Computer Algebra
Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.2-Inverse-cosine/271-5.2.2

Nasser M. Abbasi

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Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**227**]. This is test number [271].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (227)	0.00 (0)
Mathematica	98.68 (224)	1.32 (3)
Maple	95.15 (216)	4.85 (11)
Giac	70.93 (161)	29.07 (66)
Sympy	44.05 (100)	55.95 (127)
Reduce	39.21 (89)	60.79 (138)
Fricas	37.44 (85)	62.56 (142)
Maxima	33.04 (75)	66.96 (152)
Mupad	32.16 (73)	67.84 (154)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

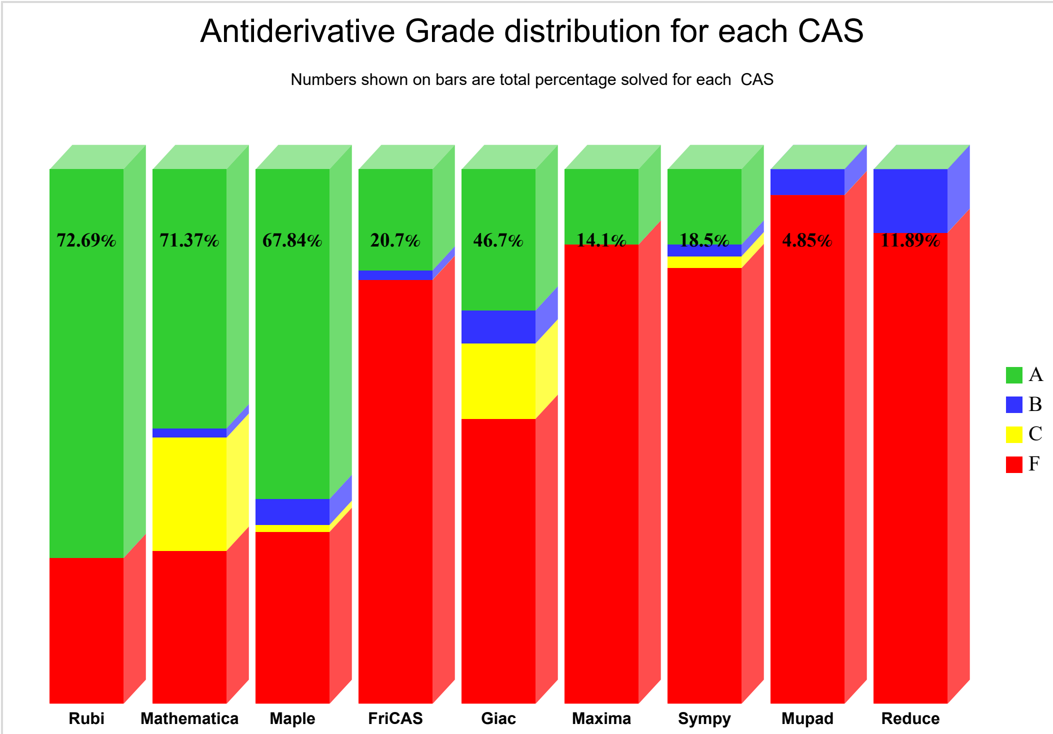
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

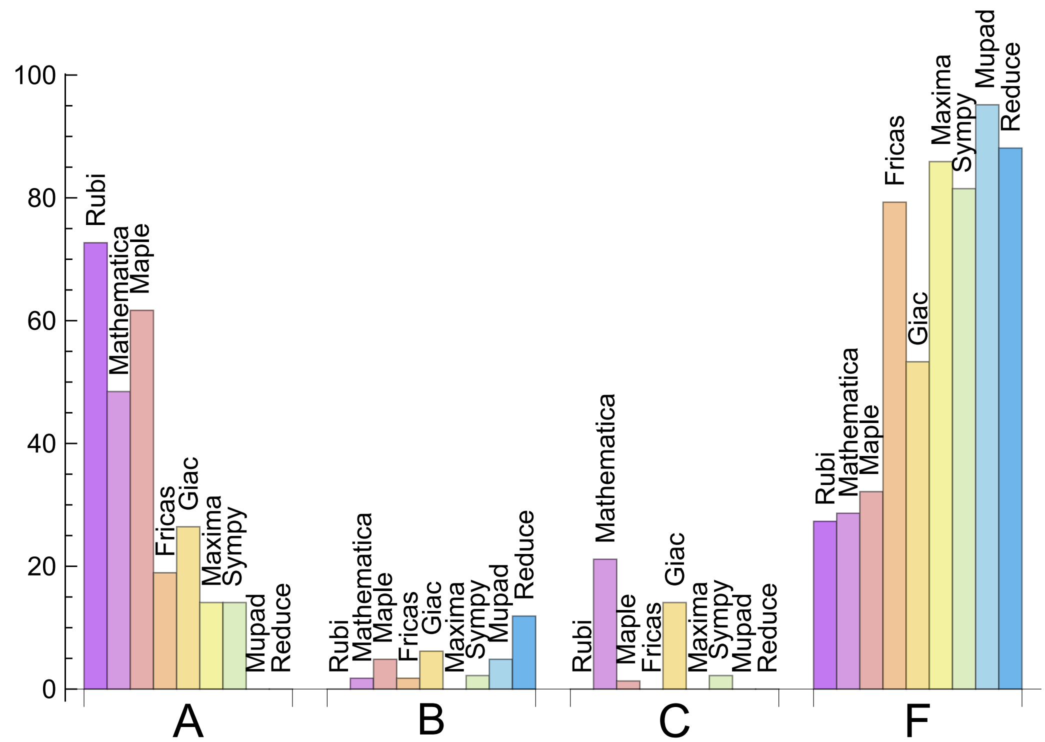
System	% A grade	% B grade	% C grade	% F grade
Rubi	72.687	0.000	0.000	27.313
Maple	61.674	4.846	1.322	32.159
Mathematica	48.458	1.762	21.145	28.634
Giac	26.432	6.167	14.097	53.304
Fricas	18.943	1.762	0.000	79.295
Maxima	14.097	0.000	0.000	85.903
Sympy	14.097	2.203	2.203	81.498
Mupad	0.000	4.846	0.000	95.154
Reduce	0.000	11.894	0.000	88.106

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	11	100.00	0.00	0.00
Giac	66	72.73	0.00	27.27
Fricas	142	44.37	0.00	55.63
Sympy	127	90.55	2.36	7.09
Maxima	152	60.53	0.00	39.47
Reduce	138	100.00	0.00	0.00
Mupad	154	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.13
Maple	0.22
Mupad	0.28
Giac	0.33
Rubi	0.53
Maxima	0.91
Mathematica	2.88
Reduce	4.72
Sympy	4.88

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	21.11	1.08	16.00	1.00
Reduce	42.24	1.86	30.00	1.22
Fricas	53.99	1.30	48.00	1.15
Sympy	55.34	1.09	17.00	1.00
Rubi	97.13	1.07	79.00	1.00
Maple	97.63	1.05	65.00	0.95
Mathematica	101.20	1.13	68.00	1.11
Maxima	109.77	5.56	69.00	1.00
Giac	183.73	1.84	57.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

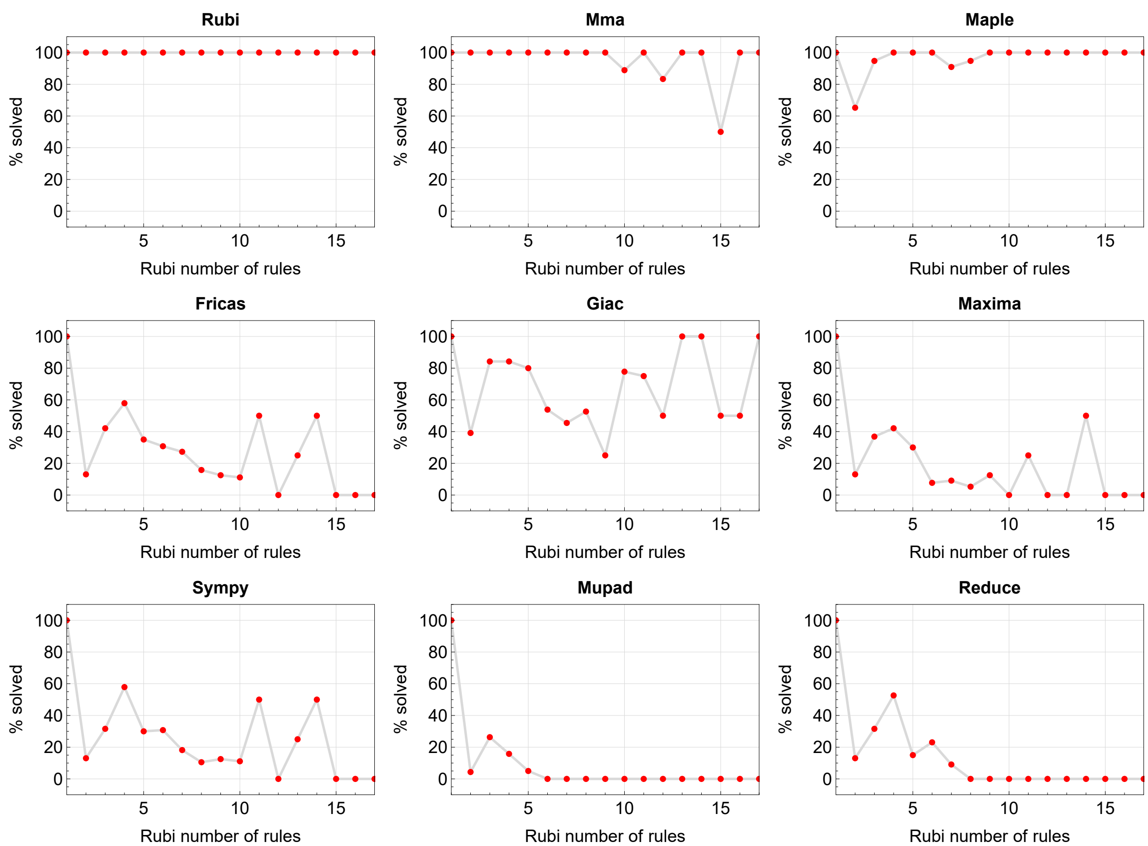


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

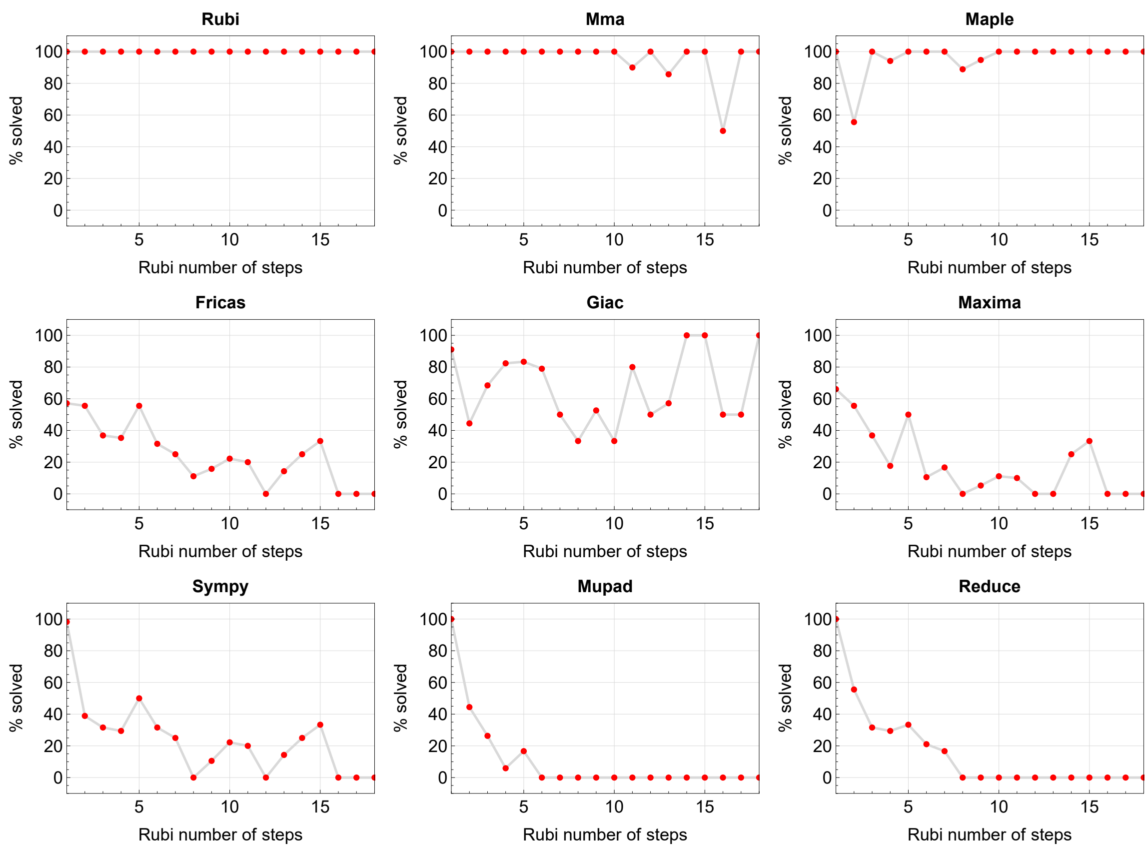


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

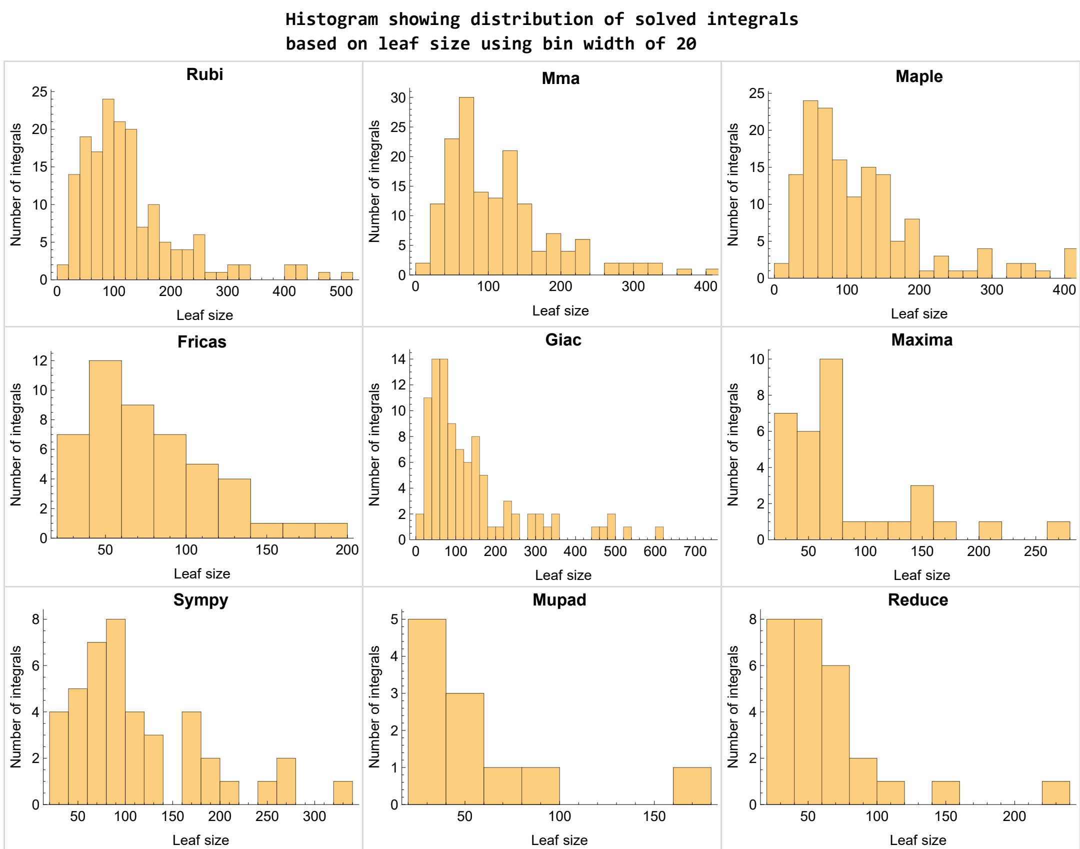


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

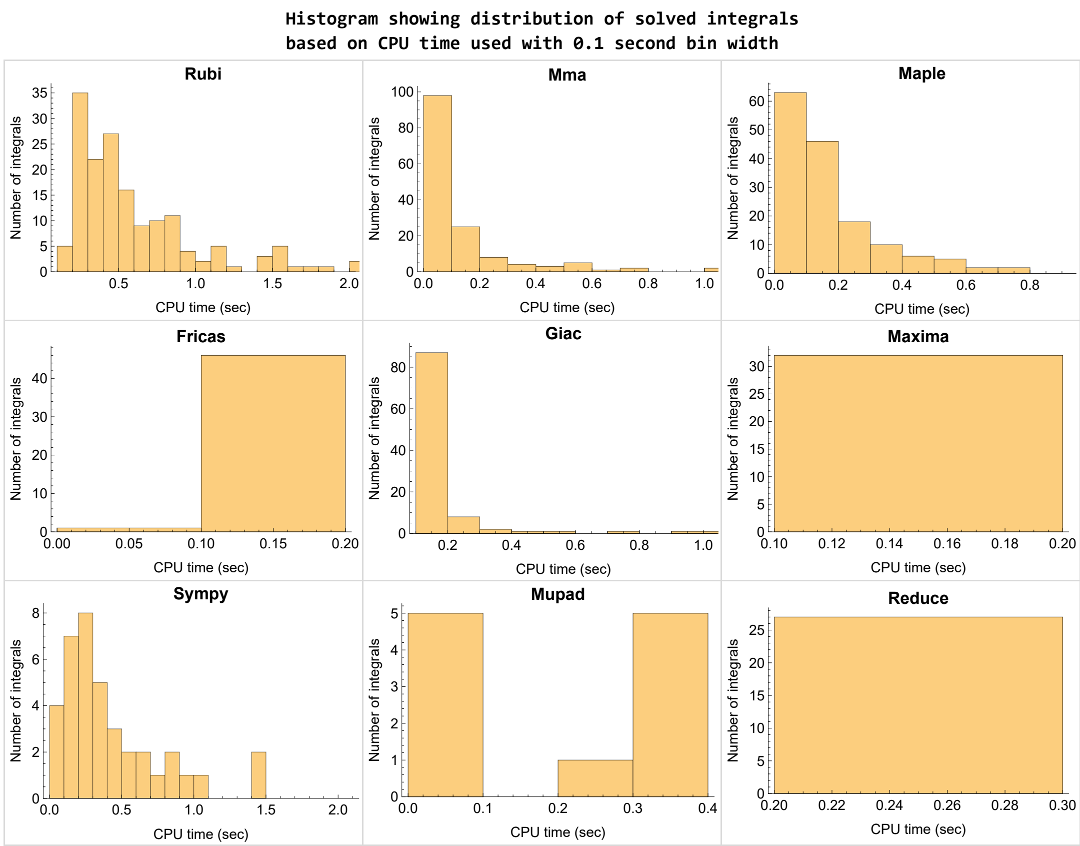


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

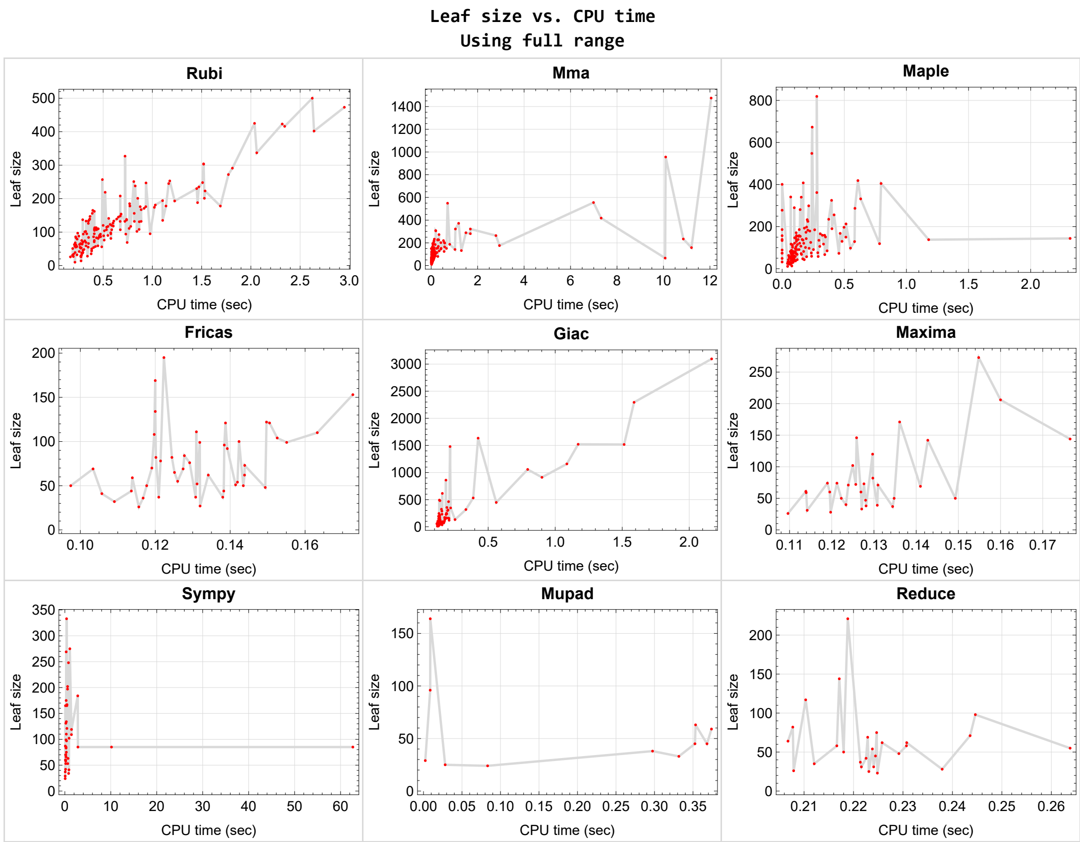


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {121}

Maple $\{150, 155\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

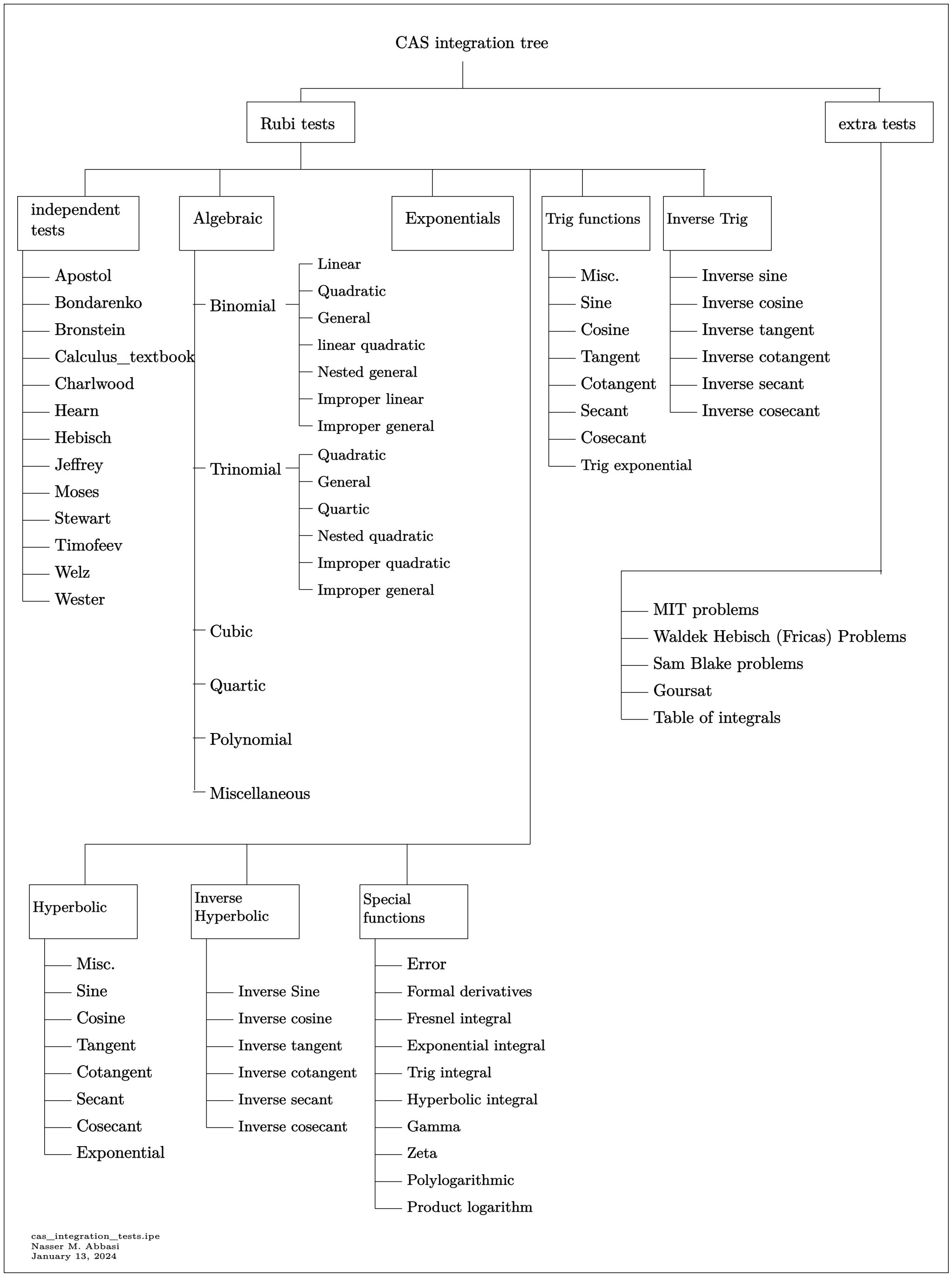
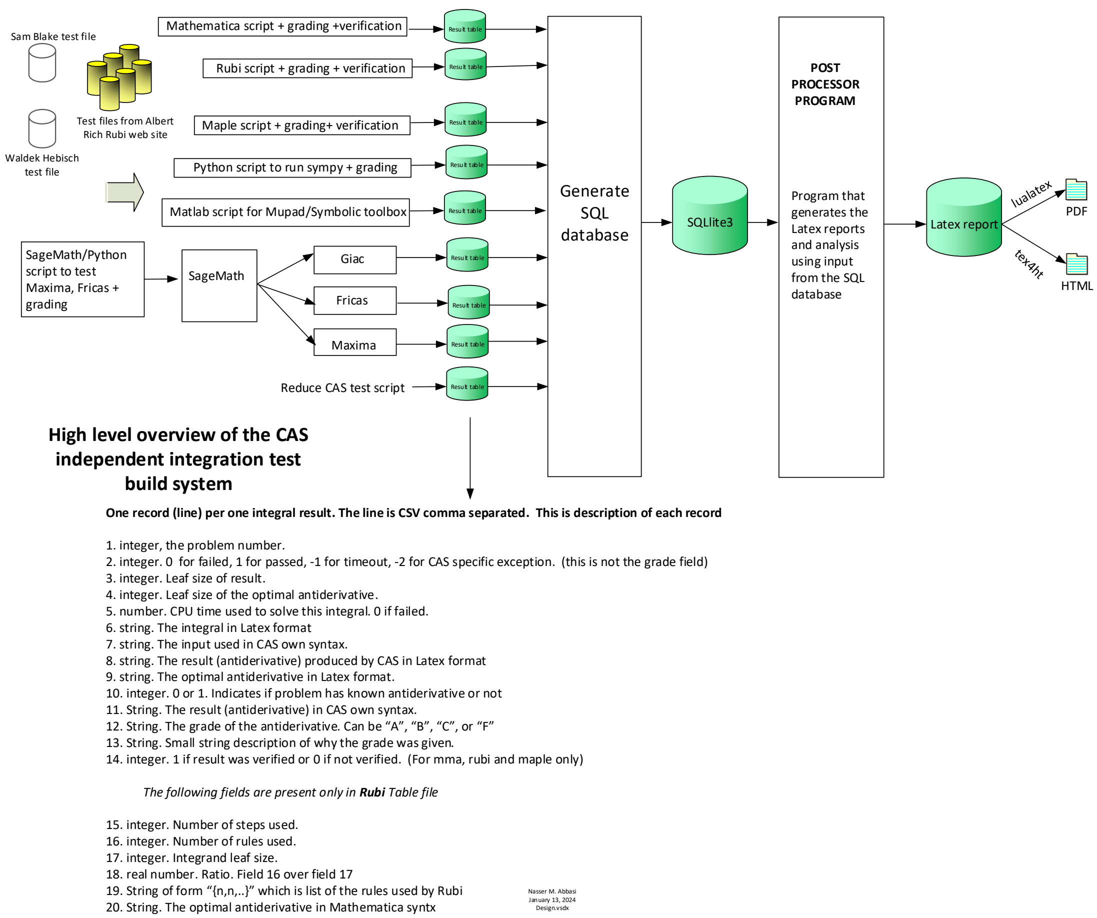


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 30
2.2 Detailed conclusion table per each integral for all CAS systems 35
2.3 Detailed conclusion table specific for Rubi results 92

2.1 List of integrals sorted by grade for each CAS

Rubi 30
Mma 31
Maple 31
Fricas 32
Maxima 32
Giac 33
Mupad 33
Sympy 34
Reduce 34

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 77, 83, 89, 95, 104, 110, 116, 122, 130, 131, 132, 133, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 163, 164, 165, 168, 169, 170, 174, 179, 184, 189, 210, 211, 212, 213, 214 }

B grade { 39, 41, 157, 209 }

C grade { 74, 75, 76, 78, 80, 81, 82, 84, 86, 87, 88, 90, 92, 93, 94, 96, 99, 100, 101, 102, 103, 105, 107, 108, 109, 111, 113, 114, 115, 117, 121, 173, 175, 178, 180, 183, 185, 188, 190, 193, 195, 198, 203, 204, 205, 206, 207, 208 }

F normal fail { 194, 199, 200 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 188, 189, 190, 193, 194, 195, 203, 204, 205, 206, 207, 208 }

B grade { 156, 157, 178, 179, 180, 183, 184, 185, 198, 199, 200 }

C grade { 130, 132, 133 }

F normal fail { 28, 39, 121, 122, 131, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 140, 141, 142, 143, 146, 148, 149, 150, 153, 154, 155, 203, 204, 205, 206, 207, 208 }

B grade { 7, 9, 145, 147 }

C grade { }

F normal fail { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 125, 126, 127, 128, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 140, 141, 142, 143, 145, 146, 147, 148, 150, 153, 155 }

B grade { }

C grade { }

F normal fail { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 121, 122, 144, 149, 151, 152, 154, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

Giac

A grade { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 140, 141, 142, 143, 148, 149, 150, 153, 155, 158, 159, 160 }

B grade { 8, 10, 19, 21, 145, 146, 147, 154, 163, 164, 165, 168, 169, 170 }

C grade { 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190 }

F normal fail { 6, 17, 18, 20, 27, 28, 29, 30, 38, 39, 40, 41, 99, 100, 101, 102, 103, 104, 105, 107, 109, 110, 111, 113, 115, 116, 117, 121, 122, 130, 131, 132, 133, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { 31, 108, 114, 144, 151, 152, 156, 157, 161, 166, 171, 196, 201, 209, 210, 211, 215, 216 }

Mupad

A grade { }

B grade { 4, 5, 7, 16, 26, 37, 142, 143, 145, 150, 155 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 140, 141, 144, 146, 147, 148, 149, 151, 152, 153, 154, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 140, 141, 142, 143, 145, 146, 147, 148, 203, 204, 205 }

B grade { 149, 150, 153, 154, 155 }

C grade { 7, 8, 9, 10, 11 }

F normal fail { 6, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 210, 211 }

F(-1) timedout fail { 86, 136, 209 }

F(-2) exception fail { 206, 207, 208, 212, 213, 214, 217, 218, 219 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 15, 16, 25, 26, 36, 37, 140, 141, 142, 143, 145, 146, 147, 149, 150, 154, 155 }

C grade { }

F normal fail { 6, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 144, 148, 151, 152, 153, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	51	72	71	50	75	67	69	0
N.S.	1	1.05	0.68	0.96	0.95	0.67	1.00	0.89	0.92	0.00
time (sec)	N/A	0.236	0.023	0.111	0.124	0.144	0.323	0.123	0.223	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	83	54	60	61	48	66	57	58	0
N.S.	1	1.20	0.78	0.87	0.88	0.70	0.96	0.83	0.84	0.00
time (sec)	N/A	0.224	0.022	0.078	0.114	0.149	0.246	0.130	0.231	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	42	52	50	41	53	47	50	0
N.S.	1	1.07	0.78	0.96	0.93	0.76	0.98	0.87	0.93	0.00
time (sec)	N/A	0.214	0.018	0.078	0.149	0.106	0.191	0.125	0.218	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	51	42	40	40	37	42	37	37	38
N.S.	1	1.13	0.93	0.89	0.89	0.82	0.93	0.82	0.82	0.84
time (sec)	N/A	0.191	0.009	0.072	0.123	0.121	0.153	0.127	0.221	0.297

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	26	26	24	26	25	24
N.S.	1	1.00	1.00	0.96	1.00	1.00	0.92	1.00	0.96	0.92
time (sec)	N/A	0.168	0.003	0.043	0.110	0.116	0.066	0.126	0.223	0.083

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	59	51	68	0	0	0	0	10	0
N.S.	1	1.16	1.00	1.33	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.326	0.011	0.156	0.000	0.000	0.000	0.000	0.234	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	26	38	82	34	48	23	25
N.S.	1	1.00	1.26	0.96	1.41	3.04	1.26	1.78	0.85	0.93
time (sec)	N/A	0.198	0.009	0.074	0.128	0.120	0.813	0.123	0.225	0.028

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	29	28	27	53	68	26	0
N.S.	1	1.00	0.91	0.85	0.82	0.79	1.56	2.00	0.76	0.00
time (sec)	N/A	0.200	0.011	0.079	0.120	0.132	0.624	0.137	0.208	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	57	67	50	60	110	109	70	42	0
N.S.	1	1.02	1.20	0.89	1.07	1.96	1.95	1.25	0.75	0.00
time (sec)	N/A	0.233	0.018	0.078	0.120	0.163	1.430	0.121	0.223	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	41	52	50	37	102	130	45	0
N.S.	1	1.09	0.71	0.90	0.86	0.64	1.76	2.24	0.78	0.00
time (sec)	N/A	0.208	0.017	0.083	0.122	0.138	0.915	0.133	0.224	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	72	73	82	122	184	101	62	0
N.S.	1	1.08	0.90	0.91	1.02	1.52	2.30	1.26	0.78	0.00
time (sec)	N/A	0.221	0.044	0.080	0.130	0.150	2.803	0.124	0.231	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	141	82	76	102	76	121	100	12	0
N.S.	1	1.18	0.68	0.63	0.85	0.63	1.01	0.83	0.10	0.00
time (sec)	N/A	0.556	0.038	0.287	0.125	0.129	0.438	0.132	0.223	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	115	74	91	0	70	97	87	12	0
N.S.	1	1.17	0.76	0.93	0.00	0.71	0.99	0.89	0.12	0.00
time (sec)	N/A	0.536	0.029	0.252	0.000	0.119	0.361	0.131	0.209	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	95	63	59	72	59	83	68	12	0
N.S.	1	1.16	0.77	0.72	0.88	0.72	1.01	0.83	0.15	0.00
time (sec)	N/A	0.410	0.033	0.237	0.126	0.114	0.256	0.131	0.226	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	66	57	43	0	51	58	55	55	0
N.S.	1	1.10	0.95	0.72	0.00	0.85	0.97	0.92	0.92	0.00
time (sec)	N/A	0.351	0.020	0.178	0.000	0.141	0.187	0.118	0.264	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	42	35	37	33	36	37	33	35	45
N.S.	1	1.20	1.00	1.06	0.94	1.03	1.06	0.94	1.00	1.29
time (sec)	N/A	0.257	0.012	0.108	0.127	0.117	0.082	0.125	0.212	0.368

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	90	73	101	0	0	0	0	12	0
N.S.	1	1.23	1.00	1.38	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.463	0.016	0.188	0.000	0.000	0.000	0.000	0.215	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	98	136	0	0	0	0	12	0
N.S.	1	1.01	1.32	1.84	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.411	0.095	0.168	0.000	0.000	0.000	0.000	0.211	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	46	43	47	39	44	0	82	12	0
N.S.	1	1.07	1.00	1.09	0.91	1.02	0.00	1.91	0.28	0.00
time (sec)	N/A	0.295	0.020	0.166	0.131	0.114	0.000	0.159	0.214	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	120	152	166	0	0	0	0	12	0
N.S.	1	0.97	1.23	1.34	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.587	0.428	0.307	0.000	0.000	0.000	0.000	0.223	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	69	82	74	62	0	171	12	0
N.S.	1	1.02	0.79	0.94	0.85	0.71	0.00	1.97	0.14	0.00
time (sec)	N/A	0.505	0.028	0.173	0.119	0.144	0.000	0.175	0.211	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	303	122	159	171	104	202	175	12	0
N.S.	1	1.51	0.61	0.79	0.85	0.52	1.00	0.87	0.06	0.00
time (sec)	N/A	1.520	0.042	0.324	0.136	0.153	0.588	0.139	0.225	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	245	115	151	0	96	167	141	12	0
N.S.	1	1.47	0.69	0.90	0.00	0.57	1.00	0.84	0.07	0.00
time (sec)	N/A	1.166	0.046	0.349	0.000	0.138	0.460	0.132	0.203	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	176	95	106	120	78	134	117	12	0
N.S.	1	1.29	0.70	0.78	0.88	0.57	0.99	0.86	0.09	0.00
time (sec)	N/A	0.876	0.035	0.303	0.130	0.121	0.317	0.137	0.229	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	117	85	58	0	69	99	83	82	0
N.S.	1	1.18	0.86	0.59	0.00	0.70	1.00	0.84	0.83	0.00
time (sec)	N/A	0.550	0.028	0.290	0.000	0.103	0.246	0.129	0.208	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	69	60	57	59	44	60	56	54	59
N.S.	1	1.15	1.00	0.95	0.98	0.73	1.00	0.93	0.90	0.98
time (sec)	N/A	0.326	0.013	0.197	0.114	0.138	0.127	0.127	0.224	0.374

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	123	101	135	0	0	0	0	12	0
N.S.	1	1.22	1.00	1.34	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.571	0.017	0.296	0.000	0.000	0.000	0.000	0.218	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	139	0	0	0	0	0	12	0
N.S.	1	1.04	1.14	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.600	0.078	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	107	92	117	0	0	0	0	12	0
N.S.	1	1.05	0.90	1.15	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.587	0.144	0.333	0.000	0.000	0.000	0.000	0.241	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	194	165	256	0	0	0	0	12	0
N.S.	1	1.01	0.86	1.33	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.103	0.579	0.418	0.000	0.000	0.000	0.000	0.214	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	181	151	190	0	0	0	0	12	0
N.S.	1	1.07	0.89	1.12	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.030	0.234	0.401	0.000	0.000	0.000	0.000	0.230	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	500	167	332	0	153	275	245	12	0
N.S.	1	1.77	0.59	1.18	0.00	0.54	0.98	0.87	0.04	0.00
time (sec)	N/A	2.623	0.057	0.632	0.000	0.173	1.082	0.133	0.230	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	416	150	197	206	134	248	212	12	0
N.S.	1	1.66	0.60	0.79	0.82	0.54	0.99	0.85	0.05	0.00
time (sec)	N/A	2.343	0.052	0.498	0.160	0.120	0.785	0.139	0.217	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	291	135	213	0	121	197	173	12	0
N.S.	1	1.47	0.68	1.08	0.00	0.61	0.99	0.87	0.06	0.00
time (sec)	N/A	1.812	0.046	0.514	0.000	0.139	0.577	0.138	0.222	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	230	114	130	146	99	165	140	12	0
N.S.	1	1.39	0.69	0.78	0.88	0.60	0.99	0.84	0.07	0.00
time (sec)	N/A	1.450	0.048	0.482	0.126	0.132	0.431	0.135	0.237	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	132	96	73	0	82	110	101	98	0
N.S.	1	1.18	0.86	0.65	0.00	0.73	0.98	0.90	0.88	0.00
time (sec)	N/A	0.886	0.028	0.457	0.000	0.124	0.311	0.131	0.245	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	85	69	67	74	55	70	65	64	63
N.S.	1	1.23	1.00	0.97	1.07	0.80	1.01	0.94	0.93	0.91
time (sec)	N/A	0.458	0.017	0.343	0.121	0.126	0.156	0.129	0.207	0.353

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	154	119	168	0	0	0	0	12	0
N.S.	1	1.29	1.00	1.41	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.682	0.017	0.467	0.000	0.000	0.000	0.000	0.211	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	185	549	0	0	0	0	0	12	0
N.S.	1	1.05	3.12	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.768	0.707	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	136	115	150	0	0	0	0	12	0
N.S.	1	1.12	0.95	1.24	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.738	0.251	0.513	0.000	0.000	0.000	0.000	0.207	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	1475	419	0	0	0	0	12	0
N.S.	1	1.00	4.85	1.38	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.520	12.051	0.610	0.000	0.000	0.000	0.000	0.221	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	48	40	40	0	0	0	47	12	0
N.S.	1	0.87	0.73	0.73	0.00	0.00	0.00	0.85	0.22	0.00
time (sec)	N/A	0.306	0.064	0.103	0.000	0.000	0.000	0.134	0.262	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	39	33	33	0	0	0	37	12	0
N.S.	1	0.91	0.77	0.77	0.00	0.00	0.00	0.86	0.28	0.00
time (sec)	N/A	0.295	0.056	0.087	0.000	0.000	0.000	0.126	0.230	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	37	31	31	0	0	0	35	12	0
N.S.	1	0.90	0.76	0.76	0.00	0.00	0.00	0.85	0.29	0.00
time (sec)	N/A	0.290	0.048	0.060	0.000	0.000	0.000	0.138	0.227	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	28	24	24	0	0	0	25	12	0
N.S.	1	0.97	0.83	0.83	0.00	0.00	0.00	0.86	0.41	0.00
time (sec)	N/A	0.272	0.044	0.057	0.000	0.000	0.000	0.130	0.224	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	20	22	0	0	0	23	12	0
N.S.	1	0.96	0.74	0.81	0.00	0.00	0.00	0.85	0.44	0.00
time (sec)	N/A	0.276	0.034	0.056	0.000	0.000	0.000	0.129	0.221	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	12	10	0
N.S.	1	1.00	1.00	0.93	0.00	0.00	0.00	0.86	0.71	0.00
time (sec)	N/A	0.276	0.015	0.078	0.000	0.000	0.000	0.129	0.206	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	0	0	10	8	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	1.00	0.80	0.00
time (sec)	N/A	0.214	0.018	0.044	0.000	0.000	0.000	0.121	0.219	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.186	0.150	0.068	0.189	0.093	0.269	0.145	0.245	0.299

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.173	0.657	0.096	0.214	0.096	0.304	0.171	0.218	0.268

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	75	86	105	0	0	0	72	12	0
N.S.	1	0.91	1.05	1.28	0.00	0.00	0.00	0.88	0.15	0.00
time (sec)	N/A	0.282	0.114	0.112	0.000	0.000	0.000	0.139	0.237	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	66	63	78	0	0	0	62	12	0
N.S.	1	0.94	0.90	1.11	0.00	0.00	0.00	0.89	0.17	0.00
time (sec)	N/A	0.271	0.116	0.085	0.000	0.000	0.000	0.137	0.238	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	64	61	81	0	0	0	60	12	0
N.S.	1	0.94	0.90	1.19	0.00	0.00	0.00	0.88	0.18	0.00
time (sec)	N/A	0.287	0.114	0.070	0.000	0.000	0.000	0.150	0.217	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	50	54	0	0	0	50	12	0
N.S.	1	0.98	0.89	0.96	0.00	0.00	0.00	0.89	0.21	0.00
time (sec)	N/A	0.265	0.102	0.057	0.000	0.000	0.000	0.139	0.215	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	50	57	0	0	0	48	12	0
N.S.	1	0.98	0.93	1.06	0.00	0.00	0.00	0.89	0.22	0.00
time (sec)	N/A	0.278	0.091	0.068	0.000	0.000	0.000	0.141	0.228	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	30	0	0	0	36	10	0
N.S.	1	1.00	0.97	0.79	0.00	0.00	0.00	0.95	0.26	0.00
time (sec)	N/A	0.282	0.065	0.088	0.000	0.000	0.000	0.129	0.231	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	0	0	0	33	8	0
N.S.	1	1.00	1.00	0.91	0.00	0.00	0.00	0.94	0.23	0.00
time (sec)	N/A	0.378	0.014	0.047	0.000	0.000	0.000	0.124	0.201	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	127	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	12.70	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.191	0.884	0.072	0.410	0.092	0.345	0.165	0.206	0.278

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	136	12	12	12	12	12
N.S.	1	1.00	1.20	1.00	13.60	1.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.196	12.683	0.123	0.521	0.094	0.365	0.221	0.202	0.278

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	132	103	121	0	0	0	86	12	0
N.S.	1	1.35	1.05	1.23	0.00	0.00	0.00	0.88	0.12	0.00
time (sec)	N/A	0.817	0.081	0.100	0.000	0.000	0.000	0.146	0.234	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	108	70	82	0	0	0	75	12	0
N.S.	1	1.30	0.84	0.99	0.00	0.00	0.00	0.90	0.14	0.00
time (sec)	N/A	0.865	0.100	0.094	0.000	0.000	0.000	0.143	0.217	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	102	65	82	0	0	0	72	12	0
N.S.	1	1.24	0.79	1.00	0.00	0.00	0.00	0.88	0.15	0.00
time (sec)	N/A	0.843	0.085	0.066	0.000	0.000	0.000	0.147	0.218	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	69	63	43	0	0	0	57	10	0
N.S.	1	1.10	1.00	0.68	0.00	0.00	0.00	0.90	0.16	0.00
time (sec)	N/A	0.744	0.026	0.082	0.000	0.000	0.000	0.143	0.217	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	54	47	43	0	0	0	43	8	0
N.S.	1	1.06	0.92	0.84	0.00	0.00	0.00	0.84	0.16	0.00
time (sec)	N/A	0.403	0.011	0.050	0.000	0.000	0.000	0.121	0.233	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	124	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	12.40	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.171	0.473	0.069	1.723	0.110	0.396	0.182	0.244	0.303

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	143	12	12	12	12	12
N.S.	1	1.00	1.20	1.00	14.30	1.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.174	6.498	0.125	1.780	0.120	0.485	0.211	0.214	0.298

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	208	159	171	0	0	0	138	12	0
N.S.	1	1.32	1.01	1.08	0.00	0.00	0.00	0.87	0.08	0.00
time (sec)	N/A	0.674	0.123	0.113	0.000	0.000	0.000	0.138	0.219	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	178	107	114	0	0	0	125	12	0
N.S.	1	1.24	0.75	0.80	0.00	0.00	0.00	0.87	0.08	0.00
time (sec)	N/A	0.772	0.176	0.082	0.000	0.000	0.000	0.139	0.219	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	174	112	117	0	0	0	121	12	0
N.S.	1	1.23	0.79	0.83	0.00	0.00	0.00	0.86	0.09	0.00
time (sec)	N/A	1.020	0.108	0.072	0.000	0.000	0.000	0.142	0.211	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	107	86	60	0	0	0	83	10	0
N.S.	1	1.10	0.89	0.62	0.00	0.00	0.00	0.86	0.10	0.00
time (sec)	N/A	0.675	0.075	0.083	0.000	0.000	0.000	0.134	0.201	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	90	71	63	0	0	0	66	8	0
N.S.	1	1.15	0.91	0.81	0.00	0.00	0.00	0.85	0.10	0.00
time (sec)	N/A	0.548	0.037	0.052	0.000	0.000	0.000	0.125	0.220	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	200	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	20.00	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.166	2.917	0.076	5.216	0.118	0.512	0.179	0.222	0.276

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	229	12	12	12	12	12
N.S.	1	1.00	1.20	1.00	22.90	1.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.178	14.938	0.135	6.483	0.114	0.677	0.289	0.208	0.276

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	120	194	143	0	0	0	247	11	0
N.S.	1	0.99	1.60	1.18	0.00	0.00	0.00	2.04	0.09	0.00
time (sec)	N/A	0.500	0.078	0.194	0.000	0.000	0.000	0.189	0.213	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	94	131	91	0	0	0	153	11	0
N.S.	1	0.99	1.38	0.96	0.00	0.00	0.00	1.61	0.12	0.00
time (sec)	N/A	0.473	0.053	0.136	0.000	0.000	0.000	0.185	0.234	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	128	96	0	0	0	165	11	0
N.S.	1	1.02	1.49	1.12	0.00	0.00	0.00	1.92	0.13	0.00
time (sec)	N/A	0.469	0.062	0.106	0.000	0.000	0.000	0.187	0.224	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	49	42	0	0	0	71	9	0
N.S.	1	0.97	0.83	0.71	0.00	0.00	0.00	1.20	0.15	0.00
time (sec)	N/A	0.422	0.026	0.074	0.000	0.000	0.000	0.169	0.229	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	69	49	0	0	0	83	7	0
N.S.	1	1.00	1.57	1.11	0.00	0.00	0.00	1.89	0.16	0.00
time (sec)	N/A	0.384	0.019	0.070	0.000	0.000	0.000	0.152	0.222	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	11	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	0.92	1.00
time (sec)	N/A	0.181	0.172	0.126	0.000	0.000	0.338	0.335	0.213	0.293

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	337	185	193	0	0	0	355	15	0
N.S.	1	1.20	0.66	0.68	0.00	0.00	0.00	1.26	0.05	0.00
time (sec)	N/A	2.060	0.086	0.179	0.000	0.000	0.000	0.192	0.253	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	201	128	121	0	0	0	225	15	0
N.S.	1	1.28	0.82	0.77	0.00	0.00	0.00	1.43	0.10	0.00
time (sec)	N/A	1.528	0.051	0.141	0.000	0.000	0.000	0.185	0.240	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	193	125	130	0	0	0	237	15	0
N.S.	1	1.31	0.85	0.88	0.00	0.00	0.00	1.61	0.10	0.00
time (sec)	N/A	1.226	0.061	0.115	0.000	0.000	0.000	0.191	0.243	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	64	64	0	0	0	107	13	0
N.S.	1	1.07	0.72	0.72	0.00	0.00	0.00	1.20	0.15	0.00
time (sec)	N/A	0.977	0.045	0.081	0.000	0.000	0.000	0.175	0.212	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	66	72	0	0	0	119	46	0
N.S.	1	1.01	0.88	0.96	0.00	0.00	0.00	1.59	0.61	0.00
time (sec)	N/A	0.511	0.019	0.073	0.000	0.000	0.000	0.209	0.224	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	15	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	1.25	1.00
time (sec)	N/A	0.186	0.173	0.122	0.000	0.000	1.332	0.395	0.208	0.277

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	402	194	233	0	0	0	463	17	0
N.S.	1	1.35	0.65	0.78	0.00	0.00	0.00	1.55	0.06	0.00
time (sec)	N/A	2.641	0.074	0.202	0.000	0.000	0.000	0.205	0.277	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	272	131	154	0	0	0	297	17	0
N.S.	1	1.33	0.64	0.75	0.00	0.00	0.00	1.45	0.08	0.00
time (sec)	N/A	1.772	0.051	0.146	0.000	0.000	0.000	0.197	0.254	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	235	128	156	0	0	0	309	17	0
N.S.	1	1.32	0.72	0.88	0.00	0.00	0.00	1.74	0.10	0.00
time (sec)	N/A	1.472	0.060	0.118	0.000	0.000	0.000	0.201	0.226	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	131	73	79	0	0	0	143	15	0
N.S.	1	1.10	0.61	0.66	0.00	0.00	0.00	1.20	0.13	0.00
time (sec)	N/A	0.867	0.062	0.084	0.000	0.000	0.000	0.178	0.224	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	69	88	0	0	0	155	14	0
N.S.	1	1.08	0.78	1.00	0.00	0.00	0.00	1.76	0.16	0.00
time (sec)	N/A	0.578	0.018	0.076	0.000	0.000	0.000	0.208	0.224	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	1.42	1.00
time (sec)	N/A	0.172	0.174	0.121	0.000	0.000	16.853	0.413	0.210	0.285

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	102	192	72	0	0	0	139	227	0
N.S.	1	0.96	1.81	0.68	0.00	0.00	0.00	1.31	2.14	0.00
time (sec)	N/A	0.323	0.069	0.153	0.000	0.000	0.000	0.177	0.305	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	130	43	0	0	0	81	169	0
N.S.	1	0.98	2.00	0.66	0.00	0.00	0.00	1.25	2.60	0.00
time (sec)	N/A	0.284	0.049	0.122	0.000	0.000	0.000	0.170	0.286	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	126	50	0	0	0	93	98	0
N.S.	1	0.99	1.77	0.70	0.00	0.00	0.00	1.31	1.38	0.00
time (sec)	N/A	0.296	0.059	0.094	0.000	0.000	0.000	0.159	0.277	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	35	75	0
N.S.	1	1.00	1.00	0.75	0.00	0.00	0.00	1.25	2.68	0.00
time (sec)	N/A	0.341	0.017	0.066	0.000	0.000	0.000	0.156	0.236	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	68	26	0	0	0	47	59	0
N.S.	1	1.00	2.19	0.84	0.00	0.00	0.00	1.52	1.90	0.00
time (sec)	N/A	0.281	0.017	0.056	0.000	0.000	0.000	0.141	0.225	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.184	0.168	0.123	0.000	0.000	0.352	0.235	0.212	0.289

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	14	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.17	1.00	1.42	1.00
time (sec)	N/A	0.179	1.761	0.144	0.000	0.000	0.473	0.250	0.232	0.278

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	165	306	183	0	0	0	0	406	0
N.S.	1	0.96	1.79	1.07	0.00	0.00	0.00	0.00	2.37	0.00
time (sec)	N/A	0.395	0.201	0.207	0.000	0.000	0.000	0.000	0.351	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	126	226	121	0	0	0	0	195	0
N.S.	1	0.99	1.78	0.95	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	0.350	0.325	0.174	0.000	0.000	0.000	0.000	0.322	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	133	233	139	0	0	0	0	288	0
N.S.	1	0.98	1.71	1.02	0.00	0.00	0.00	0.00	2.12	0.00
time (sec)	N/A	0.350	0.138	0.129	0.000	0.000	0.000	0.000	0.303	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	154	81	0	0	0	0	204	0
N.S.	1	1.04	1.69	0.89	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.334	0.283	0.106	0.000	0.000	0.000	0.000	0.276	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	159	94	0	0	0	0	129	0
N.S.	1	1.04	1.64	0.97	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.314	0.078	0.109	0.000	0.000	0.000	0.000	0.269	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	42	0	0	0	0	93	0
N.S.	1	1.00	0.80	0.76	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.348	0.034	0.079	0.000	0.000	0.000	0.000	0.248	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	66	0	0	0	0	78	0
N.S.	1	1.00	1.46	1.12	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.447	0.025	0.076	0.000	0.000	0.000	0.000	0.253	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.193	0.196	0.130	0.000	0.000	0.836	0.266	0.214	0.282

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	251	322	173	0	0	0	0	310	0
N.S.	1	1.07	1.37	0.74	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.811	1.044	0.192	0.000	0.000	0.000	0.000	0.293	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	167	203	107	0	0	0	0	214	0
N.S.	1	1.33	1.61	0.85	0.00	0.00	0.00	0.00	1.70	0.00
time (sec)	N/A	0.886	0.567	0.151	0.000	0.000	0.000	0.000	0.266	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	176	220	115	0	0	0	0	141	0
N.S.	1	1.41	1.76	0.92	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.933	0.531	0.123	0.000	0.000	0.000	0.000	0.276	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	94	61	56	0	0	0	0	99	0
N.S.	1	1.06	0.69	0.63	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.730	0.058	0.089	0.000	0.000	0.000	0.000	0.251	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	122	83	0	0	0	0	84	0
N.S.	1	1.07	1.61	1.09	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.450	0.150	0.080	0.000	0.000	0.000	0.000	0.249	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.164	0.198	0.131	0.000	0.000	6.009	0.282	0.213	0.276

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	327	418	225	0	0	0	0	310	0
N.S.	1	1.24	1.58	0.85	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.722	7.317	0.207	0.000	0.000	0.000	0.000	0.291	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	247	264	139	0	0	0	0	214	0
N.S.	1	1.30	1.39	0.73	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.935	2.788	0.161	0.000	0.000	0.000	0.000	0.276	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	253	281	154	0	0	0	0	141	0
N.S.	1	1.32	1.47	0.81	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	1.177	1.688	0.133	0.000	0.000	0.000	0.000	0.263	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	132	75	73	0	0	0	0	100	0
N.S.	1	1.11	0.63	0.61	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.721	0.067	0.089	0.000	0.000	0.000	0.000	0.251	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	118	151	110	0	0	0	0	84	0
N.S.	1	1.12	1.44	1.05	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.604	0.661	0.083	0.000	0.000	0.000	0.000	0.263	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.174	0.198	0.135	0.000	0.000	58.530	0.296	0.230	0.275

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	115	14	12	14	16	14
N.S.	1	1.00	1.17	1.00	9.58	1.17	1.00	1.17	1.33	1.17
time (sec)	N/A	0.357	0.525	1.342	0.639	0.112	5.136	0.600	0.219	0.286

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	115	14	12	14	16	14
N.S.	1	1.00	1.17	1.00	9.58	1.17	1.00	1.17	1.33	1.17
time (sec)	N/A	0.340	0.509	1.118	0.738	0.135	2.690	0.581	0.214	0.273

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	147	132	0	0	0	0	0	16	0
N.S.	1	0.98	0.88	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.367	1.301	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	54	0	0	0	0	0	14	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.217	0.032	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	16	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.33	1.17
time (sec)	N/A	0.186	0.308	0.840	0.207	0.108	0.354	0.251	0.233	0.280

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	156	14	12	14	16	14
N.S.	1	1.00	1.17	1.00	13.00	1.17	1.00	1.17	1.33	1.17
time (sec)	N/A	0.196	0.312	1.275	0.882	0.112	0.732	0.261	0.212	0.298

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	19	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	1.36	1.00
time (sec)	N/A	0.182	0.567	0.118	0.000	0.000	54.404	1.291	0.235	0.274

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	15	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	1.07	1.00
time (sec)	N/A	0.179	0.632	0.110	0.000	0.000	1.115	0.852	0.229	0.273

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	145	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	10.36	1.00
time (sec)	N/A	0.183	0.680	0.113	0.000	0.000	0.498	0.718	0.257	0.279

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	189	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	13.50	1.00
time (sec)	N/A	0.189	0.645	0.113	0.000	0.000	3.723	0.632	0.258	0.286

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	0	14	12	14	16	14
N.S.	1	1.00	1.17	1.00	0.00	1.17	1.00	1.17	1.33	1.17
time (sec)	N/A	0.181	0.504	1.344	0.000	0.135	3.578	0.677	0.219	0.289

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	162	130	287	0	0	0	0	12	0
N.S.	1	0.98	0.79	1.74	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.411	0.071	0.586	0.000	0.000	0.000	0.000	0.233	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	156	152	0	0	0	0	0	12	0
N.S.	1	0.96	0.93	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.390	0.123	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	74	138	0	0	0	0	10	0
N.S.	1	1.04	0.89	1.66	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.403	0.036	0.115	0.000	0.000	0.000	0.000	0.218	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	74	70	148	0	0	0	0	8	0
N.S.	1	0.99	0.93	1.97	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.314	0.025	0.109	0.000	0.000	0.000	0.000	0.229	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	0	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	0.00	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.199	0.222	0.115	0.000	0.170	0.347	0.175	0.223	0.281

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	0	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	0.00	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.182	0.577	0.092	0.000	0.162	0.514	0.180	0.227	0.272

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	16	0	14	16	14
N.S.	1	1.00	1.14	0.86	0.00	1.14	0.00	1.00	1.14	1.00
time (sec)	N/A	0.223	1.688	0.225	0.000	0.150	0.000	0.441	0.245	0.281

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	14	14	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.187	2.774	0.176	0.000	0.147	3.434	0.439	0.257	0.282

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	20	14	14	20	14
N.S.	1	1.00	1.14	0.86	0.00	1.43	1.00	1.00	1.43	1.00
time (sec)	N/A	0.206	1.108	0.164	0.000	0.138	1.511	0.319	0.223	0.275

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	20	14	14	68	14
N.S.	1	1.00	1.14	0.86	0.00	1.43	1.00	1.00	4.86	1.00
time (sec)	N/A	0.208	1.082	0.259	0.000	0.163	11.511	0.372	0.235	0.282

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	68	68	71	62	85	67	71	0
N.S.	1	1.16	0.89	0.89	0.93	0.82	1.12	0.88	0.93	0.00
time (sec)	N/A	0.254	0.037	0.121	0.131	0.134	0.247	0.129	0.244	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	55	60	60	54	70	56	62	0
N.S.	1	1.05	0.92	1.00	1.00	0.90	1.17	0.93	1.03	0.00
time (sec)	N/A	0.260	0.035	0.097	0.127	0.142	0.200	0.123	0.226	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	56	56	48	50	50	60	46	58	45
N.S.	1	1.10	1.10	0.94	0.98	0.98	1.18	0.90	1.14	0.88
time (sec)	N/A	0.230	0.028	0.094	0.135	0.097	0.153	0.132	0.217	0.353

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	31	32	29	31	31	29
N.S.	1	1.00	1.00	1.03	1.00	1.03	0.94	1.00	1.00	0.94
time (sec)	N/A	0.189	0.005	0.000	0.114	0.109	0.066	0.128	0.222	0.002

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	71	58	75	0	0	0	0	17	0
N.S.	1	1.13	0.92	1.19	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.409	0.018	0.235	0.000	0.000	0.000	0.000	0.240	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	37	47	92	41	347	28	33
N.S.	1	1.00	1.34	1.16	1.47	2.88	1.28	10.84	0.88	1.03
time (sec)	N/A	0.234	0.016	0.096	0.128	0.139	0.883	0.220	0.238	0.332

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	46	37	37	63	492	31	0
N.S.	1	1.00	1.13	1.18	0.95	0.95	1.62	12.62	0.79	0.00
time (sec)	N/A	0.215	0.021	0.097	0.134	0.131	0.688	0.136	0.224	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	79	61	69	121	119	1634	48	0
N.S.	1	1.00	1.27	0.98	1.11	1.95	1.92	26.35	0.77	0.00
time (sec)	N/A	0.224	0.025	0.095	0.141	0.151	1.444	0.425	0.229	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	121	125	142	111	175	143	87	0
N.S.	1	1.08	1.19	1.23	1.39	1.09	1.72	1.40	0.85	0.00
time (sec)	N/A	0.451	0.081	0.230	0.143	0.131	0.279	0.140	0.223	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	83	104	97	0	99	131	119	117	0
N.S.	1	1.09	1.37	1.28	0.00	1.30	1.72	1.57	1.54	0.00
time (sec)	N/A	0.432	0.121	0.217	0.000	0.155	0.211	0.135	0.210	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	76	74	73	65	87	75	75	96
N.S.	1	1.11	1.62	1.57	1.55	1.38	1.85	1.60	1.60	2.04
time (sec)	N/A	0.271	0.025	0.000	0.128	0.125	0.098	0.130	0.225	0.009

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	107	128	185	0	0	0	0	37	0
N.S.	1	1.16	1.39	2.01	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.534	0.077	0.262	0.000	0.000	0.000	0.000	0.226	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	134	179	0	0	0	0	49	0
N.S.	1	0.97	1.51	2.01	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.473	0.140	0.223	0.000	0.000	0.000	0.000	0.231	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	201	218	235	273	195	333	289	113	0
N.S.	1	1.18	1.28	1.38	1.61	1.15	1.96	1.70	0.66	0.00
time (sec)	N/A	0.851	0.121	0.375	0.155	0.122	0.368	0.155	0.211	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	139	185	159	0	169	269	231	221	0
N.S.	1	1.11	1.48	1.27	0.00	1.35	2.15	1.85	1.77	0.00
time (sec)	N/A	0.638	0.174	0.342	0.000	0.120	0.283	0.156	0.219	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	128	134	144	108	165	150	144	164
N.S.	1	1.06	1.62	1.70	1.82	1.37	2.09	1.90	1.82	2.08
time (sec)	N/A	0.352	0.044	0.000	0.176	0.120	0.132	0.131	0.217	0.009

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	145	204	325	0	0	0	0	57	0
N.S.	1	1.14	1.61	2.56	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.655	0.106	0.398	0.000	0.000	0.000	0.000	0.245	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	148	308	405	0	0	0	0	72	0
N.S.	1	0.98	2.04	2.68	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.674	0.194	0.796	0.000	0.000	0.000	0.000	0.213	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	105	91	102	0	0	0	172	16	0
N.S.	1	0.87	0.75	0.84	0.00	0.00	0.00	1.42	0.13	0.00
time (sec)	N/A	0.412	0.118	0.084	0.000	0.000	0.000	0.137	0.227	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	56	56	58	0	0	0	86	14	0
N.S.	1	0.89	0.89	0.92	0.00	0.00	0.00	1.37	0.22	0.00
time (sec)	N/A	0.486	0.047	0.106	0.000	0.000	0.000	0.144	0.224	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	50	46	49	0	0	0	50	12	0
N.S.	1	0.93	0.85	0.91	0.00	0.00	0.00	0.93	0.22	0.00
time (sec)	N/A	0.384	0.039	0.000	0.000	0.000	0.000	0.130	0.230	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	0	15	16
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	0.00	1.07	1.14
time (sec)	N/A	0.190	0.212	0.101	0.208	0.114	0.618	0.000	0.225	0.303

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	19	14	16	19	16
N.S.	1	1.00	1.14	1.00	1.14	1.36	1.00	1.14	1.36	1.14
time (sec)	N/A	0.190	2.528	0.241	0.222	0.134	0.547	0.763	0.220	0.309

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	139	124	147	0	0	0	615	30	0
N.S.	1	0.90	0.80	0.95	0.00	0.00	0.00	3.97	0.19	0.00
time (sec)	N/A	0.364	0.558	0.101	0.000	0.000	0.000	0.158	0.230	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	87	80	78	0	0	0	323	28	0
N.S.	1	0.96	0.88	0.86	0.00	0.00	0.00	3.55	0.31	0.00
time (sec)	N/A	0.474	0.289	0.107	0.000	0.000	0.000	0.151	0.218	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	81	72	74	0	0	0	193	26	0
N.S.	1	0.94	0.84	0.86	0.00	0.00	0.00	2.24	0.30	0.00
time (sec)	N/A	0.579	0.138	0.000	0.000	0.000	0.000	0.130	0.254	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	166	30	14	0	30	16
N.S.	1	1.00	1.14	1.00	11.86	2.14	1.00	0.00	2.14	1.14
time (sec)	N/A	0.187	6.288	0.105	0.518	0.132	1.011	0.000	0.211	0.309

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	181	36	15	16	36	16
N.S.	1	1.00	1.14	1.00	12.93	2.57	1.07	1.14	2.57	1.14
time (sec)	N/A	0.189	49.752	0.397	0.611	0.093	0.945	0.985	0.224	0.317

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	248	169	290	0	0	0	1479	44	0
N.S.	1	1.26	0.86	1.47	0.00	0.00	0.00	7.51	0.22	0.00
time (sec)	N/A	1.510	0.308	0.098	0.000	0.000	0.000	0.216	0.232	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	135	107	157	0	0	0	860	42	0
N.S.	1	1.04	0.82	1.21	0.00	0.00	0.00	6.62	0.32	0.00
time (sec)	N/A	1.104	0.205	0.116	0.000	0.000	0.000	0.185	0.218	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	139	0	0	0	481	40	0
N.S.	1	1.00	0.80	1.25	0.00	0.00	0.00	4.33	0.36	0.00
time (sec)	N/A	0.764	0.119	0.000	0.000	0.000	0.000	0.140	0.217	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	251	45	14	0	45	16
N.S.	1	1.00	1.14	1.00	17.93	3.21	1.00	0.00	3.21	1.14
time (sec)	N/A	0.195	2.200	0.169	3.411	0.135	1.638	0.000	0.225	0.324

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	284	53	15	16	53	16
N.S.	1	1.00	1.14	1.00	20.29	3.79	1.07	1.14	3.79	1.14
time (sec)	N/A	0.209	21.195	0.374	3.447	0.112	1.503	2.164	0.227	0.328

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	238	229	362	0	0	0	1057	15	0
N.S.	1	0.98	0.95	1.50	0.00	0.00	0.00	4.37	0.06	0.00
time (sec)	N/A	0.826	0.292	0.279	0.000	0.000	0.000	0.795	0.247	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	132	117	186	0	0	0	448	13	0
N.S.	1	0.96	0.85	1.36	0.00	0.00	0.00	3.27	0.09	0.00
time (sec)	N/A	0.673	0.193	0.138	0.000	0.000	0.000	0.560	0.222	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	122	186	0	0	0	531	11	0
N.S.	1	0.97	1.01	1.54	0.00	0.00	0.00	4.39	0.09	0.00
time (sec)	N/A	0.780	0.056	0.000	0.000	0.000	0.000	0.389	0.227	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	15	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	0.94	1.00
time (sec)	N/A	0.216	1.262	0.212	0.463	0.000	0.386	0.813	0.220	0.307

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	15	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	0.94	1.00
time (sec)	N/A	0.200	5.667	0.253	0.466	0.000	0.346	0.806	0.257	0.290

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	423	555	548	0	0	0	2295	39	0
N.S.	1	1.35	1.77	1.75	0.00	0.00	0.00	7.33	0.12	0.00
time (sec)	N/A	2.318	6.988	0.239	0.000	0.000	0.000	1.589	0.263	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	178	145	281	0	0	0	911	35	0
N.S.	1	1.03	0.84	1.63	0.00	0.00	0.00	5.30	0.20	0.00
time (sec)	N/A	1.690	0.420	0.145	0.000	0.000	0.000	0.901	0.274	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	156	289	278	0	0	0	1159	32	0
N.S.	1	0.98	1.82	1.75	0.00	0.00	0.00	7.29	0.20	0.00
time (sec)	N/A	0.816	1.496	0.000	0.000	0.000	0.000	1.087	0.279	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	39	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	2.44	1.00
time (sec)	N/A	0.212	0.551	0.214	0.555	0.000	11.475	0.944	0.251	0.307

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.204	5.473	0.258	0.549	0.000	2.638	1.044	200.031	0.292

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	473	956	819	0	0	0	3096	68	0
N.S.	1	1.32	2.67	2.29	0.00	0.00	0.00	8.65	0.19	0.00
time (sec)	N/A	2.949	10.094	0.279	0.000	0.000	0.000	2.167	0.345	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	223	187	408	0	0	0	1521	62	0
N.S.	1	1.03	0.87	1.89	0.00	0.00	0.00	7.04	0.29	0.00
time (sec)	N/A	1.535	0.798	0.170	0.000	0.000	0.000	1.171	0.358	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	178	372	401	0	0	0	1517	58	0
N.S.	1	0.99	2.08	2.24	0.00	0.00	0.00	8.47	0.32	0.00
time (sec)	N/A	1.138	1.183	0.000	0.000	0.000	0.000	1.514	0.308	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	68	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	4.25	1.00
time (sec)	N/A	0.214	0.605	0.211	0.649	0.000	29.826	1.029	0.294	0.286

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00	1.00
time (sec)	N/A	0.226	5.508	0.253	0.670	0.000	22.213	1.043	200.023	0.299

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	219	225	198	0	0	0	317	25	0
N.S.	1	0.98	1.01	0.89	0.00	0.00	0.00	1.42	0.11	0.00
time (sec)	N/A	0.521	0.277	0.197	0.000	0.000	0.000	0.334	0.251	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	98	85	91	0	0	0	132	23	0
N.S.	1	0.99	0.86	0.92	0.00	0.00	0.00	1.33	0.23	0.00
time (sec)	N/A	0.577	0.184	0.120	0.000	0.000	0.000	0.253	0.255	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	104	118	89	0	0	0	159	22	0
N.S.	1	1.02	1.16	0.87	0.00	0.00	0.00	1.56	0.22	0.00
time (sec)	N/A	0.513	0.054	0.000	0.000	0.000	0.000	0.201	0.245	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	25	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.56	1.00
time (sec)	N/A	0.195	1.072	0.207	0.497	0.000	0.418	0.444	0.247	0.288

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	29	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	1.81	1.00
time (sec)	N/A	0.199	5.778	0.229	0.511	0.000	0.513	0.509	0.273	0.303

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	257	273	299	0	0	0	0	39	0
N.S.	1	1.02	1.08	1.19	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.493	0.337	0.214	0.000	0.000	0.000	0.000	0.245	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	133	0	157	0	0	0	0	37	0
N.S.	1	1.02	0.00	1.21	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.608	0.000	0.137	0.000	0.000	0.000	0.000	0.235	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	138	150	157	0	0	0	0	170	0
N.S.	1	1.01	1.09	1.15	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.738	0.006	0.000	0.000	0.000	0.000	0.000	0.281	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	0	40	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	0.00	2.50	1.00
time (sec)	N/A	0.198	0.932	0.194	0.498	0.000	1.572	0.000	0.245	0.296

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	46	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	2.88	1.00
time (sec)	N/A	0.206	5.801	0.248	0.515	0.000	2.324	0.808	0.246	0.298

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	425	322	673	0	0	0	0	53	0
N.S.	1	1.46	1.10	2.30	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	2.037	1.689	0.242	0.000	0.000	0.000	0.000	0.254	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	188	0	340	0	0	0	0	51	0
N.S.	1	1.04	0.00	1.89	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.459	0.000	0.154	0.000	0.000	0.000	0.000	0.306	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	171	0	341	0	0	0	0	360	0
N.S.	1	1.05	0.00	2.09	0.00	0.00	0.00	0.00	2.21	0.00
time (sec)	N/A	0.916	0.000	0.069	0.000	0.000	0.000	0.000	0.246	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	0	55	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	0.00	3.44	1.00
time (sec)	N/A	0.205	0.934	0.210	0.605	0.000	7.523	0.000	0.236	0.292

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	63	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	3.94	1.00
time (sec)	N/A	0.206	6.021	0.243	0.620	0.000	12.997	1.775	0.271	0.290

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	137	158	144	0	100	85	0	31	0
N.S.	1	1.14	1.32	1.20	0.00	0.83	0.71	0.00	0.26	0.00
time (sec)	N/A	0.293	11.206	2.317	0.000	0.142	62.677	0.000	0.254	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	66	138	0	84	85	0	27	0
N.S.	1	1.10	0.53	1.11	0.00	0.68	0.69	0.00	0.22	0.00
time (sec)	N/A	0.357	10.073	1.179	0.000	0.128	10.147	0.000	0.225	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	113	119	0	69	85	0	23	0
N.S.	1	1.10	1.28	1.35	0.00	0.78	0.97	0.00	0.26	0.00
time (sec)	N/A	0.242	0.144	0.784	0.000	0.127	2.843	0.000	0.234	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	94	45	98	0	52	0	0	24	0
N.S.	1	1.06	0.51	1.10	0.00	0.58	0.00	0.00	0.27	0.00
time (sec)	N/A	0.298	0.032	0.550	0.000	0.131	0.000	0.000	0.247	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	93	85	0	50	0	0	34	0
N.S.	1	1.00	1.69	1.55	0.00	0.91	0.00	0.00	0.62	0.00
time (sec)	N/A	0.220	0.120	0.362	0.000	0.118	0.000	0.000	0.236	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	68	129	0	73	0	0	40	0
N.S.	1	1.06	0.54	1.03	0.00	0.58	0.00	0.00	0.32	0.00
time (sec)	N/A	0.327	0.066	0.583	0.000	0.144	0.000	0.000	0.234	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	234	0	0	0	0	0	53	0
N.S.	1	1.04	2.15	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.430	10.854	0.000	0.000	0.000	0.000	0.000	0.361	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	176	0	0	0	0	0	47	0
N.S.	1	1.04	1.61	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.427	2.941	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	113	202	0	0	0	0	0	42	0
N.S.	1	1.04	1.85	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.414	0.419	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	109	142	0	0	0	0	0	45	0
N.S.	1	1.02	1.33	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.411	1.026	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	129	0	0	0	0	0	60	0
N.S.	1	1.02	1.23	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.434	0.346	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	198	0	0	0	0	0	67	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.419	0.584	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	441	53	17	0	67	18
N.S.	1	1.00	1.11	0.89	24.50	2.94	0.94	0.00	3.72	1.00
time (sec)	N/A	0.445	43.750	0.270	4.216	0.167	72.775	0.000	0.289	0.325

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	418	44	17	0	61	18
N.S.	1	1.00	1.11	0.89	23.22	2.44	0.94	0.00	3.39	1.00
time (sec)	N/A	0.457	119.960	0.194	4.138	0.162	7.822	0.000	0.271	0.334

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	458	50	0	18	66	18
N.S.	1	1.00	1.11	0.89	25.44	2.78	0.00	1.00	3.67	1.00
time (sec)	N/A	0.421	64.272	0.298	4.128	0.126	0.000	0.532	0.283	0.335

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	489	50	0	18	86	18
N.S.	1	1.00	1.11	0.89	27.17	2.78	0.00	1.00	4.78	1.00
time (sec)	N/A	0.369	47.987	0.252	3.989	0.137	0.000	0.613	0.237	0.329

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	491	50	0	18	94	18
N.S.	1	1.00	1.11	0.89	27.28	2.78	0.00	1.00	5.22	1.00
time (sec)	N/A	0.396	30.507	0.424	4.079	0.137	0.000	0.609	0.279	0.362

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	20	15	18	20	18
N.S.	1	1.00	1.11	0.89	1.00	1.11	0.83	1.00	1.11	1.00
time (sec)	N/A	0.194	1.643	0.216	0.305	0.118	4.064	0.153	0.233	0.287

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	18	15	18	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.197	1.428	0.244	0.283	0.125	0.484	0.149	0.235	0.285

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	23	17	18	22	18
N.S.	1	1.00	1.11	0.89	1.00	1.28	0.94	1.00	1.22	1.00
time (sec)	N/A	0.190	0.841	0.240	0.352	0.106	1.372	0.136	0.255	0.290

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	31	17	18	27	18
N.S.	1	1.00	1.11	0.89	1.00	1.72	0.94	1.00	1.50	1.00
time (sec)	N/A	0.192	0.792	0.309	0.376	0.112	3.333	0.141	0.260	0.288

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	181	34	17	18	34	18
N.S.	1	1.00	1.11	0.89	10.06	1.89	0.94	1.00	1.89	1.00
time (sec)	N/A	0.190	9.645	0.227	2.047	0.128	10.055	0.157	0.253	0.318

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	181	32	17	18	32	18
N.S.	1	1.00	1.11	0.89	10.06	1.78	0.94	1.00	1.78	1.00
time (sec)	N/A	0.194	9.411	0.243	1.978	0.118	1.918	0.156	0.295	0.326

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	196	39	19	18	38	18
N.S.	1	1.00	1.11	0.89	10.89	2.17	1.06	1.00	2.11	1.00
time (sec)	N/A	0.190	25.041	0.242	1.820	0.103	4.028	0.141	0.258	0.305

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	218	51	19	18	44	18
N.S.	1	1.00	1.11	0.89	12.11	2.83	1.06	1.00	2.44	1.00
time (sec)	N/A	0.190	14.143	0.316	2.265	0.133	10.879	0.139	0.274	0.311

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [22] had the largest ratio of [1.39999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.05	8	0.500
2	A	4	4	1.20	8	0.500
3	A	5	4	1.07	8	0.500
4	A	3	3	1.13	6	0.500
5	A	2	2	1.00	4	0.500
6	A	7	6	1.16	8	0.750
7	A	5	4	1.00	8	0.500
8	A	2	2	1.00	8	0.250
9	A	6	5	1.02	8	0.625
10	A	3	3	1.09	8	0.375
11	A	7	6	1.08	8	0.750
12	A	7	7	1.18	10	0.700
13	A	6	6	1.17	10	0.600
14	A	5	5	1.16	10	0.500
15	A	4	4	1.10	8	0.500
16	A	3	3	1.20	6	0.500
17	A	8	7	1.23	10	0.700
18	A	7	6	1.01	10	0.600
19	A	3	3	1.07	10	0.300
20	A	9	8	0.97	10	0.800
21	A	5	5	1.02	10	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	15	14	1.51	10	1.400
23	A	11	11	1.47	10	1.100
24	A	10	9	1.29	10	0.900
25	A	6	6	1.18	8	0.750
26	A	4	4	1.15	6	0.667
27	A	9	8	1.22	10	0.800
28	A	8	7	1.04	10	0.700
29	A	9	8	1.05	10	0.800
30	A	13	12	1.01	10	1.200
31	A	12	11	1.07	10	1.100
32	A	13	13	1.77	10	1.300
33	A	14	14	1.66	10	1.400
34	A	10	10	1.47	10	1.000
35	A	11	11	1.39	10	1.100
36	A	7	7	1.18	8	0.875
37	A	5	5	1.23	6	0.833
38	A	10	9	1.29	10	0.900
39	A	9	8	1.05	10	0.800
40	A	10	9	1.12	10	0.900
41	A	13	12	1.00	10	1.200
42	A	4	3	0.87	10	0.300
43	A	4	3	0.91	10	0.300
44	A	4	3	0.90	10	0.300
45	A	4	3	0.97	10	0.300
46	A	4	3	0.96	10	0.300
47	A	6	5	1.00	8	0.625
48	A	4	3	1.00	6	0.500
49	N/A	1	0	1.00	10	0.000
50	N/A	1	0	1.00	10	0.000
51	A	3	2	0.91	10	0.200
52	A	3	2	0.94	10	0.200
53	A	3	2	0.94	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	2	0.98	10	0.200
55	A	3	2	0.98	10	0.200
56	A	5	4	1.00	8	0.500
57	A	5	4	1.00	6	0.667
58	N/A	1	0	1.00	10	0.000
59	N/A	1	0	1.00	10	0.000
60	A	6	5	1.35	10	0.500
61	A	9	8	1.30	10	0.800
62	A	9	8	1.24	10	0.800
63	A	9	8	1.10	8	1.000
64	A	6	5	1.06	6	0.833
65	N/A	1	0	1.00	10	0.000
66	N/A	1	0	1.00	10	0.000
67	A	5	4	1.32	10	0.400
68	A	8	7	1.24	10	0.700
69	A	9	8	1.23	10	0.800
70	A	8	7	1.10	8	0.875
71	A	7	6	1.15	6	1.000
72	N/A	1	0	1.00	10	0.000
73	N/A	1	0	1.00	10	0.000
74	A	6	5	0.99	12	0.417
75	A	6	5	0.99	12	0.417
76	A	6	5	1.02	12	0.417
77	A	6	5	0.97	10	0.500
78	A	6	5	1.00	8	0.625
79	N/A	1	0	1.00	12	0.000
80	A	15	14	1.20	12	1.167
81	A	14	13	1.28	12	1.083
82	A	11	10	1.31	12	0.833
83	A	10	9	1.07	10	0.900
84	A	7	6	1.01	8	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	N/A	1	0	1.00	12	0.000
86	A	14	13	1.35	12	1.083
87	A	11	10	1.33	12	0.833
88	A	12	11	1.32	12	0.917
89	A	9	8	1.10	10	0.800
90	A	8	7	1.08	8	0.875
91	N/A	1	0	1.00	12	0.000
92	A	4	3	0.96	12	0.250
93	A	4	3	0.98	12	0.250
94	A	4	3	0.99	12	0.250
95	A	7	6	1.00	10	0.600
96	A	5	4	1.00	8	0.500
97	N/A	1	0	1.00	12	0.000
98	N/A	1	0	1.00	12	0.000
99	A	3	2	0.96	12	0.167
100	A	3	2	0.99	12	0.167
101	A	3	2	0.98	12	0.167
102	A	3	2	1.04	12	0.167
103	A	3	2	1.04	12	0.167
104	A	6	5	1.00	10	0.500
105	A	6	5	1.00	8	0.625
106	N/A	1	0	1.00	12	0.000
107	A	6	5	1.07	12	0.417
108	A	10	9	1.33	12	0.750
109	A	10	9	1.41	12	0.750
110	A	10	9	1.06	10	0.900
111	A	7	6	1.07	8	0.750
112	N/A	1	0	1.00	12	0.000
113	A	5	4	1.24	12	0.333
114	A	9	8	1.30	12	0.667
115	A	10	9	1.32	12	0.750
116	A	9	8	1.11	10	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	A	8	7	1.12	8	0.875
118	N/A	1	0	1.00	12	0.000
119	N/A	2	0	1.00	12	0.000
120	N/A	2	0	1.00	12	0.000
121	A	2	2	0.98	12	0.167
122	A	2	2	1.00	10	0.200
123	N/A	1	0	1.00	12	0.000
124	N/A	1	0	1.00	12	0.000
125	N/A	1	0	1.00	14	0.000
126	N/A	1	0	1.00	14	0.000
127	N/A	1	0	1.00	14	0.000
128	N/A	1	0	1.00	14	0.000
129	N/A	1	0	1.00	12	0.000
130	A	4	3	0.98	10	0.300
131	A	4	3	0.96	10	0.300
132	A	7	6	1.04	8	0.750
133	A	5	4	0.99	6	0.667
134	N/A	1	0	1.00	10	0.000
135	N/A	1	0	1.00	10	0.000
136	N/A	1	0	1.00	14	0.000
137	N/A	1	0	1.00	14	0.000
138	N/A	1	0	1.00	14	0.000
139	N/A	1	0	1.00	14	0.000
140	A	4	4	1.16	12	0.333
141	A	5	4	1.05	12	0.333
142	A	3	3	1.10	10	0.300
143	A	1	1	1.00	8	0.125
144	A	7	6	1.13	12	0.500
145	A	5	4	1.00	12	0.333
146	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	6	5	1.00	12	0.417
148	A	5	5	1.08	14	0.357
149	A	4	4	1.09	12	0.333
150	A	3	3	1.11	10	0.300
151	A	8	7	1.16	14	0.500
152	A	7	6	0.97	14	0.429
153	A	9	8	1.18	14	0.571
154	A	6	6	1.11	12	0.500
155	A	3	3	1.06	10	0.300
156	A	9	8	1.14	14	0.571
157	A	8	7	0.98	14	0.500
158	A	5	4	0.87	14	0.286
159	A	11	10	0.89	12	0.833
160	A	9	8	0.93	10	0.800
161	N/A	1	0	1.00	14	0.000
162	N/A	1	0	1.00	14	0.000
163	A	3	2	0.90	14	0.143
164	A	9	8	0.96	12	0.667
165	A	9	8	0.94	10	0.800
166	N/A	1	0	1.00	14	0.000
167	N/A	1	0	1.00	14	0.000
168	A	15	14	1.26	14	1.000
169	A	14	13	1.04	12	1.083
170	A	11	10	1.00	10	1.000
171	N/A	1	0	1.00	14	0.000
172	N/A	1	0	1.00	14	0.000
173	A	6	5	0.98	16	0.312
174	A	6	5	0.96	14	0.357
175	A	11	10	0.97	12	0.833
176	N/A	1	0	1.00	16	0.000
177	N/A	1	0	1.00	16	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
178	A	18	17	1.35	16	1.062
179	A	16	15	1.03	14	1.071
180	A	13	12	0.98	12	1.000
181	N/A	1	0	1.00	16	0.000
182	N/A	1	0	1.00	16	0.000
183	A	17	16	1.32	16	1.000
184	A	9	8	1.03	14	0.571
185	A	13	12	0.99	12	1.000
186	N/A	1	0	1.00	16	0.000
187	N/A	1	0	1.00	16	0.000
188	A	5	4	0.98	16	0.250
189	A	13	12	0.99	14	0.857
190	A	11	10	1.02	12	0.833
191	N/A	1	0	1.00	16	0.000
192	N/A	1	0	1.00	16	0.000
193	A	3	2	1.02	16	0.125
194	A	11	10	1.02	14	0.714
195	A	11	10	1.01	12	0.833
196	N/A	1	0	1.00	16	0.000
197	N/A	1	0	1.00	16	0.000
198	A	17	16	1.46	16	1.000
199	A	16	15	1.04	14	1.071
200	A	13	12	1.05	12	1.000
201	N/A	1	0	1.00	16	0.000
202	N/A	1	0	1.00	16	0.000
203	A	6	5	1.14	16	0.312
204	A	9	8	1.10	16	0.500
205	A	5	4	1.10	16	0.250
206	A	8	7	1.06	16	0.438
207	A	4	3	1.00	16	0.188
208	A	9	8	1.06	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
209	A	2	2	1.04	18	0.111
210	A	2	2	1.04	18	0.111
211	A	2	2	1.04	18	0.111
212	A	2	2	1.02	18	0.111
213	A	2	2	1.02	18	0.111
214	A	2	2	1.00	18	0.111
215	N/A	2	0	1.00	18	0.000
216	N/A	2	0	1.00	18	0.000
217	N/A	2	0	1.00	18	0.000
218	N/A	2	0	1.00	18	0.000
219	N/A	2	0	1.00	18	0.000
220	N/A	1	0	1.00	18	0.000
221	N/A	1	0	1.00	18	0.000
222	N/A	1	0	1.00	18	0.000
223	N/A	1	0	1.00	18	0.000
224	N/A	1	0	1.00	18	0.000
225	N/A	1	0	1.00	18	0.000
226	N/A	1	0	1.00	18	0.000
227	N/A	1	0	1.00	18	0.000

CHAPTER 3

LISTING OF INTEGRALS

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3.52 $\int \frac{x^5}{\arccos(ax)^2}\,dx$ 448
3.53 $\int \frac{x^4}{\arccos(ax)^2}\,dx$ 453
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3.55 $\int \frac{x^2}{\arccos(ax)^2}\,dx$ 463
3.56 $\int \frac{x}{\arccos(ax)^2}\,dx$ 468
3.57 $\int \frac{1}{\arccos(ax)^2}\,dx$ 473
3.58 $\int \frac{1}{x\arccos(ax)^2}\,dx$ 478
3.59 $\int \frac{1}{x^2\arccos(ax)^2}\,dx$ 483
3.60 $\int \frac{x^4}{\arccos(ax)^3}\,dx$ 488
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3.62 $\int \frac{x^2}{\arccos(ax)^3}\,dx$ 501

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3.65 $\int \frac{1}{x\arccos(ax)^3}\,dx$ . . . 521
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3.84 $\int \arccos(ax)^{3/2}\,dx$ . . . 649
3.85 $\int \frac{\arccos(ax)^{3/2}}{x}\,dx$ . . . 655
3.86 $\int x^4\arccos(ax)^{5/2}\,dx$ . . . 660
3.87 $\int x^3\arccos(ax)^{5/2}\,dx$ . . . 672
3.88 $\int x^2\arccos(ax)^{5/2}\,dx$ . . . 682
3.89 $\int x\arccos(ax)^{5/2}\,dx$ . . . 692
3.90 $\int \arccos(ax)^{5/2}\,dx$ . . . 700
3.91 $\int \frac{\arccos(ax)^{5/2}}{x}\,dx$ . . . 707
3.92 $\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx$ . . . 712
3.93 $\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx$ . . . 719
3.94 $\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx$ . . . 725
3.95 $\int \frac{x}{\sqrt{\arccos(ax)}}\,dx$ . . . 731
3.96 $\int \frac{1}{\sqrt{\arccos(ax)}}\,dx$ . . . 737
3.97 $\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$ . . . 743
3.98 $\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$ . . . 748

3.99 $\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx$ 753
3.100 $\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx$ 759
3.101 $\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx$ 764
3.102 $\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx$ 770
3.103 $\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx$ 775
3.104 $\int \frac{x}{\arccos(ax)^{3/2}}\,dx$ 780
3.105 $\int \frac{1}{\arccos(ax)^{3/2}}\,dx$ 785
3.106 $\int \frac{1}{x\arccos(ax)^{3/2}}\,dx$ 791
3.107 $\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx$ 796
3.108 $\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx$ 803
3.109 $\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx$ 811
3.110 $\int \frac{x}{\arccos(ax)^{5/2}}\,dx$ 819
3.111 $\int \frac{1}{\arccos(ax)^{5/2}}\,dx$ 826
3.112 $\int \frac{1}{x\arccos(ax)^{5/2}}\,dx$ 832
3.113 $\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx$ 837
3.114 $\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx$ 844
3.115 $\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx$ 853
3.116 $\int \frac{x}{\arccos(ax)^{7/2}}\,dx$ 862
3.117 $\int \frac{1}{\arccos(ax)^{7/2}}\,dx$ 869
3.118 $\int \frac{1}{x\arccos(ax)^{7/2}}\,dx$ 876
3.119 $\int (bx)^m \arccos(ax)^4\,dx$ 881
3.120 $\int (bx)^m \arccos(ax)^3\,dx$ 886
3.121 $\int (bx)^m \arccos(ax)^2\,dx$ 891
3.122 $\int (bx)^m \arccos(ax)\,dx$ 897
3.123 $\int \frac{(bx)^m}{\arccos(ax)}\,dx$ 902
3.124 $\int \frac{(bx)^m}{\arccos(ax)^2}\,dx$ 907
3.125 $\int (bx)^m \arccos(ax)^{3/2}\,dx$ 912
3.126 $\int (bx)^m \sqrt{\arccos(ax)}\,dx$ 917
3.127 $\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx$ 922
3.128 $\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$ 927
3.129 $\int (bx)^m \arccos(ax)^n\,dx$ 932
3.130 $\int x^3 \arccos(ax)^n\,dx$ 937
3.131 $\int x^2 \arccos(ax)^n\,dx$ 943
3.132 $\int x \arccos(ax)^n\,dx$ 948
3.133 $\int \arccos(ax)^n\,dx$ 954

3.134 $\int \frac{\arccos(ax)^n}{x}\,dx$ . . . 959
3.135 $\int \frac{\arccos(ax)^n}{x^2}\,dx$ . . . 964
3.136 $\int (bx)^{3/2}\arccos(ax)^n\,dx$ . . . 969
3.137 $\int \sqrt{bx}\arccos(ax)^n\,dx$ . . . 974
3.138 $\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$ . . . 979
3.139 $\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$ . . . 984
3.140 $\int x^3(a+b\arccos(cx))\,dx$ . . . 989
3.141 $\int x^2(a+b\arccos(cx))\,dx$ . . . 995
3.142 $\int x(a+b\arccos(cx))\,dx$ . . . 1001
3.143 $\int (a+b\arccos(cx))\,dx$ . . . 1007
3.144 $\int \frac{a+b\arccos(cx)}{x}\,dx$ . . . 1012
3.145 $\int \frac{a+b\arccos(cx)}{x^2}\,dx$ . . . 1018
3.146 $\int \frac{a+b\arccos(cx)}{x^3}\,dx$ . . . 1024
3.147 $\int \frac{a+b\arccos(cx)}{x^4}\,dx$ . . . 1030
3.148 $\int x^2(a+b\arccos(cx))^2\,dx$ . . . 1037
3.149 $\int x(a+b\arccos(cx))^2\,dx$ . . . 1044
3.150 $\int (a+b\arccos(cx))^2\,dx$ . . . 1051
3.151 $\int \frac{(a+b\arccos(cx))^2}{x}\,dx$ . . . 1057
3.152 $\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx$ . . . 1064
3.153 $\int x^2(a+b\arccos(cx))^3\,dx$ . . . 1071
3.154 $\int x(a+b\arccos(cx))^3\,dx$ . . . 1081
3.155 $\int (a+b\arccos(cx))^3\,dx$ . . . 1089
3.156 $\int \frac{(a+b\arccos(cx))^3}{x}\,dx$ . . . 1096
3.157 $\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx$ . . . 1104
3.158 $\int \frac{x^2}{a+b\arccos(cx)}\,dx$ . . . 1112
3.159 $\int \frac{x}{a+b\arccos(cx)}\,dx$ . . . 1118
3.160 $\int \frac{1}{a+b\arccos(cx)}\,dx$ . . . 1125
3.161 $\int \frac{1}{x(a+b\arccos(cx))}\,dx$ . . . 1131
3.162 $\int \frac{1}{x^2(a+b\arccos(cx))}\,dx$ . . . 1136
3.163 $\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx$ . . . 1141
3.164 $\int \frac{x}{(a+b\arccos(cx))^2}\,dx$ . . . 1147
3.165 $\int \frac{1}{(a+b\arccos(cx))^2}\,dx$ . . . 1154
3.166 $\int \frac{1}{x(a+b\arccos(cx))^2}\,dx$ . . . 1161
3.167 $\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$ . . . 1166
3.168 $\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx$ . . . 1171
3.169 $\int \frac{x}{(a+b\arccos(cx))^3}\,dx$ . . . 1182
3.170 $\int \frac{1}{(a+b\arccos(cx))^3}\,dx$ . . . 1192

3.171 $\int \frac{1}{x(a+b\arccos(cx))^3}\,dx$ . . . 1201
3.172 $\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$ . . . 1206
3.173 $\int x^2\sqrt{a+b\arccos(cx)}\,dx$ . . . 1211
3.174 $\int x\sqrt{a+b\arccos(cx)}\,dx$ . . . 1219
3.175 $\int \sqrt{a+b\arccos(cx)}\,dx$ . . . 1226
3.176 $\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$ . . . 1234
3.177 $\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx$ . . . 1239
3.178 $\int x^2(a+b\arccos(cx))^{3/2}\,dx$ . . . 1244
3.179 $\int x(a+b\arccos(cx))^{3/2}\,dx$ . . . 1257
3.180 $\int (a+b\arccos(cx))^{3/2}\,dx$ . . . 1268
3.181 $\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx$ . . . 1277
3.182 $\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$ . . . 1282
3.183 $\int x^2(a+b\arccos(cx))^{5/2}\,dx$ . . . 1287
3.184 $\int x(a+b\arccos(cx))^{5/2}\,dx$ . . . 1302
3.185 $\int (a+b\arccos(cx))^{5/2}\,dx$ . . . 1312
3.186 $\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx$ . . . 1322
3.187 $\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$ . . . 1327
3.188 $\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx$ . . . 1332
3.189 $\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx$ . . . 1339
3.190 $\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$ . . . 1347
3.191 $\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$ . . . 1354
3.192 $\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$ . . . 1359
3.193 $\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx$ . . . 1364
3.194 $\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx$ . . . 1370
3.195 $\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$ . . . 1377
3.196 $\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$ . . . 1385
3.197 $\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$ . . . 1390
3.198 $\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx$ . . . 1395
3.199 $\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx$ . . . 1407
3.200 $\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx$ . . . 1417
3.201 $\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$ . . . 1426
3.202 $\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx$ . . . 1431
3.203 $\int (dx)^{5/2}(a+b\arccos(cx))\,dx$ . . . 1436
3.204 $\int (dx)^{3/2}(a+b\arccos(cx))\,dx$ . . . 1443
3.205 $\int \sqrt{dx}(a+b\arccos(cx))\,dx$ . . . 1451
3.206 $\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx$ . . . 1458

3.207 $\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx$ . . . 1465
3.208 $\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx$ . . . 1470
3.209 $\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx$ . . . 1478
3.210 $\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx$ . . . 1484
3.211 $\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$ . . . 1490
3.212 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx$ . . . 1496
3.213 $\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx$ . . . 1502
3.214 $\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx$ . . . 1507
3.215 $\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$ . . . 1512
3.216 $\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$ . . . 1517
3.217 $\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx$ . . . 1522
3.218 $\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx$ . . . 1527
3.219 $\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$ . . . 1532
3.220 $\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx$ . . . 1537
3.221 $\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$ . . . 1542
3.222 $\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$ . . . 1547
3.223 $\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$ . . . 1552
3.224 $\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$ . . . 1557
3.225 $\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$ . . . 1562
3.226 $\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$ . . . 1567
3.227 $\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$ . . . 1572

3.1 $\int x^4 \arccos(ax)\,dx$

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Optimal result

Integrand size = 8, antiderivative size = 75

$$\int x^4 \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{5a^5} + \frac{2\left(1-a^2x^2\right)^{3/2}}{15a^5} - \frac{\left(1-a^2x^2\right)^{5/2}}{25a^5} + \frac{1}{5}x^5\arccos(ax)$$

output
```
-1/5*(-a^2*x^2+1)^(1/2)/a^5+2/15*(-a^2*x^2+1)^(3/2)/a^5-1/25*(-a^2*x^2+1)^
(5/2)/a^5+1/5*x^5*arccos(a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int x^4 \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}(8+4a^2x^2+3a^4x^4)}{75a^5} + \frac{1}{5}x^5\arccos(ax)$$

input
```
Integrate[x^4*ArcCos[a*x],x]
```

output
```
-1/75*(Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4))/a^5 + (x^5*ArcCos[a*
x])/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5139, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arccos(ax) \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{5}a\int \frac{x^5}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^5\arccos(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{5}x^5\arccos(ax)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{10}a\int \left(\frac{\left(1-a^2x^2\right)^{3/2}}{a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}+\frac{1}{a^4\sqrt{1-a^2x^2}}\right)dx^2+\frac{1}{5}x^5\arccos(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a\left(-\frac{2\left(1-a^2x^2\right)^{5/2}}{5a^6}+\frac{4\left(1-a^2x^2\right)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+\frac{1}{5}x^5\arccos(ax)$$

input
```
Int[x^4*ArcCos[a*x],x]
```

output
```
(a*((-2*Sqrt[1 - a^2*x^2])/a^6 + (4*(1 - a^2*x^2)^(3/2))/(3*a^6) - (2*(1 -
 a^2*x^2)^(5/2))/(5*a^6)))/10 + (x^5*ArcCos[a*x])/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

method	result	si
derivativedivides	$\frac{\frac{a^5x^5\arccos(ax)}{5}-\frac{a^4x^4\sqrt{-a^2x^2+1}}{25}-\frac{4a^2x^2\sqrt{-a^2x^2+1}}{75}-\frac{8\sqrt{-a^2x^2+1}}{75}}{a^5}$	7
default	$\frac{\frac{a^5x^5\arccos(ax)}{5}-\frac{a^4x^4\sqrt{-a^2x^2+1}}{25}-\frac{4a^2x^2\sqrt{-a^2x^2+1}}{75}-\frac{8\sqrt{-a^2x^2+1}}{75}}{a^5}$	7
parts	$\frac{x^5\arccos(ax)}{5}+\frac{a\left(-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}+\frac{-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{8\sqrt{-a^2x^2+1}}{15a^4}}{a^2}\right)}{5}$	7
orering	$\frac{(27a^6x^6+4a^4x^4+16a^2x^2-32)\arccos(ax)}{75x\,a^6}-\frac{(3a^4x^4+4a^2x^2+8)(ax-1)(ax+1)\left(4x^3\arccos(ax)-\frac{x^4a}{\sqrt{-a^2x^2+1}}\right)}{75a^6x^4}$	1

input
```
int(x^4*arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccos(a*x)-1/25*a^4*x^4*(-a^2*x^2+1)^(1/2)-4/75*a^2*x^
2*(-a^2*x^2+1)^(1/2)-8/75*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int x^4 \arccos(ax)\,dx = \frac{15\,a^5x^5\arccos(ax) - (3\,a^4x^4 + 4\,a^2x^2 + 8)\sqrt{-a^2x^2+1}}{75\,a^5}$$

input
```
integrate(x^4*arccos(a*x),x, algorithm="fricas")
```

output
```
1/75*(15*a^5*x^5*arccos(a*x) - (3*a^4*x^4 + 4*a^2*x^2 + 8)*sqrt(-a^2*x^2 +
 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^4 \arccos(ax)\,dx = \begin{cases} \frac{x^5\operatorname{acos}(ax)}{5} - \frac{x^4\sqrt{-a^2x^2+1}}{25a} - \frac{4x^2\sqrt{-a^2x^2+1}}{75a^3} - \frac{8\sqrt{-a^2x^2+1}}{75a^5} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acos(a*x),x)
```

output
```
Piecewise((x**5*acos(a*x)/5 - x**4*sqrt(-a**2*x**2 + 1)/(25*a) - 4*x**2*sq
rt(-a**2*x**2 + 1)/(75*a**3) - 8*sqrt(-a**2*x**2 + 1)/(75*a**5), Ne(a, 0))
, (pi*x**5/10, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int x^4 \arccos(ax)\,dx = \frac{1}{5}\,x^5 \arccos\left(ax\right) - \frac{1}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a$$

input

```
integrate(x^4*arccos(a*x),x, algorithm="maxima")
```

output

```
1/5*x^5*arccos(a*x) - 1/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int x^4 \arccos(ax)\,dx = \frac{1}{5}\,x^5 \arccos\left(ax\right) - \frac{\sqrt{-a^2x^2+1}x^4}{25\,a} - \frac{4\sqrt{-a^2x^2+1}x^2}{75\,a^3} - \frac{8\sqrt{-a^2x^2+1}}{75\,a^5}$$

input

```
integrate(x^4*arccos(a*x),x, algorithm="giac")
```

output

```
1/5*x^5*arccos(a*x) - 1/25*sqrt(-a^2*x^2 + 1)*x^4/a - 4/75*sqrt(-a^2*x^2 +
 1)*x^2/a^3 - 8/75*sqrt(-a^2*x^2 + 1)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)\,dx = \int x^4 \operatorname{acos}(a\,x)\ dx$$

input
```
int(x^4*acos(a*x),x)
```

output
```
int(x^4*acos(a*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int x^4 \arccos(ax)\,dx = \frac{15acos(ax)\,a^5x^5 - 3\sqrt{-a^2x^2+1}\,a^4x^4 - 4\sqrt{-a^2x^2+1}\,a^2x^2 - 8\sqrt{-a^2x^2+1}}{75a^5}$$

input
```
int(x^4*acos(a*x),x)
```

output
```
(15*acos(a*x)*a**5*x**5 - 3*sqrt( - a**2*x**2 + 1)*a**4*x**4 - 4*sqrt( - a
**2*x**2 + 1)*a**2*x**2 - 8*sqrt( - a**2*x**2 + 1))/(75*a**5)
```

3.2 $\int x^3 \arccos(ax)\,dx$

Optimal result 113
Mathematica [A] (verified) 113
Rubi [A] (verified) 114
Maple [A] (verified) 115
Fricas [A] (verification not implemented) 116
Sympy [A] (verification not implemented) 116
Maxima [A] (verification not implemented) 116
Giac [A] (verification not implemented) 117
Mupad [F(-1)] 117
Reduce [B] (verification not implemented) 118

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int x^3 \arccos(ax)\,dx = -\frac{3x\sqrt{1-a^2x^2}}{32a^3} - \frac{x^3\sqrt{1-a^2x^2}}{16a} + \frac{1}{4}x^4\arccos(ax) + \frac{3\arcsin(ax)}{32a^4}$$

output
```
-3/32*x*(-a^2*x^2+1)^(1/2)/a^3-1/16*x^3*(-a^2*x^2+1)^(1/2)/a+1/4*x^4*arcco
s(a*x)+3/32*arcsin(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int x^3 \arccos(ax)\,dx = \frac{-ax\sqrt{1-a^2x^2}(3+2a^2x^2) + 8a^4x^4\arccos(ax) + 3\arcsin(ax)}{32a^4}$$

input
```
Integrate[x^3*ArcCos[a*x],x]
```

output
```
(-(a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)) + 8*a^4*x^4*ArcCos[a*x] + 3*ArcS
in[a*x])/(32*a^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5139, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax) \, dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{1}{4} a \int \frac{x^4}{\sqrt{1-a^2x^2}} dx + \frac{1}{4} x^4 \arccos(ax)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{4} a \left(\frac{3 \int \frac{x^2}{\sqrt{1-a^2x^2}} dx}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2} \right) + \frac{1}{4} x^4 \arccos(ax)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{4} a \left(\frac{3 \left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2} \right) + \frac{1}{4} x^4 \arccos(ax)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{4} a \left(\frac{3 \left(\frac{\arcsin(ax)}{2a^3} - \frac{x\sqrt{1-a^2x^2}}{2a^2} \right)}{4a^2} - \frac{x^3\sqrt{1-a^2x^2}}{4a^2} \right) + \frac{1}{4} x^4 \arccos(ax)$$

input
```
Int[x^3*ArcCos[a*x],x]
```

output
```
(x^4*ArcCos[a*x])/4 + (a*(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/2*(x*S
qrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2)))/4
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\arccos(ax)}{4}-\frac{a^3x^3\sqrt{-a^2x^2+1}}{16}-\frac{3\sqrt{-a^2x^2+1}\,ax}{32}+\frac{3\arcsin(ax)}{32}}{a^4}$	60
default	$\frac{\frac{a^4x^4\arccos(ax)}{4}-\frac{a^3x^3\sqrt{-a^2x^2+1}}{16}-\frac{3\sqrt{-a^2x^2+1}\,ax}{32}+\frac{3\arcsin(ax)}{32}}{a^4}$	60
orering	$\frac{(14a^4x^4+3a^2x^2-12)\arccos(ax)}{32a^4}-\frac{(2a^2x^2+3)(ax-1)(ax+1)\left(3x^2\arccos(ax)-\frac{x^3a}{\sqrt{-a^2x^2+1}}\right)}{32x^2a^4}$	85
parts	$\frac{x^4\arccos(ax)}{4}+\frac{a\left(-\frac{x^3\sqrt{-a^2x^2+1}}{4a^2}+\frac{-\frac{3x\sqrt{-a^2x^2+1}}{8a^2}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^2\sqrt{a^2}}}{a^2}\right)}{4}$	89

input
```
int(x^3*arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arccos(a*x)-1/16*a^3*x^3*(-a^2*x^2+1)^(1/2)-3/32*(-a^2*
x^2+1)^(1/2)*a*x+3/32*arcsin(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int x^3 \arccos(ax)\,dx = \frac{(8\,a^4x^4-3)\arccos{(ax)}-(2\,a^3x^3+3\,ax)\sqrt{-a^2x^2+1}}{32\,a^4}$$

input
```
integrate(x^3*arccos(a*x),x, algorithm="fricas")
```

output
```
1/32*((8*a^4*x^4 - 3)*arccos(a*x) - (2*a^3*x^3 + 3*a*x)*sqrt(-a^2*x^2 + 1)
)/a^4
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^3 \arccos(ax)\,dx = \begin{cases} \frac{x^4\operatorname{acos}{(ax)}}{4} - \frac{x^3\sqrt{-a^2x^2+1}}{16a} - \frac{3x\sqrt{-a^2x^2+1}}{32a^3} - \frac{3\operatorname{acos}{(ax)}}{32a^4} & \text{for}\ a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acos(a*x),x)
```

output
```
Piecewise((x**4*acos(a*x)/4 - x**3*sqrt(-a**2*x**2 + 1)/(16*a) - 3*x*sqrt(
-a**2*x**2 + 1)/(32*a**3) - 3*acos(a*x)/(32*a**4), Ne(a, 0)), (pi*x**4/8,
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int x^3 \arccos(ax)\,dx = \frac{1}{4}\,x^4\arccos{(ax)} - \frac{1}{32}\left(\frac{2\,\sqrt{-a^2x^2+1}x^3}{a^2} + \frac{3\,\sqrt{-a^2x^2+1}x}{a^4} - \frac{3\,\arcsin{(ax)}}{a^5}\right)a$$

input
```
integrate(x^3*arccos(a*x),x, algorithm="maxima")
```

output
```
1/4*x^4*arccos(a*x) - 1/32*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2
 + 1)*x/a^4 - 3*arcsin(a*x)/a^5)*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3 \arccos(ax)\, dx = \frac{1}{4}\, x^4 \arccos{(ax)} - \frac{\sqrt{-a^2x^2+1}x^3}{16\, a} - \frac{3\,\sqrt{-a^2x^2+1}x}{32\, a^3} - \frac{3\, \arccos{(ax)}}{32\, a^4}$$

input
```
integrate(x^3*arccos(a*x),x, algorithm="giac")
```

output
```
1/4*x^4*arccos(a*x) - 1/16*sqrt(-a^2*x^2 + 1)*x^3/a - 3/32*sqrt(-a^2*x^2 +
 1)*x/a^3 - 3/32*arccos(a*x)/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)\, dx = \int x^3\, \mathrm{acos}(a\,x)\; dx$$

input
```
int(x^3*acos(a*x),x)
```

output
```
int(x^3*acos(a*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int x^3 \arccos(ax)\,dx$$
$$= \frac{8acos(ax)\,a^4x^4 + 3asin(ax) - 2\sqrt{-a^2x^2+1}\,a^3x^3 - 3\sqrt{-a^2x^2+1}\,ax}{32a^4}$$

input
```
int(x^3*acos(a*x),x)
```

output
```
(8*acos(a*x)*a**4*x**4 + 3*asin(a*x) - 2*sqrt( - a**2*x**2 + 1)*a**3*x**3
- 3*sqrt( - a**2*x**2 + 1)*a*x)/(32*a**4)
```

3.3 $\int x^2 \arccos(ax)\, dx$

Optimal result 119
Mathematica [A] (verified) 119
Rubi [A] (verified) 120
Maple [A] (verified) 121
Fricas [A] (verification not implemented) 122
Sympy [A] (verification not implemented) 122
Maxima [A] (verification not implemented) 123
Giac [A] (verification not implemented) 123
Mupad [F(-1)] 123
Reduce [B] (verification not implemented) 124

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int x^2 \arccos(ax)\, dx = -\frac{\sqrt{1-a^2x^2}}{3a^3} + \frac{(1-a^2x^2)^{3/2}}{9a^3} + \frac{1}{3}x^3 \arccos(ax)$$

output
```
-1/3*(-a^2*x^2+1)^(1/2)/a^3+1/9*(-a^2*x^2+1)^(3/2)/a^3+1/3*x^3*arccos(a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^2 \arccos(ax)\, dx = -\frac{\sqrt{1-a^2x^2}(2+a^2x^2)}{9a^3} + \frac{1}{3}x^3 \arccos(ax)$$

input
```
Integrate[x^2*ArcCos[a*x],x]
```

output
```
-1/9*(Sqrt[1 - a^2*x^2]*(2 + a^2*x^2))/a^3 + (x^3*ArcCos[a*x])/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5139, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{3}a\int \frac{x^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{3}x^3\arccos(ax)$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}a\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2+\frac{1}{3}x^3\arccos(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)$$

input
```
Int[x^2*ArcCos[a*x],x]
```

output
```
(a*((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4)))/6 + (x^
3*ArcCos[a*x])/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{a^{3} x^{3} \arccos \left(a x \right)}{3}-\frac{a^{2} x^{2} \sqrt{-a^{2} x^{2}+1}}{9}-\frac{2 \sqrt{-a^{2} x^{2}+1}}{9}}{a^{3}}$	52
default	$\frac{\frac{a^{3} x^{3} \arccos \left(a x \right)}{3}-\frac{a^{2} x^{2} \sqrt{-a^{2} x^{2}+1}}{9}-\frac{2 \sqrt{-a^{2} x^{2}+1}}{9}}{a^{3}}$	52
parts	$\frac{x^{3} \arccos \left(a x \right)}{3}+\frac{a \left(-\frac{x^{2} \sqrt{-a^{2} x^{2}+1}}{3 a^{2}}-\frac{2 \sqrt{-a^{2} x^{2}+1}}{3 a^{4}}\right)}{3}$	52
orering	$\frac{\left(5 a^{4} x^{4}+2 a^{2} x^{2}-4\right) \arccos \left(a x \right)}{9 a^{4} x}-\frac{\left(a^{2} x^{2}+2\right) \left(a x -1\right) \left(a x +1\right) \left(2 x \arccos \left(a x \right)-\frac{x^{2} a}{\sqrt{-a^{2} x^{2}+1}}\right)}{9 a^{4} x^{2}}$	85

input
```
int(x^2*arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*a^3*x^3*arccos(a*x)-1/9*a^2*x^2*(-a^2*x^2+1)^(1/2)-2/9*(-a^2*x^
2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int x^2 \arccos(ax)\,dx = \frac{3\,a^3x^3\arccos{(ax)} - (a^2x^2+2)\sqrt{-a^2x^2+1}}{9\,a^3}$$

input
```
integrate(x^2*arccos(a*x),x, algorithm="fricas")
```

output
```
1/9*(3*a^3*x^3*arccos(a*x) - (a^2*x^2 + 2)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int x^2 \arccos(ax)\,dx = \begin{cases} \frac{x^3\operatorname{acos}{(ax)}}{3} - \frac{x^2\sqrt{-a^2x^2+1}}{9a} - \frac{2\sqrt{-a^2x^2+1}}{9a^3} & \text{for}\ a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acos(a*x),x)
```

output
```
Piecewise((x**3*acos(a*x)/3 - x**2*sqrt(-a**2*x**2 + 1)/(9*a) - 2*sqrt(-a*
*2*x**2 + 1)/(9*a**3), Ne(a, 0)), (pi*x**3/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int x^2 \arccos(ax)\,dx = \frac{1}{3}x^3 \arccos(ax) - \frac{1}{9}a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)$$

input
```
integrate(x^2*arccos(a*x),x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(a*x) - 1/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
+ 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int x^2 \arccos(ax)\,dx = \frac{1}{3}x^3 \arccos(ax) - \frac{\sqrt{-a^2x^2+1}x^2}{9a} - \frac{2\sqrt{-a^2x^2+1}}{9a^3}$$

input
```
integrate(x^2*arccos(a*x),x, algorithm="giac")
```

output
```
1/3*x^3*arccos(a*x) - 1/9*sqrt(-a^2*x^2 + 1)*x^2/a - 2/9*sqrt(-a^2*x^2 + 1
)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)\,dx = \begin{cases} \frac{x^3\operatorname{acos}(a\,x)}{3} - \frac{\sqrt{\frac{1}{a^2}-x^2}\left(\frac{2}{a^2}+x^2\right)}{9} & \text{if } 0 < a \\ \int x^2 \operatorname{acos}(a\,x)\,dx & \text{if } \neg 0 < a \end{cases}$$

input
```
int(x^2*acos(a*x),x)
```

output
```
piecewise(0 < a, - ((1/a^2 - x^2)^(1/2)*(2/a^2 + x^2))/9 + (x^3*acos(a*x))
/3, ~0 < a, int(x^2*acos(a*x), x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int x^2 \arccos(ax)\,dx = \frac{3acos(ax)\,a^3x^3 - \sqrt{-a^2x^2+1}\,a^2x^2 - 2\sqrt{-a^2x^2+1}}{9a^3}$$

input
```
int(x^2*acos(a*x),x)
```

output
```
(3*acos(a*x)*a**3*x**3 - sqrt( - a**2*x**2 + 1)*a**2*x**2 - 2*sqrt( - a**2
*x**2 + 1))/(9*a**3)
```

3.4 $\int x \arccos(ax)\,dx$

Optimal result 125
Mathematica [A] (verified) 125
Rubi [A] (verified) 126
Maple [A] (verified) 127
Fricas [A] (verification not implemented) 128
Sympy [A] (verification not implemented) 128
Maxima [A] (verification not implemented) 128
Giac [A] (verification not implemented) 129
Mupad [B] (verification not implemented) 129
Reduce [B] (verification not implemented) 129

Optimal result

Integrand size = 6, antiderivative size = 45

$$\int x\arccos(ax)\,dx=-\frac{x\sqrt{1-a^2x^2}}{4a}+\frac{1}{2}x^2\arccos(ax)+\frac{\arcsin(ax)}{4a^2}$$

output
```
-1/4*x*(-a^2*x^2+1)^(1/2)/a+1/2*x^2*arccos(a*x)+1/4*arcsin(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x\arccos(ax)\,dx=\frac{-ax\sqrt{1-a^2x^2}+2a^2x^2\arccos(ax)+\arcsin(ax)}{4a^2}$$

input
```
Integrate[x*ArcCos[a*x],x]
```

output
```
(-(a*x*Sqrt[1 - a^2*x^2]) + 2*a^2*x^2*ArcCos[a*x] + ArcSin[a*x])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5139, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax) \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{2}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)$$

input
```
Int[x*ArcCos[a*x],x]
```

output
```
(x^2*ArcCos[a*x])/2 + (a*(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*
a^3)))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{a^2x^2\arccos(ax)}{2}-\frac{\sqrt{-a^2x^2+1}\,ax}{4}+\frac{\arcsin(ax)}{4}}{a^2}$	40
default	$\frac{\frac{a^2x^2\arccos(ax)}{2}-\frac{\sqrt{-a^2x^2+1}\,ax}{4}+\frac{\arcsin(ax)}{4}}{a^2}$	40
orering	$\frac{(3a^2x^2-2)\arccos(ax)}{4a^2}-\frac{(ax-1)(ax+1)\left(\arccos(ax)-\frac{ax}{\sqrt{-a^2x^2+1}}\right)}{4a^2}$	57
parts	$\frac{x^2\arccos(ax)}{2}+\frac{a\left(-\frac{x\sqrt{-a^2x^2+1}}{2a^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^2\sqrt{a^2}}\right)}{2}$	63

input
```
int(x*arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*a^2*x^2*arccos(a*x)-1/4*(-a^2*x^2+1)^(1/2)*a*x+1/4*arcsin(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x \arccos(ax)\,dx = -\frac{\sqrt{-a^2x^2+1}ax - (2\,a^2x^2-1)\arccos(ax)}{4\,a^2}$$

input
```
integrate(x*arccos(a*x),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-a^2*x^2 + 1)*a*x - (2*a^2*x^2 - 1)*arccos(a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x \arccos(ax)\,dx = \begin{cases} \frac{x^2 \operatorname{acos}(ax)}{2} - \frac{x\sqrt{-a^2x^2+1}}{4a} - \frac{\operatorname{acos}(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x),x)
```

output
```
Piecewise((x**2*acos(a*x)/2 - x*sqrt(-a**2*x**2 + 1)/(4*a) - acos(a*x)/(4*
a**2), Ne(a, 0)), (pi*x**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x \arccos(ax)\,dx = \frac{1}{2}\,x^2\arccos(ax) - \frac{1}{4}\,a\left(\frac{\sqrt{-a^2x^2+1}x}{a^2} - \frac{\arcsin(ax)}{a^3}\right)$$

input
```
integrate(x*arccos(a*x),x, algorithm="maxima")
```

output
```
1/2*x^2*arccos(a*x) - 1/4*a*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x \arccos(ax)\, dx = \frac{1}{2}\, x^2 \arccos\left(ax\right) - \frac{\sqrt{-a^2x^2+1}x}{4\, a} - \frac{\arccos\left(ax\right)}{4\, a^2}$$

input
```
integrate(x*arccos(a*x),x, algorithm="giac")
```

output
```
1/2*x^2*arccos(a*x) - 1/4*sqrt(-a^2*x^2 + 1)*x/a - 1/4*arccos(a*x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x \arccos(ax)\, dx = \frac{\mathrm{acos}(a\, x)\, \left(2\, a^2\, x^2-1\right)}{4\, a^2} - \frac{x\, \sqrt{1-a^2\, x^2}}{4\, a}$$

input
```
int(x*acos(a*x),x)
```

output
```
(acos(a*x)*(2*a^2*x^2 - 1))/(4*a^2) - (x*(1 - a^2*x^2)^(1/2))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x \arccos(ax)\, dx = \frac{2acos(ax)\, a^2x^2 + asin(ax) - \sqrt{-a^2x^2+1}\, ax}{4a^2}$$

input
```
int(x*acos(a*x),x)
```

output
```
(2*acos(a*x)*a**2*x**2 + asin(a*x) - sqrt( - a**2*x**2 + 1)*a*x)/(4*a**2)
```

3.5 $\int \arccos(ax)\,dx$

Optimal result 130
Mathematica [A] (verified) 130
Rubi [A] (verified) 131
Maple [A] (verified) 132
Fricas [A] (verification not implemented) 132
Sympy [A] (verification not implemented) 132
Maxima [A] (verification not implemented) 133
Giac [A] (verification not implemented) 133
Mupad [B] (verification not implemented) 133
Reduce [B] (verification not implemented) 134

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{a} + x\arccos(ax)$$

output
```
-(-a^2*x^2+1)^(1/2)/a+x*arccos(a*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{a} + x\arccos(ax)$$

input
```
Integrate[ArcCos[a*x],x]
```

output
```
-(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)\,dx$$

$$\Big\downarrow \text{5131}$$

$$a\int \frac{x}{\sqrt{1-a^2x^2}}dx + x\arccos(ax)$$

$$\Big\downarrow \text{241}$$

$$x\arccos(ax) - \frac{\sqrt{1-a^2x^2}}{a}$$

input
```
Int[ArcCos[a*x],x]
```

output
```
-(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{\sqrt{-a^2x^2+1}}{a}+x\arccos\left(ax\right)$	25
derivativedivides	$\frac{ax\arccos(ax)-\sqrt{-a^2x^2+1}}{a}$	27
default	$\frac{ax\arccos(ax)-\sqrt{-a^2x^2+1}}{a}$	27
orering	$x\arccos\left(ax\right)+\frac{(ax-1)(ax+1)}{a\sqrt{-a^2x^2+1}}$	34

input
```
int(arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-(-a^2*x^2+1)^(1/2)/a+x*arccos(a*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx=\frac{ax\arccos\left(ax\right)-\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate(arccos(a*x),x, algorithm="fricas")
```

output
```
(a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \arccos(ax)\,dx=\begin{cases} x\operatorname{acos}\left(ax\right)-\frac{\sqrt{-a^2x^2+1}}{a} & \text{for } a\neq 0\\ \frac{\pi x}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(acos(a*x),x)
```

output
```
Piecewise((x*acos(a*x) - sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi*x/2, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = \frac{ax\arccos\left(ax\right) - \sqrt{-a^2x^2+1}}{a}$$

input
```
integrate(arccos(a*x),x, algorithm="maxima")
```

output
```
(a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = \frac{ax\arccos\left(ax\right) - \sqrt{-a^2x^2+1}}{a}$$

input
```
integrate(arccos(a*x),x, algorithm="giac")
```

output
```
(a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \arccos(ax)\,dx = x\,\mathrm{acos}(a\,x) - \frac{\sqrt{1-a^2\,x^2}}{a}$$

input
```
int(acos(a*x),x)
```

output
```
x*acos(a*x) - (1 - a^2*x^2)^(1/2)/a
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \arccos(ax)\,dx = \frac{acos(ax)\,ax - \sqrt{-a^2x^2+1}}{a}$$

input
```
int(acos(a*x),x)
```

output
```
(acos(a*x)*a*x - sqrt( - a**2*x**2 + 1))/a
```

3.6 $\int \frac{\arccos(ax)}{x} \, dx$

Optimal result 135
Mathematica [A] (verified) 135
Rubi [A] (verified) 136
Maple [A] (verified) 138
Fricas [F] 138
Sympy [F] 139
Maxima [F] 139
Giac [F] 139
Mupad [F(-1)] 140
Reduce [F] 140

Optimal result

Integrand size = 8, antiderivative size = 51

$$\int \frac{\arccos(ax)}{x} \, dx = -\frac{1}{2} i \arccos(ax)^2 + \arccos(ax) \log\left(1 + e^{2i\arccos(ax)}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)$$

output
```
-1/2*I*arccos(a*x)^2+arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-1/2*I*
polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)}{x} \, dx = -\frac{1}{2} i \arccos(ax)^2 + \arccos(ax) \log\left(1 + e^{2i\arccos(ax)}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]/x,x]
```

output
```
(-1/2*I)*ArcCos[a*x]^2 + ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (I/2
)*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x}\,dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax}d\arccos(ax)$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(ax)\tan(\arccos(ax))d\arccos(ax)$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)}{1+e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1+e^{2i\arccos(ax)}\right)de^{2i\arccos(ax)}-\frac{1}{2}i\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2$$

input
```
Int[ArcCos[a*x]/x,x]
```

output
```
(-1/2*I)*ArcCos[a*x]^2 + (2*I)*((-1/2*I)*ArcCos[a*x]*Log[1 + E^((2*I)*ArcC
os[a*x])] - PolyLog[2, -E^((2*I)*ArcCos[a*x])]/4)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{i \arccos(ax)^2}{2}+\arccos(ax)\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)-\frac{i\,\text{polylog}\left(2,-\left(ax+i\sqrt{-a^2x^2+1}\right)\right.}{2}$
default	$-\frac{i \arccos(ax)^2}{2}+\arccos(ax)\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)-\frac{i\,\text{polylog}\left(2,-\left(ax+i\sqrt{-a^2x^2+1}\right)\right.}{2}$

input
```
int(arccos(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*arccos(a*x)^2+arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-1/2*I*
polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)}{x}\,dx = \int \frac{\arccos(ax)}{x}\,dx$$

input
```
integrate(arccos(a*x)/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)}{x} \, dx = \int \frac{\operatorname{acos}(ax)}{x} \, dx$$

input
```
integrate(acos(a*x)/x,x)
```

output
```
Integral(acos(a*x)/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)}{x} \, dx = \int \frac{\arccos(ax)}{x} \, dx$$

input
```
integrate(arccos(a*x)/x,x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)}{x} \, dx = \int \frac{\arccos(ax)}{x} \, dx$$

input
```
integrate(arccos(a*x)/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x} \, dx = \int \frac{\mathrm{acos}(a\,x)}{x} \, dx$$

input
```
int(acos(a*x)/x,x)
```

output
```
int(acos(a*x)/x, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)}{x} \, dx = \int \frac{acos(ax)}{x} dx$$

input
```
int(acos(a*x)/x,x)
```

output
```
int(acos(a*x)/x,x)
```

3.7 $\int \frac{\arccos(ax)}{x^2}\,dx$

Optimal result . . . 141
Mathematica [A] (verified) . . . 141
Rubi [A] (verified) . . . 142
Maple [A] (verified) . . . 143
Fricas [B] (verification not implemented) . . . 144
Sympy [C] (verification not implemented) . . . 144
Maxima [A] (verification not implemented) . . . 145
Giac [A] (verification not implemented) . . . 145
Mupad [B] (verification not implemented) . . . 145
Reduce [B] (verification not implemented) . . . 146

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\arccos(ax)}{x^2}\,dx = -\frac{\arccos(ax)}{x} + a\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-arccos(a*x)/x+a*arctanh((-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\arccos(ax)}{x^2}\,dx = -\frac{\arccos(ax)}{x} - a\log(x) + a\log\left(1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[ArcCos[a*x]/x^2,x]
```

output
```
-(ArcCos[a*x]/x) - a*Log[x] + a*Log[1 + Sqrt[1 - a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5139, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^2}dx$$

$$\Big\downarrow \text{5139}$$

$$-a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\arccos(ax)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\arccos(ax)}{x}$$

$$\Big\downarrow \text{221}$$

$$a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}$$

input
```
Int[ArcCos[a*x]/x^2,x]
```

output
```
-(ArcCos[a*x]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{\arccos(ax)}{x}+a\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$	26
derivativedivides	$a\left(-\frac{\arccos(ax)}{ax}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$	29
default	$a\left(-\frac{\arccos(ax)}{ax}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$	29

input
```
int(arccos(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccos(a*x)/x+a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \frac{\arccos(ax)}{x^2}\,dx = \frac{ax\log\left(\sqrt{-a^2x^2+1}+1\right) - ax\log\left(\sqrt{-a^2x^2+1}-1\right) + 2\,(x-1)\arccos\left(ax\right) - 2\,x\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right)}{2\,x}$$

input
```
integrate(arccos(a*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(a*x*log(sqrt(-a^2*x^2 + 1) + 1) - a*x*log(sqrt(-a^2*x^2 + 1) - 1) + 2
*(x - 1)*arccos(a*x) - 2*x*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\arccos(ax)}{x^2}\,dx = -a\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise} \end{cases}\right) - \frac{\operatorname{acos}\left(ax\right)}{x}$$

input
```
integrate(acos(a*x)/x**2,x)
```

output
```
-a*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), Tr
ue)) - acos(a*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\arccos(ax)}{x^2}\, dx = a \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(ax)}{x}$$

input
```
integrate(arccos(a*x)/x^2,x, algorithm="maxima")
```

output
```
a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(a*x)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\arccos(ax)}{x^2}\, dx = \frac{1}{2}\, a\left(\log\left(\sqrt{-a^2x^2+1}+1\right) - \log\left(-\sqrt{-a^2x^2+1}+1\right)\right) - \frac{\arccos(ax)}{x}$$

input
```
integrate(arccos(a*x)/x^2,x, algorithm="giac")
```

output
```
1/2*a*(log(sqrt(-a^2*x^2 + 1) + 1) - log(-sqrt(-a^2*x^2 + 1) + 1)) - arcco
s(a*x)/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\arccos(ax)}{x^2}\, dx = a\, \mathrm{atanh}\left(\frac{1}{\sqrt{1-a^2\,x^2}}\right) - \frac{\mathrm{acos}(a\,x)}{x}$$

input
```
int(acos(a*x)/x^2,x)
```

output
```
a*atanh(1/(1 - a^2*x^2)^(1/2)) - acos(a*x)/x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)}{x^2}\,dx = \frac{-acos(ax) - \log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)ax}{x}$$

input
```
int(acos(a*x)/x^2,x)
```

output
```
( - (acos(a*x) + log(tan(asin(a*x)/2))*a*x))/x
```

3.8 $\int \frac{\arccos(ax)}{x^3}\,dx$

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Rubi [A] (verified) 148
Maple [A] (verified) 149
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Sympy [C] (verification not implemented) 150
Maxima [A] (verification not implemented) 150
Giac [B] (verification not implemented) 150
Mupad [F(-1)] 151
Reduce [B] (verification not implemented) 151

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\arccos(ax)}{2x^2}$$

output
```
1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*arccos(a*x)/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{ax\sqrt{1-a^2x^2} - \arccos(ax)}{2x^2}$$

input
```
Integrate[ArcCos[a*x]/x^3,x]
```

output
```
(a*x*Sqrt[1 - a^2*x^2] - ArcCos[a*x])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5139, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^3} \, dx$$

$$\Big\downarrow \; 5139$$

$$-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{2x^2}$$

$$\Big\downarrow \; 242$$

$$\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\arccos(ax)}{2x^2}$$

input
```
Int[ArcCos[a*x]/x^3,x]
```

output
```
(a*Sqrt[1 - a^2*x^2])/(2*x) - ArcCos[a*x]/(2*x^2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
parts	$\frac{a\sqrt{-a^2x^2+1}}{2x}-\frac{\arccos(ax)}{2x^2}$	29
derivativedivides	$a^2\left(-\frac{\arccos(ax)}{2a^2x^2}+\frac{\sqrt{-a^2x^2+1}}{2ax}\right)$	38
default	$a^2\left(-\frac{\arccos(ax)}{2a^2x^2}+\frac{\sqrt{-a^2x^2+1}}{2ax}\right)$	38
orering	$\frac{\left(\frac{3}{2}a^2x^3-2x\right)\arccos(ax)}{x^3}+\frac{(ax-1)(ax+1)x^2\left(-\frac{a}{\sqrt{-a^2x^2+1}\,x^3}-\frac{3\arccos(ax)}{x^4}\right)}{2}$	65

input
```
int(arccos(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(-a^2*x^2+1)^(1/2)/x-1/2*arccos(a*x)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{\sqrt{-a^2x^2+1}ax-\arccos{(ax)}}{2\,x^2}$$

input
```
integrate(arccos(a*x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(sqrt(-a^2*x^2 + 1)*a*x - arccos(a*x))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\arccos(ax)}{x^3} dx = -\frac{a\left(\begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for } |a^2x^2| > 1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases}\right)}{2} - \frac{\operatorname{acos}(ax)}{2x^2}$$

input
```
integrate(acos(a*x)/x**3,x)
```

output
```
-a*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*
x**2 + 1)/x, True))/2 - acos(a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\arccos(ax)}{x^3} dx = \frac{\sqrt{-a^2x^2+1}a}{2x} - \frac{\arccos(ax)}{2x^2}$$

input
```
integrate(arccos(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(-a^2*x^2 + 1)*a/x - 1/2*arccos(a*x)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\arccos(ax)}{x^3} dx = -\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)a - \frac{\arccos(ax)}{2x^2}$$

input
```
integrate(arccos(a*x)/x^3,x, algorithm="giac")
```

output
```
-1/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)
*abs(a) + a)/(x*abs(a)))*a - 1/2*arccos(a*x)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^3}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^3}\,dx$$

input
```
int(acos(a*x)/x^3,x)
```

output
```
int(acos(a*x)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{-acos(ax) + \sqrt{-a^2x^2+1}\,ax}{2x^2}$$

input
```
int(acos(a*x)/x^3,x)
```

output
```
( - acos(a*x) + sqrt( - a**2*x**2 + 1)*a*x)/(2*x**2)
```

3.9 $\int \frac{\arccos(ax)}{x^4}\,dx$

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Rubi [A] (verified) . . . 153
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Maxima [A] (verification not implemented) . . . 156
Giac [A] (verification not implemented) . . . 157
Mupad [F(-1)] . . . 157
Reduce [B] (verification not implemented) . . . 157

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} + \frac{1}{6}a^3\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
1/6*a*(-a^2*x^2+1)^(1/2)/x^2-1/3*arccos(a*x)/x^3+1/6*a^3*arctanh((-a^2*x^2
+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} - \frac{1}{6}a^3\log(x) + \frac{1}{6}a^3\log\left(1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[ArcCos[a*x]/x^4,x]
```

output
```
(a*Sqrt[1 - a^2*x^2])/(6*x^2) - ArcCos[a*x]/(3*x^3) - (a^3*Log[x])/6 + (a^
3*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5139, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^4}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{1}{3}a\int \frac{1}{x^3\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx^2-\frac{\arccos(ax)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{1}{6}a\left(\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\arccos(ax)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{6}a\left(-\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\arccos(ax)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{6}a\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\arccos(ax)}{3x^3}$$

input
```
Int[ArcCos[a*x]/x^4,x]
```

output
```
-1/3*ArcCos[a*x]/x^3 - (a*(-(Sqrt[1 - a^2*x^2]/x^2) - a^2*ArcTanh[Sqrt[1 -
 a^2*x^2]]))/6
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\arccos(ax)}{3x^3}-\frac{a\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{3}$	50
derivativedivides	$a^3\left(-\frac{\arccos(ax)}{3a^3x^3}+\frac{\sqrt{-a^2x^2+1}}{6a^2x^2}+\frac{\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{6}\right)$	53
default	$a^3\left(-\frac{\arccos(ax)}{3a^3x^3}+\frac{\sqrt{-a^2x^2+1}}{6a^2x^2}+\frac{\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{6}\right)$	53

input
```
int(arccos(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccos(a*x)/x^3-1/3*a*(-1/2/x^2*(-a^2*x^2+1)^(1/2)-1/2*a^2*arctanh(1/
(-a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(46) = 92$.

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.96

$$\int \frac{\arccos(ax)}{x^4}\,dx$$
$$= \frac{a^3x^3\log\left(\sqrt{-a^2x^2+1}+1\right)-a^3x^3\log\left(\sqrt{-a^2x^2+1}-1\right)-4\,x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right)+2\,\sqrt{-a^2x^2+1}a}{12\,x^3}$$

input
```
integrate(arccos(a*x)/x^4,x, algorithm="fricas")
```

output
```
1/12*(a^3*x^3*log(sqrt(-a^2*x^2 + 1) + 1) - a^3*x^3*log(sqrt(-a^2*x^2 + 1)
 - 1) - 4*x^3*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) + 2*sqrt(-a^2*x
^2 + 1)*a*x + 4*(x^3 - 1)*arccos(a*x))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{\arccos(ax)}{x^4}\,dx = -\frac{a\left(\begin{cases} -\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2} + \frac{a}{2x\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{1}{2ax^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2} - \frac{ia\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3} - \frac{\operatorname{acos}(ax)}{3x^3}$$

input
```
integrate(acos(a*x)/x**4,x)
```

output
```
-a*Piecewise((-a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
1/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (I*a**2*asin
(1/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/3 - acos(a*x)/(3*x
**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)a - \frac{\arccos(ax)}{3x^3}$$

input
```
integrate(arccos(a*x)/x^4,x, algorithm="maxima")
```

output
```
1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/
x^2)*a - 1/3*arccos(a*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{1}{12}\,a^3\left(\frac{2\,\sqrt{-a^2x^2+1}}{a^2x^2} + \log\left(\sqrt{-a^2x^2+1}+1\right) - \log\left(-\sqrt{-a^2x^2+1}+1\right)\right) - \frac{\arccos\left(ax\right)}{3\,x^3}$$

input
```
integrate(arccos(a*x)/x^4,x, algorithm="giac")
```

output
```
1/12*a^3*(2*sqrt(-a^2*x^2 + 1)/(a^2*x^2) + log(sqrt(-a^2*x^2 + 1) + 1) - l
og(-sqrt(-a^2*x^2 + 1) + 1)) - 1/3*arccos(a*x)/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^4}\,dx$$

input
```
int(acos(a*x)/x^4,x)
```

output
```
int(acos(a*x)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{-2acos(ax) + \sqrt{-a^2x^2+1}\,ax - \log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)a^3x^3}{6x^3}$$

input
```
int(acos(a*x)/x^4,x)
```

output
```
( - 2*acos(a*x) + sqrt( - a**2*x**2 + 1)*a*x - log(tan(asin(a*x)/2))*a**3*
x**3)/(6*x**3)
```

3.10 $\int \frac{\arccos(ax)}{x^5}\,dx$

Optimal result . . . 159
Mathematica [A] (verified) . . . 159
Rubi [A] (verified) . . . 160
Maple [A] (verified) . . . 161
Fricas [A] (verification not implemented) . . . 162
Sympy [C] (verification not implemented) . . . 162
Maxima [A] (verification not implemented) . . . 163
Giac [B] (verification not implemented) . . . 163
Mupad [F(-1)] . . . 164
Reduce [B] (verification not implemented) . . . 164

Optimal result

Integrand size = 8, antiderivative size = 58

$$\int \frac{\arccos(ax)}{x^5}\,dx = \frac{a\sqrt{1-a^2x^2}}{12x^3} + \frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{\arccos(ax)}{4x^4}$$

output
```
1/12*a*(-a^2*x^2+1)^(1/2)/x^3+1/6*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*arccos(a*x)
/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{\arccos(ax)}{x^5}\,dx = \frac{ax\sqrt{1-a^2x^2}(1+2a^2x^2) - 3\arccos(ax)}{12x^4}$$

input
```
Integrate[ArcCos[a*x]/x^5,x]
```

output
```
(a*x*Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2) - 3*ArcCos[a*x])/(12*x^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5139, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^5} \, dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)}{4x^4}$$

$$\Big\downarrow \text{245}$$

$$-\frac{1}{4}a\left(\frac{2}{3}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}}{3x^3}\right) - \frac{\arccos(ax)}{4x^4}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{4}a\left(-\frac{2a^2\sqrt{1-a^2x^2}}{3x} - \frac{\sqrt{1-a^2x^2}}{3x^3}\right) - \frac{\arccos(ax)}{4x^4}$$

input
```
Int[ArcCos[a*x]/x^5,x]
```

output
```
-1/4*(a*(-1/3*Sqrt[1 - a^2*x^2]/x^3 - (2*a^2*Sqrt[1 - a^2*x^2])/(3*x))) -
ArcCos[a*x]/(4*x^4)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
parts	$-\frac{\arccos(ax)}{4x^4}-\frac{a\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3}-\frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right)}{4}$	52
derivativedivides	$a^4\left(-\frac{\arccos(ax)}{4a^4x^4}+\frac{\sqrt{-a^2x^2+1}}{12a^3x^3}+\frac{\sqrt{-a^2x^2+1}}{6ax}\right)$	58
default	$a^4\left(-\frac{\arccos(ax)}{4a^4x^4}+\frac{\sqrt{-a^2x^2+1}}{12a^3x^3}+\frac{\sqrt{-a^2x^2+1}}{6ax}\right)$	58
orering	$\frac{\left(\frac{5}{6}a^4x^5-\frac{5}{12}a^2x^3-\frac{2}{3}x\right)\arccos(ax)}{x^5}+\frac{(2a^2x^2+1)(ax-1)(ax+1)x^2\left(-\frac{a}{\sqrt{-a^2x^2+1}\,x^5}-\frac{5\arccos(ax)}{x^6}\right)}{12}$	83

input
```
int(arccos(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccos(a*x)/x^4-1/4*a*(-1/3/x^3*(-a^2*x^2+1)^(1/2)-2/3*a^2/x*(-a^2*x^
2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

$$\int \frac{\arccos(ax)}{x^5} \, dx = \frac{(2\,a^3x^3 + ax)\sqrt{-a^2x^2 + 1} - 3\,\arccos{(ax)}}{12\,x^4}$$

input
```
integrate(arccos(a*x)/x^5,x, algorithm="fricas")
```

output
```
1/12*((2*a^3*x^3 + a*x)*sqrt(-a^2*x^2 + 1) - 3*arccos(a*x))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

$$\int \frac{\arccos(ax)}{x^5} \, dx = -\frac{a\left(\begin{cases} -\frac{2ia^2\sqrt{a^2x^2-1}}{3x} - \frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2| > 1 \\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x} - \frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{4} - \frac{\operatorname{acos}{(ax)}}{4x^4}$$

input
```
integrate(acos(a*x)/x**5,x)
```

output
```
-a*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/
(3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(
-a**2*x**2 + 1)/(3*x**3), True))/4 - acos(a*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\arccos(ax)}{x^5}\,dx = \frac{1}{12}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x} + \frac{\sqrt{-a^2x^2+1}}{x^3}\right)a - \frac{\arccos(ax)}{4\,x^4}$$

input
```
integrate(arccos(a*x)/x^5,x, algorithm="maxima")
```

output
```
1/12*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*a - 1/4*arccos(
a*x)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(48) = 96$.

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.24

$$\int \frac{\arccos(ax)}{x^5}\,dx = -\frac{1}{96}\left(\frac{\left(a^4 + \frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} - \frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x} + \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{x^3}}{a^2|a|}\right)a - \frac{\arccos(ax)}{4\,x^4}$$

input
```
integrate(arccos(a*x)/x^5,x, algorithm="giac")
```

output
```
-1/96*((a^4 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2
*x^2 + 1)*abs(a) + a)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x
 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*a - 1/4*arccos(a*x
)/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^5} \, dx = \int \frac{\mathrm{acos}(a\,x)}{x^5} \, dx$$

input
```
int(acos(a*x)/x^5,x)
```

output
```
int(acos(a*x)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(ax)}{x^5} \, dx = \frac{-3acos(ax) + 2\sqrt{-a^2x^2+1}\,a^3x^3 + \sqrt{-a^2x^2+1}\,ax}{12x^4}$$

input
```
int(acos(a*x)/x^5,x)
```

output
```
( - 3*acos(a*x) + 2*sqrt( - a**2*x**2 + 1)*a**3*x**3 + sqrt( - a**2*x**2 +
 1)*a*x)/(12*x**4)
```

3.11 $\int \frac{\arccos(ax)}{x^6} dx$

Optimal result 165
Mathematica [A] (verified) 165
Rubi [A] (verified) 166
Maple [A] (verified) 168
Fricas [A] (verification not implemented) 168
Sympy [C] (verification not implemented) 169
Maxima [A] (verification not implemented) 169
Giac [A] (verification not implemented) 170
Mupad [F(-1)] 170
Reduce [B] (verification not implemented) 171

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int \frac{\arccos(ax)}{x^6} dx = \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\arccos(ax)}{5x^5} + \frac{3}{40}a^5\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
1/20*a*(-a^2*x^2+1)^(1/2)/x^4+3/40*a^3*(-a^2*x^2+1)^(1/2)/x^2-1/5*arccos(a
*x)/x^5+3/40*a^5*arctanh((-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\arccos(ax)}{x^6} dx = \frac{1}{40}\left(\frac{a\sqrt{1-a^2x^2}(2+3a^2x^2)}{x^4} - \frac{8\arccos(ax)}{x^5} - 3a^5\log(x) + 3a^5\log\left(1+\sqrt{1-a^2x^2}\right)\right)$$

input
```
Integrate[ArcCos[a*x]/x^6,x]
```

output
```
((a*Sqrt[1 - a^2*x^2]*(2 + 3*a^2*x^2))/x^4 - (8*ArcCos[a*x])/x^5 - 3*a^5*L
og[x] + 3*a^5*Log[1 + Sqrt[1 - a^2*x^2]])/40
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5139, 243, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^6}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{1}{5}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{10}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}dx^2-\frac{\arccos(ax)}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$-\frac{1}{10}a\left(\frac{3}{4}a^2\int \frac{1}{x^4\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\arccos(ax)}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$-\frac{1}{10}a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\arccos(ax)}{5x^5}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{10}a\left(\frac{3}{4}a^2\left(-\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\arccos(ax)}{5x^5}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{10}a\left(\frac{3}{4}a^2\left(a^2\left(-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\right)-\frac{\sqrt{1-a^2x^2}}{x^2}\right)-\frac{\sqrt{1-a^2x^2}}{2x^4}\right)-\frac{\arccos(ax)}{5x^5}$$

input
```
Int[ArcCos[a*x]/x^6,x]
```

output
```
-1/5*ArcCos[a*x]/x^5 - (a*(-1/2*Sqrt[1 - a^2*x^2]/x^4 + (3*a^2*(-(Sqrt[1 -
 a^2*x^2]/x^2) - a^2*ArcTanh[Sqrt[1 - a^2*x^2]]))/4))/10
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$a^5\left(-\frac{\arccos(ax)}{5a^5x^5}+\frac{\sqrt{-a^2x^2+1}}{20a^4x^4}+\frac{3\sqrt{-a^2x^2+1}}{40a^2x^2}+\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{40}\right)$	73
default	$a^5\left(-\frac{\arccos(ax)}{5a^5x^5}+\frac{\sqrt{-a^2x^2+1}}{20a^4x^4}+\frac{3\sqrt{-a^2x^2+1}}{40a^2x^2}+\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{40}\right)$	73
parts	$-\frac{\arccos(ax)}{5x^5}-\frac{a\left(-\frac{\sqrt{-a^2x^2+1}}{4x^4}+\frac{3a^2\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\,\text{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{4}\right)}{5}$	73

input
```
int(arccos(a*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-1/5*arccos(a*x)/a^5/x^5+1/20/a^4/x^4*(-a^2*x^2+1)^(1/2)+3/40/a^2/x^2
*(-a^2*x^2+1)^(1/2)+3/40*arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int \frac{\arccos(ax)}{x^6}\,dx$$
$$= \frac{3\,a^5x^5\log\left(\sqrt{-a^2x^2+1}+1\right)-3\,a^5x^5\log\left(\sqrt{-a^2x^2+1}-1\right)-16\,x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right)+16\,(x^5-1)}{80\,x^5}$$

input
```
integrate(arccos(a*x)/x^6,x, algorithm="fricas")
```

output
```
1/80*(3*a^5*x^5*log(sqrt(-a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(sqrt(-a^2*x^2
+ 1) - 1) - 16*x^5*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) + 16*(x^5
- 1)*arccos(a*x) + 2*(3*a^3*x^3 + 2*a*x)*sqrt(-a^2*x^2 + 1))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.30

$$\int \frac{\arccos(ax)}{x^6}\,dx$$
$$= -\frac{a\left(\begin{cases} -\frac{3a^4\operatorname{acosh}\left(\frac{1}{ax}\right)}{8} + \frac{3a^3}{8x\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{a}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{1}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ \frac{3ia^4\operatorname{asin}\left(\frac{1}{ax}\right)}{8} - \frac{3ia^3}{8x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{ia}{8x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{i}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise} \end{cases}\right)}{5}$$
$$- \frac{\operatorname{acos}(ax)}{5x^5}$$

input
```
integrate(acos(a*x)/x**6,x)
```

output
```
-a*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**
2))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2
*x**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x
*sqrt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/5 - acos(a*x)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\arccos(ax)}{x^6}\,dx$$
$$= \frac{1}{40}\left(3\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{3\sqrt{-a^2x^2+1}a^2}{x^2} + \frac{2\sqrt{-a^2x^2+1}}{x^4}\right)a$$
$$- \frac{\arccos(ax)}{5\,x^5}$$

input
```
integrate(arccos(a*x)/x^6,x, algorithm="maxima")
```

output
```
1/40*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a^2*x^2
+ 1)*a^2/x^2 + 2*sqrt(-a^2*x^2 + 1)/x^4)*a - 1/5*arccos(a*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{\arccos(ax)}{x^6}\,dx$$
$$= \frac{3\,a^6\log\left(\sqrt{-a^2x^2+1}+1\right)-3\,a^6\log\left(-\sqrt{-a^2x^2+1}+1\right)-\frac{2\left(3\left(-a^2x^2+1\right)^{\frac{3}{2}}a^6-5\sqrt{-a^2x^2+1}a^6\right)}{a^4x^4}}{80\,a} - \frac{\arccos\left(ax\right)}{5\,x^5}$$

input
```
integrate(arccos(a*x)/x^6,x, algorithm="giac")
```

output
```
1/80*(3*a^6*log(sqrt(-a^2*x^2 + 1) + 1) - 3*a^6*log(-sqrt(-a^2*x^2 + 1) +
1) - 2*(3*(-a^2*x^2 + 1)^(3/2)*a^6 - 5*sqrt(-a^2*x^2 + 1)*a^6)/(a^4*x^4))/
a - 1/5*arccos(a*x)/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^6}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^6}\,dx$$

input
```
int(acos(a*x)/x^6,x)
```

output
```
int(acos(a*x)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{\arccos(ax)}{x^6}\,dx = \frac{-8acos(ax) + 3\sqrt{-a^2x^2+1}\,a^3x^3 + 2\sqrt{-a^2x^2+1}\,ax - 3\log\left(\tan\left(\frac{asin(ax)}{2}\right)\right)a^5x^5}{40x^5}$$

input
```
int(acos(a*x)/x^6,x)
```

output
```
( - 8*acos(a*x) + 3*sqrt( - a**2*x**2 + 1)*a**3*x**3 + 2*sqrt( - a**2*x**2
 + 1)*a*x - 3*log(tan(asin(a*x)/2))*a**5*x**5)/(40*x**5)
```

3.12 $\int x^4 \arccos(ax)^2 \, dx$

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Optimal result

Integrand size = 10, antiderivative size = 120

$$\int x^4 \arccos(ax)^2 \, dx = -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{16\sqrt{1-a^2x^2}\arccos(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} + \frac{1}{5}x^5\arccos(ax)^2$$

output
```
-16/75*x/a^4-8/225*x^3/a^2-2/125*x^5-16/75*(-a^2*x^2+1)^(1/2)*arccos(a*x)/
a^5-8/75*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3-2/25*x^4*(-a^2*x^2+1)^(1/2
)*arccos(a*x)/a+1/5*x^5*arccos(a*x)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int x^4 \arccos(ax)^2 \, dx = -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{2\sqrt{1-a^2x^2}(8+4a^2x^2+3a^4x^4)\arccos(ax)}{75a^5} + \frac{1}{5}x^5\arccos(ax)^2$$

input
```
Integrate[x^4*ArcCos[a*x]^2,x]
```

output
```
(-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 - (2*Sqrt[1 - a^2*x^2]*
(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCos[a*x])/(75*a^5) + (x^5*ArcCos[a*x]^2)/5
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5139, 5211, 15, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arccos(ax)^2 \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2}{5}a\int \frac{x^5 \arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^5 \arccos(ax)^2$$

$$\Big\downarrow \text{5211}$$

$$\frac{2}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{\int x^4dx}{5a}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)}{5a^2}\right)+\frac{1}{5}x^5 \arccos(ax)^2$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)}{5a^2}-\frac{x^5}{25a}\right)+\frac{1}{5}x^5 \arccos(ax)^2$$

$$\Big\downarrow \text{5211}$$

$$\frac{2}{5}a\left(\frac{4\left(\frac{2\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)}{5a^2}-\frac{x^5}{25a}\right)+\frac{1}{5}x^5 \arccos(ax)^2$$

$$\Big\downarrow \; 15$$

$$\frac{2}{5}a\left(\frac{4\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)}{5a^2}-\frac{x^5}{25a}\right)+\frac{1}{5}x^5\arccos(ax)^2$$

$$\Big\downarrow \; 5183$$

$$\frac{2}{5}a\left(\frac{4\left(\frac{2\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)}{5a^2}-\frac{x^5}{25a}\right)+\frac{1}{5}x^5\arccos(ax)^2$$

$$\Big\downarrow \; 24$$

$$\frac{2}{5}a\left(-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)}{5a^2}+\frac{4\left(-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}\right)}{5a^2}-\frac{x^5}{25a}\right)+\frac{1}{5}x^5\arccos(ax)^2$$

input
```
Int[x^4*ArcCos[a*x]^2,x]
```

output
```
(x^5*ArcCos[a*x]^2)/5 + (2*a*(-1/25*x^5/a - (x^4*Sqrt[1 - a^2*x^2]*ArcCos[
a*x])/(5*a^2) + (4*(-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(3*a^
2) + (2*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3*a^2)))/(5*a^2))
)/5
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\frac{a^{5}x^{5}\arccos\left(ax\right)^{2}}{5}-\frac{2\arccos\left(ax\right)\left(3a^{4}x^{4}+4a^{2}x^{2}+8\right)\sqrt{-a^{2}x^{2}+1}}{75}-\frac{2a^{5}x^{5}}{125}-\frac{8a^{3}x^{3}}{225}-\frac{16ax}{75}}{a^{5}}$
default	$\frac{\frac{a^{5}x^{5}\arccos\left(ax\right)^{2}}{5}-\frac{2\arccos\left(ax\right)\left(3a^{4}x^{4}+4a^{2}x^{2}+8\right)\sqrt{-a^{2}x^{2}+1}}{75}-\frac{2a^{5}x^{5}}{125}-\frac{8a^{3}x^{3}}{225}-\frac{16ax}{75}}{a^{5}}$
orering	$\frac{\left(549a^{6}x^{6}+200a^{4}x^{4}+1760a^{2}x^{2}-2880\right)\arccos\left(ax\right)^{2}}{1125a^{6}x}-\frac{\left(108a^{6}x^{6}+83a^{4}x^{4}+740a^{2}x^{2}-1080\right)\left(4x^{3}\arccos\left(ax\right)^{2}-\frac{2x^{4}a\ldots}{\sqrt{-\ldots}}\right.}{1125x^{4}a^{6}}$

input
```
int(x^4*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccos(a*x)^2-2/75*arccos(a*x)*(3*a^4*x^4+4*a^2*x^2+8)*
(-a^2*x^2+1)^(1/2)-2/125*a^5*x^5-8/225*a^3*x^3-16/75*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

$$\int x^4 \arccos(ax)^2 \, dx$$
$$= \frac{225\,a^5x^5\arccos\left(ax\right)^2 - 18\,a^5x^5 - 40\,a^3x^3 - 30\left(3\,a^4x^4 + 4\,a^2x^2 + 8\right)\sqrt{-a^2x^2+1}\arccos\left(ax\right) - 240\,ax}{1125\,a^5}$$

input
```
integrate(x^4*arccos(a*x)^2,x, algorithm="fricas")
```

output
```
1/1125*(225*a^5*x^5*arccos(a*x)^2 - 18*a^5*x^5 - 40*a^3*x^3 - 30*(3*a^4*x^
4 + 4*a^2*x^2 + 8)*sqrt(-a^2*x^2 + 1)*arccos(a*x) - 240*a*x)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int x^4 \arccos(ax)^2\,dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acos}^2(ax)}{5} - \frac{2x^5}{125} - \frac{2x^4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{25a} - \frac{8x^3}{225a^2} - \frac{8x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{75a^3} - \frac{16x}{75a^4} - \frac{16\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{75a^5} & \text{for } a \\ \frac{\pi^2x^5}{20} & \text{other} \end{cases}$$

input
```
integrate(x**4*acos(a*x)**2,x)
```

output
```
Piecewise((x**5*acos(a*x)**2/5 - 2*x**5/125 - 2*x**4*sqrt(-a**2*x**2 + 1)*
acos(a*x)/(25*a) - 8*x**3/(225*a**2) - 8*x**2*sqrt(-a**2*x**2 + 1)*acos(a*
x)/(75*a**3) - 16*x/(75*a**4) - 16*sqrt(-a**2*x**2 + 1)*acos(a*x)/(75*a**5
), Ne(a, 0)), (pi**2*x**5/20, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int x^4 \arccos(ax)^2\,dx$$
$$= \frac{1}{5}x^5 \arccos(ax)^2 - \frac{2}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a\arccos(ax) - \frac{2(9a^4x^5 + 20a^2x^3 + 120x)}{1125a^4}$$

input
```
integrate(x^4*arccos(a*x)^2,x, algorithm="maxima")
```

output
```
1/5*x^5*arccos(a*x)^2 - 2/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x
^2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x) - 2/1125*(9*a^4*
x^5 + 20*a^2*x^3 + 120*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int x^4 \arccos(ax)^2 \, dx = \frac{1}{5} x^5 \arccos{(ax)}^2 - \frac{2}{125} x^5 - \frac{2 \sqrt{-a^2x^2+1} x^4 \arccos{(ax)}}{25 \, a} - \frac{8 \, x^3}{225 \, a^2} - \frac{8 \sqrt{-a^2x^2+1} x^2 \arccos{(ax)}}{75 \, a^3} - \frac{16 \, x}{75 \, a^4} - \frac{16 \sqrt{-a^2x^2+1} \arccos{(ax)}}{75 \, a^5}$$

input
```
integrate(x^4*arccos(a*x)^2,x, algorithm="giac")
```

output
```
1/5*x^5*arccos(a*x)^2 - 2/125*x^5 - 2/25*sqrt(-a^2*x^2 + 1)*x^4*arccos(a*x
)/a - 8/225*x^3/a^2 - 8/75*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a^3 - 16/75*
x/a^4 - 16/75*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^2 \, dx = \int x^4 \, \mathrm{acos}(a\,x)^2 \, dx$$

input
```
int(x^4*acos(a*x)^2,x)
```

output
```
int(x^4*acos(a*x)^2, x)
```

Reduce [F]

$$\int x^4 \arccos(ax)^2 \, dx = \int acos(ax)^2 \, x^4 dx$$

input
```
int(x^4*acos(a*x)^2,x)
```

output
```
int(acos(a*x)**2*x**4,x)
```

3.13 $\int x^3 \arccos(ax)^2 \, dx$

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Reduce [F] 185

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int x^3 \arccos(ax)^2 \, dx = -\frac{3x^2}{32a^2} - \frac{x^4}{32} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{16a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{8a} - \frac{3\arccos(ax)^2}{32a^4} + \frac{1}{4}x^4\arccos(ax)^2$$

output
```
-3/32*x^2/a^2-1/32*x^4-3/16*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3-1/8*x^3*(
-a^2*x^2+1)^(1/2)*arccos(a*x)/a-3/32*arccos(a*x)^2/a^4+1/4*x^4*arccos(a*x)
^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x^3 \arccos(ax)^2 \, dx = \frac{-a^2x^2(3+a^2x^2) - 2ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arccos(ax) + (-3+8a^4x^4)\arccos(ax)^2}{32a^4}$$

input
```
Integrate[x^3*ArcCos[a*x]^2,x]
```

output
```
(-(a^2*x^2*(3 + a^2*x^2)) - 2*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos
[a*x] + (-3 + 8*a^4*x^4)*ArcCos[a*x]^2)/(32*a^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5139, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax)^2 \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{2}a\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)^2$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int x^3dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^2$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{2}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+$$
$$\frac{1}{4}x^4\arccos(ax)^2$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+$$
$$\frac{1}{4}x^4\arccos(ax)^2$$

$$\Big\downarrow \; 5153$$

$$\frac{1}{2}a\left(-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}\right)+$$
$$\frac{1}{4}x^4\arccos(ax)^2$$

input
```
Int[x^3*ArcCos[a*x]^2,x]
```

output
```
(x^4*ArcCos[a*x]^2)/4 + (a*(-1/16*x^4/a - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*
x])/(4*a^2) + (3*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) -
 ArcCos[a*x]^2/(4*a^3)))/(4*a^2)))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{a^4x^4\arccos(ax)^2}{4}-\frac{\arccos(ax)\left(2a^3x^3\sqrt{-a^2x^2+1}+3\sqrt{-a^2x^2+1}\,ax+3\arccos(ax)\right)}{16}+\frac{3\arccos(ax)^2}{32}-\frac{\left(2a^2x^2+3\right)^2}{128}}{a^4}$
default	$\frac{\frac{a^4x^4\arccos(ax)^2}{4}-\frac{\arccos(ax)\left(2a^3x^3\sqrt{-a^2x^2+1}+3\sqrt{-a^2x^2+1}\,ax+3\arccos(ax)\right)}{16}+\frac{3\arccos(ax)^2}{32}-\frac{\left(2a^2x^2+3\right)^2}{128}}{a^4}$
orering	$\frac{(37a^4x^4+21a^2x^2-60)\arccos(ax)^2}{64a^4}-\frac{(9a^4x^4+11a^2x^2-24)\left(3x^2\arccos(ax)^2-\frac{2x^3\arccos(ax)a}{\sqrt{-a^2x^2+1}}\right)}{64x^2a^4}+\frac{(a^2x^2+3)(ax-}{}$

input
```
int(x^3*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arccos(a*x)^2-1/16*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^
(1/2)+3*(-a^2*x^2+1)^(1/2)*a*x+3*arccos(a*x))+3/32*arccos(a*x)^2-1/128*(2*
a^2*x^2+3)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int x^3\arccos(ax)^2\,dx = -\frac{a^4x^4+3\,a^2x^2-(8\,a^4x^4-3)\arccos(ax)^2+2\,(2\,a^3x^3+3\,ax)\sqrt{-a^2x^2+1}\arccos(ax)}{32\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^2,x, algorithm="fricas")
```

output
```
-1/32*(a^4*x^4 + 3*a^2*x^2 - (8*a^4*x^4 - 3)*arccos(a*x)^2 + 2*(2*a^3*x^3
+ 3*a*x)*sqrt(-a^2*x^2 + 1)*arccos(a*x))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int x^3 \arccos(ax)^2\,dx$$
$$= \begin{cases} \frac{x^4 \operatorname{acos}^2(ax)}{4} - \frac{x^4}{32} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{16a^3} - \frac{3\operatorname{acos}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ \frac{\pi^2x^4}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acos(a*x)**2,x)
```

output
```
Piecewise((x**4*acos(a*x)**2/4 - x**4/32 - x**3*sqrt(-a**2*x**2 + 1)*acos(
a*x)/(8*a) - 3*x**2/(32*a**2) - 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(16*a**
3) - 3*acos(a*x)**2/(32*a**4), Ne(a, 0)), (pi**2*x**4/16, True))
```

Maxima [F]

$$\int x^3 \arccos(ax)^2\,dx = \int x^3 \arccos{(ax)}^2\,dx$$

input
```
integrate(x^3*arccos(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - a*integrate(1/2*sqr
t(a*x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/(
a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int x^3 \arccos(ax)^2 \, dx = \frac{1}{4} x^4 \arccos(ax)^2 - \frac{1}{32} x^4 - \frac{\sqrt{-a^2x^2+1} x^3 \arccos(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3\sqrt{-a^2x^2+1} x \arccos(ax)}{16a^3} - \frac{3 \arccos(ax)^2}{32a^4} + \frac{15}{256a^4}$$

input
```
integrate(x^3*arccos(a*x)^2,x, algorithm="giac")
```

output
```
1/4*x^4*arccos(a*x)^2 - 1/32*x^4 - 1/8*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/
a - 3/32*x^2/a^2 - 3/16*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^3 - 3/32*arccos
(a*x)^2/a^4 + 15/256/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^2 \, dx = \int x^3 \operatorname{acos}(a\,x)^2 \, dx$$

input
```
int(x^3*acos(a*x)^2,x)
```

output
```
int(x^3*acos(a*x)^2, x)
```

Reduce [F]

$$\int x^3 \arccos(ax)^2 \, dx = \int acos(ax)^2 x^3 dx$$

input
```
int(x^3*acos(a*x)^2,x)
```

output
```
int(acos(a*x)**2*x**3,x)
```

3.14 $\int x^2 \arccos(ax)^2 \, dx$

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Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x^2 \arccos(ax)^2 \, dx = -\frac{4x}{9a^2} - \frac{2x^3}{27} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)}{9a^3} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a} + \frac{1}{3}x^3\arccos(ax)^2$$

output
```
-4/9*x/a^2-2/27*x^3-4/9*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3-2/9*x^2*(-a^2*x
^2+1)^(1/2)*arccos(a*x)/a+1/3*x^3*arccos(a*x)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int x^2 \arccos(ax)^2 \, dx = -\frac{4x}{9a^2} - \frac{2x^3}{27} - \frac{2\sqrt{1-a^2x^2}(2+a^2x^2)\arccos(ax)}{9a^3} + \frac{1}{3}x^3\arccos(ax)^2$$

input
```
Integrate[x^2*ArcCos[a*x]^2,x]
```

output
```
(-4*x)/(9*a^2) - (2*x^3)/27 - (2*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*
x])/(9*a^3) + (x^3*ArcCos[a*x]^2)/3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5139, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)^2 \, dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{2}{3}a\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^2$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{2}{3}a\left(\frac{2\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2}{3}a\left(-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2$$

input
```
Int[x^2*ArcCos[a*x]^2,x]
```

output
```
(x^3*ArcCos[a*x]^2)/3 + (2*a*(-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a
*x])/(3*a^2) + (2*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3*a^2))
)/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{a^3x^3\arccos(ax)^2}{3}-\frac{2\arccos(ax)\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9}-\frac{2a^3x^3}{27}-\frac{4ax}{9}}{a^3}$
default	$\frac{\frac{a^3x^3\arccos(ax)^2}{3}-\frac{2\arccos(ax)\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9}-\frac{2a^3x^3}{27}-\frac{4ax}{9}}{a^3}$
orering	$\frac{(19a^4x^4+24a^2x^2-48)\arccos(ax)^2}{27a^4x}-\frac{(6a^4x^4+17a^2x^2-30)\left(2x\arccos(ax)^2-\frac{2x^2\arccos(ax)a}{\sqrt{-a^2x^2+1}}\right)}{27x^2a^4}+\frac{(a^2x^2+6)(ax-1}{}$

input
```
int(x^2*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*a^3*x^3*arccos(a*x)^2-2/9*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^
(1/2)-2/27*a^3*x^3-4/9*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int x^2\arccos(ax)^2\,dx = \frac{9\,a^3x^3\arccos\left(ax\right)^2-2\,a^3x^3-6\,(a^2x^2+2)\sqrt{-a^2x^2+1}\arccos\left(ax\right)-12\,ax}{27\,a^3}$$

input
```
integrate(x^2*arccos(a*x)^2,x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*arccos(a*x)^2 - 2*a^3*x^3 - 6*(a^2*x^2 + 2)*sqrt(-a^2*x^2
+ 1)*arccos(a*x) - 12*a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int x^2 \arccos(ax)^2\,dx = \begin{cases} \frac{x^3 \operatorname{acos}^2(ax)}{3} - \frac{2x^3}{27} - \frac{2x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a} - \frac{4x}{9a^2} - \frac{4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a^3} & \text{for } a \neq 0 \\ \frac{\pi^2x^3}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acos(a*x)**2,x)
```

output
```
Piecewise((x**3*acos(a*x)**2/3 - 2*x**3/27 - 2*x**2*sqrt(-a**2*x**2 + 1)*a
cos(a*x)/(9*a) - 4*x/(9*a**2) - 4*sqrt(-a**2*x**2 + 1)*acos(a*x)/(9*a**3),
 Ne(a, 0)), (pi**2*x**3/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(ax)^2\,dx = \frac{1}{3}\,x^3 \arccos(ax)^2 - \frac{2}{9}\,a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax) - \frac{2\,(a^2x^3+6\,x)}{27\,a^2}$$

input
```
integrate(x^2*arccos(a*x)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(a*x)^2 - 2/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^
2 + 1)/a^4)*arccos(a*x) - 2/27*(a^2*x^3 + 6*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2 \arccos(ax)^2 \, dx = \frac{1}{3} x^3 \arccos(ax)^2 - \frac{2}{27} x^3 - \frac{2\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{9a} - \frac{4x}{9a^2} - \frac{4\sqrt{-a^2x^2+1}\arccos(ax)}{9a^3}$$

input
```
integrate(x^2*arccos(a*x)^2,x, algorithm="giac")
```

output
```
1/3*x^3*arccos(a*x)^2 - 2/27*x^3 - 2/9*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/
a - 4/9*x/a^2 - 4/9*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^2 \, dx = \int x^2 \operatorname{acos}(a\,x)^2 \, dx$$

input
```
int(x^2*acos(a*x)^2,x)
```

output
```
int(x^2*acos(a*x)^2, x)
```

Reduce [F]

$$\int x^2 \arccos(ax)^2 \, dx = \int acos(ax)^2 x^2 dx$$

input
```
int(x^2*acos(a*x)^2,x)
```

output
```
int(acos(a*x)**2*x**2,x)
```

3.15 $\int x \arccos(ax)^2 \, dx$

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Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \arccos(ax)^2 \, dx = -\frac{x^2}{4} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} - \frac{\arccos(ax)^2}{4a^2} + \frac{1}{2}x^2\arccos(ax)^2$$

output
```
-1/4*x^2-1/2*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a-1/4*arccos(a*x)^2/a^2+1/2*
x^2*arccos(a*x)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x \arccos(ax)^2 \, dx = -\frac{x^2}{4} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} + \frac{(-1+2a^2x^2)\arccos(ax)^2}{4a^2}$$

input
```
Integrate[x*ArcCos[a*x]^2,x]
```

output
```
-1/4*x^2 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) + ((-1 + 2*a^2*x^2)*Arc
Cos[a*x]^2)/(4*a^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5139, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax)^2 \, dx$$

$$\Big\downarrow \; 5139$$

$$a \int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}dx + \frac{1}{2}x^2 \arccos(ax)^2$$

$$\Big\downarrow \; 5211$$

$$a \left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{\int xdx}{2a} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right) + \frac{1}{2}x^2 \arccos(ax)^2$$

$$\Big\downarrow \; 15$$

$$a \left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2} - \frac{x^2}{4a}\right) + \frac{1}{2}x^2 \arccos(ax)^2$$

$$\Big\downarrow \; 5153$$

$$a \left(-\frac{\arccos(ax)^2}{4a^3} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2} - \frac{x^2}{4a}\right) + \frac{1}{2}x^2 \arccos(ax)^2$$

input
```
Int[x*ArcCos[a*x]^2,x]
```

output
```
(x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/
(2*a^2) - ArcCos[a*x]^2/(4*a^3))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{\cos(2\arccos(ax))\arccos(ax)^2}{4}-\frac{\cos(2\arccos(ax))}{8}-\frac{\sin(2\arccos(ax))\arccos(ax)}{4}}{a^2}$
default	$\frac{\frac{\cos(2\arccos(ax))\arccos(ax)^2}{4}-\frac{\cos(2\arccos(ax))}{8}-\frac{\sin(2\arccos(ax))\arccos(ax)}{4}}{a^2}$
orering	$\frac{(7a^2x^2-6)\arccos(ax)^2}{8a^2}-\frac{(3a^2x^2-4)\left(\arccos(ax)^2-\frac{2ax\arccos(ax)}{\sqrt{-a^2x^2+1}}\right)}{8a^2}+\frac{x(ax-1)(ax+1)\left(-\frac{4\arccos(ax)a}{\sqrt{-a^2x^2+1}}-\frac{2a^3x^2 \text{ar}}{(-a^2x}\right.}{8a^2}$

input
```
int(x*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*cos(2*arccos(a*x))*arccos(a*x)^2-1/8*cos(2*arccos(a*x))-1/4*sin
(2*arccos(a*x))*arccos(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x \arccos(ax)^2 \, dx = -\frac{a^2x^2 + 2\sqrt{-a^2x^2+1}ax \arccos(ax) - (2a^2x^2-1)\arccos(ax)^2}{4a^2}$$

input
```
integrate(x*arccos(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(a^2*x^2 + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) - (2*a^2*x^2 - 1)*arc
cos(a*x)^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int x \arccos(ax)^2 \, dx = \begin{cases} \frac{x^2 \operatorname{acos}^2(ax)}{2} - \frac{x^2}{4} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{2a} - \frac{\operatorname{acos}^2(ax)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x)**2,x)
```

output
```
Piecewise((x**2*acos(a*x)**2/2 - x**2/4 - x*sqrt(-a**2*x**2 + 1)*acos(a*x)
/(2*a) - acos(a*x)**2/(4*a**2), Ne(a, 0)), (pi**2*x**2/8, True))
```

Maxima [F]

$$\int x \arccos(ax)^2\,dx = \int x \arccos(ax)^2\,dx$$

input
```
integrate(x*arccos(a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - a*integrate(sqrt(a*
x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/(a^2*
x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x \arccos(ax)^2\,dx = \frac{1}{2}\,x^2 \arccos(ax)^2 - \frac{1}{4}\,x^2 - \frac{\sqrt{-a^2x^2+1}x \arccos(ax)}{2\,a} - \frac{\arccos(ax)^2}{4\,a^2} + \frac{1}{8\,a^2}$$

input
```
integrate(x*arccos(a*x)^2,x, algorithm="giac")
```

output
```
1/2*x^2*arccos(a*x)^2 - 1/4*x^2 - 1/2*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a -
 1/4*arccos(a*x)^2/a^2 + 1/8/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^2\,dx = \int x\,\mathrm{acos}(a\,x)^2\,dx$$

input
```
int(x*acos(a*x)^2,x)
```

output
```
int(x*acos(a*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x \arccos(ax)^2 \, dx = \frac{2acos(ax)^2 \, a^2x^2 - acos(ax)^2 - 2\sqrt{-a^2x^2+1} \, acos(ax) \, ax - a^2x^2}{4a^2}$$

input
```
int(x*acos(a*x)^2,x)
```

output
```
(2*acos(a*x)**2*a**2*x**2 - acos(a*x)**2 - 2*sqrt( - a**2*x**2 + 1)*acos(a
*x)*a*x - a**2*x**2)/(4*a**2)
```

3.16 $\int \arccos(ax)^2 \, dx$

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Rubi [A] (verified) 199
Maple [A] (verified) 200
Fricas [A] (verification not implemented) 200
Sympy [A] (verification not implemented) 201
Maxima [A] (verification not implemented) 201
Giac [A] (verification not implemented) 201
Mupad [B] (verification not implemented) 202
Reduce [B] (verification not implemented) 202

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \arccos(ax)^2 \, dx = -2x - \frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2$$

output
```
-2*x-2*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a+x*arccos(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^2 \, dx = -2x - \frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2$$

input
```
Integrate[ArcCos[a*x]^2,x]
```

output
```
-2*x - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + x*ArcCos[a*x]^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^2 \, dx$$

$$\Big\downarrow \text{ 5131}$$

$$2a\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx + x\arccos(ax)^2$$

$$\Big\downarrow \text{ 5183}$$

$$2a\left(-\frac{\int 1dx}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right) + x\arccos(ax)^2$$

$$\Big\downarrow \text{ 24}$$

$$2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2} - \frac{x}{a}\right) + x\arccos(ax)^2$$

input
```
Int[ArcCos[a*x]^2,x]
```

output
```
x*ArcCos[a*x]^2 + 2*a*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{ax\arccos(ax)^2-2ax-2\arccos(ax)\sqrt{-a^2x^2+1}}{a}$	37
default	$\frac{ax\arccos(ax)^2-2ax-2\arccos(ax)\sqrt{-a^2x^2+1}}{a}$	37
orering	$x\arccos\left(ax\right)^2-\frac{2\arccos(ax)}{a\sqrt{-a^2x^2+1}}+\frac{x(ax-1)(ax+1)\left(\frac{2a^2}{-a^2x^2+1}-\frac{2\arccos(ax)a^3x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\right)}{a^2}$	86

input
```
int(arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x*arccos(a*x)^2-2*a*x-2*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \arccos(ax)^2\,dx=\frac{ax\arccos\left(ax\right)^2-2\,ax-2\,\sqrt{-a^2x^2+1}\arccos\left(ax\right)}{a}$$

input
```
integrate(arccos(a*x)^2,x, algorithm="fricas")
```

output
```
(a*x*arccos(a*x)^2 - 2*a*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \arccos(ax)^2\,dx = \begin{cases} x \operatorname{acos}^2(ax) - 2x - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{a} & \text{for } a \neq 0 \\ \frac{\pi^2 x}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)**2,x)
```

output
```
Piecewise((x*acos(a*x)**2 - 2*x - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)/a, Ne(a
, 0)), (pi**2*x/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \arccos(ax)^2\,dx = x \arccos(ax)^2 - 2\,x - \frac{2\sqrt{-a^2x^2+1}\arccos(ax)}{a}$$

input
```
integrate(arccos(a*x)^2,x, algorithm="maxima")
```

output
```
x*arccos(a*x)^2 - 2*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \arccos(ax)^2\,dx = x \arccos(ax)^2 - 2\,x - \frac{2\sqrt{-a^2x^2+1}\arccos(ax)}{a}$$

input
```
integrate(arccos(a*x)^2,x, algorithm="giac")
```

output
```
x*arccos(a*x)^2 - 2*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \arccos(ax)^2\,dx = \begin{cases} \frac{x\,\pi^2}{4} & \text{if } a=0 \\ x\,\left(\mathrm{acos}(a\,x)^2-2\right)-\frac{2\,\mathrm{acos}(a\,x)\,\sqrt{1-a^2\,x^2}}{a} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acos(a*x)^2,x)
```

output
```
piecewise(a == 0, (x*pi^2)/4, a ~= 0, x*(acos(a*x)^2 - 2) - (2*acos(a*x)*(
- a^2*x^2 + 1)^(1/2))/a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^2\,dx = \frac{acos(ax)^2\,ax - 2\sqrt{-a^2x^2+1}\,acos(ax) - 2ax}{a}$$

input
```
int(acos(a*x)^2,x)
```

output
```
(acos(a*x)**2*a*x - 2*sqrt( - a**2*x**2 + 1)*acos(a*x) - 2*a*x)/a
```

3.17 $\int \frac{\arccos(ax)^2}{x}\,dx$

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Rubi [A] (verified) . . . 204
Maple [A] (verified) . . . 206
Fricas [F] . . . 207
Sympy [F] . . . 207
Maxima [F] . . . 207
Giac [F] . . . 208
Mupad [F(-1)] . . . 208
Reduce [F] . . . 208

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int \frac{\arccos(ax)^2}{x}\,dx = -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

output
```
-1/3*I*arccos(a*x)^3+arccos(a*x)^2*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-I*ar
ccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(a*x+I*(
-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^2}{x}\,dx = -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^2/x,x]
```

output
```
(-1/3*I)*ArcCos[a*x]^3 + ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - I*
ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + PolyLog[3, -E^((2*I)*ArcC
os[a*x])]/2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5137, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x}dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{ax}d\arccos(ax)$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(ax)^2\tan(\arccos(ax))d\arccos(ax)$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1+e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3$$

$$\Big\downarrow \text{2620}$$

$$2i\left(i\int \arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right)\right)-\frac{1}{3}i\arccos(ax)^3$$

$$\Big\downarrow \text{3011}$$

$$2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int \operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{2}i\arccos(a\right.$$
$$\frac{1}{3}i\arccos(ax)^3$$

$$\Big\downarrow \text{2720}$$

$$2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)de^{2i\arccos(ax)}\right)-\right.$$
$$\frac{1}{3}i\arccos(ax)^3$$

$$\Big\downarrow \text{7143}$$

$$2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{4}\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2\log\left(1+e^{2i\,\mathrm{ar}}\right.\right.$$
$$\frac{1}{3}i\arccos(ax)^3$$

input
```
Int[ArcCos[a*x]^2/x,x]
```

output
```
(-1/3*I)*ArcCos[a*x]^3 + (2*I)*((-1/2*I)*ArcCos[a*x]^2*Log[1 + E^((2*I)*Ar
cCos[a*x])] + I*((I/2)*ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] - Po
lyLog[3, -E^((2*I)*ArcCos[a*x])]/4))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{i \arccos(ax)^3}{3}+\arccos(ax)^2\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)-i\arccos(ax)\operatorname{polylog}\left(2,\right.$
default	$-\frac{i \arccos(ax)^3}{3}+\arccos(ax)^2\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)-i\arccos(ax)\operatorname{polylog}\left(2,\right.$

input
```
int(arccos(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*arccos(a*x)^3+arccos(a*x)^2*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-I*ar
ccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(a*x+I*(
-a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x} \, dx = \int \frac{\arccos(ax)^2}{x} \, dx$$

input
```
integrate(arccos(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x} \, dx = \int \frac{\operatorname{acos}^2(ax)}{x} \, dx$$

input
```
integrate(acos(a*x)**2/x,x)
```

output
```
Integral(acos(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x} \, dx = \int \frac{\arccos(ax)^2}{x} \, dx$$

input
```
integrate(arccos(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x}\,dx = \int \frac{\arccos\left(ax\right)^2}{x}\,dx$$

input
```
integrate(arccos(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x}\,dx$$

input
```
int(acos(a*x)^2/x,x)
```

output
```
int(acos(a*x)^2/x, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x}\,dx = \int \frac{acos(ax)^2}{x}dx$$

input
```
int(acos(a*x)^2/x,x)
```

output
```
int(acos(a*x)**2/x,x)
```

3.18 $\int \frac{\arccos(ax)^2}{x^2}\,dx$

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Giac [F] . . . 213
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Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = -\frac{\arccos(ax)^2}{x} - 4ia\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + 2ia\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - 2ia\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)$$

output
```
-arccos(a*x)^2/x-4*I*a*arccos(a*x)*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+2*I*a*
polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-2*I*a*polylog(2,I*(a*x+I*(-a^2*x^
2+1)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = -\frac{\arccos(ax)\left(\arccos(ax)+2ax\left(-\log\left(1-ie^{i\arccos(ax)}\right)+\log\left(1+ie^{i\arccos(ax)}\right)\right)\right)}{x} + 2ia\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - 2ia\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^2/x^2,x]
```

output
```
-((ArcCos[a*x]*(ArcCos[a*x] + 2*a*x*(-Log[1 - I*E^(I*ArcCos[a*x])] + Log[1
 + I*E^(I*ArcCos[a*x])])))/x) + (2*I)*a*PolyLog[2, (-I)*E^(I*ArcCos[a*x])]
 - (2*I)*a*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5139, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x^2} dx$$

$$\Big\downarrow \text{5139}$$

$$-2a\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)^2}{x}$$

$$\Big\downarrow \text{5219}$$

$$2a\int \frac{\arccos(ax)}{ax}d\arccos(ax) - \frac{\arccos(ax)^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \arccos(ax)\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax) - \frac{\arccos(ax)^2}{x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\arccos(ax)^2}{x} + 2a\left(-\int \log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax) + \int \log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax) - 2i\arccos(ax)\arctan\left(e^{i\,\text{arc}}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\arccos(ax)^2}{x} + 2a\left(i\int e^{-i\arccos(ax)}\log\left(1-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)} - i\int e^{-i\arccos(ax)}\log\left(1+ie^{i\arccos(ax)}\right)de^{i\arccos(ax)} - 2i\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\arccos(ax)^2}{x}+2a\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\,\mathrm{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right)$$

input
```
Int[ArcCos[a*x]^2/x^2,x]
```

output
```
-(ArcCos[a*x]^2/x) + 2*a*((-2*I)*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + I
*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - I*PolyLog[2, I*E^(I*ArcCos[a*x])])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

method	result
derivativedivides	$a\left(-\frac{\arccos(ax)^2}{ax}-2\arccos\left(ax\right)\ln\left(1+i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)+2\arccos\left(ax\right)\ln\left(1-i\right.\right.$
default	$a\left(-\frac{\arccos(ax)^2}{ax}-2\arccos\left(ax\right)\ln\left(1+i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)+2\arccos\left(ax\right)\ln\left(1-i\right.\right.$

input
```
int(arccos(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-arccos(a*x)^2/a/x-2*arccos(a*x)*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))+2*a
rccos(a*x)*ln(1-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+2*I*dilog(1+I*(a*x+I*(-a^2*x
^2+1)^(1/2)))-2*I*dilog(1-I*(a*x+I*(-a^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x^2}\,dx=\int \frac{\arccos\left(ax\right)^2}{x^2}\,dx$$

input
```
integrate(arccos(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^2} dx = \int \frac{\operatorname{acos}^2(ax)}{x^2} dx$$

input
```
integrate(acos(a*x)**2/x**2,x)
```

output
```
Integral(acos(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x^2} dx = \int \frac{\arccos(ax)^2}{x^2} dx$$

input
```
integrate(arccos(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
(2*a*x*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-
a*x + 1), a*x)/(a^2*x^3 - x), x) - arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a
*x)^2)/x
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x^2} dx = \int \frac{\arccos(ax)^2}{x^2} dx$$

input
```
integrate(arccos(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^2}\,dx$$

input
```
int(acos(a*x)^2/x^2,x)
```

output
```
int(acos(a*x)^2/x^2, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{acos(ax)^2}{x^2}dx$$

input
```
int(acos(a*x)^2/x^2,x)
```

output
```
int(acos(a*x)**2/x**2,x)
```

3.19 $\int \frac{\arccos(ax)^2}{x^3}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2} + a^2\log(x)$$

output
```
a*(-a^2*x^2+1)^(1/2)*arccos(a*x)/x-1/2*arccos(a*x)^2/x^2+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2} + a^2\log(x)$$

input
```
Integrate[ArcCos[a*x]^2/x^3,x]
```

output
```
(a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x - ArcCos[a*x]^2/(2*x^2) + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5139, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x^3} \, dx$$

$$\Big\downarrow \text{5139}$$

$$-a \int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{5187}$$

$$-a\left(-a\int \frac{1}{x}dx - \frac{\sqrt{1-a^2x^2}\arccos(ax)}{x}\right) - \frac{\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$-a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x} - a\log(x)\right) - \frac{\arccos(ax)^2}{2x^2}$$

input
```
Int[ArcCos[a*x]^2/x^3,x]
```

output
```
-1/2*ArcCos[a*x]^2/x^2 - a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x) - a*Log[x
])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$a^{2}\left(-\frac{\arccos\left(ax\right)^{2}}{2a^{2}x^{2}}+\frac{\arccos\left(ax\right)\sqrt{-a^{2}x^{2}+1}}{ax}+\ln\left(ax\right)\right)$	47
default	$a^{2}\left(-\frac{\arccos\left(ax\right)^{2}}{2a^{2}x^{2}}+\frac{\arccos\left(ax\right)\sqrt{-a^{2}x^{2}+1}}{ax}+\ln\left(ax\right)\right)$	47

input
```
int(arccos(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arccos(a*x)^2/a^2/x^2+arccos(a*x)/a/x*(-a^2*x^2+1)^(1/2)+ln(a*x)
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{2\,a^2x^2\log\left(x\right)+2\,\sqrt{-a^2x^2+1}ax\arccos\left(ax\right)-\arccos\left(ax\right)^2}{2\,x^2}$$

input
```
integrate(arccos(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*x^2*log(x) + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) - arccos(a*x)
^2)/x^2
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \int \frac{\operatorname{acos}^2(ax)}{x^3}\,dx$$

input
```
integrate(acos(a*x)**2/x**3,x)
```

output
```
Integral(acos(a*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = a^2\log(x) + \frac{\sqrt{-a^2x^2+1}a\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2}$$

input
```
integrate(arccos(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
a^2*log(x) + sqrt(-a^2*x^2 + 1)*a*arccos(a*x)/x - 1/2*arccos(a*x)^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = -\frac{1}{2}\left(\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\arccos(ax) - 2a\log(|x|)\right)a - \frac{\arccos(ax)^2}{2x^2}$$

input
```
integrate(arccos(a*x)^2/x^3,x, algorithm="giac")
```

output
```
-1/2*((a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1
)*abs(a) + a)/(x*abs(a)))*arccos(a*x) - 2*a*log(abs(x)))*a - 1/2*arccos(a*
x)^2/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^3}\,dx$$

input
```
int(acos(a*x)^2/x^3,x)
```

output
```
int(acos(a*x)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \int \frac{acos(ax)^2}{x^3}dx$$

input
```
int(acos(a*x)^2/x^3,x)
```

output
```
int(acos(a*x)**2/x**3,x)
```

3.20 $\int \frac{\arccos(ax)^2}{x^4}\,dx$

Optimal result 220
Mathematica [A] (verified) 220
Rubi [A] (verified) 221
Maple [A] (verified) 224
Fricas [F] 224
Sympy [F] 225
Maxima [F] 225
Giac [F] 225
Mupad [F(-1)] 226
Reduce [F] 226

Optimal result

Integrand size = 10, antiderivative size = 124

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} - \frac{2}{3}ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + \frac{1}{3}ia^3\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - \frac{1}{3}ia^3\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)$$

output
```
-1/3*a^2/x+1/3*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)/x^2-1/3*arccos(a*x)^2/x^3-
2/3*I*a^3*arccos(a*x)*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+1/3*I*a^3*polylog(2
,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-1/3*I*a^3*polylog(2,I*(a*x+I*(-a^2*x^2+1)^
(1/2)))
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = -\frac{a^2x^2 - ax\sqrt{1-a^2x^2}\arccos(ax) + \arccos(ax)^2 - a^3x^3\arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right) + a^3x^3\arccos(}{3x^3}$$

input
```
Integrate[ArcCos[a*x]^2/x^4,x]
```

output
```
-1/3*(a^2*x^2 - a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + ArcCos[a*x]^2 - a^3*x^
3*ArcCos[a*x]*Log[1 - I*E^(I*ArcCos[a*x])] + a^3*x^3*ArcCos[a*x]*Log[1 + I
*E^(I*ArcCos[a*x])] - I*a^3*x^3*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + I*a^3
*x^3*PolyLog[2, I*E^(I*ArcCos[a*x])])/x^3
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5139, 5205, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{2}{3}a\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^2}{3x^3}$$

$$\Big\downarrow \text{ 5205}$$

$$-\frac{2}{3}a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{1}{2}a\int \frac{1}{x^2}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}\right)-\frac{\arccos(ax)^2}{3x^3}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2}{3}a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}\right)-\frac{\arccos(ax)^2}{3x^3}$$

$$\Big\downarrow \text{ 5219}$$

$$-\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\arccos(ax)}{ax}d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}\right)-\frac{\arccos(ax)^2}{3x^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2}{3}a\left(-\frac{1}{2}a^2\int \arccos(ax)\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}\right)-\frac{\arccos(ax)^2}{3x^3}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\arccos(ax)^2}{3x^3}-\frac{2}{3}a\left(-\frac{1}{2}a^2\left(-\int \log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+\int \log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)-2i\arccos(ax)\operatorname{arct}\right.\right.$$

$$\Big\downarrow \; 2715$$

$$-\frac{\arccos(ax)^2}{3x^3}-\frac{2}{3}a\left(-\frac{1}{2}a^2\left(i\int e^{-i\arccos(ax)}\log\left(1-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\int e^{-i\arccos(ax)}\log\left(1+ie^{i\arccos(ax)}\right)de^{i\arccos(}\right.\right.$$

$$\Big\downarrow \; 2838$$

$$-\frac{\arccos(ax)^2}{3x^3}-\frac{2}{3}a\left(-\frac{1}{2}a^2\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right)-\right.$$

input
```
Int[ArcCos[a*x]^2/x^4,x]
```

output
```
-1/3*ArcCos[a*x]^2/x^3 - (2*a*(a/(2*x) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(
2*x^2) - (a^2*((-2*I)*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + I*PolyLog[2,
 (-I)*E^(I*ArcCos[a*x])] - I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/2))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.34

method	result
derivativedivides	$a^3\left(-\frac{-\arccos(ax)\sqrt{-a^2x^2+1}\,ax+\arccos(ax)^2+a^2x^2}{3a^3x^3}-\frac{\arccos(ax)\ln\left(1+i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)}{3}+\frac{\arccos(ax)\ln\right.$
default	$a^3\left(-\frac{-\arccos(ax)\sqrt{-a^2x^2+1}\,ax+\arccos(ax)^2+a^2x^2}{3a^3x^3}-\frac{\arccos(ax)\ln\left(1+i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)}{3}+\frac{\arccos(ax)\ln\right.$

input
```
int(arccos(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3*(-arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x+arccos(a*x)^2+a^2*x^2)/a^3/
x^3-1/3*arccos(a*x)*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))+1/3*arccos(a*x)*ln(
1-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+1/3*I*dilog(1+I*(a*x+I*(-a^2*x^2+1)^(1/2))
)-1/3*I*dilog(1-I*(a*x+I*(-a^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x^4}\,dx=\int \frac{\arccos(ax)^2}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^4} dx = \int \frac{\operatorname{acos}^2(ax)}{x^4} dx$$

input
```
integrate(acos(a*x)**2/x**4,x)
```

output
```
Integral(acos(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x^4} dx = \int \frac{\arccos(ax)^2}{x^4} dx$$

input
```
integrate(arccos(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/3*(6*a*x^3*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x)/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x)^2)/x^3
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x^4} dx = \int \frac{\arccos(ax)^2}{x^4} dx$$

input
```
integrate(arccos(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^4}\,dx$$

input
```
int(acos(a*x)^2/x^4,x)
```

output
```
int(acos(a*x)^2/x^4, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{acos(ax)^2}{x^4}dx$$

input
```
int(acos(a*x)^2/x^4,x)
```

output
```
int(acos(a*x)**2/x**4,x)
```

3.21 $\int \frac{\arccos(ax)^2}{x^5}\,dx$

Optimal result 227
Mathematica [A] (verified) 227
Rubi [A] (verified) 228
Maple [A] (verified) 230
Fricas [A] (verification not implemented) 230
Sympy [F] 231
Maxima [A] (verification not implemented) 231
Giac [B] (verification not implemented) 231
Mupad [F(-1)] 232
Reduce [F] 233

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{6x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)}{3x} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{3}a^4\log(x)$$

output
```
-1/12*a^2/x^2+1/6*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)/x^3+1/3*a^3*(-a^2*x^2+1
)^(1/2)*arccos(a*x)/x-1/4*arccos(a*x)^2/x^4+1/3*a^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}(1+2a^2x^2)\arccos(ax)}{6x^3} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{3}a^4\log(x)$$

input
```
Integrate[ArcCos[a*x]^2/x^5,x]
```

output
```
-1/12*a^2/x^2 + (a*Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2)*ArcCos[a*x])/(6*x^3)
- ArcCos[a*x]^2/(4*x^4) + (a^4*Log[x])/3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5139, 5205, 15, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x^5}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{1}{2}a\int \frac{\arccos(ax)}{x^4\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5205}$$

$$-\frac{1}{2}a\left(\frac{2}{3}a^2\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{1}{3}a\int \frac{1}{x^3}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{3x^3}\right)-\frac{\arccos(ax)^2}{4x^4}$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}a\left(\frac{2}{3}a^2\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{3x^3}+\frac{a}{6x^2}\right)-\frac{\arccos(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5187}$$

$$-\frac{1}{2}a\left(\frac{2}{3}a^2\left(-a\int \frac{1}{x}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{3x^3}+\frac{a}{6x^2}\right)-\frac{\arccos(ax)^2}{4x^4}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{2}a\left(\frac{2}{3}a^2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x}-a\log(x)\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{3x^3}+\frac{a}{6x^2}\right)-\frac{\arccos(ax)^2}{4x^4}$$

input
```
Int[ArcCos[a*x]^2/x^5,x]
```

output
```
-1/4*ArcCos[a*x]^2/x^4 - (a*(a/(6*x^2) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(
3*x^3) + (2*a^2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x) - a*Log[x]))/3))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$a^{4}\left(-\frac{\arccos(ax)^{2}}{4a^{4}x^{4}}+\frac{\arccos(ax)\sqrt{-a^{2}x^{2}+1}}{6a^{3}x^{3}}-\frac{1}{12a^{2}x^{2}}+\frac{\arccos(ax)\sqrt{-a^{2}x^{2}+1}}{3ax}+\frac{\ln(ax)}{3}\right)$	82
default	$a^{4}\left(-\frac{\arccos(ax)^{2}}{4a^{4}x^{4}}+\frac{\arccos(ax)\sqrt{-a^{2}x^{2}+1}}{6a^{3}x^{3}}-\frac{1}{12a^{2}x^{2}}+\frac{\arccos(ax)\sqrt{-a^{2}x^{2}+1}}{3ax}+\frac{\ln(ax)}{3}\right)$	82

input
```
int(arccos(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/4*arccos(a*x)^2/a^4/x^4+1/6*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3/x^3
-1/12/a^2/x^2+1/3*arccos(a*x)/a/x*(-a^2*x^2+1)^(1/2)+1/3*ln(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \frac{4\,a^4x^4\log(x) - a^2x^2 + 2\,(2\,a^3x^3 + ax)\sqrt{-a^2x^2+1}\arccos(ax) - 3\,\arccos(ax)^2}{12\,x^4}$$

input
```
integrate(arccos(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*a^4*x^4*log(x) - a^2*x^2 + 2*(2*a^3*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*
arccos(a*x) - 3*arccos(a*x)^2)/x^4
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \int \frac{\operatorname{acos}^2(ax)}{x^5}\,dx$$

input
```
integrate(acos(a*x)**2/x**5,x)
```

output
```
Integral(acos(a*x)**2/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \frac{1}{12}\left(4\,a^2\log(x) - \frac{1}{x^2}\right)a^2 + \frac{1}{6}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x} + \frac{\sqrt{-a^2x^2+1}}{x^3}\right)a\arccos(ax) - \frac{\arccos(ax)^2}{4\,x^4}$$

input
```
integrate(arccos(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
1/12*(4*a^2*log(x) - 1/x^2)*a^2 + 1/6*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-
a^2*x^2 + 1)/x^3)*a*arccos(a*x) - 1/4*arccos(a*x)^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(73) = 146.

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.97

$$\int \frac{\arccos(ax)^2}{x^5}\,dx =$$

$$-\frac{1}{48}\left(4\,a^3\left(\frac{2\,a^2x^2+1}{a^2x^2}-2\,\log\left(a^2x^2\right)\right)+\left(\frac{\left(a^4+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}+\frac{\left(\sqrt{\right.}}{}}{a^2|a|}\right.\right.$$

$$-\frac{\arccos\left(ax\right)^2}{4\,x^4}$$

input
```
integrate(arccos(a*x)^2/x^5,x, algorithm="giac")
```

output
```
-1/48*(4*a^3*((2*a^2*x^2 + 1)/(a^2*x^2) - 2*log(a^2*x^2)) + ((a^4 + 9*(sqr
t(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*x^2 + 1)*abs(a) + a
)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x + (sqrt(-a^2*x^2 +
1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*arccos(a*x))*a - 1/4*arccos(a*x)^2/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^5}\,dx$$

input
```
int(acos(a*x)^2/x^5,x)
```

output
```
int(acos(a*x)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \int \frac{acos(ax)^2}{x^5}dx$$

input
```
int(acos(a*x)^2/x^5,x)
```

output
```
int(acos(a*x)**2/x**5,x)
```

3.22 $\int x^4 \arccos(ax)^3\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 201

$$\begin{aligned}\int x^4 \arccos(ax)^3\,dx &= \frac{298\sqrt{1-a^2x^2}}{375a^5} - \frac{76(1-a^2x^2)^{3/2}}{1125a^5} + \frac{6(1-a^2x^2)^{5/2}}{625a^5} \\ &\quad - \frac{16x\arccos(ax)}{25a^4} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{6}{125}x^5\arccos(ax) \\ &\quad - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3} \\ &\quad - \frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a} + \frac{1}{5}x^5\arccos(ax)^3\end{aligned}$$

output

```
298/375*(-a^2*x^2+1)^(1/2)/a^5-76/1125*(-a^2*x^2+1)^(3/2)/a^5+6/625*(-a^2*
x^2+1)^(5/2)/a^5-16/25*x*arccos(a*x)/a^4-8/75*x^3*arccos(a*x)/a^2-6/125*x^
5*arccos(a*x)-8/25*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a^5-4/25*x^2*(-a^2*x^2
+1)^(1/2)*arccos(a*x)^2/a^3-3/25*x^4*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a+1/
5*x^5*arccos(a*x)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.61

$$\int x^4 \arccos(ax)^3 \, dx$$
$$= \frac{2\sqrt{1-a^2x^2}(2072+136a^2x^2+27a^4x^4) - 30ax(120+20a^2x^2+9a^4x^4)\arccos(ax) - 225\sqrt{1-a^2x^2}(8+}{5625a^5}$$

input
```
Integrate[x^4*ArcCos[a*x]^3,x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*(2072 + 136*a^2*x^2 + 27*a^4*x^4) - 30*a*x*(120 + 20*
a^2*x^2 + 9*a^4*x^4)*ArcCos[a*x] - 225*Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 +
3*a^4*x^4)*ArcCos[a*x]^2 + 1125*a^5*x^5*ArcCos[a*x]^3)/(5625*a^5)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.51, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {5139, 5211, 5139, 243, 53, 2009, 5211, 5139, 243, 53, 2009, 5183, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arccos(ax)^3 \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{3}{5}a\int \frac{x^5 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx + \frac{1}{5}x^5 \arccos(ax)^3$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2} - \frac{2\int x^4 \arccos(ax)dx}{5a} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}\right) + \frac{1}{5}x^5 \arccos(ax)^3$$

$$\Big\downarrow \text{5139}$$

$$\frac{3}{5}a\left(-\frac{2\left(\frac{1}{5}a\int \frac{x^5}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^5\arccos(ax)\right)}{5a}+\frac{4\int \frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}\right)+\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 243

$$\frac{3}{5}a\left(\frac{4\int \frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{2\left(\frac{1}{10}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{5}x^5\arccos(ax)\right)}{5a}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}\right)+\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 53

$$\frac{3}{5}a\left(\frac{4\int \frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{2\left(\frac{1}{10}a\int \left(\frac{(1-a^2x^2)^{3/2}}{a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}+\frac{1}{a^4\sqrt{1-a^2x^2}}\right)dx^2+\frac{1}{5}x^5\arccos(ax)\right)}{5a}-\frac{x^4\sqrt{1-a^2}\cdots}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 2009

$$\frac{3}{5}a\left(\frac{4\int \frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}-\frac{2\left(\frac{1}{10}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+\frac{1}{5}x^5\cdots\right.}{5a}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 5211

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\int x^2\arccos(ax)dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}-\frac{2\left(\frac{1}{10}a\left(-\frac{2(1-\cdots}{}\right.\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 5139

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{3}a\int\frac{x^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 243

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{6}a\int\frac{x^2}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 53

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{6}a\int\left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 2009

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}\right)}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}}{5a}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\downarrow$ 5183

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\left(-\frac{2\int \arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\big\downarrow$ 5131

$$\frac{3}{5}a\left(\frac{4\left(\frac{2\left(-\frac{2\left(a\int\frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\right.}{3a}\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

$\big\downarrow$ 241

$$\frac{3}{5}a\left(-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{5a^2}-\frac{2\left(\frac{1}{10}a\left(-\frac{2(1-a^2x^2)^{5/2}}{5a^6}+\frac{4(1-a^2x^2)^{3/2}}{3a^6}-\frac{2\sqrt{1-a^2x^2}}{a^6}\right)+\frac{1}{5}x^5\arccos(ax)\right)}{5a}+\frac{4\left(\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^3$$

input
```
Int[x^4*ArcCos[a*x]^3,x]
```

output
```
(x^5*ArcCos[a*x]^3)/5 + (3*a*(-1/5*(x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a
^2 - (2*((a*((-2*Sqrt[1 - a^2*x^2])/a^6 + (4*(1 - a^2*x^2)^(3/2))/(3*a^6)
- (2*(1 - a^2*x^2)^(5/2))/(5*a^6)))/10 + (x^5*ArcCos[a*x])/5))/(5*a) + (4*
(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2 - (2*((a*((-2*Sqrt[1 - a^2
*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4)))/6 + (x^3*ArcCos[a*x])/3))/(
3*a) + (2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sqrt[1 - a^2*x
^2]/a) + x*ArcCos[a*x]))/a))/(3*a^2)))/(5*a^2)))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{a^5x^5\arccos(ax)^3}{5}-\frac{\arccos(ax)^2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{25}-\frac{6a^5x^5\arccos(ax)}{125}+\frac{2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{625}-\frac{8a^3x^3\,\mathrm{ar}}{7}}{a^5}$
default	$\frac{\frac{a^5x^5\arccos(ax)^3}{5}-\frac{\arccos(ax)^2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{25}-\frac{6a^5x^5\arccos(ax)}{125}+\frac{2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{625}-\frac{8a^3x^3\,\mathrm{ar}}{7}}{a^5}$
orering	$\frac{(9963a^8x^8+6736a^6x^6+137224a^4x^4-408128a^2x^2+248640)\arccos(ax)^3}{16875a^8x^3}-\frac{(2619a^8x^8+3658a^6x^6+76192a^4x^4-209}{}$

input
```
int(x^4*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccos(a*x)^3-1/25*arccos(a*x)^2*(3*a^4*x^4+4*a^2*x^2+8
)*(-a^2*x^2+1)^(1/2)-6/125*a^5*x^5*arccos(a*x)+2/625*(3*a^4*x^4+4*a^2*x^2+
8)*(-a^2*x^2+1)^(1/2)-8/75*a^3*x^3*arccos(a*x)+8/225*(a^2*x^2+2)*(-a^2*x^2
+1)^(1/2)+16/25*(-a^2*x^2+1)^(1/2)-16/25*a*x*arccos(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.52

$$\int x^4 \arccos(ax)^3 \, dx$$
$$= \frac{1125\, a^5 x^5 \arccos{(ax)}^3 - 30\,(9\, a^5 x^5 + 20\, a^3 x^3 + 120\, ax) \arccos{(ax)} + \left(54\, a^4 x^4 + 272\, a^2 x^2 - 225\,(3\, a^4 x^4\right.}{5625\, a^5}$$

input
```
integrate(x^4*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/5625*(1125*a^5*x^5*arccos(a*x)^3 - 30*(9*a^5*x^5 + 20*a^3*x^3 + 120*a*x)
*arccos(a*x) + (54*a^4*x^4 + 272*a^2*x^2 - 225*(3*a^4*x^4 + 4*a^2*x^2 + 8)
*arccos(a*x)^2 + 4144)*sqrt(-a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int x^4 \arccos(ax)^3 \, dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acos}^3(ax)}{5} - \frac{6x^5 \operatorname{acos}(ax)}{125} - \frac{3x^4\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25a} + \frac{6x^4\sqrt{-a^2x^2+1}}{625a} - \frac{8x^3\operatorname{acos}(ax)}{75a^2} - \frac{4x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25a^3} + \frac{272x \\ \frac{\pi^3 x^5}{40} \end{cases}$$

input
```
integrate(x**4*acos(a*x)**3,x)
```

output
```
Piecewise((x**5*acos(a*x)**3/5 - 6*x**5*acos(a*x)/125 - 3*x**4*sqrt(-a**2*
x**2 + 1)*acos(a*x)**2/(25*a) + 6*x**4*sqrt(-a**2*x**2 + 1)/(625*a) - 8*x*
*3*acos(a*x)/(75*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(25*a**3
) + 272*x**2*sqrt(-a**2*x**2 + 1)/(5625*a**3) - 16*x*acos(a*x)/(25*a**4) -
 8*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(25*a**5) + 4144*sqrt(-a**2*x**2 + 1)
/(5625*a**5), Ne(a, 0)), (pi**3*x**5/40, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.85

$$\int x^4 \arccos(ax)^3 \, dx = \frac{1}{5} x^5 \arccos(ax)^3 - \frac{1}{25} \left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6} \right) a \arccos(ax)^2 + \frac{2}{5625} a \left(\frac{27\sqrt{-a^2x^2+1}a^2x^4 + 136\sqrt{-a^2x^2+1}x^2 + \frac{2072\sqrt{-a^2x^2+1}}{a^2}}{a^4} - \frac{15(9a^4x^5 + 20a^2x^3 + 120x)\operatorname{arc}}{a^5} \right.$$

input
```
integrate(x^4*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/5*x^5*arccos(a*x)^3 - 1/25*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x
^2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x)^2 + 2/5625*a*((2
7*sqrt(-a^2*x^2 + 1)*a^2*x^4 + 136*sqrt(-a^2*x^2 + 1)*x^2 + 2072*sqrt(-a^2
*x^2 + 1)/a^2)/a^4 - 15*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccos(a*x)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int x^4 \arccos(ax)^3 \, dx = \frac{1}{5} x^5 \arccos(ax)^3 - \frac{6}{125} x^5 \arccos(ax) - \frac{3\sqrt{-a^2x^2+1}x^4 \arccos(ax)^2}{25a} + \frac{6\sqrt{-a^2x^2+1}x^4}{625a} - \frac{8x^3 \arccos(ax)}{75a^2} - \frac{4\sqrt{-a^2x^2+1}x^2 \arccos(ax)^2}{25a^3} + \frac{272\sqrt{-a^2x^2+1}x^2}{5625a^3} - \frac{16x \arccos(ax)}{25a^4} - \frac{8\sqrt{-a^2x^2+1} \arccos(ax)^2}{25a^5} + \frac{4144\sqrt{-a^2x^2+1}}{5625a^5}$$

input
```
integrate(x^4*arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/5*x^5*arccos(a*x)^3 - 6/125*x^5*arccos(a*x) - 3/25*sqrt(-a^2*x^2 + 1)*x^
4*arccos(a*x)^2/a + 6/625*sqrt(-a^2*x^2 + 1)*x^4/a - 8/75*x^3*arccos(a*x)/
a^2 - 4/25*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)^2/a^3 + 272/5625*sqrt(-a^2*x
^2 + 1)*x^2/a^3 - 16/25*x*arccos(a*x)/a^4 - 8/25*sqrt(-a^2*x^2 + 1)*arccos
(a*x)^2/a^5 + 4144/5625*sqrt(-a^2*x^2 + 1)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^3 \, dx = \int x^4 \operatorname{acos}(a\,x)^3 \, dx$$

input
```
int(x^4*acos(a*x)^3,x)
```

output
```
int(x^4*acos(a*x)^3, x)
```

Reduce [F]

$$\int x^4 \arccos(ax)^3 \, dx = \int acos(ax)^3 x^4 dx$$

input
```
int(x^4*acos(a*x)^3,x)
```

output
```
int(acos(a*x)**3*x**4,x)
```

3.23 $\int x^3 \arccos(ax)^3 \, dx$

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Mupad [F(-1)] 252
Reduce [F] 252

Optimal result

Integrand size = 10, antiderivative size = 167

$$\int x^3 \arccos(ax)^3 \, dx = \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\arccos(ax)}{32a^2} - \frac{3}{32}x^4\arccos(ax) - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} - \frac{3\arccos(ax)^3}{32a^4} + \frac{1}{4}x^4\arccos(ax)^3 - \frac{45\arcsin(ax)}{256a^4}$$

output
```
45/256*x*(-a^2*x^2+1)^(1/2)/a^3+3/128*x^3*(-a^2*x^2+1)^(1/2)/a-9/32*x^2*ar
ccos(a*x)/a^2-3/32*x^4*arccos(a*x)-9/32*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2
/a^3-3/16*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a-3/32*arccos(a*x)^3/a^4+1/
4*x^4*arccos(a*x)^3-45/256*arcsin(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$\int x^3 \arccos(ax)^3 \, dx$$
$$= \frac{3ax\sqrt{1-a^2x^2}(15+2a^2x^2) - 24a^2x^2(3+a^2x^2)\arccos(ax) - 24ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arccos(ax)^2 +}{256a^4}$$

input
```
Integrate[x^3*ArcCos[a*x]^3,x]
```

output
```
(3*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2) - 24*a^2*x^2*(3 + a^2*x^2)*ArcCo
s[a*x] - 24*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^2 + 8*(-3 +
8*a^4*x^4)*ArcCos[a*x]^3 - 45*ArcSin[a*x])/(256*a^4)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5139, 5211, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax)^3 \, dx$$
$$\Big\downarrow \text{5139}$$
$$\frac{3}{4}a\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx + \frac{1}{4}x^4 \arccos(ax)^3$$
$$\Big\downarrow \text{5211}$$
$$\frac{3}{4}a\left(\frac{3\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2} - \frac{\int x^3 \arccos(ax)dx}{2a} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}\right) + \frac{1}{4}x^4 \arccos(ax)^3$$
$$\Big\downarrow \text{5139}$$

$$\frac{3}{4}a\left(\frac{3\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\frac{1}{4}a\int\frac{x^4}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^3$$

$\Big\downarrow$ 262

$$\frac{3}{4}a\left(\frac{3\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\int\frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^3$$

$\Big\downarrow$ 262

$$\frac{3}{4}a\left(\frac{3\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^3$$

$\Big\downarrow$ 223

$$\frac{3}{4}a\left(\frac{3\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}\right)+\frac{1}{4}x^4\arccos(ax)^3$$

$\Big\downarrow$ 5211

$$\frac{3}{4}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int x\arccos(ax)dx}{a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-x\sqrt{1}\right.}{4a^2}\right.}{}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^3$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{3}{4}a\left(\frac{3\left(-\frac{\frac{1}{2}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\right.}{}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^3$$

$$\Big\downarrow \text{ 262}$$

$$\frac{3}{4}a\left(\frac{3\left(-\frac{\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos}{4a^2}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^3$$

$$\Big\downarrow \text{ 223}$$

$$\frac{3}{4}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^3$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{3}{4}a\left(-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^3}{6a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}\right)}{4a^2}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^3$$

input
```
Int[x^3*ArcCos[a*x]^3,x]
```

output
```
(x^4*ArcCos[a*x]^3)/4 + (3*a*(-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a
^2 - ((x^4*ArcCos[a*x])/4 + (a*(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/
2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3)))/(4*a^2)))/4)/(2*a) + (
3*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2 - ArcCos[a*x]^3/(6*a^3) -
((x^2*ArcCos[a*x])/2 + (a*(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2
*a^3)))/2)/a))/(4*a^2)))/4
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{a^4x^4\arccos(ax)^3}{4}-\frac{3\arccos(ax)^2\left(2a^3x^3\sqrt{-a^2x^2+1}+3\sqrt{-a^2x^2+1}\,ax+3\arccos(ax)\right)}{32}-\frac{3a^4x^4\arccos(ax)}{32}+\frac{3ax\left(2a^2x^2+3\right)\sqrt{-a^2x^2+1}}{256}}{a^4}$
default	$\frac{\frac{a^4x^4\arccos(ax)^3}{4}-\frac{3\arccos(ax)^2\left(2a^3x^3\sqrt{-a^2x^2+1}+3\sqrt{-a^2x^2+1}\,ax+3\arccos(ax)\right)}{32}-\frac{3a^4x^4\arccos(ax)}{32}+\frac{3ax\left(2a^2x^2+3\right)\sqrt{-a^2x^2+1}}{256}}{a^4}$
orering	$\frac{(350a^6x^6+399a^4x^4-1800a^2x^2+1080)\arccos(ax)^3}{512a^6x^2}-\frac{(110a^6x^6+263a^4x^4-1020a^2x^2+630)\left(3x^2\arccos(ax)^3-\frac{3x^3}{\sqrt{}}\right.}{512a^6x^4}$

input
```
int(x^3*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arccos(a*x)^3-3/32*arccos(a*x)^2*(2*a^3*x^3*(-a^2*x^2+1
)^(1/2)+3*(-a^2*x^2+1)^(1/2)*a*x+3*arccos(a*x))-3/32*a^4*x^4*arccos(a*x)+3
/256*a*x*(2*a^2*x^2+3)*(-a^2*x^2+1)^(1/2)+45/256*arccos(a*x)-9/32*a^2*x^2*
arccos(a*x)+9/64*(-a^2*x^2+1)^(1/2)*a*x+3/16*arccos(a*x)^3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.57

$$\int x^3 \arccos(ax)^3 \, dx$$
$$= \frac{8\left(8\,a^4x^4 - 3\right)\arccos\left(ax\right)^3 - 3\left(8\,a^4x^4 + 24\,a^2x^2 - 15\right)\arccos\left(ax\right) + 3\left(2\,a^3x^3 - 8\left(2\,a^3x^3 + 3\,ax\right)\arccos\right.}{256\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/256*(8*(8*a^4*x^4 - 3)*arccos(a*x)^3 - 3*(8*a^4*x^4 + 24*a^2*x^2 - 15)*a
rccos(a*x) + 3*(2*a^3*x^3 - 8*(2*a^3*x^3 + 3*a*x)*arccos(a*x)^2 + 15*a*x)*
sqrt(-a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

$$\int x^3 \arccos(ax)^3 \, dx$$
$$= \begin{cases} \frac{x^4 \operatorname{acos}^3(ax)}{4} - \frac{3x^4 \operatorname{acos}(ax)}{32} - \frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{16a} + \frac{3x^3\sqrt{-a^2x^2+1}}{128a} - \frac{9x^2\operatorname{acos}(ax)}{32a^2} - \frac{9x\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{32a^3} + \frac{45x\sqrt{}}{2} \\ \frac{\pi^3x^4}{32} \end{cases}$$

input
```
integrate(x**3*acos(a*x)**3,x)
```

output
```
Piecewise((x**4*acos(a*x)**3/4 - 3*x**4*acos(a*x)/32 - 3*x**3*sqrt(-a**2*x
**2 + 1)*acos(a*x)**2/(16*a) + 3*x**3*sqrt(-a**2*x**2 + 1)/(128*a) - 9*x**
2*acos(a*x)/(32*a**2) - 9*x*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(32*a**3) +
45*x*sqrt(-a**2*x**2 + 1)/(256*a**3) - 3*acos(a*x)**3/(32*a**4) + 45*acos(
a*x)/(256*a**4), Ne(a, 0)), (pi**3*x**4/32, True))
```

Maxima [F]

$$\int x^3 \arccos(ax)^3 \, dx = \int x^3 \arccos{(ax)}^3 \, dx$$

input
```
integrate(x^3*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*integrate(1/4*s
qrt(a*x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
^2/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\begin{aligned}\int x^3 \arccos(ax)^3 \, dx = {} & \frac{1}{4}\, x^4 \arccos{(ax)}^3 - \frac{3}{32}\, x^4 \arccos{(ax)} \\ & - \frac{3\, \sqrt{-a^2x^2+1}x^3 \arccos{(ax)}^2}{16\, a} + \frac{3\, \sqrt{-a^2x^2+1}x^3}{128\, a} \\ & - \frac{9\, x^2 \arccos{(ax)}}{32\, a^2} - \frac{9\, \sqrt{-a^2x^2+1}x \arccos{(ax)}^2}{32\, a^3} \\ & - \frac{3\, \arccos{(ax)}^3}{32\, a^4} + \frac{45\, \sqrt{-a^2x^2+1}x}{256\, a^3} + \frac{45\, \arccos{(ax)}}{256\, a^4}\end{aligned}$$

input
```
integrate(x^3*arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/4*x^4*arccos(a*x)^3 - 3/32*x^4*arccos(a*x) - 3/16*sqrt(-a^2*x^2 + 1)*x^3
*arccos(a*x)^2/a + 3/128*sqrt(-a^2*x^2 + 1)*x^3/a - 9/32*x^2*arccos(a*x)/a
^2 - 9/32*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^2/a^3 - 3/32*arccos(a*x)^3/a^4
+ 45/256*sqrt(-a^2*x^2 + 1)*x/a^3 + 45/256*arccos(a*x)/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^3 \,dx = \int x^3 \,\mathrm{acos}(a\,x)^3 \,dx$$

input
```
int(x^3*acos(a*x)^3,x)
```

output
```
int(x^3*acos(a*x)^3, x)
```

Reduce [F]

$$\int x^3 \arccos(ax)^3 \,dx = \int acos(ax)^3 \,x^3 dx$$

input
```
int(x^3*acos(a*x)^3,x)
```

output
```
int(acos(a*x)**3*x**3,x)
```

3.24 $\int x^2 \arccos(ax)^3\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 136

$$\int x^2 \arccos(ax)^3\,dx = \frac{14\sqrt{1-a^2x^2}}{9a^3} - \frac{2(1-a^2x^2)^{3/2}}{27a^3} - \frac{4x\arccos(ax)}{3a^2} - \frac{2}{9}x^3\arccos(ax) - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} + \frac{1}{3}x^3\arccos(ax)^3$$

output
```
14/9*(-a^2*x^2+1)^(1/2)/a^3-2/27*(-a^2*x^2+1)^(3/2)/a^3-4/3*x*arccos(a*x)/
a^2-2/9*x^3*arccos(a*x)-2/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a^3-1/3*x^2*(
-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a+1/3*x^3*arccos(a*x)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x^2 \arccos(ax)^3\,dx = \frac{2\sqrt{1-a^2x^2}(20+a^2x^2) - 6ax(6+a^2x^2)\arccos(ax) - 9\sqrt{1-a^2x^2}(2+a^2x^2)\arccos(ax)^2 + 9a^3x^3\arccos}{27a^3}$$

input
```
Integrate[x^2*ArcCos[a*x]^3,x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2) - 6*a*x*(6 + a^2*x^2)*ArcCos[a*x] - 9*
Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x]^2 + 9*a^3*x^3*ArcCos[a*x]^3)/(
27*a^3)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5139, 5211, 5139, 243, 53, 2009, 5183, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)^3 \, dx$$

$$\Big\downarrow \text{5139}$$

$$a \int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}} dx + \frac{1}{3} x^3 \arccos(ax)^3$$

$$\Big\downarrow \text{5211}$$

$$a \left(\frac{2 \int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}} dx}{3a^2} - \frac{2 \int x^2 \arccos(ax) dx}{3a} - \frac{x^2 \sqrt{1-a^2x^2} \arccos(ax)^2}{3a^2} \right) + \frac{1}{3} x^3 \arccos(ax)^3$$

$$\Big\downarrow \text{5139}$$

$$a \left(\frac{2 \int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}} dx}{3a^2} - \frac{2 \left(\frac{1}{3} a \int \frac{x^3}{\sqrt{1-a^2x^2}} dx + \frac{1}{3} x^3 \arccos(ax) \right)}{3a} - \frac{x^2 \sqrt{1-a^2x^2} \arccos(ax)^2}{3a^2} \right) + \frac{1}{3} x^3 \arccos(ax)^3$$

$$\Big\downarrow \text{243}$$

$$a\left(\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{6}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^3$$

$$\Big\downarrow \text{ 53}$$

$$a\left(\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{6}a\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^3$$

$$\Big\downarrow \text{ 2009}$$

$$a\left(\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}\right)+\frac{1}{3}x^3\arccos(ax)^3$$

$$\Big\downarrow \text{ 5183}$$

$$a\left(\frac{2\left(-\frac{2\int \arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}\right.}{3a}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^3$$

$$\Big\downarrow \text{ 5131}$$

$$a\left(\frac{2\left(-\frac{2\left(a\int \frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\right.\right.}{}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^3$$

$$\Big\downarrow \text{ 241}$$

$$a\left(-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{}}{}\right.\right.}{}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^3$$

input
```
Int[x^2*ArcCos[a*x]^3,x]
```

output
```
(x^3*ArcCos[a*x]^3)/3 + a*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2
- (2*((a*((-2*Sqrt[1 - a^2*x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4)))/6
 + (x^3*ArcCos[a*x])/3))/(3*a) + (2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a
^2) - (2*(-(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]))/a))/(3*a^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{a^3x^3\arccos(ax)^3}{3}-\frac{\arccos(ax)^2\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{3}+\frac{4\sqrt{-a^2x^2+1}}{3}-\frac{4ax\arccos(ax)}{3}-\frac{2a^3x^3\arccos(ax)}{9}+\frac{2\left(a^2x^2+2\right)\sqrt{-a^2x^2}\cdots}{27}\cdots}{a^3}$
default	$\frac{\frac{a^3x^3\arccos(ax)^3}{3}-\frac{\arccos(ax)^2\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{3}+\frac{4\sqrt{-a^2x^2+1}}{3}-\frac{4ax\arccos(ax)}{3}-\frac{2a^3x^3\arccos(ax)}{9}+\frac{2\left(a^2x^2+2\right)\sqrt{-a^2x^2}\cdots}{27}\cdots}{a^3}$
orering	$\frac{5\left(13a^6x^6+40a^4x^4-152a^2x^2+96\right)\arccos(ax)^3}{81a^6x^3}-\frac{\left(25a^6x^6+166a^4x^4-572a^2x^2+360\right)\left(2x\arccos(ax)^3-\frac{3x^2\arccos(ax\cdots}{\sqrt{-a^2x^2+\cdots}}\cdots\right.}{81a^6x^4}$

input
```
int(x^2*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*a^3*x^3*arccos(a*x)^3-1/3*arccos(a*x)^2*(a^2*x^2+2)*(-a^2*x^2+1
)^(1/2)+4/3*(-a^2*x^2+1)^(1/2)-4/3*a*x*arccos(a*x)-2/9*a^3*x^3*arccos(a*x)
+2/27*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int x^2 \arccos(ax)^3 \, dx$$
$$= \frac{9\, a^3x^3 \arccos{(ax)}^3 - 6\,(a^3x^3 + 6\,ax) \arccos{(ax)} + \left(2\,a^2x^2 - 9\,(a^2x^2 + 2) \arccos{(ax)}^2 + 40\right)\sqrt{-a^2x^2 + 1}}{27\,a^3}$$

input
```
integrate(x^2*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*arccos(a*x)^3 - 6*(a^3*x^3 + 6*a*x)*arccos(a*x) + (2*a^2*x
^2 - 9*(a^2*x^2 + 2)*arccos(a*x)^2 + 40)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int x^2 \arccos(ax)^3 \, dx$$
$$= \begin{cases} \frac{x^3 \operatorname{acos}^3(ax)}{3} - \frac{2x^3 \operatorname{acos}(ax)}{9} - \frac{x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a} + \frac{2x^2\sqrt{-a^2x^2+1}}{27a} - \frac{4x\operatorname{acos}(ax)}{3a^2} - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a^3} + \frac{40\sqrt{-a^2x^2+1}}{27a^3} \\ \frac{\pi^3x^3}{24} \end{cases}$$

input
```
integrate(x**2*acos(a*x)**3,x)
```

output
```
Piecewise((x**3*acos(a*x)**3/3 - 2*x**3*acos(a*x)/9 - x**2*sqrt(-a**2*x**2
 + 1)*acos(a*x)**2/(3*a) + 2*x**2*sqrt(-a**2*x**2 + 1)/(27*a) - 4*x*acos(a
*x)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(3*a**3) + 40*sqrt(-a**
2*x**2 + 1)/(27*a**3), Ne(a, 0)), (pi**3*x**3/24, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(ax)^3 \, dx = \frac{1}{3} x^3 \arccos(ax)^3 - \frac{1}{3} a \left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arccos(ax)^2 + \frac{2}{27} a \left(\frac{\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2}}{a^2} - \frac{3(a^2x^3+6x)\arccos(ax)}{a^3} \right)$$

input
```
integrate(x^2*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(a*x)^3 - 1/3*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^
2 + 1)/a^4)*arccos(a*x)^2 + 2/27*a*((sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-a^2
*x^2 + 1)/a^2)/a^2 - 3*(a^2*x^3 + 6*x)*arccos(a*x)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int x^2 \arccos(ax)^3 \, dx = \frac{1}{3} x^3 \arccos(ax)^3 - \frac{2}{9} x^3 \arccos(ax) - \frac{\sqrt{-a^2x^2+1}x^2 \arccos(ax)^2}{3a} + \frac{2\sqrt{-a^2x^2+1}x^2}{27a} - \frac{4x \arccos(ax)}{3a^2} - \frac{2\sqrt{-a^2x^2+1}\arccos(ax)^2}{3a^3} + \frac{40\sqrt{-a^2x^2+1}}{27a^3}$$

input
```
integrate(x^2*arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/3*x^3*arccos(a*x)^3 - 2/9*x^3*arccos(a*x) - 1/3*sqrt(-a^2*x^2 + 1)*x^2*a
rccos(a*x)^2/a + 2/27*sqrt(-a^2*x^2 + 1)*x^2/a - 4/3*x*arccos(a*x)/a^2 - 2
/3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a^3 + 40/27*sqrt(-a^2*x^2 + 1)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^3 \, dx = \int x^2 \operatorname{acos}(a\,x)^3 \, dx$$

input
```
int(x^2*acos(a*x)^3,x)
```

output
```
int(x^2*acos(a*x)^3, x)
```

Reduce [F]

$$\int x^2 \arccos(ax)^3 \, dx = \int a\cos(ax)^3 x^2 dx$$

input
```
int(x^2*acos(a*x)^3,x)
```

output
```
int(acos(a*x)**3*x**2,x)
```

3.25 $\int x \arccos(ax)^3 \, dx$

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Optimal result

Integrand size = 8, antiderivative size = 99

$$\int x \arccos(ax)^3 \, dx = \frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3}{4}x^2 \arccos(ax) - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} + \frac{1}{2}x^2\arccos(ax)^3 - \frac{3\arcsin(ax)}{8a^2}$$

output
```
3/8*x*(-a^2*x^2+1)^(1/2)/a-3/4*x^2*arccos(a*x)-3/4*x*(-a^2*x^2+1)^(1/2)*ar
ccos(a*x)^2/a-1/4*arccos(a*x)^3/a^2+1/2*x^2*arccos(a*x)^3-3/8*arcsin(a*x)/
a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x \arccos(ax)^3 \, dx = \frac{3ax\sqrt{1-a^2x^2} - 6a^2x^2\arccos(ax) - 6ax\sqrt{1-a^2x^2}\arccos(ax)^2 + (-2+4a^2x^2)\arccos(ax)^3 - 3\arcsin(}{8a^2}$$

input
```
Integrate[x*ArcCos[a*x]^3,x]
```

output
```
(3*a*x*Sqrt[1 - a^2*x^2] - 6*a^2*x^2*ArcCos[a*x] - 6*a*x*Sqrt[1 - a^2*x^2]
*ArcCos[a*x]^2 + (-2 + 4*a^2*x^2)*ArcCos[a*x]^3 - 3*ArcSin[a*x])/(8*a^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5139, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax)^3 \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{3}{2}a\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^3$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{2}a\left(\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int x\arccos(ax)dx}{a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^3$$

$$\Big\downarrow \text{5139}$$

$$\frac{3}{2}a\left(-\frac{\frac{1}{2}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^3$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{2}a\left(-\frac{\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^3$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}a\left(\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}\right)+\frac{1}{2}x^2\arccos(ax)^3$$

$$\Big\downarrow \text{5153}$$

$$\frac{3}{2}a\left(-\frac{\arccos(ax)^3}{6a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}\right)+\frac{1}{2}x^2\arccos(ax)^3$$

input
```
Int[x*ArcCos[a*x]^3,x]
```

output
```
(x^2*ArcCos[a*x]^3)/2 + (3*a*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2
 - ArcCos[a*x]^3/(6*a^3) - ((x^2*ArcCos[a*x])/2 + (a*(-1/2*(x*Sqrt[1 - a^2
*x^2])/a^2 + ArcSin[a*x]/(2*a^3)))/2)/a))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\frac{\cos(2\arccos(ax))\arccos(ax)^3}{4}-\frac{3\sin(2\arccos(ax))\arccos(ax)^2}{8}+\frac{3\sin(2\arccos(ax))}{16}-\frac{3\cos(2\arccos(ax))\arccos(ax)}{8}}{a^2}$
default	$\frac{\frac{\cos(2\arccos(ax))\arccos(ax)^3}{4}-\frac{3\sin(2\arccos(ax))\arccos(ax)^2}{8}+\frac{3\sin(2\arccos(ax))}{16}-\frac{3\cos(2\arccos(ax))\arccos(ax)}{8}}{a^2}$
orering	$\frac{(15a^4x^4-20a^2x^2+8)\arccos(ax)^3}{16a^4x^2}-\frac{(7a^4x^4-16a^2x^2+8)\left(\arccos(ax)^3-\frac{3x\arccos(ax)^2a}{\sqrt{-a^2x^2+1}}\right)}{16x^2a^4}+\frac{(ax-1)(ax+1)(a^2x^2-}{}$

input
```
int(x*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*cos(2*arccos(a*x))*arccos(a*x)^3-3/8*sin(2*arccos(a*x))*arccos(
a*x)^2+3/16*sin(2*arccos(a*x))-3/8*cos(2*arccos(a*x))*arccos(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int x \arccos(ax)^3 \, dx$$
$$= \frac{2 \, (2 \, a^2x^2 - 1) \arccos\left(ax\right)^3 - 3 \, (2 \, a^2x^2 - 1) \arccos\left(ax\right) - 3 \, \sqrt{-a^2x^2 + 1} \left(2 \, ax \arccos\left(ax\right)^2 - ax\right)}{8 \, a^2}$$

input
```
integrate(x*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2*x^2 - 1)*arccos(a*x)^3 - 3*(2*a^2*x^2 - 1)*arccos(a*x) - 3*s
qrt(-a^2*x^2 + 1)*(2*a*x*arccos(a*x)^2 - a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x \arccos(ax)^3 \, dx$$
$$= \begin{cases} \frac{x^2 \operatorname{acos}^3(ax)}{2} - \frac{3x^2 \operatorname{acos}(ax)}{4} - \frac{3x\sqrt{-a^2x^2+1} \operatorname{acos}^2(ax)}{4a} + \frac{3x\sqrt{-a^2x^2+1}}{8a} - \frac{\operatorname{acos}^3(ax)}{4a^2} + \frac{3 \operatorname{acos}(ax)}{8a^2} & \text{for } a \neq 0 \\ \frac{\pi^3 x^2}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x)**3,x)
```

output
```
Piecewise((x**2*acos(a*x)**3/2 - 3*x**2*acos(a*x)/4 - 3*x*sqrt(-a**2*x**2
+ 1)*acos(a*x)**2/(4*a) + 3*x*sqrt(-a**2*x**2 + 1)/(8*a) - acos(a*x)**3/(4
*a**2) + 3*acos(a*x)/(8*a**2), Ne(a, 0)), (pi**3*x**2/16, True))
```

Maxima [F]

$$\int x \arccos(ax)^3\,dx = \int x \arccos\left(ax\right)^3\,dx$$

input
```
integrate(x*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*integrate(1/2*s
qrt(a*x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
^2/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int x \arccos(ax)^3\,dx = \frac{1}{2}\,x^2\arccos\left(ax\right)^3 - \frac{3}{4}\,x^2\arccos\left(ax\right) - \frac{3\,\sqrt{-a^2x^2+1}x\arccos\left(ax\right)^2}{4\,a} - \frac{\arccos\left(ax\right)^3}{4\,a^2} + \frac{3\,\sqrt{-a^2x^2+1}x}{8\,a} + \frac{3\,\arccos\left(ax\right)}{8\,a^2}$$

input
```
integrate(x*arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/2*x^2*arccos(a*x)^3 - 3/4*x^2*arccos(a*x) - 3/4*sqrt(-a^2*x^2 + 1)*x*arc
cos(a*x)^2/a - 1/4*arccos(a*x)^3/a^2 + 3/8*sqrt(-a^2*x^2 + 1)*x/a + 3/8*ar
ccos(a*x)/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^3\,dx = \int x\,\mathrm{acos}(a\,x)^3\,dx$$

input
```
int(x*acos(a*x)^3,x)
```

output
```
int(x*acos(a*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int x \arccos(ax)^3 \, dx$$
$$= \frac{4acos(ax)^3 \, a^2x^2 - 2acos(ax)^3 - 6\sqrt{-a^2x^2+1} \, acos(ax)^2 \, ax - 6acos(ax) \, a^2x^2 - 3asin(ax) + 3\sqrt{-a^2x^2}}{8a^2}$$

input
```
int(x*acos(a*x)^3,x)
```

output
```
(4*acos(a*x)**3*a**2*x**2 - 2*acos(a*x)**3 - 6*sqrt( - a**2*x**2 + 1)*acos
(a*x)**2*a*x - 6*acos(a*x)*a**2*x**2 - 3*asin(a*x) + 3*sqrt( - a**2*x**2 +
 1)*a*x)/(8*a**2)
```

3.26 $\int \arccos(ax)^3 \, dx$

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Optimal result

Integrand size = 6, antiderivative size = 60

$$\int \arccos(ax)^3 \, dx = \frac{6\sqrt{1-a^2x^2}}{a} - 6x\arccos(ax) - \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3$$

output
```
6*(-a^2*x^2+1)^(1/2)/a-6*x*arccos(a*x)-3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/
a+x*arccos(a*x)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^3 \, dx = \frac{6\sqrt{1-a^2x^2}}{a} - 6x\arccos(ax) - \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3$$

input
```
Integrate[ArcCos[a*x]^3,x]
```

output
```
(6*Sqrt[1 - a^2*x^2])/a - 6*x*ArcCos[a*x] - (3*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]^2)/a + x*ArcCos[a*x]^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5131, 5183, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3 \, dx$$

$$\Big\downarrow \text{5131}$$

$$3a \int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}} dx + x \arccos(ax)^3$$

$$\Big\downarrow \text{5183}$$

$$3a \left(-\frac{2 \int \arccos(ax) dx}{a} - \frac{\sqrt{1-a^2x^2} \arccos(ax)^2}{a^2}\right) + x \arccos(ax)^3$$

$$\Big\downarrow \text{5131}$$

$$3a \left(-\frac{2 \left(a \int \frac{x}{\sqrt{1-a^2x^2}} dx + x \arccos(ax)\right)}{a} - \frac{\sqrt{1-a^2x^2} \arccos(ax)^2}{a^2}\right) + x \arccos(ax)^3$$

$$\Big\downarrow \text{241}$$

$$3a \left(-\frac{\sqrt{1-a^2x^2} \arccos(ax)^2}{a^2} - \frac{2 \left(x \arccos(ax) - \frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right) + x \arccos(ax)^3$$

input
```
Int[ArcCos[a*x]^3,x]
```

output
```
x*ArcCos[a*x]^3 + 3*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sq
rt[1 - a^2*x^2]/a) + x*ArcCos[a*x]))/a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{ax\arccos(ax)^3-3\arccos(ax)^2\sqrt{-a^2x^2+1}+6\sqrt{-a^2x^2+1}-6ax\arccos(ax)}{a}$
default	$\frac{ax\arccos(ax)^3-3\arccos(ax)^2\sqrt{-a^2x^2+1}+6\sqrt{-a^2x^2+1}-6ax\arccos(ax)}{a}$
orering	$x\arccos(ax)^3+\frac{3(a^2x^2-2)\arccos(ax)^2}{a\sqrt{-a^2x^2+1}}-\frac{2(ax-1)(ax+1)x\left(\frac{6\arccos(ax)a^2}{-a^2x^2+1}-\frac{3\arccos(ax)^2a^3x}{(-a^2x^2+1)^{\frac{3}{2}}}\right)}{a^2}-\frac{(ax-1)^2}{}$

input
```
int(arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x*arccos(a*x)^3-3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+6*(-a^2*x^2+1)^(
1/2)-6*a*x*arccos(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \arccos(ax)^3\,dx = \frac{ax\arccos(ax)^3 - 6\,ax\arccos(ax) - 3\sqrt{-a^2x^2+1}\left(\arccos(ax)^2-2\right)}{a}$$

input
```
integrate(arccos(a*x)^3,x, algorithm="fricas")
```

output
```
(a*x*arccos(a*x)^3 - 6*a*x*arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*(arccos(a*x)
^2 - 2))/a
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^3\,dx = \begin{cases} x\operatorname{acos}^3(ax) - 6x\operatorname{acos}(ax) - \frac{3\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{a} + \frac{6\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ \frac{\pi^3x}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)**3,x)
```

output
```
Piecewise((x*acos(a*x)**3 - 6*x*acos(a*x) - 3*sqrt(-a**2*x**2 + 1)*acos(a*
x)**2/a + 6*sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi**3*x/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \arccos(ax)^3 \, dx = x \arccos(ax)^3 - \frac{3\sqrt{-a^2x^2+1}\arccos(ax)^2}{a} - \frac{6\left(ax\arccos(ax) - \sqrt{-a^2x^2+1}\right)}{a}$$

input
```
integrate(arccos(a*x)^3,x, algorithm="maxima")
```

output
```
x*arccos(a*x)^3 - 3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a - 6*(a*x*arccos(a*x
) - sqrt(-a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \arccos(ax)^3 \, dx = x \arccos(ax)^3 - 6\,x\arccos(ax) - \frac{3\sqrt{-a^2x^2+1}\arccos(ax)^2}{a} + \frac{6\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate(arccos(a*x)^3,x, algorithm="giac")
```

output
```
x*arccos(a*x)^3 - 6*x*arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a +
 6*sqrt(-a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \arccos(ax)^3\,dx = \begin{cases} \frac{x\,\pi^3}{8} & \text{if } a = 0 \\ -x\,\left(6\,\mathrm{acos}(a\,x) - \mathrm{acos}(a\,x)^3\right) - \frac{\sqrt{1-a^2\,x^2}\,\left(3\,\mathrm{acos}(a\,x)^2-6\right)}{a} & \text{if } a \neq 0 \end{cases}$$

input
```
int(acos(a*x)^3,x)
```

output
```
piecewise(a == 0, (x*pi^3)/8, a ~= 0, - x*(6*acos(a*x) - acos(a*x)^3) - ((
- a^2*x^2 + 1)^(1/2)*(3*acos(a*x)^2 - 6))/a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \arccos(ax)^3\,dx = \frac{acos(ax)^3\,ax - 3\sqrt{-a^2x^2+1}\,acos(ax)^2 - 6acos(ax)\,ax + 6\sqrt{-a^2x^2+1}}{a}$$

input
```
int(acos(a*x)^3,x)
```

output
```
(acos(a*x)**3*a*x - 3*sqrt( - a**2*x**2 + 1)*acos(a*x)**2 - 6*acos(a*x)*a*
x + 6*sqrt( - a**2*x**2 + 1))/a
```

3.27 $\int \frac{\arccos(ax)^3}{x} \, dx$

Optimal result 274
Mathematica [A] (verified) 275
Rubi [A] (verified) 275
Maple [A] (verified) 278
Fricas [F] 279
Sympy [F] 279
Maxima [F] 279
Giac [F] 280
Mupad [F(-1)] 280
Reduce [F] 280

Optimal result

Integrand size = 10, antiderivative size = 101

$$\begin{aligned}\int \frac{\arccos(ax)^3}{x} \, dx = & -\frac{1}{4} i \arccos(ax)^4 + \arccos(ax)^3 \log\left(1 + e^{2i\arccos(ax)}\right) \\ & - \frac{3}{2} i \arccos(ax)^2 \operatorname{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) \\ & + \frac{3}{2} \arccos(ax) \operatorname{PolyLog}\left(3, -e^{2i\arccos(ax)}\right) \\ & + \frac{3}{4} i \operatorname{PolyLog}\left(4, -e^{2i\arccos(ax)}\right)\end{aligned}$$

output
```
-1/4*I*arccos(a*x)^4+arccos(a*x)^3*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-3/2*
I*arccos(a*x)^2*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3/2*arccos(a*x)*p
olylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3/4*I*polylog(4,-(a*x+I*(-a^2*x^2+
1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^3}{x}\,dx = -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}i\arccos(ax)^2\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{3}{2}\arccos(ax)\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + \frac{3}{4}i\,\text{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^3/x,x]
```

output
```
(-1/4*I)*ArcCos[a*x]^4 + ArcCos[a*x]^3*Log[1 + E^((2*I)*ArcCos[a*x])] - ((
3*I)/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + (3*ArcCos[a*x]*
PolyLog[3, -E^((2*I)*ArcCos[a*x])])/2 + ((3*I)/4)*PolyLog[4, -E^((2*I)*Arc
Cos[a*x])]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5137, 3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x}\,dx$$

$$\Big\downarrow \ 5137$$

$$-\int \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{ax}\,d\arccos(ax)$$

$$\Big\downarrow \ 3042$$

$$-\int \arccos(ax)^3\tan(\arccos(ax))\,d\arccos(ax)$$

$\Big\downarrow$ 4202

$$2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^3}{1+e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{4}i\arccos(ax)^4$$

$\Big\downarrow$ 2620

$$2i\left(\frac{3}{2}i\int \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)\right)-\frac{1}{4}i\arccos(ax)^4$$

$\Big\downarrow$ 3011

$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\right.$$
$$\frac{1}{4}i\arccos(ax)^4$$

$\Big\downarrow$ 7163

$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\int \operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos\right.\right.\right.$$
$$\frac{1}{4}i\arccos(ax)^4$$

$\Big\downarrow$ 2720

$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)de^{2i\arccos(a\right.\right.\right.$$
$$\frac{1}{4}i\arccos(ax)^4$$

$\Big\downarrow$ 7143

$$2i\left(\frac{3}{2}i\left(\frac{1}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{4}\operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(3\right.\right.\right.\right.$$
$$\frac{1}{4}i\arccos(ax)^4$$

input
```
Int[ArcCos[a*x]^3/x,x]
```

output
```
(-1/4*I)*ArcCos[a*x]^4 + (2*I)*((-1/2*I)*ArcCos[a*x]^3*Log[1 + E^((2*I)*Ar
cCos[a*x])] + ((3*I)/2)*((I/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a
*x])] - I*((-1/2*I)*ArcCos[a*x]*PolyLog[3, -E^((2*I)*ArcCos[a*x])] + PolyL
og[4, -E^((2*I)*ArcCos[a*x])]/4)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

method	result
derivativedivides	$-\frac{i \arccos(ax)^4}{4}+\arccos(ax)^3\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)-\frac{3i\arccos(ax)^2\operatorname{polylog}\left(2,-\left(ax\right.\right.}{2}$
default	$-\frac{i \arccos(ax)^4}{4}+\arccos(ax)^3\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)-\frac{3i\arccos(ax)^2\operatorname{polylog}\left(2,-\left(ax\right.\right.}{2}$

input
```
int(arccos(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*arccos(a*x)^4+arccos(a*x)^3*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-3/2*
I*arccos(a*x)^2*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3/2*arccos(a*x)*p
olylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3/4*I*polylog(4,-(a*x+I*(-a^2*x^2+
1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\arccos{(ax)}^3}{x}\,dx$$

input
```
integrate(arccos(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{x}\,dx$$

input
```
integrate(acos(a*x)**3/x,x)
```

output
```
Integral(acos(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\arccos{(ax)}^3}{x}\,dx$$

input
```
integrate(arccos(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x} \, dx = \int \frac{\arccos{(ax)}^3}{x} \, dx$$

input
```
integrate(arccos(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x} \, dx = \int \frac{\mathrm{acos}(a\,x)^3}{x} \, dx$$

input
```
int(acos(a*x)^3/x,x)
```

output
```
int(acos(a*x)^3/x, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x} \, dx = \int \frac{acos(ax)^3}{x} dx$$

input
```
int(acos(a*x)^3/x,x)
```

output
```
int(acos(a*x)**3/x,x)
```

3.28 $\int \frac{\arccos(ax)^3}{x^2}\,dx$

Optimal result 281
Mathematica [A] (verified) 282
Rubi [A] (verified) 282
Maple [F] 285
Fricas [F] 285
Sympy [F] 285
Maxima [F] 286
Giac [F] 286
Mupad [F(-1)] 286
Reduce [F] 287

Optimal result

Integrand size = 10, antiderivative size = 122

$$\begin{aligned}\int \frac{\arccos(ax)^3}{x^2}\,dx = &-\frac{\arccos(ax)^3}{x} - 6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)\\ &+ 6ia\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)\\ &- 6ia\arccos(ax)\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\\ &- 6a\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + 6a\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\end{aligned}$$

output
```
-arccos(a*x)^3/x-6*I*a*arccos(a*x)^2*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+6*I*
a*arccos(a*x)*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*I*a*arccos(a*x)*p
olylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*a*polylog(3,-I*(a*x+I*(-a^2*x^2+1
)^(1/2)))+6*a*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \frac{\arccos(ax)^3}{x^2}\,dx = -\frac{\arccos(ax)^3}{x} + 3a\Big(\arccos(ax)^2\left(\log\left(1-ie^{i\arccos(ax)}\right)-\log\left(1+ie^{i\arccos(ax)}\right)\right) + 2i\arccos(ax)\left(\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - \text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right) - 2\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + 2\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\Big)$$

input
```
Integrate[ArcCos[a*x]^3/x^2,x]
```

output
```
-(ArcCos[a*x]^3/x) + 3*a*(ArcCos[a*x]^2*(Log[1 - I*E^(I*ArcCos[a*x])] - Lo
g[1 + I*E^(I*ArcCos[a*x])]) + (2*I)*ArcCos[a*x]*(PolyLog[2, (-I)*E^(I*ArcC
os[a*x])] - PolyLog[2, I*E^(I*ArcCos[a*x])]) - 2*PolyLog[3, (-I)*E^(I*ArcC
os[a*x])] + 2*PolyLog[3, I*E^(I*ArcCos[a*x])])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5139, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x^2}\,dx$$

$$\Big\downarrow \; 5139$$

$$-3a\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)^3}{x}$$

$$\Big\downarrow \; 5219$$

$$3a\int \frac{\arccos(ax)^2}{ax}d\arccos(ax) - \frac{\arccos(ax)^3}{x}$$

$$\Big\downarrow \text{3042}$$

$$3a\int \arccos(ax)^2\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-\frac{\arccos(ax)^3}{x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\arccos(ax)^3}{x}+$$
$$3a\left(-2\int \arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+2\int \arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)-2\right.$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\arccos(ax)^3}{x}+$$
$$3a\left(2\left(i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)-2\left(i\arccos(ax)\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\arccos(ax)^3}{x}+$$
$$3a\left(2\left(i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\int e^{-i\arccos(ax)}\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}\right)-2\left(i\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arccos(ax)^3}{x}+$$
$$3a\left(-2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)+2\left(i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^3/x^2,x]
```

output
```
-(ArcCos[a*x]^3/x) + 3*a*((-2*I)*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] +
 2*(I*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - PolyLog[3, (-I)*E^(
I*ArcCos[a*x])]) - 2*(I*ArcCos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])] - Poly
Log[3, I*E^(I*ArcCos[a*x])]))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\arccos (ax)^3}{x^2} dx$$

input
```
int(arccos(a*x)^3/x^2,x)
```

output
```
int(arccos(a*x)^3/x^2,x)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^2} dx = \int \frac{\arccos (ax)^3}{x^2} dx$$

input
```
integrate(arccos(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^2} dx = \int \frac{\operatorname{acos}^3 (ax)}{x^2} dx$$

input
```
integrate(acos(a*x)**3/x**2,x)
```

output
```
Integral(acos(a*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^2}\,dx = \int \frac{\arccos(ax)^3}{x^2}\,dx$$

input
```
integrate(arccos(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
-(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*x*integrate(sqrt(a*x
+ 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^3
- x), x))/x
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^2}\,dx = \int \frac{\arccos(ax)^3}{x^2}\,dx$$

input
```
integrate(arccos(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^2}\,dx$$

input
```
int(acos(a*x)^3/x^2,x)
```

output
```
int(acos(a*x)^3/x^2, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x^2} dx = \int \frac{acos(ax)^3}{x^2} dx$$

input
```
int(acos(a*x)^3/x^2,x)
```

output
```
int(acos(a*x)**3/x**2,x)
```

3.29 $\int \frac{\arccos(ax)^3}{x^3}\,dx$

Optimal result 288
Mathematica [A] (verified) 288
Rubi [A] (verified) 289
Maple [A] (verified) 292
Fricas [F] 292
Sympy [F] 293
Maxima [F] 293
Giac [F] 293
Mupad [F(-1)] 294
Reduce [F] 294

Optimal result

Integrand size = 10, antiderivative size = 102

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = -\frac{3}{2}ia^2\arccos(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + 3a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}ia^2\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)$$

output
```
-3/2*I*a^2*arccos(a*x)^2+3/2*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/x-1/2*arcc
os(a*x)^3/x^2+3*a^2*arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-3/2*I*a
^2*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \frac{1}{2}\left(\frac{3a\left(-iax+\sqrt{1-a^2x^2}\right)\arccos(ax)^2}{x} - \frac{\arccos(ax)^3}{x^2} + 6a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - 3ia^2\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]^3/x^3,x]
```

output
```
((3*a*((-I)*a*x + Sqrt[1 - a^2*x^2])*ArcCos[a*x]^2)/x - ArcCos[a*x]^3/x^2
+ 6*a^2*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (3*I)*a^2*PolyLog[2,
-E^((2*I)*ArcCos[a*x])])/2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5139, 5187, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{3}{2}a\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 5187}$$

$$-\frac{3}{2}a\left(-2a\int \frac{\arccos(ax)}{x}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}\right)-\frac{\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 5137}$$

$$-\frac{3}{2}a\left(2a\int \frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax}d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}\right)-\frac{\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3}{2}a\left(2a\int \arccos(ax)\tan(\arccos(ax))d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}\right)-\frac{\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{ 4202}$$

$$-\frac{\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\int\frac{e^{2i\arccos(ax)}\arccos(ax)}{1+e^{2i\arccos(ax)}}d\arccos(ax)\right)\right)$$

$\big\downarrow$ 2620

$$-\frac{\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(\frac{1}{2}i\int\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos(a\right.\right.\right.$$

$\big\downarrow$ 2715

$$-\frac{\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1+e^{2i\arccos(ax)}\right)de^{2i\arccos(ax)}\right.\right.\right.$$

$\big\downarrow$ 2838

$$-\frac{\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)\log\left(1\right.\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^3/x^3,x]
```

output
```
-1/2*ArcCos[a*x]^3/x^2 - (3*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/x) + 2*
a*((I/2)*ArcCos[a*x]^2 - (2*I)*((-1/2*I)*ArcCos[a*x]*Log[1 + E^((2*I)*ArcC
os[a*x])] - PolyLog[2, -E^((2*I)*ArcCos[a*x])]/4))))/2
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

method	result
derivativedivides	$a^2 \left(-\frac{\arccos(ax)^2 \left(-3ia^2x^2 - 3\sqrt{-a^2x^2+1}\, ax + \arccos(ax)\right)}{2a^2x^2} - 3i \arccos(ax)^2 + 3\arccos(ax)\ln\left(1 -\right.\right.$
default	$a^2 \left(-\frac{\arccos(ax)^2 \left(-3ia^2x^2 - 3\sqrt{-a^2x^2+1}\, ax + \arccos(ax)\right)}{2a^2x^2} - 3i \arccos(ax)^2 + 3\arccos(ax)\ln\left(1 -\right.\right.$

input
```
int(arccos(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arccos(a*x)^2*(-3*I*a^2*x^2-3*(-a^2*x^2+1)^(1/2)*a*x+arccos(a*x)
)/a^2/x^2-3*I*arccos(a*x)^2+3*arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^
2)-3/2*I*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^3}\, dx = \int \frac{\arccos{(ax)}^3}{x^3}\, dx$$

input
```
integrate(arccos(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acos}^3(ax)}{x^3}\,dx$$

input
```
integrate(acos(a*x)**3/x**3,x)
```

output
```
Integral(acos(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{\arccos(ax)^3}{x^3}\,dx$$

input
```
integrate(arccos(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
1/2*(6*a*x^2*integrate(1/2*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^4 - x^2), x) - arctan2(sqrt(a*x + 1)*sqr
t(-a*x + 1), a*x)^3)/x^2
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{\arccos(ax)^3}{x^3}\,dx$$

input
```
integrate(arccos(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^3}\,dx$$

input
```
int(acos(a*x)^3/x^3,x)
```

output
```
int(acos(a*x)^3/x^3, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{acos(ax)^3}{x^3}dx$$

input
```
int(acos(a*x)^3/x^3,x)
```

output
```
int(acos(a*x)**3/x**3,x)
```

3.30 $\int \frac{\arccos(ax)^3}{x^4} dx$

Optimal result 295
Mathematica [A] (verified) 296
Rubi [A] (verified) 296
Maple [A] (verified) 301
Fricas [F] 301
Sympy [F] 302
Maxima [F] 302
Giac [F] 302
Mupad [F(-1)] 303
Reduce [F] 303

Optimal result

Integrand size = 10, antiderivative size = 192

$$\begin{aligned}\int \frac{\arccos(ax)^3}{x^4} dx = &-\frac{a^2 \arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} \\ &- ia^3 \arccos(ax)^2 \arctan\left(e^{i\arccos(ax)}\right) + a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right) \\ &+ ia^3 \arccos(ax)\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) \\ &- ia^3 \arccos(ax)\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right) \\ &- a^3\,\text{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) + a^3\,\text{PolyLog}\left(3, ie^{i\arccos(ax)}\right)\end{aligned}$$

output
```
-a^2*arccos(a*x)/x+1/2*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/x^2-1/3*arccos(a
*x)^3/x^3-I*a^3*arccos(a*x)^2*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+a^3*arctanh
((-a^2*x^2+1)^(1/2))+I*a^3*arccos(a*x)*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1
/2)))-I*a^3*arccos(a*x)*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-a^3*polylo
g(3,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+a^3*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/
2)))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = a^3\left(\coth^{-1}\left(\sqrt{1-a^2x^2}\right)-i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)+i\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)+\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\right)-\frac{\arccos(ax)\left(12a^2x^2+4\arccos(ax)^2-3\arccos(ax)\sin(2\arccos(ax))\right)}{12x^3}$$

input
```
Integrate[ArcCos[a*x]^3/x^4,x]
```

output
```
a^3*(ArcCoth[Sqrt[1 - a^2*x^2]] - I*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])
] + I*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - I*ArcCos[a*x]*PolyL
og[2, I*E^(I*ArcCos[a*x])] - PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + PolyLog[
3, I*E^(I*ArcCos[a*x])]) - (ArcCos[a*x]*(12*a^2*x^2 + 4*ArcCos[a*x]^2 - 3*
ArcCos[a*x]*Sin[2*ArcCos[a*x]]))/(12*x^3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {5139, 5205, 5139, 243, 73, 221, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x^4}\,dx$$

$$\Big\downarrow \text{5139}$$

$$-a\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{5205}$$

$$-a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\int \frac{\arccos(ax)}{x^2}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\right)-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{ 5139}$$

$$-a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(-a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\right)-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$-a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\right)-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{ 73}$$

$$-a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\right)-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{ 221}$$

$$-a\left(\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\right)-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{ 5219}$$

$$-a\left(-\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{ax}d\arccos(ax)-a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\right)-\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-a\left(-\frac{1}{2}a^2\int \arccos(ax)^2\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1}}{}\right.$$
$$\frac{\arccos(ax)^3}{3x^3}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\arccos(ax)^3}{3x^3}-$$
$$a\left(-\frac{1}{2}a^2\left(-2\int \arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+2\int \arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(a\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\arccos(ax)^3}{3x^3}-$$
$$a\left(-\frac{1}{2}a^2\left(2\left(i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)-2\left(i\arccos\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\arccos(ax)^3}{3x^3}-$$
$$a\left(-\frac{1}{2}a^2\left(2\left(i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\int e^{-i\arccos(ax)}\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}\right)\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arccos(ax)^3}{3x^3}-$$
$$a\left(-\frac{1}{2}a^2\left(-2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)+2\left(i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\text{PolyLog}\left(3,-ie^{i}\right.\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^3/x^4,x]
```

output
```
-1/3*ArcCos[a*x]^3/x^3 - a*(-1/2*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/x^2 - a
*(-(ArcCos[a*x]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]) - (a^2*((-2*I)*ArcCos[a
*x]^2*ArcTan[E^(I*ArcCos[a*x])] + 2*(I*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*Ar
cCos[a*x])] - PolyLog[3, (-I)*E^(I*ArcCos[a*x])]) - 2*(I*ArcCos[a*x]*PolyL
og[2, I*E^(I*ArcCos[a*x])] - PolyLog[3, I*E^(I*ArcCos[a*x])])))/2)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.33

method	result
derivativedivides	$a^3\left(-\frac{\arccos(ax)\left(-3\arccos(ax)\sqrt{-a^2x^2+1}\,ax+2\arccos(ax)^2+6a^2x^2\right)}{6a^3x^3}+\frac{\arccos(ax)^2\ln\left(1-i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)}{2}\right.$
default	$a^3\left(-\frac{\arccos(ax)\left(-3\arccos(ax)\sqrt{-a^2x^2+1}\,ax+2\arccos(ax)^2+6a^2x^2\right)}{6a^3x^3}+\frac{\arccos(ax)^2\ln\left(1-i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)}{2}\right.$

input
```
int(arccos(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/6/a^3/x^3*arccos(a*x)*(-3*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x+2*arc
cos(a*x)^2+6*a^2*x^2)+1/2*arccos(a*x)^2*ln(1-I*(a*x+I*(-a^2*x^2+1)^(1/2)))
-I*arccos(a*x)*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+polylog(3,I*(a*x+I*
(-a^2*x^2+1)^(1/2)))-1/2*arccos(a*x)^2*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))+
I*arccos(a*x)*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-polylog(3,-I*(a*x+I
*(-a^2*x^2+1)^(1/2)))-2*I*arctan(a*x+I*(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx=\int \frac{\arccos\left(ax\right)^3}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acos}^3(ax)}{x^4}\,dx$$

input
```
integrate(acos(a*x)**3/x**4,x)
```

output
```
Integral(acos(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\arccos(ax)^3}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*a*x^3*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*
sqrt(-a*x + 1), a*x)^2/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)^3)/x^3
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\arccos(ax)^3}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^4} \, dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^4} \,dx$$

input
```
int(acos(a*x)^3/x^4,x)
```

output
```
int(acos(a*x)^3/x^4, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x^4} \, dx = \int \frac{acos(ax)^3}{x^4} dx$$

input
```
int(acos(a*x)^3/x^4,x)
```

output
```
int(acos(a*x)**3/x**4,x)
```

3.31 $\int \frac{\arccos(ax)^3}{x^5}\,dx$

Optimal result 304
Mathematica [A] (verified) 305
Rubi [A] (verified) 305
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Fricas [F] 309
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Maxima [F] 310
Giac [F(-2)] 310
Mupad [F(-1)] 311
Reduce [F] 311

Optimal result

Integrand size = 10, antiderivative size = 169

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} - \frac{1}{2}ia^4\arccos(ax)^2 + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} + a^4\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{1}{2}ia^4\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)$$

output

```
1/4*a^3*(-a^2*x^2+1)^(1/2)/x-1/4*a^2*arccos(a*x)/x^2-1/2*I*a^4*arccos(a*x)
^2+1/4*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/x^3+1/2*a^3*(-a^2*x^2+1)^(1/2)*a
rccos(a*x)^2/x-1/4*arccos(a*x)^3/x^4+a^4*arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2
+1)^(1/2))^2)-1/2*I*a^4*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.89

$$\int \frac{\arccos(ax)^3}{x^5} dx = \frac{a^3x^3\sqrt{1-a^2x^2} + ax\left(-2ia^3x^3 + \sqrt{1-a^2x^2} + 2a^2x^2\sqrt{1-a^2x^2}\right)\arccos(ax)^2 - \arccos(ax)^3 + a^2x^2 \operatorname{arcco}}{4x^4}$$

input
```
Integrate[ArcCos[a*x]^3/x^5,x]
```

output
```
(a^3*x^3*Sqrt[1 - a^2*x^2] + a*x*((-2*I)*a^3*x^3 + Sqrt[1 - a^2*x^2] + 2*a
^2*x^2*Sqrt[1 - a^2*x^2])*ArcCos[a*x]^2 - ArcCos[a*x]^3 + a^2*x^2*ArcCos[a
*x]*(-1 + 4*a^2*x^2*Log[1 + E^((2*I)*ArcCos[a*x])]) - (2*I)*a^4*x^4*PolyLo
g[2, -E^((2*I)*ArcCos[a*x])])/(4*x^4)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5139, 5205, 5139, 242, 5187, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x^5} dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{3}{4}a\int \frac{\arccos(ax)^2}{x^4\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 5205}$$

$$-\frac{3}{4}a\left(\frac{2}{3}a^2\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}dx - \frac{2}{3}a\int \frac{\arccos(ax)}{x^3}dx - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{3x^3}\right) - \frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{3}{4}a\left(\frac{2}{3}a^2\int\frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-\frac{2}{3}a\left(-\frac{1}{2}a\int\frac{1}{x^2\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{2x^2}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{3x^3}\right)-\frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \; 242$$

$$-\frac{3}{4}a\left(\frac{2}{3}a^2\int\frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}dx-\frac{2}{3}a\left(\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\arccos(ax)}{2x^2}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{3x^3}\right)-\frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \; 5187$$

$$-\frac{3}{4}a\left(\frac{2}{3}a^2\left(-2a\int\frac{\arccos(ax)}{x}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}\right)-\frac{2}{3}a\left(\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\arccos(ax)}{2x^2}\right)-\frac{\sqrt{1-a^2x^2}\,\mathrm{a}}{3x}\right.$$
$$\frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \; 5137$$

$$-\frac{3}{4}a\left(\frac{2}{3}a^2\left(2a\int\frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax}d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}\right)-\frac{2}{3}a\left(\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\arccos(a}{2x^2}\right.\right.$$
$$\frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3}{4}a\left(\frac{2}{3}a^2\left(2a\int\arccos(ax)\tan(\arccos(ax))d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}\right)-\frac{2}{3}a\left(\frac{a\sqrt{1-a^2x^2}}{2x}-\mathrm{arc}\right.\right.$$
$$\frac{\arccos(ax)^3}{4x^4}$$

$$\Big\downarrow \; 4202$$

$$-\frac{\arccos(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(\frac{2}{3}a^2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\int\frac{e^{2i\arccos(ax)}\arccos(ax)}{1+e^{2i\arccos(ax)}}d\arccos(ax)\right)\right)-\frac{2}{3}a\right($$

$$\Big\downarrow \; 2620$$

$$-\frac{\arccos(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(\frac{2}{3}a^2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(\frac{1}{2}i\int\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\operatorname{arc}\right.\right.\right.\right.$$

$\downarrow$ 2715

$$-\frac{\arccos(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(\frac{2}{3}a^2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1+e^{2i\arccos(ax)}\right)de^{2i\operatorname{arcco}}\right.\right.\right.\right.$$

$\downarrow$ 2838

$$-\frac{\arccos(ax)^3}{4x^4}-$$
$$\frac{3}{4}a\left(\frac{2}{3}a^2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)\right.\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^3/x^5,x]
```

output
```
-1/4*ArcCos[a*x]^3/x^4 - (3*a*(-1/3*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/x^3
- (2*a*((a*Sqrt[1 - a^2*x^2])/(2*x) - ArcCos[a*x]/(2*x^2)))/3 + (2*a^2*(-(
(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/x) + 2*a*((I/2)*ArcCos[a*x]^2 - (2*I)*((
-1/2*I)*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - PolyLog[2, -E^((2*I)*
ArcCos[a*x])]/4))))/3))/4
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

method	result
derivativedivides	$a^4\left(-\frac{-2i\arccos(ax)^2a^4x^4-2\arccos(ax)^2\sqrt{-a^2x^2+1}\,a^3x^3-ia^4x^4-\arccos(ax)^2\sqrt{-a^2x^2+1}\,ax-a^3x^3\sqrt{-a^2x^2+1}+}{4a^4x^4}\right.$
default	$a^4\left(-\frac{-2i\arccos(ax)^2a^4x^4-2\arccos(ax)^2\sqrt{-a^2x^2+1}\,a^3x^3-ia^4x^4-\arccos(ax)^2\sqrt{-a^2x^2+1}\,ax-a^3x^3\sqrt{-a^2x^2+1}+}{4a^4x^4}\right.$

input
```
int(arccos(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/4*(-2*I*arccos(a*x)^2*a^4*x^4-2*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a
^3*x^3-I*a^4*x^4-arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a*x-a^3*x^3*(-a^2*x^2+1)
^(1/2)+arccos(a*x)^3+a^2*x^2*arccos(a*x))/a^4/x^4-I*arccos(a*x)^2+arccos(a
*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(a*x+I*(-a^2*x^2+1
)^(1/2))^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{\arccos{(ax)}^3}{x^5}\,dx$$

input
```
integrate(arccos(a*x)^3/x^5,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{\operatorname{acos}^3(ax)}{x^5}\,dx$$

input
```
integrate(acos(a*x)**3/x**5,x)
```

output
```
Integral(acos(a*x)**3/x**5, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{\arccos(ax)^3}{x^5}\,dx$$

input
```
integrate(arccos(a*x)^3/x^5,x, algorithm="maxima")
```

output
```
1/4*(12*a*x^4*integrate(1/4*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x
+ 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^6 - x^4), x) - arctan2(sqrt(a*x + 1)*sq
rt(-a*x + 1), a*x)^3)/x^4
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^3/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^5}\,dx$$

input
```
int(acos(a*x)^3/x^5,x)
```

output
```
int(acos(a*x)^3/x^5, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{acos(ax)^3}{x^5}dx$$

input
```
int(acos(a*x)^3/x^5,x)
```

output
```
int(acos(a*x)**3/x**5,x)
```

3.32 $\int x^5 \arccos(ax)^4 \, dx$

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Optimal result

Integrand size = 10, antiderivative size = 282

$$\begin{aligned}\int x^5 \arccos(ax)^4 \, dx = {} & \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} + \frac{245x\sqrt{1-a^2x^2}\arccos(ax)}{576a^5} \\ & + \frac{65x^3\sqrt{1-a^2x^2}\arccos(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{54a} \\ & + \frac{245\arccos(ax)^2}{1152a^6} - \frac{5x^2\arccos(ax)^2}{16a^4} - \frac{5x^4\arccos(ax)^2}{48a^2} \\ & - \frac{1}{18}x^6\arccos(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^3}{24a^5} \\ & - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} \\ & - \frac{5\arccos(ax)^4}{96a^6} + \frac{1}{6}x^6\arccos(ax)^4\end{aligned}$$

output
```
245/1152*x^2/a^4+65/3456*x^4/a^2+1/324*x^6+245/576*x*(-a^2*x^2+1)^(1/2)*ar
ccos(a*x)/a^5+65/864*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3+1/54*x^5*(-a^2
*x^2+1)^(1/2)*arccos(a*x)/a+245/1152*arccos(a*x)^2/a^6-5/16*x^2*arccos(a*x
)^2/a^4-5/48*x^4*arccos(a*x)^2/a^2-1/18*x^6*arccos(a*x)^2-5/24*x*(-a^2*x^2
+1)^(1/2)*arccos(a*x)^3/a^5-5/36*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a^3-
1/9*x^5*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a-5/96*arccos(a*x)^4/a^6+1/6*x^6*
arccos(a*x)^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.59

$$\int x^5 \arccos(ax)^4 \, dx$$

$$= \frac{a^2x^2(2205 + 195a^2x^2 + 32a^4x^4) + 6ax\sqrt{1-a^2x^2}(735 + 130a^2x^2 + 32a^4x^4)\arccos(ax) - 9(-245 + 360a}{}$$

input
```
Integrate[x^5*ArcCos[a*x]^4,x]
```

output
```
(a^2*x^2*(2205 + 195*a^2*x^2 + 32*a^4*x^4) + 6*a*x*Sqrt[1 - a^2*x^2]*(735
+ 130*a^2*x^2 + 32*a^4*x^4)*ArcCos[a*x] - 9*(-245 + 360*a^2*x^2 + 120*a^4*
x^4 + 64*a^6*x^6)*ArcCos[a*x]^2 - 144*a*x*Sqrt[1 - a^2*x^2]*(15 + 10*a^2*x
^2 + 8*a^4*x^4)*ArcCos[a*x]^3 + 108*(-5 + 16*a^6*x^6)*ArcCos[a*x]^4)/(1036
8*a^6)
```

Rubi [A] (verified)

Time = 2.62 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.77, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {5139, 5211, 5139, 5211, 15, 5139, 5211, 15, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \arccos(ax)^4 \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2}{3}a\int \frac{x^6 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx + \frac{1}{6}x^6 \arccos(ax)^4$$

$$\Big\downarrow \text{5211}$$

$$\frac{2}{3}a\left(\frac{5\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{6a^2} - \frac{\int x^5 \arccos(ax)^2 dx}{2a} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{6a^2}\right) + \frac{1}{6}x^6 \arccos(ax)^4$$

$$\Big\downarrow \text{5139}$$

$$\frac{2}{3}a\left(-\frac{\frac{1}{3}a\int\frac{x^6\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{6}x^6\arccos(ax)^2}{2a}+\frac{5\int\frac{x^4\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{6a^2}\right)+\frac{1}{6}x^6\arccos(ax)^4$$

$\big\downarrow$ 5211

$$\frac{2}{3}a\left(\frac{5\left(\frac{3\int\frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\int x^3\arccos(ax)^2dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}\right)}{6a^2}-\frac{\frac{1}{3}a\left(\frac{5\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}-\frac{\int x^5dx}{6a}-\frac{x^5\sqrt{1-a^2x^2}}{6a}\right.}{2a}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\big\downarrow$ 15

$$\frac{2}{3}a\left(\frac{5\left(\frac{3\int\frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\int x^3\arccos(ax)^2dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}\right)}{6a^2}-\frac{\frac{1}{3}a\left(\frac{5\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{6a^2}\right.}{2a}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\big\downarrow$ 5139

$$\frac{2}{3}a\left(\frac{5\left(\frac{3\int\frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\left(\frac{1}{2}a\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}\right)}{6a^2}-\frac{\frac{1}{3}a\left(\frac{5\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{6a^2}-\right.}{}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\big\downarrow$ 5211

$$\frac{2}{3}a\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int x^3dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}\right)+\frac{1}{4}\cdots\right.}{4a}\cdots\right.}{6a^2}\cdots\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\downarrow$ 15

$$\frac{2}{3}a\left(\frac{5\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\cdots\right.}{4a}\cdots\right.}{6a^2}\cdots\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\downarrow$ 5139

$$\frac{2}{3}a\left(\frac{5\left(\frac{3\left(-\frac{3\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}+\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\cdots}{4a^2}\cdots\right.\right.}{4a}\cdots\right.}{6a^2}\cdots\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\downarrow$ 5153

$$\frac{2}{3}a\left(-\frac{\frac{1}{3}a\left(\frac{5\left(\frac{3\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{6a^2}-\frac{x^6}{36a}\right)+\frac{1}{6}x^6\arccos(ax)^2}{2a}+\frac{5\left(-\right.}{}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\big\downarrow$ 5211

$$\frac{2}{3}a\left(-\frac{\frac{1}{3}a\left(\frac{5\left(\frac{3\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{6a^2}-\frac{x^6}{36a}\right)+\frac{1}{6}x}{2a}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\big\downarrow$ 15

$$\frac{2}{3}a\left(-\frac{\frac{1}{3}a\left(\frac{5\left(\frac{3\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)}{6a^2}-\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{6a^2}-\frac{x^6}{36a}\right)+\frac{1}{6}x^6}{2a}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

$\downarrow$ 5153

$$\frac{2}{3}a\left(-\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{6a^2}+\frac{5\left(-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}-\frac{3\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\right.\right.}{}\right.}{4a^2}\right.}{}\right.$$

$$\frac{1}{6}x^6\arccos(ax)^4$$

input
```
Int[x^5*ArcCos[a*x]^4,x]
```

output
```
(x^6*ArcCos[a*x]^4)/6 + (2*a*(-1/6*(x^5*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a
^2 - ((x^6*ArcCos[a*x]^2)/6 + (a*(-1/36*x^6/a - (x^5*Sqrt[1 - a^2*x^2]*Arc
Cos[a*x])/(6*a^2) + (5*(-1/16*x^4/a - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/
(4*a^2) + (3*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - Arc
Cos[a*x]^2/(4*a^3)))/(4*a^2)))/(6*a^2)))/3)/(2*a) + (5*(-1/4*(x^3*Sqrt[1 -
 a^2*x^2]*ArcCos[a*x]^3)/a^2 - (3*((x^4*ArcCos[a*x]^2)/4 + (a*(-1/16*x^4/a
 - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(4*a^2) + (3*(-1/4*x^2/a - (x*Sqrt[
1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3)))/(4*a^2)))/2))/
(4*a) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2 - ArcCos[a*x]^4/(
8*a^3) - (3*((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*
ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/(2*a)))/(4*a^2)))/(6*a^2))
)/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{a^6x^6\arccos(ax)^4}{6}-\frac{\arccos(ax)^3\left(8\sqrt{-a^2x^2+1}\,a^5x^5+10a^3x^3\sqrt{-a^2x^2+1}+15\sqrt{-a^2x^2+1}\,ax+15\arccos(ax)\right)}{72}-\frac{\arccos(ax)^2a^6x^6}{18}+}{}$
default	$\frac{\frac{a^6x^6\arccos(ax)^4}{6}-\frac{\arccos(ax)^3\left(8\sqrt{-a^2x^2+1}\,a^5x^5+10a^3x^3\sqrt{-a^2x^2+1}+15\sqrt{-a^2x^2+1}\,ax+15\arccos(ax)\right)}{72}-\frac{\arccos(ax)^2a^6x^6}{18}+}{}$
orering	$\frac{\left(148832a^8x^8+112095a^6x^6+1448055a^4x^4-6263460a^2x^2+4762800\right)\arccos(ax)^4}{248832a^8x^2}-\frac{\left(40800a^8x^8+62431a^6x^6+81997\right.}{}$

input
```
int(x^5*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(1/6*a^6*x^6*arccos(a*x)^4-1/72*arccos(a*x)^3*(8*(-a^2*x^2+1)^(1/2)*
a^5*x^5+10*a^3*x^3*(-a^2*x^2+1)^(1/2)+15*(-a^2*x^2+1)^(1/2)*a*x+15*arccos(
a*x))-1/18*arccos(a*x)^2*a^6*x^6+1/432*arccos(a*x)*(8*(-a^2*x^2+1)^(1/2)*a
^5*x^5+10*a^3*x^3*(-a^2*x^2+1)^(1/2)+15*(-a^2*x^2+1)^(1/2)*a*x+15*arccos(a
*x))-245/1152*arccos(a*x)^2+1/324*a^6*x^6+5/864*a^4*x^4+25/144*a^2*x^2-5/4
8*a^4*x^4*arccos(a*x)^2+5/192*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(1/2)+3*
(-a^2*x^2+1)^(1/2)*a*x+3*arccos(a*x))+5/1536*(2*a^2*x^2+3)^2-5/16*a^2*x^2*
arccos(a*x)^2+5/16*arccos(a*x)*((-a^2*x^2+1)^(1/2)*a*x+arccos(a*x))-5/32+5
/32*arccos(a*x)^4)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.54

$$\int x^5 \arccos(ax)^4 \, dx$$
$$= \frac{32 a^6x^6 + 195 a^4x^4 + 108 (16 a^6x^6 - 5) \arccos(ax)^4 + 2205 a^2x^2 - 9 (64 a^6x^6 + 120 a^4x^4 + 360 a^2x^2 - 24\ldots}{\ldots}$$

input
```
integrate(x^5*arccos(a*x)^4,x, algorithm="fricas")
```

output
```
1/10368*(32*a^6*x^6 + 195*a^4*x^4 + 108*(16*a^6*x^6 - 5)*arccos(a*x)^4 + 2
205*a^2*x^2 - 9*(64*a^6*x^6 + 120*a^4*x^4 + 360*a^2*x^2 - 245)*arccos(a*x)
^2 - 6*sqrt(-a^2*x^2 + 1)*(24*(8*a^5*x^5 + 10*a^3*x^3 + 15*a*x)*arccos(a*x
)^3 - (32*a^5*x^5 + 130*a^3*x^3 + 735*a*x)*arccos(a*x)))/a^6
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.98

$$\int x^5 \arccos(ax)^4 \, dx$$
$$= \begin{cases} \frac{x^6 \operatorname{acos}^4(ax)}{6} - \frac{x^6 \operatorname{acos}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{9a} + \frac{x^5\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{54a} - \frac{5x^4 \operatorname{acos}^2(ax)}{48a^2} + \frac{65x^4}{3456a^2} - \frac{5x^3\sqrt{-\ldots}}{\ldots} \\ \frac{\pi^4 x^6}{96} \end{cases}$$

input
```
integrate(x**5*acos(a*x)**4,x)
```

output
```
Piecewise((x**6*acos(a*x)**4/6 - x**6*acos(a*x)**2/18 + x**6/324 - x**5*sq
rt(-a**2*x**2 + 1)*acos(a*x)**3/(9*a) + x**5*sqrt(-a**2*x**2 + 1)*acos(a*x
)/(54*a) - 5*x**4*acos(a*x)**2/(48*a**2) + 65*x**4/(3456*a**2) - 5*x**3*sq
rt(-a**2*x**2 + 1)*acos(a*x)**3/(36*a**3) + 65*x**3*sqrt(-a**2*x**2 + 1)*a
cos(a*x)/(864*a**3) - 5*x**2*acos(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4)
 - 5*x**sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(24*a**5) + 245*x*sqrt(-a**2*x**2
 + 1)*acos(a*x)/(576*a**5) - 5*acos(a*x)**4/(96*a**6) + 245*acos(a*x)**2/(
1152*a**6), Ne(a, 0)), (pi**4*x**6/96, True))
```

Maxima [F]

$$\int x^5 \arccos(ax)^4\,dx = \int x^5 \arccos(ax)^4\,dx$$

input
```
integrate(x^5*arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*x^6*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 2*a*integrate(1/3*s
qrt(a*x + 1)*sqrt(-a*x + 1)*x^6*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
^3/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x^5 \arccos(ax)^4\,dx &= \frac{1}{6}x^6\arccos(ax)^4 - \frac{1}{18}x^6\arccos(ax)^2 \\ &\quad - \frac{\sqrt{-a^2x^2+1}x^5\arccos(ax)^3}{9a} + \frac{1}{324}x^6 \\ &\quad + \frac{\sqrt{-a^2x^2+1}x^5\arccos(ax)}{54a} - \frac{5x^4\arccos(ax)^2}{48a^2} \\ &\quad - \frac{5\sqrt{-a^2x^2+1}x^3\arccos(ax)^3}{36a^3} + \frac{65x^4}{3456a^2} \\ &\quad + \frac{65\sqrt{-a^2x^2+1}x^3\arccos(ax)}{864a^3} - \frac{5x^2\arccos(ax)^2}{16a^4} \\ &\quad - \frac{5\sqrt{-a^2x^2+1}x\arccos(ax)^3}{24a^5} + \frac{245x^2}{1152a^4} - \frac{5\arccos(ax)^4}{96a^6} \\ &\quad + \frac{245\sqrt{-a^2x^2+1}x\arccos(ax)}{576a^5} + \frac{245\arccos(ax)^2}{1152a^6} - \frac{9485}{82944a^6}\end{aligned}$$

input
```
integrate(x^5*arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/6*x^6*arccos(a*x)^4 - 1/18*x^6*arccos(a*x)^2 - 1/9*sqrt(-a^2*x^2 + 1)*x^
5*arccos(a*x)^3/a + 1/324*x^6 + 1/54*sqrt(-a^2*x^2 + 1)*x^5*arccos(a*x)/a
- 5/48*x^4*arccos(a*x)^2/a^2 - 5/36*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)^3/a
^3 + 65/3456*x^4/a^2 + 65/864*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a^3 - 5/1
6*x^2*arccos(a*x)^2/a^4 - 5/24*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^5 + 24
5/1152*x^2/a^4 - 5/96*arccos(a*x)^4/a^6 + 245/576*sqrt(-a^2*x^2 + 1)*x*arc
cos(a*x)/a^5 + 245/1152*arccos(a*x)^2/a^6 - 9485/82944/a^6
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \arccos(ax)^4 \, dx = \int x^5 \operatorname{acos}(a\,x)^4 \, dx$$

input
```
int(x^5*acos(a*x)^4,x)
```

output
```
int(x^5*acos(a*x)^4, x)
```

Reduce [F]

$$\int x^5 \arccos(ax)^4 \, dx = \int acos(ax)^4 x^5 dx$$

input
```
int(x^5*acos(a*x)^4,x)
```

output
```
int(acos(a*x)**4*x**5,x)
```

3.33 $\int x^4 \arccos(ax)^4 \, dx$

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Optimal result

Integrand size = 10, antiderivative size = 250

$$\begin{aligned}\int x^4 \arccos(ax)^4 \, dx = {} & \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} + \frac{16576\sqrt{1-a^2x^2}\arccos(ax)}{5625a^5} \\ & + \frac{1088x^2\sqrt{1-a^2x^2}\arccos(ax)}{5625a^3} + \frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a} \\ & - \frac{32x\arccos(ax)^2}{25a^4} - \frac{16x^3\arccos(ax)^2}{75a^2} - \frac{12}{125}x^5\arccos(ax)^2 \\ & - \frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5} - \frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3} \\ & - \frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a} + \frac{1}{5}x^5\arccos(ax)^4\end{aligned}$$

output
```
16576/5625*x/a^4+1088/16875*x^3/a^2+24/3125*x^5+16576/5625*(-a^2*x^2+1)^(1
/2)*arccos(a*x)/a^5+1088/5625*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3+24/62
5*x^4*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a-32/25*x*arccos(a*x)^2/a^4-16/75*x^3
*arccos(a*x)^2/a^2-12/125*x^5*arccos(a*x)^2-32/75*(-a^2*x^2+1)^(1/2)*arcco
s(a*x)^3/a^5-16/75*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a^3-4/25*x^4*(-a^2
*x^2+1)^(1/2)*arccos(a*x)^3/a+1/5*x^5*arccos(a*x)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.60

$$\int x^4 \arccos(ax)^4 \, dx = \frac{8ax(31080 + 680a^2x^2 + 81a^4x^4) + 120\sqrt{1 - a^2x^2}(2072 + 136a^2x^2 + 27a^4x^4)\arccos(ax) - 900ax(120 +}{843}$$

input
```
Integrate[x^4*ArcCos[a*x]^4,x]
```

output
```
(8*a*x*(31080 + 680*a^2*x^2 + 81*a^4*x^4) + 120*Sqrt[1 - a^2*x^2]*(2072 +
136*a^2*x^2 + 27*a^4*x^4)*ArcCos[a*x] - 900*a*x*(120 + 20*a^2*x^2 + 9*a^4*
x^4)*ArcCos[a*x]^2 - 4500*Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*Ar
cCos[a*x]^3 + 16875*a^5*x^5*ArcCos[a*x]^4)/(84375*a^5)
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.66, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {5139, 5211, 5139, 5211, 15, 5139, 5183, 5131, 5183, 24, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arccos(ax)^4 \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{4}{5}a\int \frac{x^5 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx + \frac{1}{5}x^5 \arccos(ax)^4$$

$$\Big\downarrow \text{5211}$$

$$\frac{4}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{5a^2} - \frac{3\int x^4 \arccos(ax)^2dx}{5a} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{5a^2}\right) + \frac{1}{5}x^5 \arccos(ax)^4$$

$$\Big\downarrow \text{5139}$$

$$\frac{4}{5}a\left(-\frac{3\left(\frac{2}{5}a\int \frac{x^5 \arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^5 \arccos(ax)^2\right)}{5a}+\frac{4\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{5a^2}\right)+$$
$$\frac{1}{5}x^5 \arccos(ax)^4$$

$$\Big\downarrow \text{5211}$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2 \arccos(ax)^2dx}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{2}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{\int x^4dx}{5a}-\frac{x^4\sqrt{1-a^2x^2}}{5}\right.\right.}{5a}\right.$$
$$\frac{1}{5}x^5 \arccos(ax)^4$$

$$\Big\downarrow \text{15}$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2 \arccos(ax)^2dx}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{2}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(a}{5a^2}\right.\right.}{5a}\right.$$
$$\frac{1}{5}x^5 \arccos(ax)^4$$

$$\Big\downarrow \text{5139}$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3 \arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{2}{5}a\left(\frac{4\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4}{}\right.\right.}{}\right.$$
$$\frac{1}{5}x^5 \arccos(ax)^4$$

$$\Big\downarrow \text{5183}$$

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(-\frac{3\int \arccos(ax)^2dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{2}{5}a\left(\cdots\right.\right.}{\cdots}\right.\cdots$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 5131

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(-\frac{3\left(2a\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(\cdots}{3a^2}\cdots\right.}{5a^2}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 5183

$$\frac{4}{5}a\left(\frac{4\left(\frac{2\left(-\frac{3\left(2a\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{\cdots}}{\cdots}\cdots\right.}{5a^2}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 24

$$\frac{4}{5}a\left(\frac{4\left(-\frac{\frac{2}{3}a\int\frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\cdots\right.\right.}{a}\cdots\right.}{3a^2}\cdots\right.}{5a^2}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 5211

$$\frac{4}{5}a\left(\frac{4\left(-\frac{\frac{2}{3}a\left(\frac{2\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(a\cdots}{a^2}\cdots\right.}{\cdots}\cdots\right.}{5a^2}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 15

$$\frac{4}{5}a\left(\frac{4\left(-\frac{\frac{2}{3}a\left(\frac{2\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\cdots\right.}{\cdots}\cdots\right.}{5a^2}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 5183

$$\frac{4}{5}a\left(\frac{4\left(-\frac{\frac{2}{3}a\left(\frac{2\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a\ldots}}{}\right.}{}\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

$\downarrow$ 24

$$\frac{4}{5}a\left(-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{5a^2}+\frac{4\left(-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\ldots\right.}{a}\right.}{3a^2}\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^4$$

input
```
Int[x^4*ArcCos[a*x]^4,x]
```

output
```
(x^5*ArcCos[a*x]^4)/5 + (4*a*(-1/5*(x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a
^2 - (3*((x^5*ArcCos[a*x]^2)/5 + (2*a*(-1/25*x^5/a - (x^4*Sqrt[1 - a^2*x^2
]*ArcCos[a*x])/(5*a^2) + (4*(-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x])/(3*a^2) + (2*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3*a^2)))
/(5*a^2)))/5))/(5*a) + (4*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2
- ((x^3*ArcCos[a*x]^2)/3 + (2*a*(-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*ArcCo
s[a*x])/(3*a^2) + (2*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3*a^
2)))/3)/a + (2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2) - (3*(x*ArcCos[a*
x]^2 + 2*a*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2)))/a))/(3*a^2)))/
(5*a^2)))/5
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{a^5x^5\arccos(ax)^4}{5}-\frac{4\arccos(ax)^3\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75}-\frac{12a^5x^5\arccos(ax)^2}{125}+\frac{8\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2}}{625}}{}$
default	$\frac{\frac{a^5x^5\arccos(ax)^4}{5}-\frac{4\arccos(ax)^3\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75}-\frac{12a^5x^5\arccos(ax)^2}{125}+\frac{8\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2}}{625}}{}$
orering	$\frac{(170181a^8x^8+190880a^6x^6+9375680a^4x^4-37873920a^2x^2+29836800)\arccos(ax)^4}{253125a^8x^3}-\frac{2(26730a^8x^8+61339a^6x^6+30}{}$

input
```
int(x^4*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccos(a*x)^4-4/75*arccos(a*x)^3*(3*a^4*x^4+4*a^2*x^2+8
)*(-a^2*x^2+1)^(1/2)-12/125*a^5*x^5*arccos(a*x)^2+8/625*arccos(a*x)*(3*a^4
*x^4+4*a^2*x^2+8)*(-a^2*x^2+1)^(1/2)+24/3125*a^5*x^5+1088/16875*a^3*x^3+16
576/5625*a*x-16/75*a^3*x^3*arccos(a*x)^2+32/225*arccos(a*x)*(a^2*x^2+2)*(-
a^2*x^2+1)^(1/2)-32/25*a*x*arccos(a*x)^2+64/25*arccos(a*x)*(-a^2*x^2+1)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.54

$$\int x^4 \arccos(ax)^4 \, dx$$
$$= \frac{16875 \, a^5x^5 \arccos{(ax)}^4 + 648 \, a^5x^5 + 5440 \, a^3x^3 - 900 \, (9 \, a^5x^5 + 20 \, a^3x^3 + 120 \, ax) \arccos{(ax)}^2 + 248640}{8437}$$

input
```
integrate(x^4*arccos(a*x)^4,x, algorithm="fricas")
```

output
```
1/84375*(16875*a^5*x^5*arccos(a*x)^4 + 648*a^5*x^5 + 5440*a^3*x^3 - 900*(9
*a^5*x^5 + 20*a^3*x^3 + 120*a*x)*arccos(a*x)^2 + 248640*a*x - 60*sqrt(-a^2
*x^2 + 1)*(75*(3*a^4*x^4 + 4*a^2*x^2 + 8)*arccos(a*x)^3 - 2*(27*a^4*x^4 +
136*a^2*x^2 + 2072)*arccos(a*x)))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.99

$$\int x^4 \arccos(ax)^4 \, dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acos}^4{(ax)}}{5} - \frac{12x^5 \operatorname{acos}^2{(ax)}}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{-a^2x^2+1} \operatorname{acos}^3{(ax)}}{25a} + \frac{24x^4\sqrt{-a^2x^2+1} \operatorname{acos}{(ax)}}{625a} - \frac{16x^3 \operatorname{acos}^2{(ax)}}{75a^2} + \frac{1088x^3}{16875a^2} - \\ \frac{\pi^4 x^5}{80} \end{cases}$$

input
```
integrate(x**4*acos(a*x)**4,x)
```

output
```
Piecewise((x**5*acos(a*x)**4/5 - 12*x**5*acos(a*x)**2/125 + 24*x**5/3125 -
 4*x**4*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(25*a) + 24*x**4*sqrt(-a**2*x**2
 + 1)*acos(a*x)/(625*a) - 16*x**3*acos(a*x)**2/(75*a**2) + 1088*x**3/(1687
5*a**2) - 16*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(75*a**3) + 1088*x**2*
sqrt(-a**2*x**2 + 1)*acos(a*x)/(5625*a**3) - 32*x*acos(a*x)**2/(25*a**4) +
 16576*x/(5625*a**4) - 32*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(75*a**5) + 16
576*sqrt(-a**2*x**2 + 1)*acos(a*x)/(5625*a**5), Ne(a, 0)), (pi**4*x**5/80,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\int x^4 \arccos(ax)^4 \, dx = \frac{1}{5} x^5 \arccos(ax)^4 - \frac{4}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right) a \arccos(ax)^3 + \frac{4}{84375}\left(2a\left(\frac{15\left(27\sqrt{-a^2x^2+1}a^2x^4 + 136\sqrt{-a^2x^2+1}x^2 + \frac{2072\sqrt{-a^2x^2+1}}{a^2}\right)\arccos(ax)}{a^5} + \frac{81a^4x^5 +$$

input
```
integrate(x^4*arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/5*x^5*arccos(a*x)^4 - 4/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x
^2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x)^3 + 4/84375*(2*a
*(15*(27*sqrt(-a^2*x^2 + 1)*a^2*x^4 + 136*sqrt(-a^2*x^2 + 1)*x^2 + 2072*sq
rt(-a^2*x^2 + 1)/a^2)*arccos(a*x)/a^5 + (81*a^4*x^5 + 680*a^2*x^3 + 31080*
x)/a^6) - 225*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccos(a*x)^2/a^5)*a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85

$$\int x^4 \arccos(ax)^4 \, dx = \frac{1}{5} x^5 \arccos(ax)^4 - \frac{12}{125} x^5 \arccos(ax)^2 - \frac{4\sqrt{-a^2x^2+1}x^4 \arccos(ax)^3}{25a} + \frac{24}{3125} x^5 + \frac{24\sqrt{-a^2x^2+1}x^4 \arccos(ax)}{625a} - \frac{16x^3 \arccos(ax)^2}{75a^2} - \frac{16\sqrt{-a^2x^2+1}x^2 \arccos(ax)^3}{75a^3} + \frac{1088x^3}{16875a^2} + \frac{1088\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{5625a^3} - \frac{32x \arccos(ax)^2}{25a^4} - \frac{32\sqrt{-a^2x^2+1}\arccos(ax)^3}{75a^5} + \frac{16576x}{5625a^4} + \frac{16576\sqrt{-a^2x^2+1}\arccos(ax)}{5625a^5}$$

input
```
integrate(x^4*arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/5*x^5*arccos(a*x)^4 - 12/125*x^5*arccos(a*x)^2 - 4/25*sqrt(-a^2*x^2 + 1)
*x^4*arccos(a*x)^3/a + 24/3125*x^5 + 24/625*sqrt(-a^2*x^2 + 1)*x^4*arccos(
a*x)/a - 16/75*x^3*arccos(a*x)^2/a^2 - 16/75*sqrt(-a^2*x^2 + 1)*x^2*arccos
(a*x)^3/a^3 + 1088/16875*x^3/a^2 + 1088/5625*sqrt(-a^2*x^2 + 1)*x^2*arccos
(a*x)/a^3 - 32/25*x*arccos(a*x)^2/a^4 - 32/75*sqrt(-a^2*x^2 + 1)*arccos(a*
x)^3/a^5 + 16576/5625*x/a^4 + 16576/5625*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^
5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^4 \, dx = \int x^4 \operatorname{acos}(a\,x)^4 \, dx$$

input
```
int(x^4*acos(a*x)^4,x)
```

output
```
int(x^4*acos(a*x)^4, x)
```

Reduce [F]

$$\int x^4 \arccos(ax)^4 \, dx = \int acos(ax)^4 x^4 dx$$

input
```
int(x^4*acos(a*x)^4,x)
```

output
```
int(acos(a*x)**4*x**4,x)
```

3.34 $\int x^3 \arccos(ax)^4\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 198

$$\int x^3 \arccos(ax)^4\,dx = \frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1-a^2x^2}\arccos(ax)}{64a^3} + \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a} + \frac{45\arccos(ax)^2}{128a^4} - \frac{9x^2\arccos(ax)^2}{16a^2} - \frac{3}{16}x^4\arccos(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} - \frac{3\arccos(ax)^4}{32a^4} + \frac{1}{4}x^4\arccos(ax)^4$$

output
```
45/128*x^2/a^2+3/128*x^4+45/64*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3+3/32*x
^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a+45/128*arccos(a*x)^2/a^4-9/16*x^2*arcc
os(a*x)^2/a^2-3/16*x^4*arccos(a*x)^2-3/8*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^
3/a^3-1/4*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a-3/32*arccos(a*x)^4/a^4+1/
4*x^4*arccos(a*x)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.68

$$\int x^3 \arccos(ax)^4 \, dx$$
$$= \frac{3a^2x^2(15+a^2x^2)+6ax\sqrt{1-a^2x^2}(15+2a^2x^2)\arccos(ax)-3(-15+24a^2x^2+8a^4x^4)\arccos(ax)^2-1}{128a^4}$$

input
```
Integrate[x^3*ArcCos[a*x]^4,x]
```

output
```
(3*a^2*x^2*(15 + a^2*x^2) + 6*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2)*ArcCo
s[a*x] - 3*(-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcCos[a*x]^2 - 16*a*x*Sqrt[1 -
a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcCos[a*x]^4)
/(128*a^4)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5139, 5211, 5139, 5211, 15, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax)^4 \, dx$$

$$\Big\downarrow \ 5139$$

$$a\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)^4$$

$$\Big\downarrow \ 5211$$

$$a\left(\frac{3\int \frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\int x^3\arccos(ax)^2dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^4$$

$$\Big\downarrow \ 5139$$

$$a\left(\frac{3\int \frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\left(\frac{1}{2}a\int \frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}\right)+$$
$$\frac{1}{4}x^4\arccos(ax)^4$$

$\Big\downarrow$ 5211

$$a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int x^3dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}\right.\right.}{4a}\right.$$
$$\frac{1}{4}x^4\arccos(ax)^4$$

$\Big\downarrow$ 15

$$a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\right.\right.}{4a}\right.$$
$$\frac{1}{4}x^4\arccos(ax)^4$$

$\Big\downarrow$ 5139

$$a\left(\frac{3\left(-\frac{3\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}+\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3}{}\right.\right.}{}\right.$$
$$\frac{1}{4}x^4\arccos(ax)^4$$

$\Big\downarrow$ 5153

$$a\left(-\frac{3\left(\frac{1}{2}a\left(\frac{3\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}+\frac{3\left(-\frac{3\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\text{arcc}\cdots\right.}{2a}\right.}{\cdots}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^4$$

$$\Big\downarrow \; 5211$$

$$a\left(-\frac{3\left(\frac{1}{2}a\left(\frac{3\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}+\frac{3\left(-\cdots\right.}{\cdots}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^4$$

$$\Big\downarrow \; 15$$

$$a\left(-\frac{3\left(\frac{1}{2}a\left(\frac{3\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}+\frac{3\left(-\frac{3\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^4$$

$$\Big\downarrow \; 5153$$

$$a\left(-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}-\frac{3\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\right.}{2a}\right.}{4a^2}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^4$$

input

```
Int[x^3*ArcCos[a*x]^4,x]
```

output

```
(x^4*ArcCos[a*x]^4)/4 + a*(-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2
- (3*((x^4*ArcCos[a*x]^2)/4 + (a*(-1/16*x^4/a - (x^3*Sqrt[1 - a^2*x^2]*Arc
Cos[a*x])/(4*a^2) + (3*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*
a^2) - ArcCos[a*x]^2/(4*a^3)))/(4*a^2)))/2))/(4*a) + (3*(-1/2*(x*Sqrt[1 -
a^2*x^2]*ArcCos[a*x]^3)/a^2 - ArcCos[a*x]^4/(8*a^3) - (3*((x^2*ArcCos[a*x]
^2)/2 + a*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos
[a*x]^2/(4*a^3))))/(2*a)))/(4*a^2))
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^4x^4\arccos(ax)^4}{4}-\frac{\arccos(ax)^3\left(2a^3x^3\sqrt{-a^2x^2+1}+3\sqrt{-a^2x^2+1}\,ax+3\arccos(ax)\right)}{8}-\frac{3a^4x^4\arccos(ax)^2}{16}+\frac{3\arccos(ax)\left(2a^3x^3\sqrt{-\right.}$
default	$\frac{a^4x^4\arccos(ax)^4}{4}-\frac{\arccos(ax)^3\left(2a^3x^3\sqrt{-a^2x^2+1}+3\sqrt{-a^2x^2+1}\,ax+3\arccos(ax)\right)}{8}-\frac{3a^4x^4\arccos(ax)^2}{16}+\frac{3\arccos(ax)\left(2a^3x^3\sqrt{-\right.}$
orering	$\frac{(781a^6x^6+1605a^4x^4-9360a^2x^2+7560)\arccos(ax)^4}{1024a^6x^2}-\frac{(285a^6x^6+1213a^4x^4-6120a^2x^2+4860)\left(3x^2\arccos(ax)^4-\right.}{1024a^6x^4}$

input
```
int(x^3*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arccos(a*x)^4-1/8*arccos(a*x)^3*(2*a^3*x^3*(-a^2*x^2+1)
^(1/2)+3*(-a^2*x^2+1)^(1/2)*a*x+3*arccos(a*x))-3/16*a^4*x^4*arccos(a*x)^2+
3/64*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(1/2)+3*(-a^2*x^2+1)^(1/2)*a*x+3*
arccos(a*x))-45/128*arccos(a*x)^2+3/512*(2*a^2*x^2+3)^2-9/16*a^2*x^2*arcco
s(a*x)^2+9/16*arccos(a*x)*((-a^2*x^2+1)^(1/2)*a*x+arccos(a*x))+9/32*a^2*x^
2-9/32+9/32*arccos(a*x)^4)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int x^3 \arccos(ax)^4 \, dx$$
$$= \frac{3\,a^4x^4 + 4\,(8\,a^4x^4 - 3)\arccos{(ax)}^4 + 45\,a^2x^2 - 3\,(8\,a^4x^4 + 24\,a^2x^2 - 15)\arccos{(ax)}^2 - 2\,\sqrt{-a^2x^2 + 1}(}{128\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^4,x, algorithm="fricas")
```

output
```
1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*arccos(a*x)^4 + 45*a^2*x^2 - 3*(8*a^4
*x^4 + 24*a^2*x^2 - 15)*arccos(a*x)^2 - 2*sqrt(-a^2*x^2 + 1)*(8*(2*a^3*x^3
 + 3*a*x)*arccos(a*x)^3 - 3*(2*a^3*x^3 + 15*a*x)*arccos(a*x)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

$$\int x^3 \arccos(ax)^4 \, dx$$
$$= \begin{cases} \frac{x^4 \operatorname{acos}^4(ax)}{4} - \frac{3x^4 \operatorname{acos}^2(ax)}{16} + \frac{3x^4}{128} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{4a} + \frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{32a} - \frac{9x^2\operatorname{acos}^2(ax)}{16a^2} + \frac{45x^2}{128a^2} - \frac{3x\sqrt{-}}{} \\ \frac{\pi^4x^4}{64} \end{cases}$$

input
```
integrate(x**3*acos(a*x)**4,x)
```

output
```
Piecewise((x**4*acos(a*x)**4/4 - 3*x**4*acos(a*x)**2/16 + 3*x**4/128 - x**
3*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(4*a) + 3*x**3*sqrt(-a**2*x**2 + 1)*ac
os(a*x)/(32*a) - 9*x**2*acos(a*x)**2/(16*a**2) + 45*x**2/(128*a**2) - 3*x*
sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(8*a**3) + 45*x*sqrt(-a**2*x**2 + 1)*aco
s(a*x)/(64*a**3) - 3*acos(a*x)**4/(32*a**4) + 45*acos(a*x)**2/(128*a**4),
Ne(a, 0)), (pi**4*x**4/64, True))
```

Maxima [F]

$$\int x^3 \arccos(ax)^4 \, dx = \int x^3 \arccos{(ax)}^4 \, dx$$

input
```
integrate(x^3*arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - a*integrate(sqrt(a*
x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^
2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87

$$\begin{aligned}\int x^3 \arccos(ax)^4 \, dx = & \frac{1}{4} x^4 \arccos{(ax)}^4 - \frac{3}{16} x^4 \arccos{(ax)}^2 \\ & - \frac{\sqrt{-a^2x^2+1}x^3 \arccos{(ax)}^3}{4a} + \frac{3}{128} x^4 \\ & + \frac{3\sqrt{-a^2x^2+1}x^3 \arccos{(ax)}}{32a} - \frac{9x^2 \arccos{(ax)}^2}{16a^2} \\ & - \frac{3\sqrt{-a^2x^2+1}x \arccos{(ax)}^3}{8a^3} + \frac{45x^2}{128a^2} - \frac{3\arccos{(ax)}^4}{32a^4} \\ & + \frac{45\sqrt{-a^2x^2+1}x \arccos{(ax)}}{64a^3} + \frac{45\arccos{(ax)}^2}{128a^4} - \frac{189}{1024a^4}\end{aligned}$$

input
```
integrate(x^3*arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/4*x^4*arccos(a*x)^4 - 3/16*x^4*arccos(a*x)^2 - 1/4*sqrt(-a^2*x^2 + 1)*x^
3*arccos(a*x)^3/a + 3/128*x^4 + 3/32*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a
- 9/16*x^2*arccos(a*x)^2/a^2 - 3/8*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^3
+ 45/128*x^2/a^2 - 3/32*arccos(a*x)^4/a^4 + 45/64*sqrt(-a^2*x^2 + 1)*x*arc
cos(a*x)/a^3 + 45/128*arccos(a*x)^2/a^4 - 189/1024/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^4 \, dx = \int x^3 \operatorname{acos}(a\,x)^4 \, dx$$

input
```
int(x^3*acos(a*x)^4,x)
```

output
```
int(x^3*acos(a*x)^4, x)
```

Reduce [F]

$$\int x^3 \arccos(ax)^4 \, dx = \int acos(ax)^4 \, x^3 dx$$

input
```
int(x^3*acos(a*x)^4,x)
```

output
```
int(acos(a*x)**4*x**3,x)
```

3.35 $\int x^2 \arccos(ax)^4 \, dx$

Optimal result 343
Mathematica [A] (verified) 344
Rubi [A] (verified) 344
Maple [A] (verified) 348
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Mupad [F(-1)] 351
Reduce [F] 351

Optimal result

Integrand size = 10, antiderivative size = 166

$$\begin{aligned}\int x^2 \arccos(ax)^4 \, dx = {} & \frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1-a^2x^2}\arccos(ax)}{27a^3} \\ & + \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{27a} - \frac{8x\arccos(ax)^2}{3a^2} \\ & - \frac{4}{9}x^3\arccos(ax)^2 - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} \\ & - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4\end{aligned}$$

output
```
160/27*x/a^2+8/81*x^3+160/27*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^3+8/27*x^2*(
-a^2*x^2+1)^(1/2)*arccos(a*x)/a-8/3*x*arccos(a*x)^2/a^2-4/9*x^3*arccos(a*x
)^2-8/9*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a^3-4/9*x^2*(-a^2*x^2+1)^(1/2)*ar
ccos(a*x)^3/a+1/3*x^3*arccos(a*x)^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

$$\int x^2 \arccos(ax)^4 \, dx$$
$$= \frac{8ax(60 + a^2x^2) + 24\sqrt{1 - a^2x^2}(20 + a^2x^2)\arccos(ax) - 36ax(6 + a^2x^2)\arccos(ax)^2 - 36\sqrt{1 - a^2x^2}(2 +}{81a^3}$$

input
```
Integrate[x^2*ArcCos[a*x]^4,x]
```

output
```
(8*a*x*(60 + a^2*x^2) + 24*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2)*ArcCos[a*x] -
36*a*x*(6 + a^2*x^2)*ArcCos[a*x]^2 - 36*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*Ar
cCos[a*x]^3 + 27*a^3*x^3*ArcCos[a*x]^4)/(81*a^3)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5139, 5211, 5139, 5183, 5131, 5183, 24, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)^4 \, dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{4}{3}a\int \frac{x^3 \arccos(ax)^3}{\sqrt{1 - a^2x^2}}dx + \frac{1}{3}x^3 \arccos(ax)^4$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{4}{3}a\left(\frac{2\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2} - \frac{\int x^2 \arccos(ax)^2 dx}{a} - \frac{x^2\sqrt{1 - a^2x^2}\arccos(ax)^3}{3a^2}\right) + \frac{1}{3}x^3 \arccos(ax)^4$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{4}{3}a\left(\frac{2\int \frac{x\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{5183}$$

$$\frac{4}{3}a\left(\frac{2\left(-\frac{3\int \arccos(ax)^2dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(a}{3a^2}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{5131}$$

$$\frac{4}{3}a\left(\frac{2\left(-\frac{3\left(2a\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{}}{}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{5183}$$

$$\frac{4}{3}a\left(\frac{2\left(-\frac{3\left(2a\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(}{a}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{24}$$

$$\frac{4}{3}a\left(-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-}}{}\right.\right.}{}\right.}{3a^2}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{5211}$$

$$\frac{4}{3}a\left(-\frac{\frac{2}{3}a\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\Big(}{}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{15}$$

$$\frac{4}{3}a\left(-\frac{\frac{2}{3}a\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\Big(-\sqrt{}}{}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{5183}$$

$$\frac{4}{3}a\left(-\frac{\frac{2}{3}a\left(\frac{2\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax}{3a^2}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^4$$

$$\Big\downarrow \text{24}$$

$$\frac{4}{3}a\left(-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}\right)}{3a^2}-\frac{\frac{2}{3}a\Big(-}{}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^4$$

input
```
Int[x^2*ArcCos[a*x]^4,x]
```

output
```
(x^3*ArcCos[a*x]^4)/3 + (4*a*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a
^2 - ((x^3*ArcCos[a*x]^2)/3 + (2*a*(-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*Ar
cCos[a*x])/(3*a^2) + (2*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3
*a^2)))/3)/a + (2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2) - (3*(x*ArcCos
[a*x]^2 + 2*a*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2)))/a))/(3*a^2)
))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{a^3x^3\arccos(ax)^4}{3}-\frac{4\arccos(ax)^3\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9}-\frac{8ax\arccos(ax)^2}{3}+\frac{160ax}{27}+\frac{16\arccos(ax)\sqrt{-a^2x^2+1}}{3}-\frac{4a^3x^3\arccos(ax)}{9}}{a^3}$
default	$\frac{\frac{a^3x^3\arccos(ax)^4}{3}-\frac{4\arccos(ax)^3\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9}-\frac{8ax\arccos(ax)^2}{3}+\frac{160ax}{27}+\frac{16\arccos(ax)\sqrt{-a^2x^2+1}}{3}-\frac{4a^3x^3\arccos(ax)}{9}}{a^3}$
orering	$\frac{(211a^6x^6+1440a^4x^4-9360a^2x^2+8640)\arccos(ax)^4}{243a^6x^3}-\frac{2(45a^6x^6+649a^4x^4-3810a^2x^2+3420)\left(2x\arccos(ax)^4-\frac{4x^2}{}\right.}{243a^6x^4}$

input
```
int(x^2*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*a^3*x^3*arccos(a*x)^4-4/9*arccos(a*x)^3*(a^2*x^2+2)*(-a^2*x^2+1
)^(1/2)-8/3*a*x*arccos(a*x)^2+160/27*a*x+16/3*arccos(a*x)*(-a^2*x^2+1)^(1/
2)-4/9*a^3*x^3*arccos(a*x)^2+8/27*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^(1/
2)+8/81*a^3*x^3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.60

$$\int x^2 \arccos(ax)^4 dx = \frac{27 a^3x^3 \arccos(ax)^4 + 8 a^3x^3 - 36 (a^3x^3 + 6 ax) \arccos(ax)^2 + 480 ax - 12 \sqrt{-a^2x^2 + 1}(3 (a^2x^2 + 2) \text{ar}}{81 a^3}$$

input
```
integrate(x^2*arccos(a*x)^4,x, algorithm="fricas")
```

output
```
1/81*(27*a^3*x^3*arccos(a*x)^4 + 8*a^3*x^3 - 36*(a^3*x^3 + 6*a*x)*arccos(a
*x)^2 + 480*a*x - 12*sqrt(-a^2*x^2 + 1)*(3*(a^2*x^2 + 2)*arccos(a*x)^3 - 2
*(a^2*x^2 + 20)*arccos(a*x)))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int x^2 \arccos(ax)^4 dx = \begin{cases} \frac{x^3 \operatorname{acos}^4(ax)}{3} - \frac{4x^3 \operatorname{acos}^2(ax)}{9} + \frac{8x^3}{81} - \frac{4x^2\sqrt{-a^2x^2+1} \operatorname{acos}^3(ax)}{9a} + \frac{8x^2\sqrt{-a^2x^2+1} \operatorname{acos}(ax)}{27a} - \frac{8x \operatorname{acos}^2(ax)}{3a^2} + \frac{160x}{27a^2} - \frac{8\sqrt{-a^2}}{} \\ \frac{\pi^4x^3}{48} \end{cases}$$

input
```
integrate(x**2*acos(a*x)**4,x)
```

output
```
Piecewise((x**3*acos(a*x)**4/3 - 4*x**3*acos(a*x)**2/9 + 8*x**3/81 - 4*x**
2*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(9*a) + 8*x**2*sqrt(-a**2*x**2 + 1)*ac
os(a*x)/(27*a) - 8*x*acos(a*x)**2/(3*a**2) + 160*x/(27*a**2) - 8*sqrt(-a**
2*x**2 + 1)*acos(a*x)**3/(9*a**3) + 160*sqrt(-a**2*x**2 + 1)*acos(a*x)/(27
*a**3), Ne(a, 0)), (pi**4*x**3/48, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(ax)^4 \, dx$$
$$= \frac{1}{3} x^3 \arccos(ax)^4 - \frac{4}{9} a \left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4} \right) \arccos(ax)^3$$
$$+ \frac{4}{81} \left(2a \left(\frac{3\left(\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2}\right) \arccos(ax)}{a^3} + \frac{a^2x^3+60x}{a^4} \right) - \frac{9(a^2x^3+6x) \arccos(ax)^2}{a^3} \right.$$

input
```
integrate(x^2*arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(a*x)^4 - 4/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^
2 + 1)/a^4)*arccos(a*x)^3 + 4/81*(2*a*(3*(sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt
(-a^2*x^2 + 1)/a^2)*arccos(a*x)/a^3 + (a^2*x^3 + 60*x)/a^4) - 9*(a^2*x^3 +
 6*x)*arccos(a*x)^2/a^3)*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int x^2 \arccos(ax)^4 \, dx = \frac{1}{3} x^3 \arccos(ax)^4 - \frac{4}{9} x^3 \arccos(ax)^2 - \frac{4\sqrt{-a^2x^2+1}x^2 \arccos(ax)^3}{9a} + \frac{8}{81} x^3 + \frac{8\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{27a} - \frac{8x \arccos(ax)^2}{3a^2} - \frac{8\sqrt{-a^2x^2+1} \arccos(ax)^3}{9a^3} + \frac{160x}{27a^2} + \frac{160\sqrt{-a^2x^2+1} \arccos(ax)}{27a^3}$$

input
```
integrate(x^2*arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/3*x^3*arccos(a*x)^4 - 4/9*x^3*arccos(a*x)^2 - 4/9*sqrt(-a^2*x^2 + 1)*x^2
*arccos(a*x)^3/a + 8/81*x^3 + 8/27*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a -
8/3*x*arccos(a*x)^2/a^2 - 8/9*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a^3 + 160/2
7*x/a^2 + 160/27*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^4 dx = \int x^2 \operatorname{acos}(a x)^4 dx$$

input
```
int(x^2*acos(a*x)^4,x)
```

output
```
int(x^2*acos(a*x)^4, x)
```

Reduce [F]

$$\int x^2 \arccos(ax)^4 dx = \int acos(ax)^4 x^2 dx$$

input
```
int(x^2*acos(a*x)^4,x)
```

output
```
int(acos(a*x)**4*x**2,x)
```

3.36 $\int x \arccos(ax)^4 \, dx$

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Optimal result

Integrand size = 8, antiderivative size = 112

$$\int x \arccos(ax)^4 \, dx = \frac{3x^2}{4} + \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{2a} + \frac{3\arccos(ax)^2}{4a^2} - \frac{3}{2}x^2\arccos(ax)^2 - \frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a} - \frac{\arccos(ax)^4}{4a^2} + \frac{1}{2}x^2\arccos(ax)^4$$

output
```
3/4*x^2+3/2*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a+3/4*arccos(a*x)^2/a^2-3/2*x
^2*arccos(a*x)^2-x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a-1/4*arccos(a*x)^4/a^
2+1/2*x^2*arccos(a*x)^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int x \arccos(ax)^4 \, dx = \frac{3a^2x^2 + 6ax\sqrt{1-a^2x^2}\arccos(ax) + (3-6a^2x^2)\arccos(ax)^2 - 4ax\sqrt{1-a^2x^2}\arccos(ax)^3 + (-1+2a^2}{4a^2}$$

input
```
Integrate[x*ArcCos[a*x]^4,x]
```

output
```
(3*a^2*x^2 + 6*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + (3 - 6*a^2*x^2)*ArcCos[
a*x]^2 - 4*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3 + (-1 + 2*a^2*x^2)*ArcCos[a
*x]^4)/(4*a^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5139, 5211, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax)^4 \, dx$$

$$\Big\downarrow \text{5139}$$

$$2a \int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}} dx + \frac{1}{2}x^2 \arccos(ax)^4$$

$$\Big\downarrow \text{5211}$$

$$2a \left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{3 \int x \arccos(ax)^2 dx}{2a} - \frac{x\sqrt{1-a^2x^2} \arccos(ax)^3}{2a^2} \right) + \frac{1}{2}x^2 \arccos(ax)^4$$

$$\Big\downarrow \text{5139}$$

$$2a \left(-\frac{3\left(a \int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}} dx + \frac{1}{2}x^2 \arccos(ax)^2\right)}{2a} + \frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}} dx}{2a^2} - \frac{x\sqrt{1-a^2x^2} \arccos(ax)^3}{2a^2} \right) + \frac{1}{2}x^2 \arccos(ax)^4$$

$$\Big\downarrow \text{5153}$$

$$2a \left(-\frac{3\left(a \int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}} dx + \frac{1}{2}x^2 \arccos(ax)^2\right)}{2a} - \frac{\arccos(ax)^4}{8a^3} - \frac{x\sqrt{1-a^2x^2} \arccos(ax)^3}{2a^2} \right) + \frac{1}{2}x^2 \arccos(ax)^4$$

$$\Big\downarrow \text{5211}$$

$$2a\left(-\frac{3\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos}{2a^2}\right.$$

$$\frac{1}{2}x^2\arccos(ax)^4$$

$$\Big\downarrow \text{15}$$

$$2a\left(-\frac{3\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(a}{2a^2}\right.$$

$$\frac{1}{2}x^2\arccos(ax)^4$$

$$\Big\downarrow \text{5153}$$

$$2a\left(-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}-\frac{3\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right.}{2a}\right.$$

$$\frac{1}{2}x^2\arccos(ax)^4$$

input
```
Int[x*ArcCos[a*x]^4,x]
```

output
```
(x^2*ArcCos[a*x]^4)/2 + 2*a*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2
- ArcCos[a*x]^4/(8*a^3) - (3*((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x*S
qrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/(2*a))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{\cos(2\arccos(ax))\arccos(ax)^4}{4}-\frac{\sin(2\arccos(ax))\arccos(ax)^3}{2}-\frac{3\cos(2\arccos(ax))\arccos(ax)^2}{4}+\frac{3\cos(2\arccos(ax))}{8}+\frac{3\sin(2\arccos}{}}{a^2}$
default	$\frac{\frac{\cos(2\arccos(ax))\arccos(ax)^4}{4}-\frac{\sin(2\arccos(ax))\arccos(ax)^3}{2}-\frac{3\cos(2\arccos(ax))\arccos(ax)^2}{4}+\frac{3\cos(2\arccos(ax))}{8}+\frac{3\sin(2\arccos}{}}{a^2}$
orering	$\frac{(31a^4x^4-60a^2x^2+40)\arccos(ax)^4}{32a^4x^2}-\frac{(15a^4x^4-52a^2x^2+40)\left(\arccos(ax)^4-\frac{4x\arccos(ax)^3a}{\sqrt{-a^2x^2+1}}\right)}{32x^2a^4}+\frac{(5a^4x^4-22a^2x^2+}{}$

input
```
int(x*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*cos(2*arccos(a*x))*arccos(a*x)^4-1/2*sin(2*arccos(a*x))*arccos(
a*x)^3-3/4*cos(2*arccos(a*x))*arccos(a*x)^2+3/8*cos(2*arccos(a*x))+3/4*sin
(2*arccos(a*x))*arccos(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int x \arccos(ax)^4 \, dx$$
$$= \frac{(2\,a^2x^2-1)\arccos{(ax)}^4+3\,a^2x^2-3\,(2\,a^2x^2-1)\arccos{(ax)}^2-2\,\left(2\,ax\arccos{(ax)}^3-3\,ax\arccos{(ax)}\right.}{4\,a^2}$$

input
```
integrate(x*arccos(a*x)^4,x, algorithm="fricas")
```

output
```
1/4*((2*a^2*x^2 - 1)*arccos(a*x)^4 + 3*a^2*x^2 - 3*(2*a^2*x^2 - 1)*arccos(
a*x)^2 - 2*(2*a*x*arccos(a*x)^3 - 3*a*x*arccos(a*x))*sqrt(-a^2*x^2 + 1))/a
^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int x \arccos(ax)^4 \, dx$$
$$= \begin{cases} \frac{x^2\operatorname{acos}^4(ax)}{2} - \frac{3x^2\operatorname{acos}^2(ax)}{2} + \frac{3x^2}{4} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{a} + \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{2a} - \frac{\operatorname{acos}^4(ax)}{4a^2} + \frac{3\operatorname{acos}^2(ax)}{4a^2} & \text{for } a \\ \frac{\pi^4x^2}{32} & \text{other} \end{cases}$$

input
```
integrate(x*acos(a*x)**4,x)
```

output
```
Piecewise((x**2*acos(a*x)**4/2 - 3*x**2*acos(a*x)**2/2 + 3*x**2/4 - x*sqrt
(-a**2*x**2 + 1)*acos(a*x)**3/a + 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(2*a)
 - acos(a*x)**4/(4*a**2) + 3*acos(a*x)**2/(4*a**2), Ne(a, 0)), (pi**4*x**2
/32, True))
```

Maxima [F]

$$\int x\arccos(ax)^4\,dx = \int x\arccos(ax)^4\,dx$$

input
```
integrate(x*arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 2*a*integrate(sqrt(
a*x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(
a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int x\arccos(ax)^4\,dx = \frac{1}{2}x^2\arccos(ax)^4 - \frac{3}{2}x^2\arccos(ax)^2 - \frac{\sqrt{-a^2x^2+1}x\arccos(ax)^3}{a} + \frac{3}{4}x^2 - \frac{\arccos(ax)^4}{4a^2} + \frac{3\sqrt{-a^2x^2+1}x\arccos(ax)}{2a} + \frac{3\arccos(ax)^2}{4a^2} - \frac{3}{8a^2}$$

input
```
integrate(x*arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/2*x^2*arccos(a*x)^4 - 3/2*x^2*arccos(a*x)^2 - sqrt(-a^2*x^2 + 1)*x*arcco
s(a*x)^3/a + 3/4*x^2 - 1/4*arccos(a*x)^4/a^2 + 3/2*sqrt(-a^2*x^2 + 1)*x*ar
ccos(a*x)/a + 3/4*arccos(a*x)^2/a^2 - 3/8/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^4 \, dx = \int x \operatorname{acos}(a\,x)^4 \, dx$$

input
```
int(x*acos(a*x)^4,x)
```

output
```
int(x*acos(a*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int x \arccos(ax)^4 \, dx$$
$$= \frac{2acos(ax)^4 a^2x^2 - acos(ax)^4 - 4\sqrt{-a^2x^2+1}\, acos(ax)^3 ax - 6acos(ax)^2 a^2x^2 + 3acos(ax)^2 + 6\sqrt{-a^2x^2}}{4a^2}$$

input
```
int(x*acos(a*x)^4,x)
```

output
```
(2*acos(a*x)**4*a**2*x**2 - acos(a*x)**4 - 4*sqrt( - a**2*x**2 + 1)*acos(a
*x)**3*a*x - 6*acos(a*x)**2*a**2*x**2 + 3*acos(a*x)**2 + 6*sqrt( - a**2*x*
*2 + 1)*acos(a*x)*a*x + 3*a**2*x**2)/(4*a**2)
```

3.37 $\int \arccos(ax)^4\,dx$

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Reduce [B] (verification not implemented) 364

Optimal result

Integrand size = 6, antiderivative size = 69

$$\int \arccos(ax)^4\,dx = 24x + \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} - 12x\arccos(ax)^2 - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4$$

output
```
24*x+24*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a-12*x*arccos(a*x)^2-4*(-a^2*x^2+1)
^(1/2)*arccos(a*x)^3/a+x*arccos(a*x)^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^4\,dx = 24x + \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} - 12x\arccos(ax)^2 - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4$$

input
```
Integrate[ArcCos[a*x]^4,x]
```

output
```
24*x + (24*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - 12*x*ArcCos[a*x]^2 - (4*Sqrt
[1 - a^2*x^2]*ArcCos[a*x]^3)/a + x*ArcCos[a*x]^4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5131, 5183, 5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^4\,dx$$

$$\Big\downarrow \text{5131}$$

$$4a\int \frac{x\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx + x\arccos(ax)^4$$

$$\Big\downarrow \text{5183}$$

$$4a\left(-\frac{3\int \arccos(ax)^2dx}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right) + x\arccos(ax)^4$$

$$\Big\downarrow \text{5131}$$

$$4a\left(-\frac{3\left(2a\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx + x\arccos(ax)^2\right)}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right) + x\arccos(ax)^4$$

$$\Big\downarrow \text{5183}$$

$$4a\left(-\frac{3\left(2a\left(-\frac{\int 1dx}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right) + x\arccos(ax)^2\right)}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right) + x\arccos(ax)^4$$

$$\Big\downarrow \text{24}$$

$$4a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2} - \frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2} - \frac{x}{a}\right) + x\arccos(ax)^2\right)}{a}\right) + x\arccos(ax)^4$$

input
```
Int[ArcCos[a*x]^4,x]
```

output
```
x*ArcCos[a*x]^4 + 4*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2) - (3*(x*Ar
cCos[a*x]^2 + 2*a*(-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2)))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{ax\arccos(ax)^4-4\arccos(ax)^3\sqrt{-a^2x^2+1}-12ax\arccos(ax)^2+24ax+24\arccos(ax)\sqrt{-a^2x^2+1}}{a}$
default	$\frac{ax\arccos(ax)^4-4\arccos(ax)^3\sqrt{-a^2x^2+1}-12ax\arccos(ax)^2+24ax+24\arccos(ax)\sqrt{-a^2x^2+1}}{a}$
orering	$x\arccos(ax)^4-\frac{8\arccos(ax)^3}{a\sqrt{-a^2x^2+1}}+\frac{(5a^2x^2-2)x\left(\frac{12\arccos(ax)^2a^2}{-a^2x^2+1}-\frac{4\arccos(ax)^3a^3x}{(-a^2x^2+1)^{\frac{3}{2}}}\right)}{a^2}+\frac{(ax-1)(ax+1)(5a^2x}{}$

input
```
int(arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(a*x*arccos(a*x)^4-4*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)-12*a*x*arccos(a*
x)^2+24*a*x+24*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \arccos(ax)^4\,dx$$
$$= \frac{ax\arccos{(ax)}^4 - 12\,ax\arccos{(ax)}^2 + 24\,ax - 4\,\sqrt{-a^2x^2+1}\left(\arccos{(ax)}^3 - 6\,\arccos{(ax)}\right)}{a}$$

input
```
integrate(arccos(a*x)^4,x, algorithm="fricas")
```

output
```
(a*x*arccos(a*x)^4 - 12*a*x*arccos(a*x)^2 + 24*a*x - 4*sqrt(-a^2*x^2 + 1)*
(arccos(a*x)^3 - 6*arccos(a*x)))/a
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \arccos(ax)^4\,dx$$
$$= \begin{cases} x\operatorname{acos}^4{(ax)} - 12x\operatorname{acos}^2{(ax)} + 24x - \frac{4\sqrt{-a^2x^2+1}\operatorname{acos}^3{(ax)}}{a} + \frac{24\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{a} & \text{for } a \neq 0 \\ \frac{\pi^4 x}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)**4,x)
```

output
```
Piecewise((x*acos(a*x)**4 - 12*x*acos(a*x)**2 + 24*x - 4*sqrt(-a**2*x**2 +
 1)*acos(a*x)**3/a + 24*sqrt(-a**2*x**2 + 1)*acos(a*x)/a, Ne(a, 0)), (pi**
4*x/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \arccos(ax)^4\,dx = x\arccos(ax)^4 - \frac{4\sqrt{-a^2x^2+1}\arccos(ax)^3}{a} - 12\left(\frac{x\arccos(ax)^2}{a} - \frac{2\left(x + \frac{\sqrt{-a^2x^2+1}\arccos(ax)}{a}\right)}{a}\right)a$$

input
```
integrate(arccos(a*x)^4,x, algorithm="maxima")
```

output
```
x*arccos(a*x)^4 - 4*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a - 12*(x*arccos(a*x)
^2/a - 2*(x + sqrt(-a^2*x^2 + 1)*arccos(a*x)/a)/a)*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \arccos(ax)^4\,dx = x\arccos(ax)^4 - 12\,x\arccos(ax)^2 - \frac{4\sqrt{-a^2x^2+1}\arccos(ax)^3}{a} + 24\,x + \frac{24\sqrt{-a^2x^2+1}\arccos(ax)}{a}$$

input
```
integrate(arccos(a*x)^4,x, algorithm="giac")
```

output
```
x*arccos(a*x)^4 - 12*x*arccos(a*x)^2 - 4*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/
a + 24*x + 24*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \arccos(ax)^4\,dx$$
$$= \begin{cases} \frac{x\,\pi^4}{16} & \text{if } a=0 \\ x\left(\mathrm{acos}(a\,x)^4-12\,\mathrm{acos}(a\,x)^2+24\right)+\frac{\sqrt{1-a^2\,x^2}\,\left(24\,\mathrm{acos}(a\,x)-4\,\mathrm{acos}(a\,x)^3\right)}{a} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acos(a*x)^4,x)
```

output
```
piecewise(a == 0, (x*pi^4)/16, a ~= 0, x*(- 12*acos(a*x)^2 + acos(a*x)^4 +
 24) + ((- a^2*x^2 + 1)^(1/2)*(24*acos(a*x) - 4*acos(a*x)^3))/a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \arccos(ax)^4\,dx$$
$$= \frac{acos(ax)^4\,ax-4\sqrt{-a^2x^2+1}\,acos(ax)^3-12acos(ax)^2\,ax+24\sqrt{-a^2x^2+1}\,acos(ax)+24ax}{a}$$

input
```
int(acos(a*x)^4,x)
```

output
```
(acos(a*x)**4*a*x - 4*sqrt( - a**2*x**2 + 1)*acos(a*x)**3 - 12*acos(a*x)**
2*a*x + 24*sqrt( - a**2*x**2 + 1)*acos(a*x) + 24*a*x)/a
```

3.38 $\int \frac{\arccos(ax)^4}{x} dx$

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Optimal result

Integrand size = 10, antiderivative size = 119

$$\begin{aligned}\int \frac{\arccos(ax)^4}{x} dx = & -\frac{1}{5}i\arccos(ax)^5 + \arccos(ax)^4 \log\left(1+e^{2i\arccos(ax)}\right) \\ & - 2i\arccos(ax)^3 \operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) \\ & + 3\arccos(ax)^2 \operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) \\ & + 3i\arccos(ax) \operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right) \\ & - \frac{3}{2}\operatorname{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)\end{aligned}$$

output
```
-1/5*I*arccos(a*x)^5+arccos(a*x)^4*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-2*I*
arccos(a*x)^3*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*arccos(a*x)^2*pol
ylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*I*arccos(a*x)*polylog(4,-(a*x+I*(-
a^2*x^2+1)^(1/2))^2)-3/2*polylog(5,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^4}{x}\,dx = -\frac{1}{5}i\arccos(ax)^5 + \arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right) - 2i\arccos(ax)^3\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + 3\arccos(ax)^2\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + 3i\arccos(ax)\,\text{PolyLog}\left(4,-e^{2i\arccos(ax)}\right) - \frac{3}{2}\,\text{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^4/x,x]
```

output
```
(-1/5*I)*ArcCos[a*x]^5 + ArcCos[a*x]^4*Log[1 + E^((2*I)*ArcCos[a*x])] - (2
*I)*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*ArcCos[a*x]^2*Pol
yLog[3, -E^((2*I)*ArcCos[a*x])] + (3*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*A
rcCos[a*x])] - (3*PolyLog[5, -E^((2*I)*ArcCos[a*x])])/2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5137, 3042, 4202, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^4}{x}dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{ax}d\arccos(ax)$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(ax)^4\tan(\arccos(ax))d\arccos(ax)$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^4}{1+e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{5}i\arccos(ax)^5$$

$$\Big\downarrow \text{2620}$$

$$2i\left(2i\int \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right)\right)-\frac{1}{5}i\arccos(ax)^5$$

$$\Big\downarrow \text{3011}$$

$$2i\left(2i\left(\frac{1}{2}i\arccos(ax)^3\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{3}{2}i\int \arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)\right.$$
$$\frac{1}{5}i\arccos(ax)^5$$

$$\Big\downarrow \text{7163}$$

$$2i\left(2i\left(\frac{1}{2}i\arccos(ax)^3\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{3}{2}i\left(i\int \arccos(ax)\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)d\arccos(ax)\right.\right.\right.$$
$$\frac{1}{5}i\arccos(ax)^5$$

$$\Big\downarrow \text{7163}$$

$$2i\left(2i\left(\frac{1}{2}i\arccos(ax)^3\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{3}{2}i\left(i\left(\frac{1}{2}i\int \operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\,\mathrm{ar}\right.\right.\right.\right.$$
$$\frac{1}{5}i\arccos(ax)^5$$

$$\Big\downarrow \text{2720}$$

$$2i\left(2i\left(\frac{1}{2}i\arccos(ax)^3\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{3}{2}i\left(i\left(\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)de^{2i\,\mathrm{arcc}}\right.\right.\right.\right.$$
$$\frac{1}{5}i\arccos(ax)^5$$

$$\Big\downarrow \text{7143}$$

$$2i\left(2i\left(\frac{1}{2}i\arccos(ax)^3\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{3}{2}i\left(i\left(\frac{1}{4}\operatorname{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)\,\mathrm{PolyLo}\right.\right.\right.\right.$$
$$\frac{1}{5}i\arccos(ax)^5$$

input
```
Int[ArcCos[a*x]^4/x,x]
```

output
```
(-1/5*I)*ArcCos[a*x]^5 + (2*I)*((-1/2*I)*ArcCos[a*x]^4*Log[1 + E^((2*I)*Ar
cCos[a*x])] + (2*I)*((I/2)*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])
] - ((3*I)/2)*((-1/2*I)*ArcCos[a*x]^2*PolyLog[3, -E^((2*I)*ArcCos[a*x])] +
 I*((-1/2*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*ArcCos[a*x])] + PolyLog[5, -
E^((2*I)*ArcCos[a*x])]/4))))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.41

method	result
derivativedivides	$-\frac{i \arccos(ax)^5}{5} + \arccos(ax)^4 \ln\left(1 + \left(ax + i\sqrt{-a^2x^2+1}\right)^2\right) - 2i \arccos(ax)^3 \operatorname{polylog}\left(\right.$
default	$-\frac{i \arccos(ax)^5}{5} + \arccos(ax)^4 \ln\left(1 + \left(ax + i\sqrt{-a^2x^2+1}\right)^2\right) - 2i \arccos(ax)^3 \operatorname{polylog}\left(\right.$

input
```
int(arccos(a*x)^4/x,x,method=_RETURNVERBOSE)
```

output
```
-1/5*I*arccos(a*x)^5+arccos(a*x)^4*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-2*I*
arccos(a*x)^3*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*arccos(a*x)^2*pol
ylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*I*arccos(a*x)*polylog(4,-(a*x+I*(-
a^2*x^2+1)^(1/2))^2)-3/2*polylog(5,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\arccos{(ax)}^4}{x}\,dx$$

input
```
integrate(arccos(a*x)^4/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^4/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\operatorname{acos}^4{(ax)}}{x}\,dx$$

input
```
integrate(acos(a*x)**4/x,x)
```

output
```
Integral(acos(a*x)**4/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\arccos{(ax)}^4}{x}\,dx$$

input
```
integrate(arccos(a*x)^4/x,x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^4/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x}\, dx = \int \frac{\arccos{(ax)}^4}{x}\, dx$$

input
```
integrate(arccos(a*x)^4/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x}\, dx = \int \frac{\mathrm{acos}(a\,x)^4}{x}\, dx$$

input
```
int(acos(a*x)^4/x,x)
```

output
```
int(acos(a*x)^4/x, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^4}{x}\, dx = \int \frac{\mathrm{acos}(ax)^4}{x} dx$$

input
```
int(acos(a*x)^4/x,x)
```

output
```
int(acos(a*x)**4/x,x)
```

3.39 $\int \frac{\arccos(ax)^4}{x^2}\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 176

$$\begin{aligned}\int \frac{\arccos(ax)^4}{x^2}\,dx &= -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)\\ &\quad + 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)\\ &\quad - 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\\ &\quad - 24a\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)\\ &\quad + 24a\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\\ &\quad - 24ia\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right) + 24ia\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)\end{aligned}$$

output
```
-arccos(a*x)^4/x-8*I*a*arccos(a*x)^3*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+12*I
*a*arccos(a*x)^2*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-12*I*a*arccos(a*
x)^2*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-24*a*arccos(a*x)*polylog(3,-I
*(a*x+I*(-a^2*x^2+1)^(1/2)))+24*a*arccos(a*x)*polylog(3,I*(a*x+I*(-a^2*x^2
+1)^(1/2)))-24*I*a*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+24*I*a*polylog
(4,I*(a*x+I*(-a^2*x^2+1)^(1/2)))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. 2(176) = 352.

Time = 0.71 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.12

$$\int \frac{\arccos(ax)^4}{x^2}\,dx = a\left(-\frac{7i\pi^4}{16}-\frac{1}{2}i\pi^3\arccos(ax)+\frac{3}{2}i\pi^2\arccos(ax)^2-2i\pi\arccos(ax)^3 + i\arccos(ax)^4-\frac{\arccos(ax)^4}{ax} + 3\pi^2\arccos(ax)\log\left(1-ie^{-i\arccos(ax)}\right) - 6\pi\arccos(ax)^2\log\left(1-ie^{-i\arccos(ax)}\right) - \frac{1}{2}\pi^3\log\left(1+ie^{-i\arccos(ax)}\right)+4\arccos(ax)^3\log\left(1+ie^{-i\arccos(ax)}\right) + \frac{1}{2}\pi^3\log\left(1+ie^{i\arccos(ax)}\right)-3\pi^2\arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right) + 6\pi\arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right) - 4\arccos(ax)^3\log\left(1+ie^{i\arccos(ax)}\right) + \frac{1}{2}\pi^3\log\left(\tan\left(\frac{1}{4}(\pi+2\arccos(ax))\right)\right) + 12i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{-i\arccos(ax)}\right) + 3i\pi(\pi-4\arccos(ax))\operatorname{PolyLog}\left(2,ie^{-i\arccos(ax)}\right) + 3i\pi^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - 12i\pi\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + 12i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + 24\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{-i\arccos(ax)}\right) - 12\pi\operatorname{PolyLog}\left(3,ie^{-i\arccos(ax)}\right)+12\pi\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) - 24\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) - 24i\operatorname{PolyLog}\left(4,-ie^{-i\arccos(ax)}\right)-24i\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]^4/x^2,x]
```

output
```
a*(((-7*I)/16)*Pi^4 - (I/2)*Pi^3*ArcCos[a*x] + ((3*I)/2)*Pi^2*ArcCos[a*x]^
2 - (2*I)*Pi*ArcCos[a*x]^3 + I*ArcCos[a*x]^4 - ArcCos[a*x]^4/(a*x) + 3*Pi^
2*ArcCos[a*x]*Log[1 - I/E^(I*ArcCos[a*x])] - 6*Pi*ArcCos[a*x]^2*Log[1 - I/
E^(I*ArcCos[a*x])] - (Pi^3*Log[1 + I/E^(I*ArcCos[a*x])])/2 + 4*ArcCos[a*x]
^3*Log[1 + I/E^(I*ArcCos[a*x])] + (Pi^3*Log[1 + I*E^(I*ArcCos[a*x])])/2 -
3*Pi^2*ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] + 6*Pi*ArcCos[a*x]^2*Log[1
 + I*E^(I*ArcCos[a*x])] - 4*ArcCos[a*x]^3*Log[1 + I*E^(I*ArcCos[a*x])] + (
Pi^3*Log[Tan[(Pi + 2*ArcCos[a*x])/4]])/2 + (12*I)*ArcCos[a*x]^2*PolyLog[2,
 (-I)/E^(I*ArcCos[a*x])] + (3*I)*Pi*(Pi - 4*ArcCos[a*x])*PolyLog[2, I/E^(I
*ArcCos[a*x])] + (3*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (12*I)*Pi
*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + (12*I)*ArcCos[a*x]^2*Pol
yLog[2, (-I)*E^(I*ArcCos[a*x])] + 24*ArcCos[a*x]*PolyLog[3, (-I)/E^(I*ArcC
os[a*x])] - 12*Pi*PolyLog[3, I/E^(I*ArcCos[a*x])] + 12*Pi*PolyLog[3, (-I)*
E^(I*ArcCos[a*x])] - 24*ArcCos[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] - (
24*I)*PolyLog[4, (-I)/E^(I*ArcCos[a*x])] - (24*I)*PolyLog[4, (-I)*E^(I*Arc
Cos[a*x])])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5139, 5219, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^4}{x^2} \, dx$$

$$\Big\downarrow \text{5139}$$

$$-4a \int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)^4}{x}$$

$$\Big\downarrow \text{5219}$$

$$4a \int \frac{\arccos(ax)^3}{ax}d\arccos(ax) - \frac{\arccos(ax)^4}{x}$$

$$\Big\downarrow \text{3042}$$

$$4a \int \arccos(ax)^3 \csc\left(\arccos(ax) + \frac{\pi}{2}\right) d\arccos(ax) - \frac{\arccos(ax)^4}{x}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\arccos(ax)^4}{x}+$$
$$4a\left(-3\int \arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)-\right.$$

$$\Big\downarrow \; 3011$$

$$-\frac{\arccos(ax)^4}{x}+$$
$$4a\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\right.$$

$$\Big\downarrow \; 7163$$

$$-\frac{\arccos(ax)^4}{x}+$$
$$4a\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(ax)\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$-\frac{\arccos(ax)^4}{x}+$$
$$4a\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\right.\right.\right.$$

$$\Big\downarrow \; 7143$$

$$-\frac{\arccos(ax)^4}{x}+$$
$$4a\left(-2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)+3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right.\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^4/x^2,x]
```

output
```
-(ArcCos[a*x]^4/x) + 4*a*((-2*I)*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] +
 3*(I*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCo
s[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + PolyLog[4, (-I)*E^(I*ArcCos[a*
x])])) - 3*(I*ArcCos[a*x]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - (2*I)*((-I)*
ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] + PolyLog[4, I*E^(I*ArcCos[a*x
])])))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\arccos(ax)^4}{x^2}dx$$

input
```
int(arccos(a*x)^4/x^2,x)
```

output
```
int(arccos(a*x)^4/x^2,x)
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x^2}\,dx = \int \frac{\arccos(ax)^4}{x^2}\,dx$$

input
```
integrate(arccos(a*x)^4/x^2,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^4/x^2, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x^2} \, dx = \int \frac{\operatorname{acos}^4(ax)}{x^2} \, dx$$

input
```
integrate(acos(a*x)**4/x**2,x)
```

output
```
Integral(acos(a*x)**4/x**2, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x^2} \, dx = \int \frac{\arccos(ax)^4}{x^2} \, dx$$

input
```
integrate(arccos(a*x)^4/x^2,x, algorithm="maxima")
```

output
```
-(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*a*x*integrate(sqrt(a*x
+ 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*x^3
- x), x))/x
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x^2} \, dx = \int \frac{\arccos(ax)^4}{x^2} \, dx$$

input
```
integrate(arccos(a*x)^4/x^2,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^4/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^4}{x^2}\,dx$$

input
```
int(acos(a*x)^4/x^2,x)
```

output
```
int(acos(a*x)^4/x^2, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^4}{x^2}\,dx = \int \frac{acos(ax)^4}{x^2}dx$$

input
```
int(acos(a*x)^4/x^2,x)
```

output
```
int(acos(a*x)**4/x**2,x)
```

3.40 $\int \frac{\arccos(ax)^4}{x^3}\,dx$

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Giac [F] . . . 386
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Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = -2ia^2\arccos(ax)^3+\frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2}+6a^2\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - 6ia^2\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + 3a^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

output
```
-2*I*a^2*arccos(a*x)^3+2*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/x-1/2*arccos(a
*x)^4/x^2+6*a^2*arccos(a*x)^2*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-6*I*a^2*a
rccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*a^2*polylog(3,-(a*x+
I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = -\frac{\arccos(ax)^4}{2x^2} - a^2\left(-2\arccos(ax)^2\left(-i\arccos(ax)+\frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax} + 3\log\left(1+e^{2i\arccos(ax)}\right)\right) + 6i\arccos(ax)\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) - 3\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]^4/x^3,x]
```

output
```
-1/2*ArcCos[a*x]^4/x^2 - a^2*(-2*ArcCos[a*x]^2*((-I)*ArcCos[a*x] + (Sqrt[1
 - a^2*x^2]*ArcCos[a*x])/(a*x) + 3*Log[1 + E^((2*I)*ArcCos[a*x])]) + (6*I)
*ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] - 3*PolyLog[3, -E^((2*I)*A
rcCos[a*x])])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5139, 5187, 5137, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^4}{x^3}\,dx$$

$$\Big\downarrow \text{5139}$$

$$-2a\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}dx - \frac{\arccos(ax)^4}{2x^2}$$

$$\Big\downarrow \text{5187}$$

$$-2a\left(-3a\int \frac{\arccos(ax)^2}{x}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}\right)-\frac{\arccos(ax)^4}{2x^2}$$

$$\Big\downarrow \text{5137}$$

$$-2a\left(3a\int \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{ax}d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}\right)-\frac{\arccos(ax)^4}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$-2a\left(3a\int \arccos(ax)^2\tan(\arccos(ax))d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}\right)-\frac{\arccos(ax)^4}{2x^2}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{\arccos(ax)^4}{2x^2}-$$
$$2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+3a\left(\frac{1}{3}i\arccos(ax)^3-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1+e^{2i\arccos(ax)}}d\arccos(ax)\right)\right)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\arccos(ax)^4}{2x^2}-$$
$$2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\int \arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\arccos(ax)^4}{2x^2}-$$
$$2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int \operatorname{Po}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\arccos(ax)^4}{2x^2}-$$
$$2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arccos(ax)^4}{2x^2}-$$
$$2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{4}\operatorname{PolyL}\right.\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^4/x^3,x]
```

output
```
-1/2*ArcCos[a*x]^4/x^2 - 2*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x) + 3*a
*((I/3)*ArcCos[a*x]^3 - (2*I)*((-1/2*I)*ArcCos[a*x]^2*Log[1 + E^((2*I)*Arc
Cos[a*x])] + I*((I/2)*ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] - Pol
yLog[3, -E^((2*I)*ArcCos[a*x])]/4))))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
derivativedivides	$a^2\left(-\frac{\arccos(ax)^3\left(-4ia^2x^2-4\sqrt{-a^2x^2+1}\,ax+\arccos(ax)\right)}{2a^2x^2}-4i\arccos(ax)^3+6\arccos(ax)^2\ln\left(1\right.\right.$
default	$a^2\left(-\frac{\arccos(ax)^3\left(-4ia^2x^2-4\sqrt{-a^2x^2+1}\,ax+\arccos(ax)\right)}{2a^2x^2}-4i\arccos(ax)^3+6\arccos(ax)^2\ln\left(1\right.\right.$

input
```
int(arccos(a*x)^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arccos(a*x)^3*(-4*I*a^2*x^2-4*(-a^2*x^2+1)^(1/2)*a*x+arccos(a*x)
)/a^2/x^2-4*I*arccos(a*x)^3+6*arccos(a*x)^2*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2)
)^2)-6*I*arccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*polylog(3,
-(a*x+I*(-a^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = \int \frac{\arccos{(ax)}^4}{x^3}\,dx$$

input
```
integrate(arccos(a*x)^4/x^3,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^4/x^3, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = \int \frac{\operatorname{acos}^4{(ax)}}{x^3}\,dx$$

input
```
integrate(acos(a*x)**4/x**3,x)
```

output
```
Integral(acos(a*x)**4/x**3, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x^3} dx = \int \frac{\arccos(ax)^4}{x^3} dx$$

input
```
integrate(arccos(a*x)^4/x^3,x, algorithm="maxima")
```

output
```
-1/2*(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*a*x^2*integrate(sqr
t(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^
2*x^4 - x^2), x))/x^2
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x^3} dx = \int \frac{\arccos(ax)^4}{x^3} dx$$

input
```
integrate(arccos(a*x)^4/x^3,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^4/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x^3} dx = \int \frac{\mathrm{acos}(a\,x)^4}{x^3} dx$$

input
```
int(acos(a*x)^4/x^3,x)
```

output
```
int(acos(a*x)^4/x^3, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^4}{x^3} \, dx = \int \frac{acos(ax)^4}{x^3} dx$$

input
```
int(acos(a*x)^4/x^3,x)
```

output
```
int(acos(a*x)**4/x**3,x)
```

3.41 $\int \frac{\arccos(ax)^4}{x^4}\,dx$

Optimal result 388
Mathematica [B] (verified) 389
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Mupad [F(-1)] 396
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Optimal result

Integrand size = 10, antiderivative size = 304

$$
\begin{aligned}
\int \frac{\arccos(ax)^4}{x^4}\,dx = & -\frac{2a^2\arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} \\
& - \frac{\arccos(ax)^4}{3x^3} - 8ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) \\
& - \frac{4}{3}ia^3\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right) \\
& + 4ia^3\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) \\
& + 2ia^3\arccos(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) \\
& - 4ia^3\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right) \\
& - 2ia^3\arccos(ax)^2\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right) \\
& - 4a^3\arccos(ax)\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) \\
& + 4a^3\arccos(ax)\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right) \\
& - 4ia^3\,\text{PolyLog}\left(4,-ie^{i\arccos(ax)}\right) + 4ia^3\,\text{PolyLog}\left(4,ie^{i\arccos(ax)}\right)
\end{aligned}
$$

output
```
-2*a^2*arccos(a*x)^2/x+2/3*a*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/x^2-1/3*arcc
os(a*x)^4/x^3-8*I*a^3*arccos(a*x)*arctan(a*x+I*(-a^2*x^2+1)^(1/2))-4/3*I*a
^3*arccos(a*x)^3*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+4*I*a^3*polylog(2,-I*(a*
x+I*(-a^2*x^2+1)^(1/2)))+2*I*a^3*arccos(a*x)^2*polylog(2,-I*(a*x+I*(-a^2*x
^2+1)^(1/2)))-4*I*a^3*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-2*I*a^3*arcc
os(a*x)^2*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-4*a^3*arccos(a*x)*polylo
g(3,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+4*a^3*arccos(a*x)*polylog(3,I*(a*x+I*(-
a^2*x^2+1)^(1/2)))-4*I*a^3*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+4*I*a^
3*polylog(4,I*(a*x+I*(-a^2*x^2+1)^(1/2)))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1475 vs. $2(304) = 608$.

Time = 12.05 (sec) , antiderivative size = 1475, normalized size of antiderivative = 4.85

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[ArcCos[a*x]^4/x^4,x]
```

output
```
a^3*(-1/6*(ArcCos[a*x]^2*(12 + ArcCos[a*x]^2)) + 4*(ArcCos[a*x]*(Log[1 - I
*E^(I*ArcCos[a*x])] - Log[1 + I*E^(I*ArcCos[a*x])]) + I*(PolyLog[2, (-I)*E
^(I*ArcCos[a*x])] - PolyLog[2, I*E^(I*ArcCos[a*x])])) + (2*((Pi^3*Log[Cot[
(Pi/2 - ArcCos[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcCos[a*x])*(Log[1 - E^(I*
(Pi/2 - ArcCos[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcCos[a*x]))]) + I*(PolyLog
[2, -E^(I*(Pi/2 - ArcCos[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcCos[a*x]))])
))/4 - (3*Pi*((Pi/2 - ArcCos[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcCos[a*x]))]
- Log[1 + E^(I*(Pi/2 - ArcCos[a*x]))]) + (2*I)*(Pi/2 - ArcCos[a*x])*(PolyL
og[2, -E^(I*(Pi/2 - ArcCos[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcCos[a*x]))
]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcCos[a*x]))] + PolyLog[3, E^(I*(Pi/2 -
 ArcCos[a*x]))])))/2 + 8*((I/64)*(Pi/2 - ArcCos[a*x])^4 + (I/4)*(Pi/2 + (-
1/2*Pi + ArcCos[a*x])/2)^4 - ((Pi/2 - ArcCos[a*x])^3*Log[1 + E^(I*(Pi/2 -
ArcCos[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2) - Log[1 +
E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + Arc
Cos[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))] + ((3
*I)/8)*(Pi/2 - ArcCos[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcCos[a*x]))] + (3
*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + Arc
Cos[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))] + (I/2)
*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))]))/4 + ((3*I)/2)
*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/...
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5139, 5205, 5139, 5219, 3042, 4669, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^4}{x^4} dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{4}{3}a\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^4}{3x^3}$$

$$\Big\downarrow \text{5205}$$

$$-\frac{4}{3}a\left(\frac{1}{2}a^2\int\frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{3}{2}a\int\frac{\arccos(ax)^2}{x^2}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}\right)-\frac{\arccos(ax)^4}{3x^3}$$

$\big\downarrow$ 5139

$$-\frac{4}{3}a\left(-\frac{3}{2}a\left(-2a\int\frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^2}{x}\right)+\frac{1}{2}a^2\int\frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}\right)-\frac{\arccos(ax)^4}{3x^3}$$

$\big\downarrow$ 5219

$$-\frac{4}{3}a\left(-\frac{1}{2}a^2\int\frac{\arccos(ax)^3}{ax}d\arccos(ax)-\frac{3}{2}a\left(2a\int\frac{\arccos(ax)}{ax}d\arccos(ax)-\frac{\arccos(ax)^2}{x}\right)-\frac{\sqrt{1-a^2x^2}\,\mathrm{arcc}}{2x^2}\right.$$
$$\frac{\arccos(ax)^4}{3x^3}$$

$\big\downarrow$ 3042

$$-\frac{4}{3}a\left(-\frac{1}{2}a^2\int\arccos(ax)^3\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-\frac{3}{2}a\left(2a\int\arccos(ax)\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\,\mathrm{ar}\right.\right.$$
$$\frac{\arccos(ax)^4}{3x^3}$$

$\big\downarrow$ 4669

$$-\frac{\arccos(ax)^4}{3x^3}-$$
$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(-3\int\arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int\arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\,\mathrm{arcc}\right.\right.$$

$\big\downarrow$ 2715

$$-\frac{\arccos(ax)^4}{3x^3}-$$
$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(-3\int\arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int\arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\,\mathrm{arcc}\right.\right.$$

$\big\downarrow$ 2838

$$-\frac{\arccos(ax)^4}{3x^3}-$$
$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(-3\int\arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int\arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\,\mathrm{arcc}\right.\right.$$

$$\Big\downarrow \; 3011$$

$$-\frac{\arccos(ax)^4}{3x^3}-$$

$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(a\right.\right.\right.$$

$$\Big\downarrow \; 7163$$

$$-\frac{\arccos(ax)^4}{3x^3}-$$

$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)d\arccos(ax)-i\operatorname{ar}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$-\frac{\arccos(ax)^4}{3x^3}-$$

$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)de^{i\operatorname{arccos}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 7143$$

$$-\frac{\arccos(ax)^4}{3x^3}-$$

$$\frac{4}{3}a\left(-\frac{1}{2}a^2\left(-2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)+3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4\right.\right.\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^4/x^4,x]
```

output
```
-1/3*ArcCos[a*x]^4/x^3 - (4*a*(-1/2*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x^2
- (3*a*(-(ArcCos[a*x]^2/x) + 2*a*((-2*I)*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*
x])] + I*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - I*PolyLog[2, I*E^(I*ArcCos[a
*x])])))/2 - (a^2*((-2*I)*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + 3*(I*A
rcCos[a*x]^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*
PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + PolyLog[4, (-I)*E^(I*ArcCos[a*x])]))
- 3*(I*ArcCos[a*x]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[
a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] + PolyLog[4, I*E^(I*ArcCos[a*x])]))))
/2))/3
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.38

method	result
derivativedivides	$a^3\left(-\frac{\arccos(ax)^2\left(-2\arccos(ax)\sqrt{-a^2x^2+1}\,ax+\arccos(ax)^2+6a^2x^2\right)}{3a^3x^3}+\frac{2\arccos(ax)^3\ln\left(1-i\left(ax+i\sqrt{-a^2x^2+1}\right.\right.}{3}\right.$
default	$a^3\left(-\frac{\arccos(ax)^2\left(-2\arccos(ax)\sqrt{-a^2x^2+1}\,ax+\arccos(ax)^2+6a^2x^2\right)}{3a^3x^3}+\frac{2\arccos(ax)^3\ln\left(1-i\left(ax+i\sqrt{-a^2x^2+1}\right.\right.}{3}\right.$

input
```
int(arccos(a*x)^4/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/a^3/x^3*arccos(a*x)^2*(-2*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x+arc
cos(a*x)^2+6*a^2*x^2)+2/3*arccos(a*x)^3*ln(1-I*(a*x+I*(-a^2*x^2+1)^(1/2)))
-2*I*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))*arccos(a*x)^2+4*arccos(a*x)*p
olylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+4*I*polylog(4,I*(a*x+I*(-a^2*x^2+1)
^(1/2)))-2/3*arccos(a*x)^3*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))+2*I*polylog(
2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))*arccos(a*x)^2-4*arccos(a*x)*polylog(3,-I*
(a*x+I*(-a^2*x^2+1)^(1/2)))-4*I*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-4
*arccos(a*x)*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))+4*arccos(a*x)*ln(1-I*(a*x+
I*(-a^2*x^2+1)^(1/2)))+4*I*dilog(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))-4*I*dilog
(1-I*(a*x+I*(-a^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\arccos\left(ax\right)^4}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^4/x^4,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^4/x^4, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\operatorname{acos}^4\left(ax\right)}{x^4}\,dx$$

input
```
integrate(acos(a*x)**4/x**4,x)
```

output
```
Integral(acos(a*x)**4/x**4, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\arccos{(ax)}^4}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^4/x^4,x, algorithm="maxima")
```

output
```
1/3*(12*a*x^3*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x
+ 1)*sqrt(-a*x + 1), a*x)^3/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sq
rt(-a*x + 1), a*x)^4)/x^3
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\arccos{(ax)}^4}{x^4}\,dx$$

input
```
integrate(arccos(a*x)^4/x^4,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^4/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)^4}{x^4}\,dx$$

input
```
int(acos(a*x)^4/x^4,x)
```

output
```
int(acos(a*x)^4/x^4, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{acos(ax)^4}{x^4}dx$$

input
```
int(acos(a*x)^4/x^4,x)
```

output
```
int(acos(a*x)**4/x**4,x)
```

3.42 $\int \frac{x^6}{\arccos(ax)}\,dx$

Optimal result . . . 398
Mathematica [A] (verified) . . . 398
Rubi [A] (verified) . . . 399
Maple [A] (verified) . . . 400
Fricas [F] . . . 401
Sympy [F] . . . 401
Maxima [F] . . . 401
Giac [A] (verification not implemented) . . . 402
Mupad [F(-1)] . . . 402
Reduce [F] . . . 402

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{x^6}{\arccos(ax)}\,dx = -\frac{5\text{Si}(\arccos(ax))}{64a^7} - \frac{9\text{Si}(3\arccos(ax))}{64a^7} - \frac{5\text{Si}(5\arccos(ax))}{64a^7} - \frac{\text{Si}(7\arccos(ax))}{64a^7}$$

output
```
-5/64*Si(arccos(a*x))/a^7-9/64*Si(3*arccos(a*x))/a^7-5/64*Si(5*arccos(a*x)
)/a^7-1/64*Si(7*arccos(a*x))/a^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\arccos(ax)}\,dx = -\frac{5\text{Si}(\arccos(ax)) + 9\text{Si}(3\arccos(ax)) + 5\text{Si}(5\arccos(ax)) + \text{Si}(7\arccos(ax))}{64a^7}$$

input
```
Integrate[x^6/ArcCos[a*x],x]
```

output
```
-1/64*(5*SinIntegral[ArcCos[a*x]] + 9*SinIntegral[3*ArcCos[a*x]] + 5*SinIn
tegral[5*ArcCos[a*x]] + SinIntegral[7*ArcCos[a*x]])/a^7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\arccos(ax)}dx$$

$$\Big\downarrow \text{ 5147}$$

$$-\frac{\int \frac{a^6x^6\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^7}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{\int \left(\frac{9\sin(3\arccos(ax))}{64\arccos(ax)}+\frac{5\sin(5\arccos(ax))}{64\arccos(ax)}+\frac{\sin(7\arccos(ax))}{64\arccos(ax)}+\frac{5\sqrt{1-a^2x^2}}{64\arccos(ax)}\right)d\arccos(ax)}{a^7}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{5}{64}\text{Si}(\arccos(ax))+\frac{9}{64}\text{Si}(3\arccos(ax))+\frac{5}{64}\text{Si}(5\arccos(ax))+\frac{1}{64}\text{Si}(7\arccos(ax))}{a^7}$$

input
```
Int[x^6/ArcCos[a*x],x]
```

output
```
-(((5*SinIntegral[ArcCos[a*x]])/64 + (9*SinIntegral[3*ArcCos[a*x]])/64 + (
5*SinIntegral[5*ArcCos[a*x]])/64 + SinIntegral[7*ArcCos[a*x]]/64)/a^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{9\,\mathrm{Si}(3\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(5\arccos(ax))}{64}-\frac{\mathrm{Si}(7\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(\arccos(ax))}{64}}{a^7}$	40
default	$\frac{-\frac{9\,\mathrm{Si}(3\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(5\arccos(ax))}{64}-\frac{\mathrm{Si}(7\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(\arccos(ax))}{64}}{a^7}$	40

input
```
int(x^6/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(-9/64*Si(3*arccos(a*x))-5/64*Si(5*arccos(a*x))-1/64*Si(7*arccos(a*x
))-5/64*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\arccos{(ax)}}\,dx$$

input
```
integrate(x^6/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(x^6/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(x**6/acos(a*x),x)
```

output
```
Integral(x**6/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\arccos{(ax)}}\,dx$$

input
```
integrate(x^6/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^6/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{\arccos(ax)}\,dx = -\frac{\text{Si}\,(7 \arccos{(ax)})}{64\,a^7} - \frac{5\ \text{Si}\,(5 \arccos{(ax)})}{64\,a^7} - \frac{9\ \text{Si}\,(3 \arccos{(ax)})}{64\,a^7} - \frac{5\ \text{Si}\,(\arccos{(ax)})}{64\,a^7}$$

input
```
integrate(x^6/arccos(a*x),x, algorithm="giac")
```

output
```
-1/64*sin_integral(7*arccos(a*x))/a^7 - 5/64*sin_integral(5*arccos(a*x))/a
^7 - 9/64*sin_integral(3*arccos(a*x))/a^7 - 5/64*sin_integral(arccos(a*x))
/a^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int(x^6/acos(a*x),x)
```

output
```
int(x^6/acos(a*x), x)
```

Reduce [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{acos\,(ax)}dx$$

input
```
int(x^6/acos(a*x),x)
```

output
```
int(x**6/acos(a*x),x)
```

3.43 $\int \frac{x^5}{\arccos(ax)} dx$

Optimal result 403
Mathematica [A] (verified) 403
Rubi [A] (verified) 404
Maple [A] (verified) 405
Fricas [F] 405
Sympy [F] 406
Maxima [F] 406
Giac [A] (verification not implemented) 406
Mupad [F(-1)] 407
Reduce [F] 407

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{x^5}{\arccos(ax)} dx = -\frac{5\text{Si}(2\arccos(ax))}{32a^6} - \frac{\text{Si}(4\arccos(ax))}{8a^6} - \frac{\text{Si}(6\arccos(ax))}{32a^6}$$

output
```
-5/32*Si(2*arccos(a*x))/a^6-1/8*Si(4*arccos(a*x))/a^6-1/32*Si(6*arccos(a*x
))/a^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\arccos(ax)} dx = -\frac{5\text{Si}(2\arccos(ax)) + 4\text{Si}(4\arccos(ax)) + \text{Si}(6\arccos(ax))}{32a^6}$$

input
```
Integrate[x^5/ArcCos[a*x],x]
```

output
```
-1/32*(5*SinIntegral[2*ArcCos[a*x]] + 4*SinIntegral[4*ArcCos[a*x]] + SinIn
tegral[6*ArcCos[a*x]])/a^6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arccos(ax)} \, dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int \frac{a^5x^5\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^6}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{5\sin(2\arccos(ax))}{32\arccos(ax)}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}+\frac{\sin(6\arccos(ax))}{32\arccos(ax)}\right)d\arccos(ax)}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{5}{32}\text{Si}(2\arccos(ax))+\frac{1}{8}\text{Si}(4\arccos(ax))+\frac{1}{32}\text{Si}(6\arccos(ax))}{a^6}$$

input
```
Int[x^5/ArcCos[a*x],x]
```

output
```
-(((5*SinIntegral[2*ArcCos[a*x]])/32 + SinIntegral[4*ArcCos[a*x]]/8 + SinI
ntegral[6*ArcCos[a*x]]/32)/a^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{5\,\mathrm{Si}(2\arccos(ax))}{32}-\frac{\mathrm{Si}(4\arccos(ax))}{8}-\frac{\mathrm{Si}(6\arccos(ax))}{32}}{a^{6}}$	33
default	$\frac{-\frac{5\,\mathrm{Si}(2\arccos(ax))}{32}-\frac{\mathrm{Si}(4\arccos(ax))}{8}-\frac{\mathrm{Si}(6\arccos(ax))}{32}}{a^{6}}$	33

input
```
int(x^5/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(-5/32*Si(2*arccos(a*x))-1/8*Si(4*arccos(a*x))-1/32*Si(6*arccos(a*x)
))
```

Fricas [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx=\int \frac{x^5}{\arccos\left(ax\right)}\,dx$$

input
```
integrate(x^5/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(x^5/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**5/acos(a*x),x)
```

output
```
Integral(x**5/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\arccos(ax)}\,dx$$

input
```
integrate(x^5/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(6\arccos(ax))}{32a^6} - \frac{\operatorname{Si}(4\arccos(ax))}{8a^6} - \frac{5\operatorname{Si}(2\arccos(ax))}{32a^6}$$

input
```
integrate(x^5/arccos(a*x),x, algorithm="giac")
```

output
```
-1/32*sin_integral(6*arccos(a*x))/a^6 - 1/8*sin_integral(4*arccos(a*x))/a^
6 - 5/32*sin_integral(2*arccos(a*x))/a^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int(x^5/acos(a*x),x)
```

output
```
int(x^5/acos(a*x), x)
```

Reduce [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{acos\left(ax\right)}dx$$

input
```
int(x^5/acos(a*x),x)
```

output
```
int(x**5/acos(a*x),x)
```

3.44 $\int \frac{x^4}{\arccos(ax)} dx$

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Mathematica [A] (verified) . . . 408
Rubi [A] (verified) . . . 409
Maple [A] (verified) . . . 410
Fricas [F] . . . 410
Sympy [F] . . . 411
Maxima [F] . . . 411
Giac [A] (verification not implemented) . . . 411
Mupad [F(-1)] . . . 412
Reduce [F] . . . 412

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{x^4}{\arccos(ax)} dx = -\frac{\text{Si}(\arccos(ax))}{8a^5} - \frac{3\text{Si}(3\arccos(ax))}{16a^5} - \frac{\text{Si}(5\arccos(ax))}{16a^5}$$

output
```
-1/8*Si(arccos(a*x))/a^5-3/16*Si(3*arccos(a*x))/a^5-1/16*Si(5*arccos(a*x))
/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\arccos(ax)} dx = -\frac{2\text{Si}(\arccos(ax)) + 3\text{Si}(3\arccos(ax)) + \text{Si}(5\arccos(ax))}{16a^5}$$

input
```
Integrate[x^4/ArcCos[a*x],x]
```

output
```
-1/16*(2*SinIntegral[ArcCos[a*x]] + 3*SinIntegral[3*ArcCos[a*x]] + SinInte
gral[5*ArcCos[a*x]])/a^5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{ 5147}$$

$$-\frac{\int \frac{a^4x^4\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^5}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{\int \left(\frac{3\sin(3\arccos(ax))}{16\arccos(ax)}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}+\frac{\sqrt{1-a^2x^2}}{8\arccos(ax)}\right)d\arccos(ax)}{a^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{8}\text{Si}(\arccos(ax))+\frac{3}{16}\text{Si}(3\arccos(ax))+\frac{1}{16}\text{Si}(5\arccos(ax))}{a^5}$$

input
```
Int[x^4/ArcCos[a*x],x]
```

output
```
-((SinIntegral[ArcCos[a*x]]/8 + (3*SinIntegral[3*ArcCos[a*x]])/16 + SinInt
egral[5*ArcCos[a*x]]/16)/a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{3\,\mathrm{Si}(3\arccos(ax))}{16}-\frac{\mathrm{Si}(5\arccos(ax))}{16}-\frac{\mathrm{Si}(\arccos(ax))}{8}}{a^5}$	31
default	$\frac{-\frac{3\,\mathrm{Si}(3\arccos(ax))}{16}-\frac{\mathrm{Si}(5\arccos(ax))}{16}-\frac{\mathrm{Si}(\arccos(ax))}{8}}{a^5}$	31

input
```
int(x^4/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(-3/16*Si(3*arccos(a*x))-1/16*Si(5*arccos(a*x))-1/8*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)}\,dx=\int \frac{x^4}{\arccos(ax)}\,dx$$

input
```
integrate(x^4/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(x^4/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)}\,dx = \int \frac{x^4}{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**4/acos(a*x),x)
```

output
```
Integral(x**4/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)}\,dx = \int \frac{x^4}{\arccos(ax)}\,dx$$

input
```
integrate(x^4/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(5\arccos(ax))}{16\,a^5} - \frac{3\operatorname{Si}(3\arccos(ax))}{16\,a^5} - \frac{\operatorname{Si}(\arccos(ax))}{8\,a^5}$$

input
```
integrate(x^4/arccos(a*x),x, algorithm="giac")
```

output
```
-1/16*sin_integral(5*arccos(a*x))/a^5 - 3/16*sin_integral(3*arccos(a*x))/a
^5 - 1/8*sin_integral(arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)} \, dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)} \,dx$$

input
```
int(x^4/acos(a*x),x)
```

output
```
int(x^4/acos(a*x), x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)} \, dx = \int \frac{x^4}{acos\left(ax\right)}dx$$

input
```
int(x^4/acos(a*x),x)
```

output
```
int(x**4/acos(a*x),x)
```

3.45 $\int \frac{x^3}{\arccos(ax)} \, dx$

Optimal result 413
Mathematica [A] (verified) 413
Rubi [A] (verified) 414
Maple [A] (verified) 415
Fricas [F] 415
Sympy [F] 416
Maxima [F] 416
Giac [A] (verification not implemented) 416
Mupad [F(-1)] 417
Reduce [F] 417

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{x^3}{\arccos(ax)} \, dx = -\frac{\text{Si}(2\arccos(ax))}{4a^4} - \frac{\text{Si}(4\arccos(ax))}{8a^4}$$

output
```
-1/4*Si(2*arccos(a*x))/a^4-1/8*Si(4*arccos(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\arccos(ax)} \, dx = -\frac{2\text{Si}(2\arccos(ax)) + \text{Si}(4\arccos(ax))}{8a^4}$$

input
```
Integrate[x^3/ArcCos[a*x],x]
```

output
```
-1/8*(2*SinIntegral[2*ArcCos[a*x]] + SinIntegral[4*ArcCos[a*x]])/a^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int \frac{a^3x^3\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{\sin(2\arccos(ax))}{4\arccos(ax)}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}\right)d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\text{Si}(2\arccos(ax))+\frac{1}{8}\text{Si}(4\arccos(ax))}{a^4}$$

input
```
Int[x^3/ArcCos[a*x],x]
```

output
```
-((SinIntegral[2*ArcCos[a*x]]/4 + SinIntegral[4*ArcCos[a*x]]/8)/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{\mathrm{Si}(2\arccos(ax))}{4}-\frac{\mathrm{Si}(4\arccos(ax))}{8}}{a^4}$	24
default	$\frac{-\frac{\mathrm{Si}(2\arccos(ax))}{4}-\frac{\mathrm{Si}(4\arccos(ax))}{8}}{a^4}$	24

input
```
int(x^3/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*Si(2*arccos(a*x))-1/8*Si(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\arccos{(ax)}}\,dx$$

input
```
integrate(x^3/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(x^3/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**3/acos(a*x),x)
```

output
```
Integral(x**3/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\arccos(ax)}\,dx$$

input
```
integrate(x^3/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(4\arccos(ax))}{8a^4} - \frac{\operatorname{Si}(2\arccos(ax))}{4a^4}$$

input
```
integrate(x^3/arccos(a*x),x, algorithm="giac")
```

output
```
-1/8*sin_integral(4*arccos(a*x))/a^4 - 1/4*sin_integral(2*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int(x^3/acos(a*x),x)
```

output
```
int(x^3/acos(a*x), x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{acos\,(ax)}dx$$

input
```
int(x^3/acos(a*x),x)
```

output
```
int(x**3/acos(a*x),x)
```

3.46 $\int \frac{x^2}{\arccos(ax)} dx$

Optimal result . . . 418
Mathematica [A] (verified) . . . 418
Rubi [A] (verified) . . . 419
Maple [A] (verified) . . . 420
Fricas [F] . . . 420
Sympy [F] . . . 421
Maxima [F] . . . 421
Giac [A] (verification not implemented) . . . 421
Mupad [F(-1)] . . . 422
Reduce [F] . . . 422

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x^2}{\arccos(ax)} dx = -\frac{\text{Si}(\arccos(ax))}{4a^3} - \frac{\text{Si}(3\arccos(ax))}{4a^3}$$

output
```
-1/4*Si(arccos(a*x))/a^3-1/4*Si(3*arccos(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\arccos(ax)} dx = -\frac{\text{Si}(\arccos(ax)) + \text{Si}(3\arccos(ax))}{4a^3}$$

input
```
Integrate[x^2/ArcCos[a*x],x]
```

output
```
-1/4*(SinIntegral[ArcCos[a*x]] + SinIntegral[3*ArcCos[a*x]])/a^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)}\,dx$$
$$\Big\downarrow \ 5147$$
$$-\frac{\int \frac{a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^3}$$
$$\Big\downarrow \ 4906$$
$$-\frac{\int \left(\frac{\sin(3\arccos(ax))}{4\arccos(ax)}+\frac{\sqrt{1-a^2x^2}}{4\arccos(ax)}\right)d\arccos(ax)}{a^3}$$
$$\Big\downarrow \ 2009$$
$$-\frac{\frac{1}{4}\text{Si}(\arccos(ax))+\frac{1}{4}\text{Si}(3\arccos(ax))}{a^3}$$

input
```
Int[x^2/ArcCos[a*x],x]
```

output
```
-((SinIntegral[ArcCos[a*x]]/4 + SinIntegral[3*ArcCos[a*x]]/4)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\mathrm{Si}(3 \arccos(ax))}{4}-\frac{\mathrm{Si}(\arccos(ax))}{4}}{a^3}$	22
default	$\frac{-\frac{\mathrm{Si}(3 \arccos(ax))}{4}-\frac{\mathrm{Si}(\arccos(ax))}{4}}{a^3}$	22

input
```
int(x^2/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/4*Si(3*arccos(a*x))-1/4*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\arccos\left(ax\right)}\,dx$$

input
```
integrate(x^2/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(x^2/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**2/acos(a*x),x)
```

output
```
Integral(x**2/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\arccos(ax)}\,dx$$

input
```
integrate(x^2/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(3\arccos(ax))}{4a^3} - \frac{\operatorname{Si}(\arccos(ax))}{4a^3}$$

input
```
integrate(x^2/arccos(a*x),x, algorithm="giac")
```

output
```
-1/4*sin_integral(3*arccos(a*x))/a^3 - 1/4*sin_integral(arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int(x^2/acos(a*x),x)
```

output
```
int(x^2/acos(a*x), x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{acos\left(ax\right)}dx$$

input
```
int(x^2/acos(a*x),x)
```

output
```
int(x**2/acos(a*x),x)
```

3.47 $\int \frac{x}{\arccos(ax)} dx$

Optimal result 423
Mathematica [A] (verified) 423
Rubi [A] (verified) 424
Maple [A] (verified) 425
Fricas [F] 426
Sympy [F] 426
Maxima [F] 426
Giac [A] (verification not implemented) 427
Mupad [F(-1)] 427
Reduce [F] 427

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{x}{\arccos(ax)} dx = -\frac{\text{Si}(2\arccos(ax))}{2a^2}$$

output

```
-1/2*Si(2*arccos(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arccos(ax)} dx = -\frac{\text{Si}(2\arccos(ax))}{2a^2}$$

input

```
Integrate[x/ArcCos[a*x],x]
```

output

```
-1/2*SinIntegral[2*ArcCos[a*x]]/a^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5147, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)}dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int \frac{ax\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \frac{\sin(2\arccos(ax))}{2\arccos(ax)}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\text{Si}(2\arccos(ax))}{2a^2}$$

input
```
Int[x/ArcCos[a*x],x]
```

output
```
-1/2*SinIntegral[2*ArcCos[a*x]]/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\operatorname{Si}(2\arccos(ax))}{2a^2}$	13
default	$-\frac{\operatorname{Si}(2\arccos(ax))}{2a^2}$	13

input
```
int(x/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Si(2*arccos(a*x))/a^2
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\arccos{(ax)}}\,dx$$

input
```
integrate(x/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(x/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(x/acos(a*x),x)
```

output
```
Integral(x/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\arccos{(ax)}}\,dx$$

input
```
integrate(x/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x}{\arccos(ax)}\,dx = -\frac{\mathrm{Si}\left(2\arccos\left(ax\right)\right)}{2\,a^2}$$

input
```
integrate(x/arccos(a*x),x, algorithm="giac")
```

output
```
-1/2*sin_integral(2*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int(x/acos(a*x),x)
```

output
```
int(x/acos(a*x), x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{acos\left(ax\right)}dx$$

input
```
int(x/acos(a*x),x)
```

output
```
int(x/acos(a*x),x)
```

3.48 $\int \frac{1}{\arccos(ax)}\,dx$

Optimal result 428
Mathematica [A] (verified) 428
Rubi [A] (verified) 429
Maple [A] (verified) 430
Fricas [F] 430
Sympy [F] 431
Maxima [F] 431
Giac [A] (verification not implemented) 431
Mupad [F(-1)] 432
Reduce [F] 432

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{1}{\arccos(ax)}\,dx=-\frac{\text{Si}(\arccos(ax))}{a}$$

output
```
-Si(arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(ax)}\,dx=-\frac{\text{Si}(\arccos(ax))}{a}$$

input
```
Integrate[ArcCos[a*x]^(-1),x]
```

output
```
-(SinIntegral[ArcCos[a*x]]/a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5135, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)}dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int \frac{\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\text{Si}(\arccos(ax))}{a}$$

input
```
Int[ArcCos[a*x]^(-1),x]
```

output
```
-(SinIntegral[ArcCos[a*x]]/a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\frac{\mathrm{Si}(\arccos(ax))}{a}$	11
default	$-\frac{\mathrm{Si}(\arccos(ax))}{a}$	11

input
```
int(1/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-Si(arccos(a*x))/a
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\arccos(ax)}\,dx$$

input
```
integrate(1/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(1/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(1/acos(a*x),x)
```

output
```
Integral(1/acos(a*x), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\arccos(ax)}\,dx$$

input
```
integrate(1/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(\arccos(ax))}{a}$$

input
```
integrate(1/arccos(a*x),x, algorithm="giac")
```

output
```
-sin_integral(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int(1/acos(a*x),x)
```

output
```
int(1/acos(a*x), x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{acos\left(ax\right)}dx$$

input
```
int(1/acos(a*x),x)
```

output
```
int(1/acos(a*x),x)
```

3.49 $\int \frac{1}{x \arccos(ax)} \, dx$

Optimal result 433
Mathematica [N/A] 433
Rubi [N/A] 434
Maple [N/A] 434
Fricas [N/A] 435
Sympy [N/A] 435
Maxima [N/A] 435
Giac [N/A] 436
Mupad [N/A] 436
Reduce [N/A] 437

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)} \, dx = \text{Int}\left(\frac{1}{x \arccos(ax)}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} \, dx = \int \frac{1}{x \arccos(ax)} \, dx$$

input
```
Integrate[1/(x*ArcCos[a*x]),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)} \, dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{x \arccos(ax)} dx$$

input
```
Int[1/(x*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos (ax)} dx$$

input
```
int(1/x/arccos(a*x),x)
```

output
```
int(1/x/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)}\,dx = \int \frac{1}{x\arccos{(ax)}}\,dx$$

input
```
integrate(1/x/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\arccos(ax)}\,dx = \int \frac{1}{x\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(1/x/acos(a*x),x)
```

output
```
Integral(1/(x*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)}\,dx = \int \frac{1}{x\arccos{(ax)}}\,dx$$

input
```
integrate(1/x/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)}\,dx = \int \frac{1}{x\arccos{(ax)}}\,dx$$

input
```
integrate(1/x/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)}\,dx = \int \frac{1}{x\,\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int(1/(x*acos(a*x)),x)
```

output
```
int(1/(x*acos(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} dx = \int \frac{1}{acos(ax)x} dx$$

input
```
int(1/x/acos(a*x),x)
```

output
```
int(1/(acos(a*x)*x),x)
```

3.50 $\int \frac{1}{x^2 \arccos(ax)} \, dx$

Optimal result . . . 438
Mathematica [N/A] . . . 438
Rubi [N/A] . . . 439
Maple [N/A] . . . 439
Fricas [N/A] . . . 440
Sympy [N/A] . . . 440
Maxima [N/A] . . . 440
Giac [N/A] . . . 441
Mupad [N/A] . . . 441
Reduce [N/A] . . . 442

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \text{Int}\left(\frac{1}{x^2 \arccos(ax)}, x\right)$$

output
```
Defer(Int)(1/x^2/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \int \frac{1}{x^2 \arccos(ax)} \, dx$$

input
```
Integrate[1/(x^2*ArcCos[a*x]),x]
```

output
```
Integrate[1/(x^2*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arccos(ax)} \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{1}{x^2 \arccos(ax)}dx$$

input
```
Int[1/(x^2*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos(ax)}dx$$

input
```
int(1/x^2/arccos(a*x),x)
```

output
```
int(1/x^2/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \arccos{(ax)}}\,dx$$

input
```
integrate(1/x^2/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x^2*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(1/x**2/acos(a*x),x)
```

output
```
Integral(1/(x**2*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \arccos{(ax)}}\,dx$$

input
```
integrate(1/x^2/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \arccos{(ax)}}\,dx$$

input
```
integrate(1/x^2/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x^2*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2\,\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int(1/(x^2*acos(a*x)),x)
```

output
```
int(1/(x^2*acos(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\, dx = \int \frac{1}{acos\left(ax\right)x^2}dx$$

input
```
int(1/x^2/acos(a*x),x)
```

output
```
int(1/(acos(a*x)*x**2),x)
```

3.51 $\int \frac{x^6}{\arccos(ax)^2} dx$

Optimal result . . . 443
Mathematica [A] (verified) . . . 443
Rubi [A] (verified) . . . 444
Maple [A] (verified) . . . 445
Fricas [F] . . . 445
Sympy [F] . . . 446
Maxima [F] . . . 446
Giac [A] (verification not implemented) . . . 446
Mupad [F(-1)] . . . 447
Reduce [F] . . . 447

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^6}{\arccos(ax)^2} dx = \frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\,\text{CosIntegral}(\arccos(ax))}{64a^7} - \frac{27\,\text{CosIntegral}(3\arccos(ax))}{64a^7} - \frac{25\,\text{CosIntegral}(5\arccos(ax))}{64a^7} - \frac{7\,\text{CosIntegral}(7\arccos(ax))}{64a^7}$$

output
```
x^6*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-5/64*Ci(arccos(a*x))/a^7-27/64*Ci(3*a
rccos(a*x))/a^7-25/64*Ci(5*arccos(a*x))/a^7-7/64*Ci(7*arccos(a*x))/a^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^6}{\arccos(ax)^2} dx = -\frac{-64a^6x^6\sqrt{1-a^2x^2} + 5\arccos(ax)\,\text{CosIntegral}(\arccos(ax)) + 27\arccos(ax)\,\text{CosIntegral}(3\arccos(ax)}{64a^7\arccos(ax)}$$

input
```
Integrate[x^6/ArcCos[a*x]^2,x]
```

output
```
-1/64*(-64*a^6*x^6*Sqrt[1 - a^2*x^2] + 5*ArcCos[a*x]*CosIntegral[ArcCos[a*
x]] + 27*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] + 25*ArcCos[a*x]*CosIntegr
al[5*ArcCos[a*x]] + 7*ArcCos[a*x]*CosIntegral[7*ArcCos[a*x]])/(a^7*ArcCos[
a*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\arccos(ax)^2}\,dx$$

$$\Big\downarrow \; 5143$$

$$\frac{\int \left(-\frac{5ax}{64\arccos(ax)}-\frac{27\cos(3\arccos(ax))}{64\arccos(ax)}-\frac{25\cos(5\arccos(ax))}{64\arccos(ax)}-\frac{7\cos(7\arccos(ax))}{64\arccos(ax)}\right)d\arccos(ax)}{a^7}+\frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{5}{64}\text{CosIntegral}(\arccos(ax))-\frac{27}{64}\text{CosIntegral}(3\arccos(ax))-\frac{25}{64}\text{CosIntegral}(5\arccos(ax))-\frac{7}{64}\text{CosIntegral}(7}{a^7}$$
$$\frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

input
```
Int[x^6/ArcCos[a*x]^2,x]
```

output
```
(x^6*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + ((-5*CosIntegral[ArcCos[a*x]])/6
4 - (27*CosIntegral[3*ArcCos[a*x]])/64 - (25*CosIntegral[5*ArcCos[a*x]])/6
4 - (7*CosIntegral[7*ArcCos[a*x]])/64)/a^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{9\sin(3\arccos(ax))}{64\arccos(ax)}-\frac{27\,\mathrm{Ci}(3\arccos(ax))}{64}+\frac{5\sin(5\arccos(ax))}{64\arccos(ax)}-\frac{25\,\mathrm{Ci}(5\arccos(ax))}{64}+\frac{\sin(7\arccos(ax))}{64\arccos(ax)}-\frac{7\,\mathrm{Ci}(7\arccos(ax))}{64}+\frac{5\sqrt{-}}{64\,\mathrm{ar}}}{a^7}$
default	$\frac{\frac{9\sin(3\arccos(ax))}{64\arccos(ax)}-\frac{27\,\mathrm{Ci}(3\arccos(ax))}{64}+\frac{5\sin(5\arccos(ax))}{64\arccos(ax)}-\frac{25\,\mathrm{Ci}(5\arccos(ax))}{64}+\frac{\sin(7\arccos(ax))}{64\arccos(ax)}-\frac{7\,\mathrm{Ci}(7\arccos(ax))}{64}+\frac{5\sqrt{-}}{64\,\mathrm{ar}}}{a^7}$

input
```
int(x^6/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(9/64/arccos(a*x)*sin(3*arccos(a*x))-27/64*Ci(3*arccos(a*x))+5/64/ar
ccos(a*x)*sin(5*arccos(a*x))-25/64*Ci(5*arccos(a*x))+1/64/arccos(a*x)*sin(
7*arccos(a*x))-7/64*Ci(7*arccos(a*x))+5/64/arccos(a*x)*(-a^2*x^2+1)^(1/2)-
5/64*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\arccos(ax)^2}\,dx$$

input
```
integrate(x^6/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^6/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\operatorname{acos}^2(ax)}\,dx$$

input
```
integrate(x**6/acos(a*x)**2,x)
```

output
```
Integral(x**6/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\arccos(ax)^2}\,dx$$

input
```
integrate(x^6/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^6 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)*integrate((7*a^2*x^7 - 6*x^5)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^
2 - a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*
x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^6}{a\arccos(ax)} - \frac{7\operatorname{Ci}(7\arccos(ax))}{64a^7} - \frac{25\operatorname{Ci}(5\arccos(ax))}{64a^7} - \frac{27\operatorname{Ci}(3\arccos(ax))}{64a^7} - \frac{5\operatorname{Ci}(\arccos(ax))}{64a^7}$$

input
```
integrate(x^6/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x^6/(a*arccos(a*x)) - 7/64*cos_integral(7*arccos(a*x))/
a^7 - 25/64*cos_integral(5*arccos(a*x))/a^7 - 27/64*cos_integral(3*arccos(
a*x))/a^7 - 5/64*cos_integral(arccos(a*x))/a^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\mathrm{acos}\,(a\,x)^2}\,dx$$

input
```
int(x^6/acos(a*x)^2,x)
```

output
```
int(x^6/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{acos\,(ax)^2}dx$$

input
```
int(x^6/acos(a*x)^2,x)
```

output
```
int(x**6/acos(a*x)**2,x)
```

3.52 $\int \frac{x^5}{\arccos(ax)^2}\,dx$

Optimal result 448
Mathematica [A] (verified) 448
Rubi [A] (verified) 449
Maple [A] (verified) 450
Fricas [F] 450
Sympy [F] 451
Maxima [F] 451
Giac [A] (verification not implemented) 451
Mupad [F(-1)] 452
Reduce [F] 452

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\,\text{CosIntegral}(2\arccos(ax))}{16a^6} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^6} - \frac{3\,\text{CosIntegral}(6\arccos(ax))}{16a^6}$$

output
```
x^5*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-5/16*Ci(2*arccos(a*x))/a^6-1/2*Ci(4*a
rccos(a*x))/a^6-3/16*Ci(6*arccos(a*x))/a^6
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = -\frac{-\frac{16a^5x^5\sqrt{1-a^2x^2}}{\arccos(ax)} + 5\,\text{CosIntegral}(2\arccos(ax)) + 8\,\text{CosIntegral}(4\arccos(ax)) + 3\,\text{CosIntegral}(6\arccos}{16a^6}$$

input
```
Integrate[x^5/ArcCos[a*x]^2,x]
```

output
```
-1/16*((-16*a^5*x^5*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + 5*CosIntegral[2*ArcCo
s[a*x]] + 8*CosIntegral[4*ArcCos[a*x]] + 3*CosIntegral[6*ArcCos[a*x]])/a^6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{\int \left(-\frac{5\cos(2\arccos(ax))}{16\arccos(ax)}-\frac{\cos(4\arccos(ax))}{2\arccos(ax)}-\frac{3\cos(6\arccos(ax))}{16\arccos(ax)}\right)d\arccos(ax)}{a^6}+\frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{5}{16}\text{CosIntegral}(2\arccos(ax))-\frac{1}{2}\text{CosIntegral}(4\arccos(ax))-\frac{3}{16}\text{CosIntegral}(6\arccos(ax))}{a^6}+\frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

input
```
Int[x^5/ArcCos[a*x]^2,x]
```

output
```
(x^5*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + ((-5*CosIntegral[2*ArcCos[a*x]])
/16 - CosIntegral[4*ArcCos[a*x]]/2 - (3*CosIntegral[6*ArcCos[a*x]])/16)/a^
6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\frac{5\sin(2\arccos(ax))}{32\arccos(ax)}-\frac{5\,\mathrm{Ci}(2\arccos(ax))}{16}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\mathrm{Ci}(4\arccos(ax))}{2}+\frac{\sin(6\arccos(ax))}{32\arccos(ax)}-\frac{3\,\mathrm{Ci}(6\arccos(ax))}{16}}{a^6}$	78
default	$\frac{\frac{5\sin(2\arccos(ax))}{32\arccos(ax)}-\frac{5\,\mathrm{Ci}(2\arccos(ax))}{16}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\mathrm{Ci}(4\arccos(ax))}{2}+\frac{\sin(6\arccos(ax))}{32\arccos(ax)}-\frac{3\,\mathrm{Ci}(6\arccos(ax))}{16}}{a^6}$	78

input
```
int(x^5/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(5/32*sin(2*arccos(a*x))/arccos(a*x)-5/16*Ci(2*arccos(a*x))+1/8/arcc
os(a*x)*sin(4*arccos(a*x))-1/2*Ci(4*arccos(a*x))+1/32/arccos(a*x)*sin(6*ar
ccos(a*x))-3/16*Ci(6*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \int \frac{x^5}{\arccos{(ax)}^2}\,dx$$

input
```
integrate(x^5/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^5/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \int \frac{x^5}{\operatorname{acos}^2(ax)}\,dx$$

input
```
integrate(x**5/acos(a*x)**2,x)
```

output
```
Integral(x**5/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \int \frac{x^5}{\arccos(ax)^2}\,dx$$

input
```
integrate(x^5/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^5 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)*integrate((6*a^2*x^6 - 5*x^4)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^
2 - a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*
x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^5}{a\arccos(ax)} - \frac{3\,\mathrm{Ci}(6\arccos(ax))}{16\,a^6} - \frac{\mathrm{Ci}(4\arccos(ax))}{2\,a^6} - \frac{5\,\mathrm{Ci}(2\arccos(ax))}{16\,a^6}$$

input
```
integrate(x^5/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x^5/(a*arccos(a*x)) - 3/16*cos_integral(6*arccos(a*x))/
a^6 - 1/2*cos_integral(4*arccos(a*x))/a^6 - 5/16*cos_integral(2*arccos(a*x
))/a^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \int \frac{x^5}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

input
```
int(x^5/acos(a*x)^2,x)
```

output
```
int(x^5/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \int \frac{x^5}{acos\left(ax\right)^2}dx$$

input
```
int(x^5/acos(a*x)^2,x)
```

output
```
int(x**5/acos(a*x)**2,x)
```

3.53 $\int \frac{x^4}{\arccos(ax)^2} \, dx$

Optimal result . . . 453
Mathematica [A] (verified) . . . 453
Rubi [A] (verified) . . . 454
Maple [A] (verified) . . . 455
Fricas [F] . . . 455
Sympy [F] . . . 456
Maxima [F] . . . 456
Giac [A] (verification not implemented) . . . 456
Mupad [F(-1)] . . . 457
Reduce [F] . . . 457

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{x^4}{\arccos(ax)^2} \, dx = \frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{8a^5} - \frac{9\,\text{CosIntegral}(3\arccos(ax))}{16a^5} - \frac{5\,\text{CosIntegral}(5\arccos(ax))}{16a^5}$$

output
```
x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-1/8*Ci(arccos(a*x))/a^5-9/16*Ci(3*arc
cos(a*x))/a^5-5/16*Ci(5*arccos(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{\arccos(ax)^2} \, dx = -\frac{-\frac{16a^4x^4\sqrt{1-a^2x^2}}{\arccos(ax)} + 2\,\text{CosIntegral}(\arccos(ax)) + 9\,\text{CosIntegral}(3\arccos(ax)) + 5\,\text{CosIntegral}(5\arccos(ax))}{16a^5}$$

input
```
Integrate[x^4/ArcCos[a*x]^2,x]
```

output
```
-1/16*((-16*a^4*x^4*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + 2*CosIntegral[ArcCos[
a*x]] + 9*CosIntegral[3*ArcCos[a*x]] + 5*CosIntegral[5*ArcCos[a*x]])/a^5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)^2}\,dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{\int \left(-\frac{ax}{8\arccos(ax)}-\frac{9\cos(3\arccos(ax))}{16\arccos(ax)}-\frac{5\cos(5\arccos(ax))}{16\arccos(ax)}\right)d\arccos(ax)}{a^5}+\frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{8}\text{CosIntegral}(\arccos(ax))-\frac{9}{16}\text{CosIntegral}(3\arccos(ax))-\frac{5}{16}\text{CosIntegral}(5\arccos(ax))}{a^5}+\frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

input
```
Int[x^4/ArcCos[a*x]^2,x]
```

output
```
(x^4*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + (-1/8*CosIntegral[ArcCos[a*x]] -
 (9*CosIntegral[3*ArcCos[a*x]])/16 - (5*CosIntegral[5*ArcCos[a*x]])/16)/a^
5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\frac{3\sin(3\arccos(ax))}{16\arccos(ax)}-\frac{9\,\mathrm{Ci}(3\arccos(ax))}{16}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}-\frac{5\,\mathrm{Ci}(5\arccos(ax))}{16}+\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)}-\frac{\mathrm{Ci}(\arccos(ax))}{8}}{a^5}$	81
default	$\frac{\frac{3\sin(3\arccos(ax))}{16\arccos(ax)}-\frac{9\,\mathrm{Ci}(3\arccos(ax))}{16}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}-\frac{5\,\mathrm{Ci}(5\arccos(ax))}{16}+\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)}-\frac{\mathrm{Ci}(\arccos(ax))}{8}}{a^5}$	81

input
```
int(x^4/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(3/16/arccos(a*x)*sin(3*arccos(a*x))-9/16*Ci(3*arccos(a*x))+1/16/arc
cos(a*x)*sin(5*arccos(a*x))-5/16*Ci(5*arccos(a*x))+1/8/arccos(a*x)*(-a^2*x
^2+1)^(1/2)-1/8*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\arccos{(ax)}^2}\,dx$$

input
```
integrate(x^4/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^4/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\operatorname{acos}^2(ax)}\,dx$$

input
```
integrate(x**4/acos(a*x)**2,x)
```

output
```
Integral(x**4/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\arccos(ax)^2}\,dx$$

input
```
integrate(x^4/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^4 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)*integrate((5*a^2*x^5 - 4*x^3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^
2 - a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*
x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^4}{a\arccos(ax)} - \frac{5\,\mathrm{Ci}(5\arccos(ax))}{16\,a^5} - \frac{9\,\mathrm{Ci}(3\arccos(ax))}{16\,a^5} - \frac{\mathrm{Ci}(\arccos(ax))}{8\,a^5}$$

input
```
integrate(x^4/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)) - 5/16*cos_integral(5*arccos(a*x))/
a^5 - 9/16*cos_integral(3*arccos(a*x))/a^5 - 1/8*cos_integral(arccos(a*x))
/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

input
```
int(x^4/acos(a*x)^2,x)
```

output
```
int(x^4/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\mathit{acos}\left(ax\right)^2}dx$$

input
```
int(x^4/acos(a*x)^2,x)
```

output
```
int(x**4/acos(a*x)**2,x)
```

3.54 $\int \frac{x^3}{\arccos(ax)^2}\,dx$

Optimal result 458
Mathematica [A] (verified) 458
Rubi [A] (verified) 459
Maple [A] (verified) 460
Fricas [F] 460
Sympy [F] 461
Maxima [F] 461
Giac [A] (verification not implemented) 461
Mupad [F(-1)] 462
Reduce [F] 462

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{2a^4} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^4}$$

output
```
x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-1/2*Ci(2*arccos(a*x))/a^4-1/2*Ci(4*ar
ccos(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = -\frac{-\frac{2a^3x^3\sqrt{1-a^2x^2}}{\arccos(ax)} + \text{CosIntegral}(2\arccos(ax)) + \text{CosIntegral}(4\arccos(ax))}{2a^4}$$

input
```
Integrate[x^3/ArcCos[a*x]^2,x]
```

output
```
-1/2*((-2*a^3*x^3*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + CosIntegral[2*ArcCos[a*
x]] + CosIntegral[4*ArcCos[a*x]])/a^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)^2}\,dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{\int \left(-\frac{\cos(2\arccos(ax))}{2\arccos(ax)}-\frac{\cos(4\arccos(ax))}{2\arccos(ax)}\right)d\arccos(ax)}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\text{CosIntegral}(2\arccos(ax))-\frac{1}{2}\text{CosIntegral}(4\arccos(ax))}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

input
```
Int[x^3/ArcCos[a*x]^2,x]
```

output
```
(x^3*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + (-1/2*CosIntegral[2*ArcCos[a*x]]
 - CosIntegral[4*ArcCos[a*x]]/2)/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)}-\frac{\operatorname{Ci}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\operatorname{Ci}(4\arccos(ax))}{2}}{a^4}$	54
default	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)}-\frac{\operatorname{Ci}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\operatorname{Ci}(4\arccos(ax))}{2}}{a^4}$	54

input
```
int(x^3/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*sin(2*arccos(a*x))/arccos(a*x)-1/2*Ci(2*arccos(a*x))+1/8/arccos
(a*x)*sin(4*arccos(a*x))-1/2*Ci(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)^2}\,dx=\int \frac{x^3}{\arccos{(ax)}^2}\,dx$$

input
```
integrate(x^3/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \int \frac{x^3}{\operatorname{acos}^2(ax)}\,dx$$

input
```
integrate(x**3/acos(a*x)**2,x)
```

output
```
Integral(x**3/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \int \frac{x^3}{\arccos(ax)^2}\,dx$$

input
```
integrate(x^3/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^3 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)*integrate((4*a^2*x^4 - 3*x^2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^
2 - a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*
x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^3}{a\arccos(ax)} - \frac{\operatorname{Ci}(4\arccos(ax))}{2a^4} - \frac{\operatorname{Ci}(2\arccos(ax))}{2a^4}$$

input
```
integrate(x^3/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)) - 1/2*cos_integral(4*arccos(a*x))/a
^4 - 1/2*cos_integral(2*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^2} \, dx = \int \frac{x^3}{\mathrm{acos}\,(a\,x)^2} \, dx$$

input
```
int(x^3/acos(a*x)^2,x)
```

output
```
int(x^3/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)^2} \, dx = \int \frac{x^3}{acos\,(ax)^2} dx$$

input
```
int(x^3/acos(a*x)^2,x)
```

output
```
int(x**3/acos(a*x)**2,x)
```

3.55 $\int \frac{x^2}{\arccos(ax)^2}\,dx$

Optimal result 463
Mathematica [A] (verified) 463
Rubi [A] (verified) 464
Maple [A] (verified) 465
Fricas [F] 465
Sympy [F] 466
Maxima [F] 466
Giac [A] (verification not implemented) 466
Mupad [F(-1)] 467
Reduce [F] 467

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{4a^3} - \frac{3\,\text{CosIntegral}(3\arccos(ax))}{4a^3}$$

output
```
x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-1/4*Ci(arccos(a*x))/a^3-3/4*Ci(3*arcc
os(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = -\frac{-\frac{4a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)} + \text{CosIntegral}(\arccos(ax)) + 3\,\text{CosIntegral}(3\arccos(ax))}{4a^3}$$

input
```
Integrate[x^2/ArcCos[a*x]^2,x]
```

output
```
-1/4*((-4*a^2*x^2*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + CosIntegral[ArcCos[a*x]
] + 3*CosIntegral[3*ArcCos[a*x]])/a^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)^2} \, dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{\int \left(-\frac{ax}{4\arccos(ax)}-\frac{3\cos(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{3}{4}\text{CosIntegral}(3\arccos(ax))}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

input
```
Int[x^2/ArcCos[a*x]^2,x]
```

output
```
(x^2*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + (-1/4*CosIntegral[ArcCos[a*x]] -
 (3*CosIntegral[3*ArcCos[a*x]])/4)/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))    Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{\sin(3\arccos(ax))}{4\arccos(ax)}-\frac{3\,\mathrm{Ci}(3\arccos(ax))}{4}+\frac{\sqrt{-a^2x^2+1}}{4\arccos(ax)}-\frac{\mathrm{Ci}(\arccos(ax))}{4}}{a^3}$	57
default	$\frac{\frac{\sin(3\arccos(ax))}{4\arccos(ax)}-\frac{3\,\mathrm{Ci}(3\arccos(ax))}{4}+\frac{\sqrt{-a^2x^2+1}}{4\arccos(ax)}-\frac{\mathrm{Ci}(\arccos(ax))}{4}}{a^3}$	57

input
```
int(x^2/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/4/arccos(a*x)*sin(3*arccos(a*x))-3/4*Ci(3*arccos(a*x))+1/4/arccos
(a*x)*(-a^2*x^2+1)^(1/2)-1/4*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^2}\,dx$$

input
```
integrate(x^2/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\operatorname{acos}^2(ax)}\,dx$$

input
```
integrate(x**2/acos(a*x)**2,x)
```

output
```
Integral(x**2/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\arccos(ax)^2}\,dx$$

input
```
integrate(x^2/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)*integrate((3*a^2*x^3 - 2*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2
- a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*x
+ 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^2}{a\arccos(ax)} - \frac{3\,\operatorname{Ci}(3\arccos(ax))}{4\,a^3} - \frac{\operatorname{Ci}(\arccos(ax))}{4\,a^3}$$

input
```
integrate(x^2/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)) - 3/4*cos_integral(3*arccos(a*x))/a
^3 - 1/4*cos_integral(arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

input
```
int(x^2/acos(a*x)^2,x)
```

output
```
int(x^2/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{acos\left(ax\right)^2}dx$$

input
```
int(x^2/acos(a*x)^2,x)
```

output
```
int(x**2/acos(a*x)**2,x)
```

3.56 $\int \frac{x}{\arccos(ax)^2}\,dx$

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Mathematica [A] (verified) . . . 468
Rubi [A] (verified) . . . 469
Maple [A] (verified) . . . 470
Fricas [F] . . . 471
Sympy [F] . . . 471
Maxima [F] . . . 471
Giac [A] (verification not implemented) . . . 472
Mupad [F(-1)] . . . 472
Reduce [F] . . . 472

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{x}{\arccos(ax)^2}\,dx = \frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{a^2}$$

output
```
x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-Ci(2*arccos(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x}{\arccos(ax)^2}\,dx = \frac{\frac{ax\sqrt{1-a^2x^2}}{\arccos(ax)} - \text{CosIntegral}(2\arccos(ax))}{a^2}$$

input
```
Integrate[x/ArcCos[a*x]^2,x]
```

output
```
((a*x*Sqrt[1 - a^2*x^2])/ArcCos[a*x] - CosIntegral[2*ArcCos[a*x]])/a^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5143, 25, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)^2} \, dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{\int -\frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int \frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int \frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\text{CosIntegral}(2\arccos(ax))}{a^2}$$

input
```
Int[x/ArcCos[a*x]^2,x]
```

output
```
(x*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{2\arccos(ax)}-\operatorname{Ci}(2\arccos(ax))}{a^2}$	30
default	$\frac{\frac{\sin(2\arccos(ax))}{2\arccos(ax)}-\operatorname{Ci}(2\arccos(ax))}{a^2}$	30

input
```
int(x/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*sin(2*arccos(a*x))/arccos(a*x)-Ci(2*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)^2} \, dx = \int \frac{x}{\arccos\left(ax\right)^2} \, dx$$

input
```
integrate(x/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^2} \, dx = \int \frac{x}{\operatorname{acos}^2\left(ax\right)} \, dx$$

input
```
integrate(x/acos(a*x)**2,x)
```

output
```
Integral(x/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)^2} \, dx = \int \frac{x}{\arccos\left(ax\right)^2} \, dx$$

input
```
integrate(x/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate((2*a^2*x^2 - 1)*s
qrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)), x) - sqrt(a*x + 1)*sqrt(-a*x + 1)*x)/(a*arctan2(sqrt(a*x + 1
)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x}{a\arccos(ax)} - \frac{\text{Ci}\,(2\arccos(ax))}{a^2}$$

input
```
integrate(x/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)) - cos_integral(2*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^2}\,dx = \int \frac{x}{\mathrm{acos}\,(a\,x)^2}\,dx$$

input
```
int(x/acos(a*x)^2,x)
```

output
```
int(x/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)^2}\,dx = \int \frac{x}{acos\,(ax)^2}dx$$

input
```
int(x/acos(a*x)^2,x)
```

output
```
int(x/acos(a*x)**2,x)
```

3.57 $\int \frac{1}{\arccos(ax)^2} dx$

Optimal result 473
Mathematica [A] (verified) 473
Rubi [A] (verified) 474
Maple [A] (verified) 475
Fricas [F] 476
Sympy [F] 476
Maxima [F] 476
Giac [A] (verification not implemented) 477
Mupad [F(-1)] 477
Reduce [F] 477

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \frac{1}{\arccos(ax)^2} dx = \frac{\sqrt{1-a^2x^2}}{a \arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{a}$$

output
```
(-a^2*x^2+1)^(1/2)/a/arccos(a*x)-Ci(arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(ax)^2} dx = \frac{\sqrt{1-a^2x^2}}{a \arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{a}$$

input
```
Integrate[ArcCos[a*x]^(-2),x]
```

output
```
Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5133, 5225, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{5133}$$

$$a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}dx+\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int \frac{ax}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\text{CosIntegral}(\arccos(ax))}{a}$$

input
```
Int[ArcCos[a*x]^(-2),x]
```

output
```
Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^{2}x^{2}+1}}{\arccos(ax)}-\operatorname{Ci}(\arccos(ax))}{a}$	32
default	$\frac{\frac{\sqrt{-a^{2}x^{2}+1}}{\arccos(ax)}-\operatorname{Ci}(\arccos(ax))}{a}$	32

input
```
int(1/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/arccos(a*x)*(-a^2*x^2+1)^(1/2)-Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)^2}\,dx = \int \frac{1}{\arccos\left(ax\right)^2}\,dx$$

input
```
integrate(1/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^2}\,dx = \int \frac{1}{\operatorname{acos}^2\left(ax\right)}\,dx$$

input
```
integrate(1/acos(a*x)**2,x)
```

output
```
Integral(acos(a*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)^2}\,dx = \int \frac{1}{\arccos\left(ax\right)^2}\,dx$$

input
```
integrate(1/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1)*s
qrt(-a*x + 1)*x/((a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
, x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{\arccos(ax)^2}\,dx = -\frac{\operatorname{Ci}(\arccos(ax))}{a} + \frac{\sqrt{-a^2x^2+1}}{a\arccos(ax)}$$

input
```
integrate(1/arccos(a*x)^2,x, algorithm="giac")
```

output
```
-cos_integral(arccos(a*x))/a + sqrt(-a^2*x^2 + 1)/(a*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^2}\,dx = \int \frac{1}{\operatorname{acos}(a\,x)^2}\,dx$$

input
```
int(1/acos(a*x)^2,x)
```

output
```
int(1/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)^2}\,dx = \int \frac{1}{\operatorname{acos}(ax)^2}dx$$

input
```
int(1/acos(a*x)^2,x)
```

output
```
int(1/acos(a*x)**2,x)
```

3.58 $\int \frac{1}{x \arccos(ax)^2} \, dx$

Optimal result 478
Mathematica [N/A] 478
Rubi [N/A] 479
Maple [N/A] 479
Fricas [N/A] 480
Sympy [N/A] 480
Maxima [N/A] 480
Giac [N/A] 481
Mupad [N/A] 481
Reduce [N/A] 482

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \text{Int}\left(\frac{1}{x \arccos(ax)^2}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \arccos(ax)^2} \, dx$$

input
```
Integrate[1/(x*ArcCos[a*x]^2),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)^2} \, dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{1}{x \arccos(ax)^2}dx$$

input
```
Int[1/(x*ArcCos[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos\left(ax\right)^{2}}dx$$

input
```
int(1/x/arccos(a*x)^2,x)
```

output
```
int(1/x/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/x/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \operatorname{acos}^2{(ax)}} \, dx$$

input
```
integrate(1/x/acos(a*x)**2,x)
```

output
```
Integral(1/(x*acos(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 12.70

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/x/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1)*s
qrt(-a*x + 1)/((a^3*x^4 - a*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x
)), x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*
x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)^2}\,dx = \int \frac{1}{x\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/x/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{acos}{(a\,x)}^2}\,dx$$

input
```
int(1/(x*acos(a*x)^2),x)
```

output
```
int(1/(x*acos(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{acos\,(ax)^2 \, x} dx$$

input
```
int(1/x/acos(a*x)^2,x)
```

output
```
int(1/(acos(a*x)**2*x),x)
```

3.59 $\int \frac{1}{x^2 \arccos(ax)^2} \, dx$

Optimal result 483
Mathematica [N/A] 483
Rubi [N/A] 484
Maple [N/A] 484
Fricas [N/A] 485
Sympy [N/A] 485
Maxima [N/A] 485
Giac [N/A] 486
Mupad [N/A] 486
Reduce [N/A] 487

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \text{Int}\left(\frac{1}{x^2 \arccos(ax)^2}, x\right)$$

output
```
Defer(Int)(1/x^2/arccos(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos(ax)^2} \, dx$$

input
```
Integrate[1/(x^2*ArcCos[a*x]^2),x]
```

output
```
Integrate[1/(x^2*ArcCos[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx$$
$$\Big\downarrow \text{5149}$$
$$\int \frac{1}{x^2 \arccos(ax)^2}dx$$

input
```
Int[1/(x^2*ArcCos[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos\left(ax\right)^2}dx$$

input
```
int(1/x^2/arccos(a*x)^2,x)
```

output
```
int(1/x^2/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \operatorname{acos}^2{(ax)}} \, dx$$

input
```
integrate(1/x**2/acos(a*x)**2,x)
```

output
```
Integral(1/(x**2*acos(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 13.60

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(a*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate((a^2*x^2 - 2)*
sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^5 - a*x^3)*arctan2(sqrt(a*x + 1)*sqrt
(-a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x^2*arctan2(sqrt(
a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^2*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \,\mathrm{acos}\left(a\,x\right)^2} \, dx$$

input
```
int(1/(x^2*acos(a*x)^2),x)
```

output
```
int(1/(x^2*acos(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{acos\left(ax\right)^2 x^2} dx$$

input
```
int(1/x^2/acos(a*x)^2,x)
```

output
```
int(1/(acos(a*x)**2*x**2),x)
```

3.60 $\int \frac{x^4}{\arccos(ax)^3} \, dx$

Optimal result 488
Mathematica [A] (verified) 488
Rubi [A] (verified) 489
Maple [A] (verified) 491
Fricas [F] 492
Sympy [F] 492
Maxima [F] 492
Giac [A] (verification not implemented) 493
Mupad [F(-1)] 493
Reduce [F] 493

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{16a^5} + \frac{27\text{Si}(3\arccos(ax))}{32a^5} + \frac{25\text{Si}(5\arccos(ax))}{32a^5}$$

output
```
1/2*x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2-2*x^3/a^2/arccos(a*x)+5/2*x^5/a
rccos(a*x)+1/16*Si(arccos(a*x))/a^5+27/32*Si(3*arccos(a*x))/a^5+25/32*Si(5
*arccos(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \frac{16a^4x^4\sqrt{1-a^2x^2} - 64a^3x^3\arccos(ax) + 80a^5x^5\arccos(ax) + 2\arccos(ax)^2\text{Si}(\arccos(ax)) + 27\arccos(a}{32a^5\arccos(ax)^2}$$

input
```
Integrate[x^4/ArcCos[a*x]^3,x]
```

output
```
(16*a^4*x^4*Sqrt[1 - a^2*x^2] - 64*a^3*x^3*ArcCos[a*x] + 80*a^5*x^5*ArcCos
[a*x] + 2*ArcCos[a*x]^2*SinIntegral[ArcCos[a*x]] + 27*ArcCos[a*x]^2*SinInt
egral[3*ArcCos[a*x]] + 25*ArcCos[a*x]^2*SinIntegral[5*ArcCos[a*x]])/(32*a^
5*ArcCos[a*x]^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5145, 5223, 5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)^3}dx$$

$$\Big\downarrow \text{5145}$$

$$\frac{5}{2}a\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx-\frac{2\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx}{a}+\frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5223}$$

$$\frac{5}{2}a\left(\frac{x^5}{a\arccos(ax)}-\frac{5\int \frac{x^4}{\arccos(ax)}dx}{a}\right)-\frac{2\left(\frac{x^3}{a\arccos(ax)}-\frac{3\int \frac{x^2}{\arccos(ax)}dx}{a}\right)}{a}+\frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{2\left(\frac{3\int \frac{a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^4}+\frac{x^3}{a\arccos(ax)}\right)}{a}+\frac{5}{2}a\left(\frac{5\int \frac{a^4x^4\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^6}+\frac{x^5}{a\arccos(ax)}\right)+\frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{5}{2}a\left(\frac{5\int\left(\frac{3\sin(3\arccos(ax))}{16\arccos(ax)}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}+\frac{\sqrt{1-a^2x^2}}{8\arccos(ax)}\right)d\arccos(ax)}{a^6}+\frac{x^5}{a\arccos(ax)}\right)-\frac{2\left(\frac{3\int\left(\frac{\sin(3\arccos(ax))}{4\arccos(ax)}+\frac{\sqrt{1-a^2x^2}}{4\arccos(ax)}\right)d\arccos(ax)}{a^4}+\frac{x^3}{a\arccos(ax)}\right)}{a}+\frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$\downarrow$ 2009

$$\frac{5}{2}a\left(\frac{5\left(\frac{1}{8}\text{Si}(\arccos(ax))+\frac{3}{16}\text{Si}(3\arccos(ax))+\frac{1}{16}\text{Si}(5\arccos(ax))\right)}{a^6}+\frac{x^5}{a\arccos(ax)}\right)-\frac{2\left(\frac{3\left(\frac{1}{4}\text{Si}(\arccos(ax))+\frac{1}{4}\text{Si}(3\arccos(ax))\right)}{a^4}+\frac{x^3}{a\arccos(ax)}\right)}{a}+\frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

input
```
Int[x^4/ArcCos[a*x]^3,x]
```

output
```
(x^4*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (2*(x^3/(a*ArcCos[a*x]) + (3
*(SinIntegral[ArcCos[a*x]]/4 + SinIntegral[3*ArcCos[a*x]]/4))/a^4))/a + (5
*a*(x^5/(a*ArcCos[a*x]) + (5*(SinIntegral[ArcCos[a*x]]/8 + (3*SinIntegral[
3*ArcCos[a*x]])/16 + SinIntegral[5*ArcCos[a*x]]/16))/a^6))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{16\arccos(ax)^2}+\frac{ax}{16\arccos(ax)}+\frac{\operatorname{Si}(\arccos(ax))}{16}+\frac{3\sin(3\arccos(ax))}{32\arccos(ax)^2}+\frac{9\cos(3\arccos(ax))}{32\arccos(ax)}+\frac{27\operatorname{Si}(3\arccos(ax))}{32}+\frac{\sin(5\arccos(ax))}{32\arccos(ax)^2}}{a^5}$
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{16\arccos(ax)^2}+\frac{ax}{16\arccos(ax)}+\frac{\operatorname{Si}(\arccos(ax))}{16}+\frac{3\sin(3\arccos(ax))}{32\arccos(ax)^2}+\frac{9\cos(3\arccos(ax))}{32\arccos(ax)}+\frac{27\operatorname{Si}(3\arccos(ax))}{32}+\frac{\sin(5\arccos(ax))}{32\arccos(ax)^2}}{a^5}$

input
```
int(x^4/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/16/arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+1/16*a*x/arccos(a*x)+1/16*Si(
arccos(a*x))+3/32/arccos(a*x)^2*sin(3*arccos(a*x))+9/32*cos(3*arccos(a*x))
/arccos(a*x)+27/32*Si(3*arccos(a*x))+1/32/arccos(a*x)^2*sin(5*arccos(a*x))
+5/32*cos(5*arccos(a*x))/arccos(a*x)+25/32*Si(5*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \int \frac{x^4}{\arccos(ax)^3} \, dx$$

input
```
integrate(x^4/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^4/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \int \frac{x^4}{\operatorname{acos}^3(ax)} \, dx$$

input
```
integrate(x**4/acos(a*x)**3,x)
```

output
```
Integral(x**4/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \int \frac{x^4}{\arccos(ax)^3} \, dx$$

input
```
integrate(x^4/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^4 - arctan2(sqrt(a*x + 1)*sqrt(-a*x
+ 1), a*x)^2*integrate((25*a^2*x^4 - 12*x^2)/arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x), x) + (5*a^2*x^5 - 4*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
1), a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \frac{5\,x^5}{2\,\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^4}{2\,a\arccos\left(ax\right)^2} - \frac{2\,x^3}{a^2\arccos\left(ax\right)} + \frac{25\,\mathrm{Si}\left(5\,\arccos\left(ax\right)\right)}{32\,a^5} + \frac{27\,\mathrm{Si}\left(3\,\arccos\left(ax\right)\right)}{32\,a^5} + \frac{\mathrm{Si}\left(\arccos\left(ax\right)\right)}{16\,a^5}$$

input
```
integrate(x^4/arccos(a*x)^3,x, algorithm="giac")
```

output
```
5/2*x^5/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)^2) - 2*x^3
/(a^2*arccos(a*x)) + 25/32*sin_integral(5*arccos(a*x))/a^5 + 27/32*sin_int
egral(3*arccos(a*x))/a^5 + 1/16*sin_integral(arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^3}\,dx$$

input
```
int(x^4/acos(a*x)^3,x)
```

output
```
int(x^4/acos(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \int \frac{x^4}{acos\left(ax\right)^3}dx$$

input
```
int(x^4/acos(a*x)^3,x)
```

output
```
int(x**4/acos(a*x)**3,x)
```

3.61 $\int \frac{x^3}{\arccos(ax)^3}\,dx$

Optimal result 494
Mathematica [A] (verified) 494
Rubi [A] (verified) 495
Maple [A] (verified) 498
Fricas [F] 498
Sympy [F] 499
Maxima [F] 499
Giac [A] (verification not implemented) 499
Mupad [F(-1)] 500
Reduce [F] 500

Optimal result

Integrand size = 10, antiderivative size = 83

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{2x^4}{\arccos(ax)} + \frac{\text{Si}(2\arccos(ax))}{2a^4} + \frac{\text{Si}(4\arccos(ax))}{a^4}$$

output
```
1/2*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2-3/2*x^2/a^2/arccos(a*x)+2*x^4/a
rccos(a*x)+1/2*Si(2*arccos(a*x))/a^4+Si(4*arccos(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \frac{\frac{a^2x^2\left(ax\sqrt{1-a^2x^2}+(-3+4a^2x^2)\arccos(ax)\right)}{\arccos(ax)^2} + \text{Si}(2\arccos(ax)) + 2\text{Si}(4\arccos(ax))}{2a^4}$$

input
```
Integrate[x^3/ArcCos[a*x]^3,x]
```

output
```
((a^2*x^2*(a*x*Sqrt[1 - a^2*x^2] + (-3 + 4*a^2*x^2)*ArcCos[a*x]))/ArcCos[a
*x]^2 + SinIntegral[2*ArcCos[a*x]] + 2*SinIntegral[4*ArcCos[a*x]])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5145, 5223, 5147, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)^3}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx}{2a}+2a\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5223}$$

$$-\frac{3\left(\frac{x^2}{a\arccos(ax)}-\frac{2\int \frac{x}{\arccos(ax)}dx}{a}\right)}{2a}+2a\left(\frac{x^4}{a\arccos(ax)}-\frac{4\int \frac{x^3}{\arccos(ax)}dx}{a}\right)+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{3\left(\frac{2\int \frac{ax\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)}{2a}+2a\left(\frac{4\int \frac{a^3x^3\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^5}+\frac{x^4}{a\arccos(ax)}\right)+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{4906}$$

$$2a\left(\frac{4\int \left(\frac{\sin(2\arccos(ax))}{4\arccos(ax)}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}\right)d\arccos(ax)}{a^5}+\frac{x^4}{a\arccos(ax)}\right)-\frac{3\left(\frac{2\int \frac{\sin(2\arccos(ax))}{2\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)}{2a}+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \textbf{27}$$

$$2a\left(\frac{4\int \left(\frac{\sin(2\arccos(ax))}{4\arccos(ax)}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}\right)d\arccos(ax)}{a^5}+\frac{x^4}{a\arccos(ax)}\right)-\frac{3\left(\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)}{2a}+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{3\left(\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)}{2a}+2a\left(\frac{4\left(\frac{1}{4}\text{Si}(2\arccos(ax))+\frac{1}{8}\text{Si}(4\arccos(ax))\right)}{a^5}+\frac{x^4}{a\arccos(ax)}\right)+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{3\left(\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)}{2a}+2a\left(\frac{4\left(\frac{1}{4}\text{Si}(2\arccos(ax))+\frac{1}{8}\text{Si}(4\arccos(ax))\right)}{a^5}+\frac{x^4}{a\arccos(ax)}\right)+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \textbf{3780}$$

$$2a\left(\frac{4\left(\frac{1}{4}\text{Si}(2\arccos(ax))+\frac{1}{8}\text{Si}(4\arccos(ax))\right)}{a^5}+\frac{x^4}{a\arccos(ax)}\right)-\frac{3\left(\frac{\text{Si}(2\arccos(ax))}{a^3}+\frac{x^2}{a\arccos(ax)}\right)}{2a}+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

input
```
Int[x^3/ArcCos[a*x]^3,x]
```

output
```
(x^3*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (3*(x^2/(a*ArcCos[a*x]) + Si
nIntegral[2*ArcCos[a*x]]/a^3))/(2*a) + 2*a*(x^4/(a*ArcCos[a*x]) + (4*(SinI
ntegral[2*ArcCos[a*x]]/4 + SinIntegral[4*ArcCos[a*x]]/8))/a^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{8\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{4\arccos(ax)}+\frac{\mathrm{Si}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{16\arccos(ax)^2}+\frac{\cos(4\arccos(ax))}{4\arccos(ax)}+\mathrm{Si}(4\arccos(ax))}{a^4}$	82
default	$\frac{\frac{\sin(2\arccos(ax))}{8\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{4\arccos(ax)}+\frac{\mathrm{Si}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{16\arccos(ax)^2}+\frac{\cos(4\arccos(ax))}{4\arccos(ax)}+\mathrm{Si}(4\arccos(ax))}{a^4}$	82

input
```
int(x^3/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*sin(2*arccos(a*x))/arccos(a*x)^2+1/4*cos(2*arccos(a*x))/arccos(
a*x)+1/2*Si(2*arccos(a*x))+1/16/arccos(a*x)^2*sin(4*arccos(a*x))+1/4*cos(4
*arccos(a*x))/arccos(a*x)+Si(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{\arccos{(ax)}^3}\,dx$$

input
```
integrate(x^3/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^3} \, dx = \int \frac{x^3}{\operatorname{acos}^3(ax)} \, dx$$

input
```
integrate(x**3/acos(a*x)**3,x)
```

output
```
Integral(x**3/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)^3} \, dx = \int \frac{x^3}{\arccos(ax)^3} \, dx$$

input
```
integrate(x^3/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^3 - 2*arctan2(sqrt(a*x + 1)*sqrt(-a*
x + 1), a*x)^2*integrate((8*a^2*x^3 - 3*x)/arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x), x) + (4*a^2*x^4 - 3*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\arccos(ax)^3} \, dx = \frac{2x^4}{\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}x^3}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{\operatorname{Si}(4\arccos(ax))}{a^4} + \frac{\operatorname{Si}(2\arccos(ax))}{2a^4}$$

input
```
integrate(x^3/arccos(a*x)^3,x, algorithm="giac")
```

output
```
2*x^4/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)^2) - 3/2*x^2
/(a^2*arccos(a*x)) + sin_integral(4*arccos(a*x))/a^4 + 1/2*sin_integral(2*
arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{\mathrm{acos}\,(a\,x)^3}\,dx$$

input
```
int(x^3/acos(a*x)^3,x)
```

output
```
int(x^3/acos(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{acos\,(ax)^3}dx$$

input
```
int(x^3/acos(a*x)^3,x)
```

output
```
int(x**3/acos(a*x)**3,x)
```

3.62 $\int \frac{x^2}{\arccos(ax)^3} \, dx$

Optimal result . . . 501
Mathematica [A] (verified) . . . 501
Rubi [A] (verified) . . . 502
Maple [A] (verified) . . . 505
Fricas [F] . . . 505
Sympy [F] . . . 506
Maxima [F] . . . 506
Giac [A] (verification not implemented) . . . 506
Mupad [F(-1)] . . . 507
Reduce [F] . . . 507

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^2}{\arccos(ax)^3} \, dx = \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{8a^3} + \frac{9\text{Si}(3\arccos(ax))}{8a^3}$$

output
```
1/2*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2-x/a^2/arccos(a*x)+3/2*x^3/arcco
s(a*x)+1/8*Si(arccos(a*x))/a^3+9/8*Si(3*arccos(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\arccos(ax)^3} \, dx = \frac{\frac{4ax\left(ax\sqrt{1-a^2x^2}+(-2+3a^2x^2)\arccos(ax)\right)}{\arccos(ax)^2} + \text{Si}(\arccos(ax)) + 9\text{Si}(3\arccos(ax))}{8a^3}$$

input
```
Integrate[x^2/ArcCos[a*x]^3,x]
```

output
```
((4*a*x*(a*x*Sqrt[1 - a^2*x^2] + (-2 + 3*a^2*x^2)*ArcCos[a*x]))/ArcCos[a*x
]^2 + SinIntegral[ArcCos[a*x]] + 9*SinIntegral[3*ArcCos[a*x]])/(8*a^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5145, 5223, 5135, 3042, 3780, 5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)^3}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx}{a}+\frac{3}{2}a\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5223}$$

$$\frac{3}{2}a\left(\frac{x^3}{a\arccos(ax)}-\frac{3\int \frac{x^2}{\arccos(ax)}dx}{a}\right)-\frac{\frac{x}{a\arccos(ax)}-\frac{\int \frac{1}{\arccos(ax)}dx}{a}}{a}+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\frac{\int \frac{\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x}{a\arccos(ax)}}{a}+\frac{3}{2}a\left(\frac{x^3}{a\arccos(ax)}-\frac{3\int \frac{x^2}{\arccos(ax)}dx}{a}\right)+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sin(\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x}{a\arccos(ax)}}{a}+\frac{3}{2}a\left(\frac{x^3}{a\arccos(ax)}-\frac{3\int \frac{x^2}{\arccos(ax)}dx}{a}\right)+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{3}{2}a\left(\frac{x^3}{a\arccos(ax)}-\frac{3\int\frac{x^2}{\arccos(ax)}dx}{a}\right)-\frac{\frac{\text{Si}(\arccos(ax))}{a^2}+\frac{x}{a\arccos(ax)}}{a}+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \ 5147$$

$$\frac{3}{2}a\left(\frac{3\int\frac{a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^4}+\frac{x^3}{a\arccos(ax)}\right)-\frac{\frac{\text{Si}(\arccos(ax))}{a^2}+\frac{x}{a\arccos(ax)}}{a}+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \ 4906$$

$$\frac{3}{2}a\left(\frac{3\int\left(\frac{\sin(3\arccos(ax))}{4\arccos(ax)}+\frac{\sqrt{1-a^2x^2}}{4\arccos(ax)}\right)d\arccos(ax)}{a^4}+\frac{x^3}{a\arccos(ax)}\right)-\frac{\frac{\text{Si}(\arccos(ax))}{a^2}+\frac{x}{a\arccos(ax)}}{a}+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{2}a\left(\frac{3\left(\frac{1}{4}\text{Si}(\arccos(ax))+\frac{1}{4}\text{Si}(3\arccos(ax))\right)}{a^4}+\frac{x^3}{a\arccos(ax)}\right)-\frac{\frac{\text{Si}(\arccos(ax))}{a^2}+\frac{x}{a\arccos(ax)}}{a}+\frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

input
```
Int[x^2/ArcCos[a*x]^3,x]
```

output
```
(x^2*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (x/(a*ArcCos[a*x]) + SinInte
gral[ArcCos[a*x]]/a^2)/a + (3*a*(x^3/(a*ArcCos[a*x]) + (3*(SinIntegral[Arc
Cos[a*x]]/4 + SinIntegral[3*ArcCos[a*x]]/4))/a^4))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)^2}+\frac{ax}{8\arccos(ax)}+\frac{\mathrm{Si}(\arccos(ax))}{8}+\frac{\sin(3\arccos(ax))}{8\arccos(ax)^2}+\frac{3\cos(3\arccos(ax))}{8\arccos(ax)}+\frac{9\,\mathrm{Si}(3\arccos(ax))}{8}}{a^3}$	82
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)^2}+\frac{ax}{8\arccos(ax)}+\frac{\mathrm{Si}(\arccos(ax))}{8}+\frac{\sin(3\arccos(ax))}{8\arccos(ax)^2}+\frac{3\cos(3\arccos(ax))}{8\arccos(ax)}+\frac{9\,\mathrm{Si}(3\arccos(ax))}{8}}{a^3}$	82

input
```
int(x^2/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/8/arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+1/8*a*x/arccos(a*x)+1/8*Si(arc
cos(a*x))+1/8/arccos(a*x)^2*sin(3*arccos(a*x))+3/8*cos(3*arccos(a*x))/arcc
os(a*x)+9/8*Si(3*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^3}\,dx$$

input
```
integrate(x^2/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\operatorname{acos}^3(ax)}\,dx$$

input
```
integrate(x**2/acos(a*x)**3,x)
```

output
```
Integral(x**2/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\arccos(ax)^3}\,dx$$

input
```
integrate(x^2/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^2 - arctan2(sqrt(a*x + 1)*sqrt(-a*x
+ 1), a*x)^2*integrate((9*a^2*x^2 - 2)/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1
), a*x), x) + (3*a^2*x^3 - 2*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
)/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \frac{3\,x^3}{2\,\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}x^2}{2\,a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{9\,\mathrm{Si}(3\,\arccos(ax))}{8\,a^3} + \frac{\mathrm{Si}(\arccos(ax))}{8\,a^3}$$

input
```
integrate(x^2/arccos(a*x)^3,x, algorithm="giac")
```

output
```
3/2*x^3/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)^2) - x/(a^
2*arccos(a*x)) + 9/8*sin_integral(3*arccos(a*x))/a^3 + 1/8*sin_integral(ar
ccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)^3}\,dx$$

input
```
int(x^2/acos(a*x)^3,x)
```

output
```
int(x^2/acos(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{acos\left(ax\right)^3}dx$$

input
```
int(x^2/acos(a*x)^3,x)
```

output
```
int(x**2/acos(a*x)**3,x)
```

3.63 $\int \frac{x}{\arccos(ax)^3} dx$

Optimal result 508
Mathematica [A] (verified) 508
Rubi [A] (verified) 509
Maple [A] (verified) 511
Fricas [F] 512
Sympy [F] 512
Maxima [F] 512
Giac [A] (verification not implemented) 513
Mupad [F(-1)] 513
Reduce [F] 514

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \frac{x}{\arccos(ax)^3} dx = \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} + \frac{\mathrm{Si}(2\arccos(ax))}{a^2}$$

output
```
1/2*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2-1/2/a^2/arccos(a*x)+x^2/arccos(a*
x)+Si(2*arccos(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arccos(ax)^3} dx = \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} + \frac{-1+2a^2x^2}{2a^2\arccos(ax)} + \frac{\mathrm{Si}(2\arccos(ax))}{a^2}$$

input
```
Integrate[x/ArcCos[a*x]^3,x]
```

output
```
(x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) + (-1 + 2*a^2*x^2)/(2*a^2*ArcCos
[a*x]) + SinIntegral[2*ArcCos[a*x]]/a^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5145, 5153, 5223, 5147, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)^3} \, dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx}{2a}+a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{5153}$$

$$a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

$$\Big\downarrow \text{5223}$$

$$a\left(\frac{x^2}{a\arccos(ax)}-\frac{2\int \frac{x}{\arccos(ax)}dx}{a}\right)+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

$$\Big\downarrow \text{5147}$$

$$a\left(\frac{2\int \frac{ax\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

$$\Big\downarrow \text{4906}$$

$$a\left(\frac{2\int \frac{\sin(2\arccos(ax))}{2\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

$$\Big\downarrow \text{27}$$

$$a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^3}+\frac{x^2}{a\arccos(ax)}\right)+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

$\Big\downarrow$ 3780

$$a\left(\frac{\text{Si}(2\arccos(ax))}{a^3}+\frac{x^2}{a\arccos(ax)}\right)+\frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{1}{2a^2\arccos(ax)}$$

input
```
Int[x/ArcCos[a*x]^3,x]
```

output
```
(x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - 1/(2*a^2*ArcCos[a*x]) + a*(x^2
/(a*ArcCos[a*x]) + SinIntegral[2*ArcCos[a*x]]/a^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\operatorname{Si}(2\arccos(ax))}{a^2}$	43
default	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\operatorname{Si}(2\arccos(ax))}{a^2}$	43

input
```
int(x/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/4*sin(2*arccos(a*x))/arccos(a*x)^2+1/2*cos(2*arccos(a*x))/arccos(
a*x)+Si(2*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\arccos\left(ax\right)^3}\,dx$$

input
```
integrate(x/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\operatorname{acos}^3\left(ax\right)}\,dx$$

input
```
integrate(x/acos(a*x)**3,x)
```

output
```
Integral(x/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\arccos\left(ax\right)^3}\,dx$$

input
```
integrate(x/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(4*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(x/arcta
n2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - sqrt(a*x + 1)*sqrt(-a*x + 1)*a
*x - (2*a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*arct
an2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x}{\arccos(ax)^3}\,dx = \frac{x^2}{\arccos{(ax)}} + \frac{\text{Si}\left(2\arccos{(ax)}\right)}{a^2} + \frac{\sqrt{-a^2x^2+1}x}{2\,a\arccos{(ax)}^2} - \frac{1}{2\,a^2\arccos{(ax)}}$$

input
```
integrate(x/arccos(a*x)^3,x, algorithm="giac")
```

output
```
x^2/arccos(a*x) + sin_integral(2*arccos(a*x))/a^2 + 1/2*sqrt(-a^2*x^2 + 1)
*x/(a*arccos(a*x)^2) - 1/2/(a^2*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)^3}\,dx$$

input
```
int(x/acos(a*x)^3,x)
```

output
```
int(x/acos(a*x)^3, x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)^3} dx = \int \frac{x}{acos(ax)^3} dx$$

input
```
int(x/acos(a*x)^3,x)
```

output
```
int(x/acos(a*x)**3,x)
```

3.64 $\int \frac{1}{\arccos(ax)^3} \, dx$

Optimal result 515
Mathematica [A] (verified) 515
Rubi [A] (verified) 516
Maple [A] (verified) 517
Fricas [F] 518
Sympy [F] 518
Maxima [F] 518
Giac [A] (verification not implemented) 519
Mupad [F(-1)] 519
Reduce [F] 520

Optimal result

Integrand size = 6, antiderivative size = 51

$$\int \frac{1}{\arccos(ax)^3} \, dx = \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} + \frac{x}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{2a}$$

output
```
1/2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2+1/2*x/arccos(a*x)+1/2*Si(arccos(a*x
))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{1}{\arccos(ax)^3} \, dx = \frac{\sqrt{1-a^2x^2} + ax\arccos(ax) + \arccos(ax)^2\text{Si}(\arccos(ax))}{2a\arccos(ax)^2}$$

input
```
Integrate[ArcCos[a*x]^(-3),x]
```

output
```
(Sqrt[1 - a^2*x^2] + a*x*ArcCos[a*x] + ArcCos[a*x]^2*SinIntegral[ArcCos[a*
x]])/(2*a*ArcCos[a*x]^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5133, 5223, 5135, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^3}dx$$

$$\Big\downarrow \text{ 5133}$$

$$\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^2}dx+\frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{ 5223}$$

$$\frac{1}{2}a\left(\frac{x}{a\arccos(ax)}-\frac{\int \frac{1}{\arccos(ax)}dx}{a}\right)+\frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{ 5135}$$

$$\frac{1}{2}a\left(\frac{\int \frac{\sqrt{1-a^2x^2}}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x}{a\arccos(ax)}\right)+\frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}a\left(\frac{\int \frac{\sin(\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x}{a\arccos(ax)}\right)+\frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{1}{2}a\left(\frac{\text{Si}(\arccos(ax))}{a^2}+\frac{x}{a\arccos(ax)}\right)+\frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}$$

input
```
Int[ArcCos[a*x]^(-3),x]
```

output
```
Sqrt[1 - a^2*x^2]/(2*a*ArcCos[a*x]^2) + (a*(x/(a*ArcCos[a*x]) + SinIntegra
l[ArcCos[a*x]]/a^2))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{2\arccos(ax)^2}+\frac{ax}{2\arccos(ax)}+\frac{\mathrm{Si}(\arccos(ax))}{2}}{a}$	43
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{2\arccos(ax)^2}+\frac{ax}{2\arccos(ax)}+\frac{\mathrm{Si}(\arccos(ax))}{2}}{a}$	43

input
```
int(1/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2/arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+1/2*a*x/arccos(a*x)+1/2*Si(arcco
s(a*x)))
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\arccos\left(ax\right)^3}\,dx$$

input
```
integrate(1/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^(-3), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\operatorname{acos}^3\left(ax\right)}\,dx$$

input
```
integrate(1/acos(a*x)**3,x)
```

output
```
Integral(acos(a*x)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\arccos\left(ax\right)^3}\,dx$$

input
```
integrate(1/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(1/arctan2(s
qrt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*
x + 1), a*x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt
(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{\arccos(ax)^3} \, dx = \frac{x}{2 \arccos{(ax)}} + \frac{\mathrm{Si}\left(\arccos{(ax)}\right)}{2\,a} + \frac{\sqrt{-a^2x^2+1}}{2\,a \arccos{(ax)}^2}$$

input
```
integrate(1/arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/2*x/arccos(a*x) + 1/2*sin_integral(arccos(a*x))/a + 1/2*sqrt(-a^2*x^2 +
1)/(a*arccos(a*x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^3} \, dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^3} \,dx$$

input
```
int(1/acos(a*x)^3,x)
```

output
```
int(1/acos(a*x)^3, x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)^3} dx = \int \frac{1}{acos(ax)^3} dx$$

input
```
int(1/acos(a*x)^3,x)
```

output
```
int(1/acos(a*x)**3,x)
```

3.65 $\int \frac{1}{x \arccos(ax)^3} dx$

Optimal result 521
Mathematica [N/A] 521
Rubi [N/A] 522
Maple [N/A] 522
Fricas [N/A] 523
Sympy [N/A] 523
Maxima [N/A] 523
Giac [N/A] 524
Mupad [N/A] 524
Reduce [N/A] 525

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)^3} dx = \operatorname{Int}\left(\frac{1}{x \arccos(ax)^3}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} dx = \int \frac{1}{x \arccos(ax)^3} dx$$

input
```
Integrate[1/(x*ArcCos[a*x]^3),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)^3} \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{1}{x \arccos(ax)^3}dx$$

input
```
Int[1/(x*ArcCos[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos \left(a x \right)^{3}}d x$$

input
```
int(1/x/arccos(a*x)^3,x)
```

output
```
int(1/x/arccos(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos{(ax)}^3} \, dx$$

input
```
integrate(1/x/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(x*arccos(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \operatorname{acos}^3{(ax)}} \, dx$$

input
```
integrate(1/x/acos(a*x)**3,x)
```

output
```
Integral(1/(x*acos(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 124, normalized size of antiderivative = 12.40

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos{(ax)}^3} \, dx$$

input
```
integrate(1/x/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(1/(x^3*a
rctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x +
 1)*a*x + arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*x^2*arctan2(sqr
t(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos{(ax)}^3} \, dx$$

input
```
integrate(1/x/arccos(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \,\mathrm{acos}{(a\,x)}^3} \, dx$$

input
```
int(1/(x*acos(a*x)^3),x)
```

output
```
int(1/(x*acos(a*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3}\,dx = \int \frac{1}{acos\left(ax\right)^{3} x}dx$$

input
```
int(1/x/acos(a*x)^3,x)
```

output
```
int(1/(acos(a*x)**3*x),x)
```

3.66 $\int \frac{1}{x^2 \arccos(ax)^3} \, dx$

Optimal result . . . 526
Mathematica [N/A] . . . 526
Rubi [N/A] . . . 527
Maple [N/A] . . . 527
Fricas [N/A] . . . 528
Sympy [N/A] . . . 528
Maxima [N/A] . . . 528
Giac [N/A] . . . 529
Mupad [N/A] . . . 529
Reduce [N/A] . . . 530

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \text{Int}\left(\frac{1}{x^2 \arccos(ax)^3}, x\right)$$

output
```
Defer(Int)(1/x^2/arccos(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 6.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos(ax)^3} \, dx$$

input
```
Integrate[1/(x^2*ArcCos[a*x]^3),x]
```

output
```
Integrate[1/(x^2*ArcCos[a*x]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{1}{x^2 \arccos(ax)^3} dx$$

input
```
Int[1/(x^2*ArcCos[a*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos\left(ax\right)^3} dx$$

input
```
int(1/x^2/arccos(a*x)^3,x)
```

output
```
int(1/x^2/arccos(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos\left(ax\right)^3} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(x^2*arccos(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{acos}^3\left(ax\right)} \, dx$$

input
```
integrate(1/x**2/acos(a*x)**3,x)
```

output
```
Integral(1/(x**2*acos(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 143, normalized size of antiderivative = 14.30

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos\left(ax\right)^3} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate((a^2*x^2
- 6)/(x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) - sqrt(a*x + 1)*
sqrt(-a*x + 1)*a*x + (a^2*x^2 - 2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a
*x))/(a^2*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(x^2*arccos(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{acos}{(a\,x)}^3} \, dx$$

input
```
int(1/(x^2*acos(a*x)^3),x)
```

output
```
int(1/(x^2*acos(a*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{acos\,(ax)^3\,x^2} dx$$

input
```
int(1/x^2/acos(a*x)^3,x)
```

output
```
int(1/(acos(a*x)**3*x**2),x)
```

3.67 $\int \frac{x^4}{\arccos(ax)^4}\,dx$

Optimal result 531
Mathematica [A] (verified) 532
Rubi [A] (verified) 532
Maple [A] (verified) 534
Fricas [F] 535
Sympy [F] 535
Maxima [F] 536
Giac [A] (verification not implemented) 536
Mupad [F(-1)] 537
Reduce [F] 537

Optimal result

Integrand size = 10, antiderivative size = 158

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2x^3}{3a^2\arccos(ax)^2} + \frac{5x^5}{6\arccos(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{48a^5} + \frac{27\,\text{CosIntegral}(3\arccos(ax))}{32a^5} + \frac{125\,\text{CosIntegral}(5\arccos(ax))}{96a^5}$$

output
```
1/3*x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3-2/3*x^3/a^2/arccos(a*x)^2+5/6*x
^5/arccos(a*x)^2+2*x^2*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)-25/6*x^4*(-a^2*x
^2+1)^(1/2)/a/arccos(a*x)+1/48*Ci(arccos(a*x))/a^5+27/32*Ci(3*arccos(a*x))
/a^5+125/96*Ci(5*arccos(a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{\arccos(ax)^4}\,dx$$
$$= \frac{32a^4x^4\sqrt{1-a^2x^2} - 64a^3x^3\arccos(ax) + 80a^5x^5\arccos(ax) + 192a^2x^2\sqrt{1-a^2x^2}\arccos(ax)^2 - 400a^4x^4}{}$$

input
```
Integrate[x^4/ArcCos[a*x]^4,x]
```

output
```
(32*a^4*x^4*Sqrt[1 - a^2*x^2] - 64*a^3*x^3*ArcCos[a*x] + 80*a^5*x^5*ArcCos
[a*x] + 192*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2 - 400*a^4*x^4*Sqrt[1 -
 a^2*x^2]*ArcCos[a*x]^2 + 2*ArcCos[a*x]^3*CosIntegral[ArcCos[a*x]] + 81*Ar
cCos[a*x]^3*CosIntegral[3*ArcCos[a*x]] + 125*ArcCos[a*x]^3*CosIntegral[5*A
rcCos[a*x]])/(96*a^5*ArcCos[a*x]^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5145, 5223, 5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)^4}\,dx$$

$$\Big\downarrow \text{5145}$$

$$\frac{5}{3}a\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx - \frac{4\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx}{3a} + \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5223}$$

$$\frac{5}{3}a\left(\frac{x^5}{2a\arccos(ax)^2}-\frac{5\int\frac{x^4}{\arccos(ax)^2}dx}{2a}\right)-\frac{4\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\int\frac{x^2}{\arccos(ax)^2}dx}{2a}\right)}{3a}+\frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5143}$$

$$\frac{5}{3}a\left(\frac{x^5}{2a\arccos(ax)^2}-\frac{5\left(\frac{\int\left(-\frac{ax}{8\arccos(ax)}-\frac{9\cos(3\arccos(ax))}{16\arccos(ax)}-\frac{5\cos(5\arccos(ax))}{16\arccos(ax)}\right)d\arccos(ax)}{a^5}+\frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)-\frac{4\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{\int\left(-\frac{ax}{4\arccos(ax)}-\frac{3\cos(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)}{3a}+\frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{5}{3}a\left(\frac{x^5}{2a\arccos(ax)^2}-\frac{5\left(\frac{-\frac{1}{8}\text{CosIntegral}(\arccos(ax))-\frac{9}{16}\text{CosIntegral}(3\arccos(ax))-\frac{5}{16}\text{CosIntegral}(5\arccos(ax))}{a^5}+\frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)-\frac{4\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{-\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{3}{4}\text{CosIntegral}(3\arccos(ax))}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)}{3a}$$

input
```
Int[x^4/ArcCos[a*x]^4,x]
```

output
```
(x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - (4*(x^3/(2*a*ArcCos[a*x]^2)
- (3*((x^2*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + (-1/4*CosIntegral[ArcCos[a
*x]] - (3*CosIntegral[3*ArcCos[a*x]])/4)/a^3))/(2*a)))/(3*a) + (5*a*(x^5/(
2*a*ArcCos[a*x]^2) - (5*((x^4*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) + (-1/8*C
osIntegral[ArcCos[a*x]] - (9*CosIntegral[3*ArcCos[a*x]])/16 - (5*CosIntegr
al[5*ArcCos[a*x]])/16)/a^5))/(2*a)))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)^3}+\frac{ax}{48\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{48\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{48}+\frac{\sin(3\arccos(ax))}{16\arccos(ax)^3}+\frac{3\cos(3\arccos(ax))}{32\arccos(ax)^2}-\frac{9\sin(3\arccos(ax))}{32\arccos(ax)}+2}{a^5}$
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)^3}+\frac{ax}{48\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{48\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{48}+\frac{\sin(3\arccos(ax))}{16\arccos(ax)^3}+\frac{3\cos(3\arccos(ax))}{32\arccos(ax)^2}-\frac{9\sin(3\arccos(ax))}{32\arccos(ax)}+2}{a^5}$

input
```
int(x^4/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/24/arccos(a*x)^3*(-a^2*x^2+1)^(1/2)+1/48*a*x/arccos(a*x)^2-1/48/a
rccos(a*x)*(-a^2*x^2+1)^(1/2)+1/48*Ci(arccos(a*x))+1/16/arccos(a*x)^3*sin(
3*arccos(a*x))+3/32*cos(3*arccos(a*x))/arccos(a*x)^2-9/32/arccos(a*x)*sin(
3*arccos(a*x))+27/32*Ci(3*arccos(a*x))+1/48/arccos(a*x)^3*sin(5*arccos(a*x
))+5/96*cos(5*arccos(a*x))/arccos(a*x)^2-25/96/arccos(a*x)*sin(5*arccos(a*
x))+125/96*Ci(5*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\arccos(ax)^4}\,dx$$

input
```
integrate(x^4/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^4/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\operatorname{acos}^4(ax)}\,dx$$

input
```
integrate(x**4/acos(a*x)**4,x)
```

output
```
Integral(x**4/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\arccos\left(ax\right)^4}\,dx$$

input
```
integrate(x^4/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*(125
*a^4*x^5 - 136*a^2*x^3 + 24*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^2 - a^
3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + (2*a^2*x^4 - (25*a^2*
x^4 - 12*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*
sqrt(-a*x + 1) + (5*a^3*x^5 - 4*a*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1
), a*x))/(a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \frac{5\,x^5}{6\,\arccos\left(ax\right)^2} - \frac{25\,\sqrt{-a^2x^2+1}x^4}{6\,a\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^4}{3\,a\arccos\left(ax\right)^3} - \frac{2\,x^3}{3\,a^2\arccos\left(ax\right)^2} + \frac{2\,\sqrt{-a^2x^2+1}x^2}{a^3\arccos\left(ax\right)} + \frac{125\,\operatorname{Ci}\left(5\,\arccos\left(ax\right)\right)}{96\,a^5} + \frac{27\,\operatorname{Ci}\left(3\,\arccos\left(ax\right)\right)}{32\,a^5} + \frac{\operatorname{Ci}\left(\arccos\left(ax\right)\right)}{48\,a^5}$$

input
```
integrate(x^4/arccos(a*x)^4,x, algorithm="giac")
```

output
```
5/6*x^5/arccos(a*x)^2 - 25/6*sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)) + 1/3*
sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)^3) - 2/3*x^3/(a^2*arccos(a*x)^2) + 2
*sqrt(-a^2*x^2 + 1)*x^2/(a^3*arccos(a*x)) + 125/96*cos_integral(5*arccos(a
*x))/a^5 + 27/32*cos_integral(3*arccos(a*x))/a^5 + 1/48*cos_integral(arcco
s(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^4}\,dx$$

input
```
int(x^4/acos(a*x)^4,x)
```

output
```
int(x^4/acos(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{acos\left(ax\right)^4}dx$$

input
```
int(x^4/acos(a*x)^4,x)
```

output
```
int(x**4/acos(a*x)**4,x)
```

3.68 $\int \frac{x^3}{\arccos(ax)^4} \, dx$

Optimal result 538
Mathematica [A] (verified) 538
Rubi [A] (verified) 539
Maple [A] (verified) 542
Fricas [F] 542
Sympy [F] 543
Maxima [F] 543
Giac [A] (verification not implemented) 543
Mupad [F(-1)] 544
Reduce [F] 544

Optimal result

Integrand size = 10, antiderivative size = 143

$$\int \frac{x^3}{\arccos(ax)^4} \, dx = \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x^2}{2a^2\arccos(ax)^2} + \frac{2x^4}{3\arccos(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{\text{CosIntegral}(2\arccos(ax))}{3a^4} + \frac{4\,\text{CosIntegral}(4\arccos(ax))}{3a^4}$$

output
```
1/3*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3-1/2*x^2/a^2/arccos(a*x)^2+2/3*x
^4/arccos(a*x)^2+x*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)-8/3*x^3*(-a^2*x^2+1)
^(1/2)/a/arccos(a*x)+1/3*Ci(2*arccos(a*x))/a^4+4/3*Ci(4*arccos(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\arccos(ax)^4} \, dx = \frac{\frac{ax\left(2a^2x^2\sqrt{1-a^2x^2}+ax(-3+4a^2x^2)\arccos(ax)-2\sqrt{1-a^2x^2}(-3+8a^2x^2)\arccos(ax)^2\right)}{\arccos(ax)^3} + 2\,\text{CosIntegral}(2\arccos(ax)) + 8\,\text{CosI}}{6a^4}$$

input
```
Integrate[x^3/ArcCos[a*x]^4,x]
```

output
```
((a*x*(2*a^2*x^2*Sqrt[1 - a^2*x^2] + a*x*(-3 + 4*a^2*x^2)*ArcCos[a*x] - 2*
Sqrt[1 - a^2*x^2]*(-3 + 8*a^2*x^2)*ArcCos[a*x]^2))/ArcCos[a*x]^3 + 2*CosIn
tegral[2*ArcCos[a*x]] + 8*CosIntegral[4*ArcCos[a*x]])/(6*a^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5145, 5223, 5143, 25, 2009, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)^4}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx}{a}+\frac{4}{3}a\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5223}$$

$$-\frac{\frac{x^2}{2a\arccos(ax)^2}-\frac{\int \frac{x}{\arccos(ax)^2}dx}{a}}{a}+\frac{4}{3}a\left(\frac{x^4}{2a\arccos(ax)^2}-\frac{2\int \frac{x^3}{\arccos(ax)^2}dx}{a}\right)+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5143}$$

$$-\frac{\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{\int -\frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}}{a}}{a}+\frac{4}{3}a\left(\frac{x^4}{2a\arccos(ax)^2}-\frac{2\left(\frac{\int \left(-\frac{\cos(2\arccos(ax))}{2\arccos(ax)}-\frac{\cos(4\arccos(ax))}{2\arccos(ax)}\right)d\arccos(ax)}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{a}\right)+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}}{a}}{a}+\frac{4}{3}a\left(\frac{x^4}{2a\arccos(ax)^2}-\frac{2\left(\frac{\int\left(-\frac{\cos(2\arccos(ax))}{2\arccos(ax)}-\frac{\cos(4\arccos(ax))}{2\arccos(ax)}\right)d\arccos(ax)}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{a}\right)+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}}{a}}{a}+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{4}{3}a\left(\frac{x^4}{2a\arccos(ax)^2}-\frac{2\left(\frac{-\frac{1}{2}\operatorname{CosIntegral}(2\arccos(ax))-\frac{1}{2}\operatorname{CosIntegral}(4\arccos(ax))}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{a}\right)$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a^2}}{a}}{a}+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{4}{3}a\left(\frac{x^4}{2a\arccos(ax)^2}-\frac{2\left(\frac{-\frac{1}{2}\operatorname{CosIntegral}(2\arccos(ax))-\frac{1}{2}\operatorname{CosIntegral}(4\arccos(ax))}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{a}\right)$$

$$\Big\downarrow \ 3783$$

$$-\frac{\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\operatorname{CosIntegral}(2\arccos(ax))}{a^2}}{a}}{a}+\frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{4}{3}a\left(\frac{x^4}{2a\arccos(ax)^2}-\frac{2\left(\frac{-\frac{1}{2}\operatorname{CosIntegral}(2\arccos(ax))-\frac{1}{2}\operatorname{CosIntegral}(4\arccos(ax))}{a^4}+\frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{a}\right)$$

input
```
Int[x^3/ArcCos[a*x]^4,x]
```

output
```
(x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - (x^2/(2*a*ArcCos[a*x]^2) - (
(x*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/a^2)/a)
/a + (4*a*(x^4/(2*a*ArcCos[a*x]^2) - (2*((x^3*Sqrt[1 - a^2*x^2])/(a*ArcCos
[a*x]) + (-1/2*CosIntegral[2*ArcCos[a*x]] - CosIntegral[4*ArcCos[a*x]]/2)/
a^4))/a))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{6\arccos(ax)}+\frac{\operatorname{Ci}(2\arccos(ax))}{3}+\frac{\sin(4\arccos(ax))}{24\arccos(ax)^3}+\frac{\cos(4\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(4\arccos(ax))}{3\arccos(ax)}}{a^4}$
default	$\frac{\frac{\sin(2\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{6\arccos(ax)}+\frac{\operatorname{Ci}(2\arccos(ax))}{3}+\frac{\sin(4\arccos(ax))}{24\arccos(ax)^3}+\frac{\cos(4\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(4\arccos(ax))}{3\arccos(ax)}}{a^4}$

input
```
int(x^3/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/12*sin(2*arccos(a*x))/arccos(a*x)^3+1/12*cos(2*arccos(a*x))/arcco
s(a*x)^2-1/6*sin(2*arccos(a*x))/arccos(a*x)+1/3*Ci(2*arccos(a*x))+1/24/arc
cos(a*x)^3*sin(4*arccos(a*x))+1/12*cos(4*arccos(a*x))/arccos(a*x)^2-1/3/ar
ccos(a*x)*sin(4*arccos(a*x))+4/3*Ci(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\arccos{(ax)}^4}\,dx$$

input
```
integrate(x^3/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^3/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\operatorname{acos}^4(ax)}\,dx$$

input
```
integrate(x**3/acos(a*x)**4,x)
```

output
```
Integral(x**3/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\arccos(ax)^4}\,dx$$

input
```
integrate(x^3/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/3*(32*
a^4*x^4 - 30*a^2*x^2 + 3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^2 - a^3)*ar
ctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + 2*(a^2*x^3 - (8*a^2*x^3 -
3*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x
 + 1) + (4*a^3*x^4 - 3*a*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/
(a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{x^3}{\arccos(ax)^4}\,dx &= \frac{2\,x^4}{3\,\arccos(ax)^2} - \frac{8\,\sqrt{-a^2x^2+1}x^3}{3\,a\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}x^3}{3\,a\arccos(ax)^3} \\ &\quad - \frac{x^2}{2\,a^2\arccos(ax)^2} + \frac{\sqrt{-a^2x^2+1}x}{a^3\arccos(ax)} \\ &\quad + \frac{4\,\operatorname{Ci}(4\,\arccos(ax))}{3\,a^4} + \frac{\operatorname{Ci}(2\,\arccos(ax))}{3\,a^4}\end{aligned}$$

input
```
integrate(x^3/arccos(a*x)^4,x, algorithm="giac")
```

output
```
2/3*x^4/arccos(a*x)^2 - 8/3*sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)) + 1/3*s
qrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)^3) - 1/2*x^2/(a^2*arccos(a*x)^2) + sq
rt(-a^2*x^2 + 1)*x/(a^3*arccos(a*x)) + 4/3*cos_integral(4*arccos(a*x))/a^4
 + 1/3*cos_integral(2*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^4} dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^4} dx$$

input
```
int(x^3/acos(a*x)^4,x)
```

output
```
int(x^3/acos(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)^4} dx = \int \frac{x^3}{\mathit{acos}\left(ax\right)^4} dx$$

input
```
int(x^3/acos(a*x)^4,x)
```

output
```
int(x**3/acos(a*x)**4,x)
```

3.69 $\int \frac{x^2}{\arccos(ax)^4}\,dx$

Optimal result 545
Mathematica [A] (verified) 545
Rubi [A] (verified) 546
Maple [A] (verified) 549
Fricas [F] 550
Sympy [F] 550
Maxima [F] 551
Giac [A] (verification not implemented) 551
Mupad [F(-1)] 552
Reduce [F] 552

Optimal result

Integrand size = 10, antiderivative size = 141

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{x^3}{2\arccos(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\arccos(ax)} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{24a^3} + \frac{9\,\text{CosIntegral}(3\arccos(ax))}{8a^3}$$

output
```
1/3*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3-1/3*x/a^2/arccos(a*x)^2+1/2*x^3
/arccos(a*x)^2+1/3*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)-3/2*x^2*(-a^2*x^2+1)
^(1/2)/a/arccos(a*x)+1/24*Ci(arccos(a*x))/a^3+9/8*Ci(3*arccos(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{\frac{8a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)^3} + \frac{4ax(-2+3a^2x^2)}{\arccos(ax)^2} - \frac{4\sqrt{1-a^2x^2}(-2+9a^2x^2)}{\arccos(ax)} - 80\,\text{CosIntegral}(\arccos(ax)) + 27(3\,\text{CosIntegral}(\text{arcc}}{24a^3}$$

input
```
Integrate[x^2/ArcCos[a*x]^4,x]
```

output
```
((8*a^2*x^2*Sqrt[1 - a^2*x^2])/ArcCos[a*x]^3 + (4*a*x*(-2 + 3*a^2*x^2))/Ar
cCos[a*x]^2 - (4*Sqrt[1 - a^2*x^2]*(-2 + 9*a^2*x^2))/ArcCos[a*x] - 80*CosI
ntegral[ArcCos[a*x]] + 27*(3*CosIntegral[ArcCos[a*x]] + CosIntegral[3*ArcC
os[a*x]]))/(24*a^3)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5145, 5223, 5133, 5143, 2009, 5225, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)^4}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{2\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx}{3a}+a\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5223}$$

$$a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\int \frac{x^2}{\arccos(ax)^2}dx}{2a}\right)-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{\int \frac{1}{\arccos(ax)^2}dx}{2a}\right)}{3a}+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5133}$$

$$-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}dx+\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}}{2a}\right)}{3a}+a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\int \frac{x^2}{\arccos(ax)^2}dx}{2a}\right)+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5143}$$

$$-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{a\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}dx+\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}}{2a}\right)}{3a}+a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{\int\left(-\frac{ax}{4\arccos(ax)}-\frac{3\cos(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{a\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}dx+\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}}{2a}\right)}{3a}+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{-\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{3}{4}\text{CosIntegral}(3\arccos(ax))}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)$$

$$\Big\downarrow \text{5225}$$

$$-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{ax}{\arccos(ax)}d\arccos(ax)}{a}}{2a}\right)}{3a}+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{-\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{3}{4}\text{CosIntegral}(3\arccos(ax))}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a}}{2a}\right)}{3a}+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{-\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{3}{4}\text{CosIntegral}(3\arccos(ax))}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)$$

$$\Big\downarrow \text{3783}$$

$$-\frac{2\left(\frac{x}{2a\arccos(ax)^2}-\frac{\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\text{CosIntegral}(\arccos(ax))}{a}}{2a}\right)}{3a}+\frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+a\left(\frac{x^3}{2a\arccos(ax)^2}-\frac{3\left(\frac{-\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{3}{4}\text{CosIntegral}(3\arccos(ax))}{a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}\right)}{2a}\right)$$

input
```
Int[x^2/ArcCos[a*x]^4,x]
```

output
```
(x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - (2*(x/(2*a*ArcCos[a*x]^2) -
(Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a)/(2*a)))/(
3*a) + a*(x^3/(2*a*ArcCos[a*x]^2) - (3*((x^2*Sqrt[1 - a^2*x^2])/(a*ArcCos[
a*x]) + (-1/4*CosIntegral[ArcCos[a*x]] - (3*CosIntegral[3*ArcCos[a*x]])/4)
/a^3))/(2*a))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{12\arccos(ax)^3}+\frac{ax}{24\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{24}+\frac{\sin(3\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(3\arccos(ax))}{8\arccos(ax)^2}-\frac{3\sin(3\arccos(ax))}{8\arccos(ax)}+\frac{9\ \mathrm{C}}{}}{a^3}$
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{12\arccos(ax)^3}+\frac{ax}{24\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{24}+\frac{\sin(3\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(3\arccos(ax))}{8\arccos(ax)^2}-\frac{3\sin(3\arccos(ax))}{8\arccos(ax)}+\frac{9\ \mathrm{C}}{}}{a^3}$

input
```
int(x^2/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/12/arccos(a*x)^3*(-a^2*x^2+1)^(1/2)+1/24*a*x/arccos(a*x)^2-1/24/a
rccos(a*x)*(-a^2*x^2+1)^(1/2)+1/24*Ci(arccos(a*x))+1/12/arccos(a*x)^3*sin(
3*arccos(a*x))+1/8*cos(3*arccos(a*x))/arccos(a*x)^2-3/8/arccos(a*x)*sin(3*
arccos(a*x))+9/8*Ci(3*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\arccos{(ax)}^4}\,dx$$

input
```
integrate(x^2/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x^2/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\operatorname{acos}^4{(ax)}}\,dx$$

input
```
integrate(x**2/acos(a*x)**4,x)
```

output
```
Integral(x**2/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\arccos{(ax)}^4}\,dx$$

input
```
integrate(x^2/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*(27*
a^2*x^3 - 20*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a
*x + 1)*sqrt(-a*x + 1), a*x)), x) + (2*a^2*x^2 - (9*a^2*x^2 - 2)*arctan2(s
qrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (3*a^3
*x^3 - 2*a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*arctan2(sqr
t(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{x^3}{2\,\arccos{(ax)}^2} - \frac{3\,\sqrt{-a^2x^2+1}x^2}{2\,a\arccos{(ax)}} + \frac{\sqrt{-a^2x^2+1}x^2}{3\,a\arccos{(ax)}^3} + \frac{9\,\mathrm{Ci}\left(3\,\arccos{(ax)}\right)}{8\,a^3} + \frac{\mathrm{Ci}\left(\arccos{(ax)}\right)}{24\,a^3} - \frac{x}{3\,a^2\arccos{(ax)}^2} + \frac{\sqrt{-a^2x^2+1}}{3\,a^3\arccos{(ax)}}$$

input
```
integrate(x^2/arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/2*x^3/arccos(a*x)^2 - 3/2*sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)) + 1/3*s
qrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)^3) + 9/8*cos_integral(3*arccos(a*x))/
a^3 + 1/24*cos_integral(arccos(a*x))/a^3 - 1/3*x/(a^2*arccos(a*x)^2) + 1/3
*sqrt(-a^2*x^2 + 1)/(a^3*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\mathrm{acos}\,(a\,x)^4}\,dx$$

input
```
int(x^2/acos(a*x)^4,x)
```

output
```
int(x^2/acos(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{acos\,(ax)^4}dx$$

input
```
int(x^2/acos(a*x)^4,x)
```

output
```
int(x**2/acos(a*x)**4,x)
```

3.70 $\int \frac{x}{\arccos(ax)^4}\,dx$

Optimal result 553
Mathematica [A] (verified) 553
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Mupad [F(-1)] 559
Reduce [F] 559

Optimal result

Integrand size = 8, antiderivative size = 97

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2} + \frac{x^2}{3\arccos(ax)^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{2\,\text{CosIntegral}(2\arccos(ax))}{3a^2}$$

output
```
1/3*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3-1/6/a^2/arccos(a*x)^2+1/3*x^2/arc
cos(a*x)^2-2/3*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)+2/3*Ci(2*arccos(a*x))/a^
2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{2ax\sqrt{1-a^2x^2} + (-1+2a^2x^2)\arccos(ax) - 4ax\sqrt{1-a^2x^2}\arccos(ax)^2 + 4\arccos(ax)^3\,\text{CosIntegral}(2\,\text{a}}{6a^2\arccos(ax)^3}$$

input
```
Integrate[x/ArcCos[a*x]^4,x]
```

output
```
(2*a*x*Sqrt[1 - a^2*x^2] + (-1 + 2*a^2*x^2)*ArcCos[a*x] - 4*a*x*Sqrt[1 - a
^2*x^2]*ArcCos[a*x]^2 + 4*ArcCos[a*x]^3*CosIntegral[2*ArcCos[a*x]])/(6*a^2
*ArcCos[a*x]^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5145, 5153, 5223, 5143, 25, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)^4}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx}{3a}+\frac{2}{3}a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5153}$$

$$\frac{2}{3}a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}-\frac{1}{6a^2\arccos(ax)^2}$$

$$\Big\downarrow \text{5223}$$

$$\frac{2}{3}a\left(\frac{x^2}{2a\arccos(ax)^2}-\frac{\int \frac{x}{\arccos(ax)^2}dx}{a}\right)+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}-\frac{1}{6a^2\arccos(ax)^2}$$

$$\Big\downarrow \text{5143}$$

$$\frac{2}{3}a\left(\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{\int -\frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}+\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}}{a}\right)+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}-\frac{1}{6a^2\arccos(ax)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}a\left(\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{\cos(2\arccos(ax))}{\arccos(ax)}d\arccos(ax)}{a^2}}{a}\right)+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}-\frac{1}{6a^2\arccos(ax)^2}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}a\left(\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int\frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a^2}}{a}\right)+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}-\frac{1}{6a^2\arccos(ax)^2}$$

$\Big\downarrow$ 3783

$$\frac{2}{3}a\left(\frac{x^2}{2a\arccos(ax)^2}-\frac{\frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\text{CosIntegral}(2\arccos(ax))}{a^2}}{a}\right)+\frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}-\frac{1}{6a^2\arccos(ax)^2}$$

input
```
Int[x/ArcCos[a*x]^4,x]
```

output
```
(x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - 1/(6*a^2*ArcCos[a*x]^2) + (2*a
*(x^2/(2*a*ArcCos[a*x]^2) - ((x*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIn
tegral[2*ArcCos[a*x]]/a^2)/a))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{6\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{6\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{3\arccos(ax)}+\frac{2\operatorname{Ci}(2\arccos(ax))}{3}}{a^2}$	60
default	$\frac{\frac{\sin(2\arccos(ax))}{6\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{6\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{3\arccos(ax)}+\frac{2\operatorname{Ci}(2\arccos(ax))}{3}}{a^2}$	60

input
```
int(x/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/6*sin(2*arccos(a*x))/arccos(a*x)^3+1/6*cos(2*arccos(a*x))/arccos(
a*x)^2-1/3*sin(2*arccos(a*x))/arccos(a*x)+2/3*Ci(2*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\arccos{(ax)}^4}\,dx$$

input
```
integrate(x/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(x/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\operatorname{acos}^4{(ax)}}\,dx$$

input
```
integrate(x/acos(a*x)**4,x)
```

output
```
Integral(x/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\arccos(ax)^4}\,dx$$

input
```
integrate(x/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*(6*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(2/3*(2*a
^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x)), x) - 2*(2*a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^2 - a*x)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (2*a^2*x^2 - 1)*arctan2(
sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{x^2}{3\arccos(ax)^2} - \frac{2\sqrt{-a^2x^2+1}x}{3a\arccos(ax)} + \frac{2\,\mathrm{Ci}(2\arccos(ax))}{3a^2} + \frac{\sqrt{-a^2x^2+1}x}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2}$$

input
```
integrate(x/arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/3*x^2/arccos(a*x)^2 - 2/3*sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)) + 2/3*cos
_integral(2*arccos(a*x))/a^2 + 1/3*sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)^3)
- 1/6/(a^2*arccos(a*x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)^4}\,dx$$

input
```
int(x/acos(a*x)^4,x)
```

output
```
int(x/acos(a*x)^4, x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{acos\left(ax\right)^4}dx$$

input
```
int(x/acos(a*x)^4,x)
```

output
```
int(x/acos(a*x)**4,x)
```

3.71 $\int \frac{1}{\arccos(ax)^4}\,dx$

Optimal result . . . 560
Mathematica [A] (verified) . . . 560
Rubi [A] (verified) . . . 561
Maple [A] (verified) . . . 563
Fricas [F] . . . 563
Sympy [F] . . . 564
Maxima [F] . . . 564
Giac [A] (verification not implemented) . . . 564
Mupad [F(-1)] . . . 565
Reduce [F] . . . 565

Optimal result

Integrand size = 6, antiderivative size = 78

$$\int \frac{1}{\arccos(ax)^4}\,dx = \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} + \frac{x}{6\arccos(ax)^2} - \frac{\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{6a}$$

output
```
1/3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3+1/6*x/arccos(a*x)^2-1/6*(-a^2*x^2+1
)^(1/2)/a/arccos(a*x)+1/6*Ci(arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{1}{\arccos(ax)^4}\,dx = \frac{2\sqrt{1-a^2x^2} + ax\arccos(ax) - \sqrt{1-a^2x^2}\arccos(ax)^2 + \arccos(ax)^3\,\text{CosIntegral}(\arccos(ax))}{6a\arccos(ax)^3}$$

input
```
Integrate[ArcCos[a*x]^(-4),x]
```

output
```
(2*Sqrt[1 - a^2*x^2] + a*x*ArcCos[a*x] - Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2 +
 ArcCos[a*x]^3*CosIntegral[ArcCos[a*x]])/(6*a*ArcCos[a*x]^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5133, 5223, 5133, 5225, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^4}\,dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{1}{3}a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^3}dx+\frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5223}$$

$$\frac{1}{3}a\left(\frac{x}{2a\arccos(ax)^2}-\frac{\int \frac{1}{\arccos(ax)^2}dx}{2a}\right)+\frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5133}$$

$$\frac{1}{3}a\left(\frac{x}{2a\arccos(ax)^2}-\frac{a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}dx+\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}}{2a}\right)+\frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{5225}$$

$$\frac{1}{3}a\left(\frac{x}{2a\arccos(ax)^2}-\frac{\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int \frac{ax}{\arccos(ax)}d\arccos(ax)}{a}}{2a}\right)+\frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\left(\frac{x}{2a\arccos(ax)^2}-\frac{\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a}}{2a}\right)+\frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

$$\Big\downarrow \; 3783$$

$$\frac{1}{3}a\left(\frac{x}{2a\arccos(ax)^2}-\frac{\frac{\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\text{CosIntegral}(\arccos(ax))}{a}}{2a}\right)+\frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}$$

input
```
Int[ArcCos[a*x]^(-4),x]
```

output
```
Sqrt[1 - a^2*x^2]/(3*a*ArcCos[a*x]^3) + (a*(x/(2*a*ArcCos[a*x]^2) - (Sqrt[
1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a)/(2*a)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{3\arccos(ax)^3}+\frac{ax}{6\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{6\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{6}}{a}$	63
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{3\arccos(ax)^3}+\frac{ax}{6\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{6\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{6}}{a}$	63

input
```
int(1/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/3/arccos(a*x)^3*(-a^2*x^2+1)^(1/2)+1/6*a*x/arccos(a*x)^2-1/6/arccos
(a*x)*(-a^2*x^2+1)^(1/2)+1/6*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\arccos\left(ax\right)^4}\,dx$$

input
```
integrate(1/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^(-4), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\operatorname{acos}^4(ax)}\,dx$$

input
```
integrate(1/acos(a*x)**4,x)
```

output
```
Integral(acos(a*x)**(-4), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\arccos(ax)^4}\,dx$$

input
```
integrate(1/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*(6*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*sqrt
(a*x + 1)*sqrt(-a*x + 1)*x/((a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x
+ 1), a*x)), x) + a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - sqrt(a*
x + 1)*sqrt(-a*x + 1)*(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - 2))/
(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{1}{\arccos(ax)^4}\,dx = \frac{\operatorname{Ci}(\arccos(ax))}{6a} + \frac{x}{6\arccos(ax)^2} - \frac{\sqrt{-a^2x^2+1}}{6a\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}}{3a\arccos(ax)^3}$$

input
```
integrate(1/arccos(a*x)^4,x, algorithm="giac")
```

output
```
1/6*cos_integral(arccos(a*x))/a + 1/6*x/arccos(a*x)^2 - 1/6*sqrt(-a^2*x^2
+ 1)/(a*arccos(a*x)) + 1/3*sqrt(-a^2*x^2 + 1)/(a*arccos(a*x)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^4}\,dx$$

input
```
int(1/acos(a*x)^4,x)
```

output
```
int(1/acos(a*x)^4, x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{acos\left(ax\right)^4}dx$$

input
```
int(1/acos(a*x)^4,x)
```

output
```
int(1/acos(a*x)**4,x)
```

3.72 $\int \frac{1}{x \arccos(ax)^4} dx$

Optimal result 566
Mathematica [N/A] 566
Rubi [N/A] 567
Maple [N/A] 567
Fricas [N/A] 568
Sympy [N/A] 568
Maxima [N/A] 568
Giac [N/A] 569
Mupad [N/A] 569
Reduce [N/A] 570

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)^4} dx = \text{Int}\left(\frac{1}{x \arccos(ax)^4}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} dx = \int \frac{1}{x \arccos(ax)^4} dx$$

input
```
Integrate[1/(x*ArcCos[a*x]^4),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]^4), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)^4} \, dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{x \arccos(ax)^4} dx$$

input
```
Int[1/(x*ArcCos[a*x]^4),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos \left(a x\right)^{4}}dx$$

input
```
int(1/x/arccos(a*x)^4,x)
```

output
```
int(1/x/arccos(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \arccos{(ax)}^4} \, dx$$

input
```
integrate(1/x/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(1/(x*arccos(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \operatorname{acos}^4{(ax)}} \, dx$$

input
```
integrate(1/x/acos(a*x)**4,x)
```

output
```
Integral(1/(x*acos(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 5.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 20.00

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \arccos{(ax)}^4} \, dx$$

input
```
integrate(1/x/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
1/6*(6*a^3*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/3*
(2*a^2*x^2 - 3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^6 - a^3*x^4)*arctan2(
sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + a*x*arctan2(sqrt(a*x + 1)*sqrt(-
a*x + 1), a*x) + 2*(a^2*x^2 + arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2
)*sqrt(a*x + 1)*sqrt(-a*x + 1))/(a^3*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^3)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \arccos{(ax)}^4} \, dx$$

input
```
integrate(1/x/arccos(a*x)^4,x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \, \mathrm{acos}{(a\,x)}^4} \, dx$$

input
```
int(1/(x*acos(a*x)^4),x)
```

output
```
int(1/(x*acos(a*x)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{acos\,(ax)^4\,x} dx$$

input
```
int(1/x/acos(a*x)^4,x)
```

output
```
int(1/(acos(a*x)**4*x),x)
```

3.73 $\int \frac{1}{x^2 \arccos(ax)^4} \, dx$

Optimal result 571
Mathematica [N/A] 571
Rubi [N/A] 572
Maple [N/A] 572
Fricas [N/A] 573
Sympy [N/A] 573
Maxima [N/A] 573
Giac [N/A] 574
Mupad [N/A] 574
Reduce [N/A] 575

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \text{Int}\left(\frac{1}{x^2 \arccos(ax)^4}, x\right)$$

output
```
Defer(Int)(1/x^2/arccos(a*x)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 14.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos(ax)^4} \, dx$$

input
```
Integrate[1/(x^2*ArcCos[a*x]^4),x]
```

output
```
Integrate[1/(x^2*ArcCos[a*x]^4), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{1}{x^2 \arccos(ax)^4} dx$$

input
```
Int[1/(x^2*ArcCos[a*x]^4),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos\left(ax\right)^4} dx$$

input
```
int(1/x^2/arccos(a*x)^4,x)
```

output
```
int(1/x^2/arccos(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^4} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral(1/(x^2*arccos(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \operatorname{acos}^4{(ax)}} \, dx$$

input
```
integrate(1/x**2/acos(a*x)**4,x)
```

output
```
Integral(1/(x**2*acos(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 6.48 (sec) , antiderivative size = 229, normalized size of antiderivative = 22.90

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^4} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^4,x, algorithm="maxima")
```

output
```
-1/6*(6*a^3*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6
*(a^4*x^4 - 20*a^2*x^2 + 24)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^7 - a^3*
x^5)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) - (2*a^2*x^2 - (a^2*x
^2 - 6)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-
a*x + 1) + (a^3*x^3 - 2*a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(
a^3*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^4} \, dx$$

input
```
integrate(1/x^2/arccos(a*x)^4,x, algorithm="giac")
```

output
```
integrate(1/(x^2*arccos(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \operatorname{acos}{(a\,x)}^4} \, dx$$

input
```
int(1/(x^2*acos(a*x)^4),x)
```

output
```
int(1/(x^2*acos(a*x)^4), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{acos\left(ax\right)^4 x^2} dx$$

input
```
int(1/x^2/acos(a*x)^4,x)
```

output
```
int(1/(acos(a*x)**4*x**2),x)
```

3.74 $\int x^4\sqrt{\arccos(ax)}\,dx$

Optimal result 576
Mathematica [C] (verified) 577
Rubi [A] (verified) 577
Maple [A] (verified) 579
Fricas [F(-2)] 580
Sympy [F] 580
Maxima [F(-2)] 580
Giac [C] (verification not implemented) 581
Mupad [F(-1)] 582
Reduce [F] 582

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int x^4\sqrt{\arccos(ax)}\,dx=\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}-\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^5}-\frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{80a^5}$$

output
```
1/5*x^5*arccos(a*x)^(1/2)-1/16*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*
arccos(a*x)^(1/2))/a^5-1/96*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arc
cos(a*x)^(1/2))/a^5-1/800*10^(1/2)*Pi^(1/2)*FresnelC(10^(1/2)/Pi^(1/2)*arc
cos(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.60

$$\int x^4\sqrt{\arccos(ax)}\,dx$$
$$= \frac{i\left(150\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2},-i\arccos(ax)\right)-150\sqrt{i\arccos(ax)}\Gamma\left(\frac{3}{2},i\arccos(ax)\right)+25\sqrt{3}\sqrt{-i\arccos(a}\right.}{}$$

input
```
Integrate[x^4*Sqrt[ArcCos[a*x]],x]
```

output
```
((I/2400)*(150*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] - 150*S
qrt[I*ArcCos[a*x]]*Gamma[3/2, I*ArcCos[a*x]] + 25*Sqrt[3]*Sqrt[(-I)*ArcCos
[a*x]]*Gamma[3/2, (-3*I)*ArcCos[a*x]] - 25*Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gam
ma[3/2, (3*I)*ArcCos[a*x]] + 3*Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (
-5*I)*ArcCos[a*x]] - 3*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (5*I)*ArcCos
[a*x]]))/(a^5*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5141, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{\arccos(ax)}\,dx$$
$$\Big\downarrow \text{5141}$$
$$\frac{1}{10}a\int \frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{5}x^5\sqrt{\arccos(ax)}$$
$$\Big\downarrow \text{5225}$$

$$\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int \frac{a^5x^5}{\sqrt{\arccos(ax)}}d\arccos(ax)}{10a^5}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^5}{\sqrt{\arccos(ax)}}d\arccos(ax)}{10a^5}$$

$\Big\downarrow$ 3793

$$\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int \left(\frac{5ax}{8\sqrt{\arccos(ax)}}+\frac{5\cos(3\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\cos(5\arccos(ax))}{16\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{10a^5}$$

$\Big\downarrow$ 2009

$$\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\frac{5}{4}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{5}{8}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{10}}\operatorname{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{10a^5}$$

input
```
Int[x^4*Sqrt[ArcCos[a*x]],x]
```

output
```
(x^5*Sqrt[ArcCos[a*x]])/5 - ((5*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos
[a*x]]])/4 + (5*Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sq
rt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/8)/(10*a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

method	result
default	$\frac{-3\sqrt{5}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-25\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-150\sqrt{2}\,\sqrt{\pi}}{2400a^5\sqrt{\mathrm{ar}}}$

input
```
int(x^4*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2400/a^5/arccos(a*x)^(1/2)*(-3*5^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2
)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))-25*3^(1/2)*2^(1/2)*
Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(
1/2))-150*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arc
cos(a*x)^(1/2))+300*a*x*arccos(a*x)+150*arccos(a*x)*cos(3*arccos(a*x))+30*
arccos(a*x)*cos(5*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4 \sqrt{\arccos(ax)}\,dx = \int x^4 \sqrt{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**4*acos(a*x)**(1/2),x)
```

output
```
Integral(x**4*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.04

$$\int x^4\sqrt{\arccos(ax)}\,dx = \frac{(i+1)\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{10}\sqrt{\arccos(ax)}\right)}{3200\,a^5} - \frac{(i-1)\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{10}\sqrt{\arccos(ax)}\right)}{3200\,a^5} + \frac{(i+1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{384\,a^5} - \frac{(i-1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{384\,a^5} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64\,a^5} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64\,a^5} + \frac{\sqrt{\arccos(ax)}e^{(5i\arccos(ax))}}{160\,a^5} + \frac{\sqrt{\arccos(ax)}e^{(3i\arccos(ax))}}{32\,a^5} + \frac{\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{16\,a^5} + \frac{\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{16\,a^5} + \frac{\sqrt{\arccos(ax)}e^{(-3i\arccos(ax))}}{32\,a^5} + \frac{\sqrt{\arccos(ax)}e^{(-5i\arccos(ax))}}{160\,a^5}$$

input
```
integrate(x^4*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/3200*I + 1/3200)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt(arcc
os(a*x)))/a^5 - (1/3200*I - 1/3200)*sqrt(10)*sqrt(pi)*erf(-(1/2*I + 1/2)*s
qrt(10)*sqrt(arccos(a*x)))/a^5 + (1/384*I + 1/384)*sqrt(6)*sqrt(pi)*erf((1
/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (1/384*I - 1/384)*sqrt(6)*sqr
t(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (1/64*I + 1/64)*
sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 - (1/64*
I - 1/64)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a
^5 + 1/160*sqrt(arccos(a*x))*e^(5*I*arccos(a*x))/a^5 + 1/32*sqrt(arccos(a*
x))*e^(3*I*arccos(a*x))/a^5 + 1/16*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a^5
 + 1/16*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a^5 + 1/32*sqrt(arccos(a*x))*
e^(-3*I*arccos(a*x))/a^5 + 1/160*sqrt(arccos(a*x))*e^(-5*I*arccos(a*x))/a^
5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt{\arccos(ax)}\, dx = \int x^4\, \sqrt{\mathrm{acos}\,(a\,x)}\, dx$$

input
```
int(x^4*acos(a*x)^(1/2),x)
```

output
```
int(x^4*acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int x^4 \sqrt{\arccos(ax)}\, dx = \int \sqrt{acos\,(ax)}\, x^4 dx$$

input
```
int(x^4*acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x))*x**4,x)
```

3.75 $\int x^3\sqrt{\arccos(ax)}\,dx$

Optimal result 583
Mathematica [C] (verified) 583
Rubi [A] (verified) 584
Maple [A] (verified) 586
Fricas [F(-2)] 586
Sympy [F] 587
Maxima [F(-2)] 587
Giac [C] (verification not implemented) 588
Mupad [F(-1)] 589
Reduce [F] 589

Optimal result

Integrand size = 12, antiderivative size = 95

$$\int x^3\sqrt{\arccos(ax)}\,dx=-\frac{3\sqrt{\arccos(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{64a^4}-\frac{\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{16a^4}$$

output
```
-3/32*arccos(a*x)^(1/2)/a^4+1/4*x^4*arccos(a*x)^(1/2)-1/128*2^(1/2)*Pi^(1/
2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^4-1/16*Pi^(1/2)*Fresne
lC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.38

$$\int x^3\sqrt{\arccos(ax)}\,dx$$
$$=\frac{i\left(4\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2},-2i\arccos(ax)\right)-4\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{3}{2},2i\arccos(ax)\right)+\sqrt{-i\arccos(ax)}\right.}{128a^4\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^3*Sqrt[ArcCos[a*x]],x]
```

output
```
((I/128)*(4*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-2*I)*ArcCos[a*x]]
- 4*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (2*I)*ArcCos[a*x]] + Sqrt[(-I)*
ArcCos[a*x]]*Gamma[3/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[3/
2, (4*I)*ArcCos[a*x]]))/(a^4*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5141, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\arccos(ax)}\,dx$$
$$\Big\downarrow \text{5141}$$
$$\frac{1}{8}a\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{4}x^4\sqrt{\arccos(ax)}$$
$$\Big\downarrow \text{5225}$$
$$\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\int \frac{a^4x^4}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^4}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^4}$$

$\Big\downarrow$ 3793

$$\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{\cos(4\arccos(ax))}{8\sqrt{\arccos(ax)}}+\frac{3}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{8a^4}$$

$\Big\downarrow$ 2009

$$\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^4}$$

input
```
Int[x^3*Sqrt[ArcCos[a*x]],x]
```

output
```
(x^4*Sqrt[ArcCos[a*x]])/4 - ((3*Sqrt[ArcCos[a*x]])/4 + (Sqrt[Pi/2]*Fresnel
C[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a
*x]])/Sqrt[Pi]])/2)/(8*a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

method	result
default	$\frac{-\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}+16\cos(2\arccos(ax))\arccos(ax)-8\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}\,\sqrt{\arccos(ax)}}{128a^4\sqrt{\arccos(ax)}}$

input
```
int(x^3*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/a^4/arccos(a*x)^(1/2)*(-FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2
))*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)+16*cos(2*arccos(a*x))*arccos(a*x)-8*
FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)*arccos(a*x)^(1/2)+4*arccos
(a*x)*cos(4*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3\sqrt{\arccos(ax)}\,dx = \int x^3\sqrt{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**3*acos(a*x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

$$\int x^3\sqrt{\arccos(ax)}\,dx = \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{512\,a^4} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{512\,a^4} + \frac{(i+1)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{64\,a^4} - \frac{(i-1)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{64\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(4i\arccos(ax))}}{64\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{16\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{16\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(-4i\arccos(ax))}}{64\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/512*I + 1/512)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/
a^4 - (1/512*I - 1/512)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(
a*x)))/a^4 + (1/64*I + 1/64)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 -
 (1/64*I - 1/64)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4 + 1/64*sqrt(
arccos(a*x))*e^(4*I*arccos(a*x))/a^4 + 1/16*sqrt(arccos(a*x))*e^(2*I*arcco
s(a*x))/a^4 + 1/16*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^4 + 1/64*sqrt(
arccos(a*x))*e^(-4*I*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{\arccos(ax)}\,dx = \int x^3\,\sqrt{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int(x^3*acos(a*x)^(1/2),x)
```

output
```
int(x^3*acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int x^3 \sqrt{\arccos(ax)}\,dx = \int \sqrt{acos\,(ax)}\,x^3 dx$$

input
```
int(x^3*acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x))*x**3,x)
```

3.76 $\int x^2 \sqrt{\arccos(ax)}\,dx$

Optimal result 590
Mathematica [C] (verified) 590
Rubi [A] (verified) 591
Maple [A] (verified) 593
Fricas [F(-2)] 593
Sympy [F] 593
Maxima [F(-2)] 594
Giac [C] (verification not implemented) 594
Mupad [F(-1)] 595
Reduce [F] 595

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^2 \sqrt{\arccos(ax)}\,dx = \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{12a^3}$$

output
```
1/3*x^3*arccos(a*x)^(1/2)-1/8*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*a
rccos(a*x)^(1/2))/a^3-1/72*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arcc
os(a*x)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int x^2\sqrt{\arccos(ax)}\,dx$$
$$= \frac{i\left(9\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{3}{2},-i\arccos(ax)\right) - 9\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{3}{2},i\arccos(ax)\right) + \sqrt{3}\left(\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{3}{2}\right.\right.\right.}{72a^3\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^2*Sqrt[ArcCos[a*x]],x]
```

output
```
((I/72)*(9*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] - 9*Sqrt[I*
ArcCos[a*x]]*Gamma[3/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*G
amma[3/2, (-3*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (3*I)*ArcCo
s[a*x]])))/(a^3*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5141, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{ 5141}$$

$$\frac{1}{6}a\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{3}x^3\sqrt{\arccos(ax)}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int \frac{a^3x^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int \left(\frac{3ax}{4\sqrt{\arccos(ax)}}+\frac{\cos(3\arccos(ax))}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{6a^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^3}$$

input
```
Int[x^2*Sqrt[ArcCos[a*x]],x]
```

output
```
(x^3*Sqrt[ArcCos[a*x]])/3 - ((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos
[a*x]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/2)/(6*a^3
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

method	result
default	$\frac{-\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-9\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+18ax\arccos(ax)+6}{72a^3\sqrt{\arccos(ax)}}$

input
```
int(x^2*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72/a^3/arccos(a*x)^(1/2)*(-3^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*Fr
esnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-9*2^(1/2)*Pi^(1/2)*arcc
os(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+18*a*x*arccos(a
*x)+6*arccos(a*x)*cos(3*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\sqrt{\arccos(ax)}\,dx = \int x^2\sqrt{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**2*acos(a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\begin{aligned}\int x^2\sqrt{\arccos(ax)}\,dx &= \frac{(i+1)\,\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{6}\sqrt{\arccos\left(ax\right)}\right)}{288\,a^3} \\ &- \frac{(i-1)\,\sqrt{6}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{6}\sqrt{\arccos\left(ax\right)}\right)}{288\,a^3} \\ &+ \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{32\,a^3} \\ &- \frac{(i-1)\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{32\,a^3} \\ &+ \frac{\sqrt{\arccos\left(ax\right)}e^{\left(3i\,\arccos\left(ax\right)\right)}}{24\,a^3} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(i\,\arccos\left(ax\right)\right)}}{8\,a^3} \\ &+ \frac{\sqrt{\arccos\left(ax\right)}e^{\left(-i\,\arccos\left(ax\right)\right)}}{8\,a^3} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(-3i\,\arccos\left(ax\right)\right)}}{24\,a^3}\end{aligned}$$

input
```
integrate(x^2*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/288*I + 1/288)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a
*x)))/a^3 - (1/288*I - 1/288)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*
sqrt(arccos(a*x)))/a^3 + (1/32*I + 1/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2
)*sqrt(2)*sqrt(arccos(a*x)))/a^3 - (1/32*I - 1/32)*sqrt(2)*sqrt(pi)*erf(-(
1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3 + 1/24*sqrt(arccos(a*x))*e^(3*
I*arccos(a*x))/a^3 + 1/8*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a^3 + 1/8*sqr
t(arccos(a*x))*e^(-I*arccos(a*x))/a^3 + 1/24*sqrt(arccos(a*x))*e^(-3*I*arc
cos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt{\arccos(ax)}\,dx = \int x^2\,\sqrt{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int(x^2*acos(a*x)^(1/2),x)
```

output
```
int(x^2*acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int x^2 \sqrt{\arccos(ax)}\,dx = \int \sqrt{acos\,(ax)}\,x^2dx$$

input
```
int(x^2*acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x))*x**2,x)
```

3.77 $\int x\sqrt{\arccos(ax)}\,dx$

Optimal result 596
Mathematica [A] (verified) 596
Rubi [A] (verified) 597
Maple [A] (verified) 598
Fricas [F(-2)] 599
Sympy [F] 599
Maxima [F(-2)] 599
Giac [C] (verification not implemented) 600
Mupad [F(-1)] 600
Reduce [F] 601

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x\sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{\arccos(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\arccos(ax)} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^2}$$

output
```
-1/4*arccos(a*x)^(1/2)/a^2+1/2*x^2*arccos(a*x)^(1/2)-1/8*Pi^(1/2)*FresnelC
(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x\sqrt{\arccos(ax)}\,dx = \frac{\frac{1}{4}\sqrt{\arccos(ax)}\cos(2\arccos(ax)) - \frac{1}{8}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

input
```
Integrate[x*Sqrt[ArcCos[a*x]],x]
```

output
```
((Sqrt[ArcCos[a*x]]*Cos[2*ArcCos[a*x]])/4 - (Sqrt[Pi]*FresnelC[(2*Sqrt[Arc
Cos[a*x]])/Sqrt[Pi]])/8)/a^2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5141, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{1}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{4a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}}{4a^2}$$

input
```
Int[x*Sqrt[ArcCos[a*x]],x]
```

output
```
(x^2*Sqrt[ArcCos[a*x]])/2 - (Sqrt[ArcCos[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/2)/(4*a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{-2\cos\left(2\arccos\left(ax\right)\right)\sqrt{\pi}\,\sqrt{\arccos\left(ax\right)}+\pi\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)}{8a^{2}\sqrt{\pi}}$	42

input
```
int(x*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^2/Pi^(1/2)*(-2*cos(2*arccos(a*x))*Pi^(1/2)*arccos(a*x)^(1/2)+Pi*Fre
snelC(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{\arccos(ax)}\,dx = \int x\sqrt{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x*acos(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int x\sqrt{\arccos(ax)}\,dx = \frac{(i+1)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{32\,a^2} - \frac{(i-1)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{32\,a^2} + \frac{\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{8\,a^2} + \frac{\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{8\,a^2}$$

input
```
integrate(x*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/32*I + 1/32)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^2 - (1/32*I - 1/
32)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^2 + 1/8*sqrt(arccos(a*x))*e
^(2*I*arccos(a*x))/a^2 + 1/8*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\arccos(ax)}\,dx = \int x\sqrt{\mathrm{acos}(a\,x)}\,dx$$

input
```
int(x*acos(a*x)^(1/2),x)
```

output
```
int(x*acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{\arccos(ax)}\,dx = \int \sqrt{acos\,(ax)}\,xdx$$

input
```
int(x*acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x))*x,x)
```

3.78 $\int \sqrt{\arccos(ax)}\,dx$

Optimal result 602
Mathematica [C] (verified) 602
Rubi [A] (verified) 603
Maple [A] (verified) 604
Fricas [F(-2)] 605
Sympy [F] 605
Maxima [F(-2)] 606
Giac [C] (verification not implemented) 606
Mupad [F(-1)] 607
Reduce [F] 607

Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \sqrt{\arccos(ax)}\,dx = x\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

output
```
x*arccos(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(
a*x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.57

$$\int \sqrt{\arccos(ax)}\,dx = \frac{i\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2},-i\arccos(ax)\right) - \sqrt{i\arccos(ax)}\Gamma\left(\frac{3}{2},i\arccos(ax)\right)\right)}{2a\sqrt{\arccos(ax)}}$$

input
```
Integrate[Sqrt[ArcCos[a*x]],x]
```

output
```
((I/2)*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] - Sqrt[I*ArcCo
s[a*x]]*Gamma[3/2, I*ArcCos[a*x]]))/(a*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5131, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{5131}$$

$$\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx + x\sqrt{\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$x\sqrt{\arccos(ax)} - \frac{\int \frac{ax}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{\arccos(ax)} - \frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a}$$

$$\Big\downarrow \text{3785}$$

$$x\sqrt{\arccos(ax)} - \frac{\int axd\sqrt{\arccos(ax)}}{a}$$

$$\Big\downarrow \text{3833}$$

$$x\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

input
```
Int[Sqrt[ArcCos[a*x]],x]
```

output
```
x*Sqrt[ArcCos[a*x]] - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/
a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2ax\arccos(ax)}{2a\sqrt{\arccos(ax)}}$	49

input
```
int(arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/arccos(a*x)^(1/2)*(-2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1
/2)/Pi^(1/2)*arccos(a*x)^(1/2))+2*a*x*arccos(a*x))
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\arccos(ax)}\,dx = \int \sqrt{\operatorname{acos}(ax)}\,dx$$

input
```
integrate(acos(a*x)**(1/2),x)
```

output
```
Integral(sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.89

$$\int \sqrt{\arccos(ax)}\,dx = \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{8a} - \frac{(i-1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{8a} + \frac{\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{2a} + \frac{\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{2a}$$

input
```
integrate(arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x))
)/a - (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arcco
s(a*x)))/a + 1/2*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a + 1/2*sqrt(arccos(a
*x))*e^(-I*arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\arccos(ax)}\,dx = \int \sqrt{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int(acos(a*x)^(1/2),x)
```

output
```
int(acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\arccos(ax)}\,dx = \int \sqrt{acos\,(ax)}dx$$

input
```
int(acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x)),x)
```

3.79 $\int \frac{\sqrt{\arccos(ax)}}{x} \, dx$

Optimal result 608
Mathematica [N/A] 608
Rubi [N/A] 609
Maple [N/A] 609
Fricas [F(-2)] 610
Sympy [N/A] 610
Maxima [F(-2)] 610
Giac [N/A] 611
Mupad [N/A] 611
Reduce [N/A] 611

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \text{Int}\left(\frac{\sqrt{\arccos(ax)}}{x}, x\right)$$

output
```
Defer(Int)(arccos(a*x)^(1/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \int \frac{\sqrt{\arccos(ax)}}{x} \, dx$$

input
```
Integrate[Sqrt[ArcCos[a*x]]/x,x]
```

output
```
Integrate[Sqrt[ArcCos[a*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{\sqrt{\arccos(ax)}}{x} dx$$

input
```
Int[Sqrt[ArcCos[a*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos (ax)}}{x} dx$$

input
```
int(arccos(a*x)^(1/2)/x,x)
```

output
```
int(arccos(a*x)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{acos}(ax)}}{x}\,dx$$

input
```
integrate(acos(a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(acos(a*x))/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \int \frac{\sqrt{\arccos\left(ax\right)}}{x}\,dx$$

input
```
integrate(arccos(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(arccos(a*x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{acos}\left(a\,x\right)}}{x}\,dx$$

input
```
int(acos(a*x)^(1/2)/x,x)
```

output
```
int(acos(a*x)^(1/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \int \frac{\sqrt{acos\left(ax\right)}}{x}dx$$

input
```
int(acos(a*x)^(1/2)/x,x)
```

output
```
int(sqrt(acos(a*x))/x,x)
```

3.80 $\int x^4 \arccos(ax)^{3/2}\,dx$

Optimal result 613
Mathematica [C] (verified) 614
Rubi [A] (verified) 614
Maple [A] (verified) 620
Fricas [F(-2)] 620
Sympy [F] 621
Maxima [F(-2)] 621
Giac [C] (verification not implemented) 621
Mupad [F(-1)] 622
Reduce [F] 622

Optimal result

Integrand size = 12, antiderivative size = 282

$$\int x^4 \arccos(ax)^{3/2}\,dx = -\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{11\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{400a^5} + \frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{25a^5} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{50a^5} + \frac{3\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5} + \frac{3\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5}$$

output
```
-4/25*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a^5-2/25*x^2*(-a^2*x^2+1)^(1/2)
*arccos(a*x)^(1/2)/a^3-3/50*x^4*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a+1/5
*x^5*arccos(a*x)^(3/2)+3/32*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arc
cos(a*x)^(1/2))/a^5+1/192*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arcco
s(a*x)^(1/2))/a^5+3/8000*10^(1/2)*Pi^(1/2)*FresnelS(10^(1/2)/Pi^(1/2)*arcc
os(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

$$\int x^4 \arccos(ax)^{3/2} \, dx =$$
$$-\frac{2250\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},-i\arccos(ax)\right)+\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},i\arccos(ax)\right)\right)+125\sqrt{3}\left(\sqrt{-i\arccos(a}\right.}{}$$

input
```
Integrate[x^4*ArcCos[a*x]^(3/2),x]
```

output
```
-1/36000*(2250*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + Sqrt
[I*ArcCos[a*x]]*Gamma[5/2, I*ArcCos[a*x]]) + 125*Sqrt[3]*(Sqrt[(-I)*ArcCos
[a*x]]*Gamma[5/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (3*
I)*ArcCos[a*x]]) + 9*Sqrt[5]*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-5*I)*Arc
Cos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (5*I)*ArcCos[a*x]]))/(a^5*Sqrt[
ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {5141, 5211, 5147, 4906, 2009, 5211, 5147, 4906, 2009, 5183, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arccos(ax)^{3/2} dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{3}{10}a\int \frac{x^5\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{10}a\left(\frac{4\int \frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{\int \frac{x^4}{\sqrt{\arccos(ax)}}dx}{10a}-\frac{x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{5a^2}\right)+$$
$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5147}$$

$$\frac{3}{10}a\left(\frac{4\int \frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\int \frac{a^4x^4\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{10a^6}-\frac{x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{5a^2}\right)+$$
$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3}{10}a\left(\frac{4\int \frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\int \left(\frac{3\sin(3\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\sin(5\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{10a^6}-\frac{x^4\sqrt{1-a^2x^2}\sqrt{\mathrm{ar}}}{5a^2}\right.$$
$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{10}a\left(\frac{4\int \frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{5a^2}+\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{10}}\mathrm{F}}{10a^6}\right.$$
$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{6a}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)}{5a^2}+\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{3\pi}{2}}}{}\right.$$
$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5147}$$

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\int\frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int\frac{a^2x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)}{5a^2}+\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\downarrow$ 4906

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\int\frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int\left(\frac{\sin(3\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)}{5a^2}+\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{}\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\downarrow$ 2009

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\int\frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)}{5a^2}+\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\downarrow$ 5183

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\left(-\frac{\int\frac{1}{\sqrt{\arccos(ax)}}dx}{2a}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4}-\frac{x^2\sqrt{1}}{}\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\downarrow$ 5135

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\left(\frac{\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4} -\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\big\downarrow$ 3042

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\left(\frac{\int \frac{\sin(\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4}\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\big\downarrow$ 3786

$$\frac{3}{10}a\left(\frac{4\left(\frac{2\left(\frac{\int \sqrt{1-a^2x^2}d\sqrt{\arccos(ax)}}{a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4} -\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

$\big\downarrow$ 3832

$$\frac{3}{10}a\left(\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos\right.\right.}{10a^6}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{3/2}$$

input
```
Int[x^4*ArcCos[a*x]^(3/2),x]
```

output
```
(x^5*ArcCos[a*x]^(3/2))/5 + (3*a*(-1/5*(x^4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[
a*x]])/a^2 + (4*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2 + (2*(
-((Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2) + (Sqrt[Pi/2]*FresnelS[Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/a^2))/(3*a^2) + ((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*
Sqrt[ArcCos[a*x]]])/2 + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]]
)/2)/(6*a^4)))/(5*a^2) + ((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]
]])/4 + (Sqrt[(3*Pi)/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sqrt[
Pi/10]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/8)/(10*a^6)))/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.68

method	result
default	$\frac{3000ax\arccos(ax)^2+125\ \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}+9\ \text{FresnelS}\left(\frac{\sqrt{2}\sqrt{5}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{5}\sqrt{2}\sqrt{\pi}\sqrt{\text{arc}}}{}$

input
```
int(x^4*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24000/a^5*(3000*a*x*arccos(a*x)^2+125*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*
arccos(a*x)^(1/2))*3^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)+9*FresnelS(2
^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))*5^(1/2)*2^(1/2)*Pi^(1/2)*arccos
(a*x)^(1/2)+1500*arccos(a*x)^2*cos(3*arccos(a*x))+300*arccos(a*x)^2*cos(5*
arccos(a*x))+2250*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^
(1/2)*arccos(a*x)^(1/2)-750*arccos(a*x)*sin(3*arccos(a*x))-90*arccos(a*x)*
sin(5*arccos(a*x))-4500*arccos(a*x)*(-a^2*x^2+1)^(1/2))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4 \arccos(ax)^{3/2}\,dx = \int x^4 \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(x**4*acos(a*x)**(3/2),x)
```

output
```
Integral(x**4*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.26

$$\int x^4 \arccos(ax)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
1/160*arccos(a*x)^(3/2)*e^(5*I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(3/2)*e
^(3*I*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a^5 + 1/
16*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(3/2)*e^(-3
*I*arccos(a*x))/a^5 + 1/160*arccos(a*x)^(3/2)*e^(-5*I*arccos(a*x))/a^5 + (
3/32000*I - 3/32000)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt(arc
cos(a*x)))/a^5 - (3/32000*I + 3/32000)*sqrt(10)*sqrt(pi)*erf(-(1/2*I + 1/2
)*sqrt(10)*sqrt(arccos(a*x)))/a^5 + (1/768*I - 1/768)*sqrt(6)*sqrt(pi)*erf
((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (1/768*I + 1/768)*sqrt(6)*
sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (3/128*I - 3/
128)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 - (
3/128*I + 3/128)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a
*x)))/a^5 + 3/1600*I*sqrt(arccos(a*x))*e^(5*I*arccos(a*x))/a^5 + 1/64*I*sq
rt(arccos(a*x))*e^(3*I*arccos(a*x))/a^5 + 3/32*I*sqrt(arccos(a*x))*e^(I*ar
ccos(a*x))/a^5 - 3/32*I*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a^5 - 1/64*I*
sqrt(arccos(a*x))*e^(-3*I*arccos(a*x))/a^5 - 3/1600*I*sqrt(arccos(a*x))*e^
(-5*I*arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^{3/2}\,dx = \int x^4\,\mathrm{acos}(a\,x)^{3/2}\,dx$$

input
```
int(x^4*acos(a*x)^(3/2),x)
```

output
```
int(x^4*acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int x^4 \arccos(ax)^{3/2}\,dx = \int \sqrt{acos\,(ax)}\;acos(ax)\,x^4dx$$

input
```
int(x^4*acos(a*x)^(3/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)*x**4,x)
```

3.81 $\int x^3 \arccos(ax)^{3/2}\,dx$

Optimal result 623
Mathematica [C] (verified) 624
Rubi [A] (verified) 624
Maple [A] (verified) 629
Fricas [F(-2)] 629
Sympy [F] 630
Maxima [F(-2)] 630
Giac [C] (verification not implemented) 631
Mupad [F(-1)] 632
Reduce [F] 632

Optimal result

Integrand size = 12, antiderivative size = 157

$$\int x^3 \arccos(ax)^{3/2}\,dx = -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a} - \frac{3\arccos(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{512a^4} + \frac{3\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{64a^4}$$

output

```
-9/64*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a^3-3/32*x^3*(-a^2*x^2+1)^(1/
2)*arccos(a*x)^(1/2)/a-3/32*arccos(a*x)^(3/2)/a^4+1/4*x^4*arccos(a*x)^(3/2
)+3/1024*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
^4+3/64*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

$$\int x^3 \arccos(ax)^{3/2} \, dx =$$
$$-\frac{8\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},-2i\arccos(ax)\right)+8\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},2i\arccos(ax)\right)+\sqrt{-i\arccos(ax)}\Gamma(}{512a^4\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^3*ArcCos[a*x]^(3/2),x]
```

output
```
-1/512*(8*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-2*I)*ArcCos[a*x]] +
8*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (2*I)*ArcCos[a*x]] + Sqrt[(-I)*Ar
cCos[a*x]]*Gamma[5/2, (-4*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2,
 (4*I)*ArcCos[a*x]])/(a^4*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5141, 5211, 5147, 4906, 2009, 5211, 5147, 4906, 27, 3042, 3786, 3832, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax)^{3/2} \, dx$$
$$\Big\downarrow \text{5141}$$
$$\frac{3}{8}a\int \frac{x^4\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)^{3/2}$$
$$\Big\downarrow \text{5211}$$
$$\frac{3}{8}a\left(\frac{3\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int \frac{x^3}{\sqrt{\arccos(ax)}}dx}{8a}-\frac{x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$\Big\downarrow$ 5147

$$\frac{3}{8}a\left(\frac{3\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\int\frac{a^3x^3\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$\Big\downarrow$ 4906

$$\frac{3}{8}a\left(\frac{\int\left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{8a^5}+\frac{3\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$\Big\downarrow$ 2009

$$\frac{3}{8}a\left(\frac{3\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\sqrt{}}{4a^2}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$\Big\downarrow$ 5211

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\arccos(ax)}}dx}{4a}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\text{Fre}}{8a^5}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$\Big\downarrow$ 5147

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int\frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}}{8a^5}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$\Big\downarrow$ 4906

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{3786}$$

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right.}{8a}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{3}{8}a\left(\frac{3\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)}{4a^2}+\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-}{8a^5}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{3}{8}a\left(\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{4a^2}+\frac{3\left(\sqrt{\pi}\right.}{}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{3/2}$$

input
```
Int[x^3*ArcCos[a*x]^(3/2),x]
```

output
```
(x^4*ArcCos[a*x]^(3/2))/4 + (3*a*(-1/4*(x^3*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[
a*x]])/a^2 + ((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/8 + (S
qrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/4)/(8*a^5) + (3*(-1/2*(x
*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2 - ArcCos[a*x]^(3/2)/(3*a^3) + (S
qrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a^3)))/(4*a^2)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.77

method	result
default	$\frac{3\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}+128\cos(2\arccos(ax))\arccos(ax)^2+32\arccos(ax)^2\cos(4\arccos(ax))+48\ \mathrm{Fresn}}{1024a^4\sqrt{\arccos(ax)}}$

input
```
int(x^3*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1024/a^4*(3*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)*arccos(a*x)^(1/2)+128*cos(2*arccos(a*x))*arccos(a*x)^2+32*arccos(a*x)^
2*cos(4*arccos(a*x))+48*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)*ar
ccos(a*x)^(1/2)-96*sin(2*arccos(a*x))*arccos(a*x)-12*arccos(a*x)*sin(4*arc
cos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \arccos(ax)^{3/2} \, dx = \int x^3 \operatorname{acos}^{\frac{3}{2}}(ax) \, dx$$

input
```
integrate(x**3*acos(a*x)**(3/2),x)
```

output
```
Integral(x**3*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.43

$$\int x^3 \arccos(ax)^{3/2}\,dx = \frac{\arccos{(ax)}^{\frac{3}{2}}e^{(4i \arccos(ax))}}{64\,a^4} + \frac{\arccos{(ax)}^{\frac{3}{2}}e^{(2i \arccos(ax))}}{16\,a^4} + \frac{\arccos{(ax)}^{\frac{3}{2}}e^{(-2i \arccos(ax))}}{16\,a^4} + \frac{\arccos{(ax)}^{\frac{3}{2}}e^{(-4i \arccos(ax))}}{64\,a^4} + \frac{(3i-3)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\,\sqrt{2}\sqrt{\arccos{(ax)}}\right)}{4096\,a^4} - \frac{(3i+3)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\,\sqrt{2}\sqrt{\arccos{(ax)}}\right)}{4096\,a^4} + \frac{(3i-3)\,\sqrt{\pi}\,\mathrm{erf}\left((i-1)\,\sqrt{\arccos{(ax)}}\right)}{256\,a^4} - \frac{(3i+3)\,\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\,\sqrt{\arccos{(ax)}}\right)}{256\,a^4} + \frac{3i\,\sqrt{\arccos{(ax)}}e^{(4i \arccos(ax))}}{512\,a^4} + \frac{3i\,\sqrt{\arccos{(ax)}}e^{(2i \arccos(ax))}}{64\,a^4} - \frac{3i\,\sqrt{\arccos{(ax)}}e^{(-2i \arccos(ax))}}{64\,a^4} - \frac{3i\,\sqrt{\arccos{(ax)}}e^{(-4i \arccos(ax))}}{512\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
1/64*arccos(a*x)^(3/2)*e^(4*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(3/2)*e^
(2*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(3/2)*e^(-2*I*arccos(a*x))/a^4 +
1/64*arccos(a*x)^(3/2)*e^(-4*I*arccos(a*x))/a^4 + (3/4096*I - 3/4096)*sqrt
(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 - (3/4096*I + 3/40
96)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 + (3/256*
I - 3/256)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 - (3/256*I + 3/256)
*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4 + 3/512*I*sqrt(arccos(a*x))*
e^(4*I*arccos(a*x))/a^4 + 3/64*I*sqrt(arccos(a*x))*e^(2*I*arccos(a*x))/a^4
 - 3/64*I*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^4 - 3/512*I*sqrt(arccos
(a*x))*e^(-4*I*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^{3/2}\,dx = \int x^3 \operatorname{acos}(a\,x)^{3/2}\,dx$$

input
```
int(x^3*acos(a*x)^(3/2),x)
```

output
```
int(x^3*acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int x^3 \arccos(ax)^{3/2}\,dx = \int \sqrt{acos\,(ax)}\,acos(ax)\,x^3dx$$

input
```
int(x^3*acos(a*x)^(3/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)*x**3,x)
```

3.82 $\int x^2 \arccos(ax)^{3/2}\, dx$

Optimal result . . . 633
Mathematica [C] (verified) . . . 634
Rubi [A] (verified) . . . 634
Maple [A] (verified) . . . 638
Fricas [F(-2)] . . . 639
Sympy [F] . . . 639
Maxima [F(-2)] . . . 639
Giac [C] (verification not implemented) . . . 640
Mupad [F(-1)] . . . 641
Reduce [F] . . . 641

Optimal result

Integrand size = 12, antiderivative size = 147

$$\int x^2 \arccos(ax)^{3/2}\, dx = -\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a} + \frac{1}{3}x^3\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{24a^3}$$

output

```
-1/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a^3-1/6*x^2*(-a^2*x^2+1)^(1/2)*a
rccos(a*x)^(1/2)/a+1/3*x^3*arccos(a*x)^(3/2)+3/16*2^(1/2)*Pi^(1/2)*Fresnel
S(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3+1/144*6^(1/2)*Pi^(1/2)*FresnelS(
6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int x^2 \arccos(ax)^{3/2}\,dx = -\frac{27\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},-i\arccos(ax)\right)+27\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},i\arccos(ax)\right)+\sqrt{3}\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2}\right.\right.}{216a^3\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^2*ArcCos[a*x]^(3/2),x]
```

output
```
-1/216*(27*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + 27*Sqrt[I
*ArcCos[a*x]]*Gamma[5/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*
Gamma[5/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (3*I)*ArcC
os[a*x]]))/(a^3*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5141, 5211, 5147, 4906, 2009, 5183, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{1}{2}a\int \frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{2}a\left(\frac{2\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{6a}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \; 5147$$

$$\frac{1}{2}a\left(\frac{2\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \frac{a^2x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \; 4906$$

$$\frac{1}{2}a\left(\frac{2\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\int \left(\frac{\sin(3\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{6a^4}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}a\left(\frac{2\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4}-\frac{x^2\sqrt{1-a^2}\cdots}{\cdots}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \; 5183$$

$$\frac{1}{2}a\left(\frac{2\left(-\frac{\int \frac{1}{\sqrt{\arccos(ax)}}dx}{2a}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\text{arc}\cdots}\right.}{6a^4}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \; 5135$$

$$\frac{1}{2}a\left(\frac{2\left(\frac{\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\cdots}\right.}{6a^4}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}a\left(\frac{2\left(\frac{\int \frac{\sin(\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\right.}{6a^4}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$\big\downarrow$ 3786

$$\frac{1}{2}a\left(\frac{2\left(\frac{\int\sqrt{1-a^2x^2}d\sqrt{\arccos(ax)}}{a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)}{3a^2}+\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\right.}{6a^4}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{3/2}$$

$\big\downarrow$ 3832

$$\frac{1}{2}a\left(\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^4}+\frac{2\left(\frac{\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}-\right.}{3a^2}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{3/2}$$

input
```
Int[x^2*ArcCos[a*x]^(3/2),x]
```

output
```
(x^3*ArcCos[a*x]^(3/2))/3 + (a*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*
x]])/a^2 + (2*(-((Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2) + (Sqrt[Pi/2]*
FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^2))/(3*a^2) + ((Sqrt[Pi/2]*Fresn
elS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/2 + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqr
t[ArcCos[a*x]]])/2)/(6*a^4)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

method	result
default	$\frac{\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}+36ax\arccos(ax)^2+27\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}+12}{144a^3\sqrt{\arccos(ax)}}$

input
```
int(x^2*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/144/a^3*(FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))*3^(1/2)*2^
(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)+36*a*x*arccos(a*x)^2+27*FresnelS(2^(1/2)/
Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)+12*arccos(a
*x)^2*cos(3*arccos(a*x))-54*arccos(a*x)*(-a^2*x^2+1)^(1/2)-6*arccos(a*x)*s
in(3*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \arccos(ax)^{3/2} \, dx = \int x^2 \operatorname{acos}^{\frac{3}{2}}(ax) \, dx$$

input
```
integrate(x**2*acos(a*x)**(3/2),x)
```

output
```
Integral(x**2*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.61

$$\int x^2 \arccos(ax)^{3/2}\, dx = \frac{\arccos(ax)^{\frac{3}{2}} e^{(3i \arccos(ax))}}{24 a^3} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(i \arccos(ax))}}{8 a^3} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(-i \arccos(ax))}}{8 a^3} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(-3i \arccos(ax))}}{24 a^3} + \frac{(i-1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{576 a^3} - \frac{(i+1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{576 a^3} + \frac{(3i-3)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64 a^3} - \frac{(3i+3)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64 a^3} + \frac{i\sqrt{\arccos(ax)}e^{(3i \arccos(ax))}}{48 a^3} + \frac{3i\sqrt{\arccos(ax)}e^{(i \arccos(ax))}}{16 a^3} - \frac{3i\sqrt{\arccos(ax)}e^{(-i \arccos(ax))}}{16 a^3} - \frac{i\sqrt{\arccos(ax)}e^{(-3i \arccos(ax))}}{48 a^3}$$

input
```
integrate(x^2*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
1/24*arccos(a*x)^(3/2)*e^(3*I*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(3/2)*e^(
I*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^3 + 1/24*a
rccos(a*x)^(3/2)*e^(-3*I*arccos(a*x))/a^3 + (1/576*I - 1/576)*sqrt(6)*sqrt
(pi)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 - (1/576*I + 1/576)*
sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 + (3/64
*I - 3/64)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a
^3 - (3/64*I + 3/64)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arcc
os(a*x)))/a^3 + 1/48*I*sqrt(arccos(a*x))*e^(3*I*arccos(a*x))/a^3 + 3/16*I*
sqrt(arccos(a*x))*e^(I*arccos(a*x))/a^3 - 3/16*I*sqrt(arccos(a*x))*e^(-I*a
rccos(a*x))/a^3 - 1/48*I*sqrt(arccos(a*x))*e^(-3*I*arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^{3/2}\,dx = \int x^2\,\mathrm{acos}(a\,x)^{3/2}\,dx$$

input
```
int(x^2*acos(a*x)^(3/2),x)
```

output
```
int(x^2*acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int x^2 \arccos(ax)^{3/2}\,dx = \int \sqrt{acos\,(ax)}\,acos(ax)\,x^2dx$$

input
```
int(x^2*acos(a*x)^(3/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)*x**2,x)
```

3.83 $\int x \arccos(ax)^{3/2} \, dx$

Optimal result . . . 642
Mathematica [A] (verified) . . . 642
Rubi [A] (verified) . . . 643
Maple [A] (verified) . . . 646
Fricas [F(-2)] . . . 646
Sympy [F] . . . 647
Maxima [F(-2)] . . . 647
Giac [C] (verification not implemented) . . . 647
Mupad [F(-1)] . . . 648
Reduce [F] . . . 648

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int x \arccos(ax)^{3/2} \, dx = -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2 \arccos(ax)^{3/2} + \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{32a^2}$$

output
```
-3/8*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a-1/4*arccos(a*x)^(3/2)/a^2+1/
2*x^2*arccos(a*x)^(3/2)+3/32*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2
))/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int x \arccos(ax)^{3/2} \, dx = \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - 2\sqrt{\arccos(ax)}(-4\arccos(ax)\cos(2\arccos(ax)) + 3}{32a^2}$$

input
```
Integrate[x*ArcCos[a*x]^(3/2),x]
```

output
```
(3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*Sqrt[ArcCos[a*x]]
*(-4*ArcCos[a*x]*Cos[2*ArcCos[a*x]] + 3*Sin[2*ArcCos[a*x]]))/(32*a^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5141, 5211, 5147, 4906, 27, 3042, 3786, 3832, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax)^{3/2} \, dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{4}a\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{\arccos(ax)}}dx}{4a}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5147}$$

$$\frac{3}{4}a\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$\Big\downarrow$ 3042

$$\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$\Big\downarrow$ 3786

$$\frac{3}{4}a\left(\frac{\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$\Big\downarrow$ 3832

$$\frac{3}{4}a\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

$\Big\downarrow$ 5153

$$\frac{3}{4}a\left(\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\frac{\arccos(ax)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}$$

input
```
Int[x*ArcCos[a*x]^(3/2),x]
```

output
```
(x^2*ArcCos[a*x]^(3/2))/2 + (3*a*(-1/2*(x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*
x]])/a^2 - ArcCos[a*x]^(3/2)/(3*a^3) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a
*x]])/Sqrt[Pi]])/(8*a^3)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{8 \cos(2 \arccos(ax)) \arccos(ax)^2+3 \text{ FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}\sqrt{\arccos(ax)}-6 \sin(2 \arccos(ax)) \arccos(ax)}{32a^2\sqrt{\arccos(ax)}}$	64

input
```
int(x*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/a^2*(8*cos(2*arccos(a*x))*arccos(a*x)^2+3*FresnelS(2*arccos(a*x)^(1/2
)/Pi^(1/2))*Pi^(1/2)*arccos(a*x)^(1/2)-6*sin(2*arccos(a*x))*arccos(a*x))/a
rccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\arccos(ax)^{3/2}\,dx=\int x\operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(x*acos(a*x)**(3/2),x)
```

output
```
Integral(x*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\arccos(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\begin{aligned}\int x\arccos(ax)^{3/2}\,dx&=\frac{\arccos(ax)^{\frac{3}{2}}e^{(2i\arccos(ax))}}{8a^2}\\&+\frac{\arccos(ax)^{\frac{3}{2}}e^{(-2i\arccos(ax))}}{8a^2}+\frac{(3i-3)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{128a^2}\\&-\frac{(3i+3)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{128a^2}\\&+\frac{3i\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{32a^2}-\frac{3i\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{32a^2}\end{aligned}$$

input
```
integrate(x*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
1/8*arccos(a*x)^(3/2)*e^(2*I*arccos(a*x))/a^2 + 1/8*arccos(a*x)^(3/2)*e^(-
2*I*arccos(a*x))/a^2 + (3/128*I - 3/128)*sqrt(pi)*erf((I - 1)*sqrt(arccos(
a*x)))/a^2 - (3/128*I + 3/128)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^
2 + 3/32*I*sqrt(arccos(a*x))*e^(2*I*arccos(a*x))/a^2 - 3/32*I*sqrt(arccos(
a*x))*e^(-2*I*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x\arccos(ax)^{3/2}\,dx=\int x\,\mathrm{acos}(a\,x)^{3/2}\,dx$$

input
```
int(x*acos(a*x)^(3/2),x)
```

output
```
int(x*acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int x\arccos(ax)^{3/2}\,dx=\int \sqrt{acos\,(ax)}\,acos(ax)\,xdx$$

input
```
int(x*acos(a*x)^(3/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)*x,x)
```

3.84 $\int \arccos(ax)^{3/2}\,dx$

Optimal result 649
Mathematica [C] (verified) 649
Rubi [A] (verified) 650
Maple [A] (verified) 652
Fricas [F(-2)] 652
Sympy [F] 653
Maxima [F(-2)] 653
Giac [C] (verification not implemented) 653
Mupad [F(-1)] 654
Reduce [F] 654

Optimal result

Integrand size = 8, antiderivative size = 75

$$\int \arccos(ax)^{3/2}\,dx = -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + x\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a}$$

output
```
-3/2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a+x*arccos(a*x)^(3/2)+3/4*2^(1/2
)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \arccos(ax)^{3/2}\,dx = -\frac{\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{5}{2},-i\arccos(ax)\right)+\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{5}{2},i\arccos(ax)\right)}{2a\sqrt{\arccos(ax)}}$$

input
```
Integrate[ArcCos[a*x]^(3/2),x]
```

output
```
-1/2*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + Sqrt[I*ArcCos[
a*x]]*Gamma[5/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5131, 5183, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{3/2}\,dx$$

$$\Big\downarrow \text{5131}$$

$$\frac{3}{2}a\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{3}{2}a\left(-\frac{\int \frac{1}{\sqrt{\arccos(ax)}}dx}{2a}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)+x\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{5135}$$

$$\frac{3}{2}a\left(\frac{\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)+x\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{\int \frac{\sin(\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)+x\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{3786}$$

$$\frac{3}{2}a\left(\frac{\int \sqrt{1-a^2x^2}d\sqrt{\arccos(ax)}}{a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)+x\arccos(ax)^{3/2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{3}{2}a\left(\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a^2}\right)+x\arccos(ax)^{3/2}$$

input
```
Int[ArcCos[a*x]^(3/2),x]
```

output
```
x*ArcCos[a*x]^(3/2) + (3*a*(-((Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2) +
 (Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^2))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{2}\left(2 \arccos(ax)^{\frac{3}{2}} \sqrt{2}\sqrt{\pi}\, ax-3\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}\sqrt{-a^{2}x^{2}+1}+3\pi \operatorname{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{4a\sqrt{\pi}}$	72

input
```
int(arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*2^(1/2)/Pi^(1/2)*(2*arccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a*x-3*2^(1/2)
*Pi^(1/2)*arccos(a*x)^(1/2)*(-a^2*x^2+1)^(1/2)+3*Pi*FresnelS(2^(1/2)/Pi^(1
/2)*arccos(a*x)^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \arccos(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \arccos(ax)^{3/2}\,dx = \int \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(acos(a*x)**(3/2),x)
```

output
```
Integral(acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \arccos(ax)^{3/2}\,dx = \frac{\arccos(ax)^{\frac{3}{2}} e^{(i\arccos(ax))}}{2a} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(-i\arccos(ax))}}{2a} + \frac{(3i-3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{16a} - \frac{(3i+3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{16a} + \frac{3i\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{4a} - \frac{3i\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{4a}$$

input
```
integrate(arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
1/2*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a + 1/2*arccos(a*x)^(3/2)*e^(-I*ar
ccos(a*x))/a + (3/16*I - 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*
sqrt(arccos(a*x)))/a - (3/16*I + 3/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)
*sqrt(2)*sqrt(arccos(a*x)))/a + 3/4*I*sqrt(arccos(a*x))*e^(I*arccos(a*x))/
a - 3/4*I*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(ax)^{3/2}\,dx = \int \mathrm{acos}(a\,x)^{3/2}\,dx$$

input
```
int(acos(a*x)^(3/2),x)
```

output
```
int(acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \arccos(ax)^{3/2}\,dx = \sqrt{acos\left(ax\right)}\,acos(ax)\,x - \frac{3\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{a^2x^2-1}dx\right)a}{2}$$

input
```
int(acos(a*x)^(3/2),x)
```

output
```
(2*sqrt(acos(a*x))*acos(a*x)*x - 3*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a
*x))*x)/(a**2*x**2 - 1),x)*a)/2
```

3.85 $\int \frac{\arccos(ax)^{3/2}}{x}\,dx$

Optimal result . . . 655
Mathematica [N/A] . . . 655
Rubi [N/A] . . . 656
Maple [N/A] . . . 656
Fricas [F(-2)] . . . 657
Sympy [N/A] . . . 657
Maxima [F(-2)] . . . 657
Giac [N/A] . . . 658
Mupad [N/A] . . . 658
Reduce [N/A] . . . 658

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \mathrm{Int}\left(\frac{\arccos(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)(arccos(a*x)^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \int \frac{\arccos(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[ArcCos[a*x]^(3/2)/x,x]
```

output
```
Integrate[ArcCos[a*x]^(3/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^{3/2}}{x} \, dx$$
$$\big\downarrow \text{5149}$$
$$\int \frac{\arccos(ax)^{3/2}}{x} dx$$

input
```
Int[ArcCos[a*x]^(3/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos \left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int(arccos(a*x)^(3/2)/x,x)
```

output
```
int(arccos(a*x)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{acos}^{\frac{3}{2}}(ax)}{x}\,dx$$

input
```
integrate(acos(a*x)**(3/2)/x,x)
```

output
```
Integral(acos(a*x)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{3/2}}{x} \, dx = \int \frac{\arccos(ax)^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate(arccos(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{3/2}}{x} \, dx = \int \frac{\mathrm{acos}(a\,x)^{3/2}}{x} \, dx$$

input
```
int(acos(a*x)^(3/2)/x,x)
```

output
```
int(acos(a*x)^(3/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\arccos(ax)^{3/2}}{x} \, dx = \int \frac{\sqrt{acos(ax)}\, acos(ax)}{x} dx$$

input
```
int(acos(a*x)^(3/2)/x,x)
```

output

```
int((sqrt(acos(a*x))*acos(a*x))/x,x)
```

3.86 $\int x^4 \arccos(ax)^{5/2}\,dx$

Optimal result 660
Mathematica [C] (verified) 661
Rubi [A] (verified) 661
Maple [A] (verified) 668
Fricas [F(-2)] 669
Sympy [F(-1)] 669
Maxima [F(-2)] 669
Giac [C] (verification not implemented) 670
Mupad [F(-1)] 671
Reduce [F] 671

Optimal result

Integrand size = 12, antiderivative size = 298

$$\int x^4 \arccos(ax)^{5/2}\,dx = -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{60a^5} + \sqrt{\frac{3\pi}{2}}\,\text{Fresn}$$

output

```
-2/5*x*arccos(a*x)^(1/2)/a^4-1/15*x^3*arccos(a*x)^(1/2)/a^2-3/100*x^5*arcc
os(a*x)^(1/2)-4/15*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a^5-2/15*x^2*(-a^2
*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a^3-1/10*x^4*(-a^2*x^2+1)^(1/2)*arccos(a*x
)^(3/2)/a+1/5*x^5*arccos(a*x)^(5/2)+15/64*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2
)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5+5/1152*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)
/Pi^(1/2)*arccos(a*x)^(1/2))/a^5+3/16000*10^(1/2)*Pi^(1/2)*FresnelC(10^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.65

$$\int x^4 \arccos(ax)^{5/2} \, dx =$$

$$-\frac{i\left(33750\sqrt{-i\arccos(ax)}\Gamma\left(\frac{7}{2},-i\arccos(ax)\right)-33750\sqrt{i\arccos(ax)}\Gamma\left(\frac{7}{2},i\arccos(ax)\right)+625\sqrt{3}\sqrt{-i\arccos(ax)}\right.}{}$$

input
```
Integrate[x^4*ArcCos[a*x]^(5/2),x]
```

output
```
((-1/540000*I)*(33750*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]]
- 33750*Sqrt[I*ArcCos[a*x]]*Gamma[7/2, I*ArcCos[a*x]] + 625*Sqrt[3]*Sqrt[(
-I)*ArcCos[a*x]]*Gamma[7/2, (-3*I)*ArcCos[a*x]] - 625*Sqrt[3]*Sqrt[I*ArcCo
s[a*x]]*Gamma[7/2, (3*I)*ArcCos[a*x]] + 27*Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*
Gamma[7/2, (-5*I)*ArcCos[a*x]] - 27*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[7/2,
 (5*I)*ArcCos[a*x]]))/(a^5*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 2.64 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.35, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5141, 5211, 5141, 5211, 5141, 5183, 5131, 5225, 3042, 3785, 3793, 2009, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arccos(ax)^{5/2} dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{1}{2}a \int \frac{x^5 \arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx + \frac{1}{5}x^5 \arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{2}a\left(\frac{4\int \frac{x^3\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{3\int x^4\sqrt{\arccos(ax)}dx}{10a}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{5a^2}\right)+\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$\big\downarrow$ 5141

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{10}a\int\frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{5}x^5\sqrt{\arccos(ax)}\right)}{10a}+\frac{4\int\frac{x^3\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{5a^2}-\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{5a^2}\right)\cdots$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$\big\downarrow$ 5211

$$\frac{1}{2}a\left(\frac{4\left(\frac{2\int\frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2\sqrt{\arccos(ax)}dx}{2a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{3a^2}\right)}{5a^2}-\frac{3\left(\frac{1}{10}a\int\frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{5}x^5\sqrt{\cdots}\right.}{10a}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$\big\downarrow$ 5141

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{10}a\int\frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{5}x^5\sqrt{\arccos(ax)}\right)}{10a}+\frac{4\left(\frac{2\int\frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\frac{1}{6}a\int\frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{3}x^3\sqrt{\cdots}}{2a}\cdots\right.}{5a^2}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$\big\downarrow$ 5183

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{10}a\int\frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{5}x^5\sqrt{\arccos(ax)}\right)}{10a}+\frac{4\left(\frac{2\left(-\frac{3\int\sqrt{\arccos(ax)}dx}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)}{3a^2}-\frac{\frac{1}{6}a\int\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5131}$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{10}a\int \frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{5}x^5\sqrt{\arccos(ax)}\right)}{10a}+\frac{4\left(\frac{2\left(-\frac{3\left(\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+x\sqrt{\arccos(ax)}\right)}{2a}-\frac{\sqrt{1-a^2x}\cdots}{}\right.}{3a^2}\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5225}$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int \frac{a^5x^5}{\sqrt{\arccos(ax)}}d\arccos(ax)}{10a^5}\right)}{10a}+\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int \frac{a^3x^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}}{2a}+\frac{2\left(-\frac{3\left(x\sqrt{\arccos(\cdots}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^5}{\sqrt{\arccos(ax)}}d\arccos(ax)}{10a^5}\right)}{10a}+\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}}{2a}+- \right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$\downarrow$ 3785

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^5}{\sqrt{\arccos(ax)}}d\arccos(ax)}{10a^5}\right)}{10a}+\frac{4\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}}{2a}+- \right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$\downarrow$ 3793

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\int\left(\frac{5ax}{8\sqrt{\arccos(ax)}}+\frac{5\cos(3\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\cos(5\arccos(ax))}{16\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{10a^5}\right)}{10a}+\frac{4\left(-\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int\left(\frac{}{4\sqrt{}}\right.}{}\right.}{}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}a\left(\frac{4\left(\frac{2\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int ax\,d\sqrt{\arccos(ax)}}{a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)}{3a^2}-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{Fres}}{6a^3}}{2a}\right.}{5a^2}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 3833$$

$$\frac{1}{2}a\left(-\frac{3\left(\frac{1}{5}x^5\sqrt{\arccos(ax)}-\frac{\frac{5}{4}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{5}{8}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos\right.}\right.}{10a^5}\right.}{10a}\right.$$

$$\frac{1}{5}x^5\arccos(ax)^{5/2}$$

input
```
Int[x^4*ArcCos[a*x]^(5/2),x]
```

output
```
(x^5*ArcCos[a*x]^(5/2))/5 + (a*(-1/5*(x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3
/2))/a^2 + (4*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/a^2 + (2*(-(
(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/a^2) - (3*(x*Sqrt[ArcCos[a*x]] - (Sq
rt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a))/(2*a)))/(3*a^2) - ((x
^3*Sqrt[ArcCos[a*x]])/3 - ((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a
*x]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/2)/(6*a^3))
/(2*a)))/(5*a^2) - (3*((x^5*Sqrt[ArcCos[a*x]])/5 - ((5*Sqrt[Pi/2]*FresnelC
[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/4 + (5*Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt
[ArcCos[a*x]]])/8 + (Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/
8)/(10*a^5)))/(10*a)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

method	result
default	$\frac{27\sqrt{5}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+18000ax\arccos(ax)^3+625\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{a}}{\sqrt{}}\right.}{}$

input
```
int(x^4*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/144000/a^5*(27*5^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))+18000*a*x*arccos(a*x)^3+625*3^(1/2)
*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arcc
os(a*x)^(1/2))+9000*arccos(a*x)^3*cos(3*arccos(a*x))+1800*arccos(a*x)^3*co
s(5*arccos(a*x))+33750*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)
/Pi^(1/2)*arccos(a*x)^(1/2))-45000*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-7500*a
rccos(a*x)^2*sin(3*arccos(a*x))-900*arccos(a*x)^2*sin(5*arccos(a*x))-67500
*a*x*arccos(a*x)-3750*arccos(a*x)*cos(3*arccos(a*x))-270*arccos(a*x)*cos(5
*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{5/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^{5/2}\, dx = \text{Timed out}$$

input
```
integrate(x**4*acos(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{5/2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.55

$$\int x^4 \arccos(ax)^{5/2} dx = \text{Too large to display}$$

input
```
integrate(x^4*arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
1/160*arccos(a*x)^(5/2)*e^(5*I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(5/2)*e
^(3*I*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(5/2)*e^(I*arccos(a*x))/a^5 + 1/
16*arccos(a*x)^(5/2)*e^(-I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(5/2)*e^(-3
*I*arccos(a*x))/a^5 + 1/160*arccos(a*x)^(5/2)*e^(-5*I*arccos(a*x))/a^5 + 1
/320*I*arccos(a*x)^(3/2)*e^(5*I*arccos(a*x))/a^5 + 5/192*I*arccos(a*x)^(3/
2)*e^(3*I*arccos(a*x))/a^5 + 5/32*I*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a^
5 - 5/32*I*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^5 - 5/192*I*arccos(a*x)^
(3/2)*e^(-3*I*arccos(a*x))/a^5 - 1/320*I*arccos(a*x)^(3/2)*e^(-5*I*arccos(
a*x))/a^5 - (3/64000*I + 3/64000)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt
(10)*sqrt(arccos(a*x)))/a^5 + (3/64000*I - 3/64000)*sqrt(10)*sqrt(pi)*erf(
-(1/2*I + 1/2)*sqrt(10)*sqrt(arccos(a*x)))/a^5 - (5/4608*I + 5/4608)*sqrt(
6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (5/4608*I -
 5/4608)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^
5 - (15/256*I + 15/256)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(ar
ccos(a*x)))/a^5 + (15/256*I - 15/256)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*
sqrt(2)*sqrt(arccos(a*x)))/a^5 - 3/3200*sqrt(arccos(a*x))*e^(5*I*arccos(a*
x))/a^5 - 5/384*sqrt(arccos(a*x))*e^(3*I*arccos(a*x))/a^5 - 15/64*sqrt(arc
cos(a*x))*e^(I*arccos(a*x))/a^5 - 15/64*sqrt(arccos(a*x))*e^(-I*arccos(a*x
))/a^5 - 5/384*sqrt(arccos(a*x))*e^(-3*I*arccos(a*x))/a^5 - 3/3200*sqrt(ar
ccos(a*x))*e^(-5*I*arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^{5/2} \, dx = \int x^4 \operatorname{acos}(a\,x)^{5/2} \, dx$$

input
```
int(x^4*acos(a*x)^(5/2),x)
```

output
```
int(x^4*acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int x^4 \arccos(ax)^{5/2} \, dx = \int \sqrt{acos\left(ax\right)}\, acos(ax)^2\, x^4 dx$$

input
```
int(x^4*acos(a*x)^(5/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)**2*x**4,x)
```

3.87 $\int x^3 \arccos(ax)^{5/2}\,dx$

Optimal result . . . 672
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Giac [C] (verification not implemented) . . . 679
Mupad [F(-1)] . . . 681
Reduce [F] . . . 681

Optimal result

Integrand size = 12, antiderivative size = 205

$$\int x^3 \arccos(ax)^{5/2}\,dx = \frac{225\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4096a^4} + \frac{15\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{256a^4}$$

output
```
225/2048*arccos(a*x)^(1/2)/a^4-45/256*x^2*arccos(a*x)^(1/2)/a^2-15/256*x^4
*arccos(a*x)^(1/2)-15/64*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a^3-5/32*x
^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a-3/32*arccos(a*x)^(5/2)/a^4+1/4*x
^4*arccos(a*x)^(5/2)+15/8192*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*
arccos(a*x)^(1/2))/a^4+15/256*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/
2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int x^3 \arccos(ax)^{5/2} \, dx =$$

$$-\frac{i\left(16\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{7}{2},-2i\arccos(ax)\right)-16\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{7}{2},2i\arccos(ax)\right)+\sqrt{-i\arccos(a}\right.}{2048a^4\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^3*ArcCos[a*x]^(5/2),x]
```

output
```
((-1/2048*I)*(16*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-2*I)*ArcCos[a
*x]] - 16*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (2*I)*ArcCos[a*x]] + Sqrt
[(-I)*ArcCos[a*x]]*Gamma[7/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Ga
mma[7/2, (4*I)*ArcCos[a*x]]))/(a^4*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5141, 5211, 5141, 5211, 5141, 5153, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax)^{5/2} \, dx$$

$$\Big\downarrow \; 5141$$

$$\frac{5}{8}a\int \frac{x^4 \arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx + \frac{1}{4}x^4 \arccos(ax)^{5/2}$$

$$\Big\downarrow \; 5211$$

$$\frac{5}{8}a\left(\frac{3\int \frac{x^2\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\int x^3\sqrt{\arccos(ax)}dx}{8a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{4a^2}\right)+$$
$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$\downarrow$ 5141

$$\frac{5}{8}a\left(\frac{3\int \frac{x^2\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\left(\frac{1}{8}a\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{4}x^4\sqrt{\arccos(ax)}\right)}{8a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{4a^2}\right)+$$
$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$\downarrow$ 5211

$$\frac{5}{8}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\sqrt{\arccos(ax)}dx}{4a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{8}a\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{4}x^4\sqrt{\text{arcc}\ldots}\right.}{8a}\right.$$
$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$\downarrow$ 5141

$$\frac{5}{8}a\left(\frac{3\left(-\frac{3\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4a}+\frac{\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{8}a\int \frac{\ldots}{\sqrt{1-a^2x^2}\ldots}\right.}{\ldots}\right.$$
$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$\downarrow$ 5153

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{8}a\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{4}x^4\sqrt{\arccos(ax)}\right)}{8a}+\frac{3\left(-\frac{3\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4a}-\text{arc}\ldots\right.}{4a^2}\right.$$
$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5225}$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\int \frac{a^4x^4}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^4}\right)}{8a}+\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4a}-\frac{\arccos(ax)^{5\cdots}}{5a^3}\cdots\right)}{4a^2}\cdots\right)$$

$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^4}\right)}{8a}+\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4a}\cdots\right)}{4a^2}\cdots\right)$$

$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{\cos(4\arccos(ax))}{8\sqrt{\arccos(ax)}}+\frac{3}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{8a^4}\right)}{8a}+\frac{3\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int\left(\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}a\left(-\frac{3\left(\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^4}\right)}{8a}-\frac{x^3\sqrt{1-a}}{}\right.$$

$$\frac{1}{4}x^4\arccos(ax)^{5/2}$$

input
```
Int[x^3*ArcCos[a*x]^(5/2),x]
```

output
```
(x^4*ArcCos[a*x]^(5/2))/4 + (5*a*(-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^
(3/2))/a^2 - (3*((x^4*Sqrt[ArcCos[a*x]])/4 - ((3*Sqrt[ArcCos[a*x]])/4 + (S
qrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sqrt[Pi]*FresnelC
[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/2)/(8*a^4)))/(8*a) + (3*(-1/2*(x*Sqrt[1
- a^2*x^2]*ArcCos[a*x]^(3/2))/a^2 - ArcCos[a*x]^(5/2)/(5*a^3) - (3*((x^2*S
qrt[ArcCos[a*x]])/2 - (Sqrt[ArcCos[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcC
os[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*a)))/(4*a^2)))/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

method	result
default	$\frac{1024 \arccos(ax)^{\frac{5}{2}} \cos(2 \arccos(ax))\sqrt{\pi}+256 \arccos(ax)^{\frac{5}{2}} \cos(4 \arccos(ax))\sqrt{\pi}-1280 \arccos(ax)^{\frac{3}{2}} \sin(2 \arccos(ax))\sqrt{\pi}-160 \arccos}{}$

input
```
int(x^3*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8192/a^4/Pi^(1/2)*(1024*arccos(a*x)^(5/2)*cos(2*arccos(a*x))*Pi^(1/2)+25
6*arccos(a*x)^(5/2)*cos(4*arccos(a*x))*Pi^(1/2)-1280*arccos(a*x)^(3/2)*sin
(2*arccos(a*x))*Pi^(1/2)-160*arccos(a*x)^(3/2)*sin(4*arccos(a*x))*Pi^(1/2)
+15*Pi*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)-960*cos(2*ar
ccos(a*x))*Pi^(1/2)*arccos(a*x)^(1/2)+480*Pi*FresnelC(2*arccos(a*x)^(1/2)/
Pi^(1/2))-60*cos(4*arccos(a*x))*Pi^(1/2)*arccos(a*x)^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \arccos(ax)^{5/2}\,dx = \int x^3 \operatorname{acos}^{\frac{5}{2}}(ax)\,dx$$

input
```
integrate(x**3*acos(a*x)**(5/2),x)
```

output
```
Integral(x**3*acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.45

$$\int x^3 \arccos(ax)^{5/2}\,dx = \frac{\arccos(ax)^{\frac{5}{2}}e^{(4i\arccos(ax))}}{64a^4} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(2i\arccos(ax))}}{16a^4} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(-2i\arccos(ax))}}{16a^4} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(-4i\arccos(ax))}}{64a^4} + \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(4i\arccos(ax))}}{512a^4} + \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(2i\arccos(ax))}}{64a^4} - \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(-2i\arccos(ax))}}{64a^4} - \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(-4i\arccos(ax))}}{512a^4} - \frac{(15i+15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32768a^4} + \frac{(15i-15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32768a^4} - \frac{(15i+15)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{1024a^4} + \frac{(15i-15)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{1024a^4} - \frac{15\sqrt{\arccos(ax)}e^{(4i\arccos(ax))}}{4096a^4} - \frac{15\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{256a^4} - \frac{15\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{256a^4} - \frac{15\sqrt{\arccos(ax)}e^{(-4i\arccos(ax))}}{4096a^4}$$

input
```
integrate(x^3*arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
1/64*arccos(a*x)^(5/2)*e^(4*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(5/2)*e^
(2*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(5/2)*e^(-2*I*arccos(a*x))/a^4 +
1/64*arccos(a*x)^(5/2)*e^(-4*I*arccos(a*x))/a^4 + 5/512*I*arccos(a*x)^(3/2
)*e^(4*I*arccos(a*x))/a^4 + 5/64*I*arccos(a*x)^(3/2)*e^(2*I*arccos(a*x))/a
^4 - 5/64*I*arccos(a*x)^(3/2)*e^(-2*I*arccos(a*x))/a^4 - 5/512*I*arccos(a*
x)^(3/2)*e^(-4*I*arccos(a*x))/a^4 - (15/32768*I + 15/32768)*sqrt(2)*sqrt(p
i)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 + (15/32768*I - 15/32768)*sq
rt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 - (15/1024*I +
15/1024)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 + (15/1024*I - 15/102
4)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4 - 15/4096*sqrt(arccos(a*x)
)*e^(4*I*arccos(a*x))/a^4 - 15/256*sqrt(arccos(a*x))*e^(2*I*arccos(a*x))/a
^4 - 15/256*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^4 - 15/4096*sqrt(arcc
os(a*x))*e^(-4*I*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^{5/2}\,dx = \int x^3\,\mathrm{acos}(a\,x)^{5/2}\,dx$$

input
```
int(x^3*acos(a*x)^(5/2),x)
```

output
```
int(x^3*acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int x^3 \arccos(ax)^{5/2}\,dx = \int \sqrt{acos\,(ax)}\,acos(ax)^2\,x^3 dx$$

input
```
int(x^3*acos(a*x)^(5/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)**2*x**3,x)
```

3.88 $\int x^2 \arccos(ax)^{5/2} dx$

Optimal result 682
Mathematica [C] (verified) 683
Rubi [A] (verified) 683
Maple [A] (verified) 688
Fricas [F(-2)] 689
Sympy [F] 689
Maxima [F(-2)] 689
Giac [C] (verification not implemented) 690
Mupad [F(-1)] 690
Reduce [F] 691

Optimal result

Integrand size = 12, antiderivative size = 178

$$\int x^2 \arccos(ax)^{5/2} dx = -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^3} + \frac{5\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{144a^3}$$

output
```
-5/6*x*arccos(a*x)^(1/2)/a^2-5/36*x^3*arccos(a*x)^(1/2)-5/9*(-a^2*x^2+1)^(
1/2)*arccos(a*x)^(3/2)/a^3-5/18*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a
+1/3*x^3*arccos(a*x)^(5/2)+15/32*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2
)*arccos(a*x)^(1/2))/a^3+5/864*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*
arccos(a*x)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int x^2 \arccos(ax)^{5/2}\,dx =$$

$$-\frac{i\left(81\sqrt{-i\arccos(ax)}\Gamma\left(\frac{7}{2},-i\arccos(ax)\right)-81\sqrt{i\arccos(ax)}\Gamma\left(\frac{7}{2},i\arccos(ax)\right)+\sqrt{3}\left(\sqrt{-i\arccos(ax)}\Gamma\right.\right.}{648a^3\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^2*ArcCos[a*x]^(5/2),x]
```

output
```
((-1/648*I)*(81*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]] - 81*S
qrt[I*ArcCos[a*x]]*Gamma[7/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a
*x]]*Gamma[7/2, (-3*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (3*I)
*ArcCos[a*x]])))/(a^3*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.32, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5141, 5211, 5141, 5183, 5131, 5225, 3042, 3785, 3793, 2009, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)^{5/2}\,dx$$

$$\downarrow \text{5141}$$

$$\frac{5}{6}a\int \frac{x^3\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$$\downarrow \text{5211}$$

$$\frac{5}{6}a\left(\frac{2\int \frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2\sqrt{\arccos(ax)}dx}{2a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$\Big\downarrow$ 5141

$$\frac{5}{6}a\left(\frac{2\int \frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\frac{1}{6}a\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{3}x^3\sqrt{\arccos(ax)}}{2a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$\Big\downarrow$ 5183

$$\frac{5}{6}a\left(\frac{2\left(-\frac{3\int \sqrt{\arccos(ax)}dx}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)}{3a^2}-\frac{\frac{1}{6}a\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{3}x^3\sqrt{\arccos(ax)}}{2a}-\frac{x^2\sqrt{1-a^2}\cdots}{}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$\Big\downarrow$ 5131

$$\frac{5}{6}a\left(\frac{2\left(-\frac{3\left(\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+x\sqrt{\arccos(ax)}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)}{3a^2}-\frac{\frac{1}{6}a\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{3}x^3\sqrt{\cdots}}{2a}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$\Big\downarrow$ 5225

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int \frac{a^3x^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}}{2a}+\frac{2\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int \frac{ax}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3}\cdots}{a^2}\right.}{3a^2}\right.$$
$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$\Big\downarrow$ 3042

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}}{2a}+\frac{2\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a}\right)}{2a}-\frac{\sqrt{1-a^2}}{}\right.}{3a^2}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3785}$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{6a^3}}{2a}+\frac{2\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int axd\sqrt{\arccos(ax)}}{a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3}}{a^2}\right.}{3a^2}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\int\left(\frac{3ax}{4\sqrt{\arccos(ax)}}+\frac{\cos(3\arccos(ax))}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{6a^3}}{2a}+\frac{2\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int axd\sqrt{\arccos(ax)}}{a}\right)}{2a}-\frac{\sqrt{1-a^2x}}{}\right.}{3a^2}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{6}a\left(\frac{2\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int axd\sqrt{\arccos(ax)}}{a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)}{3a^2}-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a\right.}}{}}{2a}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 3833$$

$$\frac{5}{6}a\left(-\frac{\frac{1}{3}x^3\sqrt{\arccos(ax)}-\frac{\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^3}}{2a}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right.}{}\right.$$

$$\frac{1}{3}x^3\arccos(ax)^{5/2}$$

input
```
Int[x^2*ArcCos[a*x]^(5/2),x]
```

output
```
(x^3*ArcCos[a*x]^(5/2))/3 + (5*a*(-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^
(3/2))/a^2 + (2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/a^2) - (3*(x*Sqrt
[ArcCos[a*x]] - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a))/(2
*a)))/(3*a^2) - ((x^3*Sqrt[ArcCos[a*x]])/3 - ((3*Sqrt[Pi/2]*FresnelC[Sqrt[
2/Pi]*Sqrt[ArcCos[a*x]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[
a*x]]])/2)/(6*a^3))/(2*a)))/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88

method	result
default	$\frac{216ax\arccos(ax)^3+72\arccos(ax)^3\cos(3\arccos(ax))+5\sqrt{3}\sqrt{2}\sqrt{\pi}\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-540\arccos(ax)^2\sqrt{}}{}$

input
```
int(x^2*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/864/a^3*(216*a*x*arccos(a*x)^3+72*arccos(a*x)^3*cos(3*arccos(a*x))+5*3^(
1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*
arccos(a*x)^(1/2))-540*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-60*arccos(a*x)^2*s
in(3*arccos(a*x))+405*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/
Pi^(1/2)*arccos(a*x)^(1/2))-810*a*x*arccos(a*x)-30*arccos(a*x)*cos(3*arcco
s(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \arccos(ax)^{5/2} \, dx = \int x^2 \operatorname{acos}^{\frac{5}{2}}(ax) \, dx$$

input
```
integrate(x**2*acos(a*x)**(5/2),x)
```

output
```
Integral(x**2*acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.74

$$\int x^2 \arccos(ax)^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
1/24*arccos(a*x)^(5/2)*e^(3*I*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(5/2)*e^(
I*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(5/2)*e^(-I*arccos(a*x))/a^3 + 1/24*a
rccos(a*x)^(5/2)*e^(-3*I*arccos(a*x))/a^3 + 5/144*I*arccos(a*x)^(3/2)*e^(3
*I*arccos(a*x))/a^3 + 5/16*I*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a^3 - 5/1
6*I*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^3 - 5/144*I*arccos(a*x)^(3/2)*e
^(-3*I*arccos(a*x))/a^3 - (5/3456*I + 5/3456)*sqrt(6)*sqrt(pi)*erf((1/2*I
- 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 + (5/3456*I - 5/3456)*sqrt(6)*sqrt(p
i)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 - (15/128*I + 15/128)
*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3 + (15/1
28*I - 15/128)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x
)))/a^3 - 5/288*sqrt(arccos(a*x))*e^(3*I*arccos(a*x))/a^3 - 15/32*sqrt(arc
cos(a*x))*e^(I*arccos(a*x))/a^3 - 15/32*sqrt(arccos(a*x))*e^(-I*arccos(a*x
))/a^3 - 5/288*sqrt(arccos(a*x))*e^(-3*I*arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^{5/2} \, dx = \int x^2 \operatorname{acos}(a\,x)^{5/2} \,dx$$

input
```
int(x^2*acos(a*x)^(5/2),x)
```

output
```
int(x^2*acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int x^2 \arccos(ax)^{5/2} \, dx = \int \sqrt{acos\,(ax)}\, acos(ax)^2 \, x^2 dx$$

input
```
int(x^2*acos(a*x)^(5/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)**2*x**2,x)
```

3.89 $\int x \arccos(ax)^{5/2} dx$

Optimal result 692
Mathematica [A] (verified) 692
Rubi [A] (verified) 693
Maple [A] (verified) 696
Fricas [F(-2)] 697
Sympy [F] 697
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Giac [C] (verification not implemented) 698
Mupad [F(-1)] 698
Reduce [F] 699

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int x \arccos(ax)^{5/2} dx = \frac{15\sqrt{\arccos(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{128a^2}$$

output
```
15/64*arccos(a*x)^(1/2)/a^2-15/32*x^2*arccos(a*x)^(1/2)-5/8*x*(-a^2*x^2+1)
^(1/2)*arccos(a*x)^(3/2)/a-1/4*arccos(a*x)^(5/2)/a^2+1/2*x^2*arccos(a*x)^(
5/2)+15/128*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int x \arccos(ax)^{5/2} dx = \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - 2\sqrt{\arccos(ax)}\left(\left(15-16\arccos(ax)^2\right)\cos(2\arccos(\right.}{128a^2}$$

input
```
Integrate[x*ArcCos[a*x]^(5/2),x]
```

output
```
(15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*Sqrt[ArcCos[a*x]
]*((15 - 16*ArcCos[a*x]^2)*Cos[2*ArcCos[a*x]] + 20*ArcCos[a*x]*Sin[2*ArcCo
s[a*x]]))/(128*a^2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5141, 5211, 5141, 5153, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax)^{5/2}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{5}{4}a\int \frac{x^2 \arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{5}{4}a\left(\frac{\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\sqrt{\arccos(ax)}dx}{4a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4a}+\frac{\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4a}-\frac{\arccos(ax)^{5/2}}{5a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)+$$
$$\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 5225$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4a}-\frac{\arccos(ax)^{5/2}}{5a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)+$$
$$\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4a}-\frac{\arccos(ax)^{5/2}}{5a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)+$$
$$\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 3793$$

$$\frac{5}{4}a\left(-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{1}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{4a^2}\right)}{4a}-\frac{\arccos(ax)^{5/2}}{5a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a^2}\right)+$$
$$\frac{1}{2}x^2\arccos(ax)^{5/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{5}{4}a\left(-\frac{\arccos(ax)^{5/2}}{5a^3}-\frac{3\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}}{4a^2}\right)}{4a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right.$$

$$\frac{1}{2}x^2\arccos(ax)^{5/2}$$

input
```
Int[x*ArcCos[a*x]^(5/2),x]
```

output
```
(x^2*ArcCos[a*x]^(5/2))/2 + (5*a*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3
/2))/a^2 - ArcCos[a*x]^(5/2)/(5*a^3) - (3*((x^2*Sqrt[ArcCos[a*x]])/2 - (Sq
rt[ArcCos[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/2)/(
4*a^2)))/(4*a)))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

method	result
default	$\frac{32 \arccos(ax)^{\frac{5}{2}} \cos(2\arccos(ax))\sqrt{\pi}-40\arccos(ax)^{\frac{3}{2}} \sin(2\arccos(ax))\sqrt{\pi}-30\cos(2\arccos(ax))\sqrt{\pi}\,\sqrt{\arccos(ax)}+15\pi\ \mathrm{FresnelC}\Big(}{128a^2\sqrt{\pi}}$

input
```
int(x*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/a^2/Pi^(1/2)*(32*arccos(a*x)^(5/2)*cos(2*arccos(a*x))*Pi^(1/2)-40*ar
ccos(a*x)^(3/2)*sin(2*arccos(a*x))*Pi^(1/2)-30*cos(2*arccos(a*x))*Pi^(1/2)
*arccos(a*x)^(1/2)+15*Pi*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x \arccos(ax)^{5/2} \, dx = \int x \operatorname{acos}^{\frac{5}{2}}(ax) \, dx$$

input
```
integrate(x*acos(a*x)**(5/2),x)
```

output
```
Integral(x*acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.20

$$\int x \arccos(ax)^{5/2} \, dx = \frac{\arccos{(ax)}^{\frac{5}{2}} e^{(2i \arccos(ax))}}{8 a^2} + \frac{\arccos{(ax)}^{\frac{5}{2}} e^{(-2i \arccos(ax))}}{8 a^2} + \frac{5i \arccos{(ax)}^{\frac{3}{2}} e^{(2i \arccos(ax))}}{32 a^2} - \frac{5i \arccos{(ax)}^{\frac{3}{2}} e^{(-2i \arccos(ax))}}{32 a^2} - \frac{(15i + 15) \sqrt{\pi} \operatorname{erf}\left((i-1) \sqrt{\arccos{(ax)}}\right)}{512 a^2} + \frac{(15i - 15) \sqrt{\pi} \operatorname{erf}\left(-(i+1) \sqrt{\arccos{(ax)}}\right)}{512 a^2} - \frac{15 \sqrt{\arccos{(ax)}} e^{(2i \arccos(ax))}}{128 a^2} - \frac{15 \sqrt{\arccos{(ax)}} e^{(-2i \arccos(ax))}}{128 a^2}$$

input
```
integrate(x*arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
1/8*arccos(a*x)^(5/2)*e^(2*I*arccos(a*x))/a^2 + 1/8*arccos(a*x)^(5/2)*e^(-
2*I*arccos(a*x))/a^2 + 5/32*I*arccos(a*x)^(3/2)*e^(2*I*arccos(a*x))/a^2 -
5/32*I*arccos(a*x)^(3/2)*e^(-2*I*arccos(a*x))/a^2 - (15/512*I + 15/512)*sq
rt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^2 + (15/512*I - 15/512)*sqrt(pi)*e
rf(-(I + 1)*sqrt(arccos(a*x)))/a^2 - 15/128*sqrt(arccos(a*x))*e^(2*I*arcco
s(a*x))/a^2 - 15/128*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^{5/2} \, dx = \int x \operatorname{acos}(a\,x)^{5/2} \, dx$$

input
```
int(x*acos(a*x)^(5/2),x)
```

output
```
int(x*acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int x \arccos(ax)^{5/2} \, dx = \int \sqrt{acos\,(ax)}\, acos(ax)^2 \, xdx$$

input
```
int(x*acos(a*x)^(5/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)**2*x,x)
```

3.90 $\int \arccos(ax)^{5/2}\,dx$

Optimal result 700
Mathematica [C] (verified) 700
Rubi [A] (verified) 701
Maple [A] (verified) 703
Fricas [F(-2)] 704
Sympy [F] 704
Maxima [F(-2)] 704
Giac [C] (verification not implemented) 705
Mupad [F(-1)] 705
Reduce [F] 706

Optimal result

Integrand size = 8, antiderivative size = 88

$$\int \arccos(ax)^{5/2}\,dx = -\frac{15}{4}x\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a} + x\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a}$$

output
```
-15/4*x*arccos(a*x)^(1/2)-5/2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a+x*arc
cos(a*x)^(5/2)+15/8*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)
^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \arccos(ax)^{5/2}\,dx = -\frac{i\left(\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{7}{2},-i\arccos(ax)\right) - \sqrt{i\arccos(ax)}\,\Gamma\left(\frac{7}{2},i\arccos(ax)\right)\right)}{2a\sqrt{\arccos(ax)}}$$

input
```
Integrate[ArcCos[a*x]^(5/2),x]
```

output
```
((-1/2*I)*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]] - Sqrt[I*Ar
cCos[a*x]]*Gamma[7/2, I*ArcCos[a*x]]))/(a*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5131, 5183, 5131, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{5/2}dx$$

$$\Big\downarrow \text{5131}$$

$$\frac{5}{2}a\int \frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{5}{2}a\left(-\frac{3\int \sqrt{\arccos(ax)}dx}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)+x\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5131}$$

$$\frac{5}{2}a\left(-\frac{3\left(\frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+x\sqrt{\arccos(ax)}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)+ x\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{5225}$$

$$\frac{5}{2}a\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int \frac{ax}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)+ x\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}a\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)+ x\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3785}$$

$$\frac{5}{2}a\left(-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\int axd\sqrt{\arccos(ax)}}{a}\right)}{2a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}\right)+x\arccos(ax)^{5/2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{5}{2}a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{a^2}-\frac{3\left(x\sqrt{\arccos(ax)}-\frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}\right)}{2a}\right)+ x\arccos(ax)^{5/2}$$

input
```
Int[ArcCos[a*x]^(5/2),x]
```

output
```
x*ArcCos[a*x]^(5/2) + (5*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/a^2) -
 (3*(x*Sqrt[ArcCos[a*x]] - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/a))/(2*a)))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

method	result
default	$\frac{\sqrt{2}\left(4 \arccos(ax)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,ax-10\arccos(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{-a^2x^2+1}-15\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\,ax+15\pi\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{8a\sqrt{\pi}}$

input
```
int(arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*2^(1/2)/Pi^(1/2)*(4*arccos(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*a*x-10*arccos
(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)-15*2^(1/2)*Pi^(1/2)*arccos
(a*x)^(1/2)*a*x+15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \arccos(ax)^{5/2}\,dx = \int \operatorname{acos}^{\frac{5}{2}}(ax)\,dx$$

input
```
integrate(acos(a*x)**(5/2),x)
```

output
```
Integral(acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arccos(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \arccos(ax)^{5/2}\,dx = \frac{\arccos(ax)^{\frac{5}{2}}e^{(i\arccos(ax))}}{2a} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(-i\arccos(ax))}}{2a} + \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(i\arccos(ax))}}{4a} - \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(-i\arccos(ax))}}{4a} - \frac{(15i+15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32a} + \frac{(15i-15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32a} - \frac{15\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{8a} - \frac{15\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{8a}$$

input
```
integrate(arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
1/2*arccos(a*x)^(5/2)*e^(I*arccos(a*x))/a + 1/2*arccos(a*x)^(5/2)*e^(-I*ar
ccos(a*x))/a + 5/4*I*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a - 5/4*I*arccos(
a*x)^(3/2)*e^(-I*arccos(a*x))/a - (15/32*I + 15/32)*sqrt(2)*sqrt(pi)*erf((
1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a + (15/32*I - 15/32)*sqrt(2)*sqrt
(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a - 15/8*sqrt(arccos(a*
x))*e^(I*arccos(a*x))/a - 15/8*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(ax)^{5/2}\,dx = \int \mathrm{acos}(a\,x)^{5/2}\,dx$$

input
```
int(acos(a*x)^(5/2),x)
```

output
```
int(acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \arccos(ax)^{5/2}\,dx = \int \sqrt{acos\,(ax)}\,acos(ax)^2\,dx$$

input
```
int(acos(a*x)^(5/2),x)
```

output
```
int(sqrt(acos(a*x))*acos(a*x)**2,x)
```

3.91 $\int \frac{\arccos(ax)^{5/2}}{x} \, dx$

Optimal result 707
Mathematica [N/A] 707
Rubi [N/A] 708
Maple [N/A] 708
Fricas [F(-2)] 709
Sympy [N/A] 709
Maxima [F(-2)] 709
Giac [N/A] 710
Mupad [N/A] 710
Reduce [N/A] 710

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \text{Int}\left(\frac{\arccos(ax)^{5/2}}{x}, x\right)$$

output
```
Defer(Int)(arccos(a*x)^(5/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \int \frac{\arccos(ax)^{5/2}}{x} \, dx$$

input
```
Integrate[ArcCos[a*x]^(5/2)/x,x]
```

output
```
Integrate[ArcCos[a*x]^(5/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx$$
$$\Big\downarrow \ 5149$$
$$\int \frac{\arccos(ax)^{5/2}}{x}dx$$

input
```
Int[ArcCos[a*x]^(5/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos \left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int(arccos(a*x)^(5/2)/x,x)
```

output
```
int(arccos(a*x)^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 16.85 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{acos}^{\frac{5}{2}}(ax)}{x}\,dx$$

input
```
integrate(acos(a*x)**(5/2)/x,x)
```

output
```
Integral(acos(a*x)**(5/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \int \frac{\arccos{(ax)}^{\frac{5}{2}}}{x} \, dx$$

input
```
integrate(arccos(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \int \frac{\mathrm{acos}(a\,x)^{5/2}}{x} \, dx$$

input
```
int(acos(a*x)^(5/2)/x,x)
```

output
```
int(acos(a*x)^(5/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \int \frac{\sqrt{acos\,(ax)}\,acos(ax)^2}{x} dx$$

input
```
int(acos(a*x)^(5/2)/x,x)
```

output
```
int((sqrt(acos(a*x))*acos(a*x)**2)/x,x)
```

3.92 $\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx$

Optimal result . . . 712
Mathematica [C] (verified) . . . 713
Rubi [A] (verified) . . . 713
Maple [A] (verified) . . . 715
Fricas [F(-2)] . . . 715
Sympy [F] . . . 715
Maxima [F(-2)] . . . 716
Giac [C] (verification not implemented) . . . 716
Mupad [F(-1)] . . . 717
Reduce [F] . . . 717

Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}$$

output
```
-1/8*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5-1/1
6*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5-1/80*1
0^(1/2)*Pi^(1/2)*FresnelS(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx =$$
$$-\frac{-10\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)-10\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},i\arccos(ax)\right)-5\sqrt{3}\sqrt{-i\arccos(ax)}$$

input
```
Integrate[x^4/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/160*(-10*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - 10*Sqrt[
I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] - 5*Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]
]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 5*Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2
, (3*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-5*I)*Ar
cCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (5*I)*ArcCos[a*x]])/(a
^5*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx$$
$$\Big\downarrow \text{ 5147}$$
$$-\frac{\int \frac{a^4x^4\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^5}$$
$$\Big\downarrow \text{ 4906}$$

$$-\frac{\int \left(\frac{3\sin(3\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\sin(5\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^5}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{4}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5}$$

input
```
Int[x^4/Sqrt[ArcCos[a*x]],x]
```

output
```
-(((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/4 + (Sqrt[(3*Pi)/2]
*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sqrt[Pi/10]*FresnelS[Sqrt[10
/Pi]*Sqrt[ArcCos[a*x]]])/8)/a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(5\sqrt{3}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)+\sqrt{5}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)+10\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(ax\right)}}{\sqrt{\pi}}\right)\right)}{80a^{5}}$	72

input
```
int(x^4/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/80/a^5*2^(1/2)*Pi^(1/2)*(5*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*ar
ccos(a*x)^(1/2))+5^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/
2))+10*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(x**4/acos(a*x)**(1/2),x)
```

output
```
Integral(x**4/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{10}\sqrt{\arccos(ax)}\right)}{320\,a^5} + \frac{(i+1)\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{10}\sqrt{\arccos(ax)}\right)}{320\,a^5} - \frac{(i-1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{64\,a^5} + \frac{(i+1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{64\,a^5} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32\,a^5} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32\,a^5}$$

input
```
integrate(x^4/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/320*I - 1/320)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt(arcco
s(a*x)))/a^5 + (1/320*I + 1/320)*sqrt(10)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt
(10)*sqrt(arccos(a*x)))/a^5 - (1/64*I - 1/64)*sqrt(6)*sqrt(pi)*erf((1/2*I
- 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (1/64*I + 1/64)*sqrt(6)*sqrt(pi)*e
rf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (1/32*I - 1/32)*sqrt(2)
*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 + (1/32*I + 1/3
2)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^4}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int(x^4/acos(a*x)^(1/2),x)
```

output
```
int(x^4/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = \frac{-2\sqrt{-a^2x^2+1}\,\sqrt{acos\left(ax\right)}\,a^4x^4+\frac{8\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{3}-\frac{4\left(\int \frac{\sqrt{acos(ax)}}{acos(ax)a^2x^2-acos(ax)}dx\right)a}{3}+\frac{4\left(\int \frac{\sqrt{acos(ax)}\,x^2}{acos(ax)a^2x^2-acos(ax)} d\right.}{3}}{a^5}$$

input
```
int(x^4/acos(a*x)^(1/2),x)
```

output
```
(2*( - 3*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a**4*x**4 + 4*sqrt( - a**2
*x**2 + 1)*sqrt(acos(a*x)) - 2*int(sqrt(acos(a*x))/(acos(a*x)*a**2*x**2 -
acos(a*x)),x)*a + 2*int((sqrt(acos(a*x))*x**2)/(acos(a*x)*a**2*x**2 - acos
(a*x)),x)*a**3 + 15*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**5)/(a**
2*x**2 - 1),x)*a**6 - 12*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)
/(a**2*x**2 - 1),x)*a**4 - 4*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x
)/(a**2*x**2 - 1),x)*a**2))/(3*a**5)
```

3.93 $\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx$

Optimal result 719
Mathematica [C] (verified) 719
Rubi [A] (verified) 720
Maple [A] (verified) 721
Fricas [F(-2)] 721
Sympy [F] 722
Maxima [F(-2)] 722
Giac [C] (verification not implemented) 722
Mupad [F(-1)] 723
Reduce [F] 724

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^4} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{4a^4}$$

output
```
-1/16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^4-
1/4*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = -\frac{-2\sqrt{2}\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-2i\arccos(ax)\right) - 2\sqrt{2}\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},2i\arccos(ax)\right) - \sqrt{-i\arccos(a}}{32a^4\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^3/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/32*(-2*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] -
2*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] - Sqrt[(-I)*Ar
cCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[1/2,
 (4*I)*ArcCos[a*x]])/(a^4*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int \frac{a^3x^3\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^4}$$

input
```
Int[x^3/Sqrt[ArcCos[a*x]],x]
```

output
```
-(((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sqrt[Pi]*Fre
snelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/4)/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{\pi}\left(\sqrt{2}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)+4\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)\right)}{16 a^{4}}$	43

input
```
int(x^3/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^4*Pi^(1/2)*(2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))
+4*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(x**3/acos(a*x)**(1/2),x)
```

output
```
Integral(x**3/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64\,a^4} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64\,a^4} - \frac{(i-1)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{16\,a^4} + \frac{(i+1)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{16\,a^4}$$

input
```
integrate(x^3/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/64*I - 1/64)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a
^4 + (1/64*I + 1/64)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x
)))/a^4 - (1/16*I - 1/16)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 + (1
/16*I + 1/16)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

input
```
int(x^3/acos(a*x)^(1/2),x)
```

output
```
int(x^3/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx$$
$$= \frac{-4\sqrt{acos\,(ax)}\,acos(ax) - 4\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,a^3x^3 - 6\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,ax - 3\left(\int \frac{\sqrt{a}}{acos(ax)}\right.}{2a^4}$$

input
```
int(x^3/acos(a*x)^(1/2),x)
```

output
```
( - 4*sqrt(acos(a*x))*acos(a*x) - 4*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))
*a**3*x**3 - 6*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a*x - 3*int((sqrt(ac
os(a*x))*x**3)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**4 + 3*int((sqrt(aco
s(a*x))*x)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**2 + 16*int((sqrt( - a**
2*x**2 + 1)*sqrt(acos(a*x))*x**4)/(a**2*x**2 - 1),x)*a**5)/(2*a**4)
```

3.94 $\int \frac{x^2}{\sqrt{\arccos(ax)}} dx$

Optimal result . . . 725
Mathematica [C] (verified) . . . 725
Rubi [A] (verified) . . . 726
Maple [A] (verified) . . . 727
Fricas [F(-2)] . . . 727
Sympy [F] . . . 728
Maxima [F(-2)] . . . 728
Giac [C] (verification not implemented) . . . 728
Mupad [F(-1)] . . . 729
Reduce [F] . . . 730

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int \frac{x^2}{\sqrt{\arccos(ax)}} dx = -\frac{\sqrt{\frac{\pi}{2}}\, \text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}}\, \text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3}$$

output

```
-1/4*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3-1/1
2*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$\int \frac{x^2}{\sqrt{\arccos(ax)}} dx = -\frac{-3\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2}, -i\arccos(ax)\right) - 3\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2}, i\arccos(ax)\right) - \sqrt{3}\left(\sqrt{-i\arccos(ax)}\Gamma\right.}{24a^3\sqrt{\arccos(ax)}}$$

input
```
Integrate[x^2/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/24*(-3*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - 3*Sqrt[I*A
rcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] - Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Ga
mma[1/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos
[a*x]]))/(a^3*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}dx$$
$$\Big\downarrow \text{5147}$$
$$-\frac{\int \frac{a^2x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}$$
$$\Big\downarrow \text{4906}$$
$$-\frac{\int \left(\frac{\sin(3\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^3}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}$$

input
```
Int[x^2/Sqrt[ArcCos[a*x]],x]
```

output
```
-(((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/2 + (Sqrt[Pi/6]*Fre
snelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/2)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\sqrt{3}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos\left(a x \right)}}{\sqrt{\pi}}\right)+3\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(a x \right)}}{\sqrt{\pi}}\right)\right)}{12 a^{3}}$	50

input
```
int(x^2/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^3*2^(1/2)*Pi^(1/2)*(3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcc
os(a*x)^(1/2))+3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(x**2/acos(a*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{48a^3} + \frac{(i+1)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{48a^3} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{16a^3} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{16a^3}$$

input
```
integrate(x^2/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/48*I - 1/48)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*
x)))/a^3 + (1/48*I + 1/48)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqr
t(arccos(a*x)))/a^3 - (1/16*I - 1/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*s
qrt(2)*sqrt(arccos(a*x)))/a^3 + (1/16*I + 1/16)*sqrt(2)*sqrt(pi)*erf(-(1/2
*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

input
```
int(x^2/acos(a*x)^(1/2),x)
```

output
```
int(x^2/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx$$
$$= \frac{-2\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,x^2 + 6\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^3}{a^2x^2-1}dx\right)a^2 - 4\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{a^2x^2-1}dx\right)}{a}$$

input
```
int(x^2/acos(a*x)^(1/2),x)
```

output
```
(2*( - sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2 + 3*int((sqrt( - a**2*x
**2 + 1)*sqrt(acos(a*x))*x**3)/(a**2*x**2 - 1),x)*a**2 - 2*int((sqrt( - a*
*2*x**2 + 1)*sqrt(acos(a*x))*x)/(a**2*x**2 - 1),x)))/a
```

3.95 $\int \frac{x}{\sqrt{\arccos(ax)}}\,dx$

Optimal result 731
Mathematica [A] (verified) 731
Rubi [A] (verified) 732
Maple [A] (verified) 734
Fricas [F(-2)] 734
Sympy [F] 734
Maxima [F(-2)] 735
Giac [C] (verification not implemented) 735
Mupad [F(-1)] 736
Reduce [F] 736

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

output
```
-1/2*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

input
```
Integrate[x/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/2*(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5147, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arccos(ax)}}dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

input
```
Int[x/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/2*(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$	21

input
```
int(x/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(x/acos(a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\,\sqrt{\pi}\,\text{erf}\left((i-1)\,\sqrt{\arccos{(ax)}}\right)}{8\,a^2} + \frac{(i+1)\,\sqrt{\pi}\,\text{erf}\left(-(i+1)\,\sqrt{\arccos{(ax)}}\right)}{8\,a^2}$$

input
```
integrate(x/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/8*I - 1/8)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^2 + (1/8*I + 1/8)
*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

input
```
int(x/acos(a*x)^(1/2),x)
```

output
```
int(x/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \frac{-\frac{4\sqrt{acos(ax)}\,acos(ax)}{3} - 2\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,ax + 4\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^2}{a^2x^2-1}dx\right)a^3}{a^2}$$

input
```
int(x/acos(a*x)^(1/2),x)
```

output
```
(2*( - 2*sqrt(acos(a*x))*acos(a*x) - 3*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*
x))*a*x + 6*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/(a**2*x**2 -
 1),x)*a**3))/(3*a**2)
```

3.96 $\int \frac{1}{\sqrt{\arccos(ax)}}\,dx$

Optimal result 737
Mathematica [C] (verified) 737
Rubi [A] (verified) 738
Maple [A] (verified) 739
Fricas [F(-2)] 740
Sympy [F] 740
Maxima [F(-2)] 740
Giac [C] (verification not implemented) 741
Mupad [F(-1)] 741
Reduce [F] 742

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

output
```
-2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = -\frac{-\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)-\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},i\arccos(ax)\right)}{2a\sqrt{\arccos(ax)}}$$

input
```
Integrate[1/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/2*(-(Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]]) - Sqrt[I*ArcC
os[a*x]]*Gamma[1/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arccos(ax)}}dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{2\int \sqrt{1-a^2x^2}d\sqrt{\arccos(ax)}}{a}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

input
```
Int[1/Sqrt[ArcCos[a*x]],x]
```

output
```
-((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)}{a}$	26

input
```
int(1/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(1/acos(a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{4\,a} + \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{4\,a}$$

input
```
integrate(1/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)
))/a + (1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arcc
os(a*x)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int(1/acos(a*x)^(1/2),x)
```

output
```
int(1/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \frac{-2\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}+2\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{a^2x^2-1}dx\right)a^2}{a}$$

input
```
int(1/acos(a*x)^(1/2),x)
```

output
```
(2*( - sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)) + int((sqrt( - a**2*x**2 + 1
)*sqrt(acos(a*x))*x)/(a**2*x**2 - 1),x)*a**2))/a
```

3.97 $\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$

Optimal result . . . 743
Mathematica [N/A] . . . 743
Rubi [N/A] . . . 744
Maple [N/A] . . . 744
Fricas [F(-2)] . . . 745
Sympy [N/A] . . . 745
Maxima [F(-2)] . . . 745
Giac [N/A] . . . 746
Mupad [N/A] . . . 746
Reduce [N/A] . . . 746

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{\arccos(ax)}},x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[ArcCos[a*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[ArcCos[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arccos(ax)}}dx$$
$$\Big\downarrow \text{5149}$$
$$\int \frac{1}{x\sqrt{\arccos(ax)}}dx$$

input
```
Int[1/(x*Sqrt[ArcCos[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\arccos\left(ax\right)}}dx$$

input
```
int(1/x/arccos(a*x)^(1/2),x)
```

output
```
int(1/x/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(1/x/acos(a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\arccos{(ax)}}}\,dx$$

input
```
integrate(1/x/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(arccos(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x*acos(a*x)^(1/2)),x)
```

output
```
int(1/(x*acos(a*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{\sqrt{acos\left(ax\right)}}{acos\left(ax\right)x}dx$$

input
```
int(1/x/acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x))/(acos(a*x)*x),x)
```

3.98 $\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$

Optimal result 748
Mathematica [N/A] 748
Rubi [N/A] 749
Maple [N/A] 749
Fricas [F(-2)] 750
Sympy [N/A] 750
Maxima [F(-2)] 750
Giac [N/A] 751
Mupad [N/A] 751
Reduce [N/A] 751

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{\arccos(ax)}},x\right)$$

output
```
Defer(Int)(1/x^2/arccos(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[ArcCos[a*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[ArcCos[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}dx$$

input
```
Int[1/(x^2*Sqrt[ArcCos[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}dx$$

input
```
int(1/x^2/arccos(a*x)^(1/2),x)
```

output
```
int(1/x^2/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(1/x**2/acos(a*x)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$$

input
```
integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x^2*sqrt(arccos(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\,\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x^2*acos(a*x)^(1/2)),x)
```

output
```
int(1/(x^2*acos(a*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{\sqrt{acos\left(ax\right)}}{acos\left(ax\right)x^2}dx$$

input
```
int(1/x^2/acos(a*x)^(1/2),x)
```

output
```
int(sqrt(acos(a*x))/(acos(a*x)*x**2),x)
```

3.99 $\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx$

Optimal result 753
Mathematica [C] (verified) 754
Rubi [A] (verified) 754
Maple [A] (verified) 756
Fricas [F(-2)] 756
Sympy [F] 757
Maxima [F(-2)] 757
Giac [F] 757
Mupad [F(-1)] 758
Reduce [F] 758

Optimal result

Integrand size = 12, antiderivative size = 171

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{5\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{9\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{\sqrt{\frac{7\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{14}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7}$$

output
```
2*x^6*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-5/32*2^(1/2)*Pi^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^7-9/32*6^(1/2)*Pi^(1/2)*FresnelC(6
^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^7-5/32*10^(1/2)*Pi^(1/2)*FresnelC(10^
(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^7-1/32*14^(1/2)*Pi^(1/2)*FresnelC(14^(
1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^7
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.79

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \frac{i\left(-10i\sqrt{1-a^2x^2}+5\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)-5\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},i a\right.\right.}{}$$

input
```
Integrate[x^6/ArcCos[a*x]^(3/2),x]
```

output
```
((I/64)*((-10*I)*Sqrt[1 - a^2*x^2] + 5*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (
-I)*ArcCos[a*x]] - 5*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + 9*Sqr
t[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 9*Sqrt[3]*Sqr
t[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] + 5*Sqrt[5]*Sqrt[(-I)*ArcCo
s[a*x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - 5*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gam
ma[1/2, (5*I)*ArcCos[a*x]] + Sqrt[7]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-7
*I)*ArcCos[a*x]] - Sqrt[7]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (7*I)*ArcCos[a*x
]] - (18*I)*Sin[3*ArcCos[a*x]] - (10*I)*Sin[5*ArcCos[a*x]] - (2*I)*Sin[7*A
rcCos[a*x]]))/(a^7*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \; 5143$$

$$\frac{2\int \left(-\frac{5ax}{64\sqrt{\arccos(ax)}}-\frac{27\cos(3\arccos(ax))}{64\sqrt{\arccos(ax)}}-\frac{25\cos(5\arccos(ax))}{64\sqrt{\arccos(ax)}}-\frac{7\cos(7\arccos(ax))}{64\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^7}+\frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$\big\downarrow$ 2009

$$\frac{2\left(-\frac{5}{32}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{9}{32}\sqrt{\frac{3\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{5}{32}\sqrt{\frac{5\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\operatorname{arcc}\right.\right.}{a^7}$$
$$\frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

input
```
Int[x^6/ArcCos[a*x]^(3/2),x]
```

output
```
(2*x^6*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) + (2*((-5*Sqrt[Pi/2]*Fresn
elC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/32 - (9*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/
Pi]*Sqrt[ArcCos[a*x]]])/32 - (5*Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sqrt[A
rcCos[a*x]]])/32 - (Sqrt[(7*Pi)/2]*FresnelC[Sqrt[14/Pi]*Sqrt[ArcCos[a*x]]]
)/32))/a^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07

method	result
default	$-\frac{9\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+5\sqrt{5}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{2}\,\sqrt{\pi}\,\sqrt{7}}{}$

input
```
int(x^6/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^7*(9*3^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)*arccos(a*x)^(1/2))+5*5^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^
(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))+2^(1/2)*Pi^(1/2
)*7^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*7^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)
^(1/2)+5*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arcc
os(a*x)^(1/2))-5*(-a^2*x^2+1)^(1/2)-9*sin(3*arccos(a*x))-5*sin(5*arccos(a*
x))-sin(7*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^6/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^6}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**6/acos(a*x)**(3/2),x)
```

output
```
Integral(x**6/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^6/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^6}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^6}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x^6/acos(a*x)^(3/2),x)
```

output
```
int(x^6/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \frac{-\frac{12acos(ax)\left(\int \frac{\sqrt{acos(ax)}}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a}{5}+\frac{2acos(ax)\left(\int \frac{\sqrt{acos(ax)}\,x^4}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a^5}{5}+2acos(ax)}{}$$

input
```
int(x^6/acos(a*x)^(3/2),x)
```

output
```
(2*( - 6*acos(a*x)*int(sqrt(acos(a*x))/(acos(a*x)**2*a**2*x**2 - acos(a*x)
**2),x)*a + acos(a*x)*int((sqrt(acos(a*x))*x**4)/(acos(a*x)**2*a**2*x**2 -
 acos(a*x)**2),x)*a**5 + 5*acos(a*x)*int((sqrt(acos(a*x))*x**2)/(acos(a*x)
**2*a**2*x**2 - acos(a*x)**2),x)*a**3 - 35*acos(a*x)*int((sqrt( - a**2*x**
2 + 1)*sqrt(acos(a*x))*x**7)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**8 + 3
0*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**5)/(acos(a*x)*a
**2*x**2 - acos(a*x)),x)*a**6 + 6*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sq
rt(acos(a*x))*x**3)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**4 + 8*acos(a*x
)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)*a**2*x**2 - ac
os(a*x)),x)*a**2 + 5*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a**6*x**6 - 2*
sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a**2*x**2 - 12*sqrt( - a**2*x**2 +
1)*sqrt(acos(a*x))))/(5*acos(a*x)*a**7)
```

3.100 $\int \frac{x^5}{\arccos(ax)^{3/2}} dx$

Optimal result 759
Mathematica [C] (verified) 759
Rubi [A] (verified) 760
Maple [A] (verified) 761
Fricas [F(-2)] 762
Sympy [F] 762
Maxima [F(-2)] 762
Giac [F] 763
Mupad [F(-1)] 763
Reduce [F] 763

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{x^5}{\arccos(ax)^{3/2}} dx = \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^6} - \frac{\sqrt{3\pi}\operatorname{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^6} - \frac{5\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^6}$$

output
```
2*x^5*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*FresnelC
(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^6-1/8*3^(1/2)*Pi^(1/2)*FresnelC(2
*3^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^6-5/8*Pi^(1/2)*FresnelC(2*arccos(a*
x)^(1/2)/Pi^(1/2))/a^6
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.78

$$\int \frac{x^5}{\arccos(ax)^{3/2}} dx = \frac{i\left(5\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-2i\arccos(ax)\right) - 5\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},2i\arccos(ax)\right)\right.}{}$$

input
```
Integrate[x^5/ArcCos[a*x]^(3/2),x]
```

output
```
((I/32)*(5*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] -
 5*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] + 8*Sqrt[(-I)
*ArcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 8*Sqrt[I*ArcCos[a*x]]*Gamma
[1/2, (4*I)*ArcCos[a*x]] + Sqrt[6]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-6*I
)*ArcCos[a*x]] - Sqrt[6]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (6*I)*ArcCos[a*x]]
 - (10*I)*Sin[2*ArcCos[a*x]] - (8*I)*Sin[4*ArcCos[a*x]] - (2*I)*Sin[6*ArcC
os[a*x]]))/(a^6*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{2\int \left(-\frac{5\cos(2\arccos(ax))}{16\sqrt{\arccos(ax)}}-\frac{\cos(4\arccos(ax))}{2\sqrt{\arccos(ax)}}-\frac{3\cos(6\arccos(ax))}{16\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^6}+\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{16}\sqrt{3\pi}\,\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{5}{16}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right.\right.}{a^6}$$

$$\frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

input
```
Int[x^5/ArcCos[a*x]^(3/2),x]
```

output
```
(2*x^5*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) + (2*(-1/2*(Sqrt[Pi/2]*Fre
snelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (Sqrt[3*Pi]*FresnelC[2*Sqrt[3/Pi]
*Sqrt[ArcCos[a*x]]])/16 - (5*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[
Pi]])/16))/a^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result
default	$\frac{-2\sqrt{\pi}\,\sqrt{3}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{6}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\arccos(ax)}-8\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}-10\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right.}{16a^{6}\sqrt{\arccos(ax)}}$

input
```
int(x^5/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a^6/arccos(a*x)^(1/2)*(-2*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*
6^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(1/2)-8*FresnelC(2*2^(1/2)/Pi^(1/2)
*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)-10*FresnelC(2*arcco
s(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)*arccos(a*x)^(1/2)+5*sin(2*arccos(a*x))+4*s
in(4*arccos(a*x))+sin(6*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^5}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**5/acos(a*x)**(3/2),x)
```

output
```
Integral(x**5/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^5}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^5/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^5}{\mathrm{acos}(a\,x)^{3/2}}\,dx$$

input
```
int(x^5/acos(a*x)^(3/2),x)
```

output
```
int(x^5/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \frac{30\sqrt{acos(ax)}\,acos(ax) - 48acos(ax)\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^6}{acos(ax)a^2x^2-acos(ax)}dx\right)a^7 + 40acos(ax)\left(\right.}{}$$

input
```
int(x^5/acos(a*x)^(3/2),x)
```

output
```
(30*sqrt(acos(a*x))*acos(a*x) - 48*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*s
qrt(acos(a*x))*x**6)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**7 + 40*acos(a
*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**4)/(acos(a*x)*a**2*x**2
 - acos(a*x)),x)*a**5 - 15*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos
(a*x)))/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a + 8*sqrt( - a**2*x**2 + 1)*
sqrt(acos(a*x))*a**5*x**5)/(4*acos(a*x)*a**6)
```

3.101 $\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx$

Optimal result 764
Mathematica [C] (verified) 764
Rubi [A] (verified) 765
Maple [A] (verified) 766
Fricas [F(-2)] 767
Sympy [F] 767
Maxima [F(-2)] 767
Giac [F] 768
Mupad [F(-1)] 768
Reduce [F] 768

Optimal result

Integrand size = 12, antiderivative size = 136

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} - \frac{3\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5}$$

output
```
2*x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-1/4*2^(1/2)*Pi^(1/2)*FresnelC
(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5-3/8*6^(1/2)*Pi^(1/2)*FresnelC(6^(
1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5-1/8*10^(1/2)*Pi^(1/2)*FresnelC(10^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \frac{i\left(-4i\sqrt{1-a^2x^2} + 2\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2}, -i\arccos(ax)\right) - 2\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2}, i\,\text{ar}\right.}{}$$

input
```
Integrate[x^4/ArcCos[a*x]^(3/2),x]
```

output
```
((I/16)*((-4*I)*Sqrt[1 - a^2*x^2] + 2*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-
I)*ArcCos[a*x]] - 2*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + 3*Sqrt
[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 3*Sqrt[3]*Sqrt
[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] + Sqrt[5]*Sqrt[(-I)*ArcCos[a
*x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/
2, (5*I)*ArcCos[a*x]] - (6*I)*Sin[3*ArcCos[a*x]] - (2*I)*Sin[5*ArcCos[a*x]
]))/(a^5*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{2\int \left(-\frac{ax}{8\sqrt{\arccos(ax)}}-\frac{9\cos(3\arccos(ax))}{16\sqrt{\arccos(ax)}}-\frac{5\cos(5\arccos(ax))}{16\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^5}+\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{3}{8}\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{8}\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos\right.\right.}{a^5}$$

$$\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

input
```
Int[x^4/ArcCos[a*x]^(3/2),x]
```

output
```
(2*x^4*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) + (2*(-1/4*(Sqrt[Pi/2]*Fre
snelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (3*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/P
i]*Sqrt[ArcCos[a*x]]])/8 - (Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCo
s[a*x]]])/8))/a^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

method	result
default	$\frac{-\sqrt{5}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-3\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-2\sqrt{2}\,\sqrt{\pi}\,\sqrt{\text{ar}}}{8a^5\sqrt{\arccos(ax)}}$

input
```
int(x^4/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^5*(-5^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)*arccos(a*x)^(1/2))-3*3^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/
2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-2*2^(1/2)*Pi^(1/2)
*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+2*(-a^2*x^
2+1)^(1/2)+3*sin(3*arccos(a*x))+sin(5*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**4/acos(a*x)**(3/2),x)
```

output
```
Integral(x**4/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^4}{\arccos\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x^4/acos(a*x)^(3/2),x)
```

output
```
int(x^4/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \frac{-\frac{4acos(ax)\left(\int \frac{\sqrt{acos(ax)}}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a}{3}+\frac{4acos(ax)\left(\int \frac{\sqrt{acos(ax)}\,x^2}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a^3}{3}-10acos(ax)$$

input
```
int(x^4/acos(a*x)^(3/2),x)
```

output

```
(2*( - 2*acos(a*x)*int(sqrt(acos(a*x))/(acos(a*x)**2*a**2*x**2 - acos(a*x)
**2),x)*a + 2*acos(a*x)*int((sqrt(acos(a*x))*x**2)/(acos(a*x)**2*a**2*x**2
 - acos(a*x)**2),x)*a**3 - 15*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(a
cos(a*x))*x**5)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**6 + 12*acos(a*x)*i
nt((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)/(acos(a*x)*a**2*x**2 - ac
os(a*x)),x)*a**4 + 4*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))
*x)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**2 + 3*sqrt( - a**2*x**2 + 1)*s
qrt(acos(a*x))*a**4*x**4 - 4*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))))/(3*a
cos(a*x)*a**5)
```

3.102 $\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx$

Optimal result 770
Mathematica [C] (verified) 770
Rubi [A] (verified) 771
Maple [A] (verified) 772
Fricas [F(-2)] 772
Sympy [F] 773
Maxima [F(-2)] 773
Giac [F] 773
Mupad [F(-1)] 774
Reduce [F] 774

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^4} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^4}$$

output
```
2*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*FresnelC
(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^4-Pi^(1/2)*FresnelC(2*arccos(a*x)
^(1/2)/Pi^(1/2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.69

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \frac{i\sqrt{2}\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-2i\arccos(ax)\right) - i\sqrt{2}\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},2i\arccos(ax)\right) +$$

input
```
Integrate[x^3/ArcCos[a*x]^(3/2),x]
```

output
```
(I*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] - I*Sqrt[
2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] + I*Sqrt[(-I)*ArcCos[
a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - I*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (4
*I)*ArcCos[a*x]] + 2*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*x]])/(4*a^4*Sqrt[
ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5143}$$

$$\frac{2\int \left(-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}-\frac{\cos(4\arccos(ax))}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

input
```
Int[x^3/ArcCos[a*x]^(3/2),x]
```

output
```
(2*x^3*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) + (2*(-1/2*(Sqrt[Pi/2]*Fre
snelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos
[a*x]])/Sqrt[Pi]])/2))/a^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

method	result
default	$\frac{-2\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}-4\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}\,\sqrt{\arccos(ax)}+2\sin(2\arccos(ax))+\sin(4\,\mathrm{arcc}}{4a^4\sqrt{\arccos(ax)}}$

input
```
int(x^3/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^4/arccos(a*x)^(1/2)*(-2*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2
))*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)-4*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1
/2))*Pi^(1/2)*arccos(a*x)^(1/2)+2*sin(2*arccos(a*x))+sin(4*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**3/acos(a*x)**(3/2),x)
```

output
```
Integral(x**3/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^3}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x^3/acos(a*x)^(3/2),x)
```

output
```
int(x^3/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \frac{12\sqrt{acos\left(ax\right)}\,acos(ax) - 3acos(ax)\left(\int \frac{\sqrt{acos(ax)}\,x^3}{acos(ax)^2a^2x^2 - acos(ax)^2}dx\right)a^4 + 3acos(ax)\left(\int\right.}{}$$

input
```
int(x^3/acos(a*x)^(3/2),x)
```

output
```
(12*sqrt(acos(a*x))*acos(a*x) - 3*acos(a*x)*int((sqrt(acos(a*x))*x**3)/(ac
os(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**4 + 3*acos(a*x)*int((sqrt(acos(
a*x))*x)/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**2 - 16*acos(a*x)*in
t((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**4)/(acos(a*x)*a**2*x**2 - aco
s(a*x)),x)*a**5 + 4*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a**3*x**3 + 6*s
qrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a*x)/(2*acos(a*x)*a**4)
```

3.103 $\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx$

Optimal result 775
Mathematica [C] (verified) 775
Rubi [A] (verified) 776
Maple [A] (verified) 777
Fricas [F(-2)] 777
Sympy [F] 778
Maxima [F(-2)] 778
Giac [F] 778
Mupad [F(-1)] 779
Reduce [F] 779

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3} - \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}$$

output
```
2*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*FresnelC
(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3-1/2*6^(1/2)*Pi^(1/2)*FresnelC(6^(
1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \frac{i\left(-2i\sqrt{1-a^2x^2} + \sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-i\arccos(ax)\right) - \sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},i\arccos\right.\right.}{}$$

input
```
Integrate[x^2/ArcCos[a*x]^(3/2),x]
```

output
```
((I/4)*((-2*I)*Sqrt[1 - a^2*x^2] + Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*
ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + Sqrt[3]*Sqr
t[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos
[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] - (2*I)*Sin[3*ArcCos[a*x]]))/(a^3*Sqr
t[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \; 5143$$

$$\frac{2\int \left(-\frac{ax}{4\sqrt{\arccos(ax)}}-\frac{3\cos(3\arccos(ax))}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

input
```
Int[x^2/ArcCos[a*x]^(3/2),x]
```

output
```
(2*x^2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) + (2*(-1/2*(Sqrt[Pi/2]*Fre
snelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]
*Sqrt[ArcCos[a*x]]])/2))/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

method	result
default	$\frac{-\sqrt{3}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(ax)}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{-a^{2}x^{2}+1}+\sin(3\,\mathrm{ar}}{2a^{3}\sqrt{\arccos(ax)}}$

input
```
int(x^2/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(-3^(1/2)*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)*arccos(a*x)^(1/2))-2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+(-a^2*x^2+1)^(1/2)+sin(3*arccos(a*x)
))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**2/acos(a*x)**(3/2),x)
```

output
```
Integral(x**2/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^2}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{acos}\,(a\,x)^{3/2}}\,dx$$

input
```
int(x^2/acos(a*x)^(3/2),x)
```

output
```
int(x^2/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \frac{-6acos(ax)\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^3}{acos(ax)a^2x^2-acos(ax)}dx\right)a^2+4acos(ax)\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)a^2x^2-acos(ax)}dx\right)+}{acos\,(ax)\,a}$$

input
```
int(x^2/acos(a*x)^(3/2),x)
```

output
```
(2*( - 3*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)/(acos
(a*x)*a**2*x**2 - acos(a*x)),x)*a**2 + 2*acos(a*x)*int((sqrt( - a**2*x**2
+ 1)*sqrt(acos(a*x))*x)/(acos(a*x)*a**2*x**2 - acos(a*x)),x) + sqrt( - a**
2*x**2 + 1)*sqrt(acos(a*x))*x**2))/(acos(a*x)*a)
```

3.104 $\int \frac{x}{\arccos(ax)^{3/2}}\,dx$

Optimal result . . . 780
Mathematica [A] (verified) . . . 780
Rubi [A] (verified) . . . 781
Maple [A] (verified) . . . 782
Fricas [F(-2)] . . . 783
Sympy [F] . . . 783
Maxima [F(-2)] . . . 783
Giac [F] . . . 784
Mupad [F(-1)] . . . 784
Reduce [F] . . . 784

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

output
```
2*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-2*Pi^(1/2)*FresnelC(2*arccos(a*
x)^(1/2)/Pi^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \frac{-2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) + \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}}{a^2}$$

input
```
Integrate[x/ArcCos[a*x]^(3/2),x]
```

output
```
(-2*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sin[2*ArcCos[a*x]]
/Sqrt[ArcCos[a*x]])/a^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5143, 25, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{2\int -\frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int \cos(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

input
```
Int[x/ArcCos[a*x]^(3/2),x]
```

output
```
(2*x*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[Pi]*FresnelC[(2*Sq
rt[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{-2\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}\,\sqrt{\arccos(ax)}+\sin(2\arccos(ax))}{a^{2}\sqrt{\arccos(ax)}}$	42

input
```
int(x/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/arccos(a*x)^(1/2)*(-2*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2
)*arccos(a*x)^(1/2)+sin(2*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/acos(a*x)**(3/2),x)
```

output
```
Integral(x/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \int \frac{x}{\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x/acos(a*x)^(3/2),x)
```

output
```
int(x/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \frac{4\sqrt{acos\left(ax\right)}\,acos(ax) - 4acos(ax)\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^2}{acos(ax)a^2x^2-acos(ax)}dx\right)a^3 + 2\sqrt{-a^2x^2+1}\,\sqrt{}}{acos\left(ax\right)a^2}$$

input
```
int(x/acos(a*x)^(3/2),x)
```

output
```
(2*(2*sqrt(acos(a*x))*acos(a*x) - 2*acos(a*x)*int((sqrt( - a**2*x**2 + 1)*
sqrt(acos(a*x))*x**2)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**3 + sqrt( -
a**2*x**2 + 1)*sqrt(acos(a*x))*a*x))/(acos(a*x)*a**2)
```

3.105 $\int \frac{1}{\arccos(ax)^{3/2}}\,dx$

Optimal result . . . 785
Mathematica [C] (verified) . . . 785
Rubi [A] (verified) . . . 786
Maple [A] (verified) . . . 788
Fricas [F(-2)] . . . 788
Sympy [F] . . . 788
Maxima [F(-2)] . . . 789
Giac [F] . . . 789
Mupad [F(-1)] . . . 789
Reduce [F] . . . 790

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx=\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

output
```
2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)-2*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = -\frac{-2\sqrt{1-a^2x^2}-i\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)+i\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},i\arccos(ax)\right)}{a\sqrt{\arccos(ax)}}$$

input
```
Integrate[ArcCos[a*x]^(-3/2),x]
```

output
```
-((-2*Sqrt[1 - a^2*x^2] - I*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[
a*x]] + I*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*
x]]))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5133, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{5133}$$

$$2a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{5225}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{ax}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int axd\sqrt{\arccos(ax)}}{a}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

input
```
Int[ArcCos[a*x]^(-3/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[2*Pi]*FresnelC[Sqrt[
2/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\sqrt{2}\left(2\pi \arccos\left(a x \right) \operatorname{FresnelC}\left(\frac{\sqrt{2}\, \sqrt{\arccos\left(a x \right)}}{\sqrt{\pi}}\right)-\sqrt{2}\, \sqrt{\pi}\, \sqrt{\arccos\left(a x \right)}\, \sqrt{-a^{2} x^{2}+1}\right)}{a \sqrt{\pi}\, \arccos\left(a x \right)}$	66

input
```
int(1/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*2^(1/2)/Pi^(1/2)/arccos(a*x)*(2*Pi*arccos(a*x)*FresnelC(2^(1/2)/Pi^(1
/2)*arccos(a*x)^(1/2))-2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)*(-a^2*x^2+1)^(1/
2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/acos(a*x)**(3/2),x)
```

output
```
Integral(acos(a*x)**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\arccos\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/acos(a*x)^(3/2),x)
```

output
```
int(1/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \frac{-2acos(ax)\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)a^2x^2-acos(ax)}dx\right)a^2+2\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}}{acos\,(ax)\,a}$$

input
```
int(1/acos(a*x)^(3/2),x)
```

output
```
(2*( - acos(a*x)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)
*a**2*x**2 - acos(a*x)),x)*a**2 + sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))))
/(acos(a*x)*a)
```

3.106 $\int \frac{1}{x \arccos(ax)^{3/2}} \, dx$

Optimal result . . . 791
Mathematica [N/A] . . . 791
Rubi [N/A] . . . 792
Maple [N/A] . . . 792
Fricas [F(-2)] . . . 793
Sympy [N/A] . . . 793
Maxima [F(-2)] . . . 793
Giac [N/A] . . . 794
Mupad [N/A] . . . 794
Reduce [N/A] . . . 794

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \text{Int}\left(\frac{1}{x \arccos(ax)^{3/2}}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \int \frac{1}{x \arccos(ax)^{3/2}} \, dx$$

input
```
Integrate[1/(x*ArcCos[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx$$

$$\Big\downarrow \; 5149$$

$$\int \frac{1}{x \arccos(ax)^{3/2}} dx$$

input
```
Int[1/(x*ArcCos[a*x]^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arccos\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/arccos(a*x)^(3/2),x)
```

output
```
int(1/x/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \int \frac{1}{x \operatorname{acos}^{\frac{3}{2}}(ax)} \, dx$$

input
```
integrate(1/x/acos(a*x)**(3/2),x)
```

output
```
Integral(1/(x*acos(a*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{3/2}}\,dx = \int \frac{1}{x \arccos{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/(x*acos(a*x)^(3/2)),x)
```

output
```
int(1/(x*acos(a*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x \arccos(ax)^{3/2}}\,dx = \int \frac{\sqrt{acos(ax)}}{acos(ax)^2\,x}dx$$

input
```
int(1/x/acos(a*x)^(3/2),x)
```

output
```
int(sqrt(acos(a*x))/(acos(a*x)**2*x),x)
```

3.107 $\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx$

Optimal result . . . 796
Mathematica [C] (verified) . . . 797
Rubi [A] (verified) . . . 797
Maple [A] (verified) . . . 800
Fricas [F(-2)] . . . 800
Sympy [F] . . . 801
Maxima [F(-2)] . . . 801
Giac [F] . . . 801
Mupad [F(-1)] . . . 802
Reduce [F] . . . 802

Optimal result

Integrand size = 12, antiderivative size = 235

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} + \frac{25\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5} - \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{25\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} - \frac{4\sqrt{\frac{2\pi}{3}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^5}$$

output
```
2/3*x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)-16/3*x^3/a^2/arccos(a*x)^(1
/2)+20/3*x^5/arccos(a*x)^(1/2)+1/6*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1
/2)*arccos(a*x)^(1/2))/a^5+3/4*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*
arccos(a*x)^(1/2))/a^5+5/12*10^(1/2)*Pi^(1/2)*FresnelS(10^(1/2)/Pi^(1/2)*a
rccos(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.37

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx =$$
$$-\frac{2\left(-\sqrt{1-a^2x^2}-e^{-i\arccos(ax)}\arccos(ax)-e^{i\arccos(ax)}\arccos(ax)+\sqrt{-i\arccos(ax)}\arccos(ax)\Gamma\left(\frac{1}{2},-i\,\mathrm{a}\right.}{}$$

input
```
Integrate[x^4/ArcCos[a*x]^(5/2),x]
```

output
```
-1/24*(2*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos
[a*x])*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*Ar
cCos[a*x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]]) -
5*ArcCos[a*x]*(E^((-5*I)*ArcCos[a*x]) + E^((5*I)*ArcCos[a*x]) - Sqrt[5]*Sq
rt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCo
s[a*x]]*Gamma[1/2, (5*I)*ArcCos[a*x]]) - 3*(3*ArcCos[a*x]*(E^((-3*I)*ArcCo
s[a*x]) + E^((3*I)*ArcCos[a*x]) - Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2
, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCo
s[a*x]]) + Sin[3*ArcCos[a*x]]) - Sin[5*ArcCos[a*x]])/(a^5*ArcCos[a*x]^(3/2
))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5145, 5223, 5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx$$

$$\Big\downarrow \; 5145$$

$$\frac{10}{3}a\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx-\frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx}{3a}+\frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{10}{3}a\left(\frac{2x^5}{a\sqrt{\arccos(ax)}}-\frac{10\int \frac{x^4}{\sqrt{\arccos(ax)}}dx}{a}\right)-\frac{8\left(\frac{2x^3}{a\sqrt{\arccos(ax)}}-\frac{6\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{a}\right)}{3a}+\frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{8\left(\frac{6\int \frac{a^2x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^4}+\frac{2x^3}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{10}{3}a\left(\frac{10\int \frac{a^4x^4\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^6}+\frac{2x^5}{a\sqrt{\arccos(ax)}}\right)+\frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{10}{3}a\left(\frac{10\int \left(\frac{3\sin(3\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\sin(5\arccos(ax))}{16\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^6}+\frac{2x^5}{a\sqrt{\arccos(ax)}}\right)-\frac{8\left(\frac{6\int \left(\frac{\sin(3\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^4}+\frac{2x^3}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{10}{3}a\left(\frac{10\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{8}\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\text{ar}\right.\right.}{a^6}\right.$$

$$\frac{8\left(\frac{6\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^4}+\frac{2x^3}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

input
```
Int[x^4/ArcCos[a*x]^(5/2),x]
```

output
```
(2*x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (8*((2*x^3)/(a*Sqrt[Ar
cCos[a*x]]) + (6*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/2 +
(Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/2))/a^4))/(3*a) + (10*
a*((2*x^5)/(a*Sqrt[ArcCos[a*x]]) + (10*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sq
rt[ArcCos[a*x]]])/4 + (Sqrt[(3*Pi)/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]
]])/8 + (Sqrt[Pi/10]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/8))/a^6))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

method	result
default	$\frac{18\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+10\sqrt{2}\,\sqrt{\pi}\,\sqrt{5}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{\pi}\,\text{Fres}}{}$

input
```
int(x^4/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/a^5*(18*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*ar
ccos(a*x)^(1/2))*arccos(a*x)^(3/2)+10*2^(1/2)*Pi^(1/2)*5^(1/2)*FresnelS(2^
(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+4*2^(1/2)*Pi^(
1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+4*a*x*
arccos(a*x)+18*arccos(a*x)*cos(3*arccos(a*x))+10*arccos(a*x)*cos(5*arccos(
a*x))+2*(-a^2*x^2+1)^(1/2)+3*sin(3*arccos(a*x))+sin(5*arccos(a*x)))/arccos
(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**4/acos(a*x)**(5/2),x)
```

output
```
Integral(x**4/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^4}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^4/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{acos}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int(x^4/acos(a*x)^(5/2),x)
```

output
```
int(x^4/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = -\frac{4acos(ax)^2\left(\int \frac{\sqrt{acos(ax)}}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right)a}{3} + \frac{4acos(ax)^2\left(\int \frac{\sqrt{acos(ax)}\,x^2}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right)a^3}{3} - \frac{10acos(ax)^2}{}$$

input
```
int(x^4/acos(a*x)^(5/2),x)
```

output
```
(2*( - 6*acos(a*x)**2*int(sqrt(acos(a*x))/(acos(a*x)**3*a**2*x**2 - acos(a
*x)**3),x)*a + 6*acos(a*x)**2*int((sqrt(acos(a*x))*x**2)/(acos(a*x)**3*a**
2*x**2 - acos(a*x)**3),x)*a**3 - 15*acos(a*x)**2*int((sqrt( - a**2*x**2 +
1)*sqrt(acos(a*x))*x**5)/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**6 +
 12*acos(a*x)**2*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)/(acos(a
*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**4 + 4*acos(a*x)**2*int((sqrt( - a**
2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*
a**2 + 3*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a**4*x**4 - 4*sqrt( - a**2
*x**2 + 1)*sqrt(acos(a*x))))/(9*acos(a*x)**2*a**5)
```

3.108 $\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx$

Optimal result 803
Mathematica [C] (verified) 803
Rubi [A] (verified) 804
Maple [A] (verified) 808
Fricas [F(-2)] 808
Sympy [F] 809
Maxima [F(-2)] 809
Giac [F(-2)] 809
Mupad [F(-1)] 810
Reduce [F] 810

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} + \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^4} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^4}$$

output
```
2/3*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)-4*x^2/a^2/arccos(a*x)^(1/2)
+16/3*x^4/arccos(a*x)^(1/2)+4/3*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/
2)*arccos(a*x)^(1/2))/a^4+4/3*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/
2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.61

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = -\frac{-4\arccos(ax)\left(e^{-4i\arccos(ax)} + e^{4i\arccos(ax)} - 2\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-4i\arccos(ax)\right) - 2\sqrt{i\arccos(ax)}\,\Gamma\left(\right.\right.}{}$$

input
```
Integrate[x^3/ArcCos[a*x]^(5/2),x]
```

output
```
-1/12*(-4*ArcCos[a*x]*(E^((-4*I)*ArcCos[a*x]) + E^((4*I)*ArcCos[a*x]) - 2*
Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 2*Sqrt[I*ArcCos[a*
x]]*Gamma[1/2, (4*I)*ArcCos[a*x]]) - 2*(2*ArcCos[a*x]*(E^((-2*I)*ArcCos[a*
x]) + E^((2*I)*ArcCos[a*x]) - Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-
2*I)*ArcCos[a*x]] - Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*
x]]) + Sin[2*ArcCos[a*x]]) - Sin[4*ArcCos[a*x]])/(a^4*ArcCos[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5145, 5223, 5147, 4906, 27, 2009, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)^{5/2}} \, dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{2\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx}{a}+\frac{8}{3}a\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$-\frac{2\left(\frac{2x^2}{a\sqrt{\arccos(ax)}}-\frac{4\int \frac{x}{\sqrt{\arccos(ax)}}dx}{a}\right)}{a}+\frac{8}{3}a\left(\frac{2x^4}{a\sqrt{\arccos(ax)}}-\frac{8\int \frac{x^3}{\sqrt{\arccos(ax)}}dx}{a}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{2\left(\frac{4\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{8}{3}a\left(\frac{8\int \frac{a^3x^3\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{8}{3}a\left(\frac{8\int \left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)-\frac{2\left(\frac{4\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{3}a\left(\frac{8\int \left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)-\frac{2\left(\frac{2\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{2\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{8}{3}a\left(\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{2\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{8}{3}a\left(\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3786$$

$$-\frac{2\left(\frac{4\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{8}{3}a\left(\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3832$$

$$\frac{8}{3}a\left(\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{4}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^5}+\frac{2x^4}{a\sqrt{\arccos(ax)}}\right)-\frac{2\left(\frac{2\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)}{a}+\frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

input
```
Int[x^3/ArcCos[a*x]^(5/2),x]
```

output
```
(2*x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (2*((2*x^2)/(a*Sqrt[Ar
cCos[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^3))/
a + (8*a*((2*x^4)/(a*Sqrt[ArcCos[a*x]]) + (8*((Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcCos[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqr
t[Pi]])/4))/a^5))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

method	result
default	$\frac{16\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+16\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+8\cos(2\arccos(ax))\arccos(ax)+}{12a^4\arccos(ax)^{\frac{3}{2}}}$

input
```
int(x^3/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/a^4*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2
))*arccos(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*ar
ccos(a*x)^(3/2)+8*cos(2*arccos(a*x))*arccos(a*x)+8*arccos(a*x)*cos(4*arcco
s(a*x))+sin(4*arccos(a*x))+2*sin(2*arccos(a*x)))/arccos(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**3/acos(a*x)**(5/2),x)
```

output
```
Integral(x**3/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int(x^3/acos(a*x)^(5/2),x)
```

output
```
int(x^3/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \frac{-9acos(ax)^2\left(\int \frac{\sqrt{acos(ax)}\,x^3}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right)a^4+9acos(ax)^2\left(\int \frac{\sqrt{acos(ax)}\,x}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right.}{}$$

input
```
int(x^3/acos(a*x)^(5/2),x)
```

output
```
( - 9*acos(a*x)**2*int((sqrt(acos(a*x))*x**3)/(acos(a*x)**3*a**2*x**2 - ac
os(a*x)**3),x)*a**4 + 9*acos(a*x)**2*int((sqrt(acos(a*x))*x)/(acos(a*x)**3
*a**2*x**2 - acos(a*x)**3),x)*a**2 - 16*acos(a*x)**2*int((sqrt( - a**2*x**
2 + 1)*sqrt(acos(a*x))*x**4)/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*a*
*5 - 12*sqrt(acos(a*x))*acos(a*x) + 4*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x
))*a**3*x**3 + 6*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a*x)/(6*acos(a*x)*
*2*a**4)
```

3.109 $\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx$

Optimal result 811
Mathematica [C] (verified) 811
Rubi [A] (verified) 812
Maple [A] (verified) 816
Fricas [F(-2)] 816
Sympy [F] 816
Maxima [F(-2)] 817
Giac [F] 817
Mupad [F(-1)] 817
Reduce [F] 818

Optimal result

Integrand size = 12, antiderivative size = 125

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} + \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}$$

output
```
2/3*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)-8/3*x/a^2/arccos(a*x)^(1/2)
+4*x^3/arccos(a*x)^(1/2)+1/3*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*ar
ccos(a*x)^(1/2))/a^3+6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x
)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = -\frac{-\sqrt{1-a^2x^2} - e^{-i\arccos(ax)}\arccos(ax) - e^{i\arccos(ax)}\arccos(ax) + \sqrt{-i\arccos(ax)}\arccos(ax)\Gamma\left(\frac{1}{2}, -i\arccos\right.}{}$$

input
```
Integrate[x^2/ArcCos[a*x]^(5/2),x]
```

output
```
-1/6*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x
])*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos
[a*x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]] - 3*Arc
Cos[a*x]*(E^((-3*I)*ArcCos[a*x]) + E^((3*I)*ArcCos[a*x]) - Sqrt[3]*Sqrt[(-
I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x
]]*Gamma[1/2, (3*I)*ArcCos[a*x]]) - Sin[3*ArcCos[a*x]])/(a^3*ArcCos[a*x]^(
3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5145, 5223, 5135, 3042, 3786, 3832, 5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{4\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx}{3a}+2a\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$2a\left(\frac{2x^3}{a\sqrt{\arccos(ax)}}-\frac{6\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{a}\right)-\frac{4\left(\frac{2x}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{1}{\sqrt{\arccos(ax)}}dx}{a}\right)}{3a}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5135}$$

$$-\frac{4\left(\frac{2\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+2a\left(\frac{2x^3}{a\sqrt{\arccos(ax)}}-\frac{6\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{a}\right)+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{4\left(\frac{2\int \frac{\sin(\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+2a\left(\frac{2x^3}{a\sqrt{\arccos(ax)}}-\frac{6\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{a}\right)+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3786$$

$$-\frac{4\left(\frac{4\int \sqrt{1-a^2x^2}d\sqrt{\arccos(ax)}}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+2a\left(\frac{2x^3}{a\sqrt{\arccos(ax)}}-\frac{6\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{a}\right)+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3832$$

$$2a\left(\frac{2x^3}{a\sqrt{\arccos(ax)}}-\frac{6\int \frac{x^2}{\sqrt{\arccos(ax)}}dx}{a}\right)-\frac{4\left(\frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 5147$$

$$2a\left(\frac{6\int \frac{a^2x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^4}+\frac{2x^3}{a\sqrt{\arccos(ax)}}\right)-\frac{4\left(\frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$2a\left(\frac{6\int \left(\frac{\sin(3\arccos(ax))}{4\sqrt{\arccos(ax)}}+\frac{\sqrt{1-a^2x^2}}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^4}+\frac{2x^3}{a\sqrt{\arccos(ax)}}\right)-\frac{4\left(\frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$2a\left(\frac{6\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^4}+\frac{2x^3}{a\sqrt{\arccos(ax)}}\right)-\frac{4\left(\frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)}{3a}+\frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

input
```
Int[x^2/ArcCos[a*x]^(5/2),x]
```

output
```
(2*x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (4*((2*x)/(a*Sqrt[ArcC
os[a*x]]) + (2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^2))/(3
*a) + 2*a*((2*x^3)/(a*Sqrt[ArcCos[a*x]]) + (6*((Sqrt[Pi/2]*FresnelS[Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/2 + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a
*x]]])/2))/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

method	result
default	$\frac{6\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+2\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+2ax\arccos(ax)+6\,\mathrm{arcc}}{6a^{3}\arccos(ax)^{\frac{3}{2}}}$

input
```
int(x^2/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^3*(6*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcc
os(a*x)^(1/2))*arccos(a*x)^(3/2)+2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1
/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+2*a*x*arccos(a*x)+6*arccos(a*x)*c
os(3*arccos(a*x))+(-a^2*x^2+1)^(1/2)+sin(3*arccos(a*x)))/arccos(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**2/acos(a*x)**(5/2),x)
```

output
```
Integral(x**2/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^2}{{\mathrm{acos}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int(x^2/acos(a*x)^(5/2),x)
```

output
```
int(x^2/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \frac{-2acos(ax)^2 \left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^3}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a^2 + \frac{4acos(ax)^2\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)}{3} + 2\sqrt{\cdots}}{acos\,(ax)^2\,a}$$

input

```
int(x^2/acos(a*x)^(5/2),x)
```

output

```
(2*( - 3*acos(a*x)**2*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)/(a
cos(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**2 + 2*acos(a*x)**2*int((sqrt(
- a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2
),x) + sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2))/(3*acos(a*x)**2*a)
```

3.110 $\int \frac{x}{\arccos(ax)^{5/2}}\,dx$

Optimal result 819
Mathematica [A] (verified) 819
Rubi [A] (verified) 820
Maple [A] (verified) 823
Fricas [F(-2)] 823
Sympy [F] 823
Maxima [F(-2)] 824
Giac [F] 824
Mupad [F(-1)] 824
Reduce [F] 825

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} + \frac{8\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^2}$$

output
```
2/3*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)-4/3/a^2/arccos(a*x)^(1/2)+8/3
*x^2/arccos(a*x)^(1/2)+8/3*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))
/a^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \frac{8\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) + \frac{4\arccos(ax)\cos(2\arccos(ax))+\sin(2\arccos(ax))}{\arccos(ax)^{3/2}}}{3a^2}$$

input
```
Integrate[x/ArcCos[a*x]^(5/2),x]
```

output
```
(8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + (4*ArcCos[a*x]*Cos[
2*ArcCos[a*x]] + Sin[2*ArcCos[a*x]])/ArcCos[a*x]^(3/2))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5145, 5153, 5223, 5147, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)^{5/2}}dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx}{3a}+\frac{4}{3}a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{4}{3}a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{4}{3}a\left(\frac{2x^2}{a\sqrt{\arccos(ax)}}-\frac{4\int \frac{x}{\sqrt{\arccos(ax)}}dx}{a}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{5147}$$

$$\frac{4}{3}a\left(\frac{4\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{4}{3}a\left(\frac{4\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{3}a\left(\frac{2\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\left(\frac{2\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{4}{3}a\left(\frac{4\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{4}{3}a\left(\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^3}+\frac{2x^2}{a\sqrt{\arccos(ax)}}\right)+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}$$

input
```
Int[x/ArcCos[a*x]^(5/2),x]
```

output
```
(2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcCos[a*x
]]) + (4*a*((2*x^2)/(a*Sqrt[ArcCos[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2*Sqrt[A
rcCos[a*x]])/Sqrt[Pi]])/a^3))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{8\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+4\cos(2\arccos(ax))\arccos(ax)+\sin(2\arccos(ax))}{3a^2\arccos(ax)^{\frac{3}{2}}}$	56

input
```
int(x/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(8*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*arccos(a*x)^(3/
2)+4*cos(2*arccos(a*x))*arccos(a*x)+sin(2*arccos(a*x)))/arccos(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x/acos(a*x)**(5/2),x)
```

output
```
Integral(x/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \int \frac{x}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \int \frac{x}{\mathrm{acos}(a\,x)^{5/2}}\,dx$$

input
```
int(x/acos(a*x)^(5/2),x)
```

output
```
int(x/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \frac{-\frac{4acos(ax)^2\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^2}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a^3}{3}-\frac{4\sqrt{acos(ax)}\,acos(ax)}{3}+\frac{2\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,ax}{3}}{acos\,(ax)^2\,a^2}$$

input
```
int(x/acos(a*x)^(5/2),x)
```

output
```
(2*( - 2*acos(a*x)**2*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/(a
cos(a*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**3 - 2*sqrt(acos(a*x))*acos(a*x
) + sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a*x))/(3*acos(a*x)**2*a**2)
```

3.111 $\int \frac{1}{\arccos(ax)^{5/2}}\,dx$

Optimal result 826
Mathematica [C] (verified) 826
Rubi [A] (verified) 827
Maple [A] (verified) 829
Fricas [F(-2)] 829
Sympy [F] 830
Maxima [F(-2)] 830
Giac [F] 830
Mupad [F(-1)] 831
Reduce [F] 831

Optimal result

Integrand size = 8, antiderivative size = 76

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} + \frac{4x}{3\sqrt{\arccos(ax)}} + \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a}$$

output
```
2/3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)+4/3*x/arccos(a*x)^(1/2)+4/3*2^(
1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = -\frac{2\left(-\sqrt{1-a^2x^2} - e^{-i\arccos(ax)}\arccos(ax) - e^{i\arccos(ax)}\arccos(ax) + \sqrt{-i\arccos(ax)}\arccos(ax)\Gamma\left(\frac{1}{2}, -i\,\text{a}\right.}{3a\arccos(ax)^{3/2}}$$

input
```
Integrate[ArcCos[a*x]^(-5/2),x]
```

output
```
(-2*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x]
)*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[
a*x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]]))/(3*a*A
rcCos[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5133, 5223, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{2}{3}a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}dx+\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{2}{3}a\left(\frac{2x}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{1}{\sqrt{\arccos(ax)}}dx}{a}\right)+\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5135}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)+\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\left(\frac{2\int \frac{\sin(\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)+\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2}{3}a\left(\frac{4\int\sqrt{1-a^2x^2}d\sqrt{\arccos(ax)}}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)+\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2}{3}a\left(\frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^2}+\frac{2x}{a\sqrt{\arccos(ax)}}\right)+\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}$$

input
```
Int[ArcCos[a*x]^(-5/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) + (2*a*((2*x)/(a*Sqrt[ArcCos
[a*x]]) + (2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^2))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sqrt{2}\left(4\pi \arccos\left(a x\right)^{2} \operatorname{FresnelS}\left(\frac{\sqrt{2}\, \sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)+2 \arccos\left(a x\right)^{\frac{3}{2}} \sqrt{2}\, \sqrt{\pi}\, a x +\sqrt{2}\, \sqrt{\pi}\, \sqrt{\arccos\left(a x\right)}\, \sqrt{-a^{2} x^{2}+1}\right)}{3 a \sqrt{\pi}\, \arccos\left(a x\right)^{2}}$	83

input
```
int(1/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*2^(1/2)/Pi^(1/2)*(4*Pi*arccos(a*x)^2*FresnelS(2^(1/2)/Pi^(1/2)*arcco
s(a*x)^(1/2))+2*arccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a*x+2^(1/2)*Pi^(1/2)*ar
ccos(a*x)^(1/2)*(-a^2*x^2+1)^(1/2))/arccos(a*x)^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/acos(a*x)**(5/2),x)
```

output
```
Integral(acos(a*x)**(-5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int(1/acos(a*x)^(5/2),x)
```

output
```
int(1/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \frac{-\frac{2acos(ax)^2\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)^2a^2x^2-acos(ax)^2}dx\right)a^2}{3}+\frac{2\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{3}}{acos\left(ax\right)^2 a}$$

input
```
int(1/acos(a*x)^(5/2),x)
```

output
```
(2*( - acos(a*x)**2*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a
*x)**2*a**2*x**2 - acos(a*x)**2),x)*a**2 + sqrt( - a**2*x**2 + 1)*sqrt(aco
s(a*x))))/(3*acos(a*x)**2*a)
```

3.112 $\int \frac{1}{x \arccos(ax)^{5/2}} \, dx$

Optimal result 832
Mathematica [N/A] 832
Rubi [N/A] 833
Maple [N/A] 833
Fricas [F(-2)] 834
Sympy [N/A] 834
Maxima [F(-2)] 834
Giac [N/A] 835
Mupad [N/A] 835
Reduce [N/A] 835

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \text{Int}\left(\frac{1}{x \arccos(ax)^{5/2}}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \arccos(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x*ArcCos[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx$$
$$\Big\downarrow \ 5149$$
$$\int \frac{1}{x \arccos(ax)^{5/2}} dx$$

input
```
Int[1/(x*ArcCos[a*x]^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arccos \left(ax\right)^{\frac{5}{2}}} dx$$

input
```
int(1/x/arccos(a*x)^(5/2),x)
```

output
```
int(1/x/arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{5/2}}\, dx = \int \frac{1}{x \operatorname{acos}^{\frac{5}{2}}(ax)}\, dx$$

input
```
integrate(1/x/acos(a*x)**(5/2),x)
```

output
```
Integral(1/(x*acos(a*x)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{5/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \arccos{(ax)}^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \operatorname{acos}{(a\,x)}^{5/2}} \, dx$$

input
```
int(1/(x*acos(a*x)^(5/2)),x)
```

output
```
int(1/(x*acos(a*x)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{\sqrt{acos(ax)}}{acos(ax)^{3} x} dx$$

input
```
int(1/x/acos(a*x)^(5/2),x)
```

output
```
int(sqrt(acos(a*x))/(acos(a*x)**3*x),x)
```

3.113 $\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx$

Optimal result 837
Mathematica [C] (verified) 838
Rubi [A] (verified) 838
Maple [A] (verified) 841
Fricas [F(-2)] 841
Sympy [F] 842
Maxima [F(-2)] 842
Giac [F] 842
Mupad [F(-1)] 843
Reduce [F] 843

Optimal result

Integrand size = 12, antiderivative size = 264

$$\begin{aligned}\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx &= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{16x^3}{15a^2\arccos(ax)^{3/2}} \\ &+\frac{4x^5}{3\arccos(ax)^{3/2}}+\frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}}-\frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} \\ &+\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^5}+\frac{5\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} \\ &-\frac{8\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^5}+\frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5}\end{aligned}$$

output
```
2/5*x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(5/2)-16/15*x^3/a^2/arccos(a*x)^(
3/2)+4/3*x^5/arccos(a*x)^(3/2)+32/5*x^2*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)
^(1/2)-40/3*x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)+1/15*2^(1/2)*Pi^(1/
2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5+9/10*6^(1/2)*Pi^(1/2)*
FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5+5/6*10^(1/2)*Pi^(1/2)*Fre
snelC(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.32 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx =$$
$$-\frac{2\left(-6\sqrt{1-a^2x^2}-2ie^{i\arccos(ax)}\arccos(ax)(-i+2\arccos(ax))-4(-i\arccos(ax))^{3/2}\arccos(ax)\Gamma\left(\frac{1}{2},-i\right.\right.}{}$$

input
```
Integrate[x^4/ArcCos[a*x]^(7/2),x]
```

output
```
-1/240*(2*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I
+ 2*ArcCos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*
ArcCos[a*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*
(I*ArcCos[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x])) - 5*A
rcCos[a*x]*(2*E^((5*I)*ArcCos[a*x])*(1 + (10*I)*ArcCos[a*x]) + 20*Sqrt[5]*
((-I)*ArcCos[a*x])^(3/2)*Gamma[1/2, (-5*I)*ArcCos[a*x]] + (2 - (20*I)*ArcC
os[a*x] + 20*Sqrt[5]*E^((5*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2
, (5*I)*ArcCos[a*x]])/E^((5*I)*ArcCos[a*x])) + 9*(-2*ArcCos[a*x]*(E^((3*I)
*ArcCos[a*x])*(1 + (6*I)*ArcCos[a*x]) + 6*Sqrt[3]*((-I)*ArcCos[a*x])^(3/2)
*Gamma[1/2, (-3*I)*ArcCos[a*x]] + (1 - (6*I)*ArcCos[a*x] + 6*Sqrt[3]*E^((3
*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcCos[a*x]])/E^((
3*I)*ArcCos[a*x])) - 2*Sin[3*ArcCos[a*x]]) - 6*Sin[5*ArcCos[a*x]])/(a^5*Ar
cCos[a*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5145, 5223, 5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx$$

$$\Big\downarrow \text{5145}$$

$$2a\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx-\frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx}{5a}+\frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{5223}$$

$$2a\left(\frac{2x^5}{3a\arccos(ax)^{3/2}}-\frac{10\int \frac{x^4}{\arccos(ax)^{3/2}}dx}{3a}\right)-\frac{8\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\int \frac{x^2}{\arccos(ax)^{3/2}}dx}{a}\right)}{5a}+\frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{5143}$$

$$2a\left(\frac{2x^5}{3a\arccos(ax)^{3/2}}-\frac{10\left(\frac{2\int \left(-\frac{ax}{8\sqrt{\arccos(ax)}}-\frac{9\cos(3\arccos(ax))}{16\sqrt{\arccos(ax)}}-\frac{5\cos(5\arccos(ax))}{16\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^5}+\frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)-\frac{8\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\int \left(-\frac{ax}{4\sqrt{\arccos(ax)}}-\frac{3\cos(3\arccos(ax))}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)}{5a}+\frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+2a\left(\frac{2x^5}{3a\arccos(ax)^{3/2}}-\frac{10\left(\frac{2\left(-\frac{1}{4}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{3}{8}\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{8}\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\text{ar}\right.\right.}{a^5}\right.}{3a}\right.$$

$$\frac{8\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)}{5a}$$

input
```
Int[x^4/ArcCos[a*x]^(7/2),x]
```

output
```
(2*x^4*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (8*((2*x^3)/(3*a*ArcCo
s[a*x]^(3/2)) - (2*((2*x^2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) + (2*(
-1/2*(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (Sqrt[(3*Pi)/2]
*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/2))/a^3))/a))/(5*a) + 2*a*((2*x^5
)/(3*a*ArcCos[a*x]^(3/2)) - (10*((2*x^4*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[
a*x]]) + (2*(-1/4*(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (3
*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/8 - (Sqrt[(5*Pi)/2
]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/8))/a^5))/(3*a))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-100\sqrt{2}\,\sqrt{\pi}\,\sqrt{5}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-108\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-8\sqrt{2}\ldots}{}$

input
```
int(x^4/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/a^5*(-100*2^(1/2)*Pi^(1/2)*5^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2
)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-108*2^(1/2)*Pi^(1/2)*3^(1/2)*Fresne
lC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-8*2^(1/2)
*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)+8
*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+108*arccos(a*x)^2*sin(3*arccos(a*x))+100
*arccos(a*x)^2*sin(5*arccos(a*x))-4*a*x*arccos(a*x)-18*arccos(a*x)*cos(3*a
rccos(a*x))-10*arccos(a*x)*cos(5*arccos(a*x))-6*(-a^2*x^2+1)^(1/2)-9*sin(3
*arccos(a*x))-3*sin(5*arccos(a*x)))/arccos(a*x)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/arccos(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^4}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x**4/acos(a*x)**(7/2),x)
```

output
```
Integral(x**4/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/arccos(a*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^4}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^4/arccos(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^4/arccos(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(x^4/acos(a*x)^(7/2),x)
```

output
```
int(x^4/acos(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \frac{-\frac{4acos(ax)^3\left(\int \frac{\sqrt{acos(ax)}}{acos(ax)^4a^2x^2-acos(ax)^4}dx\right)a}{3}+\frac{4acos(ax)^3\left(\int \frac{\sqrt{acos(ax)}\,x^2}{acos(ax)^4a^2x^2-acos(ax)^4}dx\right)a^3}{3}-2acos(ax}{}$$

input
```
int(x^4/acos(a*x)^(7/2),x)
```

output
```
(2*( - 10*acos(a*x)**3*int(sqrt(acos(a*x))/(acos(a*x)**4*a**2*x**2 - acos(
a*x)**4),x)*a + 10*acos(a*x)**3*int((sqrt(acos(a*x))*x**2)/(acos(a*x)**4*a
**2*x**2 - acos(a*x)**4),x)*a**3 - 15*acos(a*x)**3*int((sqrt( - a**2*x**2
+ 1)*sqrt(acos(a*x))*x**5)/(acos(a*x)**3*a**2*x**2 - acos(a*x)**3),x)*a**6
 + 12*acos(a*x)**3*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)/(acos
(a*x)**3*a**2*x**2 - acos(a*x)**3),x)*a**4 + 4*acos(a*x)**3*int((sqrt( - a
**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)**3*a**2*x**2 - acos(a*x)**3),x
)*a**2 + 3*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a**4*x**4 - 4*sqrt( - a*
*2*x**2 + 1)*sqrt(acos(a*x))))/(15*acos(a*x)**3*a**5)
```

3.114 $\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx$

Optimal result 844
Mathematica [C] (verified) 845
Rubi [A] (verified) 845
Maple [A] (verified) 849
Fricas [F(-2)] 850
Sympy [F] 850
Maxima [F(-2)] 851
Giac [F(-2)] 851
Mupad [F(-1)] 851
Reduce [F] 852

Optimal result

Integrand size = 12, antiderivative size = 190

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^4} + \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^4}$$

output
```
2/5*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(5/2)-4/5*x^2/a^2/arccos(a*x)^(3/
2)+16/15*x^4/arccos(a*x)^(3/2)+16/5*x*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)^(
1/2)-128/15*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)+32/15*2^(1/2)*Pi^(1
/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^4+16/15*Pi^(1/2)*Fres
nelC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx =$$
$$-\frac{-4e^{-2i\arccos(ax)}\left(1+e^{4i\arccos(ax)}(1+4i\arccos(ax))-4i\arccos(ax)\right)\arccos(ax)+\frac{16\sqrt{2}\arccos(ax)^3\Gamma\left(\frac{1}{2},-2i\arccos\right.}{\sqrt{-i\arccos(ax)}}}{}$$

input
```
Integrate[x^3/ArcCos[a*x]^(7/2),x]
```

output
```
-1/60*((-4*(1 + E^((4*I)*ArcCos[a*x])*(1 + (4*I)*ArcCos[a*x]) - (4*I)*ArcC
os[a*x])*ArcCos[a*x])/E^((2*I)*ArcCos[a*x]) + (16*Sqrt[2]*ArcCos[a*x]^3*Ga
mma[1/2, (-2*I)*ArcCos[a*x]])/Sqrt[(-I)*ArcCos[a*x]] + (16*I)*Sqrt[2]*(I*A
rcCos[a*x])^(5/2)*Gamma[1/2, (2*I)*ArcCos[a*x]] - 2*ArcCos[a*x]*(2*E^((4*I
)*ArcCos[a*x])*(1 + (8*I)*ArcCos[a*x]) + 32*((-I)*ArcCos[a*x])^(3/2)*Gamma
[1/2, (-4*I)*ArcCos[a*x]] + (2*(1 - (8*I)*ArcCos[a*x] + 16*E^((4*I)*ArcCos
[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcCos[a*x]]))/E^((4*I)*ArcC
os[a*x])) - 6*Sin[2*ArcCos[a*x]] - 3*Sin[4*ArcCos[a*x]])/(a^4*ArcCos[a*x]^
(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5145, 5223, 5143, 25, 2009, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx$$
$$\Big\downarrow \ 5145$$

$$-\frac{6\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx}{5a}+\frac{8}{5}a\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$\Big\downarrow$ 5223

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\int \frac{x}{\arccos(ax)^{3/2}}dx}{3a}\right)}{5a}+\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\int \frac{x^3}{\arccos(ax)^{3/2}}dx}{3a}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$\Big\downarrow$ 5143

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2\int -\frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)}{5a}+\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\left(\frac{2\int \left(-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}-\frac{\cos(4\arccos(ax))}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3a}\right)}{5a}+\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\left(\frac{2\int \left(-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}-\frac{\cos(4\arccos(ax))}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$\Big\downarrow$ 2009

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3a}\right)}{5a}+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)$$

$$\Big\downarrow \; 3042$$

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3a}\right)}{5a}+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)$$

$$\Big\downarrow \; 3785$$

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int\cos(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^2}\right)}{3a}\right)}{5a}+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)$$

$$\Big\downarrow \; 3833$$

$$-\frac{6\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}\right)}{3a}\right)}{5a}+\frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+$$

$$\frac{8}{5}a\left(\frac{2x^4}{3a\arccos(ax)^{3/2}}-\frac{8\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)$$

input
```
Int[x^3/ArcCos[a*x]^(7/2),x]
```

output
```
(2*x^3*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (6*((2*x^2)/(3*a*ArcCo
s[a*x]^(3/2)) - (4*((2*x*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqr
t[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^2))/(3*a)))/(5*a) + (8*a
*((2*x^4)/(3*a*ArcCos[a*x]^(3/2)) - (8*((2*x^3*Sqrt[1 - a^2*x^2])/(a*Sqrt[
ArcCos[a*x]]) + (2*(-1/2*(Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]]) - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/2))/a^4))/(3*a)
))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.73

method	result
default	$-\frac{-128\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-64\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}+32\sin(2\arccos(ax))\operatorname{arcc}}{}$

input
```
int(x^3/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60/a^4*(-128*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(
1/2))*arccos(a*x)^(5/2)-64*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))
*arccos(a*x)^(5/2)+32*sin(2*arccos(a*x))*arccos(a*x)^2+64*sin(4*arccos(a*x
))*arccos(a*x)^2-8*cos(2*arccos(a*x))*arccos(a*x)-8*arccos(a*x)*cos(4*arcc
os(a*x))-6*sin(2*arccos(a*x))-3*sin(4*arccos(a*x)))/arccos(a*x)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/arccos(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x**3/acos(a*x)**(7/2),x)
```

output
```
Integral(x**3/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/arccos(a*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/arccos(a*x)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(x^3/acos(a*x)^(7/2),x)
```

output
```
int(x^3/acos(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \frac{-15acos(ax)^3 \left(\int \frac{\sqrt{acos(ax)}\,x^3}{acos(ax)^4a^2x^2-acos(ax)^4}dx\right)a^4+15acos(ax)^3 \left(\int \frac{\sqrt{acos(ax)}\,x}{acos(ax)^4a^2x^2-acos(ax)^4}$$

input
```
int(x^3/acos(a*x)^(7/2),x)
```

output
```
( - 15*acos(a*x)**3*int((sqrt(acos(a*x))*x**3)/(acos(a*x)**4*a**2*x**2 - a
cos(a*x)**4),x)*a**4 + 15*acos(a*x)**3*int((sqrt(acos(a*x))*x)/(acos(a*x)*
*4*a**2*x**2 - acos(a*x)**4),x)*a**2 - 16*acos(a*x)**3*int((sqrt( - a**2*x
**2 + 1)*sqrt(acos(a*x))*x**4)/(acos(a*x)**3*a**2*x**2 - acos(a*x)**3),x)*
a**5 - 4*sqrt(acos(a*x))*acos(a*x) + 4*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*
x))*a**3*x**3 + 6*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a*x)/(10*acos(a*x
)**3*a**4)
```

3.115 $\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx$

Optimal result . . . 853
Mathematica [C] (verified) . . . 854
Rubi [A] (verified) . . . 854
Maple [A] (verified) . . . 859
Fricas [F(-2)] . . . 859
Sympy [F] . . . 860
Maxima [F(-2)] . . . 860
Giac [F] . . . 860
Mupad [F(-1)] . . . 861
Reduce [F] . . . 861

Optimal result

Integrand size = 12, antiderivative size = 191

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} + \frac{2\sqrt{2\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^3} + \frac{6\sqrt{6\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^3}$$

output
```
2/5*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(5/2)-8/15*x/a^2/arccos(a*x)^(3/2
)+4/5*x^3/arccos(a*x)^(3/2)+16/15*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)^(1/2)
-24/5*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)+2/15*2^(1/2)*Pi^(1/2)*Fre
snelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3+6/5*6^(1/2)*Pi^(1/2)*Fresnel
C(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx =$$
$$-\frac{-6\sqrt{1-a^2x^2}-2ie^{i\arccos(ax)}\arccos(ax)(-i+2\arccos(ax))-4(-i\arccos(ax))^{3/2}\arccos(ax)\Gamma\left(\frac{1}{2},-i\,\mathrm{arc}\right.}{}$$

input
```
Integrate[x^2/ArcCos[a*x]^(7/2),x]
```

output
```
-1/60*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*
ArcCos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcC
os[a*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*A
rcCos[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x]) - 6*ArcCos
[a*x]*(E^((3*I)*ArcCos[a*x])*(1 + (6*I)*ArcCos[a*x]) + 6*Sqrt[3]*((-I)*Arc
Cos[a*x])^(3/2)*Gamma[1/2, (-3*I)*ArcCos[a*x]] + (1 - (6*I)*ArcCos[a*x] +
6*Sqrt[3]*E^((3*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (3*I)*Arc
Cos[a*x]])/E^((3*I)*ArcCos[a*x])) - 6*Sin[3*ArcCos[a*x]])/(a^3*ArcCos[a*x]
^(5/2))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5145, 5223, 5133, 5143, 2009, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx$$

$$\Big\downarrow \; 5145$$

$$-\frac{4\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx}{5a}+\frac{6}{5}a\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 5223$$

$$\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\int \frac{x^2}{\arccos(ax)^{3/2}}dx}{a}\right)-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\int \frac{1}{\arccos(ax)^{3/2}}dx}{3a}\right)}{5a}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 5133$$

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(2a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)}{5a}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\int \frac{x^2}{\arccos(ax)^{3/2}}dx}{a}\right)+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 5143$$

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(2a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)}{5a}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\int\left(-\frac{ax}{4\sqrt{\arccos(ax)}}-\frac{3\cos(3\arccos(ax))}{4\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(2a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)}{5a}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)$$

$$\Big\downarrow \ 5225$$

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{ax}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}\right)}{3a}\right)}{5a}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)$$

$\downarrow$ 3042

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}\right)}{3a}\right)}{5a}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)$$

$\downarrow$ 3785

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int ax\,d\sqrt{\arccos(ax)}}{a}\right)}{3a}\right)}{5a}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)$$

$\downarrow$ 3833

$$-\frac{4\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{2\pi}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}\right)}{3a}\right)}{5a}+\frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{6}{5}a\left(\frac{2x^3}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^3}+\frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{a}\right)$$

input
```
Int[x^2/ArcCos[a*x]^(7/2),x]
```

output
```
(2*x^2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (4*((2*x)/(3*a*ArcCos[
a*x]^(3/2)) - (2*((2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[2*
Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a))/(3*a)))/(5*a) + (6*a*((2*x
^3)/(3*a*ArcCos[a*x]^(3/2)) - (2*((2*x^2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos
[a*x]]) + (2*(-1/2*(Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) - (
Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/2))/a^3))/a))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

method	result
default	$-\frac{-36\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-4\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}+4\arccos(ax)^{2}\sqrt{}}{30a^{3}\,\mathrm{arcc}}$

input
```
int(x^2/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30/a^3*(-36*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*
arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-4*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/P
i^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)+4*arccos(a*x)^2*(-a^2*x^2+1)^
(1/2)+36*arccos(a*x)^2*sin(3*arccos(a*x))-2*a*x*arccos(a*x)-6*arccos(a*x)*
cos(3*arccos(a*x))-3*(-a^2*x^2+1)^(1/2)-3*sin(3*arccos(a*x)))/arccos(a*x)^
(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/arccos(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x**2/acos(a*x)**(7/2),x)
```

output
```
Integral(x**2/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/arccos(a*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^2}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2/arccos(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(x^2/arccos(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(x^2/acos(a*x)^(7/2),x)
```

output
```
int(x^2/acos(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \frac{-\frac{6 acos(ax)^3\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^3}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right)a^2}{5}+\frac{4 acos(ax)^3\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right)}{5}+\frac{2\sqrt{-a^2x^2+1}}{5}}{acos\left(ax\right)^3 a}$$

input
```
int(x^2/acos(a*x)^(7/2),x)
```

output
```
(2*( - 3*acos(a*x)**3*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**3)/(a
cos(a*x)**3*a**2*x**2 - acos(a*x)**3),x)*a**2 + 2*acos(a*x)**3*int((sqrt(
- a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)**3*a**2*x**2 - acos(a*x)**3
),x) + sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2))/(5*acos(a*x)**3*a)
```

3.116 $\int \frac{x}{\arccos(ax)^{7/2}} \, dx$

Optimal result . . . 862
Mathematica [A] (verified) . . . 862
Rubi [A] (verified) . . . 863
Maple [A] (verified) . . . 866
Fricas [F(-2)] . . . 866
Sympy [F] . . . 867
Maxima [F(-2)] . . . 867
Giac [F] . . . 867
Mupad [F(-1)] . . . 868
Reduce [F] . . . 868

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int \frac{x}{\arccos(ax)^{7/2}} \, dx = \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^2}$$

output
```
2/5*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(5/2)-4/15/a^2/arccos(a*x)^(3/2)+8/
15*x^2/arccos(a*x)^(3/2)-32/15*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)+32
/15*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{x}{\arccos(ax)^{7/2}} \, dx = \frac{\frac{4\cos(2\arccos(ax))}{\arccos(ax)^{3/2}} + 32\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - \frac{(-3+16\arccos(ax)^2)\sin(2\arccos(ax))}{\arccos(ax)^{5/2}}}{15a^2}$$

input
```
Integrate[x/ArcCos[a*x]^(7/2),x]
```

output
```
((4*Cos[2*ArcCos[a*x]])/ArcCos[a*x]^(3/2) + 32*Sqrt[Pi]*FresnelC[(2*Sqrt[A
rcCos[a*x]])/Sqrt[Pi]] - ((-3 + 16*ArcCos[a*x]^2)*Sin[2*ArcCos[a*x]])/ArcC
os[a*x]^(5/2))/(15*a^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5145, 5153, 5223, 5143, 25, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arccos(ax)^{7/2}}dx$$

$$\Big\downarrow \text{ 5145}$$

$$-\frac{2\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx}{5a}+\frac{4}{5}a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{4}{5}a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5223}$$

$$\frac{4}{5}a\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\int \frac{x}{\arccos(ax)^{3/2}}dx}{3a}\right)+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5143}$$

$$\frac{4}{5}a\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2\int -\frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4}{5}a\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3a}\right)+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}a\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3a}\right)+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

$\Big\downarrow$ 3785

$$\frac{4}{5}a\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int\cos(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^2}\right)}{3a}\right)+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

$\Big\downarrow$ 3833

$$\frac{4}{5}a\left(\frac{2x^2}{3a\arccos(ax)^{3/2}}-\frac{4\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}\right)}{3a}\right)+\frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}-\frac{4}{15a^2\arccos(ax)^{3/2}}$$

input
```
Int[x/ArcCos[a*x]^(7/2),x]
```

output
```
(2*x*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - 4/(15*a^2*ArcCos[a*x]^(3
/2)) + (4*a*((2*x^2)/(3*a*ArcCos[a*x]^(3/2)) - (4*((2*x*Sqrt[1 - a^2*x^2])
/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[P
i]])/a^2))/(3*a)))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

method	result
default	$-\frac{-32\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}+16\sin(2\arccos(ax))\arccos(ax)^2-4\cos(2\arccos(ax))\arccos(ax)-3\sin(2\arccos}{15a^2\arccos(ax)^{\frac{5}{2}}}$

input
```
int(x/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/a^2*(-32*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*arccos(a*x)
^(5/2)+16*sin(2*arccos(a*x))*arccos(a*x)^2-4*cos(2*arccos(a*x))*arccos(a*x
)-3*sin(2*arccos(a*x)))/arccos(a*x)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arccos(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(x/acos(a*x)**(7/2),x)
```

output
```
Integral(x/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arccos(a*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \int \frac{x}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/arccos(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(x/arccos(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \int \frac{x}{\mathrm{acos}\,(a\,x)^{7/2}}\,dx$$

input
```
int(x/acos(a*x)^(7/2),x)
```

output
```
int(x/acos(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \frac{-\frac{4 acos(ax)^3\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^2}{acos(ax)^3 a^2x^2-acos(ax)^3}dx\right)a^3}{5}-\frac{4\sqrt{acos(ax)}\,acos(ax)}{15}+\frac{2\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,ax}{5}}{acos\,(ax)^3\,a^2}$$

input
```
int(x/acos(a*x)^(7/2),x)
```

output
```
(2*( - 6*acos(a*x)**3*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/(a
cos(a*x)**3*a**2*x**2 - acos(a*x)**3),x)*a**3 - 2*sqrt(acos(a*x))*acos(a*x
) + 3*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*a*x))/(15*acos(a*x)**3*a**2)
```

3.117 $\int \frac{1}{\arccos(ax)^{7/2}}\,dx$

Optimal result 869
Mathematica [C] (verified) 869
Rubi [A] (verified) 870
Maple [A] (verified) 872
Fricas [F(-2)] 873
Sympy [F] 873
Maxima [F(-2)] 874
Giac [F] 874
Mupad [F(-1)] 874
Reduce [F] 875

Optimal result

Integrand size = 8, antiderivative size = 105

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} + \frac{4x}{15\arccos(ax)^{3/2}} - \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a}$$

output
```
2/5*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(5/2)+4/15*x/arccos(a*x)^(3/2)-8/15*(
-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)+8/15*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.44

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = -\frac{-6\sqrt{1-a^2x^2} - 2ie^{i\arccos(ax)}\arccos(ax)(-i+2\arccos(ax)) - 4(-i\arccos(ax))^{3/2}\arccos(ax)\Gamma\left(\frac{1}{2}, -i\,\text{arc}\right.}{15a\,\text{arcc}}$$

input
```
Integrate[ArcCos[a*x]^(-7/2),x]
```

output
```
-1/15*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*
ArcCos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcC
os[a*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*A
rcCos[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x]))/(a*ArcCos
[a*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5133, 5223, 5133, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{7/2}}dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{2}{5}a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}dx+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{2}{5}a\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\int \frac{1}{\arccos(ax)^{3/2}}dx}{3a}\right)+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{5133}$$

$$\frac{2}{5}a\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(2a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3a}\right)+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \text{5225}$$

$$\frac{2}{5}a\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{ax}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}\right)}{3a}\right)+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{5}a\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a}\right)}{3a}\right)+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 3785$$

$$\frac{2}{5}a\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int axd\sqrt{\arccos(ax)}}{a}\right)}{3a}\right)+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

$$\Big\downarrow \ 3833$$

$$\frac{2}{5}a\left(\frac{2x}{3a\arccos(ax)^{3/2}}-\frac{2\left(\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}\right)}{3a}\right)+\frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}$$

input
```
Int[ArcCos[a*x]^(-7/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) + (2*a*((2*x)/(3*a*ArcCos[a*
x]^(3/2)) - (2*((2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[2*Pi
]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a))/(3*a)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05

method	result
default	$\frac{\sqrt{2}\left(8\pi \arccos(ax)^{3} \operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-4\arccos(ax)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{-a^{2}x^{2}+1}+2\arccos(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,ax+3\sqrt{2}\,\sqrt{\pi}\,\sqrt{\arccos(a}\right.}{15a\sqrt{\pi}\,\arccos(ax)^{3}}$

input
```
int(1/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/a*2^(1/2)/Pi^(1/2)*(8*Pi*arccos(a*x)^3*FresnelC(2^(1/2)/Pi^(1/2)*arcc
os(a*x)^(1/2))-4*arccos(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)+2*a
rccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a*x+3*2^(1/2)*Pi^(1/2)*arccos(a*x)^(1/2)
*(-a^2*x^2+1)^(1/2))/arccos(a*x)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

input
```
integrate(1/acos(a*x)**(7/2),x)
```

output
```
Integral(acos(a*x)**(-7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(a*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \int \frac{1}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/arccos(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \int \frac{1}{\mathrm{acos}(a\,x)^{7/2}}\,dx$$

input
```
int(1/acos(a*x)^(7/2),x)
```

output
```
int(1/acos(a*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \frac{-\frac{2acos(ax)^3\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)^3a^2x^2-acos(ax)^3}dx\right)a^2}{5}+\frac{2\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{5}}{acos\left(ax\right)^3 a}$$

input

```
int(1/acos(a*x)^(7/2),x)
```

output

```
(2*( - acos(a*x)**3*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a
*x)**3*a**2*x**2 - acos(a*x)**3),x)*a**2 + sqrt( - a**2*x**2 + 1)*sqrt(aco
s(a*x))))/(5*acos(a*x)**3*a)
```

3.118 $\int \frac{1}{x \arccos(ax)^{7/2}} dx$

Optimal result . . . 876
Mathematica [N/A] . . . 876
Rubi [N/A] . . . 877
Maple [N/A] . . . 877
Fricas [F(-2)] . . . 878
Sympy [N/A] . . . 878
Maxima [F(-2)] . . . 878
Giac [N/A] . . . 879
Mupad [N/A] . . . 879
Reduce [N/A] . . . 879

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arccos(ax)^{7/2}} dx = \text{Int}\left(\frac{1}{x \arccos(ax)^{7/2}}, x\right)$$

output
```
Defer(Int)(1/x/arccos(a*x)^(7/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arccos(ax)^{7/2}} dx = \int \frac{1}{x \arccos(ax)^{7/2}} dx$$

input
```
Integrate[1/(x*ArcCos[a*x]^(7/2)),x]
```

output
```
Integrate[1/(x*ArcCos[a*x]^(7/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx$$

$$\Big\downarrow \ 5149$$

$$\int \frac{1}{x \arccos(ax)^{7/2}}dx$$

input
```
Int[1/(x*ArcCos[a*x]^(7/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arccos \left(a x\right)^{\frac{7}{2}}}dx$$

input
```
int(1/x/arccos(a*x)^(7/2),x)
```

output
```
int(1/x/arccos(a*x)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arccos(a*x)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 58.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \int \frac{1}{x \operatorname{acos}^{\frac{7}{2}}(ax)} \, dx$$

input
```
integrate(1/x/acos(a*x)**(7/2),x)
```

output
```
Integral(1/(x*acos(a*x)**(7/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arccos(a*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{7/2}}\,dx = \int \frac{1}{x \arccos{(ax)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x/arccos(a*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/(x*arccos(a*x)^(7/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{7/2}}\,dx = \int \frac{1}{x \,\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

input
```
int(1/(x*acos(a*x)^(7/2)),x)
```

output
```
int(1/(x*acos(a*x)^(7/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x \arccos(ax)^{7/2}}\,dx = \int \frac{\sqrt{acos(ax)}}{acos(ax)^4 x}dx$$

input
```
int(1/x/acos(a*x)^(7/2),x)
```

output
```
int(sqrt(acos(a*x))/(acos(a*x)**4*x),x)
```

3.119 $\int (bx)^m \arccos(ax)^4 \, dx$

Optimal result 881
Mathematica [N/A] 881
Rubi [N/A] 882
Maple [N/A] 882
Fricas [N/A] 883
Sympy [N/A] 883
Maxima [N/A] 883
Giac [N/A] 884
Mupad [N/A] 884
Reduce [N/A] 885

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \arccos(ax)^4 \, dx = \text{Int}\left((bx)^m \arccos(ax)^4, x\right)$$

output
```
Defer(Int)((b*x)^m*arccos(a*x)^4,x)
```

Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos(ax)^4 \, dx$$

input
```
Integrate[(b*x)^m*ArcCos[a*x]^4,x]
```

output
```
Integrate[(b*x)^m*ArcCos[a*x]^4, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^4(bx)^m\,dx$$

$$\Big\downarrow \ 5139$$

$$\frac{4a\int \frac{(bx)^{m+1}\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{b(m+1)}+\frac{\arccos(ax)^4(bx)^{m+1}}{b(m+1)}$$

$$\Big\downarrow \ 5235$$

$$\frac{4a\int \frac{(bx)^{m+1}\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{b(m+1)}+\frac{\arccos(ax)^4(bx)^{m+1}}{b(m+1)}$$

input
```
Int[(b*x)^m*ArcCos[a*x]^4,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^4\,dx$$

input
```
int((b*x)^m*arccos(a*x)^4,x)
```

output
```
int((b*x)^m*arccos(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4\,dx = \int (bx)^m \arccos(ax)^4\,dx$$

input
```
integrate((b*x)^m*arccos(a*x)^4,x, algorithm="fricas")
```

output
```
integral((b*x)^m*arccos(a*x)^4, x)
```

Sympy [N/A]

Not integrable

Time = 5.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^4\,dx = \int (bx)^m \operatorname{acos}^4(ax)\,dx$$

input
```
integrate((b*x)**m*acos(a*x)**4,x)
```

output
```
Integral((b*x)**m*acos(a*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 9.58

$$\int (bx)^m \arccos(ax)^4\,dx = \int (bx)^m \arccos(ax)^4\,dx$$

input
```
integrate((b*x)^m*arccos(a*x)^4,x, algorithm="maxima")
```

output
```
(b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*(a*b^m*m + a*b
^m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m*arctan2(sqrt(a*x + 1)*sqr
t(-a*x + 1), a*x)^3/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos{(ax)}^4 \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^4,x, algorithm="giac")
```

output
```
integrate((b*x)^m*arccos(a*x)^4, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int \mathrm{acos}(a\,x)^4\,{(b\,x)}^m \, dx$$

input
```
int(acos(a*x)^4*(b*x)^m,x)
```

output
```
int(acos(a*x)^4*(b*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int (bx)^m \arccos(ax)^4 \, dx = b^m \left(\int x^m acos(ax)^4 \, dx \right)$$

input
```
int((b*x)^m*acos(a*x)^4,x)
```

output
```
b**m*int(x**m*acos(a*x)**4,x)
```

3.120 $\int (bx)^m \arccos(ax)^3 \, dx$

Optimal result . . . 886
Mathematica [N/A] . . . 886
Rubi [N/A] . . . 887
Maple [N/A] . . . 887
Fricas [N/A] . . . 888
Sympy [N/A] . . . 888
Maxima [N/A] . . . 888
Giac [N/A] . . . 889
Mupad [N/A] . . . 889
Reduce [N/A] . . . 890

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \arccos(ax)^3 \, dx = \text{Int}\left((bx)^m \arccos(ax)^3, x\right)$$

output
```
Defer(Int)((b*x)^m*arccos(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos(ax)^3 \, dx$$

input
```
Integrate[(b*x)^m*ArcCos[a*x]^3,x]
```

output
```
Integrate[(b*x)^m*ArcCos[a*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3(bx)^m\,dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{3a\int \frac{(bx)^{m+1}\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{b(m+1)}+\frac{\arccos(ax)^3(bx)^{m+1}}{b(m+1)}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{3a\int \frac{(bx)^{m+1}\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{b(m+1)}+\frac{\arccos(ax)^3(bx)^{m+1}}{b(m+1)}$$

input
```
Int[(b*x)^m*ArcCos[a*x]^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^3\,dx$$

input
```
int((b*x)^m*arccos(a*x)^3,x)
```

output
```
int((b*x)^m*arccos(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos(ax)^3 \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral((b*x)^m*arccos(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \operatorname{acos}^3(ax) \, dx$$

input
```
integrate((b*x)**m*acos(a*x)**3,x)
```

output
```
Integral((b*x)**m*acos(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 115, normalized size of antiderivative = 9.58

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos(ax)^3 \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
(b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*(a*b^m*m + a*b
^m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m*arctan2(sqrt(a*x + 1)*sqr
t(-a*x + 1), a*x)^2/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos{(ax)^3} \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^3,x, algorithm="giac")
```

output
```
integrate((b*x)^m*arccos(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3 \, dx = \int \mathrm{acos}(a\,x)^3\,(b\,x)^m \, dx$$

input
```
int(acos(a*x)^3*(b*x)^m,x)
```

output
```
int(acos(a*x)^3*(b*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int (bx)^m \arccos(ax)^3 \, dx = b^m \left(\int x^m acos(ax)^3 \, dx \right)$$

input
```
int((b*x)^m*acos(a*x)^3,x)
```

output
```
b**m*int(x**m*acos(a*x)**3,x)
```

3.121 $\int (bx)^m \arccos(ax)^2\,dx$

Optimal result . . . 891
Mathematica [C] (warning: unable to verify) . . . 892
Rubi [A] (verified) . . . 892
Maple [F] . . . 894
Fricas [F] . . . 894
Sympy [F] . . . 894
Maxima [F] . . . 895
Giac [F] . . . 895
Mupad [F(-1)] . . . 895
Reduce [F] . . . 896

Optimal result

Integrand size = 12, antiderivative size = 150

$$\begin{aligned}\int (bx)^m \arccos(ax)^2\,dx &= \frac{(bx)^{1+m}\arccos(ax)^2}{b(1+m)} \\ &+ \frac{2a(bx)^{2+m}\arccos(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{b^2(1+m)(2+m)} \\ &+ \frac{2a^2(bx)^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};a^2x^2\right)}{b^3(1+m)\left(6+5m+m^2\right)}\end{aligned}$$

output
```
(b*x)^(1+m)*arccos(a*x)^2/b/(1+m)+2*a*(b*x)^(2+m)*arccos(a*x)*hypergeom([1
/2, 1+1/2*m],[2+1/2*m],a^2*x^2)/b^2/(1+m)/(2+m)+2*a^2*(b*x)^(3+m)*hypergeo
m([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],a^2*x^2)/b^3/(1+m)/(m^2+5
*m+6)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int (bx)^m \arccos(ax)^2\,dx$$
$$= \frac{x(bx)^m \left(4\arccos(ax)^2 + ax\left(\frac{8\sqrt{1-a^2x^2}\arccos(ax)\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{4+m}{2},a^2x^2\right)}{2+m} + 2^{-m}a\sqrt{\pi}x\,\text{Gamma}(2+m\right.\right.}{4(1+m)}$$

input
```
Integrate[(b*x)^m*ArcCos[a*x]^2,x]
```

output
```
(x*(b*x)^m*(4*ArcCos[a*x]^2 + a*x*((8*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Hyperg
eometric2F1[1, (3 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m) + (a*Sqrt[Pi]*x*Gam
ma[2 + m]*HypergeometricPFQRegularized[{1, (3 + m)/2, (3 + m)/2}, {(4 + m)
/2, (5 + m)/2}, a^2*x^2])/2^m)))/(4*(1 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5139, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^2(bx)^m\,dx$$
$$\Big\downarrow \text{5139}$$
$$\frac{2a\int \frac{(bx)^{m+1}\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{b(m+1)} + \frac{\arccos(ax)^2(bx)^{m+1}}{b(m+1)}$$
$$\Big\downarrow \text{5221}$$

$$\frac{2a\left(\frac{a(bx)^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};a^2x^2\right)}{b^2(m+2)(m+3)}+\frac{\arccos(ax)(bx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{b(m+2)}\right)}{b(m+1)}+\frac{\arccos(ax)^2(bx)^{m+1}}{b(m+1)}$$

input
```
Int[(b*x)^m*ArcCos[a*x]^2,x]
```

output
```
((b*x)^(1 + m)*ArcCos[a*x]^2)/(b*(1 + m)) + (2*a*(((b*x)^(2 + m)*ArcCos[a*
x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b*(2 + m)) + (a
*(b*x)^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2
+ m/2}, a^2*x^2])/(b^2*(2 + m)*(3 + m))))/(b*(1 + m))
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int (bx)^m \arccos (ax)^2 \, dx$$

input
```
int((b*x)^m*arccos(a*x)^2,x)
```

output
```
int((b*x)^m*arccos(a*x)^2,x)
```

Fricas [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \arccos (ax)^2 \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral((b*x)^m*arccos(a*x)^2, x)
```

Sympy [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \operatorname{acos}^2 (ax) \, dx$$

input
```
integrate((b*x)**m*acos(a*x)**2,x)
```

output
```
Integral((b*x)**m*acos(a*x)**2, x)
```

Maxima [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \arccos{(ax)}^2 \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - 2*(a*b^m*m + a*b
^m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m*arctan2(sqrt(a*x + 1)*sqr
t(-a*x + 1), a*x)/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \arccos{(ax)}^2 \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate((b*x)^m*arccos(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m \arccos(ax)^2 \, dx = \int \mathrm{acos}(a\,x)^2\,{(b\,x)}^m \, dx$$

input
```
int(acos(a*x)^2*(b*x)^m,x)
```

output
```
int(acos(a*x)^2*(b*x)^m, x)
```

Reduce [F]

$$\int (bx)^m \arccos(ax)^2\,dx = b^m\left(\int x^m acos(ax)^2\,dx\right)$$

input
```
int((b*x)^m*acos(a*x)^2,x)
```

output
```
b**m*int(x**m*acos(a*x)**2,x)
```

3.122 $\int (bx)^m \arccos(ax)\,dx$

Optimal result . . . 897
Mathematica [A] (verified) . . . 897
Rubi [A] (verified) . . . 898
Maple [F] . . . 899
Fricas [F] . . . 899
Sympy [F] . . . 900
Maxima [F] . . . 900
Giac [F] . . . 900
Mupad [F(-1)] . . . 901
Reduce [F] . . . 901

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int (bx)^m \arccos(ax)\,dx = \frac{(bx)^{1+m}\arccos(ax)}{b(1+m)} + \frac{a(bx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{b^2(1+m)(2+m)}$$

output
```
(b*x)^(1+m)*arccos(a*x)/b/(1+m)+a*(b*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+
1/2*m],a^2*x^2)/b^2/(1+m)/(2+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (bx)^m \arccos(ax)\,dx = \frac{x(bx)^m\left((2+m)\arccos(ax)+ax\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},a^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[(b*x)^m*ArcCos[a*x],x]
```

output
```
(x*(b*x)^m*((2 + m)*ArcCos[a*x] + a*x*Hypergeometric2F1[1/2, 1 + m/2, 2 +
m/2, a^2*x^2]))/((1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5139, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)(bx)^m \, dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{a\int \frac{(bx)^{m+1}}{\sqrt{1-a^2x^2}}dx}{b(m+1)}+\frac{\arccos(ax)(bx)^{m+1}}{b(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{a(bx)^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{b^2(m+1)(m+2)}+\frac{\arccos(ax)(bx)^{m+1}}{b(m+1)}$$

input
```
Int[(b*x)^m*ArcCos[a*x],x]
```

output
```
((b*x)^(1 + m)*ArcCos[a*x])/(b*(1 + m)) + (a*(b*x)^(2 + m)*Hypergeometric2
F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [F]

$$\int (bx)^m \arccos(ax)\, dx$$

input
```
int((b*x)^m*arccos(a*x),x)
```

output
```
int((b*x)^m*arccos(a*x),x)
```

Fricas [F]

$$\int (bx)^m \arccos(ax)\, dx = \int (bx)^m \arccos(ax)\, dx$$

input
```
integrate((b*x)^m*arccos(a*x),x, algorithm="fricas")
```

output
```
integral((b*x)^m*arccos(a*x), x)
```

Sympy [F]

$$\int (bx)^m \arccos(ax)\,dx = \int (bx)^m \operatorname{acos}(ax)\,dx$$

input
```
integrate((b*x)**m*acos(a*x),x)
```

output
```
Integral((b*x)**m*acos(a*x), x)
```

Maxima [F]

$$\int (bx)^m \arccos(ax)\,dx = \int (bx)^m \arccos(ax)\,dx$$

input
```
integrate((b*x)^m*arccos(a*x),x, algorithm="maxima")
```

output
```
(b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - (a*b^m*m + a*b^m)*
integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m/((a^2*m + a^2)*x^2 - m - 1),
x))/(m + 1)
```

Giac [F]

$$\int (bx)^m \arccos(ax)\,dx = \int (bx)^m \arccos(ax)\,dx$$

input
```
integrate((b*x)^m*arccos(a*x),x, algorithm="giac")
```

output
```
integrate((b*x)^m*arccos(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m \arccos(ax)\,dx = \int \mathrm{acos}(a\,x)\,\left(b\,x\right)^m\,dx$$

input
```
int(acos(a*x)*(b*x)^m,x)
```

output
```
int(acos(a*x)*(b*x)^m, x)
```

Reduce [F]

$$\int (bx)^m \arccos(ax)\,dx = b^m\left(\int x^m acos(ax)\,dx\right)$$

input
```
int((b*x)^m*acos(a*x),x)
```

output
```
b**m*int(x**m*acos(a*x),x)
```

3.123 $\int \frac{(bx)^m}{\arccos(ax)}\,dx$

Optimal result 902
Mathematica [N/A] 902
Rubi [N/A] 903
Maple [N/A] 903
Fricas [N/A] 904
Sympy [N/A] 904
Maxima [N/A] 904
Giac [N/A] 905
Mupad [N/A] 905
Reduce [N/A] 906

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \text{Int}\left(\frac{(bx)^m}{\arccos(ax)},x\right)$$

output
```
Defer(Int)((b*x)^m/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos(ax)}\,dx$$

input
```
Integrate[(b*x)^m/ArcCos[a*x],x]
```

output
```
Integrate[(b*x)^m/ArcCos[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx)^m}{\arccos(ax)} \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{(bx)^m}{\arccos(ax)}dx$$

input
```
Int[(b*x)^m/ArcCos[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)}dx$$

input
```
int((b*x)^m/arccos(a*x),x)
```

output
```
int((b*x)^m/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x),x, algorithm="fricas")
```

output
```
integral((b*x)^m/arccos(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate((b*x)**m/acos(a*x),x)
```

output
```
Integral((b*x)**m/acos(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate((b*x)^m/arccos(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x),x, algorithm="giac")
```

output
```
integrate((b*x)^m/arccos(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{{\left(b\,x\right)}^m}{\mathrm{acos}\left(a\,x\right)}\,dx$$

input
```
int((b*x)^m/acos(a*x),x)
```

output
```
int((b*x)^m/acos(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = b^m\left(\int \frac{x^m}{acos\,(ax)}dx\right)$$

input
```
int((b*x)^m/acos(a*x),x)
```

output
```
b**m*int(x**m/acos(a*x),x)
```

3.124 $\int \frac{(bx)^m}{\arccos(ax)^2} dx$

Optimal result 907
Mathematica [N/A] 907
Rubi [N/A] 908
Maple [N/A] 908
Fricas [N/A] 909
Sympy [N/A] 909
Maxima [N/A] 909
Giac [N/A] 910
Mupad [N/A] 910
Reduce [N/A] 911

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{(bx)^m}{\arccos(ax)^2} dx = \text{Int}\left(\frac{(bx)^m}{\arccos(ax)^2}, x\right)$$

output
```
Defer(Int)((b*x)^m/arccos(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2} dx = \int \frac{(bx)^m}{\arccos(ax)^2} dx$$

input
```
Integrate[(b*x)^m/ArcCos[a*x]^2,x]
```

output
```
Integrate[(b*x)^m/ArcCos[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx)^m}{\arccos(ax)^2} \, dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{(bx)^m}{\arccos(ax)^2} dx$$

input
```
Int[(b*x)^m/ArcCos[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos (ax)^2} dx$$

input
```
int((b*x)^m/arccos(a*x)^2,x)
```

output
```
int((b*x)^m/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}^2}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral((b*x)^m/arccos(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\operatorname{acos}^2{(ax)}}\,dx$$

input
```
integrate((b*x)**m/acos(a*x)**2,x)
```

output
```
Integral((b*x)**m/acos(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 13.00

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}^2}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(sqrt(a*x + 1)*sqrt(-a*x + 1)*b^m*x^m - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x
+ 1), a*x)*integrate(((a^2*b^m*m + a^2*b^m)*x^2 - b^m*m)*sqrt(a*x + 1)*sqr
t(-a*x + 1)*x^m/((a^3*x^3 - a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x
)), x))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2} \, dx = \int \frac{(bx)^m}{\arccos{(ax)}^2} \, dx$$

input
```
integrate((b*x)^m/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate((b*x)^m/arccos(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2} \, dx = \int \frac{(b\,x)^m}{\mathrm{acos}\left(a\,x\right)^2} \, dx$$

input
```
int((b*x)^m/acos(a*x)^2,x)
```

output
```
int((b*x)^m/acos(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = b^m\left(\int \frac{x^m}{acos\,(ax)^2}dx\right)$$

input
```
int((b*x)^m/acos(a*x)^2,x)
```

output
```
b**m*int(x**m/acos(a*x)**2,x)
```

3.125 $\int (bx)^m \arccos(ax)^{3/2} \, dx$

Optimal result . . . 912
Mathematica [N/A] . . . 912
Rubi [N/A] . . . 913
Maple [N/A] . . . 913
Fricas [F(-2)] . . . 914
Sympy [N/A] . . . 914
Maxima [F(-2)] . . . 914
Giac [N/A] . . . 915
Mupad [N/A] . . . 915
Reduce [N/A] . . . 915

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (bx)^m \arccos(ax)^{3/2} \, dx = \text{Int}\left((bx)^m \arccos(ax)^{3/2}, x\right)$$

output
```
Defer(Int)((b*x)^m*arccos(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^m \arccos(ax)^{3/2} \, dx = \int (bx)^m \arccos(ax)^{3/2} \, dx$$

input
```
Integrate[(b*x)^m*ArcCos[a*x]^(3/2),x]
```

output
```
Integrate[(b*x)^m*ArcCos[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{3/2}(bx)^m\,dx$$

$$\Big\downarrow \text{5149}$$

$$\int \arccos(ax)^{3/2}(bx)^m dx$$

input
```
Int[(b*x)^m*ArcCos[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^m \arccos\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
int((b*x)^m*arccos(a*x)^(3/2),x)
```

output
```
int((b*x)^m*arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (bx)^m \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 54.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^{3/2}\,dx = \int (bx)^m \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate((b*x)**m*acos(a*x)**(3/2),x)
```

output
```
Integral((b*x)**m*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^m \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^{3/2} \, dx = \int (bx)^m \arccos(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x)^m*arccos(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^{3/2} \, dx = \int \text{acos}(a\,x)^{3/2} \, (b\,x)^m \, dx$$

input
```
int(acos(a*x)^(3/2)*(b*x)^m,x)
```

output
```
int(acos(a*x)^(3/2)*(b*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int (bx)^m \arccos(ax)^{3/2} \, dx = b^m \left(\int x^m \sqrt{acos(ax)}\, acos(ax) \, dx \right)$$

input
```
int((b*x)^m*acos(a*x)^(3/2),x)
```

output
```
b**m*int(x**m*sqrt(acos(a*x))*acos(a*x),x)
```

3.126 $\int (bx)^m \sqrt{\arccos(ax)}\,dx$

Optimal result 917
Mathematica [N/A] 917
Rubi [N/A] 918
Maple [N/A] 918
Fricas [F(-2)] 919
Sympy [N/A] 919
Maxima [F(-2)] 919
Giac [N/A] 920
Mupad [N/A] 920
Reduce [N/A] 920

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (bx)^m \sqrt{\arccos(ax)}\,dx = \text{Int}\left((bx)^m \sqrt{\arccos(ax)},x\right)$$

output
```
Defer(Int)((b*x)^m*arccos(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^m \sqrt{\arccos(ax)}\,dx = \int (bx)^m \sqrt{\arccos(ax)}\,dx$$

input
```
Integrate[(b*x)^m*Sqrt[ArcCos[a*x]],x]
```

output
```
Integrate[(b*x)^m*Sqrt[ArcCos[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arccos(ax)}(bx)^m\,dx$$
$$\Big\downarrow \text{5149}$$
$$\int \sqrt{\arccos(ax)}(bx)^m dx$$

input
```
Int[(b*x)^m*Sqrt[ArcCos[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^m \sqrt{\arccos(ax)}dx$$

input
```
int((b*x)^m*arccos(a*x)^(1/2),x)
```

output
```
int((b*x)^m*arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (bx)^m \sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \sqrt{\arccos(ax)}\,dx = \int (bx)^m \sqrt{\operatorname{acos}(ax)}\,dx$$

input
```
integrate((b*x)**m*acos(a*x)**(1/2),x)
```

output
```
Integral((b*x)**m*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^m \sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int (bx)^m \sqrt{\arccos(ax)}\, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x)^m*sqrt(arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int \sqrt{\mathrm{acos}(a\,x)}\,(b\,x)^m\, dx$$

input
```
int(acos(a*x)^(1/2)*(b*x)^m,x)
```

output
```
int(acos(a*x)^(1/2)*(b*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = b^m \left(\int x^m \sqrt{acos(ax)} dx \right)$$

input
```
int((b*x)^m*acos(a*x)^(1/2),x)
```

output
```
b**m*int(x**m*sqrt(acos(a*x)),x)
```

3.127 $\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} \, dx$

Optimal result 922
Mathematica [N/A] 922
Rubi [N/A] 923
Maple [N/A] 923
Fricas [F(-2)] 924
Sympy [N/A] 924
Maxima [F(-2)] 924
Giac [N/A] 925
Mupad [N/A] 925
Reduce [N/A] 925

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} \, dx = \text{Int}\left(\frac{(bx)^m}{\sqrt{\arccos(ax)}}, x\right)$$

output
```
Defer(Int)((b*x)^m/arccos(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} \, dx = \int \frac{(bx)^m}{\sqrt{\arccos(ax)}} \, dx$$

input
```
Integrate[(b*x)^m/Sqrt[ArcCos[a*x]],x]
```

output
```
Integrate[(b*x)^m/Sqrt[ArcCos[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}dx$$

input
```
Int[(b*x)^m/Sqrt[ArcCos[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(bx)^m}{\sqrt{\arccos\left(ax\right)}}dx$$

input
```
int((b*x)^m/arccos(a*x)^(1/2),x)
```

output
```
int((b*x)^m/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx = \int \frac{(bx)^m}{\sqrt{\operatorname{acos}(ax)}} dx$$

input
```
integrate((b*x)**m/acos(a*x)**(1/2),x)
```

output
```
Integral((b*x)**m/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x)^m/sqrt(arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(b\,x)^m}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

input
```
int((b*x)^m/acos(a*x)^(1/2),x)
```

output
```
int((b*x)^m/acos(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 10.36

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \frac{2b^m\left(-x^m\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}+\left(\int \frac{x^m\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{a^2x^2-1}dx\right)a^2m+\left(\int \frac{x^m\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{a^2x^2-1}dx\right)a}{a}$$

input
```
int((b*x)^m/acos(a*x)^(1/2),x)
```

output
```
(2*b**m*( - x**m*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)) + int((x**m*sqrt(
- a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(a**2*x**2 - 1),x)*a**2*m + int((x**m*
sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(a**2*x**2 - 1),x)*a**2 - int((x
**m*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(a**2*x**3 - x),x)*m))/a
```

3.128 $\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$

Optimal result 927
Mathematica [N/A] 927
Rubi [N/A] 928
Maple [N/A] 928
Fricas [F(-2)] 929
Sympy [N/A] 929
Maxima [F(-2)] 929
Giac [N/A] 930
Mupad [N/A] 930
Reduce [N/A] 930

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(bx)^m}{\arccos(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((b*x)^m/arccos(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$$

input
```
Integrate[(b*x)^m/ArcCos[a*x]^(3/2),x]
```

output
```
Integrate[(b*x)^m/ArcCos[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \; 5149$$

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}dx$$

input
```
Int[(b*x)^m/ArcCos[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(bx)^m}{\arccos\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x)^m/arccos(a*x)^(3/2),x)
```

output
```
int((b*x)^m/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}} \, dx = \int \frac{(bx)^m}{\operatorname{acos}^{\frac{3}{2}}(ax)} \, dx$$

input
```
integrate((b*x)**m/acos(a*x)**(3/2),x)
```

output
```
Integral((b*x)**m/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x)^m/arccos(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(b\,x)^m}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((b*x)^m/acos(a*x)^(3/2),x)
```

output
```
int((b*x)^m/acos(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 13.50

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \frac{2b^m\left(-acos(ax)\left(\int \frac{x^m\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x}{acos(ax)a^2x^2-acos(ax)}dx\right)a^2m - acos(ax)\left(\int \frac{x^m\sqrt{-a^2x^2+1}\,\sqrt{acos(a}}{acos(ax)a^2x^2-acos(a}\right.\right.}{acos\,(}$$

input
```
int((b*x)^m/acos(a*x)^(3/2),x)
```

output
```
(2*b**m*( - acos(a*x)*int((x**m*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x)/
(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**2*m - acos(a*x)*int((x**m*sqrt( -
a**2*x**2 + 1)*sqrt(acos(a*x))*x)/(acos(a*x)*a**2*x**2 - acos(a*x)),x)*a**
2 + acos(a*x)*int((x**m*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)
*a**2*x**3 - acos(a*x)*x),x)*m + x**m*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x
))))/(acos(a*x)*a)
```

3.129 $\int (bx)^m \arccos(ax)^n \, dx$

Optimal result 932
Mathematica [N/A] 932
Rubi [N/A] 933
Maple [N/A] 933
Fricas [N/A] 934
Sympy [N/A] 934
Maxima [F(-2)] 934
Giac [N/A] 935
Mupad [N/A] 935
Reduce [N/A] 935

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \arccos(ax)^n \, dx = \text{Int}((bx)^m \arccos(ax)^n, x)$$

output
```
Defer(Int)((b*x)^m*arccos(a*x)^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos(ax)^n \, dx$$

input
```
Integrate[(b*x)^m*ArcCos[a*x]^n,x]
```

output
```
Integrate[(b*x)^m*ArcCos[a*x]^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^m \arccos(ax)^n \, dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int (bx)^m \arccos(ax)^n dx$$

input
```
Int[(b*x)^m*ArcCos[a*x]^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^n \, dx$$

input
```
int((b*x)^m*arccos(a*x)^n,x)
```

output
```
int((b*x)^m*arccos(a*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos{(ax)}^n \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral((b*x)^m*arccos(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \operatorname{acos}^n{(ax)} \, dx$$

input
```
integrate((b*x)**m*acos(a*x)**n,x)
```

output
```
Integral((b*x)**m*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^m \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^m*arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos{(ax)}^n \, dx$$

input
```
integrate((b*x)^m*arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate((b*x)^m*arccos(a*x)^n, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int \mathrm{acos}(a\,x)^n\,{(b\,x)}^m \, dx$$

input
```
int(acos(a*x)^n*(b*x)^m,x)
```

output
```
int(acos(a*x)^n*(b*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int (bx)^m \arccos(ax)^n \, dx = b^m \left(\int x^m acos(ax)^n \, dx \right)$$

input
```
int((b*x)^m*acos(a*x)^n,x)
```

output
```
b**m*int(x**m*acos(a*x)**n,x)
```

3.130 $\int x^3 \arccos(ax)^n \, dx$

Optimal result 937
Mathematica [A] (verified) 938
Rubi [A] (verified) 938
Maple [C] (verified) 939
Fricas [F] 940
Sympy [F] 940
Maxima [F(-2)] 941
Giac [F] 941
Mupad [F(-1)] 941
Reduce [F] 942

Optimal result

Integrand size = 10, antiderivative size = 165

$$\begin{aligned}\int x^3 \arccos(ax)^n \, dx = {} & \frac{2^{-4-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-2i\arccos(ax))}{a^4} \\ & + \frac{2^{-4-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,2i\arccos(ax))}{a^4} \\ & + \frac{4^{-3-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-4i\arccos(ax))}{a^4} \\ & + \frac{4^{-3-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,4i\arccos(ax))}{a^4}\end{aligned}$$

output
```
2^(-4-n)*arccos(a*x)^n*GAMMA(1+n,-2*I*arccos(a*x))/a^4/((-I*arccos(a*x))^n
)+2^(-4-n)*arccos(a*x)^n*GAMMA(1+n,2*I*arccos(a*x))/a^4/((I*arccos(a*x))^n
)+4^(-3-n)*arccos(a*x)^n*GAMMA(1+n,-4*I*arccos(a*x))/a^4/((-I*arccos(a*x))
^n)+4^(-3-n)*arccos(a*x)^n*GAMMA(1+n,4*I*arccos(a*x))/a^4/((I*arccos(a*x))
^n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

$$\int x^3 \arccos(ax)^n\,dx$$
$$= \frac{2^{-2(3+n)} \arccos(ax)^n \left(\arccos(ax)^2\right)^{-n} \left(2^{2+n}(i\arccos(ax))^n\Gamma(1+n,-2i\arccos(ax)) + 2^{2+n}(-i\arccos(ax\right.}{}$$

input
```
Integrate[x^3*ArcCos[a*x]^n,x]
```

output
```
(ArcCos[a*x]^n*(2^(2 + n)*(I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*x
]] + 2^(2 + n)*((-I)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]] + (I*A
rcCos[a*x])^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]] + ((-I)*ArcCos[a*x])^n*Gamm
a[1 + n, (4*I)*ArcCos[a*x]]))/(2^(2*(3 + n))*a^4*(ArcCos[a*x]^2)^n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(ax)^n\,dx$$
$$\Big\downarrow \text{ 5147}$$
$$-\frac{\int a^3x^3\sqrt{1-a^2x^2}\arccos(ax)^n d\arccos(ax)}{a^4}$$
$$\Big\downarrow \text{ 4906}$$
$$-\frac{\int \left(\frac{1}{4}\sin(2\arccos(ax))\arccos(ax)^n+\frac{1}{8}\sin(4\arccos(ax))\arccos(ax)^n\right)d\arccos(ax)}{a^4}$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{-2^{-n-4}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-2i\arccos(ax))-2^{-2(n+3)}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n-$$

input
```
Int[x^3*ArcCos[a*x]^n,x]
```

output
```
-((-((2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/((-I)*Arc
Cos[a*x])^n) - (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/
(I*ArcCos[a*x])^n - (ArcCos[a*x]^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]])/(2^(2
*(3 + n))*((-I)*ArcCos[a*x])^n) - (ArcCos[a*x]^n*Gamma[1 + n, (4*I)*ArcCos
[a*x]])/(2^(2*(3 + n))*(I*ArcCos[a*x])^n))/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\pi}\left(\frac{2\arccos(ax)^{1+n}\sin(2\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{2^{\frac{1}{2}-n}\sqrt{\arccos(ax)}\ \mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},2\arccos(ax)\right)\sin(2\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{3\,2^{-\frac{3}{2}-n}\left(\frac{4}{3}+\frac{2n}{3}\right)(2\cos(2\right.}{8a^4}$

input
```
int(x^3*arccos(a*x)^n,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Pi^(1/2)/a^4*(2/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(2*arccos(a*x))-2
^(1/2-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arccos(a*x)
)*sin(2*arccos(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/
3*n)*(2*cos(2*arccos(a*x))*arccos(a*x)-sin(2*arccos(a*x)))*LommelS1(n+1/2,
1/2,2*arccos(a*x)))-2^(-5-n)*Pi^(1/2)/a^4*(2^(2+n)/Pi^(1/2)/(2+n)*arccos(a
*x)^(1+n)*sin(4*arccos(a*x))-2^(-n+1)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*Lom
melS1(n+3/2,3/2,4*arccos(a*x))*sin(4*arccos(a*x))-3*2^(-n-2)/Pi^(1/2)/(2+n
)/arccos(a*x)^(1/2)*(4/3+2/3*n)*(4*arccos(a*x)*cos(4*arccos(a*x))-sin(4*ar
ccos(a*x)))*LommelS1(n+1/2,1/2,4*arccos(a*x)))
```

Fricas [F]

$$\int x^3 \arccos(ax)^n\,dx = \int x^3 \arccos{(ax)}^n\,dx$$

input
```
integrate(x^3*arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral(x^3*arccos(a*x)^n, x)
```

Sympy [F]

$$\int x^3 \arccos(ax)^n\,dx = \int x^3 \operatorname{acos}^n{(ax)}\,dx$$

input
```
integrate(x**3*acos(a*x)**n,x)
```

output
```
Integral(x**3*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int x^3 \arccos(ax)^n \, dx = \int x^3 \arccos{(ax)}^n \, dx$$

input
```
integrate(x^3*arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate(x^3*arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^n \, dx = \int x^3 \operatorname{acos}(a\,x)^n \, dx$$

input
```
int(x^3*acos(a*x)^n,x)
```

output
```
int(x^3*acos(a*x)^n, x)
```

Reduce [F]

$$\int x^3 \arccos(ax)^n \, dx = \int acos(ax)^n \, x^3 dx$$

input
```
int(x^3*acos(a*x)^n,x)
```

output
```
int(acos(a*x)**n*x**3,x)
```

3.131 $\int x^2 \arccos(ax)^n \, dx$

Optimal result . . . 943
Mathematica [A] (verified) . . . 944
Rubi [A] (verified) . . . 944
Maple [F] . . . 945
Fricas [F] . . . 946
Sympy [F] . . . 946
Maxima [F(-2)] . . . 946
Giac [F] . . . 947
Mupad [F(-1)] . . . 947
Reduce [F] . . . 947

Optimal result

Integrand size = 10, antiderivative size = 163

$$\int x^2 \arccos(ax)^n \, dx = \frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{8a^3} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{8a^3} + \frac{3^{-1-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-3i\arccos(ax))}{8a^3} + \frac{3^{-1-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,3i\arccos(ax))}{8a^3}$$

output
```
1/8*arccos(a*x)^n*GAMMA(1+n,-I*arccos(a*x))/a^3/((-I*arccos(a*x))^n)+1/8*a
rccos(a*x)^n*GAMMA(1+n,I*arccos(a*x))/a^3/((I*arccos(a*x))^n)+1/8*3^(-1-n)
*arccos(a*x)^n*GAMMA(1+n,-3*I*arccos(a*x))/a^3/((-I*arccos(a*x))^n)+1/8*3^
(-1-n)*arccos(a*x)^n*GAMMA(1+n,3*I*arccos(a*x))/a^3/((I*arccos(a*x))^n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

$$\int x^2 \arccos(ax)^n \, dx$$
$$= \frac{\frac{1}{4}\left(\frac{1}{2}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax)) + \frac{1}{2}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax)) \right.}{}$$

input
```
Integrate[x^2*ArcCos[a*x]^n,x]
```

output
```
(((ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(2*((-I)*ArcCos[a*x])^n)
+ (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*(I*ArcCos[a*x])^n))/4 + (
3^(-1 - n)*ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-3*I)*ArcCos[a*x
]] + ((-I)*ArcCos[a*x])^n*Gamma[1 + n, (3*I)*ArcCos[a*x]]))/(8*(ArcCos[a*x
]^2)^n))/a^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5147, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(ax)^n \, dx$$
$$\Big\downarrow \; 5147$$
$$-\frac{\int a^2x^2\sqrt{1-a^2x^2}\arccos(ax)^n d\arccos(ax)}{a^3}$$
$$\Big\downarrow \; 4906$$
$$-\frac{\int \left(\frac{1}{4}\sin(3\arccos(ax))\arccos(ax)^n + \frac{1}{4}\sqrt{1-a^2x^2}\arccos(ax)^n\right) d\arccos(ax)}{a^3}$$
$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{1}{8}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-i\arccos(ax))-\frac{1}{8}3^{-n-1}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-3$$

input
```
Int[x^2*ArcCos[a*x]^n,x]
```

output
```
-((-1/8*(ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/((-I)*ArcCos[a*x])^
n - (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(8*(I*ArcCos[a*x])^n) - (3
^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]])/(8*((-I)*ArcCos[
a*x])^n) - (3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (3*I)*ArcCos[a*x]])/(8*(
I*ArcCos[a*x])^n))/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \arccos(ax)^n\,dx$$

input
```
int(x^2*arccos(a*x)^n,x)
```

output
```
int(x^2*arccos(a*x)^n,x)
```

Fricas [F]

$$\int x^2 \arccos(ax)^n\,dx = \int x^2 \arccos{(ax)}^n\,dx$$

input
```
integrate(x^2*arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral(x^2*arccos(a*x)^n, x)
```

Sympy [F]

$$\int x^2 \arccos(ax)^n\,dx = \int x^2 \operatorname{acos}^n{(ax)}\,dx$$

input
```
integrate(x**2*acos(a*x)**n,x)
```

output
```
Integral(x**2*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^n\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int x^2 \arccos(ax)^n \, dx = \int x^2 \arccos{(ax)}^n \, dx$$

input
```
integrate(x^2*arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate(x^2*arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^n \, dx = \int x^2 \operatorname{acos}(a\,x)^n \, dx$$

input
```
int(x^2*acos(a*x)^n,x)
```

output
```
int(x^2*acos(a*x)^n, x)
```

Reduce [F]

$$\int x^2 \arccos(ax)^n \, dx = \int acos(ax)^n \, x^2 dx$$

input
```
int(x^2*acos(a*x)^n,x)
```

output
```
int(acos(a*x)**n*x**2,x)
```

3.132 $\int x \arccos(ax)^n \, dx$

Optimal result 948
Mathematica [A] (verified) 948
Rubi [A] (verified) 949
Maple [C] (verified) 951
Fricas [F] 951
Sympy [F] 952
Maxima [F(-2)] 952
Giac [F] 952
Mupad [F(-1)] 953
Reduce [F] 953

Optimal result

Integrand size = 8, antiderivative size = 83

$$\int x \arccos(ax)^n \, dx = \frac{2^{-3-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-2i\arccos(ax))}{a^2} + \frac{2^{-3-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,2i\arccos(ax))}{a^2}$$

output
```
2^(-3-n)*arccos(a*x)^n*GAMMA(1+n,-2*I*arccos(a*x))/a^2/((-I*arccos(a*x))^n
)+2^(-3-n)*arccos(a*x)^n*GAMMA(1+n,2*I*arccos(a*x))/a^2/((I*arccos(a*x))^n
)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int x \arccos(ax)^n \, dx = \frac{2^{-3-n}\arccos(ax)^n\left(\arccos(ax)^2\right)^{-n}\left((i\arccos(ax))^n\Gamma(1+n,-2i\arccos(ax)) + (-i\arccos(ax))^n\Gamma(1+n,\right.}{a^2}$$

input
```
Integrate[x*ArcCos[a*x]^n,x]
```

output
```
(2^(-3 - n)*ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*
x]] + ((-I)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]]))/(a^2*(ArcCos[
a*x]^2)^n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5147, 4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(ax)^n \, dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int ax\sqrt{1-a^2x^2}\arccos(ax)^n d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \frac{1}{2}\arccos(ax)^n \sin(2\arccos(ax))d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \arccos(ax)^n \sin(2\arccos(ax))d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \arccos(ax)^n \sin(2\arccos(ax))d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{\frac{1}{2}i\int e^{-2i\arccos(ax)}\arccos(ax)^n d\arccos(ax)-\frac{1}{2}i\int e^{2i\arccos(ax)}\arccos(ax)^n d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{-2^{-n-2}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-2i\arccos(ax))-2^{-n-2}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,}{2a^2}$$

input
```
Int[x*ArcCos[a*x]^n,x]
```

output
```
-1/2*(-((2^(-2 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/((-I)*
ArcCos[a*x])^n) - (2^(-2 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]
])/(I*ArcCos[a*x])^n)/a^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{\pi}\left(\frac{2\arccos(ax)^{1+n}\sin(2\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{2^{\frac{1}{2}-n}\sqrt{\arccos(ax)}\ \mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},2\arccos(ax)\right)\sin(2\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{3\,2^{-\frac{3}{2}-n}\left(\frac{4}{3}+\frac{2n}{3}\right)(2\cos(2}{}\right.}{4a^2}$

input
```
int(x*arccos(a*x)^n,x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi^(1/2)/a^2*(2/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(2*arccos(a*x))-2
^(1/2-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arccos(a*x)
)*sin(2*arccos(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/
3*n)*(2*cos(2*arccos(a*x))*arccos(a*x)-sin(2*arccos(a*x)))*LommelS1(n+1/2,
1/2,2*arccos(a*x)))
```

Fricas [F]

$$\int x\arccos(ax)^n\,dx=\int x\arccos{(ax)}^n\,dx$$

input
```
integrate(x*arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral(x*arccos(a*x)^n, x)
```

Sympy [F]

$$\int x \arccos(ax)^n \, dx = \int x \operatorname{acos}^n(ax) \, dx$$

input
```
integrate(x*acos(a*x)**n,x)
```

output
```
Integral(x*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int x \arccos(ax)^n \, dx = \int x \arccos(ax)^n \, dx$$

input
```
integrate(x*arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate(x*arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^n \, dx = \int x \operatorname{acos}(a\, x)^n \, dx$$

input
```
int(x*acos(a*x)^n,x)
```

output
```
int(x*acos(a*x)^n, x)
```

Reduce [F]

$$\int x \arccos(ax)^n \, dx = \int acos(ax)^n \, x dx$$

input
```
int(x*acos(a*x)^n,x)
```

output
```
int(acos(a*x)**n*x,x)
```

3.133 $\int \arccos(ax)^n \, dx$

Optimal result 954
Mathematica [A] (verified) 954
Rubi [A] (verified) 955
Maple [C] (verified) 956
Fricas [F] 957
Sympy [F] 957
Maxima [F(-2)] 957
Giac [F] 958
Mupad [F(-1)] 958
Reduce [F] 958

Optimal result

Integrand size = 6, antiderivative size = 75

$$\int \arccos(ax)^n \, dx = \frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{2a} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{2a}$$

output
```
1/2*arccos(a*x)^n*GAMMA(1+n,-I*arccos(a*x))/a/((-I*arccos(a*x))^n)+1/2*arc
cos(a*x)^n*GAMMA(1+n,I*arccos(a*x))/a/((I*arccos(a*x))^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \arccos(ax)^n \, dx = \frac{\arccos(ax)^n \left(\arccos(ax)^2\right)^{-n} \left((i\arccos(ax))^n\Gamma(1+n,-i\arccos(ax)) + (-i\arccos(ax))^n\Gamma(1+n,i\arcco\right.}{2a}$$

input
```
Integrate[ArcCos[a*x]^n,x]
```

output
```
(ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-I)*ArcCos[a*x]] + ((-I)*A
rcCos[a*x])^n*Gamma[1 + n, I*ArcCos[a*x]]))/(2*a*(ArcCos[a*x]^2)^n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5135, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^n\,dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int \sqrt{1-a^2x^2}\arccos(ax)^n d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \arccos(ax)^n \sin(\arccos(ax))d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{\frac{1}{2}i\int e^{-i\arccos(ax)}\arccos(ax)^n d\arccos(ax)-\frac{1}{2}i\int e^{i\arccos(ax)}\arccos(ax)^n d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{-\frac{1}{2}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-i\arccos(ax))-\frac{1}{2}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,i\arccos(ax
$$

input
```
Int[ArcCos[a*x]^n,x]
```

output
```
-((-1/2*(ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/((-I)*ArcCos[a*x])^
n - (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*(I*ArcCos[a*x])^n))/a)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

method	result
default	$-\frac{2^{n}\sqrt{\pi}\left(\frac{\arccos(ax)^{1+n}2^{-n}\sqrt{-a^{2}x^{2}+1}}{\sqrt{\pi}\,(2+n)}-\frac{2^{-n}\sqrt{\arccos(ax)}\ \mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(ax)\right)\sqrt{-a^{2}x^{2}+1}}{\sqrt{\pi}\,(2+n)}-\frac{3\,2^{-1-n}\left(\frac{4}{3}+\frac{2n}{3}\right)\left(ax\arccos(ax)-\right.}{\sqrt{\pi}\,(2}\right.}{a}$

input
```
int(arccos(a*x)^n,x,method=_RETURNVERBOSE)
```

output
```
-2^n*Pi^(1/2)/a*(1/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*2^(-n)*(-a^2*x^2+1)^(1
/2)-2^(-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,arccos(a*x)
)*(-a^2*x^2+1)^(1/2)-3*2^(-1-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*
n)*(a*x*arccos(a*x)-(-a^2*x^2+1)^(1/2))*LommelS1(n+1/2,1/2,arccos(a*x)))
```

Fricas [F]

$$\int \arccos(ax)^n\,dx = \int \arccos{(ax)}^n\,dx$$

input
```
integrate(arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^n, x)
```

Sympy [F]

$$\int \arccos(ax)^n\,dx = \int \operatorname{acos}^n{(ax)}\,dx$$

input
```
integrate(acos(a*x)**n,x)
```

output
```
Integral(acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arccos(ax)^n\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \arccos(ax)^n\,dx = \int \arccos{(ax)}^n\,dx$$

input
```
integrate(arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(ax)^n\,dx = \int \mathrm{acos}(a\,x)^n\,dx$$

input
```
int(acos(a*x)^n,x)
```

output
```
int(acos(a*x)^n, x)
```

Reduce [F]

$$\int \arccos(ax)^n\,dx = \int acos(ax)^n\,dx$$

input
```
int(acos(a*x)^n,x)
```

output
```
int(acos(a*x)**n,x)
```

3.134 $\int \frac{\arccos(ax)^n}{x}\,dx$

Optimal result 959
Mathematica [N/A] 959
Rubi [N/A] 960
Maple [N/A] 960
Fricas [N/A] 961
Sympy [N/A] 961
Maxima [F(-2)] 961
Giac [N/A] 962
Mupad [N/A] 962
Reduce [N/A] 962

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\arccos(ax)^n}{x}\,dx = \mathrm{Int}\left(\frac{\arccos(ax)^n}{x},x\right)$$

output
```
Defer(Int)(arccos(a*x)^n/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x}\,dx = \int \frac{\arccos(ax)^n}{x}\,dx$$

input
```
Integrate[ArcCos[a*x]^n/x,x]
```

output
```
Integrate[ArcCos[a*x]^n/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{x} \, dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{\arccos(ax)^n}{x}dx$$

input
```
Int[ArcCos[a*x]^n/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\arccos \left(ax\right)^{n}}{x}dx$$

input
```
int(arccos(a*x)^n/x,x)
```

output
```
int(arccos(a*x)^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x} \, dx = \int \frac{\arccos{(ax)}^n}{x} \, dx$$

input
```
integrate(arccos(a*x)^n/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\arccos(ax)^n}{x} \, dx = \int \frac{\operatorname{acos}^n{(ax)}}{x} \, dx$$

input
```
integrate(acos(a*x)**n/x,x)
```

output
```
Integral(acos(a*x)**n/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x}\,dx = \int \frac{\arccos{(ax)}^n}{x}\,dx$$

input
```
integrate(arccos(a*x)^n/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{x}\,dx$$

input
```
int(acos(a*x)^n/x,x)
```

output
```
int(acos(a*x)^n/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x}\,dx = \int \frac{acos(ax)^n}{x}dx$$

input
```
int(acos(a*x)^n/x,x)
```

output
```
int(acos(a*x)**n/x,x)
```

3.135 $\int \frac{\arccos(ax)^n}{x^2} dx$

Optimal result 964
Mathematica [N/A] 964
Rubi [N/A] 965
Maple [N/A] 965
Fricas [N/A] 966
Sympy [N/A] 966
Maxima [F(-2)] 966
Giac [N/A] 967
Mupad [N/A] 967
Reduce [N/A] 967

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\arccos(ax)^n}{x^2} dx = \text{Int}\left(\frac{\arccos(ax)^n}{x^2}, x\right)$$

output
```
Defer(Int)(arccos(a*x)^n/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2} dx = \int \frac{\arccos(ax)^n}{x^2} dx$$

input
```
Integrate[ArcCos[a*x]^n/x^2,x]
```

output
```
Integrate[ArcCos[a*x]^n/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{x^2} \, dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{\arccos(ax)^n}{x^2}dx$$

input
```
Int[ArcCos[a*x]^n/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(ax\right)^{n}}{x^{2}}dx$$

input
```
int(arccos(a*x)^n/x^2,x)
```

output
```
int(arccos(a*x)^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{\arccos{(ax)}^n}{x^2}\,dx$$

input
```
integrate(arccos(a*x)^n/x^2,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{\operatorname{acos}^n{(ax)}}{x^2}\,dx$$

input
```
integrate(acos(a*x)**n/x**2,x)
```

output
```
Integral(acos(a*x)**n/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{\arccos{(ax)}^n}{x^2}\,dx$$

input
```
integrate(arccos(a*x)^n/x^2,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{x^2}\,dx$$

input
```
int(acos(a*x)^n/x^2,x)
```

output
```
int(acos(a*x)^n/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{acos(ax)^n}{x^2}dx$$

input
```
int(acos(a*x)^n/x^2,x)
```

output

```
int(acos(a*x)**n/x**2,x)
```

3.136 $\int (bx)^{3/2} \arccos(ax)^n \, dx$

Optimal result . . . 969
Mathematica [N/A] . . . 969
Rubi [N/A] . . . 970
Maple [N/A] . . . 970
Fricas [N/A] . . . 971
Sympy [F(-1)] . . . 971
Maxima [F(-2)] . . . 971
Giac [N/A] . . . 972
Mupad [N/A] . . . 972
Reduce [N/A] . . . 972

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \operatorname{Int}\left((bx)^{3/2} \arccos(ax)^n, x\right)$$

output
```
Defer(Int)((b*x)^(3/2)*arccos(a*x)^n,x)
```

Mathematica [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{3/2} \arccos(ax)^n \, dx$$

input
```
Integrate[(b*x)^(3/2)*ArcCos[a*x]^n,x]
```

output
```
Integrate[(b*x)^(3/2)*ArcCos[a*x]^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx)^{3/2}\arccos(ax)^n\,dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int (bx)^{3/2}\arccos(ax)^n dx$$

input
```
Int[(b*x)^(3/2)*ArcCos[a*x]^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{\frac{3}{2}}\arccos(ax)^n\,dx$$

input
```
int((b*x)^(3/2)*arccos(a*x)^n,x)
```

output
```
int((b*x)^(3/2)*arccos(a*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{\frac{3}{2}} \arccos{(ax)}^n \, dx$$

input
```
integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x)*b*x*arccos(a*x)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \text{Timed out}$$

input
```
integrate((b*x)**(3/2)*acos(a*x)**n,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{\frac{3}{2}} \arccos(ax)^n \, dx$$

input
```
integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate((b*x)^(3/2)*arccos(a*x)^n, x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int \mathrm{acos}(a\,x)^n\,(b\,x)^{3/2} \, dx$$

input
```
int(acos(a*x)^n*(b*x)^(3/2),x)
```

output
```
int(acos(a*x)^n*(b*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \sqrt{b}\left(\int \sqrt{x}\, acos(ax)^n\, xdx\right) b$$

input
```
int((b*x)^(3/2)*acos(a*x)^n,x)
```

output
```
sqrt(b)*int(sqrt(x)*acos(a*x)**n*x,x)*b
```

3.137 $\int \sqrt{bx} \arccos(ax)^n \, dx$

Optimal result 974
Mathematica [N/A] 974
Rubi [N/A] 975
Maple [N/A] 975
Fricas [N/A] 976
Sympy [N/A] 976
Maxima [F(-2)] 976
Giac [N/A] 977
Mupad [N/A] 977
Reduce [N/A] 977

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \text{Int}\left(\sqrt{bx} \arccos(ax)^n, x\right)$$

output
```
Defer(Int)((b*x)^(1/2)*arccos(a*x)^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos(ax)^n \, dx$$

input
```
Integrate[Sqrt[b*x]*ArcCos[a*x]^n,x]
```

output
```
Integrate[Sqrt[b*x]*ArcCos[a*x]^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx} \arccos(ax)^n \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \sqrt{bx} \arccos(ax)^n dx$$

input
```
Int[Sqrt[b*x]*ArcCos[a*x]^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \sqrt{bx}\, \arccos\left(ax\right)^{n} dx$$

input
```
int((b*x)^(1/2)*arccos(a*x)^n,x)
```

output
```
int((b*x)^(1/2)*arccos(a*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos{(ax)}^n \, dx$$

input
```
integrate((b*x)^(1/2)*arccos(a*x)^n,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x)*arccos(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \operatorname{acos}^n{(ax)} \, dx$$

input
```
integrate((b*x)**(1/2)*acos(a*x)**n,x)
```

output
```
Integral(sqrt(b*x)*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*x)^(1/2)*arccos(a*x)^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos{(ax)}^n \, dx$$

input
```
integrate((b*x)^(1/2)*arccos(a*x)^n,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x)*arccos(a*x)^n, x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \mathrm{acos}(a\,x)^n\,\sqrt{b\,x} \, dx$$

input
```
int(acos(a*x)^n*(b*x)^(1/2),x)
```

output
```
int(acos(a*x)^n*(b*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \sqrt{b} \left(\int \sqrt{x}\, acos(ax)^n \, dx \right)$$

input
```
int((b*x)^(1/2)*acos(a*x)^n,x)
```

output
```
sqrt(b)*int(sqrt(x)*acos(a*x)**n,x)
```

3.138 $\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$

Optimal result 979
Mathematica [N/A] 979
Rubi [N/A] 980
Maple [N/A] 980
Fricas [N/A] 981
Sympy [N/A] 981
Maxima [F(-2)] 981
Giac [N/A] 982
Mupad [N/A] 982
Reduce [N/A] 983

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \text{Int}\left(\frac{\arccos(ax)^n}{\sqrt{bx}},x\right)$$

output
```
Defer(Int)(arccos(a*x)^n/(b*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$$

input
```
Integrate[ArcCos[a*x]^n/Sqrt[b*x],x]
```

output
```
Integrate[ArcCos[a*x]^n/Sqrt[b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}} \, dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}dx$$

input
```
Int[ArcCos[a*x]^n/Sqrt[b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\arccos\left(ax\right)^n}{\sqrt{bx}}dx$$

input
```
int(arccos(a*x)^n/(b*x)^(1/2),x)
```

output
```
int(arccos(a*x)^n/(b*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{bx}}\,dx$$

input
```
integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x)*arccos(a*x)^n/(b*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\operatorname{acos}^n{(ax)}}{\sqrt{bx}}\,dx$$

input
```
integrate(acos(a*x)**n/(b*x)**(1/2),x)
```

output
```
Integral(acos(a*x)**n/sqrt(b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{bx}}\,dx$$

input
```
integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n/sqrt(b*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{\sqrt{b\,x}}\,dx$$

input
```
int(acos(a*x)^n/(b*x)^(1/2),x)
```

output
```
int(acos(a*x)^n/(b*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{x}\,acos(ax)^n}{x}dx\right)}{b}$$

input
```
int(acos(a*x)^n/(b*x)^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(x)*acos(a*x)**n)/x,x))/b
```

3.139 $\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$

Optimal result 984
Mathematica [N/A] 984
Rubi [N/A] 985
Maple [N/A] 985
Fricas [N/A] 986
Sympy [N/A] 986
Maxima [F(-2)] 986
Giac [N/A] 987
Mupad [N/A] 987
Reduce [N/A] 988

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \operatorname{Int}\left(\frac{\arccos(ax)^n}{(bx)^{3/2}},x\right)$$

output
```
Defer(Int)(arccos(a*x)^n/(b*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$$

input
```
Integrate[ArcCos[a*x]^n/(b*x)^(3/2),x]
```

output
```
Integrate[ArcCos[a*x]^n/(b*x)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}} \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}dx$$

input
```
Int[ArcCos[a*x]^n/(b*x)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\arccos\left(ax\right)^{n}}{\left(bx\right)^{\frac{3}{2}}}dx$$

input
```
int(arccos(a*x)^n/(b*x)^(3/2),x)
```

output
```
int(arccos(a*x)^n/(b*x)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}} \, dx = \int \frac{\arccos{(ax)}^n}{(bx)^{\frac{3}{2}}} \, dx$$

input
```
integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x)*arccos(a*x)^n/(b^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}} \, dx = \int \frac{\operatorname{acos}^n{(ax)}}{(bx)^{\frac{3}{2}}} \, dx$$

input
```
integrate(acos(a*x)**n/(b*x)**(3/2),x)
```

output
```
Integral(acos(a*x)**n/(b*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\arccos{(ax)}^n}{(bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n/(b*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{(b\,x)^{3/2}}\,dx$$

input
```
int(acos(a*x)^n/(b*x)^(3/2),x)
```

output
```
int(acos(a*x)^n/(b*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.86

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \frac{2\sqrt{b}\left(-\sqrt{x}\,acos(ax)^n+\left(\int \frac{\sqrt{x}\,\sqrt{-a^2x^2+1}\,acos(ax)^n}{acos(ax)a^2x^3-acos(ax)x}dx\right)anx\right)}{b^2x}$$

input
```
int(acos(a*x)^n/(b*x)^(3/2),x)
```

output
```
(2*sqrt(b)*( - sqrt(x)*acos(a*x)**n + int((sqrt(x)*sqrt( - a**2*x**2 + 1)*
acos(a*x)**n)/(acos(a*x)*a**2*x**3 - acos(a*x)*x),x)*a*n*x))/(b**2*x)
```

3.140 $\int x^3(a+b\arccos(cx))\,dx$

Optimal result 989
Mathematica [A] (verified) 989
Rubi [A] (verified) 990
Maple [A] (verified) 991
Fricas [A] (verification not implemented) 992
Sympy [A] (verification not implemented) 992
Maxima [A] (verification not implemented) 993
Giac [A] (verification not implemented) 993
Mupad [F(-1)] 994
Reduce [B] (verification not implemented) 994

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x^3(a+b\arccos(cx))\,dx=-\frac{3bx\sqrt{1-c^2x^2}}{32c^3}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{3b\arcsin(cx)}{32c^4}$$

output
```
-3/32*b*x*(-c^2*x^2+1)^(1/2)/c^3-1/16*b*x^3*(-c^2*x^2+1)^(1/2)/c+1/4*x^4*(
a+b*arccos(c*x))+3/32*b*arcsin(c*x)/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int x^3(a+b\arccos(cx))\,dx=\frac{ax^4}{4}+b\sqrt{1-c^2x^2}\left(-\frac{3x}{32c^3}-\frac{x^3}{16c}\right)+\frac{1}{4}bx^4\arccos(cx)+\frac{3b\arcsin(cx)}{32c^4}$$

input
```
Integrate[x^3*(a + b*ArcCos[c*x]),x]
```

output
```
(a*x^4)/4 + b*Sqrt[1 - c^2*x^2]*((-3*x)/(32*c^3) - x^3/(16*c)) + (b*x^4*Ar
cCos[c*x])/4 + (3*b*ArcSin[c*x])/(32*c^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5139, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5139$$

$$\frac{1}{4}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))$$

$$\Big\downarrow \; 223$$

$$\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)$$

input
```
Int[x^3*(a + b*ArcCos[c*x]),x]
```

output
```
(x^4*(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*
(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{x^4 a}{4}+\frac{b\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^3x^3\sqrt{-c^2x^2+1}}{16}-\frac{3cx\sqrt{-c^2x^2+1}}{32}+\frac{3\arcsin(cx)}{32}\right)}{c^4}$	68
derivativedivides	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^3x^3\sqrt{-c^2x^2+1}}{16}-\frac{3cx\sqrt{-c^2x^2+1}}{32}+\frac{3\arcsin(cx)}{32}\right)}{c^4}$	72
default	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^3x^3\sqrt{-c^2x^2+1}}{16}-\frac{3cx\sqrt{-c^2x^2+1}}{32}+\frac{3\arcsin(cx)}{32}\right)}{c^4}$	72
orering	$\frac{(14c^4x^4+3c^2x^2-12)(a+b\arccos(cx))}{32c^4}-\frac{(2c^2x^2+3)(cx-1)(cx+1)\left(3x^2(a+b\arccos(cx))-\frac{x^3bc}{\sqrt{-c^2x^2+1}}\right)}{32x^2c^4}$	94

input
```
int(x^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*a+b/c^4*(1/4*c^4*x^4*arccos(c*x)-1/16*c^3*x^3*(-c^2*x^2+1)^(1/2)-3
/32*c*x*(-c^2*x^2+1)^(1/2)+3/32*arcsin(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int x^3(a+b\arccos(cx))\,dx = \frac{8\,ac^4x^4+(8\,bc^4x^4-3\,b)\arccos{(cx)}-(2\,bc^3x^3+3\,bcx)\sqrt{-c^2x^2+1}}{32\,c^4}$$

input
```
integrate(x^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/32*(8*a*c^4*x^4 + (8*b*c^4*x^4 - 3*b)*arccos(c*x) - (2*b*c^3*x^3 + 3*b*c
*x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int x^3(a+b\arccos(cx))\,dx = \begin{cases} \frac{ax^4}{4}+\frac{bx^4\operatorname{acos}{(cx)}}{4}-\frac{bx^3\sqrt{-c^2x^2+1}}{16c}-\frac{3bx\sqrt{-c^2x^2+1}}{32c^3}-\frac{3b\operatorname{acos}{(cx)}}{32c^4} & \text{for}\ c\neq 0\\ \frac{x^4\left(a+\frac{\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*acos(c*x)/4 - b*x**3*sqrt(-c**2*x**2 + 1)/(16
*c) - 3*b*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 3*b*acos(c*x)/(32*c**4), Ne(c
, 0)), (x**4*(a + pi*b/2)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int x^3(a+b\arccos(cx))\,dx = \frac{1}{4}\,ax^4 + \frac{1}{32}\left(8\,x^4\arccos\left(cx\right) - \left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\,\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\,\arcsin\left(cx\right)}{c^5}\right)c\right)b$$

input

```
integrate(x^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output

```
1/4*a*x^4 + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sq
rt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int x^3(a+b\arccos(cx))\,dx = \frac{1}{4}\,bx^4\arccos\left(cx\right) + \frac{1}{4}\,ax^4 - \frac{\sqrt{-c^2x^2+1}bx^3}{16\,c} - \frac{3\,\sqrt{-c^2x^2+1}bx}{32\,c^3} - \frac{3\,b\arccos\left(cx\right)}{32\,c^4}$$

input

```
integrate(x^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output

```
1/4*b*x^4*arccos(c*x) + 1/4*a*x^4 - 1/16*sqrt(-c^2*x^2 + 1)*b*x^3/c - 3/32
*sqrt(-c^2*x^2 + 1)*b*x/c^3 - 3/32*b*arccos(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\arccos(cx))\,dx=\int x^3\,(a+b\,\mathrm{acos}(c\,x))\;dx$$

input
```
int(x^3*(a + b*acos(c*x)),x)
```

output
```
int(x^3*(a + b*acos(c*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int x^3(a+b\arccos(cx))\,dx$$
$$=\frac{8acos(cx)\,b\,c^4x^4+3asin(cx)\,b-2\sqrt{-c^2x^2+1}\,b\,c^3x^3-3\sqrt{-c^2x^2+1}\,bcx+8a\,c^4x^4}{32c^4}$$

input
```
int(x^3*(a+b*acos(c*x)),x)
```

output
```
(8*acos(c*x)*b*c**4*x**4 + 3*asin(c*x)*b - 2*sqrt( - c**2*x**2 + 1)*b*c**3
*x**3 - 3*sqrt( - c**2*x**2 + 1)*b*c*x + 8*a*c**4*x**4)/(32*c**4)
```

3.141 $\int x^2(a + b\arccos(cx))\,dx$

Optimal result 995
Mathematica [A] (verified) 995
Rubi [A] (verified) 996
Maple [A] (verified) 997
Fricas [A] (verification not implemented) 998
Sympy [A] (verification not implemented) 998
Maxima [A] (verification not implemented) 999
Giac [A] (verification not implemented) 999
Mupad [F(-1)] 999
Reduce [B] (verification not implemented) 1000

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int x^2(a+b\arccos(cx))\,dx=-\frac{b\sqrt{1-c^2x^2}}{3c^3}+\frac{b(1-c^2x^2)^{3/2}}{9c^3}+\frac{1}{3}x^3(a+b\arccos(cx))$$

output
```
-1/3*b*(-c^2*x^2+1)^(1/2)/c^3+1/9*b*(-c^2*x^2+1)^(3/2)/c^3+1/3*x^3*(a+b*ar
ccos(c*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x^2(a+b\arccos(cx))\,dx=\frac{ax^3}{3}+b\left(-\frac{2}{9c^3}-\frac{x^2}{9c}\right)\sqrt{1-c^2x^2}+\frac{1}{3}bx^3\arccos(cx)$$

input
```
Integrate[x^2*(a + b*ArcCos[c*x]),x]
```

output
```
(a*x^3)/3 + b*(-2/(9*c^3) - x^2/(9*c))*Sqrt[1 - c^2*x^2] + (b*x^3*ArcCos[c
*x])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5139, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{1}{3}bc\int \frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{6}bc\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)$$

input
```
Int[x^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/6 + (
x^3*(a + b*ArcCos[c*x]))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{x^3 a}{3}+\frac{b\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}-\frac{2\sqrt{-c^2x^2+1}}{9}\right)}{c^3}$	60
derivativedivides	$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}-\frac{2\sqrt{-c^2x^2+1}}{9}\right)}{c^3}$	64
default	$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}-\frac{2\sqrt{-c^2x^2+1}}{9}\right)}{c^3}$	64
orering	$\frac{(5c^4x^4+2c^2x^2-4)(a+b\arccos(cx))}{9c^4x}-\frac{(c^2x^2+2)(cx-1)(cx+1)\left(2x(a+b\arccos(cx))-\frac{x^2bc}{\sqrt{-c^2x^2+1}}\right)}{9c^4x^2}$	94

input
```
int(x^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/
9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int x^2(a+b\arccos(cx))\,dx=\frac{3\,bc^3x^3\arccos{(cx)}+3\,ac^3x^3-(bc^2x^2+2\,b)\sqrt{-c^2x^2+1}}{9\,c^3}$$

input
```
integrate(x^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*b*c^3*x^3*arccos(c*x) + 3*a*c^3*x^3 - (b*c^2*x^2 + 2*b)*sqrt(-c^2*x
^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int x^2(a+b\arccos(cx))\,dx=\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{acos}{(cx)}}{3}-\frac{bx^2\sqrt{-c^2x^2+1}}{9c}-\frac{2b\sqrt{-c^2x^2+1}}{9c^3} & \text{for}\ c\neq 0\\ \frac{x^3\left(a+\frac{\pi b}{2}\right)}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*acos(c*x)/3 - b*x**2*sqrt(-c**2*x**2 + 1)/(9*
c) - 2*b*sqrt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (x**3*(a + pi*b/2)/3, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int x^2(a+b\arccos(cx))\,dx = \frac{1}{3}\,ax^3+\frac{1}{9}\left(3\,x^3\arccos\left(cx\right)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)b$$

input
```
integrate(x^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqr
t(-c^2*x^2 + 1)/c^4))*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x^2(a+b\arccos(cx))\,dx = \frac{1}{3}\,bx^3\arccos\left(cx\right)+\frac{1}{3}\,ax^3-\frac{\sqrt{-c^2x^2+1}bx^2}{9\,c}-\frac{2\sqrt{-c^2x^2+1}b}{9\,c^3}$$

input
```
integrate(x^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/3*b*x^3*arccos(c*x) + 1/3*a*x^3 - 1/9*sqrt(-c^2*x^2 + 1)*b*x^2/c - 2/9*s
qrt(-c^2*x^2 + 1)*b/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))\,dx = \begin{cases} \frac{a\,x^3}{3}-b\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\,\left(\frac{2}{c^2}+x^2\right)}{9}-\frac{x^3\,\mathrm{acos}(c\,x)}{3}\right) & \text{if } 0<c \\ \int x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,dx & \text{if } \neg 0<c \end{cases}$$

input
```
int(x^2*(a + b*acos(c*x)),x)
```

output
```
piecewise(0 < c, - b*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*acos(c*
x))/3) + (a*x^3)/3, ~0 < c, int(x^2*(a + b*acos(c*x)), x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int x^2(a+b\arccos(cx))\,dx = \frac{3acos(cx)\,b\,c^3x^3-\sqrt{-c^2x^2+1}\,b\,c^2x^2-2\sqrt{-c^2x^2+1}\,b+3a\,c^3x^3}{9c^3}$$

input
```
int(x^2*(a+b*acos(c*x)),x)
```

output
```
(3*acos(c*x)*b*c**3*x**3 - sqrt( - c**2*x**2 + 1)*b*c**2*x**2 - 2*sqrt( -
c**2*x**2 + 1)*b + 3*a*c**3*x**3)/(9*c**3)
```

3.142 $\int x(a+b\arccos(cx))\,dx$

Optimal result 1001
Mathematica [A] (verified) 1001
Rubi [A] (verified) 1002
Maple [A] (verified) 1003
Fricas [A] (verification not implemented) 1004
Sympy [A] (verification not implemented) 1004
Maxima [A] (verification not implemented) 1004
Giac [A] (verification not implemented) 1005
Mupad [B] (verification not implemented) 1005
Reduce [B] (verification not implemented) 1006

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x(a+b\arccos(cx))\,dx=-\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}x^2(a+b\arccos(cx))+\frac{b\arcsin(cx)}{4c^2}$$

output
```
-1/4*b*x*(-c^2*x^2+1)^(1/2)/c+1/2*x^2*(a+b*arccos(c*x))+1/4*b*arcsin(c*x)/
c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x(a+b\arccos(cx))\,dx=\frac{ax^2}{2}-\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}bx^2\arccos(cx)+\frac{b\arcsin(cx)}{4c^2}$$

input
```
Integrate[x*(a + b*ArcCos[c*x]),x]
```

output
```
(a*x^2)/2 - (b*x*Sqrt[1 - c^2*x^2])/(4*c) + (b*x^2*ArcCos[c*x])/2 + (b*Arc
Sin[c*x])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5139, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5139$$

$$\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))$$

$$\Big\downarrow \; 262$$

$$\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))$$

$$\Big\downarrow \; 223$$

$$\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)$$

input
```
Int[x*(a + b*ArcCos[c*x]),x]
```

output
```
(x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSi
n[c*x]/(2*c^3)))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c^2x^2+1}}{4}+\frac{\arcsin(cx)}{4}\right)}{c^2}$	48
derivativedivides	$\frac{\frac{c^2x^2a}{2}+b\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c^2x^2+1}}{4}+\frac{\arcsin(cx)}{4}\right)}{c^2}$	52
default	$\frac{\frac{c^2x^2a}{2}+b\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c^2x^2+1}}{4}+\frac{\arcsin(cx)}{4}\right)}{c^2}$	52
orering	$\frac{(3c^2x^2-2)(a+b\arccos(cx))}{4c^2}-\frac{(cx-1)(cx+1)\left(a+b\arccos(cx)-\frac{cxb}{\sqrt{-c^2x^2+1}}\right)}{4c^2}$	65

input
```
int(x*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+1/4*ar
csin(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int x(a+b\arccos(cx))\,dx = \frac{2\,ac^2x^2 - \sqrt{-c^2x^2+1}bcx + (2\,bc^2x^2 - b)\arccos{(cx)}}{4\,c^2}$$

input
```
integrate(x*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*a*c^2*x^2 - sqrt(-c^2*x^2 + 1)*b*c*x + (2*b*c^2*x^2 - b)*arccos(c*x
))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int x(a+b\arccos(cx))\,dx = \begin{cases} \frac{ax^2}{2} + \frac{bx^2\operatorname{acos}{(cx)}}{2} - \frac{bx\sqrt{-c^2x^2+1}}{4c} - \frac{b\operatorname{acos}{(cx)}}{4c^2} & \text{for}\ c \neq 0 \\ \frac{x^2\left(a+\frac{\pi b}{2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*acos(c*x)/2 - b*x*sqrt(-c**2*x**2 + 1)/(4*c)
- b*acos(c*x)/(4*c**2), Ne(c, 0)), (x**2*(a + pi*b/2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int x(a+b\arccos(cx))\,dx = \frac{1}{2}\,ax^2 + \frac{1}{4}\left(2\,x^2\arccos{(cx)} - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin{(cx)}}{c^3}\right)\right)b$$

input
```
integrate(x*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(
c*x)/c^3))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int x(a+b\arccos(cx))\,dx = \frac{1}{2}\,bx^2\arccos{(cx)} + \frac{1}{2}\,ax^2 - \frac{\sqrt{-c^2x^2+1}bx}{4\,c} - \frac{b\arccos{(cx)}}{4\,c^2}$$

input
```
integrate(x*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/2*b*x^2*arccos(c*x) + 1/2*a*x^2 - 1/4*sqrt(-c^2*x^2 + 1)*b*x/c - 1/4*b*a
rccos(c*x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x(a+b\arccos(cx))\,dx = \frac{a\,x^2}{2} + \frac{b\left(\frac{\mathrm{acos}(c\,x)\,\left(2\,c^2\,x^2-1\right)}{4} - \frac{c\,x\,\sqrt{1-c^2\,x^2}}{4}\right)}{c^2}$$

input
```
int(x*(a + b*acos(c*x)),x)
```

output
```
(a*x^2)/2 + (b*((acos(c*x)*(2*c^2*x^2 - 1))/4 - (c*x*(1 - c^2*x^2)^(1/2))/
4))/c^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int x(a+b\arccos(cx))\,dx$$
$$=\frac{2acos(cx)\,b\,c^2x^2-2acos(cx)\,b-asin(cx)\,b-\sqrt{-c^2x^2+1}\,bcx+2a\,c^2x^2}{4c^2}$$

input
```
int(x*(a+b*acos(c*x)),x)
```

output
```
(2*acos(c*x)*b*c**2*x**2 - 2*acos(c*x)*b - asin(c*x)*b - sqrt( - c**2*x**2
 + 1)*b*c*x + 2*a*c**2*x**2)/(4*c**2)
```

3.143 $\int (a+b \arccos(cx)) \, dx$

Optimal result 1007
Mathematica [A] (verified) 1007
Rubi [A] (verified) 1008
Maple [A] (verified) 1008
Fricas [A] (verification not implemented) 1009
Sympy [A] (verification not implemented) 1009
Maxima [A] (verification not implemented) 1010
Giac [A] (verification not implemented) 1010
Mupad [B] (verification not implemented) 1010
Reduce [B] (verification not implemented) 1011

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a+b \arccos(cx)) \, dx = ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx \arccos(cx)$$

output
```
a*x-b*(-c^2*x^2+1)^(1/2)/c+b*x*arccos(c*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+b \arccos(cx)) \, dx = ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx \arccos(cx)$$

input
```
Integrate[a + b*ArcCos[c*x],x]
```

output
```
a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}$$

input
```
Int[a + b*ArcCos[c*x],x]
```

output
```
a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$ax+\frac{b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	32
parts	$ax+\frac{b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	32
derivativedivides	$\frac{cxa+b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	34
orering	$x(a+b\arccos(cx))+\frac{(cx-1)(cx+1)b}{c\sqrt{-c^2x^2+1}}$	39

input
```
int(a+b*arccos(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/c*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (a+b\arccos(cx))\,dx=\frac{bcx\arccos{(cx)}+acx-\sqrt{-c^2x^2+1}b}{c}$$

input
```
integrate(a+b*arccos(c*x),x, algorithm="fricas")
```

output
```
(b*c*x*arccos(c*x) + a*c*x - sqrt(-c^2*x^2 + 1)*b)/c
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (a+b\arccos(cx))\,dx=ax+b\left(\begin{cases} x\,\mathrm{acos}\,(cx)-\frac{\sqrt{-c^2x^2+1}}{c} & \text{for}\ c\neq 0\\ \frac{\pi x}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*acos(c*x),x)
```

output
```
a*x + b*Piecewise((x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (pi*x/
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a + b \arccos(cx))\,dx = ax + \frac{\left(cx \arccos(cx) - \sqrt{-c^2x^2 + 1}\right)b}{c}$$

input
```
integrate(a+b*arccos(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a + b \arccos(cx))\,dx = ax + \frac{\left(cx \arccos(cx) - \sqrt{-c^2x^2 + 1}\right)b}{c}$$

input
```
integrate(a+b*arccos(c*x),x, algorithm="giac")
```

output
```
a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (a + b \arccos(cx))\,dx = a\,x - \frac{b\,\sqrt{1 - c^2\,x^2}}{c} + b\,x\,\mathrm{acos}(c\,x)$$

input
```
int(a + b*acos(c*x),x)
```

output
```
a*x - (b*(1 - c^2*x^2)^(1/2))/c + b*x*acos(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a + b \arccos(cx))\, dx = \frac{acos(cx)\, bcx - \sqrt{-c^2x^2 + 1}\, b + acx}{c}$$

input
```
int(a+b*acos(c*x),x)
```

output
```
(acos(c*x)*b*c*x - sqrt( - c**2*x**2 + 1)*b + a*c*x)/c
```

3.144 $\int \frac{a+b\arccos(cx)}{x}\,dx$

Optimal result . . . 1012
Mathematica [A] (verified) . . . 1012
Rubi [A] (verified) . . . 1013
Maple [A] (verified) . . . 1015
Fricas [F] . . . 1015
Sympy [F] . . . 1016
Maxima [F] . . . 1016
Giac [F(-2)] . . . 1016
Mupad [F(-1)] . . . 1017
Reduce [F] . . . 1017

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{a+b\arccos(cx)}{x}\,dx = -\frac{i(a+b\arccos(cx))^2}{2b}+(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

output
```
-1/2*I*(a+b*arccos(c*x))^2/b+(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1
/2))^2)-1/2*I*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arccos(cx)}{x}\,dx = -\frac{1}{2}ib\arccos(cx)^2+b\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right)+a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

input
```
Integrate[(a + b*ArcCos[c*x])/x,x]
```

output
```
(-1/2*I)*b*ArcCos[c*x]^2 + b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] +
a*Log[x] - (I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x}dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)-\frac{i(a+b\arccos(cx))^2}{2b}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)-\frac{i(a+b\arccos(cx))^2}{2b}$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}$$

input
```
Int[(a + b*ArcCos[c*x])/x,x]
```

output
```
((-1/2*I)*(a + b*ArcCos[c*x])^2)/b + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*L
og[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

method	result
parts	$a\ln(x)+b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(c\right.\right.}{}\right.$
derivativedivides	$a\ln(cx)+b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(\right.\right.}{}\right.$
default	$a\ln(cx)+b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(\right.\right.}{}\right.$

input
```
int((a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(-1/2*I*arccos(c*x)^2+arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)-1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\int \frac{b\arccos(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx = \int \frac{a+b\operatorname{acos}(cx)}{x}\,dx$$

input
```
integrate((a+b*acos(c*x))/x,x)
```

output
```
Integral((a + b*acos(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx = \int \frac{b\arccos(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) + a*log(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x}\,dx$$

input
```
int((a + b*acos(c*x))/x,x)
```

output
```
int((a + b*acos(c*x))/x, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\left(\int \frac{acos(cx)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*acos(c*x))/x,x)
```

output
```
int(acos(c*x)/x,x)*b + log(x)*a
```

3.145 $\int \frac{a+b \arccos(cx)}{x^2} \, dx$

Optimal result 1018
Mathematica [A] (verified) 1018
Rubi [A] (verified) 1019
Maple [A] (verified) 1020
Fricas [B] (verification not implemented) 1021
Sympy [A] (verification not implemented) 1021
Maxima [A] (verification not implemented) 1022
Giac [B] (verification not implemented) 1022
Mupad [B] (verification not implemented) 1023
Reduce [B] (verification not implemented) 1023

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{a+b \arccos(cx)}{x^2} \, dx = -\frac{a+b \arccos(cx)}{x} + b c \operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
-(a+b*arccos(c*x))/x+b*c*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{a+b \arccos(cx)}{x^2} \, dx = -\frac{a}{x} - \frac{b \arccos(cx)}{x} - bc \log(x) + bc \log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[(a + b*ArcCos[c*x])/x^2,x]
```

output
```
-(a/x) - (b*ArcCos[c*x])/x - b*c*Log[x] + b*c*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5139, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2}dx$$

$$\Big\downarrow \text{5139}$$

$$-bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arccos(cx)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{b\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c}-\frac{a+b\arccos(cx)}{x}$$

$$\Big\downarrow \text{221}$$

$$b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}$$

input
```
Int[(a + b*ArcCos[c*x])/x^2,x]
```

output
```
-((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\arccos\left(cx\right)}{cx}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)\right)$	37
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\arccos\left(cx\right)}{cx}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)\right)\right)$	41
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\arccos\left(cx\right)}{cx}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)\right)\right)$	41

input
```
int((a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*c*(-1/c/x*arccos(c*x)+arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(30) = 60$.

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{a + b\arccos(cx)}{x^2}\,dx = \frac{bcx\log\left(\sqrt{-c^2x^2+1}+1\right) - bcx\log\left(\sqrt{-c^2x^2+1}-1\right) - 2\,bx\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right) + 2\,(bx-b)\arccos\left(c\right.}{2\,x}$$

input
```
integrate((a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*c*x*log(sqrt(-c^2*x^2 + 1) + 1) - b*c*x*log(sqrt(-c^2*x^2 + 1) - 1)
 - 2*b*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*(b*x - b)*arccos
(c*x) - 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{a + b\arccos(cx)}{x^2}\,dx = -\frac{a}{x} - bc\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{b\operatorname{acos}(cx)}{x}$$

input
```
integrate((a+b*acos(c*x))/x**2,x)
```

output
```
-a/x - b*c*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c
*x)), True)) - b*acos(c*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)b-\frac{a}{x}$$

input
```
integrate((a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(30) = 60$.

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 10.84

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{x^2}\,dx = &-\frac{bc\arccos(cx)}{\frac{c^2x^2-1}{(cx+1)^2}+1}+\frac{bc\log\left(\left|cx+\sqrt{-c^2x^2+1}+1\right|\right)}{\frac{c^2x^2-1}{(cx+1)^2}+1}\\&-\frac{bc\log\left(\left|-cx+\sqrt{-c^2x^2+1}-1\right|\right)}{\frac{c^2x^2-1}{(cx+1)^2}+1}\\&-\frac{ac}{\frac{c^2x^2-1}{(cx+1)^2}+1}+\frac{(c^2x^2-1)bc\arccos(cx)}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)}\\&+\frac{(c^2x^2-1)bc\log\left(\left|cx+\sqrt{-c^2x^2+1}+1\right|\right)}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)}\\&-\frac{(c^2x^2-1)bc\log\left(\left|-cx+\sqrt{-c^2x^2+1}-1\right|\right)}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)}\\&+\frac{(c^2x^2-1)ac}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)}\end{aligned}$$

input
```
integrate((a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
-b*c*arccos(c*x)/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) + b*c*log(abs(c*x + sqrt(
-c^2*x^2 + 1) + 1))/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) - b*c*log(abs(-c*x + s
qrt(-c^2*x^2 + 1) - 1))/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) - a*c/((c^2*x^2 -
1)/(c*x + 1)^2 + 1) + (c^2*x^2 - 1)*b*c*arccos(c*x)/((c*x + 1)^2*((c^2*x^2
 - 1)/(c*x + 1)^2 + 1)) + (c^2*x^2 - 1)*b*c*log(abs(c*x + sqrt(-c^2*x^2 +
1) + 1))/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2 + 1)) - (c^2*x^2 - 1)*b*c
*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x
+ 1)^2 + 1)) + (c^2*x^2 - 1)*a*c/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2 +
 1))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = b\,c\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-c^2\,x^2}}\right)-\frac{b\,\mathrm{acos}(c\,x)}{x}-\frac{a}{x}$$

input
```
int((a + b*acos(c*x))/x^2,x)
```

output
```
b*c*atanh(1/(1 - c^2*x^2)^(1/2)) - (b*acos(c*x))/x - a/x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = \frac{-acos(cx)\,b-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)bcx-a}{x}$$

input
```
int((a+b*acos(c*x))/x^2,x)
```

output
```
( - (acos(c*x)*b + log(tan(asin(c*x)/2))*b*c*x + a))/x
```

3.146 $\int \frac{a+b\arccos(cx)}{x^3}\,dx$

Optimal result 1024
Mathematica [A] (verified) 1024
Rubi [A] (verified) 1025
Maple [A] (verified) 1026
Fricas [A] (verification not implemented) 1026
Sympy [A] (verification not implemented) 1027
Maxima [A] (verification not implemented) 1027
Giac [B] (verification not implemented) 1027
Mupad [F(-1)] 1029
Reduce [B] (verification not implemented) 1029

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{a+b\arccos(cx)}{2x^2}$$

output
```
1/2*b*c*(-c^2*x^2+1)^(1/2)/x-1/2*(a+b*arccos(c*x))/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = -\frac{a}{2x^2} + \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{b\arccos(cx)}{2x^2}$$

input
```
Integrate[(a + b*ArcCos[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*c*Sqrt[1 - c^2*x^2])/(2*x) - (b*ArcCos[c*x])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5139, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3}dx$$
$$\Big\downarrow \text{5139}$$
$$-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{2x^2}$$
$$\Big\downarrow \text{242}$$
$$\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}$$

input
```
Int[(a + b*ArcCos[c*x])/x^3,x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(2*x) - (a + b*ArcCos[c*x])/(2*x^2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\arccos(cx)}{2c^2x^2}+\frac{\sqrt{-c^2x^2+1}}{2cx}\right)$	46
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arccos(cx)}{2c^2x^2}+\frac{\sqrt{-c^2x^2+1}}{2cx}\right)\right)$	50
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arccos(cx)}{2c^2x^2}+\frac{\sqrt{-c^2x^2+1}}{2cx}\right)\right)$	50
orering	$\frac{\left(\frac{3}{2}c^2x^3-2x\right)(a+b\arccos(cx))}{x^3}+\frac{(cx-1)(cx+1)x^2\left(-\frac{bc}{\sqrt{-c^2x^2+1}\,x^3}-\frac{3(a+b\arccos(cx))}{x^4}\right)}{2}$	74

input
```
int((a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arccos(c*x)+1/2/c/x*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=\frac{\sqrt{-c^2x^2+1}bcx+ax^2-b\arccos(cx)-a}{2\,x^2}$$

input
```
integrate((a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/2*(sqrt(-c^2*x^2 + 1)*b*c*x + a*x^2 - b*arccos(c*x) - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{bc\left(\begin{cases} -\frac{i\sqrt{c^2x^2-1}}{x} & \text{for } |c^2x^2|>1 \\ -\frac{\sqrt{-c^2x^2+1}}{x} & \text{otherwise} \end{cases}\right)}{2} - \frac{b\operatorname{acos}(cx)}{2x^2}$$

input
```
integrate((a+b*acos(c*x))/x**3,x)
```

output
```
-a/(2*x**2) - b*c*Piecewise((-I*sqrt(c**2*x**2 - 1)/x, Abs(c**2*x**2) > 1)
, (-sqrt(-c**2*x**2 + 1)/x, True))/2 - b*acos(c*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = \frac{1}{2}b\left(\frac{\sqrt{-c^2x^2+1}c}{x} - \frac{\arccos(cx)}{x^2}\right) - \frac{a}{2x^2}$$

input
```
integrate((a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*b*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(33) = 66$.

Time = 0.14 (sec) , antiderivative size = 492, normalized size of antiderivative = 12.62

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = -\frac{bc^2\arccos(cx)}{2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{ac^2}{2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} + \frac{(c^2x^2-1)bc^2\arccos(cx)}{(cx+1)^2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} + \frac{\sqrt{-c^2x^2+1}\,bc^2}{(cx+1)\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} + \frac{(c^2x^2-1)ac^2}{(cx+1)^2\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{(c^2x^2-1)^2bc^2\arccos(cx)}{2(cx+1)^4\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{(-c^2x^2+1)^{\frac{3}{2}}bc^2}{(cx+1)^3\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{(c^2x^2-1)^2ac^2}{2(cx+1)^4\left(\frac{2(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)}$$

input
```
integrate((a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
-1/2*b*c^2*arccos(c*x)/(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x
 + 1)^4 + 1) - 1/2*a*c^2/(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c
*x + 1)^4 + 1) + (c^2*x^2 - 1)*b*c^2*arccos(c*x)/((c*x + 1)^2*(2*(c^2*x^2
- 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) + sqrt(-c^2*x^2 + 1)*
b*c^2/((c*x + 1)*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^
4 + 1)) + (c^2*x^2 - 1)*a*c^2/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 +
(c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) - 1/2*(c^2*x^2 - 1)^2*b*c^2*arccos(c*x)/
((c*x + 1)^4*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 +
1)) - (-c^2*x^2 + 1)^(3/2)*b*c^2/((c*x + 1)^3*(2*(c^2*x^2 - 1)/(c*x + 1)^2
 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) - 1/2*(c^2*x^2 - 1)^2*a*c^2/((c*x + 1
)^4*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3}\,dx$$

input
```
int((a + b*acos(c*x))/x^3,x)
```

output
```
int((a + b*acos(c*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = \frac{-acos(cx)\,b+\sqrt{-c^2x^2+1}\,bcx-a}{2x^2}$$

input
```
int((a+b*acos(c*x))/x^3,x)
```

output
```
( - acos(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c*x - a)/(2*x**2)
```

3.147 $\int \frac{a+b\arccos(cx)}{x^4}\,dx$

Optimal result 1030
Mathematica [A] (verified) 1030
Rubi [A] (verified) 1031
Maple [A] (verified) 1033
Fricas [B] (verification not implemented) 1033
Sympy [A] (verification not implemented) 1034
Maxima [A] (verification not implemented) 1034
Giac [B] (verification not implemented) 1035
Mupad [F(-1)] 1036
Reduce [B] (verification not implemented) 1036

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx=\frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\arccos(cx)}{3x^3}+\frac{1}{6}bc^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
1/6*b*c*(-c^2*x^2+1)^(1/2)/x^2-1/3*(a+b*arccos(c*x))/x^3+1/6*b*c^3*arctanh
((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx=-\frac{a}{3x^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{b\arccos(cx)}{3x^3}-\frac{1}{6}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[(a + b*ArcCos[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 + (b*c*Sqrt[1 - c^2*x^2])/(6*x^2) - (b*ArcCos[c*x])/(3*x^3) - (
b*c^3*Log[x])/6 + (b*c^3*Log[1 + Sqrt[1 - c^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5139, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{1}{3}bc\int \frac{1}{x^3\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}bc\int \frac{1}{x^4\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arccos(cx)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{1}{6}bc\left(\frac{1}{2}c^2\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{a+b\arccos(cx)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{6}bc\left(-\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{a+b\arccos(cx)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{a+b\arccos(cx)}{3x^3}-\frac{1}{6}bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)$$

input
```
Int[(a + b*ArcCos[c*x])/x^4,x]
```

output
```
-1/3*(a + b*ArcCos[c*x])/x^3 - (b*c*(-(Sqrt[1 - c^2*x^2]/x^2) - c^2*ArcTan
h[Sqrt[1 - c^2*x^2]]))/6
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\arccos(cx)}{3c^3x^3}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6}\right)$	61
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\arccos(cx)}{3c^3x^3}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6}\right)\right)$	65
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\arccos(cx)}{3c^3x^3}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6}\right)\right)$	65

input
```
int((a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arccos(c*x)+1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)+
1/6*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(52) = 104.

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = \frac{bc^3x^3\log\left(\sqrt{-c^2x^2+1}+1\right)-bc^3x^3\log\left(\sqrt{-c^2x^2+1}-1\right)-4\,bx^3\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)+2\sqrt{-c^2x^2+1}}{12\,x^3}$$

input
```
integrate((a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/12*(b*c^3*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - b*c^3*x^3*log(sqrt(-c^2*x^2
+ 1) - 1) - 4*b*x^3*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*sqrt(
-c^2*x^2 + 1)*b*c*x + 4*(b*x^3 - b)*arccos(c*x) - 4*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = -\frac{a}{3x^3} - \frac{bc\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3} - \frac{b\operatorname{acos}(cx)}{3x^3}$$

input
```
integrate((a+b*acos(c*x))/x**4,x)
```

output
```
-a/(3*x**3) - b*c*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(
c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1
), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 -
 b*acos(c*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-c^2x^2+1}}{x^2}\right)c - \frac{2\arccos(cx)}{x^3}\right)b - \frac{a}{3x^3}$$

input
```
integrate((a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)
/x^2)*c - 2*arccos(c*x)/x^3)*b - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(52) = 104$.

Time = 0.43 (sec) , antiderivative size = 1634, normalized size of antiderivative = 26.35

$$\int \frac{a + b \arccos(cx)}{x^4} dx = \text{Too large to display}$$

input
```
integrate((a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*c^3*arccos(c*x)/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c
*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + 1/6*b*c^3*log(abs(c*x + sqr
t(-c^2*x^2 + 1) + 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*
x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/6*b*c^3*log(abs(-c*x + sqr
t(-c^2*x^2 + 1) - 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*
x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/3*a*c^3/(3*(c^2*x^2 - 1)/(
c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 +
 1) + (c^2*x^2 - 1)*b*c^3*arccos(c*x)/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x +
 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1))
+ 1/2*(c^2*x^2 - 1)*b*c^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1
)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^
2 - 1)^3/(c*x + 1)^6 + 1)) - 1/2*(c^2*x^2 - 1)*b*c^3*log(abs(-c*x + sqrt(-
c^2*x^2 + 1) - 1))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2
- 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*sqrt(-c^2*x^2
 + 1)*b*c^3/((c*x + 1)*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c
*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + (c^2*x^2 - 1)*a*c^3/((c*x
+ 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2
*x^2 - 1)^3/(c*x + 1)^6 + 1)) - (c^2*x^2 - 1)^2*b*c^3*arccos(c*x)/((c*x +
1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x
^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/2*(c^2*x^2 - 1)^2*b*c^3*log(abs(c*x + s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4}\,dx$$

input
```
int((a + b*acos(c*x))/x^4,x)
```

output
```
int((a + b*acos(c*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = \frac{-2acos(cx)\,b+\sqrt{-c^2x^2+1}\,bcx-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)b\,c^3x^3-2a}{6x^3}$$

input
```
int((a+b*acos(c*x))/x^4,x)
```

output
```
( - 2*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c*x - log(tan(asin(c*x)/2))*b
*c**3*x**3 - 2*a)/(6*x**3)
```

3.148 $\int x^2(a+b\arccos(cx))^2\,dx$

Optimal result 1037
Mathematica [A] (verified) 1037
Rubi [A] (verified) 1038
Maple [A] (verified) 1040
Fricas [A] (verification not implemented) 1041
Sympy [A] (verification not implemented) 1041
Maxima [A] (verification not implemented) 1042
Giac [A] (verification not implemented) 1042
Mupad [F(-1)] 1043
Reduce [F] 1043

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int x^2(a+b\arccos(cx))^2\,dx=-\frac{4b^2x}{9c^2}-\frac{2b^2x^3}{27}-\frac{4b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}-\frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

output
```
-4/9*b^2*x/c^2-2/27*b^2*x^3-4/9*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3
-2/9*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+1/3*x^3*(a+b*arccos(c*x)
)^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19

$$\int x^2(a+b\arccos(cx))^2\,dx =\frac{9a^2c^3x^3-6ab\sqrt{1-c^2x^2}(2+c^2x^2)-2b^2cx(6+c^2x^2)-6b\left(-3ac^3x^3+b\sqrt{1-c^2x^2}(2+c^2x^2)\right)\arccos(cx}{27c^3}$$

input
```
Integrate[x^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(9*a^2*c^3*x^3 - 6*a*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) - 2*b^2*c*x*(6 + c^
2*x^2) - 6*b*(-3*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))*ArcCos[c*x
] + 9*b^2*c^3*x^3*ArcCos[c*x]^2)/(27*c^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5139, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2}{3}bc\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5211}$$

$$\frac{2}{3}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$\frac{2}{3}bc\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{24}$$

$$\frac{2}{3}bc\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

input
```
Int[x^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*(-1/9*(b*x^3)/c - (x^2*Sqrt[1 - c^2
*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(
a + b*ArcCos[c*x]))/c^2))/(3*c^2)))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.23

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{c^3x^3\arccos(cx)^2}{3}-\frac{2\arccos(cx)\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{9}-\frac{2c^3x^3}{27}-\frac{4cx}{9}\right)}{c^3}+\frac{2ab\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2\sqrt{-c^2x^2}}{9}\right.}{c^3}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\arccos(cx)^2}{3}-\frac{2\arccos(cx)\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{9}-\frac{2c^3x^3}{27}-\frac{4cx}{9}\right)+2ab\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}-\right.}{c^3}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\arccos(cx)^2}{3}-\frac{2\arccos(cx)\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{9}-\frac{2c^3x^3}{27}-\frac{4cx}{9}\right)+2ab\left(\frac{c^3x^3\arccos(cx)}{3}-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}-\right.}{c^3}$
orering	$\frac{(19c^4x^4+24c^2x^2-48)(a+b\arccos(cx))^2}{27c^4x}-\frac{(6c^4x^4+17c^2x^2-30)\left(2x(a+b\arccos(cx))^2-\frac{2x^2(a+b\arccos(cx))bc}{\sqrt{-c^2x^2+1}}\right)}{27c^4x^2}+$

input
```
int(x^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*c^3*x^3*arccos(c*x)^2-2/9*arccos(c*x)*(c^2*x^2+2)
*(-c^2*x^2+1)^(1/2)-2/27*c^3*x^3-4/9*c*x)+2*a*b/c^3*(1/3*c^3*x^3*arccos(c*
x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arccos(cx))^2\,dx = \frac{9\,b^2c^3x^3\arccos(cx)^2+18\,abc^3x^3\arccos(cx)+(9\,a^2-2\,b^2)c^3x^3-12\,b^2cx-6\,(abc^2x^2+2\,ab+(b^2c^2x^2+$$
$$27\,c^3$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*(9*b^2*c^3*x^3*arccos(c*x)^2 + 18*a*b*c^3*x^3*arccos(c*x) + (9*a^2 -
2*b^2)*c^3*x^3 - 12*b^2*c*x - 6*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^
2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$\int x^2(a+b\arccos(cx))^2\,dx = \begin{cases} \frac{a^2x^3}{3}+\frac{2abx^3\operatorname{acos}(cx)}{3}-\frac{2abx^2\sqrt{-c^2x^2+1}}{9c}-\frac{4ab\sqrt{-c^2x^2+1}}{9c^3}+\frac{b^2x^3\operatorname{acos}^2(cx)}{3}-\frac{2b^2x^3}{27}-\frac{2b^2x^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{9c}-\frac{4b^2x}{9c^2} - \\ \frac{x^3\left(a+\frac{\pi b}{2}\right)^2}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*x**3/3 + 2*a*b*x**3*acos(c*x)/3 - 2*a*b*x**2*sqrt(-c**2*x*
*2 + 1)/(9*c) - 4*a*b*sqrt(-c**2*x**2 + 1)/(9*c**3) + b**2*x**3*acos(c*x)*
*2/3 - 2*b**2*x**3/27 - 2*b**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c) -
 4*b**2*x/(9*c**2) - 4*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c**3), Ne(c,
 0)), (x**3*(a + pi*b/2)**2/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.39

$$\int x^2(a+b\arccos(cx))^2\,dx$$
$$=\frac{1}{3}\,b^2x^3\arccos{(cx)}^2+\frac{1}{3}\,a^2x^3$$
$$+\frac{2}{9}\left(3\,x^3\arccos{(cx)}-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)ab$$
$$-\frac{2}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos{(cx)}+\frac{c^2x^3+6\,x}{c^2}\right)b^2$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*arccos(c*x)^2 + 1/3*a^2*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt
(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b - 2/27*(3*c*(sqrt(
-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 +
 6*x)/c^2)*b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int x^2(a+b\arccos(cx))^2\,dx=\frac{1}{3}\,b^2x^3\arccos{(cx)}^2+\frac{2}{3}\,abx^3\arccos{(cx)}+\frac{1}{3}\,a^2x^3-\frac{2}{27}\,b^2x^3$$
$$-\frac{2\sqrt{-c^2x^2+1}b^2x^2\arccos{(cx)}}{9\,c}-\frac{2\sqrt{-c^2x^2+1}abx^2}{9\,c}$$
$$-\frac{4\,b^2x}{9\,c^2}-\frac{4\sqrt{-c^2x^2+1}b^2\arccos{(cx)}}{9\,c^3}-\frac{4\sqrt{-c^2x^2+1}ab}{9\,c^3}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3*arccos(c*x)^2 + 2/3*a*b*x^3*arccos(c*x) + 1/3*a^2*x^3 - 2/27*b
^2*x^3 - 2/9*sqrt(-c^2*x^2 + 1)*b^2*x^2*arccos(c*x)/c - 2/9*sqrt(-c^2*x^2
+ 1)*a*b*x^2/c - 4/9*b^2*x/c^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*arccos(c*x)/c^
3 - 4/9*sqrt(-c^2*x^2 + 1)*a*b/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2,x)
```

output
```
int(x^2*(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int x^2(a+b\arccos(cx))^2\,dx$$
$$=\frac{6acos(cx)\,ab\,c^3x^3-2\sqrt{-c^2x^2+1}\,ab\,c^2x^2-4\sqrt{-c^2x^2+1}\,ab+9\left(\int acos(cx)^2\,x^2dx\right)b^2c^3+3a^2c^3x^3}{9c^3}$$

input
```
int(x^2*(a+b*acos(c*x))^2,x)
```

output
```
(6*acos(c*x)*a*b*c**3*x**3 - 2*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2 - 4*sq
rt( - c**2*x**2 + 1)*a*b + 9*int(acos(c*x)**2*x**2,x)*b**2*c**3 + 3*a**2*c
**3*x**3)/(9*c**3)
```

3.149 $\int x(a+b\arccos(cx))^2\,dx$

Optimal result 1044
Mathematica [A] (verified) 1044
Rubi [A] (verified) 1045
Maple [A] (verified) 1047
Fricas [A] (verification not implemented) 1047
Sympy [B] (verification not implemented) 1048
Maxima [F] 1048
Giac [A] (verification not implemented) 1049
Mupad [F(-1)] 1049
Reduce [B] (verification not implemented) 1049

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x(a+b\arccos(cx))^2\,dx=-\frac{1}{4}b^2x^2-\frac{bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c}-\frac{(a+b\arccos(cx))^2}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^2$$

output
```
-1/4*b^2*x^2-1/2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c-1/4*(a+b*arcco
s(c*x))^2/c^2+1/2*x^2*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x(a+b\arccos(cx))^2\,dx=\frac{cx\left(2a^2cx-b^2cx-2ab\sqrt{1-c^2x^2}\right)+2bcx\left(2acx-b\sqrt{1-c^2x^2}\right)\arccos(cx)+b^2\left(-1+2c^2x^2\right)\arccos(cx)^2}{4c^2}$$

input
```
Integrate[x*(a + b*ArcCos[c*x])^2,x]
```

output
```
(c*x*(2*a^2*c*x - b^2*c*x - 2*a*b*Sqrt[1 - c^2*x^2]) + 2*b*c*x*(2*a*c*x -
b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + b^2*(-1 + 2*c^2*x^2)*ArcCos[c*x]^2 + 2*
a*b*ArcSin[c*x])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5139, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5139}$$

$$bc\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5211}$$

$$bc\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{15}$$

$$bc\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)+\frac{1}{2}x^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5153}$$

$$bc\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)+\frac{1}{2}x^2(a+b\arccos(cx))^2$$

input
```
Int[x*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^2)/2 + b*c*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]
*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{\cos(2\arccos(cx))\arccos(cx)^2}{4}-\frac{\cos(2\arccos(cx))}{8}-\frac{\sin(2\arccos(cx))\arccos(cx)}{4}\right)}{c^2}+\frac{2ab\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-}}{}\right.}{c^2}$
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{\cos(2\arccos(cx))\arccos(cx)^2}{4}-\frac{\cos(2\arccos(cx))}{8}-\frac{\sin(2\arccos(cx))\arccos(cx)}{4}\right)+2ab\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c^2}}{4}\right.}{c^2}$
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{\cos(2\arccos(cx))\arccos(cx)^2}{4}-\frac{\cos(2\arccos(cx))}{8}-\frac{\sin(2\arccos(cx))\arccos(cx)}{4}\right)+2ab\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c^2}}{4}\right.}{c^2}$
orering	$\frac{(7c^2x^2-6)(a+b\arccos(cx))^2}{8c^2}-\frac{(3c^2x^2-4)\left((a+b\arccos(cx))^2-\frac{2x(a+b\arccos(cx))bc}{\sqrt{-c^2x^2+1}}\right)}{8c^2}+\frac{x(cx-1)(cx+1)\left(-\frac{4(a+b}{\sqrt{}}\right.}{}$

input
```
int(x*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(1/4*cos(2*arccos(c*x))*arccos(c*x)^2-1/8*cos(2*arccos
(c*x))-1/4*sin(2*arccos(c*x))*arccos(c*x))+2*a*b/c^2*(1/2*c^2*x^2*arccos(c
*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+1/4*arcsin(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int x(a+b\arccos(cx))^2\,dx$$
$$=\frac{(2\,a^2-b^2)c^2x^2+(2\,b^2c^2x^2-b^2)\arccos{(cx)}^2+2\,(2\,abc^2x^2-ab)\arccos{(cx)}-2\,(b^2cx\arccos{(cx)}+abcx}{4\,c^2}$$

input
```
integrate(x*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 - b^2)*c^2*x^2 + (2*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 + 2*(2*a*
b*c^2*x^2 - a*b)*arccos(c*x) - 2*(b^2*c*x*arccos(c*x) + a*b*c*x)*sqrt(-c^2
*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.72

$$\int x(a + b\arccos(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2x^2}{2} + abx^2\operatorname{acos}(cx) - \frac{abx\sqrt{-c^2x^2+1}}{2c} - \frac{ab\operatorname{acos}(cx)}{2c^2} + \frac{b^2x^2\operatorname{acos}^2(cx)}{2} - \frac{b^2x^2}{4} - \frac{b^2x\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{2c} - \frac{b^2\operatorname{acos}^2(cx)}{4c^2} \\ \frac{x^2\left(a+\frac{\pi b}{2}\right)^2}{2} \end{cases}$$

input
```
integrate(x*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*x**2/2 + a*b*x**2*acos(c*x) - a*b*x*sqrt(-c**2*x**2 + 1)/(
2*c) - a*b*acos(c*x)/(2*c**2) + b**2*x**2*acos(c*x)**2/2 - b**2*x**2/4 - b
**2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2*c) - b**2*acos(c*x)**2/(4*c**2), N
e(c, 0)), (x**2*(a + pi*b/2)**2/2, True))
```

Maxima [F]

$$\int x(a + b\arccos(cx))^2\,dx = \int (b\arccos(cx) + a)^2 x\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/2*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsi
n(c*x)/c^3))*a*b + 1/2*(x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 -
 2*c*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)/(c^2*x^2 - 1), x))*b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int x(a+b\arccos(cx))^2\,dx = \frac{1}{2}\,b^2x^2\arccos{(cx)}^2 + abx^2\arccos{(cx)} + \frac{1}{2}\,a^2x^2 - \frac{1}{4}\,b^2x^2 - \frac{\sqrt{-c^2x^2+1}b^2x\arccos{(cx)}}{2\,c} - \frac{\sqrt{-c^2x^2+1}abx}{2\,c} - \frac{b^2\arccos{(cx)}^2}{4\,c^2} - \frac{ab\arccos{(cx)}}{2\,c^2} + \frac{b^2}{8\,c^2}$$

input
```
integrate(x*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/2*b^2*x^2*arccos(c*x)^2 + a*b*x^2*arccos(c*x) + 1/2*a^2*x^2 - 1/4*b^2*x^
2 - 1/2*sqrt(-c^2*x^2 + 1)*b^2*x*arccos(c*x)/c - 1/2*sqrt(-c^2*x^2 + 1)*a*
b*x/c - 1/4*b^2*arccos(c*x)^2/c^2 - 1/2*a*b*arccos(c*x)/c^2 + 1/8*b^2/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^2\,dx = \int x\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,dx$$

input
```
int(x*(a + b*acos(c*x))^2,x)
```

output
```
int(x*(a + b*acos(c*x))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int x(a+b\arccos(cx))^2\,dx = \frac{2acos(cx)^2\,b^2c^2x^2 - acos(cx)^2\,b^2 - 2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx + 4acos(cx)\,ab\,c^2x^2 + 2asin(cx)\,ab - 2\surd}{4c^2}$$

input
```
int(x*(a+b*acos(c*x))^2,x)
```

output
```
(2*acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 - 2*sqrt( - c**2*x**2 +
 1)*acos(c*x)*b**2*c*x + 4*acos(c*x)*a*b*c**2*x**2 + 2*asin(c*x)*a*b - 2*s
qrt( - c**2*x**2 + 1)*a*b*c*x + 2*a**2*c**2*x**2 - b**2*c**2*x**2)/(4*c**2
)
```

3.150 $\int (a+b\arccos(cx))^2\,dx$

Optimal result 1051
Mathematica [A] (verified) 1051
Rubi [A] (verified) 1052
Maple [A] (warning: unable to verify) 1053
Fricas [A] (verification not implemented) 1053
Sympy [B] (verification not implemented) 1054
Maxima [A] (verification not implemented) 1054
Giac [A] (verification not implemented) 1055
Mupad [B] (verification not implemented) 1055
Reduce [B] (verification not implemented) 1056

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int (a+b\arccos(cx))^2\,dx=-2b^2x-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+x(a+b\arccos(cx))^2$$

output
```
-2*b^2*x-2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+x*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int (a+b\arccos(cx))^2\,dx=\left(a^2-2b^2\right)x-\frac{2ab\sqrt{1-c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1-c^2x^2}\right)\arccos(cx)}{c}+b^2x\arccos(cx)^2$$

input
```
Integrate[(a + b*ArcCos[c*x])^2,x]
```

output
```
(a^2 - 2*b^2)*x - (2*a*b*Sqrt[1 - c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 - c
^2*x^2])*ArcCos[c*x])/c + b^2*x*ArcCos[c*x]^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5131}$$

$$2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{24}$$

$$2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2$$

input
```
Int[(a + b*ArcCos[c*x])^2,x]
```

output
```
x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*Ar
cCos[c*x]))/c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{cx\,a^2+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
default	$\frac{cx\,a^2+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
parts	$x\,a^2+\frac{b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)}{c}+\frac{2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	75
orering	$x(a+b\arccos(cx))^2-\frac{2(a+b\arccos(cx))b}{c\sqrt{-c^2x^2+1}}+\frac{x(cx-1)(cx+1)\left(\frac{2b^2c^2}{-c^2x^2+1}-\frac{2(a+b\arccos(cx))b\,c^3x}{\left(-c^2x^2+1\right)^{\frac{3}{2}}}\right)}{c^2}$	103

input
```
int((a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2+b^2*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)
)+2*a*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int(a+b\arccos(cx))^2\,dx = \frac{b^2cx\arccos(cx)^2+2\,abcx\arccos(cx)+(a^2-2\,b^2)cx-2\,\sqrt{-c^2x^2+1}(b^2\arccos(cx)+ab)}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*arccos(c*x)^2 + 2*a*b*c*x*arccos(c*x) + (a^2 - 2*b^2)*c*x - 2*sqr
t(-c^2*x^2 + 1)*(b^2*arccos(c*x) + a*b))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int (a + b\arccos(cx))^2 \, dx$$
$$= \begin{cases} a^2x + 2abx\operatorname{acos}(cx) - \frac{2ab\sqrt{-c^2x^2+1}}{c} + b^2x\operatorname{acos}^2(cx) - 2b^2x - \frac{2b^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{c} & \text{for } c \neq 0 \\ x\left(a + \frac{\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*acos(c*x) - 2*a*b*sqrt(-c**2*x**2 + 1)/c + b**
2*x*acos(c*x)**2 - 2*b**2*x - 2*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/c, Ne(
c, 0)), (x*(a + pi*b/2)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int (a + b\arccos(cx))^2 \, dx = b^2x\arccos(cx)^2 - 2b^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) + a^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)ab}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arccos(c*x)^2 - 2*b^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*x
 + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx = b^2x\arccos{(cx)}^2 + 2\,abx\arccos{(cx)} + a^2x - 2\,b^2x - \frac{2\,\sqrt{-c^2x^2+1}b^2\arccos{(cx)}}{c} - \frac{2\,\sqrt{-c^2x^2+1}ab}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x - 2*b^2*x - 2*sqrt(-c^2*
x^2 + 1)*b^2*arccos(c*x)/c - 2*sqrt(-c^2*x^2 + 1)*a*b/c
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int (a+b\arccos(cx))^2\,dx = \begin{cases} x\left(a^2+\pi\,a\,b+\frac{\pi^2\,b^2}{4}\right) & \text{if } c=0 \\ a^2\,x+b^2\,x\left(\mathrm{acos}(c\,x)^2-2\right)-\frac{2\,b^2\,\mathrm{acos}(c\,x)\,\sqrt{1-c^2\,x^2}}{c}-\frac{2\,a\,b\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} & \text{if } c\neq 0 \end{cases}$$

input
```
int((a + b*acos(c*x))^2,x)
```

output
```
piecewise(c == 0, x*(a^2 + (b^2*pi^2)/4 + a*b*pi), c ~= 0, a^2*x + b^2*x*(
acos(c*x)^2 - 2) - (2*b^2*acos(c*x)*(- c^2*x^2 + 1)^(1/2))/c - (2*a*b*((-
c^2*x^2 + 1)^(1/2) - c*x*acos(c*x)))/c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx$$
$$=\frac{acos(cx)^2\,b^2cx-2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2+2acos(cx)\,abcx-2\sqrt{-c^2x^2+1}\,ab+a^2cx-2b^2cx}{c}$$

input
```
int((a+b*acos(c*x))^2,x)
```

output
```
(acos(c*x)**2*b**2*c*x - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2 + 2*acos(
c*x)*a*b*c*x - 2*sqrt( - c**2*x**2 + 1)*a*b + a**2*c*x - 2*b**2*c*x)/c
```

3.151 $\int \frac{(a+b\arccos(cx))^2}{x}\,dx$

Optimal result 1057
Mathematica [A] (verified) 1058
Rubi [A] (verified) 1058
Maple [A] (verified) 1061
Fricas [F] 1061
Sympy [F] 1062
Maxima [F] 1062
Giac [F(-2)] 1062
Mupad [F(-1)] 1063
Reduce [F] 1063

Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = -\frac{i(a+b\arccos(cx))^3}{3b} + (a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right) - ib(a+b\arccos(cx))\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + \frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)$$

output
```
-1/3*I*(a+b*arccos(c*x))^3/b+(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^
(1/2))^2)-I*b*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1
/2*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = -iab\arccos(cx)^2 - \frac{1}{3}ib^2\arccos(cx)^3 + 2ab\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + b^2\arccos(cx)^2\log\left(1+e^{2i\arccos(cx)}\right) + a^2\log(cx) - ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/x,x]
```

output
```
(-I)*a*b*ArcCos[c*x]^2 - (I/3)*b^2*ArcCos[c*x]^3 + 2*a*b*ArcCos[c*x]*Log[1
 + E^((2*I)*ArcCos[c*x])] + b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x]
)] + a^2*Log[c*x] - I*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*
x])] + (b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5137, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}\,d\arccos(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\int (a+b\arccos(cx))^2\tan(\arccos(cx))\,d\arccos(cx)$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2\right)-\frac{i(a+b\arccos(cx))^3}{3b}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{1}{2}i\,\mathrm{l}\right.\frac{i(a+b\arccos(cx))^3}{3b}$$

$$\Big\downarrow \text{2720}$$

$$2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.\frac{i(a+b\arccos(cx))^3}{3b}$$

$$\Big\downarrow \text{7143}$$

$$2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)\right)-\frac{1}{2}i\log\left(1+e^{2i\arccos}\right.\right.\frac{i(a+b\arccos(cx))^3}{3b}$$

input
```
Int[(a + b*ArcCos[c*x])^2/x,x]
```

output
```
((-1/3*I)*(a + b*ArcCos[c*x])^3)/b + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])^2
*Log[1 + E^((2*I)*ArcCos[c*x])] + I*b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2
, -E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.01

method	result
parts	$a^2 \ln(x) + b^2 \left(-\frac{i \arccos(cx)^3}{3} + \arccos(cx)^2 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - i \arccos(c$
derivativedivides	$a^2 \ln(cx) + b^2 \left(-\frac{i \arccos(cx)^3}{3} + \arccos(cx)^2 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - i \arccos($
default	$a^2 \ln(cx) + b^2 \left(-\frac{i \arccos(cx)^3}{3} + \arccos(cx)^2 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - i \arccos($

input
```
int((a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(-1/3*I*arccos(c*x)^3+arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)
^(1/2))^2)-I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*poly
log(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))-I*a*b*arccos(c*x)^2-I*a*b*polylog(2,
-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*a*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^
(1/2))^2)
```

Fricas [F]

$$\int \frac{(a + b\arccos(cx))^2}{x}\, dx = \int \frac{(b\arccos(cx) + a)^2}{x}\, dx$$

input
```
integrate((a+b*arccos(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(a+b\,\mathrm{acos}\,(cx))^2}{x}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x,x)
```

output
```
Integral((a + b*acos(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(b\arccos\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 +
 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x}\,dx$$

input
```
int((a + b*acos(c*x))^2/x,x)
```

output
```
int((a + b*acos(c*x))^2/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = 2\left(\int \frac{acos(cx)}{x}dx\right)ab+\left(\int \frac{acos(cx)^2}{x}dx\right)b^2+\log(x)\,a^2$$

input
```
int((a+b*acos(c*x))^2/x,x)
```

output
```
2*int(acos(c*x)/x,x)*a*b + int(acos(c*x)**2/x,x)*b**2 + log(x)*a**2
```

3.152 $\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result 1064
Mathematica [A] (verified) 1064
Rubi [A] (verified) 1065
Maple [A] (verified) 1067
Fricas [F] 1068
Sympy [F] 1068
Maxima [F] 1068
Giac [F(-2)] 1069
Mupad [F(-1)] 1069
Reduce [F] 1069

Optimal result

Integrand size = 14, antiderivative size = 89

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = &-\frac{(a+b\arccos(cx))^2}{x} \\ &- 4ibc(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right) \\ &+ 2ib^2c\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) \\ &- 2ib^2c\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\end{aligned}$$

output
```
-(a+b*arccos(c*x))^2/x-4*I*b*c*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)
^(1/2))+2*I*b^2*c*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*b^2*c*polyl
og(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = -\frac{a^2+2ab\left(\arccos(cx)-cx\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)+b^2\left(\arccos(cx)^2-2cx\left(\arccos(cx)\left(\log\left(1-ie^{i\arccos(c\cdots}\right.\right.\right.\right.}{x}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/x^2,x]
```

output
```
-((a^2 + 2*a*b*(ArcCos[c*x] - c*x*ArcTanh[Sqrt[1 - c^2*x^2]]) + b^2*(ArcCo
s[c*x]^2 - 2*c*x*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^
(I*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E
^(I*ArcCos[c*x])]))))/x)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5139, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5139}$$

$$-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5219}$$

$$2bc\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)-\frac{(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$2bc\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-\frac{(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{(a+b\arccos(cx))^2}{x}+$$
$$2bc\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right)(\right.$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(a+b\arccos(cx))^2}{x}+$$
$$2bc\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{(a+b\arccos(cx))^2}{x}+$$
$$2bc\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)$$

input
```
Int[(a + b*ArcCos[c*x])^2/x^2,x]
```

output
```
-((a + b*ArcCos[c*x])^2/x) + 2*b*c*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I
*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I
*E^(I*ArcCos[c*x])])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.01

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\arccos(cx)^2}{cx}-2\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)+2\arccos(cx)\ln$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arccos(cx)^2}{cx}-2\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)+2\arccos(cx)\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arccos(cx)^2}{cx}-2\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)+2\arccos(cx)\right.\right.$

input
```
int((a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arccos(c*x)^2-2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1
)^(1/2)))+2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*dilog(1+I*(
c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))+2*a*
b*c*(-1/c/x*arccos(c*x)+arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**2,x)
```

output
```
Integral((a + b*acos(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b + (2
*c*x*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x)/(c^2*x^3 - x), x) - arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)^2)*b^2/x - a^2/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/x^2,x)
```

output
```
int((a + b*acos(c*x))^2/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = \frac{-2acos(cx)\,ab+\left(\int \frac{acos(cx)^2}{x^2}dx\right)b^2x-2\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)abcx-a^2}{x}$$

input
```
int((a+b*acos(c*x))^2/x^2,x)
```

output
```
( - 2*acos(c*x)*a*b + int(acos(c*x)**2/x**2,x)*b**2*x - 2*log(tan(asin(c*x
)/2))*a*b*c*x - a**2)/x
```

3.153 $\int x^2(a+b\arccos(cx))^3\,dx$

Optimal result . . . 1071
Mathematica [A] (verified) . . . 1072
Rubi [A] (verified) . . . 1072
Maple [A] (verified) . . . 1075
Fricas [A] (verification not implemented) . . . 1076
Sympy [B] (verification not implemented) . . . 1077
Maxima [A] (verification not implemented) . . . 1078
Giac [A] (verification not implemented) . . . 1079
Mupad [F(-1)] . . . 1079
Reduce [F] . . . 1080

Optimal result

Integrand size = 14, antiderivative size = 170

$$\int x^2(a+b\arccos(cx))^3\,dx=\frac{14b^3\sqrt{1-c^2x^2}}{9c^3}-\frac{2b^3(1-c^2x^2)^{3/2}}{27c^3}-\frac{4b^2x(a+b\arccos(cx))}{3c^2}-\frac{2}{9}b^2x^3(a+b\arccos(cx))-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3}-\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c}+\frac{1}{3}x^3(a+b\arccos(cx))^3$$

output
```
14/9*b^3*(-c^2*x^2+1)^(1/2)/c^3-2/27*b^3*(-c^2*x^2+1)^(3/2)/c^3-4/3*b^2*x*
(a+b*arccos(c*x))/c^2-2/9*b^2*x^3*(a+b*arccos(c*x))-2/3*b*(-c^2*x^2+1)^(1/
2)*(a+b*arccos(c*x))^2/c^3-1/3*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^
2/c+1/3*x^3*(a+b*arccos(c*x))^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.28

$$\int x^2(a+b\arccos(cx))^3\,dx$$
$$=\frac{9a^3c^3x^3-9a^2b\sqrt{1-c^2x^2}(2+c^2x^2)-6ab^2cx(6+c^2x^2)+2b^3\sqrt{1-c^2x^2}(20+c^2x^2)-3b\big(-9a^2c^3x^3+6a}{}$$

input
```
Integrate[x^2*(a + b*ArcCos[c*x])^3,x]
```

output
```
(9*a^3*c^3*x^3 - 9*a^2*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) - 6*a*b^2*c*x*(6
+ c^2*x^2) + 2*b^3*Sqrt[1 - c^2*x^2]*(20 + c^2*x^2) - 3*b*(-9*a^2*c^3*x^3
+ 6*a*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) + 2*b^2*c*x*(6 + c^2*x^2))*ArcCos[
c*x] - 9*b^2*(-3*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))*ArcCos[c*x
]^2 + 9*b^3*c^3*x^3*ArcCos[c*x]^3)/(27*c^3)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5139, 5211, 5139, 243, 53, 2009, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \text{5139}$$

$$bc\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{5211}$$

$$bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\int x^2(a+b\arccos(cx))dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$\Big\downarrow$ 5139

$$bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}bc\int \frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)$$
$$\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$\Big\downarrow$ 243

$$bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right.$$
$$\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$\Big\downarrow$ 53

$$bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+}{3c^2}\right.$$
$$\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$\Big\downarrow$ 2009

$$bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\right.\right.}{3c}\right.$$
$$\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$\Big\downarrow$ 5183

$$bc\left(\frac{2\left(-\frac{2b\int(a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{2009}$$

$$bc\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)}{3c^2}-\frac{2b\left(\frac{1}{3}x^3\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^3$$

input
```
Int[x^2*(a + b*ArcCos[c*x])^3,x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^3)/3 + b*c*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*(a + b*Ar
cCos[c*x])^2)/c^2 - (2*b*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x
^2)^(3/2))/(3*c^4)))/6 + (x^3*(a + b*ArcCos[c*x]))/3))/(3*c) + (2*(-((Sqrt
[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c^2*x^
2])/c + b*x*ArcCos[c*x]))/c))/(3*c^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{c^3x^3\arccos(cx)^3}{3}-\frac{\arccos(cx)^2(c^2x^2+2)\sqrt{-c^2x^2+1}}{3}+\frac{4\sqrt{-c^2x^2+1}}{3}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3\arccos(cx)}{9}+\frac{2(c^2x^2}{}\right.}{}$
default	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{c^3x^3\arccos(cx)^3}{3}-\frac{\arccos(cx)^2(c^2x^2+2)\sqrt{-c^2x^2+1}}{3}+\frac{4\sqrt{-c^2x^2+1}}{3}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3\arccos(cx)}{9}+\frac{2(c^2x^2}{}\right.}{}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\frac{c^3x^3\arccos(cx)^3}{3}-\frac{\arccos(cx)^2(c^2x^2+2)\sqrt{-c^2x^2+1}}{3}+\frac{4\sqrt{-c^2x^2+1}}{3}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3\arccos(cx)}{9}+\frac{2(c^2x^2}{}\right.}{c^3}$
orering	$\frac{5(13c^6x^6+40c^4x^4-152c^2x^2+96)(a+b\arccos(cx))^3}{81c^6x^3}-\frac{(25c^6x^6+166c^4x^4-572c^2x^2+360)\left(2x(a+b\arccos(cx))^3-\frac{3x}{}\right.}{81c^6x^4}$

input
```
int(x^2*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/3*c^3*x^3*arccos(c*x)^3-1/3*arccos(c*x)^2*(c
^2*x^2+2)*(-c^2*x^2+1)^(1/2)+4/3*(-c^2*x^2+1)^(1/2)-4/3*c*x*arccos(c*x)-2/
9*c^3*x^3*arccos(c*x)+2/27*(c^2*x^2+2)*(-c^2*x^2+1)^(1/2))+3*a*b^2*(1/3*c^
3*x^3*arccos(c*x)^2-2/9*arccos(c*x)*(c^2*x^2+2)*(-c^2*x^2+1)^(1/2)-2/27*c^
3*x^3-4/9*c*x)+3*a^2*b*(1/3*c^3*x^3*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(
1/2)-2/9*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15

$$\int x^2(a+b\arccos(cx))^3\,dx$$
$$=\frac{9\,b^3c^3x^3\arccos(cx)^3+27\,ab^2c^3x^3\arccos(cx)^2+3\,(3\,a^3-2\,ab^2)c^3x^3-36\,ab^2cx+3\,((9\,a^2b-2\,b^3)c^3x^3-}{}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
1/27*(9*b^3*c^3*x^3*arccos(c*x)^3 + 27*a*b^2*c^3*x^3*arccos(c*x)^2 + 3*(3*
a^3 - 2*a*b^2)*c^3*x^3 - 36*a*b^2*c*x + 3*((9*a^2*b - 2*b^3)*c^3*x^3 - 12*
b^3*c*x)*arccos(c*x) - ((9*a^2*b - 2*b^3)*c^2*x^2 + 18*a^2*b - 40*b^3 + 9*
(b^3*c^2*x^2 + 2*b^3)*arccos(c*x)^2 + 18*(a*b^2*c^2*x^2 + 2*a*b^2)*arccos(
c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(158) = 316$.

Time = 0.37 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.96

$$\int x^2(a + b\arccos(cx))^3 \, dx$$

$$= \begin{cases} \frac{a^3x^3}{3} + a^2bx^3 \operatorname{acos}(cx) - \frac{a^2bx^2\sqrt{-c^2x^2+1}}{3c} - \frac{2a^2b\sqrt{-c^2x^2+1}}{3c^3} + ab^2x^3 \operatorname{acos}^2(cx) - \frac{2ab^2x^3}{9} - \frac{2ab^2x^2\sqrt{-c^2x^2+1} \operatorname{acos}(c}{3c} \\ \frac{x^3\left(a+\frac{\pi b}{2}\right)^3}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*acos(c*x))**3,x)
```

output
```
Piecewise((a**3*x**3/3 + a**2*b*x**3*acos(c*x) - a**2*b*x**2*sqrt(-c**2*x*
*2 + 1)/(3*c) - 2*a**2*b*sqrt(-c**2*x**2 + 1)/(3*c**3) + a*b**2*x**3*acos(
c*x)**2 - 2*a*b**2*x**3/9 - 2*a*b**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(
3*c) - 4*a*b**2*x/(3*c**2) - 4*a*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c*
*3) + b**3*x**3*acos(c*x)**3/3 - 2*b**3*x**3*acos(c*x)/9 - b**3*x**2*sqrt(
-c**2*x**2 + 1)*acos(c*x)**2/(3*c) + 2*b**3*x**2*sqrt(-c**2*x**2 + 1)/(27*
c) - 4*b**3*x*acos(c*x)/(3*c**2) - 2*b**3*sqrt(-c**2*x**2 + 1)*acos(c*x)**
2/(3*c**3) + 40*b**3*sqrt(-c**2*x**2 + 1)/(27*c**3), Ne(c, 0)), (x**3*(a +
 pi*b/2)**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.61

$$\int x^2(a+b\arccos(cx))^3\,dx = \frac{1}{3}\,b^3x^3\arccos{(cx)}^3 + ab^2x^3\arccos{(cx)}^2 + \frac{1}{3}\,a^3x^3 + \frac{1}{3}\left(3\,x^3\arccos{(cx)} - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)a^2b - \frac{2}{9}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos{(cx)} + \frac{c^2x^3+6\,x}{c^2}\right)ab^2 - \frac{1}{27}\left(9\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos{(cx)}^2 - 2\,c\left(\frac{\sqrt{-c^2x^2+1}x^2 + \frac{20\sqrt{-c^2x^2+1}}{c^2}}{c^2} - \frac{3\,(c^2x^3}{}\right.\right.$$

input
```
integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*arccos(c*x)^3 + a*b^2*x^3*arccos(c*x)^2 + 1/3*a^3*x^3 + 1/3*(3
*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^
4))*a^2*b - 2/9*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^
4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*a*b^2 - 1/27*(9*c*(sqrt(-c^2*x^2 + 1
)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x)^2 - 2*c*((sqrt(-c^2*x^2
+ 1)*x^2 + 20*sqrt(-c^2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 + 6*x)*arccos(c*x)/
c^3))*b^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.70

$$\int x^2(a+b\arccos(cx))^3\,dx = \frac{1}{3}b^3x^3\arccos(cx)^3 + ab^2x^3\arccos(cx)^2 + a^2bx^3\arccos(cx) - \frac{2}{9}b^3x^3\arccos(cx) - \frac{\sqrt{-c^2x^2+1}b^3x^2\arccos(cx)^2}{3c} + \frac{1}{3}a^3x^3 - \frac{2}{9}ab^2x^3 - \frac{2\sqrt{-c^2x^2+1}ab^2x^2\arccos(cx)}{3c} - \frac{\sqrt{-c^2x^2+1}a^2bx^2}{3c} + \frac{2\sqrt{-c^2x^2+1}b^3x^2}{27c} - \frac{4b^3x\arccos(cx)}{3c^2} - \frac{2\sqrt{-c^2x^2+1}b^3\arccos(cx)^2}{3c^3} - \frac{4ab^2x}{3c^2} - \frac{4\sqrt{-c^2x^2+1}ab^2\arccos(cx)}{3c^3} - \frac{2\sqrt{-c^2x^2+1}a^2b}{3c^3} + \frac{40\sqrt{-c^2x^2+1}b^3}{27c^3}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
1/3*b^3*x^3*arccos(c*x)^3 + a*b^2*x^3*arccos(c*x)^2 + a^2*b*x^3*arccos(c*x
) - 2/9*b^3*x^3*arccos(c*x) - 1/3*sqrt(-c^2*x^2 + 1)*b^3*x^2*arccos(c*x)^2
/c + 1/3*a^3*x^3 - 2/9*a*b^2*x^3 - 2/3*sqrt(-c^2*x^2 + 1)*a*b^2*x^2*arccos
(c*x)/c - 1/3*sqrt(-c^2*x^2 + 1)*a^2*b*x^2/c + 2/27*sqrt(-c^2*x^2 + 1)*b^3
*x^2/c - 4/3*b^3*x*arccos(c*x)/c^2 - 2/3*sqrt(-c^2*x^2 + 1)*b^3*arccos(c*x
)^2/c^3 - 4/3*a*b^2*x/c^2 - 4/3*sqrt(-c^2*x^2 + 1)*a*b^2*arccos(c*x)/c^3 -
 2/3*sqrt(-c^2*x^2 + 1)*a^2*b/c^3 + 40/27*sqrt(-c^2*x^2 + 1)*b^3/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^3\,dx = \int x^2\,(a+b\,\mathrm{acos}(c\,x))^3\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^3,x)
```

output
```
int(x^2*(a + b*acos(c*x))^3, x)
```

Reduce [F]

$$\int x^2(a+b\arccos(cx))^3\,dx$$
$$= \frac{3acos(cx)\,a^2b\,c^3x^3-\sqrt{-c^2x^2+1}\,a^2b\,c^2x^2-2\sqrt{-c^2x^2+1}\,a^2b+3\left(\int acos(cx)^3\,x^2dx\right)b^3c^3+9\left(\int acos(cx\right.}{3c^3}$$

input
```
int(x^2*(a+b*acos(c*x))^3,x)
```

output
```
(3*acos(c*x)*a**2*b*c**3*x**3 - sqrt( - c**2*x**2 + 1)*a**2*b*c**2*x**2 -
2*sqrt( - c**2*x**2 + 1)*a**2*b + 3*int(acos(c*x)**3*x**2,x)*b**3*c**3 + 9
*int(acos(c*x)**2*x**2,x)*a*b**2*c**3 + a**3*c**3*x**3)/(3*c**3)
```

3.154 $\int x(a+b\arccos(cx))^3\,dx$

Optimal result 1081
Mathematica [A] (verified) 1081
Rubi [A] (verified) 1082
Maple [A] (verified) 1085
Fricas [A] (verification not implemented) 1085
Sympy [B] (verification not implemented) 1086
Maxima [F] 1086
Giac [B] (verification not implemented) 1087
Mupad [F(-1)] 1088
Reduce [B] (verification not implemented) 1088

Optimal result

Integrand size = 12, antiderivative size = 125

$$\int x(a+b\arccos(cx))^3\,dx = \frac{3b^3x\sqrt{1-c^2x^2}}{8c} - \frac{3}{4}b^2x^2(a+b\arccos(cx)) - \frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c} - \frac{(a+b\arccos(cx))^3}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^3 - \frac{3b^3\arcsin(cx)}{8c^2}$$

output
```
3/8*b^3*x*(-c^2*x^2+1)^(1/2)/c-3/4*b^2*x^2*(a+b*arccos(c*x))-3/4*b*x*(-c^2
*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/c-1/4*(a+b*arccos(c*x))^3/c^2+1/2*x^2*(a
+b*arccos(c*x))^3-3/8*b^3*arcsin(c*x)/c^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.48

$$\int x(a+b\arccos(cx))^3\,dx$$

$$= \frac{cx\left(4a^3cx-6ab^2cx-6a^2b\sqrt{1-c^2x^2}+3b^3\sqrt{1-c^2x^2}\right)-6bcx\left(-2a^2cx+b^2cx+2ab\sqrt{1-c^2x^2}\right)\arccos(c}{}$$

input
```
Integrate[x*(a + b*ArcCos[c*x])^3,x]
```

output
```
(c*x*(4*a^3*c*x - 6*a*b^2*c*x - 6*a^2*b*Sqrt[1 - c^2*x^2] + 3*b^3*Sqrt[1 -
 c^2*x^2]) - 6*b*c*x*(-2*a^2*c*x + b^2*c*x + 2*a*b*Sqrt[1 - c^2*x^2])*ArcC
os[c*x] - 6*b^2*(a - 2*a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2
+ 2*b^3*(-1 + 2*c^2*x^2)*ArcCos[c*x]^3 + (6*a^2*b - 3*b^3)*ArcSin[c*x])/(8
*c^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5139, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \ 5139$$

$$\frac{3}{2}bc\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \ 5211$$

$$\frac{3}{2}bc\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \ 5139$$

$$\frac{3}{2}bc\left(-\frac{b\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \ 262$$

$$\frac{3}{2}bc\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\,\mathrm{a}}{2c^2}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^3$$

$\downarrow$ 223

$$\frac{3}{2}bc\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(}{2c^2}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^3$$

$\downarrow$ 5153

$$\frac{3}{2}bc\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos}{2c^2}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^3$$

input
```
Int[x*(a + b*ArcCos[c*x])^3,x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^3)/2 + (3*b*c*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x])^2)/c^2 - (a + b*ArcCos[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*Arc
Cos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3))
)/2))/c))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{\cos(2\arccos(cx))\arccos(cx)^3}{4}-\frac{3\sin(2\arccos(cx))\arccos(cx)^2}{8}+\frac{3\sin(2\arccos(cx))}{16}-\frac{3\cos(2\arccos(cx))\arccos(cx)}{8}\right)}{}$
default	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{\cos(2\arccos(cx))\arccos(cx)^3}{4}-\frac{3\sin(2\arccos(cx))\arccos(cx)^2}{8}+\frac{3\sin(2\arccos(cx))}{16}-\frac{3\cos(2\arccos(cx))\arccos(cx)}{8}\right)}{}$
parts	$\frac{x^2a^3}{2}+\frac{b^3\left(\frac{\cos(2\arccos(cx))\arccos(cx)^3}{4}-\frac{3\sin(2\arccos(cx))\arccos(cx)^2}{8}+\frac{3\sin(2\arccos(cx))}{16}-\frac{3\cos(2\arccos(cx))\arccos(cx)}{8}\right)}{c^2}$
orering	$\frac{(15c^4x^4-20c^2x^2+8)(a+b\arccos(cx))^3}{16c^4x^2}-\frac{(7c^4x^4-16c^2x^2+8)\left((a+b\arccos(cx))^3-\frac{3x(a+b\arccos(cx))^2bc}{\sqrt{-c^2x^2+1}}\right)}{16c^4x^2}+\frac{(cx-}{}$

input
```
int(x*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/4*cos(2*arccos(c*x))*arccos(c*x)^3-3/8*sin(2
*arccos(c*x))*arccos(c*x)^2+3/16*sin(2*arccos(c*x))-3/8*cos(2*arccos(c*x))
*arccos(c*x))+3*a*b^2*(1/4*cos(2*arccos(c*x))*arccos(c*x)^2-1/8*cos(2*arcc
os(c*x))-1/4*sin(2*arccos(c*x))*arccos(c*x))+3*a^2*b*(1/2*c^2*x^2*arccos(c
*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+1/4*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.35

$$\int x(a+b\arccos(cx))^3\,dx$$
$$=\frac{2\,(2\,a^3-3\,ab^2)c^2x^2+2\,(2\,b^3c^2x^2-b^3)\arccos{(cx)}^3+6\,(2\,ab^2c^2x^2-ab^2)\arccos{(cx)}^2+3\,(2\,(2\,a^2b-b^3)c}{8\,}$$

input
```
integrate(x*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^3 - 3*a*b^2)*c^2*x^2 + 2*(2*b^3*c^2*x^2 - b^3)*arccos(c*x)^3 +
 6*(2*a*b^2*c^2*x^2 - a*b^2)*arccos(c*x)^2 + 3*(2*(2*a^2*b - b^3)*c^2*x^2
- 2*a^2*b + b^3)*arccos(c*x) - 3*(2*b^3*c*x*arccos(c*x)^2 + 4*a*b^2*c*x*ar
ccos(c*x) + (2*a^2*b - b^3)*c*x)*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(116) = 232.

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.15

$$\int x(a+b\arccos(cx))^3\,dx$$
$$= \begin{cases} \frac{a^3x^2}{2} + \frac{3a^2bx^2\operatorname{acos}(cx)}{2} - \frac{3a^2bx\sqrt{-c^2x^2+1}}{4c} - \frac{3a^2b\operatorname{acos}(cx)}{4c^2} + \frac{3ab^2x^2\operatorname{acos}^2(cx)}{2} - \frac{3ab^2x^2}{4} - \frac{3ab^2x\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{2c} - 3ab \\ \frac{x^2\left(a+\frac{\pi b}{2}\right)^3}{2} \end{cases}$$

input
```
integrate(x*(a+b*acos(c*x))**3,x)
```

output
```
Piecewise((a**3*x**2/2 + 3*a**2*b*x**2*acos(c*x)/2 - 3*a**2*b*x*sqrt(-c**2
*x**2 + 1)/(4*c) - 3*a**2*b*acos(c*x)/(4*c**2) + 3*a*b**2*x**2*acos(c*x)**
2/2 - 3*a*b**2*x**2/4 - 3*a*b**2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2*c) -
3*a*b**2*acos(c*x)**2/(4*c**2) + b**3*x**2*acos(c*x)**3/2 - 3*b**3*x**2*ac
os(c*x)/4 - 3*b**3*x*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(4*c) + 3*b**3*x*sq
rt(-c**2*x**2 + 1)/(8*c) - b**3*acos(c*x)**3/(4*c**2) + 3*b**3*acos(c*x)/(
8*c**2), Ne(c, 0)), (x**2*(a + pi*b/2)**3/2, True))
```

Maxima [F]

$$\int x(a+b\arccos(cx))^3\,dx = \int (b\arccos(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/2*a^3*x^2 + 3
/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*a^
2*b - integrate(3/2*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^3*c*x^2*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x)^2 - 2*(a*b^2*c^2*x^3 - a*b^2*x)*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x)^2)/(c^2*x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(109) = 218$.

Time = 0.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.85

$$\int x(a+b\arccos(cx))^3\,dx = \frac{1}{2}\,b^3x^2\arccos(cx)^3 + \frac{3}{2}\,ab^2x^2\arccos(cx)^2 + \frac{3}{2}\,a^2bx^2\arccos(cx) - \frac{3}{4}\,b^3x^2\arccos(cx) - \frac{3\sqrt{-c^2x^2+1}b^3x\arccos(cx)^2}{4\,c} + \frac{1}{2}\,a^3x^2 - \frac{3}{4}\,ab^2x^2 - \frac{3\sqrt{-c^2x^2+1}ab^2x\arccos(cx)}{2\,c} - \frac{b^3\arccos(cx)^3}{4\,c^2} - \frac{3\sqrt{-c^2x^2+1}a^2bx}{4\,c} + \frac{3\sqrt{-c^2x^2+1}b^3x}{8\,c} - \frac{3\,ab^2\arccos(cx)^2}{4\,c^2} - \frac{3\,a^2b\arccos(cx)}{4\,c^2} + \frac{3\,b^3\arccos(cx)}{8\,c^2} + \frac{3\,ab^2}{8\,c^2}$$

input
```
integrate(x*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
1/2*b^3*x^2*arccos(c*x)^3 + 3/2*a*b^2*x^2*arccos(c*x)^2 + 3/2*a^2*b*x^2*ar
ccos(c*x) - 3/4*b^3*x^2*arccos(c*x) - 3/4*sqrt(-c^2*x^2 + 1)*b^3*x*arccos(
c*x)^2/c + 1/2*a^3*x^2 - 3/4*a*b^2*x^2 - 3/2*sqrt(-c^2*x^2 + 1)*a*b^2*x*ar
ccos(c*x)/c - 1/4*b^3*arccos(c*x)^3/c^2 - 3/4*sqrt(-c^2*x^2 + 1)*a^2*b*x/c
 + 3/8*sqrt(-c^2*x^2 + 1)*b^3*x/c - 3/4*a*b^2*arccos(c*x)^2/c^2 - 3/4*a^2*
b*arccos(c*x)/c^2 + 3/8*b^3*arccos(c*x)/c^2 + 3/8*a*b^2/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^3\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^3\,dx$$

input
```
int(x*(a + b*acos(c*x))^3,x)
```

output
```
int(x*(a + b*acos(c*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\int x(a+b\arccos(cx))^3\,dx = \frac{4acos(cx)^3\,b^3c^2x^2-2acos(cx)^3\,b^3-6\sqrt{-c^2x^2+1}\,acos(cx)^2\,b^3cx+12acos(cx)^2\,a\,b^2c^2x^2-6acos(cx)^2\,a\,\ldots}{\ldots}$$

input
```
int(x*(a+b*acos(c*x))^3,x)
```

output
```
(4*acos(c*x)**3*b**3*c**2*x**2 - 2*acos(c*x)**3*b**3 - 6*sqrt( - c**2*x**2
 + 1)*acos(c*x)**2*b**3*c*x + 12*acos(c*x)**2*a*b**2*c**2*x**2 - 6*acos(c*
x)**2*a*b**2 - 12*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b**2*c*x + 12*acos(c*
x)*a**2*b*c**2*x**2 - 6*acos(c*x)*b**3*c**2*x**2 + 6*asin(c*x)*a**2*b - 3*
asin(c*x)*b**3 - 6*sqrt( - c**2*x**2 + 1)*a**2*b*c*x + 3*sqrt( - c**2*x**2
 + 1)*b**3*c*x + 4*a**3*c**2*x**2 - 6*a*b**2*c**2*x**2)/(8*c**2)
```

3.155 $\int (a+b\arccos(cx))^3\,dx$

Optimal result 1089
Mathematica [A] (verified) 1089
Rubi [A] (verified) 1090
Maple [A] (warning: unable to verify) 1091
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Reduce [B] (verification not implemented) 1094

Optimal result

Integrand size = 10, antiderivative size = 79

$$\int (a+b\arccos(cx))^3\,dx=\frac{6b^3\sqrt{1-c^2x^2}}{c}-6b^2x(a+b\arccos(cx))-\frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+x(a+b\arccos(cx))^3$$

output
```
6*b^3*(-c^2*x^2+1)^(1/2)/c-6*b^2*x*(a+b*arccos(c*x))-3*b*(-c^2*x^2+1)^(1/2
)*(a+b*arccos(c*x))^2/c+x*(a+b*arccos(c*x))^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int (a+b\arccos(cx))^3\,dx$$
$$=\frac{a(a^2-6b^2)\,cx-3b(a^2-2b^2)\,\sqrt{1-c^2x^2}+3b\left(a^2cx-2b^2cx-2ab\sqrt{1-c^2x^2}\right)\arccos(cx)+3b^2\left(acx-b\sqrt{}\right.}{c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^3,x]
```

output
```
(a*(a^2 - 6*b^2)*c*x - 3*b*(a^2 - 2*b^2)*Sqrt[1 - c^2*x^2] + 3*b*(a^2*c*x
- 2*b^2*c*x - 2*a*b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + 3*b^2*(a*c*x - b*Sqrt
[1 - c^2*x^2])*ArcCos[c*x]^2 + b^3*c*x*ArcCos[c*x]^3)/c
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5131, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \text{ 5131}$$

$$3bc\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 5183}$$

$$3bc\left(-\frac{2b\int (a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)+x(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 2009}$$

$$3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3$$

input
```
Int[(a + b*ArcCos[c*x])^3,x]
```

output
```
x*(a + b*ArcCos[c*x])^3 + 3*b*c*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^
2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{cx\,a^3+b^3\left(\arccos(cx)^3cx-3\arccos(cx)^2\sqrt{-c^2x^2+1}+6\sqrt{-c^2x^2+1}-6cx\arccos(cx)\right)+3a\,b^2\left(\arccos(cx)^2cx-2cx-2\,\mathrm{arc}\right.}{c}$
default	$\frac{cx\,a^3+b^3\left(\arccos(cx)^3cx-3\arccos(cx)^2\sqrt{-c^2x^2+1}+6\sqrt{-c^2x^2+1}-6cx\arccos(cx)\right)+3a\,b^2\left(\arccos(cx)^2cx-2cx-2\,\mathrm{arc}\right.}{c}$
parts	$x\,a^3+\frac{b^3\left(\arccos(cx)^3cx-3\arccos(cx)^2\sqrt{-c^2x^2+1}+6\sqrt{-c^2x^2+1}-6cx\arccos(cx)\right)}{c}+\frac{3a^2b\left(cx\arccos(cx)-\sqrt{-c^2}\right.}{c}$
orering	$x(a+b\arccos(cx))^3+\frac{3(c^2x^2-2)(a+b\arccos(cx))^2b}{c\sqrt{-c^2x^2+1}}-\frac{2x(cx-1)(cx+1)\left(\frac{6(a+b\arccos(cx))b^2c^2}{-c^2x^2+1}-\frac{3(a+b\,\mathrm{arc}}{(-c}\right.}{c^2}$

input
```
int((a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^3+b^3*(arccos(c*x)^3*c*x-3*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)+6*(
-c^2*x^2+1)^(1/2)-6*c*x*arccos(c*x))+3*a*b^2*(arccos(c*x)^2*c*x-2*c*x-2*ar
ccos(c*x)*(-c^2*x^2+1)^(1/2))+3*a^2*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int (a+b\arccos(cx))^3\,dx$$
$$=\frac{b^3cx\arccos{(cx)}^3+3\,ab^2cx\arccos{(cx)}^2+3\,(a^2b-2\,b^3)cx\arccos{(cx)}+(a^3-6\,ab^2)cx-3\,\big(b^3\arccos{(cx}}{c}$$

input
```
integrate((a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
(b^3*c*x*arccos(c*x)^3 + 3*a*b^2*c*x*arccos(c*x)^2 + 3*(a^2*b - 2*b^3)*c*x
*arccos(c*x) + (a^3 - 6*a*b^2)*c*x - 3*(b^3*arccos(c*x)^2 + 2*a*b^2*arccos
(c*x) + a^2*b - 2*b^3)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.09

$$\int (a+b\arccos(cx))^3\,dx$$
$$=\begin{cases} a^3x+3a^2bx\operatorname{acos}{(cx)}-\frac{3a^2b\sqrt{-c^2x^2+1}}{c}+3ab^2x\operatorname{acos}^2{(cx)}-6ab^2x-\frac{6ab^2\sqrt{-c^2x^2+1}\operatorname{acos}{(cx)}}{c}+b^3x\operatorname{acos}^3{(cx} \\ x\left(a+\frac{\pi b}{2}\right)^3 \end{cases}$$

input
```
integrate((a+b*acos(c*x))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*x*acos(c*x) - 3*a**2*b*sqrt(-c**2*x**2 + 1)/c
 + 3*a*b**2*x*acos(c*x)**2 - 6*a*b**2*x - 6*a*b**2*sqrt(-c**2*x**2 + 1)*ac
os(c*x)/c + b**3*x*acos(c*x)**3 - 6*b**3*x*acos(c*x) - 3*b**3*sqrt(-c**2*x
**2 + 1)*acos(c*x)**2/c + 6*b**3*sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (x*(a
+ pi*b/2)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.82

$$\int (a+b\arccos(cx))^3\,dx = b^3x\arccos(cx)^3 + 3\,ab^2x\arccos(cx)^2 - 3\left(\frac{\sqrt{-c^2x^2+1}\arccos(cx)^2}{c} + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)}{c}\right)b^3 - 6\,ab^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) + a^3x + \frac{3\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)a^2b}{c}$$

input
```
integrate((a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 - 3*(sqrt(-c^2*x^2 + 1)*arcc
os(c*x)^2/c + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))/c)*b^3 - 6*a*b^2*(x
 + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^3*x + 3*(c*x*arccos(c*x) - sqrt(-
c^2*x^2 + 1))*a^2*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.90

$$\int (a+b\arccos(cx))^3\,dx = b^3x\arccos(cx)^3 + 3\,ab^2x\arccos(cx)^2 + 3\,a^2bx\arccos(cx) - 6\,b^3x\arccos(cx) - \frac{3\sqrt{-c^2x^2+1}b^3\arccos(cx)^2}{c} + a^3x - 6\,ab^2x - \frac{6\sqrt{-c^2x^2+1}ab^2\arccos(cx)}{c} - \frac{3\sqrt{-c^2x^2+1}a^2b}{c} + \frac{6\sqrt{-c^2x^2+1}b^3}{c}$$

input
```
integrate((a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 + 3*a^2*b*x*arccos(c*x) - 6*
b^3*x*arccos(c*x) - 3*sqrt(-c^2*x^2 + 1)*b^3*arccos(c*x)^2/c + a^3*x - 6*a
*b^2*x - 6*sqrt(-c^2*x^2 + 1)*a*b^2*arccos(c*x)/c - 3*sqrt(-c^2*x^2 + 1)*a
^2*b/c + 6*sqrt(-c^2*x^2 + 1)*b^3/c
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.08

$$\int (a+b\arccos(cx))^3\,dx$$
$$= \begin{cases} x\left(a^3+\frac{3\pi a^2 b}{2}+\frac{3\pi^2 a b^2}{4}+\frac{\pi^3 b^3}{8}\right) \\ a^3 x - b^3 x\left(6\,\mathrm{acos}(c\,x)-\mathrm{acos}(c\,x)^3\right) - \frac{3a^2 b\left(\sqrt{1-c^2x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} + 3\,a\,b^2\,x\left(\mathrm{acos}(c\,x)^2-2\right) - \frac{b^3\sqrt{1-c^2\,}}{} \end{cases}$$

input
```
int((a + b*acos(c*x))^3,x)
```

output
```
piecewise(c == 0, x*(a^3 + (b^3*pi^3)/8 + (3*a*b^2*pi^2)/4 + (3*a^2*b*pi)/
2), c ~= 0, a^3*x - b^3*x*(6*acos(c*x) - acos(c*x)^3) - (3*a^2*b*((- c^2*x
^2 + 1)^(1/2) - c*x*acos(c*x)))/c + 3*a*b^2*x*(acos(c*x)^2 - 2) - (b^3*(-
c^2*x^2 + 1)^(1/2)*(3*acos(c*x)^2 - 6))/c - (6*a*b^2*acos(c*x)*(- c^2*x^2
+ 1)^(1/2))/c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.82

$$\int (a+b\arccos(cx))^3\,dx$$
$$= \frac{acos(cx)^3 b^3 cx - 3\sqrt{-c^2x^2+1}\,acos(cx)^2 b^3 + 3acos(cx)^2 a b^2 cx - 6\sqrt{-c^2x^2+1}\,acos(cx)\,a b^2 + 3acos(cx}{c}$$

input
```
int((a+b*acos(c*x))^3,x)
```

output
```
(acos(c*x)**3*b**3*c*x - 3*sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**3 + 3*ac
os(c*x)**2*a*b**2*c*x - 6*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b**2 + 3*acos
(c*x)*a**2*b*c*x - 6*acos(c*x)*b**3*c*x - 3*sqrt( - c**2*x**2 + 1)*a**2*b
+ 6*sqrt( - c**2*x**2 + 1)*b**3 + a**3*c*x - 6*a*b**2*c*x)/c
```

3.156 $\int \frac{(a+b\arccos(cx))^3}{x}\,dx$

Optimal result 1096
Mathematica [A] (verified) 1097
Rubi [A] (verified) 1097
Maple [B] (verified) 1100
Fricas [F] 1101
Sympy [F] 1101
Maxima [F] 1102
Giac [F(-2)] 1102
Mupad [F(-1)] 1102
Reduce [F] 1103

Optimal result

Integrand size = 14, antiderivative size = 127

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^3}{x}\,dx = &-\frac{i(a+b\arccos(cx))^4}{4b} \\ &+ (a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right) \\ &- \frac{3}{2}ib(a+b\arccos(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) \\ &+ \frac{3}{2}b^2(a+b\arccos(cx))\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) \\ &+ \frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-1/4*I*(a+b*arccos(c*x))^4/b+(a+b*arccos(c*x))^3*ln(1+(c*x+I*(-c^2*x^2+1)^
(1/2))^2)-3/2*I*b*(a+b*arccos(c*x))^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)+3/2*b^2*(a+b*arccos(c*x))*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/
4*I*b^3*polylog(4,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \frac{1}{4}\Big(-6ia^2b\arccos(cx)^2 - 4iab^2\arccos(cx)^3 - ib^3\arccos(cx)^4 + 12a^2b\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + 12ab^2\arccos(cx)^2\log\left(1+e^{2i\arccos(cx)}\right) + 4b^3\arccos(cx)^3\log\left(1+e^{2i\arccos(cx)}\right) + 4a^3\log(cx) - 6ib(a+b\arccos(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + 6b^2(a+b\arccos(cx))\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) + 3ib^3\operatorname{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/x,x]
```

output
```
((-6*I)*a^2*b*ArcCos[c*x]^2 - (4*I)*a*b^2*ArcCos[c*x]^3 - I*b^3*ArcCos[c*x
]^4 + 12*a^2*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 12*a*b^2*ArcCo
s[c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])] + 4*b^3*ArcCos[c*x]^3*Log[1 + E^((
2*I)*ArcCos[c*x])] + 4*a^3*Log[c*x] - (6*I)*b*(a + b*ArcCos[c*x])^2*PolyLo
g[2, -E^((2*I)*ArcCos[c*x])] + 6*b^2*(a + b*ArcCos[c*x])*PolyLog[3, -E^((2
*I)*ArcCos[c*x])] + (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcCos[c*x])])/4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5137, 3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{5137}$$

$$-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{cx}d\arccos(cx)$$

$$\Big\downarrow \; 3042$$

$$-\int (a+b\arccos(cx))^3\tan(\arccos(cx))d\arccos(cx)$$

$$\Big\downarrow \; 4202$$

$$2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^3}{1+e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^4}{4b}$$

$$\Big\downarrow \; 2620$$

$$2i\left(\frac{3}{2}ib\int (a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^3\right)-\frac{i(a+b\arccos(cx))^4}{4b}$$

$$\Big\downarrow \; 3011$$

$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\int (a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\right.\right.$$
$$\frac{i(a+b\arccos(cx))^4}{4b}$$

$$\Big\downarrow \; 7163$$

$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}ib\int \operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)d\arccos(cx)-\right.\right.\right.$$
$$\frac{i(a+b\arccos(cx))^4}{4b}$$

$$\Big\downarrow \; 2720$$

$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)de\right.\right.\right.$$
$$\frac{i(a+b\arccos(cx))^4}{4b}$$

$$\Big\downarrow \; 7143$$

$$2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{4}b\operatorname{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)-\frac{1}{2}i\operatorname{PolyLog}\left(3,\right.\right.\right.\right.$$
$$\frac{i(a+b\arccos(cx))^4}{4b}$$

input
```
Int[(a + b*ArcCos[c*x])^3/x,x]
```

output
```
((-1/4*I)*(a + b*ArcCos[c*x])^4)/b + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])^3
*Log[1 + E^((2*I)*ArcCos[c*x])] + ((3*I)/2)*b*((I/2)*(a + b*ArcCos[c*x])^2
*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - I*b*((-1/2*I)*(a + b*ArcCos[c*x])*Po
lyLog[3, -E^((2*I)*ArcCos[c*x])] + (b*PolyLog[4, -E^((2*I)*ArcCos[c*x])])/
4)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(160) = 320.

Time = 0.40 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.56

method	result
parts	$a^3 \ln(x) + b^3 \left(-\frac{i \arccos(cx)^4}{4} + \arccos(cx)^3 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i \arccos(cx)}{}\right.$
derivativedivides	$a^3 \ln(cx) + b^3 \left(-\frac{i \arccos(cx)^4}{4} + \arccos(cx)^3 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i \arccos(c}{}\right.$
default	$a^3 \ln(cx) + b^3 \left(-\frac{i \arccos(cx)^4}{4} + \arccos(cx)^3 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i \arccos(c}{}\right.$

input
```
int((a+b*arccos(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(-1/4*I*arccos(c*x)^4+arccos(c*x)^3*ln(1+(c*x+I*(-c^2*x^2+1)
^(1/2))^2)-3/2*I*arccos(c*x)^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/
2*arccos(c*x)*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/4*I*polylog(4,-(c
*x+I*(-c^2*x^2+1)^(1/2))^2))+3*a*b^2*(-1/3*I*arccos(c*x)^3+arccos(c*x)^2*l
n(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^
2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))+3*a^2*b*(-1/2
*I*arccos(c*x)^2+arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^3}{x}\,dx$$

input
```
integrate((a+b*acos(c*x))**3/x,x)
```

output
```
Integral((a + b*acos(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + integrate((b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 +
 3*a*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 3*a^2*b*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^3/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^3}{x}\,dx$$

input
```
int((a + b*acos(c*x))^3/x,x)
```

output
```
int((a + b*acos(c*x))^3/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = 3\left(\int \frac{acos(cx)}{x}dx\right)a^2b + \left(\int \frac{acos(cx)^3}{x}dx\right)b^3 + 3\left(\int \frac{acos(cx)^2}{x}dx\right)a\,b^2 + \log(x)\,a^3$$

input
```
int((a+b*acos(c*x))^3/x,x)
```

output
```
3*int(acos(c*x)/x,x)*a**2*b + int(acos(c*x)**3/x,x)*b**3 + 3*int(acos(c*x)
**2/x,x)*a*b**2 + log(x)*a**3
```

3.157 $\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx$

Optimal result . . . 1104
Mathematica [B] (verified) . . . 1105
Rubi [A] (verified) . . . 1106
Maple [B] (verified) . . . 1108
Fricas [F] . . . 1109
Sympy [F] . . . 1109
Maxima [F] . . . 1110
Giac [F(-2)] . . . 1110
Mupad [F(-1)] . . . 1110
Reduce [F] . . . 1111

Optimal result

Integrand size = 14, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = &-\frac{(a+b\arccos(cx))^3}{x}\\ &-6ibc(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)\\ &+6ib^2c(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\\ &-6ib^2c(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\\ &-6b^3c\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)\\ &+6b^3c\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)\end{aligned}$$

output
```
-(a+b*arccos(c*x))^3/x-6*I*b*c*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+
1)^(1/2))+6*I*b^2*c*(a+b*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/
2)))-6*I*b^2*c*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6
*b^3*c*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+6*b^3*c*polylog(3,I*(c*x+I
*(-c^2*x^2+1)^(1/2)))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 308 vs. 2(151) = 302.

Time = 0.19 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.04

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx$$
$$= -\frac{a^3}{x} - \frac{3a^2b\arccos(cx)}{x} - 3a^2bc\log(x) + 3a^2bc\log\left(1+\sqrt{1-c^2x^2}\right)$$
$$+ 3ab^2c\Bigg(-\frac{\arccos(cx)^2}{cx} + 2\big(\arccos(cx)\left(\log\left(1-ie^{i\arccos(cx)}\right) - \log\left(1+ie^{i\arccos(cx)}\right)\right)$$
$$+ i\big(\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) - \operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\big)\big)\Bigg)$$
$$+ b^3c\Bigg(-\frac{\arccos(cx)^3}{cx} + 3\big(\arccos(cx)^2\left(\log\left(1-ie^{i\arccos(cx)}\right) - \log\left(1+ie^{i\arccos(cx)}\right)\right)$$
$$+ 2i\arccos(cx)\left(\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) - \operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)$$
$$- 2\big(\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right) - \operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)\big)\big)\Bigg)$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/x^2,x]
```

output
```
-(a^3/x) - (3*a^2*b*ArcCos[c*x])/x - 3*a^2*b*c*Log[x] + 3*a^2*b*c*Log[1 +
Sqrt[1 - c^2*x^2]] + 3*a*b^2*c*(-(ArcCos[c*x]^2/(c*x)) + 2*(ArcCos[c*x]*(L
og[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c*x])]) + I*(PolyLog[2
, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*ArcCos[c*x])]))) + b^3*c*(-(
ArcCos[c*x]^3/(c*x)) + 3*(ArcCos[c*x]^2*(Log[1 - I*E^(I*ArcCos[c*x])] - Lo
g[1 + I*E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, (-I)*E^(I*ArcC
os[c*x])] - PolyLog[2, I*E^(I*ArcCos[c*x])]) - 2*(PolyLog[3, (-I)*E^(I*Arc
Cos[c*x])] - PolyLog[3, I*E^(I*ArcCos[c*x])])))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5139, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{x^2}dx$$

$$\Big\downarrow \text{5139}$$

$$-3bc\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^3}{x}$$

$$\Big\downarrow \text{5219}$$

$$3bc\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)-\frac{(a+b\arccos(cx))^3}{x}$$

$$\Big\downarrow \text{3042}$$

$$3bc\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-\frac{(a+b\arccos(cx))^3}{x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{(a+b\arccos(cx))^3}{x}+$$
$$3bc\Bigg(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+ie^{i\arccos(cx)}\right)d$$

$$\Big\downarrow \text{3011}$$

$$-\frac{(a+b\arccos(cx))^3}{x}+$$
$$3bc\Bigg(2b\Bigg(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\Bigg)-2b\Bigg(i$$

$$\Big\downarrow \text{2720}$$

$$-\frac{(a+b\arccos(cx))^3}{x}+$$
$$3bc\Bigg(2b\Bigg(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{(a+b\arccos(cx))^3}{x}+$$
$$3bc\Big(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\Big(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\operatorname{PolyLo}$$

input
```
Int[(a + b*ArcCos[c*x])^3/x^2,x]
```

output
```
-((a + b*ArcCos[c*x])^3/x) + 3*b*c*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^
(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[
c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x]
)*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])]))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(189) = 378$.

Time = 0.80 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.68

method	result
derivativedivides	$c\left(-\frac{b^3\arccos(cx)^3+3a\,b^2\arccos(cx)^2+3\arccos(cx)a^2b+a^3}{cx}-3\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)b^3\,\mathrm{arc}\right.$
default	$c\left(-\frac{b^3\arccos(cx)^3+3a\,b^2\arccos(cx)^2+3\arccos(cx)a^2b+a^3}{cx}-3\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)b^3\,\mathrm{arc}\right.$

input
```
int((a+b*arccos(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(-(b^3*arccos(c*x)^3+3*a*b^2*arccos(c*x)^2+3*arccos(c*x)*a^2*b+a^3)/c/x-
3*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^3*arccos(c*x)^2+3*ln(1-I*(c*x+I*(-c
^2*x^2+1)^(1/2)))*b^3*arccos(c*x)^2-6*a*b^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^
2*x^2+1)^(1/2)))+6*a*b^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+6*
I*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^3*arccos(c*x)-6*polylog(3,-I*
(c*x+I*(-c^2*x^2+1)^(1/2)))*b^3-6*I*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2))
)*b^3*arccos(c*x)+6*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^3+6*I*polylo
g(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b^2-6*I*polylog(2,I*(c*x+I*(-c^2*x^2+
1)^(1/2)))*a*b^2-6*I*arctan(c*x+I*(-c^2*x^2+1)^(1/2))*a^2*b)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx=\int \frac{(b\arccos(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**3/x**2,x)
```

output
```
Integral((a + b*acos(c*x))**3/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx=\int \frac{(b\arccos(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
3*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a^2*b -
a^3/x - (b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 - x*integrate(3*
(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^3*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x)^2 + (a*b^2*c^2*x^2 - a*b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x)^2)/(c^2*x^4 - x^2), x))/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^3}{x^2}\,dx$$

input
```
int((a + b*acos(c*x))^3/x^2,x)
```

output
```
int((a + b*acos(c*x))^3/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx$$
$$= \frac{-3acos(cx)\,a^2b+\left(\int \frac{acos(cx)^3}{x^2}dx\right)b^3x+3\left(\int \frac{acos(cx)^2}{x^2}dx\right)a\,b^2x-3\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2bcx-a^3}{x}$$

input
```
int((a+b*acos(c*x))^3/x^2,x)
```

output
```
( - 3*acos(c*x)*a**2*b + int(acos(c*x)**3/x**2,x)*b**3*x + 3*int(acos(c*x)
**2/x**2,x)*a*b**2*x - 3*log(tan(asin(c*x)/2))*a**2*b*c*x - a**3)/x
```

3.158 $\int \frac{x^2}{a+b\arccos(cx)}\,dx$

Optimal result 1112
Mathematica [A] (verified) 1113
Rubi [A] (verified) 1113
Maple [A] (verified) 1115
Fricas [F] 1115
Sympy [F] 1115
Maxima [F] 1116
Giac [A] (verification not implemented) 1116
Mupad [F(-1)] 1117
Reduce [F] 1117

Optimal result

Integrand size = 14, antiderivative size = 121

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}$$

output
```
1/4*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^3+1/4*Ci(3*(a+b*arccos(c*x))/b)*s
in(3*a/b)/b/c^3-1/4*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c^3-1/4*cos(3*a/b)*
Si(3*(a+b*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = -\frac{-\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)+\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+a\right.}{4bc^3}$$

input
```
Integrate[x^2/(a + b*ArcCos[c*x]),x]
```

output
```
-1/4*(-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + Ar
cCos[c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Cos[(
3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])])/(b*c^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc^3}$$

input
```
Int[x^2/(a + b*ArcCos[c*x]),x]
```

output
```
-((-1/4*(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) - (CosIntegral[(3*(a
 + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/4 + (Cos[a/b]*SinIntegral[(a + b*ArcCo
s[c*x])/b])/4 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4)/(
b*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result	s
derivativedivides	$\frac{-\frac{\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{4b}}{c^3}$	1
default	$\frac{-\frac{\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{4b}}{c^3}$	1

input
```
int(x^2/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-1/4*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)/b+1/4*Ci(3*arccos(c*x)+3*a/
b)*sin(3*a/b)/b-1/4*Si(arccos(c*x)+a/b)*cos(a/b)/b+1/4*Ci(arccos(c*x)+a/b)
*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx=\int \frac{x^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(x^2/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx=\int \frac{x^2}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate(x**2/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \int \frac{x^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{x^2}{a+b\arccos(cx)}\,dx = & \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc^3} \\ & - \frac{\cos\left(\frac{a}{b}\right)^3 \operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc^3} \\ & - \frac{\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ & + \frac{\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ & + \frac{3\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^3} \\ & - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc^3}\end{aligned}$$

input
```
integrate(x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) - cos(a/b)
^3*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*cos_integral(3*a/b +
3*arccos(c*x))*sin(a/b)/(b*c^3) + 1/4*cos_integral(a/b + arccos(c*x))*sin(
a/b)/(b*c^3) + 3/4*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^3) -
1/4*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx=\int \frac{x^2}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int(x^2/(a + b*acos(c*x)),x)
```

output
```
int(x^2/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx=\int \frac{x^2}{acos\left(cx\right)b+a}dx$$

input
```
int(x^2/(a+b*acos(c*x)),x)
```

output
```
int(x**2/(acos(c*x)*b + a),x)
```

3.159 $\int \frac{x}{a+b\arccos(cx)}\,dx$

Optimal result . . . 1118
Mathematica [A] (verified) . . . 1118
Rubi [A] (verified) . . . 1119
Maple [A] (verified) . . . 1121
Fricas [F] . . . 1122
Sympy [F] . . . 1122
Maxima [F] . . . 1123
Giac [A] (verification not implemented) . . . 1123
Mupad [F(-1)] . . . 1123
Reduce [F] . . . 1124

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{x}{a+b\arccos(cx)}\,dx = \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{2bc^2} - \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^2}$$

output
```
1/2*Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b/c^2-1/2*cos(2*a/b)*Si(2*(a+b*ar
ccos(c*x))/b)/b/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+b\arccos(cx)}\,dx = -\frac{-\text{CosIntegral}\left(\frac{2a}{b}+2\arccos(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2bc^2}$$

input
```
Integrate[x/(a + b*ArcCos[c*x]),x]
```

output
```
-1/2*(-(CosIntegral[(2*a)/b + 2*ArcCos[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*
SinIntegral[(2*a)/b + 2*ArcCos[c*x]])/(b*c^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5147, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\arccos(cx)}dx$$
$$\Big\downarrow \textbf{5147}$$
$$-\frac{\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$
$$\Big\downarrow \textbf{25}$$
$$\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$
$$\Big\downarrow \textbf{4906}$$
$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}d(a+b\arccos(cx))}{bc^2}$$
$$\Big\downarrow \textbf{27}$$
$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{2bc^2}$$
$$\Big\downarrow \textbf{3042}$$
$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{2bc^2}$$
$$\Big\downarrow \textbf{3784}$$

$$-\frac{-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^2}$$

input
```
Int[x/(a + b*ArcCos[c*x]),x]
```

output
```
-1/2*(-(CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b
]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/(b*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{2b}}{c^{2}}$	58
default	$\frac{-\frac{\operatorname{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{2b}}{c^{2}}$	58

input
```
int(x/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)/b+1/2*Ci(2*arccos(c*x)+2*a/
b)*sin(2*a/b)/b)
```

Fricas [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(x/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{a+b\operatorname{acos}{(cx)}}\,dx$$

input
```
integrate(x/(a+b*acos(c*x)),x)
```

output
```
Integral(x/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\frac{\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc^2}-\frac{\cos\left(\frac{a}{b}\right)^2\operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^2}+\frac{\operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2bc^2}$$

input
```
integrate(x/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b*c^2) - cos(a/b)^2
*sin_integral(2*a/b + 2*arccos(c*x))/(b*c^2) + 1/2*sin_integral(2*a/b + 2*
arccos(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{acos}(c\,x)}\,dx$$

input
```
int(x/(a + b*acos(c*x)),x)
```

output
```
int(x/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{acos\,(cx)\,b+a}dx$$

input
```
int(x/(a+b*acos(c*x)),x)
```

output
```
int(x/(acos(c*x)*b + a),x)
```

3.160 $\int \frac{1}{a+b\arccos(cx)}\,dx$

Optimal result . . . 1125
Mathematica [A] (verified) . . . 1125
Rubi [A] (verified) . . . 1126
Maple [A] (verified) . . . 1128
Fricas [F] . . . 1128
Sympy [F] . . . 1129
Maxima [F] . . . 1129
Giac [A] (verification not implemented) . . . 1129
Mupad [F(-1)] . . . 1130
Reduce [F] . . . 1130

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

output
```
Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c-cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\arccos(cx)}\,dx = -\frac{-\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-1),x]
```

output
```
-((-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b +
 ArcCos[c*x]])/(b*c))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5135, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\arccos(cx)}dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \; 3783$$

$$-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcCos[c*x])^(-1),x]
```

output
```
-((-(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) + Cos[a/b]*SinIntegral[(
a + b*ArcCos[c*x])/b])/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49
default	$\frac{-\frac{\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49

input
```
int(1/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-Si(arccos(c*x)+a/b)*cos(a/b)/b+Ci(arccos(c*x)+a/b)*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{b\arccos(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate(1/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{b\arccos(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \frac{\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - cos(a/b)*sin_integral(a/b
 + arccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b \arccos(cx)} \, dx = \int \frac{1}{a + b \operatorname{acos}(c\,x)} \, dx$$

input
```
int(1/(a + b*acos(c*x)),x)
```

output
```
int(1/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{1}{a + b \arccos(cx)} \, dx = \int \frac{1}{acos(cx)\,b + a} dx$$

input
```
int(1/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b + a),x)
```

3.161 $\int \frac{1}{x(a+b\arccos(cx))}\,dx$

Optimal result 1131
Mathematica [N/A] 1131
Rubi [N/A] 1132
Maple [N/A] 1132
Fricas [N/A] 1133
Sympy [N/A] 1133
Maxima [N/A] 1133
Giac [F(-2)] 1134
Mupad [N/A] 1134
Reduce [N/A] 1135

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx=\text{Int}\left(\frac{1}{x(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx=\int \frac{1}{x(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5149}$$
$$\int \frac{1}{x(a+b\arccos(cx))}dx$$

input
```
Int[1/(x*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(1/x/(a+b*arccos(c*x)),x)
```

output
```
int(1/x/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*arccos(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{x\left(a+b\operatorname{acos}{(cx)}\right)}\,dx$$

input
```
integrate(1/x/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))),x)
```

output
```
int(1/(x*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx=\int \frac{1}{acos\,(cx)\,bx+ax}dx$$

input
```
int(1/x/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*x + a*x),x)
```

3.162 $\int \frac{1}{x^2(a+b\arccos(cx))}\,dx$

Optimal result 1136
Mathematica [N/A] 1136
Rubi [N/A] 1137
Maple [N/A] 1137
Fricas [N/A] 1138
Sympy [N/A] 1138
Maxima [N/A] 1138
Giac [N/A] 1139
Mupad [N/A] 1139
Reduce [N/A] 1140

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{1}{x^2(a+b\arccos(cx))}dx$$

input
```
Int[1/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\,(a+b\arccos\,(cx))}dx$$

input
```
int(1/x^2/(a+b*arccos(c*x)),x)
```

output
```
int(1/x^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx = \int \frac{1}{(b\arccos(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*arccos(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx = \int \frac{1}{x^2\,(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(1/x**2/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx = \int \frac{1}{(b\arccos(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{acos\,(cx)\,b\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*x**2 + a*x**2),x)
```

3.163 $\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result 1141
Mathematica [A] (verified) 1142
Rubi [A] (verified) 1142
Maple [A] (verified) 1143
Fricas [F] 1144
Sympy [F] 1144
Maxima [F] 1144
Giac [B] (verification not implemented) 1145
Mupad [F(-1)] 1146
Reduce [F] 1146

Optimal result

Integrand size = 14, antiderivative size = 155

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3} - \frac{\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3\sin\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}$$

output

```
x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-1/4*cos(a/b)*Ci((a+b*arccos(c
*x))/b)/b^2/c^3-3/4*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^3-1/4*sin(a
/b)*Si((a+b*arccos(c*x))/b)/b^2/c^3-3/4*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/
b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = -\frac{-\frac{4bc^2x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}+\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)+}{4b^2c^3}$$

input
```
Integrate[x^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/4*((-4*b*c^2*x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) + Cos[a/b]*CosI
ntegral[a/b + ArcCos[c*x]] + 3*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*
x])] + Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 3*Sin[(3*a)/b]*SinIntegra
l[3*(a/b + ArcCos[c*x])])/(b^2*c^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{ 5143}$$

$$\frac{\int \left(-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{4}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{b^2c^3}$$
$$\frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

input
```
Int[x^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^2*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (-1/4*(Cos[a/b]*CosInt
egral[(a + b*ArcCos[c*x])/b]) - (3*Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcC
os[c*x]))/b])/4 - (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/4 - (3*Sin
[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4)/(b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\sin(3\arccos(cx))}{4(a+b\arccos(cx))b}-\frac{3\left(\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)\right)}{4b^2}+\frac{\sqrt{-c^2x^2+1}}{4(a+b\arccos(cx))b}-\frac{\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)}{}}{c^3}$
default	$\frac{\frac{\sin(3\arccos(cx))}{4(a+b\arccos(cx))b}-\frac{3\left(\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)\right)}{4b^2}+\frac{\sqrt{-c^2x^2+1}}{4(a+b\arccos(cx))b}-\frac{\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)}{}}{c^3}$

input
```
int(x^2/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/4*sin(3*arccos(c*x))/(a+b*arccos(c*x))/b-3/4*(Si(3*arccos(c*x)+3*
a/b)*sin(3*a/b)+Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b))/b^2+1/4*(-c^2*x^2+1)^(
1/2)/(a+b*arccos(c*x))/b-1/4*(Si(arccos(c*x)+a/b)*sin(a/b)+Ci(arccos(c*x)+
a/b)*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(x**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x) + a*b*c)*integrate((3*c^2*x^3 - 2*x)*sqrt(c*x + 1)*sqrt(-c*x +
 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
+ a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(145) = 290$.

Time = 0.16 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.97

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-3*b*arccos(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*a
rccos(c*x) + a*b^2*c^3) - 3*b*arccos(c*x)*cos(a/b)^2*sin(a/b)*sin_integral
(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2
+ 1)*b*c^2*x^2/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*cos(a/b)^3*cos_inte
gral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*cos(a/
b)^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a
*b^2*c^3) + 9/4*b*arccos(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))
/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*b*arccos(c*x)*cos(a/b)*cos_integr
al(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*b*arccos(c*x
)*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^
2*c^3) - 1/4*b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c
^3*arccos(c*x) + a*b^2*c^3) + 9/4*a*cos(a/b)*cos_integral(3*a/b + 3*arccos
(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*cos(a/b)*cos_integral(a/b
 + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*a*sin(a/b)*sin_int
egral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*sin
(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^2}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int(x^2/(a + b*acos(c*x))^2,x)
```

output
```
int(x^2/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^2}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx$$

input
```
int(x^2/(a+b*acos(c*x))^2,x)
```

output
```
int(x**2/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.164 $\int \frac{x}{(a+b\arccos(cx))^2}\,dx$

Optimal result 1147
Mathematica [A] (verified) 1147
Rubi [A] (verified) 1148
Maple [A] (verified) 1150
Fricas [F] 1151
Sympy [F] 1151
Maxima [F] 1152
Giac [B] (verification not implemented) 1152
Mupad [F(-1)] 1153
Reduce [F] 1153

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2}-\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2}$$

output
```
x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-cos(2*a/b)*Ci(2*(a+b*arccos(c*x
))/b)/b^2/c^2-sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{bcx\sqrt{1-c^2x^2}}{a+b\arccos(cx)}-\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-\sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)}{b^2c^2}$$

input
```
Integrate[x/(a + b*ArcCos[c*x])^2,x]
```

output
```
((b*c*x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[(2*a)/b]*CosIntegral[
2*(a/b + ArcCos[c*x])] - Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/
(b^2*c^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5143, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{\int -\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\sin\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$\downarrow$ 3042

$$\frac{-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$\downarrow$ 3780

$$\frac{-\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$\downarrow$ 3783

$$\frac{-\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2}+\frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

input
```
Int[x/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (-(Cos[(2*a)/b]*CosInteg
ral[(2*(a + b*ArcCos[c*x]))/b]) - Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCo
s[c*x]))/b])/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(cx))}{2(a+b\arccos(cx))b}-\frac{\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{b^2}}{c^2}$	78
default	$\frac{\frac{\sin(2\arccos(cx))}{2(a+b\arccos(cx))b}-\frac{\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{b^2}}{c^2}$	78

input
```
int(x/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*sin(2*arccos(c*x))/(a+b*arccos(c*x))/b-(Si(2*arccos(c*x)+2*a/b)
*sin(2*a/b)+Ci(2*arccos(c*x)+2*a/b)*cos(2*a/b))/b^2)
```

Fricas [F]

$$\int \frac{x}{(a + b \arccos(cx))^2} \, dx = \int \frac{x}{(b \arccos(cx) + a)^2} \, dx$$

input
```
integrate(x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{(a + b \arccos(cx))^2} \, dx = \int \frac{x}{(a + b \operatorname{acos}(cx))^2} \, dx$$

input
```
integrate(x/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(sqrt(c*x + 1)*sqrt(-c*x + 1)*x - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
 1), c*x) + a*b*c)*integrate((2*c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)/
(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*
b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(89) = 178.

Time = 0.15 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.55

$$\begin{aligned}\int \frac{x}{(a+b\arccos(cx))^2}\,dx = &-\frac{2\,b\arccos(cx)\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\\&-\frac{2\,b\arccos(cx)\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\\&-\frac{2\,a\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\\&-\frac{2\,a\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\\&+\frac{\sqrt{-c^2x^2+1}bcx}{b^3c^2\arccos(cx)+ab^2c^2}\\&+\frac{b\arccos(cx)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\\&+\frac{a\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\end{aligned}$$

input
```
integrate(x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-2*b*arccos(c*x)*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*a
rccos(c*x) + a*b^2*c^2) - 2*b*arccos(c*x)*cos(a/b)*sin(a/b)*sin_integral(2
*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*a*cos(a/b)^2*c
os_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*a
*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x
) + a*b^2*c^2) + sqrt(-c^2*x^2 + 1)*b*c*x/(b^3*c^2*arccos(c*x) + a*b^2*c^2
) + b*arccos(c*x)*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x)
 + a*b^2*c^2) + a*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x)
 + a*b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx=\int \frac{x}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int(x/(a + b*acos(c*x))^2,x)
```

output
```
int(x/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx=\int \frac{x}{acos\left(cx\right)^{2}b^{2}+2acos\left(cx\right)ab+a^{2}}dx$$

input
```
int(x/(a+b*acos(c*x))^2,x)
```

output
```
int(x/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.165 $\int \frac{1}{(a+b\arccos(cx))^2}\,dx$

Optimal result 1154
Mathematica [A] (verified) 1154
Rubi [A] (verified) 1155
Maple [A] (verified) 1157
Fricas [F] 1158
Sympy [F] 1158
Maxima [F] 1158
Giac [B] (verification not implemented) 1159
Mupad [F(-1)] 1160
Reduce [F] 1160

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

output
```
(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-cos(a/b)*Ci((a+b*arccos(c*x))/b)/
b^2/c-sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{b\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) - \sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-2),x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[a/b]*CosIntegral[a/b + Ar
cCos[c*x]] - Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5133, 5225, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx}{b}+\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3780

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

$\Big\downarrow$ 3783

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\mathrm{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

input
```
Int[(a + b*ArcCos[c*x])^(-2),x]
```

output
```
Sqrt[1 - c^2*x^2]/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(a + b
*ArcCos[c*x])/b] + Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74
default	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74

input
```
int(1/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))/b-(Si(arccos(c*x)+a/b)*sin(a/b)+
Ci(arccos(c*x)+a/b)*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**2,x)
```

output
```
Integral((a + b*acos(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^2)*integrate
(sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x +
 1))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{1}{(a+b\arccos(cx))^2}\,dx = &-\frac{b\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}\\&-\frac{b\arccos(cx)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}\\&-\frac{a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}\\&-\frac{a\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}+\frac{\sqrt{-c^2x^2+1}\,b}{b^3c\arccos(cx)+ab^2c}\end{aligned}$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*arccos(c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*arccos(c*x)
 + a*b^2*c) - b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*
c*arccos(c*x) + a*b^2*c) - a*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3
*c*arccos(c*x) + a*b^2*c) - a*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^
3*c*arccos(c*x) + a*b^2*c) + sqrt(-c^2*x^2 + 1)*b/(b^3*c*arccos(c*x) + a*b
^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acos(c*x))^2,x)
```

output
```
int(1/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx=\int \frac{1}{acos\left(cx\right)^{2}b^{2}+2acos\left(cx\right)ab+a^{2}}dx$$

input
```
int(1/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.166 $\int \frac{1}{x(a+b\arccos(cx))^2} dx$

Optimal result 1161
Mathematica [N/A] 1161
Rubi [N/A] 1162
Maple [N/A] 1162
Fricas [N/A] 1163
Sympy [N/A] 1163
Maxima [N/A] 1163
Giac [F(-2)] 1164
Mupad [N/A] 1164
Reduce [N/A] 1165

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \mathrm{Int}\left(\frac{1}{x(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 6.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\arccos(cx))^2} \, dx$$
$$\Big\downarrow \text{5149}$$
$$\int \frac{1}{x(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

input
```
int(1/x/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate(1/x/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 11.86

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^2 x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)*integrate
(sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*x^4 - a*b*c*x^2 + (b^2*c^3*x^4 - b^
2*c*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*s
qrt(-c*x + 1))/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c
*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^2),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{acos\left(cx\right)^2 b^2x+2acos\left(cx\right)abx+a^2x}dx$$

input
```
int(1/x/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x + a**2*x),x)
```

3.167 $\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$

Optimal result 1166
Mathematica [N/A] 1166
Rubi [N/A] 1167
Maple [N/A] 1167
Fricas [N/A] 1168
Sympy [N/A] 1168
Maxima [N/A] 1168
Giac [N/A] 1169
Mupad [N/A] 1169
Reduce [N/A] 1170

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 49.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx=\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{1}{x^2(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/x^2/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\,(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate(1/x**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x**2*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 12.93

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2)*integr
ate((c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*x^5 - a*b*c*x^3 +
(b^2*c^3*x^5 - b^2*c*x^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x)
+ sqrt(c*x + 1)*sqrt(-c*x + 1))/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x) + a*b*c*x^2)
```

Giac [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^2),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{acos\,(cx)^2\,b^2x^2+2acos\,(cx)\,ab\,x^2+a^2x^2}dx$$

input
```
int(1/x^2/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*x**2 + 2*acos(c*x)*a*b*x**2 + a**2*x**2),x)
```

3.168 $\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx$

Optimal result 1171
Mathematica [A] (verified) 1172
Rubi [A] (verified) 1172
Maple [A] (verified) 1177
Fricas [F] 1178
Sympy [F] 1178
Maxima [F] 1179
Giac [B] (verification not implemented) 1179
Mupad [F(-1)] 1180
Reduce [F] 1181

Optimal result

Integrand size = 14, antiderivative size = 197

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8b^3c^3} - \frac{9\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{8b^3c^3} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^3c^3} + \frac{9\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{8b^3c^3}$$

output
```
1/2*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^2-x/b^2/c^2/(a+b*arccos(c
*x))+3/2*x^3/b^2/(a+b*arccos(c*x))-1/8*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^
3/c^3-9/8*Ci(3*(a+b*arccos(c*x))/b)*sin(3*a/b)/b^3/c^3+1/8*cos(a/b)*Si((a+
b*arccos(c*x))/b)/b^3/c^3+9/8*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^3/c^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx$$
$$=\frac{\frac{4b^2x^2\sqrt{1-c^2x^2}}{c(a+b\arccos(cx))^2}-\frac{8bx}{c^2(a+b\arccos(cx))}+\frac{12bx^3}{a+b\arccos(cx)}-\frac{\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{c^3}-\frac{9\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin}{c^3}}{8b^3}$$

input
```
Integrate[x^2/(a + b*ArcCos[c*x])^3,x]
```

output
```
((4*b^2*x^2*Sqrt[1 - c^2*x^2])/(c*(a + b*ArcCos[c*x])^2) - (8*b*x)/(c^2*(a
 + b*ArcCos[c*x])) + (12*b*x^3)/(a + b*ArcCos[c*x]) - (CosIntegral[a/b + A
rcCos[c*x]]*Sin[a/b])/c^3 - (9*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a
)/b])/c^3 + (Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]])/c^3 + (9*Cos[(3*a)/b
]*SinIntegral[3*(a/b + ArcCos[c*x])])/c^3)/(8*b^3)
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.26, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5145, 5223, 5135, 25, 3042, 3784, 25, 3042, 3780, 3783, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx$$
$$\Big\downarrow \; 5145$$
$$-\frac{\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx}{bc}+\frac{3c\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$
$$\Big\downarrow \; 5223$$

$$\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int\frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}-\frac{\frac{x}{bc(a+b\arccos(cx))}-\frac{\int\frac{1}{a+b\arccos(cx)}dx}{bc}}{bc}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\frac{\int-\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int\frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{x}{bc(a+b\arccos(cx))}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int\frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{x}{bc(a+b\arccos(cx))}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int\frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\frac{-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int-\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int\frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 3780$$

$$-\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 3783$$

$$\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{x^2}{a+b\arccos(cx)}dx}{bc}\right)}{2b}-\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 5147$$

$$\frac{3c\left(\frac{3\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3}{bc(a+b\arccos(cx))}\right)}{2b}-\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}\right)}{2b}-\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{3c\left(\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^4}\right)}{2b}-\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{3c\left(\frac{3\left(-\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\,\text{ar}}{}\right.\right.}{b^2c^4}\right.}{2b}$$

$$-\frac{\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}}{bc}+\frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

input
```
Int[x^2/(a + b*ArcCos[c*x])^3,x]
```

output
```
(x^2*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - (x/(b*c*(a + b*Arc
Cos[c*x])) + (-(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) + Cos[a/b]*Si
nIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c^2))/(b*c) + (3*c*(x^3/(b*c*(a + b
*ArcCos[c*x])) + (3*(-1/4*(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) -
(CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/4 + (Cos[a/b]*SinInt
egral[(a + b*ArcCos[c*x])/b])/4 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcC
os[c*x]))/b])/4))/(b^2*c^4)))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\sin(3\arccos(cx))}{8(a+b\arccos(cx))^2 b}+\frac{\frac{9\arccos(cx)\,\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)b}{8}-\frac{9\arccos(cx)\,\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)b}{8}+\frac{9\ \mathrm{Si}\left(3\arccos(c\right.}{}}{(a+b\arccos(cx))b^3}$
default	$\frac{\sin(3\arccos(cx))}{8(a+b\arccos(cx))^2 b}+\frac{\frac{9\arccos(cx)\,\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)b}{8}-\frac{9\arccos(cx)\,\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)b}{8}+\frac{9\ \mathrm{Si}\left(3\arccos(c\right.}{}}{(a+b\arccos(cx))b^3}$

input
```
int(x^2/(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/8*sin(3*arccos(c*x))/(a+b*arccos(c*x))^2/b+3/8*(3*arccos(c*x)*Si(
3*arccos(c*x)+3*a/b)*cos(3*a/b)*b-3*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*si
n(3*a/b)*b+3*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a-3*Ci(3*arccos(c*x)+3*a/b
)*sin(3*a/b)*a+cos(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^3+1/8*(-c^2*x^2+1
)^(1/2)/(a+b*arccos(c*x))^2/b+1/8*(arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b
)*b-arccos(c*x)*Ci(arccos(c*x)+a/b)*sin(a/b)*b+Si(arccos(c*x)+a/b)*cos(a/b
)*a-Ci(arccos(c*x)+a/b)*sin(a/b)*a+c*x*b)/(a+b*arccos(c*x))/b^3)
```

Fricas [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^3}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral(x^2/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c
*x) + a^3), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}(cx))^3}\,dx$$

input
```
integrate(x**2/(a+b*acos(c*x))**3,x)
```

output
```
Integral(x**2/(a + b*acos(c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^3}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*(3*a*c^2*x^3 + sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x^2 - 2*a*x + (3*b*c^2
*x^3 - 2*b*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 2*(b^4*c^2*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)*integrate(1/2*(9*c^2*x^2 - 2)/(b^3*
c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2*c^2), x))/(b^4*c^2*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. $2(183) = 366$.

Time = 0.22 (sec) , antiderivative size = 1479, normalized size of antiderivative = 7.51

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
3/2*b^2*c^3*x^3*arccos(c*x)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*
x) + a^2*b^3*c^3) + 3/2*a*b*c^3*x^3/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*a
rccos(c*x) + a^2*b^3*c^3) - 9/2*b^2*arccos(c*x)^2*cos(a/b)^2*cos_integral(
3*a/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arcco
s(c*x) + a^2*b^3*c^3) + 9/2*b^2*arccos(c*x)^2*cos(a/b)^3*sin_integral(3*a/
b + 3*arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*
b^3*c^3) - 9*a*b*arccos(c*x)*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x)
)*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
 + 9*a*b*arccos(c*x)*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c
^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/2*sqrt(-c^2*
x^2 + 1)*b^2*c^2*x^2/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^
2*b^3*c^3) + 9/8*b^2*arccos(c*x)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin
(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 9/
2*a^2*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arc
cos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 1/8*b^2*arccos(c*x)^
2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^
4*c^3*arccos(c*x) + a^2*b^3*c^3) - 27/8*b^2*arccos(c*x)^2*cos(a/b)*sin_int
egral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c
*x) + a^2*b^3*c^3) + 9/2*a^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x)
)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/8...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx=\int \frac{x^2}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}\,dx$$

input
```
int(x^2/(a + b*acos(c*x))^3,x)
```

output
```
int(x^2/(a + b*acos(c*x))^3, x)
```

Reduce [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{acos\,(cx)^3\,b^3+3acos\,(cx)^2\,a\,b^2+3acos\,(cx)\,a^2b+a^3}dx$$

input
```
int(x^2/(a+b*acos(c*x))^3,x)
```

output
```
int(x**2/(acos(c*x)**3*b**3 + 3*acos(c*x)**2*a*b**2 + 3*acos(c*x)*a**2*b +
 a**3),x)
```

3.169 $\int \frac{x}{(a+b\arccos(cx))^3}\,dx$

Optimal result 1182
Mathematica [A] (verified) 1183
Rubi [A] (verified) 1183
Maple [A] (verified) 1188
Fricas [F] 1188
Sympy [F] 1189
Maxima [F] 1189
Giac [B] (verification not implemented) 1189
Mupad [F(-1)] 1190
Reduce [F] 1191

Optimal result

Integrand size = 12, antiderivative size = 130

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^3c^2} + \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^3c^2}$$

output
```
1/2*x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^2-1/2/b^2/c^2/(a+b*arccos(c
*x))+x^2/b^2/(a+b*arccos(c*x))-Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^3/c^
2+cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^3/c^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx$$
$$= \frac{\frac{b^2cx\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}+\frac{b(-1+2c^2x^2)}{a+b\arccos(cx)}-2\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right)+2\cos\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(c\right.\right.}{2b^3c^2}$$

input
```
Integrate[x/(a + b*ArcCos[c*x])^3,x]
```

output
```
((b^2*c*x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2 + (b*(-1 + 2*c^2*x^2))/
(a + b*ArcCos[c*x]) - 2*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] +
2*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(2*b^3*c^2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {5145, 5153, 5223, 5147, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx$$
$$\Big\downarrow \text{5145}$$
$$-\frac{\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx}{2bc}+\frac{c\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx}{b}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$
$$\Big\downarrow \text{5153}$$
$$\frac{c\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$
$$\Big\downarrow \text{5223}$$

$$\frac{c\left(\frac{x^2}{bc(a+b\arccos(cx))}-\frac{2\int\frac{x}{a+b\arccos(cx)}dx}{bc}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{5147}$$

$$\frac{c\left(\frac{2\int-\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(\frac{x^2}{bc(a+b\arccos(cx))}-\frac{2\int\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{c\left(\frac{x^2}{bc(a+b\arccos(cx))}-\frac{2\int\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}d(a+b\arccos(cx))}{b^2c^3}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(\frac{x^2}{bc(a+b\arccos(cx))}-\frac{\int\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\left(\frac{x^2}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{c\left(\frac{-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{c\left(\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c\left(\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{c\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{c\left(\frac{\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}\right)}{b}-\frac{1}{2b^2c^2(a+b\arccos(cx))}+\frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

input
```
Int[x/(a + b*ArcCos[c*x])^3,x]
```

output
```
(x*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - 1/(2*b^2*c^2*(a + b*
ArcCos[c*x])) + (c*(x^2/(b*c*(a + b*ArcCos[c*x])) + (-(CosIntegral[(2*(a +
 b*ArcCos[c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*Arc
Cos[c*x]))/b])/(b^2*c^3)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\sin(2\arccos(cx))}{4(a+b\arccos(cx))^2 b}+\frac{2\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)b-2\arccos(cx)\,\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)b+2\,\mathrm{Si}\left(2\arccos(cx)\right.}{2(a+b\arccos(cx))b^3}}{c^2}$
default	$\frac{\frac{\sin(2\arccos(cx))}{4(a+b\arccos(cx))^2 b}+\frac{2\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)b-2\arccos(cx)\,\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)b+2\,\mathrm{Si}\left(2\arccos(cx)\right.}{2(a+b\arccos(cx))b^3}}{c^2}$

input
```
int(x/(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*sin(2*arccos(c*x))/(a+b*arccos(c*x))^2/b+1/2*(2*arccos(c*x)*Si(
2*arccos(c*x)+2*a/b)*cos(2*a/b)*b-2*arccos(c*x)*Ci(2*arccos(c*x)+2*a/b)*si
n(2*a/b)*b+2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*a-2*Ci(2*arccos(c*x)+2*a/b
)*sin(2*a/b)*a+cos(2*arccos(c*x))*b)/(a+b*arccos(c*x))/b^3)
```

Fricas [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{(b\arccos(cx)+a)^3}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral(x/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x
) + a^3), x)
```

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(cx))^3}\,dx$$

input
```
integrate(x/(a+b*acos(c*x))**3,x)
```

output
```
Integral(x/(a + b*acos(c*x))**3, x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{(b\arccos(cx)+a)^3}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*(2*a*c^2*x^2 + sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + (2*b*c^2*x^2 - b)*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 4*(b^4*c^2*arctan2(sqrt(c*x +
 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
 1), c*x) + a^2*b^2*c^2)*integrate(x/(b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x) + a*b^2), x) - a)/(b^4*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^
2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(124) = 248$.

Time = 0.19 (sec) , antiderivative size = 860, normalized size of antiderivative = 6.62

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
b^2*c^2*x^2*arccos(c*x)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) +
 a^2*b^3*c^2) - 2*b^2*arccos(c*x)^2*cos(a/b)*cos_integral(2*a/b + 2*arccos
(c*x))*sin(a/b)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3
*c^2) + 2*b^2*arccos(c*x)^2*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))
/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + a*b*c^2
*x^2/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 4*a
*b*arccos(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^5*
c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 4*a*b*arccos(
c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2
 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 2*a^2*cos(a/b)*cos_integral(2*
a/b + 2*arccos(c*x))*sin(a/b)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(
c*x) + a^2*b^3*c^2) - b^2*arccos(c*x)^2*sin_integral(2*a/b + 2*arccos(c*x)
)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 2*a^2*
cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*
a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 1/2*sqrt(-c^2*x^2 + 1)*b^2*c*x/(b^5
*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 2*a*b*arccos
(c*x)*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4
*c^2*arccos(c*x) + a^2*b^3*c^2) - 1/2*b^2*arccos(c*x)/(b^5*c^2*arccos(c*x)
^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - a^2*sin_integral(2*a/b + 2*a
rccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx=\int \frac{x}{(a+b\,\mathrm{acos}\left(c\,x\right))^3}\,dx$$

input
```
int(x/(a + b*acos(c*x))^3,x)
```

output
```
int(x/(a + b*acos(c*x))^3, x)
```

Reduce [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{acos\,(cx)^3\,b^3+3acos\,(cx)^2\,a\,b^2+3acos\,(cx)\,a^2b+a^3}dx$$

input
```
int(x/(a+b*acos(c*x))^3,x)
```

output
```
int(x/(acos(c*x)**3*b**3 + 3*acos(c*x)**2*a*b**2 + 3*acos(c*x)*a**2*b + a*
*3),x)
```

3.170 $\int \frac{1}{(a+b\arccos(cx))^3}\,dx$

Optimal result . . . 1192
Mathematica [A] (verified) . . . 1193
Rubi [A] (verified) . . . 1193
Maple [A] (verified) . . . 1197
Fricas [F] . . . 1197
Sympy [F] . . . 1198
Maxima [F] . . . 1198
Giac [B] (verification not implemented) . . . 1198
Mupad [F(-1)] . . . 1200
Reduce [F] . . . 1200

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} + \frac{x}{2b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{2b^3c} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^3c}$$

output

```
1/2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^2+1/2*x/b^2/(a+b*arccos(c*x))
-1/2*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^3/c+1/2*cos(a/b)*Si((a+b*arccos(c*
x))/b)/b^3/c
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \frac{\frac{b\left(acx+b\sqrt{1-c^2x^2}+bcx\arccos(cx)\right)}{(a+b\arccos(cx))^2}-\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{2b^3c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-3),x]
```

output
```
((b*(a*c*x + b*Sqrt[1 - c^2*x^2] + b*c*x*ArcCos[c*x]))/(a + b*ArcCos[c*x])
^2 - CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + Cos[a/b]*SinIntegral[a/b +
ArcCos[c*x]])/(2*b^3*c)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5133, 5223, 5135, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{5223}$$

$$\frac{c\left(\frac{x}{bc(a+b\arccos(cx))}-\frac{\int \frac{1}{a+b\arccos(cx)}dx}{bc}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{5135}$$

$$\frac{c\left(\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{c\left(\frac{x}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{c\left(\frac{x}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 3784$$

$$\frac{c\left(\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{c\left(\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{c\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{c\left(\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}\right)}{2b}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

input
```
Int[(a + b*ArcCos[c*x])^(-3),x]
```

output
```
Sqrt[1 - c^2*x^2]/(2*b*c*(a + b*ArcCos[c*x])^2) + (c*(x/(b*c*(a + b*ArcCos
[c*x])) + (-(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) + Cos[a/b]*SinIn
tegral[(a + b*ArcCos[c*x])/b])/(b^2*c^2)))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\sqrt{-c^2x^2+1}}{2(a+b\arccos(cx))^2 b}+\frac{\arccos(cx)\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b-\arccos(cx)\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)a}{2(a+b\arccos(cx))b^3}}{c}$
default	$\frac{\frac{\sqrt{-c^2x^2+1}}{2(a+b\arccos(cx))^2 b}+\frac{\arccos(cx)\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b-\arccos(cx)\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)a}{2(a+b\arccos(cx))b^3}}{c}$

input
```
int(1/(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2/b+1/2*(arccos(c*x)*Si(arcc
os(c*x)+a/b)*cos(a/b)*b-arccos(c*x)*Ci(arccos(c*x)+a/b)*sin(a/b)*b+Si(arcc
os(c*x)+a/b)*cos(a/b)*a-Ci(arccos(c*x)+a/b)*sin(a/b)*a+c*x*b)/(a+b*arccos(
c*x))/b^3)
```

Fricas [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x
) + a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx))^3}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**3,x)
```

output
```
Integral((a + b*acos(c*x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos\,(cx)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*(b*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*c*x + sqrt(c*x +
 1)*sqrt(-c*x + 1)*b - 2*(b^4*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
^2 + 2*a*b^3*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c)*int
egrate(1/2/(b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2), x))/(
b^4*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(101) = 202$.

Time = 0.14 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.33

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = -\frac{b^2\arccos(cx)^2\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{b^2\arccos(cx)^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{b^2cx\arccos(cx)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} - \frac{ab\arccos(cx)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c} + \frac{ab\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c} + \frac{abcx}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} - \frac{a^2\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{a^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{\sqrt{-c^2x^2+1}\,b^2}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)}$$

input
```
integrate(1/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
-1/2*b^2*arccos(c*x)^2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^5*c*arc
cos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*b^2*arccos(c*x)^2*co
s(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*ar
ccos(c*x) + a^2*b^3*c) + 1/2*b^2*c*x*arccos(c*x)/(b^5*c*arccos(c*x)^2 + 2*
a*b^4*c*arccos(c*x) + a^2*b^3*c) - a*b*arccos(c*x)*cos_integral(a/b + arcc
os(c*x))*sin(a/b)/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c
) + a*b*arccos(c*x)*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos
(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*a*b*c*x/(b^5*c*arccos(c
*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) - 1/2*a^2*cos_integral(a/b + ar
ccos(c*x))*sin(a/b)/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3
*c) + 1/2*a^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^
2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*sqrt(-c^2*x^2 + 1)*b^2/(b^5*c
*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}\,dx$$

input
```
int(1/(a + b*acos(c*x))^3,x)
```

output
```
int(1/(a + b*acos(c*x))^3, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx=\int \frac{1}{acos\left(cx\right)^{3}b^{3}+3acos\left(cx\right)^{2}a\,b^{2}+3acos\left(cx\right)a^{2}b+a^{3}}dx$$

input
```
int(1/(a+b*acos(c*x))^3,x)
```

output
```
int(1/(acos(c*x)**3*b**3 + 3*acos(c*x)**2*a*b**2 + 3*acos(c*x)*a**2*b + a*
*3),x)
```

3.171 $\int \frac{1}{x(a+b\arccos(cx))^3}\,dx$

Optimal result 1201
Mathematica [N/A] 1201
Rubi [N/A] 1202
Maple [N/A] 1202
Fricas [N/A] 1203
Sympy [N/A] 1203
Maxima [N/A] 1203
Giac [F(-2)] 1204
Mupad [N/A] 1204
Reduce [N/A] 1205

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))^3},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arccos(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x(a+b\arccos(cx))^3}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCos[c*x])^3),x]
```

output
```
Integrate[1/(x*(a + b*ArcCos[c*x])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\arccos(cx))^3}dx$$
$$\Big\downarrow \text{5149}$$
$$\int \frac{1}{x(a+b\arccos(cx))^3}dx$$

input
```
Int[1/(x*(a + b*ArcCos[c*x])^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)^{3}}dx$$

input
```
int(1/x/(a+b*arccos(c*x))^3,x)
```

output
```
int(1/x/(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos(cx)+a)^3 x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 + 3*a^2*b*x*arcc
os(c*x) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x(a+b\,\mathrm{acos}(cx))^3}\,dx$$

input
```
integrate(1/x/(a+b*acos(c*x))**3,x)
```

output
```
Integral(1/(x*(a + b*acos(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 17.93

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos(cx)+a)^3 x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + b*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x) + 2*(b^4*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2
 + 2*a*b^3*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^
2*x^2)*integrate(1/(b^3*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
 + a*b^2*c^2*x^3), x) + a)/(b^4*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)^2 + 2*a*b^3*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
a^2*b^2*c^2*x^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^3),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx$$
$$=\int \frac{1}{acos\,(cx)^3\,b^3x+3acos\,(cx)^2\,a\,b^2x+3acos\,(cx)\,a^2bx+a^3x}dx$$

input
```
int(1/x/(a+b*acos(c*x))^3,x)
```

output
```
int(1/(acos(c*x)**3*b**3*x + 3*acos(c*x)**2*a*b**2*x + 3*acos(c*x)*a**2*b*
x + a**3*x),x)
```

3.172 $\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$

Optimal result 1206
Mathematica [N/A] 1206
Rubi [N/A] 1207
Maple [N/A] 1207
Fricas [N/A] 1208
Sympy [N/A] 1208
Maxima [N/A] 1208
Giac [N/A] 1209
Mupad [N/A] 1209
Reduce [N/A] 1210

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx=\text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^3},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arccos(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 21.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx=\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^3),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+b\arccos(cx))^3} \, dx$$

$$\Big\downarrow \ 5149$$

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}dx$$

input
```
Int[1/(x^2*(a + b*ArcCos[c*x])^3),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b\arccos\left(cx\right)\right)^3}dx$$

input
```
int(1/x^2/(a+b*arccos(c*x))^3,x)
```

output
```
int(1/x^2/(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^3x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^2*arccos(c*x)^3 + 3*a*b^2*x^2*arccos(c*x)^2 + 3*a^2*b*x^
2*arccos(c*x) + a^3*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{acos}{(cx)}\right)^3}\,dx$$

input
```
integrate(1/x**2/(a+b*acos(c*x))**3,x)
```

output
```
Integral(1/(x**2*(a + b*acos(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 284, normalized size of antiderivative = 20.29

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^3x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
-1/2*(a*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + (b*c^2*x^2 - 2*b)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 2*(b^4*c^2*x^3*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x) + a^2*b^2*c^2*x^3)*integrate(1/2*(c^2*x^2 - 6)/(b^3*c^2*x
^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2*c^2*x^4), x) - 2*a)/
(b^4*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*x^
3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2*x^3)
```

Giac [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx=\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^3x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*arccos(c*x) + a)^3*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx=\int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^3}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^3),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$$
$$=\int \frac{1}{acos\left(cx\right)^3 b^3x^2+3acos\left(cx\right)^2 a\,b^2x^2+3acos\left(cx\right)a^2b\,x^2+a^3x^2}dx$$

input
```
int(1/x^2/(a+b*acos(c*x))^3,x)
```

output
```
int(1/(acos(c*x)**3*b**3*x**2 + 3*acos(c*x)**2*a*b**2*x**2 + 3*acos(c*x)*a
**2*b*x**2 + a**3*x**2),x)
```

3.173 $\int x^2\sqrt{a+b\arccos(cx)}\,dx$

Optimal result . . . 1211
Mathematica [C] (verified) . . . 1212
Rubi [A] (verified) . . . 1212
Maple [A] (verified) . . . 1214
Fricas [F(-2)] . . . 1215
Sympy [F] . . . 1215
Maxima [F] . . . 1216
Giac [C] (verification not implemented) . . . 1216
Mupad [F(-1)] . . . 1217
Reduce [F] . . . 1218

Optimal result

Integrand size = 16, antiderivative size = 242

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx=\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c^3}-\frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{12c^3}$$

output
```
1/3*x^3*(a+b*arccos(c*x))^(1/2)-1/8*b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*Fres
nelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^3-1/72*b^(1/2)*6^
(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2
)/b^(1/2))/c^3-1/8*b^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b
*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/c^3-1/72*b^(1/2)*6^(1/2)*Pi^(1/2)*Fr
esnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx =$$

$$-\frac{ibe^{-\frac{3ia}{b}}\left(-9e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+9e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\right.}{72c^3\sqrt{a+b\arc}}$$

input
```
Integrate[x^2*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
((-1/72*I)*b*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[
3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcC
os[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqrt[((-I)
*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]) + E^(
((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*ArcC
os[c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5141, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{1}{6}bc\int \frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}$$

$$\Big\downarrow \; 3793$$

$$\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int \left(\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}+\frac{3\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{6c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-$$

$$\frac{\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin}{6c^3}$$

input
```
Int[x^2*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(x^3*Sqrt[a + b*ArcCos[c*x]])/3 - ((3*Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC
[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/6]*Co
s[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/2 + (3*
Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*
Sin[a/b])/2 + (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c
*x]])/Sqrt[b]]*Sin[(3*a)/b])/2)/(6*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.50

method	result
default	$-9\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)b+9\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\sqrt{2}\right.$

input
```
int(x^2*(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72/c^3/(a+b*arccos(c*x))^(1/2)*(-9*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*ar
ccos(c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b)*b+9*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2
)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/
b)*b-2^(1/2)*Pi^(1/2)*(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(3*a/b)*Fres
nelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2^(1/2)*
Pi^(1/2)*(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(3*a/b)*FresnelS(3*2^(1/2
)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+18*arccos(c*x)*cos(-(
a+b*arccos(c*x))/b+a/b)*b+18*cos(-(a+b*arccos(c*x))/b+a/b)*a+6*arccos(c*x)
*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*b+6*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a
)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + b\arccos(cx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \sqrt{a + b\arccos(cx)}\, dx = \int x^2 \sqrt{a + b\,\mathrm{acos}\,(cx)}\, dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{b\arccos(cx)+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)*x^2, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 1057, normalized size of antiderivative = 4.37

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqr
t(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/
((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) + 1/16*sqrt(2)*sqrt(pi)*b^2*er
f(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b
*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(
abs(b)))*c^3) + 1/8*I*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos
(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))
/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) + 1/16*sqrt(2)
*sqrt(pi)*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2
*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(
abs(b)) + b*sqrt(abs(b)))*c^3) - 1/4*I*sqrt(pi)*a*sqrt(b)*erf(-1/2*sqrt(6)
*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*s
qrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/abs(b))*c^3) + 1/24
*sqrt(pi)*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I
*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b +
 I*sqrt(6)*b^2/abs(b))*c^3) + 1/4*I*sqrt(pi)*a*sqrt(b)*erf(-1/2*sqrt(6)*sq
rt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt
(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b))*c^3) + 1/24*s
qrt(pi)*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*s
qrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b...
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx=\int x^2\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^(1/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx=\int\sqrt{acos\,(cx)\,b+a}\,x^2dx$$

input
```
int(x^2*(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)*x**2,x)
```

3.174 $\int x\sqrt{a+b\arccos(cx)}\,dx$

Optimal result 1219
Mathematica [A] (verified) 1220
Rubi [A] (verified) 1220
Maple [A] (verified) 1222
Fricas [F(-2)] 1223
Sympy [F] 1223
Maxima [F] 1223
Giac [C] (verification not implemented) 1224
Mupad [F(-1)] 1224
Reduce [F] 1225

Optimal result

Integrand size = 14, antiderivative size = 137

$$\int x\sqrt{a+b\arccos(cx)}\,dx = -\frac{\sqrt{a+b\arccos(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2} - \frac{\sqrt{b}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{8c^2}$$

output
```
-1/4*(a+b*arccos(c*x))^(1/2)/c^2+1/2*x^2*(a+b*arccos(c*x))^(1/2)-1/8*b^(1/
2)*Pi^(1/2)*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2)
)/c^2-1/8*b^(1/2)*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(
1/2))*sin(2*a/b)/c^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int x\sqrt{a+b\arccos(cx)}\,dx$$
$$=\frac{2\sqrt{a+b\arccos(cx)}\cos(2\arccos(cx))-\sqrt{b}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{b}\sqrt{\pi}\,\text{FresnelS}\left(2\sqrt{\ldots}\right.}{8c^2}$$

input
```
Integrate[x*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(2*Sqrt[a + b*ArcCos[c*x]]*Cos[2*ArcCos[c*x]] - Sqrt[b]*Sqrt[Pi]*Cos[(2*a)
/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])] - Sqrt[b]*Sqr
t[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b
])/(8*c^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5141, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{1}{4}bc\int \frac{x^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^2}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^2}$$

$\Big\downarrow$ 3793

$$\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\int \left(\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}+\frac{1}{2\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{4c^2}$$

$\Big\downarrow$ 2009

$$\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos(cx)}}{4c^2}$$

input
```
Int[x*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(x^2*Sqrt[a + b*ArcCos[c*x]])/2 - (Sqrt[a + b*ArcCos[c*x]] + (Sqrt[b]*Sqrt
[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]
)/2 + (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt
[Pi])]*Sin[(2*a)/b])/2)/(4*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.36

method	result
default	$\frac{-\sqrt{\pi}\sqrt{-\frac{1}{b}}\sqrt{a+b\arccos(cx)}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{-\frac{2}{b}}\,b}\right)\cos\left(\frac{2a}{b}\right)b+\sqrt{\pi}\sqrt{-\frac{1}{b}}\sqrt{a+b\arccos(cx)}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\sqrt{a+b\arccos(c}{\sqrt{\pi}\sqrt{-\frac{2}{b}}\,b}\right.}{8c^2\sqrt{a+b\arccos(cx)}}$

input
```
int(x*(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/c^2/(a+b*arccos(c*x))^(1/2)*(-Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*
cos(2*a/b)*b+Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(2*2^(1
/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(2*a/b)*b+2*arccos
(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b+2*cos(-2*(a+b*arccos(c*x))/b+2*a
/b)*a)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+b\arccos(cx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{a+b\arccos(cx)}\,dx=\int x\sqrt{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x\sqrt{a+b\arccos(cx)}\,dx=\int\sqrt{b\arccos(cx)+a}x\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)*x, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.27

$$\int x\sqrt{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/4*I*sqrt(pi)*a*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) + 1/
16*sqrt(pi)*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos
(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) + 1/4*I*sq
rt(pi)*a*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*
x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) + 1/16*sqrt(
pi)*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) +
a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) - 1/4*I*sqrt(pi)*
a*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)*sqrt(b)
/abs(b))*e^(-2*I*a/b)/(c^2*(sqrt(b) - I*b^(3/2)/abs(b))) + 1/4*I*sqrt(pi)*
a*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) + a)*sqrt(b)
/abs(b))*e^(2*I*a/b)/(sqrt(b)*c^2*(I*b/abs(b) + 1)) + 1/8*sqrt(b*arccos(c*
x) + a)*e^(2*I*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*e^(-2*I*arcc
os(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\arccos(cx)}\,dx = \int x\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int(x*(a + b*acos(c*x))^(1/2),x)
```

output
```
int(x*(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{acos\,(cx)\,b+a}\,xdx$$

input
```
int(x*(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)*x,x)
```

3.175 $\int \sqrt{a+b\arccos(cx)}\,dx$

Optimal result . . . 1226
Mathematica [C] (verified) . . . 1227
Rubi [A] (verified) . . . 1227
Maple [A] (verified) . . . 1230
Fricas [F(-2)] . . . 1231
Sympy [F] . . . 1231
Maxima [F] . . . 1231
Giac [C] (verification not implemented) . . . 1232
Mupad [F(-1)] . . . 1232
Reduce [F] . . . 1233

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int \sqrt{a+b\arccos(cx)}\,dx = x\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{c}$$

output
```
x*(a+b*arccos(c*x))^(1/2)-1/2*b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2
^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c-1/2*b^(1/2)*2^(1/2)*Pi^
(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/
c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \sqrt{a+b\arccos(cx)}\,dx =$$
$$-\frac{ibe^{-\frac{ia}{b}}\left(-\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{2c\sqrt{a+b\arccos(cx)}}$$

input
```
Integrate[Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
((-1/2*I)*b*(-(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*
ArcCos[c*x]))/b]) + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[
3/2, (I*(a + b*ArcCos[c*x]))/b]))/(c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5131, 5225, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\arccos(cx)}\,dx$$
$$\Big\downarrow \text{5131}$$
$$\frac{1}{2}bc\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+x\sqrt{a+b\arccos(cx)}$$
$$\Big\downarrow \text{5225}$$
$$x\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3787}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{25}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3785}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}$$

$$\Big\downarrow \text{3786}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{2\sin\left(\frac{a}{b}\right)\int \sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}$$

$$\Big\downarrow \text{3832}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}$$

$$\Big\downarrow \text{3833}$$

$$\frac{x\sqrt{a+b\arccos(cx)}-\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}$$

input
```
Int[Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelC[(Sqrt[2/
Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] + Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[
2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
default	$\frac{-\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)b+\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\cdots}\right.}{2c\sqrt{a+b\arccos(cx)}}$

input
```
int((a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/(a+b*arccos(c*x))^(1/2)*(-2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)/b)*b+2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a
/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2*
arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b+2*cos(-(a+b*arccos(c*x))/b+a/b
)*a)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b \arccos(cx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{a + b \operatorname{acos}(cx)}\, dx$$

input
```
integrate((a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{b \arccos(cx) + a}\, dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.39

$$\int \sqrt{a + b \arccos(cx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqr
t(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/
((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)*b^2*erf(-
1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs
(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
 + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e
^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi
)*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)
*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b))
+ b*sqrt(abs(b)))*c) + I*sqrt(pi)*a*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
+ a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^
(I*a/b)/(c*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)
*a*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sq
rt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs
(b)) + sqrt(2)*sqrt(abs(b)))) + 1/2*sqrt(b*arccos(c*x) + a)*e^(I*arccos(c*
x))/c + 1/2*sqrt(b*arccos(c*x) + a)*e^(-I*arccos(c*x))/c
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{a + b \operatorname{acos}(c\,x)}\, dx$$

input
```
int((a + b*acos(c*x))^(1/2),x)
```

output
```
int((a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{acos\,(cx)\,b+a}dx$$

input
```
int((a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)
```

3.176 $\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$

Optimal result 1234
Mathematica [N/A] 1234
Rubi [N/A] 1235
Maple [N/A] 1235
Fricas [F(-2)] 1236
Sympy [N/A] 1236
Maxima [N/A] 1236
Giac [N/A] 1237
Mupad [N/A] 1237
Reduce [N/A] 1238

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \text{Int}\left(\frac{\sqrt{a+b\arccos(cx)}}{x},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(1/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/x,x]
```

output
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}dx$$

input
```
Int[Sqrt[a + b*ArcCos[c*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\arccos\left(cx\right)}}{x}dx$$

input
```
int((a+b*arccos(c*x))^(1/2)/x,x)
```

output
```
int((a+b*arccos(c*x))^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b\arccos(cx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a + b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{a + b\,\mathrm{acos}\,(cx)}}{x}\,dx$$

input
```
integrate((a+b*acos(c*x))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*acos(c*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{b\arccos\,(cx) + a}}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx=\int \frac{\sqrt{b\arccos{(cx)}+a}}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx=\int \frac{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}{x}\,dx$$

input
```
int((a + b*acos(c*x))^(1/2)/x,x)
```

output
```
int((a + b*acos(c*x))^(1/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}}{x}dx$$

input
```
int((a+b*acos(c*x))^(1/2)/x,x)
```

output
```
int(sqrt(acos(c*x)*b + a)/x,x)
```

3.177 $\int \frac{\sqrt{a+b \arccos (c x)}}{x^2} \, dx$

Optimal result 1239
Mathematica [N/A] 1239
Rubi [N/A] 1240
Maple [N/A] 1240
Fricas [F(-2)] 1241
Sympy [N/A] 1241
Maxima [N/A] 1241
Giac [N/A] 1242
Mupad [N/A] 1242
Reduce [N/A] 1243

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{a+b \arccos (c x)}}{x^2} \, dx=\text{Int}\left(\frac{\sqrt{a+b \arccos (c x)}}{x^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(1/2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b \arccos (c x)}}{x^2} \, dx=\int \frac{\sqrt{a+b \arccos (c x)}}{x^2} \, dx$$

input
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/x^2,x]
```

output
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2} \, dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}dx$$

input
```
Int[Sqrt[a + b*ArcCos[c*x]]/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\arccos\left(cx\right)}}{x^2}dx$$

input
```
int((a+b*arccos(c*x))^(1/2)/x^2,x)
```

output
```
int((a+b*arccos(c*x))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b \arccos(cx)}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a + b \arccos(cx)}}{x^2} \, dx = \int \frac{\sqrt{a + b \operatorname{acos}(cx)}}{x^2} \, dx$$

input
```
integrate((a+b*acos(c*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*acos(c*x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b \arccos(cx)}}{x^2} \, dx = \int \frac{\sqrt{b \arccos(cx) + a}}{x^2} \, dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx=\int \frac{\sqrt{b\arccos\left(cx\right)+a}}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx=\int \frac{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}{x^2}\,dx$$

input
```
int((a + b*acos(c*x))^(1/2)/x^2,x)
```

output
```
int((a + b*acos(c*x))^(1/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx=\int \frac{\sqrt{acos\,(cx)\,b+a}}{x^2}dx$$

input
```
int((a+b*acos(c*x))^(1/2)/x^2,x)
```

output
```
int(sqrt(acos(c*x)*b + a)/x**2,x)
```

3.178 $\int x^2(a+b\arccos(cx))^{3/2}\,dx$

Optimal result 1244
Mathematica [C] (verified) 1245
Rubi [A] (verified) 1246
Maple [B] (verified) 1253
Fricas [F(-2)] 1254
Sympy [F] 1254
Maxima [F] 1254
Giac [C] (verification not implemented) 1255
Mupad [F(-1)] 1256
Reduce [F] 1256

Optimal result

Integrand size = 16, antiderivative size = 313

$$\begin{aligned}\int x^2(a+b\arccos(cx))^{3/2}\,dx &= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} \\ &- \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c} \\ &+ \frac{1}{3}x^3(a+b\arccos(cx))^{3/2} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3} \\ &+ \frac{b^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3} \\ &- \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{8c^3} \\ &- \frac{b^{3/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{24c^3}\end{aligned}$$

output
```
-1/3*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c^3-1/6*b*x^2*(-c^2*x^2+
1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c+1/3*x^3*(a+b*arccos(c*x))^(3/2)+3/16*b^
(3/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b^(1/2))/c^3+1/144*b^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^
(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^3-3/16*b^(3/2)*2^(1/2)*P
i^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b
)/c^3-1/144*b^(3/2)*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos
(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.77

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx =$$

$$-\frac{iabe^{-\frac{3ia}{b}}\left(-9e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+9e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\sqrt{\cdots}\right.}{72c^3\sqrt{a+b\arccos\cdots}}$$

$$-\frac{\sqrt{b}\left(18\sqrt{b}\sqrt{a+b\arccos(cx)}\left(3\sqrt{1-c^2x^2}-2cx\arccos(cx)\right)-9\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)(3b\cos\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
((-1/72*I)*a*b*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamm
a[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*Ar
cCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqrt[((-
I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]) + E
^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*Ar
cCos[c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]]) - (Sqrt[b]
*(18*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*ArcCos[c
*x]) - 9*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]
*(3*b*Cos[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a
+ b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b]) - Sqrt[6*Pi]*Fres
nelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(b*Cos[(3*a)/b] + 2*a*S
in[(3*a)/b]) - Sqrt[6*Pi]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sq
rt[b]]*(2*a*Cos[(3*a)/b] - b*Sin[(3*a)/b]) + 6*Sqrt[b]*Sqrt[a + b*ArcCos[c
*x]]*(-2*ArcCos[c*x]*Cos[3*ArcCos[c*x]] + Sin[3*ArcCos[c*x]])))/(144*c^3)
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.35, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {5141, 5211, 5147, 25, 4906, 2009, 5183, 5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{1}{2}bc\int \frac{x^3\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{2}bc\left(\frac{2\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{6c}-\frac{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5147}$$

$$\frac{1}{2}bc\left(\frac{\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^4}+\frac{2\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}\sqrt{a-}}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 25

$$\frac{1}{2}bc\left(-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^4}+\frac{2\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}\sqrt{a-}}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 4906

$$\frac{1}{2}bc\left(-\frac{\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{6c^4}+\frac{2\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-}}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 2009

$$\frac{1}{2}bc\left(\frac{2\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{3c^2}+\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\sqrt{}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 5183

$$\frac{1}{2}bc\left(\frac{2\left(-\frac{b\int \frac{1}{\sqrt{a+b\arccos(cx)}}dx}{2c}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}+\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{}}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5135}$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}+\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b}}{\sqrt{}}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}bc\left(\frac{2\left(-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}+\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b}}{\sqrt{}}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}bc\left(\frac{2\left(-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}+\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b}}{\sqrt{}}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3787}$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int-\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right.}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\downarrow 25$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\downarrow 3042$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right.}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\downarrow 3785$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3786}$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{2\cos\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{1}{2}bc\left(\frac{2\left(\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{2}bc\left(\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\cdots}{6c^4}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

input
```
Int[x^2*(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^(3/2))/3 + (b*c*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*Sqrt
[a + b*ArcCos[c*x]])/c^2 + (2*(-((Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]
])/c^2) + (Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*Arc
Cos[c*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*A
rcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c^2)))/(3*c^2) + ((Sqrt[b]*Sqrt[Pi/2]*C
os[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/2 + (Sqrt[
b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/S
qrt[b]])/2 - (Sqrt[b]*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*
x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[
a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2)/(6*c^4)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(241) = 482.

Time = 0.24 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.75

method	result
default	$\frac{-\sqrt{-\frac{3}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{3a}{b}\right)\operatorname{FresnelS}\left(\frac{3\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,b}\right)b^2-\sqrt{-\frac{3}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\frac{3}{}\right.}{}$

input
```
int(x^2*(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/144/c^3/(a+b*arccos(c*x))^(1/2)*(-(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arc
cos(c*x))^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)*b^2-(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(
1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))
^(1/2)/b)*b^2-27*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos
(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^
2-27*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*Fresne
lC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2+36*arccos(
c*x)^2*cos(-(a+b*arccos(c*x))/b+a/b)*b^2+12*arccos(c*x)^2*cos(-3*(a+b*arcc
os(c*x))/b+3*a/b)*b^2+72*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*a*b+54*
arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b)*b^2+24*arccos(c*x)*cos(-3*(a+b*a
rccos(c*x))/b+3*a/b)*a*b+6*arccos(c*x)*sin(-3*(a+b*arccos(c*x))/b+3*a/b)*b
^2+36*cos(-(a+b*arccos(c*x))/b+a/b)*a^2+54*sin(-(a+b*arccos(c*x))/b+a/b)*a
*b+12*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a^2+6*sin(-3*(a+b*arccos(c*x))/b+3
*a/b)*a*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \int x^2(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \int (b\arccos(cx)+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)*x^2, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 2295, normalized size of antiderivative = 7.33

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
-1/4*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)
/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a
/b)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) + 1/8*sqrt(2)*sqrt(pi)*a
*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)
*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) +
b^2*sqrt(abs(b)))*c^3) + 1/4*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(1/2*I*sqrt(2)*
sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)
*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3)
 + 1/8*sqrt(2)*sqrt(pi)*a*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sq
rt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b
)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) - 1/2*I*sqrt(pi)*a^2*b^(3
/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(
b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b^2 + I*sqrt(6)*b
^3/abs(b))*c^3) + 1/12*sqrt(pi)*a*b^(5/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c
*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e
^(3*I*a/b)/((sqrt(6)*b^2 + I*sqrt(6)*b^3/abs(b))*c^3) + 1/8*I*sqrt(2)*sqrt
(pi)*a^2*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*s
qrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(
b)) + b*sqrt(abs(b)))*c^3) - 1/8*sqrt(2)*sqrt(pi)*a*b^2*erf(-1/2*I*sqrt(2)
*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) ...
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \int x^2\,(a+b\,\mathrm{acos}(c\,x))^{3/2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^(3/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)\,x^2dx\right)b + \left(\int \sqrt{acos\,(cx)\,b+a}\,x^2dx\right)a$$

input
```
int(x^2*(a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)*acos(c*x)*x**2,x)*b + int(sqrt(acos(c*x)*b + a)*
x**2,x)*a
```

3.179 $\int x(a+b\arccos(cx))^{3/2}\,dx$

Optimal result 1257
Mathematica [A] (verified) 1258
Rubi [A] (verified) 1258
Maple [B] (verified) 1264
Fricas [F(-2)] 1264
Sympy [F] 1265
Maxima [F] 1265
Giac [C] (verification not implemented) 1265
Mupad [F(-1)] 1266
Reduce [F] 1267

Optimal result

Integrand size = 14, antiderivative size = 172

$$\int x(a+b\arccos(cx))^{3/2}\,dx = -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c}-\frac{(a+b\arccos(cx))^{3/2}}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}+\frac{3b^{3/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2}-\frac{3b^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{32c^2}$$

output

```
-3/8*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c-1/4*(a+b*arccos(c*x)
)^(3/2)/c^2+1/2*x^2*(a+b*arccos(c*x))^(3/2)+3/32*b^(3/2)*Pi^(1/2)*cos(2*a/
b)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/c^2-3/32*b^(3/2)*P
i^(1/2)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/c^
2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.84

$$\int x(a+b\arccos(cx))^{3/2}\,dx=\frac{3b^{3/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-3b^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin$$

input
```
Integrate[x*(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(3*b^(3/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqr
t[b]*Sqrt[Pi])] - 3*b^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/
(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b] + 2*Sqrt[a + b*ArcCos[c*x]]*(4*a*Cos[2*Ar
cCos[c*x]] + 4*b*ArcCos[c*x]*Cos[2*ArcCos[c*x]] - 3*b*Sin[2*ArcCos[c*x]]))
/(32*c^2)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5141, 5211, 5147, 25, 4906, 27, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arccos(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{3}{4}bc\int\frac{x^2\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{4}bc\left(\frac{\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int \frac{x}{\sqrt{a+b\arccos(cx)}}dx}{4c}-\frac{x\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 5147

$$\frac{3}{4}bc\left(\frac{\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^3}+\frac{\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}\sqrt{a+b\text{ar}}}{2c^2}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 25

$$\frac{3}{4}bc\left(-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^3}+\frac{\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}\sqrt{a+b\text{ar}}}{2c^2}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 4906

$$\frac{3}{4}bc\left(-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^3}+\frac{\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c^2}\right)+$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 27

$$\frac{3}{4}bc\left(-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{8c^3}+\frac{\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c^2}\right)+$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3042

$$\frac{3}{4}bc\left(-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{8c^3}+\frac{\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c^2}\right)+$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 3787

$$\frac{3}{4}bc\left(\frac{-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{8c^3}+\frac{\int\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 25

$$\frac{3}{4}bc\left(\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{8c^3}+\frac{\int \frac{\sqrt{a-}}{}}{}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 3042

$$\frac{3}{4}bc\left(\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{8c^3}+\frac{\int}{}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 3785

$$\frac{3}{4}bc\left(\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{2a}{b}\right)\int \cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}}{8c^3}+\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\downarrow$ 3786

$$\frac{3}{4}bc\left(\frac{2\cos\left(\frac{2a}{b}\right)\int\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(c}}{8c^3}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3832

$$\frac{3}{4}bc\left(\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-2\sin\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}}{8c^3}+\frac{\int\frac{\sqrt{a}}{}}{}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3833

$$\frac{3}{4}bc\left(\frac{\int\frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx}{2c^2}+\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx}}{\sqrt{b}\sqrt{\pi}}\right.}{8c^3}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 5153

$$\frac{3}{4}bc\left(\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^3}-\frac{(a+b\arccos(cx}{3bc^3}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

input
```
Int[x*(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^(3/2))/2 + (3*b*c*(-1/2*(x*Sqrt[1 - c^2*x^2]*Sqrt
[a + b*ArcCos[c*x]])/c^2 - (a + b*ArcCos[c*x])^(3/2)/(3*b*c^3) + (Sqrt[b]*
Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[P
i])] - Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt
[Pi])]*Sin[(2*a)/b])/(8*c^3)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(134) = 268$.

Time = 0.14 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.63

method	result
default	$-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)b^2-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{}}{}\right.$

input
```
int(x*(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/c^2/(a+b*arccos(c*x))^(1/2)*(-3*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x
))^(1/2)*cos(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c
*x))^(1/2)/b)*b^2-3*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/
b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2
+8*arccos(c*x)^2*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b^2+16*arccos(c*x)*cos(
-2*(a+b*arccos(c*x))/b+2*a/b)*a*b+6*arccos(c*x)*sin(-2*(a+b*arccos(c*x))/b
+2*a/b)*b^2+8*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a^2+6*sin(-2*(a+b*arccos(c
*x))/b+2*a/b)*a*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \int x(a+b\operatorname{acos}(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \int (b\arccos(cx)+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)*x, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 911, normalized size of antiderivative = 5.30

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(pi)*a^2*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(
b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2)
+ 1/8*sqrt(pi)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*a
rccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2) + 1
/2*I*sqrt(pi)*a^2*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2) +
 1/8*sqrt(pi)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*ar
ccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2) - 1
/4*I*sqrt(pi)*a^2*b*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos
(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^(3/2) - I*b^(5/2)/abs(b))*c^2)
 + 1/2*I*sqrt(pi)*a^2*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqr
t(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2)
- 1/8*sqrt(pi)*a*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*a
rccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) - 3/6
4*I*sqrt(pi)*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arcco
s(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) - 1/4*I*s
qrt(pi)*a^2*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos
(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) - 1/8*sqr
t(pi)*a*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x
) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) + 3/64*I*s...
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^{3/2}\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^{3/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^(3/2),x)
```

output
```
int(x*(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)\,xdx\right)b + \left(\int \sqrt{acos\,(cx)\,b+a}\,xdx\right)a$$

input
```
int(x*(a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)*acos(c*x)*x,x)*b + int(sqrt(acos(c*x)*b + a)*x,x
)*a
```

3.180 $\int (a+b\arccos(cx))^{3/2}\,dx$

Optimal result 1268
Mathematica [C] (verified) 1269
Rubi [A] (verified) 1269
Maple [B] (verified) 1273
Fricas [F(-2)] 1274
Sympy [F] 1274
Maxima [F] 1274
Giac [C] (verification not implemented) 1275
Mupad [F(-1)] 1276
Reduce [F] 1276

Optimal result

Integrand size = 12, antiderivative size = 159

$$\int (a+b\arccos(cx))^{3/2}\,dx=-\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c}+x(a+b\arccos(cx))^{3/2}+\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}-\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2c}$$

output
```
-3/2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c+x*(a+b*arccos(c*x))^(3
/2)+3/4*b^(3/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b^(1/2))/c-3/4*b^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)
/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.82

$$\int (a+b\arccos(cx))^{3/2}\,dx = \frac{\sqrt{b}\left(2\sqrt{b}\sqrt{a+b\arccos(cx)}\left(-3\sqrt{1-c^2x^2}+2cx\arccos(cx)\right)+\frac{2ia\sqrt{b}e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(Sqrt[b]*(2*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-3*Sqrt[1 - c^2*x^2] + 2*c*x*
ArcCos[c*x]) + ((2*I)*a*Sqrt[b]*(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[
3/2, ((-I)*(a + b*ArcCos[c*x]))/b] - E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos
[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b]))/(E^((I*a)/b)*Sqrt[a + b
*ArcCos[c*x]]) + Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/
Sqrt[b]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) + Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*S
qrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b])))/(4*c)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5131, 5183, 5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{5131}$$

$$\frac{3}{2}bc\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{3}{2}bc\left(-\frac{b\int \frac{1}{\sqrt{a+b\arccos(cx)}}dx}{2c}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5135}$$

$$\frac{3}{2}bc\left(\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}bc\left(-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}bc\left(-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3787}$$

$$\frac{3}{2}bc\left(\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}bc\left(\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}bc\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \; 3785$$

$$\frac{3}{2}bc\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \; 3786$$

$$\frac{3}{2}bc\left(\frac{2\cos\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \; 3832$$

$$\frac{3}{2}bc\left(\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \; 3833$$

$$\frac{3}{2}bc\left(\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c^2}-\frac{\sqrt{1-c^2x}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

input
```
Int[(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
x*(a + b*ArcCos[c*x])^(3/2) + (3*b*c*(-((Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcC
os[c*x]])/c^2) + (Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a
+ b*ArcCos[c*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[
a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c^2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(123) = 246$.

Time = 0.00 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.75

method	result
default	$-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)b^2-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\sqrt{\right.$

input
```
int((a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/c/(a+b*arccos(c*x))^(1/2)*(-3*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arcco
s(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)/b)*b^2-3*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)
*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b
)*b^2+4*arccos(c*x)^2*cos(-(a+b*arccos(c*x))/b+a/b)*b^2+8*arccos(c*x)*cos(
-(a+b*arccos(c*x))/b+a/b)*a*b+6*arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b)*
b^2+4*cos(-(a+b*arccos(c*x))/b+a/b)*a^2+6*sin(-(a+b*arccos(c*x))/b+a/b)*a*
b)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\arccos(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\arccos(cx))^{3/2}\,dx = \int (a+b\operatorname{acos}(cx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*acos(c*x))**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\arccos(cx))^{3/2}\,dx = \int (b\arccos(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 1159, normalized size of antiderivative = 7.29

$$\int (a + b \arccos(cx))^{3/2} \, dx = \text{Too large to display}$$

input

```
integrate((a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output

```
-I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqr
t(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/
((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*sqrt(2)*sqrt(pi)*a*b^3*e
rf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(
b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*sq
rt(abs(b)))*c) + I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arcco
s(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b)
)/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*sqrt(2)
*sqrt(pi)*a*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqr
t(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a^2*b*erf(-1/2*I
*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(
c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))
*c) - 1/2*sqrt(2)*sqrt(pi)*a*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a
)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*
a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 3/8*I*sqrt(2)*sqrt(pi)*b^
3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sq
rt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*s
qrt(abs(b)))*c) - 1/2*I*sqrt(2)*sqrt(pi)*a^2*b*erf(1/2*I*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(...
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arccos(cx))^{3/2}\,dx=\int (a+b\,\mathrm{acos}(c\,x))^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^(3/2),x)
```

output
```
int((a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\arccos(cx))^{3/2}\,dx=\left(\int \sqrt{acos\,(cx)\,b+a}dx\right)a+\left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)\,dx\right)b$$

input
```
int((a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)*a + int(sqrt(acos(c*x)*b + a)*acos(c*x),x)*b
```

3.181 $\int \frac{(a+b \arccos(cx))^{3/2}}{x} \, dx$

Optimal result 1277
Mathematica [N/A] 1277
Rubi [N/A] 1278
Maple [N/A] 1278
Fricas [F(-2)] 1279
Sympy [N/A] 1279
Maxima [N/A] 1279
Giac [N/A] 1280
Mupad [N/A] 1280
Reduce [N/A] 1281

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b \arccos(cx))^{3/2}}{x} \, dx = \text{Int}\left(\frac{(a+b \arccos(cx))^{3/2}}{x}, x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b \arccos(cx))^{3/2}}{x} \, dx = \int \frac{(a+b \arccos(cx))^{3/2}}{x} \, dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/x,x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx$$

$$\Big\downarrow \ 5149$$

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}dx$$

input
```
Int[(a + b*ArcCos[c*x])^(3/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{3}{2}}}{x}dx$$

input
```
int((a+b*arccos(c*x))^(3/2)/x,x)
```

output
```
int((a+b*arccos(c*x))^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 11.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*acos(c*x))**(3/2)/x,x)
```

output
```
Integral((a + b*acos(c*x))**(3/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\int \frac{(b\arccos{(cx)}+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^{3/2}}{x}\,dx$$

input
```
int((a + b*acos(c*x))^(3/2)/x,x)
```

output
```
int((a + b*acos(c*x))^(3/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \left(\int \frac{\sqrt{acos\,(cx)\,b+a}}{x}dx\right)a + \left(\int \frac{\sqrt{acos\,(cx)\,b+a}\;acos(cx)}{x}dx\right)b$$

input
```
int((a+b*acos(c*x))^(3/2)/x,x)
```

output
```
int(sqrt(acos(c*x)*b + a)/x,x)*a + int((sqrt(acos(c*x)*b + a)*acos(c*x))/x
,x)*b
```

3.182 $\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$

Optimal result . . . 1282
Mathematica [N/A] . . . 1282
Rubi [N/A] . . . 1283
Maple [N/A] . . . 1283
Fricas [F(-2)] . . . 1284
Sympy [N/A] . . . 1284
Maxima [N/A] . . . 1284
Giac [N/A] . . . 1285
Mupad [N/A] . . . 1285
Reduce [N/A] . . . 1286

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \text{Int}\left(\frac{(a+b\arccos(cx))^{3/2}}{x^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(3/2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/x^2,x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}dx$$

input
```
Int[(a + b*ArcCos[c*x])^(3/2)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+b*arccos(c*x))^(3/2)/x^2,x)
```

output
```
int((a+b*arccos(c*x))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**(3/2)/x**2,x)
```

output
```
Integral((a + b*acos(c*x))**(3/2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx=\int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^{3/2}}{x^2}\,dx$$

input
```
int((a + b*acos(c*x))^(3/2)/x^2,x)
```

output
```
int((a + b*acos(c*x))^(3/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(acos(cx)\,b+a)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+b*acos(c*x))^(3/2)/x^2,x)
```

output
```
int((a+b*acos(c*x))^(3/2)/x^2,x)
```

3.183 $\int x^2(a+b\arccos(cx))^{5/2}\,dx$

Optimal result . . . 1287
Mathematica [C] (verified) . . . 1288
Rubi [A] (verified) . . . 1289
Maple [B] (verified) . . . 1297
Fricas [F(-2)] . . . 1298
Sympy [F] . . . 1299
Maxima [F] . . . 1299
Giac [C] (verification not implemented) . . . 1299
Mupad [F(-1)] . . . 1300
Reduce [F] . . . 1301

Optimal result

Integrand size = 16, antiderivative size = 358

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2}-\frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} -\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3}-\frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} +\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}+\frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{5b^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\cdots\right.}{144c^3}$$

output
```
-5/6*b^2*x*(a+b*arccos(c*x))^(1/2)/c^2-5/36*b^2*x^3*(a+b*arccos(c*x))^(1/2
)-5/9*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(3/2)/c^3-5/18*b*x^2*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))^(3/2)/c+1/3*x^3*(a+b*arccos(c*x))^(5/2)+15/32
*b^(5/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c
*x))^(1/2)/b^(1/2))/c^3+5/864*b^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelC
(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^3+15/32*b^(5/2)*2^(1/
2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin
(a/b)/c^3+5/864*b^(5/2)*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*ar
ccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 956, normalized size of antiderivative = 2.67

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
((-1/72*I)*a^2*b*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Ga
mma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*
ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqrt[(
(-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]) +
 E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*
ArcCos[c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]]) - (a*Sqr
t[b]*(18*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*ArcC
os[c*x]) - 9*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt
[b]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqr
t[a + b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b]) - Sqrt[6*Pi]*
FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(b*Cos[(3*a)/b] + 2
*a*Sin[(3*a)/b]) - Sqrt[6*Pi]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]]
)/Sqrt[b]]*(2*a*Cos[(3*a)/b] - b*Sin[(3*a)/b]) + 6*Sqrt[b]*Sqrt[a + b*ArcC
os[c*x]]*(-2*ArcCos[c*x]*Cos[3*ArcCos[c*x]] + Sin[3*ArcCos[c*x]])))/(72*c^
3) - (Sqrt[b]*(27*(2*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-2*Sqrt[1 - c^2*x^2]
*(a - 5*b*ArcCos[c*x]) - b*c*x*(-15 + 4*ArcCos[c*x]^2)) + Sqrt[2*Pi]*Fresn
elC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*((4*a^2 - 15*b^2)*Cos[a/
b] - 12*a*b*Sin[a/b]) + Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[
c*x]])/Sqrt[b]]*(12*a*b*Cos[a/b] + (4*a^2 - 15*b^2)*Sin[a/b])) + Sqrt[6*Pi
]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*((12*a^2 - 5*b...
```

Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.32, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5141, 5211, 5141, 5183, 5131, 5225, 3042, 3787, 25, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+b\arccos(cx))^{5/2} \, dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{5}{6}bc\int \frac{x^3(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{5}{6}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2\sqrt{a+b\arccos(cx)}dx}{2c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{5}{6}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\left(\frac{1}{6}bc\int \frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}\right)}{2c}-\frac{x^2\sqrt{1-c^2x^2}(a+\right.$$
$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{5}{6}bc\left(\frac{2\left(-\frac{3b\int \sqrt{a+b\arccos(cx)}dx}{2c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}\right)}{3c^2}-\frac{b\left(\frac{1}{6}bc\int \frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{1}{3}x^3\sqrt{a+b\arcc\right.}{2c}\right.$$
$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5131}$$

$$\frac{5}{6}bc\left(\frac{2\left(-\frac{3b\left(\frac{1}{2}bc\int\frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+x\sqrt{a+b\arccos(cx)}\right)}{2c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}\right)}{3c^2}-\frac{b\left(\frac{1}{6}bc\int\frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\,\mathrm{ar}}}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5225}$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\cos\left(\frac{a}{b}-\frac{a+b}{}\right.}{\sqrt{a+b\,\mathrm{arc}}}}{}\right.}{2c}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a}{}\right.}{\sqrt{a}}}{}\right.}{2}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\downarrow$ 3787

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\cos\left(\frac{a}{b}\right)\int \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\downarrow$ 25

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\downarrow$ 3042

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\downarrow$ 3785

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\downarrow$ 3786

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{2\sin\left(\frac{a}{b}\right)\int s}{}\right.}{}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\big\downarrow$ 3793

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\int\left(\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}+\frac{3\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{6c^3}\right)}{2c}+\frac{2\left(-\frac{3b\left(x\sqrt{}\right.}{}\right.}{}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\big\downarrow$ 2009

$$\frac{5}{6}bc\left(\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{2\sin\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}\right)}{2c}-\frac{\sqrt{1-c}}{}\right.}{3c^2}\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\big\downarrow$ 3832

$$\frac{5}{6}bc\left(\frac{2\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}\right)}{2c}-\frac{\sqrt{1-c^2}\ldots}{\ldots}\right)}{3c^2}\ldots\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

$\downarrow$ 3833

$$\frac{5}{6}bc\left(-\frac{b\left(\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx}\ldots}{\sqrt{b}}\right.\ldots}{\ldots}\right.}{2c}\ldots\right.$$

$$\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

input
```
Int[x^2*(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^(5/2))/3 + (5*b*c*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x])^(3/2))/c^2 + (2*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]
)^(3/2))/c^2) - (3*b*(x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[2*Pi]*Cos[
a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] + Sqrt[b]*Sqrt
[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2
*c)))/(2*c)))/(3*c^2) - (b*((x^3*Sqrt[a + b*ArcCos[c*x]])/3 - ((3*Sqrt[b]*
Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]
)/2 + (Sqrt[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*Arc
Cos[c*x]])/Sqrt[b]])/2 + (3*Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a
 + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/2 + (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sq
rt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2)/(6*c^3)))/(2*c
)))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(278) = 556$.

Time = 0.28 (sec) , antiderivative size = 819, normalized size of antiderivative = 2.29

method	result	size
default	Expression too large to display	819

input
```
int(x^2*(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/864/c^3*b*(36*arccos(c*x)^2*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*cos(-3*(a+b*a
rccos(c*x))/b+3*a/b)*(a+b*arccos(c*x))^(1/2)*b^2+108*arccos(c*x)^2*2^(1/2)
*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(-(a+b*arccos(c*x))/b+a/
b)*b^2+5*Pi*(-1/b)^(1/2)*(-3/b)^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/
2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^3-5*Pi*(-1/b)^(1/2)*(-3/b)^(1
/2)*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)/b)*b^3+72*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*cos(-3*(a+b*arcc
os(c*x))/b+3*a/b)*(a+b*arccos(c*x))^(1/2)*a*b+30*arccos(c*x)*2^(1/2)*Pi^(1
/2)*(-1/b)^(1/2)*sin(-3*(a+b*arccos(c*x))/b+3*a/b)*(a+b*arccos(c*x))^(1/2)
*b^2+216*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)
*cos(-(a+b*arccos(c*x))/b+a/b)*a*b+270*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)
^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(-(a+b*arccos(c*x))/b+a/b)*b^2+36*2^(1/2
)*Pi^(1/2)*(-1/b)^(1/2)*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*(a+b*arccos(c*x)
)^(1/2)*a^2-15*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*cos(-3*(a+b*arccos(c*x))/b+3*
a/b)*(a+b*arccos(c*x))^(1/2)*b^2+30*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*sin(-3*(
a+b*arccos(c*x))/b+3*a/b)*(a+b*arccos(c*x))^(1/2)*a*b+108*2^(1/2)*Pi^(1/2)
*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(-(a+b*arccos(c*x))/b+a/b)*a^2-40
5*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(-(a+b*arccos(c
*x))/b+a/b)*b^2+270*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*
sin(-(a+b*arccos(c*x))/b+a/b)*a*b+405*Pi*b^2*FresnelS(2^(1/2)/Pi^(1/2)/...
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx=\int x^2(a+b\operatorname{acos}{(cx)})^{\frac{5}{2}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx=\int (b\arccos{(cx)}+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2)*x^2, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 3096, normalized size of antiderivative = 8.65

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
1/576*(72*sqrt(pi)*a^2*b^(7/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sq
rt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/
(sqrt(6)*b^3 + I*sqrt(6)*b^4/abs(b)) + 36*I*sqrt(pi)*a*b^(9/2)*erf(-1/2*sq
rt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) +
 a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6)*b^3 + I*sqrt(6)*b^4/abs(b)) - 10*
sqrt(pi)*b^(11/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I
*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6)*b^3
+ I*sqrt(6)*b^4/abs(b)) - 72*I*sqrt(2)*sqrt(pi)*a^3*b^2*erf(-1/2*I*sqrt(2)
*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a
)*sqrt(abs(b))/b)*e^(I*a/b)/(I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b))) + 72*I
*sqrt(2)*sqrt(pi)*a^3*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(a
bs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-
I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b))) + 72*sqrt(pi)*a^2*b^(7/2)*erf(-1/2*
sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x)
 + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(6)*b^3 - I*sqrt(6)*b^4/abs(b)) -
36*I*sqrt(pi)*a*b^(9/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) +
 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(
6)*b^3 - I*sqrt(6)*b^4/abs(b)) - 10*sqrt(pi)*b^(11/2)*erf(-1/2*sqrt(6)*sqr
t(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(
b)/abs(b))*e^(-3*I*a/b)/(sqrt(6)*b^3 - I*sqrt(6)*b^4/abs(b)) - 216*I*sq...
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^{5/2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^(5/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^(5/2), x)
```

Reduce [F]

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = 2\left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)\,x^2dx\right)ab + \left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)^2\,x^2dx\right)b^2 + \left(\int \sqrt{acos\,(cx)\,b+a}\,x^2dx\right)a^2$$

input
```
int(x^2*(a+b*acos(c*x))^(5/2),x)
```

output
```
2*int(sqrt(acos(c*x)*b + a)*acos(c*x)*x**2,x)*a*b + int(sqrt(acos(c*x)*b +
 a)*acos(c*x)**2*x**2,x)*b**2 + int(sqrt(acos(c*x)*b + a)*x**2,x)*a**2
```

3.184 $\int x(a+b\arccos(cx))^{5/2}\,dx$

Optimal result . . . 1302
Mathematica [A] (verified) . . . 1303
Rubi [A] (verified) . . . 1303
Maple [B] (verified) . . . 1307
Fricas [F(-2)] . . . 1308
Sympy [F] . . . 1308
Maxima [F] . . . 1309
Giac [C] (verification not implemented) . . . 1309
Mupad [F(-1)] . . . 1310
Reduce [F] . . . 1311

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} -\frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c}-\frac{(a+b\arccos(cx))^{5/2}}{4c^2} +\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}+\frac{15b^{5/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2}+\frac{15b^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\dots}{128c^2}$$

output
```
15/64*b^2*(a+b*arccos(c*x))^(1/2)/c^2-15/32*b^2*x^2*(a+b*arccos(c*x))^(1/2
)-5/8*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(3/2)/c-1/4*(a+b*arccos(c*x
))^(5/2)/c^2+1/2*x^2*(a+b*arccos(c*x))^(5/2)+15/128*b^(5/2)*Pi^(1/2)*cos(2
*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/c^2+15/128*b^(5
/2)*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/
b)/c^2
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \frac{15b^{5/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+15b^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{}$$

input
```
Integrate[x*(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
(15*b^(5/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sq
rt[b]*Sqrt[Pi])] + 15*b^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]]
)/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b] + 2*Sqrt[a + b*ArcCos[c*x]]*((16*a^2 -
15*b^2)*Cos[2*ArcCos[c*x]] + 16*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] - 20*
a*b*Sin[2*ArcCos[c*x]] + 4*b*ArcCos[c*x]*(8*a*Cos[2*ArcCos[c*x]] - 5*b*Sin
[2*ArcCos[c*x]])))/(128*c^2)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5141, 5211, 5141, 5153, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arccos(cx))^{5/2}\,dx$$

$$\Big\downarrow \text{5141}$$

$$\frac{5}{4}bc\int \frac{x^2(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{5}{4}bc\left(\frac{\int \frac{(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{3b\int x\sqrt{a+b\arccos(cx)}dx}{4c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c^2}\right)+$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{4}bc\int \frac{x^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}\right)}{4c}+\frac{\int \frac{(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+}{2c}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{4}bc\int \frac{x^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}\right)}{4c}-\frac{(a+b\arccos(cx))^{5/2}}{5bc^3}-\frac{x\sqrt{1-c^2x^2}(a+}{2c}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5225}$$

$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^2}\right)}{4c}-\frac{(a+b\arccos(cx))^{5/2}}{5bc^3}-\frac{x\sqrt{1-c^2x}}{}\right.$$
$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^2}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{4c^2}\right)}{4c}-\frac{(a+b\arccos(cx))^{5/2}}{5bc^3}-\frac{x\sqrt{1-c\ldots}}{}\right.$$

$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5}{4}bc\left(-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\int\left(\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}+\frac{1}{2\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{4c^2}\right)}{4c}-\frac{(a+b\arccos(c\ldots}{5bc^3}\right.$$

$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{4}bc\left(-\frac{(a+b\arccos(cx))^{5/2}}{5bc^3}-\frac{3b\left(\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\ldots}{4c^2}\right.}{4c}\right.$$

$$\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

input
```
Int[x*(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^(5/2))/2 + (5*b*c*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a +
 b*ArcCos[c*x])^(3/2))/c^2 - (a + b*ArcCos[c*x])^(5/2)/(5*b*c^3) - (3*b*((
x^2*Sqrt[a + b*ArcCos[c*x]])/2 - (Sqrt[a + b*ArcCos[c*x]] + (Sqrt[b]*Sqrt[
Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])
/2 + (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[
Pi])]*Sin[(2*a)/b])/2)/(4*c^2)))/(4*c)))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(170) = 340$.

Time = 0.17 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.89

method	result
default	$\frac{15\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)b^3-15\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\ldots}{\ldots}\right.\ldots}{\ldots}$

input
```
int(x*(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/c^2/(a+b*arccos(c*x))^(1/2)*(15*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)/b)*b^3-15*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*
a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b
^3+32*arccos(c*x)^3*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b^3+96*arccos(c*x)^2
*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a*b^2+40*arccos(c*x)^2*sin(-2*(a+b*arcc
os(c*x))/b+2*a/b)*b^3+96*arccos(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a^2
*b-30*arccos(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b^3+80*arccos(c*x)*sin
(-2*(a+b*arccos(c*x))/b+2*a/b)*a*b^2+32*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*
a^3-30*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a*b^2+40*sin(-2*(a+b*arccos(c*x))
/b+2*a/b)*a^2*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \int x(a+b\operatorname{acos}(cx))^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\int(b\arccos(cx)+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2)*x, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 1521, normalized size of antiderivative = 7.04

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
3/16*sqrt(pi)*a^2*b^(9/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^4 + I*b^5/abs(b))*c^2) +
3/16*sqrt(pi)*a^2*b^(9/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^4 - I*b^5/abs(b))*c^2) +
 9/64*I*sqrt(pi)*a*b^(9/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b
*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^3 + I*b^4/abs(b))*c^2) -
 15/256*sqrt(pi)*b^(11/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^3 + I*b^4/abs(b))*c^2) -
9/64*I*sqrt(pi)*a*b^(9/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^3 - I*b^4/abs(b))*c^2) -
 15/256*sqrt(pi)*b^(11/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^3 - I*b^4/abs(b))*c^2) -
 3/4*I*sqrt(pi)*a^3*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(
b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2)
+ 3/16*sqrt(pi)*a^2*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(
b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2)
+ 3/4*I*sqrt(pi)*a^3*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt
(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2
) + 3/16*sqrt(pi)*a^2*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqr
t(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))...
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^{5/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^(5/2),x)
```

output
```
int(x*(a + b*acos(c*x))^(5/2), x)
```

Reduce [F]

$$\int x(a+b\arccos(cx))^{5/2}\,dx=2\left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)\,xdx\right)ab+\left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)^{2}\,xdx\right)b^{2}+\left(\int \sqrt{acos\,(cx)\,b+a}\,xdx\right)a^{2}$$

input
```
int(x*(a+b*acos(c*x))^(5/2),x)
```

output
```
2*int(sqrt(acos(c*x)*b + a)*acos(c*x)*x,x)*a*b + int(sqrt(acos(c*x)*b + a)
*acos(c*x)**2*x,x)*b**2 + int(sqrt(acos(c*x)*b + a)*x,x)*a**2
```

3.185 $\int (a+b\arccos(cx))^{5/2}\,dx$

Optimal result 1312
Mathematica [C] (verified) 1313
Rubi [A] (verified) 1313
Maple [B] (verified) 1318
Fricas [F(-2)] 1319
Sympy [F] 1319
Maxima [F] 1319
Giac [C] (verification not implemented) 1320
Mupad [F(-1)] 1321
Reduce [F] 1321

Optimal result

Integrand size = 12, antiderivative size = 179

$$\int (a+b\arccos(cx))^{5/2}\,dx = -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c} + x(a+b\arccos(cx))^{5/2} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c}$$

output

```
-15/4*b^2*x*(a+b*arccos(c*x))^(1/2)-5/2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c
*x))^(3/2)/c+x*(a+b*arccos(c*x))^(5/2)+15/8*b^(5/2)*2^(1/2)*Pi^(1/2)*cos(a
/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c+15/8*b^(5
/2)*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(
1/2))*sin(a/b)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.08

$$\int (a+b\arccos(cx))^{5/2}\,dx = \frac{\sqrt{b}e^{-\frac{ia}{b}}\left(\left(4a^2+15b^2\right)\left(1+e^{\frac{2ia}{b}}\right)\sqrt{2\pi}\sqrt{a+b\arccos(cx)}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
(Sqrt[b]*((4*a^2 + 15*b^2)*(1 + E^(((2*I)*a)/b))*Sqrt[2*Pi]*Sqrt[a + b*Arc
Cos[c*x]]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] - I*(4*a^
2 + 15*b^2)*(-1 + E^(((2*I)*a)/b))*Sqrt[2*Pi]*Sqrt[a + b*ArcCos[c*x]]*Fres
nelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] - 4*Sqrt[b]*(E^((I*a)/b
)*(a + b*ArcCos[c*x])*(5*(3*b*c*x + 2*a*Sqrt[1 - c^2*x^2]) + (-8*a*c*x + 1
0*b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] - 4*b*c*x*ArcCos[c*x]^2) - (2*I)*a^2*Sq
rt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b]
+ (2*I)*a^2*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, (I*
(a + b*ArcCos[c*x]))/b])))/(16*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5131, 5183, 5131, 5225, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^{5/2}\,dx$$

$$\Big\downarrow \text{5131}$$

$$\frac{5}{2}bc\int \frac{x(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}dx + x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{5}{2}bc\left(-\frac{3b\int \sqrt{a+b\arccos(cx)}dx}{2c} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}\right) + x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5131}$$

$$\frac{5}{2}bc\left(-\frac{3b\left(\frac{1}{2}bc\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx + x\sqrt{a+b\arccos(cx)}\right)}{2c} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}\right) + x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{5225}$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)} - \frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}\right)}{2c} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}\right) + x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)} - \frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}\right)}{2c} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}\right) + x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3787}$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\cos\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int-\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}\right)}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \; 25$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}\right)}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}\right.}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \; 3785$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}\right)}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \; 3786$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{2\sin\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}\right.}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{5}{2}bc\left(-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}\right.}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

$$\Big\downarrow \text{3833}$$

$$\frac{5}{2}bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{c^2}-\frac{3b\left(x\sqrt{a+b\arccos(cx)}-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{\ldots}}{2c}\right.}{2c}\right.$$

$$x(a+b\arccos(cx))^{5/2}$$

input
```
Int[(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
x*(a + b*ArcCos[c*x])^(5/2) + (5*b*c*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c
*x])^(3/2))/c^2) - (3*b*(x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[2*Pi]*C
os[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] + Sqrt[b]*S
qrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])
/(2*c)))/(2*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(139) = 278$.

Time = 0.00 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.24

method	result
default	$-\frac{b\left(4\arccos(cx)^2\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\cos\left(-\frac{a+b\arccos(cx)}{b}+\frac{a}{b}\right)b^2+8\arccos(cx)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\cos\left(-\right.\right.}{}$

input
```
int((a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c*b*(4*arccos(c*x)^2*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)*cos(-(a+b*arccos(c*x))/b+a/b)*b^2+8*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(-(a+b*arccos(c*x))/b+a/b)*a*b+10*arc
cos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(-(a+b*a
rccos(c*x))/b+a/b)*b^2+4*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(
1/2)*cos(-(a+b*arccos(c*x))/b+a/b)*a^2-15*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a
+b*arccos(c*x))^(1/2)*cos(-(a+b*arccos(c*x))/b+a/b)*b^2+10*2^(1/2)*Pi^(1/2
)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(-(a+b*arccos(c*x))/b+a/b)*a*b+1
5*Pi*b^2*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
*sin(a/b)-15*Pi*b^2*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b))*2^(1/2)/Pi^(1/2)*(-1/b)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\arccos(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\arccos(cx))^{5/2}\,dx = \int (a+b\,\mathrm{acos}\,(cx))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*acos(c*x))**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\arccos(cx))^{5/2}\,dx = \int (b\arccos(cx)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 1517, normalized size of antiderivative = 8.47

$$\int (a + b\arccos(cx))^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*sqrt(pi)*a^3*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)
/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a
/b)/((I*b^4/sqrt(abs(b)) + b^3*sqrt(abs(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a
^3*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2
)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^4/sqrt(abs(b))
 + b^3*sqrt(abs(b)))*c) - 3/2*I*sqrt(2)*sqrt(pi)*a^3*b^2*erf(-1/2*I*sqrt(2
)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) +
a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) +
 3/2*sqrt(2)*sqrt(pi)*a^2*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/s
qrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b
)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 3/2*I*sqrt(2)*sqrt(pi)*a^3
*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*
sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) +
 b^2*sqrt(abs(b)))*c) + 3/2*sqrt(2)*sqrt(pi)*a^2*b^3*erf(1/2*I*sqrt(2)*sqr
t(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sq
rt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 3/
2*I*sqrt(2)*sqrt(pi)*a^3*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt
(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(
(I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 3/2*sqrt(2)*sqrt(pi)*a^2*b^2*er
f(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqr...
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\arccos(cx))^{5/2}\,dx = \int (a + b\,\mathrm{acos}(c\,x))^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^(5/2),x)
```

output
```
int((a + b*acos(c*x))^(5/2), x)
```

Reduce [F]

$$\int (a + b\arccos(cx))^{5/2}\,dx = \left(\int \sqrt{acos\,(cx)\,b + a}dx\right)a^2 + 2\left(\int \sqrt{acos\,(cx)\,b + a}\;acos(cx)\,dx\right)ab + \left(\int \sqrt{acos\,(cx)\,b + a}\;acos(cx)^2\,dx\right)b^2$$

input
```
int((a+b*acos(c*x))^(5/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)*a**2 + 2*int(sqrt(acos(c*x)*b + a)*acos(c*x),
x)*a*b + int(sqrt(acos(c*x)*b + a)*acos(c*x)**2,x)*b**2
```

3.186 $\int \frac{(a+b\arccos(cx))^{5/2}}{x} dx$

Optimal result . . . 1322
Mathematica [N/A] . . . 1322
Rubi [N/A] . . . 1323
Maple [N/A] . . . 1323
Fricas [F(-2)] . . . 1324
Sympy [N/A] . . . 1324
Maxima [N/A] . . . 1324
Giac [N/A] . . . 1325
Mupad [N/A] . . . 1325
Reduce [N/A] . . . 1326

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x} dx = \text{Int}\left(\frac{(a+b\arccos(cx))^{5/2}}{x},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(5/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x} dx = \int \frac{(a+b\arccos(cx))^{5/2}}{x} dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(5/2)/x,x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(5/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}dx$$

input
```
Int[(a + b*ArcCos[c*x])^(5/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{5}{2}}}{x}dx$$

input
```
int((a+b*arccos(c*x))^(5/2)/x,x)
```

output
```
int((a+b*arccos(c*x))^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b\arccos(cx))^{5/2}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 29.83 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a + b\arccos(cx))^{5/2}}{x} dx = \int \frac{(a + b\,\mathrm{acos}\,(cx))^{\frac{5}{2}}}{x} dx$$

input
```
integrate((a+b*acos(c*x))**(5/2)/x,x)
```

output
```
Integral((a + b*acos(c*x))**(5/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\arccos(cx))^{5/2}}{x} dx = \int \frac{(b\arccos{(cx)} + a)^{\frac{5}{2}}}{x} dx$$

input
```
integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx=\int \frac{(b\arccos(cx)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^{5/2}}{x}\,dx$$

input
```
int((a + b*acos(c*x))^(5/2)/x,x)
```

output
```
int((a + b*acos(c*x))^(5/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \left(\int \frac{\sqrt{acos\,(cx)\,b+a}}{x}dx\right)a^2 + 2\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\;acos(cx)}{x}dx\right)ab + \left(\int \frac{\sqrt{acos\,(cx)\,b+a}\;acos(cx)^2}{x}dx\right)b^2$$

input

```
int((a+b*acos(c*x))^(5/2)/x,x)
```

output

```
int(sqrt(acos(c*x)*b + a)/x,x)*a**2 + 2*int((sqrt(acos(c*x)*b + a)*acos(c*
x))/x,x)*a*b + int((sqrt(acos(c*x)*b + a)*acos(c*x)**2)/x,x)*b**2
```

3.187 $\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$

Optimal result 1327
Mathematica [N/A] 1327
Rubi [N/A] 1328
Maple [N/A] 1328
Fricas [F(-2)] 1329
Sympy [N/A] 1329
Maxima [N/A] 1329
Giac [N/A] 1330
Mupad [N/A] 1330
Reduce [N/A] 1331

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \text{Int}\left(\frac{(a+b\arccos(cx))^{5/2}}{x^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(5/2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(5/2)/x^2,x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(5/2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2} \, dx$$
$$\Big\downarrow \ 5149$$
$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}dx$$

input
```
Int[(a + b*ArcCos[c*x])^(5/2)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a+b*arccos(c*x))^(5/2)/x^2,x)
```

output
```
int((a+b*arccos(c*x))^(5/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + b \arccos(cx))^{5/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 22.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a + b \arccos(cx))^{5/2}}{x^2} \, dx = \int \frac{(a + b \operatorname{acos}(cx))^{\frac{5}{2}}}{x^2} \, dx$$

input
```
integrate((a+b*acos(c*x))**(5/2)/x**2,x)
```

output
```
Integral((a + b*acos(c*x))**(5/2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \arccos(cx))^{5/2}}{x^2} \, dx = \int \frac{(b \arccos(cx) + a)^{\frac{5}{2}}}{x^2} \, dx$$

input
```
integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx=\int \frac{(b\arccos(cx)+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(5/2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^{5/2}}{x^2}\,dx$$

input
```
int((a + b*acos(c*x))^(5/2)/x^2,x)
```

output
```
int((a + b*acos(c*x))^(5/2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx=\int \frac{(acos(cx)\,b+a)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a+b*acos(c*x))^(5/2)/x^2,x)
```

output
```
int((a+b*acos(c*x))^(5/2)/x^2,x)
```

3.188 $\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result 1332
Mathematica [C] (verified) 1333
Rubi [A] (verified) 1333
Maple [A] (verified) 1335
Fricas [F(-2)] 1335
Sympy [F] 1336
Maxima [F] 1336
Giac [C] (verification not implemented) 1337
Mupad [F(-1)] 1338
Reduce [F] 1338

Optimal result

Integrand size = 16, antiderivative size = 223

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{2\sqrt{b}c^3}$$

output
```
-1/4*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))
^(1/2)/b^(1/2))/b^(1/2)/c^3-1/12*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1
/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^3+1/4*2^(1/2)*Pi^(
1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b
^(1/2)/c^3+1/12*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x
))^(1/2)/b^(1/2))*sin(3*a/b)/b^(1/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx$$
$$= \frac{e^{-\frac{3ia}{b}}\left(3e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+3e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\sqrt{3}\left(\sqrt{\right.\right.}{24c^3\sqrt{a+b\arccos(c}}$$

input
```
Integrate[x^2/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(3*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a
+ b*ArcCos[c*x]))/b] + 3*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*G
amma[1/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(Sqrt[((-I)*(a + b*ArcCos[c
*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] + E^(((6*I)*a)/b)*Sqrt
[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b]))/(2
4*c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx$$
$$\Big\downarrow \text{5147}$$
$$-\frac{\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc^3}$$

$$\downarrow 25$$

$$\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc^3}$$

$$\downarrow 4906$$

$$\frac{\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{bc^3}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}}{bc^3}$$

input
```
Int[x^2/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
-(((Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x
]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]])/2 - (Sqrt[b]*Sqrt[Pi/2]*FresnelC[(Sqrt[2/
Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[Pi/6]*Fr
esnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2)/(b*c
^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{\pi}\,\sqrt{2}\,\sqrt{-\frac{1}{b}}\left(3\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+3\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-\sqrt{-\frac{1}{b}}\,\sqrt{-\frac{3}{b}}\,\cos\left(\frac{3a}{b}\right)\operatorname{FresnelS}(\right.}{12c^3}$

input
```
int(x^2/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/c^3*Pi^(1/2)*2^(1/2)*(-1/b)^(1/2)*(3*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/
2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+3*sin(a/b)*FresnelC(2^(1/2)/Pi^
(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-(-1/b)^(1/2)*(-3/b)^(1/2)*co
s(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/
b)*b-(-1/b)^(1/2)*(-3/b)^(1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\mathrm{acos}\,(cx)}}\,dx$$

input
```
integrate(x**2/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x^2}{\sqrt{b\arccos(cx)+a}}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{b\arccos(cx)+a}}{2\sqrt{b}}-\frac{i\sqrt{6}\sqrt{b\arccos(cx)+a}\sqrt{b}}{2\,|b|}\right)e^{\left(\frac{3i\,a}{b}\right)}}{4\left(\sqrt{6}\sqrt{b}+\frac{i\sqrt{6}b^{\frac{3}{2}}}{|b|}\right)c^3} + \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(\frac{i\,a}{b}\right)}}{4\,c^3\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(-\frac{i\,a}{b}\right)}}{4\,c^3\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{b\arccos(cx)+a}}{2\sqrt{b}}+\frac{i\sqrt{6}\sqrt{b\arccos(cx)+a}\sqrt{b}}{2\,|b|}\right)e^{\left(-\frac{3i\,a}{b}\right)}}{4\left(\sqrt{6}\sqrt{b}-\frac{i\sqrt{6}b^{\frac{3}{2}}}{|b|}\right)c^3}$$

input

```
integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output

```
1/4*I*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sq
rt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*sqrt(b
) + I*sqrt(6)*b^(3/2)/abs(b))*c^3) + 1/4*I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqr
t(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sq
rt(abs(b))/b)*e^(I*a/b)/(c^3*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(
b)))) - 1/4*I*sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(
b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c^3*
(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - 1/4*I*sqrt(pi)*erf(-
1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(
c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*sqrt(b) - I*sqrt(6)*b^(3/
2)/abs(b))*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\mathrm{acos}\,(c\,x)}}\,dx$$

input
```
int(x^2/(a + b*acos(c*x))^(1/2),x)
```

output
```
int(x^2/(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{\sqrt{acos\,(cx)\,b+a}\,x^2}{acos\,(cx)\,b+a}dx$$

input
```
int(x^2/(a+b*acos(c*x))^(1/2),x)
```

output
```
int((sqrt(acos(c*x)*b + a)*x**2)/(acos(c*x)*b + a),x)
```

3.189 $\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result 1339
Mathematica [A] (verified) 1339
Rubi [A] (verified) 1340
Maple [A] (verified) 1343
Fricas [F(-2)] 1344
Sympy [F] 1344
Maxima [F] 1344
Giac [C] (verification not implemented) 1345
Mupad [F(-1)] 1345
Reduce [F] 1346

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx=-\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{2\sqrt{b}c^2}$$

output
```
-1/2*Pi^(1/2)*cos(2*a/b)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/
2))/b^(1/2)/c^2+1/2*Pi^(1/2)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi
^(1/2))*sin(2*a/b)/b^(1/2)/c^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx=\frac{\sqrt{\pi}\left(-\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)\right)}{2\sqrt{b}c^2}$$

input
```
Integrate[x/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(Sqrt[Pi]*(-(Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sq
rt[Pi])]) + FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(
2*a)/b]))/(2*Sqrt[b]*c^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5147, 25, 4906, 27, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}dx$$

$$\Big\downarrow \text{ 5147}$$

$$-\frac{\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2bc^2}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{2a}{b}\right)\int \cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}}{2bc^2}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{2\cos\left(\frac{2a}{b}\right)\int \sin\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{2a}{b}\right)\int \cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}}{2bc^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-2\sin\left(\frac{2a}{b}\right)\int \cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}}{2bc^2}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2bc^2}$$

input
```
Int[x/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
-1/2*(Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(
Sqrt[b]*Sqrt[Pi])] - Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])
/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(b*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\left(\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos\left(cx\right)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)+\sin\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos\left(cx\right)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)\right)}{2c^{2}}$	91

input
```
int(x/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi^(1/2)*(-1/b)^(1/2)*(cos(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b))/c^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\mathrm{acos}\,(cx)}}\,dx$$

input
```
integrate(x/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x}{\sqrt{b\arccos(cx)+a}}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}+\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(-\frac{2ia}{b}\right)}}{4c^2\left(\sqrt{b}-\frac{ib^{\frac{3}{2}}}{|b|}\right)} + \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}-\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(\frac{2ia}{b}\right)}}{4\sqrt{b}c^2\left(\frac{ib}{|b|}+1\right)}$$

input
```
integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/4*I*sqrt(pi)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x
) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/(c^2*(sqrt(b) - I*b^(3/2)/abs(b))) + 1
/4*I*sqrt(pi)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x)
+ a)*sqrt(b)/abs(b))*e^(2*I*a/b)/(sqrt(b)*c^2*(I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int(x/(a + b*acos(c*x))^(1/2),x)
```

output
```
int(x/(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a + b \arccos(cx)}}\,dx = \int \frac{\sqrt{acos\,(cx)\,b + a}\,x}{acos\,(cx)\,b + a}dx$$

input
```
int(x/(a+b*acos(c*x))^(1/2),x)
```

output
```
int((sqrt(acos(c*x)*b + a)*x)/(acos(c*x)*b + a),x)
```

3.190 $\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result 1347
Mathematica [C] (verified) 1347
Rubi [A] (verified) 1348
Maple [A] (verified) 1351
Fricas [F(-2)] 1351
Sympy [F] 1352
Maxima [F] 1352
Giac [C] (verification not implemented) 1352
Mupad [F(-1)] 1353
Reduce [F] 1353

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}c}$$

output
```
-2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/
2)/b^(1/2))/b^(1/2)/c+2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(1/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{2c\sqrt{a+b\arccos(cx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/
b] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*
ArcCos[c*x]))/b])/(2*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$\Big\downarrow$ 3787

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$\Big\downarrow$ 25

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$\Big\downarrow$ 3042

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$\Big\downarrow$ 3785

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{bc}$$

$\Big\downarrow$ 3786

$$-\frac{2\cos\left(\frac{a}{b}\right)\int \sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{bc}$$

$\Big\downarrow$ 3832

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{bc}$$

$\Big\downarrow$ 3833

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{bc}$$

input
```
Int[1/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
-((Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*
x]])/Sqrt[b]]*Sin[a/b])/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\left(\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\right)}{c}$	89

input
```
int(1/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a+b*arccos(c*x))^(1/2)/b))/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{acos}\,(cx)}}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos\,(cx)+a}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(\frac{i\,a}{b}\right)}}{c\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(-\frac{i\,a}{b}\right)}}{c\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}$$

input
```
integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*s
qrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(I*sqrt(2)*b/s
qrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b
*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(
abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int(1/(a + b*acos(c*x))^(1/2),x)
```

output
```
int(1/(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)b+a}dx$$

input
```
int(1/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b + a),x)
```

3.191 $\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result 1354
Mathematica [N/A] 1354
Rubi [N/A] 1355
Maple [N/A] 1355
Fricas [F(-2)] 1356
Sympy [N/A] 1356
Maxima [N/A] 1356
Giac [N/A] 1357
Mupad [N/A] 1357
Reduce [N/A] 1358

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{a+b\arccos(cx)}},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arccos(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}dx$$
$$\Big\downarrow \text{5149}$$
$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}dx$$

input
```
Int[1/(x*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a+b\arccos\left(cx\right)}}dx$$

input
```
int(1/x/(a+b*arccos(c*x))^(1/2),x)
```

output
```
int(1/x/(a+b*arccos(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x\sqrt{a+b\operatorname{acos}(cx)}}\,dx$$

input
```
integrate(1/x/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos(cx)+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*arccos(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos(cx)+a}x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*arccos(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^(1/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)\,bx+ax}dx$$

input
```
int(1/x/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b*x + a*x),x)
```

3.192 $\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result 1359
Mathematica [N/A] 1359
Rubi [N/A] 1360
Maple [N/A] 1360
Fricas [F(-2)] 1361
Sympy [N/A] 1361
Maxima [N/A] 1361
Giac [N/A] 1362
Mupad [N/A] 1362
Reduce [N/A] 1363

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx=\text{Int}\left(\frac{1}{x^2\sqrt{a+b\arccos(cx)}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 5.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{a+b\arccos\left(cx\right)}}dx$$

input
```
int(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

output
```
int(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x^2\sqrt{a+b\,\mathrm{acos}\,(cx)}}\,dx$$

input
```
integrate(1/x**2/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos(cx)+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos{(cx)}+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)\,b\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b*x**2 + a*x**2),x)
```

3.193 $\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result 1364
Mathematica [C] (verified) 1365
Rubi [A] (verified) 1365
Maple [A] (verified) 1367
Fricas [F(-2)] 1368
Sympy [F] 1368
Maxima [F] 1368
Giac [F] 1369
Mupad [F(-1)] 1369
Reduce [F] 1369

Optimal result

Integrand size = 16, antiderivative size = 252

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx=\frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{3/2}c^3}$$

output
```
2*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(1/2)-1/2*2^(1/2)*Pi^(1/2)*
cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2
)/c^3-1/2*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arcco
s(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3-1/2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/P
i^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(3/2)/c^3-1/2*6^(1/2)*
Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*
a/b)/b^(3/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{e^{-\frac{3ia}{b}}\left(8c^2e^{\frac{3ia}{b}}x^2\sqrt{1-c^2x^2}+ie^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-\right.}{}$$

input
```
Integrate[x^2/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(8*c^2*E^(((3*I)*a)/b)*x^2*Sqrt[1 - c^2*x^2] + I*E^(((2*I)*a)/b)*Sqrt[((-I
)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - I*E^(
((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c
*x]))/b] + I*Sqrt[3]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*I)
*(a + b*ArcCos[c*x]))/b] - I*Sqrt[3]*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos
[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b])/(4*b*c^3*E^(((3*I)*a
)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx$$

$\Big\downarrow$ 5143

$$\frac{2\int \left(-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\Big\downarrow$ 2009

$$\frac{2\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{2}}\cdots\right.}{b^2c^3}$$

$$\frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[x^2/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*x^2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (2*(-1/2*(Sqrt[b
]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b
]]) - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b
*ArcCos[c*x]])/Sqrt[b]])/2 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt
[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Fresne
lS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2))/(b^2*c^
3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\sqrt{-\frac{3}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\cos\left(\frac{3a}{b}\right)\mathrm{FresnelC}\left(\frac{3\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,b}\right)\sqrt{a+b\arccos(cx)}-\sqrt{-\frac{3}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sin\left(\frac{3a}{b}\right)\mathrm{FresnelS}\left(\frac{3\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,b}\right)}{}$

input
```
int(x^2/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^3/b*((-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi
^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)-(-3
/b)^(1/2)*Pi^(1/2)*2^(1/2)*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)+(-1/b)^(1/2)*Pi^(1
/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*ar
ccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b)+sin(-(a+b*arccos(c*x))/b+a/b)+sin(-3*(a+b*arccos(c*x))/b
+3*a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(x**2/(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}(c\,x))^{3/2}}\,dx$$

input
```
int(x^2/(a + b*acos(c*x))^(3/2),x)
```

output
```
int(x^2/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{\sqrt{acos(cx)\,b+a}\,x^2}{acos(cx)^2\,b^2+2acos(cx)\,ab+a^2}dx$$

input
```
int(x^2/(a+b*acos(c*x))^(3/2),x)
```

output
```
int((sqrt(acos(c*x)*b + a)*x**2)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a*
*2),x)
```

3.194 $\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result 1370
Mathematica [F] 1370
Rubi [A] (verified) 1371
Maple [A] (verified) 1374
Fricas [F(-2)] 1375
Sympy [F] 1375
Maxima [F] 1375
Giac [F] 1376
Mupad [F(-1)] 1376
Reduce [F] 1376

Optimal result

Integrand size = 14, antiderivative size = 130

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}c^2}$$

output
```
2*x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(1/2)-2*Pi^(1/2)*cos(2*a/b)*F
resnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^2-2*Pi^(1/2)
*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c
^2
```

Mathematica [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[x/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
Integrate[x/(a + b*ArcCos[c*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5143, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}dx$$

$$\Big\downarrow \text{5143}$$

$$\frac{2\int -\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3787}$$

$$\frac{2\left(\sin\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3042

$$\frac{2\left(-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3785

$$\frac{2\left(-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\cos\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3786

$$\frac{2\left(-2\sin\left(\frac{2a}{b}\right)\int\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}-2\cos\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3832

$$\frac{2\left(\sqrt{\pi}\left(-\sqrt{b}\right)\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-2\cos\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3833

$$\frac{2\left(\sqrt{\pi}\left(-\sqrt{b}\right)\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\right)}{b^2c^2}+\frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[x/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*x*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (2*(-(Sqrt[b]*Sqrt
[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]
) - Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi
])]*Sin[(2*a)/b]))/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21

method	result
default	$-\frac{2\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)-2\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a \cdots}}{\sqrt{\pi} \cdots}\right) \cdots}{c^2 b\sqrt{a+b\arccos(cx)}}$

input
```
int(x/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2/b*(2*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*Fresn
elC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-2*(-1/b)^(1
/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2
)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(-2*(a+b*arccos(c*x))/b+2*a/b
))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(x/(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}(c\,x))^{3/2}}\,dx$$

input
```
int(x/(a + b*acos(c*x))^(3/2),x)
```

output
```
int(x/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{\sqrt{acos(cx)\,b+a}\,x}{acos(cx)^2\,b^2+2acos(cx)\,ab+a^2}dx$$

input
```
int(x/(a+b*acos(c*x))^(3/2),x)
```

output
```
int((sqrt(acos(c*x)*b + a)*x)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2)
,x)
```

3.195 $\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result 1377
Mathematica [C] (verified) 1378
Rubi [A] (verified) 1378
Maple [A] (verified) 1382
Fricas [F(-2)] 1382
Sympy [F] 1383
Maxima [F] 1383
Giac [F] 1383
Mupad [F(-1)] 1384
Reduce [F] 1384

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c}$$

output
```
2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(1/2)-2*2^(1/2)*Pi^(1/2)*cos(a/
b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-2*
2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2)
)*sin(a/b)/b^(3/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx =$$
$$-\frac{ie^{-\frac{ia}{b}}\left(2ie^{\frac{ia}{b}}\sqrt{1-c^2x^2}-\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-3/2),x]
```

output
```
((-I)*((2*I)*E^((I*a)/b)*Sqrt[1 - c^2*x^2] - Sqrt[((-I)*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(
a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]))/(b*c*E^((I*
a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5133, 5225, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5133}$$

$$\frac{2c\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3042

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3787

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c}$$

$\Big\downarrow$ 25

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c}$$

$\Big\downarrow$ 3042

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c}$$

$\Big\downarrow$ 3785

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c}$$

$\Big\downarrow$ 3786

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(2\sin\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcCos[c*x])^(-3/2),x]
```

output
```
(2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*(Sqrt[b]*Sqrt[2*P
i]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] + Sqrt[
b]*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a
/b]))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{2\left(\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}}{\cdots}\right.\right.}{cb\sqrt{a+b\arccos(cx)}}$

input
```
int(1/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/b*((-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*Fre
snelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-(-1/b)^(1/2
)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1
/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(-(a+b*arccos(c*x))/b+a/b))
/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{\left(a+b\operatorname{acos}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^{3/2}} \,d x$$

input
```
int(1/(a + b*acos(c*x))^(3/2),x)
```

output
```
int(1/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\frac{-2acos(cx)\left(\int \frac{\sqrt{acos(cx)b+a}\,\sqrt{-c^{2}x^{2}+1}\,x}{acos(cx)b\,c^{2}x^{2}-acos(cx)b+a\,c^{2}x^{2}-a}dx\right)b\,c^{2}+2\sqrt{acos\left(cx\right)b+a}\,\sqrt{-}}{bc\left(acos\left(cx\right)b+a\right)}$$

input
```
int(1/(a+b*acos(c*x))^(3/2),x)
```

output
```
(2*( - acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(aco
s(c*x)*b*c**2*x**2 - acos(c*x)*b + a*c**2*x**2 - a),x)*b*c**2 + sqrt(acos(
c*x)*b + a)*sqrt( - c**2*x**2 + 1) - int((sqrt(acos(c*x)*b + a)*sqrt( - c*
*2*x**2 + 1)*x)/(acos(c*x)*b*c**2*x**2 - acos(c*x)*b + a*c**2*x**2 - a),x)
*a*c**2))/(b*c*(acos(c*x)*b + a))
```

3.196 $\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result 1385
Mathematica [N/A] 1385
Rubi [N/A] 1386
Maple [N/A] 1386
Fricas [F(-2)] 1387
Sympy [N/A] 1387
Maxima [N/A] 1387
Giac [F(-2)] 1388
Mupad [N/A] 1388
Reduce [N/A] 1389

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arccos(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}} \, dx$$

$$\Big\downarrow \; 5149$$

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}dx$$

input
```
Int[1/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x \left(a+b \arccos \left(c x \right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+b*arccos(c*x))^(3/2),x)
```

output
```
int(1/x/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(1/(x*(a + b*acos(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccos(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^(3/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)^{2}b^{2}x+2acos\left(cx\right)abx+a^{2}x}dx$$

input
```
int(1/x/(a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x + a**2*
x),x)
```

3.197 $\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result 1390
Mathematica [N/A] 1390
Rubi [N/A] 1391
Maple [N/A] 1391
Fricas [F(-2)] 1392
Sympy [N/A] 1392
Maxima [N/A] 1392
Giac [N/A] 1393
Mupad [N/A] 1393
Reduce [N/A] 1394

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}dx$$

input
```
Int[1/(x^2*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

output
```
int(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x^2\,(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*acos(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccos(c*x) + a)^(3/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{(b\arccos(cx)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*arccos(c*x) + a)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)^2 b^2x^2+2acos\left(cx\right)ab\,x^2+a^2x^2}dx$$

input
```
int(1/x^2/(a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*x**2 + 2*acos(c*x)*a*b*x**2 +
 a**2*x**2),x)
```

3.198 $\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result 1395
Mathematica [C] (verified) 1396
Rubi [A] (verified) 1397
Maple [B] (verified) 1403
Fricas [F(-2)] 1404
Sympy [F] 1405
Maxima [F] 1405
Giac [F] 1405
Mupad [F(-1)] 1406
Reduce [F] 1406

Optimal result

Integrand size = 16, antiderivative size = 292

$$\begin{aligned}\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx &= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}-\frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}}\\ &+\frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}}+\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3}\\ &+\frac{\sqrt{6\pi}\cos\left(\frac{3a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3}\\ &-\frac{\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c^3}\\ &-\frac{\sqrt{6\pi}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{5/2}c^3}\end{aligned}$$

output
```
2/3*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(3/2)-8/3*x/b^2/c^2/(a+b*
arccos(c*x))^(1/2)+4*x^3/b^2/(a+b*arccos(c*x))^(1/2)+1/3*2^(1/2)*Pi^(1/2)*
cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(5/2
)/c^3+6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)/b^(1/2))/b^(5/2)/c^3-1/3*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1
/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(5/2)/c^3-6^(1/2)*Pi^(1/2)
*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(
5/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx =$$

$$-\frac{-b\sqrt{1-c^2x^2}-(a+b\arccos(cx))\left(e^{-i\arccos(cx)}+e^{i\arccos(cx)}-e^{-\frac{ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx)}{b}\right.\right.}{}$$

input
```
Integrate[x^2/(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
-1/6*(-(b*Sqrt[1 - c^2*x^2]) - (a + b*ArcCos[c*x])*(E^((-I)*ArcCos[c*x]) +
 E^(I*ArcCos[c*x]) - (Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*
(a + b*ArcCos[c*x]))/b])/E^((I*a)/b) - E^((I*a)/b)*Sqrt[(I*(a + b*ArcCos[c
*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]) - 3*(a + b*ArcCos[c*x])*(E
^((-3*I)*ArcCos[c*x]) + E^((3*I)*ArcCos[c*x]) - (Sqrt[3]*Sqrt[((-I)*(a + b
*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b])/E^(((3*I)*a)
/b) - Sqrt[3]*E^(((3*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (
(3*I)*(a + b*ArcCos[c*x]))/b]) - b*Sin[3*ArcCos[c*x]])/(b^2*c^3*(a + b*Arc
Cos[c*x])^(3/2))
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.46, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5145, 5223, 5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{4\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}dx}{3bc}+\frac{2c\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}dx}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}-\frac{4\left(\frac{2x}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{1}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{3bc}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5135}$$

$$-\frac{4\left(\frac{2\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\left(\frac{2x}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{4\left(\frac{2x}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3787}$$

$$-\frac{4\left(\frac{2\left(-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int-\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4\left(\frac{2\left(\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{4\left(\frac{2\left(\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3785

$$-\frac{4\left(\frac{2\left(\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3786

$$-\frac{4\left(\frac{2\left(2\cos\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3832

$$-\frac{4\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3833

$$\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{b}-\frac{4\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \; 5147$$

$$\frac{2c\left(\frac{6\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^4}+\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}\right)}{b}-\frac{4\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^4}\right)}{b}-\frac{4\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$\frac{2c\left(\frac{2x^3}{bc\sqrt{a+b\arccos(cx)}}-\frac{6\int\left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^4}\right)}{b}-\frac{4\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2c\left(\frac{6\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\right.\right.}{b^2c^4}\right.}{b}$$

$$-\frac{4\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3bc}+\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

input
```
Int[x^2/(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
(2*x^2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - (4*((2*x)/(b
*c*Sqrt[a + b*ArcCos[c*x]]) + (2*(Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sq
rt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(
Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b]))/(b^2*c^2)))/(3*b*c
) + (2*c*((2*x^3)/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (6*((Sqrt[b]*Sqrt[Pi/2]*
Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/2 + (Sqrt
[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/
Sqrt[b]])/2 - (Sqrt[b]*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c
*x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt
[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2))/(b^2*c^4)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(236) = 472.

Time = 0.24 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.30

method	result
default	$\frac{-6\arccos(cx)\sqrt{-\frac{3}{b}}\,\sqrt{2}\,\sqrt{\pi}\,\cos\left(\frac{3a}{b}\right)\operatorname{FresnelS}\left(\frac{3\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,b}\right)\sqrt{a+b\arccos(cx)}\,b-6\arccos(cx)\sqrt{-\frac{3}{b}}\,\sqrt{2}\,\sqrt{\pi}\,\sin\left(\frac{3a}{b}\right)\text{Fresne}}{}$

input
```
int(x^2/(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/c^3/b^2*(-6*arccos(c*x)*(-3/b)^(1/2)*2^(1/2)*Pi^(1/2)*cos(3*a/b)*Fresn
elS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos
(c*x))^(1/2)*b-6*arccos(c*x)*(-3/b)^(1/2)*2^(1/2)*Pi^(1/2)*sin(3*a/b)*Fres
nelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arcco
s(c*x))^(1/2)*b-2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b)*b-2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))
^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(
1/2)/b)*b-6*(-3/b)^(1/2)*2^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi
^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)*a-6
*(-3/b)^(1/2)*2^(1/2)*Pi^(1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)*a-2*2^(1/2)*Pi
^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(
1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a-2*2^(1/2)*Pi^(1/2)*(-1/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/
2)*(a+b*arccos(c*x))^(1/2)/b)*a+2*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b
)*b+6*arccos(c*x)*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*b-sin(-(a+b*arccos(c*x
))/b+a/b)*b+2*cos(-(a+b*arccos(c*x))/b+a/b)*a-sin(-3*(a+b*arccos(c*x))/b+3
*a/b)*b+6*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a)/(a+b*arccos(c*x))^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}\,(cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral(x**2/(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*arccos(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*arccos(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{x^2}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{5/2}}\,dx$$

input
```
int(x^2/(a + b*acos(c*x))^(5/2),x)
```

output
```
int(x^2/(a + b*acos(c*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}\,x^2}{acos\left(cx\right)^3 b^3+3acos\left(cx\right)^2 a\,b^2+3acos\left(cx\right)a^2b+a^3}dx$$

input
```
int(x^2/(a+b*acos(c*x))^(5/2),x)
```

output
```
int((sqrt(acos(c*x)*b + a)*x**2)/(acos(c*x)**3*b**3 + 3*acos(c*x)**2*a*b**
2 + 3*acos(c*x)*a**2*b + a**3),x)
```

3.199 $\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result 1407
Mathematica [F] 1408
Rubi [A] (verified) 1408
Maple [B] (verified) 1414
Fricas [F(-2)] 1414
Sympy [F] 1415
Maxima [F] 1415
Giac [F] 1415
Mupad [F(-1)] 1416
Reduce [F] 1416

Optimal result

Integrand size = 14, antiderivative size = 180

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{8\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{3b^{5/2}c^2}$$

output

```
2/3*x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(3/2)-4/3/b^2/c^2/(a+b*arcc
os(c*x))^(1/2)+8/3*x^2/b^2/(a+b*arccos(c*x))^(1/2)+8/3*Pi^(1/2)*cos(2*a/b)
*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(5/2)/c^2-8/3*Pi^(
1/2)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(5/
2)/c^2
```

Mathematica [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx$$

input
```
Integrate[x/(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
Integrate[x/(a + b*ArcCos[c*x])^(5/2), x]
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5145, 5153, 5223, 5147, 25, 4906, 27, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx$$

$$\Big\downarrow \text{5145}$$

$$-\frac{2\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}dx}{3bc}+\frac{4c\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}dx}{3b}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{4c\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}dx}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{4c\left(\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}-\frac{4\int \frac{x}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5147}$$

$$\frac{4c\left(\frac{4\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4c\left(\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}-\frac{4\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{4c\left(\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}-\frac{4\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4c\left(\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4c\left(\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{ 3787}$$

$$\frac{4c\left(\frac{2\left(-\sin\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int-\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}$$

$$\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{4c\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}-$$

$$\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{4c\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}$$

$$\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3785

$$\frac{4c\left(\frac{2\left(\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}-$$

$$\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3786

$$\frac{4c\left(\frac{2\left(2\cos\left(\frac{2a}{b}\right)\int\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\big\downarrow$ 3832

$$\frac{4c\left(\frac{2\left(\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-2\sin\left(\frac{2a}{b}\right)\int\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\big\downarrow$ 3833

$$\frac{4c\left(\frac{2\left(\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\right)}{b^2c^3}+\frac{2x^2}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}-\frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

input
```
Int[x/(a + b*ArcCos[c*x])^(5/2),x]
```

output
```
(2*x*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - 4/(3*b^2*c^2*S
qrt[a + b*ArcCos[c*x]]) + (4*c*((2*x^2)/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (2
*(Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt
[b]*Sqrt[Pi])] - Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sq
rt[b]*Sqrt[Pi])]*Sin[(2*a)/b]))/(b^2*c^3)))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5145
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Simp[c*((m + 1)/(b*(n + 1)))   Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/
Sqrt[1 - c^2*x^2]), x], x] + Simp[m/(b*c*(n + 1))   Int[x^(m - 1)*((a + b*A
rcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && I
GtQ[m, 0] && LtQ[n, -2]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(142) = 284.

Time = 0.15 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.89

method	result
default	$\frac{-8\arccos(cx)\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)b-8\arccos(cx)\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin(}{}$

input
```
int(x/(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c^2/b^2*(-8*arccos(c*x)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*
cos(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2
)/b)*b-8*arccos(c*x)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a
/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b-
8*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelS(2*2^(1
/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a-8*(-1/b)^(1/2)*Pi^(
1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)
^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a+4*arccos(c*x)*cos(-2*(a+b*arccos(c*x))
/b+2*a/b)*b-sin(-2*(a+b*arccos(c*x))/b+2*a/b)*b+4*cos(-2*(a+b*arccos(c*x))
/b+2*a/b)*a)/(a+b*arccos(c*x))^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(a+b\operatorname{acos}(cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral(x/(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*arccos(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(b*arccos(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{5/2}}\,dx$$

input
```
int(x/(a + b*acos(c*x))^(5/2),x)
```

output
```
int(x/(a + b*acos(c*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}\,x}{acos\left(cx\right)^{3}b^{3}+3acos\left(cx\right)^{2}a\,b^{2}+3acos\left(cx\right)a^{2}b+a^{3}}dx$$

input
```
int(x/(a+b*acos(c*x))^(5/2),x)
```

output
```
int((sqrt(acos(c*x)*b + a)*x)/(acos(c*x)**3*b**3 + 3*acos(c*x)**2*a*b**2 +
 3*acos(c*x)*a**2*b + a**3),x)
```

3.200 $\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result 1417
Mathematica [F] 1418
Rubi [A] (verified) 1418
Maple [B] (verified) 1422
Fricas [F(-2)] 1423
Sympy [F] 1423
Maxima [F] 1424
Giac [F] 1424
Mupad [F(-1)] 1424
Reduce [F] 1425

Optimal result

Integrand size = 12, antiderivative size = 163

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c}$$

output
```
2/3*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(3/2)+4/3*x/b^2/(a+b*arccos(c
*x))^(1/2)+4/3*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*ar
ccos(c*x))^(1/2)/b^(1/2))/b^(5/2)/c-4/3*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/
Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(5/2)/c
```

Mathematica [F]

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-5/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(-5/2), x]
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5133, 5223, 5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{2c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}dx}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5223}$$

$$\frac{2c\left(\frac{2x}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{1}{\sqrt{a+b\arccos(cx)}}dx}{bc}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{5135}$$

$$\frac{2c\left(\frac{2\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2c\left(\frac{2x}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{2c\left(\frac{2x}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\downarrow$ 3787

$$\frac{2c\left(\frac{2\left(-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int-\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\downarrow$ 25

$$\frac{2c\left(\frac{2\left(\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{2c\left(\frac{2\left(\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\downarrow$ 3785

$$\frac{2c\left(\frac{2\left(\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3786

$$\frac{2c\left(\frac{2\left(2\cos\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3832

$$\frac{2c\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

$\Big\downarrow$ 3833

$$\frac{2c\left(\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c^2}+\frac{2x}{bc\sqrt{a+b\arccos(cx)}}\right)}{3b}+\frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^(-5/2),x]
```

output
```
(2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) + (2*c*((2*x)/(b*c
*Sqrt[a + b*ArcCos[c*x]]) + (2*(Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt
[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sq
rt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b]))/(b^2*c^2)))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(129) = 258$.

Time = 0.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.09

method	result
default	$\frac{-\frac{4 \arccos(cx)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)b}{3}-\frac{4 \arccos(cx)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{Fresn}}{3}}{}$

input
```
int(1/(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/c/b^2*(-2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1
/2)/b)*b-2*arccos(c*x)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/
2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)
/b)*b-2*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*Fre
snelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a-2*2^(1/2)
*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/P
i^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a+2*arccos(c*x)*cos(-(a+b*
arccos(c*x))/b+a/b)*b-sin(-(a+b*arccos(c*x))/b+a/b)*b+2*cos(-(a+b*arccos(c
*x))/b+a/b)*a)/(a+b*arccos(c*x))^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))^{5/2}}\,dx$$

input
```
int(1/(a + b*acos(c*x))^(5/2),x)
```

output
```
int(1/(a + b*acos(c*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = -\frac{2acos(cx)^2\left(\int \frac{\sqrt{acos(cx)b+a}\,\sqrt{-c^2x^2+1}\,x}{acos(cx)^2b^2c^2x^2-acos(cx)^2b^2+2acos(cx)ab\,c^2x^2-2acos(cx)ab+a^2c^2x^2-a^2}dx\right)b^2c^2}{3} - \frac{4acos}{}$$

input
```
int(1/(a+b*acos(c*x))^(5/2),x)
```

output
```
(2*( - acos(c*x)**2*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(
acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**
2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**2*c**2 - 2*acos(c*x)*in
t((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)**2*b**2*c**2
*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b +
a**2*c**2*x**2 - a**2),x)*a*b*c**2 + sqrt(acos(c*x)*b + a)*sqrt( - c**2*x*
*2 + 1) - int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)*
*2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos
(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*c**2))/(3*b*c*(acos(c*x)**2*b**
2 + 2*acos(c*x)*a*b + a**2))
```

3.201 $\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result 1426
Mathematica [N/A] 1426
Rubi [N/A] 1427
Maple [N/A] 1427
Fricas [F(-2)] 1428
Sympy [N/A] 1428
Maxima [N/A] 1428
Giac [F(-2)] 1429
Mupad [N/A] 1429
Reduce [N/A] 1430

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+b*arccos(c*x))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCos[c*x])^(5/2)),x]
```

output
```
Integrate[1/(x*(a + b*ArcCos[c*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}dx$$

input
```
Int[1/(x*(a + b*ArcCos[c*x])^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a+b*arccos(c*x))^(5/2),x)
```

output
```
int(1/x/(a+b*arccos(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 7.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral(1/(x*(a + b*acos(c*x))**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos\,(cx)+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccos(c*x) + a)^(5/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^{5/2}} \,d x$$

input
```
int(1/(x*(a + b*acos(c*x))^(5/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)^{3}b^{3}x+3acos\left(cx\right)^{2}a\,b^{2}x+3acos\left(cx\right)a^{2}bx+a^{3}x}dx$$

input
```
int(1/x/(a+b*acos(c*x))^(5/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)**3*b**3*x + 3*acos(c*x)**2*a*b**2*x +
 3*acos(c*x)*a**2*b*x + a**3*x),x)
```

3.202 $\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result 1431
Mathematica [N/A] 1431
Rubi [N/A] 1432
Maple [N/A] 1432
Fricas [F(-2)] 1433
Sympy [N/A] 1433
Maxima [N/A] 1433
Giac [N/A] 1434
Mupad [N/A] 1434
Reduce [N/A] 1435

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \mathrm{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^{5/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^(5/2)),x]
```

output
```
Integrate[1/(x^2*(a + b*ArcCos[c*x])^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}dx$$

input
```
Int[1/(x^2*(a + b*ArcCos[c*x])^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

output
```
int(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 13.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x^2\,(a+b\,\mathrm{acos}\,(cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b*acos(c*x))**(5/2),x)
```

output
```
Integral(1/(x**2*(a + b*acos(c*x))**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccos(c*x) + a)^(5/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*arccos(c*x) + a)^(5/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x^2\,(a+b\,\mathrm{acos}\,(c\,x))^{5/2}}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)^{3}b^{3}x^{2}+3acos\left(cx\right)^{2}a\,b^{2}x^{2}+3acos\left(cx\right)a^{2}b\,x^{2}+a^{3}x^{2}}dx$$

input
```
int(1/x^2/(a+b*acos(c*x))^(5/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)**3*b**3*x**2 + 3*acos(c*x)**2*a*b**2*
x**2 + 3*acos(c*x)*a**2*b*x**2 + a**3*x**2),x)
```

3.203 $\int (dx)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result 1436
Mathematica [C] (verified) 1436
Rubi [A] (verified) 1437
Maple [A] (verified) 1439
Fricas [A] (verification not implemented) 1440
Sympy [A] (verification not implemented) 1440
Maxima [F] 1441
Giac [F] 1441
Mupad [F(-1)] 1442
Reduce [F] 1442

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{20bd^{5/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{147c^{7/2}}$$

output
```
-20/147*b*d^2*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)/c^3-4/49*b*(d*x)^(5/2)*(-c^2*
x^2+1)^(1/2)/c+2/7*(d*x)^(7/2)*(a+b*arccos(c*x))/d+20/147*b*d^(5/2)*Ellipt
icF(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(7/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = \frac{2d^2\sqrt{dx}\left(-10b+4bc^2x^2+6bc^4x^4+21ac^3x^3\sqrt{1-c^2x^2}+21bc^3x^3\sqrt{1-c^2x^2}\arccos(cx\right.}{147c^3\sqrt{1-c^2x^2}}$$

input
```
Integrate[(d*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(2*d^2*Sqrt[d*x]*(-10*b + 4*b*c^2*x^2 + 6*b*c^4*x^4 + 21*a*c^3*x^3*Sqrt[1
- c^2*x^2] + 21*b*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + ((10*I)*b*Sqrt[1
 - 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sqrt[x]], -1])/S
qrt[-c^(-1)]))/(147*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5139, 262, 262, 266, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2bc\int \frac{(dx)^{7/2}}{\sqrt{1-c^2x^2}}dx}{7d}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2bc\left(\frac{5d^2\int \frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}}dx}{7c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{5/2}}{7c^2}\right)}{7d}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2bc\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2d\sqrt{1-c^2x^2}\sqrt{dx}}{3c^2}\right)}{7c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{5/2}}{7c^2}\right)}{7d}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bc\left(\frac{5d^2\left(\frac{2d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{3c^2}-\frac{2d\sqrt{1-c^2x^2}\sqrt{dx}}{3c^2}\right)}{7c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{5/2}}{7c^2}\right)}{7d}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{2bc\left(\frac{5d^2\left(\frac{2d^{3/2}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{3c^{5/2}}-\frac{2d\sqrt{1-c^2x^2}\sqrt{dx}}{3c^2}\right)}{7c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{5/2}}{7c^2}\right)}{7d}$$

input
```
Int[(d*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(2*(d*x)^(7/2)*(a + b*ArcCos[c*x]))/(7*d) + (2*b*c*((-2*d*(d*x)^(5/2)*Sqrt
[1 - c^2*x^2])/(7*c^2) + (5*d^2*((-2*d*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(3*c^2
) + (2*d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1])/(3*c^(5
/2))))/(7*c^2)))/(7*d)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\arccos(cx)}{7}+\frac{2c\left(-\frac{d^2(dx)^{\frac{5}{2}}\sqrt{-c^2x^2+1}}{7c^2}-\frac{5d^4\sqrt{dx}\,\sqrt{-c^2x^2+1}}{21c^4}+\frac{5d^4\sqrt{-cx+1}\,\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{21c^4\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{7d}\right)}{d}$
default	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\arccos(cx)}{7}+\frac{2c\left(-\frac{d^2(dx)^{\frac{5}{2}}\sqrt{-c^2x^2+1}}{7c^2}-\frac{5d^4\sqrt{dx}\,\sqrt{-c^2x^2+1}}{21c^4}+\frac{5d^4\sqrt{-cx+1}\,\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{21c^4\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{7d}\right)}{d}$
parts	$\frac{2a(dx)^{\frac{7}{2}}}{7d}+\frac{2b\left(\frac{(dx)^{\frac{7}{2}}\arccos(cx)}{7}+\frac{2c\left(-\frac{d^2(dx)^{\frac{5}{2}}\sqrt{-c^2x^2+1}}{7c^2}-\frac{5d^4\sqrt{dx}\,\sqrt{-c^2x^2+1}}{21c^4}+\frac{5d^4\sqrt{-cx+1}\,\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{21c^4\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{7d}\right)}{d}$

input
```
int((d*x)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/7*a*(d*x)^(7/2)+b*(1/7*(d*x)^(7/2)*arccos(c*x)+2/7*c/d*(-1/7/c^2*d^
2*(d*x)^(5/2)*(-c^2*x^2+1)^(1/2)-5/21/c^4*d^4*(d*x)^(1/2)*(-c^2*x^2+1)^(1/
2)+5/21/c^4*d^4/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2
)*EllipticF((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx =$$

$$-\frac{2\left(10\sqrt{-c^2d}bd^2\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)-\left(21\,bc^5d^2x^3\arccos\left(cx\right)+21\,ac^5d^2x^3-2\left(3\,bc^4d^2x^2+5\,b\right.\right.\right.}{147\,c^5}$$

input
```
integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-2/147*(10*sqrt(-c^2*d)*b*d^2*weierstrassPInverse(4/c^2, 0, x) - (21*b*c^5
*d^2*x^3*arccos(c*x) + 21*a*c^5*d^2*x^3 - 2*(3*b*c^4*d^2*x^2 + 5*b*c^2*d^2
)*sqrt(-c^2*x^2 + 1))*sqrt(d*x))/c^5
```

Sympy [A] (verification not implemented)

Time = 62.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = a\left(\begin{cases}\frac{2(dx)^{\frac{7}{2}}}{7d} & \text{for}\ d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}\frac{d^{\frac{5}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|c^2x^2e^{2i\pi}\right)}{7\Gamma\left(\frac{13}{4}\right)} & \text{for}\ d>-\infty\land d<\infty\land d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2(dx)^{\frac{7}{2}}}{7d} & \text{for}\ d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\operatorname{acos}\left(cx\right)$$

input
```
integrate((d*x)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
a*Piecewise((2*(d*x)**(7/2)/(7*d), Ne(d, 0)), (0, True)) + b*c*Piecewise((
d**(5/2)*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c**2*x**2*exp_pola
r(2*I*pi))/(7*gamma(13/4)), (d > -oo) & (d < oo) & Ne(d, 0)), (0, True)) +
 b*Piecewise((2*(d*x)**(7/2)/(7*d), Ne(d, 0)), (0, True))*acos(c*x)
```

Maxima [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx=\int (dx)^{\frac{5}{2}}\,(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/147*(42*b*c^4*d^(5/2)*x^(7/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
 - (12*b*c^4*d^2*x^(7/2) + 294*b*c^5*d^2*integrate(1/7*sqrt(c*x + 1)*sqrt(
-c*x + 1)*x^(7/2)/(c^2*x^2 - 1), x) + 28*b*c^2*d^2*x^(3/2) + 21*(2*b*d^2*a
rctan(sqrt(c)*sqrt(x)) + b*d^2*log((c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1)
))*sqrt(c))*sqrt(d))/c^4
```

Giac [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx=\int (dx)^{\frac{5}{2}}\,(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\ (d\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(d*x)^(5/2), x)
```

Reduce [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx=\frac{\sqrt{d}\,d^2\left(2\sqrt{x}\,a\,x^3+7\left(\int \sqrt{x}\,acos(cx)\,x^2dx\right)b\right)}{7}$$

input
```
int((d*x)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(2*sqrt(x)*a*x**3 + 7*int(sqrt(x)*acos(c*x)*x**2,x)*b))/7
```

3.204 $\int (dx)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result 1443
Mathematica [C] (verified) 1443
Rubi [A] (verified) 1444
Maple [A] (verified) 1447
Fricas [A] (verification not implemented) 1448
Sympy [A] (verification not implemented) 1448
Maxima [F] 1449
Giac [F] 1449
Mupad [F(-1)] 1450
Reduce [F] 1450

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx=-\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}+\frac{12bd^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}}-\frac{12bd^{3/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{25c^{5/2}}$$

output
```
-4/25*b*(d*x)^(3/2)*(-c^2*x^2+1)^(1/2)/c+2/5*(d*x)^(5/2)*(a+b*arccos(c*x))
/d+12/25*b*d^(3/2)*EllipticE(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(5/2)-12/25*
b*d^(3/2)*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx=\frac{2(dx)^{3/2}\left(5acx-2b\sqrt{1-c^2x^2}+5bcx\arccos(cx)+2b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x\right.\right.}{25c}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(2*(d*x)^(3/2)*(5*a*c*x - 2*b*Sqrt[1 - c^2*x^2] + 5*b*c*x*ArcCos[c*x] + 2*
b*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(25*c)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5139, 262, 266, 836, 27, 762, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2bc\int \frac{(dx)^{5/2}}{\sqrt{1-c^2x^2}}dx}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2bc\left(\frac{3d^2\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bc\left(\frac{6d\int \frac{dx}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{836}$$

$$\frac{2bc\left(\frac{6d\left(\frac{d\int \frac{cxd+d}{d\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}\right)}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2bc\left(\frac{6d\left(\frac{\int \frac{cxd+d}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}\right)}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{762}$$

$$\frac{2bc\left(\frac{6d\left(\frac{\int \frac{cxd+d}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{1389}$$

$$\frac{2bc\left(\frac{6d\left(\frac{d\int \frac{\sqrt{cx+1}}{\sqrt{1-cx}}d\sqrt{dx}}{c}-\frac{d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}$$

$$\Big\downarrow \text{327}$$

$$\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d}+\frac{2bc\left(\frac{6d\left(\frac{d^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{c^{3/2}}-\frac{d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{5c^2}-\frac{2d\sqrt{1-c^2x^2}(dx)^{3/2}}{5c^2}\right)}{5d}$$

input
```
Int[(d*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(2*(d*x)^(5/2)*(a + b*ArcCos[c*x]))/(5*d) + (2*b*c*((-2*d*(d*x)^(3/2)*Sqrt
[1 - c^2*x^2])/(5*c^2) + (6*d*((d^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x
])/Sqrt[d]], -1])/c^(3/2) - (d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x]])/
Sqrt[d]], -1])/c^(3/2)))/(5*c^2)))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\arccos(cx)}{5}+\frac{2c\left(-\frac{d^2(dx)^{\frac{3}{2}}\sqrt{-c^2x^2+1}}{5c^2}-\frac{3d^3\sqrt{-cx+1}\sqrt{cx+1}\left(\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\cdots\right.\right.}{5c^3\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\cdots}{5d}\cdots\right.}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\arccos(cx)}{5}+\frac{2c\left(-\frac{d^2(dx)^{\frac{3}{2}}\sqrt{-c^2x^2+1}}{5c^2}-\frac{3d^3\sqrt{-cx+1}\sqrt{cx+1}\left(\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\cdots\right.\right.}{5c^3\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\cdots}{5d}\cdots\right.}{d}$
parts	$\frac{2a(dx)^{\frac{5}{2}}}{5d}+\frac{2b\left(\frac{(dx)^{\frac{5}{2}}\arccos(cx)}{5}+\frac{2c\left(-\frac{d^2(dx)^{\frac{3}{2}}\sqrt{-c^2x^2+1}}{5c^2}-\frac{3d^3\sqrt{-cx+1}\sqrt{cx+1}\left(\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\sqrt{\cdots}\right.\right.}{5c^3\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\cdots}{5d}\cdots\right.}{d}$

input
```
int((d*x)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/5*(d*x)^(5/2)*a+b*(1/5*(d*x)^(5/2)*arccos(c*x)+2/5*c/d*(-1/5/c^2*d^
2*(d*x)^(3/2)*(-c^2*x^2+1)^(1/2)-3/5/c^3*d^3/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c
*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-Ellip
ticE((d*x)^(1/2)*(c/d)^(1/2),I)))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = \frac{2\left(6\sqrt{-c^2d}bd\text{weierstrassZeta}\left(\frac{4}{c^2},0,\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)\right)+\left(5\,bc^3dx^2\arccos\left(\right.\right.\right.}{25\,c^3}$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
2/25*(6*sqrt(-c^2*d)*b*d*weierstrassZeta(4/c^2, 0, weierstrassPInverse(4/c
^2, 0, x)) + (5*b*c^3*d*x^2*arccos(c*x) + 5*a*c^3*d*x^2 - 2*sqrt(-c^2*x^2
+ 1)*b*c^2*d*x)*sqrt(d*x))/c^3
```

Sympy [A] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = a\left(\begin{cases}\frac{2(dx)^{\frac{5}{2}}}{5d} & \text{for}\ d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}\frac{d^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle| c^2x^2e^{2i\pi}\right)}{5\Gamma\left(\frac{11}{4}\right)} & \text{for}\ d>-\infty\wedge d<\infty\wedge d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2(dx)^{\frac{5}{2}}}{5d} & \text{for}\ d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\operatorname{acos}(cx)$$

input
```
integrate((d*x)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
a*Piecewise((2*(d*x)**(5/2)/(5*d), Ne(d, 0)), (0, True)) + b*c*Piecewise((
d**(3/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c**2*x**2*exp_pola
r(2*I*pi))/(5*gamma(11/4)), (d > -oo) & (d < oo) & Ne(d, 0)), (0, True)) +
 b*Piecewise((2*(d*x)**(5/2)/(5*d), Ne(d, 0)), (0, True))*acos(c*x)
```

Maxima [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/25*(10*b*c^3*d^(3/2)*x^(5/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
- (4*b*c^3*d*x^(5/2) + 50*b*c^4*d*integrate(1/5*sqrt(c*x + 1)*sqrt(-c*x +
1)*x^(5/2)/(c^2*x^2 - 1), x) + 20*b*c*d*sqrt(x) - 5*(2*b*d*arctan(sqrt(c)*
sqrt(x)) - b*d*log((c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1)))*sqrt(c))*sqrt
(d))/c^3
```

Giac [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\ (d\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx=\frac{\sqrt{d}\,d\left(2\sqrt{x}\,a\,x^2+5\left(\int \sqrt{x}\,acos(cx)\,xdx\right)b\right)}{5}$$

input
```
int((d*x)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*(2*sqrt(x)*a*x**2 + 5*int(sqrt(x)*acos(c*x)*x,x)*b))/5
```

3.205 $\int \sqrt{dx}(a+b\arccos(cx))\,dx$

Optimal result 1451
Mathematica [C] (verified) 1452
Rubi [A] (verified) 1452
Maple [A] (verified) 1454
Fricas [A] (verification not implemented) 1455
Sympy [A] (verification not implemented) 1455
Maxima [F] 1456
Giac [F] 1456
Mupad [F(-1)] 1457
Reduce [F] 1457

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = -\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d} + \frac{4b\sqrt{d}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{9c^{3/2}}$$

output
```
-4/9*b*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)/c+2/3*(d*x)^(3/2)*(a+b*arccos(c*x))/
d+4/9*b*d^(1/2)*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = \frac{2}{9}\sqrt{dx}\left(3ax-\frac{2b\sqrt{1-c^2x^2}}{c}+3bx\arccos(cx) - \frac{2ib\sqrt{-\frac{1}{c}}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{x}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(2*Sqrt[d*x]*(3*a*x - (2*b*Sqrt[1 - c^2*x^2])/c + 3*b*x*ArcCos[c*x] - ((2*
I)*b*Sqrt[-c^(-1)]*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[
-c^(-1)]/Sqrt[x]], -1])/Sqrt[1 - c^2*x^2]))/9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5139, 262, 266, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2bc\int \frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}}dx}{3d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2bc\left(\frac{d^2\int \frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2d\sqrt{1-c^2x^2}\sqrt{dx}}{3c^2}\right)}{3d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bc\left(\frac{2d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{3c^2}-\frac{2d\sqrt{1-c^2x^2}\sqrt{dx}}{3c^2}\right)}{3d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}$$

$$\Big\downarrow \text{762}$$

$$\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{2bc\left(\frac{2d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{3c^{5/2}}-\frac{2d\sqrt{1-c^2x^2}\sqrt{dx}}{3c^2}\right)}{3d}$$

input
```
Int[Sqrt[d*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(2*(d*x)^(3/2)*(a + b*ArcCos[c*x]))/(3*d) + (2*b*c*((-2*d*Sqrt[d*x]*Sqrt[1
 - c^2*x^2])/(3*c^2) + (2*d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]], -1])/(3*c^(5/2))))/(3*d)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\arccos(cx)}{3}+\frac{2c\left(-\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{3c^2}+\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{3c^2\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	119
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\arccos(cx)}{3}+\frac{2c\left(-\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{3c^2}+\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{3c^2\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	119
parts	$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b\left(\frac{(dx)^{\frac{3}{2}}\arccos(cx)}{3}+\frac{2c\left(-\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{3c^2}+\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{3c^2\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	121

input
```
int((d*x)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/3*(d*x)^(3/2)*a+b*(1/3*(d*x)^(3/2)*arccos(c*x)+2/3*c/d*(-1/3/c^2*d^
2*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)+1/3/c^2*d^2/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c
*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = -\frac{2\left(2\sqrt{-c^2d}b\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)-\left(3\,bc^3x\arccos\left(cx\right)+3\,ac^3x-2\sqrt{-c^2x^2+1}bc^2\right)\sqrt{dx}\right)}{9\,c^3}$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-2/9*(2*sqrt(-c^2*d)*b*weierstrassPInverse(4/c^2, 0, x) - (3*b*c^3*x*arcco
s(c*x) + 3*a*c^3*x - 2*sqrt(-c^2*x^2 + 1)*b*c^2)*sqrt(d*x))/c^3
```

Sympy [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = a\left(\begin{cases}\frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}\frac{\sqrt{d}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|c^2x^2e^{2i\pi}\right)}{3\Gamma\left(\frac{9}{4}\right)} & \text{for } d>-\infty\wedge d<\infty\wedge d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\operatorname{acos}\left(cx\right)$$

input
```
integrate((d*x)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
a*Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)), (0, True)) + b*c*Piecewise((
sqrt(d)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c**2*x**2*exp_polar(
2*I*pi))/(3*gamma(9/4)), (d > -oo) & (d < oo) & Ne(d, 0)), (0, True)) + b*
Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)), (0, True))*acos(c*x)
```

Maxima [F]

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = \int \sqrt{dx}(b\arccos(cx)+a)\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/9*(6*b*c^2*sqrt(d)*x^(3/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) -
(18*b*c^3*integrate(1/3*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)/(c^2*x^2 - 1)
, x) + 4*b*c^2*x^(3/2) + 3*(2*b*arctan(sqrt(c)*sqrt(x)) + b*log((c*x - 1)/
(c*x + 2*sqrt(c)*sqrt(x) + 1)))*sqrt(c))*sqrt(d))/c^2
```

Giac [F]

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = \int \sqrt{dx}(b\arccos(cx)+a)\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*acos(c*x))*(d*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx=\frac{\sqrt{d}\,\left(2\sqrt{x}\,ax+3\left(\int \sqrt{x}\,acos(cx)\,dx\right)b\right)}{3}$$

input
```
int((d*x)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(2*sqrt(x)*a*x + 3*int(sqrt(x)*acos(c*x),x)*b))/3
```

3.206 $\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx$

Optimal result 1458
Mathematica [C] (verified) 1458
Rubi [A] (verified) 1459
Maple [A] (verified) 1461
Fricas [A] (verification not implemented) 1462
Sympy [F(-2)] 1462
Maxima [F] 1463
Giac [F] 1463
Mupad [F(-1)] 1463
Reduce [F] 1464

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} + \frac{4bE\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}} - \frac{4b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{\sqrt{c}\sqrt{d}}$$

output
```
2*(d*x)^(1/2)*(a+b*arccos(c*x))/d+4*b*EllipticE(c^(1/2)*(d*x)^(1/2)/d^(1/2
),I)/c^(1/2)/d^(1/2)-4*b*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(1/2)/
d^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2x\left(3(a+b\arccos(cx)) + 2bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)\right)}{3\sqrt{dx}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d*x],x]
```

output
```
(2*x*(3*(a + b*ArcCos[c*x]) + 2*b*c*x*Hypergeometric2F1[1/2, 3/4, 7/4, c^2
*x^2]))/(3*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5139, 266, 836, 27, 762, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2bc\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}dx}{d}+\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{4bc\int \frac{dx}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{d^2}+\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}$$

$$\Big\downarrow \text{836}$$

$$\frac{4bc\left(\frac{d\int \frac{cxd+d}{d\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}\right)}{d^2}+\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{4bc\left(\frac{\int \frac{cxd+d}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}\right)}{d^2}+\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}$$

$$\Big\downarrow \text{762}$$

$$\frac{4bc\left(\frac{\int \frac{cxd+d}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{d^2}+\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}$$

$$\Big\downarrow \; 1389$$

$$\frac{4bc\left(\frac{d\int \frac{\sqrt{cx+1}}{\sqrt{1-cx}}d\sqrt{dx}}{c}-\frac{d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{d^2}+\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}$$

$$\Big\downarrow \; 327$$

$$\frac{2\sqrt{dx}(a+b\arccos(cx))}{d}+\frac{4bc\left(\frac{d^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{c^{3/2}}-\frac{d^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{d^2}$$

input
```
Int[(a + b*ArcCos[c*x])/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*ArcCos[c*x]))/d + (4*b*c*((d^(3/2)*EllipticE[ArcSin[(S
qrt[c]*Sqrt[d*x])/Sqrt[d]], -1])/c^(3/2) - (d^(3/2)*EllipticF[ArcSin[(Sqrt
[c]*Sqrt[d*x])/Sqrt[d]], -1])/c^(3/2)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\arccos(cx)-\frac{2\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	98
default	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\arccos(cx)-\frac{2\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	98
parts	$\frac{2a\sqrt{dx}}{d}+\frac{2b\left(\sqrt{dx}\,\arccos(cx)-\frac{2\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\operatorname{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	101

input
```
int((a+b*arccos(c*x))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x)^(1/2)*a+b*((d*x)^(1/2)*arccos(c*x)-2/(c/d)^(1/2)*(-c*x+1)^(1/2)
*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-El
lipticE((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2\left(2\sqrt{-c^2d}b\text{weierstrassZeta}\left(\frac{4}{c^2},0,\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)\right)+(bc\arccos(cx)+ac)\sqrt{dx}\right)}{cd}$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2*(2*sqrt(-c^2*d)*b*weierstrassZeta(4/c^2, 0, weierstrassPInverse(4/c^2, 0
, x)) + (b*c*arccos(c*x) + a*c)*sqrt(d*x))/(c*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))/(d*x)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a + b\arccos(cx)}{\sqrt{dx}}\,dx = \int \frac{b\arccos{(cx)} + a}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
(2*b*c*sqrt(d)*sqrt(x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (2*b*c
^2*d*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(c^2*d*x^2 - d), x) +
4*b*c*sqrt(x) - (2*b*arctan(sqrt(c)*sqrt(x)) - b*log((c*x - 1)/(c*x + 2*sq
rt(c)*sqrt(x) + 1)))*sqrt(c))*sqrt(d))/(c*d)
```

Giac [F]

$$\int \frac{a + b\arccos(cx)}{\sqrt{dx}}\,dx = \int \frac{b\arccos{(cx)} + a}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arccos(cx)}{\sqrt{dx}}\,dx = \int \frac{a + b\,\mathrm{acos}(c\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*acos(c*x))/(d*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,a+\left(\int \frac{acos(cx)}{\sqrt{x}}dx\right)b}{\sqrt{d}}$$

input
```
int((a+b*acos(c*x))/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*a + int(acos(c*x)/sqrt(x),x)*b)/sqrt(d)
```

3.207 $\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx$

Optimal result 1465
Mathematica [C] (verified) 1465
Rubi [A] (verified) 1466
Maple [A] (verified) 1467
Fricas [A] (verification not implemented) 1468
Sympy [F(-2)] 1468
Maxima [F] 1468
Giac [F] 1469
Mupad [F(-1)] 1469
Reduce [F] 1469

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = -\frac{2(a+b\arccos(cx))}{d\sqrt{dx}} - \frac{4b\sqrt{c}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{d^{3/2}}$$

output
```
(-2*a-2*b*arccos(c*x))/d/(d*x)^(1/2)-4*b*c^(1/2)*EllipticF(c^(1/2)*(d*x)^(
1/2)/d^(1/2),I)/d^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \frac{2x\left(-a-b\arccos(cx)+\frac{2ib\sqrt{-\frac{1}{c}}c^2\sqrt{1-\frac{1}{c^2x^2}}x^{3/2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{1-c^2x^2}}\right)}{(dx)^{3/2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d*x)^(3/2),x]
```

output
```
(2*x*(-a - b*ArcCos[c*x] + ((2*I)*b*Sqrt[-c^(-1)]*c^2*Sqrt[1 - 1/(c^2*x^2)
]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sqrt[x]], -1])/Sqrt[1 - c^2*x^
2]))/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5139, 266, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{2bc\int \frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}dx}{d}-\frac{2(a+b\arccos(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{4bc\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{d^2}-\frac{2(a+b\arccos(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{2(a+b\arccos(cx))}{d\sqrt{dx}}-\frac{4b\sqrt{c}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{d^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcCos[c*x]))/(d*Sqrt[d*x]) - (4*b*Sqrt[c]*EllipticF[ArcSin[(Sq
rt[c]*Sqrt[d*x])/Sqrt[d]], -1])/d^(3/2)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-\frac{2c\sqrt{-cx+1}\,\sqrt{cx+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	85
default	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-\frac{2c\sqrt{-cx+1}\,\sqrt{cx+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	85
parts	$-\frac{2a}{\sqrt{dx}\,d}+\frac{2b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-\frac{2c\sqrt{-cx+1}\,\sqrt{cx+1}\,\operatorname{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	87

input
```
int((a+b*arccos(c*x))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-a/(d*x)^(1/2)+b*(-1/(d*x)^(1/2)*arccos(c*x)-2*c/d/(c/d)^(1/2)*(-c*x+
1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2
),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \frac{2\left(2\sqrt{-c^2d}bx\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)-(bc\arccos{(cx)}+ac)\sqrt{dx}\right)}{cd^2x}$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2*(2*sqrt(-c^2*d)*b*x*weierstrassPInverse(4/c^2, 0, x) - (b*c*arccos(c*x)
+ a*c)*sqrt(d*x))/(c*d^2*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))/(d*x)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \int \frac{b\arccos{(cx)}+a}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-(2*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (2*b*c*d^2*integrate(sq
rt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(c^2*d^2*x^3 - d^2*x), x) - (2*b*arctan
(1/(sqrt(c)*sqrt(x))) - b*log(-(c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1))*s
qrt(c))*sqrt(x))/(d^(3/2)*sqrt(x))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx=\int \frac{b\arccos(cx)+a}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx=\frac{\sqrt{x}\left(\int \frac{acos(cx)}{\sqrt{x}\,x}dx\right)b-2a}{\sqrt{x}\,\sqrt{d}\,d}$$

input
```
int((a+b*acos(c*x))/(d*x)^(3/2),x)
```

output
```
(sqrt(x)*int(acos(c*x)/(sqrt(x)*x),x)*b - 2*a)/(sqrt(x)*sqrt(d)*d)
```

3.208 $\int \frac{a+b \arccos(cx)}{(dx)^{5/2}} \, dx$

Optimal result 1470
Mathematica [C] (verified) 1470
Rubi [A] (verified) 1471
Maple [A] (verified) 1474
Fricas [A] (verification not implemented) 1475
Sympy [F(-2)] 1475
Maxima [F] 1475
Giac [F] 1476
Mupad [F(-1)] 1476
Reduce [F] 1477

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{a+b \arccos(cx)}{(dx)^{5/2}} \, dx = \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} + \frac{4bc^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}} - \frac{4bc^{3/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{3d^{5/2}}$$

output
```
4/3*b*c*(-c^2*x^2+1)^(1/2)/d^2/(d*x)^(1/2)-2/3*(a+b*arccos(c*x))/d/(d*x)^(
3/2)+4/3*b*c^(3/2)*EllipticE(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/d^(5/2)-4/3*b*
c^(3/2)*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/d^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{a+b \arccos(cx)}{(dx)^{5/2}} \, dx = \frac{2x\left(-3\left(a-2bcx\sqrt{1-c^2x^2}+b\arccos(cx)\right)+2bc^3x^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\right.\right.}{9(dx)^{5/2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d*x)^(5/2),x]
```

output
```
(2*x*(-3*(a - 2*b*c*x*Sqrt[1 - c^2*x^2] + b*ArcCos[c*x]) + 2*b*c^3*x^3*Hyp
ergeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(9*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5139, 264, 266, 836, 27, 762, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{2bc\int \frac{1}{(dx)^{3/2}\sqrt{1-c^2x^2}}dx}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2bc\left(-\frac{c^2\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}dx}{d^2}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2bc\left(-\frac{2c^2\int \frac{dx}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{d^3}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{836}$$

$$-\frac{2bc\left(-\frac{2c^2\left(\frac{d\int \frac{cxd+d}{d\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}\right)}{d^3}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2bc\left(-\frac{2c^2\left(\frac{\int \frac{cxd+d}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d\int \frac{1}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}\right)}{d^3}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{2bc\left(-\frac{2c^2\left(\frac{\int \frac{cxd+d}{\sqrt{1-c^2x^2}}d\sqrt{dx}}{c}-\frac{d^{3/2}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{d^3}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{1389}$$

$$-\frac{2bc\left(-\frac{2c^2\left(\frac{d\int \frac{\sqrt{cx+1}}{\sqrt{1-cx}}d\sqrt{dx}}{c}-\frac{d^{3/2}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{d^3}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}}-\frac{2bc\left(-\frac{2c^2\left(\frac{d^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{c^{3/2}}-\frac{d^{3/2}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{c^{3/2}}\right)}{d^3}-\frac{2\sqrt{1-c^2x^2}}{d\sqrt{dx}}\right)}{3d}$$

input
```
Int[(a + b*ArcCos[c*x])/(d*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcCos[c*x]))/(3*d*(d*x)^(3/2)) - (2*b*c*((-2*Sqrt[1 - c^2*x^2]
)/(d*Sqrt[d*x]) - (2*c^2*((d^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sq
rt[d]], -1])/c^(3/2) - (d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[
d]], -1])/c^(3/2)))/d^3))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\arccos(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2c\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$
default	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\arccos(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2c\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$
parts	$-\frac{2a}{3(dx)^{\frac{3}{2}}\,d}+\frac{2b\left(-\frac{\arccos(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2c\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$

input
```
int((a+b*arccos(c*x))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/3*a/(d*x)^(3/2)+b*(-1/3/(d*x)^(3/2)*arccos(c*x)-2/3*c/d*(-(-c^2*x^
2+1)^(1/2)/(d*x)^(1/2)+c/d/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*
x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-EllipticE((d*x)^(1/2)*(
c/d)^(1/2),I)))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \frac{2\left(2\sqrt{-c^2d}bcx^2\text{weierstrassZeta}\left(\frac{4}{c^2},0,\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)\right)+\left(2\sqrt{-c^2x\right.\right.}{3\,d^3x^2}$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(-c^2*d)*b*c*x^2*weierstrassZeta(4/c^2, 0, weierstrassPInverse(
4/c^2, 0, x)) + (2*sqrt(-c^2*x^2 + 1)*b*c*x - b*arccos(c*x) - a)*sqrt(d*x)
)/(d^3*x^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))/(d*x)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \int \frac{b\arccos(cx)+a}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(2*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (6*b*c*d^3*x*integr
ate(1/3*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(c^2*d^3*x^4 - d^3*x^2), x) +
 (2*b*c*x*arctan(1/(sqrt(c)*sqrt(x))) + b*c*x*log(-(c*x - 1)/(c*x + 2*sqrt
(c)*sqrt(x) + 1)))*sqrt(c))*sqrt(x))/(d^(5/2)*x^(3/2))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx=\int \frac{b\arccos(cx)+a}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{a + b \arccos(cx)}{(dx)^{5/2}} \, dx = \frac{3\sqrt{x} \left(\int \frac{acos(cx)}{\sqrt{x}\,x^2} dx\right) bx - 2a}{3\sqrt{x}\,\sqrt{d}\,d^2 x}$$

input
```
int((a+b*acos(c*x))/(d*x)^(5/2),x)
```

output
```
(3*sqrt(x)*int(acos(c*x)/(sqrt(x)*x**2),x)*b*x - 2*a)/(3*sqrt(x)*sqrt(d)*d
**2*x)
```

3.209 $\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result 1478
Mathematica [B] (verified) 1478
Rubi [A] (verified) 1479
Maple [F] 1481
Fricas [F] 1481
Sympy [F(-1)] 1481
Maxima [F] 1482
Giac [F(-2)] 1482
Mupad [F(-1)] 1483
Reduce [F] 1483

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d} + \frac{8bc(dx)^{9/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2x^2\right)}{63d^2} + \frac{16b^2c^2(dx)^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};c^2x^2\right)}{693d^3}$$

output
```
2/7*(d*x)^(7/2)*(a+b*arccos(c*x))^2/d+8/63*b*c*(d*x)^(9/2)*(a+b*arccos(c*x
))*hypergeom([1/2, 9/4],[13/4],c^2*x^2)/d^2+16/693*b^2*c^2*(d*x)^(11/2)*hy
pergeom([1, 11/4, 11/4],[13/4, 15/4],c^2*x^2)/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(109) = 218.

Time = 10.85 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{(dx)^{5/2}\left(882a^2x^3+\frac{84ab\left(-2\sqrt{1-c^2x^2}(5+3c^2x^2)+21c^3x^3\arccos(cx)+10\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)\right)}{c^3}\right.}{}$$

input
```
Integrate[(d*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
((d*x)^(5/2)*(882*a^2*x^3 + (84*a*b*(-2*Sqrt[1 - c^2*x^2]*(5 + 3*c^2*x^2)
+ 21*c^3*x^3*ArcCos[c*x] + 10*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2]))/
c^3 + (b^2*(-16*c*x*(35 + 9*c^2*x^2) - 168*Sqrt[1 - c^2*x^2]*(5 + 3*c^2*x^
2)*ArcCos[c*x] + 882*c^3*x^3*ArcCos[c*x]^2 + 840*Sqrt[1 - c^2*x^2]*ArcCos[
c*x]*Hypergeometric2F1[3/4, 1, 5/4, c^2*x^2] + (105*Sqrt[2]*c*Pi*x*Hyperge
ometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(Gamma[5/4]*Gamma[7/4])))/
c^3))/(3087*x^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5139, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \; 5139$$

$$\frac{4bc\int \frac{(dx)^{7/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{7d}+\frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d}$$

$$\Big\downarrow \; 5221$$

$$\frac{4bc\left(\frac{4bc(dx)^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};c^2x^2\right)}{99d^2}+\frac{2(dx)^{9/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2x^2\right)(a+b\arccos(cx))}{9d}\right)}{7d}+\frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d}$$

input
```
Int[(d*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(2*(d*x)^(7/2)*(a + b*ArcCos[c*x])^2)/(7*d) + (4*b*c*((2*(d*x)^(9/2)*(a +
b*ArcCos[c*x])*Hypergeometric2F1[1/2, 9/4, 13/4, c^2*x^2])/(9*d) + (4*b*c*
(d*x)^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, c^2*x^2])/(9
9*d^2)))/(7*d)
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int (dx)^{\frac{5}{2}} \left(a+b\arccos\left(cx\right)\right)^{2} dx$$

input
```
int((d*x)^(5/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int((d*x)^(5/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \int (dx)^{\frac{5}{2}} \left(b\arccos\left(cx\right)+a\right)^{2} dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*d^2*x^2*arccos(c*x)^2 + 2*a*b*d^2*x^2*arccos(c*x) + a^2*d^2*
x^2)*sqrt(d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx=\int (dx)^{\frac{5}{2}}\left(b\arccos(cx)+a\right)^2\,dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
2/7*b^2*d^(5/2)*x^(7/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 1/4
2*a^2*c^2*d^(5/2)*(4*(3*c^2*x^(7/2) + 7*x^(3/2))/c^4 + 42*arctan(sqrt(c)*s
qrt(x))/c^(11/2) + 21*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(
11/2)) + 14*a*b*c^2*d^(5/2)*integrate(1/7*x^(9/2)*arctan(sqrt(c*x + 1)*sqr
t(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 4*b^2*c*d^(5/2)*integrate(1/7*sqrt(
c*x + 1)*sqrt(-c*x + 1)*x^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))
/(c^2*x^2 - 1), x) - 1/6*a^2*d^(5/2)*(4*x^(3/2)/c^2 + 6*arctan(sqrt(c)*sqr
t(x))/c^(7/2) + 3*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(7/2)
) - 14*a*b*d^(5/2)*integrate(1/7*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x +
1)/(c*x))/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d*x)^(5/2), x)
```

Reduce [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{d}\,d^2\left(2\sqrt{x}\,a^2x^3+14\left(\int \sqrt{x}\,acos(cx)\,x^2dx\right)ab+7\left(\int \sqrt{x}\,acos(cx)^2\,x^2dx\right)b^2\right)}{7}$$

input
```
int((d*x)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(2*sqrt(x)*a**2*x**3 + 14*int(sqrt(x)*acos(c*x)*x**2,x)*a*b
+ 7*int(sqrt(x)*acos(c*x)**2*x**2,x)*b**2))/7
```

3.210 $\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result . . . 1484
Mathematica [A] (verified) . . . 1484
Rubi [A] (verified) . . . 1485
Maple [F] . . . 1486
Fricas [F] . . . 1487
Sympy [F] . . . 1487
Maxima [F] . . . 1487
Giac [F(-2)] . . . 1488
Mupad [F(-1)] . . . 1488
Reduce [F] . . . 1489

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d}+\frac{8bc(dx)^{7/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},c^2x^2\right)}{35d^2}+\frac{16b^2c^2(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};c^2x^2\right)}{315d^3}$$

output
```
2/5*(d*x)^(5/2)*(a+b*arccos(c*x))^2/d+8/35*b*c*(d*x)^(7/2)*(a+b*arccos(c*x
))*hypergeom([1/2, 7/4],[11/4],c^2*x^2)/d^2+16/315*b^2*c^2*(d*x)^(9/2)*hyp
ergeom([1, 9/4, 9/4],[11/4, 13/4],c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.61

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{(dx)^{3/2}\left(4480a^2x+\frac{128b\left(-28a\sqrt{1-c^2x^2}+70acx\arccos(cx)+35bcx\arccos(cx)^2+28a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{c}\right.}{1}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
((d*x)^(3/2)*(4480*a^2*x + (128*b*(-28*a*Sqrt[1 - c^2*x^2] + 70*a*c*x*ArcC
os[c*x] + 35*b*c*x*ArcCos[c*x]^2 + 28*a*Hypergeometric2F1[1/2, 3/4, 7/4, c
^2*x^2] + 20*b*c^2*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Hypergeometric2F1[1,
9/4, 11/4, c^2*x^2]))/c + (525*Sqrt[2]*b^2*c^2*Pi*x^3*HypergeometricPFQ[{1
, 9/4, 9/4}, {11/4, 13/4}, c^2*x^2])/(Gamma[11/4]*Gamma[13/4])))/11200
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5139, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{4bc\int \frac{(dx)^{5/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d}$$

$$\Big\downarrow \text{5221}$$

$$\frac{4bc\left(\frac{4bc(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};c^2x^2\right)}{63d^2}+\frac{2(dx)^{7/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},c^2x^2\right)(a+b\arccos(cx))}{7d}\right)}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d}$$

input
```
Int[(d*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(5*d) + (4*b*c*((2*(d*x)^(7/2)*(a +
b*ArcCos[c*x])*Hypergeometric2F1[1/2, 7/4, 11/4, c^2*x^2])/(7*d) + (4*b*c*
(d*x)^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, c^2*x^2])/(63*d
^2)))/(5*d)
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int (dx)^{\frac{3}{2}} (a + b \arccos(cx))^2 \, dx$$

input
```
int((d*x)^(3/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int((d*x)^(3/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^2\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*d*x*arccos(c*x)^2 + 2*a*b*d*x*arccos(c*x) + a^2*d*x)*sqrt(d*
x), x)
```

Sympy [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\int (dx)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^2\,dx$$

input
```
integrate((d*x)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral((d*x)**(3/2)*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^2\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
2/5*b^2*d^(3/2)*x^(5/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 1/1
0*a^2*c^2*d^(3/2)*(4*(c^2*x^(5/2) + 5*sqrt(x))/c^4 - 10*arctan(sqrt(c)*sqr
t(x))/c^(9/2) + 5*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(9/2)
) + 10*a*b*c^2*d^(3/2)*integrate(1/5*x^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-c*
x + 1)/(c*x))/(c^2*x^2 - 1), x) - 4*b^2*c*d^(3/2)*integrate(1/5*sqrt(c*x +
 1)*sqrt(-c*x + 1)*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2
*x^2 - 1), x) - 1/2*a^2*d^(3/2)*(4*sqrt(x)/c^2 - 2*arctan(sqrt(c)*sqrt(x))
/c^(5/2) + log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(5/2)) - 10*
a*b*d^(3/2)*integrate(1/5*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x
))/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,(d\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{x}\,a^2x^2+10\left(\int \sqrt{x}\,acos(cx)\,xdx\right)ab+5\left(\int \sqrt{x}\,acos(cx)^2\,xdx\right)b^2\right)}{5}$$

input
```
int((d*x)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d*(2*sqrt(x)*a**2*x**2 + 10*int(sqrt(x)*acos(c*x)*x,x)*a*b + 5*in
t(sqrt(x)*acos(c*x)**2*x,x)*b**2))/5
```

3.211 $\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$

Optimal result 1490
Mathematica [A] (verified) 1491
Rubi [A] (verified) 1491
Maple [F] 1493
Fricas [F] 1493
Sympy [F] 1493
Maxima [F] 1494
Giac [F(-2)] 1494
Mupad [F(-1)] 1495
Reduce [F] 1495

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d} + \frac{8bc(dx)^{5/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{15d^2} + \frac{16b^2c^2(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{105d^3}$$

output

```
2/3*(d*x)^(3/2)*(a+b*arccos(c*x))^2/d+8/15*b*c*(d*x)^(5/2)*(a+b*arccos(c*x
))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/d^2+16/105*b^2*c^2*(d*x)^(7/2)*hype
rgeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.85

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$$
$$=\frac{1}{27}\sqrt{dx}\left(\frac{2\left(9a^2cx-8b^2cx-12ab\sqrt{1-c^2x^2}+18abcx\arccos(cx)-12b^2\sqrt{1-c^2x^2}\arccos(cx)+9b^2cx\,\text{ar}\right.}{}\right.$$
$$\left.+\frac{3\sqrt{2}b^2\pi x\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)}\right)$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d*x]*((2*(9*a^2*c*x - 8*b^2*c*x - 12*a*b*Sqrt[1 - c^2*x^2] + 18*a*b*
c*x*ArcCos[c*x] - 12*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 9*b^2*c*x*ArcCos[
c*x]^2 + 12*a*b*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2] + 12*b^2*Sqrt[1
- c^2*x^2]*ArcCos[c*x]*Hypergeometric2F1[3/4, 1, 5/4, c^2*x^2]))/c + (3*Sq
rt[2]*b^2*Pi*x*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(Gam
ma[5/4]*Gamma[7/4])))/27
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5139, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{4bc\int \frac{(dx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d}$$

$$\Big\downarrow \text{5221}$$

$$\frac{4bc\left(\frac{4bc(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35d^2}+\frac{2(dx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)(a+b\arccos(cx))}{5d}\right)}{3d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d}$$

input
```
Int[Sqrt[d*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(3*d) + (4*b*c*((2*(d*x)^(5/2)*(a +
b*ArcCos[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(5*d) + (4*b*c*(
d*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(35*d^2
)))/(3*d)
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \sqrt{dx}\,(a+b\arccos(cx))^2\,dx$$

input
```
int((d*x)^(1/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int((d*x)^(1/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int \sqrt{dx}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x), x)
```

Sympy [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int \sqrt{dx}(a+b\operatorname{acos}(cx))^2\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d*x)*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int \sqrt{dx}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
2/3*b^2*sqrt(d)*x^(3/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 1/6
*a^2*c^2*sqrt(d)*(4*x^(3/2)/c^2 + 6*arctan(sqrt(c)*sqrt(x))/c^(7/2) + 3*lo
g((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(7/2)) + 6*a*b*c^2*sqrt(d
)*integrate(1/3*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^
2 - 1), x) - 4*b^2*c*sqrt(d)*integrate(1/3*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^
(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 1/2*a
^2*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/c^(3/2) + log((c*sqrt(x) - sqrt(c))/
(c*sqrt(x) + sqrt(c)))/c^(3/2)) - 6*a*b*sqrt(d)*integrate(1/3*sqrt(x)*arct
an(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$$
$$= \frac{\sqrt{d}\,\left(2\sqrt{x}\,a^2x+6\left(\int \sqrt{x}\,acos(cx)\,dx\right)ab+3\left(\int \sqrt{x}\,acos(cx)^2\,dx\right)b^2\right)}{3}$$

input
```
int((d*x)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*(2*sqrt(x)*a**2*x + 6*int(sqrt(x)*acos(c*x),x)*a*b + 3*int(sqrt(x
)*acos(c*x)**2,x)*b**2))/3
```

3.212 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx$

Optimal result 1496
Mathematica [A] (verified) 1497
Rubi [A] (verified) 1497
Maple [F] 1498
Fricas [F] 1499
Sympy [F(-2)] 1499
Maxima [F] 1499
Giac [F] 1500
Mupad [F(-1)] 1500
Reduce [F] 1501

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d} + \frac{8bc(dx)^{3/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)}{3d^2} + \frac{16b^2c^2(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)}{15d^3}$$

output
```
2*(d*x)^(1/2)*(a+b*arccos(c*x))^2/d+8/3*b*c*(d*x)^(3/2)*(a+b*arccos(c*x))*
hypergeom([1/2, 3/4],[7/4],c^2*x^2)/d^2+16/15*b^2*c^2*(d*x)^(5/2)*hypergeo
m([1, 5/4, 5/4],[7/4, 9/4],c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx$$
$$= \frac{3\sqrt{2}b^2c^2\pi x^3\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)+8x\,\mathrm{Gamma}\left(\frac{7}{4}\right)\mathrm{Gamma}\left(\frac{9}{4}\right)\left(3(a+b\arccos(cx))^2+4abcx\,\mathrm{Hyperge}\right.}{12\sqrt{dx}\,\mathrm{Gamma}\left(\frac{7}{4}\right)}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d*x],x]
```

output
```
(3*Sqrt[2]*b^2*c^2*Pi*x^3*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2
*x^2] + 8*x*Gamma[7/4]*Gamma[9/4]*(3*(a + b*ArcCos[c*x])^2 + 4*a*b*c*x*Hyp
ergeometric2F1[1/2, 3/4, 7/4, c^2*x^2] + 2*b^2*ArcCos[c*x]*Hypergeometric2
F1[1, 5/4, 7/4, c^2*x^2]*Sin[2*ArcCos[c*x]]))/(12*Sqrt[d*x]*Gamma[7/4]*Gam
ma[9/4])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5139, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx$$
$$\Big\downarrow \text{ 5139}$$
$$\frac{4bc\int \frac{\sqrt{dx}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{d}+\frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d}$$
$$\Big\downarrow \text{ 5221}$$

$$\frac{4bc\left(\frac{4bc(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)}{15d^2}+\frac{2(dx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)(a+b\arccos(cx))}{3d}\right)}{d}+\frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d}$$

input
```
Int[(a + b*ArcCos[c*x])^2/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*ArcCos[c*x])^2)/d + (4*b*c*((2*(d*x)^(3/2)*(a + b*ArcC
os[c*x])*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2])/(3*d) + (4*b*c*(d*x)^(
5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2])/(15*d^2)))/d
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{\left(a+b\arccos\left(cx\right)\right)^2}{\sqrt{dx}}dx$$

input
```
int((a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))**2/(d*x)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*b^2*sqrt(x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + (a^2*c^2
*sqrt(d)*(4*sqrt(x)/(c^2*d) - 2*arctan(sqrt(c)*sqrt(x))/(c^(5/2)*d) + log(
(c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(c^(5/2)*d)) + 4*a*b*c^2*sqrt
(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^
3 - d*x), x) - 8*b^2*c*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3
/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x) + a^2
*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)*d) - log((c*sqrt(x) - sqrt(c)
)/(c*sqrt(x) + sqrt(c)))/(sqrt(c)*d)) - 4*a*b*sqrt(d)*integrate(sqrt(x)*ar
ctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x))*sqrt(d))/s
qrt(d)
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,a^2+2\left(\int \frac{acos(cx)}{\sqrt{x}}dx\right)ab+\left(\int \frac{acos(cx)^2}{\sqrt{x}}dx\right)b^2}{\sqrt{d}}$$

input
```
int((a+b*acos(c*x))^2/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*a**2 + 2*int(acos(c*x)/sqrt(x),x)*a*b + int(acos(c*x)**2/sqrt(x
),x)*b**2)/sqrt(d)
```

3.213 $\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx$

Optimal result 1502
Mathematica [A] (verified) 1502
Rubi [A] (verified) 1503
Maple [F] 1504
Fricas [F] 1504
Sympy [F(-2)] 1505
Maxima [F] 1505
Giac [F] 1506
Mupad [F(-1)] 1506
Reduce [F] 1506

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = -\frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}} - \frac{8bc\sqrt{dx}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)}{d^2} - \frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{3d^3}$$

output
```
-2*(a+b*arccos(c*x))^2/d/(d*x)^(1/2)-8*b*c*(d*x)^(1/2)*(a+b*arccos(c*x))*h
ypergeom([1/4, 1/2],[5/4],c^2*x^2)/d^2-16/3*b^2*c^2*(d*x)^(3/2)*hypergeom(
[3/4, 3/4, 1],[5/4, 7/4],c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \frac{x\left(-\frac{\sqrt{2}b^2c^2\pi x^2\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)} - 2\left((a+b\arccos(cx))^2 + 4abcx\,\text{Hypergeometri}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d*x)^(3/2),x]
```

output
```
(x*(-((Sqrt[2]*b^2*c^2*Pi*x^2*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4},
 c^2*x^2])/(Gamma[5/4]*Gamma[7/4])) - 2*((a + b*ArcCos[c*x])^2 + 4*a*b*c*x
*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2] + 2*b^2*ArcCos[c*x]*Hypergeomet
ric2F1[3/4, 1, 5/4, c^2*x^2]*Sin[2*ArcCos[c*x]])))/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5139, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{4bc\int \frac{a+b\arccos(cx)}{\sqrt{dx}\sqrt{1-c^2x^2}}dx}{d}-\frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 5221}$$

$$-\frac{4bc\left(\frac{4bc(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{3d^2}+\frac{2\sqrt{dx}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)(a+b\arccos(cx))}{d}\right)}{d}-\frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d*x]) - (4*b*c*((2*Sqrt[d*x]*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2])/d + (4*b*c*(d*x)^(3/2
)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(3*d^2)))/d
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^2/(d*x)^(3/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d^2*x^2)
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))**2/(d*x)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(4*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - (a^2*c^2*sqrt(d
)*(2*arctan(sqrt(c)*sqrt(x))/(c^(3/2)*d^2) + log((c*sqrt(x) - sqrt(c))/(c*
sqrt(x) + sqrt(c)))/(c^(3/2)*d^2)) + 4*a*b*c^2*sqrt(d)*integrate(x^(5/2)*a
rctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x) + 8*
b^2*c*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c
*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x) - a^2*sqrt(d)*(2
*sqrt(c)*arctan(sqrt(c)*sqrt(x))/d^2 + sqrt(c)*log((c*sqrt(x) - sqrt(c))/(
c*sqrt(x) + sqrt(c)))/d^2 + 4/(d^2*sqrt(x))) - 4*a*b*sqrt(d)*integrate(sqr
t(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x
))*d^(3/2)*sqrt(x))/(d^(3/2)*sqrt(x))
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx=\frac{2\sqrt{x}\left(\int \frac{acos(cx)}{\sqrt{x}\,x}dx\right)ab+\sqrt{x}\left(\int \frac{acos(cx)^2}{\sqrt{x}\,x}dx\right)b^2-2a^2}{\sqrt{x}\,\sqrt{d}\,d}$$

input
```
int((a+b*acos(c*x))^2/(d*x)^(3/2),x)
```

output
```
(2*sqrt(x)*int(acos(c*x)/(sqrt(x)*x),x)*a*b + sqrt(x)*int(acos(c*x)**2/(sq
rt(x)*x),x)*b**2 - 2*a**2)/(sqrt(x)*sqrt(d)*d)
```

3.214 $\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx$

Optimal result 1507
Mathematica [A] (verified) 1507
Rubi [A] (verified) 1508
Maple [F] 1509
Fricas [F] 1509
Sympy [F(-2)] 1510
Maxima [F] 1510
Giac [F] 1511
Mupad [F(-1)] 1511
Reduce [F] 1511

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = -\frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}} + \frac{8bc(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2x^2\right)}{3d^2\sqrt{dx}} + \frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};c^2x^2\right)}{3d^3}$$

output
```
-2/3*(a+b*arccos(c*x))^2/d/(d*x)^(3/2)+8/3*b*c*(a+b*arccos(c*x))*hypergeom
([-1/4, 1/2],[3/4],c^2*x^2)/d^2/(d*x)^(1/2)+16/3*b^2*c^2*(d*x)^(1/2)*hyper
geom([1/4, 1/4, 1],[3/4, 5/4],c^2*x^2)/d^3
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.82

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \frac{x\left(-8\,\text{Gamma}\left(\frac{7}{4}\right)\text{Gamma}\left(\frac{9}{4}\right)\left(3\left(a^2-8b^2c^2x^2+2b\left(a-2bcx\sqrt{1-c^2x^2}\right)\text{arcc}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d*x)^(5/2),x]
```

output
```
(x*(-8*Gamma[7/4]*Gamma[9/4]*(3*(a^2 - 8*b^2*c^2*x^2 + 2*b*(a - 2*b*c*x*Sq
rt[1 - c^2*x^2])*ArcCos[c*x] + b^2*ArcCos[c*x]^2) - 12*a*b*c*x*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, c^2*x^2] - 4*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c
*x]*Hypergeometric2F1[1, 5/4, 7/4, c^2*x^2]) + 3*Sqrt[2]*b^2*c^4*Pi*x^4*Hy
pergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2]))/(36*(d*x)^(5/2)*Gamm
a[7/4]*Gamma[9/4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{5139, 5221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{4bc\int \frac{a+b\arccos(cx)}{(dx)^{3/2}\sqrt{1-c^2x^2}}dx}{3d}-\frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{ 5221}$$

$$-\frac{4bc\left(-\frac{4bc\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};c^2x^2\right)}{d^2}-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2x^2\right)(a+b\arccos(cx))}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcCos[c*x])^2)/(3*d*(d*x)^(3/2)) - (4*b*c*((-2*(a + b*ArcCos[c
*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, c^2*x^2])/(d*Sqrt[d*x]) - (4*b*c*Sq
rt[d*x]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, c^2*x^2])/d^2))/(3*d)
```

Defintions of rubi rules used

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(a + b\arccos(cx))^2}{(dx)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^2/(d*x)^(5/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a + b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx) + a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d^3*x^3)
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))**2/(d*x)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*((3*a^2*c^2*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)*d^3) - log((c
*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(sqrt(c)*d^3)) - 36*a*b*c^2*sqr
t(d)*integrate(1/3*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2
*d^3*x^5 - d^3*x^3), x) - 24*b^2*c*sqrt(d)*integrate(1/3*sqrt(c*x + 1)*sqr
t(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^3*x^
5 - d^3*x^3), x) - a^2*sqrt(d)*(6*c^(3/2)*arctan(sqrt(c)*sqrt(x))/d^3 - 3*
c^(3/2)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/d^3 - 4/(d^3*x^(3
/2))) + 36*a*b*sqrt(d)*integrate(1/3*sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*
x + 1)/(c*x))/(c^2*d^3*x^5 - d^3*x^3), x))*d^(5/2)*x^(3/2) + 4*b^2*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2)/(d^(5/2)*x^(3/2))
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d\,x)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \frac{6\sqrt{x}\left(\int \frac{acos(cx)}{\sqrt{x}\,x^2}dx\right)abx+3\sqrt{x}\left(\int \frac{acos(cx)^2}{\sqrt{x}\,x^2}dx\right)b^2x-2a^2}{3\sqrt{x}\,\sqrt{d}\,d^2x}$$

input
```
int((a+b*acos(c*x))^2/(d*x)^(5/2),x)
```

output
```
(6*sqrt(x)*int(acos(c*x)/(sqrt(x)*x**2),x)*a*b*x + 3*sqrt(x)*int(acos(c*x)
**2/(sqrt(x)*x**2),x)*b**2*x - 2*a**2)/(3*sqrt(x)*sqrt(d)*d**2*x)
```

3.215 $\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$

Optimal result . . . 1512
Mathematica [N/A] . . . 1512
Rubi [N/A] . . . 1513
Maple [N/A] . . . 1513
Fricas [N/A] . . . 1514
Sympy [N/A] . . . 1514
Maxima [N/A] . . . 1514
Giac [F(-2)] . . . 1515
Mupad [N/A] . . . 1515
Reduce [N/A] . . . 1516

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\text{Int}\left((dx)^{3/2}(a+b\arccos(cx))^3,x\right)$$

output
```
Defer(Int)((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 43.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$$

input
```
Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3,x]
```

output
```
Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \ 5139$$

$$\frac{6bc\int \frac{(dx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))^3}{5d}$$

$$\Big\downarrow \ 5235$$

$$\frac{6bc\int \frac{(dx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5d}+\frac{2(dx)^{5/2}(a+b\arccos(cx))^3}{5d}$$

input
```
Int[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (dx)^{\frac{3}{2}} (a+b\arccos(cx))^3\,dx$$

input
```
int((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
```

output
```
int((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^3\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*d*x*arccos(c*x)^3 + 3*a*b^2*d*x*arccos(c*x)^2 + 3*a^2*b*d*x*
arccos(c*x) + a^3*d*x)*sqrt(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 72.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^3\,dx$$

input
```
integrate((d*x)**(3/2)*(a+b*acos(c*x))**3,x)
```

output
```
Integral((d*x)**(3/2)*(a + b*acos(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 4.22 (sec) , antiderivative size = 441, normalized size of antiderivative = 24.50

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^3\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
2/5*b^3*d^(3/2)*x^(5/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/1
0*a^3*c^2*d^(3/2)*(4*(c^2*x^(5/2) + 5*sqrt(x))/c^4 - 10*arctan(sqrt(c)*sqr
t(x))/c^(9/2) + 5*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(9/2)
) + 15*a*b^2*c^2*d^(3/2)*integrate(1/5*x^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-
c*x + 1)/(c*x))^2/(c^2*x^2 - 1), x) + 15*a^2*b*c^2*d^(3/2)*integrate(1/5*x
^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 6*b^
3*c*d^(3/2)*integrate(1/5*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(5/2)*arctan(sqrt
(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 - 1), x) - 1/2*a^3*d^(3/2)*(4*s
qrt(x)/c^2 - 2*arctan(sqrt(c)*sqrt(x))/c^(5/2) + log((c*sqrt(x) - sqrt(c))
/(c*sqrt(x) + sqrt(c)))/c^(5/2)) - 15*a*b^2*d^(3/2)*integrate(1/5*x^(3/2)*
arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 - 1), x) - 15*a^2*b*
d^(3/2)*integrate(1/5*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(
c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx = \int (a+b\,\mathrm{acos}(c\,x))^3\,(d\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^3*(d*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^3*(d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx = \frac{\sqrt{d}\,d\left(2\sqrt{x}\,a^3x^2+15\left(\int \sqrt{x}\,acos(cx)\,xdx\right)a^2b+5\left(\int \sqrt{x}\,acos(cx)^3\,xdx\right)b^3+15\left(\int \sqrt{x}\right.\right.}{5}$$

input
```
int((d*x)^(3/2)*(a+b*acos(c*x))^3,x)
```

output
```
(sqrt(d)*d*(2*sqrt(x)*a**3*x**2 + 15*int(sqrt(x)*acos(c*x)*x,x)*a**2*b + 5
*int(sqrt(x)*acos(c*x)**3*x,x)*b**3 + 15*int(sqrt(x)*acos(c*x)**2*x,x)*a*b
**2))/5
```

3.216 $\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$

Optimal result 1517
Mathematica [N/A] 1517
Rubi [N/A] 1518
Maple [N/A] 1518
Fricas [N/A] 1519
Sympy [N/A] 1519
Maxima [N/A] 1519
Giac [F(-2)] 1520
Mupad [N/A] 1520
Reduce [N/A] 1521

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\operatorname{Int}\left(\sqrt{dx}(a+b\arccos(cx))^3,x\right)$$

output
```
Defer(Int)((d*x)^(1/2)*(a+b*arccos(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 119.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^3,x]
```

output
```
Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \text{5139}$$

$$\frac{2bc\int \frac{(dx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))^3}{3d}$$

$$\Big\downarrow \text{5235}$$

$$\frac{2bc\int \frac{(dx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{d}+\frac{2(dx)^{3/2}(a+b\arccos(cx))^3}{3d}$$

input
```
Int[Sqrt[d*x]*(a + b*ArcCos[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \sqrt{dx}\,(a+b\arccos(cx))^3\,dx$$

input
```
int((d*x)^(1/2)*(a+b*arccos(c*x))^3,x)
```

output
```
int((d*x)^(1/2)*(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx = \int \sqrt{dx}(b\arccos{(cx)}+a)^3\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)*sqrt(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx = \int \sqrt{dx}\left(a+b\,\mathrm{acos}\,(cx)\right)^3\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*acos(c*x))**3,x)
```

output
```
Integral(sqrt(d*x)*(a + b*acos(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 418, normalized size of antiderivative = 23.22

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx = \int \sqrt{dx}(b\arccos{(cx)}+a)^3\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
2/3*b^3*sqrt(d)*x^(3/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/6
*a^3*c^2*sqrt(d)*(4*x^(3/2)/c^2 + 6*arctan(sqrt(c)*sqrt(x))/c^(7/2) + 3*lo
g((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(7/2)) + 3*a*b^2*c^2*sqrt
(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^
2 - 1), x) + 3*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sq
rt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 2*b^3*c*sqrt(d)*integrate(sqrt(c*x
 + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/
(c^2*x^2 - 1), x) - 1/2*a^3*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/c^(3/2) + l
og((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(3/2)) - 3*a*b^2*sqrt(d)
*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 -
 1), x) - 3*a^2*b*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x
 + 1)/(c*x))/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x)^(1/2)*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx = \int (a+b\,\mathrm{acos}(c\,x))^3\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*acos(c*x))^3*(d*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^3*(d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$$
$$=\frac{\sqrt{d}\left(2\sqrt{x}\,a^3x+9\left(\int\sqrt{x}\,acos(cx)\,dx\right)a^2b+3\left(\int\sqrt{x}\,acos(cx)^3\,dx\right)b^3+9\left(\int\sqrt{x}\,acos(cx)^2\,dx\right)a\,b^2\right)}{3}$$

input
```
int((d*x)^(1/2)*(a+b*acos(c*x))^3,x)
```

output
```
(sqrt(d)*(2*sqrt(x)*a**3*x + 9*int(sqrt(x)*acos(c*x),x)*a**2*b + 3*int(sqr
t(x)*acos(c*x)**3,x)*b**3 + 9*int(sqrt(x)*acos(c*x)**2,x)*a*b**2))/3
```

3.217 $\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx$

Optimal result . . . 1522
Mathematica [N/A] . . . 1522
Rubi [N/A] . . . 1523
Maple [N/A] . . . 1523
Fricas [N/A] . . . 1524
Sympy [F(-2)] . . . 1524
Maxima [N/A] . . . 1525
Giac [N/A] . . . 1525
Mupad [N/A] . . . 1526
Reduce [N/A] . . . 1526

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \text{Int}\left(\frac{(a+b\arccos(cx))^3}{\sqrt{dx}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 64.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/Sqrt[d*x],x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^3/Sqrt[d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{6bc\int \frac{\sqrt{dx}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{d}+\frac{2\sqrt{dx}(a+b\arccos(cx))^3}{d}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{6bc\int \frac{\sqrt{dx}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{d}+\frac{2\sqrt{dx}(a+b\arccos(cx))^3}{d}$$

input
```
Int[(a + b*ArcCos[c*x])^3/Sqrt[d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}dx$$

input
```
int((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)*sqrt(d*x)/(d*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))**3/(d*x)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*b^3*sqrt(x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + (a^3*c^2
*sqrt(d)*(4*sqrt(x)/(c^2*d) - 2*arctan(sqrt(c)*sqrt(x))/(c^(5/2)*d) + log(
(c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(c^(5/2)*d)) + 6*a*b^2*c^2*sq
rt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*
d*x^3 - d*x), x) + 6*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x +
 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x) - 12*b^3*c*sqrt(d)*integra
te(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1
)/(c*x))^2/(c^2*d*x^3 - d*x), x) + a^3*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/
(sqrt(c)*d) - log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(sqrt(c)*d)
) - 6*a*b^2*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/
(c*x))^2/(c^2*d*x^3 - d*x), x) - 6*a^2*b*sqrt(d)*integrate(sqrt(x)*arctan(
sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x))*sqrt(d))/sqrt(d
)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^3/sqrt(d*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^3}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^3/(d*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^3/(d*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,a^3+3\left(\int \frac{acos(cx)}{\sqrt{x}}dx\right)a^2b+\left(\int \frac{acos(cx)^3}{\sqrt{x}}dx\right)b^3+3\left(\int \frac{acos(cx)^2}{\sqrt{x}}dx\right)a\,b^2}{\sqrt{d}}$$

input
```
int((a+b*acos(c*x))^3/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*a**3 + 3*int(acos(c*x)/sqrt(x),x)*a**2*b + int(acos(c*x)**3/sqr
t(x),x)*b**3 + 3*int(acos(c*x)**2/sqrt(x),x)*a*b**2)/sqrt(d)
```

3.218 $\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx$

Optimal result 1527
Mathematica [N/A] 1527
Rubi [N/A] 1528
Maple [N/A] 1528
Fricas [N/A] 1529
Sympy [F(-2)] 1529
Maxima [N/A] 1530
Giac [N/A] 1530
Mupad [N/A] 1531
Reduce [N/A] 1531

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \mathrm{Int}\left(\frac{(a+b\arccos(cx))^3}{(dx)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 47.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(3/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{6bc\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}\sqrt{1-c^2x^2}}dx}{d}-\frac{2(a+b\arccos(cx))^3}{d\sqrt{dx}}$$

$$\Big\downarrow \text{5235}$$

$$-\frac{6bc\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}\sqrt{1-c^2x^2}}dx}{d}-\frac{2(a+b\arccos(cx))^3}{d\sqrt{dx}}$$

input
```
Int[(a + b*ArcCos[c*x])^3/(d*x)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\arccos\left(cx\right)\right)^{3}}{\left(dx\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
```

output
```
int((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))**3/(d*x)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [N/A]

Not integrable

Time = 3.99 (sec) , antiderivative size = 489, normalized size of antiderivative = 27.17

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(4*b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 - (a^3*c^2*sqrt(d
)*(2*arctan(sqrt(c)*sqrt(x))/(c^(3/2)*d^2) + log((c*sqrt(x) - sqrt(c))/(c*
sqrt(x) + sqrt(c)))/(c^(3/2)*d^2)) + 6*a*b^2*c^2*sqrt(d)*integrate(x^(5/2)
*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^2*x^4 - d^2*x^2), x)
+ 6*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1
)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x) + 12*b^3*c*sqrt(d)*integrate(sqrt(c*x
 + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/
(c^2*d^2*x^4 - d^2*x^2), x) - a^3*sqrt(d)*(2*sqrt(c)*arctan(sqrt(c)*sqrt(x
))/d^2 + sqrt(c)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/d^2 + 4/
(d^2*sqrt(x))) - 6*a*b^2*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sq
rt(-c*x + 1)/(c*x))^2/(c^2*d^2*x^4 - d^2*x^2), x) - 6*a^2*b*sqrt(d)*integr
ate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*
x^2), x))*d^(3/2)*sqrt(x))/(d^(3/2)*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^3/(d*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^3}{\left(d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^3/(d*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^3/(d*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.78

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \frac{3\sqrt{x}\left(\int \frac{acos(cx)}{\sqrt{x}\,x}dx\right)a^2b+\sqrt{x}\left(\int \frac{acos(cx)^3}{\sqrt{x}\,x}dx\right)b^3+3\sqrt{x}\left(\int \frac{acos(cx)^2}{\sqrt{x}\,x}dx\right)a\,b^2-2}{\sqrt{x}\,\sqrt{d}\,d}$$

input
```
int((a+b*acos(c*x))^3/(d*x)^(3/2),x)
```

output
```
(3*sqrt(x)*int(acos(c*x)/(sqrt(x)*x),x)*a**2*b + sqrt(x)*int(acos(c*x)**3/
(sqrt(x)*x),x)*b**3 + 3*sqrt(x)*int(acos(c*x)**2/(sqrt(x)*x),x)*a*b**2 - 2
*a**3)/(sqrt(x)*sqrt(d)*d)
```

3.219 $\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$

Optimal result 1532
Mathematica [N/A] 1532
Rubi [N/A] 1533
Maple [N/A] 1533
Fricas [N/A] 1534
Sympy [F(-2)] 1534
Maxima [N/A] 1535
Giac [N/A] 1535
Mupad [N/A] 1536
Reduce [N/A] 1536

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \text{Int}\left(\frac{(a+b\arccos(cx))^3}{(dx)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 30.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(5/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{5139}$$

$$-\frac{2bc\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}\sqrt{1-c^2x^2}}dx}{d}-\frac{2(a+b\arccos(cx))^3}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{5235}$$

$$-\frac{2bc\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}\sqrt{1-c^2x^2}}dx}{d}-\frac{2(a+b\arccos(cx))^3}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^3/(d*x)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
```

output
```
int((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)*sqrt(d*x)/(d^3*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*acos(c*x))**3/(d*x)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 491, normalized size of antiderivative = 27.28

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(4*b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + (3*a^3*c^2*sqrt
(d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)*d^3) - log((c*sqrt(x) - sqrt(c))/(
c*sqrt(x) + sqrt(c)))/(sqrt(c)*d^3)) - 18*a*b^2*c^2*sqrt(d)*integrate(x^(5
/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^3*x^5 - d^3*x^3),
x) - 18*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x
 + 1)/(c*x))/(c^2*d^3*x^5 - d^3*x^3), x) - 12*b^3*c*sqrt(d)*integrate(sqrt
(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x)
)^2/(c^2*d^3*x^5 - d^3*x^3), x) - a^3*sqrt(d)*(6*c^(3/2)*arctan(sqrt(c)*sq
rt(x))/d^3 - 3*c^(3/2)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/d^
3 - 4/(d^3*x^(3/2))) + 18*a*b^2*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x
+ 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^3*x^5 - d^3*x^3), x) + 18*a^2*b*sqrt(d
)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^3*x^
5 - d^3*x^3), x))*d^(5/2)*x^(3/2))/(d^(5/2)*x^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^3/(d*x)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}{{\left(d\,x\right)}^{5/2}} \,d x$$

input
```
int((a + b*acos(c*x))^3/(d*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^3/(d*x)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.22

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx=\frac{9\sqrt{x}\left(\int \frac{acos(cx)}{\sqrt{x}\,x^{2}}dx\right)a^{2}bx+3\sqrt{x}\left(\int \frac{acos(cx)^{3}}{\sqrt{x}\,x^{2}}dx\right)b^{3}x+9\sqrt{x}\left(\int \frac{acos(cx)^{2}}{\sqrt{x}\,x^{2}}dx\right)a\,b^{2}x}{3\sqrt{x}\,\sqrt{d}\,d^{2}x}$$

input
```
int((a+b*acos(c*x))^3/(d*x)^(5/2),x)
```

output
```
(9*sqrt(x)*int(acos(c*x)/(sqrt(x)*x**2),x)*a**2*b*x + 3*sqrt(x)*int(acos(c
*x)**3/(sqrt(x)*x**2),x)*b**3*x + 9*sqrt(x)*int(acos(c*x)**2/(sqrt(x)*x**2
),x)*a*b**2*x - 2*a**3)/(3*sqrt(x)*sqrt(d)*d**2*x)
```

3.220 $\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx$

Optimal result 1537
Mathematica [N/A] 1537
Rubi [N/A] 1538
Maple [N/A] 1538
Fricas [N/A] 1539
Sympy [N/A] 1539
Maxima [N/A] 1539
Giac [N/A] 1540
Mupad [N/A] 1540
Reduce [N/A] 1541

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx = \text{Int}\left(\frac{(dx)^{3/2}}{a+b\arccos(cx)},x\right)$$

output
```
Defer(Int)((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx$$

input
```
Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}dx$$
$$\Big\downarrow \ 5149$$
$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}dx$$

input
```
Int[(d*x)^(3/2)/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^{\frac{3}{2}}}{a+b\arccos\left(cx\right)}dx$$

input
```
int((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{a+b\operatorname{acos}{(cx)}}\,dx$$

input
```
integrate((d*x)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral((d*x)**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/(b*arccos(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(d\,x)^{3/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*acos(c*x)),x)
```

output
```
int((d*x)^(3/2)/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,x}{acos\,(cx)\,b+a}dx\right)d$$

input
```
int((d*x)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*int((sqrt(x)*x)/(acos(c*x)*b + a),x)*d
```

3.221 $\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$

Optimal result 1542
Mathematica [N/A] 1542
Rubi [N/A] 1543
Maple [N/A] 1543
Fricas [N/A] 1544
Sympy [N/A] 1544
Maxima [N/A] 1544
Giac [N/A] 1545
Mupad [N/A] 1545
Reduce [N/A] 1546

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\text{Int}\left(\frac{\sqrt{dx}}{a+b\arccos(cx)},x\right)$$

output
```
Defer(Int)((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$$

input
```
Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$$
$$\Big\downarrow \; 5149$$
$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}dx$$

input
```
Int[Sqrt[d*x]/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{dx}}{a+b\arccos\left(cx\right)}dx$$

input
```
int((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{dx}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{dx}}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate((d*x)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(d*x)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{dx}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{dx}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{d\,x}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*acos(c*x)),x)
```

output
```
int((d*x)^(1/2)/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}}{acos\left(cx\right)b+a}dx\right)$$

input
```
int((d*x)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*int(sqrt(x)/(acos(c*x)*b + a),x)
```

3.222 $\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$

Optimal result 1547
Mathematica [N/A] 1547
Rubi [N/A] 1548
Maple [N/A] 1548
Fricas [N/A] 1549
Sympy [N/A] 1549
Maxima [N/A] 1549
Giac [N/A] 1550
Mupad [N/A] 1550
Reduce [N/A] 1551

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{\sqrt{dx}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}dx$$

input
```
Int[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{dx}\,\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(1/(d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b*d*x*arccos(c*x) + a*d*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{dx}\,(a+b\operatorname{acos}{(cx)})}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{d\,x}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d*x)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \frac{\int \frac{1}{\sqrt{x}\,acos(cx)b+\sqrt{x}\,a}dx}{\sqrt{d}}$$

input
```
int(1/(d*x)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(x)*acos(c*x)*b + sqrt(x)*a),x)/sqrt(d)
```

3.223 $\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result 1552
Mathematica [N/A] 1552
Rubi [N/A] 1553
Maple [N/A] 1553
Fricas [N/A] 1554
Sympy [N/A] 1554
Maxima [N/A] 1554
Giac [N/A] 1555
Mupad [N/A] 1555
Reduce [N/A] 1556

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx=\operatorname{Int}\left(\frac{1}{(dx)^{3/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b*d^2*x^2*arccos(c*x) + a*d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\,(d\,x)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d*x)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \frac{\int \frac{1}{\sqrt{x}\,acos(cx)bx+\sqrt{x}\,ax}dx}{\sqrt{d}\,d}$$

input
```
int(1/(d*x)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(x)*acos(c*x)*b*x + sqrt(x)*a*x),x)/(sqrt(d)*d)
```

3.224 $\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result 1557
Mathematica [N/A] 1557
Rubi [N/A] 1558
Maple [N/A] 1558
Fricas [N/A] 1559
Sympy [N/A] 1559
Maxima [N/A] 1559
Giac [N/A] 1560
Mupad [N/A] 1560
Reduce [N/A] 1561

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{(dx)^{3/2}}{(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2} \, dx$$

$$\Big\downarrow \text{5149}$$

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}dx$$

input
```
Int[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+b\arccos(cx))^2}dx$$

input
```
int((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((d*x)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((d*x)**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 181, normalized size of antiderivative = 10.06

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(sqrt(c*x + 1)*sqrt(-c*x + 1)*d^(3/2)*x^(3/2) - (b^2*c*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x) + a*b*c)*sqrt(d)*integrate(1/2*(5*c^2*d*x^2 - 3*d)
*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2
- b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x))/(b^2*c*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(d\,x)^{3/2}}{(a+b\,\mathrm{acos}(c\,x))^2}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d*x)^(3/2)/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)d$$

input
```
int((d*x)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*int((sqrt(x)*x)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*d
```

3.225 $\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$

Optimal result 1562
Mathematica [N/A] 1562
Rubi [N/A] 1563
Maple [N/A] 1563
Fricas [N/A] 1564
Sympy [N/A] 1564
Maxima [N/A] 1564
Giac [N/A] 1565
Mupad [N/A] 1565
Reduce [N/A] 1566

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{\sqrt{dx}}{(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 9.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}dx$$

input
```
Int[Sqrt[d*x]/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{dx}}{\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

input
```
int((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{dx}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{dx}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((d*x)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d*x)/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 181, normalized size of antiderivative = 10.06

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{dx}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*sqrt(d)*integ
rate(1/2*(3*c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3*x^3
 - a*b*c*x + (b^2*c^3*x^3 - b^2*c*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
 c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*sqrt(x))/(b^2*c*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{dx}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{d\,x}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d*x)^(1/2)/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{x}}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)$$

input
```
int((d*x)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*int(sqrt(x)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.226 $\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$

Optimal result . . . 1567
Mathematica [N/A] . . . 1567
Rubi [N/A] . . . 1568
Maple [N/A] . . . 1568
Fricas [N/A] . . . 1569
Sympy [N/A] . . . 1569
Maxima [N/A] . . . 1569
Giac [N/A] . . . 1570
Mupad [N/A] . . . 1570
Reduce [N/A] . . . 1571

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{\sqrt{dx}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{ 5149}$$
$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{dx}\,\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

input
```
int(1/(d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b^2*d*x*arccos(c*x)^2 + 2*a*b*d*x*arccos(c*x) + a^2*d*
x), x)
```

Sympy [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 196, normalized size of antiderivative = 10.89

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c*d*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d*x)*sqrt(
d)*integrate(1/2*(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c
^3*d*x^4 - a*b*c*d*x^2 + (b^2*c^3*d*x^4 - b^2*c*d*x^2)*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*sqrt(x
))/(b^2*c*d*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d*x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{d\,x}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d*x)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \frac{\int \frac{1}{\sqrt{x}\,acos(cx)^2b^2+2\sqrt{x}\,acos(cx)ab+\sqrt{x}\,a^2}dx}{\sqrt{d}}$$

input
```
int(1/(d*x)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt(x)*acos(c*x)**2*b**2 + 2*sqrt(x)*acos(c*x)*a*b + sqrt(x)*a**2)
,x)/sqrt(d)
```

3.227 $\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result 1572
Mathematica [N/A] 1572
Rubi [N/A] 1573
Maple [N/A] 1573
Fricas [N/A] 1574
Sympy [N/A] 1574
Maxima [N/A] 1574
Giac [N/A] 1575
Mupad [N/A] 1575
Reduce [N/A] 1576

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2} \, dx$$

$$\Big\downarrow \text{ 5149}$$

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

input
```
int(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/(b^2*d^2*x^2*arccos(c*x)^2 + 2*a*b*d^2*x^2*arccos(c*x)
+ a^2*d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 12.11

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((b^2*c*d^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d^2*x^2
)*sqrt(d)*integrate(1/2*(c^2*x^2 - 3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)
/(a*b*c^3*d^2*x^5 - a*b*c*d^2*x^3 + (b^2*c^3*d^2*x^5 - b^2*c*d^2*x^3)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1)
*sqrt(d)*sqrt(x))/(b^2*c*d^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
) + a*b*c*d^2*x^2)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(d\,x)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d*x)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \frac{\int \frac{1}{\sqrt{x}\,acos(cx)^2 b^2 x+2\sqrt{x}\,acos(cx)abx+\sqrt{x}\,a^2 x}dx}{\sqrt{d}\,d}$$

input
```
int(1/(d*x)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt(x)*acos(c*x)**2*b**2*x + 2*sqrt(x)*acos(c*x)*a*b*x + sqrt(x)*a
**2*x),x)/(sqrt(d)*d)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1577
4.2 Links to plain text integration problems used in this report for each CAS . 1595

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file