (2*(5*c^2*d + 6*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) - (6*e*(1 - c^2*x^2)^(5/2))/(5*c^6))) + (d*x^3*(a + b*ArcSin[c*x]))/3 + (e*x^5*(a + b*ArcSin[c*x]))/5`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 86 `Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5230

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{1}{5}e x^5 + \frac{1}{3}x^3 d\right) + \frac{b\left(\frac{c^3 \arcsin(cx) e x^5}{5} + \frac{\arcsin(cx) c^3 x^3 d}{3} - \frac{3e\left(-\frac{c^4 x^4 \sqrt{-c^2 x^2 + 1}}{5} - \frac{4c^2 x^2 \sqrt{-c^2 x^2 + 1}}{15} - \frac{8\sqrt{-c^2 x^2 + 1}}{15}\right)}{15c^2}\right)}{c^3}$
derivativedivides	$\frac{a\left(\frac{1}{3}d c^5 x^3 + \frac{1}{5}e c^5 x^5\right)}{c^2} + \frac{b\left(\frac{\arcsin(cx) d c^5 x^3}{3} + \frac{\arcsin(cx) e c^5 x^5}{5} - \frac{e\left(-\frac{c^4 x^4 \sqrt{-c^2 x^2 + 1}}{5} - \frac{4c^2 x^2 \sqrt{-c^2 x^2 + 1}}{15} - \frac{8\sqrt{-c^2 x^2 + 1}}{15}\right)}{5}\right)}{c^2}$
default	$\frac{a\left(\frac{1}{3}d c^5 x^3 + \frac{1}{5}e c^5 x^5\right)}{c^2} + \frac{b\left(\frac{\arcsin(cx) d c^5 x^3}{3} + \frac{\arcsin(cx) e c^5 x^5}{5} - \frac{e\left(-\frac{c^4 x^4 \sqrt{-c^2 x^2 + 1}}{5} - \frac{4c^2 x^2 \sqrt{-c^2 x^2 + 1}}{15} - \frac{8\sqrt{-c^2 x^2 + 1}}{15}\right)}{5}\right)}{c^2}$
orering	$\frac{(81x^8 e^2 c^6 + 238x^6 e c^6 d + 125c^6 d^2 x^4 + 12e^2 x^6 c^4 + 106c^4 d e x^4 + 50c^4 d^2 x^2 + 48c^2 e^2 x^4 - 176c^2 d e x^2 - 100c^2 d^2 - 96e^2 x^2 - 48d^2)}{225x(e x^2 + d)c^6}$

```
input int(x^2*(e*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/5*e*x^5+1/3*x^3*d)+b/c^3*(1/5*c^3*arcsin(c*x)*e*x^5+1/3*arcsin(c*x)*c^3*x^3*d-1/15/c^2*(3*e*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+5*d*c^2*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.89

$$\int x^2(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{45 ac^5 ex^5 + 75 ac^5 dx^3 + 15(3 bc^5 ex^5 + 5 bc^5 dx^3) \arcsin(cx) + (9 bc^4 ex^4 + 50 bc^2 d + (25 bc^4 d + 12 bc^2 e)x^2 + 24 b^2 e) \sqrt{-c^2 x^2 + 1}}{225 c^5}$$

input `integrate(x^2*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`output `1/225*(45*a*c^5*e*x^5 + 75*a*c^5*d*x^3 + 15*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3)*arcsin(c*x) + (9*b*c^4*e*x^4 + 50*b*c^2*d + (25*b*c^4*d + 12*b*c^2*e)*x^2 + 24*b^2*e)*sqrt(-c^2*x^2 + 1))/c^5`**Sympy [A] (verification not implemented)**

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.43

$$\int x^2(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{adx^3}{3} + \frac{aex^5}{5} + \frac{bdx^3 \arcsin(cx)}{3} + \frac{bex^5 \arcsin(cx)}{5} + \frac{bdx^2 \sqrt{-c^2 x^2 + 1}}{9c} + \frac{bex^4 \sqrt{-c^2 x^2 + 1}}{25c} + \frac{2bd \sqrt{-c^2 x^2 + 1}}{9c^3} + \frac{4bex^2 \sqrt{-c^2 x^2 + 1}}{75c^3} + 8b^2 e \sqrt{-c^2 x^2 + 1} \\ a \left(\frac{dx^3}{3} + \frac{ex^5}{5} \right) \end{cases}$$

input `integrate(x**2*(e*x**2+d)*(a+b*asin(c*x)),x)`output `Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*asin(c*x)/3 + b*e*x**5*asin(c*x)/5 + b*d*x**2*sqrt(-c**2*x**2 + 1)/(9*c) + b*e*x**4*sqrt(-c**2*x**2 + 1)/(25*c) + 2*b*d*sqrt(-c**2*x**2 + 1)/(9*c**3) + 4*b*e*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) + 8*b*e*sqrt(-c**2*x**2 + 1)/(75*c**5), Ne(c, 0)), (a*(d*x**3/3 + e*x**5/5), True))`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int x^2(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{1}{5} aex^5 + \frac{1}{3} adx^3 + \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2 + 1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2 + 1}}{c^4} \right) \right) bd$$

$$+ \frac{1}{75} \left(15x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2x^2 + 1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2 + 1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2 + 1}}{c^6} \right) c \right) be$$

input `integrate(x^2*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(104) = 208.

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

$$\int x^2(d + ex^2)(a + b \arcsin(cx)) dx = \frac{1}{5} aex^5 + \frac{1}{3} adx^3 + \frac{(c^2x^2 - 1)bdx \arcsin(cx)}{3c^2}$$

$$+ \frac{bdx \arcsin(cx)}{3c^2} + \frac{(c^2x^2 - 1)^2 bex \arcsin(cx)}{5c^4}$$

$$+ \frac{2(c^2x^2 - 1)bex \arcsin(cx)}{5c^4}$$

$$- \frac{(-c^2x^2 + 1)^{\frac{3}{2}}bd}{9c^3} + \frac{bex \arcsin(cx)}{5c^4}$$

$$+ \frac{\sqrt{-c^2x^2 + 1}bd}{3c^3} + \frac{(c^2x^2 - 1)^2 \sqrt{-c^2x^2 + 1}be}{25c^5}$$

$$- \frac{2(-c^2x^2 + 1)^{\frac{3}{2}}be}{15c^5} + \frac{\sqrt{-c^2x^2 + 1}be}{5c^5}$$

input `integrate(x^2*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output
$$\frac{1}{5}aex^5 + \frac{1}{3}ad^3x^3 + \frac{1}{3}(c^2x^2 - 1)b^2d^2x^2 \arcsin(cx)/c^2 + \frac{1}{3}b^2d^2x^2 \arcsin(cx)/c^2 + \frac{1}{5}(c^2x^2 - 1)^2 b^2ex^2 \arcsin(cx)/c^4 + \frac{2}{5}(c^2x^2 - 1)b^2ex^2 \arcsin(cx)/c^4 - \frac{1}{9}(-c^2x^2 + 1)^{3/2} b^2d/c^3 + \frac{1}{5}b^2ex^2 \arcsin(cx)/c^4 + \frac{1}{3}\sqrt{-c^2x^2 + 1} b^2d/c^3 + \frac{1}{25}(c^2x^2 - 1)^2 \sqrt{-c^2x^2 + 1} b^2e/c^5 - \frac{2}{15}(-c^2x^2 + 1)^{3/2} b^2e/c^5 + \frac{1}{5}\sqrt{-c^2x^2 + 1} b^2e/c^5$$

Mupad [F(-1)]

Timed out.

$$\int x^2(d + ex^2)(a + b \arcsin(cx)) dx = \int x^2(a + b \operatorname{asin}(cx))(ex^2 + d) dx$$

input `int(x^2*(a + b*asin(c*x))*(d + e*x^2),x)`

output `int(x^2*(a + b*asin(c*x))*(d + e*x^2), x)`

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int x^2(d + ex^2)(a + b \arcsin(cx)) dx = \frac{75 \operatorname{asin}(cx) b c^5 d x^3 + 45 \operatorname{asin}(cx) b c^5 e x^5 + 25 \sqrt{-c^2 x^2 + 1} b c^4 d x^2 + 9 \sqrt{-c^2 x^2 + 1} b c^4 e x^4 + 50 \sqrt{-c^2 x^2 + 1} b c^4 d x^2}{225 c^5}$$

input `int(x^2*(e*x^2+d)*(a+b*asin(c*x)),x)`

output

```
(75*asin(c*x)*b*c**5*d*x**3 + 45*asin(c*x)*b*c**5*e*x**5 + 25*sqrt(-c**2*x**2 + 1)*b*c**4*d*x**2 + 9*sqrt(-c**2*x**2 + 1)*b*c**4*e*x**4 + 50*sqrt(-c**2*x**2 + 1)*b*c**2*d + 12*sqrt(-c**2*x**2 + 1)*b*c**2*e*x**2 + 24*sqrt(-c**2*x**2 + 1)*b*e + 75*a*c**5*d*x**3 + 45*a*c**5*e*x**5)/(225*c**5)
```

3.426 $\int x(d + ex^2) (a + b \arcsin(cx)) dx$

Optimal result	3618
Mathematica [A] (verified)	3618
Rubi [A] (verified)	3619
Maple [A] (verified)	3621
Fricas [A] (verification not implemented)	3622
Sympy [A] (verification not implemented)	3622
Maxima [A] (verification not implemented)	3623
Giac [A] (verification not implemented)	3623
Mupad [F(-1)]	3624
Reduce [B] (verification not implemented)	3624

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x(d + ex^2) (a + b \arcsin(cx)) dx = \frac{b(8c^2d + 3e)x\sqrt{1 - c^2x^2}}{32c^3} + \frac{bex^3\sqrt{1 - c^2x^2}}{16c} - \frac{b(8c^4d^2 + 8c^2de + 3e^2)\arcsin(cx)}{32c^4e} + \frac{(d + ex^2)^2(a + b \arcsin(cx))}{4e}$$

output

```
1/32*b*(8*c^2*d+3*e)*x*(-c^2*x^2+1)^(1/2)/c^3+1/16*b*e*x^3*(-c^2*x^2+1)^(1/2)/c-1/32*b*(8*c^4*d^2+8*c^2*d*e+3*e^2)*arcsin(c*x)/c^4/e+1/4*(e*x^2+d)^2*(a+b*arcsin(c*x))/e
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int x(d + ex^2) (a + b \arcsin(cx)) dx = \frac{cx(8ac^3x(2d + ex^2) + b\sqrt{1 - c^2x^2}(3e + 2c^2(4d + ex^2))) + b(-8c^2d - 3e + 8c^4(2dx^2 + ex^4)) \arcsin(cx)}{32c^4}$$

input `Integrate[x*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `(c*x*(8*a*c^3*x*(2*d + e*x^2) + b*sqrt[1 - c^2*x^2]*(3*e + 2*c^2*(4*d + e*x^2))) + b*(-8*c^2*d - 3*e + 8*c^4*(2*d*x^2 + e*x^4))*ArcSin[c*x])/(32*c^4)`

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5228, 318, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x(d + ex^2)(a + b \arcsin(cx)) dx \\
 & \quad \downarrow 5228 \\
 & \frac{(d + ex^2)^2(a + b \arcsin(cx))}{4e} - \frac{bc \int \frac{(ex^2+d)^2}{\sqrt{1-c^2x^2}} dx}{4e} \\
 & \quad \downarrow 318 \\
 & \frac{(d + ex^2)^2(a + b \arcsin(cx))}{4e} - \frac{bc \left(-\int \frac{3e(2dc^2+e)x^2+d(4dc^2+e)}{4c^2\sqrt{1-c^2x^2}} dx - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2} \right)}{4e} \\
 & \quad \downarrow 25 \\
 & \frac{(d + ex^2)^2(a + b \arcsin(cx))}{4e} - \frac{bc \left(\int \frac{3e(2dc^2+e)x^2+d(4dc^2+e)}{4c^2\sqrt{1-c^2x^2}} dx - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2} \right)}{4e} \\
 & \quad \downarrow 299
 \end{aligned}$$

$$\frac{(d + ex^2)^2 (a + b \arcsin(cx))}{4e} - \frac{bc \left(\frac{(8c^4d^2 + 8c^2de + 3e^2) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{2e^2} - \frac{3ex\sqrt{1-c^2x^2}(2c^2d+e)}{2c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2} \right)}{4e}$$

↓ 223

$$\frac{(d + ex^2)^2 (a + b \arcsin(cx))}{4e} - \frac{bc \left(\frac{\arcsin(cx)(8c^4d^2 + 8c^2de + 3e^2)}{2c^3} - \frac{3ex\sqrt{1-c^2x^2}(2c^2d+e)}{2c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2} \right)}{4e}$$

input `Int[x*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `((d + e*x^2)^2*(a + b*ArcSin[c*x]))/(4*e) - (b*c*(-1/4*(e*x*Sqrt[1 - c^2*x^2]*(d + e*x^2))/c^2 + ((-3*e*(2*c^2*d + e)*x*Sqrt[1 - c^2*x^2])/(2*c^2) + ((8*c^4*d^2 + 8*c^2*d*e + 3*e^2)*ArcSin[c*x])/(2*c^3))/(4*c^2))/(4*e)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 299 `Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*x*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2*p + 3)) Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ[2*p + 3, 0]`

rule 318

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] := Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1)) Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
1))*x^2, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 5228

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
- Simp[b*(c/(2*e*(p + 1))) Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

method	result
parts	$\frac{a(e x^2+d)^2}{4e} + \frac{b \left(\frac{c^2 e \arcsin(cx) x^4}{4} + \frac{\arcsin(cx) c^2 x^2 d}{2} + \frac{c^2 \arcsin(cx) d^2}{4e} - \frac{c^4 d^2 \arcsin(cx) + e^2 \left(-\frac{c^3 x^3 \sqrt{-c^2 x^2 + 1}}{4} - \frac{3cx \sqrt{-c^2 x^2 + 1}}{8} \right)}{c^2} \right)}{c^2}$
derivativedivides	$\frac{a(c^2 e x^2 + c^2 d)^2}{4c^2 e} + \frac{b \left(\frac{\arcsin(cx) c^4 d^2}{4e} + \frac{\arcsin(cx) c^4 d x^2}{2} + \frac{e \arcsin(cx) c^4 x^4}{4} - \frac{c^4 d^2 \arcsin(cx) + e^2 \left(-\frac{c^3 x^3 \sqrt{-c^2 x^2 + 1}}{4} - \frac{3cx \sqrt{-c^2 x^2 + 1}}{8} \right)}{c^2} \right)}{c^2}$
default	$\frac{a(c^2 e x^2 + c^2 d)^2}{4c^2 e} + \frac{b \left(\frac{\arcsin(cx) c^4 d^2}{4e} + \frac{\arcsin(cx) c^4 d x^2}{2} + \frac{e \arcsin(cx) c^4 x^4}{4} - \frac{c^4 d^2 \arcsin(cx) + e^2 \left(-\frac{c^3 x^3 \sqrt{-c^2 x^2 + 1}}{4} - \frac{3cx \sqrt{-c^2 x^2 + 1}}{8} \right)}{c^2} \right)}{c^2}$
orering	$\frac{(14e^2 x^6 c^4 + 50c^4 d e x^4 + 24c^4 d^2 x^2 + 3c^2 e^2 x^4 - 31c^2 d e x^2 - 16c^2 d^2 - 12e^2 x^2 - 6d e)(a + b \arcsin(cx))}{32(e x^2 + d) c^4} - \frac{(2c^2 e x^2 + 8c^2 d + \dots)}{\dots}$

input

```
int(x*(e*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/4*a*(e*x^2+d)^2/e+b/c^2*(1/4*c^2*e*arcsin(c*x)*x^4+1/2*arcsin(c*x)*c^2*x^2*d+1/4*c^2/e*arcsin(c*x)*d^2-1/4/c^2/e*(c^4*d^2*arcsin(c*x)+e^2*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x))+2*d*c^2*e*(-1/2*c*x*(-c^2*x^2+1)^(1/2)+1/2*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int x(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{8ac^4ex^4 + 16ac^4dx^2 + (8bc^4ex^4 + 16bc^4dx^2 - 8bc^2d - 3be) \arcsin(cx) + (2bc^3ex^3 + (8bc^3d + 3bce)x)}{32c^4}$$

input

```
integrate(x*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
1/32*(8*a*c^4*e*x^4 + 16*a*c^4*d*x^2 + (8*b*c^4*e*x^4 + 16*b*c^4*d*x^2 - 8*b*c^2*d - 3*b*e)*arcsin(c*x) + (2*b*c^3*e*x^3 + (8*b*c^3*d + 3*b*c*e)*x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int x(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{adx^2}{2} + \frac{aex^4}{4} + \frac{bdx^2 \arcsin(cx)}{2} + \frac{bex^4 \arcsin(cx)}{4} + \frac{bdx\sqrt{-c^2x^2+1}}{4c} + \frac{bex^3\sqrt{-c^2x^2+1}}{16c} - \frac{bd \arcsin(cx)}{4c^2} + \frac{3bex\sqrt{-c^2x^2+1}}{32c^3} - \frac{3be \arcsin(cx)}{32c^4} \\ a\left(\frac{dx^2}{2} + \frac{ex^4}{4}\right) \end{cases}$$

input

```
integrate(x*(e*x**2+d)*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asin(c*x)/2 + b*e*x**4*asin(c*x)/4 + b*d*x*sqrt(-c**2*x**2 + 1)/(4*c) + b*e*x**3*sqrt(-c**2*x**2 + 1)/(16*c) - b*d*asin(c*x)/(4*c**2) + 3*b*e*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 3*b*e*asin(c*x)/(32*c**4), Ne(c, 0)), (a*(d*x**2/2 + e*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int x(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{1}{4} aex^4 + \frac{1}{2} adx^2 + \frac{1}{4} \left(2x^2 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2 + 1}x}{c^2} - \frac{\arcsin(cx)}{c^3} \right) \right) bd$$

$$+ \frac{1}{32} \left(8x^4 \arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2 + 1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2 + 1}x}{c^4} - \frac{3\arcsin(cx)}{c^5} \right) c \right) be$$

input `integrate(x*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`output `1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/4*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d + 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*e`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.40

$$\int x(d + ex^2)(a + b \arcsin(cx)) dx = \frac{1}{4} aex^4 + \frac{\sqrt{-c^2x^2 + 1}bdx}{4c}$$

$$+ \frac{(c^2x^2 - 1)bd \arcsin(cx)}{2c^2} - \frac{(-c^2x^2 + 1)^{\frac{3}{2}}bex}{16c^3}$$

$$+ \frac{(c^2x^2 - 1)ad}{2c^2} + \frac{bd \arcsin(cx)}{4c^2}$$

$$+ \frac{(c^2x^2 - 1)^2be \arcsin(cx)}{4c^4} + \frac{5\sqrt{-c^2x^2 + 1}bex}{32c^3}$$

$$+ \frac{(c^2x^2 - 1)be \arcsin(cx)}{2c^4} + \frac{5be \arcsin(cx)}{32c^4}$$

input `integrate(x*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```
1/4*a*e*x^4 + 1/4*sqrt(-c^2*x^2 + 1)*b*d*x/c + 1/2*(c^2*x^2 - 1)*b*d*arcsi
n(c*x)/c^2 - 1/16*(-c^2*x^2 + 1)^(3/2)*b*e*x/c^3 + 1/2*(c^2*x^2 - 1)*a*d/c
^2 + 1/4*b*d*arcsin(c*x)/c^2 + 1/4*(c^2*x^2 - 1)^2*b*e*arcsin(c*x)/c^4 + 5
/32*sqrt(-c^2*x^2 + 1)*b*e*x/c^3 + 1/2*(c^2*x^2 - 1)*b*e*arcsin(c*x)/c^4 +
5/32*b*e*arcsin(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x(d + ex^2)(a + b \arcsin(cx)) dx = \int x(a + b \operatorname{asin}(cx))(ex^2 + d) dx$$

input

```
int(x*(a + b*asin(c*x))*(d + e*x^2),x)
```

output

```
int(x*(a + b*asin(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int x(d + ex^2)(a + b \arcsin(cx)) dx$$

$$= \frac{16 \operatorname{asin}(cx) b c^4 d x^2 + 8 \operatorname{asin}(cx) b c^4 e x^4 - 8 \operatorname{asin}(cx) b c^2 d - 3 \operatorname{asin}(cx) b e + 8 \sqrt{-c^2 x^2 + 1} b c^3 d x + 2 \sqrt{-c^2 x^2 + 1} b c^3 e x^3}{32 c^4}$$

input

```
int(x*(e*x^2+d)*(a+b*asin(c*x)),x)
```

output

```
(16*asin(c*x)*b*c**4*d*x**2 + 8*asin(c*x)*b*c**4*e*x**4 - 8*asin(c*x)*b*c*
**2*d - 3*asin(c*x)*b*e + 8*sqrt(-c**2*x**2 + 1)*b*c**3*d*x + 2*sqrt(-c
**2*x**2 + 1)*b*c**3*e*x**3 + 3*sqrt(-c**2*x**2 + 1)*b*c*e*x + 16*a*c**4
*d*x**2 + 8*a*c**4*e*x**4)/(32*c**4)
```

3.427 $\int (d + ex^2) (a + b \arcsin(cx)) dx$

Optimal result	3625
Mathematica [A] (verified)	3625
Rubi [A] (verified)	3626
Maple [A] (verified)	3628
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Reduce [B] (verification not implemented)	3631

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int (d + ex^2) (a + b \arcsin(cx)) dx = \frac{b(3c^2d + e) \sqrt{1 - c^2x^2}}{3c^3} - \frac{be(1 - c^2x^2)^{3/2}}{9c^3} + dx(a + b \arcsin(cx)) + \frac{1}{3}ex^3(a + b \arcsin(cx))$$

output

```
1/3*b*(3*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3-1/9*b*e*(-c^2*x^2+1)^(3/2)/c^3+d*x*(a+b*arcsin(c*x))+1/3*e*x^3*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int (d + ex^2) (a + b \arcsin(cx)) dx = \frac{1}{9} \left(3ax(3d + ex^2) + \frac{b\sqrt{1 - c^2x^2}(2e + c^2(9d + ex^2))}{c^3} + 3bx(3d + ex^2) \arcsin(cx) \right)$$

input

```
Integrate[(d + e*x^2)*(a + b*ArcSin[c*x]),x]
```

output

$$(3ax(3d + ex^2) + (b\sqrt{1 - c^2x^2})(2e + c^2(9d + ex^2)))/c^3 + 3bx(3d + ex^2)\text{ArcSin}[cx])/9$$
Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5170, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2)(a + b \arcsin(cx)) dx$$

$$\downarrow 5170$$

$$-bc \int \frac{x(ex^2 + 3d)}{3\sqrt{1 - c^2x^2}} dx + dx(a + b \arcsin(cx)) + \frac{1}{3}ex^3(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{3}bc \int \frac{x(ex^2 + 3d)}{\sqrt{1 - c^2x^2}} dx + dx(a + b \arcsin(cx)) + \frac{1}{3}ex^3(a + b \arcsin(cx))$$

$$\downarrow 353$$

$$-\frac{1}{6}bc \int \frac{ex^2 + 3d}{\sqrt{1 - c^2x^2}} dx^2 + dx(a + b \arcsin(cx)) + \frac{1}{3}ex^3(a + b \arcsin(cx))$$

$$\downarrow 53$$

$$-\frac{1}{6}bc \int \left(\frac{3dc^2 + e}{c^2\sqrt{1 - c^2x^2}} - \frac{e\sqrt{1 - c^2x^2}}{c^2} \right) dx^2 + dx(a + b \arcsin(cx)) + \frac{1}{3}ex^3(a + b \arcsin(cx))$$

$$\downarrow 2009$$

$$dx(a + b \arcsin(cx)) + \frac{1}{3}ex^3(a + b \arcsin(cx)) - \frac{1}{6}bc \left(\frac{2e(1 - c^2x^2)^{3/2}}{3c^4} - \frac{2\sqrt{1 - c^2x^2}(3c^2d + e)}{c^4} \right)$$

input

$$\text{Int}[(d + ex^2)(a + b\text{ArcSin}[cx]), x]$$

output

$$-1/6*(b*c*((-2*(3*c^2*d + e)*\text{Sqrt}[1 - c^2*x^2])/c^4 + (2*e*(1 - c^2*x^2)^{(3/2)})/(3*c^4))) + d*x*(a + b*\text{ArcSin}[c*x]) + (e*x^3*(a + b*\text{ArcSin}[c*x]))/3$$

Defintions of rubi rules used

rule 27

$$\text{Int}[(a_*)(F_x_), x_Symbol] \rightarrow \text{Simp}[a \quad \text{Int}[F_x, x], x] \text{ ; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[F_x, (b_*)(G_x_) \text{ ; FreeQ}[b, x]]$$

rule 53

$$\text{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ ; FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$$

rule 353

$$\text{Int}[(x_)*((a_) + (b_.)*(x_)^2)^{(p_.)}*((c_) + (d_.)*(x_)^2)^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \quad \text{Subst}[\text{Int}[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] \text{ ; FreeQ}[\{a, b, c, d, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$$

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5170

$$\text{Int}[(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \quad u, x] - \text{Simp}[b*c \quad \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x]] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{ILtQ}[p + 1/2, 0])$$

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

method	result
parts	$a\left(\frac{1}{3}e x^3 + dx\right) + \frac{b\left(\frac{c \arcsin(cx)e x^3}{3} + \arcsin(cx)cxd - \frac{e\left(-\frac{e^2 x^2 \sqrt{-c^2 x^2 + 1}}{3} - \frac{2\sqrt{-c^2 x^2 + 1}}{3}\right) - 3d c^2 \sqrt{-c^2 x^2 + 1}}{3c^2}\right)}{c}$
derivativelimit	$\frac{a\left(c^3 dx + \frac{1}{3}e c^3 x^3\right)}{c^2} + \frac{b\left(\arcsin(cx)d c^3 x + \frac{\arcsin(cx)e c^3 x^3}{3} - \frac{e\left(-\frac{c^2 x^2 \sqrt{-c^2 x^2 + 1}}{3} - \frac{2\sqrt{-c^2 x^2 + 1}}{3}\right)}{3} + d c^2 \sqrt{-c^2 x^2 + 1}\right)}{c^2}$
default	$\frac{a\left(c^3 dx + \frac{1}{3}e c^3 x^3\right)}{c^2} + \frac{b\left(\arcsin(cx)d c^3 x + \frac{\arcsin(cx)e c^3 x^3}{3} - \frac{e\left(-\frac{c^2 x^2 \sqrt{-c^2 x^2 + 1}}{3} - \frac{2\sqrt{-c^2 x^2 + 1}}{3}\right)}{3} + d c^2 \sqrt{-c^2 x^2 + 1}\right)}{c^2}$
ordering	$\frac{x(5e^2 x^4 c^4 + 30c^4 d e x^2 + 9c^4 d^2 + 2c^2 e^2 x^2 - 18c^2 d e - 4e^2)(a + b \arcsin(cx))}{9(e x^2 + d)c^4} - \frac{(c^2 e x^2 + 9c^2 d + 2e)(cx - 1)(cx + 1)\left(2ex(a + b \arcsin(cx)) + d\right)}{9c^4(e x^2 + d)}$

input `int((e*x^2+d)*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)`

output `a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arcsin(c*x)*e*x^3+arcsin(c*x)*c*x*d-1/3/c^2*(e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2))-2/3*(-c^2*x^2+1)^(1/2))-3*d*c^2*(-c^2*x^2+1)^(1/2))`

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int (d + ex^2) (a + b \arcsin(cx)) dx = \frac{3ac^3ex^3 + 9ac^3dx + 3(bc^3ex^3 + 3bc^3dx) \arcsin(cx) + (bc^2ex^2 + 9bc^2d + 2be)\sqrt{-c^2x^2 + 1}}{9c^3}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*arcsin(c*x) + (b*c^2*e*x^2 + 9*b*c^2*d + 2*b*e)*sqrt(-c^2*x^2 + 1))/c^3`

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \begin{cases} adx + \frac{aex^3}{3} + bdx \arcsin(cx) + \frac{bex^3 \arcsin(cx)}{3} + \frac{bd\sqrt{-c^2x^2+1}}{c} + \frac{bex^2\sqrt{-c^2x^2+1}}{9c} + \frac{2be\sqrt{-c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input `integrate((e*x**2+d)*(a+b*asin(c*x)),x)`output `Piecewise((a*d*x + a*e*x**3/3 + b*d*x*asin(c*x) + b*e*x**3*asin(c*x)/3 + b*d*sqrt(-c**2*x**2 + 1)/c + b*e*x**2*sqrt(-c**2*x**2 + 1)/(9*c) + 2*b*e*sqrt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))`**Maxima [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \frac{1}{3} aex^3 + \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4} \right) \right) be$$

$$+ adx + \frac{(cx \arcsin(cx) + \sqrt{-c^2x^2+1})bd}{c}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`output `1/3*a*e*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d/c`

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int (d + ex^2) (a + b \arcsin(cx)) dx = \frac{1}{3} aex^3 + bdx \arcsin(cx) + adx + \frac{(c^2x^2 - 1) bex \arcsin(cx)}{3c^2} + \frac{bex \arcsin(cx)}{3c^2} + \frac{\sqrt{-c^2x^2 + 1}bd}{c} - \frac{(-c^2x^2 + 1)^{\frac{3}{2}}be}{9c^3} + \frac{\sqrt{-c^2x^2 + 1}be}{3c^3}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`output `1/3*a*e*x^3 + b*d*x*arcsin(c*x) + a*d*x + 1/3*(c^2*x^2 - 1)*b*e*x*arcsin(c*x)/c^2 + 1/3*b*e*x*arcsin(c*x)/c^2 + sqrt(-c^2*x^2 + 1)*b*d/c - 1/9*(-c^2*x^2 + 1)^(3/2)*b*e/c^3 + 1/3*sqrt(-c^2*x^2 + 1)*b*e/c^3`**Mupad [F(-1)]**

Timed out.

$$\int (d + ex^2) (a + b \arcsin(cx)) dx = \begin{cases} be \left(\frac{\sqrt{\frac{1}{c^2} - x^2} \left(\frac{2}{c^2} + x^2 \right)}{9} + \frac{x^3 \arcsin(cx)}{3} \right) + \frac{ax(e x^2 + 3d)}{3} + \frac{bd(\sqrt{1 - c^2 x^2} + cx \arcsin(cx))}{c} & \text{if } 0 < c \\ \int (a + b \arcsin(cx)) (e x^2 + d) dx & \text{if } -0 < c \end{cases}$$

input `int((a + b*asin(c*x))*(d + e*x^2),x)`output `piecewise(0 < c, b*e*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 + (x^3*asin(c*x))/3) + (a*x*(3*d + e*x^2))/3 + (b*d*((-c^2*x^2 + 1)^(1/2) + c*x*asin(c*x)))/c, ~0 < c, int((a + b*asin(c*x))*(d + e*x^2), x))`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

$$\int (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \frac{9a \sin(cx) b c^3 dx + 3a \sin(cx) b c^3 e x^3 + 9\sqrt{-c^2 x^2 + 1} b c^2 d + \sqrt{-c^2 x^2 + 1} b c^2 e x^2 + 2\sqrt{-c^2 x^2 + 1} b e + 9}{9c^3}$$

input

```
int((e*x^2+d)*(a+b*asin(c*x)),x)
```

output

```
(9*asin(c*x)*b*c**3*d*x + 3*asin(c*x)*b*c**3*e*x**3 + 9*sqrt(-c**2*x**2
+ 1)*b*c**2*d + sqrt(-c**2*x**2 + 1)*b*c**2*e*x**2 + 2*sqrt(-c**2*x**2
+ 1)*b*e + 9*a*c**3*d*x + 3*a*c**3*e*x**3)/(9*c**3)
```

$$3.428 \quad \int \frac{(d+ex^2)(a+b \arcsin(cx))}{x} dx$$

Optimal result	3632
Mathematica [A] (verified)	3633
Rubi [A] (verified)	3633
Maple [A] (verified)	3635
Fricas [F]	3635
Sympy [F]	3636
Maxima [F]	3636
Giac [F(-2)]	3636
Mupad [F(-1)]	3637
Reduce [F]	3637

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \frac{(d+ex^2)(a+b \arcsin(cx))}{x} dx = \frac{bex\sqrt{1-c^2x^2}}{4c} - \frac{be \arcsin(cx)}{4c^2} - \frac{1}{2}ibd \arcsin(cx)^2 + \frac{1}{2}ex^2(a+b \arcsin(cx)) + bd \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) - bd \arcsin(cx) \log(x) + d(a+b \arcsin(cx)) \log(x) - \frac{1}{2}ibd \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
1/4*b*e*x*(-c^2*x^2+1)^(1/2)/c-1/4*b*e*arcsin(c*x)/c^2-1/2*I*b*d*arcsin(c*x)^2+1/2*e*x^2*(a+b*arcsin(c*x))+b*d*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-b*d*arcsin(c*x)*ln(x)+d*(a+b*arcsin(c*x))*ln(x)-1/2*I*b*d*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx = \frac{1}{2}aex^2 + \frac{bex\sqrt{1 - c^2x^2}}{4c} - \frac{be \arcsin(cx)}{4c^2} + \frac{1}{2}bex^2 \arcsin(cx) + bd \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) + ad \log(x) - \frac{1}{2}ibd(\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)}))$$

input `Integrate[((d + e*x^2)*(a + b*ArcSin[c*x]))/x,x]`

output `(a*e*x^2)/2 + (b*e*x*Sqrt[1 - c^2*x^2])/(4*c) - (b*e*ArcSin[c*x])/(4*c^2) + (b*e*x^2*ArcSin[c*x])/2 + b*d*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + a*d*Log[x] - (I/2)*b*d*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])])`

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx$$

$$\downarrow 5230$$

$$-bc \int \frac{ex^2 + 2d \log(x)}{2\sqrt{1 - c^2x^2}} dx + d \log(x)(a + b \arcsin(cx)) + \frac{1}{2}ex^2(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{2}bc \int \frac{ex^2 + 2d \log(x)}{\sqrt{1 - c^2x^2}} dx + d \log(x)(a + b \arcsin(cx)) + \frac{1}{2}ex^2(a + b \arcsin(cx))$$

$$\downarrow 7293$$

$$-\frac{1}{2}bc \int \left(\frac{ex^2}{\sqrt{1-c^2x^2}} + \frac{2d \log(x)}{\sqrt{1-c^2x^2}} \right) dx + d \log(x)(a + b \arcsin(cx)) + \frac{1}{2}ex^2(a + b \arcsin(cx))$$

$$\downarrow 2009$$

$$\frac{1}{2}bc \left(\frac{e \arcsin(cx)}{2c^3} + \frac{id \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{c} + \frac{id \arcsin(cx)^2}{c} - \frac{2d \arcsin(cx) \log(1 - e^{2i \arcsin(cx)})}{c} + \frac{2d \log(x)(a + b \arcsin(cx)) + \frac{1}{2}ex^2(a + b \arcsin(cx))}{c} \right)$$

input `Int[((d + e*x^2)*(a + b*ArcSin[c*x]))/x,x]`

output `(e*x^2*(a + b*ArcSin[c*x]))/2 + d*(a + b*ArcSin[c*x])*Log[x] - (b*c*(-1/2*(e*x*Sqrt[1 - c^2*x^2])/c^2 + (e*ArcSin[c*x])/(2*c^3) + (I*d*ArcSin[c*x]^2)/c - (2*d*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c + (2*d*ArcSin[c*x]*Log[x])/c + (I*d*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c))/2`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5230 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.22

method	result
parts	$\frac{ae x^2}{2} + ad \ln(x) + b \left(-\frac{i \arcsin(cx)^2 d}{2} + d \arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1}) + d \arcsin(cx) \ln(1 - icx - \sqrt{-c^2 x^2 + 1}) \right) - ibd \operatorname{polylog}(2, -I*c*x - (-c^2*x^2+1)^{(1/2)}) - I*d*\operatorname{polylog}(2, I*c*x + (-c^2*x^2+1)^{(1/2)}) - 1/4 * e/c^2 * \arcsin(cx) * \cos(2*\arcsin(cx)) + 1/8 * e/c^2 * \sin(2*\arcsin(cx))$
derivativedivides	$\frac{ae x^2}{2} + ad \ln(cx) - \frac{ibd \arcsin(cx)^2}{2} + bd \arcsin(cx) \ln(1 - icx - \sqrt{-c^2 x^2 + 1}) - ibd \operatorname{polylog}(2, -I*c*x - (-c^2*x^2+1)^{(1/2)}) - I*d*\operatorname{polylog}(2, I*c*x + (-c^2*x^2+1)^{(1/2)}) - 1/4 * e/c^2 * \arcsin(cx) * \cos(2*\arcsin(cx)) + 1/8 * e/c^2 * \sin(2*\arcsin(cx))$
default	$\frac{ae x^2}{2} + ad \ln(cx) - \frac{ibd \arcsin(cx)^2}{2} + bd \arcsin(cx) \ln(1 - icx - \sqrt{-c^2 x^2 + 1}) - ibd \operatorname{polylog}(2, -I*c*x - (-c^2*x^2+1)^{(1/2)}) - I*d*\operatorname{polylog}(2, I*c*x + (-c^2*x^2+1)^{(1/2)}) - 1/4 * e/c^2 * \arcsin(cx) * \cos(2*\arcsin(cx)) + 1/8 * e/c^2 * \sin(2*\arcsin(cx))$

input `int((e*x^2+d)*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output `1/2*a*e*x^2+a*d*ln(x)+b*(-1/2*I*arcsin(c*x)^2*d+d*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+d*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*d*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-I*d*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-1/4*e/c^2*arcsin(c*x)*cos(2*arcsin(c*x))+1/8*e/c^2*sin(2*arcsin(c*x))`

Fricas [F]

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx = \int \frac{(ex^2 + d)(b \arcsin(cx) + a)}{x} dx$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x,x, algorithm="fricas")`

output `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsin(c*x))/x, x)`

Sympy [F]

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx))(d + ex^2)}{x} dx$$

input `integrate((e*x**2+d)*(a+b*asin(c*x))/x,x)`

output `Integral((a + b*asin(c*x))*(d + e*x**2)/x, x)`

Maxima [F]

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx = \int \frac{(ex^2 + d)(b \arcsin(cx) + a)}{x} dx$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `1/2*a*e*x^2 + a*d*log(x) + integrate((b*e*x^2 + b*d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx))(ex^2 + d)}{x} dx$$

input `int(((a + b*asin(c*x))*(d + e*x^2))/x,x)`output `int(((a + b*asin(c*x))*(d + e*x^2))/x, x)`**Reduce [F]**

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x} dx$$

$$= \frac{2a \sin(cx) b c^2 e x^2 - a \sin(cx) b e + \sqrt{-c^2 x^2 + 1} b c e x + 4 \left(\int \frac{a \sin(cx)}{x} dx \right) b c^2 d + 4 \log(x) a c^2 d + 2a c^2 e x^2}{4c^2}$$

input `int((e*x^2+d)*(a+b*asin(c*x))/x,x)`output `(2*asin(c*x)*b*c**2*e*x**2 - asin(c*x)*b*e + sqrt(-c**2*x**2 + 1)*b*c*e*x + 4*int(asin(c*x)/x,x)*b*c**2*d + 4*log(x)*a*c**2*d + 2*a*c**2*e*x**2)/(4*c**2)`

3.429 $\int \frac{(d+ex^2)(a+b \arcsin(cx))}{x^2} dx$

Optimal result	3638
Mathematica [A] (verified)	3638
Rubi [A] (verified)	3639
Maple [A] (verified)	3641
Fricas [A] (verification not implemented)	3641
Sympy [A] (verification not implemented)	3642
Maxima [A] (verification not implemented)	3642
Giac [B] (verification not implemented)	3643
Mupad [B] (verification not implemented)	3644
Reduce [B] (verification not implemented)	3644

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = \frac{be\sqrt{1 - c^2x^2}}{c} - \frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) - bcd \operatorname{arctanh}\left(\sqrt{1 - c^2x^2}\right)$$

output

```
b*e*(-c^2*x^2+1)^(1/2)/c-d*(a+b*arcsin(c*x))/x+e*x*(a+b*arcsin(c*x))-b*c*d*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = -\frac{ad}{x} + aex + \frac{be\sqrt{1 - c^2x^2}}{c} - \frac{bd \arcsin(cx)}{x} + bex \arcsin(cx) - bcd \operatorname{arctanh}\left(\sqrt{1 - c^2x^2}\right)$$

input

```
Integrate[((d + e*x^2)*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```

-((a*d)/x) + a*e*x + (b*e*Sqrt[1 - c^2*x^2])/c - (b*d*ArcSin[c*x])/x + b*e
*x*ArcSin[c*x] - b*c*d*ArcTanh[Sqrt[1 - c^2*x^2]]

```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5230, 25, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx \\
& \quad \downarrow \text{5230} \\
& -bc \int -\frac{d - ex^2}{x\sqrt{1 - c^2x^2}} dx - \frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) \\
& \quad \downarrow \text{25} \\
& bc \int \frac{d - ex^2}{x\sqrt{1 - c^2x^2}} dx - \frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) \\
& \quad \downarrow \text{354} \\
& \frac{1}{2}bc \int \frac{d - ex^2}{x^2\sqrt{1 - c^2x^2}} dx^2 - \frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) \\
& \quad \downarrow \text{90} \\
& \frac{1}{2}bc \left(d \int \frac{1}{x^2\sqrt{1 - c^2x^2}} dx^2 + \frac{2e\sqrt{1 - c^2x^2}}{c^2} \right) - \frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) \\
& \quad \downarrow \text{73} \\
& \frac{1}{2}bc \left(\frac{2e\sqrt{1 - c^2x^2}}{c^2} - \frac{2d \int \frac{1}{\frac{c^2}{c^2} - \frac{x^4}{c^2}} d\sqrt{1 - c^2x^2}}{c^2} \right) - \frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) \\
& \quad \downarrow \text{221} \\
& -\frac{d(a + b \arcsin(cx))}{x} + ex(a + b \arcsin(cx)) + \frac{1}{2}bc \left(\frac{2e\sqrt{1 - c^2x^2}}{c^2} - 2d \operatorname{arctanh}(\sqrt{1 - c^2x^2}) \right)
\end{aligned}$$

input `Int[((d + e*x^2)*(a + b*ArcSin[c*x]))/x^2,x]`

output `-((d*(a + b*ArcSin[c*x]))/x) + e*x*(a + b*ArcSin[c*x]) + (b*c*((2*e*Sqrt[1 - c^2*x^2])/c^2 - 2*d*ArcTanh[Sqrt[1 - c^2*x^2]]))/2`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 73 `Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

rule 90 `Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_] := Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))), x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)) Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 354 `Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 5230

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$c \left(\frac{a \left(ex - \frac{dc}{x} \right)}{c^2} + \frac{b \left(\arcsin(cx)ex - \frac{\arcsin(cx)dc}{x} - d c^2 \operatorname{arctanh} \left(\frac{1}{\sqrt{-c^2x^2+1}} \right) + e\sqrt{-c^2x^2+1} \right)}{c^2} \right)$	79
default	$c \left(\frac{a \left(ex - \frac{dc}{x} \right)}{c^2} + \frac{b \left(\arcsin(cx)ex - \frac{\arcsin(cx)dc}{x} - d c^2 \operatorname{arctanh} \left(\frac{1}{\sqrt{-c^2x^2+1}} \right) + e\sqrt{-c^2x^2+1} \right)}{c^2} \right)$	79
parts	$a \left(ex - \frac{d}{x} \right) + bc \left(\frac{\arcsin(cx)ex}{c} - \frac{\arcsin(cx)d}{cx} - \frac{-e\sqrt{-c^2x^2+1} + d c^2 \operatorname{arctanh} \left(\frac{1}{\sqrt{-c^2x^2+1}} \right)}{c^2} \right)$	80

input

```
int((e*x^2+d)*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output

```
c*(a/c^2*(e*c*x-d*c/x)+b/c^2*(arcsin(c*x)*e*c*x-arcsin(c*x)*d*c/x-d*c^2*arctanh(1/(-c^2*x^2+1)^(1/2))+e*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = \frac{bc^2 dx \log(\sqrt{-c^2x^2+1} + 1) - bc^2 dx \log(\sqrt{-c^2x^2+1} - 1) - 2acex^2 - 2\sqrt{-c^2x^2+1}bex + 2acd - 2}{2cx}$$

input

```
integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")
```

output

```
-1/2*(b*c^2*d*x*log(sqrt(-c^2*x^2 + 1) + 1) - b*c^2*d*x*log(sqrt(-c^2*x^2
+ 1) - 1) - 2*a*c*e*x^2 - 2*sqrt(-c^2*x^2 + 1)*b*e*x + 2*a*c*d - 2*(b*c*e*
x^2 - b*c*d)*arcsin(c*x))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = -\frac{ad}{x} + aex + bcd \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) \\ - \frac{bd \operatorname{asin}(cx)}{x} \\ + be \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right)$$

input

```
integrate((e*x**2+d)*(a+b*asin(c*x))/x**2,x)
```

output

```
-a*d/x + a*e*x + b*c*d*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1),
(I*asin(1/(c*x)), True)) - b*d*asin(c*x)/x + b*e*Piecewise((0, Eq(c, 0)),
(x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = -\left(c \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd \\ + aex + \frac{(cx \operatorname{asin}(cx) + \sqrt{-c^2x^2+1})be}{c} - \frac{ad}{x}$$

input

```
integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")
```

output

```
-(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d + a*e
*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*e/c - a*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(62) = 124$.

Time = 0.37 (sec) , antiderivative size = 1032, normalized size of antiderivative = 15.64

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = \text{Too large to display}$$

input

```
integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")
```

output

```
-1/2*b*c^6*d*x^4*arcsin(c*x)/((c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^2*x/
(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^4) - 1/2*a*c^6*d*x^4/((
c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt
(-c^2*x^2 + 1) + 1)^4) + b*c^5*d*x^3*log(abs(c)*abs(x))/((c^4*x^3/(sqrt(-c
^2*x^2 + 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1)
+ 1)^3) - b*c^5*d*x^3*log(sqrt(-c^2*x^2 + 1) + 1)/((c^4*x^3/(sqrt(-c^2*x^2
+ 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^3)
- b*c^4*d*x^2*arcsin(c*x)/((c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^2*x/(sq
rt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^2) - a*c^4*d*x^2/((c^4*x^3
/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x
^2 + 1) + 1)^2) + b*c^3*d*x*log(abs(c)*abs(x))/((c^4*x^3/(sqrt(-c^2*x^2 +
1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)) - b*
c^3*d*x*log(sqrt(-c^2*x^2 + 1) + 1)/((c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 +
c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)) - b*c^3*e*x^3/(
(c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sq
rt(-c^2*x^2 + 1) + 1)^3) - 1/2*b*c^2*d*arcsin(c*x)/(c^4*x^3/(sqrt(-c^2*x^2
+ 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1)) + 2*b*c^2*e*x^2*arcsin(c*x)/
((c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^2*x/(sqrt(-c^2*x^2 + 1) + 1))*(sq
rt(-c^2*x^2 + 1) + 1)^2) - 1/2*a*c^2*d/(c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3
+ c^2*x/(sqrt(-c^2*x^2 + 1) + 1)) + 2*a*c^2*e*x^2/((c^4*x^3/(sqrt(-c^2...
```


Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = \frac{be(\sqrt{1 - c^2 x^2} + cx \arcsin(cx))}{c} - \frac{bd \arcsin(cx)}{x} - bcd \operatorname{atanh}\left(\frac{1}{\sqrt{1 - c^2 x^2}}\right) - \frac{a(d - ex^2)}{x}$$

input `int(((a + b*asin(c*x))*(d + e*x^2))/x^2,x)`output `(b*e*((1 - c^2*x^2)^(1/2) + c*x*asin(c*x))/c - (b*d*asin(c*x))/x - b*c*d*atanh(1/(1 - c^2*x^2)^(1/2)) - (a*(d - e*x^2))/x`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^2} dx = \frac{-\arcsin(cx) bcd + \arcsin(cx) bce x^2 + \sqrt{-c^2 x^2 + 1} bex + \log\left(\tan\left(\frac{\arcsin(cx)}{2}\right)\right) b c^2 dx - acd + ace x^2}{cx}$$

input `int((e*x^2+d)*(a+b*asin(c*x))/x^2,x)`output `(- asin(c*x)*b*c*d + asin(c*x)*b*c*e*x**2 + sqrt(- c**2*x**2 + 1)*b*e*x + log(tan(asin(c*x)/2))*b*c**2*d*x - a*c*d + a*c*e*x**2)/(c*x)`

3.430 $\int \frac{(d+ex^2)(a+b \arcsin(cx))}{x^3} dx$

Optimal result	3645
Mathematica [A] (verified)	3646
Rubi [A] (verified)	3646
Maple [A] (verified)	3648
Fricas [F]	3648
Sympy [F]	3649
Maxima [F]	3649
Giac [F(-2)]	3649
Mupad [F(-1)]	3650
Reduce [F]	3650

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = -\frac{bcd\sqrt{1 - c^2x^2}}{2x} - \frac{1}{2}ibe \arcsin(cx)^2 - \frac{d(a + b \arcsin(cx))}{2x^2} + be \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) - be \arcsin(cx) \log(x) + e(a + b \arcsin(cx)) \log(x) - \frac{1}{2}ibe \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-1/2*b*c*d*(-c^2*x^2+1)^(1/2)/x-1/2*I*b*e*arcsin(c*x)^2-1/2*d*(a+b*arcsin(c*x))/x^2+b*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-b*e*arcsin(c*x)*ln(x)+e*(a+b*arcsin(c*x))*ln(x)-1/2*I*b*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.85

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = -\frac{ad}{2x^2} - \frac{bcd\sqrt{1 - c^2x^2}}{2x} - \frac{bd \arcsin(cx)}{2x^2} + be \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) + ae \log(x) - \frac{1}{2}ibe(\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)}))$$

input

```
Integrate[((d + e*x^2)*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
-1/2*(a*d)/x^2 - (b*c*d*Sqrt[1 - c^2*x^2])/(2*x) - (b*d*ArcSin[c*x])/(2*x^2) + b*e*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + a*e*Log[x] - (I/2)*b*e*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx \\ & \quad \downarrow \text{5230} \\ & -bc \int -\frac{\frac{d}{x^2} - 2e \log(x)}{2\sqrt{1 - c^2x^2}} dx - \frac{d(a + b \arcsin(cx))}{2x^2} + e \log(x)(a + b \arcsin(cx)) \\ & \quad \downarrow \text{27} \\ & \frac{1}{2}bc \int \frac{\frac{d}{x^2} - 2e \log(x)}{\sqrt{1 - c^2x^2}} dx - \frac{d(a + b \arcsin(cx))}{2x^2} + e \log(x)(a + b \arcsin(cx)) \\ & \quad \downarrow \text{7293} \end{aligned}$$

$$\frac{1}{2}bc \int \left(\frac{d}{x^2\sqrt{1-c^2x^2}} - \frac{2e \log(x)}{\sqrt{1-c^2x^2}} \right) dx - \frac{d(a+b \arcsin(cx))}{2x^2} + e \log(x)(a+b \arcsin(cx))$$

↓ 2009

$$\frac{1}{2}bc \left(-\frac{ie \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{c} - \frac{ie \arcsin(cx)^2}{c} + \frac{2e \arcsin(cx) \log(1 - e^{2i \arcsin(cx)})}{c} - \frac{2e \log(x) \arcsin(cx)}{c} \right) - \frac{d(a+b \arcsin(cx))}{2x^2} + e \log(x)(a+b \arcsin(cx)) +$$

input `Int[((d + e*x^2)*(a + b*ArcSin[c*x]))/x^3,x]`

output `-1/2*(d*(a + b*ArcSin[c*x]))/x^2 + e*(a + b*ArcSin[c*x])*Log[x] + (b*c*(-(d*Sqrt[1 - c^2*x^2])/x) - (I*e*ArcSin[c*x]^2)/c + (2*e*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c - (2*e*ArcSin[c*x]*Log[x])/c - (I*e*PolyLog[2, E^((2*I)*ArcSin[c*x])]/c))/2`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5230 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.52

method	result
derivativedivides	$c^2 \left(\frac{ae \ln(cx)}{c^2} - \frac{ad}{2c^2x^2} + \frac{b \left(-\frac{ie \arcsin(cx)^2}{2} - \frac{d(-ic^2x^2+cx\sqrt{-c^2x^2+1}+\arcsin(cx))}{2x^2} + \ln(1-icx-\sqrt{-c^2x^2+1})e \arcsin(cx) \right)}{c^2} \right)$
default	$c^2 \left(\frac{ae \ln(cx)}{c^2} - \frac{ad}{2c^2x^2} + \frac{b \left(-\frac{ie \arcsin(cx)^2}{2} - \frac{d(-ic^2x^2+cx\sqrt{-c^2x^2+1}+\arcsin(cx))}{2x^2} + \ln(1-icx-\sqrt{-c^2x^2+1})e \arcsin(cx) \right)}{c^2} \right)$
parts	$ae \ln(x) - \frac{ad}{2x^2} + b c^2 \left(-\frac{ie \arcsin(cx)^2}{2c^2} - \frac{d(-ic^2x^2+cx\sqrt{-c^2x^2+1}+\arcsin(cx))}{2c^2x^2} + \frac{e \arcsin(cx) \ln(1+icx-\sqrt{-c^2x^2+1})}{c^2} \right)$

```
input int((e*x^2+d)*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
output c^2*(a/c^2*e*ln(c*x)-1/2*a*d/c^2/x^2+b/c^2*(-1/2*I*e*arcsin(c*x)^2-1/2*d*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/x^2+ln(1-I*c*x-(-c^2*x^2+1)^(1/2))*e*arcsin(c*x)+ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*e*arcsin(c*x)-I*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))*e-I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))*e)
```

Fricas [F]

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = \int \frac{(ex^2 + d)(b \arcsin(cx) + a)}{x^3} dx$$

```
input integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")
```

```
output integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsin(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx))(d + ex^2)}{x^3} dx$$

input `integrate((e*x**2+d)*(a+b*asin(c*x))/x**3,x)`

output `Integral((a + b*asin(c*x))*(d + e*x**2)/x**3, x)`

Maxima [F]

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = \int \frac{(ex^2 + d)(b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `-1/2*b*d*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) + b*e*integrate(arctan
2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x) + a*e*log(x) - 1/2*a*d/x^2`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx))(ex^2 + d)}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d + e*x^2))/x^3,x)`output `int(((a + b*asin(c*x))*(d + e*x^2))/x^3, x)`**Reduce [F]**

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{-\arcsin(cx)bd - \sqrt{-c^2x^2 + 1}bcdx + 2\left(\int \frac{\arcsin(cx)}{x} dx\right)be x^2 + 2\log(x)ae x^2 - ad}{2x^2}$$

input `int((e*x^2+d)*(a+b*asin(c*x))/x^3,x)`output `(- asin(c*x)*b*d - sqrt(- c**2*x**2 + 1)*b*c*d*x + 2*int(asin(c*x)/x,x)*
b*e*x**2 + 2*log(x)*a*e*x**2 - a*d)/(2*x**2)`

3.431 $\int \frac{(d+ex^2)(a+b \arcsin(cx))}{x^4} dx$

Optimal result	3651
Mathematica [A] (verified)	3651
Rubi [A] (verified)	3652
Maple [A] (verified)	3654
Fricas [A] (verification not implemented)	3655
Sympy [A] (verification not implemented)	3656
Maxima [A] (verification not implemented)	3657
Giac [B] (verification not implemented)	3657
Mupad [F(-1)]	3659
Reduce [B] (verification not implemented)	3659

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{(d+ex^2)(a+b \arcsin(cx))}{x^4} dx = -\frac{bcd\sqrt{1-c^2x^2}}{6x^2} - \frac{d(a+b \arcsin(cx))}{3x^3} - \frac{e(a+b \arcsin(cx))}{x} - \frac{1}{6}bc(c^2d+6e) \operatorname{arctanh}(\sqrt{1-c^2x^2})$$

output

```
-1/6*b*c*d*(-c^2*x^2+1)^(1/2)/x^2-1/3*d*(a+b*arcsin(c*x))/x^3-e*(a+b*arcsin(c*x))/x-1/6*b*c*(c^2*d+6*e)*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)(a+b \arcsin(cx))}{x^4} dx = -\frac{ad}{3x^3} - \frac{ae}{x} - \frac{bcd\sqrt{1-c^2x^2}}{6x^2} - \frac{bd \arcsin(cx)}{3x^3} - \frac{be \arcsin(cx)}{x} - \frac{1}{6}bc^3d \operatorname{arctanh}(\sqrt{1-c^2x^2}) - bce \operatorname{arctanh}(\sqrt{1-c^2x^2})$$

input `Integrate[((d + e*x^2)*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(a*d)/x^3 - (a*e)/x - (b*c*d*Sqrt[1 - c^2*x^2])/(6*x^2) - (b*d*ArcSin[c*x])/(3*x^3) - (b*e*ArcSin[c*x])/x - (b*c^3*d*ArcTanh[Sqrt[1 - c^2*x^2]])/6 - b*c*e*ArcTanh[Sqrt[1 - c^2*x^2]]`

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5230, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx \\
 & \quad \downarrow \text{5230} \\
 & -bc \int -\frac{3ex^2 + d}{3x^3\sqrt{1 - c^2x^2}} dx - \frac{d(a + b \arcsin(cx))}{3x^3} - \frac{e(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{27} \\
 & \frac{1}{3}bc \int \frac{3ex^2 + d}{x^3\sqrt{1 - c^2x^2}} dx - \frac{d(a + b \arcsin(cx))}{3x^3} - \frac{e(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{354} \\
 & \frac{1}{6}bc \int \frac{3ex^2 + d}{x^4\sqrt{1 - c^2x^2}} dx^2 - \frac{d(a + b \arcsin(cx))}{3x^3} - \frac{e(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{87} \\
 & \frac{1}{6}bc \left(\frac{1}{2}(c^2d + 6e) \int \frac{1}{x^2\sqrt{1 - c^2x^2}} dx^2 - \frac{d\sqrt{1 - c^2x^2}}{x^2} \right) - \frac{d(a + b \arcsin(cx))}{3x^3} - \\
 & \quad \frac{e(a + b \arcsin(cx))}{x} \\
 & \quad \downarrow \text{73}
 \end{aligned}$$

$$\frac{1}{6}bc \left(-\frac{(c^2d + 6e) \int \frac{1}{c^2 - x^2} d\sqrt{1 - c^2x^2}}{c^2} - \frac{d\sqrt{1 - c^2x^2}}{x^2} \right) - \frac{d(a + b \arcsin(cx))}{3x^3} - \frac{e(a + b \arcsin(cx))}{x}$$

↓ 221

$$\frac{1}{6}bc \left(-\arctanh(\sqrt{1 - c^2x^2}) (c^2d + 6e) - \frac{d\sqrt{1 - c^2x^2}}{x^2} \right) + \frac{d(a + b \arcsin(cx))}{3x^3} - \frac{e(a + b \arcsin(cx))}{x}$$

input `Int[((d + e*x^2)*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(d*(a + b*ArcSin[c*x]))/x^3 - (e*(a + b*ArcSin[c*x]))/x + (b*c*(-((d*
Sqrt[1 - c^2*x^2])/x^2) - (c^2*d + 6*e)*ArcTanh[Sqrt[1 - c^2*x^2]]))/6`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 73 `Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Simp[p/b Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]`

rule 87 `Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p
.), x] := Simp[(-b*e - a*f)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e)) Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Intege
rQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))`

rule 221 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 354 `Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[(m - 1)/2]`

rule 5230 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

method	result
parts	$a\left(-\frac{d}{3x^3} - \frac{e}{x}\right) + b c^3 \left(-\frac{\arcsin(cx)d}{3c^3x^3} - \frac{\arcsin(cx)e}{c^3x} - \frac{-d c^2 \left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2} \right)}{3c^2} \right) + 3e \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)$
derivativedivides	$c^3 \left(\frac{a\left(-\frac{e}{cx} - \frac{d}{3cx^3}\right)}{c^2} + \frac{b \left(-\frac{\arcsin(cx)e}{cx} - \frac{\arcsin(cx)d}{3cx^3} + \frac{d c^2 \left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2} \right)}{3} \right)}{c^2} - e \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) \right)$
default	$c^3 \left(\frac{a\left(-\frac{e}{cx} - \frac{d}{3cx^3}\right)}{c^2} + \frac{b \left(-\frac{\arcsin(cx)e}{cx} - \frac{\arcsin(cx)d}{3cx^3} + \frac{d c^2 \left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2} \right)}{3} \right)}{c^2} - e \operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) \right)$

```
input int((e*x^2+d)*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
output a*(-1/3*d/x^3-e/x)+b*c^3*(-1/3*arcsin(c*x)*d/c^3/x^3-1/c^3*arcsin(c*x)*e/x
-1/3/c^2*(-d*c^2*(-1/2/c^2/x^2*(-c^2*x^2+1)^(1/2)-1/2*arctanh(1/(-c^2*x^2+
1)^(1/2)))+3*e*arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx = \frac{(bc^3d + 6 bce)x^3 \log(\sqrt{-c^2x^2 + 1} + 1) - (bc^3d + 6 bce)x^3 \log(\sqrt{-c^2x^2 + 1} - 1) + 2\sqrt{-c^2x^2 + 1}bcdx}{12x^3}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `-1/12*((b*c^3*d + 6*b*c*e)*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - (b*c^3*d + 6*b*c*e)*x^3*log(sqrt(-c^2*x^2 + 1) - 1) + 2*sqrt(-c^2*x^2 + 1)*b*c*d*x + 12*a*e*x^2 + 4*a*d + 4*(3*b*e*x^2 + b*d)*arcsin(c*x))/x^3`

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.98

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= -\frac{ad}{3x^3} - \frac{ae}{x} + \frac{bcd \left(\begin{cases} -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ \frac{ic^2 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases} \right)}{3}$$

$$+ bce \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{bd \operatorname{asin}(cx)}{3x^3} - \frac{be \operatorname{asin}(cx)}{x}$$

input `integrate((e*x**2+d)*(a+b*asin(c*x))/x**4,x)`

output `-a*d/(3*x**3) - a*e/x + b*c*d*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 + b*c*e*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*d*asin(c*x)/(3*x**3) - b*e*asin(c*x)/x`

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= -\frac{1}{6} \left(\left(c^2 \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2x^2+1}}{x^2} \right) c + \frac{2 \arcsin(cx)}{x^3} \right) bd$$

$$- \left(c \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) be - \frac{ae}{x} - \frac{ad}{3x^3}$$

input

```
integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")
```

output

```
-1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*b*d - (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*e - a*e/x - 1/3*a*d/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(75) = 150.

Time = 120.87 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.99

$$\begin{aligned}
 \int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx = & -\frac{bc^6 dx^3 \arcsin(cx)}{24(\sqrt{-c^2x^2 + 1} + 1)^3} - \frac{ac^6 dx^3}{24(\sqrt{-c^2x^2 + 1} + 1)^3} \\
 & + \frac{bc^5 dx^2}{24(\sqrt{-c^2x^2 + 1} + 1)^2} - \frac{bc^4 dx \arcsin(cx)}{8(\sqrt{-c^2x^2 + 1} + 1)} \\
 & - \frac{ac^4 dx}{8(\sqrt{-c^2x^2 + 1} + 1)} + \frac{1}{6} bc^3 d \log(|c||x|) \\
 & - \frac{1}{6} bc^3 d \log(\sqrt{-c^2x^2 + 1} + 1) \\
 & - \frac{bc^2 ex \arcsin(cx)}{2(\sqrt{-c^2x^2 + 1} + 1)} \\
 & - \frac{bc^2 d(\sqrt{-c^2x^2 + 1} + 1) \arcsin(cx)}{8x} \\
 & - \frac{ac^2 ex}{2(\sqrt{-c^2x^2 + 1} + 1)} - \frac{ac^2 d(\sqrt{-c^2x^2 + 1} + 1)}{8x} \\
 & + bce \log(|c||x|) - bce \log(\sqrt{-c^2x^2 + 1} + 1) \\
 & - \frac{bcd(\sqrt{-c^2x^2 + 1} + 1)^2}{24x^2} \\
 & - \frac{be(\sqrt{-c^2x^2 + 1} + 1) \arcsin(cx)}{2x} \\
 & - \frac{bd(\sqrt{-c^2x^2 + 1} + 1)^3 \arcsin(cx)}{24x^3} \\
 & - \frac{ae(\sqrt{-c^2x^2 + 1} + 1)}{2x} - \frac{ad(\sqrt{-c^2x^2 + 1} + 1)^3}{24x^3}
 \end{aligned}$$

input `integrate((e*x^2+d)*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output

```
-1/24*b*c^6*d*x^3*arcsin(c*x)/(sqrt(-c^2*x^2 + 1) + 1)^3 - 1/24*a*c^6*d*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + 1/24*b*c^5*d*x^2/(sqrt(-c^2*x^2 + 1) + 1)^2 - 1/8*b*c^4*d*x*arcsin(c*x)/(sqrt(-c^2*x^2 + 1) + 1) - 1/8*a*c^4*d*x/(sqrt(-c^2*x^2 + 1) + 1) + 1/6*b*c^3*d*log(abs(c)*abs(x)) - 1/6*b*c^3*d*log(sqrt(-c^2*x^2 + 1) + 1) - 1/2*b*c^2*e*x*arcsin(c*x)/(sqrt(-c^2*x^2 + 1) + 1) - 1/8*b*c^2*d*(sqrt(-c^2*x^2 + 1) + 1)*arcsin(c*x)/x - 1/2*a*c^2*e*x/(sqrt(-c^2*x^2 + 1) + 1) - 1/8*a*c^2*d*(sqrt(-c^2*x^2 + 1) + 1)/x + b*c*e*log(abs(c)*abs(x)) - b*c*e*log(sqrt(-c^2*x^2 + 1) + 1) - 1/24*b*c*d*(sqrt(-c^2*x^2 + 1) + 1)^2/x^2 - 1/2*b*e*(sqrt(-c^2*x^2 + 1) + 1)*arcsin(c*x)/x - 1/24*b*d*(sqrt(-c^2*x^2 + 1) + 1)^3*arcsin(c*x)/x^3 - 1/2*a*e*(sqrt(-c^2*x^2 + 1) + 1)/x - 1/24*a*d*(sqrt(-c^2*x^2 + 1) + 1)^3/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx))(ex^2 + d)}{x^4} dx$$

input

```
int(((a + b*asin(c*x))*(d + e*x^2))/x^4,x)
```

output

```
int(((a + b*asin(c*x))*(d + e*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{(d + ex^2)(a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{-2a \sin(cx) bd - 6a \sin(cx) be x^2 - \sqrt{-c^2 x^2 + 1} bcdx + \log\left(\tan\left(\frac{\arcsin(cx)}{2}\right)\right) b c^3 d x^3 + 6 \log\left(\tan\left(\frac{\arcsin(cx)}{2}\right)\right)}{6x^3}$$

input

```
int((e*x^2+d)*(a+b*asin(c*x))/x^4,x)
```


output

```
( - 2*asin(c*x)*b*d - 6*asin(c*x)*b*e*x**2 - sqrt( - c**2*x**2 + 1)*b*c*d*  
x + log(tan(asin(c*x)/2))*b*c**3*d*x**3 + 6*log(tan(asin(c*x)/2))*b*c*e*x*  
*3 - 2*a*d - 6*a*e*x**2)/(6*x**3)
```

3.432 $\int x^4(d + ex^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	3661
Mathematica [A] (verified)	3662
Rubi [A] (verified)	3662
Maple [A] (verified)	3664
Fricas [A] (verification not implemented)	3665
Sympy [A] (verification not implemented)	3666
Maxima [A] (verification not implemented)	3667
Giac [B] (verification not implemented)	3667
Mupad [F(-1)]	3668
Reduce [B] (verification not implemented)	3668

Optimal result

Integrand size = 21, antiderivative size = 241

$$\int x^4(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{b(63c^4d^2 + 90c^2de + 35e^2) \sqrt{1 - c^2x^2}}{315c^9} - \frac{2b(63c^4d^2 + 135c^2de + 70e^2) (1 - c^2x^2)^{3/2}}{945c^9}$$

$$+ \frac{b(21c^4d^2 + 90c^2de + 70e^2) (1 - c^2x^2)^{5/2}}{525c^9}$$

$$- \frac{2be(9c^2d + 14e) (1 - c^2x^2)^{7/2}}{441c^9} + \frac{be^2(1 - c^2x^2)^{9/2}}{81c^9}$$

$$+ \frac{1}{5}d^2x^5(a + b \arcsin(cx)) + \frac{2}{7}dex^7(a + b \arcsin(cx)) + \frac{1}{9}e^2x^9(a + b \arcsin(cx))$$

output

```
1/315*b*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(-c^2*x^2+1)^(1/2)/c^9-2/945*b*(63*c^4*d^2+135*c^2*d*e+70*e^2)*(-c^2*x^2+1)^(3/2)/c^9+1/525*b*(21*c^4*d^2+90*c^2*d*e+70*e^2)*(-c^2*x^2+1)^(5/2)/c^9-2/441*b*e*(9*c^2*d+14*e)*(-c^2*x^2+1)^(7/2)/c^9+1/81*b*e^2*(-c^2*x^2+1)^(9/2)/c^9+1/5*d^2*x^5*(a+b*arcsin(c*x))+2/7*d*e*x^7*(a+b*arcsin(c*x))+1/9*e^2*x^9*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.78

$$\int x^4 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{315ax^5(63d^2 + 90dex^2 + 35e^2x^4) + \frac{b\sqrt{1-c^2x^2}(4480e^2 + 160c^2e(81d + 14ex^2) + 24c^4(441d^2 + 270dex^2 + 70e^2x^4) + 4c^6(1323d^2x^2 + 1215d^2ex^2 + 350e^2x^4) + c^8(3969d^2x^4 + 4050d^2ex^2 + 1225e^2x^4))}{c^9} + 315bx^5(63d^2 + 90dex^2 + 35e^2x^4) \operatorname{ArcSin}[cx]}{99225}$$

input

```
Integrate[x^4*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(315*a*x^5*(63*d^2 + 90*d*e*x^2 + 35*e^2*x^4) + (b*Sqrt[1 - c^2*x^2]*(4480
*e^2 + 160*c^2*e*(81*d + 14*e*x^2) + 24*c^4*(441*d^2 + 270*d*e*x^2 + 70*e^
2*x^4) + 4*c^6*(1323*d^2*x^2 + 1215*d*e*x^4 + 350*e^2*x^6) + c^8*(3969*d^2
*x^4 + 4050*d*e*x^6 + 1225*e^2*x^8)))/c^9 + 315*b*x^5*(63*d^2 + 90*d*e*x^2
+ 35*e^2*x^4)*ArcSin[c*x])/99225
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5230, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5230$$

$$-bc \int \frac{x^5 (35e^2x^4 + 90dex^2 + 63d^2)}{315\sqrt{1-c^2x^2}} dx + \frac{1}{5}d^2x^5(a + b \arcsin(cx)) + \frac{2}{7}dex^7(a + b \arcsin(cx)) + \frac{1}{9}e^2x^9(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$\begin{aligned}
& -\frac{1}{315}bc \int \frac{x^5(35e^2x^4 + 90dex^2 + 63d^2)}{\sqrt{1-c^2x^2}} dx + \frac{1}{5}d^2x^5(a + b \arcsin(cx)) + \frac{2}{7}dex^7(a + \\
& \quad b \arcsin(cx)) + \frac{1}{9}e^2x^9(a + b \arcsin(cx)) \\
& \quad \downarrow 1578 \\
& -\frac{1}{630}bc \int \frac{x^4(35e^2x^4 + 90dex^2 + 63d^2)}{\sqrt{1-c^2x^2}} dx^2 + \frac{1}{5}d^2x^5(a + b \arcsin(cx)) + \frac{2}{7}dex^7(a + \\
& \quad b \arcsin(cx)) + \frac{1}{9}e^2x^9(a + b \arcsin(cx)) \\
& \quad \downarrow 1195 \\
& -\frac{1}{630}bc \int \left(\frac{35e^2(1-c^2x^2)^{7/2}}{c^8} - \frac{10e(9dc^2 + 14e)(1-c^2x^2)^{5/2}}{c^8} + \frac{3(21d^2c^4 + 90dec^2 + 70e^2)(1-c^2x^2)^{3/2}}{c^8} \right. \\
& \quad \left. \frac{1}{5}d^2x^5(a + b \arcsin(cx)) + \frac{2}{7}dex^7(a + b \arcsin(cx)) + \frac{1}{9}e^2x^9(a + b \arcsin(cx)) \right) \\
& \quad \downarrow 2009 \\
& \frac{1}{630}bc \left(\frac{20e(1-c^2x^2)^{7/2}(9c^2d + 14e)}{7c^{10}} - \frac{70e^2(1-c^2x^2)^{9/2}}{9c^{10}} - \frac{6(1-c^2x^2)^{5/2}(21c^4d^2 + 90c^2de + 70e^2)}{5c^{10}} + \frac{4(1-c^2x^2)^{3/2}}{3c^{10}} \right) \\
& \quad + \frac{1}{5}d^2x^5(a + b \arcsin(cx)) + \frac{2}{7}dex^7(a + b \arcsin(cx)) + \frac{1}{9}e^2x^9(a + b \arcsin(cx)) -
\end{aligned}$$

input `Int[x^4*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/630*(b*c*((-2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*Sqrt[1 - c^2*x^2])/c^10 + (4*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^(3/2))/(3*c^10) - (6*(21*c^4*d^2 + 90*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^(5/2))/(5*c^10) + (20*e*(9*c^2*d + 14*e)*(1 - c^2*x^2)^(7/2))/(7*c^10) - (70*e^2*(1 - c^2*x^2)^(9/2))/(9*c^10))) + (d^2*x^5*(a + b*ArcSin[c*x]))/5 + (2*d*e*x^7*(a + b*ArcSin[c*x]))/7 + (e^2*x^9*(a + b*ArcSin[c*x]))/9`

Definitions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 1195 $\text{Int}[((d_.) + (e_.)*(x_))^{(m_.)}*((f_.) + (g_.)*(x_))^{(n_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$
- rule 1578 $\text{Int}[(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[1/2 \ \text{Subst}[\text{Int}[x^{(m-1)/2}*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, d, e, p, q\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2]$
- rule 2009 $\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] /; \text{SumQ}[u]$
- rule 5230 $\text{Int}[((a_.) + \text{ArcSin}[(c_.)*(x_.)]*(b_.)*((f_.)*(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(p_.)}), x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (\text{GtQ}[p, 0] \ || \ (\text{IGtQ}[(m-1)/2, 0] \ \&\& \ \text{LeQ}[m + p, 0]))$

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.37

method	result
parts	$a\left(\frac{1}{9}e^2x^9 + \frac{2}{7}dex^7 + \frac{1}{5}x^5d^2\right) + \frac{b\left(\frac{c^5\arcsin(cx)e^2x^9}{9} + \frac{2c^5\arcsin(cx)dex^7}{7} + \frac{\arcsin(cx)d^2c^5x^5}{5} - \frac{35e^2\left(-\frac{c^8x^8\sqrt{-c^2x^2+1}}{9} - 8\right)}{9}\right)}{c^4}$
derivativedivides	$\frac{a\left(\frac{1}{5}d^2c^9x^5 + \frac{2}{7}dc^9ex^7 + \frac{1}{9}e^2c^9x^9\right)}{c^4} + \frac{b\left(\frac{\arcsin(cx)d^2c^9x^5}{5} + \frac{2\arcsin(cx)dc^9ex^7}{7} + \frac{\arcsin(cx)e^2c^9x^9}{9} - \frac{e^2\left(-\frac{c^8x^8\sqrt{-c^2x^2+1}}{9} - 8\right)}{9}\right)}{c^4}$
default	$\frac{a\left(\frac{1}{5}d^2c^9x^5 + \frac{2}{7}dc^9ex^7 + \frac{1}{9}e^2c^9x^9\right)}{c^4} + \frac{b\left(\frac{\arcsin(cx)d^2c^9x^5}{5} + \frac{2\arcsin(cx)dc^9ex^7}{7} + \frac{\arcsin(cx)e^2c^9x^9}{9} - \frac{e^2\left(-\frac{c^8x^8\sqrt{-c^2x^2+1}}{9} - 8\right)}{9}\right)}{c^4}$
orering	$(20825c^{10}e^3x^{12} + 76675c^{10}de^2x^{10} + 96147c^{10}d^2ex^8 + 1400c^8e^3x^{10} + 35721c^{10}d^3x^6 + 7180c^8de^2x^8 + 13824c^8d^2ex^6 + 22)$

```
input int(x^4*(e*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/9*e^2*x^9+2/7*d*e*x^7+1/5*x^5*d^2)+b/c^5*(1/9*c^5*arcsin(c*x)*e^2*x^9
+2/7*c^5*arcsin(c*x)*d*e*x^7+1/5*arcsin(c*x)*d^2*c^5*x^5-1/315/c^4*(35*e^2
*(-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c
^4*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*x
^2+1)^(1/2))+63*d^2*c^4*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c
^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+90*d*c^2*e*(-1/7*c^6*x^6*(-c^2*x
^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)
-16/35*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.91

$$\int x^4(d + ex^2)^2(a + b \arcsin(cx)) dx$$

$$= \frac{11025 ac^9 e^2 x^9 + 28350 ac^9 dex^7 + 19845 ac^9 d^2 x^5 + 315 (35 bc^9 e^2 x^9 + 90 bc^9 dex^7 + 63 bc^9 d^2 x^5) \arcsin(cx)}{c^4}$$

```
input integrate(x^4*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
1/99225*(11025*a*c^9*e^2*x^9 + 28350*a*c^9*d*e*x^7 + 19845*a*c^9*d^2*x^5 +
315*(35*b*c^9*e^2*x^9 + 90*b*c^9*d*e*x^7 + 63*b*c^9*d^2*x^5)*arcsin(c*x)
+ (1225*b*c^8*e^2*x^8 + 10584*b*c^4*d^2 + 50*(81*b*c^8*d*e + 28*b*c^6*e^2)
*x^6 + 12960*b*c^2*d*e + 3*(1323*b*c^8*d^2 + 1620*b*c^6*d*e + 560*b*c^4*e^
2)*x^4 + 4480*b*e^2 + 4*(1323*b*c^6*d^2 + 1620*b*c^4*d*e + 560*b*c^2*e^2)*
x^2)*sqrt(-c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.72

$$\int x^4 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^2x^5}{5} + \frac{2adex^7}{7} + \frac{ae^2x^9}{9} + \frac{bd^2x^5 \arcsin(cx)}{5} + \frac{2bdex^7 \arcsin(cx)}{7} + \frac{be^2x^9 \arcsin(cx)}{9} + \frac{bd^2x^4 \sqrt{-c^2x^2+1}}{25c} + \frac{2bdex^6 \sqrt{-c^2x^2+1}}{49c} + \frac{be^2x^8 \sqrt{-c^2x^2+1}}{81c} \\ a \left(\frac{d^2x^5}{5} + \frac{2dex^7}{7} + \frac{e^2x^9}{9} \right) \end{cases}$$

input

```
integrate(x**4*(e*x**2+d)**2*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d**2*x**5/5 + 2*a*d*e*x**7/7 + a*e**2*x**9/9 + b*d**2*x**5*as
in(c*x)/5 + 2*b*d*e*x**7*asin(c*x)/7 + b*e**2*x**9*asin(c*x)/9 + b*d**2*x*
**4*sqrt(-c**2*x**2 + 1)/(25*c) + 2*b*d*e*x**6*sqrt(-c**2*x**2 + 1)/(49*c)
+ b*e**2*x**8*sqrt(-c**2*x**2 + 1)/(81*c) + 4*b*d**2*x**2*sqrt(-c**2*x**2
+ 1)/(75*c**3) + 12*b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(245*c**3) + 8*b*e**2*
x**6*sqrt(-c**2*x**2 + 1)/(567*c**3) + 8*b*d**2*sqrt(-c**2*x**2 + 1)/(75*c
**5) + 16*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) + 16*b*e**2*x**4*sqrt
(-c**2*x**2 + 1)/(945*c**5) + 32*b*d*e*sqrt(-c**2*x**2 + 1)/(245*c**7) + 6
4*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(2835*c**7) + 128*b*e**2*sqrt(-c**2*x**
2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**2*x**5/5 + 2*d*e*x**7/7 + e**2*x**9/
9), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.30

$$\int x^4(d+ex^2)^2(a+b\arcsin(cx))dx = \frac{1}{9}ae^2x^9 + \frac{2}{7}adex^7 + \frac{1}{5}ad^2x^5 + \frac{1}{75}\left(15x^5\arcsin(cx) + \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd^2 + \frac{2}{245}\left(35x^7\arcsin(cx) + \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right)c\right)bd^2 + \frac{1}{2835}\left(315x^9\arcsin(cx) + \left(\frac{35\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\sqrt{-c^2x^2+1}x^2}{c^8} + \frac{128\sqrt{-c^2x^2+1}}{c^{10}}\right)c\right)bd^2$$

input `integrate(x^4*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/9*a*e^2*x^9 + 2/7*a*d*e*x^7 + 1/5*a*d^2*x^5 + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^2 + 2/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*d*e + 1/2835*(315*x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*e^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(215) = 430.

Time = 0.14 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.48

$$\int x^4(d+ex^2)^2(a+b\arcsin(cx))dx = \text{Too large to display}$$

input `integrate(x^4*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

1/9*a*e^2*x^9 + 2/7*a*d*e*x^7 + 1/5*a*d^2*x^5 + 1/5*(c^2*x^2 - 1)^2*b*d^2*
x*arcsin(c*x)/c^4 + 2/5*(c^2*x^2 - 1)*b*d^2*x*arcsin(c*x)/c^4 + 2/7*(c^2*x
^2 - 1)^3*b*d*e*x*arcsin(c*x)/c^6 + 1/5*b*d^2*x*arcsin(c*x)/c^4 + 6/7*(c^2
*x^2 - 1)^2*b*d*e*x*arcsin(c*x)/c^6 + 1/9*(c^2*x^2 - 1)^4*b*e^2*x*arcsin(c
*x)/c^8 + 1/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d^2/c^5 + 6/7*(c^2*x^2
- 1)*b*d*e*x*arcsin(c*x)/c^6 + 4/9*(c^2*x^2 - 1)^3*b*e^2*x*arcsin(c*x)/c^
8 - 2/15*(-c^2*x^2 + 1)^(3/2)*b*d^2/c^5 + 2/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x
^2 + 1)*b*d*e/c^7 + 2/7*b*d*e*x*arcsin(c*x)/c^6 + 2/3*(c^2*x^2 - 1)^2*b*e^
2*x*arcsin(c*x)/c^8 + 1/5*sqrt(-c^2*x^2 + 1)*b*d^2/c^5 + 6/35*(c^2*x^2 - 1
)^2*sqrt(-c^2*x^2 + 1)*b*d*e/c^7 + 1/81*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)
*b*e^2/c^9 + 4/9*(c^2*x^2 - 1)*b*e^2*x*arcsin(c*x)/c^8 - 2/7*(-c^2*x^2 + 1
)^(3/2)*b*d*e/c^7 + 4/63*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*e^2/c^9 + 1/
9*b*e^2*x*arcsin(c*x)/c^8 + 2/7*sqrt(-c^2*x^2 + 1)*b*d*e/c^7 + 2/15*(c^2*x
^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e^2/c^9 - 4/27*(-c^2*x^2 + 1)^(3/2)*b*e^2/c
^9 + 1/9*sqrt(-c^2*x^2 + 1)*b*e^2/c^9

```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (ex^2 + d)^2 dx$$

input

```
int(x^4*(a + b*asin(c*x))*(d + e*x^2)^2,x)
```

output

```
int(x^4*(a + b*asin(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.44

$$\int x^4 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{19845 \operatorname{asin}(cx) b c^9 d^2 x^5 + 28350 \operatorname{asin}(cx) b c^9 d e x^7 + 11025 \operatorname{asin}(cx) b c^9 e^2 x^9 + 3969 \sqrt{-c^2 x^2 + 1} b c^8 d^2 x^4 - \dots}{\dots}$$

input

```
int(x^4*(e*x^2+d)^2*(a+b*asin(c*x)),x)
```

output

```
(19845*asin(c*x)*b*c**9*d**2*x**5 + 28350*asin(c*x)*b*c**9*d*e*x**7 + 11025*asin(c*x)*b*c**9*e**2*x**9 + 3969*sqrt(-c**2*x**2 + 1)*b*c**8*d**2*x**4 + 4050*sqrt(-c**2*x**2 + 1)*b*c**8*d*e*x**6 + 1225*sqrt(-c**2*x**2 + 1)*b*c**8*e**2*x**8 + 5292*sqrt(-c**2*x**2 + 1)*b*c**6*d**2*x**2 + 4860*sqrt(-c**2*x**2 + 1)*b*c**6*d*e*x**4 + 1400*sqrt(-c**2*x**2 + 1)*b*c**6*e**2*x**6 + 10584*sqrt(-c**2*x**2 + 1)*b*c**4*d**2 + 6480*sqrt(-c**2*x**2 + 1)*b*c**4*d*e*x**2 + 1680*sqrt(-c**2*x**2 + 1)*b*c**4*e**2*x**4 + 12960*sqrt(-c**2*x**2 + 1)*b*c**2*d*e + 2240*sqrt(-c**2*x**2 + 1)*b*c**2*e**2*x**2 + 4480*sqrt(-c**2*x**2 + 1)*b*e**2 + 19845*a*c**9*d**2*x**5 + 28350*a*c**9*d*e*x**7 + 11025*a*c**9*e**2*x**9)/(99225*c**9)
```

3.433 $\int x^3(d + ex^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	3670
Mathematica [A] (verified)	3671
Rubi [A] (verified)	3671
Maple [A] (verified)	3675
Fricas [A] (verification not implemented)	3675
Sympy [A] (verification not implemented)	3676
Maxima [A] (verification not implemented)	3677
Giac [B] (verification not implemented)	3678
Mupad [F(-1)]	3680
Reduce [B] (verification not implemented)	3680

Optimal result

Integrand size = 21, antiderivative size = 241

$$\begin{aligned}
 \int x^3(d + ex^2)^2 (a + b \arcsin(cx)) dx = & \frac{b(288c^4d^2 + 320c^2de + 105e^2) x\sqrt{1 - c^2x^2}}{3072c^7} \\
 & + \frac{b(288c^4d^2 + 320c^2de + 105e^2) x^3\sqrt{1 - c^2x^2}}{4608c^5} \\
 & + \frac{be(64c^2d + 21e) x^5\sqrt{1 - c^2x^2}}{1152c^3} \\
 & + \frac{be^2x^7\sqrt{1 - c^2x^2}}{64c} \\
 & - \frac{b(288c^4d^2 + 320c^2de + 105e^2) \arcsin(cx)}{3072c^8} \\
 & + \frac{1}{4}d^2x^4(a + b \arcsin(cx)) \\
 & + \frac{1}{3}dex^6(a + b \arcsin(cx)) \\
 & + \frac{1}{8}e^2x^8(a + b \arcsin(cx))
 \end{aligned}$$

output

```
1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x*(-c^2*x^2+1)^(1/2)/c^7+1/4608
*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x^3*(-c^2*x^2+1)^(1/2)/c^5+1/1152*b*e
*(64*c^2*d+21*e)*x^5*(-c^2*x^2+1)^(1/2)/c^3+1/64*b*e^2*x^7*(-c^2*x^2+1)^(1
/2)/c-1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*arcsin(c*x)/c^8+1/4*d^2*x
^4*(a+b*arcsin(c*x))+1/3*d*e*x^6*(a+b*arcsin(c*x))+1/8*e^2*x^8*(a+b*arcsin
(c*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.79

$$\int x^3(d + ex^2)^2(a + b \arcsin(cx)) dx$$

$$= \frac{384ac^8x^4(6d^2 + 8dex^2 + 3e^2x^4) + bcx\sqrt{1 - c^2x^2}(315e^2 + 30c^2e(32d + 7ex^2) + 8c^4(108d^2 + 80dex^2 + 21e^2x^4)) + 8c^4(108d^2 + 80dex^2 + 21e^2x^4) + 16c^6(36d^2x^2 + 32d*ex^4 + 9e^2x^6) + 3b*(-288c^4d^2 - 320c^2d*e - 105e^2 + 128c^8(6d^2x^4 + 8d*ex^6 + 3e^2x^8))*\text{ArcSin}[c*x]}{9216c^8}$$

input

```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(384*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b*c*x*Sqrt[1 - c^2*x^2]*(
315*e^2 + 30*c^2*e*(32*d + 7*e*x^2) + 8*c^4*(108*d^2 + 80*d*e*x^2 + 21*e^2
*x^4) + 16*c^6*(36*d^2*x^2 + 32*d*e*x^4 + 9*e^2*x^6)) + 3*b*(-288*c^4*d^2
- 320*c^2*d*e - 105*e^2 + 128*c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8))*Arc
Sin[c*x])/ (9216*c^8)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5230, 27, 1590, 25, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d + ex^2)^2(a + b \arcsin(cx)) dx$$

↓ 5230

$$\begin{aligned}
& -bc \int \frac{x^4(3e^2x^4 + 8dex^2 + 6d^2)}{24\sqrt{1-c^2x^2}} dx + \frac{1}{4}d^2x^4(a + b \arcsin(cx)) + \frac{1}{3}dex^6(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{1}{8}e^2x^8(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow 27 \\
& -\frac{1}{24}bc \int \frac{x^4(3e^2x^4 + 8dex^2 + 6d^2)}{\sqrt{1-c^2x^2}} dx + \frac{1}{4}d^2x^4(a + b \arcsin(cx)) + \frac{1}{3}dex^6(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{1}{8}e^2x^8(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow 1590 \\
& -\frac{1}{24}bc \left(-\frac{\int -\frac{x^4(48c^2d^2 + e(64dc^2 + 21e)x^2)}{\sqrt{1-c^2x^2}} dx}{8c^2} - \frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2} \right) + \frac{1}{4}d^2x^4(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{1}{3}dex^6(a + b \arcsin(cx)) + \frac{1}{8}e^2x^8(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow 25 \\
& -\frac{1}{24}bc \left(\frac{\int \frac{x^4(48c^2d^2 + e(64dc^2 + 21e)x^2)}{\sqrt{1-c^2x^2}} dx}{8c^2} - \frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2} \right) + \frac{1}{4}d^2x^4(a + b \arcsin(cx)) + \\
& \qquad \qquad \qquad \frac{1}{3}dex^6(a + b \arcsin(cx)) + \frac{1}{8}e^2x^8(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow 363 \\
& -\frac{1}{24}bc \left(\frac{(288c^4d^2 + 320c^2de + 105e^2) \int \frac{x^4}{\sqrt{1-c^2x^2}} dx}{6c^2} - \frac{ex^5\sqrt{1-c^2x^2}(64c^2d + 21e)}{6c^2} - \frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2} \right) + \\
& \qquad \qquad \qquad \frac{1}{4}d^2x^4(a + b \arcsin(cx)) + \frac{1}{3}dex^6(a + b \arcsin(cx)) + \frac{1}{8}e^2x^8(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow 262 \\
& -\frac{1}{24}bc \left(\frac{(288c^4d^2 + 320c^2de + 105e^2) \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right)}{6c^2} - \frac{ex^5\sqrt{1-c^2x^2}(64c^2d + 21e)}{6c^2} - \frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2} \right) + \\
& \qquad \qquad \qquad \frac{1}{4}d^2x^4(a + b \arcsin(cx)) + \frac{1}{3}dex^6(a + b \arcsin(cx)) + \frac{1}{8}e^2x^8(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow 262
\end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{24}bc \left(\frac{(288c^4d^2+320c^2de+105e^2) \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right)}{6c^2} - \frac{ex^5\sqrt{1-c^2x^2}(64c^2d+21e)}{6c^2} - \frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2} \right)}{8c^2} \\
 & \frac{1}{4}d^2x^4(a+b\arcsin(cx)) + \frac{1}{3}dex^6(a+b\arcsin(cx)) + \frac{1}{8}e^2x^8(a+b\arcsin(cx)) \\
 & \quad \downarrow \text{223} \\
 & \frac{1}{4}d^2x^4(a+b\arcsin(cx)) + \frac{1}{3}dex^6(a+b\arcsin(cx)) + \frac{1}{8}e^2x^8(a+b\arcsin(cx)) - \\
 & \left(\frac{3 \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \frac{(288c^4d^2+320c^2de+105e^2)}{6c^2} - \frac{ex^5\sqrt{1-c^2x^2}(64c^2d+21e)}{6c^2} - \frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2} \right) \frac{1}{24}bc
 \end{aligned}$$

```
input Int[x^3*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]
```

```
output (d^2*x^4*(a + b*ArcSin[c*x]))/4 + (d*e*x^6*(a + b*ArcSin[c*x]))/3 + (e^2*x^8*(a + b*ArcSin[c*x]))/8 - (b*c*((-3*e^2*x^7*sqrt[1 - c^2*x^2])/(8*c^2) + (-1/6*(e*(64*c^2*d + 21*e))*x^5*sqrt[1 - c^2*x^2])/c^2 + ((288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*(-1/4*(x^3*sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/(6*c^2))/(8*c^2))/24
```

Defintions of rubi rules used

```
rule 25 Int[-(Fx_), x_Symbol] :> Simp[Identity[-1] Int[Fx, x], x]
```

```
rule 27 Int[(a_)*(Fx_), x_Symbol] :> Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[(c_)*(x_)^m*((a_) + (b_)*(x_)^2)^p], x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{m-1}*((a + b*x^2)^{p+1}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{m-2}*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 363 $\text{Int}[(e_)*(x_)^m*((a_) + (b_)*(x_)^2)^p*((c_) + (d_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*(e*x)^{m+1}*((a + b*x^2)^{p+1}/(b*e*(m+2*p+3))), x] - \text{Simp}[(a*d*(m+1) - b*c*(m+2*p+3))/(b*(m+2*p+3)) \ \text{Int}[(e*x)^m*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m+2*p+3, 0]$

rule 1590 $\text{Int}[(f_)*(x_)^m*((d_) + (e_)*(x_)^2)^q*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^p], x_Symbol] \rightarrow \text{Simp}[c^p*(f*x)^{m+4*p-1}*((d + e*x^2)^{q+1}/(e*f^{4*p-1}*(m+4*p+2*q+1))), x] + \text{Simp}[1/(e*(m+4*p+2*q+1)) \ \text{Int}[(f*x)^m*(d + e*x^2)^q*\text{ExpandToSum}[e*(m+4*p+2*q+1)*((a + b*x^2 + c*x^4)^p - c^p*x^{4*p}) - d*c^p*(m+4*p-1)*x^{4*p-2}], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, q\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{NeQ}[m+4*p+2*q+1, 0]$

rule 5230 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)*((f_)*(x_)^m*((d_) + (e_)*(x_)^2)^p], x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) \ u, x] - \text{Simp}[b*c \ \text{Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (\text{GtQ}[p, 0] \ || \ (\text{IGtQ}[(m-1)/2, 0] \ \&\& \ \text{LeQ}[m+p, 0]))$

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

method	result
parts	$a\left(\frac{1}{8}e^2x^8 + \frac{1}{3}dex^6 + \frac{1}{4}d^2x^4\right) + \frac{b\left(\frac{c^4\arcsin(cx)e^2x^8}{8} + \frac{c^4\arcsin(cx)dex^6}{3} + \frac{\arcsin(cx)d^2c^4x^4}{4} - 3e^2\left(\frac{-c^7x^7\sqrt{-c^2x^2+1}}{8} - 7e^5\right)\right)}{c^4}$
derivativelimit	$\frac{a\left(\frac{1}{4}c^8d^2x^4 + \frac{1}{3}c^8dex^6 + \frac{1}{8}e^2x^8c^8\right)}{c^4} + \frac{b\left(\frac{\arcsin(cx)d^2c^8x^4}{4} + \frac{\arcsin(cx)dc^8ex^6}{3} + \frac{\arcsin(cx)e^2c^8x^8}{8} - e^2\left(\frac{-c^7x^7\sqrt{-c^2x^2+1}}{8} - 7e^5\right)\right)}{c^4}$
default	$\frac{a\left(\frac{1}{4}c^8d^2x^4 + \frac{1}{3}c^8dex^6 + \frac{1}{8}e^2x^8c^8\right)}{c^4} + \frac{b\left(\frac{\arcsin(cx)d^2c^8x^4}{4} + \frac{\arcsin(cx)dc^8ex^6}{3} + \frac{\arcsin(cx)e^2c^8x^8}{8} - e^2\left(\frac{-c^7x^7\sqrt{-c^2x^2+1}}{8} - 7e^5\right)\right)}{c^4}$
ordering	$\frac{(2160c^8e^3x^{10} + 8240c^8dex^8 + 10944c^8d^2ex^6 + 168x^8e^3c^6 + 4032c^8d^3x^4 + 968x^6e^2c^6d + 2400x^4ec^6d^2 + 294x^6e^3c^4 + 864c^8e^5)}{921}$

```
input int(x^3*(e*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*d^2*x^4)+b/c^4*(1/8*c^4*arcsin(c*x)*e^2*x^8
+1/3*c^4*arcsin(c*x)*d*e*x^6+1/4*arcsin(c*x)*d^2*c^4*x^4-1/24/c^4*(3*e^2*(
-1/8*c^7*x^7*(-c^2*x^2+1)^(1/2)-7/48*c^5*x^5*(-c^2*x^2+1)^(1/2)-35/192*c^3
*x^3*(-c^2*x^2+1)^(1/2)-35/128*c*x*(-c^2*x^2+1)^(1/2)+35/128*arcsin(c*x))+
6*d^2*c^4*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*
arcsin(c*x))+8*d*c^2*e*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2
*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.89

$$\int x^3(d + ex^2)^2(a + b \arcsin(cx)) dx$$

$$= \frac{1152 ac^8e^2x^8 + 3072 ac^8dex^6 + 2304 ac^8d^2x^4 + 3(384 bc^8e^2x^8 + 1024 bc^8dex^6 + 768 bc^8d^2x^4 - 288 bc^4d^2)}{921}$$

```
input integrate(x^3*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```


output

```
1/9216*(1152*a*c^8*e^2*x^8 + 3072*a*c^8*d*e*x^6 + 2304*a*c^8*d^2*x^4 + 3*(
384*b*c^8*e^2*x^8 + 1024*b*c^8*d*e*x^6 + 768*b*c^8*d^2*x^4 - 288*b*c^4*d^2
- 320*b*c^2*d*e - 105*b*e^2)*arcsin(c*x) + (144*b*c^7*e^2*x^7 + 8*(64*b*c
^7*d*e + 21*b*c^5*e^2)*x^5 + 2*(288*b*c^7*d^2 + 320*b*c^5*d*e + 105*b*c^3*
e^2)*x^3 + 3*(288*b*c^5*d^2 + 320*b*c^3*d*e + 105*b*c*e^2)*x)*sqrt(-c^2*x^
2 + 1))/c^8
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.59

$$\int x^3 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^2x^4}{4} + \frac{ade^2x^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4 \arcsin(cx)}{4} + \frac{bdex^6 \arcsin(cx)}{3} + \frac{be^2x^8 \arcsin(cx)}{8} + \frac{bd^2x^3 \sqrt{-c^2x^2+1}}{16c} + \frac{bde^2x^5 \sqrt{-c^2x^2+1}}{18c} + \frac{be^2x^7 \sqrt{-c^2x^2+1}}{64c} \\ a \left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8} \right) \end{cases}$$

input

```
integrate(x**3*(e*x**2+d)**2*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asin
(c*x)/4 + b*d*e*x**6*asin(c*x)/3 + b*e**2*x**8*asin(c*x)/8 + b*d**2*x**3*s
qrt(-c**2*x**2 + 1)/(16*c) + b*d*e*x**5*sqrt(-c**2*x**2 + 1)/(18*c) + b*e
**2*x**7*sqrt(-c**2*x**2 + 1)/(64*c) + 3*b*d**2*x*sqrt(-c**2*x**2 + 1)/(32*
c**3) + 5*b*d*e*x**3*sqrt(-c**2*x**2 + 1)/(72*c**3) + 7*b*e**2*x**5*sqrt(-
c**2*x**2 + 1)/(384*c**3) - 3*b*d**2*asin(c*x)/(32*c**4) + 5*b*d*e*x*sqrt(-
c**2*x**2 + 1)/(48*c**5) + 35*b*e**2*x**3*sqrt(-c**2*x**2 + 1)/(1536*c**5
) - 5*b*d*e*asin(c*x)/(48*c**6) + 35*b*e**2*x*sqrt(-c**2*x**2 + 1)/(1024*c
**7) - 35*b*e**2*asin(c*x)/(1024*c**8), Ne(c, 0)), (a*(d**2*x**4/4 + d*e*x
**6/3 + e**2*x**8/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.18

$$\int x^3(d+ex^2)^2(a+b\arcsin(cx))dx = \frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{32}\left(8x^4\arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bd^2 + \frac{1}{144}\left(48x^6\arcsin(cx) + \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7}\right)c\right)bd^2e + \frac{1}{3072}\left(384x^8\arcsin(cx) + \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2x^2+1}x}{c^8} - \frac{105\arcsin(cx)}{c^9}\right)c\right)bd^2e^2$$

input `integrate(x^3*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^2 + 1/144*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*d*e + 1/3072*(384*x^8*arcsin(c*x) + (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*b*e^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(217) = 434$.

Time = 0.14 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.07

$$\begin{aligned}
\int x^3 (d + ex^2)^2 (a + b \arcsin(cx)) dx = & \frac{1}{8} ae^2 x^8 + \frac{1}{3} adex^6 + \frac{1}{4} ad^2 x^4 - \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} bd^2 x}{16 c^3} \\
& + \frac{(c^2 x^2 - 1)^2 bd^2 \arcsin(cx)}{4 c^4} \\
& + \frac{5 \sqrt{-c^2 x^2 + 1} bd^2 x}{32 c^3} \\
& + \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} bde x}{18 c^5} \\
& + \frac{(c^2 x^2 - 1) bd^2 \arcsin(cx)}{2 c^4} \\
& + \frac{(c^2 x^2 - 1)^3 bde \arcsin(cx)}{3 c^6} \\
& - \frac{13 (-c^2 x^2 + 1)^{\frac{3}{2}} bde x}{72 c^5} \\
& + \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1} be^2 x}{64 c^7} \\
& + \frac{5 bd^2 \arcsin(cx)}{32 c^4} + \frac{(c^2 x^2 - 1)^2 bde \arcsin(cx)}{c^6} \\
& + \frac{(c^2 x^2 - 1)^4 be^2 \arcsin(cx)}{8 c^8} \\
& + \frac{11 \sqrt{-c^2 x^2 + 1} bde x}{48 c^5} \\
& + \frac{25 (c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1} be^2 x}{384 c^7} \\
& + \frac{(c^2 x^2 - 1) bde \arcsin(cx)}{c^6} \\
& + \frac{(c^2 x^2 - 1)^3 be^2 \arcsin(cx)}{2 c^8} \\
& - \frac{163 (-c^2 x^2 + 1)^{\frac{3}{2}} be^2 x}{1536 c^7} + \frac{11 bde \arcsin(cx)}{48 c^6} \\
& + \frac{3 (c^2 x^2 - 1)^2 be^2 \arcsin(cx)}{4 c^8} \\
& + \frac{93 \sqrt{-c^2 x^2 + 1} be^2 x}{1024 c^7} \\
& + \frac{(c^2 x^2 - 1) be^2 \arcsin(cx)}{2 c^8} + \frac{93 be^2 \arcsin(cx)}{1024 c^8}
\end{aligned}$$

input `integrate(x^3*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

$$\begin{aligned} & 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 - 1/16*(-c^2*x^2 + 1)^{(3/2)}* \\ & b*d^2*x/c^3 + 1/4*(c^2*x^2 - 1)^2*b*d^2*arcsin(c*x)/c^4 + 5/32*sqrt(-c^2*x \\ & ^2 + 1)*b*d^2*x/c^3 + 1/18*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d*e*x/c^5 \\ & + 1/2*(c^2*x^2 - 1)*b*d^2*arcsin(c*x)/c^4 + 1/3*(c^2*x^2 - 1)^3*b*d*e*arcs \\ & in(c*x)/c^6 - 13/72*(-c^2*x^2 + 1)^{(3/2)}*b*d*e*x/c^5 + 1/64*(c^2*x^2 - 1)^ \\ & 3*sqrt(-c^2*x^2 + 1)*b*e^2*x/c^7 + 5/32*b*d^2*arcsin(c*x)/c^4 + (c^2*x^2 - \\ & 1)^2*b*d*e*arcsin(c*x)/c^6 + 1/8*(c^2*x^2 - 1)^4*b*e^2*arcsin(c*x)/c^8 + \\ & 11/48*sqrt(-c^2*x^2 + 1)*b*d*e*x/c^5 + 25/384*(c^2*x^2 - 1)^2*sqrt(-c^2*x^ \\ & 2 + 1)*b*e^2*x/c^7 + (c^2*x^2 - 1)*b*d*e*arcsin(c*x)/c^6 + 1/2*(c^2*x^2 - \\ & 1)^3*b*e^2*arcsin(c*x)/c^8 - 163/1536*(-c^2*x^2 + 1)^{(3/2)}*b*e^2*x/c^7 + 1 \\ & 1/48*b*d*e*arcsin(c*x)/c^6 + 3/4*(c^2*x^2 - 1)^2*b*e^2*arcsin(c*x)/c^8 + 9 \\ & 3/1024*sqrt(-c^2*x^2 + 1)*b*e^2*x/c^7 + 1/2*(c^2*x^2 - 1)*b*e^2*arcsin(c*x \\ &)/c^8 + 93/1024*b*e^2*arcsin(c*x)/c^8 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x^3 (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int x^3 (a + b \arcsin(cx)) (ex^2 + d)^2 dx$$

input `int(x^3*(a + b*asin(c*x))*(d + e*x^2)^2,x)`

output `int(x^3*(a + b*asin(c*x))*(d + e*x^2)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

$$\begin{aligned} & \int x^3 (d + ex^2)^2 (a + b \arcsin(cx)) dx \\ & = \frac{2304asin(cx) b c^8 d^2 x^4 + 3072asin(cx) b c^8 d e x^6 + 1152asin(cx) b c^8 e^2 x^8 - 864asin(cx) b c^4 d^2 - 960asin(}{ \end{aligned}$$

input `int(x^3*(e*x^2+d)^2*(a+b*asin(c*x)),x)`

output `(2304*asin(c*x)*b*c**8*d**2*x**4 + 3072*asin(c*x)*b*c**8*d*e*x**6 + 1152*a
sin(c*x)*b*c**8*e**2*x**8 - 864*asin(c*x)*b*c**4*d**2 - 960*asin(c*x)*b*c*
*2*d*e - 315*asin(c*x)*b*e**2 + 576*sqrt(-c**2*x**2 + 1)*b*c**7*d**2*x**
3 + 512*sqrt(-c**2*x**2 + 1)*b*c**7*d*e*x**5 + 144*sqrt(-c**2*x**2 + 1)
) * b*c**7*e**2*x**7 + 864*sqrt(-c**2*x**2 + 1)*b*c**5*d**2*x + 640*sqrt(
- c**2*x**2 + 1)*b*c**5*d*e*x**3 + 168*sqrt(-c**2*x**2 + 1)*b*c**5*e**2*x
5 + 960*sqrt(-c2*x**2 + 1)*b*c**3*d*e*x + 210*sqrt(-c**2*x**2 + 1)
) * b*c**3*e**2*x**3 + 315*sqrt(-c**2*x**2 + 1)*b*c*e**2*x + 2304*a*c**8*d
2*x4 + 3072*a*c**8*d*e*x**6 + 1152*a*c**8*e**2*x**8)/(9216*c**8)`

3.434 $\int x^2(d + ex^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	3682
Mathematica [A] (verified)	3683
Rubi [A] (verified)	3683
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Optimal result

Integrand size = 21, antiderivative size = 198

$$\int x^2(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{b(35c^4d^2 + 42c^2de + 15e^2) \sqrt{1 - c^2x^2}}{105c^7} - \frac{b(35c^4d^2 + 84c^2de + 45e^2) (1 - c^2x^2)^{3/2}}{315c^7}$$

$$+ \frac{be(14c^2d + 15e) (1 - c^2x^2)^{5/2}}{175c^7} - \frac{be^2(1 - c^2x^2)^{7/2}}{49c^7}$$

$$+ \frac{1}{3}d^2x^3(a + b \arcsin(cx)) + \frac{2}{5}dex^5(a + b \arcsin(cx)) + \frac{1}{7}e^2x^7(a + b \arcsin(cx))$$

output

```
1/105*b*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^(1/2)/c^7-1/315*b*(35*c^4*d^2+84*c^2*d*e+45*e^2)*(-c^2*x^2+1)^(3/2)/c^7+1/175*b*e*(14*c^2*d+15*e)*(-c^2*x^2+1)^(5/2)/c^7-1/49*b*e^2*(-c^2*x^2+1)^(7/2)/c^7+1/3*d^2*x^3*(a+b*arcsin(c*x))+2/5*d*e*x^5*(a+b*arcsin(c*x))+1/7*e^2*x^7*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int x^2(d + ex^2)^2(a + b \arcsin(cx)) dx$$

$$= \frac{105ax^3(35d^2 + 42dex^2 + 15e^2x^4) + \frac{b\sqrt{1-c^2x^2}(720e^2+24c^2e(98d+15ex^2)+2c^4(1225d^2+588dex^2+135e^2x^4)+c^6(1225d^2x^2+882dex^4+225e^2x^6))}{c^7}}{11025}$$

input

```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSin[c*x]), x]
```

output

```
(105*a*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + (b*Sqrt[1 - c^2*x^2]*(720*
e^2 + 24*c^2*e*(98*d + 15*e*x^2) + 2*c^4*(1225*d^2 + 588*d*e*x^2 + 135*e^2
*x^4) + c^6*(1225*d^2*x^2 + 882*d*e*x^4 + 225*e^2*x^6)))/c^7 + 105*b*x^3*(
35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcSin[c*x])/11025
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5230, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d + ex^2)^2(a + b \arcsin(cx)) dx$$

$$\downarrow 5230$$

$$-bc \int \frac{x^3(15e^2x^4 + 42dex^2 + 35d^2)}{105\sqrt{1-c^2x^2}} dx + \frac{1}{3}d^2x^3(a + b \arcsin(cx)) + \frac{2}{5}dex^5(a + b \arcsin(cx)) + \frac{1}{7}e^2x^7(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{105}bc \int \frac{x^3(15e^2x^4 + 42dex^2 + 35d^2)}{\sqrt{1-c^2x^2}} dx + \frac{1}{3}d^2x^3(a + b \arcsin(cx)) + \frac{2}{5}dex^5(a + b \arcsin(cx)) + \frac{1}{7}e^2x^7(a + b \arcsin(cx))$$

$$\downarrow 1578$$

$$-\frac{1}{210}bc \int \frac{x^2(15e^2x^4 + 42dex^2 + 35d^2)}{\sqrt{1-c^2x^2}} dx^2 + \frac{1}{3}d^2x^3(a + b \arcsin(cx)) + \frac{2}{5}dex^5(a + b \arcsin(cx)) + \frac{1}{7}e^2x^7(a + b \arcsin(cx))$$

$$\downarrow 1195$$

$$-\frac{1}{210}bc \int \left(-\frac{15e^2(1-c^2x^2)^{5/2}}{c^6} + \frac{3e(14dc^2 + 15e)(1-c^2x^2)^{3/2}}{c^6} + \frac{(-35d^2c^4 - 84dec^2 - 45e^2)\sqrt{1-c^2x^2}}{c^6} + \frac{1}{3}d^2x^3(a + b \arcsin(cx)) + \frac{2}{5}dex^5(a + b \arcsin(cx)) + \frac{1}{7}e^2x^7(a + b \arcsin(cx)) \right) dx$$

$$\downarrow 2009$$

$$\frac{1}{3}d^2x^3(a + b \arcsin(cx)) + \frac{2}{5}dex^5(a + b \arcsin(cx)) + \frac{1}{7}e^2x^7(a + b \arcsin(cx)) - \frac{1}{210}bc \left(-\frac{6e(1-c^2x^2)^{5/2}(14c^2d + 15e)}{5c^8} + \frac{30e^2(1-c^2x^2)^{7/2}}{7c^8} + \frac{2(1-c^2x^2)^{3/2}(35c^4d^2 + 84c^2de + 45e^2)}{3c^8} - \frac{2\sqrt{1-c^2x^2}}{c^6} \right)$$

input `Int[x^2*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/210*(b*c*((-2*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*Sqrt[1 - c^2*x^2])/c^8 + (2*(35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(1 - c^2*x^2)^(3/2))/(3*c^8) - (6*e*(14*c^2*d + 15*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (30*e^2*(1 - c^2*x^2)^(7/2))/(7*c^8))) + (d^2*x^3*(a + b*ArcSin[c*x]))/3 + (2*d*e*x^5*(a + b*ArcSin[c*x]))/5 + (e^2*x^7*(a + b*ArcSin[c*x]))/7`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 1195 `Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]`

```
rule 1578 Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5230 Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{1}{7}e^2x^7 + \frac{2}{5}dex^5 + \frac{1}{3}d^2x^3\right) + \frac{b\left(\frac{c^3\arcsin(cx)e^2x^7}{7} + \frac{2c^3\arcsin(cx)dex^5}{5} + \frac{\arcsin(cx)d^2c^3x^3}{3} - \frac{15e^2\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7} - 6\right)}{7}\right)}{c^4}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^2c^7x^3 + \frac{2}{5}dc^7ex^5 + \frac{1}{7}e^2c^7x^7\right)}{c^4} + \frac{b\left(\frac{\arcsin(cx)d^2c^7x^3}{3} + \frac{2\arcsin(cx)dc^7ex^5}{5} + \frac{\arcsin(cx)e^2c^7x^7}{7} - \frac{e^2\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7} - 6\right)}{7}\right)}{c^4}$
default	$\frac{a\left(\frac{1}{3}d^2c^7x^3 + \frac{2}{5}dc^7ex^5 + \frac{1}{7}e^2c^7x^7\right)}{c^4} + \frac{b\left(\frac{\arcsin(cx)d^2c^7x^3}{3} + \frac{2\arcsin(cx)dc^7ex^5}{5} + \frac{\arcsin(cx)e^2c^7x^7}{7} - \frac{e^2\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7} - 6\right)}{7}\right)}{c^4}$
orering	$\frac{(2925c^8e^3x^{10} + 11727c^8d^2e^2x^8 + 17199c^8d^2ex^6 + 270x^8e^3c^6 + 6125c^8d^3x^4 + 1854x^6e^2c^6d + 7938x^4ec^6d^2 + 540x^6e^3c^4 + 2925c^8e^3x^{10} + 11727c^8d^2e^2x^8 + 17199c^8d^2ex^6 + 270x^8e^3c^6 + 6125c^8d^3x^4 + 1854x^6e^2c^6d + 7938x^4ec^6d^2 + 540x^6e^3c^4 + 2925c^8e^3x^{10} + \dots)}{1}$

```
input int(x^2*(e*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*d^2*x^3)+b/c^3*(1/7*c^3*arcsin(c*x)*e^2*x^7
+2/5*c^3*arcsin(c*x)*d*e*x^5+1/3*arcsin(c*x)*d^2*c^3*x^3-1/105/c^4*(15*e^2
*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2
*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))+35*d^2*c^4*(-1/3*c^2*x^2
*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))+42*d*c^2*e*(-1/5*c^4*x^4*(-c^2
*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.94

$$\int x^2 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{1575 ac^7 e^2 x^7 + 4410 ac^7 dex^5 + 3675 ac^7 d^2 x^3 + 105 (15 bc^7 e^2 x^7 + 42 bc^7 dex^5 + 35 bc^7 d^2 x^3) \arcsin(cx) + \dots}{\dots}$$

input

```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
1/11025*(1575*a*c^7*e^2*x^7 + 4410*a*c^7*d*e*x^5 + 3675*a*c^7*d^2*x^3 + 10
5*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3)*arcsin(c*x) + (
225*b*c^6*e^2*x^6 + 2450*b*c^4*d^2 + 2352*b*c^2*d*e + 18*(49*b*c^6*d*e + 1
5*b*c^4*e^2)*x^4 + 720*b*e^2 + (1225*b*c^6*d^2 + 1176*b*c^4*d*e + 360*b*c^
2*e^2)*x^2)*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.68

$$\int x^2 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^2 x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2 x^7}{7} + \frac{bd^2 x^3 \arcsin(cx)}{3} + \frac{2bdex^5 \arcsin(cx)}{5} + \frac{be^2 x^7 \arcsin(cx)}{7} + \frac{bd^2 x^2 \sqrt{-c^2 x^2 + 1}}{9c} + \frac{2bdex^4 \sqrt{-c^2 x^2 + 1}}{25c} + \dots \\ a \left(\frac{d^2 x^3}{3} + \frac{2dex^5}{5} + \frac{e^2 x^7}{7} \right) \end{cases}$$

input

```
integrate(x**2*(e*x**2+d)**2*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a**2*x**7/7 + b*d**2*x**3*asin(c*x)/3 + 2*b*d*e*x**5*asin(c*x)/5 + b**2*x**7*asin(c*x)/7 + b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(9*c) + 2*b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(25*c) + b**2*x**6*sqrt(-c**2*x**2 + 1)/(49*c) + 2*b*d**2*sqrt(-c**2*x**2 + 1)/(9*c**3) + 8*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) + 6*b**2*x**4*sqrt(-c**2*x**2 + 1)/(245*c**3) + 16*b*d*e*sqrt(-c**2*x**2 + 1)/(75*c**5) + 8*b**2*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) + 16*b**2*sqrt(-c**2*x**2 + 1)/(245*c**7), Ne(c, 0)), (a*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.28

$$\int x^2(d + ex^2)^2(a + b \arcsin(cx)) dx = \frac{1}{7}ae^2x^7 + \frac{2}{5}adex^5 + \frac{1}{3}ad^2x^3 + \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2 + 1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2 + 1}}{c^4} \right) \right) bd^2 + \frac{2}{75} \left(15x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2x^2 + 1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2 + 1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2 + 1}}{c^6} \right) c \right) bde + \frac{1}{245} \left(35x^7 \arcsin(cx) + \left(\frac{5\sqrt{-c^2x^2 + 1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2 + 1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2 + 1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2 + 1}}{c^8} \right) c \right) b^2e^2$$

input

```
integrate(x^2*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2 + 2/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d*e + 1/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b^2*e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.17

$$\begin{aligned}
 \int x^2(d+ex^2)^2(a+b\arcsin(cx))dx = & \frac{1}{7}ae^2x^7 + \frac{2}{5}adex^5 + \frac{1}{3}ad^2x^3 \\
 & + \frac{(c^2x^2-1)bd^2x\arcsin(cx)}{3c^2} + \frac{bd^2x\arcsin(cx)}{3c^2} \\
 & + \frac{2(c^2x^2-1)^2bdex\arcsin(cx)}{5c^4} \\
 & + \frac{4(c^2x^2-1)bdex\arcsin(cx)}{5c^4} \\
 & + \frac{(c^2x^2-1)^3be^2x\arcsin(cx)}{7c^6} \\
 & - \frac{(-c^2x^2+1)^{\frac{3}{2}}bd^2}{9c^3} + \frac{2bdex\arcsin(cx)}{5c^4} \\
 & + \frac{3(c^2x^2-1)^2be^2x\arcsin(cx)}{7c^6} \\
 & + \frac{\sqrt{-c^2x^2+1}bd^2}{3c^3} \\
 & + \frac{2(c^2x^2-1)^2\sqrt{-c^2x^2+1}bde}{25c^5} \\
 & + \frac{3(c^2x^2-1)be^2x\arcsin(cx)}{7c^6} \\
 & - \frac{4(-c^2x^2+1)^{\frac{3}{2}}bde}{15c^5} \\
 & + \frac{(c^2x^2-1)^3\sqrt{-c^2x^2+1}be^2}{49c^7} \\
 & + \frac{be^2x\arcsin(cx)}{7c^6} + \frac{2\sqrt{-c^2x^2+1}bde}{5c^5} \\
 & + \frac{3(c^2x^2-1)^2\sqrt{-c^2x^2+1}be^2}{35c^7} \\
 & - \frac{(-c^2x^2+1)^{\frac{3}{2}}be^2}{7c^7} + \frac{\sqrt{-c^2x^2+1}be^2}{7c^7}
 \end{aligned}$$

input `integrate(x^2*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/3*(c^2*x^2 - 1)*b*d^2*x*
arcsin(c*x)/c^2 + 1/3*b*d^2*x*arcsin(c*x)/c^2 + 2/5*(c^2*x^2 - 1)^2*b*d*e*
x*arcsin(c*x)/c^4 + 4/5*(c^2*x^2 - 1)*b*d*e*x*arcsin(c*x)/c^4 + 1/7*(c^2*x
^2 - 1)^3*b*e^2*x*arcsin(c*x)/c^6 - 1/9*(-c^2*x^2 + 1)^(3/2)*b*d^2/c^3 + 2
/5*b*d*e*x*arcsin(c*x)/c^4 + 3/7*(c^2*x^2 - 1)^2*b*e^2*x*arcsin(c*x)/c^6 +
1/3*sqrt(-c^2*x^2 + 1)*b*d^2/c^3 + 2/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)
)*b*d*e/c^5 + 3/7*(c^2*x^2 - 1)*b*e^2*x*arcsin(c*x)/c^6 - 4/15*(-c^2*x^2 +
1)^(3/2)*b*d*e/c^5 + 1/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*e^2/c^7 +
1/7*b*e^2*x*arcsin(c*x)/c^6 + 2/5*sqrt(-c^2*x^2 + 1)*b*d*e/c^5 + 3/35*(c^2
*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e^2/c^7 - 1/7*(-c^2*x^2 + 1)^(3/2)*b*e^2/
c^7 + 1/7*sqrt(-c^2*x^2 + 1)*b*e^2/c^7
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int x^2 (a + b \operatorname{asin}(cx)) (ex^2 + d)^2 dx$$

input

```
int(x^2*(a + b*asin(c*x))*(d + e*x^2)^2,x)
```

output

```
int(x^2*(a + b*asin(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.41

$$\int x^2 (d + ex^2)^2 (a + b \arcsin(cx)) dx$$
$$= \frac{3675 a \operatorname{asin}(cx) b c^7 d^2 x^3 + 4410 a \operatorname{asin}(cx) b c^7 d e x^5 + 1575 a \operatorname{asin}(cx) b c^7 e^2 x^7 + 1225 \sqrt{-c^2 x^2 + 1} b c^6 d^2 x^2 + 8820 a^2 b c^7 d x^4 + 4410 a^2 b c^7 e x^6 + 1575 a^2 b c^7 e^2 x^8 + 1225 a^2 \sqrt{-c^2 x^2 + 1} b c^6 d x + 1225 a^2 b c^7 d x^3 + 4410 a^2 b c^7 e x^5 + 1575 a^2 b c^7 e^2 x^7 + 1225 a^2 \sqrt{-c^2 x^2 + 1} b c^6 d x^2 + 1225 a^2 b c^7 d x^4 + 4410 a^2 b c^7 e x^6 + 1575 a^2 b c^7 e^2 x^8}{1225}$$

input

```
int(x^2*(e*x^2+d)^2*(a+b*asin(c*x)),x)
```

output

```
(3675*asin(c*x)*b*c**7*d**2*x**3 + 4410*asin(c*x)*b*c**7*d*e*x**5 + 1575*a
sin(c*x)*b*c**7*e**2*x**7 + 1225*sqrt(-c**2*x**2 + 1)*b*c**6*d**2*x**2 +
882*sqrt(-c**2*x**2 + 1)*b*c**6*d*e*x**4 + 225*sqrt(-c**2*x**2 + 1)*b
*c**6*e**2*x**6 + 2450*sqrt(-c**2*x**2 + 1)*b*c**4*d**2 + 1176*sqrt(-c
**2*x**2 + 1)*b*c**4*d*e*x**2 + 270*sqrt(-c**2*x**2 + 1)*b*c**4*e**2*x**
4 + 2352*sqrt(-c**2*x**2 + 1)*b*c**2*d*e + 360*sqrt(-c**2*x**2 + 1)*b*
c**2*e**2*x**2 + 720*sqrt(-c**2*x**2 + 1)*b*e**2 + 3675*a*c**7*d**2*x**3
+ 4410*a*c**7*d*e*x**5 + 1575*a*c**7*e**2*x**7)/(11025*c**7)
```

3.435 $\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	3691
Mathematica [A] (verified)	3692
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Optimal result

Integrand size = 19, antiderivative size = 177

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx = \frac{b(24c^4d^2 + 18c^2de + 5e^2) x\sqrt{1 - c^2x^2}}{96c^5} + \frac{be(18c^2d + 5e) x^3\sqrt{1 - c^2x^2}}{144c^3} + \frac{be^2x^5\sqrt{1 - c^2x^2}}{36c} - \frac{b(2c^2d + e)(8c^4d^2 + 8c^2de + 5e^2) \arcsin(cx)}{96c^6e} + \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e}$$

output

```
1/96*b*(24*c^4*d^2+18*c^2*d*e+5*e^2)*x*(-c^2*x^2+1)^(1/2)/c^5+1/144*b*e*(1
8*c^2*d+5*e)*x^3*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*e^2*x^5*(-c^2*x^2+1)^(1/2)/
c-1/96*b*(2*c^2*d+e)*(8*c^4*d^2+8*c^2*d*e+5*e^2)*arcsin(c*x)/c^6/e+1/6*(e*
x^2+d)^3*(a+b*arcsin(c*x))/e
```


Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{cx(48ac^5x(3d^2 + 3dex^2 + e^2x^4) + b\sqrt{1 - c^2x^2}(15e^2 + 2c^2e(27d + 5ex^2) + 4c^4(18d^2 + 9dex^2 + 2e^2x^4)))}{288c^6}$$

input

```
Integrate[x*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
(c*x*(48*a*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + b*Sqrt[1 - c^2*x^2]*(15*e^2 + 2*c^2*e*(27*d + 5*e*x^2) + 4*c^4*(18*d^2 + 9*d*e*x^2 + 2*e^2*x^4))) + 3*b*(-24*c^4*d^2 - 18*c^2*d*e - 5*e^2 + 16*c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcSin[c*x])/(288*c^6)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5228, 318, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5228$$

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \int \frac{(ex^2+d)^3}{\sqrt{1-c^2x^2}} dx}{6e}$$

$$\downarrow 318$$

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \left(-\frac{\int -\frac{(ex^2+d)(5e(2dc^2+e)x^2+d(6dc^2+e))}{\sqrt{1-c^2x^2}} dx}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2} \right)}{6e}$$

$$\downarrow 25$$

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \left(\frac{\int \frac{(ex^2+d)(5e(2dc^2+e)x^2+d(6dc^2+e))}{\sqrt{1-c^2x^2}} dx}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2} \right)}{6e}$$

↓ 403

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \left(\frac{\int -\frac{e(44d^2c^4+44dec^2+15e^2)x^2+d(24d^2c^4+14dec^2+5e^2)}{\sqrt{1-c^2x^2}} dx}{4c^2}}{6c^2} - \frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2} \right)}{6e}$$

↓ 25

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \left(\frac{\int \frac{e(44d^2c^4+44dec^2+15e^2)x^2+d(24d^2c^4+14dec^2+5e^2)}{\sqrt{1-c^2x^2}} dx}{4c^2}}{6c^2} - \frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2} \right)}{6e}$$

↓ 299

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \left(\frac{\frac{3(2c^2d+e)(8c^4d^2+8c^2de+5e^2)}{2c^2} \int \frac{1}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(44c^4d^2+44c^2de+15e^2)}{2c^2}}{6c^2} - \frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2} \right)}{6e}$$

↓ 223

$$\frac{(d + ex^2)^3 (a + b \arcsin(cx))}{6e} - \frac{bc \left(\frac{\frac{3 \arcsin(cx)(2c^2d+e)(8c^4d^2+8c^2de+5e^2)}{2c^3}}{4c^2}}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(44c^4d^2+44c^2de+15e^2)}{2c^2}}{6c^2} - \frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2} \right)}{6e}$$

input `Int [x*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]`

output

$$\begin{aligned} & ((d + e*x^2)^3*(a + b*ArcSin[c*x]))/(6*e) - (b*c*(-1/6*(e*x*sqrt[1 - c^2*x \\ & ^2]*(d + e*x^2)^2)/c^2 + ((-5*e*(2*c^2*d + e)*x*sqrt[1 - c^2*x^2]*(d + e*x \\ & ^2))/(4*c^2) + (-1/2*(e*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*x*sqrt[1 - c^2* \\ & x^2])/c^2 + (3*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e + 5*e^2)*ArcSin[c*x])/ \\ & (2*c^3))/(4*c^2))/(6*c^2))/(6*e) \end{aligned}$$

Defintions of rubi rules used

rule 25

$$\text{Int}[-(F_x), x_Symbol] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[F_x, x], x]$$

rule 223

$$\text{Int}[1/\text{sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{sqrt}[a])]/\text{Rt}[-b, 2], x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$$

rule 299

$$\text{Int}[(a_) + (b_)*(x_)^2]^{(p_)*((c_) + (d_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[d*x \\ *((a + b*x^2)^{(p + 1)}/(b*(2*p + 3))), x] - \text{Simp}[(a*d - b*c*(2*p + 3))/(b*(2 \\ *p + 3)) \quad \text{Int}[(a + b*x^2)^p, x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - \\ a*d, 0] \ \&\& \ \text{NeQ}[2*p + 3, 0]$$

rule 318

$$\text{Int}[(a_) + (b_)*(x_)^2]^{(p_)*((c_) + (d_)*(x_)^2)^{(q_)}, x_Symbol] \rightarrow \text{Sim} \\ p[d*x*(a + b*x^2)^{(p + 1)*((c + d*x^2)^{(q - 1)}/(b*(2*(p + q) + 1))), x] + \text{S} \\ \text{imp}[1/(b*(2*(p + q) + 1)) \quad \text{Int}[(a + b*x^2)^p*(c + d*x^2)^{(q - 2)*\text{Simp}[c*(b \\ *c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) + \\ 1))*x^2, x], x], x] \text{ ; FreeQ}[\{a, b, c, d, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{G} \\ \text{tQ}[q, 1] \ \&\& \ \text{NeQ}[2*(p + q) + 1, 0] \ \&\& \ !\text{IGtQ}[p, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, \\ d, 2, p, q, x]$$

rule 403

$$\text{Int}[(a_) + (b_)*(x_)^2]^{(p_)*((c_) + (d_)*(x_)^2)^{(q_)*((e_) + (f_)*(\\ x_)^2), x_Symbol] \rightarrow \text{Simp}[f*x*(a + b*x^2)^{(p + 1)*((c + d*x^2)^q/(b*(2*(p + \\ q + 1) + 1))), x] + \text{Simp}[1/(b*(2*(p + q + 1) + 1)) \quad \text{Int}[(a + b*x^2)^p*(c \\ + d*x^2)^{(q - 1)*\text{Simp}[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) + \\ f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] \text{ ; FreeQ}[\{a, b, c, \\ d, e, f, p\}, x] \ \&\& \ \text{GtQ}[q, 0] \ \&\& \ \text{NeQ}[2*(p + q + 1) + 1, 0]$$

rule 5228

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))*(x_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
- Simp[b*(c/(2*e*(p + 1))) Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.43

method	result
parts	$\frac{a(e x^2+d)^3}{6e} + \frac{b \left(\frac{c^2 e^2 \arcsin(cx) x^6}{6} + \frac{c^2 e \arcsin(cx) x^4 d}{2} + \frac{\arcsin(cx) c^2 x^2 d^2}{2} + \frac{c^2 \arcsin(cx) d^3}{6e} - \frac{c^6 d^3 \arcsin(cx) + e^3 \left(-\frac{c^5 x^5 \sqrt{1-c^2 x^2}}{5} \right)}{6e} \right)}{6e}$
derivativedivides	$\frac{a(c^2 e x^2 + c^2 d)^3}{6c^4 e} + \frac{b \left(\frac{\arcsin(cx) c^6 d^3}{6e} + \frac{\arcsin(cx) c^6 d^2 x^2}{2} + \frac{e \arcsin(cx) c^6 d x^4}{2} + \frac{e^2 \arcsin(cx) c^6 x^6}{6} - \frac{c^6 d^3 \arcsin(cx) + e^3 \left(-\frac{c^5 x^5 \sqrt{1-c^2 x^2}}{5} \right)}{6e} \right)}{6c^4 e}$
default	$\frac{a(c^2 e x^2 + c^2 d)^3}{6c^4 e} + \frac{b \left(\frac{\arcsin(cx) c^6 d^3}{6e} + \frac{\arcsin(cx) c^6 d^2 x^2}{2} + \frac{e \arcsin(cx) c^6 d x^4}{2} + \frac{e^2 \arcsin(cx) c^6 x^6}{6} - \frac{c^6 d^3 \arcsin(cx) + e^3 \left(-\frac{c^5 x^5 \sqrt{1-c^2 x^2}}{5} \right)}{6e} \right)}{6c^4 e}$
orering	$\frac{(88x^8 e^3 c^6 + 380x^6 e^2 c^6 d + 684x^4 e c^6 d^2 + 10x^6 e^3 c^4 + 216c^6 d^3 x^2 + 92x^4 e^2 c^4 d - 414x^2 e c^4 d^2 + 25x^4 e^3 c^2 - 144d^3 c^4 - 319x^2 e^3 c^2 + 288(e x^2 + d)c^6)}{288(e x^2 + d)c^6}$

input

```
int(x*(e*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

output

```
1/6*a*(e*x^2+d)^3/e+b/c^2*(1/6*c^2*e^2*arcsin(c*x)*x^6+1/2*c^2*e*arcsin(c*
x)*x^4*d+1/2*arcsin(c*x)*c^2*x^2*d^2+1/6*c^2/e*arcsin(c*x)*d^3-1/6/c^4/e*(
c^6*d^3*arcsin(c*x)+e^3*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^
2*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))+3*d*c^2*e^2*(
-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x)
)+3*d^2*c^4*e*(-1/2*c*x*(-c^2*x^2+1)^(1/2)+1/2*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{48 ac^6 e^2 x^6 + 144 ac^6 dex^4 + 144 ac^6 d^2 x^2 + 3(16 bc^6 e^2 x^6 + 48 bc^6 dex^4 + 48 bc^6 d^2 x^2 - 24 bc^4 d^2 - 18 bc^2 de^2 x^5 + 18 bc^2 dex^3 + 18 bc^2 d^2 x)}{c^6}$$

input `integrate(x*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")`output `1/288*(48*a*c^6*e^2*x^6 + 144*a*c^6*d*e*x^4 + 144*a*c^6*d^2*x^2 + 3*(16*b*c^6*e^2*x^6 + 48*b*c^6*d*e*x^4 + 48*b*c^6*d^2*x^2 - 24*b*c^4*d^2 - 18*b*c^2*d*e - 5*b*e^2)*arcsin(c*x) + (8*b*c^5*e^2*x^5 + 2*(18*b*c^5*d*e + 5*b*c^3*e^2)*x^3 + 3*(24*b*c^5*d^2 + 18*b*c^3*d*e + 5*b*c*e^2)*x)*sqrt(-c^2*x^2 + 1))/c^6`**Sympy [A] (verification not implemented)**

Time = 0.49 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.69

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2 \arcsin(cx)}{2} + \frac{bdex^4 \arcsin(cx)}{2} + \frac{be^2x^6 \arcsin(cx)}{6} + \frac{bd^2x\sqrt{-c^2x^2+1}}{4c} + \frac{bdex^3\sqrt{-c^2x^2+1}}{8c} + \frac{be^2x^5}{2} \\ a\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

input `integrate(x*(e*x**2+d)**2*(a+b*asin(c*x)),x)`output `Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asin(c*x)/2 + b*d*e*x**4*asin(c*x)/2 + b*e**2*x**6*asin(c*x)/6 + b*d**2*x*sqrt(-c**2*x**2 + 1)/(4*c) + b*d*e*x**3*sqrt(-c**2*x**2 + 1)/(8*c) + b*e**2*x**5*sqrt(-c**2*x**2 + 1)/(36*c) - b*d**2*asin(c*x)/(4*c**2) + 3*b*d*e*x*sqrt(-c**2*x**2 + 1)/(16*c**3) + 5*b*e**2*x**3*sqrt(-c**2*x**2 + 1)/(144*c**3) - 3*b*d*e*asin(c*x)/(16*c**4) + 5*b*e**2*x*sqrt(-c**2*x**2 + 1)/(96*c**5) - 5*b*e**2*asin(c*x)/(96*c**6), Ne(c, 0)), (a*(d**2*x**2/2 + d*e*x**4/2 + e**2*x**6/6), True))`

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.26

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx = \frac{1}{6} ae^2 x^6 + \frac{1}{2} adex^4 + \frac{1}{2} ad^2 x^2 + \frac{1}{4} \left(2x^2 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x}{c^2} - \frac{\arcsin(cx)}{c^3} \right) \right) bd^2 + \frac{1}{16} \left(8x^4 \arcsin(cx) + \left(\frac{2\sqrt{-c^2 x^2 + 1} x^3}{c^2} + \frac{3\sqrt{-c^2 x^2 + 1} x}{c^4} - \frac{3 \arcsin(cx)}{c^5} \right) c \right) bde + \frac{1}{288} \left(48x^6 \arcsin(cx) + \left(\frac{8\sqrt{-c^2 x^2 + 1} x^5}{c^2} + \frac{10\sqrt{-c^2 x^2 + 1} x^3}{c^4} + \frac{15\sqrt{-c^2 x^2 + 1} x}{c^6} - \frac{15 \arcsin(cx)}{c^7} \right) c \right) b e^2$$

input `integrate(x*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d^2 + 1/16*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d*e + 1/288*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*e^2`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(161) = 322$.

Time = 0.13 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.98

$$\begin{aligned}
 \int x(d + ex^2)^2 (a + b \arcsin(cx)) dx = & \frac{1}{6} ae^2x^6 + \frac{1}{2} adex^4 + \frac{\sqrt{-c^2x^2 + 1}bd^2x}{4c} \\
 & + \frac{(c^2x^2 - 1)bd^2 \arcsin(cx)}{2c^2} - \frac{(-c^2x^2 + 1)^{\frac{3}{2}}bdex}{8c^3} \\
 & + \frac{(c^2x^2 - 1)ad^2}{2c^2} + \frac{bd^2 \arcsin(cx)}{4c^2} \\
 & + \frac{(c^2x^2 - 1)^2bde \arcsin(cx)}{2c^4} + \frac{5\sqrt{-c^2x^2 + 1}bdex}{16c^3} \\
 & + \frac{(c^2x^2 - 1)^2\sqrt{-c^2x^2 + 1}be^2x}{36c^5} \\
 & + \frac{(c^2x^2 - 1)bde \arcsin(cx)}{c^4} \\
 & + \frac{(c^2x^2 - 1)^3be^2 \arcsin(cx)}{6c^6} \\
 & - \frac{13(-c^2x^2 + 1)^{\frac{3}{2}}be^2x}{144c^5} + \frac{5bde \arcsin(cx)}{16c^4} \\
 & + \frac{(c^2x^2 - 1)^2be^2 \arcsin(cx)}{2c^6} \\
 & + \frac{11\sqrt{-c^2x^2 + 1}be^2x}{96c^5} \\
 & + \frac{(c^2x^2 - 1)be^2 \arcsin(cx)}{2c^6} + \frac{11be^2 \arcsin(cx)}{96c^6}
 \end{aligned}$$

input

```
integrate(x*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/4*sqrt(-c^2*x^2 + 1)*b*d^2*x/c + 1/2*(c^2*x^2 - 1)*b*d^2*arcsin(c*x)/c^2 - 1/8*(-c^2*x^2 + 1)^(3/2)*b*d*e*x/c^3 + 1/2*(c^2*x^2 - 1)*a*d^2/c^2 + 1/4*b*d^2*arcsin(c*x)/c^2 + 1/2*(c^2*x^2 - 1)^2*b*d*e*arcsin(c*x)/c^4 + 5/16*sqrt(-c^2*x^2 + 1)*b*d*e*x/c^3 + 1/36*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e^2*x/c^5 + (c^2*x^2 - 1)*b*d*e*arcsin(c*x)/c^4 + 1/6*(c^2*x^2 - 1)^3*b*e^2*arcsin(c*x)/c^6 - 13/144*(-c^2*x^2 + 1)^(3/2)*b*e^2*x/c^5 + 5/16*b*d*e*arcsin(c*x)/c^4 + 1/2*(c^2*x^2 - 1)^2*b*e^2*arcsin(c*x)/c^6 + 11/96*sqrt(-c^2*x^2 + 1)*b*e^2*x/c^5 + 1/2*(c^2*x^2 - 1)*b*e^2*arcsin(c*x)/c^6 + 11/96*b*e^2*arcsin(c*x)/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx = \int x(a + b \arcsin(cx)) (ex^2 + d)^2 dx$$

input

```
int(x*(a + b*asin(c*x))*(d + e*x^2)^2,x)
```

output

```
int(x*(a + b*asin(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.42

$$\int x(d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{144a \sin(cx) b c^6 d^2 x^2 + 144a \sin(cx) b c^6 d e x^4 + 48a \sin(cx) b c^6 e^2 x^6 - 72a \sin(cx) b c^4 d^2 - 54a \sin(cx) b c^2}{c^6}$$

input

```
int(x*(e*x^2+d)^2*(a+b*asin(c*x)),x)
```


output

```
(144*asin(c*x)*b*c**6*d**2*x**2 + 144*asin(c*x)*b*c**6*d*e*x**4 + 48*asin(c*x)*b*c**6*e**2*x**6 - 72*asin(c*x)*b*c**4*d**2 - 54*asin(c*x)*b*c**2*d*e - 15*asin(c*x)*b*e**2 + 72*sqrt(-c**2*x**2 + 1)*b*c**5*d**2*x + 36*sqrt(-c**2*x**2 + 1)*b*c**5*d*e*x**3 + 8*sqrt(-c**2*x**2 + 1)*b*c**5*e**2*x**5 + 54*sqrt(-c**2*x**2 + 1)*b*c**3*d*e*x + 10*sqrt(-c**2*x**2 + 1)*b*c**3*e**2*x**3 + 15*sqrt(-c**2*x**2 + 1)*b*c*e**2*x + 144*a*c**6*d**2*x**2 + 144*a*c**6*d*e*x**4 + 48*a*c**6*e**2*x**6)/(288*c**6)
```

3.436 $\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	3701
Mathematica [A] (verified)	3702
Rubi [A] (verified)	3702
Maple [A] (verified)	3704
Fricas [A] (verification not implemented)	3705
Sympy [A] (verification not implemented)	3706
Maxima [A] (verification not implemented)	3706
Giac [A] (verification not implemented)	3707
Mupad [F(-1)]	3708
Reduce [B] (verification not implemented)	3708

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx = \frac{b(15c^4d^2 + 10c^2de + 3e^2) \sqrt{1 - c^2x^2}}{15c^5} - \frac{2be(5c^2d + 3e)(1 - c^2x^2)^{3/2}}{45c^5} + \frac{be^2(1 - c^2x^2)^{5/2}}{25c^5} + d^2x(a + b \arcsin(cx)) + \frac{2}{3}dex^3(a + b \arcsin(cx)) + \frac{1}{5}e^2x^5(a + b \arcsin(cx))$$

output

```
1/15*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*(-c^2*x^2+1)^(1/2)/c^5-2/45*b*e*(5*c^2*d+3*e)*(-c^2*x^2+1)^(3/2)/c^5+1/25*b*e^2*(-c^2*x^2+1)^(5/2)/c^5+d^2*x*(a+b*arcsin(c*x))+2/3*d*e*x^3*(a+b*arcsin(c*x))+1/5*e^2*x^5*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{1}{225} \left(15ax(15d^2 + 10dex^2 + 3e^2x^4) + \frac{b\sqrt{1-c^2x^2}(24e^2 + 4c^2e(25d + 3ex^2) + c^4(225d^2 + 50dex^2 + 9e^2x^4))}{c^5} + 15bx(15d^2 + 10dex^2 + 3e^2x^4) \arcsin(cx) \right)$$

input `Integrate[(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `(15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 - c^2*x^2]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 + 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSin[c*x])/225`

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5170, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$\downarrow 5170$$

$$-bc \int \frac{x(3e^2x^4 + 10dex^2 + 15d^2)}{15\sqrt{1-c^2x^2}} dx + d^2x(a + b \arcsin(cx)) + \frac{2}{3}dex^3(a + b \arcsin(cx)) + \frac{1}{5}e^2x^5(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$-\frac{1}{15}bc \int \frac{x(3e^2x^4 + 10dex^2 + 15d^2)}{\sqrt{1-c^2x^2}} dx + d^2x(a + b \arcsin(cx)) + \frac{2}{3}dex^3(a + b \arcsin(cx)) + \frac{1}{5}e^2x^5(a + b \arcsin(cx))$$

↓ 1576

$$-\frac{1}{30}bc \int \frac{3e^2x^4 + 10dex^2 + 15d^2}{\sqrt{1-c^2x^2}} dx^2 + d^2x(a + b \arcsin(cx)) + \frac{2}{3}dex^3(a + b \arcsin(cx)) + \frac{1}{5}e^2x^5(a + b \arcsin(cx))$$

↓ 1140

$$-\frac{1}{30}bc \int \left(\frac{3(1-c^2x^2)^{3/2}e^2}{c^4} - \frac{2(5dc^2+3e)\sqrt{1-c^2x^2}e}{c^4} + \frac{15d^2c^4+10dec^2+3e^2}{c^4\sqrt{1-c^2x^2}} \right) dx^2 + d^2x(a + b \arcsin(cx)) + \frac{2}{3}dex^3(a + b \arcsin(cx)) + \frac{1}{5}e^2x^5(a + b \arcsin(cx))$$

↓ 2009

$$d^2x(a + b \arcsin(cx)) + \frac{2}{3}dex^3(a + b \arcsin(cx)) + \frac{1}{5}e^2x^5(a + b \arcsin(cx)) - \frac{1}{30}bc \left(\frac{4e(1-c^2x^2)^{3/2}(5c^2d+3e)}{3c^6} - \frac{6e^2(1-c^2x^2)^{5/2}}{5c^6} - \frac{2\sqrt{1-c^2x^2}(15c^4d^2+10c^2de+3e^2)}{c^6} \right)$$

input `Int[(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `-1/30*(b*c*((-2*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[1 - c^2*x^2])/c^6 + (4*e*(5*c^2*d + 3*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) - (6*e^2*(1 - c^2*x^2)^(5/2))/(5*c^6))) + d^2*x*(a + b*ArcSin[c*x]) + (2*d*e*x^3*(a + b*ArcSin[c*x]))/3 + (e^2*x^5*(a + b*ArcSin[c*x]))/5`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 1140 `Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]`

rule 1576 `Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5170 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])`

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

method	result
parts	$a\left(\frac{1}{5}e^2x^5 + \frac{2}{3}dex^3 + d^2x\right) + \frac{b\left(\frac{c\arcsin(cx)e^2x^5}{5} + \frac{2c\arcsin(cx)dex^3}{3} + \arcsin(cx)d^2cx - \frac{3e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5} - \frac{4c^2x^2}{5}\right)}{c}\right)}{c}$
derivativedivides	$\frac{a\left(d^2c^5x + \frac{2}{3}dc^5ex^3 + \frac{1}{5}e^2c^5x^5\right)}{c^4} + \frac{b\left(\arcsin(cx)d^2c^5x + \frac{2\arcsin(cx)d c^5ex^3}{3} + \frac{\arcsin(cx)e^2c^5x^5}{5} - \frac{e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5} - \frac{4c^2x^2}{5}\right)}{c}\right)}{c}$
default	$\frac{a\left(d^2c^5x + \frac{2}{3}dc^5ex^3 + \frac{1}{5}e^2c^5x^5\right)}{c^4} + \frac{b\left(\arcsin(cx)d^2c^5x + \frac{2\arcsin(cx)d c^5ex^3}{3} + \frac{\arcsin(cx)e^2c^5x^5}{5} - \frac{e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5} - \frac{4c^2x^2}{5}\right)}{c}\right)}{c}$
oring	$\frac{x(81e^3x^6c^6 + 395c^6de^2x^4 + 1275c^6d^2ex^2 + 12c^4e^3x^4 + 225c^6d^3 + 200c^4de^2x^2 - 900c^4d^2e + 48c^2e^3x^2 - 400c^2de^2 - 96e^3)}{225(e^2x^2 + d)c^6}$

```
input int((e*x^2+d)^2*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arcsin(c*x)*e^2*x^5+2/3*c*arcsin(c*x)*d*e*x^3+arcsin(c*x)*d^2*c*x-1/15/c^4*(3*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))-15*d^2*c^4*(-c^2*x^2+1)^(1/2)+10*d*c^2*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx = \frac{45 ac^5 e^2 x^5 + 150 ac^5 dex^3 + 225 ac^5 d^2 x + 15 (3 bc^5 e^2 x^5 + 10 bc^5 dex^3 + 15 bc^5 d^2 x) \arcsin(cx) + (9 bc^4 e^2 x^5 + 18 bc^4 dex^3 + 15 bc^4 d^2 x) \arcsin^2(cx)}{225 c^5}$$

```
input integrate((e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*arcsin(c*x) + (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 + 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4*d*e + 6*b*c^2*e^2)*x^2)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.60

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x \operatorname{asin}(cx) + \frac{2bdex^3 \operatorname{asin}(cx)}{3} + \frac{be^2x^5 \operatorname{asin}(cx)}{5} + \frac{bd^2\sqrt{-c^2x^2+1}}{c} + \frac{2bdex^2\sqrt{-c^2x^2+1}}{9c} + b \end{cases}$$

$$= \begin{cases} a \left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5} \right) \end{cases}$$

input

```
integrate((e*x**2+d)**2*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*asin(c*x) + 2*b*d*e*x**3*asin(c*x)/3 + b*e**2*x**5*asin(c*x)/5 + b*d**2*sqrt(-c**2*x**2 + 1)/c + 2*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(9*c) + b*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) + 4*b*d*e*sqrt(-c**2*x**2 + 1)/(9*c**3) + 4*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) + 8*b*e**2*sqrt(-c**2*x**2 + 1)/(75*c**5), Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{1}{5} ae^2x^5 + \frac{2}{3} adex^3 + \frac{2}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4} \right) \right) bde$$

$$+ \frac{1}{75} \left(15x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6} \right) c \right) be^2$$

$$+ ad^2x + \frac{(cx \arcsin(cx) + \sqrt{-c^2x^2+1})bd^2}{c}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output
$$\begin{aligned} & 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arcsin(c*x) \\ & + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d^2/c \end{aligned}$$

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.77

$$\begin{aligned} \int (d + ex^2)^2 (a + b \arcsin(cx)) dx &= \frac{1}{5} ae^2 x^5 + \frac{2}{3} adex^3 + bd^2 x \arcsin(cx) \\ &+ ad^2 x + \frac{2(c^2 x^2 - 1)bdex \arcsin(cx)}{3c^2} \\ &+ \frac{2bdex \arcsin(cx)}{3c^2} + \frac{(c^2 x^2 - 1)^2 be^2 x \arcsin(cx)}{5c^4} \\ &+ \frac{\sqrt{-c^2 x^2 + 1}bd^2}{c} + \frac{2(c^2 x^2 - 1)be^2 x \arcsin(cx)}{5c^4} \\ &- \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}}bde}{9c^3} + \frac{be^2 x \arcsin(cx)}{5c^4} \\ &+ \frac{2\sqrt{-c^2 x^2 + 1}bde}{3c^3} + \frac{(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}be^2}{25c^5} \\ &- \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}}be^2}{15c^5} + \frac{\sqrt{-c^2 x^2 + 1}be^2}{5c^5} \end{aligned}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")`

output
$$\begin{aligned} & 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + b*d^2*x*arcsin(c*x) + a*d^2*x + 2/3*(c^2*x \\ & ^2 - 1)*b*d*e*x*arcsin(c*x)/c^2 + 2/3*b*d*e*x*arcsin(c*x)/c^2 + 1/5*(c^2*x \\ & ^2 - 1)^2*b*e^2*x*arcsin(c*x)/c^4 + sqrt(-c^2*x^2 + 1)*b*d^2/c + 2/5*(c^2*x \\ & ^2 - 1)*b*e^2*x*arcsin(c*x)/c^4 - 2/9*(-c^2*x^2 + 1)^{(3/2)}*b*d*e/c^3 + 1/ \\ & 5*b*e^2*x*arcsin(c*x)/c^4 + 2/3*sqrt(-c^2*x^2 + 1)*b*d*e/c^3 + 1/25*(c^2*x \\ & ^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e^2/c^5 - 2/15*(-c^2*x^2 + 1)^{(3/2)}*b*e^2/c \\ & ^5 + 1/5*sqrt(-c^2*x^2 + 1)*b*e^2/c^5 \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (ex^2 + d)^2 dx$$

input `int((a + b*asin(c*x))*(d + e*x^2)^2,x)`output `int((a + b*asin(c*x))*(d + e*x^2)^2, x)`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.39

$$\int (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{225 \arcsin(cx) b c^5 d^2 x + 150 \arcsin(cx) b c^5 d e x^3 + 45 \arcsin(cx) b c^5 e^2 x^5 + 225 \sqrt{-c^2 x^2 + 1} b c^4 d^2 + 50 \sqrt{-c^2 x^2}}$$

input `int((e*x^2+d)^2*(a+b*asin(c*x)),x)`output `(225*asin(c*x)*b*c**5*d**2*x + 150*asin(c*x)*b*c**5*d*e*x**3 + 45*asin(c*x)*b*c**5*e**2*x**5 + 225*sqrt(-c**2*x**2 + 1)*b*c**4*d**2 + 50*sqrt(-c**2*x**2 + 1)*b*c**4*d*e*x**2 + 9*sqrt(-c**2*x**2 + 1)*b*c**4*e**2*x**4 + 100*sqrt(-c**2*x**2 + 1)*b*c**2*d*e + 12*sqrt(-c**2*x**2 + 1)*b*c**2*e**2*x**2 + 24*sqrt(-c**2*x**2 + 1)*b*e**2 + 225*a*c**5*d**2*x + 150*a*c**5*d*e*x**3 + 45*a*c**5*e**2*x**5)/(225*c**5)`

3.437 $\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x} dx$

Optimal result	3709
Mathematica [A] (verified)	3710
Rubi [A] (verified)	3710
Maple [A] (verified)	3712
Fricas [F]	3713
Sympy [F]	3713
Maxima [F]	3714
Giac [F(-2)]	3714
Mupad [F(-1)]	3714
Reduce [F]	3715

Optimal result

Integrand size = 21, antiderivative size = 229

$$\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x} dx = \frac{bdex\sqrt{1-c^2x^2}}{2c} + \frac{3be^2x\sqrt{1-c^2x^2}}{32c^3} + \frac{be^2x^3\sqrt{1-c^2x^2}}{16c} - \frac{bde \arcsin(cx)}{2c^2} - \frac{3be^2 \arcsin(cx)}{32c^4} - \frac{1}{2}ibd^2 \arcsin(cx)^2 + dex^2(a+b \arcsin(cx)) + \frac{1}{4}e^2x^4(a+b \arcsin(cx)) + bd^2 \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) - bd^2 \arcsin(cx) \log(x) + d^2(a+b \arcsin(cx)) \log(x) - \frac{1}{2}ibd^2 \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
1/2*b*d*e*x*(-c^2*x^2+1)^(1/2)/c+3/32*b*e^2*x*(-c^2*x^2+1)^(1/2)/c^3+1/16*
b*e^2*x^3*(-c^2*x^2+1)^(1/2)/c-1/2*b*d*e*arcsin(c*x)/c^2-3/32*b*e^2*arcsin
(c*x)/c^4-1/2*I*b*d^2*arcsin(c*x)^2+d*e*x^2*(a+b*arcsin(c*x))+1/4*e^2*x^4*
(a+b*arcsin(c*x))+b*d^2*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-b*d
^2*arcsin(c*x)*ln(x)+d^2*(a+b*arcsin(c*x))*ln(x)-1/2*I*b*d^2*polylog(2,(I*
c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.95

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx$$

$$= \frac{1}{4} \left(4adex^2 + ae^2x^4 + 4bdex^2 \arcsin(cx) + be^2x^4 \arcsin(cx) \right. \\ \left. + \frac{be^2 \left(cx\sqrt{1 - c^2x^2}(3 + 2c^2x^2) - 6 \arctan \left(\frac{cx}{-1 + \sqrt{1 - c^2x^2}} \right) \right)}{8c^4} \right. \\ \left. + \frac{2bde \left(cx\sqrt{1 - c^2x^2} - 2 \arctan \left(\frac{cx}{-1 + \sqrt{1 - c^2x^2}} \right) \right)}{c^2} \right. \\ \left. + 4bd^2 \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) + 4ad^2 \log(x) \right. \\ \left. - 2ibd^2 (\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)})) \right)$$

input

```
Integrate[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x,x]
```

output

```
(4*a*d*e*x^2 + a*e^2*x^4 + 4*b*d*e*x^2*ArcSin[c*x] + b*e^2*x^4*ArcSin[c*x]
+ (b*e^2*(c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2) - 6*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2]])))/(8*c^4) + (2*b*d*e*(c*x*Sqrt[1 - c^2*x^2] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2]])))/c^2 + 4*b*d^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 4*a*d^2*Log[x] - (2*I)*b*d^2*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])]))/4
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx \\
& \quad \downarrow \text{5230} \\
& -bc \int \frac{e^2 x^4 + 4dex^2 + 4d^2 \log(x)}{4\sqrt{1 - c^2 x^2}} dx + d^2 \log(x)(a + b \arcsin(cx)) + dex^2(a + b \arcsin(cx)) + \\
& \quad \frac{1}{4} e^2 x^4 (a + b \arcsin(cx)) \\
& \quad \downarrow \text{27} \\
& -\frac{1}{4} bc \int \frac{e^2 x^4 + 4dex^2 + 4d^2 \log(x)}{\sqrt{1 - c^2 x^2}} dx + d^2 \log(x)(a + b \arcsin(cx)) + dex^2(a + b \arcsin(cx)) + \\
& \quad \frac{1}{4} e^2 x^4 (a + b \arcsin(cx)) \\
& \quad \downarrow \text{7293} \\
& -\frac{1}{4} bc \int \left(\frac{e^2 x^4}{\sqrt{1 - c^2 x^2}} + \frac{4dex^2}{\sqrt{1 - c^2 x^2}} + \frac{4d^2 \log(x)}{\sqrt{1 - c^2 x^2}} \right) dx + d^2 \log(x)(a + b \arcsin(cx)) + dex^2(a + \\
& \quad b \arcsin(cx)) + \frac{1}{4} e^2 x^4 (a + b \arcsin(cx)) \\
& \quad \downarrow \text{2009} \\
& d^2 \log(x)(a + b \arcsin(cx)) + dex^2(a + b \arcsin(cx)) + \frac{1}{4} e^2 x^4 (a + b \arcsin(cx)) - \\
& \frac{1}{4} bc \left(\frac{3e^2 \arcsin(cx)}{8c^5} + \frac{2de \arcsin(cx)}{c^3} + \frac{2id^2 \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{c} + \frac{2id^2 \arcsin(cx)^2}{c} - \frac{4d^2 \arcsin(cx) \log(1 - c^2 x^2)}{c} \right)
\end{aligned}$$

input `Int[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x,x]`

output `d*e*x^2*(a + b*ArcSin[c*x]) + (e^2*x^4*(a + b*ArcSin[c*x]))/4 + d^2*(a + b*ArcSin[c*x])*Log[x] - (b*c*((-2*d*e*x*sqrt[1 - c^2*x^2])/c^2 - (3*e^2*x*sqrt[1 - c^2*x^2])/(8*c^4) - (e^2*x^3*sqrt[1 - c^2*x^2])/(4*c^2) + (2*d*e*ArcSin[c*x])/c^3 + (3*e^2*ArcSin[c*x])/(8*c^5) + ((2*I)*d^2*ArcSin[c*x]^2)/c - (4*d^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c + (4*d^2*ArcSin[c*x]*Log[x])/c + ((2*I)*d^2*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c)/4`

Defintions of rubi rules used

rule 27 `Int[(a_)*(Fx), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5230 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.05

method	result
parts	$a\left(\frac{e^2 x^4}{4} + d e x^2 + d^2 \ln(x)\right) + b\left(-\frac{i d^2 \arcsin(cx)^2}{2} + d^2 \arcsin(cx) \ln(1 + i c x + \sqrt{-c^2 x^2 - 1})\right)$
derivativedivides	$a d e x^2 + \frac{a e^2 x^4}{4} + a d^2 \ln(cx) + \frac{b\left(-\frac{i c^4 d^2 \arcsin(cx)^2}{2} + c^4 d^2 \arcsin(cx) \ln(1 - i c x - \sqrt{-c^2 x^2 + 1}) + c^4 d^2 \arcsin(cx) \ln(1 + i c x + \sqrt{-c^2 x^2 - 1})\right)}{2}$
default	$a d e x^2 + \frac{a e^2 x^4}{4} + a d^2 \ln(cx) + \frac{b\left(-\frac{i c^4 d^2 \arcsin(cx)^2}{2} + c^4 d^2 \arcsin(cx) \ln(1 - i c x - \sqrt{-c^2 x^2 + 1}) + c^4 d^2 \arcsin(cx) \ln(1 + i c x + \sqrt{-c^2 x^2 - 1})\right)}{2}$

input `int((e*x^2+d)^2*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output

```
a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(-1/2*I*d^2*arcsin(c*x)^2+d^2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+d^2*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*d^2*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-I*d^2*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))+1/32*arcsin(c*x)*e^2/c^4*cos(4*arcsin(c*x))-1/128*e^2/c^4*sin(4*arcsin(c*x))-1/8*e*arcsin(c*x)*(4*c^2*d+e)/c^4*cos(2*arcsin(c*x))+1/4*e/c^2*sin(2*arcsin(c*x))*d+1/16*e^2/c^4*sin(2*arcsin(c*x)))
```

Fricas [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(ex^2 + d)^2 (b \arcsin(cx) + a)}{x} dx$$

input

```
integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x,x, algorithm="fricas")
```

output

```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arcsin(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (d + ex^2)^2}{x} dx$$

input

```
integrate((e*x**2+d)**2*(a+b*asin(c*x))/x,x)
```

output

```
Integral((a + b*asin(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(ex^2 + d)^2 (b \arcsin(cx) + a)}{x} dx$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate((b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^2}{x} dx$$

input `int(((a + b*asin(c*x))*(d + e*x^2)^2)/x,x)`

output `int(((a + b*asin(c*x))*(d + e*x^2)^2)/x, x)`

Reduce [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x} dx$$

$$= \frac{32a \sin(cx) b c^4 d e x^2 + 8a \sin(cx) b c^4 e^2 x^4 - 16a \sin(cx) b c^2 d e - 3a \sin(cx) b e^2 + 16\sqrt{-c^2 x^2 + 1} b c^3 d e x + \dots}{32c^4}$$

input `int((e*x^2+d)^2*(a+b*asin(c*x))/x,x)`

output `(32*asin(c*x)*b*c**4*d*e*x**2 + 8*asin(c*x)*b*c**4*e**2*x**4 - 16*asin(c*x)*b*c**2*d*e - 3*asin(c*x)*b*e**2 + 16*sqrt(-c**2*x**2 + 1)*b*c**3*d*e*x + 2*sqrt(-c**2*x**2 + 1)*b*c**3*e**2*x**3 + 3*sqrt(-c**2*x**2 + 1)*b*c*e**2*x + 32*int(asin(c*x)/x,x)*b*c**4*d**2 + 32*log(x)*a*c**4*d**2 + 32*a*c**4*d*e*x**2 + 8*a*c**4*e**2*x**4)/(32*c**4)`

3.438 $\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x^2} dx$

Optimal result	3716
Mathematica [A] (verified)	3717
Rubi [A] (warning: unable to verify)	3717
Maple [A] (verified)	3720
Fricas [A] (verification not implemented)	3720
Sympy [A] (verification not implemented)	3721
Maxima [A] (verification not implemented)	3722
Giac [B] (verification not implemented)	3722
Mupad [F(-1)]	3723
Reduce [B] (verification not implemented)	3724

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x^2} dx = \frac{be(6c^2d+e)\sqrt{1-c^2x^2}}{3c^3} - \frac{be^2(1-c^2x^2)^{3/2}}{9c^3} - \frac{d^2(a+b \arcsin(cx))}{x} + 2dex(a+b \arcsin(cx)) + \frac{1}{3}e^2x^3(a+b \arcsin(cx)) - bcd^2 \operatorname{arctanh}(\sqrt{1-c^2x^2})$$

output

```
1/3*b*e*(6*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3-1/9*b*e^2*(-c^2*x^2+1)^(3/2)/c^3-d^2*(a+b*arcsin(c*x))/x+2*d*e*x*(a+b*arcsin(c*x))+1/3*e^2*x^3*(a+b*arcsin(c*x))-b*c*d^2*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx = \frac{1}{9} \left(-\frac{9ad^2}{x} + 18adex + 3ae^2x^3 + \frac{be\sqrt{1 - c^2x^2}(2e + c^2(18d + ex^2))}{c^3} + \frac{3b(-3d^2 + 6dex^2 + e^2x^4) \arcsin(cx)}{x} + 9bcd^2 \log(x) - 9bcd^2 \log\left(1 + \sqrt{1 - c^2x^2}\right) \right)$$

input `Integrate[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x^2,x]`

output `((-9*a*d^2)/x + 18*a*d*e*x + 3*a*e^2*x^3 + (b*e*Sqrt[1 - c^2*x^2]*(2*e + c^2*(18*d + e*x^2)))/c^3 + (3*b*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcSin[c*x])/x + 9*b*c*d^2*Log[x] - 9*b*c*d^2*Log[1 + Sqrt[1 - c^2*x^2]])/9`

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5230, 27, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$\downarrow \text{5230}$$

$$-bc \int -\frac{-e^2x^4 - 6dex^2 + 3d^2}{3x\sqrt{1 - c^2x^2}} dx - \frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx))$$

$$\downarrow \text{27}$$

$$\frac{1}{3}bc \int \frac{-e^2x^4 - 6dex^2 + 3d^2}{x\sqrt{1-c^2x^2}} dx - \frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx))$$

↓ 1578

$$\frac{1}{6}bc \int \frac{-e^2x^4 - 6dex^2 + 3d^2}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx))$$

↓ 1192

$$\frac{b \int \frac{-e^2x^8 + 2e(3dc^2 + e)x^4 + 3c^4d^2 - e^2 - 6c^2de}{1-x^4} d\sqrt{1-c^2x^2}}{3c^3} - \frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx))$$

↓ 25

$$\frac{b \int \frac{-e^2x^8 + 2e(3dc^2 + e)x^4 + 3c^4d^2 - e^2 - 6c^2de}{1-x^4} d\sqrt{1-c^2x^2}}{3c^3} - \frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx))$$

↓ 1467

$$\frac{b \int \left(\frac{3d^2c^4}{1-x^4} + e^2x^4 - e(6dc^2 + e) \right) d\sqrt{1-c^2x^2}}{3c^3} - \frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx))$$

↓ 2009

$$-\frac{d^2(a + b \arcsin(cx))}{x} + 2dex(a + b \arcsin(cx)) + \frac{1}{3}e^2x^3(a + b \arcsin(cx)) + \frac{b \left(-3c^4d^2 \operatorname{arctanh}(\sqrt{1-c^2x^2}) + e\sqrt{1-c^2x^2}(6c^2d + e) - \frac{1}{3}e^2x^6 \right)}{3c^3}$$

input

```
Int[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```
-((d^2*(a + b*ArcSin[c*x]))/x) + 2*d*e*x*(a + b*ArcSin[c*x]) + (e^2*x^3*(a + b*ArcSin[c*x]))/3 + (b*(-1/3*(e^2*x^6) + e*(6*c^2*d + e)*Sqrt[1 - c^2*x^2] - 3*c^4*d^2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(3*c^3)
```

Definitions of rubi rules used

- rule 25 $\text{Int}[-(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[\text{Fx}, \text{x}], \text{x}]$
- rule 27 $\text{Int}[(\text{a}_)*(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{a} \quad \text{Int}[\text{Fx}, \text{x}], \text{x}] \text{ ; FreeQ}[\text{a}, \text{x}] \ \&\& \ \text{!MatchQ}[\text{Fx}, (\text{b}_)*(\text{Gx}_)] \text{ ; FreeQ}[\text{b}, \text{x}]$
- rule 1192 $\text{Int}[(\text{d}_.) + (\text{e}_.)*(\text{x}_)^{(\text{m}_)}*((\text{f}_.) + (\text{g}_.)*(\text{x}_)^{(\text{n}_)}*((\text{a}_.) + (\text{b}_.)*(\text{x}_) + (\text{c}_.)*(\text{x}_)^2)^{(\text{p}_.)}, \text{x_Symbol}] \rightarrow \text{Simp}[2/\text{e}^{(\text{n} + 2*\text{p} + 1)} \quad \text{Subst}[\text{Int}[\text{x}^{(2*\text{m} + 1)*(\text{e}*\text{f} - \text{d}*\text{g} + \text{g}*\text{x}^2)^{\text{n}}*(\text{c}*\text{d}^2 - \text{b}*\text{d}*\text{e} + \text{a}*\text{e}^2 - (2*\text{c}*\text{d} - \text{b}*\text{e})*\text{x}^2 + \text{c}*\text{x}^4)^{\text{p}}, \text{x}], \text{x}, \text{Sqrt}[\text{d} + \text{e}*\text{x}], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{g}\}, \text{x}] \ \&\& \ \text{IGtQ}[\text{p}, 0] \ \&\& \ \text{ILtQ}[\text{n}, 0] \ \&\& \ \text{IntegerQ}[\text{m} + 1/2]$
- rule 1467 $\text{Int}[(\text{d}_.) + (\text{e}_.)*(\text{x}_)^2)^{(\text{q}_.)}*((\text{a}_.) + (\text{b}_.)*(\text{x}_)^2 + (\text{c}_.)*(\text{x}_)^4)^{(\text{p}_.)}, \text{x_Symbol}] \rightarrow \text{Int}[\text{ExpandIntegrand}[(\text{d} + \text{e}*\text{x}^2)^{\text{q}}*(\text{a} + \text{b}*\text{x}^2 + \text{c}*\text{x}^4)^{\text{p}}, \text{x}], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{b}^2 - 4*\text{a}*\text{c}, 0] \ \&\& \ \text{NeQ}[\text{c}*\text{d}^2 - \text{b}*\text{d}*\text{e} + \text{a}*\text{e}^2, 0] \ \&\& \ \text{IGtQ}[\text{p}, 0] \ \&\& \ \text{IGtQ}[\text{q}, -2]$
- rule 1578 $\text{Int}[(\text{x}_)^{(\text{m}_)}*((\text{d}_.) + (\text{e}_.)*(\text{x}_)^2)^{(\text{q}_.)}*((\text{a}_.) + (\text{b}_.)*(\text{x}_)^2 + (\text{c}_.)*(\text{x}_)^4)^{(\text{p}_.)}, \text{x_Symbol}] \rightarrow \text{Simp}[1/2 \quad \text{Subst}[\text{Int}[\text{x}^{((\text{m} - 1)/2)*(\text{d} + \text{e}*\text{x})^{\text{q}}*(\text{a} + \text{b}*\text{x} + \text{c}*\text{x}^2)^{\text{p}}, \text{x}], \text{x}, \text{x}^2], \text{x}] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{p}, \text{q}\}, \text{x}] \ \&\& \ \text{IntegerQ}[(\text{m} - 1)/2]$
- rule 2009 $\text{Int}[\text{u}_, \text{x_Symbol}] \rightarrow \text{Simp}[\text{IntSum}[\text{u}, \text{x}], \text{x}] \text{ ; SumQ}[\text{u}]$
- rule 5230 $\text{Int}[(\text{a}_.) + \text{ArcSin}[(\text{c}_.)*(\text{x}_)]*(\text{b}_.)]*((\text{f}_.)*(\text{x}_)^{(\text{m}_)}*((\text{d}_.) + (\text{e}_.)*(\text{x}_)^2)^{(\text{p}_.)}, \text{x_Symbol}] \rightarrow \text{With}[\{\text{u} = \text{IntHide}[(\text{f}*\text{x})^{\text{m}}*(\text{d} + \text{e}*\text{x}^2)^{\text{p}}, \text{x}]\}, \text{Simp}[(\text{a} + \text{b}*\text{ArcSin}[\text{c}*\text{x}]) \quad \text{u}, \text{x}] - \text{Simp}[\text{b}*\text{c} \quad \text{Int}[\text{SimplifyIntegrand}[\text{u}/\text{Sqrt}[1 - \text{c}^2*\text{x}^2], \text{x}], \text{x}], \text{x}]] \text{ ; FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{m}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{c}^2*\text{d} + \text{e}, 0] \ \&\& \ \text{IntegerQ}[\text{p}] \ \&\& \ (\text{GtQ}[\text{p}, 0] \ || \ (\text{IGtQ}[(\text{m} - 1)/2, 0] \ \&\& \ \text{LeQ}[\text{m} + \text{p}, 0]))$

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

method	result
parts	$a\left(\frac{e^2x^3}{3} + 2dex - \frac{d^2}{x}\right) + bc\left(\frac{\arcsin(cx)x^3e^2}{3c} + \frac{2\arcsin(cx)xde}{c} - \frac{\arcsin(cx)d^2}{cx} - \frac{e^2\left(-\frac{e^2x^2\sqrt{-c^2x^2+1}}{3}\right)}{3}\right)$
derivativedivides	$c\left(\frac{a\left(2c^3dex + \frac{e^2c^3x^3}{3} - \frac{c^3d^2}{x}\right)}{c^4}\right) + \frac{b\left(2\arcsin(cx)c^3dex + \frac{\arcsin(cx)e^2c^3x^3}{3} - \frac{\arcsin(cx)c^3d^2}{x} - \frac{e^2\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}\right) - 2\sqrt{-c^2x^2+1}}{3}\right)}{c^4}$
default	$c\left(\frac{a\left(2c^3dex + \frac{e^2c^3x^3}{3} - \frac{c^3d^2}{x}\right)}{c^4}\right) + \frac{b\left(2\arcsin(cx)c^3dex + \frac{\arcsin(cx)e^2c^3x^3}{3} - \frac{\arcsin(cx)c^3d^2}{x} - \frac{e^2\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}\right) - 2\sqrt{-c^2x^2+1}}{3}\right)}{c^4}$

```
input int((e*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
output a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arcsin(c*x)*x^3*e^2+2/c*arcsin(c*x)*x*d*e-arcsin(c*x)*d^2/c/x-1/3/c^4*(e^2*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2))-2/3*(-c^2*x^2+1)^(1/2))+3*c^4*d^2*arctanh(1/(-c^2*x^2+1)^(1/2))-6*c^2*d*e*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{6ac^3e^2x^4 - 9bc^4d^2x \log(\sqrt{-c^2x^2 + 1} + 1) + 9bc^4d^2x \log(\sqrt{-c^2x^2 + 1} - 1) + 36ac^3dex^2 - 18ac^3d^2}{18c^3x}$$

```
input integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")
```

output

```
1/18*(6*a*c^3*e^2*x^4 - 9*b*c^4*d^2*x*log(sqrt(-c^2*x^2 + 1) + 1) + 9*b*c^4*d^2*x*log(sqrt(-c^2*x^2 + 1) - 1) + 36*a*c^3*d*e*x^2 - 18*a*c^3*d^2 + 6*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2)*arcsin(c*x) + 2*(b*c^2*e^2*x^3 + 2*(9*b*c^2*d*e + b*e^2)*x)*sqrt(-c^2*x^2 + 1))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.33

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx = -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{bce^2 \left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right)}{3} - \frac{bd^2 \operatorname{asin}(cx)}{x} + 2bde \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right) + \frac{be^2x^3 \operatorname{asin}(cx)}{3}$$

input

```
integrate((e*x**2+d)**2*(a+b*asin(c*x))/x**2,x)
```

output

```
-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*c*e**2*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True))/3 - b*d**2*asin(c*x)/x + 2*b*d*e*Piecewise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True)) + b*e**2*x**3*asin(c*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\begin{aligned} & \int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx \\ &= \frac{1}{3} ae^2 x^3 - \left(c \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd^2 \\ &+ \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) be^2 \\ &+ 2adex + \frac{2(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1})bde}{c} - \frac{ad^2}{x} \end{aligned}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `1/3*a*e^2*x^3 - (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d^2 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*e^2 + 2*a*d*e*x + 2*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d*e/c - a*d^2/x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4243 vs. 2(114) = 228.

Time = 1.79 (sec) , antiderivative size = 4243, normalized size of antiderivative = 33.67

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx = \text{Too large to display}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output

```

-1/2*b*c^12*d^2*x^8*arcsin(c*x)/((c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*
c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3
+ c^4*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^8) - 1/2*a*c^12
*d^2*x^8/((c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2
+ 1) + 1)^5 + 3*c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^4*x/(sqrt(-c^2*x^2
+ 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^8) + b*c^11*d^2*x^7*log(abs(c)*abs(x))
/((c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1
)^5 + 3*c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^4*x/(sqrt(-c^2*x^2 + 1) + 1
))*(sqrt(-c^2*x^2 + 1) + 1)^7) - b*c^11*d^2*x^7*log(sqrt(-c^2*x^2 + 1) + 1
)/((c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1
)^5 + 3*c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^4*x/(sqrt(-c^2*x^2 + 1) + 1
))*(sqrt(-c^2*x^2 + 1) + 1)^7) - 2*b*c^10*d^2*x^6*arcsin(c*x)/((c^10*x^7/
(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^6*
x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 + c^4*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^
2*x^2 + 1) + 1)^6) - 2*a*c^10*d^2*x^6/((c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^
7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^6*x^3/(sqrt(-c^2*x^2 + 1) +
1)^3 + c^4*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^6) + 3*b*
c^9*d^2*x^5*log(abs(c)*abs(x))/((c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c
^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 3*c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3 +
c^4*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^5) - 3*b*c^9*...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \left\{ \begin{array}{l} \frac{a(-3d^2 + 6de x^2 + e^2 x^4)}{3x} + b e^2 \left(\frac{\sqrt{\frac{1}{c^2} - x^2} \left(\frac{2}{c^2} + x^2 \right)}{9} + \frac{x^3 \arcsin(cx)}{3} \right) - b c d^2 \operatorname{atanh} \left(\frac{1}{\sqrt{1 - c^2 x^2}} \right) - \frac{b d^2 \arcsin(cx)}{x} + \frac{2 b d e}{x} \\ \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^2}{x^2} dx \end{array} \right.$$

input

```
int(((a + b*asin(c*x))*(d + e*x^2)^2)/x^2,x)
```


output

```

piecewise(0 < c, (a*(- 3*d^2 + e^2*x^4 + 6*d*e*x^2))/(3*x) + b*e^2*(((1/c^
2 - x^2)^(1/2)*(2/c^2 + x^2))/9 + (x^3*asin(c*x))/3) - b*c*d^2*atanh(1/(-
c^2*x^2 + 1)^(1/2)) - (b*d^2*asin(c*x))/x + (2*b*d*e*((- c^2*x^2 + 1)^(1/2
) + c*x*asin(c*x)))/c, ~0 < c, int(((a + b*asin(c*x))*(d + e*x^2)^2)/x^2,
x))

```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.29

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{-9 \operatorname{asin}(cx) b c^3 d^2 + 18 \operatorname{asin}(cx) b c^3 d e x^2 + 3 \operatorname{asin}(cx) b c^3 e^2 x^4 + 18 \sqrt{-c^2 x^2 + 1} b c^2 d e x + \sqrt{-c^2 x^2 + 1} b c^2 d^2}{9 c^3 x}$$

input

```
int((e*x^2+d)^2*(a+b*asin(c*x))/x^2,x)
```

output

```

( - 9*asin(c*x)*b*c**3*d**2 + 18*asin(c*x)*b*c**3*d*e*x**2 + 3*asin(c*x)*b
*c**3*e**2*x**4 + 18*sqrt(- c**2*x**2 + 1)*b*c**2*d*e*x + sqrt(- c**2*x*
*2 + 1)*b*c**2*e**2*x**3 + 2*sqrt(- c**2*x**2 + 1)*b*e**2*x + 9*log(tan(a
sin(c*x)/2))*b*c**4*d**2*x - 9*a*c**3*d**2 + 18*a*c**3*d*e*x**2 + 3*a*c**3
*e**2*x**4)/(9*c**3*x)

```

3.439 $\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x^3} dx$

Optimal result	3725
Mathematica [A] (verified)	3726
Rubi [A] (verified)	3726
Maple [A] (verified)	3728
Fricas [F]	3729
Sympy [F]	3729
Maxima [F]	3730
Giac [F(-2)]	3730
Mupad [F(-1)]	3730
Reduce [F]	3731

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x^3} dx = -\frac{bcd^2\sqrt{1-c^2x^2}}{2x} + \frac{be^2x\sqrt{1-c^2x^2}}{4c} - \frac{be^2 \arcsin(cx)}{4c^2} - ibde \arcsin(cx)^2 - \frac{d^2(a+b \arcsin(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b \arcsin(cx)) + 2bde \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) - 2bde \arcsin(cx) \log(x) + 2de(a+b \arcsin(cx)) \log(x) - ibde \text{PolyLog}(2, e^{2i \arcsin(cx)})$$

output

```
-1/2*b*c*d^2*(-c^2*x^2+1)^(1/2)/x+1/4*b*e^2*x*(-c^2*x^2+1)^(1/2)/c-1/4*b*e^2*arcsin(c*x)/c^2-I*b*d*e*arcsin(c*x)^2-1/2*d^2*(a+b*arcsin(c*x))/x^2+1/2*e^2*x^2*(a+b*arcsin(c*x))+2*b*d*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-2*b*d*e*arcsin(c*x)*ln(x)+2*d*e*(a+b*arcsin(c*x))*ln(x)-I*b*d*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx = \frac{1}{2} \left(-\frac{ad^2}{x^2} + ae^2x^2 - \frac{bcd^2\sqrt{1-c^2x^2}}{x} + \frac{be^2x\sqrt{1-c^2x^2}}{2c} - 2ibde \arcsin(cx)^2 + \frac{be^2 \arctan\left(\frac{cx}{1-\sqrt{1-c^2x^2}}\right)}{c^2} + b \arcsin(cx) \left(-\frac{d^2}{x^2} + e^2x^2 + 4de \log(1 - e^{2i \arcsin(cx)}) \right) + 4ade \log(x) - 2ibde \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right)$$

input

```
Integrate[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x^3,x]
```

output

```
(-((a*d^2)/x^2) + a*e^2*x^2 - (b*c*d^2*Sqrt[1 - c^2*x^2])/x + (b*e^2*x*Sqrt[1 - c^2*x^2])/(2*c) - (2*I)*b*d*e*ArcSin[c*x]^2 + (b*e^2*ArcTan[(c*x)/(1 - Sqrt[1 - c^2*x^2])])/c^2 + b*ArcSin[c*x]*(-(d^2/x^2) + e^2*x^2 + 4*d*e*Log[1 - E^((2*I)*ArcSin[c*x])]) + 4*a*d*e*Log[x] - (2*I)*b*d*e*PolyLog[2, E^((2*I)*ArcSin[c*x])])/2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx$$

$$\begin{aligned} & \downarrow 5230 \\ -bc \int & -\frac{\frac{d^2}{x^2} - 4e \log(x)d - e^2 x^2}{2\sqrt{1-c^2 x^2}} dx - \frac{d^2(a + b \arcsin(cx))}{2x^2} + 2de \log(x)(a + b \arcsin(cx)) + \\ & \frac{1}{2}e^2 x^2(a + b \arcsin(cx)) \end{aligned}$$

$$\begin{aligned} & \downarrow 27 \\ \frac{1}{2}bc \int & \frac{\frac{d^2}{x^2} - 4e \log(x)d - e^2 x^2}{\sqrt{1-c^2 x^2}} dx - \frac{d^2(a + b \arcsin(cx))}{2x^2} + 2de \log(x)(a + b \arcsin(cx)) + \\ & \frac{1}{2}e^2 x^2(a + b \arcsin(cx)) \end{aligned}$$

$$\begin{aligned} & \downarrow 7293 \\ \frac{1}{2}bc \int & \left(\frac{d^2}{x^2 \sqrt{1-c^2 x^2}} - \frac{4e \log(x)d}{\sqrt{1-c^2 x^2}} - \frac{e^2 x^2}{\sqrt{1-c^2 x^2}} \right) dx - \frac{d^2(a + b \arcsin(cx))}{2x^2} + 2de \log(x)(a + \\ & b \arcsin(cx)) + \frac{1}{2}e^2 x^2(a + b \arcsin(cx)) \end{aligned}$$

$$\begin{aligned} & \downarrow 2009 \\ & -\frac{d^2(a + b \arcsin(cx))}{2x^2} + 2de \log(x)(a + b \arcsin(cx)) + \frac{1}{2}e^2 x^2(a + b \arcsin(cx)) + \\ \frac{1}{2}bc \left(& -\frac{e^2 \arcsin(cx)}{2c^3} - \frac{2ide \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{c} - \frac{2ide \arcsin(cx)^2}{c} + \frac{4de \arcsin(cx) \log(1 - e^{2i \arcsin(cx)})}{c} \right) \end{aligned}$$

input `Int[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x^3,x]`

output `-1/2*(d^2*(a + b*ArcSin[c*x]))/x^2 + (e^2*x^2*(a + b*ArcSin[c*x]))/2 + 2*d
e(a + b*ArcSin[c*x])*Log[x] + (b*c*(-((d^2*Sqrt[1 - c^2*x^2])/x) + (e^2*
x*Sqrt[1 - c^2*x^2])/(2*c^2) - (e^2*ArcSin[c*x])/(2*c^3) - ((2*I)*d*e*ArcS
in[c*x]^2)/c + (4*d*e*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c - (4*d
*e*ArcSin[c*x]*Log[x])/c - ((2*I)*d*e*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c
))/2`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5230 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.33

method	result
parts	$a \left(\frac{e^2 x^2}{2} + 2de \ln(x) - \frac{d^2}{2x^2} \right) - ibde \arcsin(cx)^2 + \frac{b e^2 x \sqrt{-c^2 x^2 + 1}}{4c} + \frac{b e^2 \arcsin(cx) x^2}{2} - \frac{b e^2 \arcsin(cx)}{4c}$
derivativedivides	$c^2 \left(\frac{a x^2 e^2}{2c^2} + \frac{2ade \ln(cx)}{c^2} - \frac{a d^2}{2c^2 x^2} - \frac{ibde \arcsin(cx)^2}{c^2} + \frac{b e^2 x \sqrt{-c^2 x^2 + 1}}{4c^3} + \frac{b \arcsin(cx) x^2 e^2}{2c^2} - \frac{b e^2 \arcsin(cx)}{4c^4} \right)$
default	$c^2 \left(\frac{a x^2 e^2}{2c^2} + \frac{2ade \ln(cx)}{c^2} - \frac{a d^2}{2c^2 x^2} - \frac{ibde \arcsin(cx)^2}{c^2} + \frac{b e^2 x \sqrt{-c^2 x^2 + 1}}{4c^3} + \frac{b \arcsin(cx) x^2 e^2}{2c^2} - \frac{b e^2 \arcsin(cx)}{4c^4} \right)$

input `int((e*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)`

output

```
a*(1/2*e^2*x^2+2*d*e*ln(x)-1/2*d^2/x^2)-I*b*d*e*arcsin(c*x)^2+1/4*b*e^2*x*
(-c^2*x^2+1)^(1/2)/c+1/2*b*e^2*arcsin(c*x)*x^2-1/4*b*e^2*arcsin(c*x)/c^2+1
/2*I*d^2*b*c^2-1/2*b*c*d^2*(-c^2*x^2+1)^(1/2)/x-1/2*d^2*b/x^2*arcsin(c*x)+
2*b*d*e*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*I*b*d*e*polylog(2,-I*
c*x-(-c^2*x^2+1)^(1/2))+2*b*d*e*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))
-2*I*b*d*e*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(ex^2 + d)^2 (b \arcsin(cx) + a)}{x^3} dx$$

input

```
integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")
```

output

```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arcsin(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (d + ex^2)^2}{x^3} dx$$

input

```
integrate((e*x**2+d)**2*(a+b*asin(c*x))/x**3,x)
```

output

```
Integral((a + b*asin(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(ex^2 + d)^2 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `1/2*a*e^2*x^2 - 1/2*b*d^2*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 + integrate((b*e^2*x^2 + 2*b*d*e)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^2}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d + e*x^2)^2)/x^3,x)`

output `int(((a + b*asin(c*x))*(d + e*x^2)^2)/x^3, x)`

Reduce [F]

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{-2a \sin(cx) b c^2 d^2 + 2a \sin(cx) b c^2 e^2 x^4 - a \sin(cx) b e^2 x^2 - 2\sqrt{-c^2 x^2 + 1} b c^3 d^2 x + \sqrt{-c^2 x^2 + 1} b c e^2 x^3}{4c^2 x^2}$$

input `int((e*x^2+d)^2*(a+b*asin(c*x))/x^3,x)`

output `(- 2*asin(c*x)*b*c**2*d**2 + 2*asin(c*x)*b*c**2*e**2*x**4 - asin(c*x)*b*e**2*x**2 - 2*sqrt(- c**2*x**2 + 1)*b*c**3*d**2*x + sqrt(- c**2*x**2 + 1)*b*c*e**2*x**3 + 8*int(asin(c*x)/x,x)*b*c**2*d*e*x**2 + 8*log(x)*a*c**2*d*e*x**2 - 2*a*c**2*d**2 + 2*a*c**2*e**2*x**4)/(4*c**2*x**2)`

3.440 $\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x^4} dx$

Optimal result	3732
Mathematica [A] (verified)	3733
Rubi [A] (warning: unable to verify)	3733
Maple [A] (verified)	3736
Fricas [A] (verification not implemented)	3737
Sympy [A] (verification not implemented)	3738
Maxima [A] (verification not implemented)	3739
Giac [B] (verification not implemented)	3739
Mupad [F(-1)]	3740
Reduce [B] (verification not implemented)	3741

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{(d+ex^2)^2(a+b \arcsin(cx))}{x^4} dx = \frac{be^2\sqrt{1-c^2x^2}}{c} - \frac{bcd^2\sqrt{1-c^2x^2}}{6x^2} - \frac{d^2(a+b \arcsin(cx))}{3x^3} - \frac{2de(a+b \arcsin(cx))}{x} + e^2x(a+b \arcsin(cx)) - \frac{1}{6}bcd(c^2d+12e) \operatorname{arctanh}(\sqrt{1-c^2x^2})$$

output

```
b*e^2*(-c^2*x^2+1)^(1/2)/c-1/6*b*c*d^2*(-c^2*x^2+1)^(1/2)/x^2-1/3*d^2*(a+b*arcsin(c*x))/x^3-2*d*e*(a+b*arcsin(c*x))/x+e^2*x*(a+b*arcsin(c*x))-1/6*b*c*d*(c^2*d+12*e)*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx = \frac{1}{6} \left(-\frac{2ad^2}{x^3} - \frac{12ade}{x} + 6ae^2x + 6b \left(\frac{e^2}{c} - \frac{cd^2}{6x^2} \right) \sqrt{1 - c^2x^2} - \frac{2b(d^2 + 6dex^2 - 3e^2x^4) \arcsin(cx)}{x^3} + bcd(c^2d + 12e) \log(x) - bcd(c^2d + 12e) \log \left(1 + \sqrt{1 - c^2x^2} \right) \right)$$

input `Integrate[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x^4,x]`

output `((-2*a*d^2)/x^3 - (12*a*d*e)/x + 6*a*e^2*x + 6*b*(e^2/c - (c*d^2)/(6*x^2)) *Sqrt[1 - c^2*x^2] - (2*b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcSin[c*x])/x^3 + b*c*d*(c^2*d + 12*e)*Log[x] - b*c*d*(c^2*d + 12*e)*Log[1 + Sqrt[1 - c^2*x^2]])/6`

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5230, 27, 1578, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

↓ 5230

$$-bc \int -\frac{-3e^2x^4 + 6dex^2 + d^2}{3x^3\sqrt{1 - c^2x^2}} dx - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx))$$

$$\begin{aligned}
& \downarrow 27 \\
& \frac{1}{3}bc \int \frac{-3e^2x^4 + 6dex^2 + d^2}{x^3\sqrt{1-c^2x^2}} dx - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx)) \\
& \downarrow 1578 \\
& \frac{1}{6}bc \int \frac{-3e^2x^4 + 6dex^2 + d^2}{x^4\sqrt{1-c^2x^2}} dx^2 - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx)) \\
& \downarrow 1192 \\
& \frac{b \int \frac{-3e^2x^8 - 6(c^2d-e)ex^4 + c^4d^2 - 3e^2 + 6c^2de}{(1-x^4)^2} d\sqrt{1-c^2x^2}}{3c} - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx)) \\
& \downarrow 1471 \\
& \frac{b \left(\frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)} - \frac{1}{2} \int -\frac{d^2c^4 + 12dec^2 + 6e^2x^4 - 6e^2}{1-x^4} d\sqrt{1-c^2x^2} \right)}{3c} - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx)) \\
& \downarrow 25 \\
& \frac{b \left(\frac{1}{2} \int \frac{d^2c^4 + 12dec^2 + 6e^2x^4 - 6e^2}{1-x^4} d\sqrt{1-c^2x^2} + \frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)} \right)}{3c} - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx)) \\
& \downarrow 299 \\
& \frac{b \left(\frac{1}{2} \left(c^2d(c^2d + 12e) \int \frac{1}{1-x^4} d\sqrt{1-c^2x^2} - 6e^2\sqrt{1-c^2x^2} \right) + \frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)} \right)}{3c} - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx)) \\
& \downarrow 219 \\
& \frac{b \left(\frac{1}{2} \left(c^2d \operatorname{darctanh}(\sqrt{1-c^2x^2}) (c^2d + 12e) - 6e^2\sqrt{1-c^2x^2} \right) + \frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)} \right)}{3c} - \frac{d^2(a + b \arcsin(cx))}{3x^3} - \frac{2de(a + b \arcsin(cx))}{x} + e^2x(a + b \arcsin(cx))
\end{aligned}$$

input `Int[((d + e*x^2)^2*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(d^2*(a + b*ArcSin[c*x]))/x^3 - (2*d*e*(a + b*ArcSin[c*x]))/x + e^2*x*(a + b*ArcSin[c*x]) - (b*((c^4*d^2*Sqrt[1 - c^2*x^2])/(2*(1 - x^4)) + (-6*e^2*Sqrt[1 - c^2*x^2] + c^2*d*(c^2*d + 12*e)*ArcTanh[Sqrt[1 - c^2*x^2]])/2))/(3*c)`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 219 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

rule 299 `Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*x*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2*p + 3)) Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NeQ[2*p + 3, 0]`

rule 1192 `Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[2/e^(n + 2*p + 1) Subst[Int[x^(2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]`

rule 1471

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*(d + e*x^2)^(q + 1)/(2*d*(q + 1)), x] + Simp[1/(2*d*(q
+ 1)) Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)
^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 5230

```
Int[((a_) + ArcSin[(c_)*(x)]*(b_))*((f_)*(x_)^(m_)*((d_) + (e_)*(x_)
^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.24

method	result
derivativedivides	$c^3 \left(\frac{a \left(e^2 cx - \frac{2cde}{x} - \frac{c d^2}{3x^3} \right)}{c^4} + \frac{b \left(\arcsin(cx) e^2 cx - \frac{2 \arcsin(cx) cde}{x} - \frac{\arcsin(cx) c d^2}{3x^3} + \frac{c^4 d^2 \left(-\frac{\sqrt{-c^2 x^2 + 1}}{2c^2 x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2 x^2 + 1}}\right)}{2} \right)}{3} \right)}{c^4} \right)$
default	$c^3 \left(\frac{a \left(e^2 cx - \frac{2cde}{x} - \frac{c d^2}{3x^3} \right)}{c^4} + \frac{b \left(\arcsin(cx) e^2 cx - \frac{2 \arcsin(cx) cde}{x} - \frac{\arcsin(cx) c d^2}{3x^3} + \frac{c^4 d^2 \left(-\frac{\sqrt{-c^2 x^2 + 1}}{2c^2 x^2} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2 x^2 + 1}}\right)}{2} \right)}{3} \right)}{c^4} \right)$
parts	$a \left(e^2 x - \frac{d^2}{3x^3} - \frac{2de}{x} \right) + b c^3 \left(\frac{\arcsin(cx) x e^2}{c^3} - \frac{\arcsin(cx) d^2}{3c^3 x^3} - \frac{2 \arcsin(cx) de}{c^3 x} - \frac{-3e^2 \sqrt{-c^2 x^2 + 1} - c^4 d^2}{12 c x^3} \right)$

input

```
int((e*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
c^3*(a/c^4*(e^2*c*x-2*c*d*e/x-1/3*c*d^2/x^3)+b/c^4*(arcsin(c*x)*e^2*c*x-2*arcsin(c*x)*c*d*e/x-1/3*arcsin(c*x)*c*d^2/x^3+1/3*c^4*d^2*(-1/2/c^2/x^2*(-c^2*x^2+1)^(1/2)-1/2*arctanh(1/((-c^2*x^2+1)^(1/2))))+e^2*(-c^2*x^2+1)^(1/2)-2*c^2*d*e*arctanh(1/((-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.38

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{12 ace^2 x^4 - 24 acdex^2 - (bc^4 d^2 + 12 bc^2 de)x^3 \log(\sqrt{-c^2 x^2 + 1} + 1) + (bc^4 d^2 + 12 bc^2 de)x^3 \log(\sqrt{-c^2 x^2 + 1} - 1)}{12 cx^3}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `1/12*(12*a*c*e^2*x^4 - 24*a*c*d*e*x^2 - (b*c^4*d^2 + 12*b*c^2*d*e)*x^3*log(sqrt(-c^2*x^2 + 1) + 1) + (b*c^4*d^2 + 12*b*c^2*d*e)*x^3*log(sqrt(-c^2*x^2 + 1) - 1) - 4*a*c*d^2 + 4*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2)*arcsin(c*x) - 2*(b*c^2*d^2*x - 6*b*e^2*x^3)*sqrt(-c^2*x^2 + 1))/(c*x^3)`

Sympy [A] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.73

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x$$

$$+ \frac{bcd^2 \left(\begin{cases} -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ \frac{ic^2 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases} \right)}{3}$$

$$+ 2bcde \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) - \frac{bd^2 \operatorname{asin}(cx)}{3x^3}$$

$$- \frac{2bde \operatorname{asin}(cx)}{x} + be^2 \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right)$$

input `integrate((e*x**2+d)**2*(a+b*asin(c*x))/x**4,x)`

output `-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x + b*c*d**2*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2)))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 + 2*b*c*d*e*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*d**2*asin(c*x)/(3*x**3) - 2*b*d*e*asin(c*x)/x + b*e**2*Piecewise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**2*x**2 + 1)/c, True))`

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.26

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= -\frac{1}{6} \left(\left(c^2 \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2x^2+1}}{x^2} \right) c + \frac{2 \arcsin(cx)}{x^3} \right) bd^2$$

$$- 2 \left(c \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bde$$

$$+ ae^2x + \frac{(cx \arcsin(cx) + \sqrt{-c^2x^2+1})be^2}{c} - \frac{2ade}{x} - \frac{ad^2}{3x^3}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output `-1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*b*d^2 - 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d*e + a*e^2*x + (c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2534 vs. 2(114) = 228.

Time = 1.34 (sec) , antiderivative size = 2534, normalized size of antiderivative = 20.11

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx = \text{Too large to display}$$

input `integrate((e*x^2+d)^2*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output

```

-1/24*b*c^12*d^2*x^8*arcsin(c*x)/((c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^
4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^8) - 1/24*a*c^1
2*d^2*x^8/((c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt(-c^2*x^2 +
1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^8) + 1/24*b*c^11*d^2*x^7/((c^6*x^5/(sq
rt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x
^2 + 1) + 1)^7) - 1/6*b*c^10*d^2*x^6*arcsin(c*x)/((c^6*x^5/(sqrt(-c^2*x^2
+ 1) + 1)^5 + c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)
^6) - 1/6*a*c^10*d^2*x^6/((c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(s
qrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^6) + 1/6*b*c^9*d^2*x^5*
log(abs(c)*abs(x))/((c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt(-c
^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^5) - 1/6*b*c^9*d^2*x^5*log(sq
rt(-c^2*x^2 + 1) + 1)/((c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt
(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^5) + 1/24*b*c^9*d^2*x^5/((
c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(
sqrt(-c^2*x^2 + 1) + 1)^5) - b*c^8*d*e*x^6*arcsin(c*x)/((c^6*x^5/(sqrt(-c^
2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1)
+ 1)^6) - 1/4*b*c^8*d^2*x^4*arcsin(c*x)/((c^6*x^5/(sqrt(-c^2*x^2 + 1) +
1)^5 + c^4*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^4) - a
*c^8*d*e*x^6/((c^6*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^4*x^3/(sqrt(-c^2*x^2
+ 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^6) - 1/4*a*c^8*d^2*x^4/((c^6*x^5...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^2}{x^4} dx$$

input

```
int(((a + b*asin(c*x))*(d + e*x^2)^2)/x^4,x)
```

output

```
int(((a + b*asin(c*x))*(d + e*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \frac{(d + ex^2)^2 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{-2a \sin(cx) b c d^2 - 12a \sin(cx) b c d e x^2 + 6a \sin(cx) b c e^2 x^4 - \sqrt{-c^2 x^2 + 1} b c^2 d^2 x + 6\sqrt{-c^2 x^2 + 1} b e^2 x^3}{6c x^3}$$

input

```
int((e*x^2+d)^2*(a+b*asin(c*x))/x^4,x)
```

output

```
( - 2*asin(c*x)*b*c*d**2 - 12*asin(c*x)*b*c*d*e*x**2 + 6*asin(c*x)*b*c*e**
2*x**4 - sqrt( - c**2*x**2 + 1)*b*c**2*d**2*x + 6*sqrt( - c**2*x**2 + 1)*b
*e**2*x**3 + log(tan(asin(c*x)/2))*b*c**4*d**2*x**3 + 12*log(tan(asin(c*x)
/2))*b*c**2*d*e*x**3 - 2*a*c*d**2 - 12*a*c*d*e*x**2 + 6*a*c*e**2*x**4)/(6*
c*x**3)
```

3.441 $\int x^4(d + ex^2)^3 (a + b \arcsin(cx)) dx$

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Optimal result

Integrand size = 21, antiderivative size = 341

$$\int x^4(d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{b(231c^6d^3 + 495c^4d^2e + 385c^2de^2 + 105e^3) \sqrt{1 - c^2x^2}}{1155c^{11}}$$

$$- \frac{b(462c^6d^3 + 1485c^4d^2e + 1540c^2de^2 + 525e^3) (1 - c^2x^2)^{3/2}}{3465c^{11}}$$

$$+ \frac{b(77c^6d^3 + 495c^4d^2e + 770c^2de^2 + 350e^3) (1 - c^2x^2)^{5/2}}{1925c^{11}}$$

$$- \frac{be(99c^4d^2 + 308c^2de + 210e^2) (1 - c^2x^2)^{7/2}}{1617c^{11}}$$

$$+ \frac{be^2(11c^2d + 15e) (1 - c^2x^2)^{9/2}}{297c^{11}} - \frac{be^3(1 - c^2x^2)^{11/2}}{121c^{11}}$$

$$+ \frac{1}{5}d^3x^5(a + b \arcsin(cx)) + \frac{3}{7}d^2ex^7(a + b \arcsin(cx)) + \frac{1}{3}de^2x^9(a + b \arcsin(cx)) + \frac{1}{11}e^3x^{11}(a + b \arcsin(cx))$$

output

$$\begin{aligned} & 1/1155*b*(231*c^6*d^3+495*c^4*d^2*e+385*c^2*d*e^2+105*e^3)*(-c^2*x^2+1)^(1/2)/c^11-1/3465*b*(462*c^6*d^3+1485*c^4*d^2*e+1540*c^2*d*e^2+525*e^3)*(-c^2*x^2+1)^(3/2)/c^11+1/1925*b*(77*c^6*d^3+495*c^4*d^2*e+770*c^2*d*e^2+350*e^3)*(-c^2*x^2+1)^(5/2)/c^11-1/1617*b*e*(99*c^4*d^2+308*c^2*d*e+210*e^2)*(-c^2*x^2+1)^(7/2)/c^11+1/297*b*e^2*(11*c^2*d+15*e)*(-c^2*x^2+1)^(9/2)/c^11-1/121*b*e^3*(-c^2*x^2+1)^(11/2)/c^11+1/5*d^3*x^5*(a+b*arcsin(c*x))+3/7*d^2*e*x^7*(a+b*arcsin(c*x))+1/3*d*e^2*x^9*(a+b*arcsin(c*x))+1/11*e^3*x^11*(a+b*arcsin(c*x)) \end{aligned}$$
Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.79

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{3465ax^5(231d^3 + 495d^2ex^2 + 385de^2x^4 + 105e^3x^6) + \frac{b\sqrt{1-c^2x^2}(134400e^3 + 4480c^2e^2(121d+15ex^2) + 80c^4e(9801d^2+3388d^2e^2x^2 + 630e^2x^4) + 24c^6(17787d^3 + 16335d^2ex^2 + 8470d^2e^2x^4 + 1750e^3x^6) + c^10x^4(160083d^3 + 245025d^2ex^2 + 148225d^2e^2x^4 + 33075e^3x^6) + 2c^8(106722d^3x^2 + 147015d^2e^2x^4 + 84700d^2e^2x^6 + 18375e^3x^8))}{4002075}}{4002075}$$

input

$$\text{Integrate}[x^4*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]$$

output

$$\begin{aligned} & (3465*a*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) + (b*Sqrt[1 - c^2*x^2]*(134400*e^3 + 4480*c^2*e^2*(121*d + 15*e*x^2) + 80*c^4*e*(9801*d^2 + 3388*d*e*x^2 + 630*e^2*x^4) + 24*c^6*(17787*d^3 + 16335*d^2*e*x^2 + 8470*d^2*e^2*x^4 + 1750*e^3*x^6) + c^10*x^4*(160083*d^3 + 245025*d^2*e*x^2 + 148225*d^2*e^2*x^4 + 33075*e^3*x^6) + 2*c^8*(106722*d^3*x^2 + 147015*d^2*e^2*x^4 + 84700*d^2*e^2*x^6 + 18375*e^3*x^8)))/c^11 + 3465*b*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6)*ArcSin[c*x])/4002075 \end{aligned}$$
Rubi [A] (verified)Time = 0.83 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5230, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^4(d+ex^2)^3(a+b\arcsin(cx))dx \\
 & \quad \downarrow 5230 \\
 & -bc \int \frac{x^5(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3)}{1155\sqrt{1-c^2x^2}}dx + \frac{1}{5}d^3x^5(a+b\arcsin(cx)) + \\
 & \quad \frac{3}{7}d^2ex^7(a+b\arcsin(cx)) + \frac{1}{3}de^2x^9(a+b\arcsin(cx)) + \frac{1}{11}e^3x^{11}(a+b\arcsin(cx)) \\
 & \quad \downarrow 27 \\
 & -\frac{bc \int \frac{x^5(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3)}{\sqrt{1-c^2x^2}}dx}{1155} + \frac{1}{5}d^3x^5(a+b\arcsin(cx)) + \frac{3}{7}d^2ex^7(a + \\
 & \quad b\arcsin(cx)) + \frac{1}{3}de^2x^9(a+b\arcsin(cx)) + \frac{1}{11}e^3x^{11}(a+b\arcsin(cx)) \\
 & \quad \downarrow 2331 \\
 & -\frac{bc \int \frac{x^4(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3)}{\sqrt{1-c^2x^2}}dx^2}{2310} + \frac{1}{5}d^3x^5(a+b\arcsin(cx)) + \frac{3}{7}d^2ex^7(a + \\
 & \quad b\arcsin(cx)) + \frac{1}{3}de^2x^9(a+b\arcsin(cx)) + \frac{1}{11}e^3x^{11}(a+b\arcsin(cx)) \\
 & \quad \downarrow 2123 \\
 & bc \int \left(-\frac{105e^3(1-c^2x^2)^{9/2}}{c^{10}} + \frac{35e^2(11dc^2+15e)(1-c^2x^2)^{7/2}}{c^{10}} - \frac{5e(99d^2c^4+308dec^2+210e^2)(1-c^2x^2)^{5/2}}{c^{10}} + \frac{3(77d^3c^6+495d^2ec^4+770d^2e^2c^2+231d^3e^2)}{c^{10}} \right) dx \\
 & \quad \downarrow 2310 \\
 & \frac{1}{5}d^3x^5(a+b\arcsin(cx)) + \frac{3}{7}d^2ex^7(a+b\arcsin(cx)) + \frac{1}{3}de^2x^9(a+b\arcsin(cx)) + \frac{1}{11}e^3x^{11}(a + \\
 & \quad b\arcsin(cx)) \\
 & \quad \downarrow 2009 \\
 & \frac{1}{5}d^3x^5(a + \\
 & \quad b\arcsin(cx)) + \frac{3}{7}d^2ex^7(a+b\arcsin(cx)) + \frac{1}{3}de^2x^9(a+b\arcsin(cx)) + \frac{1}{11}e^3x^{11}(a+b\arcsin(cx)) - \\
 & bc \left(-\frac{70e^2(1-c^2x^2)^{9/2}(11c^2d+15e)}{9c^{12}} + \frac{210e^3(1-c^2x^2)^{11/2}}{11c^{12}} + \frac{10e(1-c^2x^2)^{7/2}(99c^4d^2+308c^2de+210e^2)}{7c^{12}} - \frac{6(1-c^2x^2)^{5/2}(77c^6d^3+495c^4d^2e^2+231d^3e^2)}{5c^{12}} \right) \\
 & \quad \downarrow 231
 \end{aligned}$$

input `Int[x^4*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]`

output

```
-1/2310*(b*c*((-2*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2*d*e^2 + 105*e^3)*
Sqrt[1 - c^2*x^2])/c^12 + (2*(462*c^6*d^3 + 1485*c^4*d^2*e + 1540*c^2*d*e^
2 + 525*e^3)*(1 - c^2*x^2)^(3/2))/(3*c^12) - (6*(77*c^6*d^3 + 495*c^4*d^2*
e + 770*c^2*d*e^2 + 350*e^3)*(1 - c^2*x^2)^(5/2))/(5*c^12) + (10*e*(99*c^4
*d^2 + 308*c^2*d*e + 210*e^2)*(1 - c^2*x^2)^(7/2))/(7*c^12) - (70*e^2*(11*
c^2*d + 15*e)*(1 - c^2*x^2)^(9/2))/(9*c^12) + (210*e^3*(1 - c^2*x^2)^(11/2
))/(11*c^12))) + (d^3*x^5*(a + b*ArcSin[c*x]))/5 + (3*d^2*e*x^7*(a + b*Arc
Sin[c*x]))/7 + (d*e^2*x^9*(a + b*ArcSin[c*x]))/3 + (e^3*x^11*(a + b*ArcSin
[c*x]))/11
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123

```
Int[(P_x)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol]
:= Int[ExpandIntegrand[P_x*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[P_x, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331

```
Int[(P_q)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, P_q, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[P_q, x^2] && IntegerQ[(m - 1)/2]
```

rule 5230

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(x_
)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.42

method	result
parts	$a\left(\frac{1}{11}e^3x^{11} + \frac{1}{3}de^2x^9 + \frac{3}{7}d^2ex^7 + \frac{1}{5}d^3x^5\right) + \frac{b\left(\frac{c^5\arcsin(cx)e^3x^{11}}{11} + \frac{c^5\arcsin(cx)de^2x^9}{3} + \frac{3c^5\arcsin(cx)d^2e^2x^9}{7}\right)}{c^6}$
derivativedivides	$\frac{a\left(\frac{1}{5}d^3c^{11}x^5 + \frac{3}{7}d^2c^{11}ex^7 + \frac{1}{3}dc^{11}e^2x^9 + \frac{1}{11}e^3c^{11}x^{11}\right)}{c^6} + \frac{b\left(\frac{\arcsin(cx)d^3c^{11}x^5}{5} + \frac{3\arcsin(cx)d^2c^{11}ex^7}{7} + \frac{\arcsin(cx)dc^{11}e^2x^9}{3}\right)}{c^6}$
default	$\frac{a\left(\frac{1}{5}d^3c^{11}x^5 + \frac{3}{7}d^2c^{11}ex^7 + \frac{1}{3}dc^{11}e^2x^9 + \frac{1}{11}e^3c^{11}x^{11}\right)}{c^6} + \frac{b\left(\frac{\arcsin(cx)d^3c^{11}x^5}{5} + \frac{3\arcsin(cx)d^2c^{11}ex^7}{7} + \frac{\arcsin(cx)dc^{11}e^2x^9}{3}\right)}{c^6}$
orering	$(694575c^{12}e^4x^{14} + 3312400c^{12}de^3x^{12} + 6092350c^{12}d^2e^2x^{10} + 36750c^{10}e^4x^{12} + 5096520c^{12}d^3ex^8 + 226450c^{10}de^3x^{10} + \dots)$

```
input int(x^4*(e*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/11*e^3*x^11+1/3*d*e^2*x^9+3/7*d^2*e*x^7+1/5*d^3*x^5)+b/c^5*(1/11*c^5*arcsin(c*x)*e^3*x^11+1/3*c^5*arcsin(c*x)*d*e^2*x^9+3/7*c^5*arcsin(c*x)*d^2*e*x^7+1/5*arcsin(c*x)*c^5*x^5*d^3-1/1155/c^6*(105*e^3*(-1/11*c^10*x^10*(-c^2*x^2+1)^(1/2)-10/99*c^8*x^8*(-c^2*x^2+1)^(1/2)-80/693*c^6*x^6*(-c^2*x^2+1)^(1/2)-32/231*c^4*x^4*(-c^2*x^2+1)^(1/2)-128/693*c^2*x^2*(-c^2*x^2+1)^(1/2)-256/693*(-c^2*x^2+1)^(1/2))+231*d^3*c^6*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+385*d*c^2*e^2*(-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c^4*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*x^2+1)^(1/2))+495*d^2*c^4*e*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.94

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{363825 ac^{11} e^3 x^{11} + 1334025 ac^{11} d e^2 x^9 + 1715175 ac^{11} d^2 e x^7 + 800415 ac^{11} d^3 x^5 + 3465 (105 bc^{11} e^3 x^{11} + 385 b^2 c^{11} d e^2 x^9 + 495 b^2 c^{11} d^2 e x^7 + 231 b^2 c^{11} d^3 x^5) \arcsin(cx) + (33075 b^3 c^{10} e^3 x^{10} + 426888 b^3 c^6 d^3 + 1225 (121 b^3 c^{10} d e^2 + 30 b^3 c^8 e^3) x^8 + 784080 b^3 c^4 d^2 e + 25 (9801 b^3 c^{10} d^2 e + 6776 b^3 c^8 d e^2 + 1680 b^3 c^6 e^3) x^6 + 542080 b^3 c^2 d e^2 + 3 (53361 b^3 c^{10} d^3 + 98010 b^3 c^8 d^2 e + 67760 b^3 c^6 d e^2 + 16800 b^3 c^4 e^3) x^4 + 134400 b^3 e^3 + 4 (53361 b^3 c^8 d^3 + 98010 b^3 c^6 d^2 e + 67760 b^3 c^4 d e^2 + 16800 b^3 c^2 e^3) x^2) \sqrt{-c^2 x^2 + 1}}{c^{11}}$$

input `integrate(x^4*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`output `1/4002075*(363825*a*c^11*e^3*x^11 + 1334025*a*c^11*d*e^2*x^9 + 1715175*a*c^11*d^2*e*x^7 + 800415*a*c^11*d^3*x^5 + 3465*(105*b*c^11*e^3*x^11 + 385*b*c^11*d*e^2*x^9 + 495*b*c^11*d^2*e*x^7 + 231*b*c^11*d^3*x^5)*arcsin(c*x) + (33075*b*c^10*e^3*x^10 + 426888*b*c^6*d^3 + 1225*(121*b*c^10*d*e^2 + 30*b*c^8*e^3)*x^8 + 784080*b*c^4*d^2*e + 25*(9801*b*c^10*d^2*e + 6776*b*c^8*d*e^2 + 1680*b*c^6*e^3)*x^6 + 542080*b*c^2*d*e^2 + 3*(53361*b*c^10*d^3 + 98010*b*c^8*d^2*e + 67760*b*c^6*d*e^2 + 16800*b*c^4*e^3)*x^4 + 134400*b*e^3 + 4*(53361*b*c^8*d^3 + 98010*b*c^6*d^2*e + 67760*b*c^4*d*e^2 + 16800*b*c^2*e^3)*x^2)*sqrt(-c^2*x^2 + 1))/c^11`**Sympy [A] (verification not implemented)**

Time = 2.17 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.85

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^3 x^5}{5} + \frac{3ad^2 ex^7}{7} + \frac{ade^2 x^9}{3} + \frac{ae^3 x^{11}}{11} + \frac{bd^3 x^5 \arcsin(cx)}{5} + \frac{3bd^2 ex^7 \arcsin(cx)}{7} + \frac{bde^2 x^9 \arcsin(cx)}{3} + \frac{be^3 x^{11} \arcsin(cx)}{11} + \frac{bd^3 x^4 \sqrt{-c^2 x^2 + 1}}{25c} \\ a \left(\frac{d^3 x^5}{5} + \frac{3d^2 ex^7}{7} + \frac{de^2 x^9}{3} + \frac{e^3 x^{11}}{11} \right) \end{cases}$$

input `integrate(x**4*(e*x**2+d)**3*(a+b*asin(c*x)),x)`

output

```
Piecewise((a*d**3*x**5/5 + 3*a*d**2*e*x**7/7 + a*d*e**2*x**9/3 + a*e**3*x*
**11/11 + b*d**3*x**5*asin(c*x)/5 + 3*b*d**2*e*x**7*asin(c*x)/7 + b*d*e**2*
x**9*asin(c*x)/3 + b*e**3*x**11*asin(c*x)/11 + b*d**3*x**4*sqrt(-c**2*x**2
+ 1)/(25*c) + 3*b*d**2*e*x**6*sqrt(-c**2*x**2 + 1)/(49*c) + b*d*e**2*x**8
*sqrt(-c**2*x**2 + 1)/(27*c) + b*e**3*x**10*sqrt(-c**2*x**2 + 1)/(121*c) +
4*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) + 18*b*d**2*e*x**4*sqrt(-c**
2*x**2 + 1)/(245*c**3) + 8*b*d*e**2*x**6*sqrt(-c**2*x**2 + 1)/(189*c**3) +
10*b*e**3*x**8*sqrt(-c**2*x**2 + 1)/(1089*c**3) + 8*b*d**3*sqrt(-c**2*x**
2 + 1)/(75*c**5) + 24*b*d**2*e*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) + 16*b
*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(315*c**5) + 80*b*e**3*x**6*sqrt(-c**2*x
**2 + 1)/(7623*c**5) + 48*b*d**2*e*sqrt(-c**2*x**2 + 1)/(245*c**7) + 64*b*
d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(945*c**7) + 32*b*e**3*x**4*sqrt(-c**2*x*
*2 + 1)/(2541*c**7) + 128*b*d*e**2*sqrt(-c**2*x**2 + 1)/(945*c**9) + 128*b
*e**3*x**2*sqrt(-c**2*x**2 + 1)/(7623*c**9) + 256*b*e**3*sqrt(-c**2*x**2 +
1)/(7623*c**11), Ne(c, 0)), (a*(d**3*x**5/5 + 3*d**2*e*x**7/7 + d*e**2*x*
*9/3 + e**3*x**11/11), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.36

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx = \frac{1}{11} ae^3 x^{11} + \frac{1}{3} ade^2 x^9 + \frac{3}{7} ad^2 ex^7 + \frac{1}{5} ad^3 x^5$$

$$+ \frac{1}{75} \left(15 x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4\sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8\sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bd^3$$

$$+ \frac{3}{245} \left(35 x^7 \arcsin(cx) + \left(\frac{5\sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6\sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8\sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16\sqrt{-c^2 x^2 + 1}}{c^8} \right) c \right) bd^3$$

$$+ \frac{1}{945} \left(315 x^9 \arcsin(cx) + \left(\frac{35\sqrt{-c^2 x^2 + 1} x^8}{c^2} + \frac{40\sqrt{-c^2 x^2 + 1} x^6}{c^4} + \frac{48\sqrt{-c^2 x^2 + 1} x^4}{c^6} + \frac{64\sqrt{-c^2 x^2 + 1}}{c^8} \right) c \right) bd^3$$

$$+ \frac{1}{7623} \left(693 x^{11} \arcsin(cx) + \left(\frac{63\sqrt{-c^2 x^2 + 1} x^{10}}{c^2} + \frac{70\sqrt{-c^2 x^2 + 1} x^8}{c^4} + \frac{80\sqrt{-c^2 x^2 + 1} x^6}{c^6} + \frac{96\sqrt{-c^2 x^2 + 1}}{c^8} \right) c \right) bd^3$$

input

```
integrate(x^4*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```

1/11*a*e^3*x^11 + 1/3*a*d*e^2*x^9 + 3/7*a*d^2*e*x^7 + 1/5*a*d^3*x^5 + 1/75
*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)
)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^3 + 3/245*(35*x^7*arcsin(c*x)
+ (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-
c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*d^2*e + 1/945*(315*
x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x
^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 1
28*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*d*e^2 + 1/7623*(693*x^11*arcsin(c*x) + (6
3*sqrt(-c^2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(-c
^2*x^2 + 1)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(-c^2*x^2 +
1)*x^2/c^10 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c)*b*e^3

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 943 vs. $2(309) = 618$.

Time = 0.14 (sec) , antiderivative size = 943, normalized size of antiderivative = 2.77

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx = \text{Too large to display}$$

input

```

integrate(x^4*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")

```

output

```

1/11*a*e^3*x^11 + 1/3*a*d*e^2*x^9 + 3/7*a*d^2*e*x^7 + 1/5*a*d^3*x^5 + 1/5*
(c^2*x^2 - 1)^2*b*d^3*x*arcsin(c*x)/c^4 + 2/5*(c^2*x^2 - 1)*b*d^3*x*arcsin
(c*x)/c^4 + 3/7*(c^2*x^2 - 1)^3*b*d^2*e*x*arcsin(c*x)/c^6 + 1/5*b*d^3*x*ar
csin(c*x)/c^4 + 9/7*(c^2*x^2 - 1)^2*b*d^2*e*x*arcsin(c*x)/c^6 + 1/3*(c^2*x
^2 - 1)^4*b*d*e^2*x*arcsin(c*x)/c^8 + 1/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 +
1)*b*d^3/c^5 + 9/7*(c^2*x^2 - 1)*b*d^2*e*x*arcsin(c*x)/c^6 + 4/3*(c^2*x^2
- 1)^3*b*d*e^2*x*arcsin(c*x)/c^8 + 1/11*(c^2*x^2 - 1)^5*b*e^3*x*arcsin(c*
x)/c^10 - 2/15*(-c^2*x^2 + 1)^(3/2)*b*d^3/c^5 + 3/49*(c^2*x^2 - 1)^3*sqrt(
-c^2*x^2 + 1)*b*d^2*e/c^7 + 3/7*b*d^2*e*x*arcsin(c*x)/c^6 + 2*(c^2*x^2 - 1
)^2*b*d*e^2*x*arcsin(c*x)/c^8 + 5/11*(c^2*x^2 - 1)^4*b*e^3*x*arcsin(c*x)/c
^10 + 1/5*sqrt(-c^2*x^2 + 1)*b*d^3/c^5 + 9/35*(c^2*x^2 - 1)^2*sqrt(-c^2*x^
2 + 1)*b*d^2*e/c^7 + 1/27*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^9 +
4/3*(c^2*x^2 - 1)*b*d*e^2*x*arcsin(c*x)/c^8 + 10/11*(c^2*x^2 - 1)^3*b*e^3
*x*arcsin(c*x)/c^10 - 3/7*(-c^2*x^2 + 1)^(3/2)*b*d^2*e/c^7 + 4/21*(c^2*x^2
- 1)^3*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^9 + 1/121*(c^2*x^2 - 1)^5*sqrt(-c^2*x
^2 + 1)*b*e^3/c^11 + 1/3*b*d*e^2*x*arcsin(c*x)/c^8 + 10/11*(c^2*x^2 - 1)^2
*b*e^3*x*arcsin(c*x)/c^10 + 3/7*sqrt(-c^2*x^2 + 1)*b*d^2*e/c^7 + 2/5*(c^2*
x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^9 + 5/99*(c^2*x^2 - 1)^4*sqrt(-c^2
*x^2 + 1)*b*e^3/c^11 + 5/11*(c^2*x^2 - 1)*b*e^3*x*arcsin(c*x)/c^10 - 4/9*(
-c^2*x^2 + 1)^(3/2)*b*d*e^2/c^9 + 10/77*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + ...

```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx = \int x^4 (a + b \operatorname{asin}(cx)) (ex^2 + d)^3 dx$$

input

```
int(x^4*(a + b*asin(c*x))*(d + e*x^2)^3,x)
```

output

```
int(x^4*(a + b*asin(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.55

$$\int x^4 (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{800415 a \sin(cx) b c^{11} d^3 x^5 + 363825 a \sin(cx) b c^{11} e^3 x^{11} + 160083 \sqrt{-c^2 x^2 + 1} b c^{10} d^3 x^4 + 33075 \sqrt{-c^2 x^2 + 1} b c^{10} d^2 e x^6 + 148225 \sqrt{-c^2 x^2 + 1} b c^{10} d^2 e^2 x^8 + 213444 \sqrt{-c^2 x^2 + 1} b c^{10} d e^3 x^{10} + 294030 \sqrt{-c^2 x^2 + 1} b c^8 d^3 x^2 + 169400 \sqrt{-c^2 x^2 + 1} b c^8 d^2 e x^4 + 36750 \sqrt{-c^2 x^2 + 1} b c^8 d e^2 x^6 + 426888 \sqrt{-c^2 x^2 + 1} b c^8 d e^3 x^8 + 392040 \sqrt{-c^2 x^2 + 1} b c^6 d^3 x^2 + 203280 \sqrt{-c^2 x^2 + 1} b c^6 d^2 e x^4 + 42000 \sqrt{-c^2 x^2 + 1} b c^6 d e^2 x^6 + 784080 \sqrt{-c^2 x^2 + 1} b c^4 d^3 x^2 + 271040 \sqrt{-c^2 x^2 + 1} b c^4 d^2 e x^4 + 50400 \sqrt{-c^2 x^2 + 1} b c^4 d e^2 x^6 + 542080 \sqrt{-c^2 x^2 + 1} b c^2 d^3 x^2 + 67200 \sqrt{-c^2 x^2 + 1} b c^2 d^2 e x^4 + 134400 \sqrt{-c^2 x^2 + 1} b c^2 d e^2 x^6 + 134400 \sqrt{-c^2 x^2 + 1} b e^3 x^8 + 800415 a c^{11} d^3 x^5 + 1715175 a c^{11} d^2 e x^7 + 1334025 a c^{11} d e^2 x^9 + 363825 a c^{11} e^3 x^{11}}{(4002075 c^{11})}$$

input `int(x^4*(e*x^2+d)^3*(a+b*asin(c*x)),x)`

output

```
(800415*asin(c*x)*b*c**11*d**3*x**5 + 1715175*asin(c*x)*b*c**11*d**2*e*x**7 + 1334025*asin(c*x)*b*c**11*d*e**2*x**9 + 363825*asin(c*x)*b*c**11*e**3*x**11 + 160083*sqrt(-c**2*x**2 + 1)*b*c**10*d**3*x**4 + 245025*sqrt(-c**2*x**2 + 1)*b*c**10*d**2*e*x**6 + 148225*sqrt(-c**2*x**2 + 1)*b*c**10*d*e**2*x**8 + 33075*sqrt(-c**2*x**2 + 1)*b*c**10*e**3*x**10 + 213444*sqrt(-c**2*x**2 + 1)*b*c**8*d**3*x**2 + 294030*sqrt(-c**2*x**2 + 1)*b*c**8*d**2*e*x**4 + 169400*sqrt(-c**2*x**2 + 1)*b*c**8*d*e**2*x**6 + 36750*sqrt(-c**2*x**2 + 1)*b*c**8*e**3*x**8 + 426888*sqrt(-c**2*x**2 + 1)*b*c**6*d**3 + 392040*sqrt(-c**2*x**2 + 1)*b*c**6*d**2*e*x**2 + 203280*sqrt(-c**2*x**2 + 1)*b*c**6*d*e**2*x**4 + 42000*sqrt(-c**2*x**2 + 1)*b*c**6*e**3*x**6 + 784080*sqrt(-c**2*x**2 + 1)*b*c**4*d**2*e + 271040*sqrt(-c**2*x**2 + 1)*b*c**4*d*e**2*x**2 + 50400*sqrt(-c**2*x**2 + 1)*b*c**4*e**3*x**4 + 542080*sqrt(-c**2*x**2 + 1)*b*c**2*d*e**2 + 67200*sqrt(-c**2*x**2 + 1)*b*c**2*e**3*x**2 + 134400*sqrt(-c**2*x**2 + 1)*b*e**3 + 800415*a*c**11*d**3*x**5 + 1715175*a*c**11*d**2*e*x**7 + 1334025*a*c**11*d*e**2*x**9 + 363825*a*c**11*e**3*x**11)/(4002075*c**11)
```

3.442 $\int x^3(d + ex^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	3752
Mathematica [A] (verified)	3753
Rubi [A] (verified)	3753
Maple [A] (verified)	3758
Fricas [A] (verification not implemented)	3759
Sympy [A] (verification not implemented)	3759
Maxima [A] (verification not implemented)	3760
Giac [B] (verification not implemented)	3761
Mupad [F(-1)]	3762
Reduce [B] (verification not implemented)	3763

Optimal result

Integrand size = 21, antiderivative size = 335

$$\begin{aligned}
 & \int x^3(d + ex^2)^3 (a + b \arcsin(cx)) dx \\
 &= \frac{b(480c^6d^3 + 800c^4d^2e + 525c^2de^2 + 126e^3) x\sqrt{1 - c^2x^2}}{5120c^9} \\
 &+ \frac{b(480c^6d^3 + 800c^4d^2e + 525c^2de^2 + 126e^3) x^3\sqrt{1 - c^2x^2}}{7680c^7} \\
 &+ \frac{be(800c^4d^2 + 525c^2de + 126e^2) x^5\sqrt{1 - c^2x^2}}{9600c^5} \\
 &+ \frac{3be^2(25c^2d + 6e) x^7\sqrt{1 - c^2x^2}}{1600c^3} + \frac{be^3x^9\sqrt{1 - c^2x^2}}{100c} \\
 &+ \frac{b(128c^{10}d^5 - 480c^6d^3e^2 - 800c^4d^2e^3 - 525c^2de^4 - 126e^5) \arcsin(cx)}{5120c^{10}e^2} \\
 &- \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2} + \frac{(d + ex^2)^5 (a + b \arcsin(cx))}{10e^2}
 \end{aligned}$$

output

```
1/5120*b*(480*c^6*d^3+800*c^4*d^2*e+525*c^2*d*e^2+126*e^3)*x*(-c^2*x^2+1)^(1/2)/c^9+1/7680*b*(480*c^6*d^3+800*c^4*d^2*e+525*c^2*d*e^2+126*e^3)*x^3*(-c^2*x^2+1)^(1/2)/c^7+1/9600*b*e*(800*c^4*d^2+525*c^2*d*e+126*e^2)*x^5*(-c^2*x^2+1)^(1/2)/c^5+3/1600*b*e^2*(25*c^2*d+6*e)*x^7*(-c^2*x^2+1)^(1/2)/c^3+1/100*b*e^3*x^9*(-c^2*x^2+1)^(1/2)/c+1/5120*b*(128*c^10*d^5-480*c^6*d^3*e^2-800*c^4*d^2*e^3-525*c^2*d*e^4-126*e^5)*arcsin(c*x)/c^10/e^2-1/8*d*(e*x^2+d)^4*(a+b*arcsin(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arcsin(c*x))/e^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.82

$$\int x^3 (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{cx(1920ac^9x^3(10d^3 + 20d^2ex^2 + 15de^2x^4 + 4e^3x^6) + b\sqrt{1 - c^2x^2}(1890e^3 + 315c^2e^2(25d + 4ex^2) + 6c^4e(200d^2 + 875d*ex^2 + 168e^2x^4) + 8c^6(900d^3 + 1000d^2*ex^2 + 525d*ex^2 + 108e^3x^6) + 16c^8(300d^3x^2 + 400d^2*ex^4 + 225d*ex^2x^6 + 48e^3x^8))) + 15b*(-480c^6d^3 - 800c^4d^2e - 525c^2d*e^2 - 126e^3 + 128c^10x^4(10d^3 + 20d^2*ex^2 + 15d*ex^2x^4 + 4e^3x^6))*\arcsin[cx]}{(76800c^{10})}$$

input

```
Integrate[x^3*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(c*x*(1920*a*c^9*x^3*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) + b*Sqrt[1 - c^2*x^2]*(1890*e^3 + 315*c^2*e^2*(25*d + 4*e*x^2) + 6*c^4*e*(200*d^2 + 875*d*e*x^2 + 168*e^2*x^4) + 8*c^6*(900*d^3 + 1000*d^2*e*x^2 + 525*d*e^2*x^4 + 108*e^3*x^6) + 16*c^8*(300*d^3*x^2 + 400*d^2*e*x^4 + 225*d*e^2*x^6 + 48*e^3*x^8))) + 15*b*(-480*c^6*d^3 - 800*c^4*d^2*e - 525*c^2*d*e^2 - 126*e^3 + 128*c^10*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6))*ArcSin[c*x])/(76800*c^10)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5230, 27, 403, 27, 403, 25, 403, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^3 (d + ex^2)^3 (a + b \arcsin(cx)) dx \\
 & \quad \downarrow \text{5230} \\
 & -bc \int -\frac{(d - 4ex^2)(ex^2 + d)^4}{40e^2\sqrt{1 - c^2x^2}} dx + \frac{(d + ex^2)^5 (a + b \arcsin(cx))}{10e^2} - \\
 & \quad \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2} \\
 & \quad \downarrow \text{27} \\
 & \frac{bc \int \frac{(d - 4ex^2)(ex^2 + d)^4}{\sqrt{1 - c^2x^2}} dx}{40e^2} + \frac{(d + ex^2)^5 (a + b \arcsin(cx))}{10e^2} - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2} \\
 & \quad \downarrow \text{403} \\
 & bc \left(\frac{2ex\sqrt{1 - c^2x^2}(d + ex^2)^4}{5c^2} - \frac{\int -\frac{2(ex^2 + d)^3 (d(5c^2d - 2e) - e(11dc^2 + 18e)x^2)}{\sqrt{1 - c^2x^2}} dx}{10c^2} \right) \\
 & \quad \frac{40e^2}{(d + ex^2)^5 (a + b \arcsin(cx)) - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2}} + \\
 & \quad \downarrow \text{27} \\
 & bc \left(\frac{\int \frac{(ex^2 + d)^3 (d(5c^2d - 2e) - e(11dc^2 + 18e)x^2)}{\sqrt{1 - c^2x^2}} dx}{5c^2} + \frac{2ex\sqrt{1 - c^2x^2}(d + ex^2)^4}{5c^2} \right) \\
 & \quad \frac{40e^2}{(d + ex^2)^5 (a + b \arcsin(cx)) - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2}} + \\
 & \quad \downarrow \text{403} \\
 & bc \left(\frac{ex\sqrt{1 - c^2x^2}(11c^2d + 18e)(d + ex^2)^3}{8c^2} - \frac{\int -\frac{(ex^2 + d)^2 (d(40d^2c^4 - 27dec^2 - 18e^2) - e(26d^2c^4 + 201dec^2 + 126e^2)x^2)}{\sqrt{1 - c^2x^2}} dx}{5c^2} + \frac{2ex\sqrt{1 - c^2x^2}(d + ex^2)^4}{5c^2} \right) \\
 & \quad \frac{40e^2}{(d + ex^2)^5 (a + b \arcsin(cx)) - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2}} + \\
 & \quad \downarrow \text{25}
 \end{aligned}$$

$$bc \left(\frac{\int \frac{(ex^2+d)^2 (d(40d^2c^4 - 27dec^2 - 18e^2) - e(26d^2c^4 + 201dec^2 + 126e^2)x^2) dx}{\sqrt{1-c^2x^2}}}{8c^2} + \frac{ex\sqrt{1-c^2x^2}(11c^2d+18e)(d+ex^2)^3}{8c^2} + \frac{2ex\sqrt{1-c^2x^2}(d+ex^2)^4}{5c^2} \right) +$$

$$\frac{(d+ex^2)^5 (a+b \arcsin(cx))}{10e^2} - \frac{40e^2 d(d+ex^2)^4 (a+b \arcsin(cx))}{8e^2}$$

↓ 403

$$bc \left(\frac{ex\sqrt{1-c^2x^2}(26c^4d^2+201c^2de+126e^2)(d+ex^2)^2}{6c^2} - \frac{\int \frac{(ex^2+d)(e(136d^3c^6 - 1096d^2ec^4 - 1617de^2c^2 - 630e^3)x^2 + d(240d^3c^6 - 188d^2ec^4 - 309de^2c^2 - 126e^3)) dx}{\sqrt{1-c^2x^2}}}{8c^2} + \frac{ex\sqrt{1-c^2x^2}(26c^4d^2+201c^2de+126e^2)(d+ex^2)^2}{6c^2} \right) +$$

$$\frac{(d+ex^2)^5 (a+b \arcsin(cx))}{10e^2} - \frac{40e^2 d(d+ex^2)^4 (a+b \arcsin(cx))}{8e^2}$$

↓ 25

$$bc \left(\frac{\int \frac{(ex^2+d)(e(136d^3c^6 - 1096d^2ec^4 - 1617de^2c^2 - 630e^3)x^2 + d(240d^3c^6 - 188d^2ec^4 - 309de^2c^2 - 126e^3)) dx}{\sqrt{1-c^2x^2}}}{6c^2} + \frac{ex\sqrt{1-c^2x^2}(26c^4d^2+201c^2de+126e^2)(d+ex^2)^2}{6c^2} \right) +$$

$$\frac{(d+ex^2)^5 (a+b \arcsin(cx))}{10e^2} - \frac{40e^2 d(d+ex^2)^4 (a+b \arcsin(cx))}{8e^2}$$

↓ 403

$$bc \left(-\frac{\int \frac{e(1232d^4c^8 - 2536d^3ec^6 - 7758d^2e^2c^4 - 6615de^3c^2 - 1890e^4)x^2 + d(960d^4c^8 - 616d^3ec^6 - 2332d^2e^2c^4 - 2121de^3c^2 - 630e^4) dx}{\sqrt{1-c^2x^2}}}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3)}{8c^2} \right) +$$

$$\frac{(d+ex^2)^5 (a+b \arcsin(cx))}{10e^2} - \frac{40e^2 d(d+ex^2)^4 (a+b \arcsin(cx))}{8e^2}$$

↓ 25

$$bc \left(\frac{\int \frac{e(1232d^4c^8 - 2536d^3ec^6 - 7758d^2e^2c^4 - 6615de^3c^2 - 1890e^4)x^2 + d(960d^4c^8 - 616d^3ec^6 - 2332d^2e^2c^4 - 2121de^3c^2 - 630e^4)}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{5c^2} \right)$$

$$\frac{(d + ex^2)^5 (a + b \arcsin(cx))}{10e^2} - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2}$$

↓ 299

$$bc \left(\frac{15(128c^{10}d^5 - 480c^6d^3e^2 - 800c^4d^2e^3 - 525c^2de^4 - 126e^5) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{ex\sqrt{1-c^2x^2}(1232c^8d^4 - 2536c^6d^3e - 7758c^4d^2e^2 - 6615c^2de^3 - 1890e^4)}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{8c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{5c^2} \right)$$

$$\frac{(d + ex^2)^5 (a + b \arcsin(cx))}{10e^2} - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2}$$

↓ 223

$$bc \left(\frac{(d + ex^2)^5 (a + b \arcsin(cx))}{10e^2} - \frac{d(d + ex^2)^4 (a + b \arcsin(cx))}{8e^2} + \frac{15 \arcsin(cx)(128c^{10}d^5 - 480c^6d^3e^2 - 800c^4d^2e^3 - 525c^2de^4 - 126e^5)}{2c^3} - \frac{ex\sqrt{1-c^2x^2}(1232c^8d^4 - 2536c^6d^3e - 7758c^4d^2e^2 - 6615c^2de^3 - 1890e^4)}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{8c^2} - \frac{ex\sqrt{1-c^2x^2}(136c^6d^3 - 10c^4d^2e^2 - 10c^2de^3 + 10e^4)}{5c^2} \right)$$

input `Int[x^3*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]`

output

```
-1/8*(d*(d + e*x^2)^4*(a + b*ArcSin[c*x]))/e^2 + ((d + e*x^2)^5*(a + b*Arc
Sin[c*x]))/(10*e^2) + (b*c*((2*e*x*Sqrt[1 - c^2*x^2]*(d + e*x^2)^4)/(5*c^2
) + ((e*(11*c^2*d + 18*e))*x*Sqrt[1 - c^2*x^2]*(d + e*x^2)^3)/(8*c^2) + ((e
*(26*c^4*d^2 + 201*c^2*d*e + 126*e^2))*x*Sqrt[1 - c^2*x^2]*(d + e*x^2)^2)/(
6*c^2) + (-1/4*(e*(136*c^6*d^3 - 1096*c^4*d^2*e - 1617*c^2*d*e^2 - 630*e^3
))*x*Sqrt[1 - c^2*x^2]*(d + e*x^2))/c^2 + (-1/2*(e*(1232*c^8*d^4 - 2536*c^6
*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*e^3 - 1890*e^4))*x*Sqrt[1 - c^2*x^2]
)/c^2 + (15*(128*c^10*d^5 - 480*c^6*d^3*e^2 - 800*c^4*d^2*e^3 - 525*c^2*d*
e^4 - 126*e^5)*ArcSin[c*x])/(2*c^3)/(4*c^2)/(6*c^2)/(8*c^2)/(5*c^2))/(
(40*e^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3)) Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] := Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1)) Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5230

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.30

method	result
parts	$a\left(\frac{1}{10}e^3x^{10} + \frac{3}{8}de^2x^8 + \frac{1}{2}d^2ex^6 + \frac{1}{4}x^4d^3\right) + \frac{b\left(\frac{c^4 \arcsin(cx)e^3x^{10}}{10} + \frac{3c^4 \arcsin(cx)de^2x^8}{8} + \frac{c^4 \arcsin(cx)d^2e^2x^8}{2}\right)}{c^6}$
derivativedivides	$\frac{a\left(\frac{1}{4}c^{10}d^3x^4 + \frac{1}{2}c^{10}d^2ex^6 + \frac{3}{8}c^{10}de^2x^8 + \frac{1}{10}e^3x^{10}c^{10}\right)}{c^6} + \frac{b\left(\frac{\arcsin(cx)d^3c^{10}x^4}{4} + \frac{\arcsin(cx)d^2c^{10}ex^6}{2} + \frac{3 \arcsin(cx)dc^{10}e^2x^8}{8} + a\right)}{c^6}$
default	$\frac{a\left(\frac{1}{4}c^{10}d^3x^4 + \frac{1}{2}c^{10}d^2ex^6 + \frac{3}{8}c^{10}de^2x^8 + \frac{1}{10}e^3x^{10}c^{10}\right)}{c^6} + \frac{b\left(\frac{\arcsin(cx)d^3c^{10}x^4}{4} + \frac{\arcsin(cx)d^2c^{10}ex^6}{2} + \frac{3 \arcsin(cx)dc^{10}e^2x^8}{8} + a\right)}{c^6}$
oring	$(4864c^{10}e^4x^{12} + 23728c^{10}de^3x^{10} + 45200c^{10}d^2e^2x^8 + 288c^8e^4x^{10} + 40000c^{10}d^3ex^6 + 1896c^8de^3x^8 + 11200c^{10}d^4x^4 + 54400c^8d^4x^4 + 40000c^{10}d^2e^2x^8 + 288c^8e^4x^{10} + 40000c^{10}d^3ex^6 + 1896c^8de^3x^8 + 11200c^{10}d^4x^4 + 54400c^8d^4x^4)$

```
input int(x^3*(e*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/10*e^3*x^10+3/8*d*e^2*x^8+1/2*d^2*e*x^6+1/4*x^4*d^3)+b/c^4*(1/10*c^4*arcsin(c*x)*e^3*x^10+3/8*c^4*arcsin(c*x)*d*e^2*x^8+1/2*c^4*arcsin(c*x)*d^2*e*x^6+1/4*arcsin(c*x)*c^4*x^4*d^3-1/40/c^6*(4*e^3*(-1/10*c^9*x^9*(-c^2*x^2+1)^(1/2)-9/80*c^7*x^7*(-c^2*x^2+1)^(1/2)-21/160*c^5*x^5*(-c^2*x^2+1)^(1/2)-21/128*c^3*x^3*(-c^2*x^2+1)^(1/2)-63/256*c*x*(-c^2*x^2+1)^(1/2)+63/256*arcsin(c*x))+10*d^3*c^6*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x))+15*d*c^2*e^2*(-1/8*c^7*x^7*(-c^2*x^2+1)^(1/2)-7/48*c^5*x^5*(-c^2*x^2+1)^(1/2)-35/192*c^3*x^3*(-c^2*x^2+1)^(1/2)-35/128*c*x*(-c^2*x^2+1)^(1/2)+35/128*arcsin(c*x))+20*d^2*c^4*e*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95

$$\int x^3(d + ex^2)^3(a + b \arcsin(cx)) dx$$
$$= \frac{7680 ac^{10}e^3x^{10} + 28800 ac^{10}de^2x^8 + 38400 ac^{10}d^2ex^6 + 19200 ac^{10}d^3x^4 + 15(512 bc^{10}e^3x^{10} + 1920 bc^{10}de$$

input `integrate(x^3*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output

```
1/76800*(7680*a*c^10*e^3*x^10 + 28800*a*c^10*d*e^2*x^8 + 38400*a*c^10*d^2*
e*x^6 + 19200*a*c^10*d^3*x^4 + 15*(512*b*c^10*e^3*x^10 + 1920*b*c^10*d*e^2
*x^8 + 2560*b*c^10*d^2*e*x^6 + 1280*b*c^10*d^3*x^4 - 480*b*c^6*d^3 - 800*b
*c^4*d^2*e - 525*b*c^2*d*e^2 - 126*b*e^3)*arcsin(c*x) + (768*b*c^9*e^3*x^9
+ 144*(25*b*c^9*d*e^2 + 6*b*c^7*e^3)*x^7 + 8*(800*b*c^9*d^2*e + 525*b*c^7
*d*e^2 + 126*b*c^5*e^3)*x^5 + 10*(480*b*c^9*d^3 + 800*b*c^7*d^2*e + 525*b*
c^5*d*e^2 + 126*b*c^3*e^3)*x^3 + 15*(480*b*c^7*d^3 + 800*b*c^5*d^2*e + 525
*b*c^3*d*e^2 + 126*b*c*e^3)*x)*sqrt(-c^2*x^2 + 1)/c^10
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.78

$$\int x^3(d + ex^2)^3(a + b \arcsin(cx)) dx$$
$$= \begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4 \arcsin(cx)}{4} + \frac{bd^2ex^6 \arcsin(cx)}{2} + \frac{3bde^2x^8 \arcsin(cx)}{8} + \frac{be^3x^{10} \arcsin(cx)}{10} + \frac{bd^3x^3\sqrt{-c^2x^2 + 1}}{16} \\ a\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) \end{cases}$$

input `integrate(x**3*(e*x**2+d)**3*(a+b*asin(c*x)),x)`

output

```
Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x*
*10/10 + b*d**3*x**4*asin(c*x)/4 + b*d**2*e*x**6*asin(c*x)/2 + 3*b*d*e**2*
x**8*asin(c*x)/8 + b*e**3*x**10*asin(c*x)/10 + b*d**3*x**3*sqrt(-c**2*x**2
+ 1)/(16*c) + b*d**2*e*x**5*sqrt(-c**2*x**2 + 1)/(12*c) + 3*b*d*e**2*x**7
*sqrt(-c**2*x**2 + 1)/(64*c) + b*e**3*x**9*sqrt(-c**2*x**2 + 1)/(100*c) +
3*b*d**3*x*sqrt(-c**2*x**2 + 1)/(32*c**3) + 5*b*d**2*e*x**3*sqrt(-c**2*x**
2 + 1)/(48*c**3) + 7*b*d*e**2*x**5*sqrt(-c**2*x**2 + 1)/(128*c**3) + 9*b*e
**3*x**7*sqrt(-c**2*x**2 + 1)/(800*c**3) - 3*b*d**3*asin(c*x)/(32*c**4) +
5*b*d**2*e*x*sqrt(-c**2*x**2 + 1)/(32*c**5) + 35*b*d*e**2*x**3*sqrt(-c**2*
x**2 + 1)/(512*c**5) + 21*b*e**3*x**5*sqrt(-c**2*x**2 + 1)/(1600*c**5) - 5
*b*d**2*e*asin(c*x)/(32*c**6) + 105*b*d*e**2*x*sqrt(-c**2*x**2 + 1)/(1024*
c**7) + 21*b*e**3*x**3*sqrt(-c**2*x**2 + 1)/(1280*c**7) - 105*b*d*e**2*asi
n(c*x)/(1024*c**8) + 63*b*e**3*x*sqrt(-c**2*x**2 + 1)/(2560*c**9) - 63*b*e
**3*asin(c*x)/(2560*c**10), Ne(c, 0)), (a*(d**3*x**4/4 + d**2*e*x**6/2 + 3
*d*e**2*x**8/8 + e**3*x**10/10), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.27

$$\int x^3(d + ex^2)^3(a + b \arcsin(cx)) dx = \frac{1}{10} ae^3 x^{10} + \frac{3}{8} ade^2 x^8 + \frac{1}{2} ad^2 ex^6 + \frac{1}{4} ad^3 x^4$$

$$+ \frac{1}{32} \left(8x^4 \arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5} \right) c \right) bd^3$$

$$+ \frac{1}{96} \left(48x^6 \arcsin(cx) + \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7} \right) c \right) bd^3$$

$$+ \frac{1}{1024} \left(384x^8 \arcsin(cx) + \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2x^2+1}x}{c^8} - \frac{105\arcsin(cx)}{c^9} \right) c \right) bd^3$$

$$+ \frac{1}{12800} \left(1280x^{10} \arcsin(cx) + \left(\frac{128\sqrt{-c^2x^2+1}x^9}{c^2} + \frac{144\sqrt{-c^2x^2+1}x^7}{c^4} + \frac{168\sqrt{-c^2x^2+1}x^5}{c^6} + \frac{210\sqrt{-c^2x^2+1}x^3}{c^8} - \frac{210\arcsin(cx)}{c^9} \right) c \right) bd^3$$

input

```
integrate(x^3*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")
```

output

```

1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/32
*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)
*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^3 + 1/96*(48*x^6*arcsin(c*x) + (8*sqrt(
-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 +
1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*d^2*e + 1/1024*(384*x^8*arcsin(c*x) +
(48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(
-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9
)*c)*b*d*e^2 + 1/12800*(1280*x^10*arcsin(c*x) + (128*sqrt(-c^2*x^2 + 1)*x^
9/c^2 + 144*sqrt(-c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(-c^2*x^2 + 1)*x^5/c^6 +
210*sqrt(-c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(-c^2*x^2 + 1)*x/c^10 - 315*arcsi
n(c*x)/c^11)*c)*b*e^3

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(309) = 618$.

Time = 0.14 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.41

$$\int x^3(d + ex^2)^3(a + b \arcsin(cx)) dx = \text{Too large to display}$$

input

```

integrate(x^3*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")

```

output

```

1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 - 1/16
*(-c^2*x^2 + 1)^(3/2)*b*d^3*x/c^3 + 1/4*(c^2*x^2 - 1)^2*b*d^3*arcsin(c*x)/
c^4 + 5/32*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 + 1/12*(c^2*x^2 - 1)^2*sqrt(-c^2
*x^2 + 1)*b*d^2*e*x/c^5 + 1/2*(c^2*x^2 - 1)*b*d^3*arcsin(c*x)/c^4 + 1/2*(c
^2*x^2 - 1)^3*b*d^2*e*arcsin(c*x)/c^6 - 13/48*(-c^2*x^2 + 1)^(3/2)*b*d^2*
e*x/c^5 + 3/64*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d*e^2*x/c^7 + 5/32*b*d^
3*arcsin(c*x)/c^4 + 3/2*(c^2*x^2 - 1)^2*b*d^2*e*arcsin(c*x)/c^6 + 3/8*(c^2
*x^2 - 1)^4*b*d*e^2*arcsin(c*x)/c^8 + 11/32*sqrt(-c^2*x^2 + 1)*b*d^2*e*x/c
^5 + 25/128*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d*e^2*x/c^7 + 1/100*(c^2*
x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^9 + 3/2*(c^2*x^2 - 1)*b*d^2*e*arcs
in(c*x)/c^6 + 3/2*(c^2*x^2 - 1)^3*b*d*e^2*arcsin(c*x)/c^8 + 1/10*(c^2*x^2
- 1)^5*b*e^3*arcsin(c*x)/c^10 - 163/512*(-c^2*x^2 + 1)^(3/2)*b*d*e^2*x/c^7
+ 41/800*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^9 + 11/32*b*d^2*e*a
rcsin(c*x)/c^6 + 9/4*(c^2*x^2 - 1)^2*b*d*e^2*arcsin(c*x)/c^8 + 1/2*(c^2*x^
2 - 1)^4*b*e^3*arcsin(c*x)/c^10 + 279/1024*sqrt(-c^2*x^2 + 1)*b*d*e^2*x/c^
7 + 171/1600*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^9 + 3/2*(c^2*x^2
- 1)*b*d*e^2*arcsin(c*x)/c^8 + (c^2*x^2 - 1)^3*b*e^3*arcsin(c*x)/c^10 - 1
49/1280*(-c^2*x^2 + 1)^(3/2)*b*e^3*x/c^9 + 279/1024*b*d*e^2*arcsin(c*x)/c^
8 + (c^2*x^2 - 1)^2*b*e^3*arcsin(c*x)/c^10 + 193/2560*sqrt(-c^2*x^2 + 1)*b
*e^3*x/c^9 + 1/2*(c^2*x^2 - 1)*b*e^3*arcsin(c*x)/c^10 + 193/2560*b*e^3*...

```

Mupad [F(-1)]

Timed out.

$$\int x^3(d + ex^2)^3(a + b \arcsin(cx)) dx = \int x^3(a + b \arcsin(cx)) (ex^2 + d)^3 dx$$

input

```
int(x^3*(a + b*asin(c*x))*(d + e*x^2)^3,x)
```

output

```
int(x^3*(a + b*asin(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.47

$$\int x^3(d + ex^2)^3(a + b \arcsin(cx)) dx$$

$$= \frac{-1890 \operatorname{asin}(cx) b e^3 - 7200 \operatorname{asin}(cx) b c^6 d^3 + 19200 a c^{10} d^3 x^4 + 7680 a c^{10} e^3 x^{10} + 19200 \operatorname{asin}(cx) b c^{10} d^3 x^4}{1}$$

input `int(x^3*(e*x^2+d)^3*(a+b*asin(c*x)),x)`

output

```
(19200*asin(c*x)*b*c**10*d**3*x**4 + 38400*asin(c*x)*b*c**10*d**2*e*x**6 +
28800*asin(c*x)*b*c**10*d*e**2*x**8 + 7680*asin(c*x)*b*c**10*e**3*x**10 -
7200*asin(c*x)*b*c**6*d**3 - 12000*asin(c*x)*b*c**4*d**2*e - 7875*asin(c*
x)*b*c**2*d*e**2 - 1890*asin(c*x)*b*e**3 + 4800*sqrt(-c**2*x**2 + 1)*b*c
**9*d**3*x**3 + 6400*sqrt(-c**2*x**2 + 1)*b*c**9*d**2*e*x**5 + 3600*sqrt
(-c**2*x**2 + 1)*b*c**9*d*e**2*x**7 + 768*sqrt(-c**2*x**2 + 1)*b*c**9*
e**3*x**9 + 7200*sqrt(-c**2*x**2 + 1)*b*c**7*d**3*x + 8000*sqrt(-c**2*
x**2 + 1)*b*c**7*d**2*e*x**3 + 4200*sqrt(-c**2*x**2 + 1)*b*c**7*d*e**2*x
**5 + 864*sqrt(-c**2*x**2 + 1)*b*c**7*e**3*x**7 + 12000*sqrt(-c**2*x**
2 + 1)*b*c**5*d**2*e*x + 5250*sqrt(-c**2*x**2 + 1)*b*c**5*d*e**2*x**3 +
1008*sqrt(-c**2*x**2 + 1)*b*c**5*e**3*x**5 + 7875*sqrt(-c**2*x**2 + 1)
*b*c**3*d*e**2*x + 1260*sqrt(-c**2*x**2 + 1)*b*c**3*e**3*x**3 + 1890*sqr
t(-c**2*x**2 + 1)*b*c*e**3*x + 19200*a*c**10*d**3*x**4 + 38400*a*c**10*d
**2*e*x**6 + 28800*a*c**10*d*e**2*x**8 + 7680*a*c**10*e**3*x**10)/(76800*c
**10)
```


3.443 $\int x^2(d + ex^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	3764
Mathematica [A] (verified)	3765
Rubi [A] (verified)	3765
Maple [A] (verified)	3767
Fricas [A] (verification not implemented)	3768
Sympy [A] (verification not implemented)	3769
Maxima [A] (verification not implemented)	3770
Giac [B] (verification not implemented)	3770
Mupad [F(-1)]	3771
Reduce [B] (verification not implemented)	3772

Optimal result

Integrand size = 21, antiderivative size = 287

$$\int x^2(d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{b(105c^6d^3 + 189c^4d^2e + 135c^2de^2 + 35e^3)\sqrt{1 - c^2x^2}}{315c^9}$$

$$- \frac{b(105c^6d^3 + 378c^4d^2e + 405c^2de^2 + 140e^3)(1 - c^2x^2)^{3/2}}{945c^9}$$

$$+ \frac{be(63c^4d^2 + 135c^2de + 70e^2)(1 - c^2x^2)^{5/2}}{525c^9}$$

$$- \frac{be^2(27c^2d + 28e)(1 - c^2x^2)^{7/2}}{441c^9} + \frac{be^3(1 - c^2x^2)^{9/2}}{81c^9}$$

$$+ \frac{1}{3}d^3x^3(a + b \arcsin(cx)) + \frac{3}{5}d^2ex^5(a + b \arcsin(cx)) + \frac{3}{7}de^2x^7(a + b \arcsin(cx)) + \frac{1}{9}e^3x^9(a + b \arcsin(cx))$$

output

```
1/315*b*(105*c^6*d^3+189*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(-c^2*x^2+1)^(1/2)
)/c^9-1/945*b*(105*c^6*d^3+378*c^4*d^2*e+405*c^2*d*e^2+140*e^3)*(-c^2*x^2+
1)^(3/2)/c^9+1/525*b*e*(63*c^4*d^2+135*c^2*d*e+70*e^2)*(-c^2*x^2+1)^(5/2)/
c^9-1/441*b*e^2*(27*c^2*d+28*e)*(-c^2*x^2+1)^(7/2)/c^9+1/81*b*e^3*(-c^2*x^
2+1)^(9/2)/c^9+1/3*d^3*x^3*(a+b*arcsin(c*x))+3/5*d^2*e*x^5*(a+b*arcsin(c*x
))+3/7*d*e^2*x^7*(a+b*arcsin(c*x))+1/9*e^3*x^9*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.80

$$\int x^2(d + ex^2)^3(a + b \arcsin(cx)) dx$$

$$= \frac{315ax^3(105d^3 + 189d^2ex^2 + 135de^2x^4 + 35e^3x^6) + \frac{b\sqrt{1-c^2x^2}(4480e^3 + 80c^2e^2(243d + 28ex^2) + 24c^4e(1323d^2 + 405dex^2 + 70e^2x^4) + 2c^6(11025d^3 + 7938d^2ex^2 + 3645de^2x^4 + 700e^3x^6) + c^8(11025d^3x^2 + 11907d^2ex^4 + 6075de^2x^6 + 1225e^3x^8))}{c^9} + 315bx^3(105d^3 + 189d^2ex^2 + 135de^2x^4 + 35e^3x^6) \arcsin(cx)}{99225}$$

input

```
Integrate[x^2*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(315*a*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) + (b*Sqr
t[1 - c^2*x^2]*(4480*e^3 + 80*c^2*e^2*(243*d + 28*e*x^2) + 24*c^4*e*(1323*
d^2 + 405*d*e*x^2 + 70*e^2*x^4) + 2*c^6*(11025*d^3 + 7938*d^2*e*x^2 + 3645
*d*e^2*x^4 + 700*e^3*x^6) + c^8*(11025*d^3*x^2 + 11907*d^2*e*x^4 + 6075*d*
e^2*x^6 + 1225*e^3*x^8)))/c^9 + 315*b*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d
*e^2*x^4 + 35*e^3*x^6)*ArcSin[c*x])/99225
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5230, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d + ex^2)^3(a + b \arcsin(cx)) dx$$

$$\downarrow 5230$$

$$-bc \int \frac{x^3(35e^3x^6 + 135de^2x^4 + 189d^2ex^2 + 105d^3)}{315\sqrt{1-c^2x^2}} dx + \frac{1}{3}d^3x^3(a + b \arcsin(cx)) + \frac{3}{5}d^2ex^5(a + b \arcsin(cx)) + \frac{3}{7}de^2x^7(a + b \arcsin(cx)) + \frac{1}{9}e^3x^9(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$\begin{aligned}
& -\frac{1}{315}bc \int \frac{x^3(35e^3x^6 + 135de^2x^4 + 189d^2ex^2 + 105d^3)}{\sqrt{1-c^2x^2}} dx + \frac{1}{3}d^3x^3(a + b \arcsin(cx)) + \\
& \quad \frac{3}{5}d^2ex^5(a + b \arcsin(cx)) + \frac{3}{7}de^2x^7(a + b \arcsin(cx)) + \frac{1}{9}e^3x^9(a + b \arcsin(cx)) \\
& \quad \downarrow \text{2331} \\
& -\frac{1}{630}bc \int \frac{x^2(35e^3x^6 + 135de^2x^4 + 189d^2ex^2 + 105d^3)}{\sqrt{1-c^2x^2}} dx^2 + \frac{1}{3}d^3x^3(a + b \arcsin(cx)) + \\
& \quad \frac{3}{5}d^2ex^5(a + b \arcsin(cx)) + \frac{3}{7}de^2x^7(a + b \arcsin(cx)) + \frac{1}{9}e^3x^9(a + b \arcsin(cx)) \\
& \quad \downarrow \text{2123} \\
& -\frac{1}{630}bc \int \left(\frac{35e^3(1-c^2x^2)^{7/2}}{c^8} - \frac{5e^2(27dc^2 + 28e)(1-c^2x^2)^{5/2}}{c^8} + \frac{3e(63d^2c^4 + 135dec^2 + 70e^2)(1-c^2x^2)^{3/2}}{c^8} \right. \\
& \quad \left. \frac{1}{3}d^3x^3(a + b \arcsin(cx)) + \frac{3}{5}d^2ex^5(a + b \arcsin(cx)) + \frac{3}{7}de^2x^7(a + b \arcsin(cx)) + \frac{1}{9}e^3x^9(a + \right. \\
& \quad \quad \left. b \arcsin(cx)) \right) \\
& \quad \downarrow \text{2009} \\
& \frac{1}{3}d^3x^3(a + b \arcsin(cx)) + \frac{3}{5}d^2ex^5(a + b \arcsin(cx)) + \frac{3}{7}de^2x^7(a + b \arcsin(cx)) + \frac{1}{9}e^3x^9(a + \\
& \quad \quad b \arcsin(cx)) - \\
& \frac{1}{630}bc \left(\frac{10e^2(1-c^2x^2)^{7/2}(27c^2d + 28e)}{7c^{10}} - \frac{70e^3(1-c^2x^2)^{9/2}}{9c^{10}} - \frac{6e(1-c^2x^2)^{5/2}(63c^4d^2 + 135c^2de + 70e^2)}{5c^{10}} + \dots \right)
\end{aligned}$$

input `Int[x^2*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/630*(b*c*((-2*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*Sqrt[1 - c^2*x^2])/c^10 + (2*(105*c^6*d^3 + 378*c^4*d^2*e + 405*c^2*d*e^2 + 140*e^3)*(1 - c^2*x^2)^(3/2))/(3*c^10) - (6*e*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^(5/2))/(5*c^10) + (10*e^2*(27*c^2*d + 28*e)*(1 - c^2*x^2)^(7/2))/(7*c^10) - (70*e^3*(1 - c^2*x^2)^(9/2))/(9*c^10)) + (d^3*x^3*(a + b*ArcSin[c*x])/3 + (3*d^2*e*x^5*(a + b*ArcSin[c*x])/5 + (3*d*e^2*x^7*(a + b*ArcSin[c*x])/7 + (e^3*x^9*(a + b*ArcSin[c*x])/9`

Definitions of rubi rules used

- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2123 `Int[(Px_)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2])`
- rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`
- rule 5230 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

method	result
parts	$a\left(\frac{1}{9}e^3x^9 + \frac{3}{7}de^2x^7 + \frac{3}{5}d^2ex^5 + \frac{1}{3}d^3x^3\right) + \frac{b\left(\frac{c^3\arcsin(cx)e^3x^9}{9} + \frac{3c^3\arcsin(cx)de^2x^7}{7} + \frac{3c^3\arcsin(cx)d^2ex^5}{5} + \frac{\arcsin(cx)d^3x^3}{3}\right)}{c^6}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^3c^9x^3 + \frac{3}{5}d^2c^9ex^5 + \frac{3}{7}dc^9e^2x^7 + \frac{1}{9}e^3c^9x^9\right)}{c^6} + \frac{b\left(\frac{\arcsin(cx)d^3c^9x^3}{3} + \frac{3\arcsin(cx)d^2c^9ex^5}{5} + \frac{3\arcsin(cx)dc^9e^2x^7}{7} + \frac{\arcsin(cx)d^3x^3}{9}\right)}{c^6}$
default	$\frac{a\left(\frac{1}{3}d^3c^9x^3 + \frac{3}{5}d^2c^9ex^5 + \frac{3}{7}dc^9e^2x^7 + \frac{1}{9}e^3c^9x^9\right)}{c^6} + \frac{b\left(\frac{\arcsin(cx)d^3c^9x^3}{3} + \frac{3\arcsin(cx)d^2c^9ex^5}{5} + \frac{3\arcsin(cx)dc^9e^2x^7}{7} + \frac{\arcsin(cx)d^3x^3}{9}\right)}{c^6}$
orering	$(20825c^{10}e^4x^{12} + 104600c^{10}de^3x^{10} + 209466c^{10}d^2e^2x^8 + 1400c^8e^4x^{10} + 204624c^{10}d^3ex^6 + 10070c^8de^3x^8 + 55125c^{10}d^4x^4)$

```
input int(x^2*(e*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/9*e^3*x^9+3/7*d*e^2*x^7+3/5*d^2*e*x^5+1/3*d^3*x^3)+b/c^3*(1/9*c^3*arcsin(c*x)*e^3*x^9+3/7*c^3*arcsin(c*x)*d*e^2*x^7+3/5*c^3*arcsin(c*x)*d^2*e*x^5+1/3*arcsin(c*x)*c^3*x^3*d^3-1/315/c^6*(35*e^3*(-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c^4*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*x^2+1)^(1/2))+105*d^3*c^6*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))+135*d*c^2*e^2*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))+189*d^2*c^4*e*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.97

$$\int x^2(d + ex^2)^3(a + b \arcsin(cx)) dx$$

$$= \frac{11025 ac^9 e^3 x^9 + 42525 ac^9 de^2 x^7 + 59535 ac^9 d^2 ex^5 + 33075 ac^9 d^3 x^3 + 315 (35 bc^9 e^3 x^9 + 135 bc^9 de^2 x^7 + 105 b^2 c^9 d^2 ex^5 + 35 b^2 c^9 d^3 x^3 + 315 bc^9 e^3 x^9 + 135 bc^9 de^2 x^7 + 105 b^2 c^9 d^2 ex^5 + 35 b^2 c^9 d^3 x^3)}{c^6}$$

```
input integrate(x^2*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
1/99225*(11025*a*c^9*e^3*x^9 + 42525*a*c^9*d*e^2*x^7 + 59535*a*c^9*d^2*e*x^5 + 33075*a*c^9*d^3*x^3 + 315*(35*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 189*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3*x^3)*arcsin(c*x) + (1225*b*c^8*e^3*x^8 + 22050*b*c^6*d^3 + 31752*b*c^4*d^2*e + 25*(243*b*c^8*d*e^2 + 56*b*c^6*e^3)*x^6 + 19440*b*c^2*d*e^2 + 3*(3969*b*c^8*d^2*e + 2430*b*c^6*d*e^2 + 560*b*c^4*e^3)*x^4 + 4480*b*e^3 + (11025*b*c^8*d^3 + 15876*b*c^6*d^2*e + 9720*b*c^4*d*e^2 + 2240*b*c^2*e^3)*x^2)*sqrt(-c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.83

$$\int x^2 (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3 \arcsin(cx)}{3} + \frac{3bd^2ex^5 \arcsin(cx)}{5} + \frac{3bde^2x^7 \arcsin(cx)}{7} + \frac{be^3x^9 \arcsin(cx)}{9} + \frac{bd^3x^2\sqrt{-c^2x^2+1}}{9c} \\ a \left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9} \right) \end{cases}$$

input

```
integrate(x**2*(e*x**2+d)**3*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x**9/9 + b*d**3*x**3*asin(c*x)/3 + 3*b*d**2*e*x**5*asin(c*x)/5 + 3*b*d*e**2*x**7*asin(c*x)/7 + b*e**3*x**9*asin(c*x)/9 + b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(9*c) + 3*b*d**2*e*x**4*sqrt(-c**2*x**2 + 1)/(25*c) + 3*b*d*e**2*x**6*sqrt(-c**2*x**2 + 1)/(49*c) + b*e**3*x**8*sqrt(-c**2*x**2 + 1)/(81*c) + 2*b*d**3*sqrt(-c**2*x**2 + 1)/(9*c**3) + 4*b*d**2*e*x**2*sqrt(-c**2*x**2 + 1)/(25*c**3) + 18*b*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(245*c**3) + 8*b*e**3*x**6*sqrt(-c**2*x**2 + 1)/(567*c**3) + 8*b*d**2*e*sqrt(-c**2*x**2 + 1)/(25*c**5) + 24*b*d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*x**4*sqrt(-c**2*x**2 + 1)/(945*c**5) + 48*b*d*e**2*sqrt(-c**2*x**2 + 1)/(245*c**7) + 64*b*e**3*x**2*sqrt(-c**2*x**2 + 1)/(2835*c**7) + 128*b*e**3*sqrt(-c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), (a*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e**2*x**7/7 + e**3*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.34

$$\int x^2(d+ex^2)^3(a+b\arcsin(cx))dx = \frac{1}{9}ae^3x^9 + \frac{3}{7}ade^2x^7 + \frac{3}{5}ad^2ex^5 + \frac{1}{3}ad^3x^3 + \frac{1}{9}\left(3x^3\arcsin(cx) + c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^3 + \frac{1}{25}\left(15x^5\arcsin(cx) + \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd^2e + \frac{3}{245}\left(35x^7\arcsin(cx) + \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right)c\right)bd^2e + \frac{1}{2835}\left(315x^9\arcsin(cx) + \left(\frac{35\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\sqrt{-c^2x^2+1}x^2}{c^8} + \frac{128\sqrt{-c^2x^2+1}}{c^{10}}\right)c\right)bd^2e$$

input `integrate(x^2*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^3 + 1/25*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^2*e + 3/245*(35*x^7*arcsin(c*x) + (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*d^2*e + 1/2835*(315*x^9*arcsin(c*x) + (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*d^2*e`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. 2(259) = 518.

Time = 0.14 (sec) , antiderivative size = 711, normalized size of antiderivative = 2.48

$$\int x^2(d+ex^2)^3(a+b\arcsin(cx))dx = \text{Too large to display}$$

input `integrate(x^2*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/3*(c
^2*x^2 - 1)*b*d^3*x*arcsin(c*x)/c^2 + 1/3*b*d^3*x*arcsin(c*x)/c^2 + 3/5*(c
^2*x^2 - 1)^2*b*d^2*e*x*arcsin(c*x)/c^4 + 6/5*(c^2*x^2 - 1)*b*d^2*e*x*arcs
in(c*x)/c^4 + 3/7*(c^2*x^2 - 1)^3*b*d*e^2*x*arcsin(c*x)/c^6 - 1/9*(-c^2*x^
2 + 1)^(3/2)*b*d^3/c^3 + 3/5*b*d^2*e*x*arcsin(c*x)/c^4 + 9/7*(c^2*x^2 - 1)
^2*b*d*e^2*x*arcsin(c*x)/c^6 + 1/9*(c^2*x^2 - 1)^4*b*e^3*x*arcsin(c*x)/c^8
+ 1/3*sqrt(-c^2*x^2 + 1)*b*d^3/c^3 + 3/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 +
1)*b*d^2*e/c^5 + 9/7*(c^2*x^2 - 1)*b*d*e^2*x*arcsin(c*x)/c^6 + 4/9*(c^2*x
^2 - 1)^3*b*e^3*x*arcsin(c*x)/c^8 - 2/5*(-c^2*x^2 + 1)^(3/2)*b*d^2*e/c^5 +
3/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^7 + 3/7*b*d*e^2*x*arcsi
n(c*x)/c^6 + 2/3*(c^2*x^2 - 1)^2*b*e^3*x*arcsin(c*x)/c^8 + 3/5*sqrt(-c^2*x
^2 + 1)*b*d^2*e/c^5 + 9/35*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^7
+ 1/81*(c^2*x^2 - 1)^4*sqrt(-c^2*x^2 + 1)*b*e^3/c^9 + 4/9*(c^2*x^2 - 1)*b*
e^3*x*arcsin(c*x)/c^8 - 3/7*(-c^2*x^2 + 1)^(3/2)*b*d*e^2/c^7 + 4/63*(c^2*x
^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*e^3/c^9 + 1/9*b*e^3*x*arcsin(c*x)/c^8 + 3/7
*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^7 + 2/15*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*
b*e^3/c^9 - 4/27*(-c^2*x^2 + 1)^(3/2)*b*e^3/c^9 + 1/9*sqrt(-c^2*x^2 + 1)*b
*e^3/c^9

```

Mupad [F(-1)]

Timed out.

$$\int x^2(d + ex^2)^3(a + b \arcsin(cx)) dx = \int x^2(a + b \arcsin(cx))(ex^2 + d)^3 dx$$

input

```
int(x^2*(a + b*asin(c*x))*(d + e*x^2)^3,x)
```

output

```
int(x^2*(a + b*asin(c*x))*(d + e*x^2)^3, x)
```


Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.52

$$\int x^2(d + ex^2)^3(a + b \arcsin(cx)) dx$$

$$= \frac{33075 a \sin(cx) b c^9 d^3 x^3 + 59535 a \sin(cx) b c^9 d^2 e x^5 + 42525 a \sin(cx) b c^9 d e^2 x^7 + 11025 a \sin(cx) b c^9 e^3 x^9 -$$

input `int(x^2*(e*x^2+d)^3*(a+b*asin(c*x)),x)`

output

```
(33075*asin(c*x)*b*c**9*d**3*x**3 + 59535*asin(c*x)*b*c**9*d**2*e*x**5 + 4
2525*asin(c*x)*b*c**9*d*e**2*x**7 + 11025*asin(c*x)*b*c**9*e**3*x**9 + 110
25*sqrt(-c**2*x**2+1)*b*c**8*d**3*x**2 + 11907*sqrt(-c**2*x**2+1)*
b*c**8*d**2*e*x**4 + 6075*sqrt(-c**2*x**2+1)*b*c**8*d*e**2*x**6 + 1225
*sqrt(-c**2*x**2+1)*b*c**8*e**3*x**8 + 22050*sqrt(-c**2*x**2+1)*b*
c**6*d**3 + 15876*sqrt(-c**2*x**2+1)*b*c**6*d**2*e*x**2 + 7290*sqrt(-
c**2*x**2+1)*b*c**6*d*e**2*x**4 + 1400*sqrt(-c**2*x**2+1)*b*c**6*e
**3*x**6 + 31752*sqrt(-c**2*x**2+1)*b*c**4*d**2*e + 9720*sqrt(-c**2*x
**2+1)*b*c**4*d*e**2*x**2 + 1680*sqrt(-c**2*x**2+1)*b*c**4*e**3*x**4
+ 19440*sqrt(-c**2*x**2+1)*b*c**2*d*e**2 + 2240*sqrt(-c**2*x**2+1)
)*b*c**2*e**3*x**2 + 4480*sqrt(-c**2*x**2+1)*b*e**3 + 33075*a*c**9*d**
3*x**3 + 59535*a*c**9*d**2*e*x**5 + 42525*a*c**9*d*e**2*x**7 + 11025*a*c**
9*e**3*x**9)/(99225*c**9)
```

3.444 $\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	3773
Mathematica [A] (verified)	3774
Rubi [A] (verified)	3774
Maple [A] (verified)	3778
Fricas [A] (verification not implemented)	3778
Sympy [B] (verification not implemented)	3779
Maxima [A] (verification not implemented)	3780
Giac [B] (verification not implemented)	3780
Mupad [F(-1)]	3781
Reduce [B] (verification not implemented)	3781

Optimal result

Integrand size = 19, antiderivative size = 251

$$\begin{aligned}
 & \int x(d + ex^2)^3 (a + b \arcsin(cx)) dx \\
 &= \frac{b(256c^6d^3 + 288c^4d^2e + 160c^2de^2 + 35e^3) x \sqrt{1 - c^2x^2}}{1024c^7} \\
 &+ \frac{be(288c^4d^2 + 160c^2de + 35e^2) x^3 \sqrt{1 - c^2x^2}}{1536c^5} \\
 &+ \frac{be^2(32c^2d + 7e) x^5 \sqrt{1 - c^2x^2}}{384c^3} + \frac{be^3 x^7 \sqrt{1 - c^2x^2}}{64c} \\
 &- \frac{b(128c^8d^4 + 256c^6d^3e + 288c^4d^2e^2 + 160c^2de^3 + 35e^4) \arcsin(cx)}{1024c^8e} \\
 &+ \frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e}
 \end{aligned}$$

output

```

1/1024*b*(256*c^6*d^3+288*c^4*d^2*e+160*c^2*d*e^2+35*e^3)*x*(-c^2*x^2+1)^(
1/2)/c^7+1/1536*b*e*(288*c^4*d^2+160*c^2*d*e+35*e^2)*x^3*(-c^2*x^2+1)^(1/2
)/c^5+1/384*b*e^2*(32*c^2*d+7*e)*x^5*(-c^2*x^2+1)^(1/2)/c^3+1/64*b*e^3*x^7
*(-c^2*x^2+1)^(1/2)/c-1/1024*b*(128*c^8*d^4+256*c^6*d^3*e+288*c^4*d^2*e^2+
160*c^2*d*e^3+35*e^4)*arcsin(c*x)/c^8/e+1/8*(e*x^2+d)^4*(a+b*arcsin(c*x))/
e

```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92

$$\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{cx(384ac^7x(4d^3 + 6d^2ex^2 + 4de^2x^4 + e^3x^6) + b\sqrt{1 - c^2x^2}(105e^3 + 10c^2e^2(48d + 7ex^2) + 8c^4e(108d^2 + 40d^2ex^2 + 7e^2x^4) + 16c^6(48d^3 + 36d^2ex^2 + 16d^2ex^4 + 3e^3x^6))) + 3b(-256c^6d^3 - 288c^4d^2e - 160c^2de^2 - 35e^3 + 128c^8(4d^3x^2 + 6d^2ex^4 + 4de^2x^6 + e^3x^8))*\text{ArcSin}[cx]}{(3072c^8)}$$

input

```
Integrate[x*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(c*x*(384*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) + b*Sqrt[1 - c^2*x^2]*(105*e^3 + 10*c^2*e^2*(48*d + 7*e*x^2) + 8*c^4*e*(108*d^2 + 40*d*e*x^2 + 7*e^2*x^4) + 16*c^6*(48*d^3 + 36*d^2*e*x^2 + 16*d*e^2*x^4 + 3*e^3*x^6))) + 3*b*(-256*c^6*d^3 - 288*c^4*d^2*e - 160*c^2*d*e^2 - 35*e^3 + 128*c^8*(4*d^3*x^2 + 6*d^2*e*x^4 + 4*d*e^2*x^6 + e^3*x^8))*ArcSin[c*x])/(3072*c^8)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5228, 318, 25, 403, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$\downarrow 5228$$

$$\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e} - \frac{bc \int \frac{(ex^2+d)^4 dx}{\sqrt{1-c^2x^2}}}{8e}$$

$$\downarrow 318$$

$$\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e} - \frac{bc \left(-\frac{\int -\frac{(ex^2+d)^2 (7e(2dc^2+e)x^2+d(8dc^2+e))}{\sqrt{1-c^2x^2}} dx}{8c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^3}{8c^2} \right)}{8e}$$

↓ 25

$$\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e} - \frac{bc \left(\frac{\int \frac{(ex^2+d)^2 (7e(2dc^2+e)x^2+d(8dc^2+e))}{\sqrt{1-c^2x^2}} dx}{8c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^3}{8c^2} \right)}{8e}$$

↓ 403

$$\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e} - \frac{bc \left(-\frac{\int -\frac{(ex^2+d)(e(104d^2c^4+104dec^2+35e^2)x^2+d(48d^2c^4+20dec^2+7e^2))}{\sqrt{1-c^2x^2}} dx}{6c^2}}{8c^2} - \frac{7ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)^2}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^3}{8c^2} \right)}{8e}$$

↓ 25

$$\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e} - \frac{bc \left(\frac{\int \frac{(ex^2+d)(e(104d^2c^4+104dec^2+35e^2)x^2+d(48d^2c^4+20dec^2+7e^2))}{\sqrt{1-c^2x^2}} dx}{6c^2}}{8c^2} - \frac{7ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)^2}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(d+ex^2)^3}{8c^2} \right)}{8e}$$

↓ 403

$$\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{8e} - \frac{bc \left(-\frac{\int -\frac{5e(2dc^2+e)(40d^2c^4+40dec^2+21e^2)x^2+d(192d^3c^6+184d^2ec^4+132de^2c^2+35e^3)}{\sqrt{1-c^2x^2}} dx}{4c^2}}{6c^2} - \frac{ex\sqrt{1-c^2x^2}(104c^4d^2+104c^2de+35e^2)(d+ex^2)}{4c^2} - \frac{7ex\sqrt{1-c^2x^2}(d+ex^2)^3}{8c^2} \right)}{8e}$$

↓ 25

$$bc \left(\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{\int \frac{5e(2dc^2 + e)(40d^2c^4 + 40dec^2 + 21e^2)x^2 + d(192d^3c^6 + 184d^2ec^4 + 132de^2c^2 + 35e^3)}{\sqrt{1-c^2x^2}} dx - \frac{8e}{4c^2} - \frac{ex\sqrt{1-c^2x^2}(104c^4d^2 + 104c^2de + 35e^2)(d+ex^2)}{4c^2} - \frac{7ex\sqrt{1-c^2x^2}}{4c^2}}{6c^2} - \frac{8e}{8c^2} \right)$$

299

$$bc \left(\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{\frac{3(128c^8d^4 + 256c^6d^3e + 288c^4d^2e^2 + 160c^2de^3 + 35e^4)}{2c^2} \int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(40c^4d^2 + 40c^2de + 21e^2)}{2c^2} - \frac{ex\sqrt{1-c^2x^2}(104c^4d^2 + 104c^2de + 35e^2)(d+ex^2)}{4c^2}}{4c^2} - \frac{8e}{6c^2} - \frac{8e}{8c^2}}{8e}$$

223

$$bc \left(\frac{(d + ex^2)^4 (a + b \arcsin(cx))}{\frac{3 \arcsin(cx)(128c^8d^4 + 256c^6d^3e + 288c^4d^2e^2 + 160c^2de^3 + 35e^4)}{2c^3} - \frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(40c^4d^2 + 40c^2de + 21e^2)}{2c^2} - \frac{ex\sqrt{1-c^2x^2}(104c^4d^2 + 104c^2de + 35e^2)(d+ex^2)}{4c^2}}{4c^2} - \frac{8e}{6c^2} - \frac{8e}{8c^2}}{8e}$$

```
input Int [x*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]
```

```
output ((d + e*x^2)^4*(a + b*ArcSin[c*x]))/(8*e) - (b*c*(-1/8*(e*x*sqrt[1 - c^2*x^2])*(d + e*x^2)^3)/c^2 + ((-7*e*(2*c^2*d + e)*x*sqrt[1 - c^2*x^2])*(d + e*x^2)^2)/(6*c^2) + (-1/4*(e*(104*c^4*d^2 + 104*c^2*d*e + 35*e^2)*x*sqrt[1 - c^2*x^2])*(d + e*x^2))/c^2 + ((-5*e*(2*c^2*d + e)*(40*c^4*d^2 + 40*c^2*d*e + 21*e^2)*x*sqrt[1 - c^2*x^2])/(2*c^2) + (3*(128*c^8*d^4 + 256*c^6*d^3*e + 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*ArcSin[c*x])/(2*c^3))/(4*c^2))/(6*c^2))/(8*c^2))/(8*e)
```

Defintions of rubi rules used

- rule 25 $\text{Int}[-(\text{Fx}_), \text{x_Symbol}] \rightarrow \text{Simp}[\text{Identity}[-1] \quad \text{Int}[\text{Fx}, \text{x}], \text{x}]$
- rule 223 $\text{Int}[1/\text{Sqrt}[(\text{a}_) + (\text{b}_.)(\text{x}_)^2], \text{x_Symbol}] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-\text{b}, 2]*(\text{x}/\text{Sqrt}[\text{a}])]/\text{Rt}[-\text{b}, 2], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{a}, 0] \ \&\& \ \text{NegQ}[\text{b}]$
- rule 299 $\text{Int}[(\text{a}_) + (\text{b}_.)(\text{x}_)^2]^{(\text{p}_)} * ((\text{c}_) + (\text{d}_.)(\text{x}_)^2), \text{x_Symbol}] \rightarrow \text{Simp}[\text{d} * \text{x} * ((\text{a} + \text{b} * \text{x}^2)^{(\text{p} + 1)} / (\text{b} * (2 * \text{p} + 3))), \text{x}] - \text{Simp}[(\text{a} * \text{d} - \text{b} * \text{c} * (2 * \text{p} + 3)) / (\text{b} * (2 * \text{p} + 3)) \quad \text{Int}[(\text{a} + \text{b} * \text{x}^2)^{\text{p}}, \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{b} * \text{c} - \text{a} * \text{d}, 0] \ \&\& \ \text{NeQ}[2 * \text{p} + 3, 0]$
- rule 318 $\text{Int}[(\text{a}_) + (\text{b}_.)(\text{x}_)^2]^{(\text{p}_)} * ((\text{c}_) + (\text{d}_.)(\text{x}_)^2)^{(\text{q}_)}, \text{x_Symbol}] \rightarrow \text{Simp}[\text{d} * \text{x} * (\text{a} + \text{b} * \text{x}^2)^{(\text{p} + 1)} * ((\text{c} + \text{d} * \text{x}^2)^{(\text{q} - 1)} / (\text{b} * (2 * (\text{p} + \text{q}) + 1))), \text{x}] + \text{Simp}[1 / (\text{b} * (2 * (\text{p} + \text{q}) + 1)) \quad \text{Int}[(\text{a} + \text{b} * \text{x}^2)^{\text{p}} * (\text{c} + \text{d} * \text{x}^2)^{(\text{q} - 2)} * \text{Simp}[\text{c} * (\text{b} * \text{c} * (2 * (\text{p} + \text{q}) + 1) - \text{a} * \text{d}) + \text{d} * (\text{b} * \text{c} * (2 * (\text{p} + 2 * \text{q} - 1) + 1) - \text{a} * \text{d} * (2 * (\text{q} - 1) + 1)) * \text{x}^2, \text{x}], \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{p}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{b} * \text{c} - \text{a} * \text{d}, 0] \ \&\& \ \text{GtQ}[\text{q}, 1] \ \&\& \ \text{NeQ}[2 * (\text{p} + \text{q}) + 1, 0] \ \&\& \ \text{!GtQ}[\text{p}, 1] \ \&\& \ \text{IntBinomialQ}[\text{a}, \text{b}, \text{c}, \text{d}, 2, \text{p}, \text{q}, \text{x}]$
- rule 403 $\text{Int}[(\text{a}_) + (\text{b}_.)(\text{x}_)^2]^{(\text{p}_)} * ((\text{c}_) + (\text{d}_.)(\text{x}_)^2)^{(\text{q}_)} * ((\text{e}_) + (\text{f}_.)(\text{x}_)^2), \text{x_Symbol}] \rightarrow \text{Simp}[\text{f} * \text{x} * (\text{a} + \text{b} * \text{x}^2)^{(\text{p} + 1)} * ((\text{c} + \text{d} * \text{x}^2)^{\text{q}} / (\text{b} * (2 * (\text{p} + \text{q} + 1) + 1))), \text{x}] + \text{Simp}[1 / (\text{b} * (2 * (\text{p} + \text{q} + 1) + 1)) \quad \text{Int}[(\text{a} + \text{b} * \text{x}^2)^{\text{p}} * (\text{c} + \text{d} * \text{x}^2)^{(\text{q} - 1)} * \text{Simp}[\text{c} * (\text{b} * \text{e} - \text{a} * \text{f} + \text{b} * \text{e} * 2 * (\text{p} + \text{q} + 1)) + (\text{d} * (\text{b} * \text{e} - \text{a} * \text{f}) + \text{f} * 2 * \text{q} * (\text{b} * \text{c} - \text{a} * \text{d}) + \text{b} * \text{d} * \text{e} * 2 * (\text{p} + \text{q} + 1)) * \text{x}^2, \text{x}], \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{p}\}, \text{x}] \ \&\& \ \text{GtQ}[\text{q}, 0] \ \&\& \ \text{NeQ}[2 * (\text{p} + \text{q} + 1) + 1, 0]$
- rule 5228 $\text{Int}[(\text{a}_.) + \text{ArcSin}[(\text{c}_.)(\text{x}_)] * (\text{b}_.)(\text{x}_)] * ((\text{d}_) + (\text{e}_.)(\text{x}_)^2)^{(\text{p}_)}, \text{x_Symbol}] \rightarrow \text{Simp}[(\text{d} + \text{e} * \text{x}^2)^{(\text{p} + 1)} * ((\text{a} + \text{b} * \text{ArcSin}[\text{c} * \text{x}]) / (2 * \text{e} * (\text{p} + 1))), \text{x}] - \text{Simp}[\text{b} * (\text{c} / (2 * \text{e} * (\text{p} + 1))) \quad \text{Int}[(\text{d} + \text{e} * \text{x}^2)^{(\text{p} + 1)} / \text{Sqrt}[1 - \text{c}^2 * \text{x}^2], \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{p}\}, \text{x}] \ \&\& \ \text{NeQ}[\text{c}^2 * \text{d} + \text{e}, 0] \ \&\& \ \text{NeQ}[\text{p}, -1]$

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.45

method	result
parts	$\frac{a(e x^2+d)^4}{8e} + \frac{b \left(\frac{c^2 e^3 \arcsin(cx)x^8}{8} + \frac{c^2 e^2 \arcsin(cx)x^6 d}{2} + \frac{3c^2 e \arcsin(cx)x^4 d^2}{4} + \frac{\arcsin(cx)c^2 x^2 d^3}{2} + \frac{c^2 \arcsin(cx)d^4}{8e} - \frac{c^8 d^4}{8e} \right)}{8e}$
derivativedivides	$\frac{a(c^2 e x^2+c^2 d)^4}{8c^6 e} + \frac{b \left(\frac{\arcsin(cx)c^8 d^4}{8e} + \frac{\arcsin(cx)c^8 d^3 x^2}{2} + \frac{3e \arcsin(cx)c^8 d^2 x^4}{4} + \frac{e^2 \arcsin(cx)c^8 d x^6}{2} + \frac{e^3 \arcsin(cx)c^8 x^8}{8} - \frac{c^8 d^4}{8e} \right)}{8c^6 e}$
default	$\frac{a(c^2 e x^2+c^2 d)^4}{8c^6 e} + \frac{b \left(\frac{\arcsin(cx)c^8 d^4}{8e} + \frac{\arcsin(cx)c^8 d^3 x^2}{2} + \frac{3e \arcsin(cx)c^8 d^2 x^4}{4} + \frac{e^2 \arcsin(cx)c^8 d x^6}{2} + \frac{e^3 \arcsin(cx)c^8 x^8}{8} - \frac{c^8 d^4}{8e} \right)}{8c^6 e}$
orering	$(720c^8 e^4 x^{10} + 3760c^8 d e^3 x^8 + 8128c^8 d^2 e^2 x^6 + 56c^6 e^4 x^8 + 9792c^8 d^3 e x^4 + 456c^6 d e^3 x^6 + 2304c^8 d^4 x^2 + 2080c^6 d^2 e^2 x^4 + 980c^8 d^4) \arcsin(cx) + \dots$

```
input int(x*(e*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output 1/8*a*(e*x^2+d)^4/e+b/c^2*(1/8*c^2*e^3*arcsin(c*x)*x^8+1/2*c^2*e^2*arcsin(c*x)*x^6*d+3/4*c^2*e*arcsin(c*x)*x^4*d^2+1/2*arcsin(c*x)*c^2*x^2*d^3+1/8*c^2/e*arcsin(c*x)*d^4-1/8/c^6/e*(c^8*d^4*arcsin(c*x)+e^4*(-1/8*c^7*x^7*(-c^2*x^2+1)^(1/2)-7/48*c^5*x^5*(-c^2*x^2+1)^(1/2)-35/192*c^3*x^3*(-c^2*x^2+1)^(1/2)-35/128*c*x*(-c^2*x^2+1)^(1/2)+35/128*arcsin(c*x))+4*d*c^2*e^3*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))+6*d^2*c^4*e^2*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x))+4*d^3*c^6*e*(-1/2*c*x*(-c^2*x^2+1)^(1/2)+1/2*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx = \frac{384 ac^8 e^3 x^8 + 1536 ac^8 d e^2 x^6 + 2304 ac^8 d^2 e x^4 + 1536 ac^8 d^3 x^2 + 3(128 bc^8 e^3 x^8 + 512 bc^8 d e^2 x^6 + 768 bc^8 d^2 e x^4 + 512 bc^8 d^3 x^2 + 3(128 bc^8 e^3 x^8 + 512 bc^8 d e^2 x^6 + 768 bc^8 d^2 e x^4 + 512 bc^8 d^3 x^2)) \arcsin(cx)}{8e}$$

input `integrate(x*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output
$$\frac{1}{3072}*(384*a*c^8*e^3*x^8 + 1536*a*c^8*d*e^2*x^6 + 2304*a*c^8*d^2*e*x^4 + 1536*a*c^8*d^3*x^2 + 3*(128*b*c^8*e^3*x^8 + 512*b*c^8*d*e^2*x^6 + 768*b*c^8*d^2*e*x^4 + 512*b*c^8*d^3*x^2 - 256*b*c^6*d^3 - 288*b*c^4*d^2*e - 160*b*c^2*d*e^2 - 35*b*e^3)*arcsin(c*x) + (48*b*c^7*e^3*x^7 + 8*(32*b*c^7*d*e^2 + 7*b*c^5*e^3)*x^5 + 2*(288*b*c^7*d^2*e + 160*b*c^5*d*e^2 + 35*b*c^3*e^3)*x^3 + 3*(256*b*c^7*d^3 + 288*b*c^5*d^2*e + 160*b*c^3*d*e^2 + 35*b*c*e^3)*x)*sqrt(-c^2*x^2 + 1))/c^8$$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(238) = 476$.

Time = 0.90 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.92

$$\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2 \arcsin(cx)}{2} + \frac{3bd^2ex^4 \arcsin(cx)}{4} + \frac{bde^2x^6 \arcsin(cx)}{2} + \frac{be^3x^8 \arcsin(cx)}{8} + \frac{bd^3x\sqrt{-c^2x^2}}{4c} \\ a\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

input `integrate(x*(e*x**2+d)**3*(a+b*asin(c*x)),x)`

output `Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**8/8 + b*d**3*x**2*asin(c*x)/2 + 3*b*d**2*e*x**4*asin(c*x)/4 + b*d*e**2*x**6*asin(c*x)/2 + b*e**3*x**8*asin(c*x)/8 + b*d**3*x*sqrt(-c**2*x**2 + 1)/(4*c) + 3*b*d**2*e*x**3*sqrt(-c**2*x**2 + 1)/(16*c) + b*d*e**2*x**5*sqrt(-c**2*x**2 + 1)/(12*c) + b*e**3*x**7*sqrt(-c**2*x**2 + 1)/(64*c) - b*d**3*asin(c*x)/(4*c**2) + 9*b*d**2*e*x*sqrt(-c**2*x**2 + 1)/(32*c**3) + 5*b*d*e**2*x**3*sqrt(-c**2*x**2 + 1)/(48*c**3) + 7*b*e**3*x**5*sqrt(-c**2*x**2 + 1)/(384*c**3) - 9*b*d**2*e*asin(c*x)/(32*c**4) + 5*b*d*e**2*x*sqrt(-c**2*x**2 + 1)/(32*c**5) + 35*b*e**3*x**3*sqrt(-c**2*x**2 + 1)/(1536*c**5) - 5*b*d*e**2*asin(c*x)/(32*c**6) + 35*b*e**3*x*sqrt(-c**2*x**2 + 1)/(1024*c**7) - 35*b*e**3*asin(c*x)/(1024*c**8), Ne(c, 0)), (a*(d**3*x**2/2 + 3*d**2*e*x**4/4 + d*e**2*x**6/2 + e**3*x**8/8), True))`

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.37

$$\int x(d+ex^2)^3(a+b\arcsin(cx))dx = \frac{1}{8}ae^3x^8 + \frac{1}{2}ade^2x^6 + \frac{3}{4}ad^2ex^4 + \frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2x^2\arcsin(cx) + c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3}\right)\right)bd^3 + \frac{3}{32}\left(8x^4\arcsin(cx) + \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bd^2e + \frac{1}{96}\left(48x^6\arcsin(cx) + \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7}\right)c\right)bd^2e + \frac{1}{3072}\left(384x^8\arcsin(cx) + \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2x^2+1}x}{c^8} - \frac{105\arcsin(cx)}{c^9}\right)c\right)bd^2e$$

input `integrate(x*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d^3 + 3/32*(8*x^4*arcsin(c*x) + (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^2*e + 1/96*(48*x^6*arcsin(c*x) + (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*d^2*e + 1/3072*(384*x^8*arcsin(c*x) + (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*b*e^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(231) = 462.

Time = 0.14 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.38

$$\int x(d+ex^2)^3(a+b\arcsin(cx))dx = \text{Too large to display}$$

input `integrate(x*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/4*sqrt(-c^2*x^2 + 1)
*b*d^3*x/c + 1/2*(c^2*x^2 - 1)*b*d^3*arcsin(c*x)/c^2 - 3/16*(-c^2*x^2 + 1)
^(3/2)*b*d^2*e*x/c^3 + 1/2*(c^2*x^2 - 1)*a*d^3/c^2 + 1/4*b*d^3*arcsin(c*x)
/c^2 + 3/4*(c^2*x^2 - 1)^2*b*d^2*e*arcsin(c*x)/c^4 + 15/32*sqrt(-c^2*x^2 +
1)*b*d^2*e*x/c^3 + 1/12*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d*e^2*x/c^5
+ 3/2*(c^2*x^2 - 1)*b*d^2*e*arcsin(c*x)/c^4 + 1/2*(c^2*x^2 - 1)^3*b*d*e^2*
arcsin(c*x)/c^6 - 13/48*(-c^2*x^2 + 1)^(3/2)*b*d*e^2*x/c^5 + 1/64*(c^2*x^2
- 1)^3*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^7 + 15/32*b*d^2*e*arcsin(c*x)/c^4 + 3
/2*(c^2*x^2 - 1)^2*b*d*e^2*arcsin(c*x)/c^6 + 1/8*(c^2*x^2 - 1)^4*b*e^3*arc
sin(c*x)/c^8 + 11/32*sqrt(-c^2*x^2 + 1)*b*d*e^2*x/c^5 + 25/384*(c^2*x^2 -
1)^2*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^7 + 3/2*(c^2*x^2 - 1)*b*d*e^2*arcsin(c*x)
/c^6 + 1/2*(c^2*x^2 - 1)^3*b*e^3*arcsin(c*x)/c^8 - 163/1536*(-c^2*x^2 + 1)
^(3/2)*b*e^3*x/c^7 + 11/32*b*d*e^2*arcsin(c*x)/c^6 + 3/4*(c^2*x^2 - 1)^2*
b*e^3*arcsin(c*x)/c^8 + 93/1024*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^7 + 1/2*(c^2*
x^2 - 1)*b*e^3*arcsin(c*x)/c^8 + 93/1024*b*e^3*arcsin(c*x)/c^8

```

Mupad [F(-1)]

Timed out.

$$\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx = \int x(a + b \arcsin(cx)) (ex^2 + d)^3 dx$$

input

```
int(x*(a + b*asin(c*x))*(d + e*x^2)^3,x)
```

output

```
int(x*(a + b*asin(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.59

$$\int x(d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{1536a \arcsin(cx) b c^8 d^3 x^2 + 2304a \arcsin(cx) b c^8 d^2 e x^4 + 1536a \arcsin(cx) b c^8 d e^2 x^6 + 384a \arcsin(cx) b c^8 e^3 x^8 - 7680 a^2 d^3 x^2 + 2304 a^2 d^2 e x^4 + 1536 a^2 d e^2 x^6 + 384 a^2 e^3 x^8}{c^8}$$

input

```
int(x*(e*x^2+d)^3*(a+b*asin(c*x)),x)
```

output

```
(1536*asin(c*x)*b*c**8*d**3*x**2 + 2304*asin(c*x)*b*c**8*d**2*e*x**4 + 1536*asin(c*x)*b*c**8*d*e**2*x**6 + 384*asin(c*x)*b*c**8*e**3*x**8 - 768*asin(c*x)*b*c**6*d**3 - 864*asin(c*x)*b*c**4*d**2*e - 480*asin(c*x)*b*c**2*d*e**2 - 105*asin(c*x)*b*e**3 + 768*sqrt(-c**2*x**2 + 1)*b*c**7*d**3*x + 576*sqrt(-c**2*x**2 + 1)*b*c**7*d**2*e*x**3 + 256*sqrt(-c**2*x**2 + 1)*b*c**7*d*e**2*x**5 + 48*sqrt(-c**2*x**2 + 1)*b*c**7*e**3*x**7 + 864*sqrt(-c**2*x**2 + 1)*b*c**5*d**2*e*x + 320*sqrt(-c**2*x**2 + 1)*b*c**5*d*e**2*x**3 + 56*sqrt(-c**2*x**2 + 1)*b*c**5*e**3*x**5 + 480*sqrt(-c**2*x**2 + 1)*b*c**3*d*e**2*x + 70*sqrt(-c**2*x**2 + 1)*b*c**3*e**3*x**3 + 105*sqrt(-c**2*x**2 + 1)*b*c*e**3*x + 1536*a*c**8*d**3*x**2 + 2304*a*c**8*d**2*e*x**4 + 1536*a*c**8*d*e**2*x**6 + 384*a*c**8*e**3*x**8)/(3072*c**8)
```

3.445 $\int (d + ex^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	3783
Mathematica [A] (verified)	3784
Rubi [A] (verified)	3784
Maple [A] (verified)	3786
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Mupad [F(-1)]	3791
Reduce [B] (verification not implemented)	3791

Optimal result

Integrand size = 18, antiderivative size = 225

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx = \frac{b(35c^6d^3 + 35c^4d^2e + 21c^2de^2 + 5e^3) \sqrt{1 - c^2x^2}}{35c^7} - \frac{be(35c^4d^2 + 42c^2de + 15e^2) (1 - c^2x^2)^{3/2}}{105c^7} + \frac{3be^2(7c^2d + 5e) (1 - c^2x^2)^{5/2}}{175c^7} - \frac{be^3(1 - c^2x^2)^{7/2}}{49c^7} + d^3x(a + b \arcsin(cx)) + d^2ex^3(a + b \arcsin(cx)) + \frac{3}{5}de^2x^5(a + b \arcsin(cx)) + \frac{1}{7}e^3x^7(a + b \arcsin(cx))$$

```
output 1/35*b*(35*c^6*d^3+35*c^4*d^2*e+21*c^2*d*e^2+5*e^3)*(-c^2*x^2+1)^(1/2)/c^7
-1/105*b*e*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^(3/2)/c^7+3/175*b*e
^2*(7*c^2*d+5*e)*(-c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(-c^2*x^2+1)^(7/2)/c^7+
d^3*x*(a+b*arcsin(c*x))+d^2*e*x^3*(a+b*arcsin(c*x))+3/5*d*e^2*x^5*(a+b*arc
sin(c*x))+1/7*e^3*x^7*(a+b*arcsin(c*x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{105ax(35d^3 + 35d^2ex^2 + 21de^2x^4 + 5e^3x^6) + \frac{b\sqrt{1-c^2x^2}(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6))}{c^7} + 105b \arcsin(cx)(35d^3 + 35d^2ex^2 + 21de^2x^4 + 5e^3x^6)}{3675}$$

input

```
Integrate[(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```
(105*a*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + (b*sqrt[1 - c^2*x^2]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))/c^7 + 105*b*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6)*ArcSin[c*x])/3675
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5170, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$\downarrow 5170$$

$$-bc \int \frac{x(5e^3x^6 + 21de^2x^4 + 35d^2ex^2 + 35d^3)}{35\sqrt{1-c^2x^2}} dx + d^3x(a + b \arcsin(cx)) + d^2ex^3(a + b \arcsin(cx)) + \frac{3}{5}de^2x^5(a + b \arcsin(cx)) + \frac{1}{7}e^3x^7(a + b \arcsin(cx))$$

$$\downarrow 27$$

$$\begin{aligned}
& -\frac{1}{35}bc \int \frac{x(5e^3x^6 + 21de^2x^4 + 35d^2ex^2 + 35d^3)}{\sqrt{1-c^2x^2}} dx + d^3x(a + b \arcsin(cx)) + d^2ex^3(a + \\
& \quad b \arcsin(cx)) + \frac{3}{5}de^2x^5(a + b \arcsin(cx)) + \frac{1}{7}e^3x^7(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow \text{2331} \\
& -\frac{1}{70}bc \int \frac{5e^3x^6 + 21de^2x^4 + 35d^2ex^2 + 35d^3}{\sqrt{1-c^2x^2}} dx^2 + d^3x(a + b \arcsin(cx)) + d^2ex^3(a + \\
& \quad b \arcsin(cx)) + \frac{3}{5}de^2x^5(a + b \arcsin(cx)) + \frac{1}{7}e^3x^7(a + b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow \text{2389} \\
& -\frac{1}{70}bc \int \left(-\frac{5(1-c^2x^2)^{5/2}e^3}{c^6} + \frac{3(7dc^2+5e)(1-c^2x^2)^{3/2}e^2}{c^6} - \frac{(35d^2c^4+42dec^2+15e^2)\sqrt{1-c^2x^2}e}{c^6} + \frac{35d^3}{c^6} \right) \\
& \quad d^3x(a+b \arcsin(cx)) + d^2ex^3(a+b \arcsin(cx)) + \frac{3}{5}de^2x^5(a+b \arcsin(cx)) + \frac{1}{7}e^3x^7(a+b \arcsin(cx)) \\
& \qquad \qquad \qquad \downarrow \text{2009} \\
& \quad d^3x(a + b \arcsin(cx)) + d^2ex^3(a + b \arcsin(cx)) + \frac{3}{5}de^2x^5(a + b \arcsin(cx)) + \frac{1}{7}e^3x^7(a + \\
& \qquad \qquad \qquad b \arcsin(cx)) - \\
& \quad \frac{1}{70}bc \left(-\frac{6e^2(1-c^2x^2)^{5/2}(7c^2d+5e)}{5c^8} + \frac{10e^3(1-c^2x^2)^{7/2}}{7c^8} + \frac{2e(1-c^2x^2)^{3/2}(35c^4d^2+42c^2de+15e^2)}{3c^8} - \frac{2\sqrt{1-c^2x^2}}{c^8} \right)
\end{aligned}$$

input `Int[(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]`

output `-1/70*(b*c*((-2*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*Sqrt[1 - c^2*x^2])/c^8 + (2*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2)^(3/2))/(3*c^8) - (6*e^2*(7*c^2*d + 5*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (10*e^3*(1 - c^2*x^2)^(7/2))/(7*c^8))) + d^3*x*(a + b*ArcSin[c*x]) + d^2*e*x^3*(a + b*ArcSin[c*x]) + (3*d*e^2*x^5*(a + b*ArcSin[c*x]))/5 + (e^3*x^7*(a + b*ArcSin[c*x]))/7`

Defintions of rubi rules used

- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`
- rule 2389 `Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])`
- rule 5170 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])`

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{1}{7}e^3x^7 + \frac{3}{5}de^2x^5 + d^2ex^3 + d^3x\right) + \frac{b\left(\frac{c\arcsin(cx)e^3x^7}{7} + \frac{3c\arcsin(cx)de^2x^5}{5} + c\arcsin(cx)d^2ex^3 + \arcsin(cx)d^3x\right)}{c^6}$
derivativelimit	$\frac{a\left(d^3c^7x+d^2c^7ex^3+\frac{3}{5}d^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6} + \frac{b\left(\arcsin(cx)d^3c^7x+\arcsin(cx)d^2c^7ex^3+\frac{3\arcsin(cx)dc^7e^2x^5}{5}+\frac{\arcsin(cx)e^3c^7x^7}{7}\right)}{c^6}$
default	$\frac{a\left(d^3c^7x+d^2c^7ex^3+\frac{3}{5}d^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6} + \frac{b\left(\arcsin(cx)d^3c^7x+\arcsin(cx)d^2c^7ex^3+\frac{3\arcsin(cx)dc^7e^2x^5}{5}+\frac{\arcsin(cx)e^3c^7x^7}{7}\right)}{c^6}$
ordering	$\frac{x(325c^8e^4x^8+1792c^8de^3x^6+4410c^8d^2e^2x^4+30c^6e^4x^6+9800c^8d^3ex^2+294c^6de^3x^4+1225c^8d^4+2450c^6d^2e^2x^2+60c^4d^4)}{1225(e^2x^2+d)c^8}$

```
input int((e*x^2+d)^3*(a+b*arcsin(c*x)),x,method=_RETURNVERBOSE)
```

```
output a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+d^3*x)+b/c*(1/7*c*arcsin(c*x)*e^3*x^7+3/5*c*arcsin(c*x)*d*e^2*x^5+c*arcsin(c*x)*d^2*e*x^3+arcsin(c*x)*d^3*c*x-1/35/c^6*(5*e^3*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))-35*d^3*c^6*(-c^2*x^2+1)^(1/2)+21*d*c^2*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+35*d^2*c^4*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.02

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx = \frac{525 ac^7 e^3 x^7 + 2205 ac^7 de^2 x^5 + 3675 ac^7 d^2 ex^3 + 3675 ac^7 d^3 x + 105 (5 bc^7 e^3 x^7 + 21 bc^7 de^2 x^5 + 35 bc^7 d^2 ex^3 + 35 bc^7 d^3 x)}{1225(e^2x^2+d)c^8}$$

```
input integrate((e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")
```


output

```
1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 +
3675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^
2*e*x^3 + 35*b*c^7*d^3*x)*arcsin(c*x) + (75*b*c^6*e^3*x^6 + 3675*b*c^6*d^3
+ 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 + 10*b*c^4*e^3)
*x^4 + 240*b*e^3 + (1225*b*c^6*d^2*e + 588*b*c^4*d*e^2 + 120*b*c^2*e^3)*x^
2)*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.73

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x \arcsin(cx) + bd^2ex^3 \arcsin(cx) + \frac{3bde^2x^5 \arcsin(cx)}{5} + \frac{be^3x^7 \arcsin(cx)}{7} + bd^3x \arcsin(cx) \\ a \left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7} \right) \end{cases}$$

input

```
integrate((e*x**2+d)**3*(a+b*asin(c*x)),x)
```

output

```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*asin(c*x) + b*d**2*e*x**3*asin(c*x) + 3*b*d*e**2*x**5*asin(c*x)/5
+ b*e**3*x**7*asin(c*x)/7 + b*d**3*sqrt(-c**2*x**2 + 1)/c + b*d**2*e*x**2
*sqrt(-c**2*x**2 + 1)/(3*c) + 3*b*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c)
+ b*e**3*x**6*sqrt(-c**2*x**2 + 1)/(49*c) + 2*b*d**2*e*sqrt(-c**2*x**2 + 1)
)/(3*c**3) + 4*b*d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(25*c**3) + 6*b*e**3*x**
4*sqrt(-c**2*x**2 + 1)/(245*c**3) + 8*b*d*e**2*sqrt(-c**2*x**2 + 1)/(25*c
*5) + 8*b*e**3*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) + 16*b*e**3*sqrt(-c**2
*x**2 + 1)/(245*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5
/5 + e**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.30

$$\begin{aligned}
\int (d + ex^2)^3 (a + b \arcsin(cx)) dx &= \frac{1}{7} ae^3 x^7 + \frac{3}{5} ade^2 x^5 + ad^2 ex^3 \\
&+ \frac{1}{3} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bd^2 e \\
&+ \frac{1}{25} \left(15x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4\sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8\sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) bde^2 \\
&+ \frac{1}{245} \left(35x^7 \arcsin(cx) + \left(\frac{5\sqrt{-c^2 x^2 + 1} x^6}{c^2} + \frac{6\sqrt{-c^2 x^2 + 1} x^4}{c^4} + \frac{8\sqrt{-c^2 x^2 + 1} x^2}{c^6} + \frac{16\sqrt{-c^2 x^2 + 1}}{c^8} \right) c \right) bde^3 \\
&+ ad^3 x + \frac{(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1}) bd^3}{c}
\end{aligned}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```

1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arcsin(c*x) + c
*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(
15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arcsin(c*x)
+ (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-
c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*
x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d^3/c

```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(205) = 410$.

Time = 0.14 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.13

$$\begin{aligned}
 \int (d+ex^2)^3 (a+b \arcsin(cx)) dx = & \frac{1}{7} ae^3 x^7 + \frac{3}{5} ade^2 x^5 + ad^2 ex^3 + bd^3 x \arcsin(cx) + ad^3 x \\
 & + \frac{(c^2 x^2 - 1)bd^2 ex \arcsin(cx)}{c^2} + \frac{bd^2 ex \arcsin(cx)}{c^2} \\
 & + \frac{3(c^2 x^2 - 1)^2 bde^2 x \arcsin(cx)}{5c^4} \\
 & + \frac{\sqrt{-c^2 x^2 + 1}bd^3}{c} + \frac{6(c^2 x^2 - 1)bde^2 x \arcsin(cx)}{5c^4} \\
 & + \frac{(c^2 x^2 - 1)^3 be^3 x \arcsin(cx)}{7c^6} \\
 & - \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} bd^2 e}{3c^3} + \frac{3bde^2 x \arcsin(cx)}{5c^4} \\
 & + \frac{3(c^2 x^2 - 1)^2 be^3 x \arcsin(cx)}{7c^6} + \frac{\sqrt{-c^2 x^2 + 1}bd^2 e}{c^3} \\
 & + \frac{3(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}bde^2}{25c^5} \\
 & + \frac{3(c^2 x^2 - 1)be^3 x \arcsin(cx)}{7c^6} \\
 & - \frac{2(-c^2 x^2 + 1)^{\frac{3}{2}} bde^2}{5c^5} \\
 & + \frac{(c^2 x^2 - 1)^3 \sqrt{-c^2 x^2 + 1}be^3}{49c^7} \\
 & + \frac{be^3 x \arcsin(cx)}{7c^6} + \frac{3\sqrt{-c^2 x^2 + 1}bde^2}{5c^5} \\
 & + \frac{3(c^2 x^2 - 1)^2 \sqrt{-c^2 x^2 + 1}be^3}{35c^7} \\
 & - \frac{(-c^2 x^2 + 1)^{\frac{3}{2}} be^3}{7c^7} + \frac{\sqrt{-c^2 x^2 + 1}be^3}{7c^7}
 \end{aligned}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")`

output

```

1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + b*d^3*x*arcsin(c*x) + a*d^
3*x + (c^2*x^2 - 1)*b*d^2*e*x*arcsin(c*x)/c^2 + b*d^2*e*x*arcsin(c*x)/c^2
+ 3/5*(c^2*x^2 - 1)^2*b*d*e^2*x*arcsin(c*x)/c^4 + sqrt(-c^2*x^2 + 1)*b*d^3
/c + 6/5*(c^2*x^2 - 1)*b*d*e^2*x*arcsin(c*x)/c^4 + 1/7*(c^2*x^2 - 1)^3*b*e
^3*x*arcsin(c*x)/c^6 - 1/3*(-c^2*x^2 + 1)^(3/2)*b*d^2*e/c^3 + 3/5*b*d*e^2*
x*arcsin(c*x)/c^4 + 3/7*(c^2*x^2 - 1)^2*b*e^3*x*arcsin(c*x)/c^6 + sqrt(-c^
2*x^2 + 1)*b*d^2*e/c^3 + 3/25*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*d*e^2/c
^5 + 3/7*(c^2*x^2 - 1)*b*e^3*x*arcsin(c*x)/c^6 - 2/5*(-c^2*x^2 + 1)^(3/2)*
b*d*e^2/c^5 + 1/49*(c^2*x^2 - 1)^3*sqrt(-c^2*x^2 + 1)*b*e^3/c^7 + 1/7*b*e^
3*x*arcsin(c*x)/c^6 + 3/5*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^5 + 3/35*(c^2*x^2 -
1)^2*sqrt(-c^2*x^2 + 1)*b*e^3/c^7 - 1/7*(-c^2*x^2 + 1)^(3/2)*b*e^3/c^7 +
1/7*sqrt(-c^2*x^2 + 1)*b*e^3/c^7

```

Mupad [F(-1)]

Timed out.

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (ex^2 + d)^3 dx$$

input

```
int((a + b*asin(c*x))*(d + e*x^2)^3,x)
```

output

```
int((a + b*asin(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.50

$$\int (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{3675 \arcsin(cx) b c^7 d^3 x + 3675 \arcsin(cx) b c^7 d^2 e x^3 + 2205 \arcsin(cx) b c^7 d e^2 x^5 + 525 \arcsin(cx) b c^7 e^3 x^7 + 3675 a d^3 x + 3675 a d^2 e x^3 + 2205 a d e^2 x^5 + 525 a e^3 x^7}{1}$$

input

```
int((e*x^2+d)^3*(a+b*asin(c*x)),x)
```

output

```
(3675*asin(c*x)*b*c**7*d**3*x + 3675*asin(c*x)*b*c**7*d**2*e*x**3 + 2205*a
sin(c*x)*b*c**7*d*e**2*x**5 + 525*asin(c*x)*b*c**7*e**3*x**7 + 3675*sqrt(
- c**2*x**2 + 1)*b*c**6*d**3 + 1225*sqrt(- c**2*x**2 + 1)*b*c**6*d**2*e*x
**2 + 441*sqrt(- c**2*x**2 + 1)*b*c**6*d*e**2*x**4 + 75*sqrt(- c**2*x**2
+ 1)*b*c**6*e**3*x**6 + 2450*sqrt(- c**2*x**2 + 1)*b*c**4*d**2*e + 588*s
qrt(- c**2*x**2 + 1)*b*c**4*d*e**2*x**2 + 90*sqrt(- c**2*x**2 + 1)*b*c**
4*e**3*x**4 + 1176*sqrt(- c**2*x**2 + 1)*b*c**2*d*e**2 + 120*sqrt(- c**2
*x**2 + 1)*b*c**2*e**3*x**2 + 240*sqrt(- c**2*x**2 + 1)*b*e**3 + 3675*a*c
**7*d**3*x + 3675*a*c**7*d**2*e*x**3 + 2205*a*c**7*d*e**2*x**5 + 525*a*c**
7*e**3*x**7)/(3675*c**7)
```

3.446
$$\int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x} dx$$

Optimal result	3793
Mathematica [A] (verified)	3794
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Fricas [F]	3797
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Reduce [F]	3799

Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned} \int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x} dx = & \frac{3bd^2ex\sqrt{1-c^2x^2}}{4c} + \frac{9bde^2x\sqrt{1-c^2x^2}}{32c^3} \\ & + \frac{5be^3x\sqrt{1-c^2x^2}}{96c^5} + \frac{3bde^2x^3\sqrt{1-c^2x^2}}{16c} \\ & + \frac{5be^3x^3\sqrt{1-c^2x^2}}{144c^3} + \frac{be^3x^5\sqrt{1-c^2x^2}}{36c} \\ & - \frac{3bd^2e \arcsin(cx)}{4c^2} - \frac{9bde^2 \arcsin(cx)}{32c^4} \\ & - \frac{5be^3 \arcsin(cx)}{96c^6} - \frac{1}{2}ibd^3 \arcsin(cx)^2 \\ & + \frac{3}{2}d^2ex^2(a+b \arcsin(cx)) \\ & + \frac{3}{4}de^2x^4(a+b \arcsin(cx)) + \frac{1}{6}e^3x^6(a+b \arcsin(cx)) \\ & + bd^3 \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) \\ & - bd^3 \arcsin(cx) \log(x) + d^3(a+b \arcsin(cx)) \log(x) \\ & - \frac{1}{2}ibd^3 \text{PolyLog}(2, e^{2i \arcsin(cx)}) \end{aligned}$$

output

```

3/4*b*d^2*e*x*(-c^2*x^2+1)^(1/2)/c+9/32*b*d*e^2*x*(-c^2*x^2+1)^(1/2)/c^3+5
/96*b*e^3*x*(-c^2*x^2+1)^(1/2)/c^5+3/16*b*d*e^2*x^3*(-c^2*x^2+1)^(1/2)/c+5
/144*b*e^3*x^3*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*e^3*x^5*(-c^2*x^2+1)^(1/2)/c-
3/4*b*d^2*e*arcsin(c*x)/c^2-9/32*b*d*e^2*arcsin(c*x)/c^4-5/96*b*e^3*arcsin
(c*x)/c^6-1/2*I*b*d^3*arcsin(c*x)^2+3/2*d^2*e*x^2*(a+b*arcsin(c*x))+3/4*d*
e^2*x^4*(a+b*arcsin(c*x))+1/6*e^3*x^6*(a+b*arcsin(c*x))+b*d^3*arcsin(c*x)*
ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-b*d^3*arcsin(c*x)*ln(x)+d^3*(a+b*arcsin
(c*x))*ln(x)-1/2*I*b*d^3*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)

```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.90

$$\begin{aligned}
& \int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx \\
&= \frac{1}{12} \left(18ad^2ex^2 + 9ade^2x^4 + 2ae^3x^6 + 18bd^2ex^2 \arcsin(cx) + 9bde^2x^4 \arcsin(cx) \right. \\
&\quad \left. + 2be^3x^6 \arcsin(cx) \right. \\
&\quad + \frac{be^3 \left(cx\sqrt{1-c^2x^2}(15 + 10c^2x^2 + 8c^4x^4) - 30 \arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) \right)}{24c^6} \\
&\quad + \frac{9bde^2 \left(cx\sqrt{1-c^2x^2}(3 + 2c^2x^2) - 6 \arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) \right)}{8c^4} \\
&\quad + \frac{9bd^2e \left(cx\sqrt{1-c^2x^2} - 2 \arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) \right)}{c^2} \\
&\quad \left. + 12bd^3 \arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) + 12ad^3 \log(x) \right. \\
&\quad \left. - 6ibd^3 (\arcsin(cx))^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)}) \right)
\end{aligned}$$

input

```
Integrate[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x,x]
```

output

```
(18*a*d^2*e*x^2 + 9*a*d*e^2*x^4 + 2*a*e^3*x^6 + 18*b*d^2*e*x^2*ArcSin[c*x]
+ 9*b*d*e^2*x^4*ArcSin[c*x] + 2*b*e^3*x^6*ArcSin[c*x] + (b*e^3*(c*x*Sqrt[
1 - c^2*x^2])*(15 + 10*c^2*x^2 + 8*c^4*x^4) - 30*ArcTan[(c*x)/(-1 + Sqrt[1
- c^2*x^2])]))/(24*c^6) + (9*b*d*e^2*(c*x*Sqrt[1 - c^2*x^2])*(3 + 2*c^2*x^2
) - 6*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/(8*c^4) + (9*b*d^2*e*(c*x*S
qrt[1 - c^2*x^2] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/c^2 + 12*b*d
^3*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 12*a*d^3*Log[x] - (6*I)*b*
d^3*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])]))/12
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx$$

↓ 5230

$$-bc \int \frac{2e^3x^6 + 9de^2x^4 + 18d^2ex^2 + 12d^3 \log(x)}{12\sqrt{1 - c^2x^2}} dx + d^3 \log(x)(a + b \arcsin(cx)) + \frac{3}{2}d^2ex^2(a + b \arcsin(cx)) + \frac{3}{4}de^2x^4(a + b \arcsin(cx)) + \frac{1}{6}e^3x^6(a + b \arcsin(cx))$$

↓ 27

$$-\frac{1}{12}bc \int \frac{2e^3x^6 + 9de^2x^4 + 18d^2ex^2 + 12d^3 \log(x)}{\sqrt{1 - c^2x^2}} dx + d^3 \log(x)(a + b \arcsin(cx)) + \frac{3}{2}d^2ex^2(a + b \arcsin(cx)) + \frac{3}{4}de^2x^4(a + b \arcsin(cx)) + \frac{1}{6}e^3x^6(a + b \arcsin(cx))$$

↓ 7293

$$-\frac{1}{12}bc \int \left(\frac{2e^3x^6}{\sqrt{1 - c^2x^2}} + \frac{9de^2x^4}{\sqrt{1 - c^2x^2}} + \frac{18d^2ex^2}{\sqrt{1 - c^2x^2}} + \frac{12d^3 \log(x)}{\sqrt{1 - c^2x^2}} \right) dx + d^3 \log(x)(a + b \arcsin(cx)) + \frac{3}{2}d^2ex^2(a + b \arcsin(cx)) + \frac{3}{4}de^2x^4(a + b \arcsin(cx)) + \frac{1}{6}e^3x^6(a + b \arcsin(cx))$$

↓ 2009

$$d^3 \log(x)(a + b \arcsin(cx)) + \frac{3}{2} d^2 e x^2 (a + b \arcsin(cx)) + \frac{3}{4} d e^2 x^4 (a + b \arcsin(cx)) + \frac{1}{6} e^3 x^6 (a + b \arcsin(cx)) - \frac{1}{12} b c \left(\frac{5e^3 \arcsin(cx)}{8c^7} + \frac{27de^2 \arcsin(cx)}{8c^5} + \frac{9d^2 e \arcsin(cx)}{c^3} + \frac{6id^3 \text{PolyLog}(2, e^{2i \arcsin(cx)})}{c} + \frac{6id^3 \arcsin(cx)^2}{c} \right)$$

input `Int[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x,x]`

output `(3*d^2*e*x^2*(a + b*ArcSin[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcSin[c*x]))/4 + (e^3*x^6*(a + b*ArcSin[c*x]))/6 + d^3*(a + b*ArcSin[c*x])*Log[x] - (b*c*((-9*d^2*e*x*Sqrt[1 - c^2*x^2])/c^2 - (27*d*e^2*x*Sqrt[1 - c^2*x^2])/(8*c^4) - (5*e^3*x*Sqrt[1 - c^2*x^2])/(8*c^6) - (9*d*e^2*x^3*Sqrt[1 - c^2*x^2])/(4*c^2) - (5*e^3*x^3*Sqrt[1 - c^2*x^2])/(12*c^4) - (e^3*x^5*Sqrt[1 - c^2*x^2])/(3*c^2) + (9*d^2*e*ArcSin[c*x])/c^3 + (27*d*e^2*ArcSin[c*x])/(8*c^5) + (5*e^3*ArcSin[c*x])/(8*c^7) + ((6*I)*d^3*ArcSin[c*x]^2)/c - (12*d^3*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c + (12*d^3*ArcSin[c*x]*Log[x])/c + ((6*I)*d^3*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c)/12`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5230 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.96

method	result
parts	$a\left(\frac{e^3x^6}{6} + \frac{3de^2x^4}{4} + \frac{3d^2ex^2}{2} + d^3 \ln(x)\right) + b\left(-\frac{i \arcsin(cx)^2 d^3}{2} + d^3 \arcsin(cx) \ln(1 + icx + \sqrt{-c^2x^2+1})\right)$
derivativedivides	$\frac{a\left(\frac{3c^6d^2ex^2}{2} + \frac{3c^6de^2x^4}{4} + \frac{e^3x^6c^6}{6} + c^6d^3 \ln(cx)\right)}{c^6} + \frac{b\left(-\frac{ic^6d^3 \arcsin(cx)^2}{2} + c^6d^3 \arcsin(cx) \ln(1-icx-\sqrt{-c^2x^2+1})\right)}{c^6}$
default	$\frac{a\left(\frac{3c^6d^2ex^2}{2} + \frac{3c^6de^2x^4}{4} + \frac{e^3x^6c^6}{6} + c^6d^3 \ln(cx)\right)}{c^6} + \frac{b\left(-\frac{ic^6d^3 \arcsin(cx)^2}{2} + c^6d^3 \arcsin(cx) \ln(1-icx-\sqrt{-c^2x^2+1})\right)}{c^6}$

input `int((e*x^2+d)^3*(a+b*arcsin(c*x))/x,x,method=_RETURNVERBOSE)`

output `a*(1/6*e^3*x^6+3/4*d*e^2*x^4+3/2*d^2*e*x^2+d^3*ln(x))+b*(-1/2*I*arcsin(c*x)^2*d^3+d^3*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+d^3*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))-I*d^3*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-I*d^3*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-1/192*arcsin(c*x)*e^3/c^6*cos(6*arcsin(c*x))+1/1152*e^3/c^6*sin(6*arcsin(c*x))+1/32*arcsin(c*x)*e^2*(3*c^2*d+e)/c^6*cos(4*arcsin(c*x))-3/128*e^2/c^4*sin(4*arcsin(c*x))*d-1/128*e^3/c^6*sin(4*arcsin(c*x))-1/64*e*arcsin(c*x)*(48*c^4*d^2+24*c^2*d*e+5*e^2)/c^6*cos(2*arcsin(c*x))+3/8*e/c^2*sin(2*arcsin(c*x))*d^2+3/16*e^2/c^4*sin(2*arcsin(c*x))*d+5/128*e^3/c^6*sin(2*arcsin(c*x)))`

Fricas [F]

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx = \int \frac{(ex^2 + d)^3 (b \arcsin(cx) + a)}{x} dx$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x,x, algorithm="fricas")`

output `integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsin(c*x))/x, x)`

Sympy [F]

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (d + ex^2)^3}{x} dx$$

input `integrate((e*x**2+d)**3*(a+b*asin(c*x))/x,x)`

output `Integral((a + b*asin(c*x))*(d + e*x**2)**3/x, x)`

Maxima [F]

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx = \int \frac{(ex^2 + d)^3 (b \arcsin(cx) + a)}{x} dx$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x,x, algorithm="maxima")`

output `1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) + integrate((b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^3}{x} dx$$

input `int(((a + b*asin(c*x))*(d + e*x^2)^3)/x,x)`output `int(((a + b*asin(c*x))*(d + e*x^2)^3)/x, x)`**Reduce [F]**

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x} dx$$

$$= \frac{432asin(cx)bc^6d^2ex^2 + 216asin(cx)bc^6de^2x^4 + 48asin(cx)bc^6e^3x^6 - 216asin(cx)bc^4d^2e - 81asin(cx)}$$

input `int((e*x^2+d)^3*(a+b*asin(c*x))/x,x)`output `(432*asin(c*x)*b*c**6*d**2*e*x**2 + 216*asin(c*x)*b*c**6*d*e**2*x**4 + 48*asin(c*x)*b*c**6*e**3*x**6 - 216*asin(c*x)*b*c**4*d**2*e - 81*asin(c*x)*b*c**2*d*e**2 - 15*asin(c*x)*b*e**3 + 216*sqrt(-c**2*x**2 + 1)*b*c**5*d**2*e*x + 54*sqrt(-c**2*x**2 + 1)*b*c**5*d*e**2*x**3 + 8*sqrt(-c**2*x**2 + 1)*b*c**5*e**3*x**5 + 81*sqrt(-c**2*x**2 + 1)*b*c**3*d*e**2*x + 10*sqrt(-c**2*x**2 + 1)*b*c**3*e**3*x**3 + 15*sqrt(-c**2*x**2 + 1)*b*c*e**3*x + 288*int(asin(c*x)/x,x)*b*c**6*d**3 + 288*log(x)*a*c**6*d**3 + 432*a*c**6*d**2*e*x**2 + 216*a*c**6*d*e**2*x**4 + 48*a*c**6*e**3*x**6)/(288*c**6)`

3.447 $\int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x^2} dx$

Optimal result	3800
Mathematica [A] (verified)	3801
Rubi [A] (verified)	3801
Maple [A] (verified)	3803
Fricas [A] (verification not implemented)	3804
Sympy [A] (verification not implemented)	3805
Maxima [A] (verification not implemented)	3806
Giac [B] (verification not implemented)	3806
Mupad [F(-1)]	3807
Reduce [B] (verification not implemented)	3808

Optimal result

Integrand size = 21, antiderivative size = 190

$$\int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x^2} dx = \frac{be(15c^4d^2+5c^2de+e^2)\sqrt{1-c^2x^2}}{5c^5} - \frac{be^2(5c^2d+2e)(1-c^2x^2)^{3/2}}{15c^5} + \frac{be^3(1-c^2x^2)^{5/2}}{25c^5} - \frac{d^3(a+b \arcsin(cx))}{x} + 3d^2ex(a+b \arcsin(cx)) + de^2x^3(a+b \arcsin(cx)) + \frac{1}{5}e^3x^5(a+b \arcsin(cx)) - bcd^3 \operatorname{arctanh}(\sqrt{1-c^2x^2})$$

output

```
1/5*b*e*(15*c^4*d^2+5*c^2*d*e+e^2)*(-c^2*x^2+1)^(1/2)/c^5-1/15*b*e^2*(5*c^2*d+2*e)*(-c^2*x^2+1)^(3/2)/c^5+1/25*b*e^3*(-c^2*x^2+1)^(5/2)/c^5-d^3*(a+b*arcsin(c*x))/x+3*d^2*e*x*(a+b*arcsin(c*x))+d*e^2*x^3*(a+b*arcsin(c*x))+1/5*e^3*x^5*(a+b*arcsin(c*x))-b*c*d^3*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.96

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$= -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{1}{5}ae^3x^5$$

$$+ \frac{be\sqrt{1-c^2x^2}(8e^2 + 2c^2e(25d + 2ex^2) + c^4(225d^2 + 25dex^2 + 3e^2x^4))}{75c^5}$$

$$+ \frac{b(-5d^3 + 15d^2ex^2 + 5de^2x^4 + e^3x^6) \arcsin(cx)}{5x}$$

$$+ bcd^3 \log(x) - bcd^3 \log\left(1 + \sqrt{1 - c^2x^2}\right)$$

input

```
Integrate[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x^2,x]
```

output

```
-((a*d^3)/x) + 3*a*d^2*e*x + a*d*e^2*x^3 + (a*e^3*x^5)/5 + (b*e*Sqrt[1 - c^2*x^2]*(8*e^2 + 2*c^2*e*(25*d + 2*e*x^2) + c^4*(225*d^2 + 25*d*e*x^2 + 3*e^2*x^4)))/(75*c^5) + (b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^2*x^4 + e^3*x^6)*ArcSin[c*x])/(5*x) + b*c*d^3*Log[x] - b*c*d^3*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5230, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$\downarrow 5230$$

$$-bc \int -\frac{-e^3x^6 - 5de^2x^4 - 15d^2ex^2 + 5d^3}{5x\sqrt{1-c^2x^2}} dx - \frac{d^3(a + b \arcsin(cx))}{x} + 3d^2ex(a + b \arcsin(cx)) + de^2x^3(a + b \arcsin(cx)) + \frac{1}{5}e^3x^5(a + b \arcsin(cx))$$

$$\frac{1}{5}bc \int \frac{-e^3x^6 - 5de^2x^4 - 15d^2ex^2 + 5d^3}{x\sqrt{1-c^2x^2}} dx - \frac{d^3(a + b \arcsin(cx))}{x} + 3d^2ex(a + b \arcsin(cx)) + de^2x^3(a + b \arcsin(cx)) + \frac{1}{5}e^3x^5(a + b \arcsin(cx))$$

↓ 27

$$\frac{1}{10}bc \int \frac{-e^3x^6 - 5de^2x^4 - 15d^2ex^2 + 5d^3}{x^2\sqrt{1-c^2x^2}} dx^2 - \frac{d^3(a + b \arcsin(cx))}{x} + 3d^2ex(a + b \arcsin(cx)) + de^2x^3(a + b \arcsin(cx)) + \frac{1}{5}e^3x^5(a + b \arcsin(cx))$$

↓ 2331

$$\frac{1}{10}bc \int \left(\frac{5d^3}{x^2\sqrt{1-c^2x^2}} - \frac{e^3(1-c^2x^2)^{3/2}}{c^4} + \frac{e^2(5dc^2+2e)\sqrt{1-c^2x^2}}{c^4} - \frac{e(15d^2c^4+5dec^2+e^2)}{c^4\sqrt{1-c^2x^2}} \right) dx^2 - \frac{d^3(a + b \arcsin(cx))}{x} + 3d^2ex(a + b \arcsin(cx)) + de^2x^3(a + b \arcsin(cx)) + \frac{1}{5}e^3x^5(a + b \arcsin(cx))$$

↓ 2123

$$-\frac{d^3(a + b \arcsin(cx))}{x} + 3d^2ex(a + b \arcsin(cx)) + de^2x^3(a + b \arcsin(cx)) + \frac{1}{5}e^3x^5(a + b \arcsin(cx)) + \frac{1}{10}bc \left(-10d^3 \operatorname{arctanh}(\sqrt{1-c^2x^2}) - \frac{2e^2(1-c^2x^2)^{3/2}(5c^2d+2e)}{3c^6} + \frac{2e^3(1-c^2x^2)^{5/2}}{5c^6} + \frac{2e\sqrt{1-c^2x^2}(15c^4d^2+e^2)}{c^6} \right)$$

↓ 2009

input `Int[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x^2,x]`

output `-((d^3*(a + b*ArcSin[c*x]))/x) + 3*d^2*e*x*(a + b*ArcSin[c*x]) + d*e^2*x^3*(a + b*ArcSin[c*x]) + (e^3*x^5*(a + b*ArcSin[c*x]))/5 + (b*c*((2*e*(15*c^4*d^2 + 5*c^2*d*e + e^2)*Sqrt[1 - c^2*x^2])/c^6 - (2*e^2*(5*c^2*d + 2*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) + (2*e^3*(1 - c^2*x^2)^(5/2))/(5*c^6) - 10*d^3*ArcTanh[Sqrt[1 - c^2*x^2]]))/10`

Definitions of rubi rules used

- rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`
- rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`
- rule 2123 `Int[(Px_)*((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2])`
- rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`
- rule 5230 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

method	result
parts	$a\left(\frac{e^3x^5}{5} + d e^2x^3 + 3d^2ex - \frac{d^3}{x}\right) + bc\left(\frac{\arcsin(cx)e^3x^5}{5c} + \frac{\arcsin(cx)d e^2x^3}{c} + \frac{3 \arcsin(cx)d^2ex}{c} - \frac{\arcsin(cx)d^3}{cx}\right)$
derivativedivides	$c\left(\frac{a(3c^5d^2ex+c^5de^2x^3+\frac{e^3e^5x^5}{5}-\frac{e^5d^3}{x})}{c^6} + \frac{b(3 \arcsin(cx)c^5d^2ex+\arcsin(cx)c^5de^2x^3+\frac{\arcsin(cx)e^3c^5x^5}{5}-\frac{\arcsin(cx)d^3}{cx})}{c^6}\right)$
default	$c\left(\frac{a(3c^5d^2ex+c^5de^2x^3+\frac{e^3e^5x^5}{5}-\frac{e^5d^3}{x})}{c^6} + \frac{b(3 \arcsin(cx)c^5d^2ex+\arcsin(cx)c^5de^2x^3+\frac{\arcsin(cx)e^3c^5x^5}{5}-\frac{\arcsin(cx)d^3}{cx})}{c^6}\right)$

input

```
int((e*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output

```
a*(1/5*e^3*x^5+d*e^2*x^3+3*d^2*e*x-d^3/x)+b*c*(1/5/c*arcsin(c*x)*e^3*x^5+1/c*arcsin(c*x)*d*e^2*x^3+3/c*arcsin(c*x)*d^2*e*x-arcsin(c*x)*d^3/c/x-1/5/c^6*(e^3*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2))-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2))-8/15*(-c^2*x^2+1)^(1/2))+5*c^6*d^3*arctanh(1/(-c^2*x^2+1)^(1/2))+5*c^2*d*e^2*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2))-2/3*(-c^2*x^2+1)^(1/2))-15*c^4*d^2*e*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.26

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{30 ac^5 e^3 x^6 + 150 ac^5 d e^2 x^4 - 75 bc^6 d^3 x \log(\sqrt{-c^2 x^2 + 1} + 1) + 75 bc^6 d^3 x \log(\sqrt{-c^2 x^2 + 1} - 1) + 450 a d^3}{c^6}$$

input

```
integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x, algorithm="fricas")
```

output

```
1/150*(30*a*c^5*e^3*x^6 + 150*a*c^5*d*e^2*x^4 - 75*b*c^6*d^3*x*log(sqrt(-c
^2*x^2 + 1) + 1) + 75*b*c^6*d^3*x*log(sqrt(-c^2*x^2 + 1) - 1) + 450*a*c^5*
d^2*e*x^2 - 150*a*c^5*d^3 + 30*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c
^5*d^2*e*x^2 - 5*b*c^5*d^3)*arcsin(c*x) + 2*(3*b*c^4*e^3*x^5 + (25*b*c^4*d
*e^2 + 4*b*c^2*e^3)*x^3 + (225*b*c^4*d^2*e + 50*b*c^2*d*e^2 + 8*b*e^3)*x)*
sqrt(-c^2*x^2 + 1))/(c^5*x)
```

Sympy [A] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.45

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$= -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{ae^3x^5}{5} + bcd^3 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right)$$

$$- bcde^2 \left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right)$$

$$- \frac{bce^3 \left(\begin{cases} -\frac{x^4\sqrt{-c^2x^2+1}}{5c^2} - \frac{4x^2\sqrt{-c^2x^2+1}}{15c^4} - \frac{8\sqrt{-c^2x^2+1}}{15c^6} & \text{for } c^2 \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases} \right)}{5} - \frac{bd^3 \operatorname{asin}(cx)}{x}$$

$$+ 3bd^2e \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right) + bde^2x^3 \operatorname{asin}(cx) + \frac{be^3x^5 \operatorname{asin}(cx)}{5}$$

input

```
integrate((e*x**2+d)**3*(a+b*asin(c*x))/x**2,x)
```

output

```
-a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 + b*c*d**3*Piec
ewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*
c*d*e**2*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**
2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True)) - b*c*e**3*Piecewise((-x**4
*sqrt(-c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(-c**2*x**2 + 1)/(15*c**4) - 8
*sqrt(-c**2*x**2 + 1)/(15*c**6), Ne(c**2, 0)), (x**6/6, True))/5 - b*d**3*
asin(c*x)/x + 3*b*d**2*e*Piecewise((0, Eq(c, 0)), (x*asin(c*x) + sqrt(-c**
2*x**2 + 1)/c, True)) + b*d*e**2*x**3*asin(c*x) + b*e**3*x**5*asin(c*x)/5
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.27

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{1}{5} ae^3 x^5 + ade^2 x^3 - \left(c \log \left(\frac{2\sqrt{-c^2 x^2 + 1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd^3$$

$$+ \frac{1}{3} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2 x^2 + 1} x^2}{c^2} + \frac{2\sqrt{-c^2 x^2 + 1}}{c^4} \right) \right) bde^2$$

$$+ \frac{1}{75} \left(15x^5 \arcsin(cx) + \left(\frac{3\sqrt{-c^2 x^2 + 1} x^4}{c^2} + \frac{4\sqrt{-c^2 x^2 + 1} x^2}{c^4} + \frac{8\sqrt{-c^2 x^2 + 1}}{c^6} \right) c \right) be^3$$

$$+ 3ad^2 ex + \frac{3(cx \arcsin(cx) + \sqrt{-c^2 x^2 + 1})bd^2 e}{c} - \frac{ad^3}{x}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x, algorithm="maxima")`

output `1/5*a*e^3*x^5 + a*d*e^2*x^3 - (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d^3 + 1/3*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d*e^2 + 1/75*(15*x^5*arcsin(c*x) + (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*e^3 + 3*a*d^2*e*x + 3*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d^2*e/c - a*d^3/x`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10769 vs. 2(174) = 348.

Time = 11.62 (sec) , antiderivative size = 10769, normalized size of antiderivative = 56.68

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx = \text{Too large to display}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^2,x, algorithm="giac")`

output

```

-1/2*b*c^18*d^3*x^12*arcsin(c*x)/((c^16*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 +
5*c^14*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^12*x^7/(sqrt(-c^2*x^2 + 1) +
1)^7 + 10*c^10*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^8*x^3/(sqrt(-c^2*x^2
+ 1) + 1)^3 + c^6*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^12)
- 1/2*a*c^18*d^3*x^12/((c^16*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 + 5*c^14*x^
9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^12*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 10
*c^10*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^8*x^3/(sqrt(-c^2*x^2 + 1) + 1)^
3 + c^6*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^12) + b*c^17*
d^3*x^11*log(abs(c)*abs(x))/((c^16*x^11/(sqrt(-c^2*x^2 + 1) + 1)^11 + 5*c^
14*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^12*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7
+ 10*c^10*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^8*x^3/(sqrt(-c^2*x^2 + 1)
+ 1)^3 + c^6*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1) + 1)^11) - b*
c^17*d^3*x^11*log(sqrt(-c^2*x^2 + 1) + 1)/((c^16*x^11/(sqrt(-c^2*x^2 + 1)
+ 1)^11 + 5*c^14*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^12*x^7/(sqrt(-c^2*x
^2 + 1) + 1)^7 + 10*c^10*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^8*x^3/(sqrt(
-c^2*x^2 + 1) + 1)^3 + c^6*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + 1)
+ 1)^11) - 3*b*c^16*d^3*x^10*arcsin(c*x)/((c^16*x^11/(sqrt(-c^2*x^2 + 1)
+ 1)^11 + 5*c^14*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 10*c^12*x^7/(sqrt(-c^2*x
^2 + 1) + 1)^7 + 10*c^10*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + 5*c^8*x^3/(sqrt(
-c^2*x^2 + 1) + 1)^3 + c^6*x/(sqrt(-c^2*x^2 + 1) + 1))*(sqrt(-c^2*x^2 + ...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^3}{x^2} dx$$

input

```
int(((a + b*asin(c*x))*(d + e*x^2)^3)/x^2,x)
```

output

```
int(((a + b*asin(c*x))*(d + e*x^2)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.42

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^2} dx$$

$$= \frac{-75 \operatorname{asin}(cx) b c^5 d^3 + 225 \operatorname{asin}(cx) b c^5 d^2 e x^2 + 75 \operatorname{asin}(cx) b c^5 d e^2 x^4 + 15 \operatorname{asin}(cx) b c^5 e^3 x^6 + 225 \sqrt{-c^2 x^2}}{75 c^5 x}$$

input

```
int((e*x^2+d)^3*(a+b*asin(c*x))/x^2,x)
```

output

```
( - 75*asin(c*x)*b*c**5*d**3 + 225*asin(c*x)*b*c**5*d**2*e*x**2 + 75*asin(
c*x)*b*c**5*d*e**2*x**4 + 15*asin(c*x)*b*c**5*e**3*x**6 + 225*sqrt( - c**2
*x**2 + 1)*b*c**4*d**2*e*x + 25*sqrt( - c**2*x**2 + 1)*b*c**4*d*e**2*x**3
+ 3*sqrt( - c**2*x**2 + 1)*b*c**4*e**3*x**5 + 50*sqrt( - c**2*x**2 + 1)*b*
c**2*d*e**2*x + 4*sqrt( - c**2*x**2 + 1)*b*c**2*e**3*x**3 + 8*sqrt( - c**2
*x**2 + 1)*b*e**3*x + 75*log(tan(asin(c*x)/2))*b*c**6*d**3*x - 75*a*c**5*d
**3 + 225*a*c**5*d**2*e*x**2 + 75*a*c**5*d*e**2*x**4 + 15*a*c**5*e**3*x**6
)/(75*c**5*x)
```

$$3.448 \quad \int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x^3} dx$$

Optimal result	3809
Mathematica [A] (verified)	3810
Rubi [A] (verified)	3811
Maple [A] (verified)	3813
Fricas [F]	3813
Sympy [F]	3814
Maxima [F]	3814
Giac [F(-2)]	3814
Mupad [F(-1)]	3815
Reduce [F]	3815

Optimal result

Integrand size = 21, antiderivative size = 262

$$\begin{aligned} \int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x^3} dx = & -\frac{bcd^3\sqrt{1-c^2x^2}}{2x} + \frac{3be^2(8c^2d+e)x\sqrt{1-c^2x^2}}{32c^3} \\ & + \frac{be^3x^3\sqrt{1-c^2x^2}}{16c} - \frac{3be^2(8c^2d+e)\arcsin(cx)}{32c^4} \\ & - \frac{3}{2}ibd^2e\arcsin(cx)^2 - \frac{d^3(a+b\arcsin(cx))}{2x^2} \\ & + \frac{3}{2}de^2x^2(a+b\arcsin(cx)) + \frac{1}{4}e^3x^4(a+b\arcsin(cx)) \\ & + 3bd^2e\arcsin(cx)\log(1-e^{2i\arcsin(cx)}) \\ & - 3bd^2e\arcsin(cx)\log(x) \\ & + 3d^2e(a+b\arcsin(cx))\log(x) \\ & - \frac{3}{2}ibd^2e\text{PolyLog}(2, e^{2i\arcsin(cx)}) \end{aligned}$$

output

```
-1/2*b*c*d^3*(-c^2*x^2+1)^(1/2)/x+3/32*b*e^2*(8*c^2*d+e)*x*(-c^2*x^2+1)^(1/2)/c^3+1/16*b*e^3*x^3*(-c^2*x^2+1)^(1/2)/c-3/32*b*e^2*(8*c^2*d+e)*arcsin(c*x)/c^4-3/2*I*b*d^2*e*arcsin(c*x)^2-1/2*d^3*(a+b*arcsin(c*x))/x^2+3/2*d*e^2*x^2*(a+b*arcsin(c*x))+1/4*e^3*x^4*(a+b*arcsin(c*x))+3*b*d^2*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)-3*b*d^2*e*arcsin(c*x)*ln(x)+3*d^2*e*(a+b*arcsin(c*x))*ln(x)-3/2*I*b*d^2*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00

$$\begin{aligned}
& \int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx \\
&= \frac{1}{32} \left(-\frac{16ad^3}{x^2} + 48ade^2x^2 + 8ae^3x^4 - \frac{16bd^3(cx\sqrt{1-c^2x^2} + \arcsin(cx))}{x^2} \right. \\
&\quad + \frac{be^3 \left(cx\sqrt{1-c^2x^2}(3+2c^2x^2) + 8c^4x^4 \arcsin(cx) - 6 \arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) \right)}{c^4} \\
&\quad + \frac{24bde^2 \left(cx\sqrt{1-c^2x^2} + 2c^2x^2 \arcsin(cx) - 2 \arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) \right)}{c^2} \\
&\quad \left. + 96ad^2e \log(x) + 96bd^2e \left(\arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) \right. \right. \\
&\quad \left. \left. - \frac{1}{2}i(\arcsin(cx)^2 + \text{PolyLog}(2, e^{2i \arcsin(cx)})) \right) \right)
\end{aligned}$$

input `Integrate[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x^3,x]`output `((-16*a*d^3)/x^2 + 48*a*d*e^2*x^2 + 8*a*e^3*x^4 - (16*b*d^3*(c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/x^2 + (b*e^3*(c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2) + 8*c^4*x^4*ArcSin[c*x] - 6*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/c^4 + (24*b*d*e^2*(c*x*Sqrt[1 - c^2*x^2] + 2*c^2*x^2*ArcSin[c*x] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/c^2 + 96*a*d^2*e*Log[x] + 96*b*d^2*e*(ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - (I/2)*(ArcSin[c*x]^2 + PolyLog[2, E^((2*I)*ArcSin[c*x])])))/32`

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx$$

↓ 5230

$$-bc \int -\frac{-e^3 x^6 - 6de^2 x^4 - 12d^2 e \log(x)x^2 + 2d^3}{4x^2 \sqrt{1 - c^2 x^2}} dx - \frac{d^3 (a + b \arcsin(cx))}{2x^2} + 3d^2 e \log(x)(a + b \arcsin(cx)) + \frac{3}{2} de^2 x^2 (a + b \arcsin(cx)) + \frac{1}{4} e^3 x^4 (a + b \arcsin(cx))$$

↓ 27

$$\frac{1}{4} bc \int \frac{-e^3 x^6 - 6de^2 x^4 - 12d^2 e \log(x)x^2 + 2d^3}{x^2 \sqrt{1 - c^2 x^2}} dx - \frac{d^3 (a + b \arcsin(cx))}{2x^2} + 3d^2 e \log(x)(a + b \arcsin(cx)) + \frac{3}{2} de^2 x^2 (a + b \arcsin(cx)) + \frac{1}{4} e^3 x^4 (a + b \arcsin(cx))$$

↓ 7293

$$\frac{1}{4} bc \int \left(\frac{-e^3 x^6 - 6de^2 x^4 + 2d^3}{x^2 \sqrt{1 - c^2 x^2}} - \frac{12d^2 e \log(x)}{\sqrt{1 - c^2 x^2}} \right) dx - \frac{d^3 (a + b \arcsin(cx))}{2x^2} + 3d^2 e \log(x)(a + b \arcsin(cx)) + \frac{3}{2} de^2 x^2 (a + b \arcsin(cx)) + \frac{1}{4} e^3 x^4 (a + b \arcsin(cx))$$

↓ 2009

$$-\frac{d^3 (a + b \arcsin(cx))}{2x^2} + 3d^2 e \log(x)(a + b \arcsin(cx)) + \frac{3}{2} de^2 x^2 (a + b \arcsin(cx)) + \frac{1}{4} e^3 x^4 (a + b \arcsin(cx)) + \frac{1}{4} bc \left(-\frac{3e^2 \arcsin(cx) (8c^2 d + e)}{8c^5} - \frac{6id^2 e \text{PolyLog}(2, e^{2i \arcsin(cx)})}{c} - \frac{6id^2 e \arcsin(cx)^2}{c} + \frac{12d^2 e \arcsin(cx) \log(1 - c^2 x^2)}{c} \right)$$

input

```
Int[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x^3,x]
```


output

```
-1/2*(d^3*(a + b*ArcSin[c*x]))/x^2 + (3*d*e^2*x^2*(a + b*ArcSin[c*x]))/2 +
(e^3*x^4*(a + b*ArcSin[c*x]))/4 + 3*d^2*e*(a + b*ArcSin[c*x])*Log[x] + (b
*c*((-2*d^3*Sqrt[1 - c^2*x^2])/x + (3*e^2*(8*c^2*d + e)*x*Sqrt[1 - c^2*x^2
]))/(8*c^4) + (e^3*x^3*Sqrt[1 - c^2*x^2])/(4*c^2) - (3*e^2*(8*c^2*d + e)*Ar
cSin[c*x])/(8*c^5) - ((6*I)*d^2*e*ArcSin[c*x]^2)/c + (12*d^2*e*ArcSin[c*x]
*Log[1 - E^((2*I)*ArcSin[c*x])])/c - (12*d^2*e*ArcSin[c*x]*Log[x])/c - ((6
*I)*d^2*e*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c)/4
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_)] /; FreeQ[b, x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5230

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{e^3x^4}{4} + \frac{3de^2x^2}{2} + 3d^2e \ln(x) - \frac{d^3}{2x^2}\right) - 3ib d^2e \operatorname{polylog}\left(2, icx + \sqrt{-c^2x^2 + 1}\right) - \frac{bc d^3\sqrt{-c^2x^2 + 1}}{2c^2x^2}$
derivativedivides	$c^2\left(\frac{a\left(\frac{3c^4d e^2x^2}{2} + \frac{c^4e^3x^4}{4} + 3c^4d^2e \ln(cx) - \frac{c^4d^3}{2x^2}\right)}{c^6} - \frac{3ib d^2e \arcsin(cx)^2}{2c^2} - \frac{b d^3\sqrt{-c^2x^2+1}}{2cx} - \frac{b d^3 \arcsin(cx)}{2c^2x^2} + \frac{bc d^3\sqrt{-c^2x^2+1}}{2c^2x^2}\right)$
default	$c^2\left(\frac{a\left(\frac{3c^4d e^2x^2}{2} + \frac{c^4e^3x^4}{4} + 3c^4d^2e \ln(cx) - \frac{c^4d^3}{2x^2}\right)}{c^6} - \frac{3ib d^2e \arcsin(cx)^2}{2c^2} - \frac{b d^3\sqrt{-c^2x^2+1}}{2cx} - \frac{b d^3 \arcsin(cx)}{2c^2x^2} + \frac{bc d^3\sqrt{-c^2x^2+1}}{2c^2x^2}\right)$

input `int((e*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x,method=_RETURNVERBOSE)`

output `a*(1/4*e^3*x^4+3/2*d*e^2*x^2+3*d^2*e*ln(x)-1/2*d^3/x^2)-3*I*b*d^2*e*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))-1/2*b*c*d^3*(-c^2*x^2+1)^(1/2)/x-1/2*b*d^3/x^2*arcsin(c*x)-1/128*b/c^4*e^3*sin(4*arcsin(c*x))-1/8*b/c^4*e^3*arcsin(c*x)+1/32*b/c^4*arcsin(c*x)*e^3*cos(4*arcsin(c*x))+1/4*b/c^2*e^3*arcsin(c*x)*x^2-3/4*b/c^2*e^2*arcsin(c*x)*d+3/4*b/c*e^2*(-c^2*x^2+1)^(1/2)*x*d+1/2*I*b*c^2*d^3-3/2*I*b*d^2*e*arcsin(c*x)^2+3*b*d^2*e*arcsin(c*x)*ln(1-I*c*x-(-c^2*x^2+1)^(1/2))+3/2*b*e^2*arcsin(c*x)*x^2*d+1/8*b/c^3*e^3*(-c^2*x^2+1)^(1/2)*x-3*I*b*d^2*e*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+3*b*d^2*e*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))`

Fricas [F]

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(ex^2 + d)^3 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x, algorithm="fricas")`

output `integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsin(c*x))/x^3, x)`

Sympy [F]

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (d + ex^2)^3}{x^3} dx$$

input `integrate((e*x**2+d)**3*(a+b*asin(c*x))/x**3,x)`

output `Integral((a + b*asin(c*x))*(d + e*x**2)**3/x**3, x)`

Maxima [F]

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(ex^2 + d)^3 (b \arcsin(cx) + a)}{x^3} dx$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x, algorithm="maxima")`

output `1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 - 1/2*b*d^3*(sqrt(-c^2*x^2 + 1)*c/x + arcsin(c*x)/x^2) + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 + integrate((b*e^3*x^4 + 3*b*d*e^2*x^2 + 3*b*d^2*e)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/x, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^3}{x^3} dx$$

input `int(((a + b*asin(c*x))*(d + e*x^2)^3)/x^3,x)`output `int(((a + b*asin(c*x))*(d + e*x^2)^3)/x^3, x)`**Reduce [F]**

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^3} dx$$

$$= \frac{-16a \sin(cx) b c^4 d^3 + 48a \sin(cx) b c^4 d e^2 x^4 + 8a \sin(cx) b c^4 e^3 x^6 - 24a \sin(cx) b c^2 d e^2 x^2 - 3a \sin(cx) b e^3}{32c^4 x^2}$$

input `int((e*x^2+d)^3*(a+b*asin(c*x))/x^3,x)`output `(- 16*asin(c*x)*b*c**4*d**3 + 48*asin(c*x)*b*c**4*d*e**2*x**4 + 8*asin(c*x)*b*c**4*e**3*x**6 - 24*asin(c*x)*b*c**2*d*e**2*x**2 - 3*asin(c*x)*b*e**3*x**2 - 16*sqrt(-c**2*x**2 + 1)*b*c**5*d**3*x + 24*sqrt(-c**2*x**2 + 1)*b*c**3*d*e**2*x**3 + 2*sqrt(-c**2*x**2 + 1)*b*c**3*e**3*x**5 + 3*sqrt(-c**2*x**2 + 1)*b*c*e**3*x**3 + 96*int(asin(c*x)/x,x)*b*c**4*d**2*e*x**2 + 96*log(x)*a*c**4*d**2*e*x**2 - 16*a*c**4*d**3 + 48*a*c**4*d*e**2*x**4 + 8*a*c**4*e**3*x**6)/(32*c**4*x**2)`

3.449 $\int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x^4} dx$

Optimal result	3816
Mathematica [A] (verified)	3817
Rubi [A] (warning: unable to verify)	3817
Maple [A] (verified)	3821
Fricas [A] (verification not implemented)	3822
Sympy [A] (verification not implemented)	3823
Maxima [A] (verification not implemented)	3824
Giac [B] (verification not implemented)	3824
Mupad [F(-1)]	3825
Reduce [B] (verification not implemented)	3826

Optimal result

Integrand size = 21, antiderivative size = 186

$$\int \frac{(d+ex^2)^3(a+b \arcsin(cx))}{x^4} dx = \frac{be^2(9c^2d+e)\sqrt{1-c^2x^2}}{3c^3} - \frac{bcd^3\sqrt{1-c^2x^2}}{6x^2} - \frac{be^3(1-c^2x^2)^{3/2}}{9c^3} - \frac{d^3(a+b \arcsin(cx))}{3x^3} - \frac{3d^2e(a+b \arcsin(cx))}{x} + 3de^2x(a+b \arcsin(cx)) + \frac{1}{3}e^3x^3(a+b \arcsin(cx)) - \frac{1}{6}bcd^2(c^2d+18e) \operatorname{arctanh}(\sqrt{1-c^2x^2})$$

output

```
1/3*b*e^2*(9*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3-1/6*b*c*d^3*(-c^2*x^2+1)^(1/2)
)/x^2-1/9*b*e^3*(-c^2*x^2+1)^(3/2)/c^3-1/3*d^3*(a+b*arcsin(c*x))/x^3-3*d^2
*e*(a+b*arcsin(c*x))/x+3*d*e^2*x*(a+b*arcsin(c*x))+1/3*e^3*x^3*(a+b*arcsin
(c*x))-1/6*b*c*d^2*(c^2*d+18*e)*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{1}{6} \left(-\frac{2ad^3}{x^3} - \frac{18ad^2e}{x} + 18ade^2x + 2ae^3x^3 \right. \\ \left. + \frac{b\sqrt{1-c^2x^2}(-3c^4d^3 + 4e^3x^2 + 2c^2e^2x^2(27d + ex^2))}{3c^3x^2} \right. \\ \left. + \frac{2b(-d^3 - 9d^2ex^2 + 9de^2x^4 + e^3x^6) \arcsin(cx)}{x^3} + bcd^2(c^2d + 18e) \log(x) \right. \\ \left. - bcd^2(c^2d + 18e) \log\left(1 + \sqrt{1-c^2x^2}\right) \right)$$

input

```
Integrate[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x^4,x]
```

output

```
((-2*a*d^3)/x^3 - (18*a*d^2*e)/x + 18*a*d*e^2*x + 2*a*e^3*x^3 + (b*Sqrt[1 - c^2*x^2]*(-3*c^4*d^3 + 4*e^3*x^2 + 2*c^2*e^2*x^2*(27*d + e*x^2)))/(3*c^3*x^2) + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcSin[c*x])/x^3 + b*c*d^2*(c^2*d + 18*e)*Log[x] - b*c*d^2*(c^2*d + 18*e)*Log[1 + Sqrt[1 - c^2*x^2]])/6
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5230, 27, 2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

↓ 5230

$$\begin{aligned}
& -bc \int \frac{-e^3 x^6 - 9de^2 x^4 + 9d^2 ex^2 + d^3}{3x^3 \sqrt{1-c^2 x^2}} dx - \frac{d^3(a+b \arcsin(cx))}{3x^3} - \frac{3d^2 e(a+b \arcsin(cx))}{x} + \\
& \quad 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 27 \\
& \frac{1}{3} bc \int \frac{-e^3 x^6 - 9de^2 x^4 + 9d^2 ex^2 + d^3}{x^3 \sqrt{1-c^2 x^2}} dx - \frac{d^3(a+b \arcsin(cx))}{3x^3} - \frac{3d^2 e(a+b \arcsin(cx))}{x} + \\
& \quad 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 2331 \\
& \frac{1}{6} bc \int \frac{-e^3 x^6 - 9de^2 x^4 + 9d^2 ex^2 + d^3}{x^4 \sqrt{1-c^2 x^2}} dx^2 - \frac{d^3(a+b \arcsin(cx))}{3x^3} - \frac{3d^2 e(a+b \arcsin(cx))}{x} + \\
& \quad 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 2124 \\
& \frac{1}{6} bc \left(- \int \frac{-2e^3 x^4 - 18de^2 x^2 + d^2(dc^2 + 18e)}{2x^2 \sqrt{1-c^2 x^2}} dx^2 - \frac{d^3 \sqrt{1-c^2 x^2}}{x^2} \right) - \frac{d^3(a+b \arcsin(cx))}{3x^3} - \\
& \quad \frac{3d^2 e(a+b \arcsin(cx))}{x} + 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 27 \\
& \frac{1}{6} bc \left(\frac{1}{2} \int \frac{-2e^3 x^4 - 18de^2 x^2 + d^2(dc^2 + 18e)}{x^2 \sqrt{1-c^2 x^2}} dx^2 - \frac{d^3 \sqrt{1-c^2 x^2}}{x^2} \right) - \frac{d^3(a+b \arcsin(cx))}{3x^3} - \\
& \quad \frac{3d^2 e(a+b \arcsin(cx))}{x} + 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 1192 \\
& \frac{1}{6} bc \left(\frac{\int \frac{-2e^3 x^8 + 2e^2(9dc^2 + 2e)x^4 + c^6 d^3 - 2e^3 - 18c^2 de^2 + 18c^4 d^2 e}{1-x^4} d\sqrt{1-c^2 x^2}}{c^4} - \frac{d^3 \sqrt{1-c^2 x^2}}{x^2} \right) - \\
& \quad \frac{d^3(a+b \arcsin(cx))}{3x^3} - \frac{3d^2 e(a+b \arcsin(cx))}{x} + 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 25 \\
& \frac{1}{6} bc \left(- \frac{\int \frac{-2e^3 x^8 + 2e^2(9dc^2 + 2e)x^4 + c^6 d^3 - 2e^3 - 18c^2 de^2 + 18c^4 d^2 e}{1-x^4} d\sqrt{1-c^2 x^2}}{c^4} - \frac{d^3 \sqrt{1-c^2 x^2}}{x^2} \right) - \\
& \quad \frac{d^3(a+b \arcsin(cx))}{3x^3} - \frac{3d^2 e(a+b \arcsin(cx))}{x} + 3de^2 x(a+b \arcsin(cx)) + \frac{1}{3} e^3 x^3(a+b \arcsin(cx)) \\
& \quad \downarrow 1467
\end{aligned}$$

$$\frac{1}{6}bc \left(-\frac{\int (2e^3x^4 - 2e^2(9dc^2 + e) + \frac{d^3c^6 + 18d^2ec^4}{1-x^4}) d\sqrt{1-c^2x^2}}{c^4} - \frac{d^3\sqrt{1-c^2x^2}}{x^2} \right) - \frac{d^3(a + b \arcsin(cx))}{3x^3} - \frac{3d^2e(a + b \arcsin(cx))}{x} + 3de^2x(a + b \arcsin(cx)) + \frac{1}{3}e^3x^3(a + b \arcsin(cx))$$

↓ 2009

$$-\frac{d^3(a + b \arcsin(cx))}{3x^3} - \frac{3d^2e(a + b \arcsin(cx))}{x} + 3de^2x(a + b \arcsin(cx)) + \frac{1}{3}e^3x^3(a + b \arcsin(cx)) + \frac{1}{6}bc \left(\frac{-c^4d^2 \operatorname{arctanh}(\sqrt{1-c^2x^2})(c^2d + 18e) + 2e^2\sqrt{1-c^2x^2}(9c^2d + e) - \frac{2}{3}e^3x^6}{c^4} - \frac{d^3\sqrt{1-c^2x^2}}{x^2} \right)$$

input `Int[((d + e*x^2)^3*(a + b*ArcSin[c*x]))/x^4,x]`

output `-1/3*(d^3*(a + b*ArcSin[c*x]))/x^3 - (3*d^2*e*(a + b*ArcSin[c*x]))/x + 3*d*e^2*x*(a + b*ArcSin[c*x]) + (e^3*x^3*(a + b*ArcSin[c*x]))/3 + (b*c*(-((d^3*Sqrt[1 - c^2*x^2])/x^2) + ((-2*e^3*x^6)/3 + 2*e^2*(9*c^2*d + e)*Sqrt[1 - c^2*x^2] - c^4*d^2*(c^2*d + 18*e)*ArcTanh[Sqrt[1 - c^2*x^2]]/c^4))/6`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]`

rule 1192 `Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[2/e^(n + 2*p + 1) Subst[Int[x^(2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4]^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]`

rule 1467 `Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]`

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2124 `Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px,
a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
a*d))), x] + Simp[1/((m + 1)*(b*c - a*d)) Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] || !
ILtQ[n, -1])`

rule 2331 `Int[(Pq_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[1/2 S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]`

rule 5230 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_
)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))`

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.26

method	result
parts	$a \left(\frac{e^3 x^3}{3} + 3d e^2 x - \frac{d^3}{3x^3} - \frac{3d^2 e}{x} \right) + b c^3 \left(\frac{\arcsin(cx) e^3 x^3}{3c^3} + \frac{3 \arcsin(cx) x d e^2}{c^3} - \frac{\arcsin(cx) d^3}{3c^3 x^3} - \frac{3 \arcsin(cx) d^2 e}{c^3 x} \right)$
derivativedivides	$c^3 \left(\frac{a \left(3c^3 d e^2 x + \frac{e^3 c^3 x^3}{3} - \frac{c^3 d^3}{3x^3} - \frac{3c^3 d^2 e}{x} \right)}{c^6} + \frac{b \left(3 \arcsin(cx) c^3 d e^2 x + \frac{\arcsin(cx) e^3 c^3 x^3}{3} - \frac{\arcsin(cx) c^3 d^3}{3x^3} - \frac{3 \arcsin(cx) d^2 e}{x} \right)}{c^6} \right)$
default	$c^3 \left(\frac{a \left(3c^3 d e^2 x + \frac{e^3 c^3 x^3}{3} - \frac{c^3 d^3}{3x^3} - \frac{3c^3 d^2 e}{x} \right)}{c^6} + \frac{b \left(3 \arcsin(cx) c^3 d e^2 x + \frac{\arcsin(cx) e^3 c^3 x^3}{3} - \frac{\arcsin(cx) c^3 d^3}{3x^3} - \frac{3 \arcsin(cx) d^2 e}{x} \right)}{c^6} \right)$

```
input int((e*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
output a*(1/3*e^3*x^3+3*d*e^2*x-1/3*d^3/x^3-3*d^2*e/x)+b*c^3*(1/3/c^3*arcsin(c*x)*e^3*x^3+3/c^3*arcsin(c*x)*x*d*e^2-1/3*arcsin(c*x)*d^3/c^3/x^3-3/c^3*arcsin(c*x)*d^2*e/x-1/3/c^6*(e^3*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2))-2/3*(-c^2*x^2+1)^(1/2))-c^6*d^3*(-1/2/c^2/x^2*(-c^2*x^2+1)^(1/2))-1/2*arctanh(1/(-c^2*x^2+1)^(1/2)))-9*c^2*d*e^2*(-c^2*x^2+1)^(1/2)+9*c^4*d^2*e*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.32

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{12 ac^3 e^3 x^6 + 108 ac^3 d e^2 x^4 - 108 ac^3 d^2 e x^2 - 12 ac^3 d^3 - 3 (bc^6 d^3 + 18 bc^4 d^2 e) x^3 \log(\sqrt{-c^2 x^2 + 1} + 1) + 3 (bc^6 d^3 + 18 bc^4 d^2 e) x^3 \log(\sqrt{-c^2 x^2 + 1} - 1) + 12 (bc^3 e^3 x^6 + 9 bc^3 d e^2 x^4 - 9 bc^3 d^2 e x^2 - bc^3 d^3) \arcsin(cx) + 2 (2 bc^2 e^3 x^5 - 3 bc^4 d^3 x + 2 (27 bc^2 d e^2 + 2 b e^3) x^3) \sqrt{-c^2 x^2 + 1}}{c^3 x^3}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x, algorithm="fricas")`

output `1/36*(12*a*c^3*e^3*x^6 + 108*a*c^3*d*e^2*x^4 - 108*a*c^3*d^2*e*x^2 - 12*a*c^3*d^3 - 3*(b*c^6*d^3 + 18*b*c^4*d^2*e)*x^3*log(sqrt(-c^2*x^2 + 1) + 1) + 3*(b*c^6*d^3 + 18*b*c^4*d^2*e)*x^3*log(sqrt(-c^2*x^2 + 1) - 1) + 12*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e*x^2 - b*c^3*d^3)*arcsin(c*x) + 2*(2*b*c^2*e^3*x^5 - 3*b*c^4*d^3*x + 2*(27*b*c^2*d*e^2 + 2*b*e^3)*x^3)*sqrt(-c^2*x^2 + 1)/(c^3*x^3)`

Sympy [A] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.67

$$\begin{aligned}
& \int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx \\
&= -\frac{ad^3}{3x^3} - \frac{3ad^2e}{x} + 3ade^2x + \frac{ae^3x^3}{3} \\
&\quad + \frac{bcd^3 \left(\begin{cases} -\frac{c^2 \operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ \frac{ic^2 \operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases} \right)}{3} \\
&\quad + 3bcd^2e \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \left|\frac{1}{c^2x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) \\
&\quad - \frac{bce^3 \left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right)}{3} - \frac{bd^3 \operatorname{asin}(cx)}{3x^3} - \frac{3bd^2e \operatorname{asin}(cx)}{x} \\
&\quad + 3bde^2 \left(\begin{cases} 0 & \text{for } c = 0 \\ x \operatorname{asin}(cx) + \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right) + \frac{be^3x^3 \operatorname{asin}(cx)}{3}
\end{aligned}$$

input `integrate((e*x**2+d)**3*(a+b*asin(c*x))/x**4,x)`output `-a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3 + b*c*d**3*
Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(
2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/
(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 + 3*b*c*d**2*e*Piec
ewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) -
b*c*e**3*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**
2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True))/3 - b*d**3*asin(c*x)/(3*x**
3) - 3*b*d**2*e*asin(c*x)/x + 3*b*d*e**2*Piecewise((0, Eq(c, 0)), (x*asin(
c*x) + sqrt(-c**2*x**2 + 1)/c, True)) + b*e**3*x**3*asin(c*x)/3`

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.24

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{1}{3} ae^3 x^3$$

$$- \frac{1}{6} \left(\left(c^2 \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\sqrt{-c^2x^2+1}}{x^2} \right) c + \frac{2 \arcsin(cx)}{x^3} \right) bd^3$$

$$- 3 \left(c \log \left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|} \right) + \frac{\arcsin(cx)}{x} \right) bd^2 e$$

$$+ \frac{1}{9} \left(3x^3 \arcsin(cx) + c \left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4} \right) \right) be^3$$

$$+ 3ade^2x + \frac{3(cx \arcsin(cx) + \sqrt{-c^2x^2+1})bde^2}{c} - \frac{3ad^2e}{x} - \frac{ad^3}{3x^3}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x, algorithm="maxima")`

output `1/3*a*e^3*x^3 - 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c + 2*arcsin(c*x)/x^3)*b*d^3 - 3*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + arcsin(c*x)/x)*b*d^2*e + 1/9*(3*x^3*arcsin(c*x) + c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*e^3 + 3*a*d*e^2*x + 3*(c*x*arcsin(c*x) + sqrt(-c^2*x^2 + 1))*b*d*e^2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7971 vs. 2(166) = 332.

Time = 8.18 (sec) , antiderivative size = 7971, normalized size of antiderivative = 42.85

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx = \text{Too large to display}$$

input `integrate((e*x^2+d)^3*(a+b*arcsin(c*x))/x^4,x, algorithm="giac")`

output

```

-1/24*b*c^18*d^3*x^12*arcsin(c*x)/((c^12*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 +
3*c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)
^5 + c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^12) - 1/
24*a*c^18*d^3*x^12/((c^12*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 3*c^10*x^7/(sqr
t(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^6*x^3/(s
qrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^12) + 1/24*b*c^17*d^3*x
^11/((c^12*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 3*c^10*x^7/(sqrt(-c^2*x^2 + 1)
+ 1)^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^6*x^3/(sqrt(-c^2*x^2 +
1) + 1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^11) - 1/4*b*c^16*d^3*x^10*arcsin(c*x)/
((c^12*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 3*c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)
^7 + 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^6*x^3/(sqrt(-c^2*x^2 + 1) +
1)^3)*(sqrt(-c^2*x^2 + 1) + 1)^10) - 1/4*a*c^16*d^3*x^10/((c^12*x^9/(sqrt
(-c^2*x^2 + 1) + 1)^9 + 3*c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^5/
(sqrt(-c^2*x^2 + 1) + 1)^5 + c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-c^
2*x^2 + 1) + 1)^10) + 1/6*b*c^15*d^3*x^9*log(abs(c)*abs(x))/((c^12*x^9/(sq
rt(-c^2*x^2 + 1) + 1)^9 + 3*c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7 + 3*c^8*x^
5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^6*x^3/(sqrt(-c^2*x^2 + 1) + 1)^3)*(sqrt(-
c^2*x^2 + 1) + 1)^9) - 1/6*b*c^15*d^3*x^9*log(sqrt(-c^2*x^2 + 1) + 1)/((c^
12*x^9/(sqrt(-c^2*x^2 + 1) + 1)^9 + 3*c^10*x^7/(sqrt(-c^2*x^2 + 1) + 1)^7
+ 3*c^8*x^5/(sqrt(-c^2*x^2 + 1) + 1)^5 + c^6*x^3/(sqrt(-c^2*x^2 + 1) + ...

```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx = \int \frac{(a + b \arcsin(cx)) (ex^2 + d)^3}{x^4} dx$$

input

```
int(((a + b*asin(c*x))*(d + e*x^2)^3)/x^4,x)
```

output

```
int(((a + b*asin(c*x))*(d + e*x^2)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.33

$$\int \frac{(d + ex^2)^3 (a + b \arcsin(cx))}{x^4} dx$$

$$= \frac{-6a \sin(cx) b c^3 d^3 - 54a \sin(cx) b c^3 d^2 e x^2 + 54a \sin(cx) b c^3 d e^2 x^4 + 6a \sin(cx) b c^3 e^3 x^6 - 3\sqrt{-c^2 x^2 + 1} b}{18c^3 x^3}$$

input

```
int((e*x^2+d)^3*(a+b*asin(c*x))/x^4,x)
```

output

```
( - 6*asin(c*x)*b*c**3*d**3 - 54*asin(c*x)*b*c**3*d**2*e*x**2 + 54*asin(c*x)*b*c**3*d*e**2*x**4 + 6*asin(c*x)*b*c**3*e**3*x**6 - 3*sqrt(-c**2*x**2 + 1)*b*c**4*d**3*x + 54*sqrt(-c**2*x**2 + 1)*b*c**2*d*e**2*x**3 + 2*sqrt(-c**2*x**2 + 1)*b*c**2*e**3*x**5 + 4*sqrt(-c**2*x**2 + 1)*b*e**3*x**3 + 3*log(tan(asin(c*x)/2))*b*c**6*d**3*x**3 + 54*log(tan(asin(c*x)/2))*b*c**4*d**2*e*x**3 - 6*a*c**3*d**3 - 54*a*c**3*d**2*e*x**2 + 54*a*c**3*d*e**2*x**4 + 6*a*c**3*e**3*x**6)/(18*c**3*x**3)
```

3.450
$$\int \frac{x^4(a+b \arcsin(cx))}{d+ex^2} dx$$

Optimal result	3828
Mathematica [A] (verified)	3829
Rubi [A] (verified)	3830
Maple [C] (verified)	3832
Fricas [F]	3833
Sympy [F]	3833
Maxima [F(-2)]	3834
Giac [F(-2)]	3834
Mupad [F(-1)]	3834
Reduce [F]	3835

Optimal result

Integrand size = 21, antiderivative size = 653

$$\begin{aligned}
\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = & -\frac{adx}{e^2} - \frac{bd\sqrt{1-c^2x^2}}{ce^2} + \frac{b\sqrt{1-c^2x^2}}{3c^3e} - \frac{b(1-c^2x^2)^{3/2}}{9c^3e} \\
& - \frac{bdx \arcsin(cx)}{e^2} + \frac{x^3(a + b \arcsin(cx))}{3e} \\
& + \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& - \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& + \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& - \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& + \frac{ib(-d)^{3/2} \text{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& - \frac{ib(-d)^{3/2} \text{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& + \frac{ib(-d)^{3/2} \text{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\
& - \frac{ib(-d)^{3/2} \text{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}
\end{aligned}$$

output

```

-a*d*x/e^2-b*d*(-c^2*x^2+1)^(1/2)/c/e^2+1/3*b*(-c^2*x^2+1)^(1/2)/c^3/e-1/9
*b*(-c^2*x^2+1)^(3/2)/c^3/e-b*d*x*arcsin(c*x)/e^2+1/3*x^3*(a+b*arcsin(c*x)
)/e+1/2*(-d)^(3/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)
))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arcsin(c*
x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
)))/e^(5/2)+1/2*(-d)^(3/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*a
rcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d
+e)^(1/2)))/e^(5/2)+1/2*I*b*(-d)^(3/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)-1/2*I*b*(-d)^(3/2)*po
lylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/e^(5/2)+1/2*I*b*(-d)^(3/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)-1/2*I*b*(-d)^(3/2)*polylog(2,e
^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)
)

```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 515, normalized size of antiderivative = 0.79

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = -\frac{adx}{e^2} + \frac{ax^3}{3e} + \frac{ad^{3/2} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{e^{5/2}} + \frac{b\left(-\frac{4d\sqrt{e}\left(\sqrt{1-c^2x^2}+cx \arcsin(cx)\right)}{c} + \frac{4e^{3/2}\left(\sqrt{1-c^2x^2}\left(2+c^2x^2\right)+3c^3x^3 \arcsin(cx)\right)}{9c^3}\right)}{e^{5/2}} + d^{3/2}\left(-\arcsin(cx)\left(\arcsin(cx) + \frac{cx}{\sqrt{1-c^2x^2}}\right) + \frac{cx^2}{\sqrt{1-c^2x^2}}\right)$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/(d + e*x^2),x]
```

output

```

-((a*d*x)/e^2) + (a*x^3)/(3*e) + (a*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e
^(5/2) + (b*((-4*d*Sqrt[e]*(Sqrt[1 - c^2*x^2] + c*x*ArcSin[c*x]))/c + (4*e
^(3/2)*(Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) + 3*c^3*x^3*ArcSin[c*x]))/(9*c^3)
+ d^(3/2)*(-(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSi
n[c*x])))/(c*Sqrt[d] - Sqrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x
]))/(c*Sqrt[d] + Sqrt[c^2*d + e])))) - 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[
c*x]))/(-(c*Sqrt[d] + Sqrt[c^2*d + e]))] - 2*PolyLog[2, -((Sqrt[e]*E^(I*Ar
cSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))] + d^(3/2)*(ArcSin[c*x]*(ArcSi
n[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c
^2*d + e]))] + Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d
+ e]))]) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*
d + e]))] + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*
d + e]))])))/(4*e^(5/2))

```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{d^2(a + b \arcsin(cx))}{e^2(d + ex^2)} - \frac{d(a + b \arcsin(cx))}{e^2} + \frac{x^2(a + b \arcsin(cx))}{e} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2e^{5/2}} - \\
& \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2e^{5/2}} + \\
& \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2e^{5/2}} - \\
& \frac{(-d)^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2e^{5/2}} + \frac{x^3(a + b \arcsin(cx))}{e^2} - \frac{adx}{3e} + \\
& \frac{ib(-d)^{3/2} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2e^{5/2}} - \frac{ib(-d)^{3/2} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2e^{5/2}} + \\
& \frac{ib(-d)^{3/2} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2e^{5/2}} - \frac{ib(-d)^{3/2} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2e^{5/2}} - \\
& \frac{bdx \arcsin(cx)}{e^2} - \frac{bd\sqrt{1-c^2x^2}}{ce^2} - \frac{b(1-c^2x^2)^{3/2}}{9c^3e} + \frac{b\sqrt{1-c^2x^2}}{3c^3e}
\end{aligned}$$

input `Int[(x^4*(a + b*ArcSin[c*x]))/(d + e*x^2), x]`

output

```

-((a*d*x)/e^2) - (b*d*Sqrt[1 - c^2*x^2])/(c*e^2) + (b*Sqrt[1 - c^2*x^2])/(
3*c^3*e) - (b*(1 - c^2*x^2)^(3/2))/(9*c^3*e) - (b*d*x*ArcSin[c*x])/e^2 + (
x^3*(a + b*ArcSin[c*x]))/(3*e) + ((-d)^(3/2)*(a + b*ArcSin[c*x])*Log[1 - (
Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])]/(2*e^(5/2))
- ((-d)^(3/2)*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c
*Sqrt[-d] - Sqrt[c^2*d + e])]/(2*e^(5/2)) + ((-d)^(3/2)*(a + b*ArcSin[c*x
])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])]/
(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin
[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])]/(2*e^(5/2)) + ((I/2)*b*(-d)^(3/
2)*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e
]))]/e^(5/2) - ((I/2)*b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))
/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])]/e^(5/2) + ((I/2)*b*(-d)^(3/2)*PolyLog[
2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])]/e^(5/
2) - ((I/2)*b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[
-d] + Sqrt[c^2*d + e])]/e^(5/2)

```

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_)*((f_.)*(x_))^m_)*((d_ + (e_
.)*(x_)^2)^p_), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 282.73 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.58

method	result
parts	$\frac{ax^3}{3e} - \frac{adx}{e^2} + \frac{ad^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}} - \frac{bd\sqrt{-c^2x^2+1}}{c^2} - \frac{bdx \arcsin(cx)}{e^2} + \frac{b\sqrt{-c^2x^2+1}}{4c^3e} + \frac{b \arcsin(cx)x}{4c^2e} + \frac{bcd^2}{b c^6 d^2}$
derivativedivides	$\frac{-\frac{ac^5dx}{e^2} + \frac{ac^5x^3}{3e} + \frac{ac^5d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}} - \frac{bc^4\sqrt{-c^2x^2+1}d}{e^2} - \frac{bc^5 \arcsin(cx)dx}{e^2} + \frac{bc^2\sqrt{-c^2x^2+1}}{4e} + \frac{bc^3 \arcsin(cx)x}{4e}}{b c^6 d^2}$
default	$\frac{-\frac{ac^5dx}{e^2} + \frac{ac^5x^3}{3e} + \frac{ac^5d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}} - \frac{bc^4\sqrt{-c^2x^2+1}d}{e^2} - \frac{bc^5 \arcsin(cx)dx}{e^2} + \frac{bc^2\sqrt{-c^2x^2+1}}{4e} + \frac{bc^3 \arcsin(cx)x}{4e}}{b c^6 d^2}$

```
input int(x^4*(a+b*arcsin(c*x))/(e*x^2+d), x, method=_RETURNVERBOSE)
```

output

```
1/3*a/e*x^3-a*d*x/e^2+a*d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-b*d*(-
c^2*x^2+1)^(1/2)/c/e^2-b*d*x*arcsin(c*x)/e^2+1/4*b*(-c^2*x^2+1)^(1/2)/c^3/
e+1/4*b/c^2/e*arcsin(c*x)*x+1/2*b*c/e^2*d^2*sum(1/_R1/(_R1^2*e-2*c^2*d-e)*
(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c
^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/2*b*c/e
^2*d^2*sum(_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+
1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z
^4+(-4*c^2*d-2*e)*_Z^2+e))-1/36*b/c^3/e*cos(3*arcsin(c*x))-1/12*b/c^3*arcsi
n(c*x)/e*sin(3*arcsin(c*x))
```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)x^4}{ex^2 + d} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*x^4*arcsin(c*x) + a*x^4)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{d + ex^2} dx$$

input

```
integrate(x**4*(a+b*asin(c*x))/(e*x**2+d),x)
```

output

```
Integral(x**4*(a + b*asin(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x^4(a + b \arcsin(cx))}{ex^2 + d} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d + e*x^2),x)`

output `int((x^4*(a + b*asin(c*x)))/(d + e*x^2), x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{d + ex^2} dx$$

$$= \frac{3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) ad + 3\left(\int \frac{\arcsin(cx)x^4}{ex^2+d} dx\right) be^3 - 3adex + ae^2x^3}{3e^3}$$

input `int(x^4*(a+b*asin(c*x))/(e*x^2+d), x)`

output `(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + 3*int((asin(c*x)*x**4)/(d + e*x**2), x)*b*e**3 - 3*a*d*e*x + a*e**2*x**3)/(3*e**3)`

$$3.451 \quad \int \frac{x^3(a+b \arcsin(cx))}{d+ex^2} dx$$

Optimal result	3837
Mathematica [A] (verified)	3838
Rubi [A] (verified)	3839
Maple [C] (warning: unable to verify)	3840
Fricas [F]	3841
Sympy [F]	3842
Maxima [F]	3842
Giac [F(-2)]	3842
Mupad [F(-1)]	3843
Reduce [F]	3843

Optimal result

Integrand size = 21, antiderivative size = 559

$$\begin{aligned}
\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx = & \frac{bx\sqrt{1 - c^2x^2}}{4ce} - \frac{b \arcsin(cx)}{4c^2e} \\
& + \frac{x^2(a + b \arcsin(cx))}{2e} + \frac{id(a + b \arcsin(cx))^2}{2be^2} \\
& - \frac{d(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2e^2} \\
& - \frac{d(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2e^2} \\
& - \frac{d(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2e^2} \\
& - \frac{d(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2e^2} \\
& + \frac{ibd \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2e^2} \\
& + \frac{ibd \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2e^2} \\
& + \frac{ibd \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2e^2} \\
& + \frac{ibd \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2e^2}
\end{aligned}$$

output

```

1/4*b*x*(-c^2*x^2+1)^(1/2)/c/e-1/4*b*arcsin(c*x)/c^2/e+1/2*x^2*(a+b*arcsin
(c*x))/e+1/2*I*d*(a+b*arcsin(c*x))^2/b/e^2-1/2*d*(a+b*arcsin(c*x))*ln(1-e
(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2
*d*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/
2)-(c^2*d+e)^(1/2)))/e^2-1/2*d*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2
*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*d*(a+b*arcsin(c*x
))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2
)))/e^2+1/2*I*b*d*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(
1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*I*b*d*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)
^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*I*b*d*polylog(2,-e^(1/2)
*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*
d*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(
1/2)))/e^2

```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.85

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx$$

$$= \frac{2ac^2ex^2 - 2ac^2d \log(d + ex^2) + b \left(e \left(cx\sqrt{1 - c^2x^2} + 2c^2x^2 \arcsin(cx) - 2 \arctan \left(\frac{cx}{-1 + \sqrt{1 - c^2x^2}} \right) \right) + ic^2d \right)}{4c^2e^2}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x]))/(d + e*x^2),x]
```

output

```

(2*a*c^2*e*x^2 - 2*a*c^2*d*Log[d + e*x^2] + b*(e*(c*x*Sqrt[1 - c^2*x^2] +
2*c^2*x^2*ArcSin[c*x] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]) + I*c^2*d*
(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*
Sqrt[d] - Sqrt[c^2*d + e])]) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqr
t[d] + Sqrt[c^2*d + e])])) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-c
*Sqrt[d] + Sqrt[c^2*d + e])]) + 2*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))
/(c*Sqrt[d] + Sqrt[c^2*d + e]))]) + I*c^2*d*(ArcSin[c*x]*(ArcSin[c*x] + (2
*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-c*Sqrt[d] + Sqrt[c^2*d + e])])
+ Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])])) +
2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])]) +
2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])]))
/(4*c^2*e^2)

```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx \\
 & \quad \downarrow \text{5232} \\
 & \int \left(\frac{x(a + b \arcsin(cx))}{e} - \frac{dx(a + b \arcsin(cx))}{e(d + ex^2)} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{d(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^2} - \\
 & \frac{d(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^2} - \frac{d(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^2} - \\
 & \frac{d(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^2} + \frac{id(a + b \arcsin(cx))^2}{2be^2} + \frac{x^2(a + b \arcsin(cx))}{2e} + \\
 & \frac{ibd \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2e^2} + \frac{ibd \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2e^2} + \\
 & \frac{ibd \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}} \right)}{2e^2} + \frac{ibd \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}} \right)}{2e^2} - \frac{b \arcsin(cx)}{4c^2e} + \\
 & \frac{bx\sqrt{1 - c^2x^2}}{4ce}
 \end{aligned}$$

input

```
Int[(x^3*(a + b*ArcSin[c*x]))/(d + e*x^2), x]
```

output

$$\begin{aligned} & (b*x*\sqrt{1 - c^2*x^2})/(4*c*e) - (b*\text{ArcSin}[c*x])/(4*c^2*e) + (x^2*(a + b* \\ & \text{ArcSin}[c*x]))/(2*e) + ((I/2)*d*(a + b*\text{ArcSin}[c*x])^2)/(b*e^2) - (d*(a + b* \\ & \text{ArcSin}[c*x])*Log[1 - (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d \\ & + e})])/(2*e^2) - (d*(a + b*\text{ArcSin}[c*x])*Log[1 + (\sqrt{e}*E^{(I*\text{ArcSin}[c* \\ & x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e})])/(2*e^2) - (d*(a + b*\text{ArcSin}[c*x])*L \\ & og[1 - (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} + \sqrt{c^2*d + e})])/(2*e \\ & ^2) - (d*(a + b*\text{ArcSin}[c*x])*Log[1 + (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt \\ & [-d] + \sqrt{c^2*d + e})])/(2*e^2) + ((I/2)*b*d*PolyLog[2, -((\sqrt{e}*E^{(I* \\ & \text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e}))])/e^2 + ((I/2)*b*d*PolyLog \\ & [2, (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e})])/e^2 + (\\ & (I/2)*b*d*PolyLog[2, -((\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} + \sqrt{c^ \\ & 2*d + e}))])/e^2 + ((I/2)*b*d*PolyLog[2, (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c* \\ & \sqrt{-d} + \sqrt{c^2*d + e})])/e^2 \end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5232

$$\text{Int}[(a + \text{ArcSin}[c*x])*(b*x)^n*((f*x)^m*(d + e*x^2)^p], x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (f*x)^m*(d + e*x^2)^p], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x \text{ \&\& NeQ}[c^2*d + e, 0] \text{ \&\& IGtQ}[n, 0] \text{ \&\& IntegerQ}[p] \text{ \&\& IntegerQ}[m]$$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.13 (sec) , antiderivative size = 2088, normalized size of antiderivative = 3.74

method	result	size
derivativedivides	Expression too large to display	2088
default	Expression too large to display	2088
parts	Expression too large to display	2095

input

$$\text{int}(x^3*(a+b*\text{arcsin}(c*x))/(e*x^2+d), x, \text{method}=_RETURNVERBOSE)$$

output

```

1/c^4*(1/2*a*c^4/e*x^2-1/2*a*c^4*d/e^2*ln(c^2*e*x^2+c^2*d)+b*c^2*(1/4*I*(2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/2))
^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e))*d*c^2/e^3+1/2*I*(2*c^2*d-2*(c^2*
d*(c^2*d+e))^(1/2)+e)*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*
(c^2*d*(c^2*d+e))^(1/2)+e))*c^4*d^2/e^4+I*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/
2)+e)*arcsin(c*x)^2*c^4*d^2/e^4+1/2*I*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e
)*arcsin(c*x)^2*d*c^2/e^3-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^4*d^2*c^
4*ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
))*arcsin(c*x)-1/2*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^3*ln(1-e*(I*c*x
+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e))*arcsin(c*x)*
d*c^2+1/16*(I+2*arcsin(c*x))/e*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)+(-
2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/
2)*e)*c^2*d/e^3/(c^2*d+e)*ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e))*arcsin(c*x)-1/2*I*(c^2*d*(c^2*d+e))^(1/2)*c^2*d
/e^2/(c^2*d+e)*arcsin(c*x)^2-1/2*I*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4
*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^4*d^2*polylog(2,e*(I*c*x+(-c^2
*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e))/e^4/(c^2*d+e)+(-2*
(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*
e)*c^4*d^2/e^4/(c^2*d+e)*ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c
^2*d*(c^2*d+e))^(1/2)+e))*arcsin(c*x)+1/2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d...

```

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)x^3}{ex^2 + d} dx$$

input

```
integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*x^3*arcsin(c*x) + a*x^3)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{d + ex^2} dx$$

input `integrate(x**3*(a+b*asin(c*x))/(e*x**2+d), x)`

output `Integral(x**3*(a + b*asin(c*x))/(d + e*x**2), x)`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)x^3}{ex^2 + d} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d), x, algorithm="maxima")`

output `1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e*x^2 + d), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d), x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{ex^2 + d} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d + e*x^2),x)`output `int((x^3*(a + b*asin(c*x)))/(d + e*x^2), x)`**Reduce [F]**

$$\int \frac{x^3(a + b \arcsin(cx))}{d + ex^2} dx$$

$$= \frac{2a \operatorname{asin}(cx) b c^2 e x^2 - \operatorname{asin}(cx) b e + \sqrt{-c^2 x^2 + 1} b c e x - 4 \left(\int \frac{\operatorname{asin}(cx) x}{e x^2 + d} dx \right) b c^2 d e - 2 \log(e x^2 + d) a c^2 d + 2 a c^2 d}{4 c^2 e^2}$$

input `int(x^3*(a+b*asin(c*x))/(e*x^2+d),x)`output `(2*asin(c*x)*b*c**2*e*x**2 - asin(c*x)*b*e + sqrt(-c**2*x**2 + 1)*b*c*e*x - 4*int((asin(c*x)*x)/(d + e*x**2),x)*b*c**2*d*e - 2*log(d + e*x**2)*a*c**2*d + 2*a*c**2*e*x**2)/(4*c**2*e**2)`

3.452 $\int \frac{x^2(a+b \arcsin(cx))}{d+ex^2} dx$

Optimal result	3844
Mathematica [A] (verified)	3845
Rubi [A] (verified)	3846
Maple [C] (verified)	3847
Fricas [F]	3848
Sympy [F]	3849
Maxima [F(-2)]	3849
Giac [F(-2)]	3849
Mupad [F(-1)]	3850
Reduce [F]	3850

Optimal result

Integrand size = 21, antiderivative size = 579

$$\begin{aligned}
 \int \frac{x^2(a+b \arcsin(cx))}{d+ex^2} dx = & \frac{ax}{e} + \frac{b\sqrt{1-c^2x^2}}{ce} + \frac{bx \arcsin(cx)}{e} \\
 & + \frac{\sqrt{-d}(a+b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & - \frac{\sqrt{-d}(a+b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & + \frac{\sqrt{-d}(a+b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & - \frac{\sqrt{-d}(a+b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & + \frac{ib\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & - \frac{ib\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & + \frac{ib\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
 & - \frac{ib\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}
 \end{aligned}$$

output

```
a*x/e+b*(-c^2*x^2+1)^(1/2)/c/e+b*x*arcsin(c*x)/e+1/2*(-d)^(1/2)*(a+b*arcsi
n(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(
(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2
*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)+1/2*(-d)^(1/2)*(a
+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c
^2*d+e)^(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c
*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)+1/2*I*b*(
-d)^(1/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c
^2*d+e)^(1/2)))/e^(3/2)-1/2*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(I*c*x+(-c^2*
x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)+1/2*I*b*(-d)^(1/2)
*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(
1/2)))/e^(3/2)-1/2*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1
/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.79

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx$$

$$= \frac{4ac\sqrt{ex} - 4ac\sqrt{d} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) + b\left(4\sqrt{e}(\sqrt{1 - c^2x^2} + cx \arcsin(cx)) + c\sqrt{d}(\arcsin(cx) (\arcsin(cx) + \dots)\right)}{\dots}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x]))/(d + e*x^2),x]
```

output

```
(4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[e
]*Sqrt[1 - c^2*x^2] + c*x*ArcSin[c*x]) + c*Sqrt[d]*(ArcSin[c*x]*(ArcSin[c
*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d +
e]))] + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))
]) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d +
e]))] + 2*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d
+ e]))]) - c*Sqrt[d]*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*
E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e]))] + Log[1 - (Sqrt[e]*E
^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]) + 2*PolyLog[2, (Sqrt[e]*
E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e]))] + 2*PolyLog[2, (Sqrt[e]*
E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]))/(4*c*e^(3/2))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx \\
 & \quad \downarrow \text{5232} \\
 & \int \left(\frac{a + b \arcsin(cx)}{e} - \frac{d(a + b \arcsin(cx))}{e(d + ex^2)} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{-d}(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^{3/2}} - \\
 & \frac{\sqrt{-d}(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^{3/2}} + \\
 & \frac{\sqrt{-d}(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^{3/2}} - \\
 & \frac{\sqrt{-d}(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2e^{3/2}} + \frac{ax}{e} + \frac{ib\sqrt{-d} \text{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2e^{3/2}} - \\
 & \frac{ib\sqrt{-d} \text{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2e^{3/2}} + \frac{ib\sqrt{-d} \text{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-d}+\sqrt{dc^2+e}} \right)}{2e^{3/2}} - \\
 & \frac{ib\sqrt{-d} \text{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-d}+\sqrt{dc^2+e}} \right)}{2e^{3/2}} + \frac{bx \arcsin(cx)}{e} + \frac{b\sqrt{1-c^2x^2}}{ce}
 \end{aligned}$$

input

```
Int[(x^2*(a + b*ArcSin[c*x]))/(d + e*x^2),x]
```

output

$$\begin{aligned} & (a*x)/e + (b*\sqrt{1 - c^2*x^2})/(c*e) + (b*x*\text{ArcSin}[c*x])/e + (\sqrt{-d}*(a \\ & + b*\text{ArcSin}[c*x])*\text{Log}[1 - (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{ \\ & [c^2*d + e])})]/(2*e^{(3/2)}) - (\sqrt{-d}*(a + b*\text{ArcSin}[c*x])*\text{Log}[1 + (\sqrt{e} \\ &]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e})])/(2*e^{(3/2)}) + (\sqrt{ \\ & t[-d]*(a + b*\text{ArcSin}[c*x])*\text{Log}[1 - (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} \\ &] + \sqrt{c^2*d + e})])/(2*e^{(3/2)}) - (\sqrt{-d}*(a + b*\text{ArcSin}[c*x])*\text{Log}[1 + \\ & (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} + \sqrt{c^2*d + e})])/(2*e^{(3/2)}) \\ &) + ((I/2)*b*\sqrt{-d}*\text{PolyLog}[2, -((\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} \\ & - \sqrt{c^2*d + e}))]/e^{(3/2)} - ((I/2)*b*\sqrt{-d}*\text{PolyLog}[2, (\sqrt{e}*E \\ & ^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e})])]/e^{(3/2)} + ((I/2)*b*\sqrt{ \\ & rt[-d]*\text{PolyLog}[2, -((\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} + \sqrt{c^2*d \\ & + e}))]/e^{(3/2)} - ((I/2)*b*\sqrt{-d}*\text{PolyLog}[2, (\sqrt{e}*E^{(I*\text{ArcSin}[c*x] \\ &))/(I*c*\sqrt{-d} + \sqrt{c^2*d + e})])]/e^{(3/2)} \end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$$

rule 5232

$$\text{Int}[(a_. + \text{ArcSin}[c_.*(x_)]*(b_.))^n*((f_.*(x_))^m*((d_) + (e_ \\ \cdot)*(x_)^2)^p], x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (\\ f*x)^m*(d + e*x^2)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2*d + \\ e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 320.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.49

method	result
parts	$\frac{ax}{e} - \frac{ad \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b\sqrt{-c^2x^2+1}}{ce} + \frac{bx \arcsin(cx)}{e} - \frac{bcd \left(\frac{i \arcsin(cx)}{\sum_{-R1=\text{RootOf}(e-Z^4+(-4c^2d-2e)-Z^2+e)} \right)}{e}$
derivativedivides	$\frac{\frac{ac^3x}{e} - \frac{ac^3d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{bc^2\sqrt{-c^2x^2+1}}{e} + \frac{bc^3 \arcsin(cx)x}{e} + \frac{bc^4d \left(\frac{i \arcsin(cx)}{\sum_{-R1=\text{RootOf}(e-Z^4+(-4c^2d-2e)-Z^2+e)} \right)}{e}$
default	$\frac{\frac{ac^3x}{e} - \frac{ac^3d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{bc^2\sqrt{-c^2x^2+1}}{e} + \frac{bc^3 \arcsin(cx)x}{e} + \frac{bc^4d \left(\frac{i \arcsin(cx)}{\sum_{-R1=\text{RootOf}(e-Z^4+(-4c^2d-2e)-Z^2+e)} \right)}{e}$

```
input int(x^2*(a+b*arcsin(c*x))/(e*x^2+d), x, method=_RETURNVERBOSE)
```

```
output a*x/e-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*(-c^2*x^2+1)^(1/2)/c/e+b
*x*arcsin(c*x)/e-1/2*b*c/e*d*sum(1/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*
ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2)
)/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))-1/2*b*c/e*d*sum(_R1/(_R1
^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilo
g((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z
^2+e))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)x^2}{ex^2 + d} dx$$

```
input integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d), x, algorithm="fricas")
```

```
output integral((b*x^2*arcsin(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{d + ex^2} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(e*x**2+d),x)`

output `Integral(x**2*(a + b*asin(c*x))/(d + e*x**2), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{ex^2 + d} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d + e*x^2),x)`output `int((x^2*(a + b*asin(c*x)))/(d + e*x^2), x)`**Reduce [F]**

$$\int \frac{x^2(a + b \arcsin(cx))}{d + ex^2} dx$$

$$= \frac{\operatorname{asin}(cx) bce x - \sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) ac + \sqrt{-c^2 x^2 + 1} be - \left(\int \frac{\operatorname{asin}(cx)}{ex^2 + d} dx\right) bcde + ace x}{ce^2}$$

input `int(x^2*(a+b*asin(c*x))/(e*x^2+d),x)`output `(asin(c*x)*b*c*e*x - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c + sqrt(-c**2*x**2 + 1)*b*e - int(asin(c*x)/(d + e*x**2),x)*b*c*d*e + a*c*e*x)/(c*e**2)`

3.453 $\int \frac{x(a+b \arcsin(cx))}{d+ex^2} dx$

Optimal result	3851
Mathematica [A] (verified)	3852
Rubi [A] (verified)	3853
Maple [C] (warning: unable to verify)	3854
Fricas [F]	3855
Sympy [F]	3856
Maxima [F]	3856
Giac [F(-2)]	3856
Mupad [F(-1)]	3857
Reduce [F]	3857

Optimal result

Integrand size = 19, antiderivative size = 491

$$\begin{aligned}
 \int \frac{x(a+b \arcsin(cx))}{d+ex^2} dx = & -\frac{i(a+b \arcsin(cx))^2}{2be} \\
 & + \frac{(a+b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\
 & + \frac{(a+b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\
 & + \frac{(a+b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e} \\
 & + \frac{(a+b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e}
 \end{aligned}$$

output

```
-1/2*I*(a+b*arcsin(c*x))^2/b/e+1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e+1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e+1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e+1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e-1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e-1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e-1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e-1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.81

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx =$$

$$\frac{i \left(b \arcsin(cx)^2 + ib \arcsin(cx) \log \left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{c\sqrt{d} - \sqrt{c^2 d + e}} \right) + ib \arcsin(cx) \log \left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{-c\sqrt{d} + \sqrt{c^2 d + e}} \right) + ib \arcsin(cx) \right)}{e}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x]))/(d + e*x^2),x]
```

output

```
((-1/2*I)*(b*ArcSin[c*x]^2 + I*b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + I*b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + I*b*ArcSin[c*x]*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] + I*b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])]) + I*a*Log[d + e*x^2] + b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + b*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])]) + b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])]))/e
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx$$

↓ 5232

$$\int \left(\frac{a + b \arcsin(cx)}{2\sqrt{e}(\sqrt{-d} + \sqrt{ex})} - \frac{a + b \arcsin(cx)}{2\sqrt{e}(\sqrt{-d} - \sqrt{ex})} \right) dx$$

↓ 2009

$$\frac{(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2e} + \frac{(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2e} +$$

$$\frac{(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2e} + \frac{(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2e} -$$

$$\frac{i(a + b \arcsin(cx))^2}{2be} - \frac{ib \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}} \right)}{2e} - \frac{ib \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}} \right)}{2e} -$$

$$\frac{ib \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}} \right)}{2e} - \frac{ib \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}} \right)}{2e}$$

input

```
Int[(x*(a + b*ArcSin[c*x]))/(d + e*x^2),x]
```

output

$$\begin{aligned} &((-1/2*I)*(a + b*\text{ArcSin}[c*x])^2)/(b*e) + ((a + b*\text{ArcSin}[c*x])* \text{Log}[1 - (\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\text{Sqrt}[-d] - \text{Sqrt}[c^2*d + e])])/(2*e) + ((a + b \\ &* \text{ArcSin}[c*x])* \text{Log}[1 + (\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\text{Sqrt}[-d] - \text{Sqrt}[c^2*d + e])])/(2*e) + ((a + b*\text{ArcSin}[c*x])* \text{Log}[1 - (\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])}) \\ &)/(I*c*\text{Sqrt}[-d] + \text{Sqrt}[c^2*d + e])])/(2*e) + ((a + b*\text{ArcSin}[c*x])* \text{Log}[1 + (\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\text{Sqrt}[-d] + \text{Sqrt}[c^2*d + e])])/(2*e) - ((I \\ &/2)*b*\text{PolyLog}[2, -((\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\text{Sqrt}[-d] - \text{Sqrt}[c^2*d + e]))]/e - ((I/2)*b*\text{PolyLog}[2, (\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\text{Sqrt}[-d] \\ &- \text{Sqrt}[c^2*d + e])]/e - ((I/2)*b*\text{PolyLog}[2, -((\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])})/(I*c*\text{Sqrt}[-d] + \text{Sqrt}[c^2*d + e]))]/e - ((I/2)*b*\text{PolyLog}[2, (\text{Sqrt}[e]*E^{(I*\text{ArcSin}[c*x])}) \\ &)/(I*c*\text{Sqrt}[-d] + \text{Sqrt}[c^2*d + e])])]/e \end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \rightarrow \text{Simp}[\text{IntSum}[u, x], x] \text{ ; SumQ}[u]$$

rule 5232

$$\text{Int}[(a_ + \text{ArcSin}(c_*(x_))*(b_))^{(n_)}*((f_)*(x_))^{(m_)}*((d_ + (e_ \\ \cdot)(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (\\ f*x)^m*(d + e*x^2)^p, x], x] \text{ ; FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[c^2*d + \\ e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.43 (sec) , antiderivative size = 1965, normalized size of antiderivative = 4.00

method	result	size
derivativedivides	Expression too large to display	1965
default	Expression too large to display	1965
parts	Expression too large to display	1966

input

$$\text{int}(x*(a+b*\text{arcsin}(c*x))/(e*x^2+d), x, \text{method}=_RETURNVERBOSE)$$

output

```

1/c^2*(1/2*a*c^2/e*ln(c^2*e*x^2+c^2*d)+b*c^2*(-1/4*I*(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)+e))/e^2+1/2*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)/e*arcsi
n(c*x)^2+1/2*I*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*
d*(c^2*d+e))^(1/2)*e)*d*c^2*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^
2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e))/(c^2*d+e)/e^3-1/2*I/e*sum((-_R1^2*e+4*c^
2*d+2*e)/(-_R1^2*e+2*c^2*d+e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1
/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4
*c^2*d-2*e)*_Z^2+e))+1/4*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)/e*polylog(2,e
*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e))-1/2*I
*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/
2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e))*d*c^2/e^3+(2*c^2*d-2*(c^2*d*(
c^2*d+e))^(1/2)+e)/e^3*ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)+e))*arcsin(c*x)*d*c^2-1/2*I*arcsin(c*x)^2/e-1/2*I*(2*c
^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arcsin(c*x)^2/e^2+1/8*I*(c^2*d*(c^2*d+e)
)^(1/2)/d/c^2/(c^2*d+e)*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d-
2*(c^2*d*(c^2*d+e))^(1/2)+e))+1/2*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^
2*ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
))*arcsin(c*x)-(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*
d*(c^2*d+e))^(1/2)*e)*c^2*d/e^3/(c^2*d+e)*ln(1-e*(I*c*x+(-c^2*x^2+1)^(1...

```

Fricas [F]

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)x}{ex^2 + d} dx$$

input

```
integrate(x*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*x*arcsin(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{d + ex^2} dx$$

input `integrate(x*(a+b*asin(c*x))/(e*x**2+d),x)`

output `Integral(x*(a + b*asin(c*x))/(d + e*x**2), x)`

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)x}{ex^2 + d} dx$$

input `integrate(x*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="maxima")`

output `b*integrate(x*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e*x^2 + d), x) + 1/2*a*log(e*x^2 + d)/e`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{x(a + b \arcsin(cx))}{ex^2 + d} dx$$

input `int((x*(a + b*asin(c*x)))/(d + e*x^2),x)`output `int((x*(a + b*asin(c*x)))/(d + e*x^2), x)`**Reduce [F]**

$$\int \frac{x(a + b \arcsin(cx))}{d + ex^2} dx = \frac{2 \left(\int \frac{a \arcsin(cx)x}{ex^2+d} dx \right) be + \log(ex^2 + d) a}{2e}$$

input `int(x*(a+b*asin(c*x))/(e*x^2+d),x)`output `(2*int((asin(c*x)*x)/(d + e*x**2),x)*b*e + log(d + e*x**2)*a)/(2*e)`

3.454 $\int \frac{a+b \arcsin(cx)}{d+ex^2} dx$

Optimal result	3858
Mathematica [A] (verified)	3859
Rubi [A] (verified)	3860
Maple [C] (verified)	3861
Fricas [F]	3862
Sympy [F]	3863
Maxima [F(-2)]	3863
Giac [F(-2)]	3864
Mupad [F(-1)]	3864
Reduce [F]	3864

Optimal result

Integrand size = 18, antiderivative size = 541

$$\begin{aligned}
 \int \frac{a + b \arcsin(cx)}{d + ex^2} dx = & \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}
 \end{aligned}$$

output

```

1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/(-d)^(1/2)/e^(1/2)

```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.91

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx$$

$$= \frac{2a\sqrt{-d} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) - b\sqrt{d} \arcsin(cx) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right) + b\sqrt{d} \arcsin(cx) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{1}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(d + e*x^2),x]
```

output

```

(2*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - b*Sqrt[d]*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])] + b*Sqrt[d]*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/((-I)*c*Sqrt[-d] + Sqrt[c^2*d + e])] + b*Sqrt[d]*ArcSin[c*x]*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])] - b*Sqrt[d]*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/((-I)*c*Sqrt[-d] + Sqrt[c^2*d + e])] + I*b*Sqrt[d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*Sqrt[-d^2]*Sqrt[e])

```


Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5172, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx$$

$$\downarrow 5172$$

$$\int \left(\frac{\sqrt{-d}(a + b \arcsin(cx))}{2d(\sqrt{-d} - \sqrt{ex})} + \frac{\sqrt{-d}(a + b \arcsin(cx))}{2d(\sqrt{-d} + \sqrt{ex})} \right) dx$$

$$\downarrow 2009$$

$$\frac{(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2\sqrt{-d}\sqrt{e}} +$$

$$\frac{(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}} \right)}{2\sqrt{-d}\sqrt{e}} +$$

$$\frac{ib \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}} \right)}{2\sqrt{-d}\sqrt{e}} +$$

$$\frac{ib \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{ib \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}} \right)}{2\sqrt{-d}\sqrt{e}}$$

input `Int[(a + b*ArcSin[c*x])/(d + e*x^2),x]`

output

$$\begin{aligned} & ((a + b \operatorname{ArcSin}[c*x]) \operatorname{Log}[1 - (\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - ((a + b \operatorname{ArcSin}[c*x]) \operatorname{Log}[1 + (\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) + ((a + b \operatorname{ArcSin}[c*x]) \operatorname{Log}[1 - (\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - ((a + b \operatorname{ArcSin}[c*x]) \operatorname{Log}[1 + (\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) + ((I/2)*b*\operatorname{PolyLog}[2, -((\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])])]) / (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - ((I/2)*b*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) + ((I/2)*b*\operatorname{PolyLog}[2, -((\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])])]) / (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - ((I/2)*b*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[e] \operatorname{E}^{(I \operatorname{ArcSin}[c*x])}) / (I*c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) \end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{IntSum}[u, x], x] /; \operatorname{SumQ}[u]$$

rule 5172

$$\operatorname{Int}[(a_. + \operatorname{ArcSin}[c_.*(x_.)]*(b_.))^n*((d_.) + (e_.)*(x_.)^2)^p, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b \operatorname{ArcSin}[c*x])^n, (d + e*x^2)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \operatorname{NeQ}[c^2*d + e, 0] \&\& \operatorname{IntegerQ}[p] \&\& (\operatorname{GtQ}[p, 0] \mid \mid \operatorname{IGtQ}[n, 0])$$

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.89 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.44

method	result
parts	$\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \frac{bc \left(\frac{i \arcsin(cx) \ln\left(\frac{R1 - icx - \sqrt{-c^2x^2 + 1}}{R1}\right) + \operatorname{dilog}\left(\frac{R1 - icx - \sqrt{-c^2x^2 + 1}}{R1}\right)}{-R1(-R1^2 e^{-2c^2d - e})} \right)}{2}$
derivativedivides	$\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right) + bc^2}{\sqrt{de}} \left(\frac{i \arcsin(cx) \ln\left(\frac{R1 - icx - \sqrt{-c^2x^2 + 1}}{R1}\right) + \operatorname{dilog}\left(\frac{R1 - icx - \sqrt{-c^2x^2 + 1}}{R1}\right)}{-R1(-R1^2 e^{+2c^2d + e})} \right)$
default	$\frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right) + bc^2}{\sqrt{de}} \left(\frac{i \arcsin(cx) \ln\left(\frac{R1 - icx - \sqrt{-c^2x^2 + 1}}{R1}\right) + \operatorname{dilog}\left(\frac{R1 - icx - \sqrt{-c^2x^2 + 1}}{R1}\right)}{-R1(-R1^2 e^{+2c^2d + e})} \right)$

```
input int((a+b*arcsin(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
output a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/2*b*c*sum(1/_R1/(_R1^2*e-2*c^2*d-e)
)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/2*b*c
*sum(_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*
c^2*d-2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx = \int \frac{b \arcsin(cx) + a}{ex^2 + d} dx$$

```
input integrate((a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output `integral((b*arcsin(c*x) + a)/(e*x^2 + d), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{d + ex^2} dx$$

input `integrate((a+b*asin(c*x))/(e*x**2+d), x)`

output `Integral((a + b*asin(c*x))/(d + e*x**2), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d), x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{ex^2 + d} dx$$

input `int((a + b*asin(c*x))/(d + e*x^2),x)`

output `int((a + b*asin(c*x))/(d + e*x^2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{d + ex^2} dx = \frac{\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a + \left(\int \frac{\operatorname{asin}(cx)}{ex^2+d} dx\right) bde}{de}$$

input `int((a+b*asin(c*x))/(e*x^2+d),x)`

output `(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(asin(c*x)/(d + e*x* *2),x)*b*d*e)/(d*e)`

3.455 $\int \frac{a+b \arcsin(cx)}{x(d+ex^2)} dx$

Optimal result	3865
Mathematica [A] (verified)	3866
Rubi [A] (verified)	3867
Maple [C] (warning: unable to verify)	3868
Fricas [F]	3869
Sympy [F]	3870
Maxima [F]	3870
Giac [F(-2)]	3870
Mupad [F(-1)]	3871
Reduce [F]	3871

Optimal result

Integrand size = 21, antiderivative size = 518

$$\begin{aligned}
 \int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = & -\frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2d} \\
 & -\frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2d} \\
 & -\frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2d} \\
 & -\frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2d} \\
 & +\frac{(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{d} \\
 & +\frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2d} \\
 & +\frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{2d} \\
 & +\frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2d} \\
 & +\frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{2d} - \frac{ib \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{2d}
 \end{aligned}$$

output

```
-1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d-1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d-1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d-1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d+(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d+1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d+1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d-1/2*I*b*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.83

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx =$$

$$\frac{b \arcsin(cx) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{c\sqrt{d} - \sqrt{c^2 d + e}}\right) + b \arcsin(cx) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{-c\sqrt{d} + \sqrt{c^2 d + e}}\right) + b \arcsin(cx) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{c\sqrt{d} + \sqrt{c^2 d + e}}\right) + b \arcsin(cx) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{-c\sqrt{d} - \sqrt{c^2 d + e}}\right)}{d}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x*(d + e*x^2)),x]
```

output

```
-1/2*(b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c^2*d + e]))] + b*ArcSin[c*x]*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] + b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - 2*b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - 2*a*Log[x] + a*Log[d + e*x^2] - I*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] - I*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c^2*d + e]))] - I*b*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - I*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] + I*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/d
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx \\
 & \quad \downarrow \text{5232} \\
 & \int \left(\frac{a + b \arcsin(cx)}{dx} - \frac{ex(a + b \arcsin(cx))}{d(d + ex^2)} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & - \frac{(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2d} - \frac{(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2d} \\
 & - \frac{(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2d} - \frac{(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2d} + \\
 & \frac{\log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))}{d} + \frac{ib \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2d} + \\
 & \frac{ib \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2d} + \frac{ib \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}} \right)}{2d} + \\
 & \frac{ib \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}} \right)}{2d} - \frac{ib \operatorname{PolyLog} \left(2, e^{2i \arcsin(cx)} \right)}{2d}
 \end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x*(d + e*x^2)),x]`

output

```
-1/2*((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d]
] - Sqrt[c^2*d + e]))/d - ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcS
in[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))/(2*d) - ((a + b*ArcSin[c*x])*
Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))/(2*
d) - ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d]
] + Sqrt[c^2*d + e]))/(2*d) + ((a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSi
n[c*x]))]/d + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[
-d] - Sqrt[c^2*d + e]))]/d + ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]
))]/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))/d + ((I/2)*b*PolyLog[2, -((Sqrt[e]*
E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))]/d + ((I/2)*b*PolyLo
g[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))/d - ((
I/2)*b*PolyLog[2, E^((2*I)*ArcSin[c*x]))]/d
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.66

method	result
parts	$\frac{a \ln(x)}{d} - \frac{a \ln(e x^2 + d)}{2d} + b \left(\frac{\arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1})}{d} - \frac{i \operatorname{dilog}(1 + icx + \sqrt{-c^2 x^2 + 1})}{d} + \frac{i}{-R1 = R$
derivativedivides	$\frac{a \ln(cx)}{d} - \frac{a \ln(c^2 e x^2 + c^2 d)}{2d} + \frac{ib \operatorname{dilog}(icx + \sqrt{-c^2 x^2 + 1})}{d} + \frac{b \arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1})}{d} - \frac{ib \operatorname{dilog}(1 -$
default	$\frac{a \ln(cx)}{d} - \frac{a \ln(c^2 e x^2 + c^2 d)}{2d} + \frac{ib \operatorname{dilog}(icx + \sqrt{-c^2 x^2 + 1})}{d} + \frac{b \arcsin(cx) \ln(1 + icx + \sqrt{-c^2 x^2 + 1})}{d} - \frac{ib \operatorname{dilog}(1 -$

input `int((a+b*arcsin(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)`

output `a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(1/d*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-I/d*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))+1/4*I*sum((_R1^2*e-4*c^2*d-e)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/d+I*dilog(I*c*x+(-c^2*x^2+1)^(1/2))/d+1/4*I*sum((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))*e/d)`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d),x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(e*x^3 + d*x), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = \int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx$$

input `integrate((a+b*asin(c*x))/x/(e*x**2+d),x)`

output `Integral((a + b*asin(c*x))/(x*(d + e*x**2)), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d),x, algorithm="maxima")`

output `-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e*x^3 + d*x), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = \int \frac{a + b \arcsin(cx)}{x(e x^2 + d)} dx$$

input `int((a + b*asin(c*x))/(x*(d + e*x^2)),x)`

output `int((a + b*asin(c*x))/(x*(d + e*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)} dx = \frac{2 \left(\int \frac{\arcsin(cx)}{e x^3 + dx} dx \right) bd - \log(e x^2 + d) a + 2 \log(x) a}{2d}$$

input `int((a+b*asin(c*x))/x/(e*x^2+d),x)`

output `(2*int(asin(c*x)/(d*x + e*x**3),x)*b*d - log(d + e*x**2)*a + 2*log(x)*a)/(2*d)`

$$3.456 \quad \int \frac{a+b \arcsin(cx)}{x^2(d+ex^2)} dx$$

Optimal result	3873
Mathematica [A] (verified)	3874
Rubi [A] (verified)	3875
Maple [C] (warning: unable to verify)	3876
Fricas [F]	3878
Sympy [F]	3878
Maxima [F(-2)]	3878
Giac [F(-2)]	3879
Mupad [F(-1)]	3879
Reduce [F]	3879

Optimal result

Integrand size = 21, antiderivative size = 579

$$\begin{aligned}
 \int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = & -\frac{a + b \arcsin(cx)}{dx} - \frac{b \operatorname{arctanh}(\sqrt{1 - c^2 x^2})}{d} \\
 & + \frac{\sqrt{e}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & - \frac{\sqrt{e}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & + \frac{\sqrt{e}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & - \frac{\sqrt{e}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & + \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & - \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & + \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}} \\
 & - \frac{ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2(-d)^{3/2}}
 \end{aligned}$$

output

```

-(a+b*arcsin(c*x))/d/x-b*c*arctanh((-c^2*x^2+1)^(1/2))/d+1/2*e^(1/2)*(a+b*
arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*
d+e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+
(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*e^(1/
2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/
2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arcsin(c*x))*ln(1+e^(1/2)
*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)+1
/2*I*b*e^(1/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/
2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)-1/2*I*b*e^(1/2)*polylog(2,e^(1/2)*(I*c*x+(
-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*I*b*e^
(1/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d
+e)^(1/2)))/(-d)^(3/2)-1/2*I*b*e^(1/2)*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+
1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)

```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.79

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx$$

$$= \frac{-4a\sqrt{d} - 4a\sqrt{ex} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) - 4b\sqrt{d}(\arcsin(cx) + cx \operatorname{arctanh}(\sqrt{1 - c^2x^2})) + b\sqrt{ex}(\arcsin(cx) (\arcsin(cx) + cx \operatorname{arctanh}(\sqrt{1 - c^2x^2})))}{x^2(d + ex^2)}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^2*(d + e*x^2)),x]
```

output

```

(-4*a*Sqrt[d] - 4*a*Sqrt[e]*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*Sqrt[d]*(A
rcSin[c*x] + c*x*ArcTanh[Sqrt[1 - c^2*x^2]]) + b*Sqrt[e]*x*(ArcSin[c*x]*(A
rcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[
c^2*d + e]]) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d
+ e]])) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[
c^2*d + e]]) + 2*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqr
t[c^2*d + e]))]) - b*Sqrt[e]*x*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 +
(Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e]]) + Log[1 - (S
qrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]])) + 2*PolyLog[2,
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e]]) + 2*PolyLog[2,
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]])))/(4*d^(3/2)*x)

```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx \\
 & \quad \downarrow \text{5232} \\
 & \int \left(\frac{a + b \arcsin(cx)}{dx^2} - \frac{e(a + b \arcsin(cx))}{d(d + ex^2)} \right) dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{\sqrt{e}(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2(-d)^{3/2}} - \\
 & \frac{\sqrt{e}(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2(-d)^{3/2}} + \\
 & \frac{\sqrt{e}(a + b \arcsin(cx)) \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a + b \arcsin(cx)) \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}} \right)}{2(-d)^{3/2}} \\
 & \frac{a + b \arcsin(cx)}{dx} + \frac{ib\sqrt{e} \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e} \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}} \right)}{2(-d)^{3/2}} + \\
 & \frac{ib\sqrt{e} \operatorname{PolyLog} \left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}} \right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e} \operatorname{PolyLog} \left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}} \right)}{2(-d)^{3/2}} - \\
 & \frac{b \operatorname{arctanh}(\sqrt{1 - c^2x^2})}{d}
 \end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^2*(d + e*x^2)),x]`

output

$$\begin{aligned}
& -((a + b \operatorname{ArcSin}[c x]) / (d x)) - (b c \operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2 x^2]]) / d + (\operatorname{Sqrt}[e] * (a + b \operatorname{ArcSin}[c x]) * \operatorname{Log}[1 - (\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2 d + e])]) / (2 * (-d)^{(3/2)}) - (\operatorname{Sqrt}[e] * (a + b \operatorname{ArcSin}[c x]) * \operatorname{Log}[1 + (\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2 d + e])]) / (2 * (-d)^{(3/2)}) + (\operatorname{Sqrt}[e] * (a + b \operatorname{ArcSin}[c x]) * \operatorname{Log}[1 - (\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2 d + e])]) / (2 * (-d)^{(3/2)}) - (\operatorname{Sqrt}[e] * (a + b \operatorname{ArcSin}[c x]) * \operatorname{Log}[1 + (\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2 d + e])]) / (2 * (-d)^{(3/2)}) + ((I/2) * b * \operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2 d + e])])]) / (-d)^{(3/2)} - ((I/2) * b * \operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, (\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2 d + e])]) / (-d)^{(3/2)} + ((I/2) * b * \operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, -((\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2 d + e])])]) / (-d)^{(3/2)} - ((I/2) * b * \operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, (\operatorname{Sqrt}[e] * E^{(I \operatorname{ArcSin}[c x])}) / (I c \operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2 d + e])]) / (-d)^{(3/2)}
\end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{IntSum}[u, x], x] \text{ /; } \operatorname{SumQ}[u]$$

rule 5232

$$\operatorname{Int}[(a + \operatorname{ArcSin}[c x]) * (b x)^n * (f x)^m * (d + e x^2)^p, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b \operatorname{ArcSin}[c x])^n * (f x)^m * (d + e x^2)^p, x], x] \text{ /; } \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[c^2 d + e, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{IntegerQ}[p] \ \&\& \operatorname{IntegerQ}[m]$$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 379.01 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.63

method	result
parts	$-\frac{ae \arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}} - \frac{a}{dx} + bc \left(-\frac{\arcsin(cx)}{dcx} + \frac{e \left(\sum_{-R1=\text{RootOf}(e_Z^4+(-4c^2d-2e)_Z^2+e)} \right)}{\left(-R1^2 e^{-4c^2d-2e} \right)} \right)$
derivativedivides	$c \left(-\frac{a}{dcx} - \frac{ae \arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}} - \frac{b \arcsin(cx)}{cxd} - \frac{b \ln(1+icx+\sqrt{-c^2x^2+1})}{d} \right) + \frac{be \left(\sum_{-R1=\text{RootOf}(e_Z^4+(-4c^2d-2e)_Z^2+e)} \right)}{\left(-R1^2 e^{-4c^2d-2e} \right)}$
default	$c \left(-\frac{a}{dcx} - \frac{ae \arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}} - \frac{b \arcsin(cx)}{cxd} - \frac{b \ln(1+icx+\sqrt{-c^2x^2+1})}{d} \right) + \frac{be \left(\sum_{-R1=\text{RootOf}(e_Z^4+(-4c^2d-2e)_Z^2+e)} \right)}{\left(-R1^2 e^{-4c^2d-2e} \right)}$

```
input int((a+b*arcsin(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
output -a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d/x+b*c*(-arcsin(c*x)/d/c/x+1/8/d^2*e*sum((_R1^2*e-4*c^2*d-e)/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-1/8/d^2*e*sum((4*_R1^2*c^2*d+_R1^2*e-e)/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-1/d*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+1/d*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(e*x^4 + d*x^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (d + ex^2)} dx$$

input `integrate((a+b*asin(c*x))/x**2/(e*x**2+d),x)`

output `Integral((a + b*asin(c*x))/(x**2*(d + e*x**2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = \int \frac{a + b \arcsin(cx)}{x^2 (ex^2 + d)} dx$$

input `int((a + b*asin(c*x))/(x^2*(d + e*x^2)),x)`

output `int((a + b*asin(c*x))/(x^2*(d + e*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)} dx = \frac{-\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) ax + \left(\int \frac{a \arcsin(cx)}{ex^4 + dx^2} dx\right) b d^2 x - ad}{d^2 x}$$

input `int((a+b*asin(c*x))/x^2/(e*x^2+d),x)`

output `(- sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*x + int(asin(c*x)/(d*x**2 + e*x**4),x)*b*d**2*x - a*d)/(d**2*x)`

$$3.457 \quad \int \frac{a+b \arcsin(cx)}{x^3(d+ex^2)} dx$$

Optimal result	3881
Mathematica [A] (verified)	3882
Rubi [A] (verified)	3883
Maple [C] (warning: unable to verify)	3885
Fricas [F]	3886
Sympy [F]	3886
Maxima [F]	3887
Giac [F(-2)]	3887
Mupad [F(-1)]	3887
Reduce [F]	3888

Optimal result

Integrand size = 21, antiderivative size = 573

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx = & -\frac{bc\sqrt{1 - c^2x^2}}{2dx} - \frac{a + b \arcsin(cx)}{2dx^2} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2d^2} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2d^2} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2d^2} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2d^2} \\
& - \frac{e(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{d^2} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2d^2} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d - \sqrt{c^2d + e}}}\right)}{2d^2} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2d^2} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d + \sqrt{c^2d + e}}}\right)}{2d^2} + \frac{ibe \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{2d^2}
\end{aligned}$$

output

```

-1/2*b*c*(-c^2*x^2+1)^(1/2)/d/x-1/2*(a+b*arcsin(c*x))/d/x^2+1/2*e*(a+b*arc
sin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e
)^(1/2)))/d^2+1/2*e*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/
2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*e*(a+b*arcsin(c*x))*ln(1-e^(
1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*
e*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2
)+(c^2*d+e)^(1/2)))/d^2-e*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2)
)^2)/d^2-1/2*I*b*e*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)
^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*I*b*e*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+
1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*I*b*e*polylog(2,-e^(1/
2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*I*
b*e*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)
^(1/2)))/d^2+1/2*I*b*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2

```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.89

$$\begin{aligned}
& \int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx \\
&= -\frac{a}{2dx^2} - \frac{ae \log(x)}{d^2} + \frac{ae \log(d + ex^2)}{2d^2} + b \left(-\frac{cx\sqrt{1 - c^2x^2} + \arcsin(cx)}{2dx^2} \right. \\
&\quad - \frac{ie \left(\arcsin(cx) \left(\arcsin(cx) + 2i \left(\log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{c\sqrt{d} - \sqrt{c^2d+e}} \right) + \log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{c\sqrt{d} + \sqrt{c^2d+e}} \right) \right) \right) + 2 \operatorname{PolyLog} \left(2, \frac{\sqrt{e}}{-cv} \right)}{4d^2} \\
&\quad - \frac{ie \left(\arcsin(cx) \left(\arcsin(cx) + 2i \left(\log \left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-c\sqrt{d} + \sqrt{c^2d+e}} \right) + \log \left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{c\sqrt{d} + \sqrt{c^2d+e}} \right) \right) \right) + 2 \operatorname{PolyLog} \left(2, \frac{\sqrt{e}}{cv} \right)}{4d^2} \\
&\quad \left. - \frac{e(\arcsin(cx) \log(1 - e^{2i \arcsin(cx)}) - \frac{1}{2}i(\arcsin(cx))^2 + \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}))}{d^2} \right)
\end{aligned}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^3*(d + e*x^2)),x]
```

output

```

-1/2*a/(d*x^2) - (a*e*Log[x])/d^2 + (a*e*Log[d + e*x^2])/(2*d^2) + b*(-1/2
*(c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x])/(d*x^2) - ((I/4)*e*(ArcSin[c*x]*(Ar
cSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] - Sqrt[c
^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] + Sqrt[c^2*d
+ e])))) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c
^2*d + e])) + 2*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt
[c^2*d + e])))]/d^2 - ((I/4)*e*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 +
(Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c^2*d + e])) + Log[1 - (
Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*PolyLog[2,
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])) + 2*PolyLog[2,
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])))]/d^2 - (e*(Ar
cSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] - (I/2)*(ArcSin[c*x]^2 + PolyLog[
2, E^((2*I)*ArcSin[c*x])])))/d^2)

```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{e^2 x (a + b \arcsin(cx))}{d^2 (d + ex^2)} - \frac{e (a + b \arcsin(cx))}{d^2 x} + \frac{a + b \arcsin(cx)}{dx^3} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} + \\
& \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} - \\
& \frac{e \log\left(1 - e^{2i \arcsin(cx)}\right) (a + b \arcsin(cx))}{2d^2} - \frac{a + b \arcsin(cx)}{2d^2} - \\
& \frac{i b e \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^2} - \frac{i b e \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^2} - \\
& \frac{i b e \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2d^2} - \frac{i b e \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2d^2} + \\
& \frac{i b e \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{2d^2} - \frac{bc\sqrt{1-c^2x^2}}{2dx}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^3*(d + e*x^2)),x]`

output

```

-1/2*(b*c*Sqrt[1 - c^2*x^2])/(d*x) - (a + b*ArcSin[c*x])/(2*d*x^2) + (e*(a
+ b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt
[c^2*d + e])])/(2*d^2) + (e*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcS
in[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*d^2) + (e*(a + b*ArcSin[c*
x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])
/(2*d^2) + (e*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c
*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*d^2) - (e*(a + b*ArcSin[c*x])*Log[1 - E^
((2*I)*ArcSin[c*x])])/d^2 - ((I/2)*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c
*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/d^2 - ((I/2)*b*e*PolyLog[2, (Sqr
t[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/d^2 - ((I/2)*b*
e*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]
))])/d^2 - ((I/2)*b*e*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d]
+ Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*e*PolyLog[2, E^((2*I)*ArcSin[c*x])])/
d^2

```

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.32 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.73

method	result
parts	$a \left(-\frac{1}{2d x^2} - \frac{e \ln(x)}{d^2} + \frac{e \ln(e x^2 + d)}{2d^2} \right) + b c^2 \left(-\frac{-i c^2 x^2 + c x \sqrt{-c^2 x^2 + 1} + \arcsin(cx)}{2c^2 x^2 d} - \frac{e \arcsin(cx) \ln(1 + i c x)}{d^2 c^2} \right)$
derivativedivides	$c^2 \left(\frac{a e \ln(c^2 e x^2 + c^2 d)}{2c^2 d^2} - \frac{a}{2d c^2 x^2} - \frac{a e \ln(cx)}{c^2 d^2} + b c^2 \left(-\frac{-i c^2 x^2 + c x \sqrt{-c^2 x^2 + 1} + \arcsin(cx)}{2c^4 x^2 d} - \frac{i e \operatorname{dilog}(i c x)}{d^2} \right) \right)$
default	$c^2 \left(\frac{a e \ln(c^2 e x^2 + c^2 d)}{2c^2 d^2} - \frac{a}{2d c^2 x^2} - \frac{a e \ln(cx)}{c^2 d^2} + b c^2 \left(-\frac{-i c^2 x^2 + c x \sqrt{-c^2 x^2 + 1} + \arcsin(cx)}{2c^4 x^2 d} - \frac{i e \operatorname{dilog}(i c x)}{d^2} \right) \right)$

```
input int((a+b*arcsin(c*x))/x^3/(e*x^2+d), x, method=_RETURNVERBOSE)
```

output

```
a*(-1/2/d/x^2-e/d^2*ln(x)+1/2*e/d^2*ln(e*x^2+d))+b*c^2*(-1/2*(-I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/c^2/x^2/d-e/d^2/c^2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+I*e/d^2/c^2*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-1/4*I*e/d^2*sum((_R1^2*e-4*c^2*d-e)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-I*e/d^2*dilog(I*c*x+(-c^2*x^2+1)^(1/2))/c^2-1/4*I*e^2/d^2*sum((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2)
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)x^3} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*arcsin(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d + ex^2)} dx$$

input

```
integrate((a+b*asin(c*x))/x**3/(e*x**2+d),x)
```

output

```
Integral((a + b*asin(c*x))/(x**3*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")`

output `1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e*x^5 + d*x^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx = \int \frac{a + b \arcsin(cx)}{x^3 (ex^2 + d)} dx$$

input `int((a + b*asin(c*x))/(x^3*(d + e*x^2)),x)`

output `int((a + b*asin(c*x))/(x^3*(d + e*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)} dx$$

$$= \frac{2 \left(\int \frac{a \sin(cx)}{e x^5 + d x^3} dx \right) b d^2 x^2 + \log(e x^2 + d) a e x^2 - 2 \log(x) a e x^2 - a d}{2 d^2 x^2}$$

input `int((a+b*asin(c*x))/x^3/(e*x^2+d),x)`

output `(2*int(asin(c*x)/(d*x**3 + e*x**5),x)*b*d**2*x**2 + log(d + e*x**2)*a*e*x**2 - 2*log(x)*a*e*x**2 - a*d)/(2*d**2*x**2)`

$$3.458 \quad \int \frac{a+b \arcsin(cx)}{x^4(d+ex^2)} dx$$

Optimal result	3890
Mathematica [A] (verified)	3891
Rubi [A] (verified)	3892
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Fricas [F]	3895
Sympy [F]	3895
Maxima [F(-2)]	3896
Giac [F(-2)]	3896
Mupad [F(-1)]	3896
Reduce [F]	3897

Optimal result

Integrand size = 21, antiderivative size = 649

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = & -\frac{bc\sqrt{1-c^2x^2}}{6dx^2} - \frac{a + b \arcsin(cx)}{3dx^3} + \frac{e(a + b \arcsin(cx))}{d^2x} \\
& - \frac{bc^3 \operatorname{arctanh}(\sqrt{1-c^2x^2})}{6d} + \frac{bce \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d^2} \\
& + \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& - \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& + \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& - \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& + \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& - \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& + \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} \\
& - \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^i \arcsin(cx)}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}}
\end{aligned}$$

output

```

-1/6*b*c*(-c^2*x^2+1)^(1/2)/d/x^2-1/3*(a+b*arcsin(c*x))/d/x^3+e*(a+b*arcsi
n(c*x))/d^2/x-1/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))/d+b*c*e*arctanh((-c^2*
x^2+1)^(1/2))/d^2+1/2*e^(3/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*
x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*e^(3/2)*(a+
b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(5/2)+1/2*e^(3/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*
x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*e^(
3/2)*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(
1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+1/2*I*b*e^(3/2)*polylog(2,-e^(1/2)*(I*c*
x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*I*b
*e^(3/2)*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2
*d+e)^(1/2)))/(-d)^(5/2)+1/2*I*b*e^(3/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x
^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*I*b*e^(3/2)*
polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1
2)))/(-d)^(5/2)

```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 531, normalized size of antiderivative = 0.82

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = -\frac{a}{3dx^3} + \frac{ae}{d^2x} + \frac{ae^{3/2} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{d^{5/2}}$$

$$+ b \left(-\frac{e \left(-\frac{\arcsin(cx)}{x} - \operatorname{arctanh}(\sqrt{1 - c^2x^2}) \right)}{d^2} - \frac{cx\sqrt{1 - c^2x^2} + 2 \arcsin(cx) + c^3x^3 \operatorname{arctanh}(\sqrt{1 - c^2x^2})}{6dx^3} \right)$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^4*(d + e*x^2)),x]
```


output

```

-1/3*a/(d*x^3) + (a*e)/(d^2*x) + (a*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d
^(5/2) + b*(-((e*(-(ArcSin[c*x]/x) - c*ArcTanh[Sqrt[1 - c^2*x^2]]))/d^2) -
(c*x*Sqrt[1 - c^2*x^2] + 2*ArcSin[c*x] + c^3*x^3*ArcTanh[Sqrt[1 - c^2*x^2
]])/(6*d*x^3) - (e^(3/2)*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[
e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^
(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]) + 2*PolyLog[2, (Sqrt[e]*
E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])) + 2*PolyLog[2, -(Sqr
t[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]))/(4*d^(5/2)) + (e
^(3/2)*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x
]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])) + Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x])
)/(c*Sqrt[d] + Sqrt[c^2*d + e]))]) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x
]))/(c*Sqrt[d] - Sqrt[c^2*d + e])) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x
]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))))/(4*d^(5/2))

```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{e^2(a + b \arcsin(cx))}{d^2(d + ex^2)} - \frac{e(a + b \arcsin(cx))}{d^2 x^2} + \frac{a + b \arcsin(cx)}{dx^4} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2(-d)^{5/2}} - \\
& \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2(-d)^{5/2}} + \\
& \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2(-d)^{5/2}} - \\
& \frac{e^{3/2}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2(-d)^{5/2}} + \frac{e(a + b \arcsin(cx))}{d^2x} - \frac{a + b \arcsin(cx)}{3dx^3} + \\
& \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}} - \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}} + \\
& \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}} - \frac{ibe^{3/2} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}} + \\
& \frac{bce \operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2} - \frac{bc^3 \operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6d} - \frac{bc\sqrt{1-c^2x^2}}{6dx^2}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x^4*(d + e*x^2)),x]`

output `-1/6*(b*c*Sqrt[1 - c^2*x^2])/(d*x^2) - (a + b*ArcSin[c*x])/(3*d*x^3) + (e*(a + b*ArcSin[c*x]))/(d^2*x) - (b*c^3*ArcTanh[Sqrt[1 - c^2*x^2]])/(6*d) + (b*c*e*ArcTanh[Sqrt[1 - c^2*x^2]])/d^2 + (e^(3/2)*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*(-d)^(5/2)) + (e^(3/2)*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*(-d)^(5/2)) + ((I/2)*b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/(d)^(5/2) - ((I/2)*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(d)^(5/2) + ((I/2)*b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))])/(d)^(5/2) - ((I/2)*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(d)^(5/2)`

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 356.98 (sec) , antiderivative size = 491, normalized size of antiderivative = 0.76

method	result
parts	$a \left(-\frac{1}{3d x^3} + \frac{e}{d^2 x} + \frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{d^2 \sqrt{de}} \right) - \frac{b \left(4 \ln(1+icx+\sqrt{-c^2x^2+1})c^7 d^2 x^3 - 4 \ln(icx+\sqrt{-c^2x^2+1}-1)c^7 d^2 \right)}{c^3 d^2 \sqrt{de}}$
derivativedivides	$c^3 \left(\frac{a e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{c^3 d^2 \sqrt{de}} - \frac{a}{3d c^3 x^3} + \frac{ae}{c^3 d^2 x} - \frac{b \left(-4 \ln(icx+\sqrt{-c^2x^2+1}-1)c^7 d^2 x^3 + 4 \ln(1+icx+\sqrt{-c^2x^2+1})c^7 d^2 \right)}{c^3 d^2 \sqrt{de}} \right)$
default	$c^3 \left(\frac{a e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{c^3 d^2 \sqrt{de}} - \frac{a}{3d c^3 x^3} + \frac{ae}{c^3 d^2 x} - \frac{b \left(-4 \ln(icx+\sqrt{-c^2x^2+1}-1)c^7 d^2 x^3 + 4 \ln(1+icx+\sqrt{-c^2x^2+1})c^7 d^2 \right)}{c^3 d^2 \sqrt{de}} \right)$

```
input int((a+b*arcsin(c*x))/x^4/(e*x^2+d), x, method=_RETURNVERBOSE)
```

output

```
a*(-1/3/d/x^3+e/d^2/x+e^2/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/24*b/c^4*(4*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^7*d^2*x^3-4*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)*c^7*d^2*x^3+4*(-c^2*x^2+1)^(1/2)*c^5*d^2*x-24*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))*c^5*d*e*x^3+24*ln(I*c*x+(-c^2*x^2+1)^(1/2)-1)*c^5*d*e*x^3-24*arcsin(c*x)*c^4*d*e*x^2+8*c^4*d^2*arcsin(c*x)+3*sum((_R1^2*e-4*c^2*d-e)/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))*c^3*x^3*e^2-3*sum((4*_R1^2*c^2*d+_R1^2*e-e)/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))*c^3*x^3*e^2)/x^3/d^3
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)x^4} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^4/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*arcsin(c*x) + a)/(e*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = \int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx$$

input

```
integrate((a+b*asin(c*x))/x**4/(e*x**2+d),x)
```

output

```
Integral((a + b*asin(c*x))/(x**4*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(e*x^2+d),x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/x^4/(e*x^2+d),x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^4 (ex^2 + d)} dx$$

input `int((a + b*asin(c*x))/(x^4*(d + e*x^2)),x)`

output `int((a + b*asin(c*x))/(x^4*(d + e*x^2)), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^4 (d + ex^2)} dx$$

$$= \frac{3\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a e x^3 + 3 \left(\int \frac{\arcsin(cx)}{e x^6 + d x^4} dx \right) b d^3 x^3 - a d^2 + 3 a d e x^2}{3 d^3 x^3}$$

input `int((a+b*asin(c*x))/x^4/(e*x^2+d),x)`

output `(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 3*int(asin(c*x)/(d*x**4 + e*x**6),x)*b*d**3*x**3 - a*d**2 + 3*a*d*e*x**2)/(3*d**3*x**3)`

3.459
$$\int \frac{x^3(a+b \arcsin(cx))}{(d+ex^2)^2} dx$$

Optimal result	3899
Mathematica [A] (verified)	3900
Rubi [A] (verified)	3901
Maple [C] (warning: unable to verify)	3903
Fricas [F]	3904
Sympy [F]	3904
Maxima [F]	3904
Giac [F(-2)]	3905
Mupad [F(-1)]	3905
Reduce [F]	3905

Optimal result

Integrand size = 21, antiderivative size = 574

$$\begin{aligned}
\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx = & \frac{d(a + b \arcsin(cx))}{2e^2(d + ex^2)} - \frac{i(a + b \arcsin(cx))^2}{2be^2} \\
& - \frac{bc\sqrt{d} \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} \\
& - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} \\
& - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} \\
& - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} \\
& - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2}
\end{aligned}$$

output

```

1/2*d*(a+b*arcsin(c*x))/e^2/(e*x^2+d)-1/2*I*(a+b*arcsin(c*x))^2/b/e^2-1/2*
b*c*d^(1/2)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/e^2/(c^2*
d+e)^(1/2)+1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I
*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*ar
csin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+
e)^(1/2)))/e^2+1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,-e^(1/2)*(I*c*x
+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog
(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^
2-1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c
^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2

```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \frac{\frac{2ad}{d+ex^2} + 2a \log(d + ex^2) + b \left(\sqrt{d} \left(\frac{\arcsin(cx)}{\sqrt{d} + i\sqrt{ex}} - \frac{\operatorname{arctan}\left(\frac{i\sqrt{e} + c^2\sqrt{dx}}{\sqrt{c^2d + e}\sqrt{1 - c^2x^2}}\right)}{\sqrt{c^2d + e}} \right) - i\sqrt{d} \left(-\frac{\arcsin(cx)}{i\sqrt{d} + \sqrt{ex}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c^2d + e}}{\sqrt{c^2d + e}}\right)}{\sqrt{c^2d + e}} \right) \right)}{e^2}$$

input

```
Integrate[(x^3*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]
```

output

```

((2*a*d)/(d + e*x^2) + 2*a*Log[d + e*x^2] + b*(Sqrt[d]*(ArcSin[c*x]/(Sqrt[
d] + I*Sqrt[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]
*Sqrt[1 - c^2*x^2]))/Sqrt[c^2*d + e] - I*Sqrt[d]*(-(ArcSin[c*x]/(I*Sqrt[
d] + Sqrt[e]*x)) - (c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]
*Sqrt[1 - c^2*x^2]))/Sqrt[c^2*d + e] - I*(ArcSin[c*x]*(ArcSin[c*x] + (2*
I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] - Sqrt[c^2*d + e])) + L
og[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*Po
lyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2
*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]
- I*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]
))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]
))/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]
))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]
))/(c*Sqrt[d] + Sqrt[c^2*d + e])))))/(4*e^2)

```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{x(a + b \arcsin(cx))}{e(d + ex^2)} - \frac{dx(a + b \arcsin(cx))}{e(d + ex^2)^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{-\sqrt{c^2 d + e + i c \sqrt{-d}}}\right)}{2e^2} + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{-\sqrt{c^2 d + e + i c \sqrt{-d}}}\right)}{2e^2} + \\
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + i c \sqrt{-d}}}\right)}{2e^2} + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + i c \sqrt{-d}}}\right)}{2e^2} + \\
& \frac{d(a + b \arcsin(cx))}{2e^2(d + ex^2)} - \frac{i(a + b \arcsin(cx))^2}{2be^2} - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{i c \sqrt{-d} - \sqrt{dc^2 + e}}\right)}{2e^2} - \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{i c \sqrt{-d} - \sqrt{dc^2 + e}}\right)}{2e^2} - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{i \sqrt{-d} + \sqrt{dc^2 + e}}\right)}{2e^2} - \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{i \sqrt{-d} + \sqrt{dc^2 + e}}\right)}{2e^2} - \frac{bc\sqrt{d} \arctan\left(\frac{x\sqrt{c^2 d + e}}{\sqrt{d}\sqrt{1 - c^2 x^2}}\right)}{2e^2\sqrt{c^2 d + e}}
\end{aligned}$$

input

```
Int[(x^3*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]
```

output

```
(d*(a + b*ArcSin[c*x]))/(2*e^2*(d + e*x^2)) - ((I/2)*(a + b*ArcSin[c*x])^2)/(b*e^2) - (b*c*Sqrt[d]*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*e^2*Sqrt[c^2*d + e]) + ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*e^2) - ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/e^2 - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))])/e^2 - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/e^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^n_.*((f_.)*(x_.))^m_.*((d_) + (e_.)*(x_)^2)^p_., x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.78 (sec) , antiderivative size = 2101, normalized size of antiderivative = 3.66

method	result	size
derivativeldivides	Expression too large to display	2101
default	Expression too large to display	2101
parts	Expression too large to display	2113

input `int(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned}
 & 1/c^4*(1/2*a*c^6*d/e^2/(c^2*e*x^2+c^2*d)+1/2*a*c^4/e^2*\ln(c^2*e*x^2+c^2*d) \\
 & +b*c^4*(-1/8*I*(c^2*d*(c^2*d+e))^(1/2)/c^2/d/(c^2*d+e)/e*polylog(2,e*(I*c*x \\
 & x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e))+I*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*c^2 \\
 & *d*arcsin(c*x)^2/(c^2*d+e)/e^4-1/4*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2 \\
 & *d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*\ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2)) \\
 & ^2/(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e))*arcsin(c*x)/c^2/d/(c^2*d+e)/e^2+ \\
 & 1/2*I*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e) \\
 &))^(1/2)*e)*c^2*d*polylog(2,e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d-2*(c^2 \\
 & *d*(c^2*d+e))^(1/2)+e))/(c^2*d+e)/e^4+1/8*I*(2*c^4*d^2+2*(c^2*d*(c^2*d+e)) \\
 & ^2/(2*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*polylog(2,e*(I*c*x+(-c^2 \\
 & *x^2+1)^(1/2))^2/(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e))/c^2/d/(c^2*d+e)/e^ \\
 & 2-1/4*I*(c^2*d*(c^2*d+e))^(1/2)/c^2/d/(c^2*d+e)/e*arcsin(c*x)^2-(2*c^4*d^2 \\
 & +2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*c^2* \\
 & d*\ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e \\
 &))*arcsin(c*x)/(c^2*d+e)/e^4+1/4*(c^2*d*(c^2*d+e))^(1/2)/c^2/d/(c^2*d+e)/e \\
 & *arcsin(c*x)*\ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d+2*(c^2*d*(c^2*d+ \\
 & e))^(1/2)+e))+1/4*I*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(\\
 & c^2*d*(c^2*d+e))^(1/2)*e)*arcsin(c*x)^2/c^2/d/(c^2*d+e)/e^2+(2*c^2*d+2*(c^2 \\
 & *d*(c^2*d+e))^(1/2)+e)*\ln(1-e*(I*c*x+(-c^2*x^2+1)^(1/2))^2/(2*c^2*d-2*...
 \end{aligned}$$

Fricas [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^3}{(ex^2 + d)^2} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*x^3*arcsin(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(d + ex^2)^2} dx$$

input `integrate(x**3*(a+b*asin(c*x))/(e*x**2+d)**2,x)`

output `Integral(x**3*(a + b*asin(c*x))/(d + e*x**2)**2, x)`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^3}{(ex^2 + d)^2} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

output `1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(ex^2 + d)^2} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d + e*x^2)^2,x)`

output `int((x^3*(a + b*asin(c*x)))/(d + e*x^2)^2, x)`

Reduce [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \frac{2 \left(\int \frac{\operatorname{asin}(cx)x^3}{e^2x^4 + 2dex^2 + d^2} dx \right) bde^2 + 2 \left(\int \frac{\operatorname{asin}(cx)x^3}{e^2x^4 + 2dex^2 + d^2} dx \right) be^3x^2 + \log(ex^2 + d) ad + \log(ex^2 + d) aex^2 - aex^2}{2e^2(ex^2 + d)}$$

input `int(x^3*(a+b*asin(c*x))/(e*x^2+d)^2,x)`

output

```
(2*int((asin(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**2 + 2*in  
t((asin(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**3*x**2 + log(d  
+ e*x**2)*a*d + log(d + e*x**2)*a*e*x**2 - a*e*x**2)/(2*e**2*(d + e*x**2))
```

3.460 $\int \frac{x(a+b \arcsin(cx))}{(d+ex^2)^2} dx$

Optimal result	3907
Mathematica [A] (verified)	3907
Rubi [A] (verified)	3908
Maple [B] (verified)	3909
Fricas [B] (verification not implemented)	3911
Sympy [F]	3911
Maxima [F(-2)]	3912
Giac [F(-2)]	3912
Mupad [F(-1)]	3912
Reduce [B] (verification not implemented)	3913

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \frac{-a - b \arcsin(cx)}{2e(d + ex^2)} + \frac{bc \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2\sqrt{de}\sqrt{c^2d + e}}$$

output

```
1/2*(-a-b*arcsin(c*x))/e/(e*x^2+d)+1/2*b*c*arctan((c^2*d+e)^(1/2)*x/d^(1/2)
)/((-c^2*x^2+1)^(1/2))/d^(1/2)/e/(c^2*d+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = -\frac{a}{d+ex^2} + \frac{b \arcsin(cx)}{d+ex^2} - \frac{bc \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2e}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]
```

output

```
-1/2*(a/(d + e*x^2) + (b*ArcSin[c*x]))/(d + e*x^2) - (b*c*ArcTan[(Sqrt[c^2*
d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt[d]*Sqrt[c^2*d + e])/e
```


Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5228, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$\downarrow 5228$$

$$\frac{bc \int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)} dx}{2e} - \frac{a + b \arcsin(cx)}{2e(d + ex^2)}$$

$$\downarrow 291$$

$$\frac{bc \int \frac{1}{d - \frac{(-dc^2 - e)x^2}{1 - c^2x^2}} d \frac{x}{\sqrt{1 - c^2x^2}}}{2e} - \frac{a + b \arcsin(cx)}{2e(d + ex^2)}$$

$$\downarrow 218$$

$$\frac{bc \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2\sqrt{de}\sqrt{c^2d+e}} - \frac{a + b \arcsin(cx)}{2e(d + ex^2)}$$

input `Int[(x*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]`

output `-1/2*(a + b*ArcSin[c*x])/(e*(d + e*x^2)) + (b*c*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2*d + e])`

Defintions of rubi rules used

rule 218 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

rule 291 `Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

rule 5228 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x] - Simp[b*(c/(2*e*(p + 1))) Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(72) = 144$.

Time = 7.37 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.70

method	result
parts	$-\frac{a}{2e(e x^2+d)} - \frac{b c^2 \arcsin(cx)}{2e(c^2 e x^2+c^2 d)} - \frac{b c^2 \ln \left(\frac{2c^2 d+2e}{e} - \frac{2\sqrt{-c^2 de} \left(cx - \frac{\sqrt{-c^2 de}}{e} \right)}{e} + 2\sqrt{\frac{c^2 d+e}{e}} \sqrt{-\left(cx - \frac{\sqrt{-c^2 de}}{e} \right)^2 - \frac{2\sqrt{-c^2 de}}{e}} \right)}{cx - \frac{\sqrt{-c^2 de}}{e}}$
derivativedivides	$-\frac{a c^4}{2e(c^2 e x^2+c^2 d)} + b c^4 \left(-\frac{\arcsin(cx)}{2e(c^2 e x^2+c^2 d)} + \frac{\ln \left(\frac{2c^2 d+2e}{e} - \frac{2\sqrt{-c^2 de} \left(cx - \frac{\sqrt{-c^2 de}}{e} \right)}{e} + 2\sqrt{\frac{c^2 d+e}{e}} \sqrt{-\left(cx - \frac{\sqrt{-c^2 de}}{e} \right)^2 - \frac{2\sqrt{-c^2 de}}{e}} \right)}{2\sqrt{-c^2 de} \sqrt{\frac{c^2 d+e}{e}}} \right)$
default	$-\frac{a c^4}{2e(c^2 e x^2+c^2 d)} + b c^4 \left(-\frac{\arcsin(cx)}{2e(c^2 e x^2+c^2 d)} + \frac{\ln \left(\frac{2c^2 d+2e}{e} - \frac{2\sqrt{-c^2 de} \left(cx - \frac{\sqrt{-c^2 de}}{e} \right)}{e} + 2\sqrt{\frac{c^2 d+e}{e}} \sqrt{-\left(cx - \frac{\sqrt{-c^2 de}}{e} \right)^2 - \frac{2\sqrt{-c^2 de}}{e}} \right)}{2\sqrt{-c^2 de} \sqrt{\frac{c^2 d+e}{e}}} \right)$

input `int(x*(a+b*arcsin(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)`

output `-1/2*a/e/(e*x^2+d)-1/2*b*c^2/e/(c^2*e*x^2+c^2*d)*arcsin(c*x)-1/4*b*c^2/e/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e)^(1/2)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x-(-c^2*d*e)^(1/2)/e)+1/4*b*c^2/e/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x+(-c^2*d*e)^(1/2)/e)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(69) = 138$.

Time = 0.14 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.59

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \left[-\frac{4ac^2d^2 + 4ade + (bcex^2 + bcd)\sqrt{-c^2d^2 - de} \log\left(\frac{(8c^4d^2 + 8c^2de + e^2)x^4 - 2(4c^2d^2 + 3de)x^2 - 4\sqrt{-c^2d^2 - de}\sqrt{-c^2x^2 + d}}{e^2x^4 + 2dex^2 + d^2}\right)}{8(c^2d^3e + d^2e^2 + (c^2d^2e^2 + de^3)x^2)} \right. \\ \left. - \frac{2ac^2d^2 + 2ade + (bcex^2 + bcd)\sqrt{c^2d^2 + de} \arctan\left(\frac{\sqrt{c^2d^2 + de}\sqrt{-c^2x^2 + 1}((2c^2d + e)x^2 - d)}{2((c^4d^2 + c^2de)x^3 - (c^2d^2 + de)x)}\right)}{4(c^2d^3e + d^2e^2 + (c^2d^2e^2 + de^3)x^2)} + 2(bc^2d^2 + bde) \right]$$

input `integrate(x*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

output `[-1/8*(4*a*c^2*d^2 + 4*a*d*e + (b*c*e*x^2 + b*c*d)*sqrt(-c^2*d^2 - d*e)*log(((8*c^4*d^2 + 8*c^2*d*e + e^2)*x^4 - 2*(4*c^2*d^2 + 3*d*e)*x^2 - 4*sqrt(-c^2*d^2 - d*e)*sqrt(-c^2*x^2 + 1)*((2*c^2*d + e)*x^3 - d*x) + d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(b*c^2*d^2 + b*d*e)*arcsin(c*x)/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2), -1/4*(2*a*c^2*d^2 + 2*a*d*e + (b*c*e*x^2 + b*c*d)*sqrt(c^2*d^2 + d*e)*arctan(1/2*sqrt(c^2*d^2 + d*e)*sqrt(-c^2*x^2 + 1)*((2*c^2*d + e)*x^2 - d)/((c^4*d^2 + c^2*d*e)*x^3 - (c^2*d^2 + d*e)*x)) + 2*(b*c^2*d^2 + b*d*e)*arcsin(c*x)/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]`

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(d + ex^2)^2} dx$$

input `integrate(x*(a+b*asin(c*x))/(e*x**2+d)**2,x)`

output `Integral(x*(a + b*asin(c*x))/(d + e*x**2)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vector & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x(a + b \arcsin(cx))}{(ex^2 + d)^2} dx$$

input `int((x*(a + b*asin(c*x)))/(d + e*x^2)^2,x)`

output `int((x*(a + b*asin(c*x)))/(d + e*x^2)^2, x)`

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1401, normalized size of antiderivative = 16.29

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Too large to display}$$

input `int(x*(a+b*asin(c*x))/(e*x^2+d)^2,x)`

output `(- 2*asin(c*x)*b*c**3*d**3 - 2*asin(c*x)*b*c*d**2*e + 2*sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*d + 2*sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*e*x**2 + 2*sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*c**2*d**2 + 2*sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*c**2*d*e*x**2 + 2*sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*d*e + 2*sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*e**2*x**2 + sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e)*log(- sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e) + sqrt(d)*tan(asin(c*x)/2)*c)*b*d + sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e)*log(- sqrt(- 2*sqrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e) + sqrt(d)*tan(asin(c*x)/2)*c)*b*e*x**2 - sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*s...`

3.461
$$\int \frac{a+b \arcsin(cx)}{x(d+ex^2)^2} dx$$

Optimal result	3915
Mathematica [A] (verified)	3916
Rubi [A] (verified)	3917
Maple [C] (warning: unable to verify)	3919
Fricas [F]	3920
Sympy [F]	3920
Maxima [F]	3921
Giac [F(-1)]	3921
Mupad [F(-1)]	3921
Reduce [F]	3922

Optimal result

Integrand size = 21, antiderivative size = 597

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx &= \frac{a + b \arcsin(cx)}{2d(d + ex^2)} - \frac{bc \arctan\left(\frac{\sqrt{c^2 d + ex}}{\sqrt{d}\sqrt{1 - c^2 x^2}}\right)}{2d^{3/2}\sqrt{c^2 d + e}} \\
&- \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2d^2} \\
&- \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2d^2} \\
&- \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2d^2} \\
&- \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2d^2} \\
&+ \frac{(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{d^2} \\
&+ \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2d^2} \\
&+ \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2 d + e}}\right)}{2d^2} \\
&+ \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2d^2} \\
&+ \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2 d + e}}\right)}{2d^2} - \frac{ib \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{2d^2}
\end{aligned}$$

output

```

1/2*(a+b*arcsin(c*x))/d/(e*x^2+d)-1/2*b*c*arctan((c^2*d+e)^(1/2)*x/d^(1/2)
/(-c^2*x^2+1)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)-1/2*(a+b*arcsin(c*x))*ln(1-e^
(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2
*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)
-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arcsin(c*x))*ln(
1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2
+(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2+1/2*I*b*polylog(
2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^
2+1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^
2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^
2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*I*b*polylog(2,(I
*c*x+(-c^2*x^2+1)^(1/2))^2)/d^2

```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.07

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx = \frac{a}{2d^2 + 2dex^2} + \frac{a \log(x)}{d^2} - \frac{a \log(d + ex^2)}{2d^2}$$

$$+ \frac{b \left(\frac{\sqrt{d} \arcsin(cx)}{\sqrt{d} - i\sqrt{ex}} + \frac{\sqrt{d} \arcsin(cx)}{\sqrt{d} + i\sqrt{ex}} - \frac{c\sqrt{d} \arctan\left(\frac{i\sqrt{e} + c^2\sqrt{d}x}{\sqrt{c^2d + e}\sqrt{1 - c^2x^2}}\right)}{\sqrt{c^2d + e}} + \frac{ic\sqrt{d} \operatorname{arctanh}\left(\frac{\sqrt{e} + ic^2\sqrt{d}x}{\sqrt{c^2d + e}\sqrt{1 - c^2x^2}}\right)}{\sqrt{c^2d + e}} - 2 \arcsin(cx) \log \right)}{d^2}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x*(d + e*x^2)^2),x]
```

output

```

a/(2*d^2 + 2*d*e*x^2) + (a*Log[x])/d^2 - (a*Log[d + e*x^2])/(2*d^2) + (b*(
(Sqrt[d]*ArcSin[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (Sqrt[d]*ArcSin[c*x])/(Sqr
t[d] + I*Sqrt[e]*x) - (c*Sqrt[d]*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[
c^2*d + e]*Sqrt[1 - c^2*x^2])))/Sqrt[c^2*d + e] + (I*c*Sqrt[d]*ArcTanh[(Sq
rt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])))/Sqrt[c^2*d
+ e] - 2*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt
[c^2*d + e])] - 2*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqr
t[d] + Sqrt[c^2*d + e])] - 2*ArcSin[c*x]*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x
]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - 2*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*A
rcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] + 4*ArcSin[c*x]*Log[1 - E^((2*
I)*ArcSin[c*x])] + (2*I)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d]
- Sqrt[c^2*d + e])] + (2*I)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*S
qrt[d] + Sqrt[c^2*d + e])] + (2*I)*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]
))/(c*Sqrt[d] + Sqrt[c^2*d + e])] + (2*I)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin
[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - (2*I)*PolyLog[2, E^((2*I)*ArcSin[
c*x])])])/(4*d^2)

```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(-\frac{ex(a + b \arcsin(cx))}{d^2(d + ex^2)} + \frac{a + b \arcsin(cx)}{d^2x} - \frac{ex(a + b \arcsin(cx))}{d(d + ex^2)^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} - \\
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^2} + \\
& \frac{\log\left(1 - e^{2i \arcsin(cx)}\right) (a + b \arcsin(cx))}{d^2} + \frac{a + b \arcsin(cx)}{2d(d + ex^2)} + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^2} + \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^2} + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-d}+\sqrt{dc^2+e}}\right)}{2d^2} + \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-d}+\sqrt{dc^2+e}}\right)}{2d^2} - \frac{ib \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{2d^2} - \frac{bc \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(x*(d + e*x^2)^2), x]
```

output

```
(a + b*ArcSin[c*x])/(2*d*(d + e*x^2)) - (b*c*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]) - ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*d^2) + ((a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])])/d^2 + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/d^2 - ((I/2)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/d^2
```

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.19 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.78

method	result
parts	$\frac{a \ln(x)}{d^2} - \frac{a \ln(ex^2+d)}{2d^2} + \frac{a}{2d(ex^2+d)} + b \left(\frac{c^2 \arcsin(cx)}{2d(c^2ex^2+c^2d)} + \frac{i \left(\sum_{-R1=RootOf(e-Z^4+(-4c^2d-2e)-Z^2+e)} \right)}{\dots} \right)$
derivativedivides	$\frac{ac^2}{2d(c^2ex^2+c^2d)} - \frac{a \ln(c^2ex^2+c^2d)}{2d^2} + \frac{a \ln(cx)}{d^2} + \frac{bc^2 \arcsin(cx)}{2d(c^2ex^2+c^2d)} - \frac{ib \operatorname{dilog}(1+icx+\sqrt{-c^2x^2+1})}{d^2} + \frac{ib \left(-R \right)}{\dots}$
default	$\frac{ac^2}{2d(c^2ex^2+c^2d)} - \frac{a \ln(c^2ex^2+c^2d)}{2d^2} + \frac{a \ln(cx)}{d^2} + \frac{bc^2 \arcsin(cx)}{2d(c^2ex^2+c^2d)} - \frac{ib \operatorname{dilog}(1+icx+\sqrt{-c^2x^2+1})}{d^2} + \frac{ib \left(-R \right)}{\dots}$

```
input int((a+b*arcsin(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
a/d^2*ln(x)-1/2*a/d^2*ln(e*x^2+d)+1/2*a/d/(e*x^2+d)+b*(1/2*c^2*arcsin(c*x)
/d/(c^2*e*x^2+c^2*d)+1/4*I/d^2*sum((_R1^2*e-4*c^2*d-e)/(_R1^2*e-2*c^2*d-e)
*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-
c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/4*I/d^
2*sum((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2
+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z
^4+(-4*c^2*d-2*e)*_Z^2+e))*e+1/2*I*(c^2*d*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*a
rctanh(1/4*(2*e*(I*c*x+(-c^2*x^2+1)^(1/2))^2-4*c^2*d-2*e)/(c^4*d^2+c^2*d*e
)^(1/2))+I/d^2*dilog(I*c*x+(-c^2*x^2+1)^(1/2))-I/d^2*dilog(1+I*c*x+(-c^2*x
^2+1)^(1/2))+1/d^2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^2 x} dx$$

input

```
integrate((a+b*arcsin(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*arcsin(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x(d + ex^2)^2} dx$$

input

```
integrate((a+b*asin(c*x))/x/(e*x**2+d)**2,x)
```

output

```
Integral((a + b*asin(c*x))/(x*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^2 x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")`

output `1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)`

Giac [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx = \text{Timed out}$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")`

output `Timed out`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx = \int \frac{a + b \arcsin(cx)}{x(ex^2 + d)^2} dx$$

input `int((a + b*asin(c*x))/(x*(d + e*x^2)^2),x)`

output `int((a + b*asin(c*x))/(x*(d + e*x^2)^2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^2} dx$$

$$= \frac{2 \left(\int \frac{a \sin(cx)}{e^2 x^5 + 2d e x^3 + d^2 x} dx \right) b d^3 + 2 \left(\int \frac{a \sin(cx)}{e^2 x^5 + 2d e x^3 + d^2 x} dx \right) b d^2 e x^2 - \log(e x^2 + d) a d - \log(e x^2 + d) a e x^2 + 2}{2d^2 (e x^2 + d)}$$

input

```
int((a+b*asin(c*x))/x/(e*x^2+d)^2,x)
```

output

```
(2*int(asin(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**3 + 2*int(asin(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**2*e*x**2 - log(d + e*x**2)*a*d - log(d + e*x**2)*a*e*x**2 + 2*log(x)*a*d + 2*log(x)*a*e*x**2 - a*e*x**2)/(2*d**2*(d + e*x**2))
```

$$3.462 \quad \int \frac{a+b \arcsin(cx)}{x^3(d+ex^2)^2} dx$$

Optimal result	3924
Mathematica [A] (verified)	3925
Rubi [A] (verified)	3926
Maple [C] (warning: unable to verify)	3928
Fricas [F]	3929
Sympy [F]	3929
Maxima [F]	3930
Giac [F(-1)]	3930
Mupad [F(-1)]	3930
Reduce [F]	3931

Optimal result

Integrand size = 21, antiderivative size = 632

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx = & -\frac{bc\sqrt{1-c^2x^2}}{2d^2x} - \frac{a + b \arcsin(cx)}{2d^2x^2} \\
& - \frac{e(a + b \arcsin(cx))}{2d^2(d + ex^2)} + \frac{bce \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} \\
& + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} \\
& - \frac{2e(a + b \arcsin(cx)) \log(1 - e^{2i \arcsin(cx)})}{d^3} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} \\
& - \frac{ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} + \frac{ibe \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{d^3}
\end{aligned}$$

output

```

-1/2*b*c*(-c^2*x^2+1)^(1/2)/d^2/x-1/2*(a+b*arcsin(c*x))/d^2/x^2-1/2*e*(a+b
*arcsin(c*x))/d^2/(e*x^2+d)+1/2*b*c*e*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c
^2*x^2+1)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+e*(a+b*arcsin(c*x))*ln(1-e^(1/2)*
(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3+e*(a+b*ar
csin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+
e)^(1/2)))/d^3+e*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))
/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3+e*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I
*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3-2*e*(a+b*ar
csin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2/d^3-I*b*e*polylog(2,-e^(1/2)
*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3-I*b*e*po
lylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/d^3-I*b*e*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)
+(c^2*d+e)^(1/2)))/d^3-I*b*e*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/
(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3+I*b*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(
1/2))^2)/d^3

```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.09

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx =$$

$$\frac{\frac{2ad}{x^2} + \frac{2ade}{d+ex^2} + 8ae \log(x) - 4ae \log(d + ex^2) + b \left(\frac{2d(cx\sqrt{1-c^2x^2} + \arcsin(cx))}{x^2} + \sqrt{de} \left(\frac{\arcsin(cx)}{\sqrt{d+i\sqrt{e}x}} - \frac{c \arctan\left(\frac{\sqrt{c}}{\sqrt{d}}\right)}{\sqrt{d}} \right) \right)}{d^3}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^3*(d + e*x^2)^2), x]
```

output

```

-1/4*((2*a*d)/x^2 + (2*a*d*e)/(d + e*x^2) + 8*a*e*Log[x] - 4*a*e*Log[d + e
*x^2] + b*((2*d*(c*x*Sqrt[1 - c^2*x^2] + ArcSin[c*x]))/x^2 + Sqrt[d]*e*(Ar
cSin[c*x]/(Sqrt[d] + I*Sqrt[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/
(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e]) - I*Sqrt[d]*e*(-(Ar
cSin[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x
)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e]) + (2*I)*e*(ArcSin
[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d]
- Sqrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] + Sq
rt[c^2*d + e]))) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d])
+ Sqrt[c^2*d + e])] + 2*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[
d] + Sqrt[c^2*d + e])] + (2*I)*e*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[
1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1
- (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])])) + 2*PolyLog
[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 2*PolyLog
[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - (4*I)*e*
(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*Log[1 - E^((2*I)*ArcSin[c*x])]) + PolyLo
g[2, E^((2*I)*ArcSin[c*x])])))/d^3

```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{2e^2 x (a + b \arcsin(cx))}{d^3 (d + ex^2)} - \frac{2e (a + b \arcsin(cx))}{d^3 x} + \frac{e^2 x (a + b \arcsin(cx))}{d^2 (d + ex^2)^2} + \frac{a + b \arcsin(cx)}{d^2 x^3} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{d^3} + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{d^3} + \\
& \frac{e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{d^3} + \frac{e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{d^3} - \\
& \frac{2e \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))}{d^3} - \frac{e(a + b \arcsin(cx))}{2d^2(d + ex^2)} - \frac{a + b \arcsin(cx)}{2d^2x^2} - \\
& \frac{i b e \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{d^3} - \frac{i b e \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{d^3} - \\
& \frac{i b e \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{d^3} - \frac{i b e \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{d^3} + \\
& \frac{i b e \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{d^3} + \frac{b c e \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}} - \frac{bc\sqrt{1-c^2x^2}}{2d^2x}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^3*(d + e*x^2)^2), x]
```

output

```

-1/2*(b*c*Sqrt[1 - c^2*x^2])/(d^2*x) - (a + b*ArcSin[c*x])/(2*d^2*x^2) - (
e*(a + b*ArcSin[c*x])/(2*d^2*(d + e*x^2)) + (b*c*e*ArcTan[(Sqrt[c^2*d + e
]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]) + (e*(a + b
*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2
*d + e])])/d^3 + (e*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x])
)/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/d^3 + (e*(a + b*ArcSin[c*x])*Log[1 -
(Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/d^3 + (e*(a
+ b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt
[c^2*d + e])])/d^3 - (2*e*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x]
)])/d^3 - (I*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] -
Sqrt[c^2*d + e]))])/d^3 - (I*b*e*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I
*c*Sqrt[-d] - Sqrt[c^2*d + e])])/d^3 - (I*b*e*PolyLog[2, -((Sqrt[e]*E^(I*A
rcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))])/d^3 - (I*b*e*PolyLog[2, (
Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/d^3 + (I*b*e
*PolyLog[2, E^((2*I)*ArcSin[c*x])])/d^3

```

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.93 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.93

method	result
parts	$-\frac{a}{2d^2x^2} - \frac{2ae \ln(x)}{d^3} + \frac{ae \ln(ex^2+d)}{d^3} - \frac{ae}{2d^2(ex^2+d)} + b c^2 \left(-\frac{-ic^4 dx^2 - ie c^4 x^4 + \sqrt{-c^2 x^2 + 1} c^3 dx + \sqrt{-c^2 x^2 + 1} c^2 d}{2c^2 x^2 d^2} \right)$
derivativedivides	$c^2 \left(\frac{ae \ln(c^2 e x^2 + c^2 d)}{c^2 d^3} - \frac{ae}{2d^2(c^2 e x^2 + c^2 d)} - \frac{a}{2d^2 c^2 x^2} - \frac{2ae \ln(cx)}{c^2 d^3} + b c^4 \left(-\frac{-ic^4 dx^2 - ie c^4 x^4 + \sqrt{-c^2 x^2 + 1} c^3 dx + \sqrt{-c^2 x^2 + 1} c^2 d}{2c^2 x^2 d^2} \right) \right)$
default	$c^2 \left(\frac{ae \ln(c^2 e x^2 + c^2 d)}{c^2 d^3} - \frac{ae}{2d^2(c^2 e x^2 + c^2 d)} - \frac{a}{2d^2 c^2 x^2} - \frac{2ae \ln(cx)}{c^2 d^3} + b c^4 \left(-\frac{-ic^4 dx^2 - ie c^4 x^4 + \sqrt{-c^2 x^2 + 1} c^3 dx + \sqrt{-c^2 x^2 + 1} c^2 d}{2c^2 x^2 d^2} \right) \right)$

```
input int((a+b*arcsin(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*a/d^2/x^2-2*a/d^3*e*ln(x)+a/d^3*e*ln(e*x^2+d)-1/2*a/d^2*e/(e*x^2+d)+b
*c^2*(-1/2*(-I*c^4*d*x^2-I*e*c^4*x^4+(-c^2*x^2+1)^(1/2)*c^3*d*x+(-c^2*x^2+
1)^(1/2)*e*c^3*x^3+arcsin(c*x)*c^2*d+2*arcsin(c*x)*c^2*e*x^2)/c^2/x^2/d^2/
(c^2*e*x^2+c^2*d)-1/2*I*e/d^3*sum((_R1^2*e-4*c^2*d-e)/(_R1^2*e-2*c^2*d-e)*
(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c
^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-1/2*I
*e^2/d^3*sum((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-
c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=Root
Of(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-1/2*I*(c^2*d*(c^2*d+e)^(1/2)/d^3/c^
2/(c^2*d+e)*arctanh(1/4*(2*e*(I*c*x+(-c^2*x^2+1)^(1/2))^2-4*c^2*d-2*e)/(c^
4*d^2+c^2*d*e)^(1/2))*e+2*I*e/d^3/c^2*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))-2*
I*e/d^3*dilog(I*c*x+(-c^2*x^2+1)^(1/2))/c^2-2*e/d^3/c^2*arcsin(c*x)*ln(1+I
*c*x+(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^2 x^3} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*arcsin(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (d + ex^2)^2} dx$$

input

```
integrate((a+b*asin(c*x))/x**3/(e*x**2+d)**2,x)
```

output

```
Integral((a + b*asin(c*x))/(x**3*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^2 x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")`

output `-1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e*log(x)/d^3) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)`

Giac [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx = \text{Timed out}$$

input `integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")`

output `Timed out`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx = \int \frac{a + b \arcsin(cx)}{x^3 (ex^2 + d)^2} dx$$

input `int((a + b*asin(c*x))/(x^3*(d + e*x^2)^2), x)`

output `int((a + b*asin(c*x))/(x^3*(d + e*x^2)^2), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^2} dx$$

$$= \frac{2 \left(\int \frac{a \arcsin(cx)}{e^2 x^7 + 2de x^5 + d^2 x^3} dx \right) b d^4 x^2 + 2 \left(\int \frac{a \arcsin(cx)}{e^2 x^7 + 2de x^5 + d^2 x^3} dx \right) b d^3 e x^4 + 2 \log(ex^2 + d) a d e x^2 + 2 \log(ex^2 + d) a d^2 x}{2d^3 x^2 (ex^2 + d)}$$

input `int((a+b*asin(c*x))/x^3/(e*x^2+d)^2,x)`

output `(2*int(asin(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**4*x**2 + 2*int(asin(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**3*e*x**4 + 2*log(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 - 4*log(x)*a*d*e*x**2 - 4*log(x)*a*e**2*x**4 - a*d**2 + 2*a*e**2*x**4)/(2*d**3*x**2*(d + e*x**2))`

3.463
$$\int \frac{x^4(a+b \arcsin(cx))}{(d+ex^2)^2} dx$$

Optimal result	3933
Mathematica [A] (verified)	3934
Rubi [A] (verified)	3935
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Fricas [F]	3938
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Giac [F]	3939
Mupad [F(-1)]	3940
Reduce [F]	3940

Optimal result

Integrand size = 21, antiderivative size = 787

$$\begin{aligned}
\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx &= \frac{ax}{e^2} + \frac{b\sqrt{1 - c^2x^2}}{ce^2} + \frac{bx \arcsin(cx)}{e^2} - \frac{d(a + b \arcsin(cx))}{4e^{5/2}(\sqrt{-d} - \sqrt{ex})} \\
&+ \frac{d(a + b \arcsin(cx))}{4e^{5/2}(\sqrt{-d} + \sqrt{ex})} + \frac{bcd \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{5/2}\sqrt{c^2d+e}} \\
&+ \frac{bcd \operatorname{arctanh}\left(\frac{\sqrt{e+c^2}\sqrt{-dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{5/2}\sqrt{c^2d+e}} \\
&+ \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&+ \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}} \\
&- \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}
\end{aligned}$$

output

```

a*x/e^2+b*(-c^2*x^2+1)^(1/2)/c/e^2+b*x*arcsin(c*x)/e^2-1/4*d*(a+b*arcsin(c
*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*d*(a+b*arcsin(c*x))/e^(5/2)/((-d)^(
1/2)+e^(1/2)*x)+1/4*b*c*d*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1
/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/4*b*c*d*arctanh((e^(1/2)
+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(
1/2)+3/4*(-d)^(1/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/
2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e^(5/2)-3/4*(-d)^(1/2)*(a+b*arcsin(c
*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/
2))/e^(5/2)+3/4*(-d)^(1/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^
2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e^(5/2)-3/4*(-d)^(1/2)*(a+b*
arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2))/e^(5/2)+3/4*I*b*(-d)^(1/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^
2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/e^(5/2)-3/4*I*b*(-d)^(1/2)*p
olylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2
)))/e^(5/2)+3/4*I*b*(-d)^(1/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2
)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e^(5/2)-3/4*I*b*(-d)^(1/2)*polylog(2,
e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/e^(5/
2)

```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 649, normalized size of antiderivative = 0.82

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \frac{8a\sqrt{ex} + \frac{4ad\sqrt{ex}}{d+ex^2} - 12a\sqrt{d} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) + b\left(\frac{8\sqrt{e}(\sqrt{1-c^2x^2+cx} \arcsin(cx))}{c} + 2id\left(\frac{\arcsin(cx)}{\sqrt{d+i\sqrt{ex}}} - \frac{c \arctan\left(\frac{i\sqrt{e+e^2}\sqrt{d+ex}}{\sqrt{c^2d+e\sqrt{1-c^2x^2+cx}}}\right)}{\sqrt{c^2d+e}}\right)\right)}{d+ex^2}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]
```

output

```
(8*a*Sqrt[e]*x + (4*a*d*Sqrt[e]*x)/(d + e*x^2) - 12*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*((8*Sqrt[e]*(Sqrt[1 - c^2*x^2] + c*x*ArcSin[c*x]))/c + (2*I)*d*(ArcSin[c*x]/(Sqrt[d] + I*Sqrt[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))]/Sqrt[c^2*d + e]) + 2*d*(ArcSin[c*x]/(I*Sqrt[d] + Sqrt[e]*x) + (c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))]/Sqrt[c^2*d + e]) + 3*Sqrt[d]*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] - Sqrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c^2*d + e]))] + 2*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))] - 3*Sqrt[d]*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(-(c*Sqrt[d] + Sqrt[c^2*d + e]))] + Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e]))] + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]))/(8*e^(5/2))
```

Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{d^2(a + b \arcsin(cx))}{e^2(d + ex^2)^2} - \frac{2d(a + b \arcsin(cx))}{e^2(d + ex^2)} + \frac{a + b \arcsin(cx)}{e^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4e^{5/2}} - \\
& \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4e^{5/2}} + \\
& \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4e^{5/2}} - \\
& \frac{3\sqrt{-d}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4e^{5/2}} - \frac{d(a + b \arcsin(cx))}{4e^{5/2}(\sqrt{-d} - \sqrt{ex})} + \frac{d(a + b \arcsin(cx))}{4e^{5/2}(\sqrt{-d} + \sqrt{ex})} + \\
& \frac{ax}{e^2} + \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{4e^{5/2}} + \\
& \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-d}+\sqrt{dc^2+e}}\right)}{4e^{5/2}} - \frac{3ib\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-d}+\sqrt{dc^2+e}}\right)}{4e^{5/2}} + \\
& \frac{bx \arcsin(cx)}{e^2} + \frac{bcd \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4e^{5/2}\sqrt{c^2d+e}} + \frac{bcd \operatorname{arctanh}\left(\frac{c^2\sqrt{-dx}+\sqrt{e}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4e^{5/2}\sqrt{c^2d+e}} + \frac{b\sqrt{1-c^2x^2}}{ce^2}
\end{aligned}$$

input `Int[(x^4*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]`

output `(a*x)/e^2 + (b*Sqrt[1 - c^2*x^2])/(c*e^2) + (b*x*ArcSin[c*x])/e^2 - (d*(a + b*ArcSin[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a + b*ArcSin[c*x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*d*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*e^(5/2)*Sqrt[c^2*d + e]) + (b*c*d*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*e^(5/2)*Sqrt[c^2*d + e]) + (3*Sqrt[-d]*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/e^(5/2)`

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5232 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_ + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.41 (sec) , antiderivative size = 914, normalized size of antiderivative = 1.16

$$a c^4 \left(\frac{c x}{e^2} - \frac{c^2 d \left(-\frac{c x}{2(c^2 e x^2 + c^2 d)} + \frac{3 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 c \sqrt{d e}} \right)}{e^2} \right) + b c^4 \left(\frac{(-i \sqrt{-c^2 x^2 + 1} + c x)(\arcsin(c x) + i)}{2 e^2} + \frac{(i \sqrt{-c^2 x^2 + 1} + c x)(\arcsin(c x) - i)}{2 e^2} \right)$$

input `int(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^2,x)`

output

```

1/c^5*(a*c^4*(1/e^2*c*x-1/e^2*c^2*d*(-1/2*c*x/(c^2*e*x^2+c^2*d)+3/2/c/(d*e
)^(1/2)*arctan(e*x/(d*e)^(1/2))))+b*c^4*(1/2*(-I*(-c^2*x^2+1)^(1/2)+c*x)*(
arcsin(c*x)+I)/e^2+1/2*(I*(-c^2*x^2+1)^(1/2)+c*x)*(arcsin(c*x)-I)/e^2+1/2*
arcsin(c*x)/e^2*c^3*d*x/(c^2*e*x^2+c^2*d)-1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-
(c^2*d*(c^2*d+e))^(1/2)*e)*c^2*d*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)+1/2*((2*c^2*d+2
*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)
*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+
e)*e)^(1/2))*c^2*d/e^5-1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e))^(1/2)
*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(
1/2)*e)*c^2*d*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((-2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)+1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e)
))^(1/2)+e))^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(I*c*x+(
-c^2*x^2+1)^(1/2)))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^2*d
/e^5-3/4*d*c^2/e^2*sum(1/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*
c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R
1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))-3/4*d*c^2/e^2*sum(_R1/(_R1^2*e-2*c
^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I
*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e)...

```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(ex^2 + d)^2} dx$$

input

```
integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*x^4*arcsin(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(d + ex^2)^2} dx$$

input `integrate(x**4*(a+b*asin(c*x))/(e*x**2+d)**2,x)`

output `Integral(x**4*(a + b*asin(c*x))/(d + e*x**2)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(ex^2 + d)^2} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)*x^4/(e*x^2 + d)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(ex^2 + d)^2} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d + e*x^2)^2,x)`output `int((x^4*(a + b*asin(c*x)))/(d + e*x^2)^2, x)`**Reduce [F]**

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \frac{-3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) ad - 3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) ae x^2 + 2\left(\int \frac{\operatorname{asin}(cx)x^4}{e^2x^4 + 2dex^2 + d^2} dx\right) bde^3 + 2\left(\int \frac{\operatorname{asin}(cx)x^4}{e^2x^4 + 2dex^2 + d^2} dx\right) bde^3 + 2\left(\int \frac{\operatorname{asin}(cx)x^4}{e^2x^4 + 2dex^2 + d^2} dx\right) bde^3 + 2\left(\int \frac{\operatorname{asin}(cx)x^4}{e^2x^4 + 2dex^2 + d^2} dx\right) bde^3}{2e^3(e x^2 + d)}$$

input `int(x^4*(a+b*asin(c*x))/(e*x^2+d)^2,x)`output `(- 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((asin(c*x)*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**3 + 2*int((asin(c*x)*x**4)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**4*x**2 + 3*a*d*e*x + 2*a*e**2*x**3)/(2*e**3*(d + e*x**2))`

3.464
$$\int \frac{x^2(a+b \arcsin(cx))}{(d+ex^2)^2} dx$$

Optimal result	3942
Mathematica [A] (verified)	3943
Rubi [A] (verified)	3944
Maple [C] (warning: unable to verify)	3946
Fricas [F]	3947
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Maxima [F(-2)]	3948
Giac [F]	3948
Mupad [F(-1)]	3949
Reduce [F]	3949

Optimal result

Integrand size = 21, antiderivative size = 745

$$\begin{aligned}
 \int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx = & \frac{a + b \arcsin(cx)}{4e^{3/2}(\sqrt{-d} - \sqrt{ex})} - \frac{a + b \arcsin(cx)}{4e^{3/2}(\sqrt{-d} + \sqrt{ex})} \\
 & - \frac{b \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{3/2}\sqrt{c^2d+e}} - \frac{b \operatorname{arctanh}\left(\frac{\sqrt{e+c^2}\sqrt{-dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{3/2}\sqrt{c^2d+e}} \\
 & + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} \\
 & - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}
 \end{aligned}$$

output

```

1/4*(a+b*arcsin(c*x))/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)-1/4*(a+b*arcsin(c*x))
/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)-1/4*b*c*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)
/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(3/2)/(c^2*d+e)^(1/2)-1/4*b*c*arcta
nh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(3/2)/
(c^2*d+e)^(1/2)+1/4*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/
2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*(a+b*arcsin(c
*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(3/2)+1/4*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^
2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*(a+b*
arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^
2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*I*b*p
olylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2
)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2
)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*I*b*polylog(2,
e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(
1/2)/e^(3/2)

```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 603, normalized size of antiderivative = 0.81

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \frac{-\frac{4a\sqrt{ex}}{d+ex^2} + \frac{4a \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{\sqrt{d}} + b \left(-\frac{2 \arcsin(cx)}{i\sqrt{d}+\sqrt{ex}} - 2i \left(\frac{\arcsin(cx)}{\sqrt{d}+i\sqrt{ex}} - \frac{c \arctan\left(\frac{i\sqrt{e}+c^2\sqrt{dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{\sqrt{c^2d+e}} \right) - \frac{2c \operatorname{arctanh}\left(\frac{\sqrt{e}+ic^2\sqrt{d}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{\sqrt{c^2d+e}} \right)}{\dots}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]
```

output

```

((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*(-2*ArcSin[c*x])/(I*Sqrt[d] + Sqrt[e]*x) - (2*I)*(ArcSin[c*x]/(Sqrt[d]
+ I*Sqrt[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*
Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e]) - (2*c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt
[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e] - (ArcSin[c*x]
*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] - S
qrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] + Sqrt[c
^2*d + e])))) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + S
qrt[c^2*d + e])] + 2*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] +
Sqrt[c^2*d + e])])/Sqrt[d] + (ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 +
(Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (S
qrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*PolyLog[2,
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 2*PolyLog[2,
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])])/Sqrt[d]))/(8*e
^(3/2))

```

Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{a + b \arcsin(cx)}{e(d + ex^2)} - \frac{d(a + b \arcsin(cx))}{e(d + ex^2)^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{4\sqrt{-d}e^{3/2}} + \\
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{4\sqrt{-d}e^{3/2}} + \\
& \frac{a + b \arcsin(cx)}{4e^{3/2}(\sqrt{-d} - \sqrt{ex})} - \frac{a + b \arcsin(cx)}{4e^{3/2}(\sqrt{-d} + \sqrt{ex})} + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}}\right)}{4\sqrt{-d}e^{3/2}} - \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}}\right)}{4\sqrt{-d}e^{3/2}} - \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b \operatorname{arctanh}\left(\frac{\sqrt{e - c^2} \sqrt{-dx}}{\sqrt{1 - c^2 x^2} \sqrt{c^2 d + e}}\right)}{4e^{3/2} \sqrt{c^2 d + e}} - \frac{b \operatorname{arctanh}\left(\frac{c^2 \sqrt{-dx} + \sqrt{e}}{\sqrt{1 - c^2 x^2} \sqrt{c^2 d + e}}\right)}{4e^{3/2} \sqrt{c^2 d + e}}
\end{aligned}$$

input `Int[(x^2*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]`

output

$$\begin{aligned}
& (a + b \operatorname{ArcSin}[c*x]) / (4*e^{(3/2)}*(\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[e]*x)) - (a + b \operatorname{ArcSin}[c*x]) / (4*e^{(3/2)}*(\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[e]*x)) - (b*c*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e] - c^2*\operatorname{Sqrt}[-d]*x) / (\operatorname{Sqrt}[c^2*d + e]*\operatorname{Sqrt}[1 - c^2*x^2])]) / (4*e^{(3/2)}*\operatorname{Sqrt}[c^2*d + e]) - \\
& (b*c*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e] + c^2*\operatorname{Sqrt}[-d]*x) / (\operatorname{Sqrt}[c^2*d + e]*\operatorname{Sqrt}[1 - c^2*x^2])]) / (4*e^{(3/2)}*\operatorname{Sqrt}[c^2*d + e]) + ((a + b \operatorname{ArcSin}[c*x])* \operatorname{Log}[1 - (\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (4*\operatorname{Sqrt}[-d]*e^{(3/2)}) - \\
& ((a + b \operatorname{ArcSin}[c*x])* \operatorname{Log}[1 + (\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (4*\operatorname{Sqrt}[-d]*e^{(3/2)}) + ((a + b \operatorname{ArcSin}[c*x])* \operatorname{Log}[1 - (\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (4*\operatorname{Sqrt}[-d]*e^{(3/2)}) - \\
& ((a + b \operatorname{ArcSin}[c*x])* \operatorname{Log}[1 + (\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (4*\operatorname{Sqrt}[-d]*e^{(3/2)}) + ((I/4)*b*\operatorname{PolyLog}[2, -((\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (\operatorname{Sqrt}[-d]*e^{(3/2)}) - \\
& ((I/4)*b*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] - \operatorname{Sqrt}[c^2*d + e])]) / (\operatorname{Sqrt}[-d]*e^{(3/2)}) + ((I/4)*b*\operatorname{PolyLog}[2, -((\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (\operatorname{Sqrt}[-d]*e^{(3/2)}) - \\
& ((I/4)*b*\operatorname{PolyLog}[2, (\operatorname{Sqrt}[e]*E^{(I*\operatorname{ArcSin}[c*x])}) / (I*c*\operatorname{Sqrt}[-d] + \operatorname{Sqrt}[c^2*d + e])]) / (\operatorname{Sqrt}[-d]*e^{(3/2)})
\end{aligned}$$

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 417.55 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{a c^5 x}{2e(c^2 e x^2 + c^2 d)} + \frac{a c^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}} + b c^4 \left(-\frac{\arcsin(cx)cx}{2e(c^2 e x^2 + c^2 d)} - \frac{\text{RootOf}\left(e Z^4 + (-4c^2 d - 2e) Z^2 + e\right)}{\dots} \right)$
default	$-\frac{a c^5 x}{2e(c^2 e x^2 + c^2 d)} + \frac{a c^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}} + b c^4 \left(-\frac{\arcsin(cx)cx}{2e(c^2 e x^2 + c^2 d)} - \frac{\text{RootOf}\left(e Z^4 + (-4c^2 d - 2e) Z^2 + e\right)}{\dots} \right)$
parts	$-\frac{ax}{2e(e x^2 + d)} + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}} + b \left(-\frac{c^5 \arcsin(cx)x}{2e(c^2 e x^2 + c^2 d)} + \frac{\text{RootOf}\left(e Z^4 + (-4c^2 d - 2e) Z^2 + e\right)}{\dots} \right)$

```
input int(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output

```

1/c^3*(-1/2*a*c^5/e*x/(c^2*e*x^2+c^2*d)+1/2*a*c^3/e/(d*e)^(1/2)*arctan(e*x
/(d*e)^(1/2))+b*c^4*(-1/2*arcsin(c*x)*c*x/e/(c^2*e*x^2+c^2*d)-1/4/e*sum(1/
_R1/(-_R1^2*e+2*c^2*d+e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/
_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*
d-2*e)*_Z^2+e))-1/4/e*sum(_R1/(-_R1^2*e+2*c^2*d+e)*(I*arcsin(c*x)*ln((_R1-
I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),
_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d
+e))^(1/2)+e))^(1/2)*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+
(c^2*d*(c^2*d+e))^(1/2)*e)*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((-2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-1/2*(-e*(2*c^2*d-2*(c
^2*d*(c^2*d+e))^(1/2)+e))^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arct
an(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)
^(1/2))/e^4+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d
*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arc
tanh(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)
^(1/2))/e^4/(c^2*d+e)-1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)
)/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4))

```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(ex^2 + d)^2} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*x^2*arcsin(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```


Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(d + ex^2)^2} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(e*x**2+d)**2,x)`

output `Integral(x**2*(a + b*asin(c*x))/(d + e*x**2)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(ex^2 + d)^2} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)*x^2/(e*x^2 + d)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(ex^2 + d)^2} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d + e*x^2)^2,x)`output `int((x^2*(a + b*asin(c*x)))/(d + e*x^2)^2, x)`**Reduce [F]**

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

$$= \frac{\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) ad + \sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) aex^2 + 2\left(\int \frac{\operatorname{asin}(cx)x^2}{e^2x^4 + 2dex^2 + d^2} dx\right) bd^2e^2 + 2\left(\int \frac{\operatorname{asin}(cx)x^2}{e^2x^4 + 2dex^2 + d^2} dx\right) d}{2de^2(e^2x^2 + d)}$$

input `int(x^2*(a+b*asin(c*x))/(e*x^2+d)^2,x)`output `(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((asin(c*x)*x**2)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d**2*e**2 + 2*int((asin(c*x)*x**2)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**3*x**2 - a*d*e*x)/(2*d*e**2*(d + e*x**2))`

3.465
$$\int \frac{a+b \arcsin(cx)}{(d+ex^2)^2} dx$$

Optimal result	3951
Mathematica [A] (verified)	3952
Rubi [A] (verified)	3953
Maple [C] (warning: unable to verify)	3955
Fricas [F]	3956
Sympy [F]	3957
Maxima [F(-2)]	3957
Giac [F(-2)]	3957
Mupad [F(-1)]	3958
Reduce [F]	3958

Optimal result

Integrand size = 18, antiderivative size = 757

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = & -\frac{a + b \arcsin(cx)}{4d\sqrt{e}(\sqrt{-d} - \sqrt{ex})} + \frac{a + b \arcsin(cx)}{4d\sqrt{e}(\sqrt{-d} + \sqrt{ex})} \\
& + \frac{b \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}} + \frac{b \operatorname{arctanh}\left(\frac{\sqrt{e+c^2}\sqrt{-dx}}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}} \\
& - \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& - \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d} + \sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}
\end{aligned}$$

output

```

-1/4*(a+b*arcsin(c*x))/d/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*(a+b*arcsin(c*
x))/d/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)+1/4*b*c*arctanh((e^(1/2)-c^2*(-d)^(1/
2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(1/2)/(c^2*d+e)^(1/2)+1/4*b*
c*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d
/e^(1/2)/(c^2*d+e)^(1/2)-1/4*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x
^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b
*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2
*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x
+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+
1/4*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1
/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-e^(1/2)*(I*c*x
+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+
1/4*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*
d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^
2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*I*b*p
olylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2
)))/(-d)^(3/2)/e^(1/2)

```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.78

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = \frac{1}{2} \left(\frac{ax}{d^2 + dex^2} + \frac{a \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}} \right) + \frac{b \left(i\sqrt{d} \left(\frac{\arcsin(cx)}{\sqrt{d+i\sqrt{ex}}} - \frac{c \arctan\left(\frac{i\sqrt{e}+c^2\sqrt{dx}}{\sqrt{c^2d+e\sqrt{1-c^2x^2}}}\right)}{\sqrt{c^2d+e}} \right) + \sqrt{d} \left(\frac{\arcsin(cx)}{i\sqrt{d}+\sqrt{ex}} + \frac{c \operatorname{arctanh}\left(\frac{\sqrt{e}+ic^2\sqrt{dx}}{\sqrt{c^2d+e\sqrt{1-c^2x^2}}}\right)}{\sqrt{c^2d+e}} \right) \right) + i \arcsin(cx)}{2}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(d + e*x^2)^2,x]
```

output

```

((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(I*Sqrt[d]*(ArcSin[c*x]/(Sqrt[d] + I*Sqrt[e]*x) - (c*ArcTan[(I*Sqrt[
e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])))/Sqrt[c^2*d + e])
+ Sqrt[d]*(ArcSin[c*x]/(I*Sqrt[d] + Sqrt[e]*x) + (c*ArcTanh[(Sqrt[e] + I*
c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])))/Sqrt[c^2*d + e]) + I*
ArcSin[c*x]*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(-(c*Sqrt[d]) + Sqrt[c^2*
d + e])) + Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e
])]) - I*ArcSin[c*x]*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqr
t[c^2*d + e])] + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2
*d + e])]) + PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*
d + e])] - PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2
*d + e])] - PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2
*d + e])] + PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*
d + e])])]/(2*d^(3/2)*Sqrt[e])/2

```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5172, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx$$

$$\downarrow 5172$$

$$\int \left(-\frac{e(a + b \arcsin(cx))}{2d(-de - e^2x^2)} - \frac{e(a + b \arcsin(cx))}{4d(\sqrt{-d}\sqrt{e - ex})^2} - \frac{e(a + b \arcsin(cx))}{4d(\sqrt{-d}\sqrt{e + ex})^2} \right) dx$$

$$\downarrow 2009$$

$$\begin{aligned}
& -\frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& -\frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{4(-d)^{3/2}\sqrt{e}} \\
& + \frac{a + b \arcsin(cx)}{4d\sqrt{e}(\sqrt{-d} - \sqrt{ex})} + \frac{a + b \arcsin(cx)}{4d\sqrt{e}(\sqrt{-d} + \sqrt{ex})} - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \\
& \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{b \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}} + \frac{b \operatorname{arctanh}\left(\frac{c^2\sqrt{-dx}+\sqrt{e}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(d + e*x^2)^2,x]`

output

```

-1/4*(a + b*ArcSin[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSin[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*d*Sqrt[e]*Sqrt[c^2*d + e]) + (b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*d*Sqrt[e]*Sqrt[c^2*d + e]) - ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))]/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))]/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e])

```

Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5172 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || IGtQ[n, 0])]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.81 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.11

$$\frac{ac^3x}{2d(c^2ex^2+c^2d)} + \frac{ac \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}} + bc^4 \left(\frac{\arcsin(cx)x}{2cd(c^2ex^2+c^2d)} - \frac{\sqrt{(2c^2d+2\sqrt{c^2d(c^2d+e)+e})e} \left(-2\sqrt{c^2d(c^2d+e)}c^2d+2c^4d^2+2c^2de-\sqrt{c^2d}\right)}{2c^2d(c^2d+e)e^3} \right)$$

input `int((a+b*arcsin(c*x))/(e*x^2+d)^2,x)`

output

```

1/c*(1/2*a*c^3*x/d/(c^2*e*x^2+c^2*d)+1/2*a*c/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*c^4*(1/2*arcsin(c*x)/c*x/d/(c^2*e*x^2+c^2*d)-1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/c^2/d/(c^2*d+e)/e^3+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/c^2/d/e^3-1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)^(1/2)*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/c^2/d/(c^2*d+e)/e^3+1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/c^2/d/e^3+1/4/c^2/d*sum(1/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/4/c^2/d*sum(_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e)))

```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^2} dx$$

input

```
integrate((a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output

```
integral((b*arcsin(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{(d + ex^2)^2} dx$$

input `integrate((a+b*asin(c*x))/(e*x**2+d)**2,x)`

output `Integral((a + b*asin(c*x))/(d + e*x**2)**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{(ex^2 + d)^2} dx$$

input

```
int((a + b*asin(c*x))/(d + e*x^2)^2,x)
```

output

```
int((a + b*asin(c*x))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^2} dx$$

$$= \frac{\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) ad + \sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) ae x^2 + 2 \left(\int \frac{\operatorname{asin}(cx)}{e^2 x^4 + 2de x^2 + d^2} dx \right) b d^3 e + 2 \left(\int \frac{\operatorname{asin}(cx)}{e^2 x^4 + 2de x^2 + d^2} dx \right)}{2d^2 e (ex^2 + d)}$$

input

```
int((a+b*asin(c*x))/(e*x^2+d)^2,x)
```

output

```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int(asin(c*x)/(d**2 + 2*d*e*x**2 + e
**2*x**4),x)*b*d**3*e + 2*int(asin(c*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
*b*d**2*e**2*x**2 + a*d*e*x)/(2*d**2*e*(d + e*x**2))
```

$$3.466 \quad \int \frac{a+b \arcsin(cx)}{x^2(d+ex^2)^2} dx$$

Optimal result	3960
Mathematica [A] (verified)	3961
Rubi [A] (verified)	3962
Maple [C] (warning: unable to verify)	3964
Fricas [F]	3965
Sympy [F]	3966
Maxima [F(-2)]	3966
Giac [F(-1)]	3966
Mupad [F(-1)]	3967
Reduce [F]	3967

Optimal result

Integrand size = 21, antiderivative size = 795

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx = & -\frac{a + b \arcsin(cx)}{d^2 x} + \frac{\sqrt{e}(a + b \arcsin(cx))}{4d^2 (\sqrt{-d} - \sqrt{ex})} \\
& - \frac{\sqrt{e}(a + b \arcsin(cx))}{4d^2 (\sqrt{-d} + \sqrt{ex})} - \frac{bc\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{c^2 d+e}\sqrt{1-c^2 x^2}}\right)}{4d^2 \sqrt{c^2 d+e}} \\
& - \frac{bc\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{e+c^2}\sqrt{-dx}}{\sqrt{c^2 d+e}\sqrt{1-c^2 x^2}}\right)}{4d^2 \sqrt{c^2 d+e}} - \frac{bc \operatorname{arctanh}(\sqrt{1-c^2 x^2})}{d^2} \\
& - \frac{3\sqrt{e}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& + \frac{3\sqrt{e}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& - \frac{3\sqrt{e}(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& + \frac{3\sqrt{e}(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& - \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& + \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& - \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}} \\
& + \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2 d+e}}\right)}{4(-d)^{5/2}}
\end{aligned}$$

output

```

-(a+b*arcsin(c*x))/d^2/x+1/4*e^(1/2)*(a+b*arcsin(c*x))/d^2/((-d)^(1/2)-e^(
1/2)*x)-1/4*e^(1/2)*(a+b*arcsin(c*x))/d^2/((-d)^(1/2)+e^(1/2)*x)-1/4*b*c*e
^(1/2)*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/
2))/d^2/(c^2*d+e)^(1/2)-1/4*b*c*e^(1/2)*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)
/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d^2/(c^2*d+e)^(1/2)-b*c*arctanh((-c^2
*x^2+1)^(1/2))/d^2-3/4*e^(1/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2
*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*e^(1/2)*(a
+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c
^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*e^(1/2)*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c
*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*e
^(1/2)*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)
^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*I*b*e^(1/2)*polylog(2,-e^(1/2)*(I*c
*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*I*
b*e^(1/2)*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(5/2)-3/4*I*b*e^(1/2)*polylog(2,-e^(1/2)*(I*c*x+(-c^2*
x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*I*b*e^(1/2)
*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1
/2)))/(-d)^(5/2)

```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 672, normalized size of antiderivative = 0.85

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx$$

$$= \frac{-\frac{8a\sqrt{d}}{x} - \frac{4a\sqrt{d}ex}{d+ex^2} - 12a\sqrt{e} \arctan\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) + b\left(-2i\sqrt{d}\sqrt{e}\left(\frac{\arcsin(cx)}{\sqrt{d+i\sqrt{ex}}} - \frac{c \arctan\left(\frac{i\sqrt{e}+c^2\sqrt{d}x}{\sqrt{c^2d+e\sqrt{1-c^2x^2}}}\right)}{\sqrt{c^2d+e}}\right)\right) + 2\sqrt{d}\sqrt{e}\left(\right)}{}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^2*(d + e*x^2)^2),x]
```

output

```

((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(S
qrt[e]*x)/Sqrt[d]] + b*((-2*I)*Sqrt[d]*Sqrt[e]*(ArcSin[c*x]/(Sqrt[d] + I*S
qrt[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1
- c^2*x^2])))/Sqrt[c^2*d + e]) + 2*Sqrt[d]*Sqrt[e]*(-(ArcSin[c*x]/(I*Sqrt[
d] + Sqrt[e]*x)) - (c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]
*Sqrt[1 - c^2*x^2])))/Sqrt[c^2*d + e]) - (8*Sqrt[d]*(ArcSin[c*x] + c*x*Arc
Tanh[Sqrt[1 - c^2*x^2]]))/x + 3*Sqrt[e]*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*
(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] - Sqrt[c^2*d + e])) + Log[
1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*PolyL
og[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2*Po
lyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))] -
3*Sqrt[e]*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[
c*x]))]/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])) + Log[1 - (Sqrt[e]*E^(I*ArcSin[c*
x]))]/(c*Sqrt[d] + Sqrt[c^2*d + e])))) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[
c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[
c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])))]/(8*d^(5/2))

```

Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx$$

$$\downarrow \text{5232}$$

$$\int \left(-\frac{e(a + b \arcsin(cx))}{d^2 (d + ex^2)} + \frac{a + b \arcsin(cx)}{d^2 x^2} - \frac{e(a + b \arcsin(cx))}{d (d + ex^2)^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
 & \frac{3\sqrt{e} \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d-\sqrt{dc^2+e}}}\right) (a + b \arcsin(cx))}{4(-d)^{5/2}} + \\
 & \frac{3\sqrt{e} \log\left(\frac{e^{i \arcsin(cx)}\sqrt{e}}{ic\sqrt{-d-\sqrt{dc^2+e}}} + 1\right) (a + b \arcsin(cx))}{4(-d)^{5/2}} - \\
 & \frac{3\sqrt{e} \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-dc+\sqrt{dc^2+e}}}\right) (a + b \arcsin(cx))}{4(-d)^{5/2}} + \\
 & \frac{3\sqrt{e} \log\left(\frac{e^{i \arcsin(cx)}\sqrt{e}}{i\sqrt{-dc+\sqrt{dc^2+e}}} + 1\right) (a + b \arcsin(cx))}{4(-d)^{5/2}} - \frac{a + b \arcsin(cx)}{d^2 x} + \frac{\sqrt{e}(a + b \arcsin(cx))}{4d^2(\sqrt{-d} - \sqrt{ex})} - \\
 & \frac{\sqrt{e}(a + b \arcsin(cx))}{4d^2(\sqrt{ex} + \sqrt{-d})} - \frac{bc\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)}{4d^2\sqrt{dc^2+e}} - \frac{bc\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{-dxc^2+\sqrt{e}}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)}{4d^2\sqrt{dc^2+e}} - \\
 & \frac{bc \operatorname{arctanh}(\sqrt{1-c^2x^2})}{d^2} - \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d-\sqrt{dc^2+e}}}\right)}{4(-d)^{5/2}} + \\
 & \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d-\sqrt{dc^2+e}}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-dc+\sqrt{dc^2+e}}}\right)}{4(-d)^{5/2}} + \\
 & \frac{3ib\sqrt{e} \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{i\sqrt{-dc+\sqrt{dc^2+e}}}\right)}{4(-d)^{5/2}}
 \end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^2*(d + e*x^2)^2), x]
```


output

```

-((a + b*ArcSin[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcSin[c*x]))/(4*d^2*(Sqr
t[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcSin[c*x]))/(4*d^2*(Sqrt[-d] + Sqr
t[e]*x)) - (b*c*Sqrt[e]*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e
]*Sqrt[1 - c^2*x^2])])/(4*d^2*Sqrt[c^2*d + e]) - (b*c*Sqrt[e]*ArcTanh[(Sqr
t[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*d^2*Sqrt[c
^2*d + e]) - (b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/d^2 - (3*Sqrt[e]*(a + b*ArcS
in[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d +
e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(
I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqr
t[e]*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d]
+ Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSin[c*x])*Log[
1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(4*(-d)
^(5/2)) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I
*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLo
g[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(-d)^(
5/2) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*
Sqrt[-d] + Sqrt[c^2*d + e]))])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2
, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(-d)^(5/2
)

```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.72 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.21

Expression too large to display

input `int((a+b*arcsin(c*x))/x^2/(e*x^2+d)^2,x)`

output `c*(-1/2*a/d^2*e*c*x/(c^2*e*x^2+c^2*d)-3/2*a/c/d^2*e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d^2/c/x+b*c^4*(-1/2/c^5/x*arcsin(c*x)*(3*c^2*e*x^2+2*c^2*d)/d^2/(c^2*e*x^2+c^2*d)-1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/c^4/d^2/e^2-1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/c^4/d^2/e^2-3/16/d^3/c^6*e*sum((4*_R1^2*c^2*d+_R1^2*e-e)/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+3/16/d^3/c^6*e*sum((_R1^2*e-4*c^2*d-e)/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))-1/d^2/c^4*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+1/d^2/c^4*ln(I*c*x+(-c^2*x^2+1)^(1/2))-1)+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/c^4/(c^2*d+e)/e^2+1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)^(1/2)*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((-2*c^2*d+2*(c^2*d*(c^2*d...`

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^2 x^2} dx$$

input `integrate((a+b*arcsin(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (d + ex^2)^2} dx$$

input `integrate((a+b*asin(c*x))/x**2/(e*x**2+d)**2,x)`

output `Integral((a + b*asin(c*x))/(x**2*(d + e*x**2)**2), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx = \text{Timed out}$$

input `integrate((a+b*arcsin(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")`

output `Timed out`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^2 (ex^2 + d)^2} dx$$

input `int((a + b*asin(c*x))/(x^2*(d + e*x^2)^2),x)`output `int((a + b*asin(c*x))/(x^2*(d + e*x^2)^2), x)`**Reduce [F]**

$$\int \frac{a + b \arcsin(cx)}{x^2 (d + ex^2)^2} dx$$

$$= \frac{-3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) adx - 3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) aex^3 + 2\left(\int \frac{\operatorname{asin}(cx)}{e^2x^6 + 2dex^4 + d^2x^2} dx\right) b d^4x + 2\left(\int \frac{\operatorname{asin}(cx)}{e^2x^6 + 2dex^4 + d^2x^2} dx\right) b d^4x + 2\left(\int \frac{\operatorname{asin}(cx)}{e^2x^6 + 2dex^4 + d^2x^2} dx\right) b d^4x}{2d^3x(ex^2 + d)}$$

input `int((a+b*asin(c*x))/x^2/(e*x^2+d)^2,x)`output `(- 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*x - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 2*int(asin(c*x)/(d**2*x**2 + 2*d*e*x**4 + e**2*x**6),x)*b*d**4*x + 2*int(asin(c*x)/(d**2*x**2 + 2*d*e*x**4 + e**2*x**6),x)*b*d**3*e*x**3 - 2*a*d**2 - 3*a*d*e*x**2)/(2*d**3*x*(d + e*x**2))`

$$3.467 \quad \int \frac{x^5(a+b \arcsin(cx))}{(d+ex^2)^3} dx$$

Optimal result	3969
Mathematica [A] (warning: unable to verify)	3970
Rubi [A] (verified)	3971
Maple [C] (warning: unable to verify)	3973
Fricas [F]	3974
Sympy [F]	3975
Maxima [F]	3975
Giac [F(-2)]	3975
Mupad [F(-1)]	3976
Reduce [F]	3976

Optimal result

Integrand size = 21, antiderivative size = 705

$$\begin{aligned}
\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = & \frac{bcdx\sqrt{1 - c^2x^2}}{8e^2(c^2d + e)(d + ex^2)} - \frac{d^2(a + b \arcsin(cx))}{4e^3(d + ex^2)^2} \\
& + \frac{d(a + b \arcsin(cx))}{e^3(d + ex^2)} - \frac{i(a + b \arcsin(cx))^2}{2be^3} \\
& - \frac{bc\sqrt{d} \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^3\sqrt{c^2d + e}} \\
& + \frac{bc\sqrt{d}(2c^2d + e) \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8e^3(c^2d + e)^{3/2}} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3} \\
& + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3} \\
& - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} \\
& - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} \\
& - \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3} \\
& - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i \arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3}
\end{aligned}$$

output

```

1/8*b*c*d*x*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/(e*x^2+d)-1/4*d^2*(a+b*arcsin
(c*x))/e^3/(e*x^2+d)^2+d*(a+b*arcsin(c*x))/e^3/(e*x^2+d)-1/2*I*(a+b*arcsin
(c*x))^2/b/e^3-b*c*d^(1/2)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(
1/2))/e^3/(c^2*d+e)^(1/2)+1/8*b*c*d^(1/2)*(2*c^2*d+e)*arctan((c^2*d+e)^(1/
2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/e^3/(c^2*d+e)^(3/2)+1/2*(a+b*arcsin(c*x))
*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))
/e^3+1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-
d)^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(
-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arcsin(c
*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/e^3-1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(
1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(
1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-e^(1/2)*(I*
c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I*b*poly
log(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))
/e^3

```

Mathematica [A] (warning: unable to verify)

Time = 4.72 (sec) , antiderivative size = 973, normalized size of antiderivative = 1.38

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(x^5*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]
```

output

```

((-4*a*d^2)/(d + e*x^2)^2 + (16*a*d)/(d + e*x^2) + 8*a*Log[d + e*x^2] + b*
((c*d*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
+ (c*d*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) +
(7*Sqrt[d]*ArcSin[c*x])/(Sqrt[d] - I*Sqrt[e]*x) - (d*ArcSin[c*x])/(Sqrt[d]
+ I*Sqrt[e]*x)^2 + (7*Sqrt[d]*ArcSin[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (d*A
rcSin[c*x])/(I*Sqrt[d] + Sqrt[e]*x)^2 - (8*I)*ArcSin[c*x]^2 - (7*c*Sqrt[d]
*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/
Sqrt[c^2*d + e] + ((7*I)*c*Sqrt[d]*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sq
rt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e] + 8*ArcSin[c*x]*Log[1 +
(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 8*ArcSin[c*x]
*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] +
8*ArcSin[c*x]*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d
+ e])] + 8*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sq
rt[c^2*d + e])] + (I*c^3*d^(3/2)*Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*S
qrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]
*x)))]/(c^2*d + e)^(3/2) - (I*c^3*d^(3/2)*Log[(e*Sqrt[c^2*d + e]*(Sqrt[e]
+ I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt[d]
]*Sqrt[e]*x)))]/(c^2*d + e)^(3/2) - (8*I)*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[
c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] - (8*I)*PolyLog[2, (Sqrt[e]*E^(I*Arc
Sin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] - (8*I)*PolyLog[2, -(Sqrt...

```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{d^2 x(a + b \arcsin(cx))}{e^2 (d + ex^2)^3} - \frac{2dx(a + b \arcsin(cx))}{e^2 (d + ex^2)^2} + \frac{x(a + b \arcsin(cx))}{e^2 (d + ex^2)} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{-\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{2e^3} + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{-\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{2e^3} + \\
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{2e^3} + \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{e} e^{i \arcsin(cx)}}{\sqrt{c^2 d + e + ic\sqrt{-d}}}\right)}{2e^3} - \\
& \frac{d^2(a + b \arcsin(cx))}{4e^3(d + ex^2)^2} + \frac{d(a + b \arcsin(cx))}{e^3(d + ex^2)} - \frac{i(a + b \arcsin(cx))^2}{2be^3} - \\
& \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}}\right)}{2e^3} - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{ic\sqrt{-d} - \sqrt{dc^2 + e}}\right)}{2e^3} - \\
& \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e} e^{i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}}\right)}{2e^3} - \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e} e^{i \arcsin(cx)}}{i\sqrt{-dc} + \sqrt{dc^2 + e}}\right)}{2e^3} + \\
& \frac{bc\sqrt{d}(2c^2d + e) \arctan\left(\frac{x\sqrt{c^2d + e}}{\sqrt{d}\sqrt{1 - c^2x^2}}\right)}{8e^3(c^2d + e)^{3/2}} - \frac{bc\sqrt{d} \arctan\left(\frac{x\sqrt{c^2d + e}}{\sqrt{d}\sqrt{1 - c^2x^2}}\right)}{e^3\sqrt{c^2d + e}} + \frac{bcdx\sqrt{1 - c^2x^2}}{8e^2(c^2d + e)(d + ex^2)}
\end{aligned}$$

input

```
Int[(x^5*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]
```

output

```
(b*c*d*x*Sqrt[1 - c^2*x^2])/(8*e^2*(c^2*d + e)*(d + e*x^2)) - (d^2*(a + b*
ArcSin[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcSin[c*x]))/(e^3*(d + e*
x^2)) - ((I/2)*(a + b*ArcSin[c*x])^2)/(b*e^3) - (b*c*Sqrt[d]*ArcTan[(Sqrt[
c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(e^3*Sqrt[c^2*d + e]) + (b*c*S
qrt[d]*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2]
)))/(8*e^3*(c^2*d + e)^(3/2)) + ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I
*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSin
[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]
)))/(2*e^3) + ((a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*
c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSin[c*x])*Log[1 + (Sq
rt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*e^3) - ((I/
2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d +
e]))])/e^3 - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d]
- Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*
x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (Sqrt[e]
]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/e^3
```

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5232 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.83 (sec) , antiderivative size = 3508, normalized size of antiderivative = 4.98

method	result	size
derivativedivides	Expression too large to display	3508
default	Expression too large to display	3508
parts	Expression too large to display	3513

input `int(x^5*(a+b*arcsin(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```

1/c^6*(a*c^6*(1/2/e^3*ln(c^2*e*x^2+c^2*d)+c^2*d/e^3/(c^2*e*x^2+c^2*d)-1/4*
c^4*d^2/e^3/(c^2*e*x^2+c^2*d)^2)+b*c^6*(5/8*I*(2*c^4*d^2+2*(c^2*d*(c^2*d+e
))^2/(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e))/(c^4*d^2+2*c^2*d
*e+e^2)/e^3-I/e^3/(c^2*d+e)*c^2*d*arcsin(c*x)^2-1/2*I/e^3/(c^2*d+e)*c^2*d*
sum((-_R1^2*e+4*c^2*d+2*e)/(-_R1^2*e+2*c^2*d+e)*(I*arcsin(c*x)*ln((_R1-I*c
*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1
=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/8*c^2*d*(6*c^4*d^2*arcsin(c*x)+8*
arcsin(c*x)*c^4*d*e*x^2-I*c^4*d^2-2*I*c^4*d*e*x^2-I*e^2*c^4*x^4+(-c^2*x^2+
1)^(1/2)*c^3*d*e*x+(-c^2*x^2+1)^(1/2)*e^2*c^3*x^3+6*c^2*d*e*arcsin(c*x)+8*
arcsin(c*x)*e^2*c^2*x^2)/e^3/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-I/e^2/(c^2*d+e)
*arcsin(c*x)^2-1/2*I/e^2/(c^2*d+e)*sum((-_R1^2*e+4*c^2*d+2*e)/(-_R1^2*e+2*
c^2*d+e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-
I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+
(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*d^2*c^4*ln(1-e*(I*c*x+(-c^2*x^2+1)^(
1/2))^2/(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e))*arcsin(c*x)/e^5/(c^2*d+e)+I
*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1
/2)*e)*c^4*d^2*arcsin(c*x)^2/(c^4*d^2+2*c^2*d*e+e^2)/e^5-3/4*I*(c^2*d*(c^2
*d+e))^(1/2)/e^3/(c^2*d+e)^2*c^2*d*arctanh(1/4*(4*c^2*d-2*e*(I*c*x+(-c^2*x
^2+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-1/8*I*(c^2*d*(c^2*d+e))^(1...

```

Fricas [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^5}{(ex^2 + d)^3} dx$$

input

```
integrate(x^5*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral((b*x^5*arcsin(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^5(a + b \operatorname{asin}(cx))}{(d + ex^2)^3} dx$$

input `integrate(x**5*(a+b*asin(c*x))/(e*x**2+d)**3,x)`

output `Integral(x**5*(a + b*asin(c*x))/(d + e*x**2)**3, x)`

Maxima [F]

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^5}{(ex^2 + d)^3} dx$$

input `integrate(x^5*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="maxima")`

output `1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2 + d)/e^3) + b*integrate(x^5*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^5*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^5(a + b \arcsin(cx))}{(ex^2 + d)^3} dx$$

input `int((x^5*(a + b*asin(c*x)))/(d + e*x^2)^3,x)`output `int((x^5*(a + b*asin(c*x)))/(d + e*x^2)^3, x)`**Reduce [F]**

$$\int \frac{x^5(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)x^5}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d^2 e^3 + 8 \left(\int \frac{\arcsin(cx)x^5}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d e^4 x^2 + 4 \left(\int \frac{\arcsin(cx)x^5}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) a d^3 (e^2 x^4 + 2de^2 x^2 + d^2)}$$

input `int(x^5*(a+b*asin(c*x))/(e*x^2+d)^3,x)`output `(4*int((asin(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**2*e**3 + 8*int((asin(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d*e**4*x**2 + 4*int((asin(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*e**5*x**4 + 2*log(d + e*x**2)*a*d**2 + 4*log(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 + a*d**2 - 2*a*e**2*x**4)/(4*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))`

3.468 $\int \frac{x^3(a+b \arcsin(cx))}{(d+ex^2)^3} dx$

Optimal result	3977
Mathematica [A] (verified)	3977
Rubi [A] (verified)	3978
Maple [B] (verified)	3981
Fricas [B] (verification not implemented)	3982
Sympy [F]	3982
Maxima [F]	3983
Giac [F(-2)]	3983
Mupad [F(-1)]	3984
Reduce [F]	3984

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{x^3(a+b \arcsin(cx))}{(d+ex^2)^3} dx = -\frac{bcx\sqrt{1-c^2x^2}}{8e(c^2d+e)(d+ex^2)} - \frac{b \arcsin(cx)}{4de^2} + \frac{x^4(a+b \arcsin(cx))}{4d(d+ex^2)^2} + \frac{bc(2c^2d+3e) \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8\sqrt{d}e^2(c^2d+e)^{3/2}}$$

```
output -1/8*b*c*x*(-c^2*x^2+1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)-1/4*b*arcsin(c*x)/d/e^2+1/4*x^4*(a+b*arcsin(c*x))/d/(e*x^2+d)^2+1/8*b*c*(2*c^2*d+3*e)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(1/2)/e^2/(c^2*d+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b \arcsin(cx))}{(d+ex^2)^3} dx = \frac{-\frac{bcex\sqrt{1-c^2x^2}(d+ex^2)}{c^2d+e}+2a(d+2ex^2)}{(d+ex^2)^2} - \frac{2b(d+2ex^2) \arcsin(cx)}{(d+ex^2)^2} + \frac{bc(2c^2d+3e) \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}(c^2d+e)^{3/2}}$$

input `Integrate[(x^3*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]`

output `(-(((b*c*e*x*Sqrt[1 - c^2*x^2]*(d + e*x^2))/(c^2*d + e) + 2*a*(d + 2*e*x^2)))/(d + e*x^2)^2) - (2*b*(d + 2*e*x^2)*ArcSin[c*x])/(d + e*x^2)^2 + (b*c*(2*c^2*d + 3*e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt[d]*(c^2*d + e)^(3/2))/(8*e^2)`

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5230, 27, 372, 398, 223, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx \\
 & \quad \downarrow \text{5230} \\
 & \frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - bc \int \frac{x^4}{4d\sqrt{1 - c^2x^2}(ex^2 + d)^2} dx \\
 & \quad \downarrow \text{27} \\
 & \frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - \frac{bc \int \frac{x^4}{\sqrt{1 - c^2x^2}(ex^2 + d)^2} dx}{4d} \\
 & \quad \downarrow \text{372} \\
 & \frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - \frac{bc \left(\frac{dx\sqrt{1 - c^2x^2}}{2e(c^2d + e)(d + ex^2)} - \frac{\int \frac{d - 2(dc^2 + e)x^2}{\sqrt{1 - c^2x^2}(ex^2 + d)} dx}{2e(c^2d + e)} \right)}{4d} \\
 & \quad \downarrow \text{398}
 \end{aligned}$$

$$\frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - \frac{bc \left(\frac{\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}}{\frac{d(2c^2d+3e) \int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)} dx}{e}} - \frac{2(c^2d+e) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{e} \right)}{4d}$$

↓ 223

$$\frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - \frac{bc \left(\frac{\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}}{\frac{d(2c^2d+3e) \int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)} dx}{e}} - \frac{2 \arcsin(cx)(c^2d+e)}{ce} \right)}{4d}$$

↓ 291

$$\frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - \frac{bc \left(\frac{\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}}{\frac{d(2c^2d+3e) \int \frac{1}{(-dc^2-e)x^2} d \frac{x}{\sqrt{1-c^2x^2}}}{e}} - \frac{2 \arcsin(cx)(c^2d+e)}{ce} \right)}{4d}$$

↓ 218

$$\frac{x^4(a + b \arcsin(cx))}{4d(d + ex^2)^2} - \frac{bc \left(\frac{\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}}{\frac{\sqrt{d}(2c^2d+3e) \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e\sqrt{c^2d+e}}} - \frac{2 \arcsin(cx)(c^2d+e)}{ce} \right)}{4d}$$

input `Int[(x^3*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]`

output `(x^4*(a + b*ArcSin[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*((d*x*sqrt[1 - c^2*x^2])/(2*e*(c^2*d + e)*(d + e*x^2)) - ((-2*(c^2*d + e)*ArcSin[c*x])/(c*e) + (sqrt[d]*(2*c^2*d + 3*e)*ArcTan[(sqrt[c^2*d + e]*x)/(sqrt[d]*sqrt[1 - c^2*x^2])])/(e*sqrt[c^2*d + e])))/(2*e*(c^2*d + e)))/(4*d)`

Defintions of rubi rules used

- rule 27 $\text{Int}[(a_*)(Fx_), x_Symbol] \rightarrow \text{Simp}[a \text{ Int}[Fx, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[Fx, (b_*)(Gx_)] /; \text{FreeQ}[b, x]$
- rule 218 $\text{Int}[((a_) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$
- rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_*)(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$
- rule 291 $\text{Int}[1/(\text{Sqrt}[(a_) + (b_*)(x_)^2]*((c_) + (d_*)(x_)^2)), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$
- rule 372 $\text{Int}[((e_*)(x_))^{(m_)*((a_) + (b_*)(x_)^2)^{(p_)*((c_) + (d_*)(x_)^2)^{(q_)}}, x_Symbol] \rightarrow \text{Simp}[(-a)*e^3*(e*x)^{(m-3)}*(a + b*x^2)^{(p+1)}*((c + d*x^2)^{(q+1)}/(2*b*(b*c - a*d)*(p+1))), x] + \text{Simp}[e^4/(2*b*(b*c - a*d)*(p+1)) \text{Int}[(e*x)^{(m-4)}*(a + b*x^2)^{(p+1)}*(c + d*x^2)^q*\text{Simp}[a*c*(m-3) + (a*d*(m+2*q-1) + 2*b*c*(p+1))*x^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 3] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, e, m, 2, p, q, x]$
- rule 398 $\text{Int}[((e_) + (f_*)(x_)^2)/(((a_) + (b_*)(x_)^2)*\text{Sqrt}[(c_) + (d_*)(x_)^2]), x_Symbol] \rightarrow \text{Simp}[f/b \text{ Int}[1/\text{Sqrt}[c + d*x^2], x], x] + \text{Simp}[(b*e - a*f)/b \text{ Int}[1/((a + b*x^2)*\text{Sqrt}[c + d*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x]$
- rule 5230 $\text{Int}[((a_.) + \text{ArcSin}[c_*(x_)]*(b_.))*((f_*)(x_))^{(m_)*((d_) + (e_*)(x_)^2)^{(p_)}}, x_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Simp}[(a + b*\text{ArcSin}[c*x]) u, x] - \text{Simp}[b*c \text{ Int}[\text{SimplifyIntegrand}[u/\text{Sqrt}[1 - c^2*x^2], x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (\text{GtQ}[p, 0] \ || \ (\text{IGtQ}[(m-1)/2, 0] \ \&\& \ \text{LeQ}[m + p, 0]))$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(133) = 266$.

Time = 19.65 (sec) , antiderivative size = 1020, normalized size of antiderivative = 6.67

method	result	size
parts	Expression too large to display	1020
derivativedivides	Expression too large to display	1038
default	Expression too large to display	1038

input `int(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(1/4*c^8*arcsin(c*x)/e^2
*d/(c^2*e*x^2+c^2*d)^2-1/2*c^6*arcsin(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*c^6/e
^2*(1/4/e*(-1/(c^2*d+e)*e/(c*x-(-c^2*d*e)^(1/2)/e)*(-(c*x-(-c^2*d*e)^(1/2)
/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2)-(-c
^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*e)
^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e)
^(1/2)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2)
))/((c*x-(-c^2*d*e)^(1/2)/e))) +1/4/e*(-1/(c^2*d+e)*e/(c*x+(-c^2*d*e)^(1/2)/
e)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)
/e)+(c^2*d+e)/e)^(1/2)+(-c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d+e)/e)^(1/2)*ln((
2*(c^2*d+e)/e+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)
)^(1/2)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)
^(1/2)/e)+(c^2*d+e)/e)^(1/2))/((c*x+(-c^2*d*e)^(1/2)/e))) -3/4/(-c^2*d*e)^(1/
2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d
*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e)^(1/2)/e)^2-2*(-c^2*d*
e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/((c*x-(-c^2*d*e)^(1
/2)/e))+3/4/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e+2*(-c^2
*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x+(-c^2*
d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)
^(1/2))/((c*x+(-c^2*d*e)^(1/2)/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(133) = 266$.

Time = 0.25 (sec) , antiderivative size = 921, normalized size of antiderivative = 6.02

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="fricas")`

output

```
[-1/32*(8*a*c^4*d^4 + 16*a*c^2*d^3*e + 8*a*d^2*e^2 + 16*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*log(((8*c^4*d^2 + 8*c^2*d*e + e^2)*x^4 - 2*(4*c^2*d^2 + 3*d*e)*x^2 - 4*sqrt(-c^2*d^2 - d*e)*sqrt(-c^2*x^2 + 1))*((2*c^2*d + e)*x^3 - d*x) + d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arcsin(c*x) + 4*sqrt(-c^2*x^2 + 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*arctan(1/2*sqrt(c^2*d^2 + d*e)*sqrt(-c^2*x^2 + 1))*((2*c^2*d + e)*x^2 - d)/((c^4*d^2 + c^2*d*e)*x^3 - (c^2*d^2 + d*e)*x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arcsin(c*x) + 2*sqrt(-c^2*x^2 + 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

Sympy [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^3(a + b \operatorname{asin}(cx))}{(d + ex^2)^3} dx$$

input `integrate(x**3*(a+b*asin(c*x))/(e*x**2+d)**3,x)`

output `Integral(x**3*(a + b*asin(c*x))/(d + e*x**2)**3, x)`

Maxima [F]

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^3}{(ex^2 + d)^3} dx$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="maxima")`

output `-1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*((2*e*x^2 + d)*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)*integrate(1/4*(2*c*e*x^2 + c*d)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))/(c^4*e^4*x^8 - c^2*d^2*e^2*x^2 + (2*c^4*d*e^3 - c^2*e^4)*x^6 + (c^4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(log(c*x + 1) + log(-c*x + 1))), x)*b/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)`

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate(x^3*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^3(a + b \arcsin(cx))}{(ex^2 + d)^3} dx$$

input `int((x^3*(a + b*asin(c*x)))/(d + e*x^2)^3,x)`output `int((x^3*(a + b*asin(c*x)))/(d + e*x^2)^3, x)`**Reduce [F]**

$$\int \frac{x^3(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)x^3}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d^3 + 8 \left(\int \frac{\arcsin(cx)x^3}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d^2 e x^2 + 4 \left(\int \frac{\arcsin(cx)x^3}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right)}{4d(e^2x^4 + 2dex^2 + d^2)}$$

input `int(x^3*(a+b*asin(c*x))/(e*x^2+d)^3,x)`output `(4*int((asin(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3 + 8*int((asin(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**2*e*x**2 + 4*int((asin(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d*e**2*x**4 + a*x**4)/(4*d*(d**2 + 2*d*e*x**2 + e**2*x**4))`

3.469
$$\int \frac{x(a+b \arcsin(cx))}{(d+ex^2)^3} dx$$

Optimal result	3985
Mathematica [A] (verified)	3985
Rubi [A] (verified)	3986
Maple [B] (verified)	3988
Fricas [B] (verification not implemented)	3989
Sympy [F]	3990
Maxima [F]	3990
Giac [F(-2)]	3991
Mupad [F(-1)]	3991
Reduce [F]	3992

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \frac{bcx\sqrt{1 - c^2x^2}}{8d(c^2d + e)(d + ex^2)} - \frac{a + b \arcsin(cx)}{4e(d + ex^2)^2} + \frac{bc(2c^2d + e) \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{3/2}e(c^2d + e)^{3/2}}$$

output

$1/8*b*c*x*(-c^2*x^2+1)^{(1/2)}/d/(c^2*d+e)/(e*x^2+d)-1/4*(a+b*\arcsin(c*x))/e/(e*x^2+d)^2+1/8*b*c*(2*c^2*d+e)*\arctan((c^2*d+e)^{(1/2)}*x/d^{(1/2)}/(-c^2*x^2+1)^{(1/2)})/d^{(3/2)}/e/(c^2*d+e)^{(3/2)}$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \frac{1}{8} \left(\frac{-\frac{2a}{e} + \frac{bcx\sqrt{1-c^2x^2}(d+ex^2)}{d(c^2d+e)}}{(d + ex^2)^2} - \frac{2b \arcsin(cx)}{e(d + ex^2)^2} + \frac{bc(2c^2d + e) \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{3/2}e(c^2d + e)^{3/2}} \right)$$

input `Integrate[(x*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]`

output `((((-2*a)/e + (b*c*x*sqrt[1 - c^2*x^2]*(d + e*x^2))/(d*(c^2*d + e)))/(d + e*x^2)^2 - (2*b*ArcSin[c*x])/(e*(d + e*x^2)^2) + (b*c*(2*c^2*d + e)*ArcTan[(sqrt[c^2*d + e]*x)/(sqrt[d]*sqrt[1 - c^2*x^2])])/(d^(3/2)*e*(c^2*d + e)^(3/2)))/8`

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5228, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx \\
 & \quad \downarrow 5228 \\
 & \frac{bc \int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)^2} dx}{4e} - \frac{a + b \arcsin(cx)}{4e(d + ex^2)^2} \\
 & \quad \downarrow 296 \\
 & \frac{bc \left(\frac{(2c^2d+e) \int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)} dx}{2d(c^2d+e)} + \frac{ex\sqrt{1-c^2x^2}}{2d(c^2d+e)(d+ex^2)} \right)}{4e} - \frac{a + b \arcsin(cx)}{4e(d + ex^2)^2} \\
 & \quad \downarrow 291 \\
 & \frac{bc \left(\frac{(2c^2d+e) \int \frac{1}{(-dc^2-e)x^2} d \frac{x}{\sqrt{1-c^2x^2}}}{2d(c^2d+e)} + \frac{ex\sqrt{1-c^2x^2}}{2d(c^2d+e)(d+ex^2)} \right)}{4e} - \frac{a + b \arcsin(cx)}{4e(d + ex^2)^2} \\
 & \quad \downarrow 218
 \end{aligned}$$

$$\frac{bc \left(\frac{(2c^2d+e) \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{3/2}(c^2d+e)^{3/2}} + \frac{ex\sqrt{1-c^2x^2}}{2d(c^2d+e)(d+ex^2)} \right)}{4e} - \frac{a + b \arcsin(cx)}{4e(d+ex^2)^2}$$

input `Int[(x*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]`

output `-1/4*(a + b*ArcSin[c*x])/(e*(d + e*x^2)^2) + (b*c*((e*x*Sqrt[1 - c^2*x^2])/(2*d*(c^2*d + e)*(d + e*x^2)) + ((2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(3/2)*(c^2*d + e)^(3/2)))/(4*e)`

Defintions of rubi rules used

rule 218 `Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

rule 291 `Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*((c_) + (d_)*(x_)^2)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

rule 296 `Int[((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol] := Simp[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d)), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d)) Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]`

rule 5228 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x] - Simp[b*(c/(2*e*(p + 1))) Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(115) = 230.

Time = 19.00 (sec) , antiderivative size = 998, normalized size of antiderivative = 7.50

method	result
	$b \frac{c^6 \arcsin(cx)}{4e(c^2 e x^2 + c^2 d)^2} + \frac{e \sqrt{-\left(cx - \frac{\sqrt{-c^2 de}}{e}\right)^2 - \frac{2\sqrt{-c^2 de} \left(cx - \frac{\sqrt{-c^2 de}}{e}\right) + \frac{c^2 d + e}{e}}}{(c^2 d + e) \left(cx - \frac{\sqrt{-c^2 de}}{e}\right)} - \frac{\sqrt{-c^2 de} \ln \left \frac{e \sqrt{-\left(cx - \frac{\sqrt{-c^2 de}}{e}\right)^2 - \frac{2\sqrt{-c^2 de} \left(cx - \frac{\sqrt{-c^2 de}}{e}\right) + \frac{c^2 d + e}{e}}}{(c^2 d + e) \left(cx - \frac{\sqrt{-c^2 de}}{e}\right)} + \frac{c^2 d + e}{e} \right }{c^6}$
parts	$-\frac{a}{4e(e x^2 + d)^2} +$
derivativedivides	Expression too large to display
default	Expression too large to display

input `int(x*(a+b*arcsin(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```

-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arcsin(c*x)+1/4
*c^6/e*(-1/4/d/c^2/e*(-1/(c^2*d+e)*e/(c*x-(c^2*d*e)^(1/2)/e)*(-(c*x-(c^2
*d*e)^(1/2)/e)^2-2*(c^2*d*e)^(1/2)/e*(c*x-(c^2*d*e)^(1/2)/e)+(c^2*d+e)/e
)^(1/2)-(-c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2
*(c^2*d*e)^(1/2)/e*(c*x-(c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-
(c^2*d*e)^(1/2)/e)^2-2*(c^2*d*e)^(1/2)/e*(c*x-(c^2*d*e)^(1/2)/e)+(c^2*d
+e)/e)^(1/2))/((c*x-(c^2*d*e)^(1/2)/e)))-1/4/d/c^2/e*(-1/(c^2*d+e)*e/(c*x+
(c^2*d*e)^(1/2)/e)*(-(c*x+(c^2*d*e)^(1/2)/e)^2+2*(c^2*d*e)^(1/2)/e*(c*x
+(c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2)+(-c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d
+e)/e)^(1/2)*ln((2*(c^2*d+e)/e+2*(c^2*d*e)^(1/2)/e*(c*x+(c^2*d*e)^(1/2)/
e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x+(c^2*d*e)^(1/2)/e)^2+2*(c^2*d*e)^(1/2)/e
*(c*x+(c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/((c*x+(c^2*d*e)^(1/2)/e)))-1
/4/d/c^2/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(c^2*d*
e)^(1/2)/e*(c*x-(c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(c^2*d*e
)^(1/2)/e)^2-2*(c^2*d*e)^(1/2)/e*(c*x-(c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1
/2))/((c*x-(c^2*d*e)^(1/2)/e))+1/4/d/c^2/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1
/2)*ln((2*(c^2*d+e)/e+2*(c^2*d*e)^(1/2)/e*(c*x+(c^2*d*e)^(1/2)/e)+2*((c^
2*d+e)/e)^(1/2)*(-(c*x+(c^2*d*e)^(1/2)/e)^2+2*(c^2*d*e)^(1/2)/e*(c*x+(c
^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/((c*x+(c^2*d*e)^(1/2)/e)))

```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 783, normalized size of antiderivative = 5.89

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

input

```
integrate(x*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output

```

[-1/32*(8*a*c^4*d^4 + 16*a*c^2*d^3*e + 8*a*d^2*e^2 + (2*b*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*log(((8*c^4*d^2 + 8*c^2*d*e + e^2)*x^4 - 2*(4*c^2*d^2 + 3*d*e)*x^2 - 4*sqrt(-c^2*d^2 - d*e)*sqrt(-c^2*x^2 + 1))*((2*c^2*d + e)*x^3 - d*x) + d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arcsin(c*x) - 4*sqrt(-c^2*x^2 + 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (2*b*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*arctan(1/2*sqrt(c^2*d^2 + d*e)*sqrt(-c^2*x^2 + 1)*((2*c^2*d + e)*x^2 - d)/((c^4*d^2 + c^2*d*e)*x^3 - (c^2*d^2 + d*e)*x)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arcsin(c*x) - 2*sqrt(-c^2*x^2 + 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]

```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(d + ex^2)^3} dx$$

input

```
integrate(x*(a+b*asin(c*x))/(e*x**2+d)**3,x)
```

output

```
Integral(x*(a + b*asin(c*x))/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x}{(ex^2 + d)^3} dx$$

input

```
integrate(x*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output

```
-1/4*(4*(c*e^3*x^4 + 2*c*d*e^2*x^2 + c*d^2*e)*integrate(1/4*e^(1/2*log(c*x
+ 1) + 1/2*log(-c*x + 1))/(c^4*e^3*x^8 - c^2*d^2*e*x^2 + (2*c^4*d*e^2 - c
^2*e^3)*x^6 + (c^4*d^2*e - 2*c^2*d*e^2)*x^4 + (c^2*e^3*x^6 + (2*c^2*d*e^2
- e^3)*x^4 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(-c*x
+ 1))), x) + arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))*b/(e^3*x^4 + 2*d
*e^2*x^2 + d^2*e) - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(x*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x(a + b \operatorname{asin}(cx))}{(ex^2 + d)^3} dx$$

input

```
int((x*(a + b*asin(c*x)))/(d + e*x^2)^3,x)
```

output

```
int((x*(a + b*asin(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$= \frac{4 \left(\int \frac{\arcsin(cx)x}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d^2 e + 8 \left(\int \frac{\arcsin(cx)x}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d e^2 x^2 + 4 \left(\int \frac{\arcsin(cx)x}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d e^2 x^2 + 4 \left(\int \frac{\arcsin(cx)x}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx \right) b d e^2 x^2}{4e(e^2x^4 + 2dex^2 + d^2)}$$

input

```
int(x*(a+b*asin(c*x))/(e*x^2+d)^3,x)
```

output

```
(4*int((asin(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)
*b*d**2*e + 8*int((asin(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e
**3*x**6),x)*b*d*e**2*x**2 + 4*int((asin(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*
d*e**2*x**4 + e**3*x**6),x)*b*e**3*x**4 - a)/(4*e*(d**2 + 2*d*e*x**2 + e**
2*x**4))
```

$$3.470 \quad \int \frac{a+b \arcsin(cx)}{x(d+ex^2)^3} dx$$

Optimal result	3994
Mathematica [A] (warning: unable to verify)	3995
Rubi [A] (verified)	3996
Maple [C] (warning: unable to verify)	3998
Fricas [F]	3999
Sympy [F]	4000
Maxima [F]	4000
Giac [F(-1)]	4000
Mupad [F(-1)]	4001
Reduce [F]	4001

Optimal result

Integrand size = 21, antiderivative size = 727

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = & -\frac{bcex\sqrt{1-c^2x^2}}{8d^2(c^2d+e)(d+ex^2)} + \frac{a+b\arcsin(cx)}{4d(d+ex^2)^2} + \frac{a+b\arcsin(cx)}{2d^2(d+ex^2)} \\
& -\frac{bc\arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}} - \frac{bc(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{5/2}(c^2d+e)^{3/2}} \\
& -\frac{(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} \\
& -\frac{(a+b\arcsin(cx))\log\left(1+\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} \\
& -\frac{(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} \\
& -\frac{(a+b\arcsin(cx))\log\left(1+\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} \\
& +\frac{(a+b\arcsin(cx))\log(1-e^{2i\arcsin(cx)})}{d^3} \\
& +\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} \\
& +\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} \\
& +\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} \\
& +\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{ib\operatorname{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{2d^3}
\end{aligned}$$

output

```

-1/8*b*c*e*x*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d+e)/(e*x^2+d)+1/4*(a+b*arcsin(c*x))/d/(e*x^2+d)^2+1/2*(a+b*arcsin(c*x))/d^2/(e*x^2+d)-1/2*b*c*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-1/8*b*c*(2*c^2*d+e)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)-1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d^3-1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d^3-1/2*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d^3-1/2*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d^3+(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d^3+1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2))/d^3+1/2*I*b*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d^3+1/2*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))/d^3-1/2*I*b*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d^3

```

Mathematica [A] (warning: unable to verify)

Time = 3.70 (sec) , antiderivative size = 1022, normalized size of antiderivative = 1.41

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x*(d + e*x^2)^3),x]
```


output

```
(-((b*c*d*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*
x))) - (b*c*d*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]
*x)) + (4*a*d^2)/(d + e*x^2)^2 + (8*a*d)/(d + e*x^2) + (b*d*ArcSin[c*x])/
(Sqrt[d] - I*Sqrt[e]*x)^2 + (5*b*Sqrt[d]*ArcSin[c*x])/(Sqrt[d] - I*Sqrt[e]*
x) + (b*d*ArcSin[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (5*b*Sqrt[d]*ArcSin[c*x]
)/(Sqrt[d] + I*Sqrt[e]*x) - (5*b*c*Sqrt[d]*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]
]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/Sqrt[c^2*d + e] + ((5*I)*b*c*Sq
rt[d]*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^
2]))/Sqrt[c^2*d + e] - 8*b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x])
)]/(c*Sqrt[d] - Sqrt[c^2*d + e])] - 8*b*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*Ar
cSin[c*x]))/(-c*Sqrt[d]) + Sqrt[c^2*d + e])] - 8*b*ArcSin[c*x]*Log[1 - (
Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - 8*b*ArcSin[c*x
]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] + 16*
b*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])] + 16*a*Log[x] - 8*a*Log[d + e
*x^2] - (I*b*c^3*d^(3/2)*Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x
+ Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x)))]/(
c^2*d + e)^(3/2) + (I*b*c^3*d^(3/2)*Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^
2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt[d]*Sqrt
[e]*x)))]/(c^2*d + e)^(3/2) + (8*I)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]
)))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + (8*I)*b*PolyLog[2, (Sqrt[e]*E^(I*Ar...
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx$$

↓ 5232

$$\int \left(-\frac{ex(a + b \arcsin(cx))}{d^3(d + ex^2)} + \frac{a + b \arcsin(cx)}{d^3x} - \frac{ex(a + b \arcsin(cx))}{d^2(d + ex^2)^2} - \frac{ex(a + b \arcsin(cx))}{d(d + ex^2)^3} \right) dx$$

↓ 2009

$$\begin{aligned}
& \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^3} - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^3} \\
& - \frac{(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^3} - \frac{(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^3} + \\
& \frac{\log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))}{d^3} + \frac{a + b \arcsin(cx)}{2d^2 (d + ex^2)} + \frac{a + b \arcsin(cx)}{4d (d + ex^2)^2} + \\
& \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^3} + \\
& \frac{ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2d^3} + \frac{ib \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2d^3} - \\
& \frac{ib \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{2d^3} - \frac{bc(2c^2d + e) \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{5/2} (c^2d + e)^{3/2}} - \frac{bc \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2} \sqrt{c^2d + e}} \\
& - \frac{bcex\sqrt{1-c^2x^2}}{8d^2 (c^2d + e) (d + ex^2)}
\end{aligned}$$

input `Int[(a + b*ArcSin[c*x])/(x*(d + e*x^2)^3), x]`

output

```

-1/8*(b*c*e*x*sqrt[1 - c^2*x^2])/(d^2*(c^2*d + e)*(d + e*x^2)) + (a + b*ArcSin[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcSin[c*x])/(2*d^2*(d + e*x^2)) - (b*c*ArcTan[(sqrt[c^2*d + e]*x)/(sqrt[d]*sqrt[1 - c^2*x^2])])/(2*d^(5/2)*sqrt[c^2*d + e]) - (b*c*(2*c^2*d + e)*ArcTan[(sqrt[c^2*d + e]*x)/(sqrt[d]*sqrt[1 - c^2*x^2])])/(8*d^(5/2)*(c^2*d + e)^(3/2)) - ((a + b*ArcSin[c*x])*Log[1 - (sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] - sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSin[c*x])*Log[1 + (sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] - sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSin[c*x])*Log[1 - (sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] + sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSin[c*x])*Log[1 + (sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] + sqrt[c^2*d + e])])/(2*d^3) + ((a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])])/d^3 + ((I/2)*b*PolyLog[2, -((sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] - sqrt[c^2*d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] - sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, -((sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] + sqrt[c^2*d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*sqrt[-d] + sqrt[c^2*d + e])])/d^3 - ((I/2)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/d^3

```

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 5232 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]`

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.48 (sec) , antiderivative size = 1130, normalized size of antiderivative = 1.55

method	result	size
parts	Expression too large to display	1130
derivativedivides	Expression too large to display	1174
default	Expression too large to display	1174

input `int((a+b*arcsin(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```

a/d^3*ln(x)+1/4*a/d/(e*x^2+d)^2-1/2*a/d^3*ln(e*x^2+d)+1/2*a/d^2/(e*x^2+d)+
b*(1/8*c^2*(6*c^4*d^2*arcsin(c*x)+4*arcsin(c*x)*c^4*d*e*x^2+I*c^4*d^2+2*I*
c^4*d*e*x^2+I*e^2*c^4*x^4-(-c^2*x^2+1)^(1/2)*c^3*d*e*x-(-c^2*x^2+1)^(1/2)*
e^2*c^3*x^3+6*c^2*d*e*arcsin(c*x)+4*arcsin(c*x)*e^2*c^2*x^2)/d^2/(c^2*e*x^
2+c^2*d)^2/(c^2*d+e)+5/8*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*arctanh
(1/4*(2*e*(I*c*x+(-c^2*x^2+1)^(1/2))^2-4*c^2*d-2*e)/(c^4*d^2+c^2*d*e)^(1/2)
))*e-I/(c^2*d+e)*c^2/d^2*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))+I/(c^2*d+e)*c^2
/d^2*dilog(I*c*x+(-c^2*x^2+1)^(1/2))+1/(c^2*d+e)*c^2/d^2*arcsin(c*x)*ln(1+
I*c*x+(-c^2*x^2+1)^(1/2))+1/4*I/(c^2*d+e)*c^2/d^2*sum((_R1^2-1)/(_R1^2*e-2
*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1
-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))
*e+1/(c^2*d+e)/d^3*e*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+1/4*I/(c^2
*d+e)/d^3*e*sum((_R1^2*e-4*c^2*d-e)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln(
(_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_
R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/4*I/(c^2*d+e)/d^3*e^2*sum
((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(
1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-
4*c^2*d-2*e)*_Z^2+e))+3/4*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^2*c^2*ar
ctanh(1/4*(2*e*(I*c*x+(-c^2*x^2+1)^(1/2))^2-4*c^2*d-2*e)/(c^4*d^2+c^2*d*e)
^(1/2))-I/(c^2*d+e)/d^3*e*dilog(1+I*c*x+(-c^2*x^2+1)^(1/2))+I/(c^2*d+e)...

```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^3 x} dx$$

input

```
integrate((a+b*arcsin(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output

```
integral((b*arcsin(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x(d + ex^2)^3} dx$$

input `integrate((a+b*asin(c*x))/x/(e*x**2+d)**3,x)`

output `Integral((a + b*asin(c*x))/(x*(d + e*x**2)**3), x)`

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^3 x} dx$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")`

output `1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)/d^3 + 4*log(x)/d^3) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)`

Giac [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = \text{Timed out}$$

input `integrate((a+b*arcsin(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")`

output `Timed out`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x(e x^2 + d)^3} dx$$

input `int((a + b*asin(c*x))/(x*(d + e*x^2)^3),x)`output `int((a + b*asin(c*x))/(x*(d + e*x^2)^3), x)`**Reduce [F]**

$$\int \frac{a + b \arcsin(cx)}{x(d + ex^2)^3} dx$$

$$= \frac{4 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^7 + 3d e^2 x^5 + 3d^2 e x^3 + d^3} dx \right) b d^5 + 8 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^7 + 3d e^2 x^5 + 3d^2 e x^3 + d^3} dx \right) b d^4 e x^2 + 4 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^7 + 3d e^2 x^5 + 3d^2 e x^3 + d^3} dx \right)}$$

input `int((a+b*asin(c*x))/x/(e*x^2+d)^3,x)`output `(4*int(asin(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*b*d**5 + 8*int(asin(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*b*d**4*e*x**2 + 4*int(asin(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*b*d**3*e**2*x**4 - 2*log(d + e*x**2)*a*d**2 - 4*log(d + e*x**2)*a*d*e*x**2 - 2*log(d + e*x**2)*a*e**2*x**4 + 4*log(x)*a*d**2 + 8*log(x)*a*d*e*x**2 + 4*log(x)*a*e**2*x**4 + 2*a*d**2 - a*e**2*x**4)/(4*d**3*(d**2 + 2*d*e*x**2 + e**2*x**4))`

$$3.471 \quad \int \frac{a+b \arcsin(cx)}{x^3(d+ex^2)^3} dx$$

Optimal result	4003
Mathematica [A] (warning: unable to verify)	4004
Rubi [A] (verified)	4005
Maple [C] (warning: unable to verify)	4007
Fricas [F]	4008
Sympy [F(-1)]	4009
Maxima [F]	4009
Giac [F(-1)]	4010
Mupad [F(-1)]	4010
Reduce [F]	4010

Optimal result

Integrand size = 21, antiderivative size = 783

$$\begin{aligned}
\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = & -\frac{bc\sqrt{1-c^2x^2}}{2d^3x} + \frac{bce^2x\sqrt{1-c^2x^2}}{8d^3(c^2d+e)(d+ex^2)} - \frac{a+b\arcsin(cx)}{2d^3x^2} \\
& - \frac{e(a+b\arcsin(cx))}{4d^2(d+ex^2)^2} - \frac{e(a+b\arcsin(cx))}{d^3(d+ex^2)} \\
& + \frac{bce \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{7/2}\sqrt{c^2d+e}} + \frac{bce(2c^2d+e) \arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{7/2}(c^2d+e)^{3/2}} \\
& + \frac{3e(a+b\arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4} \\
& + \frac{3e(a+b\arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4} \\
& + \frac{3e(a+b\arcsin(cx)) \log\left(1 - \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4} \\
& + \frac{3e(a+b\arcsin(cx)) \log\left(1 + \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4} \\
& - \frac{3e(a+b\arcsin(cx)) \log\left(1 - e^{2i\arcsin(cx)}\right)}{d^4} \\
& - \frac{3ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4} \\
& - \frac{3ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4} \\
& - \frac{3ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4} \\
& - \frac{3ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4} \\
& + \frac{3ibe \operatorname{PolyLog}\left(2, e^{2i\arcsin(cx)}\right)}{2d^4}
\end{aligned}$$

output

```

-1/2*b*c*(-c^2*x^2+1)^(1/2)/d^3/x+1/8*b*c*e^2*x*(-c^2*x^2+1)^(1/2)/d^3/(c^
2*d+e)/(e*x^2+d)-1/2*(a+b*arcsin(c*x))/d^3/x^2-1/4*e*(a+b*arcsin(c*x))/d^2
/(e*x^2+d)^2-e*(a+b*arcsin(c*x))/d^3/(e*x^2+d)+b*c*e*arctan((c^2*d+e)^(1/2)
)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(7/2)/(c^2*d+e)^(1/2)+1/8*b*c*e*(2*c^2*d
+e)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(7/2)/(c^2*d+e)
^(3/2)+3/2*e*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*
c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4+3/2*e*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I
*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4+3/2*e*(a+b*
arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/d^4+3/2*e*(a+b*arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)
^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^4-3*e*(a+b*arcsin(c*x))*ln(1-(I
*c*x+(-c^2*x^2+1)^(1/2))^2)/d^4+3/2*I*b*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2)
)^2)/d^4-3/2*I*b*e*polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(
-d)^(1/2)+(c^2*d+e)^(1/2)))/d^4-3/2*I*b*e*polylog(2,-e^(1/2)*(I*c*x+(-c^2*
x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4-3/2*I*b*e*polylog(2,e
^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4-3/2
*I*b*e*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d
+e)^(1/2)))/d^4

```

Mathematica [A] (warning: unable to verify)

Time = 5.30 (sec) , antiderivative size = 1065, normalized size of antiderivative = 1.36

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(x^3*(d + e*x^2)^3),x]
```

output

```

((-8*a*d)/x^2 - (4*a*d^2*e)/(d + e*x^2)^2 - (16*a*d*e)/(d + e*x^2) - 48*a*
e*Log[x] + 24*a*e*Log[d + e*x^2] + b*((-8*c*d*Sqrt[1 - c^2*x^2])/x + (c*d*
e^(3/2)*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) + (c*d*
e^(3/2)*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - (8*d*A
rcSin[c*x])/x^2 - (9*Sqrt[d]*e*ArcSin[c*x])/(Sqrt[d] - I*Sqrt[e]*x) - (d*e
*ArcSin[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 - (9*Sqrt[d]*e*ArcSin[c*x])/(Sqrt[
d] + I*Sqrt[e]*x) + (d*e*ArcSin[c*x])/(I*Sqrt[d] + Sqrt[e]*x)^2 + (9*c*Sqr
t[d]*e*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^
2])])/Sqrt[c^2*d + e] - ((9*I)*c*Sqrt[d]*e*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d
]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e] + 24*e*ArcSin[c
*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt[c^2*d + e])] + 2
4*e*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d] + Sqrt[c
^2*d + e])] + 24*e*ArcSin[c*x]*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt
[d] + Sqrt[c^2*d + e])] + 24*e*ArcSin[c*x]*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*
x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])] - 48*e*ArcSin[c*x]*Log[1 - E^((2*I)*Ar
cSin[c*x])] + (I*c^3*d^(3/2)*e*Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqr
t[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x
)))]/(c^2*d + e)^(3/2) - (I*c^3*d^(3/2)*e*Log[(e*Sqrt[c^2*d + e]*(Sqrt[e]
+ I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt[d
]*Sqrt[e]*x)))]/(c^2*d + e)^(3/2) - (24*I)*e*PolyLog[2, (Sqrt[e]*E^(I*A...

```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 783, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx$$

↓ 5232

$$\int \left(\frac{3e^2 x (a + b \arcsin(cx))}{d^4 (d + ex^2)} - \frac{3e (a + b \arcsin(cx))}{d^4 x} + \frac{2e^2 x (a + b \arcsin(cx))}{d^3 (d + ex^2)^2} + \frac{a + b \arcsin(cx)}{d^3 x^3} + \frac{e^2 x (a + b \arcsin(cx))}{d^2 (d + ex^2)} \right) dx$$

↓ 2009

$$\begin{aligned}
& \frac{3e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^4} + \\
& \frac{3e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{-\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^4} + \\
& \frac{3e(a + b \arcsin(cx)) \log\left(1 - \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^4} + \frac{3e(a + b \arcsin(cx)) \log\left(1 + \frac{\sqrt{ee^i \arcsin(cx)}}{\sqrt{c^2d+e+ic\sqrt{-d}}}\right)}{2d^4} - \\
& \frac{3e \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx))}{d^4} - \frac{e(a + b \arcsin(cx))}{d^3(d + ex^2)} - \frac{a + b \arcsin(cx)}{2d^3x^2} - \\
& \frac{e(a + b \arcsin(cx))}{4d^2(d + ex^2)^2} - \frac{3ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^4} - \frac{3ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{2d^4} - \\
& \frac{3ibe \operatorname{PolyLog}\left(2, -\frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2d^4} - \frac{3ibe \operatorname{PolyLog}\left(2, \frac{\sqrt{ee^i \arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{2d^4} + \\
& \frac{3ibe \operatorname{PolyLog}\left(2, e^{2i \arcsin(cx)}\right)}{2d^4} + \frac{bce(2c^2d + e) \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{7/2}(c^2d + e)^{3/2}} + \\
& \frac{bce \arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{7/2}\sqrt{c^2d + e}} + \frac{bce^2x\sqrt{1-c^2x^2}}{8d^3(c^2d + e)(d + ex^2)} - \frac{bc\sqrt{1-c^2x^2}}{2d^3x}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(x^3*(d + e*x^2)^3), x]
```

output

```

-1/2*(b*c*Sqrt[1 - c^2*x^2])/(d^3*x) + (b*c*e^2*x*Sqrt[1 - c^2*x^2])/(8*d^
3*(c^2*d + e)*(d + e*x^2)) - (a + b*ArcSin[c*x])/(2*d^3*x^2) - (e*(a + b*A
rcSin[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcSin[c*x]))/(d^3*(d + e*x
^2)) + (b*c*e*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(d^
(7/2)*Sqrt[c^2*d + e]) + (b*c*e*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)/(
Sqrt[d]*Sqrt[1 - c^2*x^2])])/(8*d^(7/2)*(c^2*d + e)^(3/2)) + (3*e*(a + b*A
rcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d
+ e])])/(2*d^4) + (3*e*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c
*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(2*d^4) + (3*e*(a + b*ArcSin[c*x]
)*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(
2*d^4) + (3*e*(a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c
*Sqrt[-d] + Sqrt[c^2*d + e])])/(2*d^4) - (3*e*(a + b*ArcSin[c*x])*Log[1 -
E^((2*I)*ArcSin[c*x])]/d^4 - (((3*I)/2)*b*e*PolyLog[2, -(Sqrt[e]*E^(I*Ar
cSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])]/d^4 - (((3*I)/2)*b*e*PolyL
og[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])]/d^4 -
(((3*I)/2)*b*e*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + S
qrt[c^2*d + e])])]/d^4 - (((3*I)/2)*b*e*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*
x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])]/d^4 + (((3*I)/2)*b*e*PolyLog[2, E^
((2*I)*ArcSin[c*x])]/d^4

```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232

```
Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)*((f_)*(x_))^(m_)*((d_) + (e_
)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.61 (sec) , antiderivative size = 1344, normalized size of antiderivative = 1.72

method	result	size
parts	Expression too large to display	1344
derivativedivides	Expression too large to display	1395
default	Expression too large to display	1395

input `int((a+b*arcsin(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```
-1/2*a/d^3/x^2-3*a/d^4*e*ln(x)+3/2*a/d^4*e*ln(e*x^2+d)-1/4*a/d^2*e/(e*x^2+d)^2-a/d^3*e/(e*x^2+d)+b*c^2*(-1/8*(-8*I*c^8*d^2*e*x^4-6*I*c^6*d*e^2*x^4-4*I*c^8*d*e^2*x^6+4*(-c^2*x^2+1)^(1/2)*c^7*d^3*x+8*(-c^2*x^2+1)^(1/2)*c^7*d^2*e*x^3+4*(-c^2*x^2+1)^(1/2)*c^7*d*e^2*x^5-3*I*e^3*c^6*x^6-4*I*c^8*d^3*x^2-3*I*c^6*d^2*e*x^2+4*c^6*d^3*arcsin(c*x)+18*arcsin(c*x)*c^6*d^2*e*x^2+12*arcsin(c*x)*c^6*d*e^2*x^4+4*(-c^2*x^2+1)^(1/2)*c^5*d^2*e*x+7*(-c^2*x^2+1)^(1/2)*c^5*d*e^2*x^3+3*(-c^2*x^2+1)^(1/2)*e^3*c^5*x^5+4*c^4*d^2*e*arcsin(c*x)+18*arcsin(c*x)*c^4*d*e^2*x^2+12*arcsin(c*x)*e^3*c^4*x^4)/c^2/x^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2/d^3-3/(c^2*d+e)/d^3*e*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-9/8*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^4/c^2*arctanh(1/4*(2*e*(I*c*x+(-c^2*x^2+1)^(1/2))^2-4*c^2*d-2*e)/(c^4*d^2+c^2*d*e)^(1/2))*e^2-3/(c^2*d+e)*e^2/d^4/c^2*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-3/4*I/(c^2*d+e)*e^2/d^4*sum((_R1^2*e-4*c^2*d-e)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-3/4*I/(c^2*d+e)*e^3/d^4*sum((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))/c^2-3/4*I/(c^2*d+e)*e^2/d^3*sum((_R1^2-1)/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^...
```

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^3 x^3} dx$$

input `integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")`

output

```
integral((b*arcsin(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = \text{Timed out}$$

input

```
integrate((a+b*asin(c*x))/x**3/(e*x**2+d)**3,x)
```

output

Timed out

Maxima [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^3 x^3} dx$$

input

```
integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

output

```
-1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x^2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + b*integrate(arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = \text{Timed out}$$

input `integrate((a+b*arcsin(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")`

output Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{x^3 (ex^2 + d)^3} dx$$

input `int((a + b*asin(c*x))/(x^3*(d + e*x^2)^3),x)`

output `int((a + b*asin(c*x))/(x^3*(d + e*x^2)^3), x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{x^3 (d + ex^2)^3} dx$$

$$= \frac{4 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^9 + 3d e^2 x^7 + 3d^2 e x^5 + d^3 x^3} dx \right) b d^6 x^2 + 8 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^9 + 3d e^2 x^7 + 3d^2 e x^5 + d^3 x^3} dx \right) b d^5 e x^4 + 4 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^9 + 3d e^2 x^7 + 3d^2 e x^5} dx \right)}$$

input `int((a+b*asin(c*x))/x^3/(e*x^2+d)^3,x)`

output

```
(4*int(asin(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7 + e**3*x**9),x)
)*b*d**6*x**2 + 8*int(asin(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7
+ e**3*x**9),x)*b*d**5*e*x**4 + 4*int(asin(c*x)/(d**3*x**3 + 3*d**2*e*x**
5 + 3*d*e**2*x**7 + e**3*x**9),x)*b*d**4*e**2*x**6 + 6*log(d + e*x**2)*a*d
**2*e*x**2 + 12*log(d + e*x**2)*a*d*e**2*x**4 + 6*log(d + e*x**2)*a*e**3*x
**6 - 12*log(x)*a*d**2*e*x**2 - 24*log(x)*a*d*e**2*x**4 - 12*log(x)*a*e**3
*x**6 - 2*a*d**3 - 6*a*d**2*e*x**2 + 3*a*e**3*x**6)/(4*d**4*x**2*(d**2 + 2
*d*e*x**2 + e**2*x**4))
```


3.472 $\int \frac{x^4(a+b \arcsin(cx))}{(d+ex^2)^3} dx$

Optimal result 4012
 Mathematica [A] (warning: unable to verify) 4013
 Rubi [A] (verified) 4014
 Maple [C] (warning: unable to verify) 4017
 Fricas [F] 4017
 Sympy [F] 4018
 Maxima [F(-2)] 4018
 Giac [F] 4019
 Mupad [F(-1)] 4019
 Reduce [F] 4019

Optimal result

Integrand size = 21, antiderivative size = 1082

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

output

```

1/16*b*c*(-d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)-e^(1/2)*x
)+1/16*b*c*(-d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)+e^(1/2)
*x)-1/16*(-d)^(1/2)*(a+b*arcsin(c*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)^2+5/1
6*(a+b*arcsin(c*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(-d)^(1/2)*(a+b*ar
csin(c*x))/e^(5/2)/((-d)^(1/2)+e^(1/2)*x)^2-5/16*(a+b*arcsin(c*x))/e^(5/2)
/((-d)^(1/2)+e^(1/2)*x)+1/16*b*c^3*d*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c
^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(3/2)-5/16*b*c*arctanh
((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c
^2*d+e)^(1/2)+1/16*b*c^3*d*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1
/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(3/2)-5/16*b*c*arctanh((e^(1/2)+
c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(1
/2)+3/16*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-
d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arcsin(c*x))*ln(1+
e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(
1/2)/e^(5/2)+3/16*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)
))/I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arcsin(c
*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(5/2)-3/16*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/
2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)+3/16*I*b*polylog(
2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))...

```

Mathematica [A] (warning: unable to verify)

Time = 3.77 (sec) , antiderivative size = 1014, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(x^4*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]
```

output

```

(((−I)*b*c*Sqrt[d]*Sqrt[e]*Sqrt[1 − c^2*x^2])/((c^2*d + e)*((−I)*Sqrt[d] +
Sqrt[e]*x)) + (I*b*c*Sqrt[d]*Sqrt[e]*Sqrt[1 − c^2*x^2])/((c^2*d + e)*(I*S
qrt[d] + Sqrt[e]*x)) + (4*a*d*Sqrt[e]*x)/(d + e*x^2)^2 − (10*a*Sqrt[e]*x)/
(d + e*x^2) + (I*b*Sqrt[d]*ArcSin[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (I*b*S
qrt[d]*ArcSin[c*x])/(I*Sqrt[d] + Sqrt[e]*x)^2 − (5*b*ArcSin[c*x])/(I*Sqrt[
d] + Sqrt[e]*x) + (6*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] − (5*I)*b*(Arc
Sin[c*x])/(Sqrt[d] + I*Sqrt[e]*x) − (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(
Sqrt[c^2*d + e]*Sqrt[1 − c^2*x^2])])/Sqrt[c^2*d + e] − (5*b*c*ArcTanh[(Sq
rt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 − c^2*x^2])])/Sqrt[c^2*d
+ e] + ((3*I)*b*ArcSin[c*x]*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(−(c*Sqrt
[d] + Sqrt[c^2*d + e])) + Log[1 − (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d]
+ Sqrt[c^2*d + e])))/Sqrt[d] − ((3*I)*b*ArcSin[c*x]*(Log[1 + (Sqrt[e]*E^(
I*ArcSin[c*x]))]/(c*Sqrt[d] − Sqrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*Arc
Sin[c*x]))]/(c*Sqrt[d] + Sqrt[c^2*d + e])))/Sqrt[d] + (b*c^3*d*(Log[4] + L
og[(e*Sqrt[c^2*d + e]*(Sqrt[e] − I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1
− c^2*x^2])]/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x)))])/(c^2*d + e)^(3/2) + (b*c^3
*d*(Log[4] + Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*
d + e]*Sqrt[1 − c^2*x^2])]/(c^3*(d − I*Sqrt[d]*Sqrt[e]*x)))])/(c^2*d + e)^(
3/2) + (3*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] − Sqrt[c^2*
d + e]))/Sqrt[d] − (3*b*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))]/(−(c*Sq...

```

Rubi [A] (verified)

Time = 3.81 (sec) , antiderivative size = 1082, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{d^2(a + b \arcsin(cx))}{e^2(d + ex^2)^3} - \frac{2d(a + b \arcsin(cx))}{e^2(d + ex^2)^2} + \frac{a + b \arcsin(cx)}{e^2(d + ex^2)} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& \frac{b \operatorname{darctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c^3}{16e^{5/2}(dc^2+e)^{3/2}} + \frac{b \operatorname{darctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c^3}{16e^{5/2}(dc^2+e)^{3/2}} - \\
& \frac{5b \operatorname{arctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c}{16e^{5/2}\sqrt{dc^2+e}} - \frac{5b \operatorname{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c}{16e^{5/2}\sqrt{dc^2+e}} + \\
& \frac{b\sqrt{-d}\sqrt{1-c^2x^2}c}{16e^2(dc^2+e)(\sqrt{-d}-\sqrt{ex})} + \frac{b\sqrt{-d}\sqrt{1-c^2x^2}c}{16e^2(dc^2+e)(\sqrt{ex}+\sqrt{-d})} + \frac{5(a+b\arcsin(cx))}{16e^{5/2}(\sqrt{-d}-\sqrt{ex})} - \\
& \frac{5(a+b\arcsin(cx))}{16e^{5/2}(\sqrt{ex}+\sqrt{-d})} - \frac{\sqrt{-d}(a+b\arcsin(cx))}{16e^{5/2}(\sqrt{-d}-\sqrt{ex})^2} + \frac{\sqrt{-d}(a+b\arcsin(cx))}{16e^{5/2}(\sqrt{ex}+\sqrt{-d})^2} + \\
& \frac{3(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\arcsin(cx))\log\left(\frac{e^{i\arcsin(cx)}\sqrt{e}}{ic\sqrt{-d}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} + \\
& \frac{3(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{e}e^{i\arcsin(cx)}}{i\sqrt{-d}c+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\arcsin(cx))\log\left(\frac{e^{i\arcsin(cx)}\sqrt{e}}{i\sqrt{-d}c+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}} + \\
& \frac{3ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} + \\
& \frac{3ib \operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i\arcsin(cx)}}{i\sqrt{-d}c+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib \operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i\arcsin(cx)}}{i\sqrt{-d}c+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}
\end{aligned}$$

input

```
Int[(x^4*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]
```

output

$$\begin{aligned}
& (b*c*\sqrt{-d}*\sqrt{1 - c^2*x^2})/(16*e^2*(c^2*d + e)*(\sqrt{-d} - \sqrt{e}*x \\
&)) + (b*c*\sqrt{-d}*\sqrt{1 - c^2*x^2})/(16*e^2*(c^2*d + e)*(\sqrt{-d} + \sqrt{e}*x \\
&)) - (\sqrt{-d}*(a + b*\text{ArcSin}[c*x]))/(16*e^{5/2}*(\sqrt{-d} - \sqrt{e}*x \\
&)^2) + (5*(a + b*\text{ArcSin}[c*x]))/(16*e^{5/2}*(\sqrt{-d} - \sqrt{e}*x)) + (\sqrt{-d}*(a + b*\text{ArcSin}[c*x]))/(16*e^{5/2}*(\sqrt{-d} + \sqrt{e}*x)^2) - (5*(a + b*\text{ArcSin}[c*x]))/(16*e^{5/2}*(\sqrt{-d} + \sqrt{e}*x)) + (b*c^3*d*\text{ArcTanh}[(\sqrt{e} - c^2*\sqrt{-d}*x)/(\sqrt{c^2*d + e}*\sqrt{1 - c^2*x^2})])/(16*e^{5/2}*(c^2*d + e)^{3/2}) - (5*b*c*\text{ArcTanh}[(\sqrt{e} - c^2*\sqrt{-d}*x)/(\sqrt{c^2*d + e}*\sqrt{1 - c^2*x^2})])/(16*e^{5/2}*\sqrt{c^2*d + e}) + (b*c^3*d*\text{ArcTanh}[(\sqrt{e} + c^2*\sqrt{-d}*x)/(\sqrt{c^2*d + e}*\sqrt{1 - c^2*x^2})])/(16*e^{5/2}*(c^2*d + e)^{3/2}) - (5*b*c*\text{ArcTanh}[(\sqrt{e} + c^2*\sqrt{-d}*x)/(\sqrt{c^2*d + e}*\sqrt{1 - c^2*x^2})])/(16*e^{5/2}*\sqrt{c^2*d + e}) + (3*(a + b*\text{ArcSin}[c*x])*Log[1 - (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e})])/(16*\sqrt{-d}*e^{5/2}) - (3*(a + b*\text{ArcSin}[c*x])*Log[1 + (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e})])/(16*\sqrt{-d}*e^{5/2}) + (3*(a + b*\text{ArcSin}[c*x])*Log[1 - (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} + \sqrt{c^2*d + e})])/(16*\sqrt{-d}*e^{5/2}) - (3*(a + b*\text{ArcSin}[c*x])*Log[1 + (\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} + \sqrt{c^2*d + e})])/(16*\sqrt{-d}*e^{5/2}) + (((3*I)/16)*b*PolyLog[2, -((\sqrt{e}*E^{(I*\text{ArcSin}[c*x])})/(I*c*\sqrt{-d} - \sqrt{c^2*d + e}))])/(\sqrt{-d}*e^{5/2}) - (((3*I)/16)*b*Pol...
\end{aligned}$$

Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x_Symbol] \text{ :> Simp[IntSum}[u, x], x] \text{ /; SumQ}[u]$$

rule 5232

$$\text{Int}[\{(a_.) + \text{ArcSin}[(c_.)*(x_)]*(b_.)\}^{(n_.)}\{(f_.)*(x_)\}^{(m_.)}\{(d_) + (e_.)*(x_)^2\}^{(p_.)}, x_Symbol] \text{ :> Int[ExpandIntegrand}[(a + b*\text{ArcSin}[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.80 (sec) , antiderivative size = 1752, normalized size of antiderivative = 1.62

Expression too large to display

input `int(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^3,x)`

output

```
1/c^5*(a*c^6*((-5/8/e*c^3*x^3-3/8/e^2*c^3*d*x)/(c^2*e*x^2+c^2*d)^2+3/8/e^2
/c/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))+b*c^6*(-1/8*(5*arcsin(c*x)*d*c^5*e
*x^3+3*arcsin(c*x)*d^2*c^5*x-c^4*d*e*x^2*(-c^2*x^2+1)^(1/2)-d^2*c^4*(-c^2*
x^2+1)^(1/2)+5*arcsin(c*x)*c^3*x^3*e^2+3*arcsin(c*x)*c^3*x*d*e)/e^2/(c^2*d
+e)/(c^2*e*x^2+c^2*d)^2+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2
)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(
1/2)*e)*c^2*d*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^5-5/8*((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(I*c*
x+(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4
/(c^2*d+e)+3/16/(c^2*d+e)/e*sum(_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln(
_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_
R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))-1/2*(-e*(2*c^2*d-2*(c^2*d*(
c^2*d+e))^(1/2)+e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*c^2*d*arct
an(e*(I*c*x+(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)
^(1/2))/e^5/(c^2*d+e)-1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*
(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2))
/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*c^2*d/e^5/(c^2*d+e)+5/8*
((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2
))*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(I*c*x...
```

Fricas [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(ex^2 + d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="fricas")`

output `integral((b*x^4*arcsin(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(d + ex^2)^3} dx$$

input `integrate(x**4*(a+b*asin(c*x))/(e*x**2+d)**3,x)`

output `Integral(x**4*(a + b*asin(c*x))/(d + e*x**2)**3, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^4}{(ex^2 + d)^3} dx$$

input `integrate(x^4*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)*x^4/(e*x^2 + d)^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^4(a + b \operatorname{asin}(cx))}{(ex^2 + d)^3} dx$$

input `int((x^4*(a + b*asin(c*x)))/(d + e*x^2)^3,x)`

output `int((x^4*(a + b*asin(c*x)))/(d + e*x^2)^3, x)`

Reduce [F]

$$\int \frac{x^4(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$= \frac{3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a d^2 + 6\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a d e x^2 + 3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a e^2 x^4 + 8 \left(\int \frac{\operatorname{asin}}{e^3 x^6 + 3d e^2 x} \right)}{8d}$$

input `int(x^4*(a+b*asin(c*x))/(e*x^2+d)^3,x)`

output

```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d)
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int((asin(c*x)*x**4)/(d**3 + 3*d**2*e*x
**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3 + 16*int((asin(c*x)*x**4)/
(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**2*e**4*x**2 + 8
*int((asin(c*x)*x**4)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x
)*b*d*e**5*x**4 - 3*a*d**2*e*x - 5*a*d*e**2*x**3)/(8*d*e**3*(d**2 + 2*d*e*
x**2 + e**2*x**4))
```

3.473 $\int \frac{x^2(a+b \arcsin(cx))}{(d+ex^2)^3} dx$

Optimal result	4021
Mathematica [A] (warning: unable to verify)	4022
Rubi [A] (verified)	4023
Maple [C] (warning: unable to verify)	4026
Fricas [F]	4027
Sympy [F]	4027
Maxima [F(-2)]	4027
Giac [F]	4028
Mupad [F(-1)]	4028
Reduce [F]	4028

Optimal result

Integrand size = 21, antiderivative size = 1092

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

output

```

1/16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(1/2)/e/(c^2*d+e)/((-d)^(1/2)-e^(1/2)*x)+
1/16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(1/2)/e/(c^2*d+e)/((-d)^(1/2)+e^(1/2)*x)-
1/16*(a+b*arcsin(c*x))/(-d)^(1/2)/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)^2-1/16*(a
+b*arcsin(c*x))/d/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(a+b*arcsin(c*x))/(-
d)^(1/2)/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)^2+1/16*(a+b*arcsin(c*x))/d/e^(3/2)
/((-d)^(1/2)+e^(1/2)*x)-1/16*b*c^3*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2
*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(3/2)/(c^2*d+e)^(3/2)+1/16*b*c*arctanh((
e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(3/2)/(c
^2*d+e)^(1/2)-1/16*b*c^3*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)
)/(-c^2*x^2+1)^(1/2))/e^(3/2)/(c^2*d+e)^(3/2)+1/16*b*c*arctanh((e^(1/2)+c^
2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(3/2)/(c^2*d+e)^(1
/2)-1/16*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-
d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsin(c*x))*ln(1+
e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(3/2)-1/16*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)
))/I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arcsin(c
*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/
2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(
2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))...

```

Mathematica [A] (warning: unable to verify)

Time = 4.02 (sec) , antiderivative size = 1014, normalized size of antiderivative = 0.93

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]
```

output

```

(((2*I)*b*c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d]
+ Sqrt[e]*x)) - ((2*I)*b*c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[d]*(c^2*d + e)
*(I*Sqrt[d] + Sqrt[e]*x)) - (8*a*Sqrt[e]*x)/(d + e*x^2)^2 + (4*a*Sqrt[e]*x
)/(d^2 + d*e*x^2) + ((2*I)*b*ArcSin[c*x])/(Sqrt[d]*(Sqrt[d] - I*Sqrt[e]*x)
^2) - ((2*I)*b*ArcSin[c*x])/(Sqrt[d]*(Sqrt[d] + I*Sqrt[e]*x)^2) + (4*a*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + ((2*I)*b*(ArcSin[c*x]/(Sqrt[d] + I*Sqr
t[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 -
c^2*x^2])])/Sqrt[c^2*d + e]))/d + (2*b*(ArcSin[c*x]/(I*Sqrt[d] + Sqrt[e]*x
) + (c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x
^2])])/Sqrt[c^2*d + e]))/d - (2*b*c^3*(Log[4] + Log[(e*Sqrt[c^2*d + e]*(Sqr
t[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*
Sqrt[d]*Sqrt[e]*x))]))/(c^2*d + e)^(3/2) - (2*b*c^3*(Log[4] + Log[(e*Sqrt[
c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])
))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(c^2*d + e)^(3/2) - (b*(ArcSin[c*x]*(
ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] - Sqrt
[c^2*d + e]]) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*
d + e]])) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt
[c^2*d + e]]) + 2*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sq
rt[c^2*d + e]))))/d^(3/2) + (b*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 +
(Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e]]) + Log[1 ...

```

Rubi [A] (verified)

Time = 3.16 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$\downarrow \text{5232}$$

$$\int \left(\frac{a + b \arcsin(cx)}{e(d + ex^2)^2} - \frac{d(a + b \arcsin(cx))}{e(d + ex^2)^3} \right) dx$$

$$\downarrow \text{2009}$$

$$\begin{aligned}
& - \frac{\operatorname{barctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c^3}{16e^{3/2}(dc^2+e)^{3/2}} - \frac{\operatorname{barctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c^3}{16e^{3/2}(dc^2+e)^{3/2}} + \frac{\operatorname{barctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c}{16de^{3/2}\sqrt{dc^2+e}} + \\
& \frac{\operatorname{barctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right) c}{16de^{3/2}\sqrt{dc^2+e}} + \frac{b\sqrt{1-c^2x^2}c}{16\sqrt{-de}(dc^2+e)(\sqrt{-d}-\sqrt{ex})} + \\
& \frac{b\sqrt{1-c^2x^2}c}{16\sqrt{-de}(dc^2+e)(\sqrt{ex}+\sqrt{-d})} - \frac{a+b\arcsin(cx)}{16de^{3/2}(\sqrt{-d}-\sqrt{ex})} + \frac{a+b\arcsin(cx)}{16de^{3/2}(\sqrt{ex}+\sqrt{-d})} - \\
& \frac{a+b\arcsin(cx)}{16\sqrt{-de}^{3/2}(\sqrt{-d}-\sqrt{ex})^2} + \frac{a+b\arcsin(cx)}{16\sqrt{-de}^{3/2}(\sqrt{ex}+\sqrt{-d})^2} - \\
& \frac{(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\arcsin(cx))\log\left(\frac{e^{i\arcsin(cx)}\sqrt{e}}{ic\sqrt{-d}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}} - \\
& \frac{(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{e}e^{i\arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\arcsin(cx))\log\left(\frac{e^{i\arcsin(cx)}\sqrt{e}}{i\sqrt{-dc}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}} - \\
& \frac{ib\operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \\
& \frac{ib\operatorname{PolyLog}\left(2, -\frac{\sqrt{e}e^{i\arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2, \frac{\sqrt{e}e^{i\arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}
\end{aligned}$$

input

```
Int[(x^2*(a + b*ArcSin[c*x]))/(d + e*x^2)^3,x]
```

output

```
(b*c*Sqrt[1 - c^2*x^2])/(16*Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] - Sqrt[e]*x))
+ (b*c*Sqrt[1 - c^2*x^2])/(16*Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] + Sqrt[e]*
x)) - (a + b*ArcSin[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)^2) -
(a + b*ArcSin[c*x])/(16*d*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcSin
[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)^2) + (a + b*ArcSin[c*x]
)/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c^3*ArcTanh[(Sqrt[e] - c^2*Sq
rt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(3/2)*(c^2*d + e)^(3
/2)) + (b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c
^2*x^2])])/(16*d*e^(3/2)*Sqrt[c^2*d + e]) - (b*c^3*ArcTanh[(Sqrt[e] + c^2*
Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(3/2)*(c^2*d + e)^(
3/2)) + (b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 -
c^2*x^2])])/(16*d*e^(3/2)*Sqrt[c^2*d + e]) - ((a + b*ArcSin[c*x])*Log[1 -
(Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(16*(-d)^(
3/2)*e^(3/2)) + ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(
I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSin
[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e]
)])/((16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*A
rcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) -
((I/16)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^
2*d + e]))])/((-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2, (Sqrt[e]*E^(I*...
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)^n_.)*((f_.)*(x_)^m_.)*((d_) + (e_
.)*(x_)^2)^p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.56 (sec) , antiderivative size = 1224, normalized size of antiderivative = 1.12

method	result	size
parts	Expression too large to display	1224
derivativeldivides	Expression too large to display	1231
default	Expression too large to display	1231

input `int(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))+b/c^3*(1/8*c^4*(arcsin(c*x)*d*c^5*e*x^3-arcsin(c*x)*d^2*c^5*x-c^4*d*e*x^2*(-c^2*x^2+1)^(1/2)-d^2*c^4*(-c^2*x^2+1)^(1/2)+arcsin(c*x)*c^3*x^3*e^2-arcsin(c*x)*c^3*x*d*e)/e/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1/8*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^4*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/d/e^3+1/8*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*c^4/(c^2*d+e)/d/e^3-1/8*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^4*d^2+2*(c^2*d*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*c^4*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/d/e^3+1/8*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(I*c*x+(-c^2*x^2+1)^(1/2)))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^4/(c^2*d+e)/d/e^3+1/16/(c^2*d+e)/d*c^4*sum(1/_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))+1/16/(c^2*d+e)/d*c^4*sum(_R1/(_R1^2*e-2*c^2*d-e)*(I*arcsin(c*x)*ln((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-I*c*x-(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(ex^2 + d)^3} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="fricas")`

output `integral((b*x^2*arcsin(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(d + ex^2)^3} dx$$

input `integrate(x**2*(a+b*asin(c*x))/(e*x**2+d)**3,x)`

output `Integral(x**2*(a + b*asin(c*x))/(d + e*x**2)**3, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{(b \arcsin(cx) + a)x^2}{(ex^2 + d)^3} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)*x^2/(e*x^2 + d)^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))}{(ex^2 + d)^3} dx$$

input `int((x^2*(a + b*asin(c*x)))/(d + e*x^2)^3,x)`

output `int((x^2*(a + b*asin(c*x)))/(d + e*x^2)^3, x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))}{(d + ex^2)^3} dx$$

$$= \frac{\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) a d^2 + 2\sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) a d e x^2 + \sqrt{e} \sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e} \sqrt{d}}\right) a e^2 x^4 + 8 \left(\int \frac{\operatorname{asin}(cx)}{e^3 x^6 + 3d e^2 x^4 + 3d^2 e x^2 + d^3} dx \right)}{8d^2 e^2}$$

input `int(x^2*(a+b*asin(c*x))/(e*x^2+d)^3,x)`

output

```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 2*sqrt(e)*sqrt(d)*
atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*a*e**2*x**4 + 8*int((asin(c*x)*x**2)/(d**3 + 3*d**2*e*x**2
+ 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2 + 16*int((asin(c*x)*x**2)/(d**
3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**2 + 8*int
((asin(c*x)*x**2)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*
d**2*e**4*x**4 - a*d**2*e*x + a*d*e**2*x**3)/(8*d**2*e**2*(d**2 + 2*d*e*x*
*2 + e**2*x**4))
```

$$3.474 \quad \int \frac{a+b \arcsin(cx)}{(d+ex^2)^3} dx$$

Optimal result	4030
Mathematica [A] (warning: unable to verify)	4031
Rubi [A] (verified)	4032
Maple [C] (warning: unable to verify)	4035
Fricas [F]	4036
Sympy [F]	4036
Maxima [F(-2)]	4036
Giac [F(-2)]	4037
Mupad [F(-1)]	4037
Reduce [F]	4037

Optimal result

Integrand size = 18, antiderivative size = 1092

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \text{Too large to display}$$

output

```

1/16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)-e^(1/2)*x)+1/
16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)+e^(1/2)*x)-1/16
*(a+b*arcsin(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)^2-3/16*(a+b*a
rcsin(c*x))/d^2/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(a+b*arcsin(c*x))/(-d)
^(3/2)/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)^2+3/16*(a+b*arcsin(c*x))/d^2/e^(1/2)
/((-d)^(1/2)+e^(1/2)*x)+1/16*b*c^3*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2
*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(1/2)/(c^2*d+e)^(3/2)+3/16*b*c*arctanh
((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d^2/e^(1/2)
/(c^2*d+e)^(1/2)+1/16*b*c^3*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(
1/2)/(-c^2*x^2+1)^(1/2))/d/e^(1/2)/(c^2*d+e)^(3/2)+3/16*b*c*arctanh((e^(1
/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d^2/e^(1/2)/(c^2
*d+e)^(1/2)+3/16*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))
/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arcsin(c*x
))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcsin(c*x))*ln(1-e^(1/2)*(I*c*x+(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*
arcsin(c*x))*ln(1+e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,e^(1/2)*(I*c*x+(-c^2*x^
2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*
polylog(2,-e^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)...

```

Mathematica [A] (warning: unable to verify)

Time = 4.39 (sec) , antiderivative size = 1033, normalized size of antiderivative = 0.95

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])/(d + e*x^2)^3,x]
```

output

```

((8*a*d^(3/2)*x)/(d + e*x^2)^2 + (12*a*Sqrt[d]*x)/(d + e*x^2) + (12*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((6*I)*b*Sqrt[d]*(ArcSin[c*x]/(Sqrt[d] + I*Sqrt[e]*x) - (c*ArcTan[(I*Sqrt[e] + c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e]))/Sqrt[e] - (6*b*Sqrt[d]*(-(ArcSin[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*ArcTanh[(Sqrt[e] + I*c^2*Sqrt[d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/Sqrt[c^2*d + e]))/Sqrt[e] + (2*I)*b*d*(-((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))) - ArcSin[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - (I*c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x)))]))/(Sqrt[e]*(c^2*d + e)^(3/2))) + 2*b*d*((I*c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) + (I*ArcSin[c*x])/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x)))]))/(Sqrt[e]*(c^2*d + e)^(3/2))) - (3*b*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))]/(c*Sqrt[d] - Sqrt[c^2*d + e])) + Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e]))]) + 2*PolyLog[2, (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2*PolyLog[2, -(Sqrt[e]*E^(I*ArcSin[c*x]))/(c*Sqrt[d] + Sqrt[c^2*d + e])])]/Sqrt[e] + (3*b*(ArcSin[c*x]*(ArcSin[c*x] + (2*I)*(Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(-(c*Sqrt[d]) + ...

```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5172, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx$$

↓ 5172

$$\int \left(-\frac{3e(a + b \arcsin(cx))}{8d^2(-de - e^2x^2)} - \frac{3e(a + b \arcsin(cx))}{16d^2(\sqrt{-d}\sqrt{e} - ex)^2} - \frac{3e(a + b \arcsin(cx))}{16d^2(\sqrt{-d}\sqrt{e} + ex)^2} - \frac{e^{3/2}(a + b \arcsin(cx))}{8(-d)^{3/2}(\sqrt{-d}\sqrt{e} - ex)^3} - \frac{e^{3/2}(a + b \arcsin(cx))}{8(-d)^{3/2}(\sqrt{-d}\sqrt{e} + ex)^3} \right) dx$$

↓ 2009

$$\begin{aligned}
& \frac{\operatorname{barctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16d\sqrt{e}(dc^2+e)^{3/2}} + \frac{\operatorname{barctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16d\sqrt{e}(dc^2+e)^{3/2}} + \frac{3\operatorname{barctanh}\left(\frac{\sqrt{e-c^2}\sqrt{-dx}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16d^2\sqrt{e}\sqrt{dc^2+e}} + \\
& \frac{3\operatorname{barctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16d^2\sqrt{e}\sqrt{dc^2+e}} + \frac{b\sqrt{1-c^2x^2}c}{16(-d)^{3/2}(dc^2+e)(\sqrt{-d}-\sqrt{ex})} + \\
& \frac{b\sqrt{1-c^2x^2}c}{16(-d)^{3/2}(dc^2+e)(\sqrt{ex}+\sqrt{-d})} - \frac{3(a+b\arcsin(cx))}{16d^2\sqrt{e}(\sqrt{-d}-\sqrt{ex})(a+b\arcsin(cx))} + \frac{3(a+b\arcsin(cx))}{16d^2\sqrt{e}(\sqrt{ex}+\sqrt{-d})(a+b\arcsin(cx))} - \\
& \frac{16(-d)^{3/2}\sqrt{e}(\sqrt{-d}-\sqrt{ex})^2}{3(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{ee^i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)} + \frac{16(-d)^{3/2}\sqrt{e}(\sqrt{ex}+\sqrt{-d})^2}{3(a+b\arcsin(cx))\log\left(\frac{e^i\arcsin(cx)\sqrt{e}}{ic\sqrt{-d}-\sqrt{dc^2+e}}+1\right)} + \\
& \frac{3(a+b\arcsin(cx))\log\left(1-\frac{\sqrt{ee^i\arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3(a+b\arcsin(cx))\log\left(\frac{e^i\arcsin(cx)\sqrt{e}}{i\sqrt{-dc}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}} + \\
& \frac{3ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{ee^i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{\sqrt{ee^i\arcsin(cx)}}{ic\sqrt{-d}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \\
& \frac{3ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{ee^i\arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\operatorname{PolyLog}\left(2,\frac{\sqrt{ee^i\arcsin(cx)}}{i\sqrt{-dc}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])/(d + e*x^2)^3,x]
```

output

```
(b*c*Sqrt[1 - c^2*x^2])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] - Sqrt[e]*x))
+ (b*c*Sqrt[1 - c^2*x^2])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] + Sqrt[e]*
x)) - (a + b*ArcSin[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)^2)
- (3*(a + b*ArcSin[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a +
b*ArcSin[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)^2) + (3*(a +
b*ArcSin[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c^3*ArcTanh[(
Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*d*Sqrt
[e]*(c^2*d + e)^(3/2)) + (3*b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c
^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]) + (b*c^3*A
rcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(1
6*d*Sqrt[e]*(c^2*d + e)^(3/2)) + (3*b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)
/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]) +
(3*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] -
Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSin[c*x])*Log[1
+ (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e])])/(16*(-d)
^(5/2)*Sqrt[e]) + (3*(a + b*ArcSin[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcSin[c*x]
)))/(I*c*Sqrt[-d] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b
ArcSin[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcSin[c*x]))/(I*c*Sqrt[-d] + Sqrt[c^2*
d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, -((Sqrt[e]*E
(I*ArcSin[c*x]))/(I*c*Sqrt[-d] - Sqrt[c^2*d + e]))])/(16*(-d)^(5/2)*Sqrt[e]...
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5172

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.17 (sec) , antiderivative size = 1772, normalized size of antiderivative = 1.62

method	result	size
parts	Expression too large to display	1772
derivativedivides	Expression too large to display	1793
default	Expression too large to display	1793

input `int((a+b*arcsin(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)`

output

$$\begin{aligned} & 1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^{(1/2)}*\arctan(e \\ & *x/(d*e)^{(1/2)})+b/c*(1/8*c^2*(5*\arcsin(c*x)*d^2*c^5*x+3*\arcsin(c*x)*d*c^5* \\ & e*x^3+d^2*c^4*(-c^2*x^2+1)^{(1/2)}+c^4*d*e*x^2*(-c^2*x^2+1)^{(1/2)}+5*\arcsin(c \\ & *x)*c^3*x*d*e+3*\arcsin(c*x)*c^3*x^3*e^2)/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2 \\ & -1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)}*(-2*(c^2*d*(c^2*d+e)) \\ & ^{(1/2)}*c^2*d+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^{(1/2)}*e)*c^4*\arctanh(e* \\ & (I*c*x+(-c^2*x^2+1)^{(1/2)})/((2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)} \\ &)/(c^2*d+e)^2/d/e^3+3/8*((2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)}*(2 \\ & *c^2*d-2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*c^2*\arctanh(e*(I*c*x+(-c^2*x^2+1)^{(1/2)} \\ &))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)}/(c^2*d+e)/d^2/e^2+3/16 \\ & /(c^2*d+e)/d^2*c^2*e*\sum(_R1/(_R1^2*e-2*c^2*d-e)*(I*\arcsin(c*x)*\ln((_R1-I* \\ & c*x-(-c^2*x^2+1)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-I*c*x-(-c^2*x^2+1)^{(1/2)})/_R1)),_R \\ & 1=\operatorname{RootOf}(e*_Z^4+(-4*c^2*d-2*e)*_Z^2+e)+1/2*(-e*(2*c^2*d-2*(c^2*d*(c^2*d+e) \\ &))^{(1/2)}+e)^{(1/2)}*(2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*\arctan(e*(I*c*x+(\\ & -c^2*x^2+1)^{(1/2)})/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}-e)*e)^{(1/2)})*c^4/(\\ & c^2*d+e)/d/e^3+1/2*((2*c^2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)}*(2*c^2* \\ & d-2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*\arctanh(e*(I*c*x+(-c^2*x^2+1)^{(1/2)})/((2*c^ \\ & 2*d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)})*c^4/(c^2*d+e)/d/e^3-3/8*((2*c^2 \\ & *d+2*(c^2*d*(c^2*d+e))^{(1/2)}+e)*e)^{(1/2)}*(-2*(c^2*d*(c^2*d+e))^{(1/2)}*c^2*d \\ & +2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^{(1/2)}*e)*c^2*\arctanh(e*(I*c*x+(-\dots \end{aligned}$$

Fricas [F]

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \int \frac{b \arcsin(cx) + a}{(ex^2 + d)^3} dx$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F]

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{(d + ex^2)^3} dx$$

input `integrate((a+b*asin(c*x))/(e*x**2+d)**3,x)`

output `Integral((a + b*asin(c*x))/(d + e*x**2)**3, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input `integrate((a+b*arcsin(c*x))/(e*x^2+d)^3,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx = \int \frac{a + b \operatorname{asin}(cx)}{(ex^2 + d)^3} dx$$

input `int((a + b*asin(c*x))/(d + e*x^2)^3,x)`

output `int((a + b*asin(c*x))/(d + e*x^2)^3, x)`

Reduce [F]

$$\int \frac{a + b \arcsin(cx)}{(d + ex^2)^3} dx$$

$$= \frac{3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a d^2 + 6\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a d e x^2 + 3\sqrt{e}\sqrt{d} \operatorname{atan}\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) a e^2 x^4 + 8 \left(\int \frac{\operatorname{asin}}{e^3 x^6 + 3d e^2 x} \right)}{8d^3}$$

input `int((a+b*asin(c*x))/(e*x^2+d)^3,x)`

output

```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d)
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int(asin(c*x)/(d**3 + 3*d**2*e*x**2 + 3
*d*e**2*x**4 + e**3*x**6),x)*b*d**5*e + 16*int(asin(c*x)/(d**3 + 3*d**2*e*
x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2*x**2 + 8*int(asin(c*x)/(d
**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**4 + 5*a
*d**2*e*x + 3*a*d*e**2*x**3)/(8*d**3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

$$3.475 \quad \int \frac{x \arcsin(2x)}{1-4x^2} dx$$

Optimal result	4039
Mathematica [B] (verified)	4040
Rubi [A] (verified)	4040
Maple [A] (verified)	4043
Fricas [F]	4043
Sympy [F]	4043
Maxima [F]	4044
Giac [F]	4044
Mupad [F(-1)]	4044
Reduce [F]	4045

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x \arcsin(2x)}{1-4x^2} dx = \frac{1}{8}i \arcsin(2x)^2 - \frac{1}{4} \arcsin(2x) \log(1 + e^{2i \arcsin(2x)}) + \frac{1}{8}i \operatorname{PolyLog}(2, -e^{2i \arcsin(2x)})$$

output

```
1/8*I*arcsin(2*x)^2-1/4*arcsin(2*x)*ln(1+(2*I*x+(-4*x^2+1)^(1/2))^2)+1/8*I
*polylog(2,-(2*I*x+(-4*x^2+1)^(1/2))^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 213 vs. $2(54) = 108$.

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.94

$$\int \frac{x \arcsin(2x)}{1 - 4x^2} dx = \frac{1}{8} \left(-2i\pi \arcsin(2x) + i \arcsin(2x)^2 - 4\pi \log(1 + e^{-i \arcsin(2x)}) \right. \\ \left. - \pi \log(1 - ie^{i \arcsin(2x)}) - 2 \arcsin(2x) \log(1 - ie^{i \arcsin(2x)}) \right. \\ \left. + \pi \log(1 + ie^{i \arcsin(2x)}) - 2 \arcsin(2x) \log(1 + ie^{i \arcsin(2x)}) \right. \\ \left. + 4\pi \log\left(\cos\left(\frac{1}{2} \arcsin(2x)\right)\right) \right. \\ \left. - \pi \log\left(-\cos\left(\frac{1}{4}(\pi + 2 \arcsin(2x))\right)\right) \right. \\ \left. + \pi \log\left(\sin\left(\frac{1}{4}(\pi + 2 \arcsin(2x))\right)\right) \right. \\ \left. + 2i \operatorname{PolyLog}(2, -ie^{i \arcsin(2x)}) + 2i \operatorname{PolyLog}(2, ie^{i \arcsin(2x)}) \right)$$

input `Integrate[(x*ArcSin[2*x])/(1 - 4*x^2),x]`

output `((-2*I)*Pi*ArcSin[2*x] + I*ArcSin[2*x]^2 - 4*Pi*Log[1 + E^((-I)*ArcSin[2*x])]) - Pi*Log[1 - I*E^(I*ArcSin[2*x])] - 2*ArcSin[2*x]*Log[1 - I*E^(I*ArcSin[2*x])] + Pi*Log[1 + I*E^(I*ArcSin[2*x])] - 2*ArcSin[2*x]*Log[1 + I*E^(I*ArcSin[2*x])] + 4*Pi*Log[Cos[ArcSin[2*x]/2]] - Pi*Log[-Cos[(Pi + 2*ArcSin[2*x])/4]] + Pi*Log[Sin[(Pi + 2*ArcSin[2*x])/4]] + (2*I)*PolyLog[2, (-I)*E^(I*ArcSin[2*x])] + (2*I)*PolyLog[2, I*E^(I*ArcSin[2*x])]/8`

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(2x)}{1 - 4x^2} dx$$

↓ 5180

$$\frac{1}{4} \int \frac{2x \arcsin(2x)}{\sqrt{1 - 4x^2}} d \arcsin(2x)$$

↓ 3042

$$\frac{1}{4} \int \arcsin(2x) \tan(\arcsin(2x)) d \arcsin(2x)$$

↓ 4202

$$\frac{1}{4} \left(\frac{1}{2} i \arcsin(2x)^2 - 2i \int \frac{e^{2i \arcsin(2x)} \arcsin(2x)}{1 + e^{2i \arcsin(2x)}} d \arcsin(2x) \right)$$

↓ 2620

$$\frac{1}{4} \left(\frac{1}{2} i \arcsin(2x)^2 - 2i \left(\frac{1}{2} i \int \log \left(1 + e^{2i \arcsin(2x)} \right) d \arcsin(2x) - \frac{1}{2} i \arcsin(2x) \log \left(1 + e^{2i \arcsin(2x)} \right) \right) \right)$$

↓ 2715

$$\frac{1}{4} \left(\frac{1}{2} i \arcsin(2x)^2 - 2i \left(\frac{1}{4} \int e^{-2i \arcsin(2x)} \log \left(1 + e^{2i \arcsin(2x)} \right) d e^{2i \arcsin(2x)} - \frac{1}{2} i \arcsin(2x) \log \left(1 + e^{2i \arcsin(2x)} \right) \right) \right)$$

↓ 2838

$$\frac{1}{4} \left(\frac{1}{2} i \arcsin(2x)^2 - 2i \left(-\frac{1}{4} \text{PolyLog} \left(2, -e^{2i \arcsin(2x)} \right) - \frac{1}{2} i \arcsin(2x) \log \left(1 + e^{2i \arcsin(2x)} \right) \right) \right)$$

input `Int[(x*ArcSin[2*x])/(1 - 4*x^2),x]`

output `((I/2)*ArcSin[2*x]^2 - (2*I)*((-1/2*I)*ArcSin[2*x]*Log[1 + E^((2*I)*ArcSin[2*x])] - PolyLog[2, -E^((2*I)*ArcSin[2*x])/4])/4`

Definitions of rubi rules used

rule 2620 $\text{Int}[\frac{((F_{-})^{(g_{-})} * (e_{-}) + (f_{-}) * (x_{-})))^{(n_{-})} * ((c_{-}) + (d_{-}) * (x_{-}))^{(m_{-})}}{((a_{-}) + (b_{-}) * (F_{-})^{(g_{-})} * (e_{-}) + (f_{-}) * (x_{-}))^{(n_{-})}}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\frac{(c + d*x)^m}{(b*f*g*n*\text{Log}[F])} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x] - \text{Simp}[d*(m/(b*f*g*n*\text{Log}[F])) \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + b*((F^{(g*(e + f*x)))^n/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_{-}) + (b_{-}) * (F_{-})^{(e_{-})} * ((c_{-}) + (d_{-}) * (x_{-}))^{(n_{-})}], x_{\text{Symbol}}] \rightarrow \text{Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_{-}) * ((d_{-}) + (e_{-}) * (x_{-})^{(n_{-})})]/(x_{-}), x_{\text{Symbol}}] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c)*e*x^n]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_{-}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[\frac{((c_{-}) + (d_{-}) * (x_{-}))^{(m_{-})} * \tan[(e_{-}) + (f_{-}) * (x_{-})], x_{\text{Symbol}}] \rightarrow \text{Simp}[I * \frac{(c + d*x)^{(m+1)}}{(d*(m+1))}, x] - \text{Simp}[2*I \text{Int}[(c + d*x)^m * (E^{(2*I*(e + f*x))}) / (1 + E^{(2*I*(e + f*x))})], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0]$

rule 5180 $\text{Int}[\frac{((a_{-}) + \text{ArcSin}[(c_{-}) * (x_{-})] * (b_{-}))^{(n_{-})} * (x_{-})}{(d_{-}) + (e_{-}) * (x_{-})^2}, x_{\text{Symbol}}] \rightarrow \text{Simp}[-e^{(-1)} \text{Subst}[\text{Int}[(a + b*x)^n * \text{Tan}[x], x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0]$

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{i \arcsin(2x)^2}{8} - \frac{\arcsin(2x) \ln\left(1 + (2ix + \sqrt{-4x^2+1})^2\right)}{4} + \frac{i \operatorname{polylog}\left(2, -(2ix + \sqrt{-4x^2+1})^2\right)}{8}$	59
default	$\frac{i \arcsin(2x)^2}{8} - \frac{\arcsin(2x) \ln\left(1 + (2ix + \sqrt{-4x^2+1})^2\right)}{4} + \frac{i \operatorname{polylog}\left(2, -(2ix + \sqrt{-4x^2+1})^2\right)}{8}$	59

input `int(x*arcsin(2*x)/(-4*x^2+1),x,method=_RETURNVERBOSE)`

output `1/8*I*arcsin(2*x)^2-1/4*arcsin(2*x)*ln(1+(2*I*x+(-4*x^2+1)^(1/2))^2)+1/8*I*
*polylog(2,-(2*I*x+(-4*x^2+1)^(1/2))^2)`

Fricas [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2} dx = \int -\frac{x \arcsin(2x)}{4x^2-1} dx$$

input `integrate(x*arcsin(2*x)/(-4*x^2+1),x, algorithm="fricas")`

output `integral(-x*arcsin(2*x)/(4*x^2 - 1), x)`

Sympy [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2} dx = -\int \frac{x \operatorname{asin}(2x)}{4x^2-1} dx$$

input `integrate(x*asin(2*x)/(-4*x**2+1),x)`

output `-Integral(x*asin(2*x)/(4*x**2 - 1), x)`

Maxima [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2} dx = \int -\frac{x \arcsin(2x)}{4x^2-1} dx$$

input `integrate(x*arcsin(2*x)/(-4*x^2+1),x, algorithm="maxima")`

output `-1/8*arctan2(2*x, sqrt(2*x + 1)*sqrt(-2*x + 1))*log(2*x + 1) - 1/8*arctan2(2*x, sqrt(2*x + 1)*sqrt(-2*x + 1))*log(-2*x + 1) - integrate(1/4*(e^(1/2*log(2*x + 1) + 1/2*log(-2*x + 1))*log(2*x + 1) + e^(1/2*log(2*x + 1) + 1/2*log(-2*x + 1))*log(-2*x + 1))/(16*x^4 - 4*x^2 + (4*x^2 - 1)*e^(log(2*x + 1) + log(-2*x + 1))), x)`

Giac [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2} dx = \int -\frac{x \arcsin(2x)}{4x^2-1} dx$$

input `integrate(x*arcsin(2*x)/(-4*x^2+1),x, algorithm="giac")`

output `integrate(-x*arcsin(2*x)/(4*x^2 - 1), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(2x)}{1-4x^2} dx = -\int \frac{x \operatorname{asin}(2x)}{4x^2-1} dx$$

input `int(-(x*asin(2*x))/(4*x^2 - 1),x)`

output `-int((x*asin(2*x))/(4*x^2 - 1), x)`

Reduce [F]

$$\int \frac{x \arcsin(2x)}{1 - 4x^2} dx = - \left(\int \frac{\arcsin(2x) x}{4x^2 - 1} dx \right)$$

input `int(x*asin(2*x)/(-4*x^2+1),x)`

output `- int((asin(2*x)*x)/(4*x**2 - 1),x)`

3.476 $\int \frac{x \arcsin(2x)}{1+4x^2} dx$

Optimal result	4046
Mathematica [A] (verified)	4047
Rubi [A] (verified)	4048
Maple [A] (verified)	4049
Fricas [F]	4050
Sympy [F]	4050
Maxima [F]	4050
Giac [F]	4051
Mupad [F(-1)]	4051
Reduce [F]	4051

Optimal result

Integrand size = 15, antiderivative size = 245

$$\begin{aligned}
 \int \frac{x \arcsin(2x)}{1+4x^2} dx = & -\frac{1}{8}i \arcsin(2x)^2 + \frac{1}{8} \arcsin(2x) \log \left(1 - \left(1 - \sqrt{2} \right) e^{i \arcsin(2x)} \right) \\
 & + \frac{1}{8} \arcsin(2x) \log \left(1 + \left(1 - \sqrt{2} \right) e^{i \arcsin(2x)} \right) \\
 & + \frac{1}{8} \arcsin(2x) \log \left(1 - \left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \\
 & + \frac{1}{8} \arcsin(2x) \log \left(1 + \left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \\
 & - \frac{1}{8}i \operatorname{PolyLog} \left(2, -\left(\left(1 - \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right) \\
 & - \frac{1}{8}i \operatorname{PolyLog} \left(2, \left(1 - \sqrt{2} \right) e^{i \arcsin(2x)} \right) \\
 & - \frac{1}{8}i \operatorname{PolyLog} \left(2, -\left(\left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right) \\
 & - \frac{1}{8}i \operatorname{PolyLog} \left(2, \left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right)
 \end{aligned}$$

output

```
-1/8*I*arcsin(2*x)^2+1/8*arcsin(2*x)*ln(1-(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))+1/8*arcsin(2*x)*ln(1+(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))+1/8*arcsin(2*x)*ln(1-(1+2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))+1/8*arcsin(2*x)*ln(1+(1+2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*I*polylog(2,-(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*I*polylog(2,(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*I*polylog(2,-(1+2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*I*polylog(2,(1+2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.87

$$\int \frac{x \arcsin(2x)}{1+4x^2} dx = -\frac{1}{8}i \left(\arcsin(2x)^2 + i \arcsin(2x) \log \left(1 - \left(-1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right. \\ \left. + i \arcsin(2x) \log \left(1 + \left(-1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right. \\ \left. + i \arcsin(2x) \log \left(1 - \left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right. \\ \left. + i \arcsin(2x) \log \left(1 + \left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right. \\ \left. + \text{PolyLog} \left(2, - \left(\left(-1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right) \right. \\ \left. + \text{PolyLog} \left(2, \left(-1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right. \\ \left. + \text{PolyLog} \left(2, - \left(\left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right) \right. \\ \left. + \text{PolyLog} \left(2, \left(1 + \sqrt{2} \right) e^{i \arcsin(2x)} \right) \right)$$

input

```
Integrate[(x*ArcSin[2*x])/(1+4*x^2),x]
```

output

```
(-1/8*I)*(ArcSin[2*x]^2 + I*ArcSin[2*x]*Log[1 - (-1 + Sqrt[2])*E^(I*ArcSin[2*x])] + I*ArcSin[2*x]*Log[1 + (-1 + Sqrt[2])*E^(I*ArcSin[2*x])] + I*ArcSin[2*x]*Log[1 - (1 + Sqrt[2])*E^(I*ArcSin[2*x])] + I*ArcSin[2*x]*Log[1 + (1 + Sqrt[2])*E^(I*ArcSin[2*x])] + PolyLog[2, -((-1 + Sqrt[2])*E^(I*ArcSin[2*x]))] + PolyLog[2, (-1 + Sqrt[2])*E^(I*ArcSin[2*x])] + PolyLog[2, -((1 + Sqrt[2])*E^(I*ArcSin[2*x]))] + PolyLog[2, (1 + Sqrt[2])*E^(I*ArcSin[2*x])])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(2x)}{4x^2 + 1} dx$$

↓ 5232

$$\int \left(\frac{\arcsin(2x)}{4(2x+i)} - \frac{\arcsin(2x)}{4(-2x+i)} \right) dx$$

↓ 2009

$$\begin{aligned} & -\frac{1}{8}i \operatorname{PolyLog}\left(2, -\left((1-\sqrt{2})e^{i\arcsin(2x)}\right)\right) - \frac{1}{8}i \operatorname{PolyLog}\left(2, (1-\sqrt{2})e^{i\arcsin(2x)}\right) - \\ & \frac{1}{8}i \operatorname{PolyLog}\left(2, -\left((1+\sqrt{2})e^{i\arcsin(2x)}\right)\right) - \frac{1}{8}i \operatorname{PolyLog}\left(2, (1+\sqrt{2})e^{i\arcsin(2x)}\right) - \\ & \frac{1}{8}i \arcsin(2x)^2 + \frac{1}{8} \arcsin(2x) \log\left(1 - (1-\sqrt{2})e^{i\arcsin(2x)}\right) + \\ & \frac{1}{8} \arcsin(2x) \log\left(1 + (1-\sqrt{2})e^{i\arcsin(2x)}\right) + \frac{1}{8} \arcsin(2x) \log\left(1 - (1+\sqrt{2})e^{i\arcsin(2x)}\right) + \\ & \frac{1}{8} \arcsin(2x) \log\left(1 + (1+\sqrt{2})e^{i\arcsin(2x)}\right) \end{aligned}$$

input `Int[(x*ArcSin[2*x])/(1 + 4*x^2),x]`

output `(-1/8*I)*ArcSin[2*x]^2 + (ArcSin[2*x]*Log[1 - (1 - Sqrt[2])*E^(I*ArcSin[2*x])])/8 + (ArcSin[2*x]*Log[1 + (1 - Sqrt[2])*E^(I*ArcSin[2*x])])/8 + (ArcSin[2*x]*Log[1 - (1 + Sqrt[2])*E^(I*ArcSin[2*x])])/8 + (ArcSin[2*x]*Log[1 + (1 + Sqrt[2])*E^(I*ArcSin[2*x])])/8 - (I/8)*PolyLog[2, -((1 - Sqrt[2])*E^(I*ArcSin[2*x]))] - (I/8)*PolyLog[2, (1 - Sqrt[2])*E^(I*ArcSin[2*x])] - (I/8)*PolyLog[2, -((1 + Sqrt[2])*E^(I*ArcSin[2*x]))] - (I/8)*PolyLog[2, (1 + Sqrt[2])*E^(I*ArcSin[2*x])]`

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5232 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_ + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{i \arcsin(2x)^2}{8} + \frac{\arcsin(2x) \ln\left(\frac{3+2\sqrt{2}-(2ix+\sqrt{-4x^2+1})^2}{3+2\sqrt{2}}\right)}{8} + \frac{\arcsin(2x) \ln\left(\frac{-3+2\sqrt{2}+(2ix+\sqrt{-4x^2+1})^2}{-3+2\sqrt{2}}\right)}{8} - \dots$
default	$-\frac{i \arcsin(2x)^2}{8} + \frac{\arcsin(2x) \ln\left(\frac{3+2\sqrt{2}-(2ix+\sqrt{-4x^2+1})^2}{3+2\sqrt{2}}\right)}{8} + \frac{\arcsin(2x) \ln\left(\frac{-3+2\sqrt{2}+(2ix+\sqrt{-4x^2+1})^2}{-3+2\sqrt{2}}\right)}{8} - \dots$

```
input int(x*arcsin(2*x)/(4*x^2+1),x,method=_RETURNVERBOSE)
```

```
output -1/8*I*arcsin(2*x)^2+1/8*arcsin(2*x)*ln((3+2*2^(1/2)-(2*I*x+(-4*x^2+1)^(1/2))^2)/(3+2*2^(1/2)))+1/8*arcsin(2*x)*ln((-3+2*2^(1/2)+(2*I*x+(-4*x^2+1)^(1/2))^2)/(-3+2*2^(1/2)))-1/16*I*dilog((3+2*2^(1/2)-(2*I*x+(-4*x^2+1)^(1/2))^2)/(3+2*2^(1/2)))-1/16*I*dilog((-3+2*2^(1/2)+(2*I*x+(-4*x^2+1)^(1/2))^2)/(-3+2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x \arcsin(2x)}{1 + 4x^2} dx = \int \frac{x \arcsin(2x)}{4x^2 + 1} dx$$

input `integrate(x*arcsin(2*x)/(4*x^2+1),x, algorithm="fricas")`

output `integral(x*arcsin(2*x)/(4*x^2 + 1), x)`

Sympy [F]

$$\int \frac{x \arcsin(2x)}{1 + 4x^2} dx = \int \frac{x \operatorname{asin}(2x)}{4x^2 + 1} dx$$

input `integrate(x*asin(2*x)/(4*x**2+1),x)`

output `Integral(x*asin(2*x)/(4*x**2 + 1), x)`

Maxima [F]

$$\int \frac{x \arcsin(2x)}{1 + 4x^2} dx = \int \frac{x \arcsin(2x)}{4x^2 + 1} dx$$

input `integrate(x*arcsin(2*x)/(4*x^2+1),x, algorithm="maxima")`

output `integrate(x*arcsin(2*x)/(4*x^2 + 1), x)`

Giac [F]

$$\int \frac{x \arcsin(2x)}{1+4x^2} dx = \int \frac{x \arcsin(2x)}{4x^2+1} dx$$

input `integrate(x*arcsin(2*x)/(4*x^2+1),x, algorithm="giac")`

output `integrate(x*arcsin(2*x)/(4*x^2 + 1), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(2x)}{1+4x^2} dx = \int \frac{x \operatorname{asin}(2x)}{4x^2+1} dx$$

input `int((x*asin(2*x))/(4*x^2 + 1),x)`

output `int((x*asin(2*x))/(4*x^2 + 1), x)`

Reduce [F]

$$\int \frac{x \arcsin(2x)}{1+4x^2} dx = \int \frac{\operatorname{asin}(2x) x}{4x^2+1} dx$$

input `int(x*asin(2*x)/(4*x^2+1),x)`

output `int((asin(2*x)*x)/(4*x**2 + 1),x)`

3.477 $\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx$

Optimal result	4052
Mathematica [A] (verified)	4053
Rubi [A] (verified)	4054
Maple [F]	4058
Fricas [F]	4059
Sympy [F(-1)]	4059
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Reduce [F]	4061

Optimal result

Integrand size = 23, antiderivative size = 484

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx$$

$$= \frac{be(3c^2de(7+m)^2(12+7m+m^2) + 3c^4d^2(35+12m+m^2)^2 + e^2(360+342m+119m^2+18m^3+m^4))}{c^5f^2(3+m)^2(5+m)^2(7+m)^2} + \frac{be^2(3c^2d(7+m)^2 + e(30+11m+m^2))(fx)^{4+m}\sqrt{1-c^2x^2}}{c^3f^4(5+m)^2(7+m)^2} + \frac{be^3(fx)^{6+m}\sqrt{1-c^2x^2}}{cf^6(7+m)^2}$$

$$+ \frac{d^3(fx)^{1+m}(a + b \arcsin(cx))}{f(1+m)} + \frac{3d^2e(fx)^{3+m}(a + b \arcsin(cx))}{f^3(3+m)}$$

$$+ \frac{3de^2(fx)^{5+m}(a + b \arcsin(cx))}{f^5(5+m)} + \frac{e^3(fx)^{7+m}(a + b \arcsin(cx))}{f^7(7+m)}$$

$$+ \frac{b\left(\frac{c^6d^3(3+m)(5+m)(7+m)}{1+m} + \frac{e(2+m)(3c^2de(7+m)^2(12+7m+m^2) + 3c^4d^2(35+12m+m^2)^2 + e^2(360+342m+119m^2+18m^3+m^4))}{(3+m)(5+m)(7+m)}\right)}{c^5f^2(2+m)(3+m)(5+m)(7+m)}$$

output

```

b*e*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*
m^3+119*m^2+342*m+360))*(f*x)^(2+m)*(-c^2*x^2+1)^(1/2)/c^5/f^2/(3+m)^2/(5+
m)^2/(7+m)^2+b*e^2*(3*c^2*d*(7+m)^2+e*(m^2+11*m+30))*(f*x)^(4+m)*(-c^2*x^2
+1)^(1/2)/c^3/f^4/(5+m)^2/(7+m)^2+b*e^3*(f*x)^(6+m)*(-c^2*x^2+1)^(1/2)/c/f
^6/(7+m)^2+d^3*(f*x)^(1+m)*(a+b*arcsin(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(
a+b*arcsin(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arcsin(c*x))/f^5/(5+m)
+e^3*(f*x)^(7+m)*(a+b*arcsin(c*x))/f^7/(7+m)-b*(c^6*d^3*(3+m)*(5+m)*(7+m)/
(1+m)+e*(2+m)*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^
2*(m^4+18*m^3+119*m^2+342*m+360))/(3+m)/(5+m)/(7+m))*(f*x)^(2+m)*hypergeom
([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)/c^5/f^2/(2+m)/(3+m)/(5+m)/(7+m)

```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.64

$$\begin{aligned}
& \int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx \\
&= x(fx)^m \left(\frac{ad^3}{1+m} + \frac{3ad^2ex^2}{3+m} + \frac{3ade^2x^4}{5+m} + \frac{ae^3x^6}{7+m} + \frac{bd^3 \arcsin(cx)}{1+m} \right. \\
&\quad + \frac{3bd^2ex^2 \arcsin(cx)}{3+m} + \frac{3bde^2x^4 \arcsin(cx)}{5+m} + \frac{be^3x^6 \arcsin(cx)}{7+m} \\
&\quad - \frac{bcd^3x \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 1 + \frac{m}{2}, 2 + \frac{m}{2}, c^2x^2\right)}{2 + 3m + m^2} \\
&\quad - \frac{3bcd^2ex^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 2 + \frac{m}{2}, 3 + \frac{m}{2}, c^2x^2\right)}{12 + 7m + m^2} \\
&\quad - \frac{3bcde^2x^5 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 3 + \frac{m}{2}, 4 + \frac{m}{2}, c^2x^2\right)}{(5+m)(6+m)} \\
&\quad \left. - \frac{bce^3x^7 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 4 + \frac{m}{2}, 5 + \frac{m}{2}, c^2x^2\right)}{(7+m)(8+m)} \right)
\end{aligned}$$

input

```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]
```

output

```

x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcSin[c*x])/(1 + m) + (3*b*d^2*e*x^2*
ArcSin[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcSin[c*x])/(5 + m) + (b*e^3*x^6*Ar
cSin[c*x])/(7 + m) - (b*c*d^3*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c
^2*x^2])/(2 + 3*m + m^2) - (3*b*c*d^2*e*x^3*Hypergeometric2F1[1/2, 2 + m/2
, 3 + m/2, c^2*x^2])/(12 + 7*m + m^2) - (3*b*c*d*e^2*x^5*Hypergeometric2F1
[1/2, 3 + m/2, 4 + m/2, c^2*x^2])/((5 + m)*(6 + m)) - (b*c*e^3*x^7*Hyperge
ometric2F1[1/2, 4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)))

```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5230, 27, 2340, 25, 1590, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d + ex^2)^3 (fx)^m (a + b \arcsin(cx)) dx \\
 & \quad \downarrow 5230 \\
 & -bc \int \frac{(fx)^{m+1} \left(\frac{e^3 x^6}{m+7} + \frac{3de^2 x^4}{m+5} + \frac{3d^2 ex^2}{m+3} + \frac{d^3}{m+1} \right)}{f \sqrt{1 - c^2 x^2}} dx + \frac{d^3 (fx)^{m+1} (a + b \arcsin(cx))}{f(m+1)} + \\
 & \quad \frac{3d^2 e (fx)^{m+3} (a + b \arcsin(cx))}{f^3(m+3)} + \frac{3de^2 (fx)^{m+5} (a + b \arcsin(cx))}{f^5(m+5)} + \\
 & \quad \frac{e^3 (fx)^{m+7} (a + b \arcsin(cx))}{f^7(m+7)} \\
 & \quad \downarrow 27 \\
 & -bc \int \frac{(fx)^{m+1} \left(\frac{e^3 x^6}{m+7} + \frac{3de^2 x^4}{m+5} + \frac{3d^2 ex^2}{m+3} + \frac{d^3}{m+1} \right)}{\sqrt{1 - c^2 x^2}} dx + \frac{d^3 (fx)^{m+1} (a + b \arcsin(cx))}{f(m+1)} + \\
 & \quad \frac{3d^2 e (fx)^{m+3} (a + b \arcsin(cx))}{f^3(m+3)} + \frac{3de^2 (fx)^{m+5} (a + b \arcsin(cx))}{f^5(m+5)} + \\
 & \quad \frac{e^3 (fx)^{m+7} (a + b \arcsin(cx))}{f^7(m+7)} \\
 & \quad \downarrow 2340
 \end{aligned}$$

$$bc \left(\frac{\int \frac{(fx)^{m+1} \left(\frac{e^2(3c^2d(m+7)^2 + e(m^2+11m+30))x^4}{(m+5)(m+7)} + \frac{3c^2d^2e(m+7)x^2}{m+3} + \frac{c^2d^3(m+7)}{m+1} \right)}{c^2(m+7)} dx - \frac{e^3\sqrt{1-c^2x^2}(fx)^{m+6}}{c^2f^5(m+7)^2} \right) +$$

$$\frac{d^3(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a+b\arcsin(cx))}{f^7(m+7)}$$

↓ 25

$$bc \left(\frac{\int \frac{(fx)^{m+1} \left(\frac{e^2(3c^2d(m+7)^2 + e(m^2+11m+30))x^4}{(m+5)(m+7)} + \frac{3c^2d^2e(m+7)x^2}{m+3} + \frac{c^2d^3(m+7)}{m+1} \right)}{c^2(m+7)} dx - \frac{e^3\sqrt{1-c^2x^2}(fx)^{m+6}}{c^2f^5(m+7)^2} \right) +$$

$$\frac{d^3(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a+b\arcsin(cx))}{f^7(m+7)}$$

↓ 1590

$$bc \left(\frac{\int \frac{(fx)^{m+1} \left(\frac{d^3(m+5)(m+7)c^4}{m+1} + \frac{e(3d^2(m^2+12m+35)^2c^4 + 3de(m+7)^2(m^2+7m+12)c^2 + e^2(m^4+18m^3+119m^2+342m+360))x^2}{(m+3)(m+5)(m+7)} \right)}{c^2(m+5)} dx - \frac{e^2\sqrt{1-c^2x^2}}{c^2(m+7)} \right) +$$

$$\frac{d^3(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a+b\arcsin(cx))}{f^7(m+7)}$$

↓ 25

$$bc \left(\frac{f^{m+1} \left(\frac{d^3(m+5)(m+7)c^4}{m+1} + \frac{e(3d^2(m^2+12m+35))^2 c^4 + 3de(m+7)^2(m^2+7m+12)c^2 + e^2(m^4+18m^3+119m^2+342m+360)}{(m+3)(m+5)(m+7)} \right) x^2}{\frac{\sqrt{1-c^2x^2}}{c^2(m+5)}} dx - \frac{e^2 \sqrt{1-c^2x^2}}{c^2(m+7)} \right)$$

$$\frac{d^3(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a + b \arcsin(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a + b \arcsin(cx))}{f^7(m+7)}$$

↓ 363

$$bc \left(\frac{\left(\frac{c^4 d^3(m+5)(m+7)}{m+1} + \frac{e(m+2)(3c^4 d^2(m^2+12m+35))^2 + 3c^2 de(m+7)^2(m^2+7m+12) + e^2(m^4+18m^3+119m^2+342m+360)}{c^2(m+3)^2(m+5)(m+7)} \right) \int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}} dx - \frac{e^2 \sqrt{1-c^2x^2}}{c^2(m+5)} \right)$$

$$\frac{d^3(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a + b \arcsin(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a + b \arcsin(cx))}{f^7(m+7)}$$

↓ 278

$$\frac{d^3(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5}(a + b \arcsin(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a + b \arcsin(cx))}{f^7(m+7)}$$

$$bc \left(\frac{(fx)^{m+2} \left(\frac{c^4 d^3(m+5)(m+7)}{m+1} + \frac{e(m+2)(3c^4 d^2(m^2+12m+35))^2 + 3c^2 de(m+7)^2(m^2+7m+12) + e^2(m^4+18m^3+119m^2+342m+360)}{c^2(m+3)^2(m+5)(m+7)} \right)}{f(m+2)} \right) \text{Hypergeometric2} \\ c^2(m+5)$$

input `Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSin[c*x]),x]`

output

```
(d^3*(f*x)^(1 + m)*(a + b*ArcSin[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
m)*(a + b*ArcSin[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcS
in[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcSin[c*x]))/(f^7*(7
+ m)) - (b*c*(-((e^3*(f*x)^(6 + m)*Sqrt[1 - c^2*x^2])/(c^2*f^5*(7 + m)^2))
+ (-((e^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f*x)^(4 + m)*Sqrt[1
- c^2*x^2])/(c^2*f^3*(5 + m)^2*(7 + m))) + (-((e*(3*c^2*d*e*(7 + m)^2*(12
+ 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342*m + 119*m^2
+ 18*m^3 + m^4))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2])/(c^2*f*(3 + m)^2*(5 + m
*(7 + m))) + (((c^4*d^3*(5 + m)*(7 + m))/(1 + m) + (e*(2 + m)*(3*c^2*d*e*(
7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342
*m + 119*m^2 + 18*m^3 + m^4)))/(c^2*(3 + m)^2*(5 + m)*(7 + m)))*(f*x)^(2 +
m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f*(2 + m)))/(c
^2*(5 + m))/(c^2*(7 + m)))/f
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x
_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3)),
x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^(
q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
+ 1)) Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340

```
Int[(Pq)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1)) Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5230

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [F]

$$\int (fx)^m (ex^2 + d)^3 (a + b \arcsin(cx)) dx$$

input

```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsin(c*x)),x)
```

output

```
int((f*x)^m*(e*x^2+d)^3*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx = \int (ex^2 + d)^3 (b \arcsin(cx) + a)(fx)^m dx$$

input `integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsin(c*x))*(f*x)^m, x)`

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx = \text{Timed out}$$

input `integrate((f*x)**m*(e*x**2+d)**3*(a+b*asin(c*x)),x)`

output `Timed out`

Maxima [F]

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx = \int (ex^2 + d)^3 (b \arcsin(cx) + a)(fx)^m dx$$

input `integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3 + 9*b
*e^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^7 + 3*(b*d*e^2*f^m*m^3 + 1
1*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^5 + 3*(b*d^2*e*f^
m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f^m)*x^3 + (b*d
^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + 71*b*d^3*f^m*m + 105*b*d^3*f^m)*x)*x^m*arc
tan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (m^4 + 16*m^3 + 86*m^2 + 176*m +
105)*integrate(-(b*c*e^3*f^m*m^3 + 9*b*c*e^3*f^m*m^2 + 23*b*c*e^3*f^m*m
+ 15*b*c*e^3*f^m)*x^7 + 3*(b*c*d*e^2*f^m*m^3 + 11*b*c*d*e^2*f^m*m^2 + 31*b
*c*d*e^2*f^m*m + 21*b*c*d*e^2*f^m)*x^5 + 3*(b*c*d^2*e*f^m*m^3 + 13*b*c*d^2
*e*f^m*m^2 + 47*b*c*d^2*e*f^m*m + 35*b*c*d^2*e*f^m)*x^3 + (b*c*d^3*f^m*m^3
+ 15*b*c*d^3*f^m*m^2 + 71*b*c*d^3*f^m*m + 105*b*c*d^3*f^m)*x)*sqrt(c*x +
1)*sqrt(-c*x + 1)*x^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 +
176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m^3 + 86*
m^2 + 176*m + 105)
```

Giac [F]

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx = \int (ex^2 + d)^3 (b \arcsin(cx) + a)(fx)^m dx$$

input

```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
integrate((e*x^2 + d)^3*(b*arcsin(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (fx)^m (ex^2 + d)^3 dx$$

input

```
int((a + b*asin(c*x))*(f*x)^m*(d + e*x^2)^3,x)
```

output

```
int((a + b*asin(c*x))*(f*x)^m*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int (fx)^m (d + ex^2)^3 (a + b \arcsin(cx)) dx = \text{Too large to display}$$

input `int((f*x)^m*(e*x^2+d)^3*(a+b*asin(c*x)),x)`

output

```
(f**m*(x**m*a*d**3*m**3*x + 15*x**m*a*d**3*m**2*x + 71*x**m*a*d**3*m*x + 105*x**m*a*d**3*x + 3*x**m*a*d**2*e*m**3*x**3 + 39*x**m*a*d**2*e*m**2*x**3 + 141*x**m*a*d**2*e*m*x**3 + 105*x**m*a*d**2*e*x**3 + 3*x**m*a*d*e**2*m**3*x**5 + 33*x**m*a*d*e**2*m**2*x**5 + 93*x**m*a*d*e**2*m*x**5 + 63*x**m*a*d*e**2*x**5 + x**m*a*e**3*m**3*x**7 + 9*x**m*a*e**3*m**2*x**7 + 23*x**m*a*e**3*m*x**7 + 15*x**m*a*e**3*x**7 + int(x**m*asin(c*x)*x**6,x)*b*e**3*m**4 + 16*int(x**m*asin(c*x)*x**6,x)*b*e**3*m**3 + 86*int(x**m*asin(c*x)*x**6,x)*b*e**3*m**2 + 176*int(x**m*asin(c*x)*x**6,x)*b*e**3*m + 105*int(x**m*asin(c*x)*x**6,x)*b*e**3 + 3*int(x**m*asin(c*x)*x**4,x)*b*d*e**2*m**4 + 48*int(x**m*asin(c*x)*x**4,x)*b*d*e**2*m**3 + 258*int(x**m*asin(c*x)*x**4,x)*b*d*e**2*m**2 + 528*int(x**m*asin(c*x)*x**4,x)*b*d*e**2*m + 315*int(x**m*asin(c*x)*x**4,x)*b*d*e**2 + 3*int(x**m*asin(c*x)*x**2,x)*b*d**2*e*m**4 + 48*int(x**m*asin(c*x)*x**2,x)*b*d**2*e*m**3 + 258*int(x**m*asin(c*x)*x**2,x)*b*d**2*e*m**2 + 528*int(x**m*asin(c*x)*x**2,x)*b*d**2*e*m + 315*int(x**m*asin(c*x)*x**2,x)*b*d**2*e + int(x**m*asin(c*x),x)*b*d**3*m**4 + 16*int(x**m*asin(c*x),x)*b*d**3*m**3 + 86*int(x**m*asin(c*x),x)*b*d**3*m**2 + 176*int(x**m*asin(c*x),x)*b*d**3*m + 105*int(x**m*asin(c*x),x)*b*d**3))/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.478 $\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx$

Optimal result	4062
Mathematica [A] (verified)	4063
Rubi [A] (verified)	4063
Maple [F]	4066
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Sympy [F]	4067
Maxima [F]	4067
Giac [F]	4068
Mupad [F(-1)]	4068
Reduce [F]	4069

Optimal result

Integrand size = 23, antiderivative size = 293

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{be(2c^2d(5+m)^2 + e(12+7m+m^2))(fx)^{2+m}\sqrt{1-c^2x^2}}{c^3f^2(3+m)^2(5+m)^2}$$

$$+ \frac{be^2(fx)^{4+m}\sqrt{1-c^2x^2}}{cf^4(5+m)^2} + \frac{d^2(fx)^{1+m}(a+b\arcsin(cx))}{f(1+m)}$$

$$+ \frac{2de(fx)^{3+m}(a+b\arcsin(cx))}{f^3(3+m)} + \frac{e^2(fx)^{5+m}(a+b\arcsin(cx))}{f^5(5+m)}$$

$$- \frac{b\left(\frac{c^4d^2(3+m)(5+m)}{1+m} + \frac{e(2+m)(2c^2d(5+m)^2 + e(12+7m+m^2))}{(3+m)(5+m)}\right)(fx)^{2+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{c^3f^2(2+m)(3+m)(5+m)}$$

output

```
b*e*(2*c^2*d*(5+m)^2+e*(m^2+7*m+12))*(f*x)^(2+m)*(-c^2*x^2+1)^(1/2)/c^3/f^2/(3+m)^2/(5+m)^2+b*e^2*(f*x)^(4+m)*(-c^2*x^2+1)^(1/2)/c/f^4/(5+m)^2+d^2*(f*x)^(1+m)*(a+b*arcsin(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arcsin(c*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arcsin(c*x))/f^5/(5+m)-b*(c^4*d^2*(3+m)*(5+m)/(1+m)+e*(2+m)*(2*c^2*d*(5+m)^2+e*(m^2+7*m+12))/(3+m)/(5+m))*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2*m], c^2*x^2)/c^3/f^2/(2+m)/(3+m)/(5+m)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.76

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= x(fx)^m \left(\frac{ad^2}{1+m} + \frac{2adex^2}{3+m} + \frac{ae^2x^4}{5+m} + \frac{bd^2 \arcsin(cx)}{1+m} + \frac{2bdex^2 \arcsin(cx)}{3+m} \right. \\ \left. + \frac{be^2x^4 \arcsin(cx)}{5+m} - \frac{bcd^2x \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 1 + \frac{m}{2}, 2 + \frac{m}{2}, c^2x^2\right)}{2 + 3m + m^2} \right. \\ \left. - \frac{2bcdex^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 2 + \frac{m}{2}, 3 + \frac{m}{2}, c^2x^2\right)}{12 + 7m + m^2} \right. \\ \left. - \frac{bce^2x^5 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 3 + \frac{m}{2}, 4 + \frac{m}{2}, c^2x^2\right)}{(5+m)(6+m)} \right)$$

input

```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]
```

output

```
x*(f*x)^m*((a*d^2)/(1+m) + (2*a*d*e*x^2)/(3+m) + (a*e^2*x^4)/(5+m) +
(b*d^2*ArcSin[c*x])/(1+m) + (2*b*d*e*x^2*ArcSin[c*x])/(3+m) + (b*e^2*x^4*ArcSin[c*x])/(5+m) - (b*c*d^2*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2])/(2 + 3*m + m^2) - (2*b*c*d*e*x^3*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2])/(12 + 7*m + m^2) - (b*c*e^2*x^5*Hypergeometric2F1[1/2, 3 + m/2, 4 + m/2, c^2*x^2])/((5+m)*(6+m)))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5230, 27, 1590, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2)^2 (fx)^m (a + b \arcsin(cx)) dx$$

↓ 5230

$$\begin{aligned}
 & -bc \int \frac{(fx)^{m+1} \left(\frac{e^2 x^4}{m+5} + \frac{2dex^2}{m+3} + \frac{d^2}{m+1} \right)}{f\sqrt{1-c^2x^2}} dx + \frac{d^2(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \\
 & \quad \frac{2de(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} \\
 & \quad \downarrow 27 \\
 & - \frac{bc \int \frac{(fx)^{m+1} \left(\frac{e^2 x^4}{m+5} + \frac{2dex^2}{m+3} + \frac{d^2}{m+1} \right)}{\sqrt{1-c^2x^2}} dx}{f} + \frac{d^2(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \\
 & \quad \frac{2de(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} \\
 & \quad \downarrow 1590 \\
 & bc \left(- \frac{\int \frac{(fx)^{m+1} \left(\frac{c^2(m+5)d^2}{m+1} + \frac{e(2c^2d(m+5)^2 + e(m^2+7m+12))x^2}{(m+3)(m+5)} \right)}{\sqrt{1-c^2x^2}} dx}{c^2(m+5)} - \frac{e^2\sqrt{1-c^2x^2}(fx)^{m+4}}{c^2f^3(m+5)^2} \right) + \\
 & \quad \frac{d^2(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \frac{2de(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} \\
 & \quad \downarrow 25 \\
 & bc \left(\frac{\int \frac{(fx)^{m+1} \left(\frac{c^2(m+5)d^2}{m+1} + \frac{e(2c^2d(m+5)^2 + e(m^2+7m+12))x^2}{(m+3)(m+5)} \right)}{\sqrt{1-c^2x^2}} dx}{c^2(m+5)} - \frac{e^2\sqrt{1-c^2x^2}(fx)^{m+4}}{c^2f^3(m+5)^2} \right) + \\
 & \quad \frac{d^2(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \frac{2de(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} \\
 & \quad \downarrow 363 \\
 & bc \left(\frac{\left(\frac{c^2d^2(m+5)}{m+1} + \frac{e(m+2)(2c^2d(m+5)^2 + e(m^2+7m+12))}{c^2(m+3)^2(m+5)} \right) \int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}} dx - \frac{e\sqrt{1-c^2x^2}(fx)^{m+2}(2c^2d(m+5)^2 + e(m^2+7m+12))}{c^2f(m+3)^2(m+5)}}{c^2(m+5)} - \frac{e^2\sqrt{1-c^2x^2}(fx)^{m+4}}{c^2f^3(m+5)^2} \right) + \\
 & \quad \frac{d^2(fx)^{m+1}(a+b\arcsin(cx))}{f(m+1)} + \frac{2de(fx)^{m+3}(a+b\arcsin(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a+b\arcsin(cx))}{f^5(m+5)} \\
 & \quad \downarrow 278
 \end{aligned}$$

$$\frac{d^2(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{2de(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a + b \arcsin(cx))}{f^5(m+5)} -$$

$$bc \left(\frac{(fx)^{m+2} \left(\frac{c^2 d^2 (m+5)}{m+1} + \frac{e(m+2)(2c^2 d(m+5)^2 + e(m^2 + 7m + 12))}{c^2 (m+3)^2 (m+5)} \right)}{f(m+2)} \operatorname{Hypergeometric2F1} \left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2 \right) - \frac{e \sqrt{1-c^2 x^2} (fx)^{m+2} (2c^2 d(m+5)^2 + e(m^2 + 7m + 12))}{c^2 f(m+3)^2 (m+5)} \right)$$

$$f$$

input `Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSin[c*x]),x]`

output `(d^2*(f*x)^(1 + m)*(a + b*ArcSin[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a + b*ArcSin[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSin[c*x]))/(f^5*(5 + m)) - (b*c*(-((e^2*(f*x)^(4 + m)*Sqrt[1 - c^2*x^2])/(c^2*f^3*(5 + m)^2)) + (-((e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2])/(c^2*f*(3 + m)^2*(5 + m))) + (((c^2*d^2*(5 + m))/(1 + m) + (e*(2 + m)*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2)))/(c^2*(3 + m)^2*(5 + m)))*(f*x)^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f*(2 + m)))/(c^2*(5 + m)))/f`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 27 `Int[(a_)*(Fx_), x_Symbol] := Simp[a Int[Fx, x], x] /; FreeQ[a, x] && !MatchQ[Fx, (b_)*(Gx_)] /; FreeQ[b, x]`

rule 278 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

rule 363

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x
_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^(
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^(
q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
+ 1)) Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5230

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_
)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [F]

$$\int (fx)^m (ex^2 + d)^2 (a + b \arcsin(cx)) dx$$

input

```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsin(c*x)),x)
```

output

```
int((f*x)^m*(e*x^2+d)^2*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int (ex^2 + d)^2 (b \arcsin(cx) + a)(fx)^m dx$$

input `integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="fricas")`

output `integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arcsin(c*x))*(f*x)^m, x)`

Sympy [F]

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int (fx)^m (a + b \arcsin(cx)) (d + ex^2)^2 dx$$

input `integrate((f*x)**m*(e*x**2+d)**2*(a+b*asin(c*x)),x)`

output `Integral((f*x)**m*(a + b*asin(c*x))*(d + e*x**2)**2, x)`

Maxima [F]

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int (ex^2 + d)^2 (b \arcsin(cx) + a)(fx)^m dx$$

input `integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output

```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + (((b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^5 + 2*
(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^3 + (b*d^2*f^m*m^2 + 8*b*d
^2*f^m*m + 15*b*d^2*f^m)*x)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1))
+ (m^3 + 9*m^2 + 23*m + 15)*integrate(-((b*c*e^2*f^m*m^2 + 4*b*c*e^2*f^m*
m + 3*b*c*e^2*f^m)*x^5 + 2*(b*c*d*e*f^m*m^2 + 6*b*c*d*e*f^m*m + 5*b*c*d*e*
f^m)*x^3 + (b*c*d^2*f^m*m^2 + 8*b*c*d^2*f^m*m + 15*b*c*d^2*f^m)*x)*sqrt(c*
x + 1)*sqrt(-c*x + 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)
*x^2 + 9*m^2 + 23*m + 15), x)/(m^3 + 9*m^2 + 23*m + 15)
```

Giac [F]

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int (ex^2 + d)^2 (b \arcsin(cx) + a)(fx)^m dx$$

input

```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsin(c*x)),x, algorithm="giac")
```

output

```
integrate((e*x^2 + d)^2*(b*arcsin(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (fx)^m (ex^2 + d)^2 dx$$

input

```
int((a + b*asin(c*x))*(f*x)^m*(d + e*x^2)^2,x)
```

output

```
int((a + b*asin(c*x))*(f*x)^m*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int (fx)^m (d + ex^2)^2 (a + b \arcsin(cx)) dx$$

$$= \frac{f^m (x^m a d^2 m^2 x + 8x^m a d^2 m x + 15x^m a d^2 x + 2x^m a d e m^2 x^3 + 12x^m a d e m x^3 + 10x^m a d e x^3 + x^m a e^2 m^2 x^5 + 4x^m a e^2 m x^5 + 3x^m a e^2 x^5 + \int(x^m \arcsin(cx) x^{4,x}) b e^{2,m} + 9 \int(x^m \arcsin(cx) x^{4,x}) b e^{2,m} + 23 \int(x^m \arcsin(cx) x^{4,x}) b e^{2,m} + 15 \int(x^m \arcsin(cx) x^{4,x}) b e^{2,m} + 2 \int(x^m \arcsin(cx) x^{2,x}) b d e^{m,3} + 18 \int(x^m \arcsin(cx) x^{2,x}) b d e^{m,2} + 46 \int(x^m \arcsin(cx) x^{2,x}) b d e^{m,1} + 30 \int(x^m \arcsin(cx) x^{2,x}) b d e^{m,0} + \int(x^m \arcsin(cx), x) b d^{2,m,3} + 9 \int(x^m \arcsin(cx), x) b d^{2,m,2} + 23 \int(x^m \arcsin(cx), x) b d^{2,m,1} + 15 \int(x^m \arcsin(cx), x) b d^{2,m,0})}{(m^3 + 9m^2 + 23m + 15)}$$

input `int((f*x)^m*(e*x^2+d)^2*(a+b*asin(c*x)),x)`

output `(f**m*(x**m*a*d**2*m**2*x + 8*x**m*a*d**2*m*x + 15*x**m*a*d**2*x + 2*x**m*a*d*e*m**2*x**3 + 12*x**m*a*d*e*m*x**3 + 10*x**m*a*d*e*x**3 + x**m*a*e**2*m**2*x**5 + 4*x**m*a*e**2*m*x**5 + 3*x**m*a*e**2*x**5 + int(x**m*asin(c*x)*x**4,x)*b*e**2*m**3 + 9*int(x**m*asin(c*x)*x**4,x)*b*e**2*m**2 + 23*int(x**m*asin(c*x)*x**4,x)*b*e**2*m + 15*int(x**m*asin(c*x)*x**4,x)*b*e**2 + 2*int(x**m*asin(c*x)*x**2,x)*b*d*e**m**3 + 18*int(x**m*asin(c*x)*x**2,x)*b*d*e**m**2 + 46*int(x**m*asin(c*x)*x**2,x)*b*d*e**m + 30*int(x**m*asin(c*x)*x**2,x)*b*d*e + int(x**m*asin(c*x),x)*b*d**2*m**3 + 9*int(x**m*asin(c*x),x)*b*d**2*m**2 + 23*int(x**m*asin(c*x),x)*b*d**2*m + 15*int(x**m*asin(c*x),x)*b*d**2))/(m**3 + 9*m**2 + 23*m + 15)`

3.479 $\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx$

Optimal result	4070
Mathematica [A] (verified)	4071
Rubi [A] (verified)	4071
Maple [F]	4073
Fricas [F]	4073
Sympy [F]	4074
Maxima [F]	4074
Giac [F]	4074
Mupad [F(-1)]	4075
Reduce [F]	4075

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \frac{be(fx)^{2+m}\sqrt{1-c^2x^2}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}(a+b\arcsin(cx))}{f(1+m)} + \frac{e(fx)^{3+m}(a+b\arcsin(cx))}{f^3(3+m)}$$

$$- \frac{b\left(\frac{e(2+m)}{3+m} + \frac{c^2d(3+m)}{1+m}\right)(fx)^{2+m} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2x^2\right)}{cf^2(2+m)(3+m)}$$

output

```
b*e*(f*x)^(2+m)*(-c^2*x^2+1)^(1/2)/c/f^2/(3+m)^2+d*(f*x)^(1+m)*(a+b*arcsin
(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arcsin(c*x))/f^3/(3+m)-b*(e*(2+m)/(3+m)+
c^2*d*(3+m)/(1+m))*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)
/c/f^2/(2+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= x(fx)^m \left(-\frac{bcdx \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 1 + \frac{m}{2}, 2 + \frac{m}{2}, c^2x^2\right)}{2 + 3m + m^2} + \frac{\frac{d(3+m)+e(1+m)x^2}{1+m}(a+b \arcsin(cx)) - \frac{bcex^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, 2 + \frac{m}{2}, 3 + \frac{m}{2}, c^2x^2\right)}{4+m}}{3 + m} \right)$$

input `Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `x*(f*x)^m*(-((b*c*d*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2])/((2 + 3*m + m^2))) + (((d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcSin[c*x]))/(1 + m) - (b*c*e*x^3*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2])/(4 + m))/(3 + m))`

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5230, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2) (fx)^m (a + b \arcsin(cx)) dx$$

$$\downarrow \text{5230}$$

$$-bc \int \frac{(fx)^{m+1} (e(m+1)x^2 + d(m+3))}{f(m^2 + 4m + 3) \sqrt{1 - c^2x^2}} dx + \frac{d(fx)^{m+1} (a + b \arcsin(cx))}{f(m+1)} + \frac{e(fx)^{m+3} (a + b \arcsin(cx))}{f^3(m+3)}$$

$$\begin{aligned}
& \downarrow 27 \\
& -\frac{bc \int \frac{(fx)^{m+1} (e(m+1)x^2 + d(m+3))}{\sqrt{1-c^2x^2}} dx}{f(m^2 + 4m + 3)} + \frac{d(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{e(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} \\
& \downarrow 363 \\
& -\frac{bc \left(\left(\frac{e(m+1)(m+2)}{c^2(m+3)} + d(m+3) \right) \int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}} dx - \frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+2}}{c^2 f(m+3)} \right)}{f(m^2 + 4m + 3)} + \\
& \quad \frac{d(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{e(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} \\
& \downarrow 278 \\
& \frac{d(fx)^{m+1}(a + b \arcsin(cx))}{f(m+1)} + \frac{e(fx)^{m+3}(a + b \arcsin(cx))}{f^3(m+3)} - \\
& \frac{bc \left(\frac{(fx)^{m+2} \left(\frac{e(m+1)(m+2)}{c^2(m+3)} + d(m+3) \right) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, c^2x^2\right)}{f(m+2)} - \frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+2}}{c^2 f(m+3)} \right)}{f(m^2 + 4m + 3)}
\end{aligned}$$

input `Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSin[c*x]),x]`

output `(d*(f*x)^(1 + m)*(a + b*ArcSin[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a + b*ArcSin[c*x]))/(f^3*(3 + m)) - (b*c*(-((e*(1 + m)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]))/(c^2*f*(3 + m))) + (((e*(1 + m)*(2 + m))/(c^2*(3 + m)) + d*(3 + m))*(f*x)^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2]))/(f*(2 + m)))/(f*(3 + 4*m + m^2))`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 278 `Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

rule 363

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5230

```
Int(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [F]

$$\int (fx)^m (ex^2 + d) (a + b \arcsin(cx)) dx$$

input

```
int((f*x)^m*(e*x^2+d)*(a+b*arcsin(c*x)),x)
```

output

```
int((f*x)^m*(e*x^2+d)*(a+b*arcsin(c*x)),x)
```

Fricas [F]

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx = \int (ex^2 + d)(b \arcsin(cx) + a)(fx)^m dx$$

input

```
integrate((f*x)^m*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="fricas")
```

output

```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsin(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx = \int (fx)^m (a + b \operatorname{asin}(cx)) (d + ex^2) dx$$

input `integrate((f*x)**m*(e*x**2+d)*(a+b*asin(c*x)),x)`

output `Integral((f*x)**m*(a + b*asin(c*x))*(d + e*x**2), x)`

Maxima [F]

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx = \int (ex^2 + d)(b \arcsin(cx) + a)(fx)^m dx$$

input `integrate((f*x)^m*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="maxima")`

output `a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m + b*e*f^m)*x^3 + (b*d*f^m*m + 3*b*d*f^m)*x)*x^m*arctan2(c*x, sqrt(c*x + 1)*sqrt(-c*x + 1)) + (m^2 + 4*m + 3)*integrate(((b*c*e*f^m*m + b*c*e*f^m)*x^3 + (b*c*d*f^m*m + 3*b*c*d*f^m)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)`

Giac [F]

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx = \int (ex^2 + d)(b \arcsin(cx) + a)(fx)^m dx$$

input `integrate((f*x)^m*(e*x^2+d)*(a+b*arcsin(c*x)),x, algorithm="giac")`

output `integrate((e*x^2 + d)*(b*arcsin(c*x) + a)*(f*x)^m, x)`

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx = \int (a + b \arcsin(cx)) (fx)^m (ex^2 + d) dx$$

input `int((a + b*asin(c*x))*(f*x)^m*(d + e*x^2),x)`output `int((a + b*asin(c*x))*(f*x)^m*(d + e*x^2), x)`**Reduce [F]**

$$\int (fx)^m (d + ex^2) (a + b \arcsin(cx)) dx$$

$$= \frac{f^m (x^m a d m x + 3 x^m a d x + x^m a e m x^3 + x^m a e x^3 + (\int x^m \arcsin(cx) x^2 dx) b e m^2 + 4 (\int x^m \arcsin(cx) x^2 dx) b e m}{m^2 + 4m + 3}$$

input `int((f*x)^m*(e*x^2+d)*(a+b*asin(c*x)),x)`output `(f**m*(x**m*a*d*m*x + 3*x**m*a*d*x + x**m*a*e*m*x**3 + x**m*a*e*x**3 + int(x**m*asin(c*x)*x**2,x)*b*e*m**2 + 4*int(x**m*asin(c*x)*x**2,x)*b*e*m + 3*int(x**m*asin(c*x)*x**2,x)*b*e + int(x**m*asin(c*x),x)*b*d*m**2 + 4*int(x**m*asin(c*x),x)*b*d*m + 3*int(x**m*asin(c*x),x)*b*d))/(m**2 + 4*m + 3)`

3.480 $\int \frac{(fx)^m(a+b \arcsin(cx))}{d+ex^2} dx$

Optimal result	4076
Mathematica [N/A]	4076
Rubi [N/A]	4077
Maple [N/A]	4077
Fricas [N/A]	4078
Sympy [N/A]	4078
Maxima [N/A]	4078
Giac [F(-2)]	4079
Mupad [N/A]	4079
Reduce [N/A]	4080

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b \arcsin(cx))}{d+ex^2} dx = \text{Int}\left(\frac{(fx)^m(a+b \arcsin(cx))}{d+ex^2}, x\right)$$

output `Defer(Int)((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d), x)`

Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b \arcsin(cx))}{d+ex^2} dx = \int \frac{(fx)^m(a+b \arcsin(cx))}{d+ex^2} dx$$

input `Integrate[((f*x)^m*(a + b*ArcSin[c*x]))/(d + e*x^2), x]`

output `Integrate[((f*x)^m*(a + b*ArcSin[c*x]))/(d + e*x^2), x]`

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{d + ex^2} dx$$

↓ 5234

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{d + ex^2} dx$$

input `Int[((f*x)^m*(a + b*ArcSin[c*x]))/(d + e*x^2),x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{ex^2 + d} dx$$

input `int((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d),x)`

output `int((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d),x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)(fx)^m}{ex^2 + d} dx$$

input `integrate((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)*(f*x)^m/(e*x^2 + d), x)`

Sympy [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(fx)^m (a + b \operatorname{asin}(cx))}{d + ex^2} dx$$

input `integrate((f*x)**m*(a+b*asin(c*x))/(e*x**2+d),x)`

output `Integral((f*x)**m*(a + b*asin(c*x))/(d + e*x**2), x)`

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(b \arcsin(cx) + a)(fx)^m}{ex^2 + d} dx$$

input `integrate((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)*(f*x)^m/(e*x^2 + d), x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(fx)^m(a + b \arcsin(cx))}{d + ex^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve cteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a + b \arcsin(cx))}{d + ex^2} dx = \int \frac{(a + b \arcsin(cx)) (fx)^m}{ex^2 + d} dx$$

input `int(((a + b*asin(c*x))*(f*x)^m)/(d + e*x^2),x)`

output `int(((a + b*asin(c*x))*(f*x)^m)/(d + e*x^2), x)`

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{(fx)^m(a + b \arcsin(cx))}{d + ex^2} dx = f^m \left(\left(\int \frac{x^m}{ex^2 + d} dx \right) a + \left(\int \frac{x^m \arcsin(cx)}{ex^2 + d} dx \right) b \right)$$

input `int((f*x)^m*(a+b*asin(c*x))/(e*x^2+d),x)`output `f**m*(int(x**m/(d + e*x**2),x)*a + int((x**m*asin(c*x))/(d + e*x**2),x)*b)`

3.481
$$\int \frac{(fx)^m(a+b \arcsin(cx))}{(d+ex^2)^2} dx$$

Optimal result	4081
Mathematica [N/A]	4081
Rubi [N/A]	4082
Maple [N/A]	4082
Fricas [N/A]	4083
Sympy [F(-1)]	4083
Maxima [N/A]	4083
Giac [F(-2)]	4084
Mupad [N/A]	4084
Reduce [N/A]	4084

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b \arcsin(cx))}{(d+ex^2)^2} dx = \text{Int}\left(\frac{(fx)^m(a+b \arcsin(cx))}{(d+ex^2)^2}, x\right)$$

output

```
Defer(Int)((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b \arcsin(cx))}{(d+ex^2)^2} dx = \int \frac{(fx)^m(a+b \arcsin(cx))}{(d+ex^2)^2} dx$$

input

```
Integrate[((f*x)^m*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]
```

output

```
Integrate[((f*x)^m*(a + b*ArcSin[c*x]))/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

↓ 5234

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{(d + ex^2)^2} dx$$

input `Int[((f*x)^m*(a + b*ArcSin[c*x]))/(d + e*x^2)^2,x]`

output `$Aborted`

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{(ex^2 + d)^2} dx$$

input `int((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d)^2,x)`

output `int((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d)^2,x)`

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

input `integrate((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

output `integral((b*arcsin(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Timed out}$$

input `integrate((f*x)**m*(a+b*asin(c*x))/(e*x**2+d)**2,x)`

output `Timed out`

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(b \arcsin(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

input `integrate((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

output `integrate((b*arcsin(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)`

Giac [F(-2)]

Exception generated.

$$\int \frac{(fx)^m(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \text{Exception raised: RuntimeError}$$

input `integrate((f*x)^m*(a+b*arcsin(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value`

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a + b \arcsin(cx))}{(d + ex^2)^2} dx = \int \frac{(a + b \arcsin(cx)) (fx)^m}{(ex^2 + d)^2} dx$$

input `int(((a + b*asin(c*x))*(f*x)^m)/(d + e*x^2)^2,x)`

output `int(((a + b*asin(c*x))*(f*x)^m)/(d + e*x^2)^2, x)`

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.83

$$\int \frac{(fx)^m(a + b \arcsin(cx))}{(d + ex^2)^2} dx = f^m \left(\left(\int \frac{x^m}{e^2x^4 + 2dex^2 + d^2} dx \right) a + \left(\int \frac{x^m \arcsin(cx)}{e^2x^4 + 2dex^2 + d^2} dx \right) b \right)$$

input `int((f*x)^m*(a+b*asin(c*x))/(e*x^2+d)^2,x)`

output `f**m*(int(x**m/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*a + int((x**m*asin(c*x))
/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b)`

3.482 $\int x^2 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$

Optimal result	4086
Mathematica [A] (verified)	4087
Rubi [A] (verified)	4087
Maple [C] (verified)	4091
Fricas [F]	4092
Sympy [F]	4093
Maxima [F(-2)]	4093
Giac [F]	4094
Mupad [F(-1)]	4094
Reduce [F]	4094

Optimal result

Integrand size = 35, antiderivative size = 351

$$\int x^2 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \frac{b^2 x \sqrt{d + cdx} \sqrt{e - cex}}{64c^2} - \frac{1}{32} b^2 x^3 \sqrt{d + cdx} \sqrt{e - cex}$$

$$- \frac{b^2 \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)}{64c^3 \sqrt{1 - c^2 x^2}} + \frac{bx^2 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))}{8c \sqrt{1 - c^2 x^2}}$$

$$- \frac{bcx^4 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))}{8 \sqrt{1 - c^2 x^2}}$$

$$- \frac{x \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2}{8c^2}$$

$$+ \frac{1}{4} x^3 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 + \frac{\sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^3}{24bc^3 \sqrt{1 - c^2 x^2}}$$

output

$$\frac{1}{64} b^2 x (c d x + d)^{1/2} (-c e x + e)^{1/2} / c^2 - \frac{1}{32} b^2 x^3 (c d x + d)^{1/2} (-c e x + e)^{1/2} - \frac{1}{64} b^2 (c d x + d)^{1/2} (-c e x + e)^{1/2} \arcsin(c x) / c^3 / (-c^2 x^2 + 1)^{1/2} + \frac{1}{8} b x^2 (c d x + d)^{1/2} (-c e x + e)^{1/2} (a + b \arcsin(c x)) / c / (-c^2 x^2 + 1)^{1/2} - \frac{1}{8} b c x^4 (c d x + d)^{1/2} (-c e x + e)^{1/2} (a + b \arcsin(c x)) / (-c^2 x^2 + 1)^{1/2} - \frac{1}{8} x (c d x + d)^{1/2} (-c e x + e)^{1/2} (a + b \arcsin(c x))^2 / c^2 + \frac{1}{4} x^3 (c d x + d)^{1/2} (-c e x + e)^{1/2} (a + b \arcsin(c x))^2 + \frac{1}{24} (c d x + d)^{1/2} (-c e x + e)^{1/2} (a + b \arcsin(c x))^3 / c^3 / (-c^2 x^2 + 1)^{1/2}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.85

$$\int x^2 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \frac{32b^2 \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)^3 - 96a^2 \sqrt{d} \sqrt{e} \sqrt{1 - c^2 x^2} \arctan\left(\frac{cx \sqrt{d + cdx} \sqrt{e - cex}}{\sqrt{d} \sqrt{e} (-1 + c^2 x^2)}\right) - 12b \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)^2 (-4a + b \sin[4 \arcsin(cx)]) + 3 \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)^2 (-4a + b \sin[4 \arcsin(cx)]) + 3 \sqrt{d + cdx} \sqrt{e - cex} (32a^2 c x \sqrt{1 - c^2 x^2} (-1 + 2c^2 x^2) - 4a b \cos[4 \arcsin(cx)] + b^2 \sin[4 \arcsin(cx)])}{(768c^3 \sqrt{1 - c^2 x^2})}$$

input

```
Integrate[x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2,x]
```

output

```
(32*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^3 - 96*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 12*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]*(b*Cos[4*ArcSin[c*x]] + 4*a*Sin[4*ArcSin[c*x]]) - 24*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^2*(-4*a + b*Sin[4*ArcSin[c*x]]) + 3*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(32*a^2*c*x*Sqrt[1 - c^2*x^2]*(-1 + 2*c^2*x^2) - 4*a*b*Cos[4*ArcSin[c*x]] + b^2*Sin[4*ArcSin[c*x]]))/(768*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.80, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {5238, 5198, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{cdx + d} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5238$$

$$\frac{\sqrt{cdx + d} \sqrt{e - cex} \int x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 dx}{\sqrt{1 - c^2 x^2}}$$

$$\downarrow 5198$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \int x^3(a + b \arcsin(cx))dx + \frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5138

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \left(\frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{4}bc \int \frac{x^4}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

↓ 262

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \left(\frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \int \frac{x^2}{\sqrt{1-c^2x^2}} dx}{4c^2} - \frac{x^3\sqrt{1-c^2x^2}}{4c^2} \right) \right) \right)}{\sqrt{1 - c^2x^2}}$$

↓ 262

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \left(\frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} - \frac{x^3}{4c^2} \right) \right) \right)}{\sqrt{1 - c^2x^2}}$$

↓ 223

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{1}{4}x^4(a + b \arcsin(cx)) - \frac{1}{4}bc \int \frac{x^4}{\sqrt{1-c^2x^2}} dx \right) \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5210

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} + \frac{b \int x(a+b \arcsin(cx))dx}{c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right) + \frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5138

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right) + \frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

↓ 262

$$\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c^2} \right) \right)$$

↓ 223

$$\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4} \left(\frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right) \right)$$

↓ 5152

$$\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 + \frac{1}{4} \left(\frac{(a+b \arcsin(cx))^3}{6bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx) - x\sqrt{1-c^2x^2}}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right) \right)$$

input

```
Int[x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2,x]
```

output

```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/4 - (b*c*((x^4*(a + b*ArcSin[c*x]))/4 - (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4))/2 + (-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2 + (a + b*ArcSin[c*x])^3/(6*b*c^3) + (b*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c)/4)/Sqrt[1 - c^2*x^2]
```

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5198 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^n/(f*(m+2))), x] + (\text{Simp}[(1/(m+2))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^m*((a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m+2)))*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(f*x)^{(m+1)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IGtQ}[m, -2] \ || \ \text{EqQ}[n, 1])$

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)
)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.)
)*(x_)^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.34 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.16

method	result
default	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \left(2x^3 c^2 \sqrt{-de(c^2x^2-1)} \sqrt{c^2de} + \arctan \left(\frac{\sqrt{c^2de} x}{\sqrt{-de(c^2x^2-1)}} \right) de - \sqrt{c^2de} \sqrt{-de(c^2x^2-1)} x \right)}{8 \sqrt{-de(c^2x^2-1)} c^2 \sqrt{c^2de}} + b^2 \left(-\sqrt{d(cx+1)} \sqrt{-e(cx-1)} \right)$
parts	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \left(2x^3 c^2 \sqrt{-de(c^2x^2-1)} \sqrt{c^2de} + \arctan \left(\frac{\sqrt{c^2de} x}{\sqrt{-de(c^2x^2-1)}} \right) de - \sqrt{c^2de} \sqrt{-de(c^2x^2-1)} x \right)}{8 \sqrt{-de(c^2x^2-1)} c^2 \sqrt{c^2de}} + b^2 \left(-\sqrt{d(cx+1)} \sqrt{-e(cx-1)} \right)$

input

```
int(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RET
URNVERBOSE)
```


output

```

1/8*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*x^3*c^2*(-d*e*(c^2*x^2-1))
^(1/2)*(c^2*d*e)^(1/2)+arctan((c^2*d*e)^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2))*
d*e-(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2)/c
^2/(c^2*d*e)^(1/2)+b^2*(-1/24*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x
^2+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^3+1/512*(-e*(c*x-1))^(1/2)*(d*(c*x
+1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/
2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(4*I*arcsin(c*x)+8*arcsi
n(c*x)^2-1)/c^3/(c^2*x^2-1)+1/512*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(8*
I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c
^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-4*I*arcsin(c*x)+8*arcsin(c*x)^2-1)/c^
3/(c^2*x^2-1))+2*a*b*(-1/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^3/(c^2*x^2-1)*arcsin(c*x)^2+1/256*(-e*(c*x-1))^(1/2)*(d*(c*x+1
))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)
*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))/c^3/(c^2
*x^2-1)+1/256*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)
*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1
)^(1/2)+4*c*x)*(-I+4*arcsin(c*x))/c^3/(c^2*x^2-1))

```

Fricas [F]

$$\int x^2 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \int \sqrt{cdx + d} \sqrt{-cex + e} (b \arcsin(cx) + a)^2 x^2 dx$$

input

```

integrate(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algo
rithm="fricas")

```

output

```

integral((b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(c*
d*x + d)*sqrt(-c*e*x + e), x)

```

Sympy [F]

$$\int x^2 \sqrt{d + cx} \sqrt{e - cx} (a + b \arcsin(cx))^2 dx$$

$$= \int x^2 \sqrt{d(cx + 1)} \sqrt{-e(cx - 1)} (a + b \arcsin(cx))^2 dx$$

input `integrate(x**2*(c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(x**2*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*asin(c*x))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int x^2 \sqrt{d + cx} \sqrt{e - cx} (a + b \arcsin(cx))^2 dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int x^2 \sqrt{d + cx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \int \sqrt{cdx + d} \sqrt{-cex + e} (b \arcsin(cx) + a)^2 x^2 dx$$

input `integrate(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorith="giac")`

output `integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arcsin(c*x) + a)^2*x^2, x)`

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{d + cx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \int x^2 (a + b \arcsin(cx))^2 \sqrt{d + cx} \sqrt{e - cex} dx$$

input `int(x^2*(a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)`

output `int(x^2*(a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)`

Reduce [F]

$$\int x^2 \sqrt{d + cx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{e} \sqrt{d} \left(-2a \sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 + 2\sqrt{cx+1} \sqrt{-cx+1} a^2 c^3 x^3 - \sqrt{cx+1} \sqrt{-cx+1} a^2 cx + 16 \left(\int \sqrt{cx+1} dx \right) \right)}{8c^3}$$

input `int(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*asin(c*x))^2,x)`

output

```
(sqrt(e)*sqrt(d)*(- 2*asin(sqrt(- c*x + 1)/sqrt(2))*a**2 + 2*sqrt(c*x +
1)*sqrt(- c*x + 1)*a**2*c**3*x**3 - sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2*c
*x + 16*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)*x**2,x)*a*b*c**3 + 8*
int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)**2*x**2,x)*b**2*c**3))/(8*c**
3)
```

3.483 $\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx$

Optimal result	4096
Mathematica [A] (verified)	4097
Rubi [A] (verified)	4097
Maple [B] (verified)	4100
Fricas [A] (verification not implemented)	4101
Sympy [F]	4102
Maxima [F(-2)]	4102
Giac [F]	4102
Mupad [F(-1)]	4103
Reduce [F]	4103

Optimal result

Integrand size = 33, antiderivative size = 225

$$\begin{aligned} & \int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \frac{4b^2\sqrt{d+cdx}\sqrt{e-cex}}{9c^2} + \frac{2b^2\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{27c^2} \\ &+ \frac{2bx\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))}{3c\sqrt{1-c^2x^2}} \\ &- \frac{2bcx^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))}{9\sqrt{1-c^2x^2}} \\ &- \frac{\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)(a+b\arcsin(cx))^2}{3c^2} \end{aligned}$$

output

```
4/9*b^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2+2/27*b^2*(c*d*x+d)^(1/2)*(-c*
e*x+e)^(1/2)*(-c^2*x^2+1)/c^2+2/3*b*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+
b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-2/9*b*c*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)
^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/3*(c*d*x+d)^(1/2)*(-c*e*x+e)
^(1/2)*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx$$

$$= \frac{\sqrt{d+cdx}\sqrt{e-cex}\left(6abcx\sqrt{1-c^2x^2}(-3+c^2x^2) + 9a^2(-1+c^2x^2)^2 - 2b^2(7-8c^2x^2+c^4x^4) + 6b(bcx\sqrt{1-c^2x^2}(-3+c^2x^2) + 3a(-1+c^2x^2)^2)\arcsin[cx] + 9b^2(-1+c^2x^2)^2\arcsin[cx]^2\right)}{27c^2(-1+c^2x^2)}$$

input

```
Integrate[x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2,x]
```

output

```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(6*a*b*c*x*Sqrt[1 - c^2*x^2]*(-3 + c^2*x^2) + 9*a^2*(-1 + c^2*x^2)^2 - 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*Sqrt[1 - c^2*x^2]*(-3 + c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcSin[c*x] + 9*b^2*(-1 + c^2*x^2)^2*ArcSin[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {5238, 5182, 5154, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cdx+d}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx$$

$$\downarrow \text{5238}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex} \int x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{5182}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex} \left(\frac{2b \int (1-c^2x^2)(a+b\arcsin(cx)) dx}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2}{3c^2} \right)}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{5154}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{2b \left(-bc \int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) \right)}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{3c^2} \right)}{\sqrt{1 - c^2x^2}}$$

↓ 27

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{2b \left(-\frac{1}{3}bc \int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) \right)}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{3c^2} \right)}{\sqrt{1 - c^2x^2}}$$

↓ 353

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{2b \left(-\frac{1}{6}bc \int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}} dx^2 - \frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) \right)}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{3c^2} \right)}{\sqrt{1 - c^2x^2}}$$

↓ 53

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{2b \left(-\frac{1}{6}bc \int \left(\sqrt{1-c^2x^2} + \frac{2}{\sqrt{1-c^2x^2}} \right) dx^2 - \frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) \right)}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{3c^2} \right)}{\sqrt{1 - c^2x^2}}$$

↓ 2009

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{2b \left(-\frac{1}{3}c^2x^3(a+b \arcsin(cx)) + x(a+b \arcsin(cx)) - \frac{1}{6}bc \left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2} - \frac{4\sqrt{1-c^2x^2}}{c^2} \right) \right)}{3c} - \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{3c^2} \right)}{\sqrt{1 - c^2x^2}}$$

input `Int[x*sqrt[d + c*d*x]*sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2,x]`

output

```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-1/3*((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[
c*x])^2)/c^2 + (2*b*(-1/6*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x
^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSin[c*x]) - (c^2*x^3*(a + b*ArcSin[c*x]
))/3))/(3*c))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !Ma
tchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2)^(q_), x_Symbol]
:= Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5154

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol]
:= With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x]
- Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5182

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_
), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```


rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[(-d^2)*(g/e)^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(191) = 382.

Time = 4.13 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.88

method	result
orering	$\frac{(19c^6x^6 - 71c^4x^4 + 48c^2x^2 - 14)\sqrt{cdx+d}\sqrt{-cxe+e}(a+b\arcsin(cx))^2}{27c^4(cx-1)x^2(cx+1)} - \frac{2(3c^4x^4 - 16c^2x^2 + 7)\left(\sqrt{cdx+d}\sqrt{-cxe+e}(a+b\arcsin(cx))\right)}{216c^2(c^2x^2-1)}$
default	$\frac{a^2\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(c^2x^2-1)}{3c^2} + b^2\left(\frac{\sqrt{-e(cx-1)}\sqrt{d(cx+1)}(4e^4x^4 - 5c^2x^2 - 4i\sqrt{-c^2x^2+1}x^3c^3 + 3i\sqrt{-c^2x^2+1}cx+1)}{216c^2(c^2x^2-1)}\right)$
parts	$\frac{a^2\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(c^2x^2-1)}{3c^2} + b^2\left(\frac{\sqrt{-e(cx-1)}\sqrt{d(cx+1)}(4e^4x^4 - 5c^2x^2 - 4i\sqrt{-c^2x^2+1}x^3c^3 + 3i\sqrt{-c^2x^2+1}cx+1)}{216c^2(c^2x^2-1)}\right)$

input

```
int(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```

1/27*(19*c^6*x^6-71*c^4*x^4+48*c^2*x^2-14)/c^4/(c*x-1)/x^2/(c*x+1)*(c*d*x+
d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2-2/27*(3*c^4*x^4-16*c^2*x^2+7
)/c^4/x^2*((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2+1/2*x/(c*d
*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c*d-1/2*x*(c*d*x+d)^(1/2)
/(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c*e+2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(
1/2)*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/27*(c^2*x^2-7)/c^4*(c*x-1
)/x*(c*x+1)*(1/(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c*d-(c
*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c*e+4*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2)-1/4*x/(c*d*x+d)^(
3/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c^2*d^2-1/2*x/(c*d*x+d)^(1/2)/(-
c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c^2*d*e+2*x/(c*d*x+d)^(1/2)*(-c*e*x+e)^(
1/2)*(a+b*arcsin(c*x))*c^2*d*b/(-c^2*x^2+1)^(1/2)-1/4*x*(c*d*x+d)^(1/2)/(-
c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2*c^2*e^2-2*x*(c*d*x+d)^(1/2)/(-c*e*x+e)
^(1/2)*(a+b*arcsin(c*x))*c^2*e*b/(-c^2*x^2+1)^(1/2)+2*x*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*b^2*c^2/(-c^2*x^2+1)+2*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)
*(a+b*arcsin(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))

```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx$$

$$= \frac{((9a^2 - 2b^2)c^4x^4 - 2(9a^2 - 8b^2)c^2x^2 + 9(b^2c^4x^4 - 2b^2c^2x^2 + b^2))\arcsin(cx)^2 + 9a^2 - 14b^2 + 18(abc^4$$

input

```

integrate(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algori
thm="fricas")

```

output

```

1/27*((9*a^2 - 2*b^2)*c^4*x^4 - 2*(9*a^2 - 8*b^2)*c^2*x^2 + 9*(b^2*c^4*x^4
- 2*b^2*c^2*x^2 + b^2)*arcsin(c*x)^2 + 9*a^2 - 14*b^2 + 18*(a*b*c^4*x^4 -
2*a*b*c^2*x^2 + a*b)*arcsin(c*x) + 6*(a*b*c^3*x^3 - 3*a*b*c*x + (b^2*c^3*
x^3 - 3*b^2*c*x)*arcsin(c*x))*sqrt(-c^2*x^2 + 1))*sqrt(c*d*x + d)*sqrt(-c
e*x + e)/(c^4*x^2 - c^2)

```

Sympy [F]

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx$$

$$= \int x\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\arcsin(cx))^2 dx$$

input `integrate(x*(c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(x*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*asin(c*x))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx = \text{Exception raised: ValueError}$$

input `integrate(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx$$

$$= \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2 x dx$$

input `integrate(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arcsin(c*x) + a)^2*x, x)`

Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \int x(a+b\arcsin(cx))^2\sqrt{d+cdx}\sqrt{e-cex} dx \end{aligned}$$

input `int(x*(a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2),x)`

output `int(x*(a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)`

Reduce [F]

$$\begin{aligned} & \int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \frac{\sqrt{e}\sqrt{d}(\sqrt{cx+1}\sqrt{-cx+1}a^2c^2x^2 - \sqrt{cx+1}\sqrt{-cx+1}a^2 + 6(\int\sqrt{cx+1}\sqrt{-cx+1}\arcsin(cx)xdx)ab + b^2c^2x^2)}{3c^2} \end{aligned}$$

input `int(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(e)*sqrt(d)*(sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2*c**2*x**2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2 + 6*int(sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)*x,x)*a*b*c**2 + 3*int(sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)**2*x,x)*b**2*c**2)/(3*c**2)`

3.484 $\int \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$

Optimal result	4104
Mathematica [A] (verified)	4105
Rubi [A] (verified)	4105
Maple [C] (verified)	4108
Fricas [F]	4109
Sympy [F]	4109
Maxima [F(-2)]	4109
Giac [F]	4110
Mupad [F(-1)]	4110
Reduce [F]	4111

Optimal result

Integrand size = 32, antiderivative size = 222

$$\int \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= -\frac{1}{4} b^2 x \sqrt{d + cdx} \sqrt{e - cex} + \frac{b^2 \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)}{4c\sqrt{1 - c^2x^2}}$$

$$- \frac{bcx^2 \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))}{2\sqrt{1 - c^2x^2}}$$

$$+ \frac{1}{2} x \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 + \frac{\sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^3}{6bc\sqrt{1 - c^2x^2}}$$

output

```
-1/4*b^2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+1/4*b^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arcsin(c*x)/c/(-c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+1/2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2+1/6*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.30

$$\int \sqrt{d + cdx} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$= \frac{4b^2 \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)^3 - 12a^2 \sqrt{d} \sqrt{e} \sqrt{1 - c^2 x^2} \arctan\left(\frac{cx \sqrt{d + cdx} \sqrt{e - cex}}{\sqrt{d} \sqrt{e} (-1 + c^2 x^2)}\right) + 6b \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)^2 + 6ab \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx) + 3a^2 \sqrt{d + cdx} \sqrt{e - cex}}{c^2 \sqrt{d + cdx} \sqrt{e - cex}}$$

input

```
Integrate[Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2,x]
```

output

```
(4*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^3 - 12*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]*(b*Cos[2*ArcSin[c*x]] + 2*a*Sin[2*ArcSin[c*x]]) + 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^2*(2*a + b*Sin[2*ArcSin[c*x]]) + 3*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(4*a^2*c*x*Sqrt[1 - c^2*x^2] + 2*a*b*Cos[2*ArcSin[c*x]] - b^2*Sin[2*ArcSin[c*x]]))/(24*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5178, 5156, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx + d} \sqrt{e - cex} (a + b \arcsin(cx))^2 dx$$

$$\downarrow 5178$$

$$\frac{\sqrt{cdx + d} \sqrt{e - cex} \int \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 dx}{\sqrt{1 - c^2 x^2}}$$

$$\downarrow 5156$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - bc \int x(a + b \arcsin(cx)) dx + \frac{1}{2} x \sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5138

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(-bc \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) \right) - \frac{1}{2} bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}}$$

↓ 262

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(-bc \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right) \right) + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}}$$

↓ 223

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2 - bc \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\int \frac{1}{\sqrt{1-c^2x^2}} dx - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right) \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5152

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} x \sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2 - bc \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right) \right)}{\sqrt{1 - c^2x^2}}$$

input `Int[Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2,x]`

output `(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/2 + (a + b*ArcSin[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2]))/c^2 + ArcSin[c*x]/(2*c^3)))/2))/Sqrt[1 - c^2*x^2]`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /;$ $\text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1})/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)})/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ $\text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /;$ $\text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5156 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[x*\text{Sqrt}[d + e*x^2]*((a + b*\text{ArcSin}[c*x])^{(n/2)}), x] + (\text{Simp}[(1/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[(a + b*\text{ArcSin}[c*x])^n/\text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2)*\text{Simp}[\text{Sqrt}[d + e*x^2]/\text{Sqrt}[1 - c^2*x^2]] \ \text{Int}[x*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /;$ $\text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0]$

rule 5178 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_) + (e_)*(x_))^{(p_)}*((f_) + (g_)*(x_))^{(q_)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^2)^q) \ \text{Int}[(d + e*x)^{(p-q)}*(1 - c^2*x^2)^q*(a + b*\text{ArcSin}[c*x])^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{EqQ}[e*f + d*g, 0] \ \&\& \ \text{EqQ}[c^2*d^2 - e^2, 0] \ \&\& \ \text{HalfIntegerQ}[p, q] \ \&\& \ \text{GeQ}[p - q, 0]$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.86

method	result
default	$-\frac{a^2\sqrt{cdx+d}(-cxe+e)^{\frac{3}{2}}}{2ce} + \frac{a^2\sqrt{-cxe+e}\sqrt{cdx+d}}{2c} + \frac{a^2de\sqrt{(cdx+d)(-cxe+e)}\arctan\left(\frac{\sqrt{c^2de}x}{\sqrt{-c^2d^2e+de}}\right)}{2\sqrt{-cxe+e}\sqrt{cdx+d}\sqrt{c^2de}} + b^2\left(-\frac{\sqrt{-e(cx+d)}}{\sqrt{cdx+d}}\right)$
parts	$-\frac{a^2\sqrt{cdx+d}(-cxe+e)^{\frac{3}{2}}}{2ce} + \frac{a^2\sqrt{-cxe+e}\sqrt{cdx+d}}{2c} + \frac{a^2de\sqrt{(cdx+d)(-cxe+e)}\arctan\left(\frac{\sqrt{c^2de}x}{\sqrt{-c^2d^2e+de}}\right)}{2\sqrt{-cxe+e}\sqrt{cdx+d}\sqrt{c^2de}} + b^2\left(-\frac{\sqrt{-e(cx+d)}}{\sqrt{cdx+d}}\right)$

input `int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNV
ERBOSE)`

output
$$\begin{aligned} & -1/2*a^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)+1/2*a^2/c*(-c*e*x+e)^(1/2)*(\\ & c*d*x+d)^(1/2)+1/2*a^2*d*e*((c*d*x+d)*(-c*e*x+e))^(1/2)/(-c*e*x+e)^(1/2)/(\\ & c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e) \\ & ^{(1/2)})+b^2*(-1/6*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/ \\ & c/(c^2*x^2-1)*arcsin(c*x)^3+1/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-2* \\ & I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*I*ar \\ & csin(c*x)+2*arcsin(c*x)^2-1)/c/(c^2*x^2-1)+1/16*(-e*(c*x-1))^(1/2)*(d*(c*x \\ & +1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)- \\ & 2*c*x)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))/c/(c^2*x^2-1)+2*a*b*(-1/4*(-e* \\ & (c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arcsin(c \\ & *x)^2+1/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x \\ & ^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(I+2*arcsin(c*x))/c/(c^2*x^2- \\ & 1)+1/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c \\ & ^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcsin(c*x))/c/(c^2*x^2-1) \end{aligned}$$

Fricas [F]

$$\begin{aligned} & \int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2 dx \end{aligned}$$

input `integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm m="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)`

Sympy [F]

$$\begin{aligned} & \int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \int \sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\arcsin(cx))^2 dx \end{aligned}$$

input `integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*asin(c*x))**2,x)`

output `Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*asin(c*x))**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx = \text{Exception raised: ValueError}$$

input `integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm m="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\begin{aligned} & \int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2 dx \end{aligned}$$

input

```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2,x, algorithm
m="giac")
```

output

```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arcsin(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2 dx \\ &= \int (a+b\arcsin(cx))^2 \sqrt{d+cdx}\sqrt{e-cex} dx \end{aligned}$$

input

```
int((a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2),x)
```

output

```
int((a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx} \sqrt{e-cex} (a+b \arcsin(cx))^2 dx$$

$$= \frac{\sqrt{e} \sqrt{d} \left(-2a \sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 + \sqrt{cx+1} \sqrt{-cx+1} a^2 cx + 4 \left(\int \sqrt{cx+1} \sqrt{-cx+1} a \sin(cx) dx \right) abc + 2 \right)}{2c}$$

input `int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(e)*sqrt(d)*(- 2*asin(sqrt(- c*x + 1)/sqrt(2))*a**2 + sqrt(c*x + 1) *sqrt(- c*x + 1)*a**2*c*x + 4*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x),x)*a*b*c + 2*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)**2,x)*b**2*c) / (2*c)`

3.485 $\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{x} dx$

Optimal result	4112
Mathematica [A] (verified)	4113
Rubi [A] (verified)	4114
Maple [B] (verified)	4117
Fricas [F]	4118
Sympy [F]	4119
Maxima [F(-2)]	4119
Giac [F]	4119
Mupad [F(-1)]	4120
Reduce [F]	4121

Optimal result

Integrand size = 35, antiderivative size = 432

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{x} dx$$

$$= -2b^2\sqrt{d+cdx}\sqrt{e-cex} - \frac{2abcx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{1-c^2x^2}}$$

$$- \frac{2b^2cx\sqrt{d+cdx}\sqrt{e-cex} \arcsin(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2$$

$$- \frac{2\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$+ \frac{2ib\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{2ib\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{2b^2\sqrt{d+cdx}\sqrt{e-cex} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$+ \frac{2b^2\sqrt{d+cdx}\sqrt{e-cex} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

output

```

-2*b^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)-2*a*b*c*x*(c*d*x+d)^(1/2)*(-c*e*x+
e)^(1/2)/(-c^2*x^2+1)^(1/2)-2*b^2*c*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arc
sin(c*x)/(-c^2*x^2+1)^(1/2)+(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c
*x))^2-2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*
x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*(c*d*x+d)^(1/2)*(-c*e*x+e)^(
1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(
1/2)-2*I*b*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I
*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*(c*d*x+d)^(1/2)*(-c*e*x+
e)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*(c*
d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^
2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x} dx = a^2\sqrt{d+cdx}\sqrt{e-cex}$$

$$+ a^2\sqrt{d}\sqrt{e}\log(cx) - a^2\sqrt{d}\sqrt{e}\log\left(de + \sqrt{d}\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\right)$$

$$- \frac{2ab\sqrt{d+cdx}\sqrt{e-cex}(cx - \sqrt{1-c^2x^2}\arcsin(cx) - \arcsin(cx)\log(1 - e^{i\arcsin(cx)}) + \arcsin(cx)\log(1 - e^{i\arcsin(cx)}))}{\sqrt{1-c^2x^2}}$$

$$- \frac{b^2\sqrt{d+cdx}\sqrt{e-cex}(2\sqrt{1-c^2x^2} + 2cx\arcsin(cx) - \sqrt{1-c^2x^2}\arcsin(cx)^2 - \arcsin(cx)^2\log(1 - e^{i\arcsin(cx)}))}{\sqrt{1-c^2x^2}}$$

input

```
Integrate[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2)/x,x]
```

output

```

a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x] + a^2*Sqrt[d]*Sqrt[e]*Log[c*x] - a^2*S
qrt[d]*Sqrt[e]*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]]
- (2*a*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(c*x - Sqrt[1 - c^2*x^2]*ArcSin[c
*x] - ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) + ArcSin[c*x]*Log[1 + E^(I*Ar
cSin[c*x])]) - I*PolyLog[2, -E^(I*ArcSin[c*x])] + I*PolyLog[2, E^(I*ArcSin[
c*x])])]/Sqrt[1 - c^2*x^2] - (b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*Sqrt[
1 - c^2*x^2] + 2*c*x*ArcSin[c*x] - Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2 - ArcSi
n[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] + ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*
x])]) - (2*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] + (2*I)*ArcSin[c*x
]*PolyLog[2, E^(I*ArcSin[c*x])] + 2*PolyLog[3, -E^(I*ArcSin[c*x])] - 2*Pol
yLog[3, E^(I*ArcSin[c*x])])]/Sqrt[1 - c^2*x^2]

```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5238, 5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx + d}\sqrt{e - cex}(a + b \arcsin(cx))^2}{x} dx$$

$$\downarrow \text{5238}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \int \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2}{x} dx}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{5198}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2x^2}} dx - 2bc \int (a + b \arcsin(cx)) dx + \sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2x^2}} dx + \sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 - 2bc \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2x^2}}{c} \right) \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{5218}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{(a + b \arcsin(cx))^2}{cx} d \arcsin(cx) + \sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 - 2bc \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2x^2}}{c} \right) \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{3042}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(\int (a + b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx) + \sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 - 2bc \left(ax + \frac{bx \arcsin(cx)}{c} + \frac{b\sqrt{1 - c^2x^2}}{c} \right) \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{4671}$$

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(-2b \int (a + b \arcsin(cx)) \log(1 - e^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a + b \arcsin(cx)) \log(1 + e^{i \arcsin(cx)}) d \arcsin(cx) \right)}{\sqrt{1 - c^2x^2}}$$

↓ 3011

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx) \right)}{}$$

↓ 2720

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a + b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) \right)}{}$$

↓ 7143

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a + b \arcsin(cx))^2 + \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 - 2bc(ax + bx \arcsin(cx)) \right)}{}$$

input `Int[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2)/x,x]`

output `(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2 - 2*b*c*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]) - 2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])])/Sqrt[1 - c^2*x^2]`

Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 $\text{Int}[\text{Log}[1 + (e_.) * ((F_.)^{((c_.) * (a_.) + (b_.) * (x_)))})^{(n_.)}] * ((f_.) + (g_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-f + g*x)^m * (\text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n]) / (b*c*n*\text{Log}[F]), x] + \text{Simp}[g*(m/(b*c*n*\text{Log}[F])) \text{Int}[(f + g*x)^{(m - 1)} * \text{PolyLog}[2, (-e) * (F^{(c*(a + b*x))})^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.) * (x_)] * ((c_.) + (d_.) * (x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2*(c + d*x)^m * (\text{ArcTanh}[E^{(I*(e + f*x))}] / f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 + E^{(I*(e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 5198 $\text{Int}[((a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.))^{(n_.)} * ((f_.) * (x_))^{(m_.)} * \text{Sqrt}[(d_.) + (e_.) * (x_)^2], x_Symbol] \rightarrow \text{Simp}[(f*x)^{(m + 1)} * \text{Sqrt}[d + e*x^2] * ((a + b*\text{ArcSin}[c*x])^n / (f*(m + 2))), x] + (\text{Simp}[(1/(m + 2)) * \text{Simp}[\text{Sqrt}[d + e*x^2] / \text{Sqrt}[1 - c^2*x^2]] \text{Int}[(f*x)^m * ((a + b*\text{ArcSin}[c*x])^n / \text{Sqrt}[1 - c^2*x^2]), x], x] - \text{Simp}[b*c*(n/(f*(m + 2))) * \text{Simp}[\text{Sqrt}[d + e*x^2] / \text{Sqrt}[1 - c^2*x^2]] \text{Int}[(f*x)^{(m + 1)} * (a + b*\text{ArcSin}[c*x])^{(n - 1)}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& (\text{IGtQ}[m, -2] || \text{EqQ}[n, 1])$

rule 5218 $\text{Int}[(((a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.))^{(n_.)} * (x_)^{(m_.)}) / \text{Sqrt}[(d_.) + (e_.) * (x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/c^{(m + 1)}) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] \text{Subst}[\text{Int}[(a + b*x)^n * \text{Sin}[x]^m, x], x, \text{ArcSin}[c*x]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$

rule 5238 $\text{Int}[((a_.) + \text{ArcSin}[(c_.) * (x_)] * (b_.))^{(n_.)} * ((h_.) * (x_))^{(m_.)} * ((d_.) + (e_.) * (x_))^{(p_.)} * ((f_.) + (g_.) * (x_))^{(q_.)}, x_Symbol] \rightarrow \text{Simp}[((-d^2) * (g/e))^{\text{IntPart}[q]} * (d + e*x)^{\text{FracPart}[q]} * ((f + g*x)^{\text{FracPart}[q]} / (1 - c^2*x^2)^{\text{FracPart}[q]}) \text{Int}[(h*x)^m * (d + e*x)^{(p - q)} * (1 - c^2*x^2)^q * (a + b*\text{ArcSin}[c*x])^n], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, n\}, x] \&\& \text{EqQ}[e*f + d*g, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0] \&\& \text{HalfIntegerQ}[p, q] \&\& \text{GeQ}[p - q, 0]$

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(420) = 840$.

Time = 3.60 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.07

method	result
default	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \left(-de \ln \left(\frac{2\sqrt{de} \sqrt{-de(c^2x^2-1)} + 2de}{x} \right) + \sqrt{de} \sqrt{-de(c^2x^2-1)} \right)}{\sqrt{de} \sqrt{-de(c^2x^2-1)}} + b^2 \left(\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} (c^2x^2 - 1)}{\sqrt{de} \sqrt{-de(c^2x^2-1)}} \right)$
parts	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \left(-de \ln \left(\frac{2\sqrt{de} \sqrt{-de(c^2x^2-1)} + 2de}{x} \right) + \sqrt{de} \sqrt{-de(c^2x^2-1)} \right)}{\sqrt{de} \sqrt{-de(c^2x^2-1)}} + b^2 \left(\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} (c^2x^2 - 1)}{\sqrt{de} \sqrt{-de(c^2x^2-1)}} \right)$

input

```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output

```

a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-d*e*ln(2*((d*e)^(1/2)*(-d*e*(c^
2*x^2-1))^(1/2)+d*e)/x)+(d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))/(d*e)^(1/2)/
(-d*e*(c^2*x^2-1))^(1/2)+b^2*(1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(c^
2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))/(c^2*x
^2-1)+1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c
^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))/(c^2*x^2-1)-(-e*(c*x-1))^(1/2)
*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)*(arcsin(c*x)^2*ln(1-(I*c*x+(-c^2*x^2
+1)^(1/2))^(1/2))+arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-arc
sin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x
-(-c^2*x^2+1)^(1/2))-4*I*arcsin(c*x)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(
1/2))-4*I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-2*poly
log(3,-I*c*x-(-c^2*x^2+1)^(1/2))+8*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^(1
/2))+8*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2)))/(c^2*x^2-1))+2*a*b*(1
/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-
1)*(arcsin(c*x)+I)/(c^2*x^2-1)+1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I
*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)/(c^2*x^2-1)+I*(-c^2*x^2
+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*arcsin(c*x)*ln(1-(I*c*x+
(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(
1/2))-I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-polylog(2,-I*c*x-(-c^2*
x^2+1)^(1/2))+2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2...

```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x} dx$$

$$= \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2}{x} dx$$

input

```

integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x,x, algori
thm="fricas")

```

output

```

integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/x, x)

```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x} dx$$

$$= \int \frac{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\arcsin(cx))^2}{x} dx$$

input `integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*asin(c*x))**2/x,x)`

output `Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*asin(c*x))**2/x, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x} dx = \text{Exception raised: ValueError}$$

input `integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x} dx$$

$$= \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2}{x} dx$$

input `integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output `integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arcsin(c*x) + a)^2/x, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x} dx$$

$$= \int \frac{(a+b\arcsin(cx))^2 \sqrt{d+cdx}\sqrt{e-cex}}{x} dx$$

input `int(((a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x,x)`

output `int(((a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x, x)`

Reduce [F]

$$\begin{aligned}
& \int \frac{\sqrt{d+cx}\sqrt{e-cx}(a+b\arcsin(cx))^2}{x} dx \\
&= \sqrt{e}\sqrt{d} \left(\sqrt{cx+1}\sqrt{-cx+1}a^2 + 2 \left(\int \frac{\sqrt{cx+1}\sqrt{-cx+1}a\sin(cx)}{x} dx \right) ab \right. \\
&\quad \left. + \left(\int \frac{\sqrt{cx+1}\sqrt{-cx+1}a\sin(cx)^2}{x} dx \right) b^2 \right. \\
&\quad - \log \left(-\sqrt{2} + \tan \left(\frac{a\sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2} \right) - 1 \right) a^2 \\
&\quad + \log \left(-\sqrt{2} + \tan \left(\frac{a\sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2} \right) + 1 \right) a^2 \\
&\quad - \log \left(\sqrt{2} + \tan \left(\frac{a\sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2} \right) - 1 \right) a^2 \\
&\quad \left. + \log \left(\sqrt{2} + \tan \left(\frac{a\sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2} \right) + 1 \right) a^2 \right)
\end{aligned}$$

input `int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*asin(c*x))^2/x,x)`

output `sqrt(e)*sqrt(d)*(sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2 + 2*int((sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x))/x,x)*a*b + int((sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)**2)/x,x)*b**2 - log(-sqrt(2) + tan(asin(sqrt(-c*x + 1)/sqrt(2))/2) - 1)*a**2 + log(-sqrt(2) + tan(asin(sqrt(-c*x + 1)/sqrt(2))/2) + 1)*a**2 - log(sqrt(2) + tan(asin(sqrt(-c*x + 1)/sqrt(2))/2) - 1)*a**2 + log(sqrt(2) + tan(asin(sqrt(-c*x + 1)/sqrt(2))/2) + 1)*a**2)`

3.486 $\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{x^2} dx$

Optimal result	4122
Mathematica [A] (verified)	4123
Rubi [A] (verified)	4123
Maple [B] (verified)	4127
Fricas [F]	4128
Sympy [F]	4128
Maxima [F(-2)]	4129
Giac [F]	4129
Mupad [F(-1)]	4130
Reduce [F]	4130

Optimal result

Integrand size = 35, antiderivative size = 257

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{x^2} dx$$

$$= -\frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{x} - \frac{ic\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}}$$

$$- \frac{c\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^3}{3b\sqrt{1-c^2x^2}}$$

$$+ \frac{2bc\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{ib^2c\sqrt{d+cdx}\sqrt{e-cex} \text{PolyLog}(2, e^{2i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

output

```
-(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x-I*c*(c*d*x+d)^(1/2)
)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(1/2)-1/3*c*(c*d*x+d)^(
1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*(c*d
*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1
/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*polylo
g(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx$$

$$= \frac{-3a^2\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2} - 3ib\sqrt{d+cdx}\sqrt{e-cex}(-iacx+bcx-ib\sqrt{1-c^2x^2})\arcsin(cx)^2 - \dots}{\dots}$$

input

```
Integrate[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```
(-3*a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] - (3*I)*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((-I)*a*c*x + b*c*x - I*b*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 - b^2*c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^3 + 3*a^2*c*Sqrt[d]*Sqrt[e]*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]*(-(a*Sqrt[1 - c^2*x^2]) + b*c*x*Log[1 - E^((2*I)*ArcSin[c*x])]) + 6*a*b*c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Log[c*x] - (3*I)*b^2*c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(3*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {5238, 5196, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx$$

$$\downarrow \text{5238}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}}{\sqrt{1-c^2x^2}} \int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x^2} dx$$

↓ 5196

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \int \frac{a+b \arcsin(cx)}{x} dx - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 5136

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \int \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{cx} d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 3042

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \int -((a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2})) d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 25

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) - 2bc \int (a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 4200

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \left(2i \int - \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 25

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \left(-2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 2620

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{2} i b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 2715

$$\frac{\sqrt{cdx + d}\sqrt{e - cex} \left(c^2 \left(- \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) + 2bc \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} d \arcsin(cx) \right) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

↓ 2838

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int\frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x}+2bc\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+\right.\right.}{\left.\left.\sqrt{1-c^2x^2}\right)}\right)$$

↓ 5152

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x}+2bc\left(-2i\left(\frac{1}{2}i\log(1-e^{2i\arcsin(cx)})\right)(a+b\arcsin(cx))+\frac{1}{4}b\text{PolyLog}\right.\right.}{\left.\left.\sqrt{1-c^2x^2}\right)}\right)$$

input `Int[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcSin[c*x])^2)/x^2,x]`

output `(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/x) - (c*(a + b*ArcSin[c*x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])]) + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4)))/Sqrt[1 - c^2*x^2]`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5196 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*((f_.)*(x_)^(m_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]`

rule 5238 `Int[((a_.) + ArcSin[(c_.)*(x_)*(b_.)]^(n_.)*((h_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(p_.)*((f_) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((-d^2)*(g/e))^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(245) = 490$.

Time = 3.86 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.67

method	result
default	$\frac{a^2 \left(-\arctan\left(\frac{\sqrt{c^2 d e x}}{\sqrt{-d e (c^2 x^2 - 1)}}\right) x c^2 d e - \sqrt{c^2 d e} \sqrt{-d e (c^2 x^2 - 1)} \right) \sqrt{d(c x + 1)} \sqrt{-e(c x - 1)}}{\sqrt{c^2 d e} \sqrt{-d e (c^2 x^2 - 1)} x} + b^2 \left(\frac{\sqrt{-e(c x - 1)} \sqrt{d(c x + 1)} \sqrt{-c^2 d e}}{3 c^2 x^2 - 3} \right)$
parts	$\frac{a^2 \left(-\arctan\left(\frac{\sqrt{c^2 d e x}}{\sqrt{-d e (c^2 x^2 - 1)}}\right) x c^2 d e - \sqrt{c^2 d e} \sqrt{-d e (c^2 x^2 - 1)} \right) \sqrt{d(c x + 1)} \sqrt{-e(c x - 1)}}{\sqrt{c^2 d e} \sqrt{-d e (c^2 x^2 - 1)} x} + b^2 \left(\frac{\sqrt{-e(c x - 1)} \sqrt{d(c x + 1)} \sqrt{-c^2 d e}}{3 c^2 x^2 - 3} \right)$

input

```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output

```
a^2*(-arctan((c^2*d*e)^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2))*x*c^2*d*e-(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c^2*d*e)^(1/2)/(-d*e*(c^2*x^2-1))^(1/2)/x+b^2*(1/3*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^3/c/(c^2*x^2-1)-(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)^2/x/(c^2*x^2-1)+2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+arcsin(c*x)^2+polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))+2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2)))/c/(c^2*x^2-1)+2*a*b*(1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)*arcsin(c*x)^2/c/(c^2*x^2-1)+2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arcsin(c*x)*c/(c^2*x^2-1)-(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)/x/(c^2*x^2-1)-(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)*ln((I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))^2-1)*c/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx$$

$$= \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2}{x^2} dx$$

input `integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x, algorithm="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/x^2, x)`

Sympy [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx$$

$$= \int \frac{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\arcsin(cx))^2}{x^2} dx$$

input `integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*asin(c*x))**2/x**2,x)`

output `Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*asin(c*x))**2/x**2, x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx = \text{Exception raised: ValueError}$$

input

```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x, algo
rithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\begin{aligned} & \int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx \\ &= \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arcsin(cx)+a)^2}{x^2} dx \end{aligned}$$

input

```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/x^2,x, algo
rithm="giac")
```

output

```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arcsin(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx$$

$$= \int \frac{(a+b\arcsin(cx))^2 \sqrt{d+cdx}\sqrt{e-cex}}{x^2} dx$$

input `int(((a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x^2,x)`output `int(((a + b*asin(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x^2, x)`**Reduce [F]**

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arcsin(cx))^2}{x^2} dx$$

$$= \frac{\sqrt{e}\sqrt{d}\left(2a\sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2cx - \sqrt{cx+1}\sqrt{-cx+1}a^2 + 2\left(\int \frac{\sqrt{cx+1}\sqrt{-cx+1}\arcsin(cx)}{x^2} dx\right)abx + \left(\int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{x}\right)\right)}{x}$$

input `int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*asin(c*x))^2/x^2,x)`output `(sqrt(e)*sqrt(d)*(2*asin(sqrt(-c*x + 1)/sqrt(2))*a**2*c*x - sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2 + 2*int((sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x))/x**2,x)*a*b*x + int((sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)**2)/x**2,x)*b**2*x))/x`

3.487 $\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2 dx$

Optimal result	4131
Mathematica [A] (verified)	4132
Rubi [A] (verified)	4133
Maple [C] (verified)	4138
Fricas [F]	4139
Sympy [F(-1)]	4140
Maxima [F(-2)]	4140
Giac [F]	4140
Mupad [F(-1)]	4141
Reduce [F]	4141

Optimal result

Integrand size = 35, antiderivative size = 509

$$\begin{aligned}
 & \int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2 dx = \\
 & -\frac{7b^2dex\sqrt{d+cdx}\sqrt{e-cex}}{1152c^2} - \frac{43b^2dex^3\sqrt{d+cdx}\sqrt{e-cex}}{1728} \\
 & + \frac{1}{108}b^2c^2dex^5\sqrt{d+cdx}\sqrt{e-cex} + \frac{7b^2de\sqrt{d+cdx}\sqrt{e-cex} \arcsin(cx)}{1152c^3\sqrt{1-c^2x^2}} \\
 & + \frac{bdex^2\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{16c\sqrt{1-c^2x^2}} \\
 & - \frac{7bcdex^4\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{48\sqrt{1-c^2x^2}} \\
 & + \frac{bc^3dex^6\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{18\sqrt{1-c^2x^2}} \\
 & - \frac{dex\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{16c^2} \\
 & + \frac{1}{8}dex^3\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2 \\
 & + \frac{1}{6}dex^3\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)(a+b \arcsin(cx))^2 \\
 & + \frac{de\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^3}{48bc^3\sqrt{1-c^2x^2}}
 \end{aligned}$$

output

```
-7/1152*b^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2-43/1728*b^2*d*e*x^3
*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+1/108*b^2*c^2*d*e*x^5*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)+7/1152*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arcsin(c*x)
/c^3/(-c^2*x^2+1)^(1/2)+1/16*b*d*e*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a
+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-7/48*b*c*d*e*x^4*(c*d*x+d)^(1/2)*(-c*
e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+1/18*b*c^3*d*e*x^6*(c*d*
x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/16*d*e*
x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/c^2+1/8*d*e*x^3*(c*
d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2+1/6*d*e*x^3*(c*d*x+d)^(1
/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2+1/48*d*e*(c*d*x+d)^(
1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.89

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \frac{288b^2de\sqrt{d + cdx}\sqrt{e - cex} \arcsin(cx)^3 - 864a^2d^{3/2}e^{3/2}\sqrt{1 - c^2x^2} \arctan\left(\frac{cx\sqrt{d + cdx}}{\sqrt{1 - c^2x^2}}\right) - 12bd\sqrt{d + cdx}\sqrt{e - cex} \arcsin(cx) + 9b^2\cos(2\arcsin(cx)) + 9b^2\cos(4\arcsin(cx)) + 2b^2\cos(6\arcsin(cx)) - 36a\sin(2\arcsin(cx)) + 36a\sin(4\arcsin(cx)) + 12a^2\sin(6\arcsin(cx)) - 72bd\sqrt{d + cdx}\sqrt{e - cex} \arcsin(cx)^2(-12a - 3b\sin(2\arcsin(cx)) + 3b\sin(4\arcsin(cx)) + b\sin(6\arcsin(cx))) + d\sqrt{d + cdx}\sqrt{e - cex}(-864a^2cx\sqrt{1 - c^2x^2} + 4032a^2c^3x^3\sqrt{1 - c^2x^2} - 2304a^2c^5x^5\sqrt{1 - c^2x^2} + 216ab\cos(2\arcsin(cx)) - 108ab\cos(4\arcsin(cx)) - 24ab\cos(6\arcsin(cx)) - 108b^2\sin(2\arcsin(cx)) + 27b^2\sin(4\arcsin(cx)) + 4b^2\sin(6\arcsin(cx)))}{(13824c^3\sqrt{1 - c^2x^2})}$$

input

```
Integrate[x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(288*b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^3 - 864*a^2*d^(3/2)
*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/
(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 12*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*
x]*ArcSin[c*x]*(-18*b*Cos[2*ArcSin[c*x]] + 9*b*Cos[4*ArcSin[c*x]] + 2*b*Co
s[6*ArcSin[c*x]] - 36*a*SIn[2*ArcSin[c*x]] + 36*a*SIn[4*ArcSin[c*x]] + 12*
a*SIn[6*ArcSin[c*x]]) - 72*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*
x]^2*(-12*a - 3*b*SIn[2*ArcSin[c*x]] + 3*b*SIn[4*ArcSin[c*x]] + b*SIn[6*Ar
cSin[c*x]]) + d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-864*a^2*c*x*Sqrt[1 - c
^2*x^2] + 4032*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] - 2304*a^2*c^5*x^5*Sqrt[1 - c
^2*x^2] + 216*a*b*Cos[2*ArcSin[c*x]] - 108*a*b*Cos[4*ArcSin[c*x]] - 24*a*b
*Cos[6*ArcSin[c*x]] - 108*b^2*SIn[2*ArcSin[c*x]] + 27*b^2*SIn[4*ArcSin[c*x
]] + 4*b^2*SIn[6*ArcSin[c*x]]))/(13824*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.90, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5238, 5202, 5192, 27, 363, 262, 262, 223, 5198, 5138, 262, 262, 223, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx + d)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx$$

$$\downarrow \text{5238}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \int x^2(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))^2 dx}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{5202}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 dx - \frac{1}{3}bc \int x^3(1 - c^2x^2)(a + b \arcsin(cx)) dx + \frac{1}{6}x^3(1 - c^2x^2)(a + b \arcsin(cx))^2 \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{5192}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 dx - \frac{1}{3}bc \left(-bc \int \frac{x^4(3 - 2c^2x^2)}{12\sqrt{1 - c^2x^2}} dx - \frac{1}{6}c^2x^6(a + b \arcsin(cx))^2 \right) \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{27}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 dx - \frac{1}{3}bc \left(-\frac{1}{12}bc \int \frac{x^4(3 - 2c^2x^2)}{\sqrt{1 - c^2x^2}} dx - \frac{1}{6}c^2x^6(a + b \arcsin(cx))^2 \right) \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{363}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 dx - \frac{1}{3}bc \left(-\frac{1}{12}bc \left(\frac{4}{3} \int \frac{x^4}{\sqrt{1 - c^2x^2}} dx + \frac{1}{3}x^5\sqrt{1 - c^2x^2} \right) - \frac{1}{6}c^2x^6(a + b \arcsin(cx))^2 \right) \right)}{\sqrt{1 - c^2x^2}}$$

$$\downarrow \text{262}$$

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 dx - \frac{1}{3} bc \left(-\frac{1}{12} bc \left(\frac{4}{3} \left(\frac{3 \int \frac{x^2}{\sqrt{1 - c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1 - c^2 x^2}}{4c^2} \right) + \frac{1}{3} \right) \right) \right)$$

↓ 262

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 dx - \frac{1}{3} bc \left(-\frac{1}{12} bc \left(\frac{4}{3} \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1 - c^2 x^2}} dx}{2c^2} - \frac{x \sqrt{1 - c^2 x^2}}{2c^2} \right)}{4c^2} - x^3 \right) \right) \right) \right)$$

↓ 223

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \int x^2 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx))^2 dx + \frac{1}{6} x^3 (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2 - \frac{1}{3} bc \left(-\frac{1}{6} \right) \right)$$

↓ 5198

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{2} bc \int x^3 (a + b \arcsin(cx)) dx + \frac{1}{4} x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right)$$

↓ 5138

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{2} bc \left(\frac{1}{4} x^4 (a + b \arcsin(cx)) - \frac{1}{4} bc \int \frac{x^4}{\sqrt{1 - c^2 x^2}} dx \right) + \frac{1}{4} x^3 \sqrt{1 - c^2 x^2} (a + b \arcsin(cx)) \right) \right)$$

↓ 262

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \int \frac{x^2 (a + b \arcsin(cx))^2}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{2} bc \left(\frac{1}{4} x^4 (a + b \arcsin(cx)) - \frac{1}{4} bc \left(\frac{3 \int \frac{x^2}{\sqrt{1 - c^2 x^2}} dx}{4c^2} - \frac{x^3 \sqrt{1 - c^2 x^2}}{4c^2} \right) \right) \right) \right)$$

↓ 262

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - \frac{1}{2}bc \left(\frac{1}{4}x^4(a+b\arcsin(cx)) - \frac{1}{4}bc \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx - x\sqrt{1-c^2x^2}}{2c^2} \right)}{4c^2} \right) \right) \right)$$

↓ 223

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \int \frac{x^2(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{4}x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{1}{4}x^4(a+b\arcsin(cx)) \right) \right)$$

↓ 5210

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \left(\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{b \int x(a+b\arcsin(cx)) dx}{c} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} \right) + \frac{1}{4}x^3\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2 \right)$$

↓ 5138

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \left(\frac{b \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c} + \frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} \right) \right)$$

↓ 262

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \left(\frac{b \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} + \frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} \right) \right)$$

↓ 223

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{2} \left(\frac{1}{4} \left(\frac{\int \frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b\arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right) \right)$$

↓ 5152

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{1}{6}x^3(1 - c^2x^2)^{3/2}(a + b\arcsin(cx))^2 + \frac{1}{2} \left(\frac{1}{4}x^3\sqrt{1 - c^2x^2}(a + b\arcsin(cx))^2 + \frac{1}{4} \left(\frac{a+b\arcsin(cx)}{6} \right) \right) \right)$$

input `Int[x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2,x]`

output `(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x^3*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/6 - (b*c*((x^4*(a + b*ArcSin[c*x])))/4 - (c^2*x^6*(a + b*ArcSin[c*x]))/6 - (b*c*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2))/3))/12))/3 + ((x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/4 - (b*c*((x^4*(a + b*ArcSin[c*x])))/4 - (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2))/4))/2 + (-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2 + (a + b*ArcSin[c*x])^3/(6*b*c^3) + (b*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c)/4)/2))/Sqrt[1 - c^2*x^2]`

Defintions of rubi rules used

rule 27 `Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]`

rule 223 `Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

rule 262 `Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/(b*(m + 2*p + 1)) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]`

rule 363

```
Int[((e._)*(x._))^(m._)*((a._) + (b._)*(x._)^2)^(p._)*((c._) + (d._)*(x._)^2), x
_Symbol] := Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3)) Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5138

```
Int[((a._) + ArcSin[(c._)*(x._)]*(b._))^(n._)*((d._)*(x._))^(m._), x_Symbol]
:= Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5152

```
Int[((a._) + ArcSin[(c._)*(x._)]*(b._))^(n._)/Sqrt[(d._) + (e._)*(x._)^2], x_S
ymbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

rule 5192

```
Int[((a._) + ArcSin[(c._)*(x._)]*(b._))*((f._)*(x._))^(m._)*((d._) + (e._)*(x._
^2)^(p._), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5198

```
Int[((a._) + ArcSin[(c._)*(x._)]*(b._))^(n._)*((f._)*(x._))^(m._)*Sqrt[(d._) +
(e._)*(x._)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
- c^2*x^2]] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[
(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5202

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcS
in[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]
```

rule 5210

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSin[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
+ 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] + S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.73 (sec) , antiderivative size = 1464, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	1464
parts	Expression too large to display	1464

input

```
int(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RET
URNVERBOSE)
```

output

```

1/48*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*d*e*(-8*x^5*c^4*(-d*e*(c^2*x
^2-1))^(1/2)*(c^2*d*e)^(1/2)+14*x^3*c^2*(-d*e*(c^2*x^2-1))^(1/2)*(c^2*d*e)
^(1/2)+3*arctan((c^2*d*e)^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2))*d*e-3*(c^2*d*e)
^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x/c^2/(-d*e*(c^2*x^2-1))^(1/2)/(c^2*d*e)
^(1/2)+b^2*(-1/48*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/
c^3/(c^2*x^2-1)*arcsin(c*x)^3*d*e-1/6912*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1
/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x
^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)
^(1/2)-6*c*x)*(6*I*arcsin(c*x)+18*arcsin(c*x)^2-1)*d*e/c^3/(c^2*x^2-1)+1/
256*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2
*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))*d
*e/c^3/(c^2*x^2-1)+1/27648*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*c^2*x^2
-c*x*(-c^2*x^2+1)^(1/2)-I)*(132*I*arcsin(c*x)+144*arcsin(c*x)^2-23)*cos(5*
arcsin(c*x))*d*e/c^3/(c^2*x^2-1)-1/27648*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1
/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(84*I*arcsin(c*x)+288*arcsin(c*x)
^2-31)*sin(5*arcsin(c*x))*d*e/c^3/(c^2*x^2-1)-1/1024*(-e*(c*x-1))^(1/2)*(d
*(c*x+1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(4*I*arcsin(c*x)+16*a
rcsin(c*x)^2-5)*cos(3*arcsin(c*x))*d*e/c^3/(c^2*x^2-1)+3/1024*(-e*(c*x-1))
^(1/2)*(d*(c*x+1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(I+4*arcsin(
c*x))*sin(3*arcsin(c*x))*d*e/c^3/(c^2*x^2-1))+2*a*b*(-1/32*(-e*(c*x-1))...

```

Fricas [F]

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \int (cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2 x^2 dx$$

input

```

integrate(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algo
rithm="fricas")

```

output

```

integral(-(a^2*c^2*d*e*x^4 - a^2*d*e*x^2 + (b^2*c^2*d*e*x^4 - b^2*d*e*x^2)
*arcsin(c*x)^2 + 2*(a*b*c^2*d*e*x^4 - a*b*d*e*x^2)*arcsin(c*x))*sqrt(c*d*x
+ d)*sqrt(-c*e*x + e), x)

```


Sympy [F(-1)]

Timed out.

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x**2*(c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \int (cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2 x^2 dx$$

input `integrate(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output

```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arcsin(c*x) + a)^2*x^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \int x^2(a + b \arcsin(cx))^2 (d + cdx)^{3/2} (e - cex)^{3/2} dx$$

input

```
int(x^2*(a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2),x)
```

output

```
int(x^2*(a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int x^2(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \frac{\sqrt{e} \sqrt{d} de \left(-6 \arcsin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 - 8\sqrt{cx+1} \sqrt{-cx+1} a^2 c^5 x^5 + 14\sqrt{cx+1} \right)}{48c^3}$$

input

```
int(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*asin(c*x))^2,x)
```

output

```
(sqrt(e)*sqrt(d)*d*e*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 8*sqrt(c*
x + 1)*sqrt( - c*x + 1)*a**2*c**5*x**5 + 14*sqrt(c*x + 1)*sqrt( - c*x + 1)
*a**2*c**3*x**3 - 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x - 96*int(sqrt(
c*x + 1)*sqrt( - c*x + 1)*asin(c*x)*x**4,x)*a*b*c**5 + 96*int(sqrt(c*x + 1)
)*sqrt( - c*x + 1)*asin(c*x)*x**2,x)*a*b*c**3 - 48*int(sqrt(c*x + 1)*sqrt(
 - c*x + 1)*asin(c*x)**2*x**4,x)*b**2*c**5 + 48*int(sqrt(c*x + 1)*sqrt( -
c*x + 1)*asin(c*x)**2*x**2,x)*b**2*c**3)/(48*c**3)
```

3.488 $\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2 dx$

Optimal result	4142
Mathematica [A] (verified)	4143
Rubi [A] (verified)	4143
Maple [B] (verified)	4146
Fricas [A] (verification not implemented)	4147
Sympy [F(-1)]	4148
Maxima [F(-2)]	4148
Giac [F]	4148
Mupad [F(-1)]	4149
Reduce [F]	4149

Optimal result

Integrand size = 33, antiderivative size = 338

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2 dx = \frac{16b^2de\sqrt{d+cdx}\sqrt{e-cex}}{75c^2} + \frac{8b^2de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{225c^2} + \frac{2b^2de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)^2}{125c^2} + \frac{2bdex\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{5c\sqrt{1-c^2x^2}} - \frac{4bcdex^3\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{15\sqrt{1-c^2x^2}} + \frac{2bc^3dex^5\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{25\sqrt{1-c^2x^2}} - \frac{de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)^2(a+b \arcsin(cx))^2}{5c^2}$$

output

```
16/75*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2+8/225*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)/c^2+2/125*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)^2/c^2+2/5*b*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/c/(-c^2*x^2+1)^(1/2)-4/15*b*c*d*e*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+2/25*b*c^3*d*e*x^5*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)-1/5*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)^2*(a+b*arcsin(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.61

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx =$$

$$\frac{de\sqrt{d + cdx}\sqrt{e - cex}\left(225a^2(-1 + c^2x^2)^3 + 30abcx\sqrt{1 - c^2x^2}(15 - 10c^2x^2 + 3c^4x^4) + 2b^2(149 - 187c^2x^2)\right)}{c^2(-1 + c^2x^2)}$$

input

```
Integrate[x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
-1/1125*(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(225*a^2*(-1 + c^2*x^2)^3 + 30*a*b*c*x*Sqrt[1 - c^2*x^2]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 - 187*c^2*x^2 + 47*c^4*x^4 - 9*c^6*x^6) + 30*b*(15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[1 - c^2*x^2]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcSin[c*x] + 225*b^2*(-1 + c^2*x^2)^3*ArcSin[c*x]^2))/(c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {5238, 5182, 5154, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx + d)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx$$

$$\downarrow 5238$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \int x(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))^2 dx}{\sqrt{1 - c^2x^2}}$$

$$\downarrow 5182$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\frac{2b \int (1 - c^2x^2)^2 (a + b \arcsin(cx)) dx}{5c} - \frac{(1 - c^2x^2)^{5/2} (a + b \arcsin(cx))^2}{5c^2} \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5154

$$\frac{de\sqrt{cdx} + d\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \left(\frac{2b \left(-bc \int \frac{x(3c^4x^4 - 10c^2x^2 + 15)}{15\sqrt{1 - c^2x^2}} dx + \frac{1}{5}c^4x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2x^3(a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5c} \right) - (1 -$$

↓ 27

$$\frac{de\sqrt{cdx} + d\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \left(\frac{2b \left(-\frac{1}{15}bc \int \frac{x(3c^4x^4 - 10c^2x^2 + 15)}{\sqrt{1 - c^2x^2}} dx + \frac{1}{5}c^4x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2x^3(a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5c} \right) - (1 -$$

↓ 1576

$$\frac{de\sqrt{cdx} + d\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \left(\frac{2b \left(-\frac{1}{30}bc \int \frac{3c^4x^4 - 10c^2x^2 + 15}{\sqrt{1 - c^2x^2}} dx^2 + \frac{1}{5}c^4x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2x^3(a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5c} \right) - (1 -$$

↓ 1140

$$\frac{de\sqrt{cdx} + d\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \left(\frac{2b \left(-\frac{1}{30}bc \int \left(3(1 - c^2x^2)^{3/2} + 4\sqrt{1 - c^2x^2} + \frac{8}{\sqrt{1 - c^2x^2}} \right) dx^2 + \frac{1}{5}c^4x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2x^3(a + b \arcsin(cx)) + x(a + b \arcsin(cx)) \right)}{5c} \right) - (1 -$$

↓ 2009

$$\frac{de\sqrt{cdx} + d\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \left(\frac{2b \left(\frac{1}{5}c^4x^5(a + b \arcsin(cx)) - \frac{2}{3}c^2x^3(a + b \arcsin(cx)) + x(a + b \arcsin(cx)) - \frac{1}{30}bc \left(-\frac{6(1 - c^2x^2)^{5/2}}{5c^2} - \frac{8(1 - c^2x^2)^{3/2}}{3c^2} \right) \right)}{5c} \right) - (1 -$$

input

```
Int [x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-1/5*((1 - c^2*x^2)^(5/2)*(a + b*ArcSin[c*x])^2)/c^2 + (2*b*(-1/30*(b*c*((-16*Sqrt[1 - c^2*x^2]))/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2))) + x*(a + b*ArcSin[c*x]) - (2*c^2*x^3*(a + b*ArcSin[c*x]))/3 + (c^4*x^5*(a + b*ArcSin[c*x]))/5)/(5*c))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 1140

```
Int[((d_) + (e_)*(x_)^(m_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[1/2 Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5154

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5182

```
Int[((a_) + ArcSin[(c_)*(x_)])*(b_))^(n_)*(x_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(290) = 580.

Time = 4.11 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.01

method	result
orering	$\frac{(549c^8x^8 - 1982c^6x^6 + 4355c^4x^4 - 1420c^2x^2 + 298)(cdx+d)^{\frac{3}{2}}(-cxe+e)^{\frac{3}{2}}(a+b\arcsin(cx))^2}{1125c^4x^2(cx-1)^2(cx+1)^2} - \frac{2(54c^6x^6 - 217c^4x^4 + 672c^2x^2 - 149)}{...}$
default	Expression too large to display
parts	Expression too large to display

input

```
int(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RETUR
NVERBOSE)
```

output

```

1/1125*(549*c^8*x^8-1982*c^6*x^6+4355*c^4*x^4-1420*c^2*x^2+298)/c^4/x^2/(c
*x-1)^2/(c*x+1)^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2-2/1
125*(54*c^6*x^6-217*c^4*x^4+672*c^2*x^2-149)/c^4/(c*x-1)/x^2/(c*x+1)*((c*d
*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2+3/2*x*(c*d*x+d)^(1/2)*(-c
*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2*c*d-3/2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(1/
2)*(a+b*arcsin(c*x))^2*c*e+2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsi
n(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/1125*(9*c^4*x^4-38*c^2*x^2+149)/c^4/x*(3
*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2*c*d-3*(c*d*x+d)^(3/2
)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2*c*e+4*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3
/2)*(a+b*arcsin(c*x))*b*c/(-c^2*x^2+1)^(1/2)+3/4*x/(c*d*x+d)^(1/2)*(-c*e*x
+e)^(3/2)*(a+b*arcsin(c*x))^2*c^2*d^2-9/2*c^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*(a+b*arcsin(c*x))^2+6*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*a
rcsin(c*x))*c^2*d*b/(-c^2*x^2+1)^(1/2)+3/4*x*(c*d*x+d)^(3/2)/(-c*e*x+e)^(1
/2)*(a+b*arcsin(c*x))^2*c^2*e^2-6*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*
arcsin(c*x))*c^2*e*b/(-c^2*x^2+1)^(1/2)+2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/
2)*b^2*c^2/(-c^2*x^2+1)+2*x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin
(c*x))*b*c^3/(-c^2*x^2+1)^(3/2)

```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.89

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx =$$

$$\frac{(9(25a^2 - 2b^2)c^6 dex^6 - (675a^2 - 94b^2)c^4 dex^4 + (675a^2 - 374b^2)c^2 dex^2 - (225a^2 - 298b^2)de + 225c^6 dex^6 - 3b^2c^4 dex^4 + 3b^2c^2 dex^2 - b^2d)e \arcsin(cx)^2 + 450(a*b*c^6 dex^6 - 3a*b*c^4 dex^4 + 3a*b*c^2 dex^2 - a*b*d)e \arcsin(cx) + 30(3a*b*c^5 dex^5 - 10a*b*c^3 dex^3 + 15a*b*c dex + (3b^2c^5 dex^5 - 10b^2c^3 dex^3 + 15b^2c dex) \arcsin(cx)) \sqrt{-c^2x^2 + 1} \sqrt{c dx + d} \sqrt{-cex + e}}{(c^4x^2 - c^2)}$$

input

```

integrate(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algori
thm="fricas")

```

output

```

-1/1125*(9*(25*a^2 - 2*b^2)*c^6*d*e*x^6 - (675*a^2 - 94*b^2)*c^4*d*e*x^4 +
(675*a^2 - 374*b^2)*c^2*d*e*x^2 - (225*a^2 - 298*b^2)*d*e + 225*(b^2*c^6*
d*e*x^6 - 3*b^2*c^4*d*e*x^4 + 3*b^2*c^2*d*e*x^2 - b^2*d*e)*arcsin(c*x)^2 +
450*(a*b*c^6*d*e*x^6 - 3*a*b*c^4*d*e*x^4 + 3*a*b*c^2*d*e*x^2 - a*b*d*e)*a
rcsin(c*x) + 30*(3*a*b*c^5*d*e*x^5 - 10*a*b*c^3*d*e*x^3 + 15*a*b*c*d*e*x +
(3*b^2*c^5*d*e*x^5 - 10*b^2*c^3*d*e*x^3 + 15*b^2*c*d*e*x)*arcsin(c*x))*sq
rt(-c^2*x^2 + 1))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*x^2 - c^2)

```


Sympy [F(-1)]

Timed out.

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate(x*(c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \text{Exception raised: ValueError}$$

input `integrate(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \int (cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2 x dx$$

input `integrate(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm="giac")`

output `integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arcsin(c*x) + a)^2*x, x)`

Mupad [F(-1)]

Timed out.

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \int x(a + b \arcsin(cx))^2 (d + cdx)^{3/2} (e - cex)^{3/2} dx$$

input `int(x*(a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)`

output `int(x*(a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)`

Reduce [F]

$$\int x(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = \frac{\sqrt{e} \sqrt{d} d e (-\sqrt{cx + 1} \sqrt{-cx + 1} a^2 c^4 x^4 + 2\sqrt{cx + 1} \sqrt{-cx + 1} a^2 c^2 x^2 - \sqrt{cx + 1} \sqrt{-cx + 1} a^2 c^2 x^2 - \sqrt{cx + 1} \sqrt{-cx + 1} a^2 c^2 x^2)}{\dots}$$

input `int(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(e)*sqrt(d)*d*e*(-sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2*c**4*x**4 + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2*c**2*x**2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*a**2 - 10*int(sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)*x**3,x)*a*b*c**4 + 10*int(sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)*x,x)*a*b*c**2 - 5*int(sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)**2*x**3,x)*b**2*c**4 + 5*int(sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(c*x)**2*x,x)*b**2*c**2))/(5*c**2)`

3.489 $\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2 dx$

Optimal result	4150
Mathematica [A] (verified)	4151
Rubi [A] (verified)	4151
Maple [C] (verified)	4155
Fricas [F]	4156
Sympy [F(-1)]	4157
Maxima [F(-2)]	4157
Giac [F]	4157
Mupad [F(-1)]	4158
Reduce [F]	4158

Optimal result

Integrand size = 32, antiderivative size = 362

$$\int (d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2 dx = -\frac{1}{32}b^2x(d+cdx)^{3/2}(e-cex)^{3/2} - \frac{15b^2x(d+cdx)^{3/2}(e-cex)^{3/2}}{64(1-c^2x^2)} + \frac{9b^2(d+cdx)^{3/2}(e-cex)^{3/2} \arcsin(cx)}{64c(1-c^2x^2)^{3/2}}$$

output

```
-1/32*b^2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)-15*b^2*x*(c*d*x+d)^(3/2)*(-c*
e*x+e)^(3/2)/(-64*c^2*x^2+64)+9/64*b^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*ar
csin(c*x)/c/(-c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2
)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(3/2)+1/8*b*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3
/2)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c+1/4*x*(c*d*x+d)^(3/2)*(-c*e*x+e
)^(3/2)*(a+b*arcsin(c*x))^2+3*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcs
in(c*x))^2/(-8*c^2*x^2+8)+1/8*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin
(c*x))^3/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.03

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{32b^2 de \sqrt{d + cdx} \sqrt{e - cex} \arcsin(cx)^3 - 96a^2 d^{3/2} e^{3/2} \sqrt{1 - c^2 x^2} \arctan\left(\frac{cx}{\sqrt{1 - c^2 x^2}}\right) + \dots}{(1 - c^2 x^2)^{3/2}}$$

input

```
Integrate[(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
(32*b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^3 - 96*a^2*d^(3/2)*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 8*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^2*(12*a + 8*b*Sin[2*ArcSin[c*x]] + b*Sin[4*ArcSin[c*x]]) + d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(160*a^2*c*x*Sqrt[1 - c^2*x^2] - 64*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 64*a*b*Cos[2*ArcSin[c*x]] + 4*a*b*Cos[4*ArcSin[c*x]] - 32*b^2*Sin[2*ArcSin[c*x]] - b^2*Sin[4*ArcSin[c*x]]) + 4*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]*(16*b*Cos[2*ArcSin[c*x]] + b*Cos[4*ArcSin[c*x]] + 4*a*(8*Sin[2*ArcSin[c*x]] + Sin[4*ArcSin[c*x]])))/(256*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.77, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {5178, 5158, 5156, 5138, 262, 223, 5152, 5182, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx + d)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx$$

↓ 5178

$$\frac{(cdx + d)^{3/2} (e - cex)^{3/2} \int (1 - c^2 x^2)^{3/2} (a + b \arcsin(cx))^2 dx}{(1 - c^2 x^2)^{3/2}}$$

↓ 5158

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \int x(1 - c^2x^2) (a + b \arcsin(cx))dx + \frac{3}{4} \int \sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2 dx + \frac{1}{4}x \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 5156

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \int x(1 - c^2x^2) (a + b \arcsin(cx))dx + \frac{3}{4} \left(\frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - bc \int x(a + b \arcsin(cx)) \right) \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 5138

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \int x(1 - c^2x^2) (a + b \arcsin(cx))dx + \frac{3}{4} \left(-bc \left(\frac{1}{2}x^2(a + b \arcsin(cx)) \right) - \frac{1}{2}bc \int \frac{x}{\sqrt{1-c^2x^2}} \right) \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 262

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \int x(1 - c^2x^2) (a + b \arcsin(cx))dx + \frac{3}{4} \left(-bc \left(\frac{1}{2}x^2(a + b \arcsin(cx)) \right) - \frac{1}{2}bc \left(\int \frac{x}{\sqrt{1-c^2x^2}} \right) \right) \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 223

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \int x(1 - c^2x^2) (a + b \arcsin(cx))dx + \frac{3}{4} \left(\frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1 - c^2x^2} (a + b \arcsin(cx)) \right) \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 5152

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \int x(1 - c^2x^2) (a + b \arcsin(cx))dx + \frac{1}{4}x(1 - c^2x^2)^{3/2} (a + b \arcsin(cx))^2 + \frac{3}{4} \left(\frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right) \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 5182

$$\frac{(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \left(\frac{b \int (1-c^2x^2)^{3/2} dx}{4c} - \frac{(1-c^2x^2)^2(a+b \arcsin(cx))}{4c^2} \right) + \frac{1}{4}x(1 - c^2x^2)^{3/2} (a + b \arcsin(cx)) \right)}{(1 - c^2x^2)^{3/2}}$$

↓ 211

$$(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \left(\frac{b \left(\frac{3}{4} \int \sqrt{1-c^2x^2} dx + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b \arcsin(cx))}{4c^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) \right)$$

↓ 211

$$(cdx + d)^{3/2}(e - cex)^{3/2} \left(-\frac{1}{2}bc \left(\frac{b \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-c^2x^2}} dx + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b \arcsin(cx))}{4c^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) \right)$$

↓ 223

$$(cdx + d)^{3/2}(e - cex)^{3/2} \left(\frac{1}{4}x(1-c^2x^2)^{3/2} (a + b \arcsin(cx))^2 - \frac{1}{2}bc \left(\frac{b \left(\frac{3}{4} \left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} \right)}{4c} - \frac{(1-c^2x^2)^2(a+b \arcsin(cx))}{4c^2} \right) + \frac{1}{4}x(1-c^2x^2)^{3/2} (a + b \arcsin(cx)) \right)$$

input

```
Int[(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2,x]
```

output

```
((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*((x*(1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/4 + (3*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/2 + (a + b*ArcSin[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/4 - (b*c*(-1/4*((1 - c^2*x^2)^2*(a + b*ArcSin[c*x]))/c^2 + (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/2)/(1 - c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262 $\text{Int}[\text{((c_.)*(x_))}^m \text{((a_) + (b_.)*(x_)^2)}^p, x_Symbol] \text{:> Simp}[c*(c*x)^{m-1} \text{((a + b*x^2)}^{p+1} / (b*(m+2*p+1))), x] - \text{Simp}[a*c^2 \text{((m-1)/(b*(m+2*p+1))) Int}[(c*x)^{m-2} \text{(a + b*x^2)}^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{GtQ}[m, 2-1] \&\& \text{NeQ}[m+2*p+1, 0] \&\& \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[\text{((a_.) + ArcSin[(c_.)*(x_)]*(b_.))}^n \text{((d_.)*(x_))}^m, x_Symbol] \text{:> Simp}[(d*x)^{m+1} \text{((a + b*ArcSin[c*x])}^n / (d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \text{Int}[(d*x)^{m+1} \text{((a + b*ArcSin[c*x])}^{n-1} / \text{Sqrt}[1 - c^2*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[\text{((a_.) + ArcSin[(c_.)*(x_)]*(b_.))}^n / \text{Sqrt}[(d_) + (e_.)*(x_)^2], x_Symbol] \text{:> Simp}[(1/(b*c*(n+1))) * \text{Simp}[\text{Sqrt}[1 - c^2*x^2] / \text{Sqrt}[d + e*x^2]] * (a + b*ArcSin[c*x])^{n+1}, x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[n, -1]$

rule 5156 $\text{Int}[\text{((a_.) + ArcSin[(c_.)*(x_)]*(b_.))}^n \text{Sqrt}[(d_) + (e_.)*(x_)^2], x_Symbol] \text{:> Simp}[x * \text{Sqrt}[d + e*x^2] * \text{((a + b*ArcSin[c*x])}^{n/2}), x] + (\text{Simp}[(1/2) * \text{Simp}[\text{Sqrt}[d + e*x^2] / \text{Sqrt}[1 - c^2*x^2]] \text{Int}[(a + b*ArcSin[c*x])^n / \text{Sqrt}[1 - c^2*x^2], x], x] - \text{Simp}[b*c*(n/2) * \text{Simp}[\text{Sqrt}[d + e*x^2] / \text{Sqrt}[1 - c^2*x^2]] \text{Int}[x * (a + b*ArcSin[c*x])^{n-1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0]$

rule 5158 $\text{Int}[\text{((a_.) + ArcSin[(c_.)*(x_)]*(b_.))}^n \text{((d_) + (e_.)*(x_)^2)}^p, x_Symbol] \text{:> Simp}[x * (d + e*x^2)^p * \text{((a + b*ArcSin[c*x])}^{n/(2*p+1)}), x] + (\text{Simp}[2*d*(p/(2*p+1)) \text{Int}[(d + e*x^2)^{p-1} * (a + b*ArcSin[c*x])^n, x], x] - \text{Simp}[b*c*(n/(2*p+1)) * \text{Simp}[(d + e*x^2)^p / (1 - c^2*x^2)^p] \text{Int}[x * (1 - c^2*x^2)^{p-1/2} * (a + b*ArcSin[c*x])^{n-1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{GtQ}[n, 0] \&\& \text{GtQ}[p, 0]$

rule 5178 $\text{Int}[\text{((a_.) + ArcSin[(c_.)*(x_)]*(b_.))}^n \text{((d_) + (e_.)*(x_))}^p * ((f_) + (g_.)*(x_))^q, x_Symbol] \text{:> Simp}[(d + e*x)^q * ((f + g*x)^q / (1 - c^2*x^2)^q) \text{Int}[(d + e*x)^{p-q} * (1 - c^2*x^2)^q * (a + b*ArcSin[c*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x] \&\& \text{EqQ}[e*f + d*g, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0] \&\& \text{HalfIntegerQ}[p, q] \&\& \text{GeQ}[p - q, 0]$

rule 5182

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 1101, normalized size of antiderivative = 3.04

method	result	size
default	Expression too large to display	1101
parts	Expression too large to display	1101

input

```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x,method=_RETURNV  
ERBOSE)
```


output

```

-1/4*a^2/c/e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)-1/4*a^2*d/c/e*(c*d*x+d)^(1/2)
)*(-c*e*x+e)^(5/2)+1/8*a^2*d/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+3/8*a^2*d*
e/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+3/8*a^2*d^2*e^2*((c*d*x+d)*(-c*e*x+e)
)^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(
1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(-1/8*(-e*(c*x-1))^(1/2)*(d*(c*x+1))
)^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arcsin(c*x)^3*d*e-1/512*(-e*(c*x-1
))^(1/2)*(d*(c*x+1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*
(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(4*I*arc
sin(c*x)+8*arcsin(c*x)^2-1)*d*e/c/(c^2*x^2-1)+1/16*(-e*(c*x-1))^(1/2)*(d*(
c*x+1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/
2)-2*c*x)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))*d*e/c/(c^2*x^2-1)-1/512*(-e*
(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(68*
I*arcsin(c*x)+56*arcsin(c*x)^2-31)*cos(3*arcsin(c*x))*d*e/c/(c^2*x^2-1)+3/
512*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2
-1)*(20*I*arcsin(c*x)+24*arcsin(c*x)^2-11)*sin(3*arcsin(c*x))*d*e/c/(c^2*x
^2-1))+2*a*b*(-3/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2
)/c/(c^2*x^2-1)*arcsin(c*x)^2*d*e-1/256*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/
2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c
^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(I+4*arcsin(c*x))*d*e/c/(c^2*x^2
-1)+1/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x...

```

Fricas [F]

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \int (cdx + d)^{\frac{3}{2}} (-cex + e)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 dx$$

input

```

integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorith
m="fricas")

```

output

```

integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arcsin(
c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arcsin(c*x))*sqrt(c*d*x + d)*sqrt(-
c*e*x + e), x)

```

Sympy [F(-1)]

Timed out.

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Timed out}$$

input `integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*asin(c*x))**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \text{Exception raised: ValueError}$$

input `integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm m="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \int (cdx + d)^{\frac{3}{2}} (-cex + e)^{\frac{3}{2}} (b \arcsin(cx) + a)^2 dx$$

input `integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2,x, algorithm m="giac")`

output `integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arcsin(c*x) + a)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \int (a + b \arcsin(cx))^2 (d + cdx)^{3/2} (e - cex)^{3/2} dx$$

input `int((a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)`

output `int((a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)`

Reduce [F]

$$\int (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arcsin(cx))^2 dx = \frac{\sqrt{e} \sqrt{d} de \left(-6 \arcsin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 - 2\sqrt{cx+1} \sqrt{-cx+1} a^2 c^3 x^3 + 5\sqrt{cx+1} \right)}{8c}$$

input `int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*asin(c*x))^2,x)`

output `(sqrt(e)*sqrt(d)*d*e*(- 6*asin(sqrt(- c*x + 1)/sqrt(2))*a**2 - 2*sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2*c**3*x**3 + 5*sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2*c*x - 16*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)*x**2,x)*a*b*c**3 + 16*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x),x)*a*b*c - 8*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)**2,x)*b**2*c))/(8*c)`

$$3.490 \quad \int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2}{x} dx$$

Optimal result	4160
Mathematica [A] (verified)	4161
Rubi [A] (verified)	4162
Maple [B] (verified)	4167
Fricas [F]	4168
Sympy [F(-1)]	4169
Maxima [F(-2)]	4169
Giac [F]	4169
Mupad [F(-1)]	4170
Reduce [F]	4170

Optimal result

Integrand size = 35, antiderivative size = 647

$$\begin{aligned}
& \int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \\
& -\frac{22}{9}b^2de\sqrt{d + cdx}\sqrt{e - cex} - \frac{2abcdex\sqrt{d + cdx}\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \\
& -\frac{2}{27}b^2de\sqrt{d + cdx}\sqrt{e - cex}(1 - c^2x^2) \\
& -\frac{2b^2cdex\sqrt{d + cdx}\sqrt{e - cex} \arcsin(cx)}{\sqrt{1 - c^2x^2}} \\
& -\frac{2bcdex\sqrt{d + cdx}\sqrt{e - cex}(a + b \arcsin(cx))}{3\sqrt{1 - c^2x^2}} \\
& +\frac{2bc^3dex^3\sqrt{d + cdx}\sqrt{e - cex}(a + b \arcsin(cx))}{9\sqrt{1 - c^2x^2}} \\
& +de\sqrt{d + cdx}\sqrt{e - cex}(a + b \arcsin(cx))^2 \\
& +\frac{1}{3}de\sqrt{d + cdx}\sqrt{e - cex}(1 - c^2x^2)(a + b \arcsin(cx))^2 \\
& -\frac{2de\sqrt{d + cdx}\sqrt{e - cex}(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{1 - c^2x^2}} \\
& +\frac{2ibde\sqrt{d + cdx}\sqrt{e - cex}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{1 - c^2x^2}} \\
& -\frac{2ibde\sqrt{d + cdx}\sqrt{e - cex}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{1 - c^2x^2}} \\
& -\frac{2b^2de\sqrt{d + cdx}\sqrt{e - cex} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{1 - c^2x^2}} \\
& +\frac{2b^2de\sqrt{d + cdx}\sqrt{e - cex} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{1 - c^2x^2}}
\end{aligned}$$

output

```

-22/9*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)-2*a*b*c*d*e*x*(c*d*x+d)^(1/2)
*(-c*e*x+e)^(1/2)/(-c^2*x^2+1)^(1/2)-2/27*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*(-c^2*x^2+1)-2*b^2*c*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arc
sin(c*x)/(-c^2*x^2+1)^(1/2)-2/3*b*c*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)
*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+2/9*b*c^3*d*e*x^3*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+d*e*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2+1/3*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)
)*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2-2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*
(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2
*I*b*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c
*x-(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*I*b*d*e*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2
+1)^(1/2)-2*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*polylog(3,-I*c*x-(-c^
2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1
/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)

```

Mathematica [A] (verified)

Time = 4.47 (sec) , antiderivative size = 632, normalized size of antiderivative = 0.98

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx =$$

$$-\frac{1}{3}a^2de\sqrt{d + cdx}\sqrt{e - cex}(-4 + c^2x^2)$$

$$+ \frac{2abde\sqrt{d + cdx}\sqrt{e - cex}(-3cx + c^3x^3 + 3(1 - c^2x^2)^{3/2} \arcsin(cx))}{9\sqrt{1 - c^2x^2}}$$

$$+ a^2d^{3/2}e^{3/2} \log(cx) - a^2d^{3/2}e^{3/2} \log\left(de + \sqrt{d}\sqrt{e}\sqrt{d + cdx}\sqrt{e - cex}\right) - \frac{2abde\sqrt{d + cdx}\sqrt{e - cex}(cx - \sqrt{1 - c^2x^2})}{9\sqrt{1 - c^2x^2}}$$

input

```

Integrate[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2)/x,x]

```

output

```

-1/3*(a^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-4 + c^2*x^2)) + (2*a*b*d*e
*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-3*c*x + c^3*x^3 + 3*(1 - c^2*x^2)^(3/2)
*ArcSin[c*x]))/(9*Sqrt[1 - c^2*x^2]) + a^2*d^(3/2)*e^(3/2)*Log[c*x] - a^2*
d^(3/2)*e^(3/2)*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]]
- (2*a*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(c*x - Sqrt[1 - c^2*x^2]*Arc
Sin[c*x] - ArcSin[c*x]*Log[1 - E^(I*ArcSin[c*x])]) + ArcSin[c*x]*Log[1 + E^
(I*ArcSin[c*x])]) - I*PolyLog[2, -E^(I*ArcSin[c*x])] + I*PolyLog[2, E^(I*Ar
cSin[c*x])])]/Sqrt[1 - c^2*x^2] - (b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
*(2*Sqrt[1 - c^2*x^2] + 2*c*x*ArcSin[c*x] - Sqrt[1 - c^2*x^2]*ArcSin[c*x]^
2 - ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] + ArcSin[c*x]^2*Log[1 + E^(I*
ArcSin[c*x])]) - (2*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] + (2*I)*A
rcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] + 2*PolyLog[3, -E^(I*ArcSin[c*x])
] - 2*PolyLog[3, E^(I*ArcSin[c*x])])/Sqrt[1 - c^2*x^2] + (b^2*d*e*Sqrt[d
+ c*d*x]*Sqrt[e - c*e*x]*(27*Sqrt[1 - c^2*x^2]*(-2 + ArcSin[c*x]^2) + (-2
+ 9*ArcSin[c*x]^2)*Cos[3*ArcSin[c*x]] - 6*ArcSin[c*x]*(9*c*x + Sin[3*ArcSi
n[c*x]])))/(108*Sqrt[1 - c^2*x^2])

```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.50, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5238, 5202, 5154, 27, 353, 53, 2009, 5198, 2009, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx + d)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx$$

$$\downarrow \text{5238}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex}}{\sqrt{1 - c^2x^2}} \int \frac{(1 - c^2x^2)^{3/2}(a + b \arcsin(cx))^2}{x} dx$$

$$\downarrow \text{5202}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(-\frac{2}{3}bc \int (1 - c^2x^2) (a + b \arcsin(cx)) dx + \int \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2}{x} dx + \frac{1}{3}(1 - c^2x^2)^{3/2} (a \right)}{\sqrt{1 - c^2x^2}}$$

↓ 5154

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2}{3}bc \left(-bc \int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right) \right)}{\sqrt{1-c^2x^2}}$$

↓ 27

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2}{3}bc \left(-\frac{1}{3}bc \int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}} dx - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right) \right)}{\sqrt{1-c^2x^2}}$$

↓ 353

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2}{3}bc \left(-\frac{1}{6}bc \int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}} dx^2 - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right) \right)}{\sqrt{1-c^2x^2}}$$

↓ 53

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x} dx - \frac{2}{3}bc \left(-\frac{1}{6}bc \int \left(\sqrt{1-c^2x^2} + \frac{2}{\sqrt{1-c^2x^2}} \right) dx^2 - \frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right) \right)}{\sqrt{1-c^2x^2}}$$

↓ 2009

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))^2}{x} dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2 - \frac{2}{3}bc \left(-\frac{1}{3}c^2x^3(a+b\arcsin(cx)) + x(a+b\arcsin(cx)) \right) \right)}{\sqrt{1-c^2x^2}}$$

↓ 5198

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx - 2bc \int (a+b\arcsin(cx)) dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2 + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}}$$

↓ 2009

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(\int \frac{(a+b\arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx + \frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2 + \sqrt{1-c^2x^2}(a+b\arcsin(cx)) \right)}{\sqrt{1-c^2x^2}}$$

↓ 5218

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\int\frac{(a+b\arcsin(cx))^2}{cx}d\arcsin(cx)+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2+\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)$$

↓ 3042

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\int(a+b\arcsin(cx))^2\csc(\arcsin(cx))d\arcsin(cx)+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2+\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)$$

↓ 4671

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2b\int(a+b\arcsin(cx))\log(1-e^{i\arcsin(cx)})d\arcsin(cx)+2b\int(a+b\arcsin(cx))\log(1+e^{i\arcsin(cx)})d\arcsin(cx)\right)$$

↓ 3011

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(2b(i\operatorname{PolyLog}(2,-e^{i\arcsin(cx)})(a+b\arcsin(cx))-ib\int\operatorname{PolyLog}(2,-e^{i\arcsin(cx)})d\arcsin(cx)\right)$$

↓ 2720

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(2b(i\operatorname{PolyLog}(2,-e^{i\arcsin(cx)})(a+b\arcsin(cx))-b\int e^{-i\arcsin(cx)}\operatorname{PolyLog}(2,-e^{i\arcsin(cx)})d\arcsin(cx)\right)$$

↓ 7143

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2\operatorname{arctanh}(e^{i\arcsin(cx)})(a+b\arcsin(cx))^2+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2+\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)$$

input

```
Int[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2)/x,x]
```

output

```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2 + ((1 - c^2*x^2)^(3/2)*(a + b*ArcSin[c*x])^2)/3 - 2*b*c*(a*x + (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcSin[c*x]) - (2*b*c*(-1/6*(b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2))) + x*(a + b*ArcSin[c*x]) - (c^2*x^3*(a + b*ArcSin[c*x]))/3))/3 - 2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])]) - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])]))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(F_x_), x_Symbol] := Simp[a Int[F_x, x], x] /; FreeQ[a, x] && !MatchQ[F_x, (b_)*(G_x_) /; FreeQ[b, x]]
```

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] := Simp[1/2 Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5154 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcSin[c*x]) u, x] - Simp[b*c Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]`

rule 5198 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*Sqrt[(d_.) + (e_.)*(x_)^2], x_Symbol] := Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^m*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x] - Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2] Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])`

rule 5202 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1)) Int[(f*x)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1]`

rule 5218

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_)^(m_.))*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1325 vs. $2(605) = 1210$.

Time = 4.45 (sec) , antiderivative size = 1326, normalized size of antiderivative = 2.05

method	result	size
default	Expression too large to display	1326
parts	Expression too large to display	1326

input

```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x,x,method=_RETUR
NVERBOSE)
```

output

```

-1/3*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*d*e*(x^2*c^2*(d*e)^(1/2)*(-d
*e*(c^2*x^2-1))^(1/2)+3*d*e*ln(2*((d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)+d*e
)/x)-4*(d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)/(d*e)^(1/2)/(-d*e*(c^2*x^2-1)
)^(1/2)+b^2*(-1/216*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(4*c^4*x^4-5*c^2*
x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*(6*I*arcs
in(c*x)+9*arcsin(c*x)^2-2)*d*e/(c^2*x^2-1)+5/8*(-e*(c*x-1))^(1/2)*(d*(c*x+
1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin
(c*x))*d*e/(c^2*x^2-1)-(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(
1/2)*(arcsin(c*x)^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+arcsin(c*x)^2*ln
(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)
)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))-4*I*arcsin(c
*x)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-4*I*arcsin(c*x)*polylog(2,
-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-2*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))+
8*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+8*polylog(3,-(I*c*x+(-c^2*x^
2+1)^(1/2))^(1/2))*d*e/(c^2*x^2-1)+1/27*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1
/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(33*I*arcsin(c*x)+18*arcsin(c*x)^
2-34)*cos(2*arcsin(c*x))*d*e/(c^2*x^2-1)+1/108*(-e*(c*x-1))^(1/2)*(d*(c*x+
1))^(1/2)*(I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)-I)*(138*I*arcsin(c*x)+63*arcsi
n(c*x)^2-134)*sin(2*arcsin(c*x))*d*e/(c^2*x^2-1)+2*a*b*(-1/72*(-e*(c*x-1)
)^(1/2)*(d*(c*x+1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x...

```

Fricas [F]

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \int \frac{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2}{x} dx$$

input

```

integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x,x, algori
thm="fricas")

```

output

```

integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arcsin(
c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arcsin(c*x))*sqrt(c*d*x + d)*sqrt(-
c*e*x + e)/x, x)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \text{Timed out}$$

input `integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*asin(c*x))**2/x,x)`

output Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \text{Exception raised: ValueError}$$

input `integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="maxima")`

output Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [F]

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \int \frac{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2}{x} dx$$

input `integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x,x, algorithm="giac")`

output `integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arcsin(c*x) + a)^2/x, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \int \frac{(a + b \arcsin(cx))^2 (d + cdx)^{3/2} (e - cex)^{3/2}}{x} dx$$

input `int(((a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x,x)`

output `int(((a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x, x)`

Reduce [F]

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x} dx = \frac{\sqrt{e} \sqrt{d} de \left(-\sqrt{cx + 1} \sqrt{-cx + 1} a^2 c^2 x^2 + 4\sqrt{cx + 1} \sqrt{-cx + 1} a b \arcsin(cx) + 4\sqrt{cx + 1} \sqrt{-cx + 1} b^2 \arcsin^2(cx) \right)}{x}$$

input `int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*asin(c*x))^2/x,x)`

output `(sqrt(e)*sqrt(d)*d*e*(- sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2*c**2*x**2 + 4 *sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2 + 6*int((sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x))/x,x)*a*b + 3*int((sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)**2)/x,x)*b**2 - 6*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)*x,x)*a*b*c**2 - 3*int(sqrt(c*x + 1)*sqrt(- c*x + 1)*asin(c*x)**2*x,x)*b**2*c**2 - 3*log(- sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) - 1)*a**2 + 3*log(- sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) + 1)*a**2 - 3*log(sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) - 1)*a**2 + 3*log(sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) + 1)*a**2))/3`

3.491
$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2}{x^2} dx$$

Optimal result	4171
Mathematica [A] (verified)	4172
Rubi [A] (verified)	4173
Maple [A] (verified)	4179
Fricas [F]	4180
Sympy [F(-1)]	4180
Maxima [F(-2)]	4180
Giac [F]	4181
Mupad [F(-1)]	4181
Reduce [F]	4182

Optimal result

Integrand size = 35, antiderivative size = 505

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b \arcsin(cx))^2}{x^2} dx = \frac{1}{4}b^2c^2dex\sqrt{d+cdx}\sqrt{e-cex}$$

$$- \frac{5b^2cde\sqrt{d+cdx}\sqrt{e-cex} \arcsin(cx)}{4\sqrt{1-c^2x^2}} + \frac{3bc^3dex^2\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))}{2\sqrt{1-c^2x^2}}$$

$$+ bcde\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}(a+b \arcsin(cx))$$

$$- \frac{3}{2}c^2dex\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2$$

$$- \frac{icde\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}}$$

$$- \frac{de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)(a+b \arcsin(cx))^2}{x}$$

$$- \frac{cde\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx))^3}{2b\sqrt{1-c^2x^2}}$$

$$+ \frac{2bcde\sqrt{d+cdx}\sqrt{e-cex}(a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

$$- \frac{ib^2cde\sqrt{d+cdx}\sqrt{e-cex} \text{PolyLog}(2, e^{2i \arcsin(cx)})}{\sqrt{1-c^2x^2}}$$

output

```

1/4*b^2*c^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)-5/4*b^2*c*d*e*(c*d*x+d)
^(1/2)*(-c*e*x+e)^(1/2)*arcsin(c*x)/(-c^2*x^2+1)^(1/2)+3/2*b*c^3*d*e*x^2*(
c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))/(-c^2*x^2+1)^(1/2)+b*c*d
*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))-3
/2*c^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2-I*c*d*e*
(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^2/(-c^2*x^2+1)^(1/2)-d*
e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/x-1/2*
c*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))^3/b/(-c^2*x^2+1)^(
1/2)+2*b*c*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I
*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*d*e*(c*d*x+d)^(1/2)
*(-c*e*x+e)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/
2)

```

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.07

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \frac{-8a^2de\sqrt{d + cdx}\sqrt{e - cex}\sqrt{1 - c^2x^2} - 4a^2c^2dex^2\sqrt{d + cdx}\sqrt{e - cex}\sqrt{1 - c^2x^2}}{x^2}$$

input

```

Integrate[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2)/x^2,
x]

```

output

```

(-8*a^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] - 4*a^2*c^2*
d*e*x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] - 4*b^2*c*d*e*x*
Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]^3 + 12*a^2*c*d^(3/2)*e^(3/2)*x
*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*S
qrt[e]*(-1 + c^2*x^2))] - 2*a*b*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Co
s[2*ArcSin[c*x]] + 16*a*b*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Log[c*x]
- (8*I)*b^2*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*PolyLog[2, E^((2*I)*A
rcSin[c*x])] + b^2*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sin[2*ArcSin[c*
x]] - 2*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcSin[c*x]*(8*a*Sqrt[1 - c^
2*x^2] + b*c*x*Cos[2*ArcSin[c*x]] - 8*b*c*x*Log[1 - E^((2*I)*ArcSin[c*x])]
+ 2*a*c*x*Sin[2*ArcSin[c*x]]) - 2*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*A
rcSin[c*x]^2*(6*a*c*x + (4*I)*b*c*x + 4*b*Sqrt[1 - c^2*x^2] + b*c*x*Sin[2*
ArcSin[c*x]])/(8*x*Sqrt[1 - c^2*x^2])

```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.63, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5238, 5200, 5156, 5138, 262, 223, 5152, 5188, 211, 223, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx + d)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx$$

$$\downarrow \text{5238}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \int \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{x^2} dx}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{5200}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(-3c^2 \int \sqrt{1-c^2x^2}(a + b \arcsin(cx))^2 dx + 2bc \int \frac{(1-c^2x^2)(a+b \arcsin(cx))}{x} dx - \frac{(1-c^2x^2)^{3/2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{5156}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(-3c^2 \left(\frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx - bc \int x(a + b \arcsin(cx)) dx + \frac{1}{2} x \sqrt{1-c^2x^2}(a + b \arcsin(cx)) \right) \right)}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{5138}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(-3c^2 \left(-bc \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right) + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx + \frac{1}{2} x \sqrt{1-c^2x^2} \right) \right)}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{262}$$

$$\frac{de\sqrt{cdx + d}\sqrt{e - cex} \left(-3c^2 \left(-bc \left(\frac{1}{2} x^2 (a + b \arcsin(cx)) - \frac{1}{2} bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right) \right) + \frac{1}{2} \int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx \right)}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{223}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx-3c^2\left(\frac{1}{2}\int\frac{(a+b\arcsin(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)\right)}{\sqrt{1-c^2x^2}}$$

↓ 5152

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\int\frac{(1-c^2x^2)(a+b\arcsin(cx))}{x}dx-\frac{(1-c^2x^2)^{3/2}(a+b\arcsin(cx))^2}{x}-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arcsin(cx))\right)\right)}{\sqrt{1-c^2x^2}}$$

↓ 5188

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)-\frac{(1-c^2x^2)^3}{x}\right)}{\sqrt{1-c^2x^2}}$$

↓ 211

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\left(\int\frac{a+b\arcsin(cx)}{x}dx-\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)\right)}{\sqrt{1-c^2x^2}}$$

↓ 223

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\left(\int\frac{a+b\arcsin(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)\right)}{\sqrt{1-c^2x^2}}$$

↓ 5136

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\left(\int\frac{\sqrt{1-c^2x^2}(a+b\arcsin(cx))}{cx}d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))-\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}\right)\right)\right)}{\sqrt{1-c^2x^2}}$$

↓ 3042

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(2bc\left(\int-\left((a+b\arcsin(cx))\tan\left(\arcsin(cx)+\frac{\pi}{2}\right)\right)d\arcsin(cx)+\frac{1}{2}(1-c^2x^2)(a+b\arcsin(cx))\right)\right)}{\sqrt{1-c^2x^2}}$$

↓ 25

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(2bc \left(-\int (a + b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) + \frac{1}{2}(1 - c^2x^2) (a + b \arcsin(cx)) \right) \right)$$

↓ 4200

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(2bc \left(2i \int -\frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2}(1 - c^2x^2) (a + b \arcsin(cx)) - \frac{i(a + b \arcsin(cx))}{2} \right) \right)$$

↓ 25

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(2bc \left(-2i \int \frac{e^{2i \arcsin(cx)}(a + b \arcsin(cx))}{1 - e^{2i \arcsin(cx)}} d \arcsin(cx) + \frac{1}{2}(1 - c^2x^2) (a + b \arcsin(cx)) - \frac{i(a + b \arcsin(cx))}{2} \right) \right)$$

↓ 2620

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(2bc \left(-2i \left(\frac{1}{2}i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{2}ib \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) \right) \right)$$

↓ 2715

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(2bc \left(-2i \left(\frac{1}{2}i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) - \frac{1}{4}b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) \right) \right)$$

↓ 2838

$$de\sqrt{cdx + d}\sqrt{e - cex} \left(2bc \left(\frac{1}{2}(1 - c^2x^2) (a + b \arcsin(cx)) - 2i \left(\frac{1}{2}i \log(1 - e^{2i \arcsin(cx)}) (a + b \arcsin(cx)) + \frac{1}{4}b \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) \right) \right) \right)$$

input

```
Int[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcSin[c*x])^2)/x^2,x]
```

output

```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-(((1 - c^2*x^2)^(3/2)*(a + b*ArcSin
[c*x])^2)/x) - 3*c^2*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/2 + (a +
b*ArcSin[c*x])^3/(6*b*c) - b*c*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*
(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)) + 2*b*c*(((1 - c^2*x
^2)*(a + b*ArcSin[c*x]))/2 - ((I/2)*(a + b*ArcSin[c*x])^2)/b - (b*c*((x*Sq
rt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 - (2*I)*((I/2)*(a + b*ArcSin[c*
x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])
/4)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

rule 211

```
Int[((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1)) Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1))) Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715 `Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]`

rule 4200 `Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5138 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Simp[b*c*(n
/(d*(m + 1))) Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]`

rule 5152 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
+ b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]`

rule 5156

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSin[c*x])^n/2), x] + (Simp[(1/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2], x], x] - Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]] Int[x*(a + b*ArcSin[c*x])^(n - 1), x], x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5188

```
Int[(((a_.) + ArcSin[(c_.)*(x_)])*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.)/(x_), x_Symbol]
:> Simp[(d + e*x^2)^p*((a + b*ArcSin[c*x])/(2*p)), x] + (Simp[d Int[(d + e*x^2)^(p - 1)*((a + b*ArcSin[c*x])/x), x], x] - Simp[b*c*(d^p/(2*p)) Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5200

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcSin[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1))) Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((h_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(p_.)*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol]
:> Simp[((-d^2)*(g/e))^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.25

method	result
default	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} de \left(3 \arctan \left(\frac{\sqrt{c^2 de} x}{\sqrt{-de(c^2 x^2 - 1)}} \right) x c^2 de + x^2 c^2 \sqrt{-de(c^2 x^2 - 1)} \sqrt{c^2 de + 2\sqrt{c^2 de} \sqrt{-de(c^2 x^2 - 1)}} \right)}{2\sqrt{c^2 de} \sqrt{-de(c^2 x^2 - 1)} x} + \dots$
parts	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} de \left(3 \arctan \left(\frac{\sqrt{c^2 de} x}{\sqrt{-de(c^2 x^2 - 1)}} \right) x c^2 de + x^2 c^2 \sqrt{-de(c^2 x^2 - 1)} \sqrt{c^2 de + 2\sqrt{c^2 de} \sqrt{-de(c^2 x^2 - 1)}} \right)}{2\sqrt{c^2 de} \sqrt{-de(c^2 x^2 - 1)} x} + \dots$

input

```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x,method=_RET
URNVERBOSE)
```

output

```
-1/2*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*d*e*(3*arctan((c^2*d*e)^(1/2)
)*x/(-d*e*(c^2*x^2-1))^(1/2))*x*c^2*d*e+x^2*c^2*(-d*e*(c^2*x^2-1))^(1/2)*(
c^2*d*e)^(1/2)+2*(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))/(c^2*d*e)^(1/2)
/(-d*e*(c^2*x^2-1))^(1/2)/x+1/4*b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(
-c^2*x^2+1)^(1/2)/x/(c^2*x^2-1)*(2*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)*c^2*x^
2-2*c^3*x^3*arcsin(c*x)+2*arcsin(c*x)^3*c*x+4*I*arcsin(c*x)^2*x*c-c^2*x^2*
(-c^2*x^2+1)^(1/2)-8*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))*c*
x-8*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))*c*x-8*arcsin(c*x)*l
n(1+I*c*x+(-c^2*x^2+1)^(1/2))*c*x+8*I*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))
*x*c+16*I*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))*x*c+16*I*polylog(2,-
(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))*x*c+4*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)+c
*x*arcsin(c*x)*d*e+1/4*a*b*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2
+1)^(1/2)*(4*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-2*c^3*x^3+6*arcsin(c*x
)^2*c*x+8*I*arcsin(c*x)*x*c-8*ln((I*c*x+(-c^2*x^2+1)^(1/2))^2-1)*x*c+8*arc
sin(c*x)*(-c^2*x^2+1)^(1/2)+c*x)*d*e/x/(c^2*x^2-1)
```


Fricas [F]

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2}{x^2} dx$$

input `integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x, algorith="fricas")`

output `integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arcsin(c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arcsin(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/x^2, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \text{Timed out}$$

input `integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*asin(c*x))**2/x**2,x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \text{Exception raised: ValueError}$$

input `integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x, algorith="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}(b \arcsin(cx) + a)^2}{x^2} dx$$

input

```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcsin(c*x))^2/x^2,x, algo
rithm="giac")
```

output

```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arcsin(c*x) + a)^2/x^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \int \frac{(a + b \arcsin(cx))^2 (d + cdx)^{3/2} (e - cex)^{3/2}}{x^2} dx$$

input

```
int(((a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x^2,x)
```

output

```
int(((a + b*asin(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d + cdx)^{3/2}(e - cex)^{3/2}(a + b \arcsin(cx))^2}{x^2} dx = \frac{\sqrt{e} \sqrt{d} de \left(6 \operatorname{asin}\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 cx - \sqrt{cx+1} \sqrt{-cx+1}\right)}{x^2}$$

input `int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*asin(c*x))^2/x^2,x)`

output `(sqrt(e)*sqrt(d)*d*e*(6*asin(sqrt(-c*x+1)/sqrt(2))*a**2*c*x - sqrt(c*x+1)*sqrt(-c*x+1)*a**2*c**2*x**2 - 2*sqrt(c*x+1)*sqrt(-c*x+1)*a**2 + 4*int((sqrt(c*x+1)*sqrt(-c*x+1)*asin(c*x))/x**2,x)*a*b*x + 2*int((sqrt(c*x+1)*sqrt(-c*x+1)*asin(c*x)**2)/x**2,x)*b**2*x - 4*int(sqrt(c*x+1)*sqrt(-c*x+1)*asin(c*x),x)*a*b*c**2*x - 2*int(sqrt(c*x+1)*sqrt(-c*x+1)*asin(c*x)**2,x)*b**2*c**2*x))/(2*x)`

3.492 $\int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx$

Optimal result	4183
Mathematica [A] (verified)	4184
Rubi [A] (verified)	4184
Maple [C] (verified)	4187
Fricas [F]	4188
Sympy [F(-1)]	4189
Maxima [F(-2)]	4189
Giac [F]	4189
Mupad [F(-1)]	4190
Reduce [F]	4190

Optimal result

Integrand size = 35, antiderivative size = 250

$$\int \frac{x^2(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx = \frac{b^2x(1-c^2x^2)}{4c^2\sqrt{d+cdx}\sqrt{e-cex}} - \frac{b^2\sqrt{1-c^2x^2} \arcsin(cx)}{4c^3\sqrt{d+cdx}\sqrt{e-cex}} + \frac{bx^2\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{2c\sqrt{d+cdx}\sqrt{e-cex}} - \frac{x(1-c^2x^2)(a+b \arcsin(cx))^2}{2c^2\sqrt{d+cdx}\sqrt{e-cex}} + \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{6bc^3\sqrt{d+cdx}\sqrt{e-cex}}$$

output

```
1/4*b^2*x*(-c^2*x^2+1)/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/4*b^2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)/c^3/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/2*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/2*x*(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/6*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c^3/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx$$

$$= \frac{12b\sqrt{d}\sqrt{e}(a\sqrt{1 - c^2x^2} + bcx(-1 + c^2x^2)) \arcsin(cx)^2 + 4b^2\sqrt{d}\sqrt{e}\sqrt{1 - c^2x^2} \arcsin(cx)^3 - 12a^2\sqrt{d + cd}}$$

input `Integrate[(x^2*(a + b*ArcSin[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]`

output `(12*b*Sqrt[d]*Sqrt[e]*(a*Sqrt[1 - c^2*x^2] + b*c*x*(-1 + c^2*x^2))*ArcSin[c*x]^2 + 4*b^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^3 - 12*a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 3*Sqrt[d]*Sqrt[e]*(a*b*Sqrt[1 - c^2*x^2] + 2*b^2*c*x*(-1 + c^2*x^2) + a^2*(4*c*x - 4*c^3*x^3) + a*b*Cos[3*ArcSin[c*x]]) - 3*b*Sqrt[d]*Sqrt[e]*ArcSin[c*x]*(2*a*c*x + b*Sqrt[1 - c^2*x^2] + b*Cos[3*ArcSin[c*x]] + 2*a*Sin[3*ArcSin[c*x]]))/(24*c^3*Sqrt[d]*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5238, 5210, 5138, 262, 223, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{cdx + d}\sqrt{e - cex}} dx$$

$$\downarrow 5238$$

$$\frac{\sqrt{1 - c^2x^2} \int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}} dx}{\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow 5210$$

$$\frac{\sqrt{1-c^2x^2} \left(\frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{2c^2} + \frac{b \int x(a+b \arcsin(cx)) dx}{c} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

↓ 5138

$$\frac{\sqrt{1-c^2x^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \int \frac{x^2}{\sqrt{1-c^2x^2}} dx \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

↓ 262

$$\frac{\sqrt{1-c^2x^2} \left(\frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}} dx}{2c^2} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} + \frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} \right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

↓ 223

$$\frac{\sqrt{1-c^2x^2} \left(\frac{\int \frac{(a+b \arcsin(cx))^2 dx}{\sqrt{1-c^2x^2}}}{2c^2} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

↓ 5152

$$\frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^3}{6bc^3} - \frac{x\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{2c^2} + \frac{b \left(\frac{1}{2}x^2(a+b \arcsin(cx)) - \frac{1}{2}bc \left(\frac{\arcsin(cx)}{2c^3} - \frac{x\sqrt{1-c^2x^2}}{2c^2} \right) \right)}{c} \right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

input `Int[(x^2*(a + b*ArcSin[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]`

output `(Sqrt[1 - c^2*x^2]*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/c^2 + (a + b*ArcSin[c*x])^3/(6*b*c^3) + (b*((x^2*(a + b*ArcSin[c*x]))/2 - (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3))))/2)/c)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Definitions of rubi rules used

rule 223 $\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

rule 262 $\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[c*(c*x)^{(m-1)}*((a + b*x^2)^{(p+1)}/(b*(m+2*p+1))), x] - \text{Simp}[a*c^2*((m-1)/(b*(m+2*p+1))) \ \text{Int}[(c*x)^{(m-2)}*(a + b*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{GtQ}[m, 2-1] \ \&\& \ \text{NeQ}[m+2*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, 2, m, p, x]$

rule 5138 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((d_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^n/(d*(m+1))), x] - \text{Simp}[b*c*(n/(d*(m+1))) \ \text{Int}[(d*x)^{(m+1)}*((a + b*\text{ArcSin}[c*x])^{(n-1)}/\text{Sqrt}[1 - c^2*x^2]), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

rule 5152 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}/\text{Sqrt}[(d_) + (e_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(1/(b*c*(n+1)))*\text{Simp}[\text{Sqrt}[1 - c^2*x^2]/\text{Sqrt}[d + e*x^2]]*(a + b*\text{ArcSin}[c*x])^{(n+1)}, x] /;$ $\text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[n, -1]$

rule 5210 $\text{Int}[(a_) + \text{ArcSin}[c_*(x_)]*(b_)^{(n_)}*((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[f*(f*x)^{(m-1)}*(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(e*(m+2*p+1))), x] + (\text{Simp}[f^2*((m-1)/(c^2*(m+2*p+1))) \ \text{Int}[(f*x)^{(m-2)}*(d + e*x^2)^p*(a + b*\text{ArcSin}[c*x])^n, x], x] + \text{Simp}[b*f*(n/(c*(m+2*p+1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \ \text{Int}[(f*x)^{(m-1)}*(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x]) /;$ $\text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 1] \ \&\& \ \text{NeQ}[m+2*p+1, 0]$

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[(-d^2)*(g/e)^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 1006, normalized size of antiderivative = 4.02

method	result
default	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \left(\arctan \left(\frac{\sqrt{c^2 d e x}}{\sqrt{-de(c^2 x^2 - 1)}} \right) de - \sqrt{c^2 de} \sqrt{-de(c^2 x^2 - 1)} x \right)}{2c^2 \sqrt{-de(c^2 x^2 - 1)} ed \sqrt{c^2 de}} + b^2 \left(-\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2 x^2}}{6de c^3 (c^2 x^2 - 1)} \right)$
parts	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \left(\arctan \left(\frac{\sqrt{c^2 d e x}}{\sqrt{-de(c^2 x^2 - 1)}} \right) de - \sqrt{c^2 de} \sqrt{-de(c^2 x^2 - 1)} x \right)}{2c^2 \sqrt{-de(c^2 x^2 - 1)} ed \sqrt{c^2 de}} + b^2 \left(-\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2 x^2}}{6de c^3 (c^2 x^2 - 1)} \right)$

input

```
int(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RET URNVERBOSE)
```


output

```

1/2*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(arctan((c^2*d*e)^(1/2)*x/(-d
*e*(c^2*x^2-1))^(1/2))*d*e-(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x)/c^2
/(-d*e*(c^2*x^2-1))^(1/2)/e/d/(c^2*d*e)^(1/2)+b^2*(-1/6*(-e*(c*x-1))^(1/2)
*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c^3/(c^2*x^2-1)*arcsin(c*x)^3+1/
32*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2
*x^2-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(2*I*arcsin(c*x)+2*arcsin(c*x)^2-1)/c^3/e
/(c*x-1)/d/(c*x+1)-1/32*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(4*I*(-c^2*x^
2+1)^(1/2)*x^2*c^2+4*c^3*x^3+2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^2*
x^2+1)^(1/2)-3*c*x-1)*(2*arcsin(c*x)^2-1-2*I*arcsin(c*x))/d/e/c^3/(c^2*x^2
-1)+1/8*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+I)*
arcsin(c*x)*cos(2*arcsin(c*x))/c^3/e/(c*x-1)/d/(c*x+1)+1/16*(-e*(c*x-1))^(
1/2)*(d*(c*x+1))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+I)*(2*arcsin(c*x)^2-1)*si
n(2*arcsin(c*x))/c^3/e/(c*x-1)/d/(c*x+1)+2*a*b*(-1/4*(-e*(c*x-1))^(1/2)*
d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c^3/(c^2*x^2-1)*arcsin(c*x)^2+1/32
*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x
^2-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(I+2*arcsin(c*x))/c^3/e/(c*x-1)/d/(c*x+1)-1
/32*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4
*c^3*x^3+2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^2*x^2+1)^(1/2)-3*c*x-1
)*(-I+2*arcsin(c*x))/d/e/c^3/(c^2*x^2-1)+1/16*(-e*(c*x-1))^(1/2)*(d*(c*x+1
))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+I)*cos(2*arcsin(c*x))/c^3/e/(c*x-1)/...

```

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{\sqrt{cdx + d}\sqrt{-cex + e}} dx$$

input

```

integrate(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="fricas")

```

output

```

integral(-(b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(c
*d*x + d)*sqrt(-c*e*x + e)/(c^2*d*e*x^2 - d*e), x)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \text{Timed out}$$

input `integrate(x**2*(a+b*asin(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)`

output `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{\sqrt{cdx + d}\sqrt{-cex + e}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)^2*x^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{x^2(a + b \sin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx$$

input `int((x^2*(a + b*asin(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)`

output `int((x^2*(a + b*asin(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)`

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx$$

$$= \frac{-2 \operatorname{asin}\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 - \sqrt{cx+1}\sqrt{-cx+1} a^2 cx + 4 \left(\int \frac{\operatorname{asin}(cx)x^2}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) ab c^3 + 2 \left(\int \frac{\operatorname{asin}(cx)^2 x^2}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) b^2 c^3}{2\sqrt{e}\sqrt{d}c^3}$$

input `int(x^2*(a+b*asin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)`

output `(- 2*asin(sqrt(- c*x + 1)/sqrt(2))*a**2 - sqrt(c*x + 1)*sqrt(- c*x + 1) *a**2*c*x + 4*int((asin(c*x)*x**2)/(sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*a*b *c**3 + 2*int((asin(c*x)**2*x**2)/(sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*b**2 *c**3)/(2*sqrt(e)*sqrt(d)*c**3)`

3.493 $\int \frac{x(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx$

Optimal result	4191
Mathematica [A] (verified)	4191
Rubi [A] (verified)	4192
Maple [C] (verified)	4193
Fricas [A] (verification not implemented)	4194
Sympy [F]	4194
Maxima [F(-2)]	4195
Giac [F]	4195
Mupad [F(-1)]	4195
Reduce [F]	4196

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \frac{2b^2(1 - c^2x^2)}{c^2\sqrt{d + cdx}\sqrt{e - cex}} + \frac{2bx\sqrt{1 - c^2x^2}(a + b \arcsin(cx))}{c\sqrt{d + cdx}\sqrt{e - cex}} - \frac{(1 - c^2x^2)(a + b \arcsin(cx))^2}{c^2\sqrt{d + cdx}\sqrt{e - cex}}$$

output

```
2*b^2*(-c^2*x^2+1)/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \frac{\sqrt{d + cdx}\sqrt{e - cex}(2abcx\sqrt{1 - c^2x^2} + a^2(-1 + c^2x^2) - 2b^2(-1 + c^2x^2) + 2b(bcx\sqrt{1 - c^2x^2} + a(-1 - cx))}{c^2de(-1 + cx)(1 + cx)}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output

$$-\left(\left(\sqrt{d + cdx} \sqrt{e - cex} \left(2abcx\sqrt{1 - c^2x^2} + a^2(-1 + c^2x^2) - 2b^2(-1 + c^2x^2) + 2b(bc x \sqrt{1 - c^2x^2} + a(-1 + c^2x^2))\right) \operatorname{ArcSin}[cx] + b^2(-1 + c^2x^2) \operatorname{ArcSin}[cx]^2\right)\right) / (c^2 d e (-1 + cx)(1 + cx))$$

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5238, 5182, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{cdx + d}\sqrt{e - cex}} dx \\ & \quad \downarrow \text{5238} \\ & \frac{\sqrt{1 - c^2x^2} \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}} dx}{\sqrt{cdx + d}\sqrt{e - cex}} \\ & \quad \downarrow \text{5182} \\ & \frac{\sqrt{1 - c^2x^2} \left(\frac{2b \int (a + b \arcsin(cx)) dx}{c} - \frac{\sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2}{c^2} \right)}{\sqrt{cdx + d}\sqrt{e - cex}} \\ & \quad \downarrow \text{2009} \\ & \frac{\sqrt{1 - c^2x^2} \left(\frac{2b \left(ax + bx \arcsin(cx) + \frac{b\sqrt{1 - c^2x^2}}{c} \right)}{c} - \frac{\sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2}{c^2} \right)}{\sqrt{cdx + d}\sqrt{e - cex}} \end{aligned}$$

input

$$\text{Int}[(x*(a + b*ArcSin[c*x]))^2]/(\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x]),x]$$

output

$$\left(\sqrt{1 - c^2x^2} * \left(-\left(\sqrt{1 - c^2x^2} * (a + b*ArcSin[c*x])^2\right) / c^2 + (2*b*(a*x + (b*\text{Sqrt}[1 - c^2x^2])) / c + b*x*ArcSin[c*x])\right) / c\right) / (\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x])$$

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 5182 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

```
rule 5238 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(p_.))*((f_) + (g_.)*(x_)^(q_.), x_Symbol] := Simp[((-d^2)*(g/e))^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.67

method	result
default	$-\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)}}{c^2 ed} + b^2 \left(-\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} (c^2 x^2 - i\sqrt{-c^2 x^2 + 1} cx - 1) (\arcsin(cx))^2 - 2 + 2i \arcsin(cx)}{2e(cx+1)c^2 d(cx-1)} - \dots \right)$
parts	$-\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)}}{c^2 ed} + b^2 \left(-\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} (c^2 x^2 - i\sqrt{-c^2 x^2 + 1} cx - 1) (\arcsin(cx))^2 - 2 + 2i \arcsin(cx)}{2e(cx+1)c^2 d(cx-1)} - \dots \right)$
orering	$\frac{(c^4 x^4 - 4c^2 x^2 + 2)(a + b \arcsin(cx))^2}{c^4 x^2 \sqrt{cdx+d} \sqrt{-cxe+e}} + \frac{2(cx-1)(cx+1) \left(\frac{(a+b \arcsin(cx))^2}{\sqrt{cdx+d} \sqrt{-cxe+e}} + \frac{2x(a+b \arcsin(cx))bc}{\sqrt{cdx+d} \sqrt{-cxe+e} \sqrt{-c^2 x^2 + 1}} - \frac{x(a+b \arcsin(cx))^2 cd}{2(cdx+d)^{\frac{3}{2}} \sqrt{-cxe+e}} + \dots \right)}{c^4 x^2}$

```
input int(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2), x, method=_RETURNVERBOSE)
```

output

```
-a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/c^2/e/d+b^2*(-1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)^2-2+2*I*arcsin(c*x))/e/(c*x+1)/c^2/d/(c*x-1)-1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)^2-2-2*I*arcsin(c*x))/c^2/d/e/(c^2*x^2-1))+2*a*b*(-1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(c^2*x^2-I*c*x*(-c^2*x^2+1)^(1/2)-1)*(arcsin(c*x)+I)/e/(c*x+1)/c^2/d/(c*x-1)-1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arcsin(c*x)-I)/c^2/d/e/(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \frac{((a^2 - 2b^2)c^2x^2 + (b^2c^2x^2 - b^2) \arcsin(cx)^2 - a^2 + 2b^2 + 2(abc^2x^2 - ab) \arcsin(cx) + 2(b^2cx \arcsin(cx) - c^2dex^2 - c^2de)}{c^4dex^2 - c^2de}$$

input

```
integrate(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="fricas")
```

output

```
-((a^2 - 2*b^2)*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsin(c*x)^2 - a^2 + 2*b^2 + 2*(a*b*c^2*x^2 - a*b)*arcsin(c*x) + 2*(b^2*c*x*arcsin(c*x) + a*b*c*x)*sqrt(-c^2*x^2 + 1))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*d*e*x^2 - c^2*d*e)
```

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d(cx + 1)}\sqrt{-e(cx - 1)}} dx$$

input

```
integrate(x*(a+b*asin(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output

```
Integral(x*(a + b*asin(c*x))**2/(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \text{Exception raised: ValueError}$$

input `integrate(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{\sqrt{cdx + d}\sqrt{-cex + e}} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)^2*x/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx$$

input `int((x*(a + b*asin(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)`

output `int((x*(a + b*asin(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)`

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx$$

$$= \frac{-\sqrt{cx + 1}\sqrt{-cx + 1}a^2 + 2\left(\int \frac{asin(cx)x}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) abc^2 + \left(\int \frac{asin(cx)^2x}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) b^2c^2}{\sqrt{e}\sqrt{d}c^2}$$

input `int(x*(a+b*asin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)`

output `(- sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2 + 2*int((asin(c*x)*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*a*b*c**2 + int((asin(c*x)**2*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*b**2*c**2)/(sqrt(e)*sqrt(d)*c**2)`

3.494 $\int \frac{(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} dx$

Optimal result	4197
Mathematica [B] (verified)	4197
Rubi [A] (verified)	4198
Maple [B] (verified)	4199
Fricas [F]	4200
Sympy [F]	4200
Maxima [F(-2)]	4200
Giac [F]	4201
Mupad [F(-1)]	4201
Reduce [F]	4202

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^3}{3bc\sqrt{d + cdx}\sqrt{e - cex}}$$

output 1/3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(55) = 110.

Time = 2.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.89

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \frac{\frac{3ab\sqrt{1-c^2x^2} \arcsin(cx)^2}{\sqrt{d+cdx}\sqrt{e-cex}} + \frac{b^2\sqrt{1-c^2x^2} \arcsin(cx)^3}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{3a^2 \arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)}{\sqrt{d}\sqrt{e}}}{3c}$$

input Integrate[(a + b*ArcSin[c*x])^2/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]

output

$$\begin{aligned} & ((3*a*b*\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c*x]^2)/(\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x]) \\ & + (b^2*\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c*x]^3)/(\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x]) \\ & - (3*a^2*\text{ArcTan}[(c*x*\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x])]/(\text{Sqrt}[d]*\text{Sqrt}[e]*(- \\ & 1 + c^2*x^2)))]/(\text{Sqrt}[d]*\text{Sqrt}[e]))/(3*c) \end{aligned}$$
Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5178, 5152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{(a + b \arcsin(cx))^2}{\sqrt{cdx + d}\sqrt{e - cex}} dx \\ & \quad \downarrow \text{5178} \\ & \frac{\sqrt{1 - c^2x^2} \int \frac{(a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}} dx}{\sqrt{cdx + d}\sqrt{e - cex}} \\ & \quad \downarrow \text{5152} \\ & \frac{\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^3}{3bc\sqrt{cdx + d}\sqrt{e - cex}} \end{aligned}$$

input

$$\text{Int}[(a + b*\text{ArcSin}[c*x])^2/(\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x]),x]$$

output

$$(\text{Sqrt}[1 - c^2*x^2]*(a + b*\text{ArcSin}[c*x])^3)/(3*b*c*\text{Sqrt}[d + c*d*x]*\text{Sqrt}[e - c*e*x])$$

Definitions of rubi rules used

rule 5152

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

rule 5178

```
Int[((a_.) + ArcSin[(c_.)*(x_.)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_.)*((f_) + (g_.)*(x_))^(q_), x_Symbol]
:= Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^2)^q) Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x]
/; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(47) = 94$.

Time = 2.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.58

method	result
default	$\frac{a^2 \sqrt{cdx+d}(-cxe+e) \arctan\left(\frac{\sqrt{c^2de}x}{\sqrt{-c^2dx^2e+de}}\right)}{\sqrt{-cxe+e}\sqrt{cdx+d}\sqrt{c^2de}} - \frac{b^2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2x^2+1} \arcsin(cx)^3}{3dec(c^2x^2-1)} - \frac{ab\sqrt{-e(cx-1)} \sqrt{d(cx+1)}}{dec}$
parts	$\frac{a^2 \sqrt{cdx+d}(-cxe+e) \arctan\left(\frac{\sqrt{c^2de}x}{\sqrt{-c^2dx^2e+de}}\right)}{\sqrt{-cxe+e}\sqrt{cdx+d}\sqrt{c^2de}} - \frac{b^2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2x^2+1} \arcsin(cx)^3}{3dec(c^2x^2-1)} - \frac{ab\sqrt{-e(cx-1)} \sqrt{d(cx+1)}}{dec}$

input

```
int((a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
a^2*((c*d*x+d)*(-c*e*x+e))^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))-1/3*b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c/(c^2*x^2-1)*arcsin(c*x)^3-a*b*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c/(c^2*x^2-1)*arcsin(c*x)^2
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{cdx + d}\sqrt{-cex + e}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm m="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^2*d*e*x^2 - d*e), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(a + b \arcsin(cx))^2}{\sqrt{d}(cx + 1)\sqrt{-e}(cx - 1)} dx$$

input `integrate((a+b*asin(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)`

output `Integral((a + b*asin(c*x))**2/(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm m="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{cdx + d}\sqrt{-cex + e}} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output

```
integrate((b*arcsin(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cdx}\sqrt{e - cex}} dx$$

input

```
int((a + b*asin(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output

```
int((a + b*asin(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{\sqrt{d + cx} \sqrt{e - cx}} dx$$

$$= \frac{-2a \sin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 + 2\left(\int \frac{\arcsin(cx)}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) abc + \left(\int \frac{\arcsin(cx)^2}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) b^2 c}{\sqrt{e} \sqrt{d} c}$$

input `int((a+b*asin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)`

output `(- 2*asin(sqrt(- c*x + 1)/sqrt(2))*a**2 + 2*int(asin(c*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*a*b*c + int(asin(c*x)**2/(sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*b**2*c)/(sqrt(e)*sqrt(d)*c)`

3.495 $\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}} dx$

Optimal result	4203
Mathematica [A] (verified)	4204
Rubi [A] (verified)	4204
Maple [B] (verified)	4207
Fricas [F]	4208
Sympy [F]	4208
Maxima [F(-2)]	4208
Giac [F]	4209
Mupad [F(-1)]	4209
Reduce [F]	4210

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}} dx = -\frac{2\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{\sqrt{d + cdx}\sqrt{e - cex}} + \frac{2ib\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{\sqrt{d + cdx}\sqrt{e - cex}} - \frac{2ib\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{\sqrt{d + cdx}\sqrt{e - cex}} - \frac{2b^2\sqrt{1 - c^2x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{\sqrt{d + cdx}\sqrt{e - cex}} + \frac{2b^2\sqrt{1 - c^2x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{\sqrt{d + cdx}\sqrt{e - cex}}$$

output

```
-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))
)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*
x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-
2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1
/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-
I*c*x-(-c^2*x^2+1)^(1/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*(-c^2*x^2
+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(
1/2)
```


Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.17

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}} dx = \frac{a^2 \log(cx)}{\sqrt{d}\sqrt{e}} - \frac{a^2 \log\left(de + \sqrt{d}\sqrt{e}\sqrt{d + cdx}\sqrt{e - cex}\right)}{\sqrt{d}\sqrt{e}}$$

$$+ \frac{2ab\sqrt{1 - c^2x^2}(\arcsin(cx) (\log(1 - e^{i \arcsin(cx)}) - \log(1 + e^{i \arcsin(cx)})) + i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - i \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{\sqrt{d + cdx}\sqrt{e - cex}}$$

$$+ \frac{b^2\sqrt{1 - c^2x^2}(\arcsin(cx)^2 \log(1 - e^{i \arcsin(cx)}) - \arcsin(cx)^2 \log(1 + e^{i \arcsin(cx)}) + 2i \arcsin(cx) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) - 2i \arcsin(cx) \operatorname{PolyLog}(2, e^{i \arcsin(cx)}))}{\sqrt{d + cdx}\sqrt{e - cex}}$$

input `Integrate[(a + b*ArcSin[c*x])^2/(x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]`

output `(a^2*Log[c*x])/(Sqrt[d]*Sqrt[e]) - (a^2*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]) + (2*a*b*Sqrt[1 - c^2*x^2]*(ArcSin[c*x]*(Log[1 - E^(I*ArcSin[c*x])] - Log[1 + E^(I*ArcSin[c*x])]) + I*PolyLog[2, -E^(I*ArcSin[c*x])] - I*PolyLog[2, E^(I*ArcSin[c*x])]))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]) + (b^2*Sqrt[1 - c^2*x^2]*(ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])] - ArcSin[c*x]^2*Log[1 + E^(I*ArcSin[c*x])] + (2*I)*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] - (2*I)*ArcSin[c*x]*PolyLog[2, E^(I*ArcSin[c*x])]) - 2*PolyLog[3, -E^(I*ArcSin[c*x])] + 2*PolyLog[3, E^(I*ArcSin[c*x])]))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.52, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5238, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{cdx + d}\sqrt{e - cex}} dx$$

↓ 5238

$$\begin{aligned}
& \frac{\sqrt{1-c^2x^2} \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx}{\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{5218} \\
& \frac{\sqrt{1-c^2x^2} \int \frac{(a+b \arcsin(cx))^2}{cx} d \arcsin(cx)}{\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{3042} \\
& \frac{\sqrt{1-c^2x^2} \int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx)}{\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{4671} \\
& \frac{\sqrt{1-c^2x^2} (-2b \int (a+b \arcsin(cx)) \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{3011} \\
& \frac{\sqrt{1-c^2x^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) d \arcsin(cx))}{\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{2720} \\
& \frac{\sqrt{1-c^2x^2} (2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) de^{i \arcsin(cx)}) - 2b(i \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{i \arcsin(cx)} \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) de^{i \arcsin(cx)})}{\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{7143} \\
& \frac{\sqrt{1-c^2x^2} (-2 \operatorname{arctanh}(e^{i \arcsin(cx)}) (a+b \arcsin(cx))^2 + 2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \operatorname{PolyLog}(2, e^{i \arcsin(cx)}) (a+b \arcsin(cx)))}{\sqrt{cdx+d}\sqrt{e-cex}}
\end{aligned}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output

```
(Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] +
2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -
E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*
x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
)
```

Defintions of rubi rules used

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x)) *(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 3011 `Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4671 `Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5218 `Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]`

rule 5238 `Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(p_)*((f_.) + (g_.)*(x_))^(q_), x_Symbol] := Simp[((-d^2)*(g/e))^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]`

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(295) = 590.

Time = 3.08 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.09

method	result
default	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \ln\left(\frac{2\sqrt{de} \sqrt{-de(c^2x^2-1)+2de}}{x}\right)}{\sqrt{-de(c^2x^2-1)} \sqrt{de}} - \frac{b^2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2x^2+1} \left(\arcsin(cx)^2 \ln\left(1-\sqrt{icx+1}\right)\right)}{\sqrt{-de(c^2x^2-1)} \sqrt{de}}$
parts	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)} \ln\left(\frac{2\sqrt{de} \sqrt{-de(c^2x^2-1)+2de}}{x}\right)}{\sqrt{-de(c^2x^2-1)} \sqrt{de}} - \frac{b^2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2x^2+1} \left(\arcsin(cx)^2 \ln\left(1-\sqrt{icx+1}\right)\right)}{\sqrt{-de(c^2x^2-1)} \sqrt{de}}$

input

```
int((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*ln(2*((d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)+d*e)/x)/(-d*e*(c^2*x^2-1))^(1/2)/(d*e)^(1/2)-b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)*(arcsin(c*x)^2*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+arcsin(c*x)^2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-arcsin(c*x)^2*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))+2*I*arcsin(c*x)*polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))-4*I*arcsin(c*x)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-4*I*arcsin(c*x)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-2*polylog(3,-I*c*x+(-c^2*x^2+1)^(1/2))+8*polylog(3,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+8*polylog(3,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2)))/d/e/(c^2*x^2-1)+2*I*a*b*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))-I*arcsin(c*x)*ln(1+I*c*x+(-c^2*x^2+1)^(1/2))-polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))+2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2)))/d/e/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{cdx + d}\sqrt{-cex + ex}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="fricas")`

output `integral(-(b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^2*d*e*x^3 - d*e*x), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}} dx = \int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d}(cx + 1)\sqrt{-e}(cx - 1)} dx$$

input `integrate((a+b*asin(c*x))**2/x/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)`

output `Integral((a + b*asin(c*x))**2/(x*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d + cdx} \sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{cdx + d} \sqrt{-cex + ex}} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="giac")
```

output

```
integrate((b*arcsin(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d + cdx} \sqrt{e - cex}} dx = \int \frac{(a + b \arcsin(cx))^2}{x \sqrt{d + cdx} \sqrt{e - cex}} dx$$

input

```
int((a + b*asin(c*x))^2/(x*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x\sqrt{d+cx}\sqrt{e-cx}} dx$$

$$= \frac{2\left(\int \frac{\arcsin(cx)}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) ab + \left(\int \frac{\arcsin(cx)^2}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right) b^2 - \log\left(-\sqrt{2} + \tan\left(\frac{\arcsin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2}\right) - 1\right) a^2 + \log\left(\dots\right)}{1}$$

input

```
int((a+b*asin(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output

```
(2*int(asin(c*x)/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x),x)*a*b + int(asin(c*x)
**2/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x),x)*b**2 - log(-sqrt(2) + tan(asin
(sqrt(-c*x + 1)/sqrt(2))/2) - 1)*a**2 + log(-sqrt(2) + tan(asin(sqrt(
-c*x + 1)/sqrt(2))/2) + 1)*a**2 - log(sqrt(2) + tan(asin(sqrt(-c*x + 1)
/sqrt(2))/2) - 1)*a**2 + log(sqrt(2) + tan(asin(sqrt(-c*x + 1)/sqrt(2))/
2) + 1)*a**2)/(sqrt(e)*sqrt(d))
```

3.496 $\int \frac{(a+b \arcsin(cx))^2}{x^2 \sqrt{d+cdx} \sqrt{e-cex}} dx$

Optimal result	4211
Mathematica [A] (verified)	4212
Rubi [A] (verified)	4212
Maple [B] (verified)	4215
Fricas [F]	4216
Sympy [F]	4216
Maxima [F(-2)]	4217
Giac [F]	4217
Mupad [F(-1)]	4217
Reduce [F]	4218

Optimal result

Integrand size = 35, antiderivative size = 214

$$\int \frac{(a+b \arcsin(cx))^2}{x^2 \sqrt{d+cdx} \sqrt{e-cex}} dx = -\frac{ic\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{(1-c^2x^2)(a+b \arcsin(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2bc\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1-e^{2i \arcsin(cx)})}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{ib^2c\sqrt{1-c^2x^2} \text{PolyLog}(2, e^{2i \arcsin(cx)})}{\sqrt{d+cdx}\sqrt{e-cex}}$$

output

```
-I*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-(-c^2*x^2+1)*(a+b*arcsin(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```


Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cdx} \sqrt{e - cex}} dx$$

$$= \frac{b^2(-1 + c^2x^2 - icx\sqrt{1 - c^2x^2}) \arcsin(cx)^2 + 2b \arcsin(cx) (-a + ac^2x^2 + bcx\sqrt{1 - c^2x^2}) \log(1 - e^{2i \arcsin(cx)})}{x\sqrt{d + cdx}\sqrt{e - cex}}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output

```
(b^2*(-1 + c^2*x^2 - I*c*x*Sqrt[1 - c^2*x^2])*ArcSin[c*x]^2 + 2*b*ArcSin[c*x]*(-a + a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2]*Log[1 - E^((2*I)*ArcSin[c*x])]) + a*(-a + a*c^2*x^2 + 2*b*c*x*Sqrt[1 - c^2*x^2]*Log[c*x]) - I*b^2*c*x*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5238, 5186, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{cdx + d} \sqrt{e - cex}} dx$$

$$\downarrow \text{5238}$$

$$\frac{\sqrt{1 - c^2x^2} \int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{1 - c^2x^2}} dx}{\sqrt{cdx + d} \sqrt{e - cex}}$$

$$\downarrow \text{5186}$$

$$\frac{\sqrt{1 - c^2x^2} \left(2bc \int \frac{a + b \arcsin(cx)}{x} dx - \frac{\sqrt{1 - c^2x^2} (a + b \arcsin(cx))^2}{x} \right)}{\sqrt{cdx + d} \sqrt{e - cex}}$$

$$\frac{\sqrt{1-c^2x^2} \left(2bc \int \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))}{cx} d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 5136$$

$$\frac{\sqrt{1-c^2x^2} \left(2bc \int -((a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2})) d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 3042$$

$$\frac{\sqrt{1-c^2x^2} \left(-2bc \int (a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 25$$

$$\frac{\sqrt{1-c^2x^2} \left(-2bc \int (a+b \arcsin(cx)) \tan(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 4200$$

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} + 2bc \left(2i \int -\frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 25$$

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} + 2bc \left(-2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1-e^{2i \arcsin(cx)}} d \arcsin(cx) - \frac{i(a+b \arcsin(cx))^2}{2b} \right) \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 2620$$

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} + 2bc \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{2} i b \int \log(1 - e^{2i \arcsin(cx)}) \right) \right) \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 2715$$

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} + 2bc \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \int e^{-2i \arcsin(cx)} \log \right) \right) \right)}{\sqrt{cdx + d\sqrt{e-cex}}} \quad \downarrow \quad 2838$$

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{x} + 2bc \left(-2i \left(\frac{1}{2} i \log(1 - e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) + \frac{1}{4} b \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right) \right) \right)}{\sqrt{cdx + d\sqrt{e-cex}}}$$

input `Int[(a + b*ArcSin[c*x])^2/(x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]`

output `(Sqrt[1 - c^2*x^2]*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcSin[c*x])^2)/x) + 2*b*c*((-1/2*I)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcSin[c*x])*Log[1 - E^((2*I)*ArcSin[c*x])] + (b*PolyLog[2, E^((2*I)*ArcSin[c*x])])/4)))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_))^(n_)]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4200 `Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

rule 5136 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]`

rule 5186 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b *ArcSin[c*x])^n/(d*f*(m + 1))), x] - Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b *ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]`

rule 5238 `Int[((a_.) + ArcSin[(c_.)*(x_)])*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[((-d^2)*(g/e))^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b *ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(210) = 420$.

Time = 2.43 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.52

method	result
default	$-\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)}}{dex} + b^2 \left(-\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} (i\sqrt{-c^2x^2+1} cx + c^2x^2 - 1) \arcsin(cx)^2}{(c^2x^2-1)xde} + \frac{2i\sqrt{-c^2x^2+1} \sqrt{d(cx-1)}}{(c^2x^2-1)xde} \right)$
parts	$-\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)}}{dex} + b^2 \left(-\frac{\sqrt{-e(cx-1)} \sqrt{d(cx+1)} (i\sqrt{-c^2x^2+1} cx + c^2x^2 - 1) \arcsin(cx)^2}{(c^2x^2-1)xde} + \frac{2i\sqrt{-c^2x^2+1} \sqrt{d(cx-1)}}{(c^2x^2-1)xde} \right)$

input `int((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2), x, method=_RET URNVERBOSE)`

output

```
-a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d/e/x+b^2*(-(-e*(c*x-1))^(1/2)*(
d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)^2/(c^2*x
^2-1)/x/d/e+2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d/
e/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcs
in(c*x)*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+I*c*x+(-
c^2*x^2+1)^(1/2))+arcsin(c*x)^2+polylog(2,-I*c*x+(-c^2*x^2+1)^(1/2))+2*pol
ylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,-(I*c*x+(-c^2*x^2+1)
^(1/2))^(1/2)))*)+2*a*b*(2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x
-1))^(1/2)/d/e/(c^2*x^2-1)*arcsin(c*x)*c-(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1
/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*arcsin(c*x)/(c^2*x^2-1)/x/d/e-(-e
*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/(c^2*x^2-1)*ln((I
*c*x+(-c^2*x^2+1)^(1/2))^2-1)*c)
```

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cdx} \sqrt{e - cex}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{cdx + d} \sqrt{-cex + ex^2}} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="fricas")
```

output

```
integral(-(b^2*arcsin(c*x))^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^2*d*e*x^4 - d*e*x^2), x)
```

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cdx} \sqrt{e - cex}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d(cx + 1)} \sqrt{-e(cx - 1)}} dx$$

input

```
integrate((a+b*asin(c*x))**2/x**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output

```
Integral((a + b*asin(c*x))**2/(x**2*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cx} \sqrt{e - cx}} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e`

Giac [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cx} \sqrt{e - cx}} dx = \int \frac{(b \arcsin(cx) + a)^2}{\sqrt{cdx + d} \sqrt{-cex + ex^2}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="giac")`

output `integrate((b*arcsin(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*x^2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cx} \sqrt{e - cx}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cx} \sqrt{e - cx}} dx$$

input `int((a + b*asin(c*x))^2/(x^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)`

output `int((a + b*asin(c*x))^2/(x^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2 \sqrt{d + cdx} \sqrt{e - cex}} dx$$

$$= \frac{-\sqrt{cx + 1} \sqrt{-cx + 1} a^2 + 2 \left(\int \frac{a \sin(cx)}{\sqrt{cx+1} \sqrt{-cx+1} x^2} dx \right) abx + \left(\int \frac{a \sin(cx)^2}{\sqrt{cx+1} \sqrt{-cx+1} x^2} dx \right) b^2 x}{\sqrt{e} \sqrt{d} x}$$

input `int((a+b*asin(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)`

output `(- sqrt(c*x + 1)*sqrt(- c*x + 1)*a**2 + 2*int(asin(c*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)*x**2),x)*a*b*x + int(asin(c*x)**2/(sqrt(c*x + 1)*sqrt(- c*x + 1)*x**2),x)*b**2*x)/(sqrt(e)*sqrt(d)*x)`

3.497 $\int \frac{x^2(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx$

Optimal result	4219
Mathematica [B] (verified)	4220
Rubi [A] (verified)	4220
Maple [B] (verified)	4224
Fricas [F]	4225
Sympy [F]	4225
Maxima [F(-2)]	4225
Giac [F]	4226
Mupad [F(-1)]	4226
Reduce [F]	4227

Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{x^2(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx = \frac{x(a+b \arcsin(cx))^2}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{i\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{c^3de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{\sqrt{1-c^2x^2}(a+b \arcsin(cx))^3}{3bc^3de\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2b\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{c^3de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{c^3de\sqrt{d+cdx}\sqrt{e-cex}}$$

output

```
x*(a+b*arcsin(c*x))^2/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/3*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^3/b/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```


Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 636 vs. $2(295) = 590$.

Time = 3.15 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.16

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{3a^2c\sqrt{d}ex + 3a^2\sqrt{e}\sqrt{d + cdx}\sqrt{e - cex} \arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e(-1+c^2x^2)}}\right) + 3ab\sqrt{d}}$$

input

```
Integrate[(x^2*(a + b*ArcSin[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output

```
(3*a^2*c*Sqrt[d]*e*x + 3*a^2*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 3*a*b*Sqrt[d]*e*(2*c*x*ArcSin[c*x] + Sqrt[1 - c^2*x^2]*(-ArcSin[c*x]^2 + 2*(Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]) + Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]))) + b^2*Sqrt[d]*e*((6*I)*Pi*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + 3*c*x*ArcSin[c*x]^2 - (3*I)*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2 - Sqrt[1 - c^2*x^2]*ArcSin[c*x]^3 + 12*Pi*Sqrt[1 - c^2*x^2]*Log[1 + E^((-I)*ArcSin[c*x])] + 3*Pi*Sqrt[1 - c^2*x^2]*Log[1 - I*E^(I*ArcSin[c*x])] + 6*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - 3*Pi*Sqrt[1 - c^2*x^2]*Log[1 + I*E^(I*ArcSin[c*x])] + 6*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - 12*Pi*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2]] + 3*Pi*Sqrt[1 - c^2*x^2]*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] - 3*Pi*Sqrt[1 - c^2*x^2]*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (6*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (6*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])]))/(3*c^3*d^(3/2)*e^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5238, 5206, 5152, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{x^2(a+b \arcsin(cx))^2}{(cdx+d)^{3/2}(e-cex)^{3/2}} dx \\
& \quad \downarrow \text{5238} \\
& \frac{\sqrt{1-c^2x^2} \int \frac{x^2(a+b \arcsin(cx))^2}{(1-c^2x^2)^{3/2}} dx}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{5206} \\
& \frac{\sqrt{1-c^2x^2} \left(-\frac{2b \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c} - \frac{\int \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} dx}{c^2} + \frac{x(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} \right)}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{5152} \\
& \frac{\sqrt{1-c^2x^2} \left(-\frac{2b \int \frac{x(a+b \arcsin(cx))}{1-c^2x^2} dx}{c} - \frac{(a+b \arcsin(cx))^3}{3bc^3} + \frac{x(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} \right)}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{5180} \\
& \frac{\sqrt{1-c^2x^2} \left(-\frac{2b \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^3} - \frac{(a+b \arcsin(cx))^3}{3bc^3} + \frac{x(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} \right)}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{3042} \\
& \frac{\sqrt{1-c^2x^2} \left(-\frac{2b \int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c^3} - \frac{(a+b \arcsin(cx))^3}{3bc^3} + \frac{x(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} \right)}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{4202} \\
& \frac{\sqrt{1-c^2x^2} \left(-\frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{c^3} - \frac{(a+b \arcsin(cx))^3}{3bc^3} + \frac{x(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} \right)}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{2620} \\
& \frac{\sqrt{1-c^2x^2} \left(-\frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)})(a+b \arcsin(cx)) \right) \right)}{c^3} - \frac{(a+b \arcsin(cx))^3}{3bc^3} \right)}{de\sqrt{cdx+d}\sqrt{e-cex}} \\
& \quad \downarrow \text{2715}
\end{aligned}$$

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de^{2i \arcsin(cx)} - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{c^3} \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 2838

$$\frac{\sqrt{1-c^2x^2} \left(-\frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right) \right)}{c^3} \right)}{de\sqrt{cdx} + d\sqrt{e-cex}} - \frac{(a+b \arcsin(cx))^3}{3bc^3}$$

input `Int[(x^2*(a + b*ArcSin[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]`

output `(Sqrt[1 - c^2*x^2]*((x*(a + b*ArcSin[c*x])^2)/(c^2*Sqrt[1 - c^2*x^2]) - (a + b*ArcSin[c*x])^3/(3*b*c^3) - (2*b*((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4))/c^3)/(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Defintions of rubi rules used

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4202 `Int[((c_) + (d_)*(x_)^(m_))*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x)))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5152 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)/Sqrt[(d_) + (e_)*(x_)^2], x_Symbol] := Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && NeQ[n, -1]`

rule 5180 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*(x_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5206 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1))) Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IGtQ[m, 1]`

rule 5238 `Int[((a_) + ArcSin[(c_)*(x_)])*(b_)^(n_)*((h_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_)*((f_) + (g_)*(x_)^q), x_Symbol] := Simp[((-d^2)*(g/e))^IntPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPart[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]`

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(283) = 566.

Time = 6.33 (sec) , antiderivative size = 725, normalized size of antiderivative = 2.46

method	result
default	$\frac{a^2 \left(-\arctan\left(\frac{\sqrt{c^2 d e} x}{\sqrt{-d e (c x - 1)(c x + 1)}}\right) x^2 c^2 d e + \arctan\left(\frac{\sqrt{c^2 d e} x}{\sqrt{-d e (c x - 1)(c x + 1)}}\right) e d - \sqrt{c^2 d e} \sqrt{-d e (c^2 x^2 - 1)} x \right) \sqrt{d (c x + 1)} \sqrt{-e (c x - 1)}}{d^2 e^2 (c x + 1) \sqrt{c^2 d e} (c x - 1) \sqrt{-d e (c^2 x^2 - 1)} c^2} +$
parts	$\frac{a^2 \left(-\arctan\left(\frac{\sqrt{c^2 d e} x}{\sqrt{-d e (c x - 1)(c x + 1)}}\right) x^2 c^2 d e + \arctan\left(\frac{\sqrt{c^2 d e} x}{\sqrt{-d e (c x - 1)(c x + 1)}}\right) e d - \sqrt{c^2 d e} \sqrt{-d e (c^2 x^2 - 1)} x \right) \sqrt{d (c x + 1)} \sqrt{-e (c x - 1)}}{d^2 e^2 (c x + 1) \sqrt{c^2 d e} (c x - 1) \sqrt{-d e (c^2 x^2 - 1)} c^2} +$

input `int(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETURNVERBOSE)`

output `a^2*(-arctan((c^2*d*e)^(1/2)*x/(-d*e*(c*x-1)*(c*x+1))^(1/2))*x^2*c^2*d*e+arctan((c^2*d*e)^(1/2)*x/(-d*e*(c*x-1)*(c*x+1))^(1/2))*e*d-(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/(c*x+1)/(c^2*d*e)^(1/2)/(c*x-1)/(-d*e*(c^2*x^2-1))^(1/2)/c^2+1/3*b^2*(arcsin(c*x)^3*c^2*x^2+3*I*arcsin(c*x)^2*x^2*c^2-6*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x^2*c^2-6*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x^2*c^2+6*I*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x^2*c^2+6*I*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x^2*c^2+3*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)*c*x-arcsin(c*x)^3-3*I*arcsin(c*x)^2+6*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))+6*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-6*I*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))-6*I*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-2*c^2*x^2+1)/c^3/d^2/e^2+a*b*(arcsin(c*x)^2*x^2*c^2+2*I*arcsin(c*x)*x^2*c^2-2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)*x^2*c^2+2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c*x-arcsin(c*x)^2-2*I*arcsin(c*x)+2*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2))*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-2*c^2*x^2+1)/c^3/d^2/e^2`

Fricas [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="fricas")`

output `integral((b^2*x^2*arcsin(c*x)^2 + 2*a*b*x^2*arcsin(c*x) + a^2*x^2)*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x)`

Sympy [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{x^2(a + b \operatorname{asin}(cx))^2}{(d(cx + 1))^{\frac{3}{2}}(-e(cx - 1))^{\frac{3}{2}}} dx$$

input `integrate(x**2*(a+b*asin(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)`

output `Integral(x**2*(a + b*asin(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input

```
integrate(x^2*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="giac")
```

output

```
integrate((b*arcsin(c*x) + a)^2*x^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx$$

input

```
int((x^2*(a + b*asin(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output

```
int((x^2*(a + b*asin(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{2\sqrt{cx+1}\sqrt{-cx+1} \operatorname{asin}\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right) a^2 - 2\sqrt{cx+1}\sqrt{-cx+1} \left(\int \frac{\sqrt{cx+1}\sqrt{-cx+1}}{\sqrt{cx+1}\sqrt{-cx+1}} dx\right)}{(d + cdx)^{3/2}(e - cex)^{3/2}}$$

input `int(x^2*(a+b*asin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)`

output `(2*sqrt(c*x + 1)*sqrt(-c*x + 1)*asin(sqrt(-c*x + 1)/sqrt(2))*a**2 - 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*int((asin(c*x)*x**2)/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt(-c*x + 1)),x)*a*b*c**3 - sqrt(c*x + 1)*sqrt(-c*x + 1)*int((asin(c*x)**2*x**2)/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt(-c*x + 1)),x)*b**2*c**3 + a**2*c*x)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c**3*d*e)`

3.498 $\int \frac{x(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx$

Optimal result	4228
Mathematica [A] (verified)	4229
Rubi [A] (verified)	4229
Maple [A] (verified)	4232
Fricas [F]	4232
Sympy [F]	4233
Maxima [F]	4233
Giac [F]	4233
Mupad [F(-1)]	4234
Reduce [F]	4234

Optimal result

Integrand size = 33, antiderivative size = 244

$$\int \frac{x(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx = \frac{(a+b \arcsin(cx))^2}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} + \frac{4ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, -ie^{i \arcsin(cx)})}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2ib^2\sqrt{1-c^2x^2} \text{PolyLog}(2, ie^{i \arcsin(cx)})}{c^2de\sqrt{d+cdx}\sqrt{e-cex}}$$

output

```
(a+b*arcsin(c*x))^2/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.86

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{a^2 + 2ab \arcsin(cx) + ib^2\pi\sqrt{1 - c^2x^2} \arcsin(cx) + b^2 \arcsin(cx)^2 - b^2\pi\sqrt{1 - c^2x^2}}{(d + cdx)^{3/2}(e - cex)^{3/2}}$$

input

```
Integrate[(x*(a + b*ArcSin[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),
x]
```

output

```
(a^2 + 2*a*b*ArcSin[c*x] + I*b^2*Pi*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + b^2*Ar
cSin[c*x]^2 - b^2*Pi*Sqrt[1 - c^2*x^2]*Log[1 - I*E^(I*ArcSin[c*x])] - 2*b^
2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - b^2*Pi*Sqrt
[1 - c^2*x^2]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b^2*Sqrt[1 - c^2*x^2]*ArcSi
n[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] + b^2*Pi*Sqrt[1 - c^2*x^2]*Log[-Cos[(P
i + 2*ArcSin[c*x])/4]] + 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] -
Sin[ArcSin[c*x]/2]] - 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin
[ArcSin[c*x]/2]] + b^2*Pi*Sqrt[1 - c^2*x^2]*Log[Sin[(Pi + 2*ArcSin[c*x])/4
]] - (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2*I
)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c^2*d*e*Sqrt[d +
c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {5238, 5182, 5164, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a + b \arcsin(cx))^2}{(cdx + d)^{3/2}(e - cex)^{3/2}} dx$$

↓ 5238

$$\frac{\sqrt{1 - c^2x^2} \int \frac{x(a + b \arcsin(cx))^2}{(1 - c^2x^2)^{3/2}} dx}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\begin{aligned}
 & \downarrow 5182 \\
 & \frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} - \frac{2b \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx}{c} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}} \\
 & \downarrow 5164 \\
 & \frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} - \frac{2b \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx)}{c^2} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}} \\
 & \downarrow 3042 \\
 & \frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} - \frac{2b \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx)}{c^2} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}} \\
 & \downarrow 4669 \\
 & \frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} - \frac{2b(-b \int \log(1-ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1+ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx))}{c^2} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}} \\
 & \downarrow 2715 \\
 & \frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} - \frac{2b(ib \int e^{-i \arcsin(cx)} \log(1-ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1+ie^{i \arcsin(cx)}) de^{i \arcsin(cx)}}{c^2} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}} \\
 & \downarrow 2838 \\
 & \frac{\sqrt{1-c^2x^2} \left(\frac{(a+b \arcsin(cx))^2}{c^2\sqrt{1-c^2x^2}} - \frac{2b(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}))}{c^2} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}
 \end{aligned}$$

input `Int[(x*(a + b*ArcSin[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]`

output `(Sqrt[1 - c^2*x^2]*((a + b*ArcSin[c*x])^2/(c^2*Sqrt[1 - c^2*x^2]) - (2*b*(-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]))/c^2))/(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Defintions of rubi rules used

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^{(n_)}], x_Symbol]$
 $\text{:> Simp}[1/(d*e*n*\text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^{n}], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x_Symbol] \text{:> Simp}[-\text{PolyLog}[2,$
 $(-c)*e*x^n/n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \text{:> Int}[\text{DeactivateTrig}[u, x], x] /;$ $\text{FunctionOfTrigOfLinear}$
 $\text{Q}[u, x]$

rule 4669 $\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^{(m_)}, x_Symbol]$
 $\text{:> Simp}[-2*(c + d*x)^m*(\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]/f), x] + (-\text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x],$
 $x] + \text{Simp}[d*(m/f) \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x]) /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

rule 5164 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)} / ((d_) + (e_)*(x_)^2), x_Symbol]$
 $\text{:> Simp}[1/(c*d) \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sec}[x], x], x, \text{ArcSin}[c*x]], x]$
 $/;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0]$

rule 5182 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*(x_)*((d_) + (e_)*(x_)^2)^{(p_)}, x_Symbol]$
 $\text{:> Simp}[(d + e*x^2)^{(p+1)}*((a + b*\text{ArcSin}[c*x])^n/(2*e*(p + 1))), x] + \text{Simp}[b*(n/(2*c*(p + 1)))*\text{Simp}[(d + e*x^2)^p/(1 - c^2*x^2)^p] \text{Int}[(1 - c^2*x^2)^{(p+1/2)}*(a + b*\text{ArcSin}[c*x])^{(n-1)}, x], x] /;$ $\text{FreeQ}\{a,$
 $b, c, d, e, p\}, x\} \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[p, -1]$

rule 5238 $\text{Int}[(a_) + \text{ArcSin}[(c_)*(x_)]*(b_)]^{(n_)}*((h_)*(x_))^{(m_)}*((d_) + (e_)*(x_))^{(p_)}*((f_) + (g_)*(x_))^{(q_)}, x_Symbol]$
 $\text{:> Simp}[(d^2*(g/e))^n \text{IntPart}[q]*(d + e*x)^{\text{FracPart}[q]}*((f + g*x)^{\text{FracPart}[q]}/(1 - c^2*x^2)^{\text{FracPart}[q]}) \text{Int}[(h*x)^m*(d + e*x)^{(p-q)}*(1 - c^2*x^2)^q*(a + b*\text{ArcSin}[c*x])^n,$
 $x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g, h, m, n\}, x\} \ \&\& \ \text{EqQ}[e*f + d*g, 0] \ \&\& \ \text{EqQ}[c^2*d^2 - e^2, 0] \ \&\& \ \text{HalfIntegerQ}[p, q] \ \&\& \ \text{GeQ}[p - q, 0]$

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.57

method	result
default	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)}}{(-cx-1)(cx-1)e^2 d^2 c^2} + b^2 \left(-\frac{\arcsin(cx)^2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)}}{d^2 e^2 (c^2 x^2 - 1) c^2} - \frac{2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2 x^2 + 1} (\arcsin(cx) \ln(1 + I * (I * c * x + (-c^2 * x^2 + 1)^{1/2}))) - \arcsin(cx) * \ln(1 - I * (I * c * x + (-c^2 * x^2 + 1)^{1/2}))) - I * \text{polylog}(2, -I * (I * c * x + (-c^2 * x^2 + 1)^{1/2}))) + I * \text{polylog}(2, I * (I * c * x + (-c^2 * x^2 + 1)^{1/2})))}{d^2 e^2 (c^2 x^2 - 1) / c^2} - 2 * a * b * (-(-c^2 * x^2 + 1)^{1/2} * \ln(I * c * x + (-c^2 * x^2 + 1)^{1/2} + I) + (-c^2 * x^2 + 1)^{1/2} * \ln(I * c * x + (-c^2 * x^2 + 1)^{1/2} - I) + \arcsin(cx)) * (-e * (c * x - 1))^{1/2} * (d * (c * x + 1))^{1/2} / d^2 e^2 (c^2 * x^2 - 1) / c^2 \right)$
parts	$\frac{a^2 \sqrt{d(cx+1)} \sqrt{-e(cx-1)}}{(-cx-1)(cx-1)e^2 d^2 c^2} + b^2 \left(-\frac{\arcsin(cx)^2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)}}{d^2 e^2 (c^2 x^2 - 1) c^2} - \frac{2 \sqrt{-e(cx-1)} \sqrt{d(cx+1)} \sqrt{-c^2 x^2 + 1} (\arcsin(cx) \ln(1 + I * (I * c * x + (-c^2 * x^2 + 1)^{1/2}))) - \arcsin(cx) * \ln(1 - I * (I * c * x + (-c^2 * x^2 + 1)^{1/2}))) - I * \text{polylog}(2, -I * (I * c * x + (-c^2 * x^2 + 1)^{1/2}))) + I * \text{polylog}(2, I * (I * c * x + (-c^2 * x^2 + 1)^{1/2})))}{d^2 e^2 (c^2 x^2 - 1) / c^2} - 2 * a * b * (-(-c^2 * x^2 + 1)^{1/2} * \ln(I * c * x + (-c^2 * x^2 + 1)^{1/2} + I) + (-c^2 * x^2 + 1)^{1/2} * \ln(I * c * x + (-c^2 * x^2 + 1)^{1/2} - I) + \arcsin(cx)) * (-e * (c * x - 1))^{1/2} * (d * (c * x + 1))^{1/2} / d^2 e^2 (c^2 * x^2 - 1) / c^2 \right)$

input `int(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETURNVERBOSE)`

output `a^2/(-c*x-1)/(c*x-1)/e^2/d^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/c^2+b^2*(-arcsin(c*x)^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)/d^2/e^2/(c^2*x^2-1)/c^2-2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)*(arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))-arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))))-I*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))+I*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))))/d^2/e^2/(c^2*x^2-1)/c^2-2*a*b*(-(-c^2*x^2+1)^(1/2)*ln(I*c*x+(-c^2*x^2+1)^(1/2)+I)+(-c^2*x^2+1)^(1/2)*ln(I*c*x+(-c^2*x^2+1)^(1/2)-I)+arcsin(c*x))*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)/d^2/e^2/(c^2*x^2-1)/c^2`

Fricas [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(cdx + d)^{3/2}(-cex + e)^{3/2}} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,algorithm="fricas")`

output `integral((b^2*x*arcsin(c*x)^2 + 2*a*b*x*arcsin(c*x) + a^2*x)*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x)`

Sympy [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{x(a + b \arcsin(cx))^2}{(d(cx + 1))^{\frac{3}{2}}(-e(cx - 1))^{\frac{3}{2}}} dx$$

input `integrate(x*(a+b*asin(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)`

output `Integral(x*(a + b*asin(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="maxima")`

output `sqrt(d)*sqrt(e)*integrate((b^2*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2 + 2*a*b*x*arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x) + a^2/(sqrt(-c^2*d*e*x^2 + d*e)*c^2*d*e)`

Giac [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2 x}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate(x*(a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="giac")`

output

```
integrate((b*arcsin(c*x) + a)^2*x/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{x(a + b \operatorname{asin}(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx$$

input

```
int((x*(a + b*asin(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output

```
int((x*(a + b*asin(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{-2\sqrt{cx + 1}\sqrt{-cx + 1} \left(\int \frac{\operatorname{asin}(cx)x}{\sqrt{cx+1}\sqrt{-cx+1}c^2x^2 - \sqrt{cx+1}\sqrt{-cx+1}} dx \right) ab c^2 - \sqrt{cx}}{\sqrt{e}\sqrt{d}\sqrt{cx + 1}\sqrt{-c}}$$

input

```
int(x*(a+b*asin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output

```
( - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*int((asin(c*x)*x)/(sqrt(c*x + 1)*sqrt
( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c**2 - sqr
t(c*x + 1)*sqrt( - c*x + 1)*int((asin(c*x)**2*x)/(sqrt(c*x + 1)*sqrt( - c*
x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c**2 + a**2)/(s
qrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*d*e)
```

3.499
$$\int \frac{(a+b \arcsin(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} dx$$

Optimal result	4235
Mathematica [B] (verified)	4236
Rubi [A] (verified)	4236
Maple [B] (verified)	4239
Fricas [F]	4240
Sympy [F]	4241
Maxima [F]	4241
Giac [F]	4241
Mupad [F(-1)]	4242
Reduce [F]	4242

Optimal result

Integrand size = 32, antiderivative size = 231

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{x(a + b \arcsin(cx))^2}{de\sqrt{d + cdx}\sqrt{e - cex}} - \frac{i\sqrt{1 - c^2x^2}(a + b \arcsin(cx))^2}{cde\sqrt{d + cdx}\sqrt{e - cex}} + \frac{2b\sqrt{1 - c^2x^2}(a + b \arcsin(cx)) \log(1 + e^{2i \arcsin(cx)})}{cde\sqrt{d + cdx}\sqrt{e - cex}} - \frac{ib^2\sqrt{1 - c^2x^2} \text{PolyLog}(2, -e^{2i \arcsin(cx)})}{cde\sqrt{d + cdx}\sqrt{e - cex}}$$

output

```
x*(a+b*arcsin(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/c/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/c/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```


Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 550 vs. $2(231) = 462$.

Time = 2.82 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.38

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{a^2cx + 2abcx \arcsin(cx) + 2ib^2\pi\sqrt{1 - c^2x^2} \arcsin(cx) + b^2cx \arcsin(cx)^2}{(d + cdx)^{3/2}(e - cex)^{3/2}}$$

input `Integrate[(a + b*ArcSin[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]`

output `(a^2*c*x + 2*a*b*c*x*ArcSin[c*x] + (2*I)*b^2*Pi*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + b^2*c*x*ArcSin[c*x]^2 - I*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2 + 4*b^2*Pi*Sqrt[1 - c^2*x^2]*Log[1 + E^((-I)*ArcSin[c*x])] + b^2*Pi*Sqrt[1 - c^2*x^2]*Log[1 - I*E^(I*ArcSin[c*x])] + 2*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])] - b^2*Pi*Sqrt[1 - c^2*x^2]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*b^2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - 4*b^2*Pi*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2]] + b^2*Pi*Sqrt[1 - c^2*x^2]*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]] + 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]] + 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]] - b^2*Pi*Sqrt[1 - c^2*x^2]*Log[Sin[(Pi + 2*ArcSin[c*x])/4]] - (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])])/(c*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])`

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.61, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5178, 5160, 5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{(cdx + d)^{3/2}(e - cex)^{3/2}} dx$$

$$\begin{aligned} & \downarrow \text{5178} \\ & \frac{(1 - c^2 x^2)^{3/2} \int \frac{(a+b \arcsin(cx))^2}{(1-c^2 x^2)^{3/2}} dx}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{5160} \\ & \frac{(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - 2bc \int \frac{x(a+b \arcsin(cx))}{1-c^2 x^2} dx \right)}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{5180} \\ & \frac{(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \int \frac{cx(a+b \arcsin(cx))}{\sqrt{1-c^2 x^2}} d \arcsin(cx)}{c} \right)}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{3042} \\ & \frac{(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \int (a+b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c} \right)}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{4202} \\ & \frac{(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)} (a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{c} \right)}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{2620} \\ & \frac{(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} b \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{c} \right)}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{2715} \\ & \frac{(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) d e^{2i \arcsin(cx)} - \frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{c} \right)}{(cdx + d)^{3/2}(e - cex)^{3/2}} \\ & \downarrow \text{2838} \end{aligned}$$

$$(1 - c^2 x^2)^{3/2} \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) \right) (a+b \arcsin(cx)) - \frac{1}{4} b \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) \right)}{c} \right)$$

$$(cdx + d)^{3/2} (e - cex)^{3/2}$$

input `Int[(a + b*ArcSin[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]`

output `((1 - c^2*x^2)^(3/2)*((x*(a + b*ArcSin[c*x])^2)/Sqrt[1 - c^2*x^2] - (2*b*((I/2)*(a + b*ArcSin[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x])])/4)))/c)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))`

Defintions of rubi rules used

rule 2620 `Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Simp[d*(m/(b*f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

rule 2715 `Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

```
rule 4202 Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[
I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I Int[(c + d*x)^m*(E^(2*I*(
e + f*x)))/(1 + E^(2*I*(e + f*x)))], x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

```
rule 5160 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*((a + b*ArcSin[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]
```

```
rule 5178 Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^(p_))*((f_)
+ (g_.)*(x_))^(q_), x_Symbol] := Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q) Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

```
rule 5180 Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(227) = 454.

Time = 3.25 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.05

method	result
default	$a^2 \left(-\frac{1}{cde\sqrt{cdx+d}\sqrt{-cxe+e}} + \frac{\sqrt{cdx+d}}{ce d^2\sqrt{-cxe+e}} \right) - \frac{b^2 \left(\arcsin(cx)^2 + 2 \operatorname{polylog} \left(2, -i \left(icx + \sqrt{-c^2x^2+1} \right) \right) + 2 \operatorname{polylog} \left(2, i \left(icx + \sqrt{-c^2x^2+1} \right) \right) \right)}{2cd}$
parts	$a^2 \left(-\frac{1}{cde\sqrt{cdx+d}\sqrt{-cxe+e}} + \frac{\sqrt{cdx+d}}{ce d^2\sqrt{-cxe+e}} \right) - \frac{b^2 \left(\arcsin(cx)^2 + 2 \operatorname{polylog} \left(2, -i \left(icx + \sqrt{-c^2x^2+1} \right) \right) + 2 \operatorname{polylog} \left(2, i \left(icx + \sqrt{-c^2x^2+1} \right) \right) \right)}{2cd}$

input `int((a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)`

output `a^2*(-1/c/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/c/e/d^2/(-c*e*x+e)^(1/2)*
(c*d*x+d)^(1/2))-b^2*(arcsin(c*x)^2+2*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/
2)))+2*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))-2*I*(-c^2*x^2+1)^(1/2)*poly
log(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x*c-2*I*(-c^2*x^2+1)^(1/2)*polylog(2,
I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x*c-2*I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+
1)^(1/2)))*x^2*c^2-2*I*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x^2*
c^2+2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x*
c+2*(-c^2*x^2+1)^(1/2)*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x*c-
2*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*x^2*c^2-2*polylog(2,I*(I*c*x+(-
c^2*x^2+1)^(1/2)))*x^2*c^2+2*I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2)
)))+2*I*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2)))*arcsin(c*x))*(-I*(-c^2*x^2+1)^(
1/2)+c*x)*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)/c/d^2/e^2/(c^2*x^2-1)+2*a*b
*(I*arcsin(c*x)*c^2*x^2-ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)*x^2*c^2+arcsin(
c*x)*(-c^2*x^2+1)^(1/2)*c*x-I*arcsin(c*x)+ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^
2))*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/e^2/(c^4
*x^4-2*c^2*x^2+1)`

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm
m="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{(d(cx + 1))^{\frac{3}{2}}(-e(cx - 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2)), x)`

Maxima [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm m="maxima")`

output `-b^2*integrate(arctan2(c*x, sqrt(c*x + 1))*sqrt(-c*x + 1))^2/((c^2*d*e*x^2 - d*e)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/(sqrt(d)*sqrt(e)) + 2*a*b*x*arcsin(c*x)/(sqrt(-c^2*d*e*x^2 + d*e)*d*e) + a^2*x/(sqrt(-c^2*d*e*x^2 + d*e)*d*e) - a*b*sqrt(1/(d*e))*log(x^2 - 1/c^2)/(c*d*e)`

Giac [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate((a+b*arcsin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm m="giac")`

output `integrate((b*arcsin(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx$$

input `int((a + b*asin(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)`

output `int((a + b*asin(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{-2\sqrt{cx + 1} \sqrt{-cx + 1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{cx+1} \sqrt{-cx+1} c^2 x^2 - \sqrt{cx+1} \sqrt{-cx+1}} dx \right) ab - \sqrt{cx + 1}}{\sqrt{e} \sqrt{d} \sqrt{cx + 1} \sqrt{-cx + 1}}$$

input `int((a+b*asin(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)`

output `(- 2*sqrt(c*x + 1)*sqrt(- c*x + 1)*int(asin(c*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*a*b - sqrt(c*x + 1)*sqrt(- c*x + 1)*int(asin(c*x)**2/(sqrt(c*x + 1)*sqrt(- c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt(- c*x + 1)),x)*b**2 + a**2*x)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt(- c*x + 1)*d*e)`

3.500 $\int \frac{(a+b \arcsin(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}} dx$

Optimal result	4243
Mathematica [A] (verified)	4244
Rubi [A] (verified)	4245
Maple [A] (warning: unable to verify)	4249
Fricas [F]	4250
Sympy [F]	4251
Maxima [F(-2)]	4251
Giac [F]	4251
Mupad [F(-1)]	4252
Reduce [F]	4252

Optimal result

Integrand size = 35, antiderivative size = 548

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}} dx &= \frac{(a+b \arcsin(cx))^2}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{4ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \arctan(e^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &- \frac{2\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2 \operatorname{arctanh}(e^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{2ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, -e^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &- \frac{2ib^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{2ib^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, ie^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &- \frac{2ib\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \operatorname{PolyLog}(2, e^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &- \frac{2b^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(3, -e^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{2b^2\sqrt{1-c^2x^2} \operatorname{PolyLog}(3, e^{i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \end{aligned}$$

output

```
(a+b*arcsin(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctan(I*c*x+(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2*arctanh(I*c*x+(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,-I*c*x-(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2)))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*polylog(2,I*c*x+(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-I*c*x-(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,I*c*x+(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.60

$$\int \frac{(a + b \arcsin(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \text{Too large to display}$$

input

```
Integrate[(a + b*ArcSin[c*x])^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output

```
(-((a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(-1 + c^2*x^2)) + a^2*Sqrt[d]*Sqr
t[e]*Log[c*x] - a^2*Sqrt[d]*Sqrt[e]*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d + c*d
*x]*Sqrt[e - c*e*x]] + (2*a*b*d*e*(ArcSin[c*x] + Sqrt[1 - c^2*x^2]*ArcSin[
c*x]*Log[1 - E^(I*ArcSin[c*x])]) - Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 + E^
(I*ArcSin[c*x])]) + Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c
*x]/2]]) - Sqrt[1 - c^2*x^2]*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]]) +
I*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcSin[c*x])] - I*Sqrt[1 - c^2*x^2]*
PolyLog[2, E^(I*ArcSin[c*x])])/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]) + (b^2*d
*e*(I*Pi*Sqrt[1 - c^2*x^2]*ArcSin[c*x] + ArcSin[c*x]^2 + Sqrt[1 - c^2*x^2]
*ArcSin[c*x]^2*Log[1 - E^(I*ArcSin[c*x])]) - Pi*Sqrt[1 - c^2*x^2]*Log[1 - I
*E^(I*ArcSin[c*x])]) - 2*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - I*E^(I*ArcSi
n[c*x])] - Pi*Sqrt[1 - c^2*x^2]*Log[1 + I*E^(I*ArcSin[c*x])] + 2*Sqrt[1 -
c^2*x^2]*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])] - Sqrt[1 - c^2*x^2]*ArcS
in[c*x]^2*Log[1 + E^(I*ArcSin[c*x])] + Pi*Sqrt[1 - c^2*x^2]*Log[-Cos[(Pi +
2*ArcSin[c*x])/4]] + Pi*Sqrt[1 - c^2*x^2]*Log[Sin[(Pi + 2*ArcSin[c*x])/4]
] + (2*I)*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*PolyLog[2, -E^(I*ArcSin[c*x])] - (
2*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] + (2*I)*Sqrt[1 -
c^2*x^2]*PolyLog[2, I*E^(I*ArcSin[c*x])] - (2*I)*Sqrt[1 - c^2*x^2]*ArcSin
[c*x]*PolyLog[2, E^(I*ArcSin[c*x])] - 2*Sqrt[1 - c^2*x^2]*PolyLog[3, -E^(I
*ArcSin[c*x])] + 2*Sqrt[1 - c^2*x^2]*PolyLog[3, E^(I*ArcSin[c*x])])]/(S...
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {5238, 5208, 5164, 3042, 4669, 2715, 2838, 5218, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x(cdx + d)^{3/2}(e - cex)^{3/2}} dx$$

$$\downarrow \text{5238}$$

$$\frac{\sqrt{1 - c^2x^2} \int \frac{(a + b \arcsin(cx))^2}{x(1 - c^2x^2)^{3/2}} dx}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow \text{5208}$$

$$\frac{\sqrt{1-c^2x^2} \left(-2bc \int \frac{a+b \arcsin(cx)}{1-c^2x^2} dx + \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx + \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 5164

$$\frac{\sqrt{1-c^2x^2} \left(-2b \int \frac{a+b \arcsin(cx)}{\sqrt{1-c^2x^2}} d \arcsin(cx) + \int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx + \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 3042

$$\frac{\sqrt{1-c^2x^2} \left(\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx - 2b \int (a+b \arcsin(cx)) \csc(\arcsin(cx) + \frac{\pi}{2}) d \arcsin(cx) + \frac{(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 4669

$$\frac{\sqrt{1-c^2x^2} \left(-2b(-b \int \log(1 - ie^{i \arcsin(cx)}) d \arcsin(cx) + b \int \log(1 + ie^{i \arcsin(cx)}) d \arcsin(cx) - 2i \arctan(e^{i \arcsin(cx)}) \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 2715

$$\frac{\sqrt{1-c^2x^2} \left(-2b(ib \int e^{-i \arcsin(cx)} \log(1 - ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} - ib \int e^{-i \arcsin(cx)} \log(1 + ie^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 2838

$$\frac{\sqrt{1-c^2x^2} \left(\int \frac{(a+b \arcsin(cx))^2}{x\sqrt{1-c^2x^2}} dx - 2b(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 5218

$$\frac{\sqrt{1-c^2x^2} \left(\int \frac{(a+b \arcsin(cx))^2}{cx} d \arcsin(cx) - 2b(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 3042

$$\frac{\sqrt{1-c^2x^2} \left(\int (a+b \arcsin(cx))^2 \csc(\arcsin(cx)) d \arcsin(cx) - 2b(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - \right)}{d\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 4671

$$\frac{\sqrt{1-c^2x^2} \left(-2b \int (a+b \arcsin(cx)) \log(1-e^{i \arcsin(cx)}) d \arcsin(cx) + 2b \int (a+b \arcsin(cx)) \log(1+e^{i \arcsin(cx)}) \right)}{}$$

↓ 3011

$$\frac{\sqrt{1-c^2x^2} \left(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - ib \int \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) d \arcsin(cx)) - 2b(i \right)}{}$$

↓ 2720

$$\frac{\sqrt{1-c^2x^2} \left(2b(i \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) (a+b \arcsin(cx)) - b \int e^{-i \arcsin(cx)} \operatorname{PolyLog}(2, -e^{i \arcsin(cx)}) de^{i \arcsin(cx)} \right)}{}$$

↓ 7143

$$\frac{\sqrt{1-c^2x^2} \left(-2b(-2i \arctan(e^{i \arcsin(cx)}) (a+b \arcsin(cx)) + ib \operatorname{PolyLog}(2, -ie^{i \arcsin(cx)}) - ib \operatorname{PolyLog}(2, ie^{i \arcsin(cx)}) \right)}{}$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output

```
(Sqrt[1 - c^2*x^2]*((a + b*ArcSin[c*x])^2/Sqrt[1 - c^2*x^2] - 2*(a + b*ArcSin[c*x])^2*ArcTanh[E^(I*ArcSin[c*x])] - 2*b*((-2*I)*(a + b*ArcSin[c*x])*ArcTan[E^(I*ArcSin[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcSin[c*x])] - I*b*PolyLog[2, I*E^(I*ArcSin[c*x])]) + 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, -E^(I*ArcSin[c*x])] - b*PolyLog[3, -E^(I*ArcSin[c*x])]) - 2*b*(I*(a + b*ArcSin[c*x])*PolyLog[2, E^(I*ArcSin[c*x])] - b*PolyLog[3, E^(I*ArcSin[c*x])])))/(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F]) Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720 `Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x] Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x)) *(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

rule 2838 `Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

rule 3011 `Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := Simp[(-f + g*x)^m*(PolyLog[2, (-e)*(F^(c*(a + b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F])) Int[(f + g*x)^(m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4669 `Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

rule 4671 `Int[csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f) Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

rule 5164 `Int[((a_) + ArcSin[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[1/(c*d) Subst[Int[(a + b*x)^n*Sec[x], x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]`

rule 5208

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)
)*(x_)^(2)^(p_), x_Symbol] := Simp[(-f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSin[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
  Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]  Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5218

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] := Simp[(1/c^(m + 1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*
x^2]]  Subst[Int[(a + b*x)^n*Sin[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_.)
*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] := Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])  Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (warning: unable to verify)

Time = 4.00 (sec) , antiderivative size = 1083, normalized size of antiderivative = 1.98

method	result	size
default	Expression too large to display	1083
parts	Expression too large to display	1083

input

```
int((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETUR
NVERBOSE)
```


Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(d(cx + 1))^{\frac{3}{2}}(-e(cx - 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))**2/x/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/(x*(d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}} dx$$

input `integrate((a+b*arcsin(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="giac")`

output `integrate((b*arcsin(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*x), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx$$

input `int((a + b*asin(c*x))^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)`

output `int((a + b*asin(c*x))^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)`

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{-2\sqrt{cx+1}\sqrt{-cx+1} \left(\int \frac{\operatorname{asin}(cx)}{\sqrt{cx+1}\sqrt{-cx+1}c^2x^3 - \sqrt{cx+1}\sqrt{-cx+1}x} dx \right) ab - \sqrt{cx}}$$

input `int((a+b*asin(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2), x)`

output `(- 2*sqrt(c*x + 1)*sqrt(- c*x + 1)*int(asin(c*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)*c**2*x**3 - sqrt(c*x + 1)*sqrt(- c*x + 1)*x), x)*a*b - sqrt(c*x + 1)*sqrt(- c*x + 1)*int(asin(c*x)**2/(sqrt(c*x + 1)*sqrt(- c*x + 1)*c**2*x**3 - sqrt(c*x + 1)*sqrt(- c*x + 1)*x), x)*b**2 - sqrt(c*x + 1)*sqrt(- c*x + 1)*log(- sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) - 1)*a**2 + sqrt(c*x + 1)*sqrt(- c*x + 1)*log(- sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) + 1)*a**2 - sqrt(c*x + 1)*sqrt(- c*x + 1)*log(sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) - 1)*a**2 + sqrt(c*x + 1)*sqrt(- c*x + 1)*log(sqrt(2) + tan(asin(sqrt(- c*x + 1)/sqrt(2))/2) + 1)*a**2 + a**2)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt(- c*x + 1)*d*e)`

3.501
$$\int \frac{(a+b \arcsin(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}} dx$$

Optimal result	4253
Mathematica [A] (warning: unable to verify)	4254
Rubi [A] (verified)	4255
Maple [A] (verified)	4260
Fricas [F]	4261
Sympy [F]	4261
Maxima [F(-2)]	4261
Giac [F]	4262
Mupad [F(-1)]	4262
Reduce [F]	4263

Optimal result

Integrand size = 35, antiderivative size = 396

$$\begin{aligned} \int \frac{(a+b \arcsin(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}} dx = & -\frac{(a+b \arcsin(cx))^2}{dex\sqrt{d+cdx}\sqrt{e-cex}} \\ & + \frac{2c^2x(a+b \arcsin(cx))^2}{de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2ic\sqrt{1-c^2x^2}(a+b \arcsin(cx))^2}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ & - \frac{4bc\sqrt{1-c^2x^2}(a+b \arcsin(cx))\operatorname{arctanh}(e^{2i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ & + \frac{4bc\sqrt{1-c^2x^2}(a+b \arcsin(cx)) \log(1+e^{2i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ & - \frac{ib^2c\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \\ & - \frac{ib^2c\sqrt{1-c^2x^2} \operatorname{PolyLog}(2, e^{2i \arcsin(cx)})}{de\sqrt{d+cdx}\sqrt{e-cex}} \end{aligned}$$

output

```

-(a+b*arcsin(c*x))^2/d/e/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*c^2*x*(a+b*arcsin(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-4*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*arctanh((I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arcsin(c*x))*ln(1+(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)

```

Mathematica [A] (warning: unable to verify)

Time = 3.79 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.42

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{c \csc\left(\frac{1}{2} \arcsin(cx)\right) \sec\left(\frac{1}{2} \arcsin(cx)\right) (-2a^2 + 4a^2c^2x^2 - 4ab \arcsin(cx))}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}}$$

input

```

Integrate[(a + b*ArcSin[c*x])^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x]

```

output

```

(c*Csc[ArcSin[c*x]/2]*Sec[ArcSin[c*x]/2]*(-2*a^2 + 4*a^2*c^2*x^2 - 4*a*b*ArcSin[c*x]*Cos[2*ArcSin[c*x]] - 2*b^2*ArcSin[c*x]^2*Cos[2*ArcSin[c*x]] + (2*I)*b^2*Pi*ArcSin[c*x]*Sin[2*ArcSin[c*x]] - (2*I)*b^2*ArcSin[c*x]^2*SIn[2*ArcSin[c*x]] + 4*b^2*Pi*Log[1 + E^((-I)*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] + b^2*Pi*Log[1 - I*E^(I*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] + 2*b^2*ArcSin[c*x]*Log[1 - I*E^(I*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] - b^2*Pi*Log[1 + I*E^(I*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] + 2*b^2*ArcSin[c*x]*Log[1 + I*E^(I*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] + 2*b^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] + 2*a*b*Log[c*x]*Sin[2*ArcSin[c*x]] - 4*b^2*Pi*Log[Cos[ArcSin[c*x]/2]]*Sin[2*ArcSin[c*x]] + b^2*Pi*Log[-Cos[(Pi + 2*ArcSin[c*x])/4]]*Sin[2*ArcSin[c*x]] + 2*a*b*Log[Cos[ArcSin[c*x]/2] - Sin[ArcSin[c*x]/2]]*Sin[2*ArcSin[c*x]] + 2*a*b*Log[Cos[ArcSin[c*x]/2] + Sin[ArcSin[c*x]/2]]*Sin[2*ArcSin[c*x]] - b^2*Pi*Log[Sin[(Pi + 2*ArcSin[c*x])/4]]*Sin[2*ArcSin[c*x]] - (2*I)*b^2*PolyLog[2, (-I)*E^(I*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] - (2*I)*b^2*PolyLog[2, I*E^(I*ArcSin[c*x])]*Sin[2*ArcSin[c*x]] - I*b^2*PolyLog[2, E^((2*I)*ArcSin[c*x])]*Sin[2*ArcSin[c*x]]))/(4*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])

```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.62, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5238, 5204, 5160, 5180, 3042, 4202, 2620, 2715, 2838, 5184, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(cdx + d)^{3/2}(e - cex)^{3/2}} dx$$

$$\downarrow \text{5238}$$

$$\frac{\sqrt{1 - c^2x^2} \int \frac{(a + b \arcsin(cx))^2}{x^2(1 - c^2x^2)^{3/2}} dx}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow \text{5204}$$

$$\frac{\sqrt{1 - c^2x^2} \left(2c^2 \int \frac{(a + b \arcsin(cx))^2}{(1 - c^2x^2)^{3/2}} dx + 2bc \int \frac{a + b \arcsin(cx)}{x(1 - c^2x^2)} dx - \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2x^2}} \right)}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow \text{5160}$$

$$\frac{\sqrt{1 - c^2x^2} \left(2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}} - 2bc \int \frac{x(a + b \arcsin(cx))}{1 - c^2x^2} dx \right) + 2bc \int \frac{a + b \arcsin(cx)}{x(1 - c^2x^2)} dx - \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2x^2}} \right)}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow \text{5180}$$

$$\frac{\sqrt{1 - c^2x^2} \left(2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}} - \frac{2b \int \frac{cx(a + b \arcsin(cx))}{\sqrt{1 - c^2x^2}} d \arcsin(cx)}{c} \right) + 2bc \int \frac{a + b \arcsin(cx)}{x(1 - c^2x^2)} dx - \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2x^2}} \right)}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow \text{3042}$$

$$\frac{\sqrt{1 - c^2x^2} \left(2bc \int \frac{a + b \arcsin(cx)}{x(1 - c^2x^2)} dx + 2c^2 \left(\frac{x(a + b \arcsin(cx))^2}{\sqrt{1 - c^2x^2}} - \frac{2b \int (a + b \arcsin(cx)) \tan(\arcsin(cx)) d \arcsin(cx)}{c} \right) - \frac{(a + b \arcsin(cx))^2}{x\sqrt{1 - c^2x^2}} \right)}{de\sqrt{cdx + d}\sqrt{e - cex}}$$

$$\downarrow \text{4202}$$

$$\frac{\sqrt{1-c^2x^2} \left(2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \int \frac{e^{2i \arcsin(cx)}(a+b \arcsin(cx))}{1+e^{2i \arcsin(cx)}} d \arcsin(cx) \right)}{c} \right) + 2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 2620

$$\frac{\sqrt{1-c^2x^2} \left(2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{2} ib \int \log(1+e^{2i \arcsin(cx)}) d \arcsin(cx) - \frac{1}{2} \right) \right)}{c} \right) \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 2715

$$\frac{\sqrt{1-c^2x^2} \left(2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(\frac{1}{4} b \int e^{-2i \arcsin(cx)} \log(1+e^{2i \arcsin(cx)}) de \right) \right)}{c} \right) \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 2838

$$\frac{\sqrt{1-c^2x^2} \left(2bc \int \frac{a+b \arcsin(cx)}{x(1-c^2x^2)} dx + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{c} \right) \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 5184

$$\frac{\sqrt{1-c^2x^2} \left(2bc \int \frac{a+b \arcsin(cx)}{cx\sqrt{1-c^2x^2}} d \arcsin(cx) + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{c} \right) \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 4919

$$\frac{\sqrt{1-c^2x^2} \left(4bc \int (a+b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \left(-\frac{1}{2} i \log(1+e^{2i \arcsin(cx)}) (a+b \arcsin(cx)) \right) \right)}{c} \right) \right)}{de\sqrt{cdx} + d\sqrt{e-cex}}$$

↓ 3042

$$\sqrt{1 - c^2 x^2} \left(4bc \int (a + b \arcsin(cx)) \csc(2 \arcsin(cx)) d \arcsin(cx) + 2c^2 \left(\frac{x(a+b \arcsin(cx))^2}{\sqrt{1-c^2 x^2}} - \frac{2b \left(\frac{i(a+b \arcsin(cx))^2}{2b} - 2i \right)}{d\sqrt{cdx} + d\sqrt{e - ce x}} \right) \right)$$

↓ 4671

$$\sqrt{1 - c^2 x^2} \left(4bc \left(-\frac{1}{2} b \int \log(1 - e^{2i \arcsin(cx)}) d \arcsin(cx) + \frac{1}{2} b \int \log(1 + e^{2i \arcsin(cx)}) d \arcsin(cx) - (\operatorname{arctanh}(e^{2i \arcsin(cx)})) \right) \right)$$

↓ 2715

$$\sqrt{1 - c^2 x^2} \left(4bc \left(\frac{1}{4} ib \int e^{-2i \arcsin(cx)} \log(1 - e^{2i \arcsin(cx)}) d e^{2i \arcsin(cx)} - \frac{1}{4} ib \int e^{-2i \arcsin(cx)} \log(1 + e^{2i \arcsin(cx)}) d e^{2i \arcsin(cx)} \right) \right)$$

↓ 2838

$$\sqrt{1 - c^2 x^2} \left(4bc \left(-(\operatorname{arctanh}(e^{2i \arcsin(cx)})) (a + b \arcsin(cx)) \right) + \frac{1}{4} ib \operatorname{PolyLog}(2, -e^{2i \arcsin(cx)}) - \frac{1}{4} ib \operatorname{PolyLog}(2, e^{2i \arcsin(cx)}) \right)$$

input

```
Int[(a + b*ArcSin[c*x])^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output

```
(Sqrt[1 - c^2*x^2]*(-(a + b*ArcSin[c*x])^2/(x*Sqrt[1 - c^2*x^2])) + 2*c^2*
*((x*(a + b*ArcSin[c*x])^2)/Sqrt[1 - c^2*x^2] - (2*b*((I/2)*(a + b*ArcSin
[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcSin[c*x])*Log[1 + E^((2*I)*ArcSin
[c*x]]) - (b*PolyLog[2, -E^((2*I)*ArcSin[c*x]))/4]))/c) + 4*b*c*(-((a + b*
ArcSin[c*x])*ArcTanh[E^((2*I)*ArcSin[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)
)*ArcSin[c*x]]) - (I/4)*b*PolyLog[2, E^((2*I)*ArcSin[c*x])]))/(d*e*Sqrt[d
+ c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 2620 $\text{Int}[\frac{((F_)^{((g_.) * (e_.) + (f_.) * (x_.))})^{(n_.)} * ((c_.) + (d_.) * (x_.))^{(m_.)}}{((a_.) + (b_.) * (F_)^{((g_.) * (e_.) + (f_.) * (x_.))})^{(n_.)}}, x_Symbol] \rightarrow \text{Simp}[\frac{((c + d*x)^m / (b*f*g*n * \text{Log}[F])) * \text{Log}[1 + b * ((F^{(g*(e + f*x)))^n / a})], x] - \text{Simp}[d * (m / (b*f*g*n * \text{Log}[F])) \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + b * ((F^{(g*(e + f*x)))^n / a})], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

rule 2715 $\text{Int}[\text{Log}[(a_) + (b_.) * (F_)^{((e_.) * ((c_.) + (d_.) * (x_.))})^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[1 / (d * e * n * \text{Log}[F]) \text{Subst}[\text{Int}[\text{Log}[a + b*x] / x, x], x, (F^{(e * (c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

rule 2838 $\text{Int}[\text{Log}[(c_.) * ((d_) + (e_.) * (x_)^{(n_.)})] / (x_), x_Symbol] \rightarrow \text{Simp}[-\text{PolyLog}[2, (-c) * e * x^n] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c * d, 1]$

rule 3042 $\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4202 $\text{Int}[\frac{((c_.) + (d_.) * (x_.))^{(m_.)} * \tan[(e_.) + (f_.) * (x_.)]}{(c + d*x)^{(m+1)} / (d * (m+1))}, x_Symbol] \rightarrow \text{Simp}[I * ((c + d*x)^{(m+1)} / (d * (m+1))), x] - \text{Simp}[2 * I \text{Int}[(c + d*x)^m * (E^{(2 * I * (e + f*x))} / (1 + E^{(2 * I * (e + f*x))}))], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 4671 $\text{Int}[\text{csc}[(e_.) + (f_.) * (x_.)] * ((c_.) + (d_.) * (x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-2 * (c + d*x)^m * (\text{ArcTanh}[E^{(I * (e + f*x))}] / f), x] + (-\text{Simp}[d * (m / f) \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 - E^{(I * (e + f*x))}], x], x] + \text{Simp}[d * (m / f) \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + E^{(I * (e + f*x))}], x], x]) /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0]$

rule 4919 $\text{Int}[\text{Csc}[(a_.) + (b_.) * (x_.)]^{(n_.)} * ((c_.) + (d_.) * (x_.))^{(m_.)} * \text{Sec}[(a_.) + (b_.) * (x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[2^n \text{Int}[(c + d*x)^m * \text{Csc}[2*a + 2*b*x]^n], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IntegerQ}[n] \&\& \text{RationalQ}[m]$

rule 5160

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] := Simp[x*((a + b*ArcSin[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]] Int[x*((a + b*ArcSin[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
+ e, 0] && GtQ[n, 0]
```

rule 5180

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[-e^(-1) Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5184

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[1/d Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, ArcSi
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5204

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_)^(m_.))*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] := Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
) Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n, x], x] - Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p] Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5238

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_)^(m_.))*((d_) + (e_.
)*(x_)^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol] := Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q]) Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcSin[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```


Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.83

method	result
default	$\frac{a^2(-2c^2x^2+1)\sqrt{d(cx+1)}\sqrt{-e(cx-1)}}{d^2e^2(cx+1)(cx-1)x} + b^2 \left(-\frac{\arcsin(cx)^2(2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1)\sqrt{-e(cx-1)}\sqrt{d(cx+1)}}{(c^2x^2-1)xe^2d^2} + \frac{2i\sqrt{-c^2x^2+1}\sqrt{-e(cx-1)}\sqrt{d(cx+1)}}{e^2d^2} \right)$
parts	$\frac{a^2(-2c^2x^2+1)\sqrt{d(cx+1)}\sqrt{-e(cx-1)}}{d^2e^2(cx+1)(cx-1)x} + b^2 \left(-\frac{\arcsin(cx)^2(2i\sqrt{-c^2x^2+1}cx+2c^2x^2-1)\sqrt{-e(cx-1)}\sqrt{d(cx+1)}}{(c^2x^2-1)xe^2d^2} + \frac{2i\sqrt{-c^2x^2+1}\sqrt{-e(cx-1)}\sqrt{d(cx+1)}}{e^2d^2} \right)$

input `int((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETURNVERBOSE)`

output `a^2*(-2*c^2*x^2+1)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/(c*x+1)/(c*x-1)/x+b^2*(-arcsin(c*x)^2*(2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)/(c^2*x^2-1)/x/e^2/d^2+2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/(c^2*x^2-1)*(I*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+I*arcsin(c*x)*ln(1+I*(I*c*x+(-c^2*x^2+1)^(1/2))))+2*arcsin(c*x)^2+2*polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,-(I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))+polylog(2,-I*(I*c*x+(-c^2*x^2+1)^(1/2))))+polylog(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))))*c)+2*a*b*(2*I*arcsin(c*x)*x^3*c^3-ln((I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))^4-1)*x^3*c^3+2*arcsin(c*x)*(-c^2*x^2+1)^(1/2)*c^2*x^2-2*I*arcsin(c*x)*c*x+ln((I*c*x+(-c^2*x^2+1)^(1/2))^(1/2))^4-1)*x*c-arcsin(c*x)*(-c^2*x^2+1)^(1/2)*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^4*x^4-2*c^2*x^2+1)/x/e^2/d^2`

Fricas [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}x^2} dx$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="fricas")`

output `integral((b^2*arcsin(c*x)^2 + 2*a*b*arcsin(c*x) + a^2)*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*d^2*e^2*x^6 - 2*c^2*d^2*e^2*x^4 + d^2*e^2*x^2), x)`

Sympy [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(a + b \arcsin(cx))^2}{x^2(d(cx + 1))^{\frac{3}{2}}(-e(cx - 1))^{\frac{3}{2}}} dx$$

input `integrate((a+b*asin(c*x))**2/x**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)`

output `Integral((a + b*asin(c*x))**2/(x**2*(d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \text{Exception raised: ValueError}$$

input `integrate((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorithm="maxima")`

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(b \arcsin(cx) + a)^2}{(cdx + d)^{\frac{3}{2}}(-cex + e)^{\frac{3}{2}}x^2} dx$$

input

```
integrate((a+b*arcsin(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="giac")
```

output

```
integrate((b*arcsin(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*x^2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \int \frac{(a + b \operatorname{asin}(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx$$

input

```
int((a + b*asin(c*x))^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output

```
int((a + b*asin(c*x))^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a + b \arcsin(cx))^2}{x^2(d + cdx)^{3/2}(e - cex)^{3/2}} dx = \frac{-2\sqrt{cx+1}\sqrt{-cx+1} \left(\int \frac{\arcsin(cx)}{\sqrt{cx+1}\sqrt{-cx+1}c^2x^4 - \sqrt{cx+1}\sqrt{-cx+1}x^2} dx \right) abx - \sqrt{e}\sqrt{d}\sqrt{cx}}{\sqrt{e}\sqrt{d}\sqrt{cx}}$$

input `int((a+b*asin(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)`

output `(- 2*sqrt(c*x + 1)*sqrt(- c*x + 1)*int(asin(c*x)/(sqrt(c*x + 1)*sqrt(- c*x + 1)*c**2*x**4 - sqrt(c*x + 1)*sqrt(- c*x + 1)*x**2),x)*a*b*x - sqrt(c*x + 1)*sqrt(- c*x + 1)*int(asin(c*x)**2/(sqrt(c*x + 1)*sqrt(- c*x + 1)*c**2*x**4 - sqrt(c*x + 1)*sqrt(- c*x + 1)*x**2),x)*b**2*x + 2*a**2*c**2*x**2 - a**2)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt(- c*x + 1)*d*e*x)`

CHAPTER 4

APPENDIX

4.1	Listing of Grading functions	4264
4.2	Links to plain text integration problems used in this report for each CAS .	4282

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*                               Small rewrite of logic in main function to make it*)
(*                               match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCountOptimal},
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal."}
        ]
      ,(*ELSE*)
      finalresult={"C","Result contains complex when optimal does not."}
    ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal."}
    ]
  ]
  ,(*ELSE*)(*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order of result is higher than in optimal."}
  ]
  ,

```

```

        finalresult={"F","Contains unresolved integral."}
    ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],

```

```

    Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 5]],
    If [AppellFunctionQ [Head [expn]],
        Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 6]],
    If [Head [expn] === RootSum,
        Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 7]],
    If [Head [expn] === Integrate || Head [expn] === Int,
        Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 8]],
    9]]]]]]]]]]]]

```

```

ElementaryFunctionQ [func_] :=
  MemberQ [{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ [func_] :=
  MemberQ [{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ [func_] :=
  MemberQ [{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```

AppellFunctionQ [func_] :=
  MemberQ [{AppellF1}, func]

```


Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022  add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B","result has leaf size over 500,000. Avoiding possible recursion issue";
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result    := ExpnType(result);
      ExpnType_optimal   := ExpnType(optimal);

      if debug then
          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#     is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal

```

```

#   antiderivative
#   "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf

        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex
    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well
    fi;

```

```

        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of
                            convert(leaf_count_result,string)," $ vs. $2(",
                            convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;
end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function

```

```

# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+'') or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9

```

```

    end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.

```

```
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc;
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]
```

```

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')

```

```

    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""

```



```

else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType

#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arcsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:

```

```

    if m:
        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
    if m:
        print ("func ", func , " is special_function")
    else:
        print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']  #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False

```

```

if debug:
    print ("Enter is_atom, expn=",expn)

if not hasattr(expn, 'parent'):
    return False

#thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print ("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)

```

```

    return expnType(expn.operands()[0]) #expnType(expn.args[0])
elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
    if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print ("Enter grade_antiderivative, result=",result)
    print ("Enter grade_antiderivative, optimal=",optimal)
    print ("type(anti)=", type(result))
    print ("type(optimal)=", type(optimal))

```

```

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result - 2*leaf_count_optimal)
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_result - expnType_optimal)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file