Computer Algebra Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.2-Inverse-cosine/272-5.2.3

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**117**]. This is test number [272].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (117)	0.00 (0)
Mathematica	99.15 (116)	0.85 (1)
Maple	96.58 (113)	3.42 (4)
Giac	60.68 (71)	39.32 (46)
Fricas	47.01 (55)	52.99 (62)
Maxima	47.01 (55)	52.99 (62)
Sympy	47.01 (55)	52.99 (62)
Reduce	40.17 (47)	59.83 (70)
Mupad	28.21 (33)	71.79 (84)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

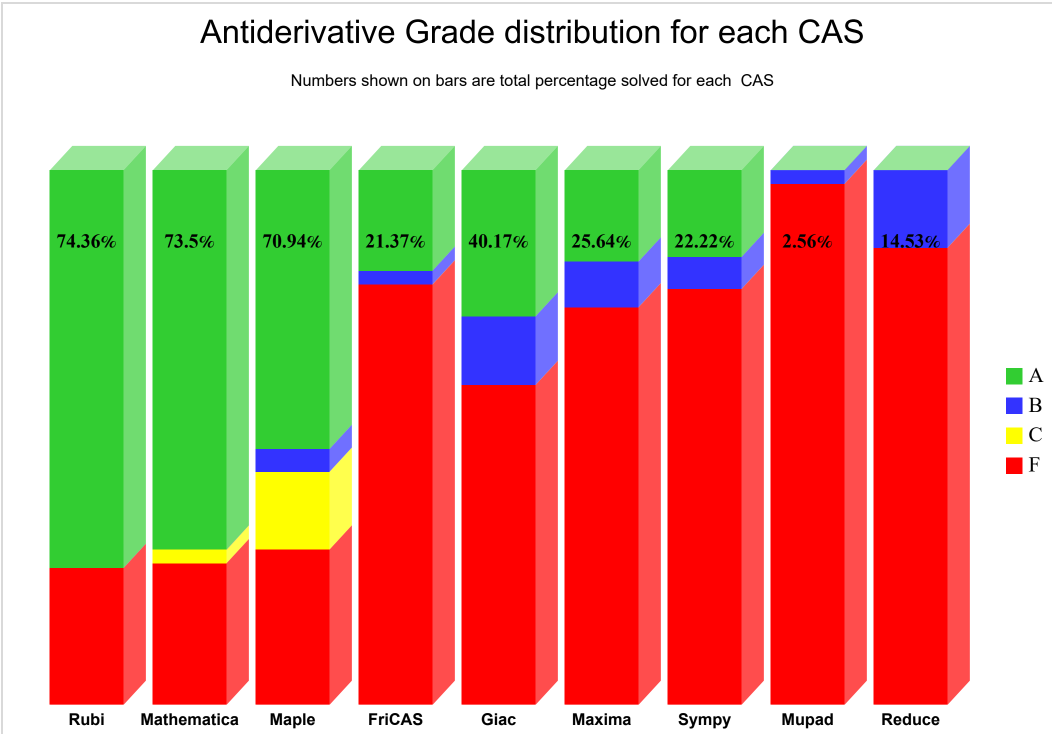
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

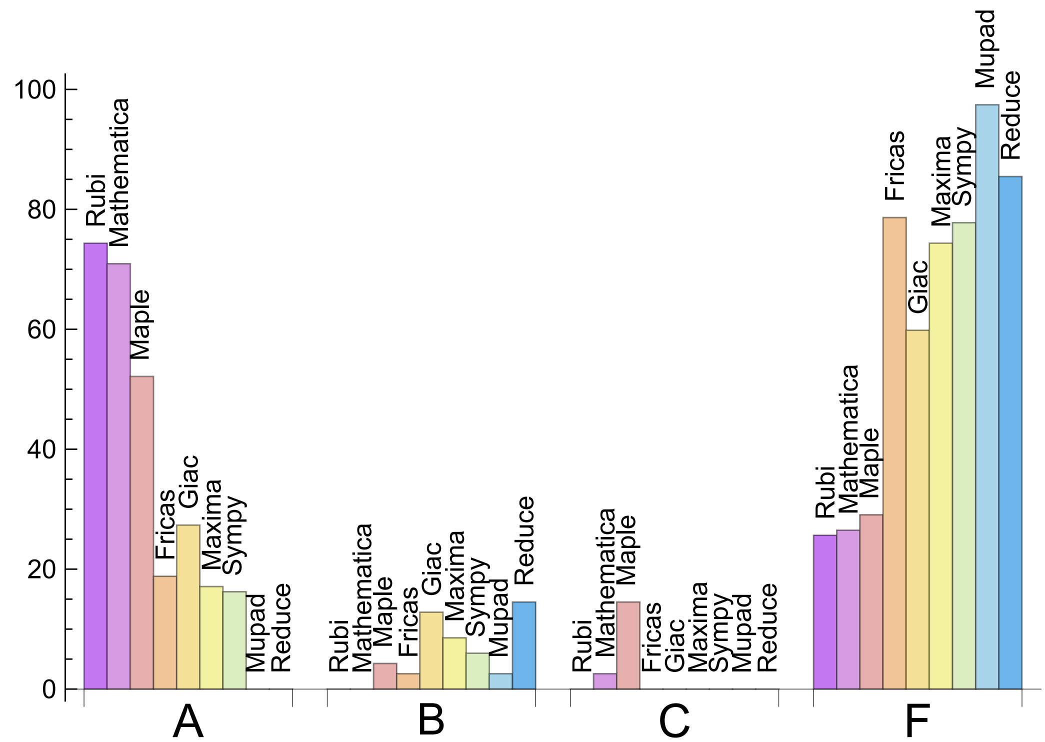
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.359	0.000	0.000	25.641
Mathematica	70.940	0.000	2.564	26.496
Maple	52.137	4.274	14.530	29.060
Giac	27.350	12.821	0.000	59.829
Fricas	18.803	2.564	0.000	78.632
Maxima	17.094	8.547	0.000	74.359
Sympy	16.239	5.983	0.000	77.778
Mupad	0.000	2.564	0.000	97.436
Reduce	0.000	14.530	0.000	85.470

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Giac	46	2.17	2.17	95.65
Maxima	62	87.10	0.00	12.90
Sympy	62	90.32	9.68	0.00
Fricas	62	100.00	0.00	0.00
Reduce	70	100.00	0.00	0.00
Mupad	84	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.13
Mupad	0.35
Maxima	0.51
Rubi	0.58
Maple	0.73
Giac	3.06
Mathematica	4.33
Sympy	7.10
Reduce	17.31

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	24.97	1.06	22.00	1.00
Reduce	96.72	2.95	61.00	1.60
Mathematica	139.75	1.00	94.50	1.02
Rubi	141.92	0.96	113.00	1.00
Fricas	157.15	2.26	72.00	1.52
Sympy	167.00	1.55	36.00	1.30
Maxima	225.20	4.41	141.00	1.36
Maple	256.58	1.51	142.00	1.23
Giac	287.44	1.55	75.00	1.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

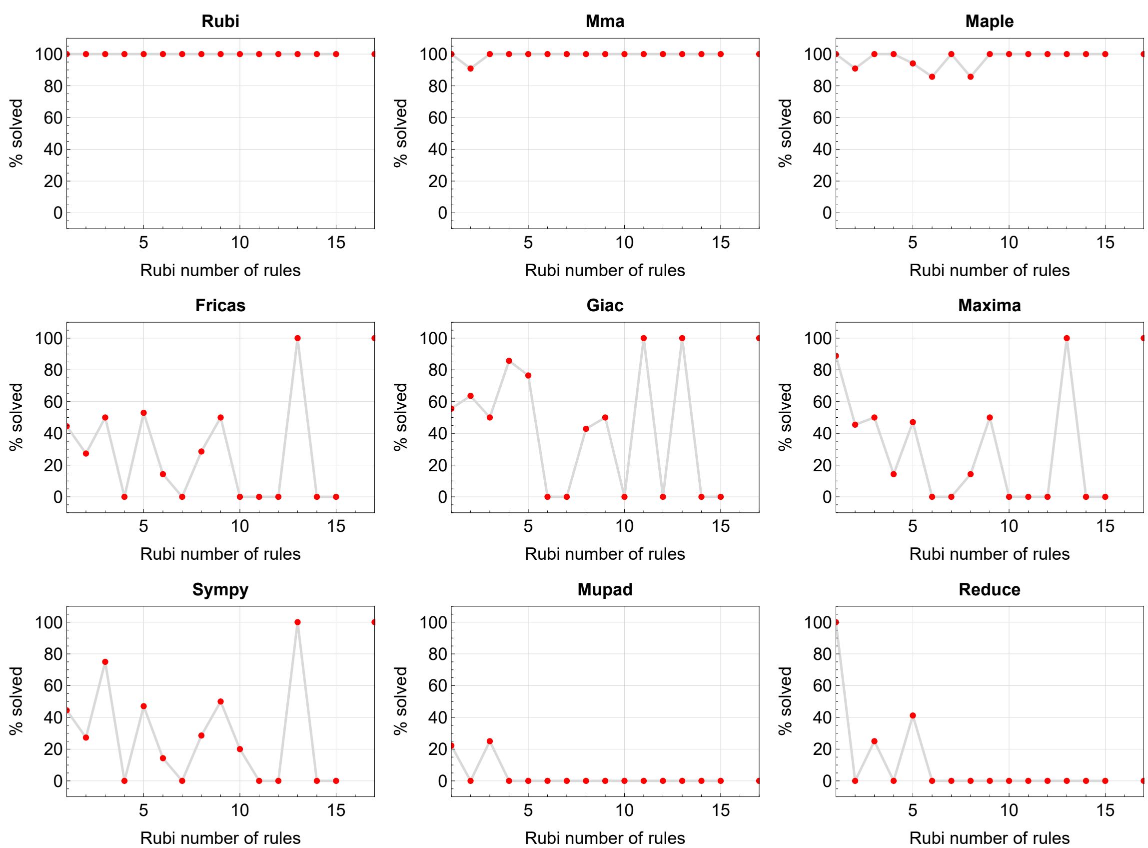


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

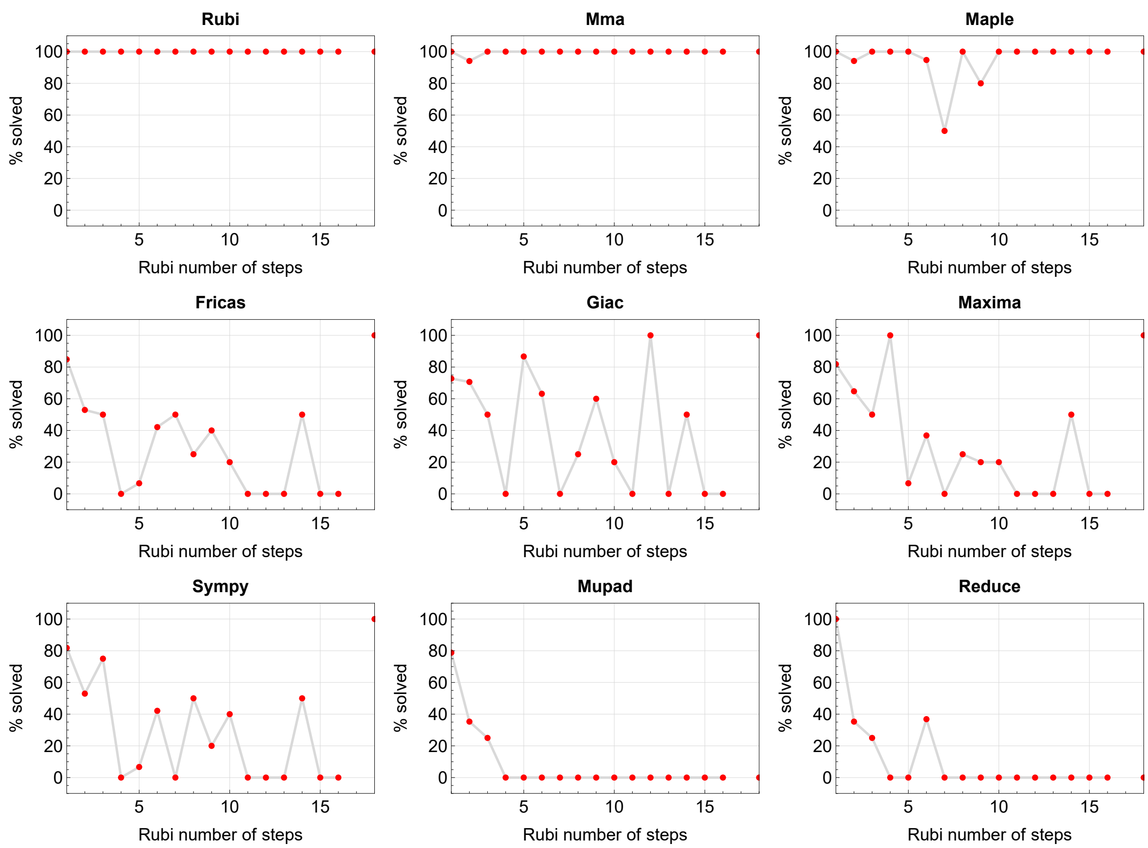


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

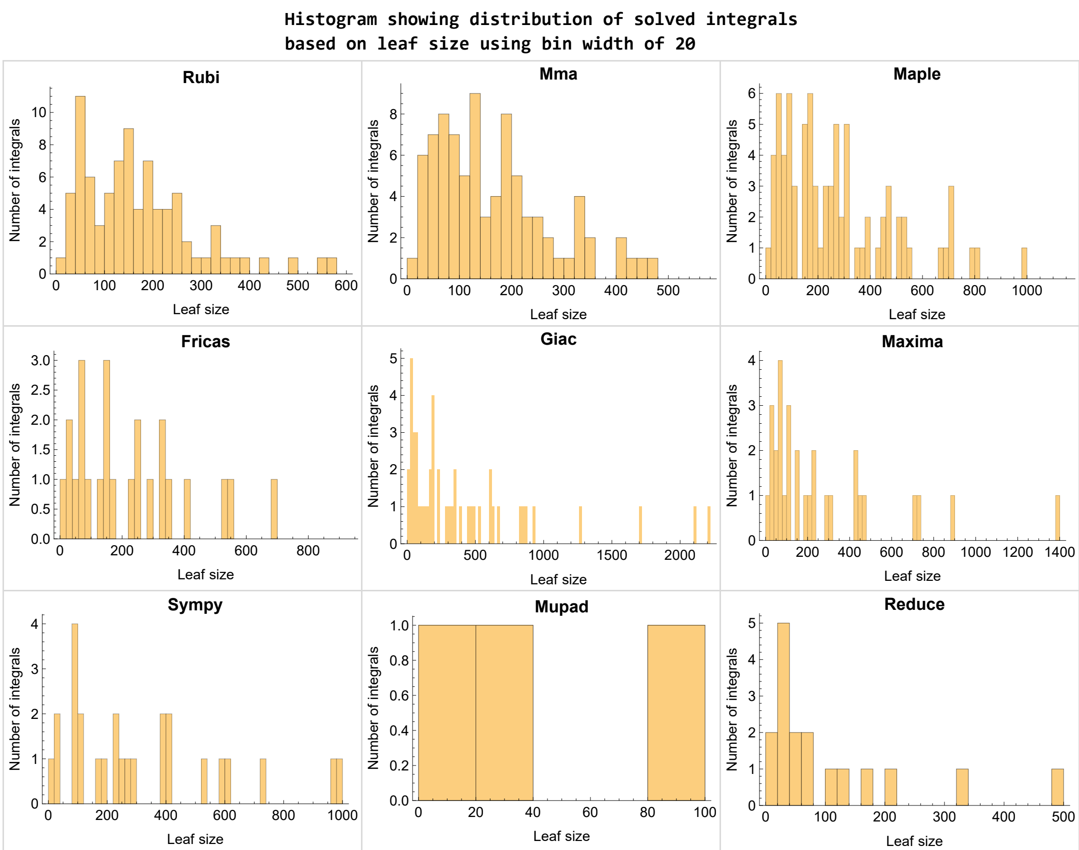


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

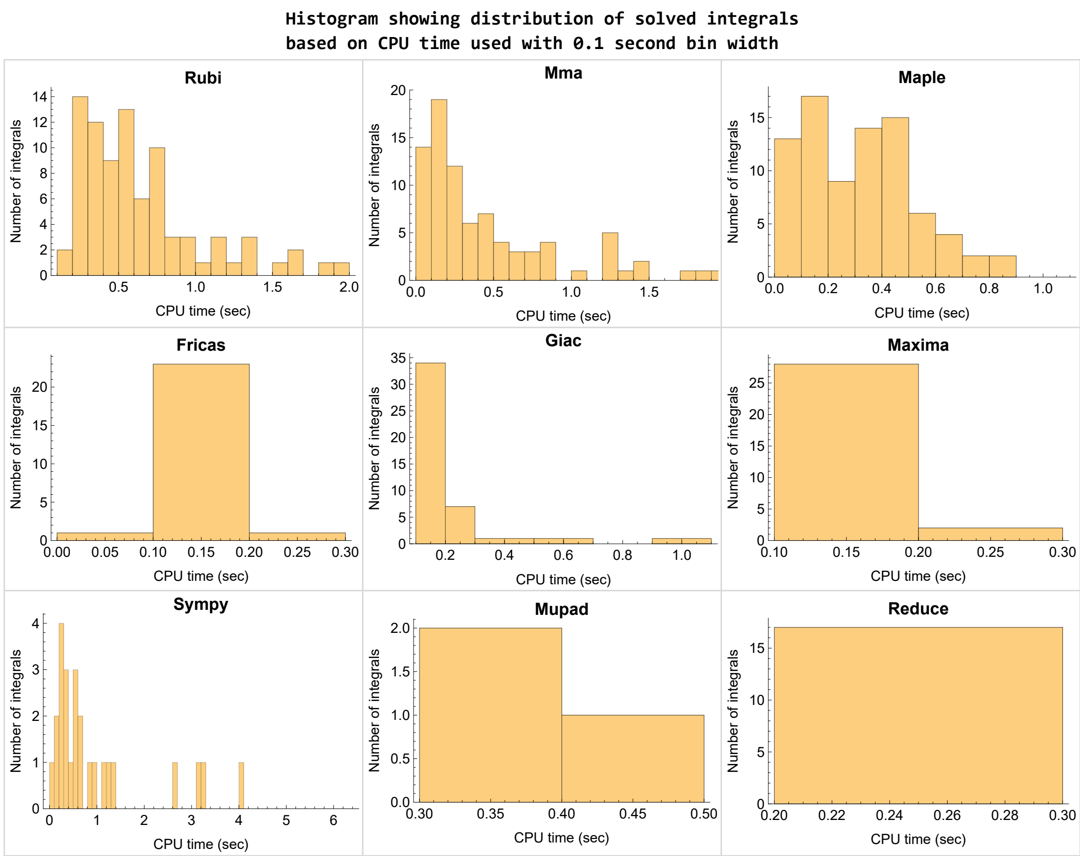


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

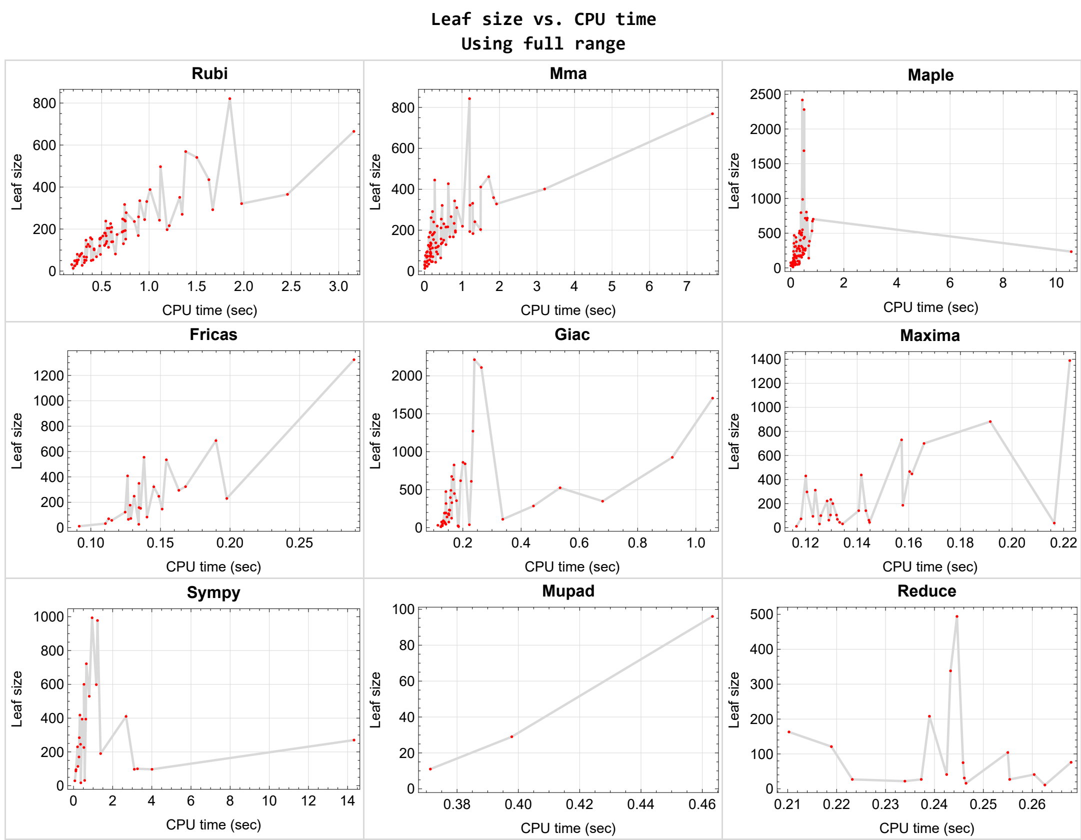


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{20, 21, 25, 26, 30, 31, 35, 36, 72, 73, 78, 79, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{71\}$

Mathematica $\{104, 105\}$

Maple {89}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

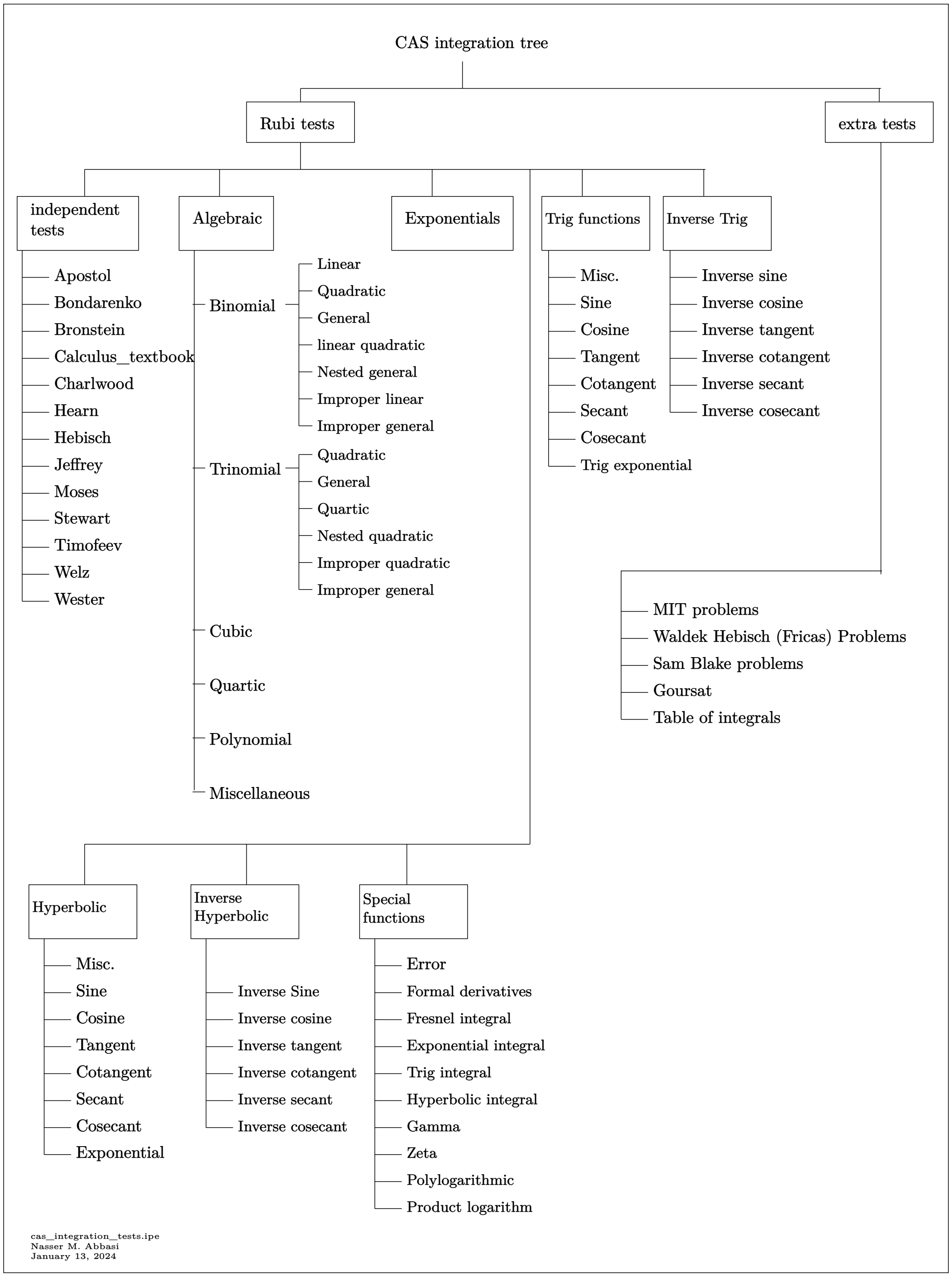
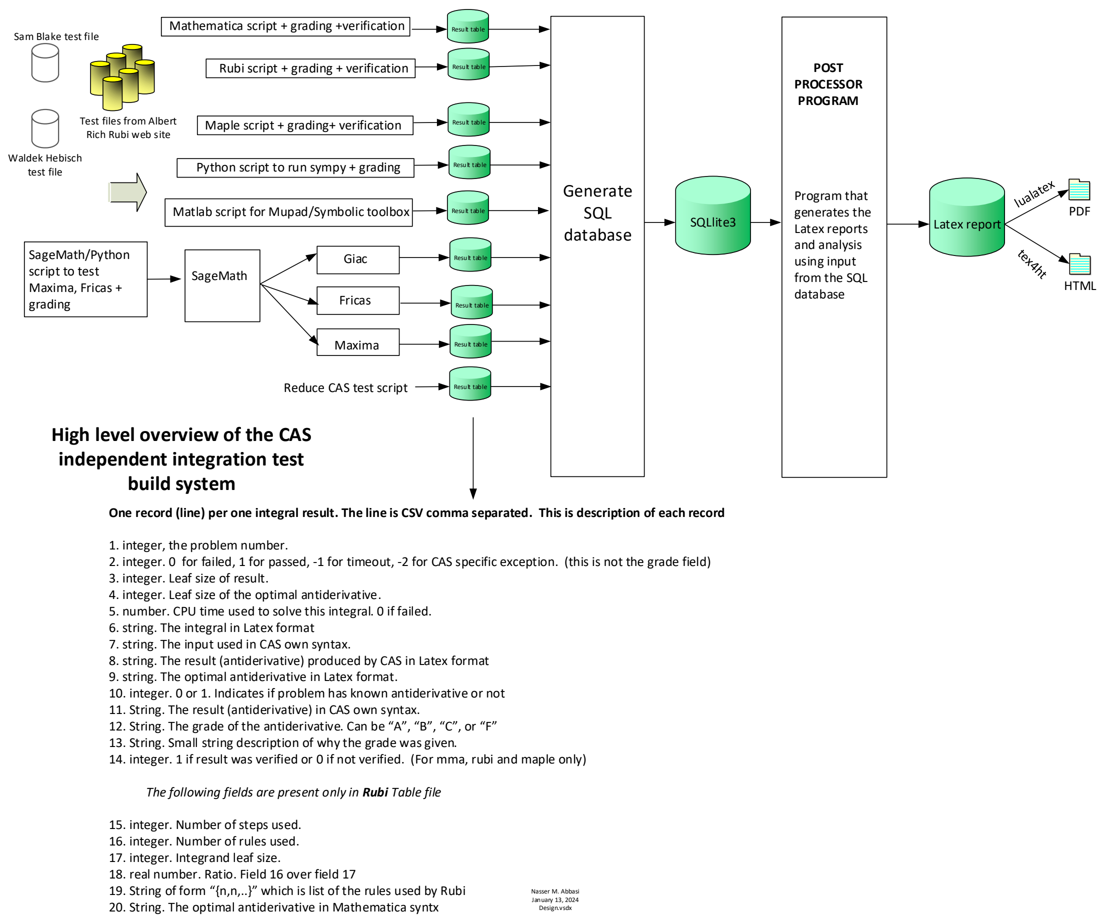


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 103, 104, 105 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 96, 97, 98 }

B grade { }

C grade { 103, 104, 105 }

F normal fail { 90 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50, 54, 61, 65, 66, 67, 71, 77, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93, 96, 97, 98 }

B grade { 40, 46, 48, 60, 62 }

C grade { 51, 52, 53, 55, 56, 57, 58, 59, 63, 64, 68, 69, 70, 74, 75, 76, 85 }

F normal fail { 90, 103, 104, 105 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 7, 8, 9, 12, 13, 14, 49, 50, 71, 77, 80, 81, 82, 83, 84, 86, 87, 88, 89 }

B grade { 103, 104, 105 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 85, 90, 91, 92, 93, 96, 97, 98 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 2, 3, 41, 42, 49, 50, 54, 55, 56, 65, 77, 80, 81, 82, 83, 84, 86, 87, 88, 89 }

B grade { 1, 7, 8, 9, 12, 13, 14, 40, 46, 60 }

C grade { }

F normal fail { 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 43, 44, 45, 47, 48, 51, 52, 53, 57, 58, 59, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 74, 75, 76, 91, 92, 93, 96, 97, 98, 104, 105 }

F(-1) timedout fail { }

F(-2) exception fail { 85, 90, 101, 102, 103, 106, 107, 108 }

Giac

A grade { 1, 2, 3, 7, 8, 9, 17, 18, 19, 23, 24, 29, 49, 50, 54, 60, 65, 68, 69, 70, 80, 81, 82, 83, 84, 86, 87, 88, 89, 91, 92, 93 }

B grade { 12, 13, 14, 22, 27, 28, 32, 33, 34, 74, 75, 76, 96, 97, 98 }

C grade { }

F normal fail { 66 }

F(-1) timedout fail { 36 }

F(-2) exception fail { 4, 5, 6, 10, 11, 15, 16, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 67, 71, 77, 85, 90, 100, 102, 103, 104, 105, 107, 108, 109 }

Mupad

A grade { }

B grade { 50, 84, 89 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 80, 81, 82, 83, 85, 86, 87, 88, 90, 91, 92, 93, 96, 97, 98, 103, 104, 105 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 7, 8, 9, 37, 38, 39, 49, 50, 80, 81, 82, 83, 84, 86, 87, 88 }

B grade { 12, 13, 14, 40, 44, 46, 89 }

C grade { }

F normal fail { 4, 5, 10, 11, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 41, 42, 45, 47, 48, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 85, 90, 91, 92, 93, 96, 97, 98, 103, 104 }

F(-1) timedout fail { 6, 43, 51, 57, 100, 105 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 40, 46, 50, 54, 60, 65, 71, 77, 80, 81, 82, 83, 84, 89 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 42, 43, 44, 45, 47, 48, 49, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 66, 67, 68, 69, 70, 74, 75, 76, 85, 86, 87, 88, 90, 91, 92, 93, 96, 97, 98, 103, 104, 105 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	167	119	162	312	158	226	192	163	0
N.S.	1	0.95	0.68	0.93	1.78	0.90	1.29	1.10	0.93	0.00
time (sec)	N/A	0.513	0.132	0.147	0.124	0.135	0.530	0.137	0.210	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	127	95	119	200	122	170	140	121	0
N.S.	1	0.97	0.73	0.91	1.53	0.93	1.30	1.07	0.92	0.00
time (sec)	N/A	0.357	0.075	0.113	0.131	0.125	0.279	0.145	0.219	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	67	80	100	72	95	76	76	0
N.S.	1	1.03	0.87	1.04	1.30	0.94	1.23	0.99	0.99	0.00
time (sec)	N/A	0.278	0.061	0.093	0.126	0.129	0.125	0.137	0.268	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	66	107	140	0	0	0	0	54	0
N.S.	1	0.78	1.26	1.65	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.341	0.154	0.677	0.000	0.000	0.000	0.000	0.279	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	122	220	189	0	0	0	0	148	0
N.S.	1	0.84	1.52	1.30	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.531	0.340	0.457	0.000	0.000	0.000	0.000	0.293	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	174	321	252	0	0	0	0	262	0
N.S.	1	0.88	1.62	1.27	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.663	0.471	0.541	0.000	0.000	0.000	0.000	0.294	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	351	241	384	730	323	529	449	279	0
N.S.	1	1.18	0.81	1.29	2.45	1.08	1.78	1.51	0.94	0.00
time (sec)	N/A	1.324	1.344	0.706	0.157	0.168	0.804	0.171	0.348	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	245	193	275	466	247	394	329	213	0
N.S.	1	1.12	0.88	1.26	2.13	1.13	1.80	1.50	0.97	0.00
time (sec)	N/A	0.953	1.208	0.322	0.160	0.149	0.438	0.162	0.339	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	141	138	173	234	146	230	185	145	0
N.S.	1	1.10	1.08	1.35	1.83	1.14	1.80	1.45	1.13	0.00
time (sec)	N/A	0.554	0.163	0.279	0.130	0.151	0.203	0.151	0.277	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	122	215	314	0	0	0	0	85	0
N.S.	1	0.70	1.24	1.80	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.531	0.571	0.306	0.000	0.000	0.000	0.000	0.275	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	197	401	423	0	0	0	0	233	0
N.S.	1	0.78	1.60	1.69	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	1.190	3.203	0.490	0.000	0.000	0.000	0.000	0.276	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	665	411	715	1389	535	978	841	423	0
N.S.	1	1.49	0.92	1.60	3.11	1.20	2.19	1.89	0.95	0.00
time (sec)	N/A	3.160	1.499	0.545	0.222	0.154	1.224	0.208	0.324	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	435	331	508	882	408	722	618	332	0
N.S.	1	1.32	1.00	1.54	2.67	1.24	2.19	1.87	1.01	0.00
time (sec)	N/A	1.631	1.284	0.415	0.192	0.126	0.655	0.192	0.249	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	236	240	313	447	248	418	355	238	0
N.S.	1	1.20	1.22	1.60	2.28	1.27	2.13	1.81	1.21	0.00
time (sec)	N/A	0.845	0.245	0.401	0.161	0.131	0.327	0.178	0.283	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	186	427	549	0	0	0	0	114	0
N.S.	1	0.69	1.58	2.03	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.717	0.635	0.441	0.000	0.000	0.000	0.000	0.270	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	321	769	804	0	0	0	0	312	0
N.S.	1	0.74	1.78	1.87	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.976	7.678	0.589	0.000	0.000	0.000	0.000	0.244	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	51	43	42	0	0	0	59	64	0
N.S.	1	0.76	0.64	0.63	0.00	0.00	0.00	0.88	0.96	0.00
time (sec)	N/A	0.336	0.297	0.167	0.000	0.000	0.000	0.139	0.236	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	40	34	35	0	0	0	44	46	0
N.S.	1	0.80	0.68	0.70	0.00	0.00	0.00	0.88	0.92	0.00
time (sec)	N/A	0.318	0.115	0.086	0.000	0.000	0.000	0.142	0.246	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	23	22	0	0	0	25	28	0
N.S.	1	0.93	0.79	0.76	0.00	0.00	0.00	0.86	0.97	0.00
time (sec)	N/A	0.294	0.072	0.079	0.000	0.000	0.000	0.130	0.267	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	25	24	22	24	27	22
N.S.	1	1.00	1.10	1.00	1.25	1.20	1.10	1.20	1.35	1.10
time (sec)	N/A	0.197	1.517	0.310	0.225	0.098	0.754	0.549	0.247	0.320

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	23	36	36	23	36	22
N.S.	1	1.00	1.10	1.00	1.15	1.80	1.80	1.15	1.80	1.10
time (sec)	N/A	0.202	7.095	0.597	0.250	0.106	0.979	2.088	0.269	0.325

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	79	83	107	0	0	0	169	64	0
N.S.	1	0.84	0.88	1.14	0.00	0.00	0.00	1.80	0.68	0.00
time (sec)	N/A	0.488	0.351	0.135	0.000	0.000	0.000	0.152	0.284	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	68	70	83	0	0	0	124	46	0
N.S.	1	0.88	0.91	1.08	0.00	0.00	0.00	1.61	0.60	0.00
time (sec)	N/A	0.445	0.224	0.093	0.000	0.000	0.000	0.161	0.245	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	55	61	0	0	0	73	28	0
N.S.	1	0.98	1.02	1.13	0.00	0.00	0.00	1.35	0.52	0.00
time (sec)	N/A	0.408	0.139	0.080	0.000	0.000	0.000	0.152	0.269	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	154	24	26	24	31	22
N.S.	1	1.00	1.10	1.00	7.70	1.20	1.30	1.20	1.55	1.10
time (sec)	N/A	0.328	4.464	0.305	0.480	0.118	0.866	0.596	0.248	0.331

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	201	36	41	23	42	22
N.S.	1	1.00	1.10	1.00	10.05	1.80	2.05	1.15	2.10	1.10
time (sec)	N/A	0.337	12.991	0.577	0.572	0.103	1.432	2.432	0.254	0.325

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	208	183	188	0	0	0	674	80	0
N.S.	1	0.77	0.68	0.70	0.00	0.00	0.00	2.51	0.30	0.00
time (sec)	N/A	0.607	1.293	0.122	0.000	0.000	0.000	0.162	0.270	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	158	139	142	0	0	0	396	58	0
N.S.	1	0.79	0.69	0.71	0.00	0.00	0.00	1.97	0.29	0.00
time (sec)	N/A	0.492	0.478	0.096	0.000	0.000	0.000	0.157	0.242	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	106	93	94	0	0	0	178	36	0
N.S.	1	0.85	0.74	0.75	0.00	0.00	0.00	1.42	0.29	0.00
time (sec)	N/A	0.420	0.159	0.082	0.000	0.000	0.000	0.150	0.253	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	29	39	36	28	40	26
N.S.	1	1.00	1.08	1.00	1.21	1.62	1.50	1.17	1.67	1.08
time (sec)	N/A	0.207	2.657	0.279	0.236	0.118	1.540	3.280	0.232	0.331

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	27	67	61	27	58	26
N.S.	1	1.00	1.08	1.00	1.12	2.79	2.54	1.12	2.42	1.08
time (sec)	N/A	0.214	31.639	0.822	0.249	0.125	2.742	177.321	0.218	0.341

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	243	461	468	0	0	0	2110	136	0
N.S.	1	0.80	1.52	1.54	0.00	0.00	0.00	6.96	0.45	0.00
time (sec)	N/A	0.741	1.711	0.128	0.000	0.000	0.000	0.264	0.258	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	193	233	354	0	0	0	1271	100	0
N.S.	1	0.82	0.99	1.51	0.00	0.00	0.00	5.41	0.43	0.00
time (sec)	N/A	0.749	0.792	0.118	0.000	0.000	0.000	0.235	0.268	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	139	122	238	0	0	0	610	64	0
N.S.	1	0.89	0.78	1.52	0.00	0.00	0.00	3.89	0.41	0.00
time (sec)	N/A	0.605	0.371	0.104	0.000	0.000	0.000	0.229	0.256	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	237	71	73	28	74	26
N.S.	1	1.00	1.08	1.00	9.88	2.96	3.04	1.17	3.08	1.08
time (sec)	N/A	0.437	26.206	0.292	0.645	0.111	3.586	8.326	0.259	0.334

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	344	119	121	0	112	26
N.S.	1	1.00	1.08	1.00	14.33	4.96	5.04	0.00	4.67	1.08
time (sec)	N/A	0.440	57.709	1.316	0.919	0.101	14.611	0.000	0.290	0.326

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	190	157	203	0	0	270	0	150	0
N.S.	1	1.07	0.88	1.14	0.00	0.00	1.52	0.00	0.84	0.00
time (sec)	N/A	0.730	0.442	0.495	0.000	0.000	14.328	0.000	0.237	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	131	114	152	0	0	190	0	101	0
N.S.	1	1.07	0.93	1.25	0.00	0.00	1.56	0.00	0.83	0.00
time (sec)	N/A	0.546	0.300	0.410	0.000	0.000	1.383	0.000	0.257	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	98	0	0	97	0	51	0
N.S.	1	1.00	1.04	1.44	0.00	0.00	1.43	0.00	0.75	0.00
time (sec)	N/A	0.314	0.170	0.264	0.000	0.000	4.008	0.000	0.245	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	49	44	0	97	0	27	0
N.S.	1	1.00	1.00	1.96	1.76	0.00	3.88	0.00	1.08	0.00
time (sec)	N/A	0.219	0.019	0.249	0.133	0.000	3.104	0.000	0.223	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	89	93	62	0	0	0	80	0
N.S.	1	1.00	1.68	1.75	1.17	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.254	0.150	0.318	0.145	0.000	0.000	0.000	0.212	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	134	167	141	0	0	0	197	0
N.S.	1	1.04	1.20	1.49	1.26	0.00	0.00	0.00	1.76	0.00
time (sec)	N/A	0.374	0.176	0.331	0.143	0.000	0.000	0.000	0.226	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	181	176	244	0	0	0	0	355	0
N.S.	1	1.08	1.05	1.45	0.00	0.00	0.00	0.00	2.11	0.00
time (sec)	N/A	0.531	0.207	0.345	0.000	0.000	0.000	0.000	0.244	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	258	203	293	0	0	410	0	166	0
N.S.	1	1.20	0.94	1.36	0.00	0.00	1.91	0.00	0.77	0.00
time (sec)	N/A	0.889	1.496	0.442	0.000	0.000	2.683	0.000	0.234	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	120	129	179	0	0	0	0	82	0
N.S.	1	0.97	1.04	1.44	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.480	0.572	0.379	0.000	0.000	0.000	0.000	0.232	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	69	105	0	100	0	41	0
N.S.	1	1.00	1.00	2.76	4.20	0.00	4.00	0.00	1.64	0.00
time (sec)	N/A	0.214	0.025	0.327	0.132	0.000	3.267	0.000	0.260	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	113	167	269	0	0	0	0	142	0
N.S.	1	0.91	1.35	2.17	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.572	0.678	0.494	0.000	0.000	0.000	0.000	0.249	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	216	310	1687	0	0	0	0	361	0
N.S.	1	0.96	1.37	7.46	0.00	0.00	0.00	0.00	1.60	0.00
time (sec)	N/A	1.213	0.860	0.495	0.000	0.000	0.000	0.000	0.232	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	31	27	13	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	0.91	0.79	0.38	0.00
time (sec)	N/A	0.238	0.006	0.006	0.125	0.134	0.568	0.127	0.222	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	17	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.31	0.85	0.85	0.85
time (sec)	N/A	0.200	0.008	0.083	0.116	0.092	0.373	0.125	0.263	0.371

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	278	266	689	0	0	0	0	150	0
N.S.	1	1.06	1.02	2.63	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.759	0.709	0.605	0.000	0.000	0.000	0.000	0.260	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	204	211	479	0	0	0	0	101	0
N.S.	1	1.10	1.14	2.59	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.560	0.441	0.381	0.000	0.000	0.000	0.000	0.223	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	280	0	0	0	0	51	0
N.S.	1	1.00	1.15	2.41	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.340	0.446	0.281	0.000	0.000	0.000	0.000	0.241	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	86	44	0	0	24	27	0
N.S.	1	1.00	1.02	1.76	0.90	0.00	0.00	0.49	0.55	0.00
time (sec)	N/A	0.215	0.081	0.164	0.145	0.000	0.000	0.183	0.255	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	177	62	0	0	0	80	0
N.S.	1	1.00	0.96	2.21	0.78	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.243	0.164	0.319	0.129	0.000	0.000	0.000	0.232	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	113	472	141	0	0	0	197	0
N.S.	1	1.03	0.73	3.06	0.92	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.382	0.214	0.413	0.141	0.000	0.000	0.000	0.279	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	238	154	2280	0	0	0	0	355	0
N.S.	1	1.06	0.68	10.13	0.00	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	0.544	0.241	0.501	0.000	0.000	0.000	0.000	0.253	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	331	328	987	0	0	0	0	166	0
N.S.	1	1.12	1.11	3.33	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.976	1.921	0.447	0.000	0.000	0.000	0.000	0.260	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	168	219	531	0	0	0	0	82	0
N.S.	1	0.88	1.14	2.77	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.547	1.013	0.335	0.000	0.000	0.000	0.000	0.254	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	64	142	105	0	0	38	41	0
N.S.	1	1.00	1.31	2.90	2.14	0.00	0.00	0.78	0.84	0.00
time (sec)	N/A	0.229	0.433	0.212	0.130	0.000	0.000	0.222	0.242	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	140	167	399	0	0	0	0	142	0
N.S.	1	0.68	0.81	1.95	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.617	0.769	0.359	0.000	0.000	0.000	0.000	0.312	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	270	322	2418	0	0	0	0	361	0
N.S.	1	0.84	1.00	7.53	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	1.349	1.213	0.437	0.000	0.000	0.000	0.000	0.232	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	365	138	533	0	0	0	0	53	0
N.S.	1	1.04	0.39	1.52	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.460	0.301	0.799	0.000	0.000	0.000	0.000	0.238	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	169	87	260	0	0	0	0	23	0
N.S.	1	0.79	0.40	1.21	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.887	0.171	0.466	0.000	0.000	0.000	0.000	0.227	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	52	69	0	0	14	16	0
N.S.	1	1.00	1.00	1.24	1.64	0.00	0.00	0.33	0.38	0.00
time (sec)	N/A	0.241	0.056	0.309	0.132	0.000	0.000	0.185	0.246	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	152	181	278	0	0	0	0	52	0
N.S.	1	0.64	0.77	1.18	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.754	0.234	0.553	0.000	0.000	0.000	0.000	0.280	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	292	216	673	0	0	0	0	68	0
N.S.	1	0.75	0.56	1.74	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.672	0.621	0.818	0.000	0.000	0.000	0.000	0.216	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	204	197	319	0	0	0	525	94	0
N.S.	1	0.47	0.46	0.74	0.00	0.00	0.00	1.22	0.22	0.00
time (sec)	N/A	0.582	0.824	0.677	0.000	0.000	0.000	0.534	0.225	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	152	151	244	0	0	0	285	61	0
N.S.	1	0.52	0.51	0.83	0.00	0.00	0.00	0.97	0.21	0.00
time (sec)	N/A	0.480	0.514	0.217	0.000	0.000	0.000	0.442	0.259	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	101	94	170	0	0	0	110	27	0
N.S.	1	0.60	0.56	1.01	0.00	0.00	0.00	0.65	0.16	0.00
time (sec)	N/A	0.421	0.320	0.200	0.000	0.000	0.000	0.337	0.234	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	46	57	0	57	0	0	22	0
N.S.	1	1.11	1.00	1.24	0.00	1.24	0.00	0.00	0.48	0.00
time (sec)	N/A	0.235	0.184	0.187	0.000	0.115	0.000	0.000	0.234	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	81	27	26	88	26
N.S.	1	1.00	1.08	0.92	1.00	3.12	1.04	1.00	3.38	1.00
time (sec)	N/A	0.231	9.441	0.431	0.356	0.106	3.416	0.207	0.281	0.317

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	108	27	26	129	26
N.S.	1	1.00	1.08	0.92	1.00	4.15	1.04	1.00	4.96	1.00
time (sec)	N/A	0.237	6.436	1.724	0.308	0.110	8.011	0.237	0.245	0.307

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	238	343	717	0	0	0	1706	136	0
N.S.	1	0.56	0.80	1.68	0.00	0.00	0.00	3.99	0.32	0.00
time (sec)	N/A	0.750	0.804	0.623	0.000	0.000	0.000	1.058	0.257	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	186	231	501	0	0	0	926	89	0
N.S.	1	0.65	0.80	1.75	0.00	0.00	0.00	3.23	0.31	0.00
time (sec)	N/A	0.600	0.515	0.319	0.000	0.000	0.000	0.919	0.238	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	130	118	279	0	0	0	348	41	0
N.S.	1	0.80	0.73	1.72	0.00	0.00	0.00	2.15	0.25	0.00
time (sec)	N/A	0.730	0.348	0.247	0.000	0.000	0.000	0.680	0.245	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	59	38	70	0	0	27	0
N.S.	1	1.00	1.00	1.28	0.83	1.52	0.00	0.00	0.59	0.00
time (sec)	N/A	0.240	0.211	0.103	0.216	0.113	0.000	0.000	0.237	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	239	133	29	26	144	26
N.S.	1	1.00	1.08	0.92	9.19	5.12	1.12	1.00	5.54	1.00
time (sec)	N/A	0.418	16.069	0.461	0.758	0.111	12.520	0.313	0.257	0.328

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	317	178	29	26	216	26
N.S.	1	1.00	1.08	0.92	12.19	6.85	1.12	1.00	8.31	1.00
time (sec)	N/A	0.400	17.416	2.282	0.924	0.133	27.216	0.355	0.248	0.325

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	261	440	430	322	598	475	494	0
N.S.	1	1.00	0.82	1.39	1.36	1.02	1.89	1.50	1.56	0.00
time (sec)	N/A	0.742	0.176	0.195	0.120	0.145	1.163	0.142	0.245	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	226	188	305	297	230	394	318	338	0
N.S.	1	1.00	0.84	1.36	1.32	1.02	1.75	1.41	1.50	0.00
time (sec)	N/A	0.595	0.148	0.144	0.121	0.198	0.627	0.142	0.243	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	152	126	194	186	152	245	192	208	0
N.S.	1	1.01	0.84	1.29	1.24	1.01	1.63	1.28	1.39	0.00
time (sec)	N/A	0.397	0.103	0.141	0.158	0.136	0.358	0.138	0.239	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	84	91	100	94	83	114	91	104	0
N.S.	1	1.04	1.12	1.23	1.16	1.02	1.41	1.12	1.28	0.00
time (sec)	N/A	0.290	0.055	0.122	0.123	0.140	0.224	0.133	0.255	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	31	32	29	31	31	29
N.S.	1	1.00	1.00	1.03	1.00	1.03	0.94	1.00	1.00	0.94
time (sec)	N/A	0.185	0.006	0.005	0.134	0.110	0.067	0.114	0.246	0.398

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	843	234	0	0	0	0	46	0
N.S.	1	1.00	1.56	0.43	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.504	1.201	10.564	0.000	0.000	0.000	0.000	0.269	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	445	702	700	555	994	826	487	0
N.S.	1	1.00	0.78	1.23	1.23	0.98	1.75	1.45	0.86	0.00
time (sec)	N/A	1.387	0.272	0.845	0.166	0.138	0.948	0.170	0.270	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	291	443	438	349	600	491	324	0
N.S.	1	1.00	0.87	1.32	1.31	1.04	1.79	1.47	0.97	0.00
time (sec)	N/A	0.902	0.214	0.519	0.142	0.135	0.533	0.159	0.256	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	166	221	222	177	284	232	184	0
N.S.	1	1.00	1.06	1.42	1.42	1.13	1.82	1.49	1.18	0.00
time (sec)	N/A	0.482	0.116	0.452	0.128	0.128	0.295	0.153	0.251	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	76	74	73	65	87	75	75	96
N.S.	1	1.11	1.62	1.57	1.55	1.38	1.85	1.60	1.60	2.04
time (sec)	N/A	0.321	0.026	0.000	0.118	0.127	0.120	0.127	0.246	0.463

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	821	821	0	0	0	0	0	0	74	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.852	0.000	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	254	311	0	0	0	635	57	0
N.S.	1	1.00	0.65	0.80	0.00	0.00	0.00	1.64	0.15	0.00
time (sec)	N/A	1.011	0.448	0.181	0.000	0.000	0.000	0.167	0.262	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	125	152	0	0	0	229	33	0
N.S.	1	1.00	0.69	0.84	0.00	0.00	0.00	1.27	0.18	0.00
time (sec)	N/A	0.539	0.163	0.151	0.000	0.000	0.000	0.155	0.253	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	50	46	49	0	0	0	50	12	0
N.S.	1	0.93	0.85	0.91	0.00	0.00	0.00	0.93	0.22	0.00
time (sec)	N/A	0.390	0.010	0.046	0.000	0.000	0.000	0.130	0.246	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	29	17	22	31	22
N.S.	1	1.00	1.10	1.00	1.10	1.45	0.85	1.10	1.55	1.10
time (sec)	N/A	0.223	0.654	1.053	0.214	0.095	3.101	0.404	0.288	0.319

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	53	19	22	59	22
N.S.	1	1.00	1.10	1.00	1.10	2.65	0.95	1.10	2.95	1.10
time (sec)	N/A	0.206	3.332	4.570	0.210	0.092	96.612	6.498	0.315	0.316

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	359	796	0	0	0	2213	99	0
N.S.	1	1.00	0.72	1.60	0.00	0.00	0.00	4.45	0.20	0.00
time (sec)	N/A	1.121	1.842	0.384	0.000	0.000	0.000	0.240	0.253	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	190	375	0	0	0	860	61	0
N.S.	1	1.00	0.77	1.51	0.00	0.00	0.00	3.47	0.25	0.00
time (sec)	N/A	0.721	0.825	0.210	0.000	0.000	0.000	0.200	0.253	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	81	72	74	0	0	0	193	26	0
N.S.	1	0.94	0.84	0.86	0.00	0.00	0.00	2.24	0.30	0.00
time (sec)	N/A	0.646	0.142	0.006	0.000	0.000	0.000	0.142	0.249	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	319	57	19	22	64	22
N.S.	1	1.00	1.10	1.00	15.95	2.85	0.95	1.10	3.20	1.10
time (sec)	N/A	0.221	34.800	1.228	2.183	0.120	39.741	0.810	0.260	0.323

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	438	98	0	0	115	22
N.S.	1	1.00	1.10	1.00	21.90	4.90	0.00	0.00	5.75	1.10
time (sec)	N/A	0.222	71.276	4.392	3.021	0.121	0.000	0.000	0.356	0.335

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	61	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	3.05	1.00
time (sec)	N/A	0.220	4.999	2.157	0.000	0.124	9.138	0.277	0.267	0.537

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	0	48	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	0.00	2.40	1.00
time (sec)	N/A	0.206	2.832	1.768	0.000	0.115	4.676	0.000	0.289	0.494

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	73	0	0	294	0	0	123	0
N.S.	1	1.00	1.03	0.00	0.00	4.14	0.00	0.00	1.73	0.00
time (sec)	N/A	0.267	0.079	0.000	0.000	0.163	0.000	0.000	0.289	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	147	190	0	0	686	0	0	276	0
N.S.	1	1.01	1.30	0.00	0.00	4.70	0.00	0.00	1.89	0.00
time (sec)	N/A	0.335	0.157	0.000	0.000	0.190	0.000	0.000	0.320	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	242	189	0	0	1324	0	0	461	0
N.S.	1	1.07	0.84	0.00	0.00	5.86	0.00	0.00	2.04	0.00
time (sec)	N/A	1.110	0.266	0.000	0.000	0.289	0.000	0.000	0.347	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	22	89	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	1.00	4.05	1.00
time (sec)	N/A	0.213	16.112	1.287	0.000	0.122	8.998	0.369	0.321	0.387

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	0	76	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	0.00	3.45	1.00
time (sec)	N/A	0.213	8.327	1.145	0.000	0.105	5.154	0.000	0.325	0.399

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	54	20	0	222	22
N.S.	1	1.00	1.09	0.91	0.00	2.45	0.91	0.00	10.09	1.00
time (sec)	N/A	0.217	3.074	2.135	0.000	0.134	5.033	0.000	0.340	0.366

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	130	65	20	0	488	22
N.S.	1	1.00	1.09	0.91	5.91	2.95	0.91	0.00	22.18	1.00
time (sec)	N/A	0.221	7.822	2.062	0.384	0.153	58.250	0.000	0.371	0.401

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	19	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.221	0.973	1.921	0.206	0.110	0.363	0.265	200.042	0.300

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	39	20	22	29	22
N.S.	1	1.00	1.09	0.91	1.00	1.77	0.91	1.00	1.32	1.00
time (sec)	N/A	0.232	0.873	1.407	0.205	0.125	0.636	0.207	0.642	0.313

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	63	20	22	63	22
N.S.	1	1.00	1.09	0.91	1.00	2.86	0.91	1.00	2.86	1.00
time (sec)	N/A	0.227	1.543	1.949	0.243	0.097	1.732	0.248	0.679	0.300

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	87	20	22	107	22
N.S.	1	1.00	1.09	0.91	1.00	3.95	0.91	1.00	4.86	1.00
time (sec)	N/A	0.244	3.519	1.971	0.257	0.118	7.646	0.286	0.883	0.300

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	237	36	20	22	22	22
N.S.	1	1.00	1.09	0.91	10.77	1.64	0.91	1.00	1.00	1.00
time (sec)	N/A	0.216	6.535	1.877	1.480	0.114	0.655	0.383	200.019	0.327

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	365	67	22	22	22	22
N.S.	1	1.00	1.09	0.91	16.59	3.05	1.00	1.00	1.00	1.00
time (sec)	N/A	0.218	12.519	1.379	1.610	0.116	1.227	0.277	200.027	0.340

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	456	108	22	22	22	22
N.S.	1	1.00	1.09	0.91	20.73	4.91	1.00	1.00	1.00	1.00
time (sec)	N/A	0.219	26.706	1.930	3.282	0.125	4.679	0.366	200.023	0.335

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	578	149	22	22	187	22
N.S.	1	1.00	1.09	0.91	26.27	6.77	1.00	1.00	8.50	1.00
time (sec)	N/A	0.220	53.527	2.057	4.439	0.129	26.584	0.415	0.782	0.325

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [93] had the largest ratio of [.800000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.95	22	0.227
2	A	6	5	0.97	22	0.227
3	A	6	5	1.03	20	0.250
4	A	6	5	0.78	22	0.227
5	A	9	8	0.84	22	0.364
6	A	11	10	0.88	22	0.455
7	A	9	9	1.18	24	0.375
8	A	8	8	1.12	24	0.333
9	A	5	5	1.10	22	0.227
10	A	7	6	0.70	24	0.250
11	A	11	10	0.78	24	0.417
12	A	18	17	1.49	24	0.708
13	A	14	13	1.32	24	0.542
14	A	10	9	1.20	22	0.409
15	A	8	7	0.69	24	0.292
16	A	16	15	0.74	24	0.625
17	A	5	4	0.76	20	0.200
18	A	5	4	0.80	20	0.200
19	A	5	4	0.93	18	0.222
20	N/A	1	0	1.00	20	0.000
21	N/A	1	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	5	4	0.84	20	0.200
23	A	5	4	0.88	20	0.200
24	A	5	4	0.98	18	0.222
25	N/A	2	0	1.00	20	0.000
26	N/A	2	0	1.00	20	0.000
27	A	6	5	0.77	24	0.208
28	A	6	5	0.79	24	0.208
29	A	6	5	0.85	22	0.227
30	N/A	1	0	1.00	24	0.000
31	N/A	1	0	1.00	24	0.000
32	A	5	4	0.80	24	0.167
33	A	5	4	0.82	24	0.167
34	A	5	4	0.89	22	0.182
35	N/A	2	0	1.00	24	0.000
36	N/A	2	0	1.00	24	0.000
37	A	8	8	1.07	24	0.333
38	A	6	6	1.07	24	0.250
39	A	3	3	1.00	24	0.125
40	A	1	1	1.00	24	0.042
41	A	2	2	1.00	24	0.083
42	A	4	4	1.04	24	0.167
43	A	6	6	1.08	24	0.250
44	A	10	10	1.20	26	0.385
45	A	5	5	0.97	26	0.192
46	A	1	1	1.00	26	0.038
47	A	10	9	0.91	26	0.346
48	A	13	12	0.96	26	0.462
49	A	3	3	1.00	14	0.214
50	A	1	1	1.00	21	0.048
51	A	8	8	1.06	24	0.333
52	A	6	6	1.10	24	0.250
53	A	3	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	1	1	1.00	24	0.042
55	A	2	2	1.00	24	0.083
56	A	4	4	1.03	24	0.167
57	A	6	6	1.06	24	0.250
58	A	10	10	1.12	26	0.385
59	A	5	5	0.88	26	0.192
60	A	1	1	1.00	26	0.038
61	A	10	9	0.68	26	0.346
62	A	13	12	0.84	26	0.462
63	A	14	14	1.04	22	0.636
64	A	6	6	0.79	22	0.273
65	A	1	1	1.00	22	0.045
66	A	11	10	0.64	22	0.455
67	A	15	14	0.75	22	0.636
68	A	5	4	0.47	26	0.154
69	A	5	4	0.52	26	0.154
70	A	5	4	0.60	26	0.154
71	A	1	1	1.11	26	0.038
72	N/A	1	0	1.00	26	0.000
73	N/A	1	0	1.00	26	0.000
74	A	6	5	0.56	26	0.192
75	A	6	5	0.65	26	0.192
76	A	12	11	0.80	26	0.423
77	A	1	1	1.00	26	0.038
78	N/A	2	0	1.00	26	0.000
79	N/A	2	0	1.00	26	0.000
80	A	6	5	1.00	18	0.278
81	A	6	5	1.00	18	0.278
82	A	6	5	1.01	18	0.278
83	A	6	5	1.04	16	0.312
84	A	1	1	1.00	8	0.125
85	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	20	0.100
87	A	2	2	1.00	20	0.100
88	A	2	2	1.00	18	0.111
89	A	3	3	1.11	10	0.300
90	A	2	2	1.00	20	0.100
91	A	2	2	1.00	20	0.100
92	A	2	2	1.00	18	0.111
93	A	9	8	0.93	10	0.800
94	N/A	1	0	1.00	20	0.000
95	N/A	1	0	1.00	20	0.000
96	A	2	2	1.00	20	0.100
97	A	2	2	1.00	18	0.111
98	A	9	8	0.94	10	0.800
99	N/A	1	0	1.00	20	0.000
100	N/A	1	0	1.00	20	0.000
101	N/A	1	0	1.00	20	0.000
102	N/A	1	0	1.00	20	0.000
103	A	6	5	1.00	20	0.250
104	A	7	6	1.01	20	0.300
105	A	9	8	1.07	20	0.400
106	N/A	1	0	1.00	22	0.000
107	N/A	1	0	1.00	22	0.000
108	N/A	1	0	1.00	22	0.000
109	N/A	1	0	1.00	22	0.000
110	N/A	1	0	1.00	22	0.000
111	N/A	1	0	1.00	22	0.000
112	N/A	1	0	1.00	22	0.000
113	N/A	1	0	1.00	22	0.000
114	N/A	1	0	1.00	22	0.000
115	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	N/A	1	0	1.00	22	0.000
117	N/A	1	0	1.00	22	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\begin{aligned}\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx &= -\frac{16bd^3\sqrt{1-c^2x^2}}{35c}-\frac{8bd^3(1-c^2x^2)^{3/2}}{105c}-\frac{6bd^3(1-c^2x^2)^{5/2}}{175c}-\frac{bd^3(1-c^2x^2)^{7/2}}{49c}\\ &\quad +d^3x(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))\end{aligned}$$

output
```
-16/35*b*d^3*(-c^2*x^2+1)^(1/2)/c-8/105*b*d^3*(-c^2*x^2+1)^(3/2)/c-6/175*b
*d^3*(-c^2*x^2+1)^(5/2)/c-1/49*b*d^3*(-c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arcco
s(c*x))-c^2*d^3*x^3*(a+b*arccos(c*x))+3/5*c^4*d^3*x^5*(a+b*arccos(c*x))-1/
7*c^6*d^3*x^7*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx = -\frac{d^3\left(b\sqrt{1-c^2x^2}(2161-757c^2x^2+351c^4x^4-75c^6x^6)+105acx(-35+35c^2x^2-21c^4x^4+5c^6x^6)+10\right.}{3675c}$$

input
```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/3675*(d^3*(b*Sqrt[1 - c^2*x^2]*(2161 - 757*c^2*x^2 + 351*c^4*x^4 - 75*c
^6*x^6) + 105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 105*b*c*
x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCos[c*x]))/c
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5155, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5155}$$

$$bc\int \frac{d^3x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}bcd^3\int \frac{x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{70}bcd^3\int \frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{70}bcd^3\int \left(5\left(1-c^2x^2\right)^{5/2}+6\left(1-c^2x^2\right)^{3/2}+8\sqrt{1-c^2x^2}+\frac{16}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))+\frac{1}{70}bcd^3\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^2}-\frac{12(1-c^2x^2)^{5/2}}{5c^2}-\frac{16(1-c^2x^2)^{3/2}}{3c^2}-\frac{32\sqrt{1-c^2x^2}}{c^2}\right)$$

input
```
Int[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^3*((-32*Sqrt[1 - c^2*x^2])/c^2 - (16*(1 - c^2*x^2)^(3/2))/(3*c^2) -
 (12*(1 - c^2*x^2)^(5/2))/(5*c^2) - (10*(1 - c^2*x^2)^(7/2))/(7*c^2)))/70
+ d^3*x*(a + b*ArcCos[c*x]) - c^2*d^3*x^3*(a + b*ArcCos[c*x]) + (3*c^4*d^3
*x^5*(a + b*ArcCos[c*x]))/5 - (c^6*d^3*x^7*(a + b*ArcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

method	result
parts	$-d^3a\left(\frac{1}{7}c^6x^7-\frac{3}{5}c^4x^5+c^2x^3-x\right)-\frac{d^3b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{3\arccos(cx)c^5x^5}{5}+c^3x^3\arccos(cx)-cx\arccos(c\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{3\arccos(cx)c^5x^5}{5}+c^3x^3\arccos(cx)-cx\arccos(cx)+\frac{2161\sqrt{-c^2x}}{3675}\right.}{c}$
default	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{3\arccos(cx)c^5x^5}{5}+c^3x^3\arccos(cx)-cx\arccos(cx)+\frac{2161\sqrt{-c^2x}}{3675}\right.}{c}$
orering	$\frac{x(325c^6x^6-1437c^4x^4+2739c^2x^2-5547)(-c^2dx^2+d)^3(a+b\arccos(cx))}{1225(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{(75c^6x^6-351c^4x^4+757c^2x^2-2161)\left(-6\right.}{3675}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/7*c^6*x^7-3/5*c^4*x^5+c^2*x^3-x)-d^3*b/c*(1/7*arccos(c*x)*c^7*x^
7-3/5*arccos(c*x)*c^5*x^5+c^3*x^3*arccos(c*x)-c*x*arccos(c*x)+2161/3675*(-
c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)+117/1225*c^4*x^4*(-c^
2*x^2+1)^(1/2)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.90

$$\int \left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx =$$
$$-\frac{525\,ac^7d^3x^7-2205\,ac^5d^3x^5+3675\,ac^3d^3x^3-3675\,acd^3x+105\left(5\,bc^7d^3x^7-21\,bc^5d^3x^5+35\,bc^3d^3x^3\right.}{3675\,c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 367
5*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 -
 35*b*c*d^3*x)*arccos(c*x) - (75*b*c^6*d^3*x^6 - 351*b*c^4*d^3*x^4 + 757*b
*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.29

$$\int \left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx$$
$$=\begin{cases} -\frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}-ac^2d^3x^3+ad^3x-\frac{bc^6d^3x^7\operatorname{acos}(cx)}{7}+\frac{bc^5d^3x^6\sqrt{-c^2x^2+1}}{49}+\frac{3bc^4d^3x^5\operatorname{acos}(cx)}{5}-\frac{117bc^3d^3x^4\sqrt{-}}{1225} \\ d^3x\left(a+\frac{\pi b}{2}\right) \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 - a*c**2*d**3*x**3 +
 a*d**3*x - b*c**6*d**3*x**7*acos(c*x)/7 + b*c**5*d**3*x**6*sqrt(-c**2*x**
2 + 1)/49 + 3*b*c**4*d**3*x**5*acos(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(-c*
*2*x**2 + 1)/1225 - b*c**2*d**3*x**3*acos(c*x) + 757*b*c*d**3*x**2*sqrt(-c
**2*x**2 + 1)/3675 + b*d**3*x*acos(c*x) - 2161*b*d**3*sqrt(-c**2*x**2 + 1)
/(3675*c), Ne(c, 0)), (d**3*x*(a + pi*b/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(155) = 310.

Time = 0.12 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.78

$$\int \left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx=-\frac{1}{7}ac^6d^3x^7+\frac{3}{5}ac^4d^3x^5$$
$$-\frac{1}{245}\left(35x^7\arccos(cx)-\left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2}+\frac{6\sqrt{-c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{-c^2x^2+1}x^2}{c^6}+\frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$+\frac{1}{25}\left(15x^5\arccos(cx)-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^4d^3$$
$$-ac^2d^3x^3-\frac{1}{3}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^2d^3$$
$$+ad^3x+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arccos(c*x) - (5*sq
rt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2
+ 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcc
os(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arccos(
c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d^
3 + a*d^3*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\begin{aligned}\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx =& -\frac{1}{7}\,bc^6d^3x^7\arccos(cx) - \frac{1}{7}\,ac^6d^3x^7 \\ &+ \frac{1}{49}\,\sqrt{-c^2x^2+1}bc^5d^3x^6 + \frac{3}{5}\,bc^4d^3x^5\arccos(cx) \\ &+ \frac{3}{5}\,ac^4d^3x^5 - \frac{117}{1225}\,\sqrt{-c^2x^2+1}bc^3d^3x^4 \\ &- bc^2d^3x^3\arccos(cx) - ac^2d^3x^3 \\ &+ \frac{757}{3675}\,\sqrt{-c^2x^2+1}bcd^3x^2 + bd^3x\arccos(cx) \\ &+ ad^3x - \frac{2161\,\sqrt{-c^2x^2+1}bd^3}{3675\,c}\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/7*b*c^6*d^3*x^7*arccos(c*x) - 1/7*a*c^6*d^3*x^7 + 1/49*sqrt(-c^2*x^2 +
1)*b*c^5*d^3*x^6 + 3/5*b*c^4*d^3*x^5*arccos(c*x) + 3/5*a*c^4*d^3*x^5 - 117
/1225*sqrt(-c^2*x^2 + 1)*b*c^3*d^3*x^4 - b*c^2*d^3*x^3*arccos(c*x) - a*c^2
*d^3*x^3 + 757/3675*sqrt(-c^2*x^2 + 1)*b*c*d^3*x^2 + b*d^3*x*arccos(c*x) +
 a*d^3*x - 2161/3675*sqrt(-c^2*x^2 + 1)*b*d^3/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{d^3\left(-525acos(cx)\,b\,c^7x^7+2205acos(cx)\,b\,c^5x^5-3675acos(cx)\,b\,c^3x^3+3675acos(cx)\,bcx+75\sqrt{-c^2x^2+\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - 525*acos(c*x)*b*c**7*x**7 + 2205*acos(c*x)*b*c**5*x**5 - 3675*ac
os(c*x)*b*c**3*x**3 + 3675*acos(c*x)*b*c*x + 75*sqrt( - c**2*x**2 + 1)*b*c
**6*x**6 - 351*sqrt( - c**2*x**2 + 1)*b*c**4*x**4 + 757*sqrt( - c**2*x**2
+ 1)*b*c**2*x**2 - 2161*sqrt( - c**2*x**2 + 1)*b - 525*a*c**7*x**7 + 2205*
a*c**5*x**5 - 3675*a*c**3*x**3 + 3675*a*c*x))/(3675*c)
```

3.2 $\int (d-c^2dx^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int (d-c^2dx^2)^2 (a+b\arccos(cx))\,dx = -\frac{8bd^2\sqrt{1-c^2x^2}}{15c} - \frac{4bd^2(1-c^2x^2)^{3/2}}{45c} - \frac{bd^2(1-c^2x^2)^{5/2}}{25c} + d^2x(a+b\arccos(cx)) - \frac{2}{3}c^2d^2x^3(a+b\arccos(cx)) + \frac{1}{5}c^4d^2x^5(a+b\arccos(cx))$$

output

```
-8/15*b*d^2*(-c^2*x^2+1)^(1/2)/c-4/45*b*d^2*(-c^2*x^2+1)^(3/2)/c-1/25*b*d^
2*(-c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*arccos(c*x))-2/3*c^2*d^2*x^3*(a+b*arccos
(c*x))+1/5*c^4*d^2*x^5*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{d^2\left(b\sqrt{1-c^2x^2}(-149+38c^2x^2-9c^4x^4)+15acx(15-10c^2x^2+3c^4x^4)+15bcx(15-10c^2x^2+3c^4x^4)\,\mathrm{ar}\right.}{225c}$$

input
```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(b*Sqrt[1 - c^2*x^2]*(-149 + 38*c^2*x^2 - 9*c^4*x^4) + 15*a*c*x*(15 -
 10*c^2*x^2 + 3*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)*ArcCos[c
*x]))/(225*c)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5155, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5155$$

$$bc\int \frac{d^2x\left(3c^4x^4-10c^2x^2+15\right)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}bcd^2\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \; 1576$$

$$\frac{1}{30}bcd^2\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{30}bcd^2\int \left(3\left(1-c^2x^2\right)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))+\frac{1}{30}bcd^2\left(-\frac{6\left(1-c^2x^2\right)^{5/2}}{5c^2}-\frac{8\left(1-c^2x^2\right)^{3/2}}{3c^2}-\frac{16\sqrt{1-c^2x^2}}{c^2}\right)$$

input
```
Int[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c^2) -
(6*(1 - c^2*x^2)^(5/2))/(5*c^2)))/30 + d^2*x*(a + b*ArcCos[c*x]) - (2*c^2*
d^2*x^3*(a + b*ArcCos[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcCos[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

method	result
parts	$d^2a\left(\frac{1}{5}c^4x^5-\frac{2}{3}c^2x^3+x\right)+\frac{d^2b\left(\frac{\arccos(cx)c^5x^5}{5}-\frac{2c^3x^3\arccos(cx)}{3}+cx\arccos(cx)-\frac{149\sqrt{-c^2x^2+1}}{225}+\frac{38c^2x^2\sqrt{-}}{225}\right.}{c}$
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\arccos(cx)c^5x^5}{5}-\frac{2c^3x^3\arccos(cx)}{3}+cx\arccos(cx)-\frac{149\sqrt{-c^2x^2+1}}{225}+\frac{38c^2x^2\sqrt{-c^2x^2+1}}{225}-c\right.}{c}$
default	$\frac{d^2a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\arccos(cx)c^5x^5}{5}-\frac{2c^3x^3\arccos(cx)}{3}+cx\arccos(cx)-\frac{149\sqrt{-c^2x^2+1}}{225}+\frac{38c^2x^2\sqrt{-c^2x^2+1}}{225}-c\right.}{c}$
orering	$\frac{x(81c^4x^4-302c^2x^2+821)(-c^2dx^2+d)^2(a+b\arccos(cx))}{225(cx-1)(cx+1)(c^2x^2-1)}-\frac{(9c^4x^4-38c^2x^2+149)\left(-4(-c^2dx^2+d)(a+b\arccos(cx\right.}{225c^2(cx-1)(cx+1)}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/5*c^4*x^5-2/3*c^2*x^3+x)+d^2*b/c*(1/5*arccos(c*x)*c^5*x^5-2/3*c^3
*x^3*arccos(c*x)+c*x*arccos(c*x)-149/225*(-c^2*x^2+1)^(1/2)+38/225*c^2*x^2
*(-c^2*x^2+1)^(1/2)-1/25*c^4*x^4*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{45\,ac^5d^2x^5-150\,ac^3d^2x^3+225\,acd^2x+15\,(3\,bc^5d^2x^5-10\,bc^3d^2x^3+15\,bcd^2x)\arccos{(cx)}-(9\,bc^4d^2x^4}{225\,c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*
d^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*arccos(c*x) - (9*b*c^4*d^2*x^4
- 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.30

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^2x^5}{5}-\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\operatorname{acos}{(cx)}}{5}-\frac{bc^3d^2x^4\sqrt{-c^2x^2+1}}{25}-\frac{2bc^2d^2x^3\operatorname{acos}{(cx)}}{3}+\frac{38bcd^2x^2\sqrt{-c^2x^2+1}}{225}+bd^2x\,a \\ d^2x\left(a+\frac{\pi b}{2}\right)\end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**5/5 - 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d
**2*x**5*acos(c*x)/5 - b*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)/25 - 2*b*c**2
*d**2*x**3*acos(c*x)/3 + 38*b*c*d**2*x**2*sqrt(-c**2*x**2 + 1)/225 + b*d**
2*x*acos(c*x) - 149*b*d**2*sqrt(-c**2*x**2 + 1)/(225*c), Ne(c, 0)), (d**2*
x*(a + pi*b/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.53

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx = \frac{1}{5}\,ac^4d^2x^5 + \frac{1}{75}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^4d^2 - \frac{2}{3}\,ac^2d^2x^3 - \frac{2}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^2d^2 + ad^2x + \frac{\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c
^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2
 - 2/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/
c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arccos(c*x) -
sqrt(-c^2*x^2 + 1))*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx = \frac{1}{5}\,bc^4d^2x^5\arccos\left(cx\right) + \frac{1}{5}\,ac^4d^2x^5 - \frac{1}{25}\,\sqrt{-c^2x^2+1}bc^3d^2x^4 - \frac{2}{3}\,bc^2d^2x^3\arccos\left(cx\right) - \frac{2}{3}\,ac^2d^2x^3 + \frac{38}{225}\,\sqrt{-c^2x^2+1}bcd^2x^2 + bd^2x\arccos\left(cx\right) + ad^2x - \frac{149\,\sqrt{-c^2x^2+1}bd^2}{225\,c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/5*b*c^4*d^2*x^5*arccos(c*x) + 1/5*a*c^4*d^2*x^5 - 1/25*sqrt(-c^2*x^2 + 1
)*b*c^3*d^2*x^4 - 2/3*b*c^2*d^2*x^3*arccos(c*x) - 2/3*a*c^2*d^2*x^3 + 38/2
25*sqrt(-c^2*x^2 + 1)*b*c*d^2*x^2 + b*d^2*x*arccos(c*x) + a*d^2*x - 149/22
5*sqrt(-c^2*x^2 + 1)*b*d^2/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{d^2\left(45acos(cx)\,b\,c^5x^5-150acos(cx)\,b\,c^3x^3+225acos(cx)\,bcx-9\sqrt{-c^2x^2+1}\,b\,c^4x^4+38\sqrt{-c^2x^2+1}\,b\,c^2\right.}{225c}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(45*acos(c*x)*b*c**5*x**5 - 150*acos(c*x)*b*c**3*x**3 + 225*acos(c*x
)*b*c*x - 9*sqrt( - c**2*x**2 + 1)*b*c**4*x**4 + 38*sqrt( - c**2*x**2 + 1)
*b*c**2*x**2 - 149*sqrt( - c**2*x**2 + 1)*b + 45*a*c**5*x**5 - 150*a*c**3*
x**3 + 225*a*c*x))/(225*c)
```

3.3 $\int (d-c^2dx^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{2bd\sqrt{1-c^2x^2}}{3c}-\frac{bd(1-c^2x^2)^{3/2}}{9c} + dx(a+b\arccos(cx))-\frac{1}{3}c^2dx^3(a+b\arccos(cx))$$

output
```
-2/3*b*d*(-c^2*x^2+1)^(1/2)/c-1/9*b*d*(-c^2*x^2+1)^(3/2)/c+d*x*(a+b*arccos
(c*x))-1/3*c^2*d*x^3*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx = \frac{d\left(b\sqrt{1-c^2x^2}(-7+c^2x^2)+a(9cx-3c^3x^3)-3bcx(-3+c^2x^2)\arccos(cx)\right)}{9c}$$

input
```
Integrate[(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*(b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + a*(9*c*x - 3*c^3*x^3) - 3*b*c*x*(
-3 + c^2*x^2)*ArcCos[c*x]))/(9*c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5155}$$

$$bc\int \frac{dx\left(3-c^2x^2\right)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd\int \frac{x\left(3-c^2x^2\right)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{6}bcd\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}bcd\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))+\frac{1}{6}bcd\left(-\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)$$

input
```
Int[(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2)))/6 +
 d*x*(a + b*ArcCos[c*x]) - (c^2*d*x^3*(a + b*ArcCos[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result	size
parts	$-da\left(\frac{1}{3}c^2x^3-x\right)-\frac{db\left(\frac{c^3x^3\arccos(cx)}{3}-cx\arccos(cx)-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}+\frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	80
derivativedivides	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\arccos(cx)}{3}-cx\arccos(cx)-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}+\frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	82
default	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\arccos(cx)}{3}-cx\arccos(cx)-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}+\frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	82
orering	$\frac{x(5c^2x^2-23)(-c^2d\,x^2+d)(a+b\arccos(cx))}{9c^2x^2-9}-\frac{(c^2x^2-7)\left(-2d\,c^2x(a+b\arccos(cx))-\frac{(-c^2d\,x^2+d)bc}{\sqrt{-c^2x^2+1}}\right)}{9c^2}$	102

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/3*c^2*x^3-x)-d*b/c*(1/3*c^3*x^3*arccos(c*x)-c*x*arccos(c*x)-1/9*c^
2*x^2*(-c^2*x^2+1)^(1/2)+7/9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx$$
$$=-\frac{3\,ac^3dx^3-9\,acdx+3\,(bc^3dx^3-3\,bcdx)\arccos(cx)-(bc^2dx^2-7\,bd)\sqrt{-c^2x^2+1}}{9\,c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*arccos(c*x)
- (b*c^2*d*x^2 - 7*b*d)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int (d - c^2dx^2) (a + b \arccos(cx)) \, dx = \begin{cases} -\frac{ac^2dx^3}{3} + adx - \frac{bc^2dx^3 \operatorname{acos}(cx)}{3} + \frac{bcdx^2\sqrt{-c^2x^2+1}}{9} + bdx \operatorname{acos}(cx) - \frac{7bd\sqrt{-c^2x^2+1}}{9c} & \text{for } c \neq 0 \\ dx\left(a + \frac{\pi b}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**2*d*x**3/3 + a*d*x - b*c**2*d*x**3*acos(c*x)/3 + b*c*d*x*
*2*sqrt(-c**2*x**2 + 1)/9 + b*d*x*acos(c*x) - 7*b*d*sqrt(-c**2*x**2 + 1)/(
9*c), Ne(c, 0)), (d*x*(a + pi*b/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int (d - c^2dx^2) (a + b \arccos(cx)) \, dx = -\frac{1}{3} ac^2dx^3 - \frac{1}{9}\left(3x^3 \arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^2d + adx + \frac{\left(cx \arccos(cx) - \sqrt{-c^2x^2+1}\right)bd}{c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2
+ 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arccos(c*x) - sqrt(-c^
2*x^2 + 1))*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (d-c^2dx^2)\,(a+b\arccos(cx))\,dx = -\frac{1}{3}\,bc^2dx^3\arccos{(cx)} - \frac{1}{3}\,ac^2dx^3 + \frac{1}{9}\,\sqrt{-c^2x^2+1}bcdx^2 + bdx\arccos{(cx)} + adx - \frac{7\,\sqrt{-c^2x^2+1}bd}{9\,c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/3*b*c^2*d*x^3*arccos(c*x) - 1/3*a*c^2*d*x^3 + 1/9*sqrt(-c^2*x^2 + 1)*b*
c*d*x^2 + b*d*x*arccos(c*x) + a*d*x - 7/9*sqrt(-c^2*x^2 + 1)*b*d/c
```

Mupad [F(-1)]

Timed out.

$$\int (d-c^2dx^2)\,(a+b\arccos(cx))\,dx = \begin{cases} b\,c^2\,d\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\,\left(\frac{2}{c^2}+x^2\right)}{9}-\frac{x^3\,\mathrm{acos}(c\,x)}{3}\right)-\frac{b\,d\,\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c}-\frac{a\,d\,x\,(c^2\,x^2-3)}{3} & \text{if}\ \ 0<c\\ \int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx & \text{if}\ \ \neg 0<c \end{cases}$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
piecewise(0 < c, b*c^2*d*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*aco
s(c*x))/3) - (b*d*((- c^2*x^2 + 1)^(1/2) - c*x*acos(c*x)))/c - (a*d*x*(c^2
*x^2 - 3))/3, ~0 < c, int((a + b*acos(c*x))*(d - c^2*d*x^2), x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (d - c^2 d x^2) (a + b \arccos(cx)) \, dx$$
$$= \frac{d\left(-3 acos(cx)\, b c^3 x^3 + 9 acos(cx)\, bcx + \sqrt{-c^2x^2+1}\, b c^2 x^2 - 7\sqrt{-c^2x^2+1}\, b - 3a c^3 x^3 + 9acx\right)}{9c}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - 3*acos(c*x)*b*c**3*x**3 + 9*acos(c*x)*b*c*x + sqrt( - c**2*x**2 + 1
)*b*c**2*x**2 - 7*sqrt( - c**2*x**2 + 1)*b - 3*a*c**3*x**3 + 9*a*c*x))/(9*
c)
```

3.4 $\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \frac{(2a+b\pi-b(\pi-2\arccos(cx)))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{cd} - \frac{ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{cd} + \frac{ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{cd}$$

output
```
(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/c/d-I*b*
polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/c/d+I*b*polylog(2,c*x+I*(-c^2*x^2+1)^
(1/2))/c/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \frac{-2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2b\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)-a\log(1-cx)+a\log(1+cx)}{2cd}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2),x]
```

output
```
(-2*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 2*b*ArcCos[c*x]*Log[1 + E^(
I*ArcCos[c*x])] - a*Log[1 - c*x] + a*Log[1 + c*x] - (2*I)*b*PolyLog[2, -E^
(I*ArcCos[c*x])] + (2*I)*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}dx$$

$$\Big\downarrow \text{ 5165}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\text{arc}}{cd}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{cd}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{cd}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2),x]
```

output
```
-((-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(
I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(c*d))
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{\frac{a\ \mathrm{arctanh}(cx)}{d}-\frac{b\left(-\mathrm{arctanh}(cx)\arccos(cx)-i\,\mathrm{arctanh}(cx)\left(\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)\right)+i\,\mathrm{dilog}\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)\right.}{d}}{c}$
default	$\frac{\frac{a\ \mathrm{arctanh}(cx)}{d}-\frac{b\left(-\mathrm{arctanh}(cx)\arccos(cx)-i\,\mathrm{arctanh}(cx)\left(\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)\right)+i\,\mathrm{dilog}\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)\right.}{d}}{c}$
parts	$-\frac{a\ln(cx-1)}{2dc}+\frac{a\ln(cx+1)}{2dc}-\frac{b\left(-\mathrm{arctanh}(cx)\arccos(cx)-i\,\mathrm{arctanh}(cx)\left(\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right.\right.\right.}{dc}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d*arctanh(c*x)-b/d*(-arctanh(c*x)*arccos(c*x)-I*arctanh(c*x)*(ln(1-
I*(c*x+1)/(-c^2*x^2+1)^(1/2))-ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)))+I*dilog(
1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-I*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx=\int -\frac{b\arccos(cx)+a}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = -\frac{\int \frac{a}{c^2x^2-1}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**2 - 1), x) + Integral(b*acos(c*x)/(c**2*x**2 - 1), x
))/d
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \int -\frac{b\arccos\,(cx)+a}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 1/2*(2*c*d*integrate(1/2
*sqrt(c*x + 1)*sqrt(-c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/(c^2*d*x^2 -
d), x) - (log(c*x + 1) - log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x))*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arccos(cx)}{d - c^2 dx^2} \, dx = \int \frac{a + b \operatorname{acos}(c\,x)}{d - c^2\,d\,x^2} \, dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{a + b \arccos(cx)}{d - c^2 dx^2} \, dx = \frac{-2\left(\int \frac{acos(cx)}{c^2x^2-1}dx\right) bc - \log(c^2x - c)\,a + \log(c^2x + c)\,a}{2cd}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acos(c*x)/(c**2*x**2 - 1),x)*b*c - log(c**2*x - c)*a + log(c**2*
x + c)*a)/(2*c*d)
```

3.5 $\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx &= \frac{b}{2cd^2\sqrt{1-c^2x^2}} + \frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)} \\ &+ \frac{(2a+b\pi-b(\pi-2\arccos(cx)))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{2cd^2} \\ &- \frac{ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2cd^2} + \frac{ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2cd^2}\end{aligned}$$

output
```
1/2*b/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)+1/
2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/c/d^2-
1/2*I*b*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/c/d^2+1/2*I*b*polylog(2,c*x+I
*(-c^2*x^2+1)^(1/2))/c/d^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.52

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx$$
$$=\frac{\frac{b\sqrt{1-c^2x^2}}{c-c^2x}+\frac{b\sqrt{1-c^2x^2}}{c+c^2x}-\frac{2ax}{-1+c^2x^2}+\frac{b\arccos(cx)}{c-c^2x}-\frac{b\arccos(cx)}{c+c^2x}-\frac{2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)}{c}+\frac{2b\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)}{c}\cdots}{4d^2}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^2,x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(c - c^2*x) + (b*Sqrt[1 - c^2*x^2])/(c + c^2*x) - (
2*a*x)/(-1 + c^2*x^2) + (b*ArcCos[c*x])/(c - c^2*x) - (b*ArcCos[c*x])/(c +
 c^2*x) - (2*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])])/c + (2*b*ArcCos[c*x
]*Log[1 + E^(I*ArcCos[c*x])])/c - (a*Log[1 - c*x])/c + (a*Log[1 + c*x])/c
- ((2*I)*b*PolyLog[2, -E^(I*ArcCos[c*x])])/c + ((2*I)*b*PolyLog[2, E^(I*Ar
cCos[c*x])])/c)/(4*d^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5163, 27, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{\int \frac{a+b\arccos(cx)}{d(1-c^2x^2)}dx}{2d}+\frac{bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2d^2}+\frac{bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2d^2}+\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,\text{arc}}{2cd^2}$$
$$\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{2cd^2}$$
$$\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2cd^2}+$$
$$\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^2,x]
```

output
```
b/(2*c*d^2*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*d^2*(1 - c^2*x^
2)) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2,
-E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}+\frac{i\,\mathrm{polylog}}{}\right.}{}}{c}$
default	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}+\frac{i\,\mathrm{polylog}}{}\right.}{}}{c}$
parts	$\frac{a\left(-\frac{1}{4c(cx-1)}-\frac{\ln(cx-1)}{4c}-\frac{1}{4c(cx+1)}+\frac{\ln(cx+1)}{4c}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}+\frac{i}{}\right.}{}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^2*(-1/4/(c*x-1)-1/4*ln(c*x-1)-1/4/(c*x+1)+1/4*ln(c*x+1))+b/d^2*(-
1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)-1/2*arccos(c*x)*ln(1-
c*x-I*(-c^2*x^2+1)^(1/2))+1/2*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+1/2*ar
ccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/2*I*polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{\left(d - c^2dx^2\right)^2}\,dx = \frac{\int \frac{a}{c^4x^4 - 2c^2x^2 + 1}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^4x^4 - 2c^2x^2 + 1}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*acos(c*x)/(c**4
*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\arccos(cx)}{\left(d - c^2dx^2\right)^2}\,dx = \int \frac{b\arccos\left(cx\right) + a}{\left(c^2dx^2 - d\right)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d
^2)) - 1/4*((2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x +
 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 4*(c^3*d^2*x^2 - c*d^2)*
integrate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*
x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x))*b/(c^3*d^2*x^2 - c*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\arccos(cx)}{\left(d - c^2dx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx$$
$$=\frac{4\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)b\,c^3x^2-4\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)bc-\log(c^2x-c)\,a\,c^2x^2+\log(c^2x-c)\,a+\log(c^2x+c}{4c\,d^2\,(c^2x^2-1)}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acos(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3*x**2 - 4*int(acos
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c - log(c**2*x - c)*a*c**2*x**2 +
 log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a - 2*a
*c*x)/(4*c*d**2*(c**2*x**2 - 1))
```

3.6 $\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \frac{b}{12cd^3(1-c^2x^2)^{3/2}} + \frac{3b}{8cd^3\sqrt{1-c^2x^2}} + \frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2} + \frac{3x(a+b\arccos(cx))}{8d^3(1-c^2x^2)} + \frac{3(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{8cd^3} - \frac{3ib\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{8cd^3} + \frac{3ib\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{8cd^3}$$

output
```
1/12*b/c/d^3/(-c^2*x^2+1)^(3/2)+3/8*b/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b*
arccos(c*x))/d^3/(-c^2*x^2+1)^2+3/8*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)+3
/8*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/c/d^3
-3/8*I*b*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/c/d^3+3/8*I*b*polylog(2,c*x+
I*(-c^2*x^2+1)^(1/2))/c/d^3
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.62

$$\int \frac{a + b\arccos(cx)}{(d - c^2dx^2)^3} dx$$
$$= \frac{\frac{4ax}{(-1+c^2x^2)^2} - \frac{6ax}{-1+c^2x^2} + \frac{b\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{3c(1+cx)^2} + \frac{3b\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{c+c^2x} + \frac{3b\left(\sqrt{1-c^2x^2}+\arccos(cx)\right)}{c-c^2x} + \frac{b\left((2-cx\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^3,x]
```

output
```
((4*a*x)/(-1 + c^2*x^2)^2 - (6*a*x)/(-1 + c^2*x^2) + (b*((2 + c*x)*Sqrt[1
- c^2*x^2] - 3*ArcCos[c*x]))/(3*c*(1 + c*x)^2) + (3*b*(Sqrt[1 - c^2*x^2] -
 ArcCos[c*x]))/(c + c^2*x) + (3*b*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(c -
c^2*x) + (b*((2 - c*x)*Sqrt[1 - c^2*x^2] + 3*ArcCos[c*x]))/(3*c*(-1 + c*x)
^2) - (3*a*Log[1 - c*x])/c + (3*a*Log[1 + c*x])/c - (((3*I)/2)*b*(ArcCos[c
*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])]) + 4*PolyLog[2, -E^(I*
ArcCos[c*x])]))/c + (((3*I)/2)*b*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 -
 E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])]))/c)/(16*d^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5163, 27, 241, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\arccos(cx)}{(d - c^2dx^2)^3} dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{3\int \frac{a+b\arccos(cx)}{d^2(1-c^2x^2)^2}dx}{4d} + \frac{bc\int \frac{x}{(1-c^2x^2)^{5/2}}dx}{4d^3} + \frac{x(a + b\arccos(cx))}{4d^3(1 - c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^2}dx}{4d^3}+\frac{bc\int \frac{x}{\left(1-c^2x^2\right)^{5/2}}dx}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \; 241$$

$$\frac{3\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^2}dx}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 5163$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{\left(1-c^2x^2\right)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2\left(1-c^2x^2\right)}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 241$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2\left(1-c^2x^2\right)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 5165$$

$$\frac{3\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2\left(1-c^2x^2\right)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2\left(1-c^2x^2\right)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 4671$$

$$\frac{3\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(a+b\arccos(cx}{2\left(1-c^2x^2\right)}\right.}{4d^3}$$

$$\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 2715

$$\frac{3\left(-\frac{-ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,\text{arcc}}{2c}\right.}{4d^3}$$
$$\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 2838

$$\frac{3\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2}}\right.}{4d^3}$$
$$\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^3,x]
```

output
```
b/(12*c*d^3*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x]))/(4*d^3*(1 - c^2
*x^2)^2) + (3*(b/(2*c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 -
 c^2*x^2)) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*Poly
Log[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c)))/(4
*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

method	result
derivativedivides	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\arccos(cx)+9c^2x^2\sqrt{-c^2x^2+1}-15cx}{24c^4x^4-48c^2x^2+}\right.}{}}{}$
default	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\arccos(cx)+9c^2x^2\sqrt{-c^2x^2+1}-15cx}{24c^4x^4-48c^2x^2+}\right.}{}}{}$
parts	$-\frac{a\left(-\frac{1}{16c(cx-1)^2}+\frac{3}{16c(cx-1)}+\frac{3\ln(cx-1)}{16c}+\frac{1}{16c(cx+1)^2}+\frac{3}{16c(cx+1)}-\frac{3\ln(cx+1)}{16c}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\arccos(cx)+9c^2x^2\sqrt{-c^2x^2}}{24c^4x^4-}\right.}{}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a/d^3*(-1/16/(c*x-1)^2+3/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^2+3/
16/(c*x+1)-3/16*ln(c*x+1))-b/d^3*(1/24*(9*c^3*x^3*arccos(c*x)+9*c^2*x^2*(-
c^2*x^2+1)^(1/2)-15*c*x*arccos(c*x)-11*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*
x^2+1)+3/8*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/8*arccos(c*x)*ln(1
+c*x+I*(-c^2*x^2+1)^(1/2))-3/8*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3/8*I
*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*a*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c
*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 1/16*((6*c^3*x^3 - 10*c*x - 3*
(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-
c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 16*(c^5*d^3*x^4 - 2
*c^3*d^3*x^2 + c*d^3)*integrate(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2
*c^2*x^2 + 1)*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x + 1))*sq
rt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 -
d^3), x))*b/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{-16\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^5x^4+32\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^3x^2-16\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)bc}{}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**5
*x**4 + 32*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
c**3*x**2 - 16*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x
)*b*c - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c**2*x - c)*a*c**2*x**2 - 3*
log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c*
*2*x**2 + 3*log(c**2*x + c)*a - 6*a*c**3*x**3 + 10*a*c*x)/(16*c*d**3*(c**4
*x**4 - 2*c**2*x**2 + 1))
```

3.7 $\int (d - c^2 d x^2)^3 (a + b \arccos(cx))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$\begin{aligned}
&\int (d - c^2 d x^2)^3 (a + b \arccos(cx))^2 \, dx \\
&= -\frac{4322 b^2 d^3 x}{3675} + \frac{1514 b^2 c^2 d^3 x^3}{11025} - \frac{234 b^2 c^4 d^3 x^5}{6125} + \frac{2}{343} b^2 c^6 d^3 x^7 \\
&\quad - \frac{32 b d^3 \sqrt{1 - c^2 x^2} (a + b \arccos(cx))}{35c} - \frac{16 b d^3 (1 - c^2 x^2)^{3/2} (a + b \arccos(cx))}{105c} \\
&\quad - \frac{12 b d^3 (1 - c^2 x^2)^{5/2} (a + b \arccos(cx))}{175c} - \frac{2 b d^3 (1 - c^2 x^2)^{7/2} (a + b \arccos(cx))}{49c} \\
&\quad + \frac{16}{35} d^3 x (a + b \arccos(cx))^2 + \frac{8}{35} d^3 x (1 - c^2 x^2) (a + b \arccos(cx))^2 + \frac{6}{35} d^3 x (1 - c^2 x^2)^2 (a + b \arccos(cx))^2 + \frac{1}{7}
\end{aligned}$$

output

```
-4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3-234/6125*b^2*c^4*d^3*x^5+2
/343*b^2*c^6*d^3*x^7-32/35*b*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c-16
/105*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c-12/175*b*d^3*(-c^2*x^2+1
)^(5/2)*(a+b*arccos(c*x))/c-2/49*b*d^3*(-c^2*x^2+1)^(7/2)*(a+b*arccos(c*x)
)/c+16/35*d^3*x*(a+b*arccos(c*x))^2+8/35*d^3*x*(-c^2*x^2+1)*(a+b*arccos(c*
x))^2+6/35*d^3*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2+1/7*d^3*x*(-c^2*x^2+1)
^3*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.81

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(-11025a^2cx(-35+35c^2x^2-21c^4x^4+5c^6x^6)+210ab\sqrt{1-c^2x^2}(-2161+757c^2x^2-351c^4x^4+75c\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*(-11025*a^2*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 210*a*b
*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6) + 2*b^
2*c*x*(-226905 + 26495*c^2*x^2 - 7371*c^4*x^4 + 1125*c^6*x^6) + 210*b*(-10
5*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*
(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6))*ArcCos[c*x] - 11025*b^2*
c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCos[c*x]^2))/(385875*c)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5159, 27, 5159, 5159, 5131, 5183, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d\int d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d^3\int \left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5159$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d^3\left(\frac{2}{5}bc\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}\int \left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5159$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d^3\left(\frac{2}{5}bc\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int (a+b\,\mathrm{arcc}\right.\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5131$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5183$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)d}{3c}\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{b\int \left(1-c^2x^2\right)^3dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^2}\right)+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 24$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{b\int \left(1-c^2x^2\right)^3dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^2}\right)+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 210$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}bc\left(-\frac{b\int(1-c^2x^2)\,dx}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{b\int(-c^6x^6+3c^4x^4-3c^2x^2+1)\,dx}{7c}-\frac{(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{7c^2}\right)+$$
$$\frac{1}{7}d^3x(1-c^2x^2)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}d^3x(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{6}{7}d^3\left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\text{ar}}{c^2}\right.\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{7c^2}-\frac{b\left(-\frac{1}{7}c^6x^7+\frac{3c^4x^5}{5}-c^2x^3+x\right)}{7c}\right)$$

input
```
Int[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*x*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/7 + (2*b*c*d^3*(-1/7*(b*(x -
 c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/c - ((1 - c^2*x^2)^(7/2)*(a + b*A
rcCos[c*x]))/(7*c^2)))/7 + (6*d^3*((x*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^
2)/5 + (2*b*c*(-1/5*(b*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/c - ((1 - c^2*x^
2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2)))/5 + (4*((x*(1 - c^2*x^2)*(a + b*Ar
cCos[c*x])^2)/3 + (2*b*c*(-1/3*(b*(x - (c^2*x^3)/3))/c - ((1 - c^2*x^2)^(3
/2)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcCos[c*x])^2 + 2*b*c
*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2)))/3))/5))/7
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-d^3a^2\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^3\sqrt{-c^2x^2+1}}{49}\right.}{}$
default	$\frac{-d^3a^2\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^3\sqrt{-c^2x^2+1}}{49}\right.}{}$
parts	$-d^3a^2\left(\frac{1}{7}c^6x^7-\frac{3}{5}c^4x^5+c^2x^3-x\right)-\frac{d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-\right.}{49}\right.}{}$
orering	$\frac{x\left(47625c^8x^8-271212c^6x^6+741678c^4x^4-3539900c^2x^2+128625\right)\left(-c^2d\,x^2+d\right)^3\left(a+b\arccos(cx)\right)^2}{128625(cx-1)^2(cx+1)^2\left(c^2x^2-1\right)^2}-\frac{\left(20250c^8x^8-1\right.}{}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d^3*a^2*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b^2*(1/35*arccos(c
*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-2/49*arccos(c*x)*(c^2*x^2-1
)^3*(-c^2*x^2+1)^(1/2)-2/1715*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x+12/
175*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)+4/875*(3*c^4*x^4-10*c^2*x
^2+15)*c*x-16/105*arccos(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/315*(c^2*x
^2-3)*c*x+32/35*c*x+32/35*arccos(c*x)*(-c^2*x^2+1)^(1/2))-2*d^3*a*b*(1/7*a
rccos(c*x)*c^7*x^7-3/5*arccos(c*x)*c^5*x^5+c^3*x^3*arccos(c*x)-c*x*arccos(
c*x)+2161/3675*(-c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)+117/
1225*c^4*x^4*(-c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.08

$$\int \left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)^2dx =$$
$$-\frac{1125\left(49a^2-2b^2\right)c^7d^3x^7-189\left(1225a^2-78b^2\right)c^5d^3x^5+35\left(11025a^2-1514b^2\right)c^3d^3x^3-105\left(3675a\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/385875*(1125*(49*a^2 - 2*b^2)*c^7*d^3*x^7 - 189*(1225*a^2 - 78*b^2)*c^5
*d^3*x^5 + 35*(11025*a^2 - 1514*b^2)*c^3*d^3*x^3 - 105*(3675*a^2 - 4322*b^
2)*c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 - 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^
3*x^3 - 35*b^2*c*d^3*x)*arccos(c*x)^2 + 22050*(5*a*b*c^7*d^3*x^7 - 21*a*b*
c^5*d^3*x^5 + 35*a*b*c^3*d^3*x^3 - 35*a*b*c*d^3*x)*arccos(c*x) - 210*(75*a
*b*c^6*d^3*x^6 - 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 - 2161*a*b*d^3
+ (75*b^2*c^6*d^3*x^6 - 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 - 2161*b
^2*d^3)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.78

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$=\begin{cases} -\frac{a^2c^6d^3x^7}{7}+\frac{3a^2c^4d^3x^5}{5}-a^2c^2d^3x^3+a^2d^3x-\frac{2abc^6d^3x^7\operatorname{acos}(cx)}{7}+\frac{2abc^5d^3x^6\sqrt{-c^2x^2+1}}{49}+\frac{6abc^4d^3x^5\operatorname{acos}(cx)}{5}-234a \\ d^3x\left(a+\frac{\pi b}{2}\right)^2 \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 - a**2*c**2*d*
*3*x**3 + a**2*d**3*x - 2*a*b*c**6*d**3*x**7*acos(c*x)/7 + 2*a*b*c**5*d**3
*x**6*sqrt(-c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*acos(c*x)/5 - 234*a*b
*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)/1225 - 2*a*b*c**2*d**3*x**3*acos(c*x)
 + 1514*a*b*c*d**3*x**2*sqrt(-c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*acos(c*x)
 - 4322*a*b*d**3*sqrt(-c**2*x**2 + 1)/(3675*c) - b**2*c**6*d**3*x**7*acos(
c*x)**2/7 + 2*b**2*c**6*d**3*x**7/343 + 2*b**2*c**5*d**3*x**6*sqrt(-c**2*x
**2 + 1)*acos(c*x)/49 + 3*b**2*c**4*d**3*x**5*acos(c*x)**2/5 - 234*b**2*c*
*4*d**3*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)
/1225 - b**2*c**2*d**3*x**3*acos(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025
+ 1514*b**2*c*d**3*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/3675 + b**2*d**3*x*
acos(c*x)**2 - 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(-c**2*x**2 + 1)
*acos(c*x)/(3675*c), Ne(c, 0)), (d**3*x*(a + pi*b/2)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(263) = 526$.

Time = 0.16 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.45

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/7*b^2*c^6*d^3*x^7*arccos(c*x)^2 - 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3
*x^5*arccos(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 - 2/245*(35*x^7*arccos(c*x) - (5*
sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^
2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 + 2/25725*(105*
(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2
*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arccos(c*x) + (75*c^6*x^7
 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^6*d^3 - b^2*c^2*d^3*x^3*
arccos(c*x)^2 + 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 +
 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 -
 2/375*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)
/c^4)*b^2*c^4*d^3 - a^2*c^2*d^3*x^3 - 2/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^
2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 + 2/9*(3*c*(sq
rt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^
3 + 6*x)/c^2)*b^2*c^2*d^3 + b^2*d^3*x*arccos(c*x)^2 - 2*b^2*d^3*(x + sqrt(
-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*d^3*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*
x^2 + 1))*a*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.51

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx = -\frac{1}{7}\,b^2c^6d^3x^7\arccos(cx)^2 - \frac{2}{7}\,abc^6d^3x^7\arccos(cx) - \frac{1}{7}\,a^2c^6d^3x^7 + \frac{2}{343}\,b^2c^6d^3x^7 + \frac{2}{49}\,\sqrt{-c^2x^2+1}b^2c^5d^3x^6\arccos(cx) + \frac{2}{49}\,\sqrt{-c^2x^2+1}abc^5d^3x^6 + \frac{3}{5}\,b^2c^4d^3x^5\arccos(cx)^2 + \frac{6}{5}\,abc^4d^3x^5\arccos(cx) + \frac{3}{5}\,a^2c^4d^3x^5 - \frac{234}{6125}\,b^2c^4d^3x^5 - \frac{234}{1225}\,\sqrt{-c^2x^2+1}b^2c^3d^3x^4\arccos(cx) - \frac{234}{1225}\,\sqrt{-c^2x^2+1}abc^3d^3x^4 - b^2c^2d^3x^3\arccos(cx)^2 - 2\,abc^2d^3x^3\arccos(cx) - a^2c^2d^3x^3 + \frac{1514}{11025}\,b^2c^2d^3x^3 + \frac{1514}{3675}\,\sqrt{-c^2x^2+1}b^2cd^3x^2\arccos(cx) + \frac{1514}{3675}\,\sqrt{-c^2x^2+1}abcd^3x^2 + b^2d^3x\arccos(cx)^2 + 2\,abd^3x\arccos(cx) + a^2d^3x - \frac{4322}{3675}\,b^2d^3x - \frac{4322\,\sqrt{-c^2x^2+1}b^2d^3\arccos(cx)}{3675\,c} - \frac{4322\,\sqrt{-c^2x^2+1}abd^3}{3675\,c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/7*b^2*c^6*d^3*x^7*arccos(c*x)^2 - 2/7*a*b*c^6*d^3*x^7*arccos(c*x) - 1/7
*a^2*c^6*d^3*x^7 + 2/343*b^2*c^6*d^3*x^7 + 2/49*sqrt(-c^2*x^2 + 1)*b^2*c^5
*d^3*x^6*arccos(c*x) + 2/49*sqrt(-c^2*x^2 + 1)*a*b*c^5*d^3*x^6 + 3/5*b^2*c
^4*d^3*x^5*arccos(c*x)^2 + 6/5*a*b*c^4*d^3*x^5*arccos(c*x) + 3/5*a^2*c^4*d
^3*x^5 - 234/6125*b^2*c^4*d^3*x^5 - 234/1225*sqrt(-c^2*x^2 + 1)*b^2*c^3*d^
3*x^4*arccos(c*x) - 234/1225*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^3*x^4 - b^2*c^2*
d^3*x^3*arccos(c*x)^2 - 2*a*b*c^2*d^3*x^3*arccos(c*x) - a^2*c^2*d^3*x^3 +
1514/11025*b^2*c^2*d^3*x^3 + 1514/3675*sqrt(-c^2*x^2 + 1)*b^2*c*d^3*x^2*ar
ccos(c*x) + 1514/3675*sqrt(-c^2*x^2 + 1)*a*b*c*d^3*x^2 + b^2*d^3*x*arccos(
c*x)^2 + 2*a*b*d^3*x*arccos(c*x) + a^2*d^3*x - 4322/3675*b^2*d^3*x - 4322/
3675*sqrt(-c^2*x^2 + 1)*b^2*d^3*arccos(c*x)/c - 4322/3675*sqrt(-c^2*x^2 +
1)*a*b*d^3/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(3675acos(cx)^2b^2cx-7350\sqrt{-c^2x^2+1}\,acos(cx)\,b^2-1050acos(cx)\,ab\,c^7x^7+4410acos(cx)\,ab\,c^5x^5-\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output

```
(d**3*(3675*acos(c*x)**2*b**2*c*x - 7350*sqrt( - c**2*x**2 + 1)*acos(c*x)*
b**2 - 1050*acos(c*x)*a*b*c**7*x**7 + 4410*acos(c*x)*a*b*c**5*x**5 - 7350*
acos(c*x)*a*b*c**3*x**3 + 7350*acos(c*x)*a*b*c*x + 150*sqrt( - c**2*x**2 +
 1)*a*b*c**6*x**6 - 702*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 + 1514*sqrt(
- c**2*x**2 + 1)*a*b*c**2*x**2 - 4322*sqrt( - c**2*x**2 + 1)*a*b - 3675*in
t(acos(c*x)**2*x**6,x)*b**2*c**7 + 11025*int(acos(c*x)**2*x**4,x)*b**2*c**
5 - 11025*int(acos(c*x)**2*x**2,x)*b**2*c**3 - 525*a**2*c**7*x**7 + 2205*a
**2*c**5*x**5 - 3675*a**2*c**3*x**3 + 3675*a**2*c*x - 7350*b**2*c*x))/(367
5*c)
```

3.8 $\int (d-c^2dx^2)^2 (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int (d-c^2dx^2)^2 (a+b\arccos(cx))^2 \, dx = -\frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3-\frac{2}{125}b^2c^4d^2x^5 - \frac{16bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{15c} - \frac{8bd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{45c} - \frac{2bd^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{25c} + \frac{8}{15}d^2x(a+b\arccos(cx))^2 + \frac{4}{15}d^2x(1-c^2x^2)(a+b\arccos(cx))^2 + \frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arccos(cx))^2$$

output
```
-298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3-2/125*b^2*c^4*d^2*x^5-16/15*b*d^
2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c-8/45*b*d^2*(-c^2*x^2+1)^(3/2)*(a+
b*arccos(c*x))/c-2/25*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))/c+8/15*d^
2*x*(a+b*arccos(c*x))^2+4/15*d^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+1/5*d^
2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.88

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{d^2\left(225a^2cx(15-10c^2x^2+3c^4x^4)-30ab\sqrt{1-c^2x^2}(149-38c^2x^2+9c^4x^4)-2b^2cx(2235-190c^2x^2+2\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(225*a^2*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 - c^2*x^2]
*(149 - 38*c^2*x^2 + 9*c^4*x^4) - 2*b^2*c*x*(2235 - 190*c^2*x^2 + 27*c^4*x
^4) - 30*b*(-15*a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*
(149 - 38*c^2*x^2 + 9*c^4*x^4))*ArcCos[c*x] + 225*b^2*c*x*(15 - 10*c^2*x^2
 + 3*c^4*x^4)*ArcCos[c*x]^2))/(3375*c)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5159, 27, 5159, 5131, 5183, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \; 5159$$

$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}d\int d\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}d^2\int \left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\downarrow \text{5159}$$

$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+$$
$$\frac{4}{5}d^2\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int(a+b\arccos(cx))^2dx+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2\right)$$
$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\downarrow \text{5131}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(2bc\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}x(\right.$$
$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\downarrow \text{5183}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int\left(1-c^2x^2\right)dx}{3c}-\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{b\int\left(1-c^2x^2\right)^2dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+$$
$$b\arccos(cx))^2$$

$$\downarrow \text{24}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}bc\left(-\frac{b\int\left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2b\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{b\int\left(1-c^2x^2\right)^2dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+$$
$$b\arccos(cx))^2$$

$$\downarrow \text{210}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}bc\left(-\frac{b\int\left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2b\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{b\int\left(c^4x^4-2c^2x^2+1\right)dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}\right)+$$
$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\downarrow \text{2009}$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{4}{5}d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}-\frac{b\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)}{5c}\right)$$

input
```
Int[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*x*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*d^2*(-1/5*(b*(x -
 (2*c^2*x^3)/3 + (c^4*x^5)/5))/c - ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]
))/(5*c^2)))/5 + (4*d^2*((x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*
c*(-1/3*(b*(x - (c^2*x^3)/3))/c - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])
)/(3*c^2)))/3 + (2*(x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1
- c^2*x^2]*(a + b*ArcCos[c*x]))/c^2)))/3))/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{d^2a^2\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\left(3c^4x^4-10c^2x\right.}{375}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\left(3c^4x^4-10c^2x\right.}{375}\right.}{}$
parts	$d^2a^2\left(\frac{1}{5}c^4x^5-\frac{2}{3}c^2x^3+x\right)+\frac{d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\left(3c\right.}{}\right.}{}$
orering	$\frac{x\left(1647c^6x^6-8677c^4x^4+51845c^2x^2-3375\right)\left(-c^2d\,x^2+d\right)^2\left(a+b\arccos(cx)\right)^2}{3375(cx-1)(cx+1)\left(c^2x^2-1\right)^2}-\frac{\left(324c^6x^6-2035c^4x^4+18450c^2x^2-22\right.}{}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^2*a^2*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b^2*(1/15*arccos(c*x)^2*(3*
c^4*x^4-10*c^2*x^2+15)*c*x-2/25*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/
2)-2/375*(3*c^4*x^4-10*c^2*x^2+15)*c*x+8/45*arccos(c*x)*(c^2*x^2-1)*(-c^2*
x^2+1)^(1/2)+8/135*(c^2*x^2-3)*c*x-16/15*c*x-16/15*arccos(c*x)*(-c^2*x^2+1
)^(1/2))+2*d^2*a*b*(1/5*arccos(c*x)*c^5*x^5-2/3*c^3*x^3*arccos(c*x)+c*x*ar
ccos(c*x)-149/225*(-c^2*x^2+1)^(1/2)+38/225*c^2*x^2*(-c^2*x^2+1)^(1/2)-1/2
5*c^4*x^4*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{27\,(25\,a^2-2\,b^2)c^5d^2x^5-10\,(225\,a^2-38\,b^2)c^3d^2x^3+15\,(225\,a^2-298\,b^2)cd^2x+225\,(3\,b^2c^5d^2x^5-10\,b^2c}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 - 2*b^2)*c^5*d^2*x^5 - 10*(225*a^2 - 38*b^2)*c^3*d^2*x^
3 + 15*(225*a^2 - 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 - 10*b^2*c^3*d
^2*x^3 + 15*b^2*c*d^2*x)*arccos(c*x)^2 + 450*(3*a*b*c^5*d^2*x^5 - 10*a*b*c
^3*d^2*x^3 + 15*a*b*c*d^2*x)*arccos(c*x) - 30*(9*a*b*c^4*d^2*x^4 - 38*a*b*
c^2*d^2*x^2 + 149*a*b*d^2 + (9*b^2*c^4*d^2*x^4 - 38*b^2*c^2*d^2*x^2 + 149*
b^2*d^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.80

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^5}{5}-\frac{2a^2c^2d^2x^3}{3}+a^2d^2x+\frac{2abc^4d^2x^5\operatorname{acos}(cx)}{5}-\frac{2abc^3d^2x^4\sqrt{-c^2x^2+1}}{25}-\frac{4abc^2d^2x^3\operatorname{acos}(cx)}{3}+\frac{76abcd^2x^2\sqrt{-c^2x^2+1}}{225} \\ d^2x\left(a+\frac{\pi b}{2}\right)^2 \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**5/5 - 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
 2*a*b*c**4*d**2*x**5*acos(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(-c**2*x**2 +
 1)/25 - 4*a*b*c**2*d**2*x**3*acos(c*x)/3 + 76*a*b*c*d**2*x**2*sqrt(-c**2*
x**2 + 1)/225 + 2*a*b*d**2*x*acos(c*x) - 298*a*b*d**2*sqrt(-c**2*x**2 + 1)
/(225*c) + b**2*c**4*d**2*x**5*acos(c*x)**2/5 - 2*b**2*c**4*d**2*x**5/125
- 2*b**2*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/25 - 2*b**2*c**2*d*
*2*x**3*acos(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 + 76*b**2*c*d**2*x**2*
sqrt(-c**2*x**2 + 1)*acos(c*x)/225 + b**2*d**2*x*acos(c*x)**2 - 298*b**2*d
**2*x/225 - 298*b**2*d**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(225*c), Ne(c, 0)
), (d**2*x*(a + pi*b/2)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(193) = 386$.

Time = 0.16 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.13

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{1}{5}\,b^2c^4d^2x^5\arccos\left(cx\right)^2 + \frac{1}{5}\,a^2c^4d^2x^5 - \frac{2}{3}\,b^2c^2d^2x^3\arccos\left(cx\right)^2$$
$$+ \frac{2}{75}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abc^4d^2$$
$$- \frac{2}{1125}\left(15\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos\left(cx\right) + \frac{9\,c^4x^5 + 20\,c^2x^3 +}{c^4}\right.$$
$$- \frac{2}{3}\,a^2c^2d^2x^3 - \frac{4}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abc^2d^2$$
$$+ \frac{4}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos\left(cx\right) + \frac{c^2x^3+6\,x}{c^2}\right)b^2c^2d^2$$
$$+ b^2d^2x\arccos\left(cx\right)^2 - 2\,b^2d^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos\left(cx\right)}{c}\right)$$
$$+ a^2d^2x + \frac{2\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*c^4*d^2*x^5*arccos(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 - 2/3*b^2*c^2*d^2*
x^3*arccos(c*x)^2 + 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c
^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d
^2 - 2/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c
^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 1
20*x)/c^4)*b^2*c^4*d^2 - 2/3*a^2*c^2*d^2*x^3 - 4/9*(3*x^3*arccos(c*x) - c*
(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 + 4/2
7*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x)
 + (c^2*x^3 + 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arccos(c*x)^2 - 2*b^2*d^2*
(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*d^2*x + 2*(c*x*arccos(c*x) -
sqrt(-c^2*x^2 + 1))*a*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.50

$$\begin{aligned}\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx =& \frac{1}{5}\,b^2c^4d^2x^5\arccos\left(cx\right)^2+\frac{2}{5}\,abc^4d^2x^5\arccos\left(cx\right)\\&+\frac{1}{5}\,a^2c^4d^2x^5-\frac{2}{125}\,b^2c^4d^2x^5\\&-\frac{2}{25}\,\sqrt{-c^2x^2+1}b^2c^3d^2x^4\arccos\left(cx\right)\\&-\frac{2}{25}\,\sqrt{-c^2x^2+1}abc^3d^2x^4\\&-\frac{2}{3}\,b^2c^2d^2x^3\arccos\left(cx\right)^2\\&-\frac{4}{3}\,abc^2d^2x^3\arccos\left(cx\right)\\&-\frac{2}{3}\,a^2c^2d^2x^3+\frac{76}{675}\,b^2c^2d^2x^3\\&+\frac{76}{225}\,\sqrt{-c^2x^2+1}b^2cd^2x^2\arccos\left(cx\right)\\&+\frac{76}{225}\,\sqrt{-c^2x^2+1}abcd^2x^2+b^2d^2x\arccos\left(cx\right)^2\\&+2\,abd^2x\arccos\left(cx\right)+a^2d^2x-\frac{298}{225}\,b^2d^2x\\&-\frac{298\,\sqrt{-c^2x^2+1}b^2d^2\arccos\left(cx\right)}{225\,c}\\&-\frac{298\,\sqrt{-c^2x^2+1}abd^2}{225\,c}\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/5*b^2*c^4*d^2*x^5*arccos(c*x)^2 + 2/5*a*b*c^4*d^2*x^5*arccos(c*x) + 1/5*
a^2*c^4*d^2*x^5 - 2/125*b^2*c^4*d^2*x^5 - 2/25*sqrt(-c^2*x^2 + 1)*b^2*c^3*
d^2*x^4*arccos(c*x) - 2/25*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^2*x^4 - 2/3*b^2*c^
2*d^2*x^3*arccos(c*x)^2 - 4/3*a*b*c^2*d^2*x^3*arccos(c*x) - 2/3*a^2*c^2*d^
2*x^3 + 76/675*b^2*c^2*d^2*x^3 + 76/225*sqrt(-c^2*x^2 + 1)*b^2*c*d^2*x^2*a
rccos(c*x) + 76/225*sqrt(-c^2*x^2 + 1)*a*b*c*d^2*x^2 + b^2*d^2*x*arccos(c*
x)^2 + 2*a*b*d^2*x*arccos(c*x) + a^2*d^2*x - 298/225*b^2*d^2*x - 298/225*s
qrt(-c^2*x^2 + 1)*b^2*d^2*arccos(c*x)/c - 298/225*sqrt(-c^2*x^2 + 1)*a*b*d
^2/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^2\left(225acos(cx)^2\,b^2cx-450\sqrt{-c^2x^2+1}\,acos(cx)\,b^2+90acos(cx)\,ab\,c^5x^5-300acos(cx)\,ab\,c^3x^3+450ac\right.}{}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(d**2*(225*acos(c*x)**2*b**2*c*x - 450*sqrt( - c**2*x**2 + 1)*acos(c*x)*b*
*2 + 90*acos(c*x)*a*b*c**5*x**5 - 300*acos(c*x)*a*b*c**3*x**3 + 450*acos(c
*x)*a*b*c*x - 18*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 + 76*sqrt( - c**2*x*
*2 + 1)*a*b*c**2*x**2 - 298*sqrt( - c**2*x**2 + 1)*a*b + 225*int(acos(c*x)
**2*x**4,x)*b**2*c**5 - 450*int(acos(c*x)**2*x**2,x)*b**2*c**3 + 45*a**2*c
**5*x**5 - 150*a**2*c**3*x**3 + 225*a**2*c*x - 450*b**2*c*x))/(225*c)
```

3.9 $\int (d-c^2dx^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int (d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3 - \frac{4bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c} - \frac{2bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{9c} + \frac{2}{3}dx(a+b\arccos(cx))^2 + \frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^2$$

output
```
-14/9*b^2*d*x+2/27*b^2*c^2*d*x^3-4/3*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*
x))/c-2/9*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c+2/3*d*x*(a+b*arccos(c
*x))^2+1/3*d*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08

$$\int (d - c^2dx^2) (a + b\arccos(cx))^2 dx$$
$$= \frac{d\left(2b^2cx(-21 + c^2x^2) + 6ab\sqrt{1 - c^2x^2}(-7 + c^2x^2) - 9a^2cx(-3 + c^2x^2) + 6b\left(b\sqrt{1 - c^2x^2}(-7 + c^2x^2) + \right.\right.}{27c}$$

input
```
Integrate[(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*(2*b^2*c*x*(-21 + c^2*x^2) + 6*a*b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) - 9
*a^2*c*x*(-3 + c^2*x^2) + 6*b*(b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + a*(9*c
*x - 3*c^3*x^3))*ArcCos[c*x] - 9*b^2*c*x*(-3 + c^2*x^2)*ArcCos[c*x]^2))/(2
7*c)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5159, 5131, 5183, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - c^2dx^2) (a + b\arccos(cx))^2 dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{2}{3}bcd\int x\sqrt{1 - c^2x^2}(a + b\arccos(cx))dx + \frac{2}{3}d\int (a + b\arccos(cx))^2dx + \frac{1}{3}dx(1 - c^2x^2)(a + b\arccos(cx))^2$$

$$\Big\downarrow \text{5131}$$

$$\frac{2}{3}d\left(2bc\int \frac{x(a + b\arccos(cx))}{\sqrt{1 - c^2x^2}}dx + x(a + b\arccos(cx))^2\right) + \frac{2}{3}bcd\int x\sqrt{1 - c^2x^2}(a + b\arccos(cx))dx + \frac{1}{3}dx(1 - c^2x^2)(a + b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$\frac{2}{3}d\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bcd\left(-\frac{b\int(1-c^2x^2)\,dx}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{24}$$

$$\frac{2}{3}bcd\left(-\frac{b\int(1-c^2x^2)\,dx}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bcd\left(-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c^2}-\frac{b\left(x-\frac{c^2x^3}{3}\right)}{3c}\right)$$

input
```
Int[(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*d*(-1/3*(b*(x - (c^2*
x^3)/3))/c - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*d*
(x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x]))/c^2)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-d\,a^2\left(\frac{1}{3}c^3x^3-cx\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{3}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{3}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}}{9}-\frac{2\left(c^2\right.}{}\right.}{c}$
default	$\frac{-d\,a^2\left(\frac{1}{3}c^3x^3-cx\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{3}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{3}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}}{9}-\frac{2\left(c^2\right.}{}\right.}{c}$
parts	$-d\,a^2\left(\frac{1}{3}c^2x^3-x\right)-\frac{d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{3}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{3}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+}}{9}\right.}{c}$
orering	$\frac{x(19c^4x^4-166c^2x^2+27)\left(-c^2d\,x^2+d\right)(a+b\arccos(cx))^2}{27(c^2x^2-1)^2}-\frac{(2c^4x^4-29c^2x^2+7)\left(-2d\,c^2x(a+b\arccos(cx))^2-\frac{2\left(-c^2\right.}{}\right.}{9c^2(c^2x^2-1)}$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*a^2*(1/3*c^3*x^3-c*x)-d*b^2*(1/3*arccos(c*x)^2*(c^2*x^2-3)*c*x+4/3
*c*x+4/3*arccos(c*x)*(-c^2*x^2+1)^(1/2)-2/9*arccos(c*x)*(c^2*x^2-1)*(-c^2*
x^2+1)^(1/2)-2/27*(c^2*x^2-3)*c*x)-2*d*a*b*(1/3*c^3*x^3*arccos(c*x)-c*x*ar
ccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)+7/9*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx = -\frac{(9\,a^2-2\,b^2)c^3dx^3-3\,(9\,a^2-14\,b^2)cdx+9\,(b^2c^3dx^3-3\,b^2cdx)\arccos{(cx)}^2+18\,(abc^3dx^3-3\,abcdx)}{27\,c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/27*((9*a^2 - 2*b^2)*c^3*d*x^3 - 3*(9*a^2 - 14*b^2)*c*d*x + 9*(b^2*c^3*d
*x^3 - 3*b^2*c*d*x)*arccos(c*x)^2 + 18*(a*b*c^3*d*x^3 - 3*a*b*c*d*x)*arcco
s(c*x) - 6*(a*b*c^2*d*x^2 - 7*a*b*d + (b^2*c^2*d*x^2 - 7*b^2*d)*arccos(c*x
))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.80

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}-\frac{a^2c^2dx^3}{3}+a^2dx-\frac{2abc^2dx^3\operatorname{acos}(cx)}{3}+\frac{2abcdx^2\sqrt{-c^2x^2+1}}{9}+2abdx\operatorname{acos}(cx)-\frac{14abd\sqrt{-c^2x^2+1}}{9c}-\frac{b^2c^2dx^3\operatorname{acos}^2(cx)}{3}\\ dx\left(a+\frac{\pi b}{2}\right)^2\end{cases}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**2*d*x**3/3 + a**2*d*x - 2*a*b*c**2*d*x**3*acos(c*x)/3
+ 2*a*b*c*d*x**2*sqrt(-c**2*x**2 + 1)/9 + 2*a*b*d*x*acos(c*x) - 14*a*b*d*s
qrt(-c**2*x**2 + 1)/(9*c) - b**2*c**2*d*x**3*acos(c*x)**2/3 + 2*b**2*c**2*
d*x**3/27 + 2*b**2*c*d*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/9 + b**2*d*x*ac
os(c*x)**2 - 14*b**2*d*x/9 - 14*b**2*d*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c
), Ne(c, 0)), (d*x*(a + pi*b/2)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(111) = 222$.

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.83

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx$$
$$=-\frac{1}{3}b^2c^2dx^3\arccos(cx)^2-\frac{1}{3}a^2c^2dx^3$$
$$-\frac{2}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abc^2d$$
$$+\frac{2}{27}\left(3c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos(cx)+\frac{c^2x^3+6x}{c^2}\right)b^2c^2d$$
$$+b^2dx\arccos(cx)^2-2b^2d\left(x+\frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right)$$
$$+a^2dx+\frac{2\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)abd}{c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/3*b^2*c^2*d*x^3*arccos(c*x)^2 - 1/3*a^2*c^2*d*x^3 - 2/9*(3*x^3*arccos(c
*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d
 + 2/27*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcco
s(c*x) + (c^2*x^3 + 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arccos(c*x)^2 - 2*b^2*d*
(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*d*x + 2*(c*x*arccos(c*x) - sq
rt(-c^2*x^2 + 1))*a*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.45

$$\int (d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{1}{3}b^2c^2dx^3\arccos(cx)^2 - \frac{2}{3}abc^2dx^3\arccos(cx) - \frac{1}{3}a^2c^2dx^3 + \frac{2}{27}b^2c^2dx^3 + \frac{2}{9}\sqrt{-c^2x^2+1}b^2cdx^2\arccos(cx) + \frac{2}{9}\sqrt{-c^2x^2+1}abcdx^2 + b^2dx\arccos(cx)^2 + 2abdx\arccos(cx) + a^2dx - \frac{14}{9}b^2dx - \frac{14\sqrt{-c^2x^2+1}b^2d\arccos(cx)}{9c} - \frac{14\sqrt{-c^2x^2+1}abd}{9c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/3*b^2*c^2*d*x^3*arccos(c*x)^2 - 2/3*a*b*c^2*d*x^3*arccos(c*x) - 1/3*a^2
*c^2*d*x^3 + 2/27*b^2*c^2*d*x^3 + 2/9*sqrt(-c^2*x^2 + 1)*b^2*c*d*x^2*arcco
s(c*x) + 2/9*sqrt(-c^2*x^2 + 1)*a*b*c*d*x^2 + b^2*d*x*arccos(c*x)^2 + 2*a*
b*d*x*arccos(c*x) + a^2*d*x - 14/9*b^2*d*x - 14/9*sqrt(-c^2*x^2 + 1)*b^2*d
*arccos(c*x)/c - 14/9*sqrt(-c^2*x^2 + 1)*a*b*d/c
```

Mupad [F(-1)]

Timed out.

$$\int (d - c^2dx^2) (a + b\arccos(cx))^2 \, dx = \int (a + b\,\mathrm{acos}(c\,x))^2 \left(d - c^2\,d\,x^2\right) \, dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int (d - c^2dx^2) (a + b\arccos(cx))^2 \, dx$$
$$= \frac{d\left(9acos(cx)^2 b^2cx - 18\sqrt{-c^2x^2+1}\, acos(cx)\, b^2 - 6acos(cx)\, ab\, c^3x^3 + 18acos(cx)\, abcx + 2\sqrt{-c^2x^2+1}\, a}{9c}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(d*(9*acos(c*x)**2*b**2*c*x - 18*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2 - 6
*acos(c*x)*a*b*c**3*x**3 + 18*acos(c*x)*a*b*c*x + 2*sqrt( - c**2*x**2 + 1)
*a*b*c**2*x**2 - 14*sqrt( - c**2*x**2 + 1)*a*b - 9*int(acos(c*x)**2*x**2,x
)*b**2*c**3 - 3*a**2*c**3*x**3 + 9*a**2*c*x - 18*b**2*c*x))/(9*c)
```

3.10 $\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx &= \frac{(2a+b\pi-b(\pi-2\arccos(cx)))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{2cd}\\ &\quad-\frac{ib(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{cd}\\ &\quad+\frac{ib(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{cd}\\ &\quad+\frac{2b^2\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{cd}-\frac{2b^2\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{cd}\end{aligned}$$

output
```
1/2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/c/
d-I*b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))
/c/d+I*b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2
))/c/d+2*b^2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/c/d-2*b^2*polylog(3,c*x+
I*(-c^2*x^2+1)^(1/2))/c/d
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-4ab\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)-2b^2\arccos(cx)^2\log\left(1-e^{i\arccos(cx)}\right)+4ab\arccos(cx)\log\left(1+e^{i\,\mathrm{ar}}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2),x]
```

output
```
(-4*a*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 2*b^2*ArcCos[c*x]^2*Log[1
 - E^(I*ArcCos[c*x])] + 4*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 2*b
^2*ArcCos[c*x]^2*Log[1 + E^(I*ArcCos[c*x])] - a^2*Log[1 - c*x] + a^2*Log[1
 + c*x] - (4*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] + (4*
I)*b*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] + 4*b^2*PolyLog[3,
-E^(I*ArcCos[c*x])] - 4*b^2*PolyLog[3, E^(I*ArcCos[c*x])])/(2*c*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5165, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \; 5165$$

$$-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,d\arccos(cx)}{cd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))\,d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{-2b\int(a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos}{cd}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\operatorname{PolyLog}\right.}{}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\operatorname{PolyLog}\left(3,\right.\right.}{cd}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2),x]
```

output
```
-((-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*Arc
Cos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])
]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[
3, E^(I*ArcCos[c*x])]))/(c*d))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{\frac{a^2 \operatorname{arctanh}(cx)}{d}-\frac{b^2\left(\arccos(cx)^2 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-2i \arccos(cx) \operatorname{polylog}\left(2,cx+i\sqrt{-c^2x^2+1}\right)+2 \operatorname{polylog}\left(3,cx+i\sqrt{-c^2x^2+1}\right.}{}}{}$
default	$\frac{\frac{a^2 \operatorname{arctanh}(cx)}{d}-\frac{b^2\left(\arccos(cx)^2 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-2i \arccos(cx) \operatorname{polylog}\left(2,cx+i\sqrt{-c^2x^2+1}\right)+2 \operatorname{polylog}\left(3,cx+i\sqrt{-c^2x^2+1}\right.}{}}{}$
parts	$-\frac{a^2 \ln(cx-1)}{2dc}+\frac{a^2 \ln(cx+1)}{2dc}-\frac{b^2\left(\arccos(cx)^2 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-2i \arccos(cx) \operatorname{polylog}\left(2,cx+i\sqrt{-c^2x^2+\right.}{}$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/d*arctanh(c*x)-b^2/d*(arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2)
)-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*polylog(3,c*x+I*(-
c^2*x^2+1)^(1/2))-arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*arccos(
c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*polylog(3,-c*x-I*(-c^2*x^2+1)^
(1/2)))-2*a*b/d*(-arctanh(c*x)*arccos(c*x)-I*arctanh(c*x)*(ln(1-I*(c*x+1)/
(-c^2*x^2+1)^(1/2))-ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)))+I*dilog(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))-I*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^2*d*x^2 - d), x
)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^2}{c^2x^2-1}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^2x^2-1}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2/(c**2*x**2 - 1), x) + Integral(b**2*acos(c*x)**2/(c**2*x**
2 - 1), x) + Integral(2*a*b*acos(c*x)/(c**2*x**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) + 1/2*((b^2*log(c*x + 1)
 - b^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - 2*c*d
*integrate(((b^2*log(c*x + 1) - b^2*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 2*a*b*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x))/(c^2*d*x^2 - d), x))/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-4\left(\int \frac{acos(cx)}{c^2x^2-1}dx\right)abc-2\left(\int \frac{acos(cx)^2}{c^2x^2-1}dx\right)b^2c-\log(c^2x-c)\,a^2+\log(c^2x+c)\,a^2}{2cd}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - 4*int(acos(c*x)/(c**2*x**2 - 1),x)*a*b*c - 2*int(acos(c*x)**2/(c**2*x*
*2 - 1),x)*b**2*c - log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2)/(2*c*d)
```

3.11 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 251

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx &= \frac{b(a+b\arccos(cx))}{cd^2\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)} \\ &\quad+\frac{(2a+b\pi-b(\pi-2\arccos(cx)))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{4cd^2}+\frac{b^2\operatorname{arctanh}(cx)}{cd^2} \\ &\quad-\frac{ib(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2cd^2} \\ &\quad+\frac{ib(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2cd^2} \\ &\quad+\frac{b^2\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{cd^2}-\frac{b^2\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{cd^2}\end{aligned}$$

output
```
b*(a+b*arccos(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))^2/d^2
/(-c^2*x^2+1)+1/4*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^2*arctanh(c*x+I*(-c^2*x^
2+1)^(1/2))/c/d^2+b^2*arctanh(c*x)/c/d^2-1/2*I*b*(2*a+b*Pi-b*(Pi-2*arccos(
c*x)))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/c/d^2+1/2*I*b*(2*a+b*Pi-b*(Pi-
2*arccos(c*x)))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/c/d^2+b^2*polylog(3,-c
*x-I*(-c^2*x^2+1)^(1/2))/c/d^2-b^2*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/c/d
^2
```

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{-\frac{4a^2x}{-1+c^2x^2}-\frac{2a^2\log(1-cx)}{c}+\frac{2a^2\log(1+cx)}{c}+\frac{4ab\left(\frac{\sqrt{1-c^2x^2}}{1-cx}+\frac{\sqrt{1-c^2x^2}}{1+cx}+\frac{\arccos(cx)}{1-cx}-\frac{\arccos(cx)}{1+cx}-2\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2\,\mathrm{a}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^2,x]
```

output
```
((-4*a^2*x)/(-1 + c^2*x^2) - (2*a^2*Log[1 - c*x])/c + (2*a^2*Log[1 + c*x])
/c + (4*a*b*(Sqrt[1 - c^2*x^2]/(1 - c*x) + Sqrt[1 - c^2*x^2]/(1 + c*x) + A
rcCos[c*x]/(1 - c*x) - ArcCos[c*x]/(1 + c*x) - 2*ArcCos[c*x]*Log[1 - E^(I*
ArcCos[c*x])] + 2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] - (2*I)*PolyLog[2
, -E^(I*ArcCos[c*x])] + (2*I)*PolyLog[2, E^(I*ArcCos[c*x])]))/c + (b^2*(4*
ArcCos[c*x]*Cot[ArcCos[c*x]/2] + ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^2 - 4*Ar
cCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c*x])]) - 8*
Log[Tan[ArcCos[c*x]/2]] - (8*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])
] - PolyLog[2, E^(I*ArcCos[c*x])]) + 8*(PolyLog[3, -E^(I*ArcCos[c*x])] - P
olyLog[3, E^(I*ArcCos[c*x])]) - ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^2 + 4*Arc
Cos[c*x]*Tan[ArcCos[c*x]/2]))/c)/(8*d^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5163, 27, 5165, 3042, 4671, 3011, 2720, 5183, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \text{5163}$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{2d}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 27

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 5165

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 3042

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 4671

$$-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos}{2cd^2}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 3011

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\right.}{}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 2720

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\big\downarrow$ 5183

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\frac{bc\left(\frac{b\int\frac{1}{1-c^2x^2}dx}{c}+\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}\right)}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\downarrow$ 219

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\frac{bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\downarrow$ 7143

$$\frac{bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)}{d^2}-$$

$$\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,-\right.\right.}{2cd^2}$$

$$\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(2*d^2*(1 - c^2*x^2)) + (b*c*((a + b*ArcCos[c*x]
)/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2))/d^2 - (-2*(a + b*ArcCos
[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2
, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*
ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCos[c*x]
)]))/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2 \cdots}\right.}{2}\cdots\right.}{\cdots}$
default	$\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2 \cdots}\right.}{2}\cdots\right.}{\cdots}$
parts	$\frac{a^2\left(-\frac{1}{4c(cx-1)}-\frac{\ln(cx-1)}{4c}-\frac{1}{4c(cx+1)}+\frac{\ln(cx+1)}{4c}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1-cx- \cdots\right.}{2}\cdots\right.}{\cdots}$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/d^2*(-1/4/(c*x-1)-1/4*ln(c*x-1)-1/4/(c*x+1)+1/4*ln(c*x+1))+b^2/d^
2*(-1/2/(c^2*x^2-1)*arccos(c*x)*(c*x*arccos(c*x)+2*(-c^2*x^2+1)^(1/2))-1/2
*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x)*polylog(2,c*x+
I*(-c^2*x^2+1)^(1/2))-polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+1/2*arccos(c*x)^
2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))+polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+2*arctanh(c*x+I*(-c^2*x^2+1
)^(1/2)))+2*a*b/d^2*(-1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)
-1/2*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+1/2*I*polylog(2,c*x+I*(-c^
2*x^2+1)^(1/2))+1/2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/2*I*polyl
og(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^4*d^2*x^4 - 2*c^
2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a^2}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*acos(c*x)
**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*acos(c*x)/(c**4*x**
4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a + b\arccos(cx))^2}{(d - c^2dx^2)^2}\,dx = \int \frac{(b\arccos{(cx)} + a)^2}{(c^2dx^2 - d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c
*d^2)) - 1/4*((2*b^2*c*x - (b^2*c^2*x^2 - b^2)*log(c*x + 1) + (b^2*c^2*x^2
 - b^2)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 4*(c
^3*d^2*x^2 - c*d^2)*integrate(-1/2*((2*b^2*c*x - (b^2*c^2*x^2 - b^2)*log(c
*x + 1) + (b^2*c^2*x^2 - b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 4*a*b*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x))/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x))/(c^3*d^2*x^2
 - c*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\arccos(cx))^2}{(d - c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{8\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^3x^2-8\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)abc+4\left(\int \frac{acos(cx)^2}{c^4x^4-2c^2x^2+1}dx\right)b^2c^3x^2-4\left(\int \frac{acos(cx)^2}{c^4x^4-2c^2x^2+1}\right.}{4c\,d^2\,(c^2x^2-}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(8*int(acos(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**3*x**2 - 8*int(ac
os(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c + 4*int(acos(c*x)**2/(c**4*
x**4 - 2*c**2*x**2 + 1),x)*b**2*c**3*x**2 - 4*int(acos(c*x)**2/(c**4*x**4
- 2*c**2*x**2 + 1),x)*b**2*c - log(c**2*x - c)*a**2*c**2*x**2 + log(c**2*x
 - c)*a**2 + log(c**2*x + c)*a**2*c**2*x**2 - log(c**2*x + c)*a**2 - 2*a**
2*c*x)/(4*c*d**2*(c**2*x**2 - 1))
```

3.12 $\int (d-c^2dx^2)^3 (a+b\arccos(cx))^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 446

$$\int (d-c^2dx^2)^3 (a+b\arccos(cx))^3 \, dx = \frac{413312b^3d^3\sqrt{1-c^2x^2}}{128625c} + \frac{30256b^3d^3(1-c^2x^2)^{3/2}}{385875c} + \frac{2664b^3d^3(1-c^2x^2)^{5/2}}{214375c} + \frac{6b^3d^3(1-c^2x^2)^{7/2}}{2401c} - \frac{4322b^2d^3x(a+b\arccos(cx))}{1225} + \frac{1514b^2c^2d^3x^3(a+b\arccos(cx))}{3675} - \frac{702b^2c^4d^3x^5(a+b\arccos(cx))}{6125} + \frac{6}{343}b^2c^6d^3x^7(a+b\arccos(cx)) - \frac{48bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{35c} - \frac{8bd^3(1-c^2x^2)^{3/2}(a+b\arccos(c\ldots}{35c}$$

output
```
413312/128625*b^3*d^3*(-c^2*x^2+1)^(1/2)/c+30256/385875*b^3*d^3*(-c^2*x^2+
1)^(3/2)/c+2664/214375*b^3*d^3*(-c^2*x^2+1)^(5/2)/c+6/2401*b^3*d^3*(-c^2*x
^2+1)^(7/2)/c-4322/1225*b^2*d^3*x*(a+b*arccos(c*x))+1514/3675*b^2*c^2*d^3*
x^3*(a+b*arccos(c*x))-702/6125*b^2*c^4*d^3*x^5*(a+b*arccos(c*x))+6/343*b^2
*c^6*d^3*x^7*(a+b*arccos(c*x))-48/35*b*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(
c*x))^2/c-8/35*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c-18/175*b*d^3
*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c-3/49*b*d^3*(-c^2*x^2+1)^(7/2)*(a
+b*arccos(c*x))^2/c+16/35*d^3*x*(a+b*arccos(c*x))^3+8/35*d^3*x*(-c^2*x^2+1
)*(a+b*arccos(c*x))^3+6/35*d^3*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^3+1/7*d^
3*x*(-c^2*x^2+1)^3*(a+b*arccos(c*x))^3
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.92

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^3\,dx$$
$$= \frac{d^3\left(2b^3\sqrt{1-c^2x^2}(22329151-747937c^2x^2+134541c^4x^4-16875c^6x^6)-385875a^3cx(-35+35c^2x^2-2\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^3,x]
```

output
```
(d^3*(2*b^3*Sqrt[1 - c^2*x^2]*(22329151 - 747937*c^2*x^2 + 134541*c^4*x^4
- 16875*c^6*x^6) - 385875*a^3*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x
^6) + 11025*a^2*b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 7
5*c^6*x^6) + 210*a*b^2*c*x*(-226905 + 26495*c^2*x^2 - 7371*c^4*x^4 + 1125*
c^6*x^6) + 105*b*(-11025*a^2*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^
6) + 210*a*b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6
*x^6) + 2*b^2*c*x*(-226905 + 26495*c^2*x^2 - 7371*c^4*x^4 + 1125*c^6*x^6))
*ArcCos[c*x] + 11025*b^2*(-105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^
6*x^6) + b*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x
^6))*ArcCos[c*x]^2 - 385875*b^3*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6
*x^6)*ArcCos[c*x]^3))/(13505625*c)
```

Rubi [A] (verified)

Time = 3.16 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5159, 27, 5159, 5159, 5131, 5183, 2009, 5155, 27, 353, 53, 1576, 1140, 2009, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \; 5159$$

$$\frac{3}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2dx+\frac{6}{7}d\int d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2dx+\frac{6}{7}d^3\int\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2dx+\frac{6}{7}d^3\left(\frac{3}{5}bc\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{4}{5}\int\left(1-c^2x^2\right)(a+b\arccos(cx))^3dx+\frac{1}{5}x\left(1-c^2x^2\right)^2(a+\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2dx+\frac{6}{7}d^3\left(\frac{3}{5}bc\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{4}{5}\left(bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{2}{3}\int(a+b\,\mathrm{arcc}\right.\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{5131}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(3bc\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^3\right)+bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\right.\right.$$
$$\frac{3}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{5183}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(3bc\left(-\frac{2b\int(a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)+x(a+b\arccos(cx))^3\right)+bc\left(-\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(-\frac{2b\int\left(1-c^2x^2\right)^3(a+b\arccos(cx))dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2}\right)+$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\int\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+\right.\right.$$
$$\frac{3}{7}bcd^3\left(-\frac{2b\int\left(1-c^2x^2\right)^3(a+b\arccos(cx))dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2}\right)+$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 5155}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\left(bc\int\frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\,\text{arc}}{3c^2}\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(-\frac{2b\left(bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a+\right.}{7c}\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\left(\frac{1}{3}bc\int\frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\,\text{ar}}{3c^2}\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{7c}\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 353}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int\frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\,\text{ar}}{3c^2}\right.\right.\right.$$
$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{7c}\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{7c}\right.$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{7c}\right.$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{7c}\right.$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{7c}\right.$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}}{c^2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 2331}$$

$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{70}bc\int \frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a+\right.}{7c}\right.$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\text{ar}}{c^2}\right.\right.\right.\right.$$

$\downarrow$ 2389

$$\frac{3}{7}bcd^3\left(-\frac{2b\left(\frac{1}{70}bc\int\left(5\left(1-c^2x^2\right)^{5/2}+6\left(1-c^2x^2\right)^{3/2}+8\sqrt{1-c^2x^2}+\frac{16}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{7}c^6x^7(a+b\arccos(cx))\right.}{7c}\right.$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\text{ar}}{c^2}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^3+$$

$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\text{ar}}{c^2}\right.\right.\right.\right.$$

$$\frac{3}{7}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2}-\frac{2b\left(-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a\right.}{}\right.$$

input
```
Int[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^3,x]
```

output
```
(d^3*x*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^3)/7 + (3*b*c*d^3*(-1/7*((1 - c
^2*x^2)^(7/2)*(a + b*ArcCos[c*x])^2)/c^2 - (2*b*((b*c*((-32*Sqrt[1 - c^2*x
^2])/c^2 - (16*(1 - c^2*x^2)^(3/2))/(3*c^2) - (12*(1 - c^2*x^2)^(5/2))/(5*
c^2) - (10*(1 - c^2*x^2)^(7/2))/(7*c^2)))/70 + x*(a + b*ArcCos[c*x]) - c^2
*x^3*(a + b*ArcCos[c*x]) + (3*c^4*x^5*(a + b*ArcCos[c*x]))/5 - (c^6*x^7*(a
 + b*ArcCos[c*x]))/7))/(7*c)))/7 + (6*d^3*((x*(1 - c^2*x^2)^2*(a + b*ArcCo
s[c*x])^3)/5 + (3*b*c*(-1/5*((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/c^
2 - (2*b*((b*c*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c
^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2)))/30 + x*(a + b*ArcCos[c*x]) - (2*c^
2*x^3*(a + b*ArcCos[c*x]))/3 + (c^4*x^5*(a + b*ArcCos[c*x]))/5))/(5*c)))/5
 + (4*((x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^3)/3 + b*c*(-1/3*((1 - c^2*x^2
)^(3/2)*(a + b*ArcCos[c*x])^2)/c^2 - (2*b*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^
2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3
*(a + b*ArcCos[c*x]))/3))/(3*c)) + (2*(x*(a + b*ArcCos[c*x])^3 + 3*b*c*(-(
(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c
^2*x^2])/c + b*x*ArcCos[c*x]))/c)))/3))/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-d^3a^3\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b^3\left(\frac{\arccos(cx)^3\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{3\arccos(cx)^2\left(c^2x^2-1\right)^3\sqrt{-c^2x^2+1}}{49}\right.}{}$
default	$\frac{-d^3a^3\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b^3\left(\frac{\arccos(cx)^3\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{3\arccos(cx)^2\left(c^2x^2-1\right)^3\sqrt{-c^2x^2+1}}{49}\right.}{}$
parts	$-d^3a^3\left(\frac{1}{7}c^6x^7-\frac{3}{5}c^4x^5+c^2x^3-x\right)-\frac{d^3b^3\left(\frac{\arccos(cx)^3\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{3\arccos(cx)^2\left(c^2x^2\right.}{49}\right.}{}$
orering	$\frac{x\left(6215625c^8x^8-37489212c^6x^6+126346014c^4x^4-1949470892c^2x^2-879660415\right)\left(-c^2d\,x^2+d\right)^3\left(a+b\arccos(cx)\right)^3}{13505625(cx-1)(cx+1)\left(c^2x^2-1\right)^3}-$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d^3*a^3*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b^3*(1/35*arccos(c
*x)^3*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-3/49*arccos(c*x)^2*(c^2*x^2
-1)^3*(-c^2*x^2+1)^(1/2)-6/1715*arccos(c*x)*(5*c^6*x^6-21*c^4*x^4+35*c^2*x
^2-35)*c*x+6/2401*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-2664/214375*(c^2*x^2-1)
^2*(-c^2*x^2+1)^(1/2)+30256/385875*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-413312/1
28625*(-c^2*x^2+1)^(1/2)+18/175*arccos(c*x)^2*(c^2*x^2-1)^2*(-c^2*x^2+1)^(
1/2)+12/875*arccos(c*x)*(3*c^4*x^4-10*c^2*x^2+15)*c*x-8/35*arccos(c*x)^2*(
c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/105*arccos(c*x)*(c^2*x^2-3)*c*x+48/35*arc
cos(c*x)^2*(-c^2*x^2+1)^(1/2)+96/35*c*x*arccos(c*x))-3*d^3*a*b^2*(1/35*arc
cos(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-2/49*arccos(c*x)*(c^2*
x^2-1)^3*(-c^2*x^2+1)^(1/2)-2/1715*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*
x+12/175*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)+4/875*(3*c^4*x^4-10*
c^2*x^2+15)*c*x-16/105*arccos(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/315*(
c^2*x^2-3)*c*x+32/35*c*x+32/35*arccos(c*x)*(-c^2*x^2+1)^(1/2))-3*d^3*a^2*b
*(1/7*arccos(c*x)*c^7*x^7-3/5*arccos(c*x)*c^5*x^5+c^3*x^3*arccos(c*x)-c*x*
arccos(c*x)+2161/3675*(-c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(-c^2*x^2+1)^(1/
2)+117/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.20

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^3\,dx =$$
$$-\frac{39375\,(49\,a^3-6\,ab^2)c^7d^3x^7-6615\,(1225\,a^3-234\,ab^2)c^5d^3x^5+3675\,(3675\,a^3-1514\,ab^2)c^3d^3x^3-11}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
-1/13505625*(39375*(49*a^3 - 6*a*b^2)*c^7*d^3*x^7 - 6615*(1225*a^3 - 234*a
*b^2)*c^5*d^3*x^5 + 3675*(3675*a^3 - 1514*a*b^2)*c^3*d^3*x^3 - 11025*(1225
*a^3 - 4322*a*b^2)*c*d^3*x + 385875*(5*b^3*c^7*d^3*x^7 - 21*b^3*c^5*d^3*x^
5 + 35*b^3*c^3*d^3*x^3 - 35*b^3*c*d^3*x)*arccos(c*x)^3 + 1157625*(5*a*b^2*
c^7*d^3*x^7 - 21*a*b^2*c^5*d^3*x^5 + 35*a*b^2*c^3*d^3*x^3 - 35*a*b^2*c*d^3
*x)*arccos(c*x)^2 + 105*(1125*(49*a^2*b - 2*b^3)*c^7*d^3*x^7 - 189*(1225*a
^2*b - 78*b^3)*c^5*d^3*x^5 + 35*(11025*a^2*b - 1514*b^3)*c^3*d^3*x^3 - 105
*(3675*a^2*b - 4322*b^3)*c*d^3*x)*arccos(c*x) - (16875*(49*a^2*b - 2*b^3)*
c^6*d^3*x^6 - 81*(47775*a^2*b - 3322*b^3)*c^4*d^3*x^4 + (8345925*a^2*b - 1
495874*b^3)*c^2*d^3*x^2 - (23825025*a^2*b - 44658302*b^3)*d^3 + 11025*(75*
b^3*c^6*d^3*x^6 - 351*b^3*c^4*d^3*x^4 + 757*b^3*c^2*d^3*x^2 - 2161*b^3*d^3
)*arccos(c*x)^2 + 22050*(75*a*b^2*c^6*d^3*x^6 - 351*a*b^2*c^4*d^3*x^4 + 75
7*a*b^2*c^2*d^3*x^2 - 2161*a*b^2*d^3)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(422) = 844$.

Time = 1.22 (sec) , antiderivative size = 978, normalized size of antiderivative = 2.19

$$\int \left(d - c^2dx^2\right)^3 (a + b\arccos(cx))^3\, dx = \text{Too large to display}$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**3,x)
```

output
```
Piecewise((-a**3*c**6*d**3*x**7/7 + 3*a**3*c**4*d**3*x**5/5 - a**3*c**2*d*
*3*x**3 + a**3*d**3*x - 3*a**2*b*c**6*d**3*x**7*acos(c*x)/7 + 3*a**2*b*c**
5*d**3*x**6*sqrt(-c**2*x**2 + 1)/49 + 9*a**2*b*c**4*d**3*x**5*acos(c*x)/5
- 351*a**2*b*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)/1225 - 3*a**2*b*c**2*d**3
*x**3*acos(c*x) + 757*a**2*b*c*d**3*x**2*sqrt(-c**2*x**2 + 1)/1225 + 3*a**
2*b*d**3*x*acos(c*x) - 2161*a**2*b*d**3*sqrt(-c**2*x**2 + 1)/(1225*c) - 3*
a*b**2*c**6*d**3*x**7*acos(c*x)**2/7 + 6*a*b**2*c**6*d**3*x**7/343 + 6*a*b
**2*c**5*d**3*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x)/49 + 9*a*b**2*c**4*d**3*
x**5*acos(c*x)**2/5 - 702*a*b**2*c**4*d**3*x**5/6125 - 702*a*b**2*c**3*d**
3*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/1225 - 3*a*b**2*c**2*d**3*x**3*acos(
c*x)**2 + 1514*a*b**2*c**2*d**3*x**3/3675 + 1514*a*b**2*c*d**3*x**2*sqrt(-
c**2*x**2 + 1)*acos(c*x)/1225 + 3*a*b**2*d**3*x*acos(c*x)**2 - 4322*a*b**2
*d**3*x/1225 - 4322*a*b**2*d**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/(1225*c) -
b**3*c**6*d**3*x**7*acos(c*x)**3/7 + 6*b**3*c**6*d**3*x**7*acos(c*x)/343 +
 3*b**3*c**5*d**3*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/49 - 6*b**3*c**5*
d**3*x**6*sqrt(-c**2*x**2 + 1)/2401 + 3*b**3*c**4*d**3*x**5*acos(c*x)**3/5
 - 702*b**3*c**4*d**3*x**5*acos(c*x)/6125 - 351*b**3*c**3*d**3*x**4*sqrt(-
c**2*x**2 + 1)*acos(c*x)**2/1225 + 29898*b**3*c**3*d**3*x**4*sqrt(-c**2*x*
*2 + 1)/1500625 - b**3*c**2*d**3*x**3*acos(c*x)**3 + 1514*b**3*c**2*d**3*x
**3*acos(c*x)/3675 + 757*b**3*c*d**3*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(395) = 790$.

Time = 0.22 (sec) , antiderivative size = 1389, normalized size of antiderivative = 3.11

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^3\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
-1/7*b^3*c^6*d^3*x^7*arccos(c*x)^3 - 3/7*a*b^2*c^6*d^3*x^7*arccos(c*x)^2 -
 1/7*a^3*c^6*d^3*x^7 + 3/5*b^3*c^4*d^3*x^5*arccos(c*x)^3 + 9/5*a*b^2*c^4*d
^3*x^5*arccos(c*x)^2 + 3/5*a^3*c^4*d^3*x^5 - b^3*c^2*d^3*x^3*arccos(c*x)^3
 - 3/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2
*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c
^8)*c)*a^2*b*c^6*d^3 + 2/8575*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(
-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 +
1)/c^8)*c*arccos(c*x) + (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/
c^6)*a*b^2*c^6*d^3 + 1/900375*(11025*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqr
t(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2
+ 1)/c^8)*c*arccos(c*x)^2 - 2*c*((1125*sqrt(-c^2*x^2 + 1)*c^4*x^6 + 3996*s
qrt(-c^2*x^2 + 1)*c^2*x^4 + 15128*sqrt(-c^2*x^2 + 1)*x^2 + 206656*sqrt(-c^
2*x^2 + 1)/c^2)/c^6 - 105*(75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x
)*arccos(c*x)/c^7))*b^3*c^6*d^3 - 3*a*b^2*c^2*d^3*x^3*arccos(c*x)^2 + 3/25
*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1
)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a^2*b*c^4*d^3 - 2/125*(15*(3*sqrt
(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 +
1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*a*b^2*c^4*d^
3 - 1/1875*(225*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c
^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x)^2 - 2*c*((27*sqrt(-c^2*x^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(395) = 790$.

Time = 0.21 (sec) , antiderivative size = 841, normalized size of antiderivative = 1.89

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
-1/7*b^3*c^6*d^3*x^7*arccos(c*x)^3 - 3/7*a*b^2*c^6*d^3*x^7*arccos(c*x)^2 -
 3/7*a^2*b*c^6*d^3*x^7*arccos(c*x) + 6/343*b^3*c^6*d^3*x^7*arccos(c*x) + 3
/49*sqrt(-c^2*x^2 + 1)*b^3*c^5*d^3*x^6*arccos(c*x)^2 - 1/7*a^3*c^6*d^3*x^7
 + 6/343*a*b^2*c^6*d^3*x^7 + 6/49*sqrt(-c^2*x^2 + 1)*a*b^2*c^5*d^3*x^6*arc
cos(c*x) + 3/5*b^3*c^4*d^3*x^5*arccos(c*x)^3 + 3/49*sqrt(-c^2*x^2 + 1)*a^2
*b*c^5*d^3*x^6 - 6/2401*sqrt(-c^2*x^2 + 1)*b^3*c^5*d^3*x^6 + 9/5*a*b^2*c^4
*d^3*x^5*arccos(c*x)^2 + 9/5*a^2*b*c^4*d^3*x^5*arccos(c*x) - 702/6125*b^3*
c^4*d^3*x^5*arccos(c*x) - 351/1225*sqrt(-c^2*x^2 + 1)*b^3*c^3*d^3*x^4*arcc
os(c*x)^2 + 3/5*a^3*c^4*d^3*x^5 - 702/6125*a*b^2*c^4*d^3*x^5 - 702/1225*sq
rt(-c^2*x^2 + 1)*a*b^2*c^3*d^3*x^4*arccos(c*x) - b^3*c^2*d^3*x^3*arccos(c*
x)^3 - 351/1225*sqrt(-c^2*x^2 + 1)*a^2*b*c^3*d^3*x^4 + 29898/1500625*sqrt(
-c^2*x^2 + 1)*b^3*c^3*d^3*x^4 - 3*a*b^2*c^2*d^3*x^3*arccos(c*x)^2 - 3*a^2*
b*c^2*d^3*x^3*arccos(c*x) + 1514/3675*b^3*c^2*d^3*x^3*arccos(c*x) + 757/12
25*sqrt(-c^2*x^2 + 1)*b^3*c*d^3*x^2*arccos(c*x)^2 - a^3*c^2*d^3*x^3 + 1514
/3675*a*b^2*c^2*d^3*x^3 + 1514/1225*sqrt(-c^2*x^2 + 1)*a*b^2*c*d^3*x^2*arc
cos(c*x) + b^3*d^3*x*arccos(c*x)^3 + 757/1225*sqrt(-c^2*x^2 + 1)*a^2*b*c*d
^3*x^2 - 1495874/13505625*sqrt(-c^2*x^2 + 1)*b^3*c*d^3*x^2 + 3*a*b^2*d^3*x
*arccos(c*x)^2 + 3*a^2*b*d^3*x*arccos(c*x) - 4322/1225*b^3*d^3*x*arccos(c*
x) - 2161/1225*sqrt(-c^2*x^2 + 1)*b^3*d^3*arccos(c*x)^2/c + a^3*d^3*x - 43
22/1225*a*b^2*d^3*x - 4322/1225*sqrt(-c^2*x^2 + 1)*a*b^2*d^3*arccos(c*x...
```

Mupad [F(-1)]

Timed out.

$$\int (d-c^2dx^2)^3 (a+b\arccos(cx))^3\,dx = \int (a+b\,\mathrm{acos}(c\,x))^3 \left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int((a + b*acos(c*x))^3*(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))^3*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \left(d - c^2 d x^2\right)^3 (a + b \arccos(cx))^3 \, dx$$
$$= \frac{d^3\left(-2161\sqrt{-c^2x^2+1}\, a^2 b - 175 a^3 c^7 x^7 + 735 a^3 c^5 x^5 - 1225 a^3 c^3 x^3 - 11025 \left(\int acos(cx)^2\, x^2 dx\right) a\, b^2 c^3 - 3\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^3,x)
```

output
```
(d**3*(1225*acos(c*x)**3*b**3*c*x - 3675*sqrt( - c**2*x**2 + 1)*acos(c*x)*
*2*b**3 + 3675*acos(c*x)**2*a*b**2*c*x - 7350*sqrt( - c**2*x**2 + 1)*acos(
c*x)*a*b**2 - 525*acos(c*x)*a**2*b*c**7*x**7 + 2205*acos(c*x)*a**2*b*c**5*
x**5 - 3675*acos(c*x)*a**2*b*c**3*x**3 + 3675*acos(c*x)*a**2*b*c*x - 7350*
acos(c*x)*b**3*c*x + 75*sqrt( - c**2*x**2 + 1)*a**2*b*c**6*x**6 - 351*sqrt
( - c**2*x**2 + 1)*a**2*b*c**4*x**4 + 757*sqrt( - c**2*x**2 + 1)*a**2*b*c*
*2*x**2 - 2161*sqrt( - c**2*x**2 + 1)*a**2*b + 7350*sqrt( - c**2*x**2 + 1)
*b**3 - 1225*int(acos(c*x)**3*x**6,x)*b**3*c**7 + 3675*int(acos(c*x)**3*x*
*4,x)*b**3*c**5 - 3675*int(acos(c*x)**3*x**2,x)*b**3*c**3 - 3675*int(acos(
c*x)**2*x**6,x)*a*b**2*c**7 + 11025*int(acos(c*x)**2*x**4,x)*a*b**2*c**5 -
 11025*int(acos(c*x)**2*x**2,x)*a*b**2*c**3 - 175*a**3*c**7*x**7 + 735*a**
3*c**5*x**5 - 1225*a**3*c**3*x**3 + 1225*a**3*c*x - 7350*a*b**2*c*x))/(122
5*c)
```

3.13 $\int (d-c^2dx^2)^2 (a+b\arccos(cx))^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}\int (d-c^2dx^2)^2 (a+b\arccos(cx))^3 \, dx &= \frac{4144b^3d^2\sqrt{1-c^2x^2}}{1125c}+\frac{272b^3d^2(1-c^2x^2)^{3/2}}{3375c}\\ &+\frac{6b^3d^2(1-c^2x^2)^{5/2}}{625c}-\frac{298}{75}b^2d^2x(a+b\arccos(cx))+\frac{76}{225}b^2c^2d^2x^3(a+b\arccos(cx))\\ &-\frac{6}{125}b^2c^4d^2x^5(a+b\arccos(cx))-\frac{8bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{5c}\\ &-\frac{4bd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{15c}-\frac{3bd^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{25c}\\ &+\frac{8}{15}d^2x(a+b\arccos(cx))^3+\frac{4}{15}d^2x(1-c^2x^2)(a+b\arccos(cx))^3+\frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arccos(cx))^3\end{aligned}$$

output
```
4144/1125*b^3*d^2*(-c^2*x^2+1)^(1/2)/c+272/3375*b^3*d^2*(-c^2*x^2+1)^(3/2)
/c+6/625*b^3*d^2*(-c^2*x^2+1)^(5/2)/c-298/75*b^2*d^2*x*(a+b*arccos(c*x))+7
6/225*b^2*c^2*d^2*x^3*(a+b*arccos(c*x))-6/125*b^2*c^4*d^2*x^5*(a+b*arccos(
c*x))-8/5*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/c-4/15*b*d^2*(-c^2*
x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c-3/25*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arcc
os(c*x))^2/c+8/15*d^2*x*(a+b*arccos(c*x))^3+4/15*d^2*x*(-c^2*x^2+1)*(a+b*a
rccos(c*x))^3+1/5*d^2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^3
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^3\,dx$$
$$=\frac{d^2\left(1125a^3cx(15-10c^2x^2+3c^4x^4)-225a^2b\sqrt{1-c^2x^2}(149-38c^2x^2+9c^4x^4)-30ab^2cx(2235-190c^2x^2\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^3,x]
```

output
```
(d^2*(1125*a^3*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) - 225*a^2*b*Sqrt[1 - c^2*
x^2]*(149 - 38*c^2*x^2 + 9*c^4*x^4) - 30*a*b^2*c*x*(2235 - 190*c^2*x^2 + 2
7*c^4*x^4) + 2*b^3*Sqrt[1 - c^2*x^2]*(31841 - 842*c^2*x^2 + 81*c^4*x^4) -
15*b*(-225*a^2*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 30*a*b*Sqrt[1 - c^2*x^2
]*(149 - 38*c^2*x^2 + 9*c^4*x^4) + 2*b^2*c*x*(2235 - 190*c^2*x^2 + 27*c^4*
x^4))*ArcCos[c*x] - 225*b^2*(-15*a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + b*S
qrt[1 - c^2*x^2]*(149 - 38*c^2*x^2 + 9*c^4*x^4))*ArcCos[c*x]^2 + 1125*b^3*
c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)*ArcCos[c*x]^3))/(16875*c)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.32, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {5159, 27, 5159, 5131, 5183, 2009, 5155, 27, 353, 53, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{4}{5}d\int d\left(1-c^2x^2\right)(a+b\arccos(cx))^3dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{4}{5}d^2\int \left(1-c^2x^2\right)(a+b\arccos(cx))^3dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$\downarrow$ 5159

$$\frac{3}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{4}{5}d^2\left(bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{2}{3}\int (a+b\arccos(cx))^3dx+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^3\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$\downarrow$ 5131

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(3bc\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^3\right)+bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}x(\right.$$
$$\frac{3}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$\downarrow$ 5183

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(3bc\left(-\frac{2b\int (a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)+x(a+b\arccos(cx))^3\right)+bc\left(-\frac{2b}{}\right.\right.$$
$$\frac{3}{5}bcd^2\left(-\frac{2b\int \left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2}\right)+$$
$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$\downarrow$ 2009

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\int \left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\operatorname{arc}\right.$$
$$\frac{3}{5}bcd^2\left(-\frac{2b\int \left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2}\right)+$$
$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$\downarrow$ 5155

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\left(bc\int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos}{3c^2}\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{2b\left(bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx)\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \; 27$$

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\left(\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos}{3c^2}\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{2b\left(\frac{1}{15}bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \; 353$$

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos}{3c^2}\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{2b\left(\frac{1}{15}bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \; 53$$

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2}{}\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{2b\left(\frac{1}{15}bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \; 1576$$

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2}{}\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{2b\left(\frac{1}{30}bc\int\frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(ca\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \; 1140$$

$$\frac{4}{5}d^2\left(bc\left(-\frac{2b\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2}{}\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{2b\left(\frac{1}{30}bc\int\left(3\left(1-c^2x^2\right)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\,\mathrm{arc}\right.}{5c}\right.$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^3+$$

$$\frac{4}{5}d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b}{}\right.}{c}\right.\right.\right.$$

$$\frac{3}{5}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2}-\frac{2b\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\,\mathrm{a}\right.}{5c}\right.$$

input
```
Int[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^3,x]
```

output
```
(d^2*x*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^3)/5 + (3*b*c*d^2*(-1/5*((1 - c
^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/c^2 - (2*b*((b*c*((-16*Sqrt[1 - c^2*x
^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^
2)))/30 + x*(a + b*ArcCos[c*x]) - (2*c^2*x^3*(a + b*ArcCos[c*x]))/3 + (c^4
*x^5*(a + b*ArcCos[c*x]))/5))/(5*c)))/5 + (4*d^2*((x*(1 - c^2*x^2)*(a + b*
ArcCos[c*x])^3)/3 + b*c*(-1/3*((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/
c^2 - (2*b*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*
c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a + b*ArcCos[c*x]))/3))/(3*c)
) + (2*(x*(a + b*ArcCos[c*x])^3 + 3*b*c*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCo
s[c*x])^2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/
c)))/3))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{d^2a^3\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b^3\left(\frac{\arccos(cx)^3\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{3\arccos(cx)^2\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\arccos(cx)\left(3c\right.}{}\right.}{}$
default	$\frac{d^2a^3\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b^3\left(\frac{\arccos(cx)^3\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{3\arccos(cx)^2\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\arccos(cx)\left(3c\right.}{}\right.}{}$
parts	$d^2a^3\left(\frac{1}{5}c^4x^5-\frac{2}{3}c^2x^3+x\right)+\frac{d^2b^3\left(\frac{\arccos(cx)^3\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{3\arccos(cx)^2\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\,a}{}\right.}{}$
orering	$\frac{x\left(29889c^6x^6-179507c^4x^4+2768347c^2x^2+1732471\right)\left(-c^2d\,x^2+d\right)^2\left(a+b\arccos(cx)\right)^3}{50625\left(c^2x^2-1\right)^3}-\frac{\left(7857c^6x^6-60788c^4x^4+144\right.}{}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^2*a^3*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b^3*(1/15*arccos(c*x)^3*(3*
c^4*x^4-10*c^2*x^2+15)*c*x-3/25*arccos(c*x)^2*(c^2*x^2-1)^2*(-c^2*x^2+1)^(
1/2)-2/125*arccos(c*x)*(3*c^4*x^4-10*c^2*x^2+15)*c*x+6/625*(c^2*x^2-1)^2*(
-c^2*x^2+1)^(1/2)-272/3375*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+4144/1125*(-c^2*
x^2+1)^(1/2)+4/15*arccos(c*x)^2*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+8/45*arccos
(c*x)*(c^2*x^2-3)*c*x-8/5*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)-16/5*c*x*arccos
(c*x))+3*d^2*a*b^2*(1/15*arccos(c*x)^2*(3*c^4*x^4-10*c^2*x^2+15)*c*x-2/25*
arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)-2/375*(3*c^4*x^4-10*c^2*x^2+1
5)*c*x+8/45*arccos(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+8/135*(c^2*x^2-3)*c
*x-16/15*c*x-16/15*arccos(c*x)*(-c^2*x^2+1)^(1/2))+3*d^2*a^2*b*(1/5*arccos
(c*x)*c^5*x^5-2/3*c^3*x^3*arccos(c*x)+c*x*arccos(c*x)-149/225*(-c^2*x^2+1)
^(1/2)+38/225*c^2*x^2*(-c^2*x^2+1)^(1/2)-1/25*c^4*x^4*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.24

$$\int \left(d-c^2dx^2\right)^2 \left(a+b\arccos(cx)\right)^3 \, dx$$

$$= \frac{135\,(25\,a^3-6\,ab^2)c^5d^2x^5-150\,(75\,a^3-38\,ab^2)c^3d^2x^3+225\,(75\,a^3-298\,ab^2)cd^2x+1125\,(3\,b^3c^5d^2x^5-}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
1/16875*(135*(25*a^3 - 6*a*b^2)*c^5*d^2*x^5 - 150*(75*a^3 - 38*a*b^2)*c^3*
d^2*x^3 + 225*(75*a^3 - 298*a*b^2)*c*d^2*x + 1125*(3*b^3*c^5*d^2*x^5 - 10*
b^3*c^3*d^2*x^3 + 15*b^3*c*d^2*x)*arccos(c*x)^3 + 3375*(3*a*b^2*c^5*d^2*x^
5 - 10*a*b^2*c^3*d^2*x^3 + 15*a*b^2*c*d^2*x)*arccos(c*x)^2 + 15*(27*(25*a^
2*b - 2*b^3)*c^5*d^2*x^5 - 10*(225*a^2*b - 38*b^3)*c^3*d^2*x^3 + 15*(225*a
^2*b - 298*b^3)*c*d^2*x)*arccos(c*x) - (81*(25*a^2*b - 2*b^3)*c^4*d^2*x^4
- 2*(4275*a^2*b - 842*b^3)*c^2*d^2*x^2 + (33525*a^2*b - 63682*b^3)*d^2 + 2
25*(9*b^3*c^4*d^2*x^4 - 38*b^3*c^2*d^2*x^2 + 149*b^3*d^2)*arccos(c*x)^2 +
450*(9*a*b^2*c^4*d^2*x^4 - 38*a*b^2*c^2*d^2*x^2 + 149*a*b^2*d^2)*arccos(c*
x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(311) = 622$.

Time = 0.66 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.19

$$\int \left(d - c^2 d x^2\right)^2 (a + b \arccos(cx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**3,x)
```

output
```
Piecewise((a**3*c**4*d**2*x**5/5 - 2*a**3*c**2*d**2*x**3/3 + a**3*d**2*x +
 3*a**2*b*c**4*d**2*x**5*acos(c*x)/5 - 3*a**2*b*c**3*d**2*x**4*sqrt(-c**2*
x**2 + 1)/25 - 2*a**2*b*c**2*d**2*x**3*acos(c*x) + 38*a**2*b*c*d**2*x**2*s
qrt(-c**2*x**2 + 1)/75 + 3*a**2*b*d**2*x*acos(c*x) - 149*a**2*b*d**2*sqrt(
-c**2*x**2 + 1)/(75*c) + 3*a*b**2*c**4*d**2*x**5*acos(c*x)**2/5 - 6*a*b**2
*c**4*d**2*x**5/125 - 6*a*b**2*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c*
x)/25 - 2*a*b**2*c**2*d**2*x**3*acos(c*x)**2 + 76*a*b**2*c**2*d**2*x**3/22
5 + 76*a*b**2*c*d**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/75 + 3*a*b**2*d**
2*x*acos(c*x)**2 - 298*a*b**2*d**2*x/75 - 298*a*b**2*d**2*sqrt(-c**2*x**2
+ 1)*acos(c*x)/(75*c) + b**3*c**4*d**2*x**5*acos(c*x)**3/5 - 6*b**3*c**4*d
**2*x**5*acos(c*x)/125 - 3*b**3*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c
*x)**2/25 + 6*b**3*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)/625 - 2*b**3*c**2*d
**2*x**3*acos(c*x)**3/3 + 76*b**3*c**2*d**2*x**3*acos(c*x)/225 + 38*b**3*c
*d**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/75 - 1684*b**3*c*d**2*x**2*sq
rt(-c**2*x**2 + 1)/16875 + b**3*d**2*x*acos(c*x)**3 - 298*b**3*d**2*x*acos
(c*x)/75 - 149*b**3*d**2*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(75*c) + 63682*
b**3*d**2*sqrt(-c**2*x**2 + 1)/(16875*c), Ne(c, 0)), (d**2*x*(a + pi*b/2)*
*3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(292) = 584$.

Time = 0.19 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.67

$$\int \left(d - c^2 d x^2\right)^2 (a + b \arccos(cx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output

```
1/5*b^3*c^4*d^2*x^5*arccos(c*x)^3 + 3/5*a*b^2*c^4*d^2*x^5*arccos(c*x)^2 +
1/5*a^3*c^4*d^2*x^5 - 2/3*b^3*c^2*d^2*x^3*arccos(c*x)^3 - 2*a*b^2*c^2*d^2*
x^3*arccos(c*x)^2 + 1/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c
^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a^2*b*c^4
*d^2 - 2/375*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/
c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 +
120*x)/c^4)*a*b^2*c^4*d^2 - 1/5625*(225*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*
sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x)^2 - 2
*c*((27*sqrt(-c^2*x^2 + 1)*c^2*x^4 + 136*sqrt(-c^2*x^2 + 1)*x^2 + 2072*sqr
t(-c^2*x^2 + 1)/c^2)/c^4 - 15*(9*c^4*x^5 + 20*c^2*x^3 + 120*x)*arccos(c*x)
/c^5))*b^3*c^4*d^2 - 2/3*a^3*c^2*d^2*x^3 + b^3*d^2*x*arccos(c*x)^3 - 2/3*(
3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c
^4))*a^2*b*c^2*d^2 + 4/9*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^
2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*a*b^2*c^2*d^2 + 2/27*(9*c*(
sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x)^2 - 2*c
*((sqrt(-c^2*x^2 + 1)*x^2 + 20*sqrt(-c^2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 +
6*x)*arccos(c*x)/c^3))*b^3*c^2*d^2 + 3*a*b^2*d^2*x*arccos(c*x)^2 - 3*(sqrt
(-c^2*x^2 + 1)*arccos(c*x)^2/c + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))/
c)*b^3*d^2 - 6*a*b^2*d^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^3*d^2*
x + 3*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a^2*b*d^2/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(292) = 584$.

Time = 0.19 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.87

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^3\,dx = \frac{1}{5}\,b^3c^4d^2x^5\arccos(cx)^3+\frac{3}{5}\,ab^2c^4d^2x^5\arccos(cx)^2 + \frac{3}{5}\,a^2bc^4d^2x^5\arccos(cx) - \frac{6}{125}\,b^3c^4d^2x^5\arccos(cx) - \frac{3}{25}\,\sqrt{-c^2x^2+1}b^3c^3d^2x^4\arccos(cx)^2 + \frac{1}{5}\,a^3c^4d^2x^5 - \frac{6}{125}\,ab^2c^4d^2x^5 - \frac{6}{25}\,\sqrt{-c^2x^2+1}ab^2c^3d^2x^4\arccos(cx) - \frac{2}{3}\,b^3c^2d^2x^3\arccos(cx)^3 - \frac{3}{25}\,\sqrt{-c^2x^2+1}a^2bc^3d^2x^4 + \frac{6}{625}\,\sqrt{-c^2x^2+1}b^3c^3d^2x^4 - 2\,ab^2c^2d^2x^3\arccos(cx)^2 - 2\,a^2bc^2d^2x^3\arccos(cx) + \frac{76}{225}\,b^3c^2d^2x^3\arccos(cx) + \frac{38}{75}\,\sqrt{-c^2x^2+1}b^3cd^2x^2\arccos(cx)^2 - \frac{2}{3}\,a^3c^2d^2x^3 + \frac{76}{225}\,ab^2c^2d^2x^3 + \frac{76}{75}\,\sqrt{-c^2x^2+1}ab^2cd^2x^2\arccos(cx) + b^3d^2x\arccos(cx)^3 + \frac{38}{75}\,\sqrt{-c^2x^2+1}a^2bcd^2x^2 - \frac{1684}{16875}\,\sqrt{-c^2x^2+1}b^3cd^2x^2 + 3\,ab^2d^2x\arccos(cx)^2 + 3\,a^2bd^2x\arccos(cx) - \frac{298}{75}\,b^3d^2x\arccos(cx) - \frac{149\,\sqrt{-c^2x^2+1}b^3d^2\arccos(cx)^2}{75\,c} + a^3d^2x - \frac{298}{75}\,ab^2d^2x - \frac{298\,\sqrt{-c^2x^2+1}ab^2d^2\arccos(cx)}{75\,c} - \frac{149\,\sqrt{-c^2x^2+1}a^2bd^2}{75\,c} + \frac{63682\,\sqrt{-c^2x^2+1}b^3d^2}{16875\,c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
1/5*b^3*c^4*d^2*x^5*arccos(c*x)^3 + 3/5*a*b^2*c^4*d^2*x^5*arccos(c*x)^2 +
3/5*a^2*b*c^4*d^2*x^5*arccos(c*x) - 6/125*b^3*c^4*d^2*x^5*arccos(c*x) - 3/
25*sqrt(-c^2*x^2 + 1)*b^3*c^3*d^2*x^4*arccos(c*x)^2 + 1/5*a^3*c^4*d^2*x^5
- 6/125*a*b^2*c^4*d^2*x^5 - 6/25*sqrt(-c^2*x^2 + 1)*a*b^2*c^3*d^2*x^4*arcc
os(c*x) - 2/3*b^3*c^2*d^2*x^3*arccos(c*x)^3 - 3/25*sqrt(-c^2*x^2 + 1)*a^2*
b*c^3*d^2*x^4 + 6/625*sqrt(-c^2*x^2 + 1)*b^3*c^3*d^2*x^4 - 2*a*b^2*c^2*d^2
*x^3*arccos(c*x)^2 - 2*a^2*b*c^2*d^2*x^3*arccos(c*x) + 76/225*b^3*c^2*d^2*
x^3*arccos(c*x) + 38/75*sqrt(-c^2*x^2 + 1)*b^3*c*d^2*x^2*arccos(c*x)^2 - 2
/3*a^3*c^2*d^2*x^3 + 76/225*a*b^2*c^2*d^2*x^3 + 76/75*sqrt(-c^2*x^2 + 1)*a
*b^2*c*d^2*x^2*arccos(c*x) + b^3*d^2*x*arccos(c*x)^3 + 38/75*sqrt(-c^2*x^2
 + 1)*a^2*b*c*d^2*x^2 - 1684/16875*sqrt(-c^2*x^2 + 1)*b^3*c*d^2*x^2 + 3*a*
b^2*d^2*x*arccos(c*x)^2 + 3*a^2*b*d^2*x*arccos(c*x) - 298/75*b^3*d^2*x*arc
cos(c*x) - 149/75*sqrt(-c^2*x^2 + 1)*b^3*d^2*arccos(c*x)^2/c + a^3*d^2*x -
 298/75*a*b^2*d^2*x - 298/75*sqrt(-c^2*x^2 + 1)*a*b^2*d^2*arccos(c*x)/c -
149/75*sqrt(-c^2*x^2 + 1)*a^2*b*d^2/c + 63682/16875*sqrt(-c^2*x^2 + 1)*b^3
*d^2/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^2(a+b\arccos(cx))^3\,dx=\int (a+b\,\mathrm{acos}(c\,x))^3\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int((a + b*acos(c*x))^3*(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^3*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^3\, dx$$
$$= \frac{d^2\left(75acos(cx)^3 b^3cx - 225\sqrt{-c^2x^2+1}\, acos(cx)^2 b^3 + 225acos(cx)^2 a\, b^2cx - 450\sqrt{-c^2x^2+1}\, acos(cx)\, a\, b \ldots\right.}{}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^3,x)
```

output
```
(d**2*(75*acos(c*x)**3*b**3*c*x - 225*sqrt( - c**2*x**2 + 1)*acos(c*x)**2*
b**3 + 225*acos(c*x)**2*a*b**2*c*x - 450*sqrt( - c**2*x**2 + 1)*acos(c*x)*
a*b**2 + 45*acos(c*x)*a**2*b*c**5*x**5 - 150*acos(c*x)*a**2*b*c**3*x**3 +
225*acos(c*x)*a**2*b*c*x - 450*acos(c*x)*b**3*c*x - 9*sqrt( - c**2*x**2 +
1)*a**2*b*c**4*x**4 + 38*sqrt( - c**2*x**2 + 1)*a**2*b*c**2*x**2 - 149*sqr
t( - c**2*x**2 + 1)*a**2*b + 450*sqrt( - c**2*x**2 + 1)*b**3 + 75*int(acos
(c*x)**3*x**4,x)*b**3*c**5 - 150*int(acos(c*x)**3*x**2,x)*b**3*c**3 + 225*
int(acos(c*x)**2*x**4,x)*a*b**2*c**5 - 450*int(acos(c*x)**2*x**2,x)*a*b**2
*c**3 + 15*a**3*c**5*x**5 - 50*a**3*c**3*x**3 + 75*a**3*c*x - 450*a*b**2*c
*x))/(75*c)
```

3.14 $\int (d-c^2dx^2)(a+b\arccos(cx))^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (d-c^2dx^2)(a+b\arccos(cx))^3\,dx = \frac{40b^3d\sqrt{1-c^2x^2}}{9c}+\frac{2b^3d(1-c^2x^2)^{3/2}}{27c}-\frac{14}{3}b^2dx(a+b\arccos(cx))+\frac{2}{9}b^2c^2dx^3(a+b\arccos(cx))-\frac{2bd\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}-\frac{bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c}+\frac{2}{3}dx(a+b\arccos(cx))^3+\frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^3$$

output
```
40/9*b^3*d*(-c^2*x^2+1)^(1/2)/c+2/27*b^3*d*(-c^2*x^2+1)^(3/2)/c-14/3*b^2*d
*x*(a+b*arccos(c*x))+2/9*b^2*c^2*d*x^3*(a+b*arccos(c*x))-2*b*d*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))^2/c-1/3*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))
^2/c+2/3*d*x*(a+b*arccos(c*x))^3+1/3*d*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.22

$$\int (d-c^2dx^2)(a+b\arccos(cx))^3\,dx = \frac{d\left(-2b^3\sqrt{1-c^2x^2}(-61+c^2x^2)+6ab^2cx(-21+c^2x^2)+9a^2b\sqrt{1-c^2x^2}(-7+c^2x^2)-9a^3cx(-3+c^2x^2\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^3,x]
```

output
```
(d*(-2*b^3*Sqrt[1 - c^2*x^2]*(-61 + c^2*x^2) + 6*a*b^2*c*x*(-21 + c^2*x^2)
 + 9*a^2*b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) - 9*a^3*c*x*(-3 + c^2*x^2) + 3
*b*(2*b^2*c*x*(-21 + c^2*x^2) + 6*a*b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) - 9
*a^2*c*x*(-3 + c^2*x^2))*ArcCos[c*x] + 9*b^2*(b*Sqrt[1 - c^2*x^2]*(-7 + c^
2*x^2) + a*(9*c*x - 3*c^3*x^3))*ArcCos[c*x]^2 - 9*b^3*c*x*(-3 + c^2*x^2)*A
rcCos[c*x]^3))/(27*c)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5159, 5131, 5183, 2009, 5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d-c^2dx^2)(a+b\arccos(cx))^3\,dx$$

$$\Big\downarrow \text{ 5159}$$

$$bcd\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{2}{3}d\int (a+b\arccos(cx))^3dx+\frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^3$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{2}{3}d\left(3bc\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^3\right)+bcd\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3$$

$\downarrow$ 5183

$$\frac{2}{3}d\left(3bc\left(-\frac{2b\int(a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)+x(a+b\arccos(cx))^3\right)+bcd\left(-\frac{2b\int\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3$$

$\downarrow$ 2009

$$bcd\left(-\frac{2b\int\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}d\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3\right)$$

$\downarrow$ 5155

$$bcd\left(-\frac{2b\left(bc\int\frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right.$$

$$\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}d\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3\right)$$

$\downarrow$ 27

$$bcd\left(-\frac{2b\left(\frac{1}{3}bc\int\frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx}{3c^2}\right.$$

$$\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3+\frac{2}{3}d\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3\right)$$

$$\Big\downarrow \text{353}$$

$$bcd\left(-\frac{2b\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx}{3c^2}\right.$$

$$\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3+$$

$$\frac{2}{3}d\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3\right)$$

$$\Big\downarrow \text{53}$$

$$bcd\left(-\frac{2b\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\left(1-c^2x^2\right)^{3/2}\right.$$

$$\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3+$$

$$\frac{2}{3}d\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^3+$$

$$\frac{2}{3}d\left(3bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)+x(a+b\arccos(cx))^3\right)+$$

$$bcd\left(-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x}{3c^2}\right.\right.}{3c}\right.$$

input
```
Int[(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^3,x]
```

output
```
(d*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^3)/3 + b*c*d*(-1/3*((1 - c^2*x^2)^(
3/2)*(a + b*ArcCos[c*x])^2)/c^2 - (2*b*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 -
 (2*(1 - c^2*x^2)^(3/2))/(3*c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a
 + b*ArcCos[c*x]))/3))/(3*c)) + (2*d*(x*(a + b*ArcCos[c*x])^3 + 3*b*c*(-((
Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c^
2*x^2])/c + b*x*ArcCos[c*x]))/c)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-d\,a^3\left(\frac{1}{3}c^3x^3-cx\right)-d\,b^3\left(\frac{\arccos(cx)^3\left(c^2x^2-3\right)cx}{3}+2\arccos(cx)^2\sqrt{-c^2x^2+1}-\frac{40\sqrt{-c^2x^2+1}}{9}+4cx\arccos(cx)-\frac{\arccos(cx)}{}\right.}{}$
default	$\frac{-d\,a^3\left(\frac{1}{3}c^3x^3-cx\right)-d\,b^3\left(\frac{\arccos(cx)^3\left(c^2x^2-3\right)cx}{3}+2\arccos(cx)^2\sqrt{-c^2x^2+1}-\frac{40\sqrt{-c^2x^2+1}}{9}+4cx\arccos(cx)-\frac{\arccos(cx)}{}\right.}{}$
parts	$-d\,a^3\left(\frac{1}{3}c^2x^3-x\right)-\frac{d\,b^3\left(\frac{\arccos(cx)^3\left(c^2x^2-3\right)cx}{3}+2\arccos(cx)^2\sqrt{-c^2x^2+1}-\frac{40\sqrt{-c^2x^2+1}}{9}+4cx\arccos(cx)-\right.}{c}$
orering	$\frac{5x\left(13c^4x^4-194c^2x^2-179\right)\left(-c^2d\,x^2+d\right)\left(a+b\arccos(cx)\right)^3}{81\left(c^2x^2-1\right)^2}-\frac{\left(25c^4x^4-683c^2x^2-242\right)\left(-2d\,c^2x\left(a+b\arccos(cx)\right)^3-\right.}{81c^2\left(c^2x^2-1\right)}$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*a^3*(1/3*c^3*x^3-c*x)-d*b^3*(1/3*arccos(c*x)^3*(c^2*x^2-3)*c*x+2*a
rccos(c*x)^2*(-c^2*x^2+1)^(1/2)-40/9*(-c^2*x^2+1)^(1/2)+4*c*x*arccos(c*x)-
1/3*arccos(c*x)^2*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-2/9*arccos(c*x)*(c^2*x^2-
3)*c*x+2/27*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2))-3*d*a*b^2*(1/3*arccos(c*x)^2*(
c^2*x^2-3)*c*x+4/3*c*x+4/3*arccos(c*x)*(-c^2*x^2+1)^(1/2)-2/9*arccos(c*x)*
(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-2/27*(c^2*x^2-3)*c*x)-3*d*a^2*b*(1/3*c^3*x^
3*arccos(c*x)-c*x*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)+7/9*(-c^2*x^2
+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.27

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^3\,dx =$$
$$-\frac{3\,(3\,a^3-2\,ab^2)c^3dx^3-9\,(3\,a^3-14\,ab^2)cdx+9\,(b^3c^3dx^3-3\,b^3cdx)\arccos{(cx)}^3+27\,(ab^2c^3dx^3-3\,ab$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

output
```
-1/27*(3*(3*a^3 - 2*a*b^2)*c^3*d*x^3 - 9*(3*a^3 - 14*a*b^2)*c*d*x + 9*(b^3
*c^3*d*x^3 - 3*b^3*c*d*x)*arccos(c*x)^3 + 27*(a*b^2*c^3*d*x^3 - 3*a*b^2*c*
d*x)*arccos(c*x)^2 + 3*((9*a^2*b - 2*b^3)*c^3*d*x^3 - 3*(9*a^2*b - 14*b^3)
*c*d*x)*arccos(c*x) - ((9*a^2*b - 2*b^3)*c^2*d*x^2 + 9*(b^3*c^2*d*x^2 - 7*
b^3*d)*arccos(c*x)^2 - (63*a^2*b - 122*b^3)*d + 18*(a*b^2*c^2*d*x^2 - 7*a*
b^2*d)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(184) = 368$.

Time = 0.33 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.13

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^3\,dx$$
$$=\begin{cases}-\frac{a^3c^2dx^3}{3}+a^3dx-a^2bc^2dx^3\operatorname{acos}{(cx)}+\frac{a^2bcdx^2\sqrt{-c^2x^2+1}}{3}+3a^2bdx\operatorname{acos}{(cx)}-\frac{7a^2bd\sqrt{-c^2x^2+1}}{3c}-ab^2c^2dx^3\\ dx\left(a+\frac{\pi b}{2}\right)^3\end{cases}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**3,x)
```

output
```
Piecewise((-a**3*c**2*d*x**3/3 + a**3*d*x - a**2*b*c**2*d*x**3*acos(c*x) +
 a**2*b*c*d*x**2*sqrt(-c**2*x**2 + 1)/3 + 3*a**2*b*d*x*acos(c*x) - 7*a**2*
b*d*sqrt(-c**2*x**2 + 1)/(3*c) - a*b**2*c**2*d*x**3*acos(c*x)**2 + 2*a*b**
2*c**2*d*x**3/9 + 2*a*b**2*c*d*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/3 + 3*a
*b**2*d*x*acos(c*x)**2 - 14*a*b**2*d*x/3 - 14*a*b**2*d*sqrt(-c**2*x**2 + 1
)*acos(c*x)/(3*c) - b**3*c**2*d*x**3*acos(c*x)**3/3 + 2*b**3*c**2*d*x**3*a
cos(c*x)/9 + b**3*c*d*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/3 - 2*b**3*c*
d*x**2*sqrt(-c**2*x**2 + 1)/27 + b**3*d*x*acos(c*x)**3 - 14*b**3*d*x*acos(
c*x)/3 - 7*b**3*d*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(3*c) + 122*b**3*d*sqr
t(-c**2*x**2 + 1)/(27*c), Ne(c, 0)), (d*x*(a + pi*b/2)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(173) = 346$.

Time = 0.16 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.28

$$\int (d - c^2dx^2) (a + b\arccos(cx))^3 dx$$
$$= -\frac{1}{3} b^3c^2dx^3 \arccos(cx)^3 - ab^2c^2dx^3 \arccos(cx)^2 - \frac{1}{3} a^3c^2dx^3 + b^3dx \arccos(cx)^3$$
$$- \frac{1}{3} \left(3 x^3 \arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right) a^2bc^2d$$
$$+ \frac{2}{9} \left(3 c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right) \arccos(cx) + \frac{c^2x^3+6x}{c^2}\right) ab^2c^2d$$
$$+ \frac{1}{27} \left(9 c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right) \arccos(cx)^2 - 2 c\left(\frac{\sqrt{-c^2x^2+1}x^2 + \frac{20\sqrt{-c^2x^2+1}}{c^2}}{c^2} - \frac{3(c^2x^3\right.\right.$$
$$+ 3 ab^2dx \arccos(cx)^2$$
$$- 3 \left(\frac{\sqrt{-c^2x^2+1} \arccos(cx)^2}{c} + \frac{2\left(cx \arccos(cx) - \sqrt{-c^2x^2+1}\right)}{c}\right) b^3d$$
$$- 6 ab^2d\left(x + \frac{\sqrt{-c^2x^2+1} \arccos(cx)}{c}\right)$$
$$+ a^3dx + \frac{3\left(cx \arccos(cx) - \sqrt{-c^2x^2+1}\right)a^2bd}{c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

output
```
-1/3*b^3*c^2*d*x^3*arccos(c*x)^3 - a*b^2*c^2*d*x^3*arccos(c*x)^2 - 1/3*a^3
*c^2*d*x^3 + b^3*d*x*arccos(c*x)^3 - 1/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^2
*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a^2*b*c^2*d + 2/9*(3*c*(sqr
t(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3
 + 6*x)/c^2)*a*b^2*c^2*d + 1/27*(9*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(
-c^2*x^2 + 1)/c^4)*arccos(c*x)^2 - 2*c*((sqrt(-c^2*x^2 + 1)*x^2 + 20*sqrt(
-c^2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 + 6*x)*arccos(c*x)/c^3))*b^3*c^2*d + 3
*a*b^2*d*x*arccos(c*x)^2 - 3*(sqrt(-c^2*x^2 + 1)*arccos(c*x)^2/c + 2*(c*x*
arccos(c*x) - sqrt(-c^2*x^2 + 1))/c)*b^3*d - 6*a*b^2*d*(x + sqrt(-c^2*x^2
+ 1)*arccos(c*x)/c) + a^3*d*x + 3*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a
^2*b*d/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(173) = 346$.

Time = 0.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^3\,dx =& -\frac{1}{3}\,b^3c^2dx^3\arccos(cx)^3 - ab^2c^2dx^3\arccos(cx)^2 \\ & - a^2bc^2dx^3\arccos(cx) + \frac{2}{9}\,b^3c^2dx^3\arccos(cx) \\ & + \frac{1}{3}\,\sqrt{-c^2x^2+1}b^3cdx^2\arccos(cx)^2 - \frac{1}{3}\,a^3c^2dx^3 \\ & + \frac{2}{9}\,ab^2c^2dx^3 + \frac{2}{3}\,\sqrt{-c^2x^2+1}ab^2cdx^2\arccos(cx) \\ & + b^3dx\arccos(cx)^3 + \frac{1}{3}\,\sqrt{-c^2x^2+1}a^2bcdx^2 \\ & - \frac{2}{27}\,\sqrt{-c^2x^2+1}b^3cdx^2 + 3\,ab^2dx\arccos(cx)^2 \\ & + 3\,a^2bdx\arccos(cx) - \frac{14}{3}\,b^3dx\arccos(cx) \\ & - \frac{7\,\sqrt{-c^2x^2+1}b^3d\arccos(cx)^2}{3\,c} + a^3dx \\ & - \frac{14}{3}\,ab^2dx - \frac{14\,\sqrt{-c^2x^2+1}ab^2d\arccos(cx)}{3\,c} \\ & - \frac{7\,\sqrt{-c^2x^2+1}a^2bd}{3\,c} + \frac{122\,\sqrt{-c^2x^2+1}b^3d}{27\,c}\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

output
```
-1/3*b^3*c^2*d*x^3*arccos(c*x)^3 - a*b^2*c^2*d*x^3*arccos(c*x)^2 - a^2*b*c
^2*d*x^3*arccos(c*x) + 2/9*b^3*c^2*d*x^3*arccos(c*x) + 1/3*sqrt(-c^2*x^2 +
 1)*b^3*c*d*x^2*arccos(c*x)^2 - 1/3*a^3*c^2*d*x^3 + 2/9*a*b^2*c^2*d*x^3 +
2/3*sqrt(-c^2*x^2 + 1)*a*b^2*c*d*x^2*arccos(c*x) + b^3*d*x*arccos(c*x)^3 +
 1/3*sqrt(-c^2*x^2 + 1)*a^2*b*c*d*x^2 - 2/27*sqrt(-c^2*x^2 + 1)*b^3*c*d*x^
2 + 3*a*b^2*d*x*arccos(c*x)^2 + 3*a^2*b*d*x*arccos(c*x) - 14/3*b^3*d*x*arc
cos(c*x) - 7/3*sqrt(-c^2*x^2 + 1)*b^3*d*arccos(c*x)^2/c + a^3*d*x - 14/3*a
*b^2*d*x - 14/3*sqrt(-c^2*x^2 + 1)*a*b^2*d*arccos(c*x)/c - 7/3*sqrt(-c^2*x
^2 + 1)*a^2*b*d/c + 122/27*sqrt(-c^2*x^2 + 1)*b^3*d/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^3\,dx=\int (a+b\,\mathrm{acos}(c\,x))^3\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int((a + b*acos(c*x))^3*(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))^3*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^3\,dx$$
$$=\frac{d\left(3acos(cx)^3\,b^3cx-9\sqrt{-c^2x^2+1}\,acos(cx)^2\,b^3+9acos(cx)^2\,a\,b^2cx-18\sqrt{-c^2x^2+1}\,acos(cx)\,a\,b^2-3ac\right.}{}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^3,x)
```

output
```
(d*(3*acos(c*x)**3*b**3*c*x - 9*sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**3 +
 9*acos(c*x)**2*a*b**2*c*x - 18*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b**2 -
3*acos(c*x)*a**2*b*c**3*x**3 + 9*acos(c*x)*a**2*b*c*x - 18*acos(c*x)*b**3*
c*x + sqrt( - c**2*x**2 + 1)*a**2*b*c**2*x**2 - 7*sqrt( - c**2*x**2 + 1)*a
**2*b + 18*sqrt( - c**2*x**2 + 1)*b**3 - 3*int(acos(c*x)**3*x**2,x)*b**3*c
**3 - 9*int(acos(c*x)**2*x**2,x)*a*b**2*c**3 - a**3*c**3*x**3 + 3*a**3*c*x
 - 18*a*b**2*c*x))/(3*c)
```

3.15 $\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 270

$$\begin{aligned}
\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx &= \frac{(2a+b\pi-b(\pi-2\arccos(cx)))^3\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{4cd} \\
&\quad - \frac{3ib(2a+b\pi-b(\pi-2\arccos(cx)))^2\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{4cd} \\
&\quad + \frac{3ib(2a+b\pi-b(\pi-2\arccos(cx)))^2\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{4cd} \\
&\quad + \frac{3b^2(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{cd} \\
&\quad - \frac{3b^2(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{cd} \\
&\quad + \frac{6ib^3\operatorname{PolyLog}\left(4,-e^{i\arccos(cx)}\right)}{cd} - \frac{6ib^3\operatorname{PolyLog}\left(4,e^{i\arccos(cx)}\right)}{cd}
\end{aligned}$$

output
```
1/4*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^3*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/c/
d-3/4*I*b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^2*polylog(2,-c*x-I*(-c^2*x^2+1)^
(1/2))/c/d+3/4*I*b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^2*polylog(2,c*x+I*(-c^2
*x^2+1)^(1/2))/c/d+3*b^2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*polylog(3,-c*x-I*
(-c^2*x^2+1)^(1/2))/c/d-3*b^2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*polylog(3,c*
x+I*(-c^2*x^2+1)^(1/2))/c/d+6*I*b^3*polylog(4,-c*x-I*(-c^2*x^2+1)^(1/2))/c
/d-6*I*b^3*polylog(4,c*x+I*(-c^2*x^2+1)^(1/2))/c/d
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.58

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx = \frac{ib^3\pi^4 - 2ib^3\arccos(cx)^4 - 8b^3\arccos(cx)^3\log\left(1-e^{-i\arccos(cx)}\right) - 24a^2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right) -$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/(d - c^2*d*x^2),x]
```

output
```
(I*b^3*Pi^4 - (2*I)*b^3*ArcCos[c*x]^4 - 8*b^3*ArcCos[c*x]^3*Log[1 - E^((-I
)*ArcCos[c*x])] - 24*a^2*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 24*a*b
^2*ArcCos[c*x]^2*Log[1 - E^(I*ArcCos[c*x])] + 24*a^2*b*ArcCos[c*x]*Log[1 +
 E^(I*ArcCos[c*x])] + 24*a*b^2*ArcCos[c*x]^2*Log[1 + E^(I*ArcCos[c*x])] +
8*b^3*ArcCos[c*x]^3*Log[1 + E^(I*ArcCos[c*x])] - 4*a^3*Log[1 - c*x] + 4*a^
3*Log[1 + c*x] - (24*I)*b^3*ArcCos[c*x]^2*PolyLog[2, E^((-I)*ArcCos[c*x])]
 - (24*I)*b*(a + b*ArcCos[c*x])^2*PolyLog[2, -E^(I*ArcCos[c*x])] + (24*I)*
a^2*b*PolyLog[2, E^(I*ArcCos[c*x])] + (48*I)*a*b^2*ArcCos[c*x]*PolyLog[2,
E^(I*ArcCos[c*x])] - 48*b^3*ArcCos[c*x]*PolyLog[3, E^((-I)*ArcCos[c*x])] +
 48*a*b^2*PolyLog[3, -E^(I*ArcCos[c*x])] + 48*b^3*ArcCos[c*x]*PolyLog[3, -
E^(I*ArcCos[c*x])] - 48*a*b^2*PolyLog[3, E^(I*ArcCos[c*x])] + (48*I)*b^3*P
olyLog[4, E^((-I)*ArcCos[c*x])] + (48*I)*b^3*PolyLog[4, -E^(I*ArcCos[c*x])
])/(8*c*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5165, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \text{ 5165}$$

$$-\frac{\int \frac{(a+b\arccos(cx))^3}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int (a+b\arccos(cx))^3\csc(\arccos(cx))d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{-3b\int (a+b\arccos(cx))^2\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+3b\int (a+b\arccos(cx))^2\log\left(1+e^{i\arccos(cx)}\right)d\,\mathrm{arcc}}{cd}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3b\left(i\,\mathrm{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int (a+b\arccos(cx))\,\mathrm{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(\right.}{}$$

$$\Big\downarrow \text{ 7163}$$

$$-\frac{3b\left(i\,\mathrm{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\left(ib\int \mathrm{PolyLog}\left(3,-e^{i\arccos(cx)}\right)d\arccos(cx)-i\,\mathrm{PolyLog}\right.\right.}{}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{3b\left(i\,\mathrm{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\left(b\int e^{-i\arccos(cx)}\,\mathrm{PolyLog}\left(3,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-\right.\right.}{}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^3+3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\left(b\,\text{PolyLo}\right.\right.}{}$$

input
```
Int[(a + b*ArcCos[c*x])^3/(d - c^2*d*x^2),x]
```

output
```
-((-2*(a + b*ArcCos[c*x])^3*ArcTanh[E^(I*ArcCos[c*x])] + 3*b*(I*(a + b*Arc
Cos[c*x])^2*PolyLog[2, -E^(I*ArcCos[c*x])] - (2*I)*b*((-I)*(a + b*ArcCos[c
*x])*PolyLog[3, -E^(I*ArcCos[c*x])] + b*PolyLog[4, -E^(I*ArcCos[c*x])])) -
 3*b*(I*(a + b*ArcCos[c*x])^2*PolyLog[2, E^(I*ArcCos[c*x])] - (2*I)*b*((-I
)*(a + b*ArcCos[c*x])*PolyLog[3, E^(I*ArcCos[c*x])] + b*PolyLog[4, E^(I*Ar
cCos[c*x])])))/(c*d))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.03

method	result
derivativedivides	$\frac{a^3 \operatorname{arctanh}(cx)}{d}-\frac{b^3\left(\arccos(cx)^3 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\arccos(cx)^3 \ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)+6 \arccos(cx) \operatorname{polylog}\left(3,cx+i\sqrt{-c^2\right.\right.}{}$
default	$\frac{a^3 \operatorname{arctanh}(cx)}{d}-\frac{b^3\left(\arccos(cx)^3 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\arccos(cx)^3 \ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)+6 \arccos(cx) \operatorname{polylog}\left(3,cx+i\sqrt{-c^2\right.\right.}{}$
parts	$-\frac{a^3 \ln(cx-1)}{2dc}+\frac{a^3 \ln(cx+1)}{2dc}-\frac{b^3\left(\arccos(cx)^3 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\arccos(cx)^3 \ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)+6\right.}{}$

input
```
int((a+b*arccos(c*x))^3/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^3/d*arctanh(c*x)-b^3/d*(arccos(c*x)^3*ln(1-c*x-I*(-c^2*x^2+1)^(1/2)
)-arccos(c*x)^3*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+6*arccos(c*x)*polylog(3,c*x
+I*(-c^2*x^2+1)^(1/2))-6*arccos(c*x)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))-
3*I*arccos(c*x)^2*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3*I*arccos(c*x)^2*po
lylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+6*I*polylog(4,c*x+I*(-c^2*x^2+1)^(1/2))
-6*I*polylog(4,-c*x-I*(-c^2*x^2+1)^(1/2)))-3*a*b^2/d*(arccos(c*x)^2*ln(1-c
*x-I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2
))+2*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))-arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^
2+1)^(1/2))+2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*polylog
(3,-c*x-I*(-c^2*x^2+1)^(1/2)))-3*a^2*b/d*(-arctanh(c*x)*arccos(c*x)-I*arct
anh(c*x)*(ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-ln(1+I*(c*x+1)/(-c^2*x^2+1)^(
1/2)))+I*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-I*dilog(1-I*(c*x+1)/(-c^2*x
^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos{(cx)}+a)^3}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
 + a^3)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^3}{c^2x^2-1}\,dx + \int \frac{b^3\operatorname{acos}^3{(cx)}}{c^2x^2-1}\,dx + \int \frac{3ab^2\operatorname{acos}^2{(cx)}}{c^2x^2-1}\,dx + \int \frac{3a^2b\operatorname{acos}{(cx)}}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**3/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**3/(c**2*x**2 - 1), x) + Integral(b**3*acos(c*x)**3/(c**2*x**
2 - 1), x) + Integral(3*a*b**2*acos(c*x)**2/(c**2*x**2 - 1), x) + Integral
(3*a**2*b*acos(c*x)/(c**2*x**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos{(cx)}+a)^3}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) + 1/2*((b^3*log(c*x + 1)
 - b^3*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 - 2*c*d
*integrate(3/2*(2*a*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a
^2*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + (b^3*log(c*x + 1) - b^3*
log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x)^2)/(c^2*d*x^2 - d), x))/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^3}{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^3/(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))^3/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^3}{d-c^2dx^2}\,dx = \frac{-6\left(\int \frac{acos(cx)}{c^2x^2-1}dx\right)a^2bc-2\left(\int \frac{acos(cx)^3}{c^2x^2-1}dx\right)b^3c-6\left(\int \frac{acos(cx)^2}{c^2x^2-1}dx\right)a\,b^2c-\log(c^2x-c)\,a^3+\log(c^2x+c)\,a^3}{2cd}$$

input
```
int((a+b*acos(c*x))^3/(-c^2*d*x^2+d),x)
```

output
```
( - 6*int(acos(c*x)/(c**2*x**2 - 1),x)*a**2*b*c - 2*int(acos(c*x)**3/(c**2
*x**2 - 1),x)*b**3*c - 6*int(acos(c*x)**2/(c**2*x**2 - 1),x)*a*b**2*c - lo
g(c**2*x - c)*a**3 + log(c**2*x + c)*a**3)/(2*c*d)
```

3.16 $\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 431

$$
\begin{aligned}
\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx
&= \frac{3b(a+b\arccos(cx))^2}{2cd^2\sqrt{1-c^2x^2}} + \frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)} \\
&\quad + \frac{3b^2(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{cd^2} \\
&\quad + \frac{(2a+b\pi-b(\pi-2\arccos(cx)))^3\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{8cd^2} \\
&\quad - \frac{3ib^3\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{cd^2} \\
&\quad - \frac{3ib(2a+b\pi-b(\pi-2\arccos(cx)))^2\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{8cd^2} \\
&\quad + \frac{3ib^3\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{cd^2} \\
&\quad + \frac{3ib(2a+b\pi-b(\pi-2\arccos(cx)))^2\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{8cd^2} \\
&\quad + \frac{3b^2(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{2cd^2} \\
&\quad - \frac{3b^2(2a+b\pi-b(\pi-2\arccos(cx)))\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{2cd^2} \\
&\quad + \frac{3ib^3\operatorname{PolyLog}\left(4,-e^{i\arccos(cx)}\right)}{cd^2} - \frac{3ib^3\operatorname{PolyLog}\left(4,e^{i\arccos(cx)}\right)}{cd^2}
\end{aligned}
$$

output
```
3/2*b*(a+b*arccos(c*x))^2/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))
^3/d^2/(-c^2*x^2+1)+3*b^2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*arctanh(c*x+I*(-
c^2*x^2+1)^(1/2))/c/d^2+1/8*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^3*arctanh(c*x+
I*(-c^2*x^2+1)^(1/2))/c/d^2-3*I*b^3*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/c
/d^2-3/8*I*b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^2*polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))/c/d^2+3*I*b^3*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/c/d^2+3/8*I*b*
(2*a+b*Pi-b*(Pi-2*arccos(c*x)))^2*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/c/d^
2+3/2*b^2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*polylog(3,-c*x-I*(-c^2*x^2+1)^(1
/2))/c/d^2-3/2*b^2*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*polylog(3,c*x+I*(-c^2*x
^2+1)^(1/2))/c/d^2+3*I*b^3*polylog(4,-c*x-I*(-c^2*x^2+1)^(1/2))/c/d^2-3*I*
b^3*polylog(4,c*x+I*(-c^2*x^2+1)^(1/2))/c/d^2
```

Mathematica [A] (verified)

Time = 7.68 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^3/(d - c^2*d*x^2)^2,x]
```

output
```
-1/2*(a^3*x)/(d^2*(-1 + c^2*x^2)) - (a^3*Log[1 - c*x])/(4*c*d^2) + (a^3*Lo
g[1 + c*x])/(4*c*d^2) + (3*a^2*b*((Sqrt[1 - c^2*x^2] - ArcCos[c*x])/(4*(1
+ c*x)) + (Sqrt[1 - c^2*x^2] + ArcCos[c*x])/(4*(1 - c*x)) + ((-1/2*I)*ArcC
os[c*x]^2 + 2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] - (2*I)*PolyLog[2, -E
^(I*ArcCos[c*x])])/4 + (-2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + (2*I)*
(ArcCos[c*x]^2/4 + PolyLog[2, E^(I*ArcCos[c*x])]))/4))/(c*d^2) + (3*a*b^2*
(4*ArcCos[c*x]*Cot[ArcCos[c*x]/2] + ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^2 - 4
*ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c*x])]) -
 8*Log[Tan[ArcCos[c*x]/2]] - (8*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*
x])] - PolyLog[2, E^(I*ArcCos[c*x])]) - 8*(-PolyLog[3, -E^(I*ArcCos[c*x])]
 + PolyLog[3, E^(I*ArcCos[c*x])]) - ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^2 + 4
*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/(8*c*d^2) + (b^3*(I*Pi^4 - (2*I)*ArcCos[
c*x]^4 + 12*ArcCos[c*x]^2*Cot[ArcCos[c*x]/2] + 2*ArcCos[c*x]^3*Csc[ArcCos[
c*x]/2]^2 - 8*ArcCos[c*x]^3*Log[1 - E^((-I)*ArcCos[c*x])] - 48*ArcCos[c*x]
*Log[1 - E^(I*ArcCos[c*x])] + 48*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] +
8*ArcCos[c*x]^3*Log[1 + E^(I*ArcCos[c*x])] - (24*I)*ArcCos[c*x]^2*PolyLog[
2, E^((-I)*ArcCos[c*x])] - (24*I)*(2 + ArcCos[c*x]^2)*PolyLog[2, -E^(I*Arc
Cos[c*x])] + (48*I)*PolyLog[2, E^(I*ArcCos[c*x])] - 48*ArcCos[c*x]*PolyLog
[3, E^((-I)*ArcCos[c*x])] + 48*ArcCos[c*x]*PolyLog[3, -E^(I*ArcCos[c*x])]
+ (48*I)*PolyLog[4, E^((-I)*ArcCos[c*x])] + (48*I)*PolyLog[4, -E^(I*Arc...
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.74, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5163, 27, 5165, 3042, 4671, 3011, 5183, 5165, 3042, 4671, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{ 5163}$$

$$\frac{3bc\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{\int \frac{(a+b\arccos(cx))^3}{d(1-c^2x^2)}dx}{2d}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3bc\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{\int \frac{(a+b\arccos(cx))^3}{1-c^2x^2}dx}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 5165$$

$$\frac{3bc\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{2d^2}-\frac{\int \frac{(a+b\arccos(cx))^3}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 3042$$

$$\frac{3bc\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{2d^2}-\frac{\int (a+b\arccos(cx))^3\csc(\arccos(cx))d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 4671$$

$$-\frac{-3b\int (a+b\arccos(cx))^2\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+3b\int (a+b\arccos(cx))^2\log\left(1+e^{i\arccos(cx)}\right)d\arcc}{2cd^2}$$
$$\frac{3bc\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 3011$$

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int (a+b\arccos(cx))\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(\right.}{}$$
$$\frac{3bc\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 5183$$

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int (a+b\arccos(cx))\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(\right.}{}$$
$$\frac{3bc\left(\frac{2b\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c}+\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 5165$$

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(\right.}{}$$

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^2}\right)}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$\downarrow$ 3042

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(\right.}{}$$

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^2}\right)}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$\downarrow$ 4671

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos\right.}{c^2}\right.}{2d^2}$$

$$\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx\right.}{}$$

$$\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$\downarrow$ 2715

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arcta}\right.}{c^2}\right.}{2d^2}$$

$$\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx\right.}{}$$

$$\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$\downarrow$ 2838

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\int(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(\right.}{}$$

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2}\right)}{2d^2}+$$

$$\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\left(ib\int \text{PolyLog}\left(3,-e^{i\arccos(cx)}\right)d\arccos(cx)-i\,\text{PolyLog}\right.\right.}{}$$

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2}\right)}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\left(b\int e^{-i\arccos(cx)}\text{PolyLog}\left(3,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-\right.\right.}{}$$

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2}\right)}{2d^2}+\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3bc\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2}\right)}{2d^2}-$$

$$\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^3+3b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2ib\left(b\,\text{PolyLog}\right.\right.}{}$$

$$\frac{x(a+b\arccos(cx))^3}{2d^2(1-c^2x^2)}$$

input
```
Int[(a + b*ArcCos[c*x])^3/(d - c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcCos[c*x])^3)/(2*d^2*(1 - c^2*x^2)) + (3*b*c*((a + b*ArcCos[c*
x])^2/(c^2*Sqrt[1 - c^2*x^2]) - (2*b*(-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*
ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*A
rcCos[c*x])]))/c^2))/(2*d^2) - (-2*(a + b*ArcCos[c*x])^3*ArcTanh[E^(I*ArcC
os[c*x])] + 3*b*(I*(a + b*ArcCos[c*x])^2*PolyLog[2, -E^(I*ArcCos[c*x])] -
(2*I)*b*((-I)*(a + b*ArcCos[c*x])*PolyLog[3, -E^(I*ArcCos[c*x])] + b*PolyL
og[4, -E^(I*ArcCos[c*x])])) - 3*b*(I*(a + b*ArcCos[c*x])^2*PolyLog[2, E^(I
*ArcCos[c*x])] - (2*I)*b*((-I)*(a + b*ArcCos[c*x])*PolyLog[3, E^(I*ArcCos[
c*x])] + b*PolyLog[4, E^(I*ArcCos[c*x])])))/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 804, normalized size of antiderivative = 1.87

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^3\left(-\frac{\arccos(cx)^2\left(cx\arccos(cx)+3\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^3\ln\left(1-cx-i\sqrt{-\cdots}\right.}{2}\cdots\right.}{\cdots}$
default	$\frac{a^3\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^3\left(-\frac{\arccos(cx)^2\left(cx\arccos(cx)+3\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^3\ln\left(1-cx-i\sqrt{-\cdots}\right.}{2}\cdots\right.}{\cdots}$
parts	$\frac{a^3\left(-\frac{1}{4c(cx-1)}-\frac{\ln(cx-1)}{4c}-\frac{1}{4c(cx+1)}+\frac{\ln(cx+1)}{4c}\right)}{d^2}+\frac{b^3\left(-\frac{\arccos(cx)^2\left(cx\arccos(cx)+3\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^3\ln\left(1-cx\cdots\right.}{2}\cdots\right.}{\cdots}$

input
```
int((a+b*arccos(c*x))^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^3/d^2*(-1/4/(c*x-1)-1/4*ln(c*x-1)-1/4/(c*x+1)+1/4*ln(c*x+1))+b^3/d^
2*(-1/2/(c^2*x^2-1)*arccos(c*x)^2*(c*x*arccos(c*x)+3*(-c^2*x^2+1)^(1/2))-1
/2*arccos(c*x)^3*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+3/2*I*polylog(2,c*x+I*(-c^
2*x^2+1)^(1/2))*arccos(c*x)^2-3*arccos(c*x)*polylog(3,c*x+I*(-c^2*x^2+1)^(
1/2))-3*I*polylog(4,c*x+I*(-c^2*x^2+1)^(1/2))+1/2*arccos(c*x)^3*ln(1+c*x+I
*(-c^2*x^2+1)^(1/2))-3/2*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))*arccos(c*x
)^2+3*arccos(c*x)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+3*I*polylog(4,-c*x-
I*(-c^2*x^2+1)^(1/2))-3*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+3*I*pol
ylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/
2))-3*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)))+3*a*b^2/d^2*(-1/2/(c^2*x^2-1
)*arccos(c*x)*(c*x*arccos(c*x)+2*(-c^2*x^2+1)^(1/2))-1/2*arccos(c*x)^2*ln(
1-c*x-I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/
2))-polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+1/2*arccos(c*x)^2*ln(1+c*x+I*(-c^2
*x^2+1)^(1/2))-I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(
3,-c*x-I*(-c^2*x^2+1)^(1/2))+2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2)))+3*a^2*b/
d^2*(-1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)-1/2*arccos(c*x)
*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+1/2*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+
1/2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/2*I*polylog(2,-c*x-I*(-c^
2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos{(cx)}+a)^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
+ a^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a^3}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b^3\operatorname{acos}^3(cx)}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{3ab^2\operatorname{acos}^2(cx)}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{3a^2b\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**3/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**3*acos(c*x)
**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(3*a*b**2*acos(c*x)**2/(c*
*4*x**4 - 2*c**2*x**2 + 1), x) + Integral(3*a**2*b*acos(c*x)/(c**4*x**4 -
2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a^3*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c
*d^2)) - 1/4*((2*b^3*c*x - (b^3*c^2*x^2 - b^3)*log(c*x + 1) + (b^3*c^2*x^2
 - b^3)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 4*(c
^3*d^2*x^2 - c*d^2)*integrate(-3/4*(4*a*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x)^2 + 4*a^2*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + (2*b
^3*c*x - (b^3*c^2*x^2 - b^3)*log(c*x + 1) + (b^3*c^2*x^2 - b^3)*log(-c*x +
 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x))/(c^3*d^2*x^2 - c*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^3}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))^3/(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^3/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^3}{(d-c^2dx^2)^2}\,dx = \frac{12\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)a^2b\,c^3x^2-12\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)a^2bc+4\left(\int \frac{acos(cx)^3}{c^4x^4-2c^2x^2+1}dx\right)b^3c^3x^2-4\left(\int \frac{acos(cx}{c^4x^4-2c^2}\right.}{}$$

input
```
int((a+b*acos(c*x))^3/(-c^2*d*x^2+d)^2,x)
```

output

```
(12*int(acos(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a**2*b*c**3*x**2 - 12*i
nt(acos(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a**2*b*c + 4*int(acos(c*x)**
3/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**3*c**3*x**2 - 4*int(acos(c*x)**3/(c*
*4*x**4 - 2*c**2*x**2 + 1),x)*b**3*c + 12*int(acos(c*x)**2/(c**4*x**4 - 2*
c**2*x**2 + 1),x)*a*b**2*c**3*x**2 - 12*int(acos(c*x)**2/(c**4*x**4 - 2*c*
*2*x**2 + 1),x)*a*b**2*c - log(c**2*x - c)*a**3*c**2*x**2 + log(c**2*x - c
)*a**3 + log(c**2*x + c)*a**3*c**2*x**2 - log(c**2*x + c)*a**3 - 2*a**3*c*
x)/(4*c*d**2*(c**2*x**2 - 1))
```

3.17 $\int \frac{(c-a^2cx^2)^3}{\arccos(ax)} dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)} dx = -\frac{35c^3\text{Si}(\arccos(ax))}{64a} + \frac{21c^3\text{Si}(3\arccos(ax))}{64a} - \frac{7c^3\text{Si}(5\arccos(ax))}{64a} + \frac{c^3\text{Si}(7\arccos(ax))}{64a}$$

output
```
-35/64*c^3*Si(arccos(a*x))/a+21/64*c^3*Si(3*arccos(a*x))/a-7/64*c^3*Si(5*a
rccos(a*x))/a+1/64*c^3*Si(7*arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)} dx = \frac{c^3(-35\text{Si}(\arccos(ax)) + 21\text{Si}(3\arccos(ax)) - 7\text{Si}(5\arccos(ax)) + \text{Si}(7\arccos(ax)))}{64a}$$

input
```
Integrate[(c - a^2*c*x^2)^3/ArcCos[a*x],x]
```

output
```
(c^3*(-35*SinIntegral[ArcCos[a*x]] + 21*SinIntegral[3*ArcCos[a*x]] - 7*Sin
Integral[5*ArcCos[a*x]] + SinIntegral[7*ArcCos[a*x]]))/(64*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{c^3\int \frac{(1-a^2x^2)^{7/2}}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^3\int \frac{\sin(\arccos(ax))^7}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{c^3\int \left(-\frac{21\sin(3\arccos(ax))}{64\arccos(ax)}+\frac{7\sin(5\arccos(ax))}{64\arccos(ax)}-\frac{\sin(7\arccos(ax))}{64\arccos(ax)}+\frac{35\sqrt{1-a^2x^2}}{64\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\left(\frac{35}{64}\text{Si}(\arccos(ax))-\frac{21}{64}\text{Si}(3\arccos(ax))+\frac{7}{64}\text{Si}(5\arccos(ax))-\frac{1}{64}\text{Si}(7\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)^3/ArcCos[a*x],x]
```

output
```
-((c^3*((35*SinIntegral[ArcCos[a*x]])/64 - (21*SinIntegral[3*ArcCos[a*x]])
/64 + (7*SinIntegral[5*ArcCos[a*x]])/64 - SinIntegral[7*ArcCos[a*x]]/64))/
a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{c^3(21\ \mathrm{Si}(3\arccos(ax))-7\ \mathrm{Si}(5\arccos(ax))+\mathrm{Si}(7\arccos(ax))-35\ \mathrm{Si}(\arccos(ax)))}{64a}$	42
default	$\frac{c^3(21\ \mathrm{Si}(3\arccos(ax))-7\ \mathrm{Si}(5\arccos(ax))+\mathrm{Si}(7\arccos(ax))-35\ \mathrm{Si}(\arccos(ax)))}{64a}$	42

input
```
int((-a^2*c*x^2+c)^3/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(21*Si(3*arccos(a*x))-7*Si(5*arccos(a*x))+Si(7*arccos(a*x))-35*
Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccos(a*x),
 x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = -c^3\left(\int \frac{3a^2x^2}{\operatorname{acos}(ax)}\,dx + \int \left(-\frac{3a^4x^4}{\operatorname{acos}(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acos}(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acos}(ax)}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**3/acos(a*x),x)
```

output
```
-c**3*(Integral(3*a**2*x**2/acos(a*x), x) + Integral(-3*a**4*x**4/acos(a*x
), x) + Integral(a**6*x**6/acos(a*x), x) + Integral(-1/acos(a*x), x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*c*x^2 - c)^3/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \frac{c^3\,\mathrm{Si}\,(7\arccos(ax))}{64\,a} - \frac{7\,c^3\,\mathrm{Si}\,(5\arccos(ax))}{64\,a} + \frac{21\,c^3\,\mathrm{Si}\,(3\arccos(ax))}{64\,a} - \frac{35\,c^3\,\mathrm{Si}\,(\arccos(ax))}{64\,a}$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x),x, algorithm="giac")
```

output
```
1/64*c^3*sin_integral(7*arccos(a*x))/a - 7/64*c^3*sin_integral(5*arccos(a*
x))/a + 21/64*c^3*sin_integral(3*arccos(a*x))/a - 35/64*c^3*sin_integral(a
rccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \int \frac{(c-a^2\,c\,x^2)^3}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int((c - a^2*c*x^2)^3/acos(a*x),x)
```

output
```
int((c - a^2*c*x^2)^3/acos(a*x), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = c^3\left(-\left(\int \frac{x^6}{acos\,(ax)}dx\right)a^6 + 3\left(\int \frac{x^4}{acos\,(ax)}dx\right)a^4 - 3\left(\int \frac{x^2}{acos\,(ax)}dx\right)a^2 + \int \frac{1}{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^3/acos(a*x),x)
```

output
```
c**3*( - int(x**6/acos(a*x),x)*a**6 + 3*int(x**4/acos(a*x),x)*a**4 - 3*int
(x**2/acos(a*x),x)*a**2 + int(1/acos(a*x),x))
```

3.18 $\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = -\frac{5c^2\text{Si}(\arccos(ax))}{8a}+\frac{5c^2\text{Si}(3\arccos(ax))}{16a}-\frac{c^2\text{Si}(5\arccos(ax))}{16a}$$

output
```
-5/8*c^2*Si(arccos(a*x))/a+5/16*c^2*Si(3*arccos(a*x))/a-1/16*c^2*Si(5*arcc
os(a*x))/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = -\frac{c^2(10\text{Si}(\arccos(ax))-5\text{Si}(3\arccos(ax))+\text{Si}(5\arccos(ax)))}{16a}$$

input
```
Integrate[(c - a^2*c*x^2)^2/ArcCos[a*x],x]
```

output
```
-1/16*(c^2*(10*SinIntegral[ArcCos[a*x]] - 5*SinIntegral[3*ArcCos[a*x]] + S
inIntegral[5*ArcCos[a*x]]))/a
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{c^2\int \frac{(1-a^2x^2)^{5/2}}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2\int \frac{\sin(\arccos(ax))^5}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{c^2\int \left(-\frac{5\sin(3\arccos(ax))}{16\arccos(ax)}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}+\frac{5\sqrt{1-a^2x^2}}{8\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\left(\frac{5}{8}\text{Si}(\arccos(ax))-\frac{5}{16}\text{Si}(3\arccos(ax))+\frac{1}{16}\text{Si}(5\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)^2/ArcCos[a*x],x]
```

output
```
-((c^2*((5*SinIntegral[ArcCos[a*x]])/8 - (5*SinIntegral[3*ArcCos[a*x]])/16
 + SinIntegral[5*ArcCos[a*x]]/16))/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{c^2(5\,\mathrm{Si}(3\arccos(ax))-\mathrm{Si}(5\arccos(ax))-10\,\mathrm{Si}(\arccos(ax)))}{16a}$	35
default	$\frac{c^2(5\,\mathrm{Si}(3\arccos(ax))-\mathrm{Si}(5\arccos(ax))-10\,\mathrm{Si}(\arccos(ax)))}{16a}$	35

input
```
int((-a^2*c*x^2+c)^2/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(5*Si(3*arccos(a*x))-Si(5*arccos(a*x))-10*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx=\int \frac{(a^2cx^2-c)^2}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx=c^2\left(\int \left(-\frac{2a^2x^2}{\operatorname{acos}(ax)}\right)dx+\int \frac{a^4x^4}{\operatorname{acos}(ax)}\,dx+\int \frac{1}{\operatorname{acos}(ax)}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**2/acos(a*x),x)
```

output
```
c**2*(Integral(-2*a**2*x**2/acos(a*x), x) + Integral(a**4*x**4/acos(a*x),
x) + Integral(1/acos(a*x), x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx=\int \frac{(a^2cx^2-c)^2}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 - c)^2/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = -\frac{c^2\operatorname{Si}(5\arccos(ax))}{16a} + \frac{5c^2\operatorname{Si}(3\arccos(ax))}{16a} - \frac{5c^2\operatorname{Si}(\arccos(ax))}{8a}$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="giac")
```

output
```
-1/16*c^2*sin_integral(5*arccos(a*x))/a + 5/16*c^2*sin_integral(3*arccos(a
*x))/a - 5/8*c^2*sin_integral(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = \int \frac{(c-a^2\,c\,x^2)^2}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int((c - a^2*c*x^2)^2/acos(a*x),x)
```

output
```
int((c - a^2*c*x^2)^2/acos(a*x), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = c^2\left(\left(\int \frac{x^4}{acos\,(ax)}dx\right)a^4 - 2\left(\int \frac{x^2}{acos\,(ax)}dx\right)a^2 + \int \frac{1}{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^2/acos(a*x),x)
```

output
```
c**2*(int(x**4/acos(a*x),x)*a**4 - 2*int(x**2/acos(a*x),x)*a**2 + int(1/ac
os(a*x),x))
```

3.19 $\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=-\frac{3c\text{Si}(\arccos(ax))}{4a}+\frac{c\text{Si}(3\arccos(ax))}{4a}$$

output
```
-3/4*c*Si(arccos(a*x))/a+1/4*c*Si(3*arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=\frac{c(-3\text{Si}(\arccos(ax))+\text{Si}(3\arccos(ax)))}{4a}$$

input
```
Integrate[(c - a^2*c*x^2)/ArcCos[a*x],x]
```

output
```
(c*(-3*SinIntegral[ArcCos[a*x]] + SinIntegral[3*ArcCos[a*x]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-a^2cx^2}{\arccos(ax)}dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{c\int \frac{(1-a^2x^2)^{3/2}}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c\int \frac{\sin(\arccos(ax))^3}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{c\int \left(\frac{3\sqrt{1-a^2x^2}}{4\arccos(ax)}-\frac{\sin(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\left(\frac{3}{4}\text{Si}(\arccos(ax))-\frac{1}{4}\text{Si}(3\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)/ArcCos[a*x],x]
```

output
```
-((c*((3*SinIntegral[ArcCos[a*x]])/4 - SinIntegral[3*ArcCos[a*x]]/4))/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{c(\mathrm{Si}(3\arccos(ax))-3\ \mathrm{Si}(\arccos(ax)))}{4a}$	22
default	$\frac{c(\mathrm{Si}(3\arccos(ax))-3\ \mathrm{Si}(\arccos(ax)))}{4a}$	22

input
```
int((-a^2*c*x^2+c)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(Si(3*arccos(a*x))-3*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=\int -\frac{a^2cx^2-c}{\arccos{(ax)}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=-c\left(\int \frac{a^2x^2}{\operatorname{acos}{(ax)}}\,dx+\int \left(-\frac{1}{\operatorname{acos}{(ax)}}\right)\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)/acos(a*x),x)
```

output
```
-c*(Integral(a**2*x**2/acos(a*x), x) + Integral(-1/acos(a*x), x))
```

Maxima [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=\int -\frac{a^2cx^2-c}{\arccos{(ax)}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*c*x^2 - c)/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = \frac{c\,\mathrm{Si}\,(3\,\arccos\,(ax))}{4\,a} - \frac{3\,c\,\mathrm{Si}\,(\arccos\,(ax))}{4\,a}$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x),x, algorithm="giac")
```

output
```
1/4*c*sin_integral(3*arccos(a*x))/a - 3/4*c*sin_integral(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = \int \frac{c-a^2\,c\,x^2}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int((c - a^2*c*x^2)/acos(a*x),x)
```

output
```
int((c - a^2*c*x^2)/acos(a*x), x)
```

Reduce [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = c\left(-\left(\int \frac{x^2}{acos\,(ax)}dx\right)a^2 + \int \frac{1}{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)/acos(a*x),x)
```

output
```
c*( - int(x**2/acos(a*x),x)*a**2 + int(1/acos(a*x),x))
```

3.20 $\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \text{Int}\left(\frac{1}{(c-a^2cx^2)\arccos(ax)},x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)}\,dx$$

$$\Big\downarrow \; 5175$$

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)}dx$$

input
```
Int[1/((c - a^2*c*x^2)*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right) \arccos \left(a x \right)}d x$$

input
```
int(1/(-a^2*c*x^2+c)/arccos(a*x),x)
```

output
```
int(1/(-a^2*c*x^2+c)/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx=\int -\frac{1}{(a^2cx^2-c)\arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-1/((a^2*c*x^2 - c)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx=-\frac{\int \frac{1}{a^2x^2\operatorname{acos}(ax)-\operatorname{acos}(ax)}\,dx}{c}$$

input
```
integrate(1/(-a**2*c*x**2+c)/acos(a*x),x)
```

output
```
-Integral(1/(a**2*x**2*acos(a*x) - acos(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx=\int -\frac{1}{(a^2cx^2-c)\arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x),x, algorithm="maxima")
```

output
```
-integrate(1/((a^2*c*x^2 - c)*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(-1/((a^2*c*x^2 - c)*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int \frac{1}{\mathrm{acos}(a\,x)\,\left(c-a^2\,c\,x^2\right)}\,dx$$

input
```
int(1/(acos(a*x)*(c - a^2*c*x^2)),x)
```

output
```
int(1/(acos(a*x)*(c - a^2*c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = -\frac{\int \frac{1}{acos(ax)a^2x^2-acos(ax)}dx}{c}$$

input
```
int(1/(-a^2*c*x^2+c)/acos(a*x),x)
```

output
```
( - int(1/(acos(a*x)*a**2*x**2 - acos(a*x)),x))/c
```

3.21 $\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx = \text{Int}\left(\frac{1}{(c-a^2cx^2)^2 \arccos(ax)}, x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)^2/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 7.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx = \int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)^2}dx$$

input
```
Int[1/((c - a^2*c*x^2)^2*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c\right)^{2} \arccos \left(a x\right)}d x$$

input
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x),x)
```

output
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{acos}(ax)-2a^2x^2 \operatorname{acos}(ax)+\operatorname{acos}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(-a**2*c*x**2+c)**2/acos(a*x),x)
```

output
```
Integral(1/(a**4*x**4*acos(a*x) - 2*a**2*x**2*acos(a*x) + acos(a*x)), x)/c
**2
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos(ax)} \, dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx = \int \frac{1}{\mathrm{acos}(a\,x)\,\left(c-a^2\,c\,x^2\right)^2} \, dx$$

input
```
int(1/(acos(a*x)*(c - a^2*c*x^2)^2),x)
```

output
```
int(1/(acos(a*x)*(c - a^2*c*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \frac{\int \frac{1}{acos(ax)a^4x^4-2acos(ax)a^2x^2+acos(ax)}dx}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^2/acos(a*x),x)
```

output
```
int(1/(acos(a*x)*a**4*x**4 - 2*acos(a*x)*a**2*x**2 + acos(a*x)),x)/c**2
```

3.22 $\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \frac{c^3(1-a^2x^2)^{7/2}}{a\arccos(ax)} - \frac{35c^3\,\mathrm{CosIntegral}(\arccos(ax))}{64a} + \frac{63c^3\,\mathrm{CosIntegral}(3\arccos(ax))}{64a} - \frac{35c^3\,\mathrm{CosIntegral}(5\arccos(ax))}{64a} + \frac{7c^3\,\mathrm{CosIntegral}(7\arccos(ax))}{64a}$$

output
```
c^3*(-a^2*x^2+1)^(7/2)/a/arccos(a*x)-35/64*c^3*Ci(arccos(a*x))/a+63/64*c^3
*Ci(3*arccos(a*x))/a-35/64*c^3*Ci(5*arccos(a*x))/a+7/64*c^3*Ci(7*arccos(a*
x))/a
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \frac{c^3\left(64(1-a^2x^2)^{7/2} - 35\arccos(ax)\,\mathrm{CosIntegral}(\arccos(ax)) + 63\arccos(ax)\,\mathrm{CosIntegral}(3\arccos(ax))\right.}{64a\arccos(ax)}$$

input
```
Integrate[(c - a^2*c*x^2)^3/ArcCos[a*x]^2,x]
```

output
```
(c^3*(64*(1 - a^2*x^2)^(7/2) - 35*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] + 6
3*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] - 35*ArcCos[a*x]*CosIntegral[5*Ar
cCos[a*x]] + 7*ArcCos[a*x]*CosIntegral[7*ArcCos[a*x]]))/(64*a*ArcCos[a*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c-a^2cx^2\right)^3}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{5167}$$

$$7ac^3\int \frac{x\left(1-a^2x^2\right)^{5/2}}{\arccos(ax)}dx+\frac{c^3\left(1-a^2x^2\right)^{7/2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{a\arccos(ax)}-\frac{7c^3\int \frac{ax\left(1-a^2x^2\right)^3}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{4906}$$

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{a\arccos(ax)}-\frac{7c^3\int \left(\frac{5ax}{64\arccos(ax)}-\frac{9\cos(3\arccos(ax))}{64\arccos(ax)}+\frac{5\cos(5\arccos(ax))}{64\arccos(ax)}-\frac{\cos(7\arccos(ax))}{64\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(1-a^2x^2\right)^{7/2}}{a\arccos(ax)}-\frac{7c^3\left(\frac{5}{64}\text{CosIntegral}(\arccos(ax))-\frac{9}{64}\text{CosIntegral}(3\arccos(ax))+\frac{5}{64}\text{CosIntegral}(5\arccos(ax))-\frac{1}{64}\text{CosIntegral}\right.}{a}$$

input
```
Int[(c - a^2*c*x^2)^3/ArcCos[a*x]^2,x]
```

output
```
(c^3*(1 - a^2*x^2)^(7/2))/(a*ArcCos[a*x]) - (7*c^3*((5*CosIntegral[ArcCos[
a*x]])/64 - (9*CosIntegral[3*ArcCos[a*x]])/64 + (5*CosIntegral[5*ArcCos[a*
x]])/64 - CosIntegral[7*ArcCos[a*x]]/64))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{c^3\left(63\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)+7\ \mathrm{Ci}(7\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(\arccos(ax)\right.}{64a\arccos(ax)}$
default	$\frac{c^3\left(63\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)+7\ \mathrm{Ci}(7\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(\arccos(ax)\right.}{64a\arccos(ax)}$

input
```
int((-a^2*c*x^2+c)^3/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(63*Ci(3*arccos(a*x))*arccos(a*x)-35*Ci(5*arccos(a*x))*arccos(a
*x)+7*Ci(7*arccos(a*x))*arccos(a*x)-35*Ci(arccos(a*x))*arccos(a*x)+35*(-a^
2*x^2+1)^(1/2)-21*sin(3*arccos(a*x))+7*sin(5*arccos(a*x))-sin(7*arccos(a*x
)))/arccos(a*x)
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccos(a*x)^
2, x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = -c^3\left(\int \frac{3a^2x^2}{\operatorname{acos}^2(ax)}\,dx + \int \left(-\frac{3a^4x^4}{\operatorname{acos}^2(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acos}^2(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acos}^2(ax)}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**3/acos(a*x)**2,x)
```

output
```
-c**3*(Integral(3*a**2*x**2/acos(a*x)**2, x) + Integral(-3*a**4*x**4/acos(
a*x)**2, x) + Integral(a**6*x**6/acos(a*x)**2, x) + Integral(-1/acos(a*x)*
*2, x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(7*(a^5*c^3*x^5 - 2
*a^3*c^3*x^3 + a*c^3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/arctan2(sqrt(a*x + 1)
*sqrt(-a*x + 1), a*x), x) - (a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 -
 c^3)*sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1
), a*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(84) = 168$.

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.80

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = -\frac{\sqrt{-a^2x^2+1}a^5c^3x^6}{\arccos(ax)} + \frac{3\sqrt{-a^2x^2+1}a^3c^3x^4}{\arccos(ax)} - \frac{3\sqrt{-a^2x^2+1}ac^3x^2}{\arccos(ax)} + \frac{7c^3\operatorname{Ci}(7\arccos(ax))}{64a} - \frac{35c^3\operatorname{Ci}(5\arccos(ax))}{64a} + \frac{63c^3\operatorname{Ci}(3\arccos(ax))}{64a} - \frac{35c^3\operatorname{Ci}(\arccos(ax))}{64a} + \frac{\sqrt{-a^2x^2+1}c^3}{a\arccos(ax)}$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x)^2,x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*a^5*c^3*x^6/arccos(a*x) + 3*sqrt(-a^2*x^2 + 1)*a^3*c^3
*x^4/arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*a*c^3*x^2/arccos(a*x) + 7/64*c^3*c
os_integral(7*arccos(a*x))/a - 35/64*c^3*cos_integral(5*arccos(a*x))/a + 6
3/64*c^3*cos_integral(3*arccos(a*x))/a - 35/64*c^3*cos_integral(arccos(a*x
))/a + sqrt(-a^2*x^2 + 1)*c^3/(a*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \int \frac{(c-a^2\,c\,x^2)^3}{\operatorname{acos}(a\,x)^2}\,dx$$

input
```
int((c - a^2*c*x^2)^3/acos(a*x)^2,x)
```

output
```
int((c - a^2*c*x^2)^3/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = c^3\left(-\left(\int \frac{x^6}{acos\,(ax)^2}dx\right)a^6+3\left(\int \frac{x^4}{acos\,(ax)^2}dx\right)a^4 - 3\left(\int \frac{x^2}{acos\,(ax)^2}dx\right)a^2+\int \frac{1}{acos\,(ax)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)^3/acos(a*x)^2,x)
```

output
```
c**3*( - int(x**6/acos(a*x)**2,x)*a**6 + 3*int(x**4/acos(a*x)**2,x)*a**4 -
 3*int(x**2/acos(a*x)**2,x)*a**2 + int(1/acos(a*x)**2,x))
```

3.23 $\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)} - \frac{5c^2\,\text{CosIntegral}(\arccos(ax))}{8a} + \frac{15c^2\,\text{CosIntegral}(3\arccos(ax))}{16a} - \frac{5c^2\,\text{CosIntegral}(5\arccos(ax))}{16a}$$

output
```
c^2*(-a^2*x^2+1)^(5/2)/a/arccos(a*x)-5/8*c^2*Ci(arccos(a*x))/a+15/16*c^2*C
i(3*arccos(a*x))/a-5/16*c^2*Ci(5*arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \frac{c^2\left(16(1-a^2x^2)^{5/2} - 10\arccos(ax)\,\text{CosIntegral}(\arccos(ax)) + 15\arccos(ax)\,\text{CosIntegral}(3\arccos(ax))\right.}{16a\arccos(ax)}$$

input
```
Integrate[(c - a^2*c*x^2)^2/ArcCos[a*x]^2,x]
```

output
```
(c^2*(16*(1 - a^2*x^2)^(5/2) - 10*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] + 1
5*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] - 5*ArcCos[a*x]*CosIntegral[5*Arc
Cos[a*x]]))/(16*a*ArcCos[a*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{5167}$$

$$5ac^2\int \frac{x(1-a^2x^2)^{3/2}}{\arccos(ax)}dx+\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}-\frac{5c^2\int \frac{ax(1-a^2x^2)^2}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{4906}$$

$$\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}-\frac{5c^2\int \left(\frac{ax}{8\arccos(ax)}-\frac{3\cos(3\arccos(ax))}{16\arccos(ax)}+\frac{\cos(5\arccos(ax))}{16\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}-\frac{5c^2\left(\frac{1}{8}\text{CosIntegral}(\arccos(ax))-\frac{3}{16}\text{CosIntegral}(3\arccos(ax))+\frac{1}{16}\text{CosIntegral}(5\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)^2/ArcCos[a*x]^2,x]
```

output
```
(c^2*(1 - a^2*x^2)^(5/2))/(a*ArcCos[a*x]) - (5*c^2*(CosIntegral[ArcCos[a*x
]]/8 - (3*CosIntegral[3*ArcCos[a*x]])/16 + CosIntegral[5*ArcCos[a*x]]/16))
/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{c^2\left(15\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-5\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)-10\ \mathrm{Ci}(\arccos(ax))\arccos(ax)+10\sqrt{-a^2x^2+1}-5\,\mathrm{s}\right.}{16a\arccos(ax)}$
default	$\frac{c^2\left(15\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-5\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)-10\ \mathrm{Ci}(\arccos(ax))\arccos(ax)+10\sqrt{-a^2x^2+1}-5\,\mathrm{s}\right.}{16a\arccos(ax)}$

input
```
int((-a^2*c*x^2+c)^2/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(15*Ci(3*arccos(a*x))*arccos(a*x)-5*Ci(5*arccos(a*x))*arccos(a*
x)-10*Ci(arccos(a*x))*arccos(a*x)+10*(-a^2*x^2+1)^(1/2)-5*sin(3*arccos(a*x
))+sin(5*arccos(a*x)))/arccos(a*x)
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx=\int \frac{(a^2cx^2-c)^2}{\arccos{(ax)}^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx=c^2\left(\int\left(-\frac{2a^2x^2}{\operatorname{acos}^2{(ax)}}\right)dx+\int\frac{a^4x^4}{\operatorname{acos}^2{(ax)}}\,dx+\int\frac{1}{\operatorname{acos}^2{(ax)}}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**2/acos(a*x)**2,x)
```

output
```
c**2*(Integral(-2*a**2*x**2/acos(a*x)**2, x) + Integral(a**4*x**4/acos(a*x
)**2, x) + Integral(acos(a*x)**(-2), x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \int \frac{(a^2cx^2-c)^2}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(5*(a^3*c^2*x^3 -
a*c^2*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x), x) - (a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*x +
 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.61

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}a^3c^2x^4}{\arccos(ax)} - \frac{2\sqrt{-a^2x^2+1}ac^2x^2}{\arccos(ax)} - \frac{5\,c^2\,\mathrm{Ci}\,(5\arccos(ax))}{16\,a} + \frac{15\,c^2\,\mathrm{Ci}\,(3\arccos(ax))}{16\,a} - \frac{5\,c^2\,\mathrm{Ci}\,(\arccos(ax))}{8\,a} + \frac{\sqrt{-a^2x^2+1}c^2}{a\arccos(ax)}$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*a^3*c^2*x^4/arccos(a*x) - 2*sqrt(-a^2*x^2 + 1)*a*c^2*x^
2/arccos(a*x) - 5/16*c^2*cos_integral(5*arccos(a*x))/a + 15/16*c^2*cos_int
egral(3*arccos(a*x))/a - 5/8*c^2*cos_integral(arccos(a*x))/a + sqrt(-a^2*x
^2 + 1)*c^2/(a*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx=\int \frac{{\left(c-a^2\,c\,x^2\right)}^2}{{\mathrm{acos}\left(a\,x\right)}^2}\,dx$$

input
```
int((c - a^2*c*x^2)^2/acos(a*x)^2,x)
```

output
```
int((c - a^2*c*x^2)^2/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx=c^2\left(\left(\int \frac{x^4}{acos\left(ax\right)^2}dx\right)a^4-2\left(\int \frac{x^2}{acos\left(ax\right)^2}dx\right)a^2+\int \frac{1}{acos\left(ax\right)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)^2/acos(a*x)^2,x)
```

output
```
c**2*(int(x**4/acos(a*x)**2,x)*a**4 - 2*int(x**2/acos(a*x)**2,x)*a**2 + in
t(1/acos(a*x)**2,x))
```

3.24 $\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)} - \frac{3c\,\text{CosIntegral}(\arccos(ax))}{4a} + \frac{3c\,\text{CosIntegral}(3\arccos(ax))}{4a}$$

output
```
c*(-a^2*x^2+1)^(3/2)/a/arccos(a*x)-3/4*c*Ci(arccos(a*x))/a+3/4*c*Ci(3*arcc
os(a*x))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \frac{c\left(4(1-a^2x^2)^{3/2} - 3\arccos(ax)\,\text{CosIntegral}(\arccos(ax)) + 3\arccos(ax)\,\text{CosIntegral}(3\arccos(ax))\right)}{4a\arccos(ax)}$$

input
```
Integrate[(c - a^2*c*x^2)/ArcCos[a*x]^2,x]
```

output
```
(c*(4*(1 - a^2*x^2)^(3/2) - 3*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] + 3*Arc
Cos[a*x]*CosIntegral[3*ArcCos[a*x]]))/(4*a*ArcCos[a*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{5167}$$

$$3ac\int \frac{x\sqrt{1-a^2x^2}}{\arccos(ax)}dx+\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}-\frac{3c\int \frac{ax(1-a^2x^2)}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{4906}$$

$$\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}-\frac{3c\int \left(\frac{ax}{4\arccos(ax)}-\frac{\cos(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}-\frac{3c\left(\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{1}{4}\text{CosIntegral}(3\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)/ArcCos[a*x]^2,x]
```

output
```
(c*(1 - a^2*x^2)^(3/2))/(a*ArcCos[a*x]) - (3*c*(CosIntegral[ArcCos[a*x]]/4
 - CosIntegral[3*ArcCos[a*x]]/4))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{c\left(3\,\mathrm{Ci}(3\arccos(ax))\arccos(ax)-3\,\mathrm{Ci}(\arccos(ax))\arccos(ax)+3\sqrt{-a^2x^2+1}-\sin(3\arccos(ax))\right)}{4a\arccos(ax)}$	61
default	$\frac{c\left(3\,\mathrm{Ci}(3\arccos(ax))\arccos(ax)-3\,\mathrm{Ci}(\arccos(ax))\arccos(ax)+3\sqrt{-a^2x^2+1}-\sin(3\arccos(ax))\right)}{4a\arccos(ax)}$	61

input
```
int((-a^2*c*x^2+c)/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(3*Ci(3*arccos(a*x))*arccos(a*x)-3*Ci(arccos(a*x))*arccos(a*x)+3*(
-a^2*x^2+1)^(1/2)-sin(3*arccos(a*x)))/arccos(a*x)
```

Fricas [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \int -\frac{a^2cx^2-c}{\arccos\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = -c\left(\int \frac{a^2x^2}{\operatorname{acos}^2\left(ax\right)}\,dx + \int \left(-\frac{1}{\operatorname{acos}^2\left(ax\right)}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)/acos(a*x)**2,x)
```

output
```
-c*(Integral(a**2*x**2/acos(a*x)**2, x) + Integral(-1/acos(a*x)**2, x))
```

Maxima [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \int -\frac{a^2cx^2-c}{\arccos\left(ax\right)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(3*a^2*c*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1
)*sqrt(-a*x + 1)*x/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - (a^2*c
*x^2 - c)*sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = -\frac{\sqrt{-a^2x^2+1}acx^2}{\arccos(ax)} + \frac{3\,c\,\mathrm{Ci}\,(3\,\arccos(ax))}{4\,a} - \frac{3\,c\,\mathrm{Ci}\,(\arccos(ax))}{4\,a} + \frac{\sqrt{-a^2x^2+1}c}{a\arccos(ax)}$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*a*c*x^2/arccos(a*x) + 3/4*c*cos_integral(3*arccos(a*x)
)/a - 3/4*c*cos_integral(arccos(a*x))/a + sqrt(-a^2*x^2 + 1)*c/(a*arccos(a
*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \int \frac{c-a^2\,c\,x^2}{\mathrm{acos}\,(a\,x)^2}\,dx$$

input
```
int((c - a^2*c*x^2)/acos(a*x)^2,x)
```

output
```
int((c - a^2*c*x^2)/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = c\left(-\left(\int \frac{x^2}{acos\,(ax)^2}dx\right)a^2+\int \frac{1}{acos\,(ax)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)/acos(a*x)^2,x)
```

output
```
c*( - int(x**2/acos(a*x)**2,x)*a**2 + int(1/acos(a*x)**2,x))
```

3.25 $\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \text{Int}\left(\frac{1}{(c-a^2cx^2)\arccos(ax)^2},x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]^2),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^2\left(c-a^2cx^2\right)}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{ac\sqrt{1-a^2x^2}\arccos(ax)}-\frac{a\int \frac{x}{(1-a^2x^2)^{3/2}\arccos(ax)}dx}{c}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{ac\sqrt{1-a^2x^2}\arccos(ax)}-\frac{a\int \frac{x}{(1-a^2x^2)^{3/2}\arccos(ax)}dx}{c}$$

input
```
Int[1/((c - a^2*c*x^2)*ArcCos[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right) \arccos \left(a x \right)^{2}}d x$$

input
```
int(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x)
```

output
```
int(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-1/((a^2*c*x^2 - c)*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = -\frac{\int \frac{1}{a^2x^2\operatorname{acos}^2{(ax)}-\operatorname{acos}^2{(ax)}}\,dx}{c}$$

input
```
integrate(1/(-a**2*c*x**2+c)/acos(a*x)**2,x)
```

output
```
-Integral(1/(a**2*x**2*acos(a*x)**2 - acos(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.70

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^4*c*x^2 - a^2*c)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x/((a^4*c*x^4 - 2*a^2*c*x^2 + c)*arctan2(sqr
t(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x + 1))/((a^3
*c*x^2 - a*c)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*c*x^2 - c)*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int \frac{1}{\operatorname{acos}(a\,x)^2\,\left(c-a^2\,c\,x^2\right)}\,dx$$

input
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)),x)
```

output
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = -\frac{\int \frac{1}{acos(ax)^2a^2x^2-acos(ax)^2}dx}{c}$$

input
```
int(1/(-a^2*c*x^2+c)/acos(a*x)^2,x)
```

output
```
( - int(1/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2),x))/c
```

3.26 $\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \text{Int}\left(\frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2}, x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]^2),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^2 (c-a^2cx^2)^2} \, dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{ac^2 (1-a^2x^2)^{3/2} \arccos(ax)}-\frac{3a\int \frac{x}{(1-a^2x^2)^{5/2}\arccos(ax)}dx}{c^2}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{ac^2 (1-a^2x^2)^{3/2} \arccos(ax)}-\frac{3a\int \frac{x}{(1-a^2x^2)^{5/2}\arccos(ax)}dx}{c^2}$$

input
```
Int[1/((c - a^2*c*x^2)^2*ArcCos[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^2 \arccos\left(ax\right)^2}dx$$

input
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x)
```

output
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos(ax)^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2}\,dx = \frac{\int \frac{1}{a^4x^4\operatorname{acos}^2(ax)-2a^2x^2\operatorname{acos}^2(ax)+\operatorname{acos}^2(ax)}\,dx}{c^2}$$

input
```
integrate(1/(-a**2*c*x**2+c)**2/acos(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**4*acos(a*x)**2 - 2*a**2*x**2*acos(a*x)**2 + acos(a*x)*
*2), x)/c**2
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 10.05

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(3*(a^6*c^2*x^4 - 2*a^4*c^2*x^2 + a^2*c^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x/((a^6*c^2*x^6 - 3*a^4
*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
), x) + sqrt(a*x + 1)*sqrt(-a*x + 1))/((a^5*c^2*x^4 - 2*a^3*c^2*x^2 + a*c^
2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \int \frac{1}{\mathrm{acos}\,(a\,x)^2\,(c-a^2\,c\,x^2)^2} \, dx$$

input
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)^2),x)
```

output
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \frac{\int \frac{1}{acos(ax)^2a^4x^4-2acos(ax)^2a^2x^2+acos(ax)^2}dx}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^2/acos(a*x)^2,x)
```

output
```
int(1/(acos(a*x)**2*a**4*x**4 - 2*acos(a*x)**2*a**2*x**2 + acos(a*x)**2),x
)/c**2
```

3.27 $\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 269

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx = {} & \frac{35d^3 \operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{64bc} \\ & - \frac{21d^3 \operatorname{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{64bc} \\ & + \frac{7d^3 \operatorname{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{64bc} \\ & - \frac{d^3 \operatorname{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)\sin\left(\frac{7a}{b}\right)}{64bc} \\ & - \frac{35d^3\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{64bc} \\ & + \frac{21d^3\cos\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64bc} \\ & - \frac{7d^3\cos\left(\frac{5a}{b}\right)\operatorname{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64bc} \\ & + \frac{d^3\cos\left(\frac{7a}{b}\right)\operatorname{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64bc}\end{aligned}$$

output
```
35/64*d^3*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c-21/64*d^3*Ci(3*(a+b*arccos(
c*x))/b)*sin(3*a/b)/b/c+7/64*d^3*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c-
1/64*d^3*Ci(7*(a+b*arccos(c*x))/b)*sin(7*a/b)/b/c-35/64*d^3*cos(a/b)*Si((a
+b*arccos(c*x))/b)/b/c+21/64*d^3*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b/c-
7/64*d^3*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c+1/64*d^3*cos(7*a/b)*Si(7
*(a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.68

$$\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx$$
$$= \frac{d^3\left(35\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-21\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)+7\,\text{CosIntegr}\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^3/(a + b*ArcCos[c*x]),x]
```

output
```
(d^3*(35*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] - 21*CosIntegral[3*(a/b +
 ArcCos[c*x])]*Sin[(3*a)/b] + 7*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*
a)/b] - CosIntegral[7*(a/b + ArcCos[c*x])]*Sin[(7*a)/b] - 35*Cos[a/b]*SinI
ntegral[a/b + ArcCos[c*x]] + 21*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c
*x])] - 7*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] + Cos[(7*a)/b]*S
inIntegral[7*(a/b + ArcCos[c*x])]))/(64*b*c)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5169, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^3}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \; 5169$$

$$-\frac{d^3\int -\frac{\sin^7\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \; 25$$

$$\frac{d^3\int \frac{\sin^7\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^3\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^7}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \; 3793$$

$$\frac{d^3\int \left(-\frac{\sin\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{7\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{21\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{35\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+}{bc}$$

$$\Big\downarrow \; 2009$$

$$-\frac{d^3\left(-\frac{35}{64}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{21}{64}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{7}{64}\sin\left(\frac{5a}{b}\right)\text{CosIntegra}\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^3/(a + b*ArcCos[c*x]),x]
```

output
```
-((d^3*((-35*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/64 + (21*CosInte
gral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/64 - (7*CosIntegral[(5*(a +
b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/64 + (CosIntegral[(7*(a + b*ArcCos[c*x]))
/b]*Sin[(7*a)/b])/64 + (35*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/64
 - (21*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/64 + (7*Cos[(5
*a)/b]*SinIntegral[(5*(a + b*ArcCos[c*x]))/b])/64 - (Cos[(7*a)/b]*SinInteg
ral[(7*(a + b*ArcCos[c*x]))/b])/64))/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{d^3\left(\mathrm{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos\left(\frac{7a}{b}\right)-\mathrm{Ci}\left(7\arccos(cx)+\frac{7a}{b}\right)\sin\left(\frac{7a}{b}\right)-7\ \mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)+7\ \mathrm{Ci}\left(5\arccos(cx)\right.\right.}{}$
default	$\frac{d^3\left(\mathrm{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos\left(\frac{7a}{b}\right)-\mathrm{Ci}\left(7\arccos(cx)+\frac{7a}{b}\right)\sin\left(\frac{7a}{b}\right)-7\ \mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)+7\ \mathrm{Ci}\left(5\arccos(cx)\right.\right.}{}$

input
```
int((-c^2*d*x^2+d)^3/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/64/c*d^3*(Si(7*arccos(c*x)+7*a/b)*cos(7*a/b)-Ci(7*arccos(c*x)+7*a/b)*sin
(7*a/b)-7*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)+7*Ci(5*arccos(c*x)+5*a/b)*sin
(5*a/b)+21*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)-21*Ci(3*arccos(c*x)+3*a/b)*s
in(3*a/b)-35*Si(arccos(c*x)+a/b)*cos(a/b)+35*Ci(arccos(c*x)+a/b)*sin(a/b))
/b
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx=\int -\frac{(c^2dx^2-d)^3}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)/(b*arccos(c*
x) + a), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx=-d^3\left(\int\frac{3c^2x^2}{a+b\,\mathrm{acos}\,(cx)}\,dx+\int\left(-\frac{3c^4x^4}{a+b\,\mathrm{acos}\,(cx)}\right)dx+\int\frac{c^6x^6}{a+b\,\mathrm{acos}\,(cx)}\,dx+\int\left(-\frac{1}{a+b\,\mathrm{acos}\,(cx)}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3/(a+b*acos(c*x)),x)
```

output
```
-d**3*(Integral(3*c**2*x**2/(a + b*acos(c*x)), x) + Integral(-3*c**4*x**4/
(a + b*acos(c*x)), x) + Integral(c**6*x**6/(a + b*acos(c*x)), x) + Integra
l(-1/(a + b*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)^3}{a + b\arccos(cx)}\,dx = \int -\frac{(c^2dx^2 - d)^3}{b\arccos(cx) + a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)^3/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(253) = 506$.

Time = 0.16 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.51

$$\int \frac{(d - c^2dx^2)^3}{a + b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-d^3*cos(a/b)^6*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b*c) + d^3*c
os(a/b)^7*sin_integral(7*a/b + 7*arccos(c*x))/(b*c) + 5/4*d^3*cos(a/b)^4*c
os_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b*c) + 7/4*d^3*cos(a/b)^4*cos
_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c) - 7/4*d^3*cos(a/b)^5*sin_i
ntegral(7*a/b + 7*arccos(c*x))/(b*c) - 7/4*d^3*cos(a/b)^5*sin_integral(5*a
/b + 5*arccos(c*x))/(b*c) - 3/8*d^3*cos(a/b)^2*cos_integral(7*a/b + 7*arcc
os(c*x))*sin(a/b)/(b*c) - 21/16*d^3*cos(a/b)^2*cos_integral(5*a/b + 5*arcc
os(c*x))*sin(a/b)/(b*c) - 21/16*d^3*cos(a/b)^2*cos_integral(3*a/b + 3*arcc
os(c*x))*sin(a/b)/(b*c) + 7/8*d^3*cos(a/b)^3*sin_integral(7*a/b + 7*arccos
(c*x))/(b*c) + 35/16*d^3*cos(a/b)^3*sin_integral(5*a/b + 5*arccos(c*x))/(b
*c) + 21/16*d^3*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b*c) + 1/6
4*d^3*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b*c) + 7/64*d^3*cos_in
tegral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c) + 21/64*d^3*cos_integral(3*a/
b + 3*arccos(c*x))*sin(a/b)/(b*c) + 35/64*d^3*cos_integral(a/b + arccos(c*
x))*sin(a/b)/(b*c) - 7/64*d^3*cos(a/b)*sin_integral(7*a/b + 7*arccos(c*x))
/(b*c) - 35/64*d^3*cos(a/b)*sin_integral(5*a/b + 5*arccos(c*x))/(b*c) - 63
/64*d^3*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c) - 35/64*d^3*cos
(a/b)*sin_integral(a/b + arccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx=\int \frac{(d-c^2\,d\,x^2)^3}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((d - c^2*d*x^2)^3/(a + b*acos(c*x)),x)
```

output
```
int((d - c^2*d*x^2)^3/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3}{a+b\arccos(cx)}\,dx = d^3\left(-\left(\int \frac{x^6}{acos\,(cx)\,b+a}dx\right)c^6+3\left(\int \frac{x^4}{acos\,(cx)\,b+a}dx\right)c^4 - 3\left(\int \frac{x^2}{acos\,(cx)\,b+a}dx\right)c^2+\int \frac{1}{acos\,(cx)\,b+a}dx\right)$$

input
```
int((-c^2*d*x^2+d)^3/(a+b*acos(c*x)),x)
```

output
```
d**3*( - int(x**6/(acos(c*x)*b + a),x)*c**6 + 3*int(x**4/(acos(c*x)*b + a)
,x)*c**4 - 3*int(x**2/(acos(c*x)*b + a),x)*c**2 + int(1/(acos(c*x)*b + a),
x))
```

3.28 $\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx = \frac{5d^2\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8bc} - \frac{5d^2\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16bc} + \frac{d^2\,\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16bc} - \frac{5d^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc} + \frac{5d^2\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc} - \frac{d^2\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc}$$

output
```
5/8*d^2*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c-5/16*d^2*Ci(3*(a+b*arccos(c*x
))/b)*sin(3*a/b)/b/c+1/16*d^2*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c-5/8
*d^2*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c+5/16*d^2*cos(3*a/b)*Si(3*(a+b*ar
ccos(c*x))/b)/b/c-1/16*d^2*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx = \frac{d^2\left(10\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-5\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)+\text{CosIntegral}\left(\right.\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(10*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] - 5*CosIntegral[3*(a/b +
ArcCos[c*x])]*Sin[(3*a)/b] + CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*a)/
b] - 10*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 5*Cos[(3*a)/b]*SinIntegr
al[3*(a/b + ArcCos[c*x])] - Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])
]))/(16*b*c)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5169, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{d^2\int -\frac{\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\int \frac{\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{d^2\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^5}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \textbf{3793}$$

$$\frac{d^2\int \left(\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{5\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{5\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{d^2\left(-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegra}\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^2/(a + b*ArcCos[c*x]),x]
```

output
```
-((d^2*((-5*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/8 + (5*CosIntegra
l[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/16 - (CosIntegral[(5*(a + b*Arc
Cos[c*x]))/b]*Sin[(5*a)/b])/16 + (5*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x
])/b])/8 - (5*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 + (C
os[(5*a)/b]*SinIntegral[(5*(a + b*ArcCos[c*x]))/b])/16))/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{d^{2}\left(\operatorname{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)-\operatorname{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)-5\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)+5\operatorname{Ci}(3\arccos(c\right.}{16cb}$
default	$-\frac{d^{2}\left(\operatorname{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)-\operatorname{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)-5\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)+5\operatorname{Ci}(3\arccos(c\right.}{16cb}$

input
```
int((-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16/c*d^2*(Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)-Ci(5*arccos(c*x)+5*a/b)*si
n(5*a/b)-5*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)+5*Ci(3*arccos(c*x)+3*a/b)*si
n(3*a/b)+10*Si(arccos(c*x)+a/b)*cos(a/b)-10*Ci(arccos(c*x)+a/b)*sin(a/b))/
b
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(c^2dx^2-d)^2}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx = d^2\left(\int \left(-\frac{2c^2x^2}{a+b\operatorname{acos}(cx)}\right)dx + \int \frac{c^4x^4}{a+b\operatorname{acos}(cx)}\,dx + \int \frac{1}{a+b\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2/(a+b*acos(c*x)),x)
```

output
```
d**2*(Integral(-2*c**2*x**2/(a + b*acos(c*x)), x) + Integral(c**4*x**4/(a
+ b*acos(c*x)), x) + Integral(1/(a + b*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(c^2dx^2-d)^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 - d)^2/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(189) = 378$.

Time = 0.16 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.97

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx = \frac{d^2\cos\left(\frac{a}{b}\right)^4\operatorname{Ci}\left(\frac{5a}{b}+5\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc} - \frac{d^2\cos\left(\frac{a}{b}\right)^5\operatorname{Si}\left(\frac{5a}{b}+5\arccos(cx)\right)}{bc} - \frac{3d^2\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{5a}{b}+5\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc} - \frac{5d^2\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc} + \frac{5d^2\cos\left(\frac{a}{b}\right)^3\operatorname{Si}\left(\frac{5a}{b}+5\arccos(cx)\right)}{4bc} + \frac{5d^2\cos\left(\frac{a}{b}\right)^3\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc} + \frac{d^2\operatorname{Ci}\left(\frac{5a}{b}+5\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{16bc} + \frac{5d^2\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{16bc} + \frac{5d^2\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{8bc} - \frac{5d^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{5a}{b}+5\arccos(cx)\right)}{16bc} - \frac{15d^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{16bc} - \frac{5d^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{8bc}$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
d^2*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c) - d^2*co
s(a/b)^5*sin_integral(5*a/b + 5*arccos(c*x))/(b*c) - 3/4*d^2*cos(a/b)^2*co
s_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c) - 5/4*d^2*cos(a/b)^2*cos_
integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c) + 5/4*d^2*cos(a/b)^3*sin_in
tegral(5*a/b + 5*arccos(c*x))/(b*c) + 5/4*d^2*cos(a/b)^3*sin_integral(3*a/
b + 3*arccos(c*x))/(b*c) + 1/16*d^2*cos_integral(5*a/b + 5*arccos(c*x))*si
n(a/b)/(b*c) + 5/16*d^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c)
 + 5/8*d^2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - 5/16*d^2*cos(a
/b)*sin_integral(5*a/b + 5*arccos(c*x))/(b*c) - 15/16*d^2*cos(a/b)*sin_int
egral(3*a/b + 3*arccos(c*x))/(b*c) - 5/8*d^2*cos(a/b)*sin_integral(a/b + a
rccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx=\int \frac{(d-c^2\,d\,x^2)^2}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((d - c^2*d*x^2)^2/(a + b*acos(c*x)),x)
```

output
```
int((d - c^2*d*x^2)^2/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2}{a+b\arccos(cx)}\,dx=d^2\left(\left(\int \frac{x^4}{acos\,(cx)\,b+a}dx\right)c^4-2\left(\int \frac{x^2}{acos\,(cx)\,b+a}dx\right)c^2+\int \frac{1}{acos\,(cx)\,b+a}dx\right)$$

input
```
int((-c^2*d*x^2+d)^2/(a+b*acos(c*x)),x)
```

output
```
d**2*(int(x**4/(acos(c*x)*b + a),x)*c**4 - 2*int(x**2/(acos(c*x)*b + a),x)
*c**2 + int(1/(acos(c*x)*b + a),x))
```

3.29 $\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=\frac{3d\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc}-\frac{d\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc}-\frac{3d\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc}+\frac{d\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc}$$

output
```
3/4*d*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c-1/4*d*Ci(3*(a+b*arccos(c*x))/b)
*sin(3*a/b)/b/c-3/4*d*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c+1/4*d*cos(3*a/b
)*Si(3*(a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx$$
$$= \frac{d\left(3\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)-3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\right.\right.}{4bc}$$

input
```
Integrate[(d - c^2*d*x^2)/(a + b*ArcCos[c*x]),x]
```

output
```
(d*(3*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] - CosIntegral[3*(a/b + ArcCo
s[c*x])]*Sin[(3*a)/b] - 3*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Cos[(3
*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])]))/(4*b*c)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5169, 25, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx$$
$$\Big\downarrow \; 5169$$
$$-\frac{d\int -\frac{\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$
$$\Big\downarrow \; 25$$
$$\frac{d\int \frac{\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$
$$\Big\downarrow \; 3042$$

$$\frac{d\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^3}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$\frac{d\int \left(\frac{3\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}-\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx}{b}\right.\right.}{bc}$$

input
```
Int[(d - c^2*d*x^2)/(a + b*ArcCos[c*x]),x]
```

output
```
-((d*((-3*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/4 + (CosIntegral[(3
*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/4 + (3*Cos[a/b]*SinIntegral[(a + b*
ArcCos[c*x])/b])/4 - (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])
/4))/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{d\left(\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)-3\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)+3\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\mathrm{si}\right.}{4cb}$
default	$\frac{d\left(\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)-3\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)+3\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\mathrm{si}\right.}{4cb}$

input
```
int((-c^2*d*x^2+d)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*d*(Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)-Ci(3*arccos(c*x)+3*a/b)*sin(3*
a/b)-3*Si(arccos(c*x)+a/b)*cos(a/b)+3*Ci(arccos(c*x)+a/b)*sin(a/b))/b
```

Fricas [F]

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=\int -\frac{c^2dx^2-d}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*d*x^2 - d)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=-d\left(\int \frac{c^2x^2}{a+b\operatorname{acos}(cx)}\,dx+\int \left(-\frac{1}{a+b\operatorname{acos}(cx)}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)/(a+b*acos(c*x)),x)
```

output
```
-d*(Integral(c**2*x**2/(a + b*acos(c*x)), x) + Integral(-1/(a + b*acos(c*x
)), x))
```

Maxima [F]

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=\int -\frac{c^2dx^2-d}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=&-\frac{d\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc}\\&+\frac{d\cos\left(\frac{a}{b}\right)^3\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc}\\&+\frac{d\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc}\\&+\frac{3d\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc}\\&-\frac{3d\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc}\\&-\frac{3d\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc}\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-d*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c) + d*cos(a
/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b*c) + 1/4*d*cos_integral(3*a/b
 + 3*arccos(c*x))*sin(a/b)/(b*c) + 3/4*d*cos_integral(a/b + arccos(c*x))*s
in(a/b)/(b*c) - 3/4*d*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c) -
 3/4*d*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=\int \frac{d-c^2\,d\,x^2}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((d - c^2*d*x^2)/(a + b*acos(c*x)),x)
```

output
```
int((d - c^2*d*x^2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{d-c^2dx^2}{a+b\arccos(cx)}\,dx=d\left(-\left(\int \frac{x^2}{acos\,(cx)\,b+a}dx\right)c^2+\int \frac{1}{acos\,(cx)\,b+a}dx\right)$$

input
```
int((-c^2*d*x^2+d)/(a+b*acos(c*x)),x)
```

output
```
d*( - int(x**2/(acos(c*x)*b + a),x)*c**2 + int(1/(acos(c*x)*b + a),x))
```

3.30 $\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx = \operatorname{Int}\left(\frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}dx$$

input
```
Int[1/((d - c^2*d*x^2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-c^2d\,x^2+d)\,(a+b\arccos\,(cx))}dx$$

input
```
int(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx = \int -\frac{1}{(c^2dx^2-d)(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-1/(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx = -\frac{\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{acos}(cx)-b\operatorname{acos}(cx)}\,dx}{d}$$

input
```
integrate(1/(-c**2*d*x**2+d)/(a+b*acos(c*x)),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acos(c*x) - b*acos(c*x)), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx = \int -\frac{1}{(c^2dx^2-d)(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-integrate(1/((c^2*d*x^2 - d)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 3.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))}\,dx = \int -\frac{1}{(c^2dx^2-d)(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(-1/((c^2*d*x^2 - d)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))}\,dx = -\frac{\int \frac{1}{acos(cx)b\,c^2x^2-acos(cx)b+a\,c^2x^2-a}dx}{d}$$

input
```
int(1/(-c^2*d*x^2+d)/(a+b*acos(c*x)),x)
```

output
```
( - int(1/(acos(c*x)*b*c**2*x**2 - acos(c*x)*b + a*c**2*x**2 - a),x))/d
```

3.31 $\int \frac{1}{(d-c^2dx^2)^2(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(d-c^2dx^2)^2\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d-c^2dx^2)^2\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 31.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)^2\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(d-c^2dx^2)^2\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d-c^2dx^2\right)^2 (a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{\left(d-c^2dx^2\right)^2 (a+b\arccos(cx))}dx$$

input
```
Int[1/((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-c^2d\,x^2+d\right)^2 (a+b\arccos\left(cx\right))}dx$$

input
```
int(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x)
```

output
```
int(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)}\,dx=\int \frac{1}{\left(c^2dx^2-d\right)^2\left(b\arccos\left(cx\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b
*c^2*d^2*x^2 + b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)}\,dx=\frac{\int \frac{1}{ac^4x^4-2ac^2x^2+a+bc^4x^4\operatorname{acos}\left(cx\right)-2bc^2x^2\operatorname{acos}\left(cx\right)+b\operatorname{acos}\left(cx\right)}\,dx}{d^2}$$

input
```
integrate(1/(-c**2*d*x**2+d)**2/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(a*c**4*x**4 - 2*a*c**2*x**2 + a + b*c**4*x**4*acos(c*x) - 2*b*
c**2*x**2*acos(c*x) + b*acos(c*x)), x)/d**2
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(d-c^2dx^2\right)^2 (a+b\arccos(cx))}\,dx = \int \frac{1}{\left(c^2dx^2-d\right)^2 (b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((c^2*d*x^2 - d)^2*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 177.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(d-c^2dx^2\right)^2 (a+b\arccos(cx))}\,dx = \int \frac{1}{\left(c^2dx^2-d\right)^2 (b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((c^2*d*x^2 - d)^2*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d-c^2dx^2\right)^2 (a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{1}{(d-c^2dx^2)^2 (a+b\arccos(cx))}\,dx = \frac{\int \frac{1}{acos(cx)b\,c^4x^4-2acos(cx)b\,c^2x^2+acos(cx)b+a\,c^4x^4-2a\,c^2x^2+a}dx}{d^2}$$

input
```
int(1/(-c^2*d*x^2+d)^2/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*c**4*x**4 - 2*acos(c*x)*b*c**2*x**2 + acos(c*x)*b + a*c
**4*x**4 - 2*a*c**2*x**2 + a),x)/d**2
```

3.32 $\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 303

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx &= \frac{d^3(1-c^2x^2)^{7/2}}{bc(a+b\arccos(cx))} \\ &- \frac{35d^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{64b^2c} \\ &+ \frac{63d^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64b^2c} \\ &- \frac{35d^3\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64b^2c} \\ &+ \frac{7d^3\cos\left(\frac{7a}{b}\right)\text{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64b^2c} \\ &- \frac{35d^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{64b^2c} \\ &+ \frac{63d^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64b^2c} \\ &- \frac{35d^3\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64b^2c} \\ &+ \frac{7d^3\sin\left(\frac{7a}{b}\right)\text{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64b^2c}\end{aligned}$$

output
```
d^3*(-c^2*x^2+1)^(7/2)/b/c/(a+b*arccos(c*x))-35/64*d^3*cos(a/b)*Ci((a+b*ar
ccos(c*x))/b)/b^2/c+63/64*d^3*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c-3
5/64*d^3*cos(5*a/b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c+7/64*d^3*cos(7*a/b)*Ci
(7*(a+b*arccos(c*x))/b)/b^2/c-35/64*d^3*sin(a/b)*Si((a+b*arccos(c*x))/b)/b
^2/c+63/64*d^3*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c-35/64*d^3*sin(5*
a/b)*Si(5*(a+b*arccos(c*x))/b)/b^2/c+7/64*d^3*sin(7*a/b)*Si(7*(a+b*arccos(
c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.52

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx = -\frac{d^3\left(-64b\sqrt{1-c^2x^2}+192bc^2x^2\sqrt{1-c^2x^2}-192bc^4x^4\sqrt{1-c^2x^2}+64bc^6x^6\sqrt{1-c^2x^2}+35(a+b\arccos\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^3/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/64*(d^3*(-64*b*Sqrt[1 - c^2*x^2] + 192*b*c^2*x^2*Sqrt[1 - c^2*x^2] - 19
2*b*c^4*x^4*Sqrt[1 - c^2*x^2] + 64*b*c^6*x^6*Sqrt[1 - c^2*x^2] + 35*(a + b
*ArcCos[c*x])*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] - 63*(a + b*ArcCos[c
*x])*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*x])] + 35*a*Cos[(5*a)/b]*C
osIntegral[5*(a/b + ArcCos[c*x])] + 35*b*ArcCos[c*x]*Cos[(5*a)/b]*CosInteg
ral[5*(a/b + ArcCos[c*x])] - 7*a*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcCos[
c*x])] - 7*b*ArcCos[c*x]*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcCos[c*x])] +
 35*a*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 35*b*ArcCos[c*x]*Sin[a/b]*
SinIntegral[a/b + ArcCos[c*x]] - 63*a*Sin[(3*a)/b]*SinIntegral[3*(a/b + Ar
cCos[c*x])] - 63*b*ArcCos[c*x]*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*
x])] + 35*a*Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] + 35*b*ArcCos[
c*x]*Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] - 7*a*Sin[(7*a)/b]*Si
nIntegral[7*(a/b + ArcCos[c*x])] - 7*b*ArcCos[c*x]*Sin[(7*a)/b]*SinIntegra
l[7*(a/b + ArcCos[c*x])]))/(b^2*c*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{7cd^3\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}dx}{b}+\frac{d^3(1-c^2x^2)^{7/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$\frac{d^3(1-c^2x^2)^{7/2}}{bc(a+b\arccos(cx))}-\frac{7d^3\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{4906}$$

$$\frac{d^3(1-c^2x^2)^{7/2}}{bc(a+b\arccos(cx))}-$$
$$\frac{7d^3\int\left(-\frac{\cos\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{5\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{9\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{5\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+}{b^2c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(1-c^2x^2)^{7/2}}{bc(a+b\arccos(cx))}-$$
$$7d^3\left(\frac{5}{64}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{9}{64}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{5}{64}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}$$

input
```
Int[(d - c^2*d*x^2)^3/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*(1 - c^2*x^2)^(7/2))/(b*c*(a + b*ArcCos[c*x])) - (7*d^3*((5*Cos[a/b]*
CosIntegral[(a + b*ArcCos[c*x])/b])/64 - (9*Cos[(3*a)/b]*CosIntegral[(3*(a
 + b*ArcCos[c*x]))/b])/64 + (5*Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcCos[c
*x]))/b])/64 - (Cos[(7*a)/b]*CosIntegral[(7*(a + b*ArcCos[c*x]))/b])/64 +
(5*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/64 - (9*Sin[(3*a)/b]*SinIn
tegral[(3*(a + b*ArcCos[c*x]))/b])/64 + (5*Sin[(5*a)/b]*SinIntegral[(5*(a
+ b*ArcCos[c*x]))/b])/64 - (Sin[(7*a)/b]*SinIntegral[(7*(a + b*ArcCos[c*x]
))/b])/64))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{d^3\left(7\arccos(cx)\operatorname{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\sin\left(\frac{7a}{b}\right)b+7\arccos(cx)\operatorname{Ci}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos\left(\frac{7a}{b}\right)b-35\arccos(cx)\operatorname{Si}\left(5\arccos\right.\right.}{}$
default	$\frac{d^3\left(7\arccos(cx)\operatorname{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\sin\left(\frac{7a}{b}\right)b+7\arccos(cx)\operatorname{Ci}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos\left(\frac{7a}{b}\right)b-35\arccos(cx)\operatorname{Si}\left(5\arccos\right.\right.}{}$

input
```
int((-c^2*d*x^2+d)^3/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/c*d^3*(7*arccos(c*x)*Si(7*arccos(c*x)+7*a/b)*sin(7*a/b)*b+7*arccos(c*
x)*Ci(7*arccos(c*x)+7*a/b)*cos(7*a/b)*b-35*arccos(c*x)*Si(5*arccos(c*x)+5*
a/b)*sin(5*a/b)*b-35*arccos(c*x)*Ci(5*arccos(c*x)+5*a/b)*cos(5*a/b)*b+63*a
rccos(c*x)*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+63*arccos(c*x)*Ci(3*arccos
(c*x)+3*a/b)*cos(3*a/b)*b-35*arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b-35
*arccos(c*x)*Ci(arccos(c*x)+a/b)*cos(a/b)*b+7*Si(7*arccos(c*x)+7*a/b)*sin(
7*a/b)*a+7*Ci(7*arccos(c*x)+7*a/b)*cos(7*a/b)*a-35*Si(5*arccos(c*x)+5*a/b)
*sin(5*a/b)*a-35*Ci(5*arccos(c*x)+5*a/b)*cos(5*a/b)*a+63*Si(3*arccos(c*x)+
3*a/b)*sin(3*a/b)*a+63*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a-35*Si(arccos(c
*x)+a/b)*sin(a/b)*a-35*Ci(arccos(c*x)+a/b)*cos(a/b)*a+35*(-c^2*x^2+1)^(1/2
)*b-sin(7*arccos(c*x))*b+7*sin(5*arccos(c*x))*b-21*sin(3*arccos(c*x))*b)/(
a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx=\int -\frac{(c^2dx^2-d)^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)/(b^2*arccos(
c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx = -d^3\left(\int \frac{3c^2x^2}{a^2+2ab\operatorname{acos}(cx)+b^2\operatorname{acos}^2(cx)}\,dx + \int \left(-\frac{3c^4x^4}{a^2+2ab\operatorname{acos}(cx)+b^2\operatorname{acos}^2(cx)}\right)dx + \int \frac{c^6x^6}{a^2+2ab\operatorname{acos}(cx)+b^2\operatorname{acos}^2(cx)}\,dx + \int \left(-\frac{1}{a^2+2ab\operatorname{acos}(cx)+b^2\operatorname{acos}^2(cx)}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3/(a+b*acos(c*x))**2,x)
```

output
```
-d**3*(Integral(3*c**2*x**2/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2),
x) + Integral(-3*c**4*x**4/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x
) + Integral(c**6*x**6/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x) +
Integral(-1/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx = \int -\frac{(c^2dx^2-d)^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)*sqrt(c*x + 1)*sqrt(-
c*x + 1) - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*inte
grate(7*(c^5*d^3*x^5 - 2*c^3*d^3*x^3 + c*d^3*x)*sqrt(c*x + 1)*sqrt(-c*x +
1)/(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b), x))/(b^2*c*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2110 vs. $2(285) = 570$.

Time = 0.26 (sec) , antiderivative size = 2110, normalized size of antiderivative = 6.96

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-sqrt(-c^2*x^2 + 1)*b*c^6*d^3*x^6/(b^3*c*arccos(c*x) + a*b^2*c) + 7*b*d^3*
arccos(c*x)*cos(a/b)^7*cos_integral(7*a/b + 7*arccos(c*x))/(b^3*c*arccos(c
*x) + a*b^2*c) + 7*b*d^3*arccos(c*x)*cos(a/b)^6*sin(a/b)*sin_integral(7*a/
b + 7*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 3*sqrt(-c^2*x^2 + 1)*b*
c^4*d^3*x^4/(b^3*c*arccos(c*x) + a*b^2*c) + 7*a*d^3*cos(a/b)^7*cos_integra
l(7*a/b + 7*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 7*a*d^3*cos(a/b)^
6*sin(a/b)*sin_integral(7*a/b + 7*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*
c) - 49/4*b*d^3*arccos(c*x)*cos(a/b)^5*cos_integral(7*a/b + 7*arccos(c*x))
/(b^3*c*arccos(c*x) + a*b^2*c) - 35/4*b*d^3*arccos(c*x)*cos(a/b)^5*cos_int
egral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 35/4*b*d^3*ar
ccos(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arccos(c*x))/(b^3*c*a
rccos(c*x) + a*b^2*c) - 35/4*b*d^3*arccos(c*x)*cos(a/b)^4*sin(a/b)*sin_int
egral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 49/4*a*d^3*co
s(a/b)^5*cos_integral(7*a/b + 7*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c)
 - 35/4*a*d^3*cos(a/b)^5*cos_integral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos
(c*x) + a*b^2*c) - 35/4*a*d^3*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*a
rccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 35/4*a*d^3*cos(a/b)^4*sin(a/b)
*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 49/8*
b*d^3*arccos(c*x)*cos(a/b)^3*cos_integral(7*a/b + 7*arccos(c*x))/(b^3*c*ar
ccos(c*x) + a*b^2*c) + 175/16*b*d^3*arccos(c*x)*cos(a/b)^3*cos_integral...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx=\int \frac{{\left(d-c^2\,d\,x^2\right)}^3}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((d - c^2*d*x^2)^3/(a + b*acos(c*x))^2,x)
```

output
```
int((d - c^2*d*x^2)^3/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3}{(a+b\arccos(cx))^2}\,dx=d^3\left(-\left(\int \frac{x^6}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)c^6+3\left(\int \frac{x^4}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)c^4-3\left(\int \frac{x^2}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)c^2+\int \frac{1}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)$$

input
```
int((-c^2*d*x^2+d)^3/(a+b*acos(c*x))^2,x)
```

output
```
d**3*( - int(x**6/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**6 + 3
*int(x**4/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**4 - 3*int(x**
2/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**2 + int(1/(acos(c*x)*
*2*b**2 + 2*acos(c*x)*a*b + a**2),x))
```

3.33 $\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 235

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx =& \frac{d^2(1-c^2x^2)^{5/2}}{bc(a+b\arccos(cx))}-\frac{5d^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c}\\&+\frac{15d^2\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c}\\&-\frac{5d^2\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c}\\&-\frac{5d^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c}\\&+\frac{15d^2\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c}\\&-\frac{5d^2\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c}\end{aligned}$$

output
```
d^2*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccos(c*x))-5/8*d^2*cos(a/b)*Ci((a+b*arcc
os(c*x))/b)/b^2/c+15/16*d^2*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c-5/1
6*d^2*cos(5*a/b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c-5/8*d^2*sin(a/b)*Si((a+b*
arccos(c*x))/b)/b^2/c+15/16*d^2*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c
-5/16*d^2*sin(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.99

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx$$
$$= \frac{d^2\left(\frac{16b\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \frac{32bc^2x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)} + \frac{16bc^4x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - 10\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) + 15\cos\left(\frac{3a}{b}\right)\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*((16*b*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (32*b*c^2*x^2*Sqrt[1
- c^2*x^2])/(a + b*ArcCos[c*x]) + (16*b*c^4*x^4*Sqrt[1 - c^2*x^2])/(a + b*
ArcCos[c*x]) - 10*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + 15*Cos[(3*a)/b
]*CosIntegral[3*(a/b + ArcCos[c*x])] - 5*Cos[(5*a)/b]*CosIntegral[5*(a/b +
 ArcCos[c*x])] - 10*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 15*Sin[(3*a)
/b]*SinIntegral[3*(a/b + ArcCos[c*x])] - 5*Sin[(5*a)/b]*SinIntegral[5*(a/b
 + ArcCos[c*x])]))/(16*b^2*c)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{5167}$$
$$\frac{5cd^2\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}dx}{b}+\frac{d^2(1-c^2x^2)^{5/2}}{bc(a+b\arccos(cx))}$$
$$\Big\downarrow \text{5225}$$

$$\frac{d^2(1-c^2x^2)^{5/2}}{bc(a+b\arccos(cx))}-\frac{5d^2\int\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\downarrow$ 4906

$$\frac{d^2(1-c^2x^2)^{5/2}}{bc(a+b\arccos(cx))}-\frac{5d^2\int\left(\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c}$$

$\downarrow$ 2009

$$\frac{d^2(1-c^2x^2)^{5/2}}{bc(a+b\arccos(cx))}-\frac{5d^2\left(\frac{1}{8}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{16}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{16}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b}$$

input
```
Int[(d - c^2*d*x^2)^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(1 - c^2*x^2)^(5/2))/(b*c*(a + b*ArcCos[c*x])) - (5*d^2*((Cos[a/b]*Co
sIntegral[(a + b*ArcCos[c*x])/b])/8 - (3*Cos[(3*a)/b]*CosIntegral[(3*(a +
b*ArcCos[c*x]))/b])/16 + (Cos[(5*a)/b]*CosIntegral[(5*(a + b*ArcCos[c*x]))
/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 - (3*Sin[(3*a)/b
]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Sin[(5*a)/b]*SinIntegral[(
5*(a + b*ArcCos[c*x]))/b])/16))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{d^2\left(5\arccos(cx)\,\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)b+5\arccos(cx)\,\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)b-15\arccos(cx)\,\mathrm{Si}(3\,\mathrm{arc}\right.}{}$
default	$-\frac{d^2\left(5\arccos(cx)\,\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)b+5\arccos(cx)\,\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)b-15\arccos(cx)\,\mathrm{Si}(3\,\mathrm{arc}\right.}{}$

input
```
int((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/c*d^2*(5*arccos(c*x)*Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)*b+5*arccos(c
*x)*Ci(5*arccos(c*x)+5*a/b)*cos(5*a/b)*b-15*arccos(c*x)*Si(3*arccos(c*x)+3
*a/b)*sin(3*a/b)*b-15*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)*b+10*
arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b+10*arccos(c*x)*Ci(arccos(c*x)+a
/b)*cos(a/b)*b+5*Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)*a+5*Ci(5*arccos(c*x)+5
*a/b)*cos(5*a/b)*a-15*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a-15*Ci(3*arccos(
c*x)+3*a/b)*cos(3*a/b)*a+10*Si(arccos(c*x)+a/b)*sin(a/b)*a+10*Ci(arccos(c*
x)+a/b)*cos(a/b)*a-10*(-c^2*x^2+1)^(1/2)*b-sin(5*arccos(c*x))*b+5*sin(3*ar
ccos(c*x))*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(c^2dx^2-d)^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)/(b^2*arccos(c*x)^2 + 2*a*b*ar
ccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx = d^2\left(\int \left(-\frac{2c^2x^2}{a^2+2ab\,\mathrm{acos}\,(cx)+b^2\,\mathrm{acos}^2\,(cx)}\right)dx + \int \frac{c^4x^4}{a^2+2ab\,\mathrm{acos}\,(cx)+b^2\,\mathrm{acos}^2\,(cx)}\,dx + \int \frac{1}{a^2+2ab\,\mathrm{acos}\,(cx)+b^2\,\mathrm{acos}^2\,(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2/(a+b*acos(c*x))**2,x)
```

output
```
d**2*(Integral(-2*c**2*x**2/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2),
x) + Integral(c**4*x**4/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x) +
 Integral(1/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)^2}{(a + b\arccos(cx))^2}\,dx = \int \frac{(c^2dx^2 - d)^2}{(b\arccos(cx) + a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*c
*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate(5*(c^3*d^2*
x^3 - c*d^2*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(b^2*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x) + a*b), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
 c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1271 vs. $2(221) = 442$.

Time = 0.23 (sec) , antiderivative size = 1271, normalized size of antiderivative = 5.41

$$\int \frac{(d - c^2dx^2)^2}{(a + b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
sqrt(-c^2*x^2 + 1)*b*c^4*d^2*x^4/(b^3*c*arccos(c*x) + a*b^2*c) - 5*b*d^2*a
rccos(c*x)*cos(a/b)^5*cos_integral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*
x) + a*b^2*c) - 5*b*d^2*arccos(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b
 + 5*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 5*a*d^2*cos(a/b)^5*cos_i
ntegral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 5*a*d^2*cos
(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*x) +
a*b^2*c) + 25/4*b*d^2*arccos(c*x)*cos(a/b)^3*cos_integral(5*a/b + 5*arccos
(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 15/4*b*d^2*arccos(c*x)*cos(a/b)^3*c
os_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 15/4*b*
d^2*arccos(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(5*a/b + 5*arccos(c*x))/(b
^3*c*arccos(c*x) + a*b^2*c) + 15/4*b*d^2*arccos(c*x)*cos(a/b)^2*sin(a/b)*s
in_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 2*sqrt(
-c^2*x^2 + 1)*b*c^2*d^2*x^2/(b^3*c*arccos(c*x) + a*b^2*c) + 25/4*a*d^2*cos
(a/b)^3*cos_integral(5*a/b + 5*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c)
+ 15/4*a*d^2*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arccos(
c*x) + a*b^2*c) + 15/4*a*d^2*cos(a/b)^2*sin(a/b)*sin_integral(5*a/b + 5*ar
ccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 15/4*a*d^2*cos(a/b)^2*sin(a/b)*
sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 25/16*
b*d^2*arccos(c*x)*cos(a/b)*cos_integral(5*a/b + 5*arccos(c*x))/(b^3*c*arcc
os(c*x) + a*b^2*c) - 45/16*b*d^2*arccos(c*x)*cos(a/b)*cos_integral(3*a/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{\left(d-c^2\,d\,x^2\right)^2}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((d - c^2*d*x^2)^2/(a + b*acos(c*x))^2,x)
```

output
```
int((d - c^2*d*x^2)^2/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^2}\,dx = d^2\left(\left(\int \frac{x^4}{acos(cx)^2 b^2+2acos(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{x^2}{acos(cx)^2 b^2+2acos(cx)\,ab+a^2}dx\right)c^2 + \int \frac{1}{acos(cx)^2 b^2+2acos(cx)\,ab+a^2}dx\right)$$

input
```
int((-c^2*d*x^2+d)^2/(a+b*acos(c*x))^2,x)
```

output
```
d**2*(int(x**4/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**4 - 2*in
t(x**2/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**2 + int(1/(acos(
c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x))
```

3.34 $\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^2}\,dx=\frac{d(1-c^2x^2)^{3/2}}{bc(a+b\arccos(cx))}-\frac{3d\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c}+\frac{3d\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c}-\frac{3d\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c}+\frac{3d\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c}$$

output
```
d*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccos(c*x))-3/4*d*cos(a/b)*Ci((a+b*arccos(c
*x))/b)/b^2/c+3/4*d*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c-3/4*d*sin(a
/b)*Si((a+b*arccos(c*x))/b)/b^2/c+3/4*d*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/
b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^2}\,dx$$
$$= \frac{d\left(\frac{4b(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}+3\left(-\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\right.\right.}{4b^2c}$$

input
```
Integrate[(d - c^2*d*x^2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*((4*b*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]) + 3*(-(Cos[a/b]*CosInteg
ral[a/b + ArcCos[c*x]]) + Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*x])]
- Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Sin[(3*a)/b]*SinIntegral[3*(a/
b + ArcCos[c*x])])))/(4*b^2*c)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \text{ 5167}$$
$$\frac{3cd\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}dx}{b}+\frac{d(1-c^2x^2)^{3/2}}{bc(a+b\arccos(cx))}$$
$$\Big\downarrow \text{ 5225}$$
$$\frac{d(1-c^2x^2)^{3/2}}{bc(a+b\arccos(cx))}-\frac{3d\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 4906

$$\frac{d(1-c^2x^2)^{3/2}}{bc(a+b\arccos(cx))}-\frac{3d\int\left(\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 2009

$$\frac{d(1-c^2x^2)^{3/2}}{bc(a+b\arccos(cx))}-$$

$$\frac{3d\left(\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)\right.}{b^2c}$$

input
```
Int[(d - c^2*d*x^2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*(1 - c^2*x^2)^(3/2))/(b*c*(a + b*ArcCos[c*x])) - (3*d*((Cos[a/b]*CosInt
egral[(a + b*ArcCos[c*x])/b])/4 - (Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcC
os[c*x]))/b])/4 + (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/4 - (Sin[(
3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.52

method	result
derivativedivides	$d\left(3\arccos(cx)\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)b+3\arccos(cx)\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)b-3\arccos(cx)\operatorname{Si}(\arccos(cx$
default	$d\left(3\arccos(cx)\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)b+3\arccos(cx)\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)b-3\arccos(cx)\operatorname{Si}(\arccos(cx$

input
```
int((-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c*d*(3*arccos(c*x)*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+3*arccos(c*x)*
Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)*b-3*arccos(c*x)*Si(arccos(c*x)+a/b)*sin
(a/b)*b-3*arccos(c*x)*Ci(arccos(c*x)+a/b)*cos(a/b)*b+3*Si(3*arccos(c*x)+3*
a/b)*sin(3*a/b)*a+3*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a-3*Si(arccos(c*x)+
a/b)*sin(a/b)*a-3*Ci(arccos(c*x)+a/b)*cos(a/b)*a+3*(-c^2*x^2+1)^(1/2)*b-si
n(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^2}\,dx=\int -\frac{c^2dx^2-d}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(c^2*d*x^2 - d)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x
)
```

Sympy [F]

$$\int \frac{d - c^2 d x^2}{(a + b \arccos(cx))^2} \, dx = -d \left(\int \frac{c^2 x^2}{a^2 + 2ab \operatorname{acos}(cx) + b^2 \operatorname{acos}^2(cx)} \, dx + \int \left(-\frac{1}{a^2 + 2ab \operatorname{acos}(cx) + b^2 \operatorname{acos}^2(cx)} \right) dx \right)$$

input
```
integrate((-c**2*d*x**2+d)/(a+b*acos(c*x))**2,x)
```

output
```
-d*(Integral(c**2*x**2/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x) +
Integral(-1/(a**2 + 2*a*b*acos(c*x) + b**2*acos(c*x)**2), x))
```

Maxima [F]

$$\int \frac{d - c^2 d x^2}{(a + b \arccos(cx))^2} \, dx = \int -\frac{c^2 d x^2 - d}{(b \arccos(cx) + a)^2} \, dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 3*(b^2*c^2*d*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^2*d)*integrate(sqrt(c*x + 1)*sqrt(-
c*x + 1)*x/(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b), x))/(b^
2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(147) = 294$.

Time = 0.23 (sec) , antiderivative size = 610, normalized size of antiderivative = 3.89

$$\int \frac{d - c^2 d x^2}{(a + b \arccos(cx))^2} \, dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
3*b*d*arccos(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c*ar
ccos(c*x) + a*b^2*c) + 3*b*d*arccos(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(
3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - sqrt(-c^2*x^2 + 1)*
b*c^2*d*x^2/(b^3*c*arccos(c*x) + a*b^2*c) + 3*a*d*cos(a/b)^3*cos_integral(
3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 3*a*d*cos(a/b)^2*si
n(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) -
 9/4*b*d*arccos(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c*a
rccos(c*x) + a*b^2*c) - 3/4*b*d*arccos(c*x)*cos(a/b)*cos_integral(a/b + ar
ccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 3/4*b*d*arccos(c*x)*sin(a/b)*si
n_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 3/4*b*d*
arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c*arccos(c*x) +
a*b^2*c) - 9/4*a*d*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c*arc
cos(c*x) + a*b^2*c) - 3/4*a*d*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^
3*c*arccos(c*x) + a*b^2*c) - 3/4*a*d*sin(a/b)*sin_integral(3*a/b + 3*arcco
s(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 3/4*a*d*sin(a/b)*sin_integral(a/b
+ arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + sqrt(-c^2*x^2 + 1)*b*d/(b^3
*c*arccos(c*x) + a*b^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d - c^2dx^2}{(a + b\arccos(cx))^2}\,dx = \int \frac{d - c^2\,d\,x^2}{\left(a + b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((d - c^2*d*x^2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d - c^2*d*x^2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^2}\,dx = d\left(-\left(\int \frac{x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{1}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)$$

input
```
int((-c^2*d*x^2+d)/(a+b*acos(c*x))^2,x)
```

output
```
d*( - int(x**2/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**2 + int(
1/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x))
```

3.35 $\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 26.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{bcd\sqrt{1-c^2x^2}(a+b\arccos(cx))}-\frac{c\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx}{bd}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{bcd\sqrt{1-c^2x^2}(a+b\arccos(cx))}-\frac{c\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx}{bd}$$

input
```
Int[1/((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-c^2d\,x^2+d)\,(a+b\arccos{(cx)})^2}dx$$

input
```
int(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx = \int -\frac{1}{(c^2dx^2-d)(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2
 + 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.04

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx = -\frac{\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\operatorname{acos}{(cx)}-2ab\operatorname{acos}{(cx)}+b^2c^2x^2\operatorname{acos}^2{(cx)}-b^2\operatorname{acos}^2{(cx)}}\,dx}{d}$$

input
```
integrate(1/(-c**2*d*x**2+d)/(a+b*acos(c*x))**2,x)
```

output
```
-Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acos(c*x) - 2*a*b*aco
s(c*x) + b**2*c**2*x**2*acos(c*x)**2 - b**2*acos(c*x)**2), x)/d
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 237, normalized size of antiderivative = 9.88

$$\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))^2}\,dx = \int -\frac{1}{(c^2dx^2-d)(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((a*b*c^4*d*x^2 - a*b*c^2*d + (b^2*c^4*d*x^2 - b^2*c^2*d)*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x))*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a*b
*c^4*d*x^4 - 2*a*b*c^2*d*x^2 + a*b*d + (b^2*c^4*d*x^4 - 2*b^2*c^2*d*x^2 +
b^2*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqr
t(-c*x + 1))/(a*b*c^3*d*x^2 - a*b*c*d + (b^2*c^3*d*x^2 - b^2*c*d)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 8.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{(d-c^2dx^2)(a+b\arccos(cx))^2}\,dx = \int -\frac{1}{(c^2dx^2-d)(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(-1/((c^2*d*x^2 - d)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\left(d-c^2\,d\,x^2\right)}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.08

$$\int \frac{1}{(d-c^2dx^2)\,(a+b\arccos(cx))^2}\,dx = -\frac{\int \frac{1}{acos(cx)^2b^2c^2x^2-acos(cx)^2b^2+2acos(cx)ab\,c^2x^2-2acos(cx)ab+a^2c^2x^2-a^2}dx}{d}$$

input
```
int(1/(-c^2*d*x^2+d)/(a+b*acos(c*x))^2,x)
```

output
```
( - int(1/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a
*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x))/d
```

3.36 $\int \frac{1}{\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{1}{\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 57.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{bcd^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} - \frac{3c\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}dx}{bd^2}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{bcd^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} - \frac{3c\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}dx}{bd^2}$$

input
```
Int[1/((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-c^2d\,x^2+d)^2 (a+b\arccos(cx))^2}dx$$

input
```
int(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.96

$$\int \frac{1}{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x
^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a
*b*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 14.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 5.04

$$\int \frac{1}{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}\,dx = \frac{\int \frac{1}{a^2c^4x^4-2a^2c^2x^2+a^2+2abc^4x^4\operatorname{acos}(cx)-4abc^2x^2\operatorname{acos}(cx)+2ab\operatorname{acos}(cx)+b^2c^4x^4\operatorname{acos}^2(cx)-2b^2c^2x^2\operatorname{acos}^2(cx)+b^2\operatorname{acos}^2(cx)}\,dx}{d^2}$$

input
```
integrate(1/(-c**2*d*x**2+d)**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(a**2*c**4*x**4 - 2*a**2*c**2*x**2 + a**2 + 2*a*b*c**4*x**4*aco
s(c*x) - 4*a*b*c**2*x**2*acos(c*x) + 2*a*b*acos(c*x) + b**2*c**4*x**4*acos
(c*x)**2 - 2*b**2*c**2*x**2*acos(c*x)**2 + b**2*acos(c*x)**2), x)/d**2
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 344, normalized size of antiderivative = 14.33

$$\int \frac{1}{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(3*(a*b*c^6*d^2*x^4 - 2*a*b*c^4*d^2*x^2 + a*b*c^2*d^2 + (b^2*c^6*d^2*x^4 -
 2*b^2*c^4*d^2*x^2 + b^2*c^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x))*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a*b*c^6*d^2*x^6 - 3*a*b*c^4*
d^2*x^4 + 3*a*b*c^2*d^2*x^2 - a*b*d^2 + (b^2*c^6*d^2*x^6 - 3*b^2*c^4*d^2*x
^4 + 3*b^2*c^2*d^2*x^2 - b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b*c^5*d^2*x^4 - 2*a*b*c^3*d^2*x
^2 + a*b*c*d^2 + (b^2*c^5*d^2*x^4 - 2*b^2*c^3*d^2*x^2 + b^2*c*d^2)*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.67

$$\int \frac{1}{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}\,dx = \frac{\int \frac{1}{acos(cx)^2b^2c^4x^4-2acos(cx)^2b^2c^2x^2+acos(cx)^2b^2+2acos(cx)ab\,c^4x^4-4acos(cx)ab\,c^2x^2+2acos(cx)ab+a^2c^4x^4-2a^2c^2x^2+a^2}dx}{d^2}$$

input
```
int(1/(-c^2*d*x^2+d)^2/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*c**4*x**4 - 2*acos(c*x)**2*b**2*c**2*x**2 + acos(
c*x)**2*b**2 + 2*acos(c*x)*a*b*c**4*x**4 - 4*acos(c*x)*a*b*c**2*x**2 + 2*a
cos(c*x)*a*b + a**2*c**4*x**4 - 2*a**2*c**2*x**2 + a**2),x)/d**2
```

3.37 $\int (\pi - c^2\pi x^2)^{5/2} (a + b\arccos(cx))\, dx$

Optimal result

Integrand size = 24, antiderivative size = 178

$$\int (\pi - c^2\pi x^2)^{5/2} (a + b\arccos(cx))\, dx = \frac{5}{32}bc\pi^{5/2}x^2 - \frac{5b\pi^{5/2}(1-c^2x^2)^2}{96c} - \frac{b\pi^{5/2}(1-c^2x^2)^3}{36c} + \frac{5}{16}\pi^2 x\sqrt{\pi - c^2\pi x^2}(a+b\arccos(cx)) + \frac{5}{24}\pi x(\pi - c^2\pi x^2)^{3/2}(a+b\arccos(cx)) + \frac{1}{6}x(\pi - c^2\pi x^2)^{5/2}(a+b\arccos$$

output

```
5/32*b*c*Pi^(5/2)*x^2-5/96*b*Pi^(5/2)*(-c^2*x^2+1)^2/c-1/36*b*Pi^(5/2)*(-c
^2*x^2+1)^3/c+5/16*Pi^2*x*(-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x))+5/24*Pi
*x*(-Pi*c^2*x^2+Pi)^(3/2)*(a+b*arccos(c*x))+1/6*x*(-Pi*c^2*x^2+Pi)^(5/2)*(
a+b*arccos(c*x))-5/32*Pi^(5/2)*(a+b*arccos(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.88

$$\int (\pi - c^2\pi x^2)^{5/2} (a + b\arccos(cx))\, dx = \frac{\pi^{5/2}\left(1584acx\sqrt{1-c^2x^2} - 1248ac^3x^3\sqrt{1-c^2x^2} + 384ac^5x^5\sqrt{1-c^2x^2} - 360b\arccos(c\right.}{}$$

input
```
Integrate[(Pi - c^2*Pi*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(Pi^(5/2)*(1584*a*c*x*Sqrt[1 - c^2*x^2] - 1248*a*c^3*x^3*Sqrt[1 - c^2*x^2]
 + 384*a*c^5*x^5*Sqrt[1 - c^2*x^2] - 360*b*ArcCos[c*x]^2 + 720*a*ArcSin[c*
x] + 270*b*Cos[2*ArcCos[c*x]] - 27*b*Cos[4*ArcCos[c*x]] + 2*b*Cos[6*ArcCos
[c*x]] + 12*b*ArcCos[c*x]*(45*Sin[2*ArcCos[c*x]] - 9*Sin[4*ArcCos[c*x]] +
Sin[6*ArcCos[c*x]])))/(2304*c)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5159, 241, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{5}{6}\pi\int \left(\pi-c^2\pi x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}\pi^{5/2}bc\int x\left(1-c^2x^2\right)^2dx+\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{241}$$

$$\frac{5}{6}\pi\int \left(\pi-c^2\pi x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

$$\Big\downarrow \text{5159}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int \sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))dx+\frac{1}{4}\pi^{3/2}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))\right.$$
$$\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

$$\Big\downarrow \text{244}$$

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int\sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))dx+\frac{1}{4}\pi^{3/2}bc\int\left(x-c^2x^3\right)dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)$$
$$\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

$\downarrow$ 2009

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\int\sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}\pi^{3/2}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)+$$
$$\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

$\downarrow$ 5157

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/}\right.$$
$$\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

$\downarrow$ 15

$$\frac{5}{6}\pi\left(\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))+\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a\right.$$
$$\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

$\downarrow$ 5153

$$\frac{1}{6}x\left(\pi-\pi c^2x^2\right)^{5/2}(a+b\arccos(cx))+$$
$$\frac{5}{6}\pi\left(\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))-\frac{\sqrt{\pi}(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}\right.\right.$$
$$\frac{\pi^{5/2}b\left(1-c^2x^2\right)^3}{36c}$$

input

```
Int[(Pi - c^2*Pi*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/36*(b*Pi^(5/2)*(1 - c^2*x^2)^3)/c + (x*(Pi - c^2*Pi*x^2)^(5/2)*(a + b*A
rcCos[c*x]))/6 + (5*Pi*((b*c*Pi^(3/2)*(x^2/2 - (c^2*x^4)/4))/4 + (x*(Pi -
c^2*Pi*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*Pi*((b*c*Sqrt[Pi]*x^2)/4 + (
x*Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]))/2 - (Sqrt[Pi]*(a + b*ArcCos[c
*x])^2)/(4*b*c)))/4))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14

method	result
default	$\frac{ax\left(-\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{6}+\frac{5a\pi x\left(-\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{-\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\arctan\left(\frac{\sqrt{\pi c^2}\,x}{\sqrt{-\pi c^2x^2+\pi}}\right)}{16\sqrt{\pi c^2}}-\frac{b\pi^{\frac{5}{2}}\left(-48\sqrt{-c^2x^2+1}\ a\right.}{}$
parts	$\frac{ax\left(-\pi c^2x^2+\pi\right)^{\frac{5}{2}}}{6}+\frac{5a\pi x\left(-\pi c^2x^2+\pi\right)^{\frac{3}{2}}}{24}+\frac{5a\pi^2x\sqrt{-\pi c^2x^2+\pi}}{16}+\frac{5a\pi^3\arctan\left(\frac{\sqrt{\pi c^2}\,x}{\sqrt{-\pi c^2x^2+\pi}}\right)}{16\sqrt{\pi c^2}}-\frac{b\pi^{\frac{5}{2}}\left(-48\sqrt{-c^2x^2+1}\ a\right.}{}$

input
```
int((-Pi*c^2*x^2+Pi)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(-Pi*c^2*x^2+Pi)^(5/2)+5/24*a*Pi*x*(-Pi*c^2*x^2+Pi)^(3/2)+5/16*a*P
i^2*x*(-Pi*c^2*x^2+Pi)^(1/2)+5/16*a*Pi^3/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1
/2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/288*b*Pi^(5/2)*(-48*(-c^2*x^2+1)^(1/2)*arc
cos(c*x)*x^5*c^5-8*c^6*x^6+156*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+39*c
^4*x^4-198*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-99*c^2*x^2+45*arccos(c*x)^2)
/c
```

Fricas [F]

$$\int \left(\pi - c^2\pi x^2\right)^{5/2}(a + b\arccos(cx))\,dx = \int \left(\pi - \pi c^2x^2\right)^{\frac{5}{2}}(b\arccos(cx) + a)\,dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(pi^2*a*c^4*x^4 - 2*pi^2*a*c^2*x^2 + pi^2*a
 + (pi^2*b*c^4*x^4 - 2*pi^2*b*c^2*x^2 + pi^2*b)*arccos(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

$$\int \left(\pi - c^2\pi x^2\right)^{5/2} (a + b\arccos(cx))\, dx = \begin{cases} \frac{\pi^{\frac{5}{2}} a c^4 x^5 \sqrt{-c^2x^2+1}}{6} - \frac{13\pi^{\frac{5}{2}} a c^2 x^3 \sqrt{-c^2x^2+1}}{24} + \frac{11\pi^{\frac{5}{2}} a x \sqrt{-c^2x^2+1}}{16} - \frac{5\pi^{\frac{5}{2}} a \operatorname{acos}(cx)}{16c} + \frac{\pi^{\frac{5}{2}} b c^5 x^6}{36} + \frac{\pi^{\frac{5}{2}} bc \cdots \\ \pi^{\frac{5}{2}} x \left(a + \frac{\pi b}{2}\right) \end{cases}$$

input
```
integrate((-pi*c**2*x**2+pi)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((pi**(5/2)*a*c**4*x**5*sqrt(-c**2*x**2 + 1)/6 - 13*pi**(5/2)*a*c
**2*x**3*sqrt(-c**2*x**2 + 1)/24 + 11*pi**(5/2)*a*x*sqrt(-c**2*x**2 + 1)/1
6 - 5*pi**(5/2)*a*acos(c*x)/(16*c) + pi**(5/2)*b*c**5*x**6/36 + pi**(5/2)*
b*c**4*x**5*sqrt(-c**2*x**2 + 1)*acos(c*x)/6 - 13*pi**(5/2)*b*c**3*x**4/96
 - 13*pi**(5/2)*b*c**2*x**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/24 + 11*pi**(5/
2)*b*c*x**2/32 + 11*pi**(5/2)*b*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/16 - 5*pi
**(5/2)*b*acos(c*x)**2/(32*c), Ne(c, 0)), (pi**(5/2)*x*(a + pi*b/2), True)
)
```

Maxima [F]

$$\int \left(\pi - c^2\pi x^2\right)^{5/2} (a + b\arccos(cx))\, dx = \int \left(\pi - \pi c^2x^2\right)^{\frac{5}{2}} (b\arccos(cx) + a)\, dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
sqrt(pi)*b*integrate((pi^2*c^4*x^4 - 2*pi^2*c^2*x^2 + pi^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/48*(15*p
i^2*sqrt(pi - pi*c^2*x^2)*x + 10*pi*(pi - pi*c^2*x^2)^(3/2)*x + 8*(pi - pi
*c^2*x^2)^(5/2)*x + 15*pi^(5/2)*arcsin(c*x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi - c^2\pi x^2\right)^{5/2} (a + b \arccos(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-pi*c^2*x^2+pi)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi - c^2\pi x^2\right)^{5/2} (a + b \arccos(cx))\, dx = \int (a + b\,\mathrm{acos}(c\,x))\,\left(\Pi - \Pi\,c^2\,x^2\right)^{5/2}\, dx$$

input
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(\pi - c^2\pi x^2\right)^{5/2} (a + b \arccos(cx))\, dx = \frac{\sqrt{\pi}\,\pi^2\left(15 asin(cx)\, a + 8\sqrt{-c^2x^2+1}\, a\, c^5x^5 - 26\sqrt{-c^2x^2+1}\, a\, c^3x^3 + 33\sqrt{-c^2x^2+1}\, ac\right.}{}$$

input
```
int((-Pi*c^2*x^2+Pi)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(pi)*pi**2*(15*asin(c*x)*a + 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 - 2
6*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 33*sqrt( - c**2*x**2 + 1)*a*c*x + 4
8*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**5 - 96*int(sqrt( - c**
2*x**2 + 1)*acos(c*x)*x**2,x)*b*c**3 + 48*int(sqrt( - c**2*x**2 + 1)*acos(
c*x),x)*b*c))/(48*c)
```

3.38 $\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b \arccos(cx))\, dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b \arccos(cx))\, dx = \frac{3}{16} b c \pi^{3/2} x^2 - \frac{b\pi^{3/2}\left(1 - c^2x^2\right)^2}{16c} + \frac{3}{8}\pi x \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx)) + \frac{1}{4} x \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx)) - \frac{3\pi^{3/2}(a + b\arccos(cx))^2}{16bc}$$

output

```
3/16*b*c*Pi^(3/2)*x^2-1/16*b*Pi^(3/2)*(-c^2*x^2+1)^2/c+3/8*Pi*x*(-Pi*c^2*x
^2+Pi)^(1/2)*(a+b*arccos(c*x))+1/4*x*(-Pi*c^2*x^2+Pi)^(3/2)*(a+b*arccos(c*
x))-3/16*Pi^(3/2)*(a+b*arccos(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b \arccos(cx))\, dx = -\frac{\pi^{3/2}\left(-80acx\sqrt{1 - c^2x^2} + 32ac^3x^3\sqrt{1 - c^2x^2} + 24b\arccos(cx)^2 - 48a\arcsin(cx) - 16b\cos(2\arccos(cx))\right)}{128c}$$

input

```
Integrate[(Pi - c^2*Pi*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/128*(Pi^(3/2)*(-80*a*c*x*Sqrt[1 - c^2*x^2] + 32*a*c^3*x^3*Sqrt[1 - c^2*
x^2] + 24*b*ArcCos[c*x]^2 - 48*a*ArcSin[c*x] - 16*b*Cos[2*ArcCos[c*x]] + b
*Cos[4*ArcCos[c*x]] + 4*b*ArcCos[c*x]*(-8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCo
s[c*x]])))/c
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi-\pi c^2x^2\right)^{3/2} (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{4}\pi\int\sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))dx+\frac{1}{4}\pi^{3/2}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4}\pi\int\sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))dx+\frac{1}{4}\pi^{3/2}bc\int\left(x-c^2x^3\right)dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}\pi\int\sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}\pi^{3/2}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)$$

$$\Big\downarrow \text{5157}$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}\pi^{3/2}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))+\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}\pi^{3/2}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\pi\left(\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))-\frac{\sqrt{\pi}(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}\sqrt{\pi}bcx^2\right)+\frac{1}{4}\pi^{3/2}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)$$

input
```
Int[(Pi - c^2*Pi*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*Pi^(3/2)*(x^2/2 - (c^2*x^4)/4))/4 + (x*(Pi - c^2*Pi*x^2)^(3/2)*(a + b
*ArcCos[c*x]))/4 + (3*Pi*((b*c*Sqrt[Pi]*x^2)/4 + (x*Sqrt[Pi - c^2*Pi*x^2]*
(a + b*ArcCos[c*x]))/2 - (Sqrt[Pi]*(a + b*ArcCos[c*x])^2)/(4*b*c)))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.25

method	result
default	$\frac{a x \left(-\pi c^{2} x^{2}+\pi \right)^{\frac{3}{2}}}{4}+\frac{3 a \pi x \sqrt{-\pi c^{2} x^{2}+\pi}}{8}+\frac{3 a \,\pi^{2} \arctan\left(\frac{\sqrt{\pi c^{2}}\, x}{\sqrt{-\pi c^{2} x^{2}+\pi}}\right)}{8 \sqrt{\pi c^{2}}}-\frac{b \,\pi^{\frac{3}{2}} \left(16 \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x \right) x^{3} c^{3}+4 c^{4} x^{4}-40 \sqrt{\cdots}\right.}{\cdots}$
parts	$\frac{a x \left(-\pi c^{2} x^{2}+\pi \right)^{\frac{3}{2}}}{4}+\frac{3 a \pi x \sqrt{-\pi c^{2} x^{2}+\pi}}{8}+\frac{3 a \,\pi^{2} \arctan\left(\frac{\sqrt{\pi c^{2}}\, x}{\sqrt{-\pi c^{2} x^{2}+\pi}}\right)}{8 \sqrt{\pi c^{2}}}-\frac{b \,\pi^{\frac{3}{2}} \left(16 \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x \right) x^{3} c^{3}+4 c^{4} x^{4}-40 \sqrt{\cdots}\right.}{\cdots}$

input
```
int((-Pi*c^2*x^2+Pi)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(-Pi*c^2*x^2+Pi)^(3/2)+3/8*a*Pi*x*(-Pi*c^2*x^2+Pi)^(1/2)+3/8*a*Pi^
2/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1/2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/64*b*Pi
^(3/2)*(16*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+4*c^4*x^4-40*(-c^2*x^2+1
)^(1/2)*arccos(c*x)*x*c-20*c^2*x^2+12*arccos(c*x)^2+25)/c
```

Fricas [F]

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx))\, dx = \int \left(\pi - \pi c^2 x^2\right)^{\frac{3}{2}} (b\arccos(cx) + a)\, dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(pi - pi*c^2*x^2)*(pi*a*c^2*x^2 - pi*a + (pi*b*c^2*x^2 - pi*
b)*arccos(c*x)), x)
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.56

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx))\, dx = \begin{cases} -\frac{\pi^{\frac{3}{2}}ac^2x^3\sqrt{-c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}}ax\sqrt{-c^2x^2+1}}{8} - \frac{3\pi^{\frac{3}{2}}a\operatorname{acos}(cx)}{8c} - \frac{\pi^{\frac{3}{2}}bc^3x^4}{16} - \frac{\pi^{\frac{3}{2}}bc^2x^3\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{4} \cdots \\ \pi^{\frac{3}{2}}x\left(a + \frac{\pi b}{2}\right) \end{cases}$$

input
```
integrate((-pi*c**2*x**2+pi)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-pi**(3/2)*a*c**2*x**3*sqrt(-c**2*x**2 + 1)/4 + 5*pi**(3/2)*a*x
*sqrt(-c**2*x**2 + 1)/8 - 3*pi**(3/2)*a*acos(c*x)/(8*c) - pi**(3/2)*b*c**3
*x**4/16 - pi**(3/2)*b*c**2*x**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/4 + 5*pi**
(3/2)*b*c*x**2/16 + 5*pi**(3/2)*b*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/8 - 3*p
i**(3/2)*b*acos(c*x)**2/(16*c), Ne(c, 0)), (pi**(3/2)*x*(a + pi*b/2), True
))
```

Maxima [F]

$$\int \left(\pi - c^2\pi x^2\right)^{3/2}\left(a + b\arccos(cx)\right)dx = \int \left(\pi - \pi c^2x^2\right)^{\frac{3}{2}}\left(b\arccos\left(cx\right) + a\right)dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
sqrt(pi)*b*integrate((pi - pi*c^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/8*(3*pi*sqrt(pi - pi*c^2*x^2)
*x + 2*(pi - pi*c^2*x^2)^(3/2)*x + 3*pi^(3/2)*arcsin(c*x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi - c^2\pi x^2\right)^{3/2}\left(a + b\arccos(cx)\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate((-pi*c^2*x^2+pi)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi - c^2\pi x^2\right)^{3/2}\left(a + b\arccos(cx)\right)dx = \int \left(a + b\,\mathrm{acos}(c\,x)\right)\,\left(\Pi - \Pi\,c^2\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx))\, dx = \frac{\sqrt{\pi}\,\pi\left(3asin(cx)\,a - 2\sqrt{-c^2x^2+1}\,a\,c^3x^3 + 5\sqrt{-c^2x^2+1}\,acx - 8\left(\int \sqrt{-c^2x^2+1}\,acos(\right.}{8c}$$

input
```
int((-Pi*c^2*x^2+Pi)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(pi)*pi*(3*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 5*sqr
t( - c**2*x**2 + 1)*a*c*x - 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)
*b*c**3 + 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b*c))/(8*c)
```

3.39 $\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \frac{1}{4}bc\sqrt{\pi}x^2 + \frac{1}{2}x\sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx)) - \frac{\sqrt{\pi}(a + b\arccos(cx))^2}{4bc}$$

output
```
1/4*b*c*Pi^(1/2)*x^2+1/2*x*(-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x))-1/4*Pi
^(1/2)*(a+b*arccos(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \frac{\sqrt{\pi}\left(4acx\sqrt{1 - c^2x^2} - 2b\arccos(cx)^2 + 4a\arcsin(cx) + b\cos(2\arccos(cx)) + 2b\arccos(cx)\sin(2\arccos(c\right.}{8c}$$

input
```
Integrate[Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[Pi]*(4*a*c*x*Sqrt[1 - c^2*x^2] - 2*b*ArcCos[c*x]^2 + 4*a*ArcSin[c*x]
 + b*Cos[2*ArcCos[c*x]] + 2*b*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/(8*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}\sqrt{\pi}bc\int xdx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))+\frac{1}{4}\sqrt{\pi}bcx^2$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))-\frac{\sqrt{\pi}(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}\sqrt{\pi}bcx^2$$

input
```
Int[Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*Sqrt[Pi]*x^2)/4 + (x*Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]))/2 - (
Sqrt[Pi]*(a + b*ArcCos[c*x])^2)/(4*b*c)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{ax\sqrt{-\pi\,c^{2}x^{2}+\pi}}{2}+\frac{a\pi\arctan\left(\frac{\sqrt{\pi\,c^{2}}\,x}{\sqrt{-\pi\,c^{2}x^{2}+\pi}}\right)}{2\sqrt{\pi\,c^{2}}}-\frac{b\sqrt{\pi}\left(-2\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-c^{2}x^{2}+\arccos\left(cx\right)^{2}+1\right)}{4c}$	98
parts	$\frac{ax\sqrt{-\pi\,c^{2}x^{2}+\pi}}{2}+\frac{a\pi\arctan\left(\frac{\sqrt{\pi\,c^{2}}\,x}{\sqrt{-\pi\,c^{2}x^{2}+\pi}}\right)}{2\sqrt{\pi\,c^{2}}}-\frac{b\sqrt{\pi}\left(-2\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-c^{2}x^{2}+\arccos\left(cx\right)^{2}+1\right)}{4c}$	98

input
```
int((-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(-Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1/
2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/4*b*Pi^(1/2)*(-2*(-c^2*x^2+1)^(1/2)*arccos(
c*x)*x*c-c^2*x^2+arccos(c*x)^2+1)/c
```

Fricas [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b \arccos(cx)) \, dx = \int \sqrt{\pi - \pi c^2x^2}(b \arccos(cx) + a) \, dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(b*arccos(c*x) + a), x)
```

Sympy [A] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int \sqrt{\pi - c^2\pi x^2}(a + b \arccos(cx)) \, dx = \begin{cases} \frac{\sqrt{\pi}a\left(\frac{cx\sqrt{-c^2x^2+1}}{2} + \frac{\operatorname{asin}(cx)}{2}\right) + \sqrt{\pi}b\left(\frac{c^2x^2}{4} + \left(\frac{cx\sqrt{-c^2x^2+1}}{2} + \frac{\operatorname{asin}(cx)}{2}\right)\operatorname{acos}(cx) + \frac{\operatorname{asin}^2(cx)}{4}\right)}{c} & \text{for } c \neq 0 \\ \sqrt{\pi}x\left(a + \frac{\pi b}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-pi*c**2*x**2+pi)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Piecewise(((sqrt(pi)*a*(c*x*sqrt(-c**2*x**2 + 1)/2 + asin(c*x)/2) + sqrt(p
i)*b*(c**2*x**2/4 + (c*x*sqrt(-c**2*x**2 + 1)/2 + asin(c*x)/2)*acos(c*x) +
 asin(c*x)**2/4))/c, Ne(c, 0)), (sqrt(pi)*x*(a + pi*b/2), True))
```

Maxima [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b \arccos(cx)) \, dx = \int \sqrt{\pi - \pi c^2x^2}(b \arccos(cx) + a) \, dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
sqrt(pi)*b*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x), x) + 1/2*(sqrt(pi - pi*c^2*x^2)*x + sqrt(pi)*arcsin(c*
x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \int (a + b\,\mathrm{acos}(c\,x))\,\sqrt{\Pi - \Pi\,c^2\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx$$
$$= \frac{\sqrt{\pi}\left(asin(cx)\,a + \sqrt{-c^2x^2+1}\,acx + 2\left(\int \sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((-Pi*c^2*x^2+Pi)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(pi)*(asin(c*x)*a + sqrt( - c**2*x**2 + 1)*a*c*x + 2*int(sqrt( - c**2
*x**2 + 1)*acos(c*x),x)*b*c))/(2*c)
```

3.40 $\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx=-\frac{(a+b\arccos(cx))^2}{2bc\sqrt{\pi}}$$

output
```
-1/2*(a+b*arccos(c*x))^2/b/c/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx=-\frac{(a+b\arccos(cx))^2}{2bc\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[Pi - c^2*Pi*x^2],x]
```

output
```
-1/2*(a + b*ArcCos[c*x])^2/(b*c*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-\pi c^2x^2}}\,dx$$

$$\Big\downarrow \; 5153$$

$$-\frac{(a+b\arccos(cx))^2}{2\sqrt{\pi}bc}$$

input
```
Int[(a + b*ArcCos[c*x])/Sqrt[Pi - c^2*Pi*x^2],x]
```

output
```
-1/2*(a + b*ArcCos[c*x])^2/(b*c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

method	result	size
default	$\frac{a \arctan\left(\frac{\sqrt{\pi c^2}\, x}{\sqrt{-\pi c^2 x^2+\pi}}\right)}{\sqrt{\pi c^2}}-\frac{b \arccos(cx)^2}{2c\sqrt{\pi}}$	49
parts	$\frac{a \arctan\left(\frac{\sqrt{\pi c^2}\, x}{\sqrt{-\pi c^2 x^2+\pi}}\right)}{\sqrt{\pi c^2}}-\frac{b \arccos(cx)^2}{2c\sqrt{\pi}}$	49

input
```
int((a+b*arccos(c*x))/(-Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1/2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/2*b/c/P
i^(1/2)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx=\int \frac{b\arccos(cx)+a}{\sqrt{\pi-\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/sqrt(pi - pi*c^2*x^2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(20) = 40.

Time = 3.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.88

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx$$
$$=\begin{cases} a\left(\begin{cases}\frac{\log\left(-2\pi c^2x+2\sqrt{\pi}\sqrt{-c^2}\sqrt{-\pi c^2x^2+\pi}\right)}{\sqrt{\pi}\sqrt{-c^2}} & \text{for}\ \pi c^2\neq 0\\ \frac{x}{\sqrt{\pi}} & \text{otherwise}\end{cases}\right) & \text{for}\ b=0\\ \frac{x\left(a+\frac{\pi b}{2}\right)}{\sqrt{\pi}} & \text{for}\ c=0\\ -\frac{(a+b\,\mathrm{acos}\,(cx))^2}{2\sqrt{\pi}bc} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*acos(c*x))/(-pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((a*Piecewise((log(-2*pi*c**2*x + 2*sqrt(pi)*sqrt(-c**2)*sqrt(-pi
*c**2*x**2 + pi))/(sqrt(pi)*sqrt(-c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi), Tr
ue)), Eq(b, 0)), (x*(a + pi*b/2)/sqrt(pi), Eq(c, 0)), (-(a + b*acos(c*x))*
*2/(2*sqrt(pi)*b*c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(21) = 42$.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx = \frac{b\arccos(cx)\arcsin(cx)}{\sqrt{\pi}c} + \frac{b\arcsin(cx)^2}{2\sqrt{\pi}c} + \frac{a\arcsin(cx)}{\sqrt{\pi}c}$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima")
```

output
```
b*arccos(c*x)*arcsin(c*x)/(sqrt(pi)*c) + 1/2*b*arcsin(c*x)^2/(sqrt(pi)*c)
+ a*arcsin(c*x)/(sqrt(pi)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{\Pi-\Pi\,c^2\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{a+b\arccos(cx)}{\sqrt{\pi-c^2\pi x^2}}\,dx=\frac{\sqrt{\pi}\left(-acos(cx)^2\,b+2asin(cx)\,a\right)}{2c\pi}$$

input
```
int((a+b*acos(c*x))/(-Pi*c^2*x^2+Pi)^(1/2),x)
```

output
```
(sqrt(pi)*( - acos(c*x)**2*b + 2*asin(c*x)*a))/(2*c*pi)
```

3.41 $\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-c^2\pi x^2}} - \frac{b\log(1-c^2x^2)}{2c\pi^{3/2}}$$

output
```
x*(a+b*arccos(c*x))/Pi/(-Pi*c^2*x^2+Pi)^(1/2)-1/2*b*ln(-c^2*x^2+1)/c/Pi^(3
/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \frac{-2acx\sqrt{1-c^2x^2} - 2bcx\sqrt{1-c^2x^2}\arccos(cx) + (b-bc^2x^2)\log(1-c^2x^2)}{2c\pi^{3/2}(-1+c^2x^2)}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(Pi - c^2*Pi*x^2)^(3/2),x]
```

output
```
(-2*a*c*x*Sqrt[1 - c^2*x^2] - 2*b*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + (b -
 b*c^2*x^2)*Log[1 - c^2*x^2])/(2*c*Pi^(3/2)*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(\pi-\pi c^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 5161$$

$$\frac{bc\int \frac{x}{1-c^2x^2}dx}{\pi^{3/2}}+\frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-\pi c^2x^2}}$$

$$\Big\downarrow \; 240$$

$$\frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{b\log\left(1-c^2x^2\right)}{2\pi^{3/2}c}$$

input
```
Int[(a + b*ArcCos[c*x])/(Pi - c^2*Pi*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(Pi*Sqrt[Pi - c^2*Pi*x^2]) - (b*Log[1 - c^2*x^2])/
(2*c*Pi^(3/2))
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{ax}{\pi\sqrt{-\pi\,c^2x^2+\pi}}-\frac{b\left(\ln\left(-c^2x^2+1\right)x^2c^2+2\sqrt{-c^2x^2+1}\,\arccos\left(cx\right)xc-\ln\left(-c^2x^2+1\right)\right)}{2c\,\pi^{\frac{3}{2}}\left(c^2x^2-1\right)}$	93
parts	$\frac{ax}{\pi\sqrt{-\pi\,c^2x^2+\pi}}-\frac{b\left(\ln\left(-c^2x^2+1\right)x^2c^2+2\sqrt{-c^2x^2+1}\,\arccos\left(cx\right)xc-\ln\left(-c^2x^2+1\right)\right)}{2c\,\pi^{\frac{3}{2}}\left(c^2x^2-1\right)}$	93

input
```
int((a+b*arccos(c*x))/(-Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/Pi*x/(-Pi*c^2*x^2+Pi)^(1/2)-1/2*b/c/Pi^(3/2)*(ln(-c^2*x^2+1)*x^2*c^2+2*(
-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-ln(-c^2*x^2+1))/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(\pi-\pi c^2x^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(b*arccos(c*x) + a)/(pi^2*c^4*x^4 - 2*pi^2*
c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{3/2}}\,dx=\frac{\int \frac{a}{-c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx+\int \frac{b\operatorname{acos}\left(cx\right)}{-c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*acos(c*x))/(-pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a/(-c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c**2*x**2 + 1)), x) +
 Integral(b*acos(c*x)/(-c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c**2*x**2 +
 1)), x))/pi**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}\,dx=\frac{bx\arccos{(cx)}}{\pi\sqrt{\pi-\pi c^2x^2}}+\frac{ax}{\pi\sqrt{\pi-\pi c^2x^2}}+\frac{b\log\left(x^2-\frac{1}{c^2}\right)}{2\,\pi^{\frac{3}{2}}c}$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arccos(c*x)/(pi*sqrt(pi - pi*c^2*x^2)) + a*x/(pi*sqrt(pi - pi*c^2*x^2)
) + 1/2*b*log(x^2 - 1/c^2)/(pi^(3/2)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{\left(\Pi-\Pi\,c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{3/2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b+ax}{\sqrt{\pi}\,\sqrt{-c^2x^2+1}\,\pi}$$

input
```
int((a+b*acos(c*x))/(-Pi*c^2*x^2+Pi)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2
 - sqrt( - c**2*x**2 + 1)),x)*b + a*x)/(sqrt(pi)*sqrt( - c**2*x**2 + 1)*pi
)
```

3.42 $\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \frac{b}{6c\pi^{5/2}(1-c^2x^2)} + \frac{x(a+b\arccos(cx))}{3\pi(\pi-c^2\pi x^2)^{3/2}} + \frac{2x(a+b\arccos(cx))}{3\pi^2\sqrt{\pi-c^2\pi x^2}} - \frac{b\log(1-c^2x^2)}{3c\pi^{5/2}}$$

output
```
1/6*b/c/Pi^(5/2)/(-c^2*x^2+1)+1/3*x*(a+b*arccos(c*x))/Pi/(-Pi*c^2*x^2+Pi)^
(3/2)+2/3*x*(a+b*arccos(c*x))/Pi^2/(-Pi*c^2*x^2+Pi)^(1/2)-1/3*b*ln(-c^2*x^
2+1)/c/Pi^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \frac{b-bc^2x^2+6acx\sqrt{1-c^2x^2}-4ac^3x^3\sqrt{1-c^2x^2}-2bcx\sqrt{1-c^2x^2}(-3+2c^2x^2)\,\mathrm{ar}}{6c\pi^{5/2}(-1+c^2x^2)^2}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(Pi - c^2*Pi*x^2)^(5/2),x]
```

output
```
(b - b*c^2*x^2 + 6*a*c*x*Sqrt[1 - c^2*x^2] - 4*a*c^3*x^3*Sqrt[1 - c^2*x^2]
 - 2*b*c*x*Sqrt[1 - c^2*x^2]*(-3 + 2*c^2*x^2)*ArcCos[c*x] - 2*b*(-1 + c^2*
x^2)^2*Log[1 - c^2*x^2])/(6*c*Pi^(5/2)*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(\pi-\pi c^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{2\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}dx}{3\pi}+\frac{bc\int \frac{x}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\arccos(cx))}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{3/2}}dx}{3\pi}+\frac{x(a+b\arccos(cx))}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}+\frac{b}{6\pi^{5/2}c\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{5161}$$

$$\frac{2\left(\frac{bc\int \frac{x}{1-c^2x^2}dx}{\pi^{3/2}}+\frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-\pi c^2x^2}}\right)}{3\pi}+\frac{x(a+b\arccos(cx))}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}+\frac{b}{6\pi^{5/2}c\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\arccos(cx))}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{b\log\left(1-c^2x^2\right)}{2\pi^{3/2}c}\right)}{3\pi}+\frac{b}{6\pi^{5/2}c\left(1-c^2x^2\right)}$$

input
```
Int[(a + b*ArcCos[c*x])/(Pi - c^2*Pi*x^2)^(5/2),x]
```

output
```
b/(6*c*Pi^(5/2)*(1 - c^2*x^2)) + (x*(a + b*ArcCos[c*x]))/(3*Pi*(Pi - c^2*P
i*x^2)^(3/2)) + (2*((x*(a + b*ArcCos[c*x]))/(Pi*Sqrt[Pi - c^2*Pi*x^2]) - (
b*Log[1 - c^2*x^2])/(2*c*Pi^(3/2))))/(3*Pi)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.49

method	result
default	$a\left(\frac{x}{3\pi(-\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{-\pi c^2x^2+\pi}}\right)-\frac{b\left(2\ln(-c^2x^2+1)x^4c^4+4\sqrt{-c^2x^2+1}\,\arccos(cx)x^3c^3-4\ln(-c^2x^2+1)x^2c^2-6\sqrt{}\right.}{6c\,\pi^{\frac{5}{2}}(c^2x^2-1)^2}$
parts	$a\left(\frac{x}{3\pi(-\pi c^2x^2+\pi)^{\frac{3}{2}}}+\frac{2x}{3\pi^2\sqrt{-\pi c^2x^2+\pi}}\right)-\frac{b\left(2\ln(-c^2x^2+1)x^4c^4+4\sqrt{-c^2x^2+1}\,\arccos(cx)x^3c^3-4\ln(-c^2x^2+1)x^2c^2-6\sqrt{}\right.}{6c\,\pi^{\frac{5}{2}}(c^2x^2-1)^2}$

input
```
int((a+b*arccos(c*x))/(-Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/Pi*x/(-Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/6*
b/c/Pi^(5/2)*(2*ln(-c^2*x^2+1)*x^4*c^4+4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^
3*c^3-4*ln(-c^2*x^2+1)*x^2*c^2-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+c^2*x^
2+2*ln(-c^2*x^2+1)-1)/(c^2*x^2-1)^2
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(\pi-\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(pi - pi*c^2*x^2)*(b*arccos(c*x) + a)/(pi^3*c^6*x^6 - 3*pi^3
*c^4*x^4 + 3*pi^3*c^2*x^2 - pi^3), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{\left(\pi - c^2\pi x^2\right)^{5/2}}\,dx = \frac{\int \frac{a}{c^4x^4\sqrt{-c^2x^2+1}-2c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^4x^4\sqrt{-c^2x^2+1}-2c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*acos(c*x))/(-pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a/(c**4*x**4*sqrt(-c**2*x**2 + 1) - 2*c**2*x**2*sqrt(-c**2*x**2
+ 1) + sqrt(-c**2*x**2 + 1)), x) + Integral(b*acos(c*x)/(c**4*x**4*sqrt(-c
**2*x**2 + 1) - 2*c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c**2*x**2 + 1)),
x))/pi**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.26

$$\int \frac{a + b\arccos(cx)}{\left(\pi - c^2\pi x^2\right)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{1}{\pi^{\frac{5}{2}}c^4x^2 - \pi^{\frac{5}{2}}c^2} + \frac{2\,\log\left(cx+1\right)}{\pi^{\frac{5}{2}}c^2} + \frac{2\,\log\left(cx-1\right)}{\pi^{\frac{5}{2}}c^2}\right) + \frac{1}{3}\,b\left(\frac{x}{\pi\left(\pi - \pi c^2x^2\right)^{\frac{3}{2}}} + \frac{2\,x}{\pi^2\sqrt{\pi - \pi c^2x^2}}\right)\arccos\left(cx\right) + \frac{1}{3}\,a\left(\frac{x}{\pi\left(\pi - \pi c^2x^2\right)^{\frac{3}{2}}} + \frac{2\,x}{\pi^2\sqrt{\pi - \pi c^2x^2}}\right)$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b*c*(1/(pi^(5/2)*c^4*x^2 - pi^(5/2)*c^2) + 2*log(c*x + 1)/(pi^(5/2)*c
^2) + 2*log(c*x - 1)/(pi^(5/2)*c^2)) + 1/3*b*(x/(pi*(pi - pi*c^2*x^2)^(3/2
)) + 2*x/(pi^2*sqrt(pi - pi*c^2*x^2)))*arccos(c*x) + 1/3*a*(x/(pi*(pi - pi
*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi - pi*c^2*x^2)))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(\Pi-\Pi\,c^2\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^2x^2-3\sqrt{-c^2x^2+1}\,(}{3\sqrt{\pi}\,\sqrt{-c^2x^2+1}\,\pi^2\,(c^2x^}$$

input
```
int((a+b*acos(c*x))/(-Pi*c^2*x^2+Pi)^(5/2),x)
```

output

```
(3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b*c**2*x
**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*
x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b +
 2*a*c**2*x**3 - 3*a*x)/(3*sqrt(pi)*sqrt( - c**2*x**2 + 1)*pi**2*(c**2*x**
2 - 1))
```

3.43 $\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{7/2}}\,dx = \frac{b}{20c\pi^{7/2}(1-c^2x^2)^2}+\frac{2b}{15c\pi^{7/2}(1-c^2x^2)} + \frac{x(a+b\arccos(cx))}{5\pi(\pi-c^2\pi x^2)^{5/2}}+\frac{4x(a+b\arccos(cx))}{15\pi^2(\pi-c^2\pi x^2)^{3/2}} + \frac{8x(a+b\arccos(cx))}{15\pi^3\sqrt{\pi-c^2\pi x^2}}-\frac{4b\log(1-c^2x^2)}{15c\pi^{7/2}}$$

output
```
1/20*b/c/Pi^(7/2)/(-c^2*x^2+1)^2+2/15*b/c/Pi^(7/2)/(-c^2*x^2+1)+1/5*x*(a+b
*arccos(c*x))/Pi/(-Pi*c^2*x^2+Pi)^(5/2)+4/15*x*(a+b*arccos(c*x))/Pi^2/(-Pi
*c^2*x^2+Pi)^(3/2)+8/15*x*(a+b*arccos(c*x))/Pi^3/(-Pi*c^2*x^2+Pi)^(1/2)-4/
15*b*ln(-c^2*x^2+1)/c/Pi^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{7/2}}\,dx = \frac{-11b+19bc^2x^2-8bc^4x^4-60acx\sqrt{1-c^2x^2}+80ac^3x^3\sqrt{1-c^2x^2}-32ac^5x^5\sqrt{1-}}{60c\pi^7}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(Pi - c^2*Pi*x^2)^(7/2),x]
```

output
```
(-11*b + 19*b*c^2*x^2 - 8*b*c^4*x^4 - 60*a*c*x*Sqrt[1 - c^2*x^2] + 80*a*c^
3*x^3*Sqrt[1 - c^2*x^2] - 32*a*c^5*x^5*Sqrt[1 - c^2*x^2] - 4*b*c*x*Sqrt[1
- c^2*x^2]*(15 - 20*c^2*x^2 + 8*c^4*x^4)*ArcCos[c*x] - 16*b*(-1 + c^2*x^2)
^3*Log[1 - c^2*x^2])/(60*c*Pi^(7/2)*(-1 + c^2*x^2)^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5163, 241, 5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\left(\pi-\pi c^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \; 5163$$

$$\frac{4\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{5/2}}dx}{5\pi}+\frac{bc\int \frac{x}{\left(1-c^2x^2\right)^3}dx}{5\pi^{7/2}}+\frac{x(a+b\arccos(cx))}{5\pi\left(\pi-\pi c^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 241$$

$$\frac{4\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{5/2}}dx}{5\pi}+\frac{x(a+b\arccos(cx))}{5\pi\left(\pi-\pi c^2x^2\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \; 5163$$

$$\frac{4\left(\frac{2\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{3/2}}dx}{3\pi}+\frac{bc\int \frac{x}{\left(1-c^2x^2\right)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\arccos(cx))}{3\pi(\pi-\pi c^2x^2)^{3/2}}\right)}{5\pi}+\frac{x(a+b\arccos(cx))}{5\pi\left(\pi-\pi c^2x^2\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\left(\frac{2\int \frac{a+b\arccos(cx)}{\left(\pi-c^2\pi x^2\right)^{3/2}}dx}{3\pi}+\frac{x(a+b\arccos(cx))}{3\pi(\pi-\pi c^2x^2)^{3/2}}+\frac{b}{6\pi^{5/2}c(1-c^2x^2)}\right)}{5\pi}+\frac{x(a+b\arccos(cx))}{5\pi\left(\pi-\pi c^2x^2\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{5161}$$

$$\frac{4\left(\frac{2\left(\frac{bc\int \frac{x}{1-c^2x^2}dx}{\pi^{3/2}}+\frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-\pi c^2x^2}}\right)}{3\pi}+\frac{x(a+b\arccos(cx))}{3\pi(\pi-\pi c^2x^2)^{3/2}}+\frac{b}{6\pi^{5/2}c(1-c^2x^2)}\right)}{5\pi}+\frac{x(a+b\arccos(cx))}{5\pi\left(\pi-\pi c^2x^2\right)^{5/2}}+\frac{b}{20\pi^{7/2}c\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\arccos(cx))}{5\pi\left(\pi-\pi c^2x^2\right)^{5/2}}+\frac{4\left(\frac{x(a+b\arccos(cx))}{3\pi(\pi-\pi c^2x^2)^{3/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{b\log\left(1-c^2x^2\right)}{2\pi^{3/2}c}\right)}{3\pi}+\frac{b}{6\pi^{5/2}c(1-c^2x^2)}\right)}{5\pi}+\frac{b}{20\pi^{7/2}c\left(1-c^2x^2\right)^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(Pi - c^2*Pi*x^2)^(7/2),x]
```

output
```
b/(20*c*Pi^(7/2)*(1 - c^2*x^2)^2) + (x*(a + b*ArcCos[c*x]))/(5*Pi*(Pi - c^
2*Pi*x^2)^(5/2)) + (4*(b/(6*c*Pi^(5/2)*(1 - c^2*x^2)) + (x*(a + b*ArcCos[c
*x]))/(3*Pi*(Pi - c^2*Pi*x^2)^(3/2)) + (2*((x*(a + b*ArcCos[c*x]))/(Pi*Sqr
t[Pi - c^2*Pi*x^2]) - (b*Log[1 - c^2*x^2])/(2*c*Pi^(3/2))))/(3*Pi)))/(5*Pi
)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.45

method	result
default	$a\left(\frac{x}{5\pi(-\pi c^2x^2+\pi)^{\frac{5}{2}}}+\frac{\frac{4x}{15\pi\left(-\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{8x}{15\pi^2\sqrt{-\pi c^2x^2+\pi}}}{\pi}\right)-\frac{b\left(16\ln(-c^2x^2+1)x^6c^6+32\sqrt{-c^2x^2+1}\,\arccos(cx)x^5c^5-\right.}{}$
parts	$a\left(\frac{x}{5\pi(-\pi c^2x^2+\pi)^{\frac{5}{2}}}+\frac{\frac{4x}{15\pi\left(-\pi c^2x^2+\pi\right)^{\frac{3}{2}}}+\frac{8x}{15\pi^2\sqrt{-\pi c^2x^2+\pi}}}{\pi}\right)-\frac{b\left(16\ln(-c^2x^2+1)x^6c^6+32\sqrt{-c^2x^2+1}\,\arccos(cx)x^5c^5-\right.}{}$

input
```
int((a+b*arccos(c*x))/(-Pi*c^2*x^2+Pi)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5/Pi*x/(-Pi*c^2*x^2+Pi)^(5/2)+4/5/Pi*(1/3/Pi*x/(-Pi*c^2*x^2+Pi)^(3/2)
+2/3/Pi^2*x/(-Pi*c^2*x^2+Pi)^(1/2)))-1/60*b/c/Pi^(7/2)*(16*ln(-c^2*x^2+1)*
x^6*c^6+32*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^5*c^5-48*ln(-c^2*x^2+1)*x^4*c^
4-80*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+8*c^4*x^4+48*ln(-c^2*x^2+1)*x^
2*c^2+60*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-19*c^2*x^2-16*ln(-c^2*x^2+1)+1
1)/(c^2*x^2-1)^3
```

Fricas [F]

$$\int \frac{a + b\arccos(cx)}{(\pi - c^2\pi x^2)^{7/2}}\,dx = \int \frac{b\arccos(cx) + a}{(\pi - \pi c^2x^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(b*arccos(c*x) + a)/(pi^4*c^8*x^8 - 4*pi^4*
c^6*x^6 + 6*pi^4*c^4*x^4 - 4*pi^4*c^2*x^2 + pi^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arccos(cx)}{(\pi - c^2\pi x^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(-pi*c**2*x**2+pi)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{7/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(\pi-\pi c^2x^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(3*x/(pi*(pi - pi*c^2*x^2)^(5/2)) + 4*x/(pi^2*(pi - pi*c^2*x^2)^(3/
2)) + 8*x/(pi^3*sqrt(pi - pi*c^2*x^2))) - b*integrate(arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x)/((pi^3*c^6*x^6 - 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 -
pi^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(pi)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))/(-pi*c^2*x^2+pi)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{7/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{{(\Pi-\Pi\,c^2\,x^2)}^{7/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(7/2),x)
```

output
```
int((a + b*acos(c*x))/(Pi - Pi*c^2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(\pi-c^2\pi x^2)^{7/2}}\,dx = \frac{-15\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^6x^6-3\sqrt{-c^2x^2+1}\,c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^4x^4}{}$$

input
```
int((a+b*acos(c*x))/(-Pi*c^2*x^2+Pi)^(7/2),x)
```

output
```
( - 15*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**6*x
**6 - 3*sqrt( - c**2*x**2 + 1)*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*c**2*x
**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**4*x**4 + 30*sqrt( - c**2*x**2 + 1)*i
nt(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*
c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x
)*b*c**2*x**2 - 15*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2
 + 1)*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*c**4*x**4 + 3*sqrt( - c**2*x**2
 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b + 8*a*c**4*x**5 - 20*a*c**2
*x**3 + 15*a*x)/(15*sqrt(pi)*sqrt( - c**2*x**2 + 1)*pi**3*(c**4*x**4 - 2*c
**2*x**2 + 1))
```

3.44 $\int (\pi - c^2\pi x^2)^{3/2} (a + b \arccos(cx))^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 215

$$\int (\pi - c^2\pi x^2)^{3/2} (a + b \arccos(cx))^2 \, dx = -\frac{15}{64} b^2 \pi^{3/2} x \sqrt{1 - c^2 x^2} - \frac{1}{32} b^2 \pi^{3/2} x (1 - c^2 x^2)^{3/2} + \frac{3}{8} b c \pi^{3/2} x^2 (a + b \arccos(cx)) - \frac{b \pi^{3/2} (1 - c^2 x^2)^2 (a + b \arccos(cx))}{8c} + \frac{3}{8} \pi x \sqrt{\pi - c^2$$

output
```
-15/64*b^2*Pi^(3/2)*x*(-c^2*x^2+1)^(1/2)-1/32*b^2*Pi^(3/2)*x*(-c^2*x^2+1)^
(3/2)+3/8*b*c*Pi^(3/2)*x^2*(a+b*arccos(c*x))-1/8*b*Pi^(3/2)*(-c^2*x^2+1)^2
*(a+b*arccos(c*x))/c+3/8*Pi*x*(-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x))^2+1
/4*x*(-Pi*c^2*x^2+Pi)^(3/2)*(a+b*arccos(c*x))^2-1/8*Pi^(3/2)*(a+b*arccos(c
*x))^3/b/c+9/64*b^2*Pi^(3/2)*arcsin(c*x)/c
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int (\pi - c^2\pi x^2)^{3/2} (a + b \arccos(cx))^2 \, dx = \frac{\pi^{3/2} \left(160 a^2 c x \sqrt{1 - c^2 x^2} - 64 a^2 c^3 x^3 \sqrt{1 - c^2 x^2} - 32 b^2 \arccos(cx)^3 + 96 a^2 \arcsin(cx) +$$

input
```
Integrate[(Pi - c^2*Pi*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Pi^(3/2)*(160*a^2*c*x*Sqrt[1 - c^2*x^2] - 64*a^2*c^3*x^3*Sqrt[1 - c^2*x^2
] - 32*b^2*ArcCos[c*x]^3 + 96*a^2*ArcSin[c*x] + 64*a*b*Cos[2*ArcCos[c*x]]
- 4*a*b*Cos[4*ArcCos[c*x]] - 32*b^2*Sin[2*ArcCos[c*x]] + b^2*Sin[4*ArcCos[
c*x]] - 8*b*ArcCos[c*x]^2*(12*a - 8*b*Sin[2*ArcCos[c*x]] + b*Sin[4*ArcCos[
c*x]]) - 4*b*ArcCos[c*x]*(-16*b*Cos[2*ArcCos[c*x]] + b*Cos[4*ArcCos[c*x]]
+ 4*a*(-8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]]))))/(256*c)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\pi-\pi c^2x^2\right)^{3/2} (a+b\arccos(cx))^2 dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{1}{2}\pi^{3/2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\pi\int \sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5157}$$

$$\frac{1}{2}\pi^{3/2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\sqrt{\pi}bc\int x(a+b\arccos(cx))dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2\right)+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{2}\pi^{3/2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+$$
$$\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a$$
$$\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\downarrow 262$$

$$\frac{1}{2}\pi^{3/2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+$$
$$\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}$$
$$\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\downarrow 223$$

$$\frac{3}{4}\pi\left(\frac{1}{2}\sqrt{\pi}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x$$
$$\frac{1}{2}\pi^{3/2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\downarrow 5153$$

$$\frac{1}{2}\pi^{3/2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+$$
$$\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2-\right.$$
$$\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\downarrow 5183$$

$$\frac{1}{2}\pi^{3/2}bc\left(-\frac{b\int\left(1-c^2x^2\right)^{3/2}dx}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)+$$
$$\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2-\right.$$
$$\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\downarrow 211$$

$$\frac{1}{2}\pi^{3/2}bc\left(-\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2-\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2\right.$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\pi^{3/2}bc\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2-\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2\right.$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\pi^{3/2}bc\left(-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}\right)+\frac{3}{4}\pi\left(\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2-\frac{1}{4}x\left(\pi-\pi c^2x^2\right)^{3/2}(a+b\arccos(cx))^2\right.$$

input
```
Int[(Pi - c^2*Pi*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(Pi - c^2*Pi*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + (3*Pi*((x*Sqrt[Pi -
c^2*Pi*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[Pi]*(a + b*ArcCos[c*x])^3)/(6
*b*c) + b*c*Sqrt[Pi]*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 -
 c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/4 + (b*c*Pi^(3/2)*(-1/4*((1 -
c^2*x^2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*(
(x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.36

method	result
default	$\frac{a^2 x(-\pi c^2x^2+\pi)^{\frac{3}{2}}}{4}+\frac{3a^2\pi x\sqrt{-\pi c^2x^2+\pi}}{8}+\frac{3a^2\pi^2\arctan\left(\frac{\sqrt{\pi c^2}\,x}{\sqrt{-\pi c^2x^2+\pi}}\right)}{8\sqrt{\pi c^2}}-\frac{b^2\pi^{\frac{3}{2}}\left(16\sqrt{-c^2x^2+1}\,\arccos(cx)^2x^3c^3+8c^4x^4\,\mathrm{a}\right.}{}$
parts	$\frac{a^2 x(-\pi c^2x^2+\pi)^{\frac{3}{2}}}{4}+\frac{3a^2\pi x\sqrt{-\pi c^2x^2+\pi}}{8}+\frac{3a^2\pi^2\arctan\left(\frac{\sqrt{\pi c^2}\,x}{\sqrt{-\pi c^2x^2+\pi}}\right)}{8\sqrt{\pi c^2}}-\frac{b^2\pi^{\frac{3}{2}}\left(16\sqrt{-c^2x^2+1}\,\arccos(cx)^2x^3c^3+8c^4x^4\,\mathrm{a}\right.}{}$

input
```
int((-Pi*c^2*x^2+Pi)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x*(-Pi*c^2*x^2+Pi)^(3/2)+3/8*a^2*Pi*x*(-Pi*c^2*x^2+Pi)^(1/2)+3/8*a
^2*Pi^2/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1/2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/6
4*b^2*Pi^(3/2)*(16*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2*x^3*c^3+8*c^4*x^4*arcc
os(c*x)-2*c^3*x^3*(-c^2*x^2+1)^(1/2)-40*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2*x
*c-40*c^2*x^2*arccos(c*x)+17*c*x*(-c^2*x^2+1)^(1/2)+8*arccos(c*x)^3+17*arc
cos(c*x))/c-1/32*a*b*Pi^(3/2)*(16*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+4
*c^4*x^4-40*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-20*c^2*x^2+12*arccos(c*x)^2
+25)/c
```

Fricas [F]

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx))^2\,dx = \int \left(\pi - \pi c^2x^2\right)^{\frac{3}{2}} (b\arccos{(cx)} + a)^2\,dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(pi - pi*c^2*x^2)*(pi*a^2*c^2*x^2 - pi*a^2 + (pi*b^2*c^2*x^2
 - pi*b^2)*arccos(c*x)^2 + 2*(pi*a*b*c^2*x^2 - pi*a*b)*arccos(c*x)), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(197) = 394$.

Time = 2.68 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.91

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx))^2\,dx = \begin{cases} -\frac{\pi^{\frac{3}{2}} a^2 c^2 x^3 \sqrt{-c^2x^2+1}}{4} + \frac{5\pi^{\frac{3}{2}} a^2 x \sqrt{-c^2x^2+1}}{8} - \frac{3\pi^{\frac{3}{2}} a^2 \operatorname{acos}{(cx)}}{8c} - \frac{\pi^{\frac{3}{2}} abc^3x^4}{8} - \frac{\pi^{\frac{3}{2}} abc^2x^3\sqrt{-c^2x^2+1}\operatorname{acc}}{2} \\ \pi^{\frac{3}{2}} x\left(a + \frac{\pi b}{2}\right)^2 \end{cases}$$

input
```
integrate((-pi*c**2*x**2+pi)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-pi**(3/2)*a**2*c**2*x**3*sqrt(-c**2*x**2 + 1)/4 + 5*pi**(3/2)*
a**2*x*sqrt(-c**2*x**2 + 1)/8 - 3*pi**(3/2)*a**2*acos(c*x)/(8*c) - pi**(3/
2)*a*b*c**3*x**4/8 - pi**(3/2)*a*b*c**2*x**3*sqrt(-c**2*x**2 + 1)*acos(c*x
)/2 + 5*pi**(3/2)*a*b*c*x**2/8 + 5*pi**(3/2)*a*b*x*sqrt(-c**2*x**2 + 1)*ac
os(c*x)/4 - 3*pi**(3/2)*a*b*acos(c*x)**2/(8*c) - pi**(3/2)*b**2*c**3*x**4*
acos(c*x)/8 - pi**(3/2)*b**2*c**2*x**3*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/4
 + pi**(3/2)*b**2*c**2*x**3*sqrt(-c**2*x**2 + 1)/32 + 5*pi**(3/2)*b**2*c*x
**2*acos(c*x)/8 + 5*pi**(3/2)*b**2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/8 -
 17*pi**(3/2)*b**2*x*sqrt(-c**2*x**2 + 1)/64 - pi**(3/2)*b**2*acos(c*x)**3
/(8*c) - 17*pi**(3/2)*b**2*acos(c*x)/(64*c), Ne(c, 0)), (pi**(3/2)*x*(a +
pi*b/2)**2, True))
```

Maxima [F]

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b \arccos(cx))^2 \, dx = \int \left(\pi - \pi c^2 x^2\right)^{\frac{3}{2}} (b \arccos(cx) + a)^2 \, dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
1/8*(3*pi*sqrt(pi - pi*c^2*x^2)*x + 2*(pi - pi*c^2*x^2)^(3/2)*x + 3*pi^(3/
2)*arcsin(c*x)/c)*a^2 + sqrt(pi)*integrate(-((pi*b^2*c^2*x^2 - pi*b^2)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(pi*a*b*c^2*x^2 - pi*a*b)*ar
ctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b \arccos(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-pi*c^2*x^2+pi)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b \arccos(cx))^2 \, dx = \int (a + b \operatorname{acos}(c\,x))^2 \left(\Pi - \Pi\,c^2\,x^2\right)^{3/2} \, dx$$

input
```
int((a + b*acos(c*x))^2*(Pi - Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(Pi - Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(\pi - c^2\pi x^2\right)^{3/2} (a + b\arccos(cx))^2\, dx = \frac{\sqrt{\pi}\,\pi\left(3asin(cx)\,a^2 - 2\sqrt{-c^2x^2+1}\,a^2c^3x^3 + 5\sqrt{-c^2x^2+1}\,a^2cx - 16\left(\int \sqrt{-c^2x^2+1}\,a\right.\right.}{}$$

input
```
int((-Pi*c^2*x^2+Pi)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(pi)*pi*(3*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 +
 5*sqrt( - c**2*x**2 + 1)*a**2*c*x - 16*int(sqrt( - c**2*x**2 + 1)*acos(c*
x)*x**2,x)*a*b*c**3 + 16*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b*c - 8
*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt( -
 c**2*x**2 + 1)*acos(c*x)**2,x)*b**2*c))/(8*c)
```

3.45 $\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 124

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2\,dx = -\frac{1}{4}b^2\sqrt{\pi}x\sqrt{1-c^2x^2} + \frac{1}{2}bc\sqrt{\pi}x^2(a+b\arccos(cx)) + \frac{1}{2}x\sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2 - \frac{\sqrt{\pi}(a + b\arccos(cx))^3}{6bc} + \frac{b^2\sqrt{\pi}\arcsin(cx)}{4c}$$

output
```
-1/4*b^2*Pi^(1/2)*x*(-c^2*x^2+1)^(1/2)+1/2*b*c*Pi^(1/2)*x^2*(a+b*arccos(c*
x))+1/2*x*(-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x))^2-1/6*Pi^(1/2)*(a+b*arc
cos(c*x))^3/b/c+1/4*b^2*Pi^(1/2)*arcsin(c*x)/c
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2\,dx = \frac{\sqrt{\pi}\left(-4b^2\arccos(cx)^3 + 6b\arccos(cx)(b\cos(2\arccos(cx)) + 2a\sin(2\arccos(cx))) + 6b\arccos(cx)^2(-2a\right.}{}$$

input
```
Integrate[Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[Pi]*(-4*b^2*ArcCos[c*x]^3 + 6*b*ArcCos[c*x]*(b*Cos[2*ArcCos[c*x]] +
2*a*Sin[2*ArcCos[c*x]]) + 6*b*ArcCos[c*x]^2*(-2*a + b*Sin[2*ArcCos[c*x]])
+ 3*(4*a^2*c*x*Sqrt[1 - c^2*x^2] + 4*a^2*ArcSin[c*x] + 2*a*b*Cos[2*ArcCos[
c*x]] - b^2*Sin[2*ArcCos[c*x]])))/(24*c)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5157, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\sqrt{\pi}bc\int x(a+b\arccos(cx))dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\sqrt{\pi}bc\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\sqrt{\pi}bc\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5153}$$

$$\sqrt{\pi}bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{\pi-\pi c^2x^2}(a+b\arccos(cx))^2-\frac{\sqrt{\pi}(a+b\arccos(cx))^3}{6bc}$$

input
```
Int[Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[Pi]*(a + b*ArcCo
s[c*x])^3)/(6*b*c) + b*c*Sqrt[Pi]*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/
2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.44

method	result
default	$\frac{a^2 x\sqrt{-\pi c^2x^2+\pi}}{2}+\frac{a^2\pi\arctan\left(\frac{\sqrt{\pi c^2}\,x}{\sqrt{-\pi c^2x^2+\pi}}\right)}{2\sqrt{\pi c^2}}-\frac{b^2\sqrt{\pi}\left(-6\sqrt{-c^2x^2+1}\arccos(cx)^2xc-6c^2x^2\arccos(cx)+2\arccos(cx)^3+3cx\right.}{12c}$
parts	$\frac{a^2 x\sqrt{-\pi c^2x^2+\pi}}{2}+\frac{a^2\pi\arctan\left(\frac{\sqrt{\pi c^2}\,x}{\sqrt{-\pi c^2x^2+\pi}}\right)}{2\sqrt{\pi c^2}}-\frac{b^2\sqrt{\pi}\left(-6\sqrt{-c^2x^2+1}\arccos(cx)^2xc-6c^2x^2\arccos(cx)+2\arccos(cx)^3+3cx\right.}{12c}$

input
```
int((-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(-Pi*c^2*x^2+Pi)^(1/2)+1/2*a^2*Pi/(Pi*c^2)^(1/2)*arctan((Pi*c^2)
^(1/2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/12*b^2*Pi^(1/2)*(-6*(-c^2*x^2+1)^(1/2)*
arccos(c*x)^2*x*c-6*c^2*x^2*arccos(c*x)+2*arccos(c*x)^3+3*c*x*(-c^2*x^2+1)
^(1/2)+3*arccos(c*x))/c-1/2*a*b*Pi^(1/2)*(-2*(-c^2*x^2+1)^(1/2)*arccos(c*x
)*x*c-c^2*x^2+arccos(c*x)^2+1)/c
```

Fricas [F]

$$\int \sqrt{\pi-c^2\pi x^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{\pi-\pi c^2x^2}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2), x)
```

Sympy [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2\,dx = \sqrt{\pi}\left(\int a^2\sqrt{-c^2x^2+1}\,dx + \int b^2\sqrt{-c^2x^2+1}\operatorname{acos}^2(cx)\,dx + \int 2ab\sqrt{-c^2x^2+1}\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-pi*c**2*x**2+pi)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
sqrt(pi)*(Integral(a**2*sqrt(-c**2*x**2 + 1), x) + Integral(b**2*sqrt(-c**
2*x**2 + 1)*acos(c*x)**2, x) + Integral(2*a*b*sqrt(-c**2*x**2 + 1)*acos(c*
x), x))
```

Maxima [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2\,dx = \int \sqrt{\pi - \pi c^2x^2}(b\arccos(cx) + a)^2\,dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
1/2*(sqrt(pi - pi*c^2*x^2)*x + sqrt(pi)*arcsin(c*x)/c)*a^2 + sqrt(pi)*inte
grate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2 \, dx = \int (a + b\,\mathrm{acos}(c\,x))^2\,\sqrt{\Pi - \Pi\,c^2\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(Pi - Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(Pi - Pi*c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))^2 \, dx = \frac{\sqrt{\pi}\,\left(asin(cx)\,a^2 + \sqrt{-c^2x^2+1}\,a^2cx + 4\left(\int \sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)abc + 2\left(\int \sqrt{-c^2x^2+1}\,acos(cx)^2\,d\right.\right.}{2c}$$

input
```
int((-Pi*c^2*x^2+Pi)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(pi)*(asin(c*x)*a**2 + sqrt( - c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt(
- c**2*x**2 + 1)*acos(c*x),x)*a*b*c + 2*int(sqrt( - c**2*x**2 + 1)*acos(c*
x)**2,x)*b**2*c))/(2*c)
```

3.46 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 25

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx=-\frac{(a+b\arccos(cx))^3}{3bc\sqrt{\pi}}$$

output
```
-1/3*(a+b*arccos(c*x))^3/b/c/Pi^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx=-\frac{(a+b\arccos(cx))^3}{3bc\sqrt{\pi}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[Pi - c^2*Pi*x^2],x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^3/(b*c*Sqrt[Pi])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-\pi c^2x^2}}\,dx$$

$$\Big\downarrow \; 5153$$

$$-\frac{(a+b\arccos(cx))^3}{3\sqrt{\pi}bc}$$

input
```
Int[(a + b*ArcCos[c*x])^2/Sqrt[Pi - c^2*Pi*x^2],x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^3/(b*c*Sqrt[Pi])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(21) = 42$.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

method	result	size
default	$\frac{a^2 \arctan\left(\frac{\sqrt{\pi\,c^2}\,x}{\sqrt{-\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi\,c^2}} - \frac{b^2 \arccos(cx)^3}{3c\sqrt{\pi}} - \frac{ab \arccos(cx)^2}{c\sqrt{\pi}}$	69
parts	$\frac{a^2 \arctan\left(\frac{\sqrt{\pi\,c^2}\,x}{\sqrt{-\pi\,c^2x^2+\pi}}\right)}{\sqrt{\pi\,c^2}} - \frac{b^2 \arccos(cx)^3}{3c\sqrt{\pi}} - \frac{ab \arccos(cx)^2}{c\sqrt{\pi}}$	69

input
```
int((a+b*arccos(c*x))^2/(-Pi*c^2*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1/2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/3*b^2
/c/Pi^(1/2)*arccos(c*x)^3-a*b/c/Pi^(1/2)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{\pi-\pi c^2x^2}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/sqrt(pi - pi*c^2*x^
2), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(20) = 40$.

Time = 3.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.00

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi - c^2\pi x^2}}\,dx = \begin{cases} a^2\left(\begin{cases} \frac{\log\left(-2\pi c^2x+2\sqrt{\pi}\sqrt{-c^2}\sqrt{-\pi c^2x^2+\pi}\right)}{\sqrt{\pi}\sqrt{-c^2}} & \text{for}\ \pi c^2 \neq 0 \\ \frac{x}{\sqrt{\pi}} & \text{otherwise} \end{cases}\right) & \text{for}\ b = 0 \\ \frac{x\left(a+\frac{\pi b}{2}\right)^2}{\sqrt{\pi}} & \text{for}\ c = 0 \\ -\frac{(a+b\operatorname{acos}(cx))^3}{3\sqrt{\pi}bc} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*acos(c*x))**2/(-pi*c**2*x**2+pi)**(1/2),x)
```

output
```
Piecewise((a**2*Piecewise((log(-2*pi*c**2*x + 2*sqrt(pi)*sqrt(-c**2)*sqrt(
-pi*c**2*x**2 + pi))/(sqrt(pi)*sqrt(-c**2)), Ne(pi*c**2, 0)), (x/sqrt(pi),
 True)), Eq(b, 0)), (x*(a + pi*b/2)**2/sqrt(pi), Eq(c, 0)), (-(a + b*acos(
c*x))**3/(3*sqrt(pi)*b*c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(21) = 42$.

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.20

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi - c^2\pi x^2}}\,dx = \frac{b^2\arccos(cx)^2\arcsin(cx)}{\sqrt{\pi}c} + \frac{1}{3}\left(\frac{3\arccos(cx)\arcsin(cx)^2}{\sqrt{\pi}c} + \frac{\arcsin(cx)^3}{\sqrt{\pi}c}\right)b^2 + \frac{2ab\arccos(cx)\arcsin(cx)}{\sqrt{\pi}c} + \frac{ab\arcsin(cx)^2}{\sqrt{\pi}c} + \frac{a^2\arcsin(cx)}{\sqrt{\pi}c}$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(1/2),x, algorithm="maxima"
)
```

output
```
b^2*arccos(c*x)^2*arcsin(c*x)/(sqrt(pi)*c) + 1/3*(3*arccos(c*x)*arcsin(c*x
)^2/(sqrt(pi)*c) + arcsin(c*x)^3/(sqrt(pi)*c))*b^2 + 2*a*b*arccos(c*x)*arc
sin(c*x)/(sqrt(pi)*c) + a*b*arcsin(c*x)^2/(sqrt(pi)*c) + a^2*arcsin(c*x)/(
sqrt(pi)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{\Pi-\Pi\,c^2\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(Pi - Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2/(Pi - Pi*c^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{\pi-c^2\pi x^2}}\,dx = \frac{\sqrt{\pi}\left(-acos(cx)^3 b^2-3acos(cx)^2 ab+3asin(cx)\,a^2\right)}{3c\pi}$$

input
```
int((a+b*acos(c*x))^2/(-Pi*c^2*x^2+Pi)^(1/2),x)
```

output
```
(sqrt(pi)*( - acos(c*x)**3*b**2 - 3*acos(c*x)**2*a*b + 3*asin(c*x)*a**2))/
(3*c*pi)
```

3.47 $\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 124

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx=\frac{i(a+b\arccos(cx))^2}{c\pi^{3/2}}+\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-c^2\pi x^2}}-\frac{b(2a+b\pi-b(\pi-2\arccos(cx)))\log\left(1-e^{2i\arccos(cx)}\right)}{c\pi^{3/2}}+\frac{ib^2\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{c\pi^{3/2}}$$

output
```
I*(a+b*arccos(c*x))^2/c/Pi^(3/2)+x*(a+b*arccos(c*x))^2/Pi/(-Pi*c^2*x^2+Pi)
^(1/2)-b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2
)/c/Pi^(3/2)+I*b^2*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/Pi^(3/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx=\frac{b^2\left(cx+i\sqrt{1-c^2x^2}\right)\arccos(cx)^2+2b\arccos(cx)\left(acx-b\sqrt{1-c^2x^2}\log\left(1-e\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(Pi - c^2*Pi*x^2)^(3/2),x]
```

output
```
(b^2*(c*x + I*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2*b*ArcCos[c*x]*(a*c*x -
b*Sqrt[1 - c^2*x^2]*Log[1 - E^((2*I)*ArcCos[c*x])]) + a*(a*c*x - b*Sqrt[1
- c^2*x^2]*Log[-1 + c^2*x^2]) + I*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I
)*ArcCos[c*x])])/(c*Pi^(3/2)*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-\pi c^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{5161}$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{\pi^{3/2}}+\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\pi^{3/2}c}+\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}$$

$$\Big\downarrow \text{4200}$$

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$$\Big\downarrow \text{25}$$

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$\big\downarrow$ 2620

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

$\big\downarrow$ 2715

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+\cdots}{}\right.}{\pi^{3/2}c}$$

$\big\downarrow$ 2838

$$\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(Pi - c^2*Pi*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(Pi*Sqrt[Pi - c^2*Pi*x^2]) - (2*b*(((-1/2*I)*(a
+ b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*
ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/(c*Pi^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.17

method	result
default	$\frac{a^2 x}{\pi\sqrt{-\pi c^2x^2+\pi}}+b^2\left(-\frac{\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\arccos(cx)^2}{\pi^{\frac{3}{2}}c(c^2x^2-1)}+\frac{2i\left(i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)+i\arccos(cx)\ln\left(1-\right.\right.}{}\right.$
parts	$\frac{a^2 x}{\pi\sqrt{-\pi c^2x^2+\pi}}+b^2\left(-\frac{\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\arccos(cx)^2}{\pi^{\frac{3}{2}}c(c^2x^2-1)}+\frac{2i\left(i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)+i\arccos(cx)\ln\left(1-\right.\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/(-Pi*c^2*x^2+Pi)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/Pi*x/(-Pi*c^2*x^2+Pi)^(1/2)+b^2*(-1/Pi^(3/2)*(I*c^2*x^2+c*x*(-c^2*x^2+
1)^(1/2)-I)*arccos(c*x)^2/c/(c^2*x^2-1)+2*I*(I*arccos(c*x)*ln(1+c*x+I*(-c^
2*x^2+1)^(1/2))+I*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2
+polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
/c/Pi^(3/2))-a*b/c/Pi^(3/2)*(ln(-c^2*x^2+1)*x^2*c^2+2*(-c^2*x^2+1)^(1/2)*a
rccos(c*x)*x*c-ln(-c^2*x^2+1))/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(\pi-\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(pi^2*c^4*x^4 - 2*pi^2*c^2*x^2 + pi^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \frac{\int \frac{a^2}{-c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{-c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx + \int \frac{2ab\operatorname{acos}(\ldots}{-c^2x^2\sqrt{-c^2x^2+1}+\ldots}}{\pi^{\frac{3}{2}}}$$

input
```
integrate((a+b*acos(c*x))**2/(-pi*c**2*x**2+pi)**(3/2),x)
```

output
```
(Integral(a**2/(-c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c**2*x**2 + 1)), x
) + Integral(b**2*acos(c*x)**2/(-c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c*
*2*x**2 + 1)), x) + Integral(2*a*b*acos(c*x)/(-c**2*x**2*sqrt(-c**2*x**2 +
 1) + sqrt(-c**2*x**2 + 1)), x))/pi**(3/2)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(\pi-\pi c^2x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(3/2),x, algorithm="maxima"
)
```

output
```
2*a*b*x*arccos(c*x)/(pi*sqrt(pi - pi*c^2*x^2)) - b^2*integrate(-arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((pi - pi*c^2*x^2)*sqrt(c*x + 1)*sqrt(-
c*x + 1)), x)/sqrt(pi) + a^2*x/(pi*sqrt(pi - pi*c^2*x^2)) + a*b*log(x^2 -
1/c^2)/(pi^(3/2)*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(\Pi-\Pi\,c^2\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(Pi - Pi*c^2*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2/(Pi - Pi*c^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab-\sqrt{-c^2x^2+1}\left(\int \frac{acos(}{\sqrt{-c^2x^2+1}\,c^2x}\right.}{\sqrt{\pi}\sqrt{-c^2x^2+1}\,\pi}$$

input
```
int((a+b*acos(c*x))^2/(-Pi*c^2*x^2+Pi)^(3/2),x)
```

output

```
( - 2*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*2 - sqrt( - c**2*x**2 + 1)),x)*a*b - sqrt( - c**2*x**2 + 1)*int(acos(c*x)
**2/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2 +
a**2*x)/(sqrt(pi)*sqrt( - c**2*x**2 + 1)*pi)
```

3.48 $\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 226

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx &= \frac{b^2x}{3\pi^{5/2}\sqrt{1-c^2x^2}}+\frac{b(a+b\arccos(cx))}{3c\pi^{5/2}\left(1-c^2x^2\right)}\\ &+\frac{2i(a+b\arccos(cx))^2}{3c\pi^{5/2}}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-c^2\pi x^2\right)^{3/2}}+\frac{2x(a+b\arccos(cx))^2}{3\pi^2\sqrt{\pi-c^2\pi x^2}}\\ &-\frac{2b(2a+b\pi-b(\pi-2\arccos(cx)))\log\left(1-e^{2i\arccos(cx)}\right)}{3c\pi^{5/2}}\\ &+\frac{2ib^2\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3c\pi^{5/2}}\end{aligned}$$

output
```
1/3*b^2*x/Pi^(5/2)/(-c^2*x^2+1)^(1/2)+1/3*b*(a+b*arccos(c*x))/c/Pi^(5/2)/(
-c^2*x^2+1)+2/3*I*(a+b*arccos(c*x))^2/c/Pi^(5/2)+1/3*x*(a+b*arccos(c*x))^2
/Pi/(-Pi*c^2*x^2+Pi)^(3/2)+2/3*x*(a+b*arccos(c*x))^2/Pi^2/(-Pi*c^2*x^2+Pi)
^(1/2)-2/3*b*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2
))^2)/c/Pi^(5/2)+2/3*I*b^2*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/Pi^(5
/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx =$$

$$-\frac{-3a^2cx-b^2cx+2a^2c^3x^3+b^2c^3x^3-ab\sqrt{1-c^2x^2}+b^2\left(-3cx+2c^3x^3-2i\sqrt{1-c^2x^2}+2ic^2x^2\sqrt{1-c^2x\right.}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(Pi - c^2*Pi*x^2)^(5/2),x]
```

output
```
-1/3*(-3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 - c^
2*x^2] + b^2*(-3*c*x + 2*c^3*x^3 - (2*I)*Sqrt[1 - c^2*x^2] + (2*I)*c^2*x^2
*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + b*ArcCos[c*x]*(-6*a*c*x + 4*a*c^3*x^3
- b*Sqrt[1 - c^2*x^2] + 4*b*(1 - c^2*x^2)^(3/2)*Log[1 - E^((2*I)*ArcCos[c*
x])]) + 2*a*b*Sqrt[1 - c^2*x^2]*Log[-1 + c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 -
 c^2*x^2]*Log[-1 + c^2*x^2] - (2*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^(
(2*I)*ArcCos[c*x])])/(c*Pi^(5/2)*(1 - c^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-\pi c^2x^2)^{5/2}}dx$$

$$\Big\downarrow \ 5163$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{3/2}}dx}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \ 5161$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{2bc\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{\pi^{3/2}}+\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{2b\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\pi^{3/2}c}+\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{4200}$$

$$\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arc}{2}\right.}{\pi^{3/2}c}\right.}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{2bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 5183$$

$$\frac{2bc\left(\frac{b\int\frac{1}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)}{3\pi^{5/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2\left(\frac{x(a+b\arccos(cx))^2}{\pi\sqrt{\pi-\pi c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{\pi^{3/2}c}\right)}{3\pi}+\frac{x(a+b\arccos(cx))^2}{3\pi\left(\pi-\pi c^2x^2\right)^{3/2}}+\frac{2bc\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3\pi^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(Pi - c^2*Pi*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(3*Pi*(Pi - c^2*Pi*x^2)^(3/2)) + (2*b*c*((b*x)/(
2*c*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*c^2*(1 - c^2*x^2))))/(3*Pi
^(5/2)) + (2*((x*(a + b*ArcCos[c*x])^2)/(Pi*Sqrt[Pi - c^2*Pi*x^2]) - (2*b*
(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log
[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/(
c*Pi^(3/2))))/(3*Pi)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(216) = 432$.

Time = 0.50 (sec) , antiderivative size = 1687, normalized size of antiderivative = 7.46

method	result	size
default	Expression too large to display	1687
parts	Expression too large to display	1687

input
```
int((a+b*arccos(c*x))^2/(-Pi*c^2*x^2+Pi)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arc
cos(c*x)^2*x-b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^4*(-c^2*x^
2+1)^(1/2)*x^5+7/3*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^2*(-
c^2*x^2+1)^(1/2)*x^3+16/3*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)
*c*arccos(c*x)*x^2-4/3*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^
7*arccos(c*x)*x^8+16/3*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^
5*arccos(c*x)*x^6-8*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^3*a
rccos(c*x)*x^4+2/3*I*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^7*
x^8-10/3*I*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^5*x^6+6*I*b^
2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c^3*x^4-14/3*I*b^2/Pi^(5/2)
/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*c*x^2+8/3*I*b^2/Pi^(5/2)/(3*c^4*x^4-7
*c^2*x^2+4)/(c^2*x^2-1)/c*arccos(c*x)^2-4/3*b^2/c/Pi^(5/2)*arccos(c*x)*ln(
1+c*x+I*(-c^2*x^2+1)^(1/2))-4/3*b^2/c/Pi^(5/2)*arccos(c*x)*ln(1-c*x-I*(-c^
2*x^2+1)^(1/2))+4/3*I*b^2/c/Pi^(5/2)*arccos(c*x)^2+4/3*I*b^2/c/Pi^(5/2)*po
lylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+4/3*I*b^2/c/Pi^(5/2)*polylog(2,c*x+I*(-
c^2*x^2+1)^(1/2))+a^2*(1/3/Pi*x/(-Pi*c^2*x^2+Pi)^(3/2)+2/3/Pi^2*x/(-Pi*c^2
*x^2+Pi)^(1/2))-4/3*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)*(-c^2
*x^2+1)^(1/2)*x-4/3*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)/c*arc
cos(c*x)+4/3*I*b^2/Pi^(5/2)/(3*c^4*x^4-7*c^2*x^2+4)/(c^2*x^2-1)/c-1/3*a*b/
c/Pi^(5/2)*(2*ln(-c^2*x^2+1)*x^4*c^4+4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(\pi-\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(pi - pi*c^2*x^2)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a
^2)/(pi^3*c^6*x^6 - 3*pi^3*c^4*x^4 + 3*pi^3*c^2*x^2 - pi^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx=\frac{\int \frac{a^2}{c^4x^4\sqrt{-c^2x^2+1}-2c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x^2+1}}\,dx+\int \frac{b^2\operatorname{acos}^2(cx)}{c^4x^4\sqrt{-c^2x^2+1}-2c^2x^2\sqrt{-c^2x^2+1}+\sqrt{-c^2x \cdots}}\cdots}{\pi^{\frac{5}{2}}}$$

input
```
integrate((a+b*acos(c*x))**2/(-pi*c**2*x**2+pi)**(5/2),x)
```

output
```
(Integral(a**2/(c**4*x**4*sqrt(-c**2*x**2 + 1) - 2*c**2*x**2*sqrt(-c**2*x*
*2 + 1) + sqrt(-c**2*x**2 + 1)), x) + Integral(b**2*acos(c*x)**2/(c**4*x**
4*sqrt(-c**2*x**2 + 1) - 2*c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c**2*x**
2 + 1)), x) + Integral(2*a*b*acos(c*x)/(c**4*x**4*sqrt(-c**2*x**2 + 1) - 2
*c**2*x**2*sqrt(-c**2*x**2 + 1) + sqrt(-c**2*x**2 + 1)), x))/pi**(5/2)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(\pi-\pi c^2x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a*b*c*(1/(pi^(5/2)*c^4*x^2 - pi^(5/2)*c^2) + 2*log(c*x + 1)/(pi^(5/2)
*c^2) + 2*log(c*x - 1)/(pi^(5/2)*c^2)) + 2/3*a*b*(x/(pi*(pi - pi*c^2*x^2)^
(3/2)) + 2*x/(pi^2*sqrt(pi - pi*c^2*x^2)))*arccos(c*x) + 1/3*a^2*(x/(pi*(p
i - pi*c^2*x^2)^(3/2)) + 2*x/(pi^2*sqrt(pi - pi*c^2*x^2))) + b^2*integrate
(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((pi^2*c^4*x^4 - 2*pi^2*c^2*
x^2 + pi^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(pi)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-pi*c^2*x^2+pi)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(\pi-c^2\pi x^2)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(\Pi-\Pi\,c^2\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(Pi - Pi*c^2*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2/(Pi - Pi*c^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a + b\arccos(cx))^2}{(\pi - c^2\pi x^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^2x^2 - 6\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acos(c*x))^2/(-Pi*c^2*x^2+Pi)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*c**2
*x**2 - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a
*b + 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x*
*2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2
 + 1)),x)*b**2 + 2*a**2*c**2*x**3 - 3*a**2*x)/(3*sqrt(pi)*sqrt( - c**2*x**
2 + 1)*pi**2*(c**2*x**2 - 1))
```

3.49 $\int \sqrt{1-x^2} \arccos(x)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arccos(x)\, dx = \frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2} \arccos(x) - \frac{\arccos(x)^2}{4}$$

output
```
1/4*x^2+1/2*x*(-x^2+1)^(1/2)*arccos(x)-1/4*arccos(x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arccos(x)\, dx = \frac{1}{4}\left(x^2 + 2x\sqrt{1-x^2} \arccos(x) - \arccos(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\arccos(x)\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{1-x^2}\arccos(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\arccos(x)+\frac{x^2}{4}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2}\sqrt{1-x^2}x\arccos(x)-\frac{\arccos(x)^2}{4}+\frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-x\sqrt{-x^2+1}+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$	33

input
```
int((-x^2+1)^(1/2)*arccos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2$$

input
```
integrate((-x^2+1)^(1/2)*arccos(x),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}\right) \operatorname{acos}(x) + \frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate((-x**2+1)**(1/2)*acos(x),x)
```

output
```
x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*acos(x) + asin(x)**2/4
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{4} x^2 + \frac{1}{2} \left(\sqrt{-x^2+1}x + \arcsin(x)\right) \arccos(x) + \frac{1}{4} \arcsin(x)^2$$

input
```
integrate((-x^2+1)^(1/2)*arccos(x),x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{2} \sqrt{-x^2+1}x \arccos(x) + \frac{1}{4} x^2 - \frac{1}{4} \arccos(x)^2 - \frac{1}{8}$$

input
```
integrate((-x^2+1)^(1/2)*arccos(x),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(acos(x)*(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \sqrt{-x^2+1}\,acos(x)\,dx$$

input
```
int((-x^2+1)^(1/2)*acos(x),x)
```

output
```
int(sqrt( - x**2 + 1)*acos(x),x)
```

3.50 $\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2a\arccos(ax)^2}$$

output
```
1/2/a/arccos(a*x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2a\arccos(ax)^2}$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3),x]
```

output
```
1/(2*a*ArcCos[a*x]^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2a\arccos(ax)^2}$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3),x]
```

output
```
1/(2*a*ArcCos[a*x]^2)
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{1}{2a\arccos(ax)^2}$	12
default	$\frac{1}{2a\arccos(ax)^2}$	12

input
```
int(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/arccos(a*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\arccos{(ax)}^2}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/2/(a*arccos(a*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \begin{cases} \frac{1}{2a\operatorname{acos}^2{(ax)}} & \text{for } a \neq 0 \\ \frac{8x}{\pi^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/acos(a*x)**3,x)
```

output
```
Piecewise((1/(2*a*acos(a*x)**2), Ne(a, 0)), (8*x/pi**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\arccos{(ax)}^2}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2/(a*arccos(a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\arccos{(ax)}^2}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/2/(a*arccos(a*x)^2)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\,\mathrm{acos}{(a\,x)}^2}$$

input
```
int(1/(acos(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
1/(2*a*acos(a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2acos\left(ax\right)^2 a}$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/acos(a*x)^3,x)
```

output
```
1/(2*acos(a*x)**2*a)
```

3.51 $\int (d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 262

$$\int (d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx = \frac{5bcd^2x^2\sqrt{d-c^2dx^2}}{32\sqrt{1-c^2x^2}} - \frac{5bd^2(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}{96c} - \frac{bd^2(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c} + \frac{5}{16}d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx)) + \frac{5}{24}dx(d-c^2dx^2)^{3/2}(a+b\arccos(cx)) + \frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arccos$$

output
```
5/32*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/96*b*d^2*(-c^2*
x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c-1/36*b*d^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*
x^2+d)^(1/2)/c+5/16*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+5/24*d*x*
(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))+1/6*x*(-c^2*d*x^2+d)^(5/2)*(a+b*arc
cos(c*x))-5/32*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*x^2+
1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = \frac{d^2\left(-360b\sqrt{d-c^2dx^2}\arccos(cx)^2-720a\sqrt{d}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\sqrt{d-c\right)}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(-360*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 720*a*Sqrt[d]*Sqrt[1 - c^
2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d
 - c^2*d*x^2]*(1584*a*c*x*Sqrt[1 - c^2*x^2] - 1248*a*c^3*x^3*Sqrt[1 - c^2*
x^2] + 384*a*c^5*x^5*Sqrt[1 - c^2*x^2] + 270*b*Cos[2*ArcCos[c*x]] - 27*b*C
os[4*ArcCos[c*x]] + 2*b*Cos[6*ArcCos[c*x]]) + 12*b*Sqrt[d - c^2*d*x^2]*Arc
Cos[c*x]*(45*Sin[2*ArcCos[c*x]] - 9*Sin[4*ArcCos[c*x]] + Sin[6*ArcCos[c*x]
])))/(2304*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5159, 241, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{241}$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 5159

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 244

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 2009

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx}}{4\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 5157

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{4}x(d-\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 15

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{1}{4}x(d-c^2dx\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx}{4bc\sqrt{1-c^2x^2}}\right.\right.$$
$$\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/36*(b*d^2*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/c + (x*(d - c^2*d*x^
2)^(5/2)*(a + b*ArcCos[c*x]))/6 + (5*d*((b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2
- (c^2*x^4)/4))/(4*Sqrt[1 - c^2*x^2]) + (x*(d - c^2*d*x^2)^(3/2)*(a + b*Ar
cCos[c*x]))/4 + (3*d*((b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2])
+ (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])))/4))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.63

method	result
default	$\frac{ax\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{6}+\frac{5adx\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{24}+\frac{5a\,d^{2}x\sqrt{-c^{2}d\,x^{2}+d}}{16}+\frac{5a\,d^{3}\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{16\sqrt{c^{2}d}}+b\left(\frac{5\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{-c^{2}\cdots}}{32c(c^{2}\cdots}\right.$
parts	$\frac{ax\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{6}+\frac{5adx\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{24}+\frac{5a\,d^{2}x\sqrt{-c^{2}d\,x^{2}+d}}{16}+\frac{5a\,d^{3}\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{16\sqrt{c^{2}d}}+b\left(\frac{5\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{-c^{2}\cdots}}{32c(c^{2}\cdots}\right.$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*
(-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))+b*(5/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*
x^2-1)*arccos(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*
x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+1)^(1/2)*x^4
*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+6*arcc
os(c*x))*d^2/c/(c^2*x^2-1)+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d
^2/c/(c^2*x^2-1)+5/4608*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+
c^2*x^2-1)*(5*I+24*arccos(c*x))*cos(5*arccos(c*x))*d^2/c/(c^2*x^2-1)+1/460
8*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(29*I+96*arc
cos(c*x))*sin(5*arccos(c*x))*d^2/c/(c^2*x^2-1)-9/512*(-d*(c^2*x^2-1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(3*I+8*arccos(c*x))*cos(3*arccos(c
*x))*d^2/c/(c^2*x^2-1)-3/512*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x
^2+1)^(1/2)-I)*(11*I+16*arccos(c*x))*sin(3*arccos(c*x))*d^2/c/(c^2*x^2-1))
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int (-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/48*(8*(-c^2*
d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(-c^2*d*x^2 +
d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d^2\left(15asin(cx)\,a+8\sqrt{-c^2x^2+1}\,a\,c^5x^5-26\sqrt{-c^2x^2+1}\,a\,c^3x^3+33\sqrt{-c^2x^2+1}\,ac\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 - 26*
sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 33*sqrt( - c**2*x**2 + 1)*a*c*x + 48*
int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**5 - 96*int(sqrt( - c**2*
x**2 + 1)*acos(c*x)*x**2,x)*b*c**3 + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*
x),x)*b*c))/(48*c)
```

3.52 $\int (d-c^2dx^2)^{3/2} (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int (d-c^2dx^2)^{3/2} (a+b\arccos(cx))\,dx = \frac{3bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} - \frac{bd(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}{16c} + \frac{3}{8}dx\sqrt{d-c^2dx^2}(a+b\arccos(cx)) + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arccos(cx)) - \frac{3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}$$

output
```
3/16*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*d*(-c^2*x^2+
1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c
*x))+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-3/16*d*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.14

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\frac{-24bd\sqrt{d-c^2dx^2}\arccos(cx)^2-48ad^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+d\sqrt{d-c^2d}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-24*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 48*a*d^(3/2)*Sqrt[1 - c^2*x^2
]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + d*Sqrt[d -
c^2*d*x^2]*(16*a*c*x*(5 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + 16*b*Cos[2*ArcCos
[c*x]] - b*Cos[4*ArcCos[c*x]]) - 4*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-8
*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]]))/(128*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 244}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \textbf{5157}$$

$$\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \textbf{15}$$

$$\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \textbf{5153}$$

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/(4*Sqrt[1 - c^2*x^2]) +
(x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*d*((b*c*x^2*Sqrt[d -
c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c
*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*
x^2])))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.59

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{-c^2dx^2+d}}{8}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2d}{16(c^2x^2-1)c}-\right.$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{-c^2dx^2+d}}{8}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2d}{16(c^2x^2-1)c}-\right.$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(3/16*(-d*(c^2*x^
2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*d-1/256*(-d*(c^
2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x
-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d/
(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2
+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d/(c^2*x^2-1)/c-
3/256*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(5*I+12
*arccos(c*x))*cos(3*arccos(c*x))*d/(c^2*x^2-1)/c-1/256*(-d*(c^2*x^2-1))^(1
/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(17*I+28*arccos(c*x))*sin(3*arcco
s(c*x))*d/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\int \left(-d(cx-1)(cx+1)\right)^{\frac{3}{2}}(a+b\,\mathrm{acos}\,(cx))\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate(-(c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x +
 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*arcsin(c*x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d\left(3asin(cx)\,a-2\sqrt{-c^2x^2+1}\,a\,c^3x^3+5\sqrt{-c^2x^2+1}\,acx-8\left(\int\sqrt{-c^2x^2+1}\,acos(c\right.\right.}{8c}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 5*sqrt(
 - c**2*x**2 + 1)*a*c*x - 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b
*c**3 + 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b*c))/(8*c)
```

3.53 $\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}$$

output
```
1/4*b*c*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))-1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c/(-
c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{1}{8}\left(4ax\sqrt{d-c^2dx^2}-\frac{4a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{c}+\frac{b\sqrt{d-c^2dx^2}(-2\arccos(cx)^2+\cos(2\arccos(cx))+2\arccos(cx)\sin(2\arccos(cx)))}{c\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(4*a*x*Sqrt[d - c^2*d*x^2] - (4*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))])/c + (b*Sqrt[d - c^2*d*x^2]*(-2*ArcCos[c*x]^2 +
 Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/(c*Sqrt[1 - c^2*x
^2]))/8
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output

```
(b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCos[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4
*b*c*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.41

method	result
default	$\frac{ax\sqrt{-c^2d\,x^2+d}}{2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{4(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\,\left(2c^3x^3-2cx+2i\right.}{}\right.$
parts	$\frac{ax\sqrt{-c^2d\,x^2+d}}{2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{4(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\,\left(2c^3x^3-2cx+2i\right.}{}\right.$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/
(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^
2*x^2-1)/c*arccos(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*I*
(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))/(c^2*x^
2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*
x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acos}{(cx)})\;dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x), x) + 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/
c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(asin(cx)\,a+\sqrt{-c^2x^2+1}\,acx+2\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(asin(c*x)*a + sqrt( - c**2*x**2 + 1)*a*c*x + 2*int(sqrt( - c**2*
x**2 + 1)*acos(c*x),x)*b*c))/(2*c)
```

3.54 $\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

output
```
-1/2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}\arccos(cx)(2a+b\arccos(cx))}{2c\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/2*(Sqrt[1 - c^2*x^2]*ArcCos[c*x]*(2*a + b*ArcCos[c*x]))/(c*Sqrt[d - c^2
*d*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \ 5153$$

$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/2*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(b*c*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2cd(c^2x^2-1)}$	86
parts	$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2cd(c^2x^2-1)}$	86

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\operatorname{acos}{(cx)}}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \frac{b\arccos(cx)\arcsin(cx)}{c\sqrt{d}} + \frac{b\arcsin(cx)^2}{2\,c\sqrt{d}} + \frac{a\arcsin(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*arcsin(c*x)^2/(c*sqrt(d)) +
a*arcsin(c*x)/(c*sqrt(d))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = -\frac{b\arccos(cx)^2+2\,a\arccos(cx)}{2\,c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*arccos(c*x)^2 + 2*a*arccos(c*x))/(c*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx=\frac{\sqrt{d}\,\left(-acos(cx)^2\,b+2asin(cx)\,a\right)}{2cd}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*( - acos(c*x)**2*b + 2*asin(c*x)*a))/(2*c*d)
```

3.55 $\int \frac{a+b \arccos(cx)}{(d-c^2dx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}} \, dx = \frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2cd\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*(-c^2*x^2+1)^(1/2)*ln(-c^
2*x^2+1)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}} \, dx = \frac{\sqrt{d-c^2dx^2}\left(-2acx-2bcx\arccos(cx)+b\sqrt{1-c^2x^2}\log(-1+c^2x^2)\right)}{2cd^2(-1+c^2x^2)}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-2*a*c*x - 2*b*c*x*ArcCos[c*x] + b*Sqrt[1 - c^2*x^2]
*Log[-1 + c^2*x^2]))/(2*c*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5161}$$

$$\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*Log
[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

method	result
default	$\frac{ax}{d\sqrt{-c^2d\,x^2+d}}-\frac{ib\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)}{c\,d^2(c^2x^2-1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x}{d^2(c^2x^2-1)}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln\Big((}{c\,d^2(c^2x^2-}$
parts	$\frac{ax}{d\sqrt{-c^2d\,x^2+d}}-\frac{ib\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)}{c\,d^2(c^2x^2-1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x}{d^2(c^2x^2-1)}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln\Big((}{c\,d^2(c^2x^2-}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d*x/(-c^2*d*x^2+d)^(1/2)-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c
/d^2/(c^2*x^2-1)*arccos(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d^2/(c^2
*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2
*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}(cx)}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \frac{bx\arccos(cx)}{\sqrt{-c^2dx^2+d}d} + \frac{ax}{\sqrt{-c^2dx^2+d}d} + \frac{b\log\left(x^2-\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arccos(c*x)/(sqrt(-c^2*d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d) +
1/2*b*log(x^2 - 1/c^2)/(c*d^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b+ax}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2
 - sqrt( - c**2*x**2 + 1)),x)*b + a*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.56 $\int \frac{a+b \arccos(cx)}{\left(d-c^2dx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{a+b \arccos(cx)}{\left(d-c^2dx^2\right)^{5/2}} \, dx = \frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}} + \frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

output
```
1/6*b/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arccos(c*x)
)/d/(-c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arccos(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-
1/3*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{a+b \arccos(cx)}{\left(d-c^2dx^2\right)^{5/2}} \, dx = \frac{\sqrt{d-c^2dx^2}\left(6acx-4ac^3x^3+b\sqrt{1-c^2x^2}+b(6cx-4c^3x^3)\arccos(cx)-2b(1-\right.}{6cd^3\left(-1+c^2x^2\right)^2}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(6*a*c*x - 4*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] + b*(6*c
*x - 4*c^3*x^3)*ArcCos[c*x] - 2*b*(1 - c^2*x^2)^(3/2)*Log[-1 + c^2*x^2]))/
(6*c*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{2\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{2\left(\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

output
```
b/(6*c*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcCos[c*x])
)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*((x*(a + b*ArcCos[c*x]))/(d*Sqrt[d - c^
2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x
^2])))/(3*d)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.06

method	result
default	$a\left(\frac{x}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2dx^2+d}}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\left(2c^3x^3-3cx+2i\sqrt{-c^2x^2+1}\,x^2c^2-2i\sqrt{-c^2x^2+1}\right)\left(8i\ln\left(\left(cx+i\right.\right.\right.}{}$
parts	$a\left(\frac{x}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2dx^2+d}}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\left(2c^3x^3-3cx+2i\sqrt{-c^2x^2+1}\,x^2c^2-2i\sqrt{-c^2x^2+1}\right)\left(8i\ln\left(\left(cx+i\right.\right.\right.}{}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/d*x/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d*
(c^2*x^2-1))^(1/2)*(2*c^3*x^3-3*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-2*I*(-c
^2*x^2+1)^(1/2))*(8*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*x^6*c^6+8*ln((c*x
+I*(-c^2*x^2+1)^(1/2))^2-1)*(-c^2*x^2+1)^(1/2)*x^5*c^5-24*I*ln((c*x+I*(-c^
2*x^2+1)^(1/2))^2-1)*x^4*c^4+2*I*x^4*c^4-20*ln((c*x+I*(-c^2*x^2+1)^(1/2))^
2-1)*(-c^2*x^2+1)^(1/2)*x^3*c^3+2*c^3*x^3*(-c^2*x^2+1)^(1/2)+24*I*ln((c*x+
I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2+6*c^2*x^2*arccos(c*x)-4*I*c^2*x^2+12*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*(-c^2*x^2+1)^(1/2)*x*c-3*c*x*(-c^2*x^2+1)
^(1/2)-8*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)-8*arccos(c*x)+2*I)/d^3/(3*c^
6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^
3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\,(cx)}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2-c^2d^{\frac{5}{2}}}+\frac{2\,\log\,(cx+1)}{c^2d^{\frac{5}{2}}}+\frac{2\,\log\,(cx-1)}{c^2d^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}d}\right)\arccos\,(cx) + \frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}d}\right)$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b*c*(1/(c^4*d^(5/2)*x^2 - c^2*d^(5/2)) + 2*log(c*x + 1)/(c^2*d^(5/2))
 + 2*log(c*x - 1)/(c^2*d^(5/2))) + 1/3*b*(2*x/(sqrt(-c^2*d*x^2 + d)*d^2) +
 x/((-c^2*d*x^2 + d)^(3/2)*d))*arccos(c*x) + 1/3*a*(2*x/(sqrt(-c^2*d*x^2 +
 d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^2x^2-3\sqrt{-c^2x^2+1}\,(}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d^2\,(c^2x^2}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b*c**2*x
**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*
x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b +
 2*a*c**2*x**3 - 3*a*x)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**2
- 1))
```

3.57 $\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 225

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \frac{b}{20cd^3(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}} + \frac{2b}{15cd^3\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x(a+b\arccos(cx))}{5d(d-c^2dx^2)^{5/2}} + \frac{4x(a+b\arccos(cx))}{15d^2(d-c^2dx^2)^{3/2}} + \frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d-c^2dx^2}} - \frac{4b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{15cd^3\sqrt{d-c^2dx^2}}$$

output
```
1/20*b/c/d^3/(-c^2*x^2+1)^(3/2)/(-c^2*d*x^2+d)^(1/2)+2/15*b/c/d^3/(-c^2*x^
2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/5*x*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(
5/2)+4/15*x*(a+b*arccos(c*x))/d^2/(-c^2*d*x^2+d)^(3/2)+8/15*x*(a+b*arccos(
c*x))/d^3/(-c^2*d*x^2+d)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c/
d^3/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.68

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(-60acx+80ac^3x^3-32ac^5x^5-11b\sqrt{1-c^2x^2}+8bc^2x^2\sqrt{1-c^2x^2}-\right.}{60cd^4\left(-1+\right.}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(7/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-60*a*c*x + 80*a*c^3*x^3 - 32*a*c^5*x^5 - 11*b*Sqrt[
1 - c^2*x^2] + 8*b*c^2*x^2*Sqrt[1 - c^2*x^2] - 4*b*c*x*(15 - 20*c^2*x^2 +
8*c^4*x^4)*ArcCos[c*x] + 16*b*(1 - c^2*x^2)^(5/2)*Log[-1 + c^2*x^2]))/(60*
c*d^4*(-1 + c^2*x^2)^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5163, 241, 5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{4\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}dx}{5d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^3}dx}{5d^3\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{5d\left(d-c^2dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}dx}{5d}+\frac{x(a+b\arccos(cx))}{5d\left(d-c^2dx^2\right)^{5/2}}+\frac{b}{20cd^3\left(1-c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5163}$$

$$\frac{4\left(\frac{2\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^{3/2}}dx}{3d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{\left(1-c^2x^2\right)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}\right)}{5d}+\frac{x(a+b\arccos(cx))}{5d\left(d-c^2dx^2\right)^{5/2}}+\frac{b}{20cd^3\left(1-c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\left(\frac{2\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}\right)}{5d}+\frac{x(a+b\arccos(cx))}{5d\left(d-c^2dx^2\right)^{5/2}}+\frac{b}{20cd^3\left(1-c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{4\left(\frac{2\left(\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}\right)}{5d}+\frac{x(a+b\arccos(cx))}{5d\left(d-c^2dx^2\right)^{5/2}}+\frac{b}{20cd^3\left(1-c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{4\left(\frac{x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}\right)}{5d}+\frac{x(a+b\arccos(cx))}{5d\left(d-c^2dx^2\right)^{5/2}}+\frac{b}{20cd^3\left(1-c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(7/2),x]
```

output
```
b/(20*c*d^3*(1 - c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcCos[c*
x]))/(5*d*(d - c^2*d*x^2)^(5/2)) + (4*(b/(6*c*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d
 - c^2*d*x^2]) + (x*(a + b*ArcCos[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*
((x*(a + b*ArcCos[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*Lo
g[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])))/(3*d)))/(5*d)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 2280, normalized size of antiderivative = 10.13

method	result	size
default	Expression too large to display	2280
parts	Expression too large to display	2280

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-176/15*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6
-517*c^4*x^4+287*c^2*x^2-64)/c*(-c^2*x^2+1)^(1/2)-22*I*b*(-d*(c^2*x^2-1))^
(1/2)/d^4/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64
)*x-64*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6-
517*c^4*x^4+287*c^2*x^2-64)*arccos(c*x)*x+64/3*I*b*(-d*(c^2*x^2-1))^(1/2)/
d^4/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c^7*
(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^8-280/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(4
0*c^10*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c^5*(-c^2*
x^2+1)^(1/2)*arccos(c*x)*x^6+784/5*I*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c^10
*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c^3*(-c^2*x^2+1)
^(1/2)*arccos(c*x)*x^4-1784/15*I*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c^10*x^1
0-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c*(-c^2*x^2+1)^(1/2)
*arccos(c*x)*x^2+541/3*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c^10*x^10-215*c^8*
x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c^2*arccos(c*x)*x^3+22*I*b*(-d
*(c^2*x^2-1))^(1/2)/d^4/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+
287*c^2*x^2-64)*(-c^2*x^2+1)*x-128/15*I*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c
^10*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c^12*x^13+176
/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^4/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6-51
7*c^4*x^4+287*c^2*x^2-64)*c^10*x^11-2552/15*I*b*(-d*(c^2*x^2-1))^(1/2)/d^4
/(40*c^10*x^10-215*c^8*x^8+469*c^6*x^6-517*c^4*x^4+287*c^2*x^2-64)*c^8*...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \int \frac{b\arccos(cx)+a}{(-c^2dx^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^8*d^4*x^8 - 4*c^6*d^4
*x^6 + 6*c^4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \int \frac{b\arccos(cx)+a}{(-c^2dx^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(8*x/(sqrt(-c^2*d*x^2 + d)*d^3) + 4*x/((-c^2*d*x^2 + d)^(3/2)*d^2)
+ 3*x/((-c^2*d*x^2 + d)^(5/2)*d)) - b*integrate(arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)/((c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)*sqrt
(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^{7/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(7/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{7/2}}\,dx = \frac{-15\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^6x^6-3\sqrt{-c^2x^2+1}\,c^4x^4+3\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^4x^4}{}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(7/2),x)
```

output
```
( - 15*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**6*x
**6 - 3*sqrt( - c**2*x**2 + 1)*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*c**2*x
**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**4*x**4 + 30*sqrt( - c**2*x**2 + 1)*i
nt(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*
c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x
)*b*c**2*x**2 - 15*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2
 + 1)*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*c**4*x**4 + 3*sqrt( - c**2*x**2
 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b + 8*a*c**4*x**5 - 20*a*c**2
*x**3 + 15*a*x)/(15*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**3*(c**4*x**4 - 2*c**
2*x**2 + 1))
```

3.58 $\int \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 296

$$\int \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2 \, dx = -\frac{15}{64} b^2 d x \sqrt{d-c^2 d x^2}-\frac{1}{32} b^2 x \left(d-c^2 d x^2\right)^{3/2} +\frac{3 b c d x^2 \sqrt{d-c^2 d x^2} (a+b \arccos (c x))}{8 \sqrt{1-c^2 x^2}}-\frac{b d \left(1-c^2 x^2\right)^{3/2} \sqrt{d-c^2 d x^2} (a+b \arccos (c x))}{8 c} +\frac{3}{8} d x \sqrt{d-c^2 d x^2} (a+b \arccos (c x))^2+\frac{1}{4} x \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2-\frac{d \sqrt{d-c^2 d x^2} (a+b \arccos (c x))^3}{8 b c \sqrt{1-c^2 x^2}}$$

output

```
-15/64*b^2*d*x*(-c^2*d*x^2+d)^(1/2)-1/32*b^2*x*(-c^2*d*x^2+d)^(3/2)+3/8*b*
c*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/8*b*d*
(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c+3/8*d*x*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(
c*x))^2-1/8*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1
/2)+9/64*b^2*d*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.11

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{-32b^2d\sqrt{d-c^2dx^2}\arccos(cx)^3-96a^2d^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-8bd\sqrt{d-}}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-32*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 96*a^2*d^(3/2)*Sqrt[1 - c^2
*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 8*b*d*S
qrt[d - c^2*d*x^2]*ArcCos[c*x]^2*(12*a - 8*b*Sin[2*ArcCos[c*x]] + b*Sin[4*
ArcCos[c*x]]) + d*Sqrt[d - c^2*d*x^2]*(160*a^2*c*x*Sqrt[1 - c^2*x^2] - 64*
a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 64*a*b*Cos[2*ArcCos[c*x]] - 4*a*b*Cos[4*Ar
cCos[c*x]] - 32*b^2*Sin[2*ArcCos[c*x]] + b^2*Sin[4*ArcCos[c*x]]) - 4*b*d*S
qrt[d - c^2*d*x^2]*ArcCos[c*x]*(-16*b*Cos[2*ArcCos[c*x]] + b*Cos[4*ArcCos[
c*x]] + 4*a*(-8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])))/(256*c*Sqrt[1 -
 c^2*x^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5157}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx)\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5153}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 5183$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\int (1-c^2x^2)^{3/2}dx}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 211$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 211$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 223$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])
^3)/(6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcC
os[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))
/2))/Sqrt[1 - c^2*x^2]))/4 + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^
2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqr
t[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 987, normalized size of antiderivative = 3.33

method	result
default	$\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3a^2dx\sqrt{-c^2dx^2+d}}{8}+\frac{3a^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3d}{8(c^2x^2-1)c}-\right.$
parts	$\frac{a^2x(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3a^2dx\sqrt{-c^2dx^2+d}}{8}+\frac{3a^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3d}{8(c^2x^2-1)c}-\right.$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^
2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/8*(-d*
(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*d-1/512*
(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^
4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c
*x)+8*arccos(c*x)^2-1)*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*
x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*arccos(c
*x)^2-1+2*I*arccos(c*x))*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I
*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arcco
s(c*x)^2-1-2*I*arccos(c*x))*d/(c^2*x^2-1)/c-1/512*(-d*(c^2*x^2-1))^(1/2)*(
-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-1
2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-4*I*arccos(c*x)+8*arccos(c*x)^2-1)
*d/(c^2*x^2-1)/c)+2*a*b*(3/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c
^2*x^2-1)/c*arccos(c*x)^2*d-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3
*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I
*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1
))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2
*c*x)*(-I+2*arccos(c*x))*d/(c^2*x^2-1)/c-3/256*(-d*(c^2*x^2-1))^(1/2)*(-I*
(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(5*I+12*arccos(c*x))*cos(3*arccos(c*x))*
d/(c^2*x^2-1)/c-1/256*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1...
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx=\int (-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))^2\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a^2 + sqrt(d)*integrate(-((b^2*c^2*d*x^2 - b^2*d)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{d}\,d\left(3asin(cx)\,a^2-2\sqrt{-c^2x^2+1}\,a^2c^3x^3+5\sqrt{-c^2x^2+1}\,a^2cx-16\left(\int\sqrt{-c^2x^2+1}\,a\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 + 5
*sqrt( - c**2*x**2 + 1)*a**2*c*x - 16*int(sqrt( - c**2*x**2 + 1)*acos(c*x)
*x**2,x)*a*b*c**3 + 16*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b*c - 8*i
nt(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt( - c
**2*x**2 + 1)*acos(c*x)**2,x)*b**2*c))/(8*c)
```

3.59 $\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = -\frac{1}{4}b^2x\sqrt{d-c^2dx^2} + \frac{bcx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}} + \frac{b^2\sqrt{d-c^2dx^2}\arcsin(cx)}{4c\sqrt{1-c^2x^2}}$$

output
```
-1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*b*c*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
os(c*x))/(-c^2*x^2+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2
-1/6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)+1/4*b
^2*(-c^2*d*x^2+d)^(1/2)*arcsin(c*x)/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \frac{1}{2}a^2x\sqrt{d-c^2dx^2} - \frac{a^2\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{2c} - \frac{b^2\sqrt{d-c^2dx^2}(4\arccos(cx)^3-6\arccos(cx)\cos(2\arccos(cx))+(3-6\arccos(cx)^2)\sin(2\arccos(cx)))}{24c\sqrt{1-c^2x^2}} + \frac{ab\sqrt{d-c^2dx^2}(\cos(2\arccos(cx))+2\arccos(cx)(-\arccos(cx)+\sin(2\arccos(cx))))}{4c\sqrt{1-c^2x^2}}$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(a^2*x*Sqrt[d - c^2*d*x^2])/2 - (a^2*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*c) - (b^2*Sqrt[d - c^2*d*x^2]*(4*ArcCos[
c*x]^3 - 6*ArcCos[c*x]*Cos[2*ArcCos[c*x]] + (3 - 6*ArcCos[c*x]^2)*Sin[2*Ar
cCos[c*x]]))/(24*c*Sqrt[1 - c^2*x^2]) + (a*b*Sqrt[d - c^2*d*x^2]*(Cos[2*Ar
cCos[c*x]] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(4*c*Sqrt
[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5157, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5139$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 262$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 223$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5153$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^3)/(6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(
(x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSi
n[c*x]/(2*c^3)))/2))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.77

method	result
default	$\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{6(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\,\left(2c^3x^3-2cx\right.}{}\right.$
parts	$\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{6(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\,\left(2c^3x^3-2cx\right.}{}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*a^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/
2)/(c^2*x^2-1)/c*arccos(c*x)^3+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*
x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*arccos(c*x)^2-1+
2*I*arccos(c*x))/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1
-2*I*arccos(c*x))/(c^2*x^2-1)/c)+2*a*b*(1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x
^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3
*x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arcco
s(c*x))/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)
*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(c^2*x^2
-1)/c)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
), x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^2\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos\,(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a^2 + sqrt(d)*integra
te((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$= \frac{\sqrt{d}\,\left(asin(cx)\,a^2+\sqrt{-c^2x^2+1}\,a^2cx+4\left(\int \sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)abc+2\left(\int \sqrt{-c^2x^2+1}\,acos(cx)^2\,dx\right)\ldots\right)}{2c}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*(asin(c*x)*a**2 + sqrt( - c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt( -
 c**2*x**2 + 1)*acos(c*x),x)*a*b*c + 2*int(sqrt( - c**2*x**2 + 1)*acos(c*x
)**2,x)*b**2*c))/(2*c)
```

3.60 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d-c^2dx^2}}$$

output
```
-1/3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}\arccos(cx)\left(3a^2+3ab\arccos(cx)+b^2\arccos(cx)^2\right)}{3c\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*ArcCos[c*x]*(3*a^2 + 3*a*b*ArcCos[c*x] + b^2*ArcCo
s[c*x]^2))/(c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \text{5153}$$
$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^3)/(b*c*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(43) = 86.

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.90

method	result	size
default	$\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d\, x^{2}+d}}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos \left(c x \right)^{3}}{3 c d \left(c^{2} x^{2}-1\right)}+\frac{a b \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos \left(c x \right)^{2}}{c d \left(c^{2} x^{2}-1\right)}$	142
parts	$\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d\, x^{2}+d}}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos \left(c x \right)^{3}}{3 c d \left(c^{2} x^{2}-1\right)}+\frac{a b \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos \left(c x \right)^{2}}{c d \left(c^{2} x^{2}-1\right)}$	142

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d
*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^3+a*b*(
-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{b^2\arccos{(cx)}^2\arcsin{(cx)}}{c\sqrt{d}} + \frac{1}{3}\left(\frac{3\arccos{(cx)}\arcsin{(cx)}^2}{c\sqrt{d}} + \frac{\arcsin{(cx)}^3}{c\sqrt{d}}\right)b^2 + \frac{2ab\arccos{(cx)}\arcsin{(cx)}}{c\sqrt{d}} + \frac{ab\arcsin{(cx)}^2}{c\sqrt{d}} + \frac{a^2\arcsin{(cx)}}{c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b^2*arccos(c*x)^2*arcsin(c*x)/(c*sqrt(d)) + 1/3*(3*arccos(c*x)*arcsin(c*x)
^2/(c*sqrt(d)) + arcsin(c*x)^3/(c*sqrt(d)))*b^2 + 2*a*b*arccos(c*x)*arcsin
(c*x)/(c*sqrt(d)) + a*b*arcsin(c*x)^2/(c*sqrt(d)) + a^2*arcsin(c*x)/(c*sqr
t(d))
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -\frac{b^2\arccos{(cx)}^3 + 3\,ab\arccos{(cx)}^2 + 3\,a^2\arccos{(cx)}}{3\,c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(b^2*arccos(c*x)^3 + 3*a*b*arccos(c*x)^2 + 3*a^2*arccos(c*x))/(c*sqrt
(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d}\,\left(-acos(cx)^3\,b^2-3acos(cx)^2\,ab+3asin(cx)\,a^2\right)}{3cd}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*( - acos(c*x)**3*b**2 - 3*acos(c*x)**2*a*b + 3*asin(c*x)*a**2))/(
3*c*d)
```

3.61 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}} + \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cd\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2}(2a+b\pi-b(\pi-2\arccos(cx)))\log\left(1-e^{2i\arccos(cx)}\right)}{cd\sqrt{d-c^2dx^2}} + \frac{ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{cd\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+I*(-c^2*x^2+1)^(1/2)*(a+b*arc
cos(c*x))^2/c/d/(-c^2*d*x^2+d)^(1/2)-b*(-c^2*x^2+1)^(1/2)*(2*a+b*Pi-b*(Pi-
2*arccos(c*x)))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/d/(-c^2*d*x^2+d)^(1/2
)+I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/d/(-c
^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{b^2\left(cx+i\sqrt{1-c^2x^2}\right)\arccos(cx)^2+2b\arccos(cx)\left(acx-b\sqrt{1-c^2x^2}\log\left(1-e\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(b^2*(c*x + I*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2*b*ArcCos[c*x]*(a*c*x -
b*Sqrt[1 - c^2*x^2]*Log[1 - E^((2*I)*ArcCos[c*x])]) + a*(a*c*x - b*Sqrt[1
- c^2*x^2]*Log[1 - c^2*x^2]) + I*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)
*ArcCos[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 5161}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5181}$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4200$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2620$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\text{a}}{}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(}\right.\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]
*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Lo
g[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/
(c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.95

method	result
default	$\frac{a^2 x}{d\sqrt{-c^2 d\,x^2+d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{(c^2x^2-1)c\,d^2}-\frac{2i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,\left(i\arccos(cx)\ln\left(1+c\right.\right.}{}\right.$
parts	$\frac{a^2 x}{d\sqrt{-c^2 d\,x^2+d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{(c^2x^2-1)c\,d^2}-\frac{2i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,\left(i\arccos(cx)\ln\left(1+c\right.\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*x/(-c^2*d*x^2+d)^(1/2)+b^2*(-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)
^(1/2)+c*x)*arccos(c*x)^2/(c^2*x^2-1)/c/d^2-2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^
2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x
)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2+polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/(c^2*x^2-1)/c/d^2)-2*I*a*b*
(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/(c^2*x^2-1)*arccos(c*x)-2*
a*b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d^2/(c^2*x^2-1)*x+2*a*b*(-d*(c^2*x^
2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln((c*x+I*(-c^2*x^2+1)^(1
/2))^2-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
2*a*b*x*arccos(c*x)/(sqrt(-c^2*d*x^2 + d)*d) - b^2*integrate(arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x +
 1)), x)/sqrt(d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d) + a*b*log(x^2 - 1/c^2)/(
c*d^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab-\sqrt{-c^2x^2+1}\left(\int \frac{acos(}{\sqrt{-c^2x^2+1}\,c^2x}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*2 - sqrt( - c**2*x**2 + 1)),x)*a*b - sqrt( - c**2*x**2 + 1)*int(acos(c*x)
**2/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2 +
a**2*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.62 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 321

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{b^2x}{3d^2\sqrt{d-c^2dx^2}}+\frac{b(a+b\arccos(cx))}{3cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x(a+b\arccos(cx))^2}{3d\,(d-c^2dx^2)^{3/2}}+\frac{2x(a+b\arccos(cx))^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{2i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(2a+b\pi-b(\pi-2\arccos(cx)))\log\left(1-e^{2i\arccos(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}} + \frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

output

```
1/3*b^2*x/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*arccos(c*x))/c/d^2/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^
(3/2)+2/3*x*(a+b*arccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+2/3*I*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))^2/c/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*b*(-c^2*x^2+1)^
(1/2)*(2*a+b*Pi-b*(Pi-2*arccos(c*x)))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c
/d^2/(-c^2*d*x^2+d)^(1/2)+2/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+I*(-
c^2*x^2+1)^(1/2))^2)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{-3a^2cx-b^2cx+2a^2c^3x^3+b^2c^3x^3-ab\sqrt{1-c^2x^2}+b^2\left(-3cx+2c^3x^3-2i\sqrt{1\right.}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(-3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 - c^2*x^2
] + b^2*(-3*c*x + 2*c^3*x^3 - (2*I)*Sqrt[1 - c^2*x^2] + (2*I)*c^2*x^2*Sqrt
[1 - c^2*x^2])*ArcCos[c*x]^2 + b*ArcCos[c*x]*(-6*a*c*x + 4*a*c^3*x^3 - b*S
qrt[1 - c^2*x^2] + 4*b*(1 - c^2*x^2)^(3/2)*Log[1 - E^((2*I)*ArcCos[c*x])])
 + 2*a*b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 - c^2*x
^2]*Log[1 - c^2*x^2] - (2*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^((2*I)*A
rcCos[c*x])])/(3*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.84, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 5181$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 4200$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 25

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2620

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx)}{2b}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}$$

$$\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2715

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}$$

$$\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+$$

$$\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5183

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{b\int \frac{1}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 - c^
2*x^2]*((b*x)/(2*c*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*c^2*(1 - c^
2*x^2))))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*(a + b*ArcCos[c*x])^2)/(d*S
qrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x]
)^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] +
 (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])))/(3
*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2417 vs. $2(305) = 610$.

Time $= 0.44$ (sec) , antiderivative size $= 2418$, normalized size of antiderivative $= 7.53$

method	result	size
default	Expression too large to display	2418
parts	Expression too large to display	2418

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*x+17
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*
arccos(c*x)^2*x^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4
+11*c^2*x^2-4)*c^4*(-c^2*x^2+1)*x^5-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*
c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*(-c^2*x^2+1)*x^3-4/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c*(-c^2*x^2+1)^(1/2)*ar
ccos(c*x)-2*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*
x^2-4)*arccos(c*x)*x+4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^
4*x^4+11*c^2*x^2-4)/c*(-c^2*x^2+1)^(1/2)-14/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)
/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c*(-c^2*x^2+1)^(1/2)*arccos(c*x)^
2*x^2+4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)*c^4*(-c^2*x^2+1)*arccos(c*x)*x^5+2*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(
3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2*x^
4-10/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)*c^2*(-c^2*x^2+1)*arccos(c*x)*x^3-2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6
*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4*arccos(c*x)^2*x^5+a^2*(1/3/d*x/(-c^2*d*x
^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))+4/3*b^2*(-c^2*x^2+1)^(1/2)*(-d
*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^
(1/2))+4/3*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c
*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-1/3*a*b*(-d*(c^2*x^2-1))^(1...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 - c^2*d^(5/2)) + 2*log(c*x + 1)/(c^2*d^(5/2
)) + 2*log(c*x - 1)/(c^2*d^(5/2))) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2 + d)*d^
2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccos(c*x) + 1/3*a^2*(2*x/(sqrt(-c^2*d
*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqr
t(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^2x^2-6\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*c**2
*x**2 - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a
*b + 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x*
*2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2
 + 1)),x)*b**2 + 2*a**2*c**2*x**3 - 3*a**2*x)/(3*sqrt(d)*sqrt( - c**2*x**2
 + 1)*d**2*(c**2*x**2 - 1))
```

3.63 $\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 351

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=-\frac{45acx^2\sqrt{c-a^2cx^2}}{128\sqrt{1-a^2x^2}}$$
$$+\frac{3c(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}}{128a}-\frac{45}{64}cx\sqrt{c-a^2cx^2}\arccos(ax)$$
$$-\frac{3}{32}x\left(c-a^2cx^2\right)^{3/2}\arccos(ax)-\frac{27c\sqrt{c-a^2cx^2}\arccos(ax)^2}{128a\sqrt{1-a^2x^2}}+\frac{9acx^2\sqrt{c-a^2cx^2}\arccos(ax)^2}{16\sqrt{1-a^2x^2}}-\frac{3c(1-a^2x^2}{}$$

output
```
-45/128*a*c*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)+3/128*c*(-a^2*x^2+
1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a-45/64*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)
-3/32*x*(-a^2*c*x^2+c)^(3/2)*arccos(a*x)-27/128*c*(-a^2*c*x^2+c)^(1/2)*arc
cos(a*x)^2/a/(-a^2*x^2+1)^(1/2)+9/16*a*c*x^2*(-a^2*c*x^2+c)^(1/2)*arccos(a
*x)^2/(-a^2*x^2+1)^(1/2)-3/16*c*(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)*ar
ccos(a*x)^2/a+3/8*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3+1/4*x*(-a^2*c*x^2
+c)^(3/2)*arccos(a*x)^3-3/32*c*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^4/a/(-a^2*
x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.39

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx =$$
$$-\frac{c\sqrt{c-a^2cx^2}(96\arccos(ax)^4-3(-64\cos(2\arccos(ax))+\cos(4\arccos(ax)))+24\arccos(ax)^2(-16\cos(2$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3,x]
```

output
```
-1/1024*(c*Sqrt[c - a^2*c*x^2]*(96*ArcCos[a*x]^4 - 3*(-64*Cos[2*ArcCos[a*x
]] + Cos[4*ArcCos[a*x]]) + 24*ArcCos[a*x]^2*(-16*Cos[2*ArcCos[a*x]] + Cos[
4*ArcCos[a*x]]) - 12*ArcCos[a*x]*(-32*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*
x]]) + 32*ArcCos[a*x]^3*(-8*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*x]])))/(a*
Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5159, 5157, 5139, 5153, 5183, 5159, 244, 2009, 5157, 15, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \; 5159$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{3}{4}c\int \sqrt{c-a^2cx^2}\arccos(ax)^3dx+\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5157$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+$$
$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\int x\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int\frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}\right)+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5139

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+$$
$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int\frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5153

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5183

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\int\left(1-a^2x^2\right)^{3/2}\arccos(ax)dx}{2a}-\frac{\left(1-a^2x^2\right)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5159

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\int \sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}a\int x(1-a^2x^2)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)}{2a}-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 244

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\int \sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}a\int (x-a^2x^3)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)}{2a}-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 2009

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\int \sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 5157

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{a\int xdx}{2}+\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}-\frac{(1-a^2x^2)^2}{4a}\right.}{4\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 15

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)+\frac{ax^2}{4}\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}-\frac{(1-a^2x^2)^2\text{arc}}{4a^2}\right.}{4\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5153$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-c}}{4\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 5211$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx}}{8a\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-c}}{4\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 15$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-}}{4\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \; 5153$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-}}{4\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}+\frac{3a\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\right.}{2\sqrt{1-a^2x^2}}\right.$$

input
```
Int[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3,x]
```

output
```
(x*(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3)/4 + (3*c*((x*Sqrt[c - a^2*c*x^2]*A
rcCos[a*x]^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^4)/(8*a*Sqrt[1 - a^2*x^
2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x
*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/(2*Sqrt
[1 - a^2*x^2])))/4 + (3*a*c*Sqrt[c - a^2*c*x^2]*(-1/4*((1 - a^2*x^2)^2*Arc
Cos[a*x]^2)/a^2 - ((a*(x^2/2 - (a^2*x^4)/4))/4 + (x*(1 - a^2*x^2)^(3/2)*Ar
cCos[a*x])/4 + (3*((a*x^2)/4 + (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/2 - ArcCo
s[a*x]^2/(4*a)))/4)/(2*a)))/(4*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.52

method	result
default	$\frac{3\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4 c}{32a(a^2x^2-1)} - \frac{\sqrt{-c(a^2x^2-1)}\,\left(8a^5x^5-12a^3x^3+8i\sqrt{-a^2x^2+1}\,a^4x^4+4ax-8i\sqrt{-a^2x^2+1}\,a^2x^2+i\sqrt{}\right.}{2048a(a^2x^2-1)}$

input
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/(a^2*x^2-1)*arccos(a*x)^4
*c-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*I*(-a^2*x^2+1)^(1
/2)*a^4*x^4+4*a*x-8*I*(-a^2*x^2+1)^(1/2)*a^2*x^2+I*(-a^2*x^2+1)^(1/2))*(24
*I*arccos(a*x)^2+32*arccos(a*x)^3-3*I-12*arccos(a*x))*c/a/(a^2*x^2-1)+1/32
*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-I*
(-a^2*x^2+1)^(1/2))*(6*I*arccos(a*x)^2+4*arccos(a*x)^3-3*I-6*arccos(a*x))*
c/a/(a^2*x^2-1)+1/32*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2*x
^2+2*a^3*x^3+I*(-a^2*x^2+1)^(1/2)-2*a*x)*(-6*I*arccos(a*x)^2+4*arccos(a*x)
^3+3*I-6*arccos(a*x))*c/a/(a^2*x^2-1)-1/2048*(-c*(a^2*x^2-1))^(1/2)*(-8*I*
(-a^2*x^2+1)^(1/2)*a^4*x^4+8*a^5*x^5+8*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-12*a^3
*x^3-I*(-a^2*x^2+1)^(1/2)+4*a*x)*(-24*I*arccos(a*x)^2+32*arccos(a*x)^3+3*I
-12*arccos(a*x))*c/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3, x)
```

Sympy [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \left(-c(ax-1)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{acos}^3\left(ax\right)dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*acos(a*x)**3,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*acos(a*x)**3, x)
```

Maxima [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*arccos(a*x)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \mathrm{acos}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int (c-a^2cx^2)^{3/2}\arccos(ax)^3\,dx = \sqrt{c}\,c\left(-\left(\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,x^2dx\right)a^2 + \int \sqrt{-a^2x^2+1}\,acos(ax)^3\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)*acos(a*x)^3,x)
```

output
```
sqrt(c)*c*( - int(sqrt( - a**2*x**2 + 1)*acos(a*x)**3*x**2,x)*a**2 + int(s
qrt( - a**2*x**2 + 1)*acos(a*x)**3,x))
```

3.64 $\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 215

$$\begin{aligned}\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = &-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{1-a^2x^2}}-\frac{3}{4}x\sqrt{c-a^2cx^2}\arccos(ax)\\&-\frac{3\sqrt{c-a^2cx^2}\arccos(ax)^2}{8a\sqrt{1-a^2x^2}}\\&+\frac{3ax^2\sqrt{c-a^2cx^2}\arccos(ax)^2}{4\sqrt{1-a^2x^2}}\\&+\frac{1}{2}x\sqrt{c-a^2cx^2}\arccos(ax)^3-\frac{\sqrt{c-a^2cx^2}\arccos(ax)^4}{8a\sqrt{1-a^2x^2}}\end{aligned}$$

output
```
-3/8*a*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-3/4*x*(-a^2*c*x^2+c)^(1
/2)*arccos(a*x)-3/8*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^2/a/(-a^2*x^2+1)^(1/2
)+3/4*a*x^2*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^2/(-a^2*x^2+1)^(1/2)+1/2*x*(-
a^2*c*x^2+c)^(1/2)*arccos(a*x)^3-1/8*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^4/a/
(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.40

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx =$$
$$-\frac{\sqrt{c(1-a^2x^2)}((3-6\arccos(ax)^2)\cos(2\arccos(ax))+2\arccos(ax)\,(\arccos(ax)^3+(3-2\arccos(ax)^2}{16a\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^3,x]
```

output
```
-1/16*(Sqrt[c*(1 - a^2*x^2)]*((3 - 6*ArcCos[a*x]^2)*Cos[2*ArcCos[a*x]] + 2
*ArcCos[a*x]*(ArcCos[a*x]^3 + (3 - 2*ArcCos[a*x]^2)*Sin[2*ArcCos[a*x]])))/
(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5157, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\sqrt{c-a^2cx^2}\,dx$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{3a\sqrt{c-a^2cx^2}\int x\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$\Big\downarrow$ 5211

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$\Big\downarrow$ 15

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$\Big\downarrow$ 5153

$$-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}+\frac{3a\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

input

```
Int[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^3,x]
```

output

```
(x*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a*x]
^4)/(8*a*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCos[a*x]^2
)/2 + a*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a
*x]^2/(4*a^3))))/(2*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4}{8a(a^2x^2-1)} + \frac{\sqrt{-c(a^2x^2-1)}\,\left(2a^3x^3-2ax+2i\sqrt{-a^2x^2+1}\,a^2x^2-i\sqrt{-a^2x^2+1}\right)\left(6i\arccos(ax)^2+4\,\mathrm{ar}\right.}{32a(a^2x^2-1)}$

input
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/(a^2*x^2-1)*arccos(a*x)^4+
1/32*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*I*(-a^2*x^2+1)^(1/2)*a^2*x^
2-I*(-a^2*x^2+1)^(1/2))*(6*I*arccos(a*x)^2+4*arccos(a*x)^3-3*I-6*arccos(a*
x))/a/(a^2*x^2-1)+1/32*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2
*x^2+2*a^3*x^3+I*(-a^2*x^2+1)^(1/2)-2*a*x)*(-6*I*arccos(a*x)^2+4*arccos(a*
x)^3+3*I-6*arccos(a*x))/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \int \sqrt{-a^2cx^2+c}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3, x)
```

Sympy [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \int \sqrt{-c\,(ax-1)\,(ax+1)}\operatorname{acos}^3\,(ax)\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*acos(a*x)**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acos(a*x)**3, x)
```

Maxima [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \int \sqrt{-a^2cx^2+c}\arccos\,(ax)^3\;dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx=\int \mathrm{acos}(a\,x)^3\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx=\sqrt{c}\left(\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*acos(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt( - a**2*x**2 + 1)*acos(a*x)**3,x)
```

3.65 $\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}} dx = -\frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

output
```
-1/4*(-a^2*x^2+1)^(1/2)*arccos(a*x)^4/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}} dx = -\frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[ArcCos[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/4*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^4)/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{5153\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/4*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^4)/(a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4}{4ac(a^2x^2-1)}$	52

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/c/(a^2*x^2-1)*arccos(a*x)^
4
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\arccos{(ax)}^3}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(acos(a*x)**3/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \frac{\arccos(ax)^3\arcsin(ax)}{a\sqrt{c}} + \frac{3\arccos(ax)^2\arcsin(ax)^2}{2a\sqrt{c}} + \frac{\frac{4\arccos(ax)\arcsin(ax)^3}{a} + \frac{\arcsin(ax)^4}{a}}{4\sqrt{c}}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
arccos(a*x)^3*arcsin(a*x)/(a*sqrt(c)) + 3/2*arccos(a*x)^2*arcsin(a*x)^2/(a
*sqrt(c)) + 1/4*(4*arccos(a*x)*arcsin(a*x)^3/a + arcsin(a*x)^4/a)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.33

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\arccos(ax)^4}{4a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*arccos(a*x)^4/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.38

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{c}\,acos(ax)^4}{4ac}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c)*acos(a*x)**4)/(4*a*c)
```

3.66 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 236

$$\begin{aligned}\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx &= \frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{i\sqrt{1-a^2x^2}\arccos(ax)^3}{ac\sqrt{c-a^2cx^2}} \\ &- \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)}{ac\sqrt{c-a^2cx^2}} \\ &+ \frac{3i\sqrt{1-a^2x^2}\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)}{ac\sqrt{c-a^2cx^2}} \\ &- \frac{3\sqrt{1-a^2x^2}\operatorname{PolyLog}\left(3,e^{2i\arccos(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
x*arccos(a*x)^3/c/(-a^2*c*x^2+c)^(1/2)+I*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/
a/c/(-a^2*c*x^2+c)^(1/2)-3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*ln(1-(a*x+I*(-
a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x^2+c)^(1/2)+3*I*(-a^2*x^2+1)^(1/2)*arcco
s(a*x)*polylog(2,(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x^2+c)^(1/2)-3/
2*(-a^2*x^2+1)^(1/2)*polylog(3,(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x
^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.77

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{-i\pi^3\sqrt{1-a^2x^2}-8ax\arccos(ax)^3+8i\sqrt{1-a^2x^2}\arccos(ax)^3+24\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1-e^{-2ia\cdots}\right.}{8ac\sqrt{c(1\cdots}}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/8*((-I)*Pi^3*Sqrt[1 - a^2*x^2] - 8*a*x*ArcCos[a*x]^3 + (8*I)*Sqrt[1 - a
^2*x^2]*ArcCos[a*x]^3 + 24*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2*Log[1 - E^((-2*
I)*ArcCos[a*x])] + (24*I)*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*PolyLog[2, E^((-2*
I)*ArcCos[a*x])] + 12*Sqrt[1 - a^2*x^2]*PolyLog[3, E^((-2*I)*ArcCos[a*x])]
)/(a*c*Sqrt[c*(1 - a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5161, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5161}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)^2}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int -\arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\sqrt{1-a^2x^2}\int \arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{4200}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\int \arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{3}i\,\text{a}\ldots}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int \text{PolyLog}\ldots\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\,\text{arcco}\ldots}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{x \arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3\ldots\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(x*ArcCos[a*x]^3)/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*((-1/3*I)
*ArcCos[a*x]^3 - (2*I)*((I/2)*ArcCos[a*x]^2*Log[1 - E^((2*I)*ArcCos[a*x])]
 - I*((I/2)*ArcCos[a*x]*PolyLog[2, E^((2*I)*ArcCos[a*x])] - PolyLog[3, E^(
(2*I)*ArcCos[a*x])]/4))))/(a*c*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-i\sqrt{-a^2x^2+1}+ax\right)\arccos(ax)^3}{c^2a(a^2x^2-1)} - \frac{\sqrt{-a^2x^2+1}\,\sqrt{-c(a^2x^2-1)}\left(2i\arccos(ax)^3-3\arccos(ax)^2\ln\left(1+ax+i\sqrt{-\right.\right.}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-I*(-a^2*x^2+1)^(1/2)+a*x)*arccos(a*x)^3/c^2/a/(a
^2*x^2-1)-(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*I*arccos(a*x)^3-3*a
rccos(a*x)^2*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*arccos(a*x)^2*ln(1-a*x-I*(-a
^2*x^2+1)^(1/2))+6*I*arccos(a*x)*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))+6*I*
arccos(a*x)*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-6*polylog(3,-a*x-I*(-a^2*x
^2+1)^(1/2))-6*polylog(3,a*x+I*(-a^2*x^2+1)^(1/2)))/c^2/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos{(ax)}^3}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(acos(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x))/(sqrt(c)*c)
```

3.67 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 387

$$\begin{aligned}\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{x\arccos(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\arccos(ax)^2}{2ac^2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}} \\ &+ \frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}} + \frac{2x\arccos(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{2i\sqrt{1-a^2x^2}\arccos(ax)^3}{3ac^2\sqrt{c-a^2cx^2}} \\ &- \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\log\left(1-a^2x^2\right)}{2ac^2\sqrt{c-a^2cx^2}} \\ &+ \frac{2i\sqrt{1-a^2x^2}\arccos(ax)\,\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}} \\ &- \frac{\sqrt{1-a^2x^2}\,\text{PolyLog}\left(3,e^{2i\arccos(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
x*arccos(a*x)/c^2/(-a^2*c*x^2+c)^(1/2)+1/2*arccos(a*x)^2/a/c^2/(-a^2*x^2+1
)^(1/2)/(-a^2*c*x^2+c)^(1/2)+1/3*x*arccos(a*x)^3/c/(-a^2*c*x^2+c)^(3/2)+2/
3*x*arccos(a*x)^3/c^2/(-a^2*c*x^2+c)^(1/2)+2/3*I*(-a^2*x^2+1)^(1/2)*arccos
(a*x)^3/a/c^2/(-a^2*c*x^2+c)^(1/2)-2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*ln(1
-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*c*x^2+c)^(1/2)-1/2*(-a^2*x^2+1)
^(1/2)*ln(-a^2*x^2+1)/a/c^2/(-a^2*c*x^2+c)^(1/2)+2*I*(-a^2*x^2+1)^(1/2)*ar
ccos(a*x)*polylog(2,(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*c*x^2+c)^(1/
2)-(-a^2*x^2+1)^(1/2)*polylog(3,(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*
c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.56

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(1-a^2x^2)^{3/2}\left(-i\pi^3-\frac{12ax\arccos(ax)}{\sqrt{1-a^2x^2}}+\frac{6\arccos(ax)^2}{-1+a^2x^2}+8i\arccos(ax)^3-\frac{4ax\arccos(ax)^3}{(1-a^2x^2)^{3/2}}-\frac{8ax\arccos(ax)^3}{\sqrt{1-a^2x^2}}+24\,\mathrm{arc}\right.}{}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/12*((1 - a^2*x^2)^(3/2)*((-I)*Pi^3 - (12*a*x*ArcCos[a*x])/Sqrt[1 - a^2*
x^2] + (6*ArcCos[a*x]^2)/(-1 + a^2*x^2) + (8*I)*ArcCos[a*x]^3 - (4*a*x*Arc
Cos[a*x]^3)/(1 - a^2*x^2)^(3/2) - (8*a*x*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2]
+ 24*ArcCos[a*x]^2*Log[1 - E^((-2*I)*ArcCos[a*x])] + 6*Log[1 - a^2*x^2] +
(24*I)*ArcCos[a*x]*PolyLog[2, E^((-2*I)*ArcCos[a*x])] + 12*PolyLog[3, E^((
-2*I)*ArcCos[a*x])]))/(a*c*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.75, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 5183, 5161, 240, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)^2}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int -\arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{3\sqrt{1-a^2x^2}\int \arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\int \arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int \operatorname{PolyLog}\left(2,e^{2i\arccos\cdots}\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}\cdots\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^{3/2}}dx}{a}+\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}\right)}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}\cdots\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{a\int \frac{x}{1-a^2x^2}dx+\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}}{a}+\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}\right)}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}\cdots\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\text{PolyL}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}+\frac{\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}}{a}\right)}{c^2\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}+\frac{\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}}{a}\right)}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2i\arccos(a}\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(x*ArcCos[a*x]^3)/(3*c*(c - a^2*c*x^2)^(3/2)) + (a*Sqrt[1 - a^2*x^2]*(ArcC
os[a*x]^2/(2*a^2*(1 - a^2*x^2)) + ((x*ArcCos[a*x])/Sqrt[1 - a^2*x^2] - Log
[1 - a^2*x^2]/(2*a))/a))/(c^2*Sqrt[c - a^2*c*x^2]) + (2*((x*ArcCos[a*x]^3)
/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*((-1/3*I)*ArcCos[a*x]^3 -
(2*I)*((I/2)*ArcCos[a*x]^2*Log[1 - E^((2*I)*ArcCos[a*x])] - I*((I/2)*ArcCo
s[a*x]*PolyLog[2, E^((2*I)*ArcCos[a*x])] - PolyLog[3, E^((2*I)*ArcCos[a*x]
)]/4))))/(a*c*Sqrt[c - a^2*c*x^2])))/(3*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(-2 i \sqrt{-a^{2} x^{2}+1}\, a^{2} x^{2}+2 a^{3} x^{3}+2 i \sqrt{-a^{2} x^{2}+1}-3 a x\right) \arccos\left(a x\right) \left(-6 i \arccos\left(a x\right) a^{4} x^{4}+6 \sqrt{-a^{2} x^{2}+1}\, \arccos\left(a x\right) a}{}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2+2*a^3*x^3+2*I
*(-a^2*x^2+1)^(1/2)-3*a*x)*arccos(a*x)*(-6*I*arccos(a*x)*a^4*x^4+6*(-a^2*x
^2+1)^(1/2)*arccos(a*x)*a^3*x^3-6*I*(-a^2*x^2+1)^(1/2)*a^3*x^3-6*a^4*x^4+6
*a^2*x^2*arccos(a*x)^2+12*I*arccos(a*x)*a^2*x^2-9*arccos(a*x)*(-a^2*x^2+1)
^(1/2)*a*x+6*I*(-a^2*x^2+1)^(1/2)*a*x+18*a^2*x^2-8*arccos(a*x)^2-6*I*arcco
s(a*x)-12)/c^3/(3*a^6*x^6-10*a^4*x^4+11*a^2*x^2-4)/a+(-c*(a^2*x^2-1))^(1/2
)*(-a^2*x^2+1)^(1/2)/a/c^3/(a^2*x^2-1)*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-2*(-
c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/c^3/(a^2*x^2-1)*ln(a*x+I*(-a^2*x
^2+1)^(1/2))+(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/c^3/(a^2*x^2-1)*l
n(I*(-a^2*x^2+1)^(1/2)+a*x-1)-2/3*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2
)*(2*I*arccos(a*x)^3-3*arccos(a*x)^2*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*arcc
os(a*x)^2*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+6*I*arccos(a*x)*polylog(2,-a*x-I*
(-a^2*x^2+1)^(1/2))+6*I*arccos(a*x)*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-6*
polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))-6*polylog(3,a*x+I*(-a^2*x^2+1)^(1/2))
)/a/c^3/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{acos}^3(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(acos(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2 +
 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.68 $\int \frac{\left(d-c^2dx^2\right)^{5/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 430

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{15d^2\sqrt{d-c^2dx^2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc\sqrt{1-c^2x^2}} - \frac{3d^2\sqrt{d-c^2dx^2}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc\sqrt{1-c^2x^2}} + \frac{d^2\sqrt{d-c^2dx^2}\cos\left(\frac{6a}{b}\right)\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc\sqrt{1-c^2x^2}} - \frac{5d^2\sqrt{d-c^2dx^2}\log(a+b\arccos(cx))}{16bc\sqrt{1-c^2x^2}} + \frac{15d^2\sqrt{d-c^2dx^2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc\sqrt{1-c^2x^2}} - \frac{3d^2\sqrt{d-c^2dx^2}\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc\sqrt{1-c^2x^2}} + \frac{d^2\sqrt{d-c^2dx^2}\sin\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc\sqrt{1-c^2x^2}}$$

output
```
15/32*d^2*(-c^2*d*x^2+d)^(1/2)*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c/(-
c^2*x^2+1)^(1/2)-3/16*d^2*(-c^2*d*x^2+d)^(1/2)*cos(4*a/b)*Ci(4*(a+b*arccos
(c*x))/b)/b/c/(-c^2*x^2+1)^(1/2)+1/32*d^2*(-c^2*d*x^2+d)^(1/2)*cos(6*a/b)*
Ci(6*(a+b*arccos(c*x))/b)/b/c/(-c^2*x^2+1)^(1/2)-5/16*d^2*(-c^2*d*x^2+d)^(
1/2)*ln(a+b*arccos(c*x))/b/c/(-c^2*x^2+1)^(1/2)+15/32*d^2*(-c^2*d*x^2+d)^(
1/2)*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c/(-c^2*x^2+1)^(1/2)-3/16*d^2*
(-c^2*d*x^2+d)^(1/2)*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c/(-c^2*x^2+1)
^(1/2)+1/32*d^2*(-c^2*d*x^2+d)^(1/2)*sin(6*a/b)*Si(6*(a+b*arccos(c*x))/b)/
b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.46

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(15\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-6\cos\left(\frac{4a}{b}\right)\text{CosInteg}\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)/(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(15*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x]
)] - 6*Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcCos[c*x])] + Cos[(6*a)/b]*CosI
ntegral[6*(a/b + ArcCos[c*x])] - 18*Log[a + b*ArcCos[c*x]] + 8*Log[8*(a +
b*ArcCos[c*x])] + 15*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] - 6*S
in[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])] + Sin[(6*a)/b]*SinIntegral[
6*(a/b + ArcCos[c*x])]))/(32*b*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.47, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{ 5169}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int \frac{\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^6}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int \left(-\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{3\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{15\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{5}{16(a+b\arccos(cx}\right.}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\left(-\frac{15}{32}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{3}{16}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\cos\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)/(a + b*ArcCos[c*x]),x]
```

output
```
-((d^2*Sqrt[d - c^2*d*x^2]*((-15*Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos
[c*x]))/b])/32 + (3*Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/1
6 - (Cos[(6*a)/b]*CosIntegral[(6*(a + b*ArcCos[c*x]))/b])/32 + (5*Log[a +
b*ArcCos[c*x]])/16 - (15*Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/
b])/32 + (3*Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/16 - (Sin
[(6*a)/b]*SinIntegral[(6*(a + b*ArcCos[c*x]))/b])/32))/(b*c*Sqrt[1 - c^2*x
^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.74

method	result
default	$\frac{\sqrt{-d(c^2x^2-1)}\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\left(20i\sqrt{-c^2x^2+1}\,\ln(a+b\arccos(cx))+20\ln(a+b\arccos(cx))cx+\text{expIntegral}_1\left(6i\arccos(c\right.\right.}{}$

input
```
int((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/64*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(20*I*(-c
^2*x^2+1)^(1/2)*ln(a+b*arccos(c*x))+20*ln(a+b*arccos(c*x))*c*x+Ei(1,6*I*ar
ccos(c*x)+6*I*a/b)*exp(I*(b*arccos(c*x)+6*a)/b)+Ei(1,-6*I*arccos(c*x)-6*I*
a/b)*exp(-I*(-b*arccos(c*x)+6*a)/b)-6*Ei(1,4*I*arccos(c*x)+4*I*a/b)*exp(I*
(b*arccos(c*x)+4*a)/b)+15*Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(I*(b*arccos(c*
x)+2*a)/b)+15*Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(-I*(-b*arccos(c*x)+2*a)/b
)-6*Ei(1,-4*I*arccos(c*x)-4*I*a/b)*exp(-I*(-b*arccos(c*x)+4*a)/b))*d^2/c/(
c^2*x^2-1)/b
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)/(b*arcco
s(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.22

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/32*(32*d^(5/2)*cos(a/b)^6*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^2) +
32*d^(5/2)*cos(a/b)^5*sin(a/b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*c^2)
 - 48*d^(5/2)*cos(a/b)^4*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^2) - 48*
d^(5/2)*cos(a/b)^4*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^2) - 32*d^(5/2
)*cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*c^2) - 48*d^(
5/2)*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c^2) + 18*
d^(5/2)*cos(a/b)^2*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^2) + 48*d^(5/2
)*cos(a/b)^2*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^2) + 30*d^(5/2)*cos(
a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^2) + 6*d^(5/2)*cos(a/b)*si
n(a/b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*c^2) + 24*d^(5/2)*cos(a/b)*s
in(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c^2) + 30*d^(5/2)*cos(a/b)*
sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b*c^2) - d^(5/2)*cos_integra
l(6*a/b + 6*arccos(c*x))/(b*c^2) - 6*d^(5/2)*cos_integral(4*a/b + 4*arccos
(c*x))/(b*c^2) - 15*d^(5/2)*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^2) -
10*d^(5/2)*log(b*arccos(c*x) + a)/(b*c^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{(d-c^2\,d\,x^2)^{5/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((d - c^2*d*x^2)^(5/2)/(a + b*acos(c*x)),x)
```

output
```
int((d - c^2*d*x^2)^(5/2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{a+b\arccos(cx)}\,dx=\sqrt{d}\,d^2\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx+\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)\,b+a}dx\right)c^4-2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*d**2*(int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x) + int((sqrt(
 - c**2*x**2 + 1)*x**4)/(acos(c*x)*b + a),x)*c**4 - 2*int((sqrt( - c**2*x*
*2 + 1)*x**2)/(acos(c*x)*b + a),x)*c**2)
```

3.69 $\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 294

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = \frac{d\sqrt{d-c^2dx^2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc\sqrt{1-c^2x^2}} - \frac{d\sqrt{d-c^2dx^2}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc\sqrt{1-c^2x^2}} - \frac{3d\sqrt{d-c^2dx^2}\log(a+b\arccos(cx))}{8bc\sqrt{1-c^2x^2}} + \frac{d\sqrt{d-c^2dx^2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc\sqrt{1-c^2x^2}} - \frac{d\sqrt{d-c^2dx^2}\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc\sqrt{1-c^2x^2}}$$

output
```
1/2*d*(-c^2*d*x^2+d)^(1/2)*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c/(-c^2*
x^2+1)^(1/2)-1/8*d*(-c^2*d*x^2+d)^(1/2)*cos(4*a/b)*Ci(4*(a+b*arccos(c*x))/
b)/b/c/(-c^2*x^2+1)^(1/2)-3/8*d*(-c^2*d*x^2+d)^(1/2)*ln(a+b*arccos(c*x))/b
/c/(-c^2*x^2+1)^(1/2)+1/2*d*(-c^2*d*x^2+d)^(1/2)*sin(2*a/b)*Si(2*(a+b*arcc
os(c*x))/b)/b/c/(-c^2*x^2+1)^(1/2)-1/8*d*(-c^2*d*x^2+d)^(1/2)*sin(4*a/b)*S
i(4*(a+b*arccos(c*x))/b)/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.51

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = \frac{d\sqrt{d-c^2dx^2}\left(4\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\right.\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)/(a + b*ArcCos[c*x]),x]
```

output
```
(d*Sqrt[d - c^2*d*x^2]*(4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])]
- Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcCos[c*x])] - 4*Log[a + b*ArcCos[c*x
]] + Log[8*(a + b*ArcCos[c*x])] + 4*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcC
os[c*x])] - Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])]))/(8*b*c*Sqrt[
1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{ 5169}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \frac{\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^4}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \left(\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}+\frac{3}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\sqrt{d-c^2dx^2}\left(-\frac{1}{2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{2}\sin\left(\frac{2a}{b}\right.\right.}{bc\sqrt{1-c^2x^2}}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)/(a + b*ArcCos[c*x]),x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*(-1/2*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[
c*x]))/b]) + (Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/8 + (3*
Log[a + b*ArcCos[c*x]])/8 - (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x
]))/b])/2 + (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8))/(b*c
*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\sqrt{-d(c^2x^2-1)}\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\left(-6i\sqrt{-c^2x^2+1}\,\ln(a+b\arccos(cx))-6\ln(a+b\arccos(cx))cx+\mathrm{expIntegral}_1\left(4i\arccos\right.\right.}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(-6*I*(-
c^2*x^2+1)^(1/2)*ln(a+b*arccos(c*x))-6*ln(a+b*arccos(c*x))*c*x+Ei(1,4*I*ar
ccos(c*x)+4*I*a/b)*exp(I*(b*arccos(c*x)+4*a)/b)+Ei(1,-4*I*arccos(c*x)-4*I*
a/b)*exp(-I*(-b*arccos(c*x)+4*a)/b)-4*Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(I*
(b*arccos(c*x)+2*a)/b)-4*Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(-I*(-b*arccos(
c*x)+2*a)/b))*d/c/(c^2*x^2-1)/b
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}}{b\arccos\,(cx)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.97

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\frac{1}{8}\left(\frac{8\,d^{\frac{3}{2}}\cos\left(\frac{a}{b}\right)^4\mathrm{Ci}\left(\frac{4\,a}{b}+4\,\arccos\,(cx)\right)}{bc^2}+\frac{8\,d^{\frac{3}{2}}\cos\left(\frac{a}{b}\right)^3\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{4\,a}{b}+4\,\arccos\,(cx)\right)}{bc^2}-\frac{8\,d^{\frac{3}{2}}\cos\left(\frac{a}{b}\right)^2\,\mathrm{C}}{}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/8*(8*d^(3/2)*cos(a/b)^4*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^2) + 8
*d^(3/2)*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c^2) -
 8*d^(3/2)*cos(a/b)^2*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^2) - 8*d^(3
/2)*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^2) - 4*d^(3/2)*cos
(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c^2) - 8*d^(3/2)*cos
(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b*c^2) + d^(3/2)*cos_i
ntegral(4*a/b + 4*arccos(c*x))/(b*c^2) + 4*d^(3/2)*cos_integral(2*a/b + 2*
arccos(c*x))/(b*c^2) + 3*d^(3/2)*log(b*arccos(c*x) + a)/(b*c^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{\left(d-c^2\,d\,x^2\right)^{3/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d - c^2*d*x^2)^(3/2)/(a + b*acos(c*x)),x)
```

output
```
int((d - c^2*d*x^2)^(3/2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{a+b\arccos(cx)}\,dx = \sqrt{d}\,d\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*d*(int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x) - int((sqrt( -
c**2*x**2 + 1)*x**2)/(acos(c*x)*b + a),x)*c**2)
```

3.70 $\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx = \frac{\sqrt{d-c^2dx^2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}\log(a+b\arccos(cx))}{2bc\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc\sqrt{1-c^2x^2}}$$

output
```
1/2*(-c^2*d*x^2+d)^(1/2)*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c/(-c^2*x^
2+1)^(1/2)-1/2*(-c^2*d*x^2+d)^(1/2)*ln(a+b*arccos(c*x))/b/c/(-c^2*x^2+1)^(
1/2)+1/2*(-c^2*d*x^2+d)^(1/2)*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c/(-c
^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx$$
$$= \frac{\sqrt{d\left(1-c^2x^2\right)}\left(\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-\log(a+b\arccos(cx))+\sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\text{arc}\right.\right.\right.}{2bc\sqrt{1-c^2x^2}}$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d*(1 - c^2*x^2)]*(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] -
Log[a + b*ArcCos[c*x]] + Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])]))
/(2*b*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.60, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx$$
$$\Big\downarrow \text{5169}$$
$$-\frac{\sqrt{d-c^2dx^2}\int \frac{\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\sqrt{d-c^2dx^2}\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^2}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$
$$\Big\downarrow \text{3793}$$

$$-\frac{\sqrt{d-c^2dx^2}\int\left(\frac{1}{2(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(-\frac{1}{2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{2}\log(a+b\arccos(cx\right.}{bc\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(-1/2*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*
x]))/b]) + Log[a + b*ArcCos[c*x]]/2 - (Sin[(2*a)/b]*SinIntegral[(2*(a + b*
ArcCos[c*x]))/b])/2))/(b*c*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

method	result
default	$\frac{\sqrt{-d(c^2x^2-1)}\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\left(2i\sqrt{-c^2x^2+1}\,\ln(a+b\arccos(cx))+2\ln(a+b\arccos(cx))cx+\operatorname{expIntegral}_1(2i\arccos(cx)\right.}{4c(c^2x^2-1)b}$

input
```
int((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(2*I*(-c^2
*x^2+1)^(1/2)*ln(a+b*arccos(c*x))+2*ln(a+b*arccos(c*x))*c*x+Ei(1,2*I*arcco
s(c*x)+2*I*a/b)*exp(I*(b*arccos(c*x)+2*a)/b)+Ei(1,-2*I*arccos(c*x)-2*I*a/b
)*exp(-I*(-b*arccos(c*x)+2*a)/b))/c/(c^2*x^2-1)/b
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2dx^2+d}}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}{a+b\operatorname{acos}\left(cx\right)}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2dx^2+d}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx$$
$$= \frac{1}{2}c\sqrt{d}\left(\frac{2\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^2} + \frac{2\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^2} - \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arcc}\right.}{bc^2}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/2*c*sqrt(d)*(2*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^2) +
2*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b*c^2) - cos_inte
gral(2*a/b + 2*arccos(c*x))/(b*c^2) - log(b*arccos(c*x) + a)/(b*c^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{d-c^2\,d\,x^2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((d - c^2*d*x^2)^(1/2)/(a + b*acos(c*x)),x)
```

output
```
int((d - c^2*d*x^2)^(1/2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}}{a+b\arccos(cx)}\,dx=\sqrt{d}\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x)
```

3.71 $\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = -\frac{\sqrt{1-c^2x^2}\log(a+b\arccos(cx))}{bc\sqrt{d-c^2dx^2}}$$

output
```
-(-c^2*x^2+1)^(1/2)*ln(a+b*arccos(c*x))/b/c/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = -\frac{\sqrt{1-c^2x^2}\log(a+b\arccos(cx))}{bc\sqrt{d-c^2dx^2}}$$

input
```
Integrate[1/(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-((Sqrt[1 - c^2*x^2]*Log[a + b*ArcCos[c*x]])/(b*c*Sqrt[d - c^2*d*x^2]))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {5151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \text{5151}$$

$$-\frac{\sqrt{1-c^2x^2}\log(a+b\arccos(cx))}{b^2c^2\sqrt{d}\sqrt{d-c^2dx^2}}$$

input
```
Int[1/(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-((Sqrt[1 - c^2*x^2]*Log[a + b*ArcCos[c*x]])/(b^2*c^2*Sqrt[d]*Sqrt[d - c^2
*d*x^2]))
```

Defintions of rubi rules used

rule 5151
```
Int[1/(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> Simp[(-(b*c)^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(Log[a +
 b*ArcCos[c*x]]/(b*c*Sqrt[d])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln(a+b\arccos(cx))}{cd(c^2x^2-1)b}$	57

input
```
int(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*ln(a+b*arccos(c*
x))/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = \frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}\log\left(\frac{2\,(b\arccos(cx)+a)}{b}\right)}{bc^3dx^2-bcd}$$

input
```
integrate(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*log(2*(b*arccos(c*x) + a)/b)/(b*c^
3*d*x^2 - b*c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(1/(-c**2*d*x**2+d)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}\,dx=-\frac{\sqrt{d}\log(acos(cx)\,b+a)}{bcd}$$

input
```
int(1/(-c^2*d*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
( - sqrt(d)*log(acos(c*x)*b + a))/(b*c*d)
```

3.72 $\int \frac{1}{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2d\,x^2+d)^{\frac{3}{2}}(a+b\arccos(cx))}dx$$

input
```
int(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.12

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)/(a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (
b*c^4*d^2*x^4 - 2*b*c^2*d^2*x^2 + b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(1/(-c**2*d*x**2+d)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}\,dx = -\frac{\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx}{\sqrt{d}\,d}$$

input
```
int(1/(-c^2*d*x^2+d)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
( - int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**2 - sqrt( - c**2*x**
2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**
2 + 1)*a),x))/(sqrt(d)*d)
```

3.73 $\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}dx$$

$$\Big\downarrow \; 5175$$

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.15

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)/(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*
d^3*x^2 - a*d^3 + (b*c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d
^3)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(1/(-c**2*d*x**2+d)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.96

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}\,dx = \frac{\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,c^4x^4-2\sqrt{-c^2x^2+1}\,acos(cx)b\,c^2x^2+\sqrt{-c^2x^2+1}\,acos(cx)b+\sqrt{-c}}}{\sqrt{d}\,d^2}$$

input
```
int(1/(-c^2*d*x^2+d)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**4 - 2*sqrt( - c**2*x**2
+ 1)*acos(c*x)*b*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( -
c**2*x**2 + 1)*a*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + sqrt(
- c**2*x**2 + 1)*a),x)/(sqrt(d)*d**2)
```

3.74 $\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 428

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx &= \frac{d^2(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))} \\
&+ \frac{15d^2\sqrt{d-c^2dx^2}\,\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{16b^2c\sqrt{1-c^2x^2}} \\
&- \frac{3d^2\sqrt{d-c^2dx^2}\,\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{4b^2c\sqrt{1-c^2x^2}} \\
&+ \frac{3d^2\sqrt{d-c^2dx^2}\,\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)\sin\left(\frac{6a}{b}\right)}{16b^2c\sqrt{1-c^2x^2}} \\
&- \frac{15d^2\sqrt{d-c^2dx^2}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{16b^2c\sqrt{1-c^2x^2}} \\
&+ \frac{3d^2\sqrt{d-c^2dx^2}\cos\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{4b^2c\sqrt{1-c^2x^2}} \\
&- \frac{3d^2\sqrt{d-c^2dx^2}\cos\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{16b^2c\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
d^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(a+b*arccos(c*x))+15/16*d^
2*(-c^2*d*x^2+d)^(1/2)*Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^2/c/(-c^2*x^
2+1)^(1/2)-3/4*d^2*(-c^2*d*x^2+d)^(1/2)*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/
b)/b^2/c/(-c^2*x^2+1)^(1/2)+3/16*d^2*(-c^2*d*x^2+d)^(1/2)*Ci(6*(a+b*arccos
(c*x))/b)*sin(6*a/b)/b^2/c/(-c^2*x^2+1)^(1/2)-15/16*d^2*(-c^2*d*x^2+d)^(1/
2)*cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^2/c/(-c^2*x^2+1)^(1/2)+3/4*d^2*(
-c^2*d*x^2+d)^(1/2)*cos(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c/(-c^2*x^2+1
)^(1/2)-3/16*d^2*(-c^2*d*x^2+d)^(1/2)*cos(6*a/b)*Si(6*(a+b*arccos(c*x))/b)
/b^2/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.80

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{d^2\sqrt{d-c^2dx^2}\left(-16b+48bc^2x^2-48bc^4x^4+16bc^6x^6-15(a+b\arccos(cx))\,\mathrm{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx\right.\right.\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/16*(d^2*Sqrt[d - c^2*d*x^2]*(-16*b + 48*b*c^2*x^2 - 48*b*c^4*x^4 + 16*b
*c^6*x^6 - 15*(a + b*ArcCos[c*x])*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[(
2*a)/b] + 12*(a + b*ArcCos[c*x])*CosIntegral[4*(a/b + ArcCos[c*x])]*Sin[(4
*a)/b] - 3*a*CosIntegral[6*(a/b + ArcCos[c*x])]*Sin[(6*a)/b] - 3*b*ArcCos[
c*x]*CosIntegral[6*(a/b + ArcCos[c*x])]*Sin[(6*a)/b] + 15*a*Cos[(2*a)/b]*S
inIntegral[2*(a/b + ArcCos[c*x])] + 15*b*ArcCos[c*x]*Cos[(2*a)/b]*SinInteg
ral[2*(a/b + ArcCos[c*x])] - 12*a*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcCos
[c*x])] - 12*b*ArcCos[c*x]*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])]
 + 3*a*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])] + 3*b*ArcCos[c*x]*C
os[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])]))/(b^2*c*Sqrt[1 - c^2*x^2]*
(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5167, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{6cd^2\sqrt{d-c^2dx^2}\int \frac{x\left(1-c^2x^2\right)^2}{a+b\arccos(cx)}dx}{b\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{5/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{5/2}}{bc(a+b\arccos(cx))}-\frac{6d^2\sqrt{d-c^2dx^2}\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6d^2\sqrt{d-c^2dx^2}\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{5/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6d^2\sqrt{d-c^2dx^2}\int \left(\frac{\sin\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{5\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}$$
$$\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{5/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{5/2}}{bc(a+b\arccos(cx))}-$$
$$\frac{6d^2\sqrt{d-c^2dx^2}\left(-\frac{5}{32}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\sin\left(\frac{6}{}\right.\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[1 - c^2*x^2]*(d - c^2*d*x^2)^(5/2))/(b*c*(a + b*ArcCos[c*x])) - (6*d
^2*Sqrt[d - c^2*d*x^2]*((-5*CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*
a)/b])/32 + (CosIntegral[(4*(a + b*ArcCos[c*x]))/b]*Sin[(4*a)/b])/8 - (Cos
Integral[(6*(a + b*ArcCos[c*x]))/b]*Sin[(6*a)/b])/32 + (5*Cos[(2*a)/b]*Sin
Integral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Cos[(4*a)/b]*SinIntegral[(4*(a
+ b*ArcCos[c*x]))/b])/8 + (Cos[(6*a)/b]*SinIntegral[(6*(a + b*ArcCos[c*x])
)/b])/32))/(b^2*c*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
 + 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{-d(c^2x^2-1)}\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\left(-7i\sin(5\arccos(cx))b+21i\sin(3\arccos(cx))b+24i\,\text{expIntegral}_1\left(4i\arccos(cx)+\frac{4ia}{b}\right)e^{i(}\right.}{}$

input
```
int((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/64*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(-7*I*sin
(5*arccos(c*x))*b+21*I*sin(3*arccos(c*x))*b+24*I*Ei(1,4*I*arccos(c*x)+4*I*
a/b)*exp(I*(b*arccos(c*x)+4*a)/b)*b*arccos(c*x)+6*I*Ei(1,-6*I*arccos(c*x)-
6*I*a/b)*exp(-I*(-b*arccos(c*x)+6*a)/b)*b*arccos(c*x)+24*I*(-c^2*x^2+1)^(1
/2)*b*c^2*x^2-30*I*Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(I*(b*arccos(c*x)+2*a)
/b)*b*arccos(c*x)+30*I*Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(-I*(-b*arccos(c*
x)+2*a)/b)*b*arccos(c*x)+30*I*Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(-I*(-b*ar
ccos(c*x)+2*a)/b)*a-6*I*Ei(1,6*I*arccos(c*x)+6*I*a/b)*exp(I*(b*arccos(c*x)
+6*a)/b)*b*arccos(c*x)-24*I*Ei(1,-4*I*arccos(c*x)-4*I*a/b)*exp(-I*(-b*arcc
os(c*x)+4*a)/b)*b*arccos(c*x)-6*I*Ei(1,6*I*arccos(c*x)+6*I*a/b)*exp(I*(b*a
rccos(c*x)+6*a)/b)*a-24*I*Ei(1,-4*I*arccos(c*x)-4*I*a/b)*exp(-I*(-b*arccos
(c*x)+4*a)/b)*a+64*I*(-c^2*x^2+1)^(1/2)*b*c^6*x^6-80*I*(-c^2*x^2+1)^(1/2)*
b*c^4*x^4-5*cos(5*arccos(c*x))*b+9*cos(3*arccos(c*x))*b-12*c*x*b-36*I*(-c^
2*x^2+1)^(1/2)*b+6*I*Ei(1,-6*I*arccos(c*x)-6*I*a/b)*exp(-I*(-b*arccos(c*x)
+6*a)/b)*a+24*I*Ei(1,4*I*arccos(c*x)+4*I*a/b)*exp(I*(b*arccos(c*x)+4*a)/b)
*a-30*I*Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(I*(b*arccos(c*x)+2*a)/b)*a+56*b*
c^3*x^3+64*b*c^7*x^7-112*b*c^5*x^5)*d^2/c/(c^2*x^2-1)/b^2/(a+b*arccos(c*x)
)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)/(b^2*arc
cos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^6*d^2*x^6 - 3*c^4*d^2*x^4 + 3*c^2*d^2*x^2 - d^2 - (b^2*c*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate(6*(c^5*d^2*x^5 - 2*c^3*d^
2*x^3 + c*d^2*x)/(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b), x
))*sqrt(d)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(388) = 776$.

Time = 1.06 (sec) , antiderivative size = 1706, normalized size of antiderivative = 3.99

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input

```
integrate((-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output

```
-1/16*(16*b*c^6*d^(5/2)*x^6/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 48*b*c^4*d
^(5/2)*x^4/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 96*b*d^(5/2)*arccos(c*x)*co
s(a/b)^5*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x)
 + a*b^2*c^2) + 96*b*d^(5/2)*arccos(c*x)*cos(a/b)^6*sin_integral(6*a/b + 6
*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 96*a*d^(5/2)*cos(a/b)^5*
cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*
c^2) + 96*a*d^(5/2)*cos(a/b)^6*sin_integral(6*a/b + 6*arccos(c*x))/(b^3*c^
2*arccos(c*x) + a*b^2*c^2) + 96*b*d^(5/2)*arccos(c*x)*cos(a/b)^3*cos_integ
ral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 96
*b*d^(5/2)*arccos(c*x)*cos(a/b)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(
a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 144*b*d^(5/2)*arccos(c*x)*cos(a/b
)^4*sin_integral(6*a/b + 6*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2)
- 96*b*d^(5/2)*arccos(c*x)*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/
(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 96*a*d^(5/2)*cos(a/b)^3*cos_integral(6
*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 96*a*d^
(5/2)*cos(a/b)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^2*arc
cos(c*x) + a*b^2*c^2) - 144*a*d^(5/2)*cos(a/b)^4*sin_integral(6*a/b + 6*ar
ccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 96*a*d^(5/2)*cos(a/b)^4*sin
_integral(4*a/b + 4*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 48*b*
c^2*d^(5/2)*x^2/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 18*b*d^(5/2)*arccos...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(d-c^2\,d\,x^2)^{5/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((d - c^2*d*x^2)^(5/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d - c^2*d*x^2)^(5/2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \sqrt{d}\,d^2\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*d**2*(int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*
a*b + a**2),x) + int((sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)**2*b**2 + 2*
acos(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(
c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**2)
```

3.75 $\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 287

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \frac{d(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))} + \frac{d\sqrt{d-c^2dx^2}\,\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2c\sqrt{1-c^2x^2}} - \frac{d\sqrt{d-c^2dx^2}\,\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{2b^2c\sqrt{1-c^2x^2}} - \frac{d\sqrt{d-c^2dx^2}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c\sqrt{1-c^2x^2}} + \frac{d\sqrt{d-c^2dx^2}\cos\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{2b^2c\sqrt{1-c^2x^2}}$$

output
```
d*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(a+b*arccos(c*x))+d*(-c^2*d*
x^2+d)^(1/2)*Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^2/c/(-c^2*x^2+1)^(1/2)
-1/2*d*(-c^2*d*x^2+d)^(1/2)*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/b)/b^2/c/(-c
^2*x^2+1)^(1/2)-d*(-c^2*d*x^2+d)^(1/2)*cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b
)/b^2/c/(-c^2*x^2+1)^(1/2)+1/2*d*(-c^2*d*x^2+d)^(1/2)*cos(4*a/b)*Si(4*(a+b
*arccos(c*x))/b)/b^2/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.80

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{d\sqrt{1-c^2x^2}(-1+c^2x^2)\sqrt{-d(-1+c^2x^2)}}{bc(a+b\arccos(cx))} + \frac{d\sqrt{-d(1-c^2x^2)^2}\sqrt{d-c^2dx^2}\left(2\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right)-\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(c\right.\right.}{2b^2c\sqrt{1-c^2x^2}\sqrt{(-1+c^2x^2)(c}}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
-((d*Sqrt[1 - c^2*x^2]*(-1 + c^2*x^2)*Sqrt[-(d*(-1 + c^2*x^2))])/(b*c*(a +
 b*ArcCos[c*x]))) + (d*Sqrt[-(d*(1 - c^2*x^2)^2)]*Sqrt[d - c^2*d*x^2]*(2*C
osIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] - CosIntegral[4*(a/b + ArcC
os[c*x])]*Sin[(4*a)/b] - 2*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])]
 + Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])]))/(2*b^2*c*Sqrt[1 - c^2
*x^2]*Sqrt[(-1 + c^2*x^2)*(d - c^2*d*x^2)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5167, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \ 5167$$

$$\frac{4cd\sqrt{d-c^2dx^2}\int \frac{x(1-c^2x^2)}{a+b\arccos(cx)}dx}{b\sqrt{1-c^2x^2}} + \frac{\sqrt{1-c^2x^2}(d-c^2dx^2)^{3/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \ 5225$$

$$\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{3/2}}{bc(a+b\arccos(cx))}-\frac{4d\sqrt{d-c^2dx^2}\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{4d\sqrt{d-c^2dx^2}\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{3/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 4906$$

$$\frac{4d\sqrt{d-c^2dx^2}\int \left(\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{3/2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{1-c^2x^2}\left(d-c^2dx^2\right)^{3/2}}{bc(a+b\arccos(cx))}-\frac{4d\sqrt{d-c^2dx^2}\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\right.}{b^2c\sqrt{1-c^2x^2}}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[1 - c^2*x^2]*(d - c^2*d*x^2)^(3/2))/(b*c*(a + b*ArcCos[c*x])) - (4*d
*Sqrt[d - c^2*d*x^2]*(-1/4*(CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*
a)/b]) + (CosIntegral[(4*(a + b*ArcCos[c*x]))/b]*Sin[(4*a)/b])/8 + (Cos[(2
*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/4 - (Cos[(4*a)/b]*SinIntegr
al[(4*(a + b*ArcCos[c*x]))/b])/8))/(b^2*c*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.75

method	result
default	$\frac{\sqrt{-d(c^2x^2-1)}\left(ic^2x^2+cx\sqrt{-c^2x^2+1}-i\right)\left(5i\sin(3\arccos(cx))b-16b\,c^5x^5-16i\sqrt{-c^2x^2+1}\,b\,c^4x^4+20b\,c^3x^3-11i\sqrt{-c^2x^2+1}\,b+8i\,e\right.}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(5*I*sin(
3*arccos(c*x))*b-16*b*c^5*x^5-16*I*(-c^2*x^2+1)^(1/2)*b*c^4*x^4+20*b*c^3*x
^3-11*I*(-c^2*x^2+1)^(1/2)*b+8*I*Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(-I*(-b
*arccos(c*x)+2*a)/b)*b*arccos(c*x)+8*I*Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(
-I*(-b*arccos(c*x)+2*a)/b)*a+4*I*Ei(1,4*I*arccos(c*x)+4*I*a/b)*exp(I*(b*ar
ccos(c*x)+4*a)/b)*b*arccos(c*x)-8*I*Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(I*(b
*arccos(c*x)+2*a)/b)*b*arccos(c*x)+4*I*Ei(1,4*I*arccos(c*x)+4*I*a/b)*exp(I
*(b*arccos(c*x)+4*a)/b)*a-4*I*exp(-I*(-b*arccos(c*x)+4*a)/b)*Ei(1,-4*I*arc
cos(c*x)-4*I*a/b)*b*arccos(c*x)-8*I*Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(I*(b
*arccos(c*x)+2*a)/b)*a-4*I*exp(-I*(-b*arccos(c*x)+4*a)/b)*Ei(1,-4*I*arccos
(c*x)-4*I*a/b)*a+12*I*(-c^2*x^2+1)^(1/2)*b*c^2*x^2-7*c*x*b+3*cos(3*arccos(
c*x))*b)*d/c/(c^2*x^2-1)/b^2/(a+b*arccos(c*x))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(c^4*d*x^4 - 2*c^2*d*x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x) + a*b*c)*integrate(4*(c^3*d*x^3 - c*d*x)/(b^2*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x) + a*b), x) + d)*sqrt(d)/(b^2*c*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 926 vs. 2(263) = 526.

Time = 0.92 (sec) , antiderivative size = 926, normalized size of antiderivative = 3.23

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c^4*d^(3/2)*x^4/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 8*b*d^(3/2)*a
rccos(c*x)*cos(a/b)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^
2*arccos(c*x) + a*b^2*c^2) + 8*b*d^(3/2)*arccos(c*x)*cos(a/b)^4*sin_integr
al(4*a/b + 4*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 8*a*d^(3/2)*
cos(a/b)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*
x) + a*b^2*c^2) + 8*a*d^(3/2)*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x
))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 4*b*c^2*d^(3/2)*x^2/(b^3*c^2*arccos
(c*x) + a*b^2*c^2) + 4*b*d^(3/2)*arccos(c*x)*cos(a/b)*cos_integral(4*a/b +
 4*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 4*b*d^(3/2)*a
rccos(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c^2*
arccos(c*x) + a*b^2*c^2) - 8*b*d^(3/2)*arccos(c*x)*cos(a/b)^2*sin_integral
(4*a/b + 4*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 4*b*d^(3/2)*ar
ccos(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c
*x) + a*b^2*c^2) + 4*a*d^(3/2)*cos(a/b)*cos_integral(4*a/b + 4*arccos(c*x)
)*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 4*a*d^(3/2)*cos(a/b)*cos_in
tegral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) -
 8*a*d^(3/2)*cos(a/b)^2*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^2*arcco
s(c*x) + a*b^2*c^2) - 4*a*d^(3/2)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos
(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + b*d^(3/2)*arccos(c*x)*sin_integ
ral(4*a/b + 4*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 2*b*d^(3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(d-c^2\,d\,x^2)^{3/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((d - c^2*d*x^2)^(3/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d - c^2*d*x^2)^(3/2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \sqrt{d}\,d\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*d*(int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b
 + a**2),x) - int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 + 2*aco
s(c*x)*a*b + a**2),x)*c**2)
```

3.76 $\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 162

$$\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))} + \frac{\sqrt{d-c^2dx^2}\,\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2c\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c\sqrt{1-c^2x^2}}$$

output
```
(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(a+b*arccos(c*x))+(-c^2*d*x^2+
d)^(1/2)*Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^2/c/(-c^2*x^2+1)^(1/2)-(-c
^2*d*x^2+d)^(1/2)*cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^2/c/(-c^2*x^2+1)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx =$$
$$-\frac{\sqrt{d-c^2dx^2}\left(b(-1+c^2x^2)-(a+b\arccos(cx))\operatorname{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right)+(a+b\operatorname{arcc}\right.}{b^2c\sqrt{1-c^2x^2}(a+b\arccos(cx))}$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(b*(-1 + c^2*x^2) - (a + b*ArcCos[c*x])*CosIntegral
[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] + (a + b*ArcCos[c*x])*Cos[(2*a)/b]*Si
nIntegral[2*(a/b + ArcCos[c*x])]))/(b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x])))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.80, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {5167, 5147, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \ 5167$$

$$\frac{2c\sqrt{d-c^2dx^2}\int \frac{x}{a+b\arccos(cx)}dx}{b\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \ 5147$$

$$\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}-$$
$$\frac{2\sqrt{d-c^2dx^2}\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 25$$

$$\frac{2\sqrt{d-c^2dx^2}\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow 4906$$

$$\frac{2\sqrt{d-c^2dx^2}\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c\sqrt{1-c^2x^2}}+\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow 3784$$

$$\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}-\frac{\sqrt{d-c^2dx^2}\left(-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(c\right.}{b^2c\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}-\frac{\sqrt{d-c^2dx^2}\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))\right)}{b^2c\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 3042$$

$$\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}-$$
$$\frac{\sqrt{d-c^2dx^2}\left(\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx$$
$$b^2c\sqrt{1-c^2x^2}$$

$\Big\downarrow$ 3780

$$\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}-\frac{\sqrt{d-c^2dx^2}\left(\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))\right)}{b^2c\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 3783

$$\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}{bc(a+b\arccos(cx))}-\frac{\sqrt{d-c^2dx^2}\left(\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\right)}{b^2c\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Sqrt[
d - c^2*d*x^2]*(-(CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b]) + C
os[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b]))/(b^2*c*Sqrt[1 - c^2*x
^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(-2ib\,c^3x^3+2x^2c^2\sqrt{-c^2x^2+1}\,b+\text{expIntegral}_1\left(-2i\arccos(cx)-\frac{2ia}{b}\right)\mathrm{e}^{-\frac{i(-b\arccos(}{b}}\right.}{}$

input
```
int((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(-2*I*b*
c^3*x^3+2*x^2*c^2*(-c^2*x^2+1)^(1/2)*b+Ei(1,-2*I*arccos(c*x)-2*I*a/b)*exp(
-I*(-b*arccos(c*x)+2*a)/b)*b*arccos(c*x)-Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp
(I*(b*arccos(c*x)+2*a)/b)*b*arccos(c*x)+2*I*c*x*b+Ei(1,-2*I*arccos(c*x)-2*
I*a/b)*exp(-I*(-b*arccos(c*x)+2*a)/b)*a-Ei(1,2*I*arccos(c*x)+2*I*a/b)*exp(
I*(b*arccos(c*x)+2*a)/b)*a-2*(-c^2*x^2+1)^(1/2)*b)/c/(c^2*x^2-1)/b^2/(a+b*
arccos(c*x))
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
), x)
```

Sympy [F]

$$\int \frac{\sqrt{d - c^2dx^2}}{(a + b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}}{(a + b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d - c^2dx^2}}{(a + b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2dx^2 + d}}{(b\arccos{(cx)} + a)^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^2 - 2*(b^2*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^
2)*integrate(x/(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b), x)
- 1)*sqrt(d)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(150) = 300$.

Time = 0.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{d - c^2dx^2}}{(a + b\arccos(cx))^2}\,dx = -\left(\frac{bc^2x^2}{b^3c^2\arccos{(cx)} + ab^2c^2} - \frac{2\,b\arccos{(cx)}\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{2\,a}{b} + 2\,\arccos{(cx)}\right)\sin\left(\frac{a}{b}\right)}{b^3c^2\arccos{(cx)} + ab^2c^2} + \frac{2\,b\arccos{(cx)}\,\mathrm{co}}{b^3c^2\,\mathrm{a}}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-(b*c^2*x^2/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*b*arccos(c*x)*cos(a/b)*c
os_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c
^2) + 2*b*arccos(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*
c^2*arccos(c*x) + a*b^2*c^2) - 2*a*cos(a/b)*cos_integral(2*a/b + 2*arccos(
c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 2*a*cos(a/b)^2*sin_inte
gral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - b*arccos(c
*x)*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2)
- a*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2)
- b/(b^3*c^2*arccos(c*x) + a*b^2*c^2))*c*sqrt(d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{d-c^2\,d\,x^2}}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((d - c^2*d*x^2)^(1/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d - c^2*d*x^2)^(1/2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}}{(a+b\arccos(cx))^2}\,dx = \sqrt{d}\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)^2\,b^2+2acos(cx)\,ab+a^2}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b +
a**2),x)
```

3.77 $\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}}{bc\sqrt{d-c^2dx^2}(a+b\arccos(cx))}$$

output
```
(-c^2*x^2+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}}{bc\sqrt{d-c^2dx^2}(a+b\arccos(cx))}$$

input
```
Integrate[1/(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Sqrt[1 - c^2*x^2]/(b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \ 5153$$

$$\frac{\sqrt{1-c^2x^2}}{bc\sqrt{d-c^2dx^2}(a+b\arccos(cx))}$$

input
```
Int[1/(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Sqrt[1 - c^2*x^2]/(b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

method	result	size
default	$-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}}{cd(c^2x^2-1)(a+b\arccos(cx))b}$	59

input
```
int(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)/(a+b*arccos(c*x
))/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = -\frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}}{abc^3dx^2-abcd+(b^2c^3dx^2-b^2cd)\arccos{(cx)}}$$

input
```
integrate(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
-sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)/(a*b*c^3*d*x^2 - a*b*c*d + (b^2*c
^3*d*x^2 - b^2*c*d)*arccos(c*x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(1/(-c**2*d*x**2+d)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = \frac{\sqrt{d}}{b^2cd\arctan\left(\sqrt{cx+1}\sqrt{-cx+1},cx\right)+abcd}$$

input
```
integrate(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
sqrt(d)/(b^2*c*d*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-c^2*d*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}\,dx = -\frac{\sqrt{d}\,acos(cx)}{acd\,(acos\,(cx)\,b+a)}$$

input
```
int(1/(-c^2*d*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
( - sqrt(d)*acos(c*x))/(a*c*d*(acos(c*x)*b + a))
```

3.78 $\int \frac{1}{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{1}{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}-\frac{2c\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))}dx}{bd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5235}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}-\frac{2c\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))}dx}{bd\sqrt{d-c^2dx^2}}$$

input
```
Int[1/((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 5.12

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)/(a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d
^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*
b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 12.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(1/(-c**2*d*x**2+d)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 239, normalized size of antiderivative = 9.19

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(2*(a*b*c^4*d^2*x^2 - a*b*c^2*d^2 + (b^2*c^4*d^2*x^2 - b^2*c^2*d^2)*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(x/(a*b*c^4*d^2*x^4 - 2*a*
b*c^2*d^2*x^2 + a*b*d^2 + (b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + 1)*sqrt(d)/(a*b*c^3*d^2*
x^2 - a*b*c*d^2 + (b^2*c^3*d^2*x^2 - b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.54

$$\int \frac{1}{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=$$
$$-\frac{\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)^2b^2c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)^2b^2+2\sqrt{-c^2x^2+1}\,acos(cx)ab\,c^2x^2-2\sqrt{-c^2x^2+1}\,acos(cx)ab+\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x}}}{\sqrt{d}\,d}$$

input
```
int(1/(-c^2*d*x^2+d)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
( - int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 - sqrt( - c*
*2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c*
*2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*
a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x))/(sqrt(d)*d)
```

3.79 $\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}-\frac{4c\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^3(a+b\arccos(cx))}dx}{bd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}-\frac{4c\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^3(a+b\arccos(cx))}dx}{bd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[1/((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 6.85

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)/(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^
2*c^2*d^3*x^2 - a^2*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2
*d^3*x^2 - b^2*d^3)*arccos(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4
 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 27.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate(1/(-c**2*d*x**2+d)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 317, normalized size of antiderivative = 12.19

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(4*(a*b*c^6*d^3*x^4 - 2*a*b*c^4*d^3*x^2 + a*b*c^2*d^3 + (b^2*c^6*d^3*x^4 -
 2*b^2*c^4*d^3*x^2 + b^2*c^2*d^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x))*integrate(x/(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 -
 a*b*d^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*
d^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + 1)*sqrt(d)/(a*b*c^5
*d^3*x^4 - 2*a*b*c^3*d^3*x^2 + a*b*c*d^3 + (b^2*c^5*d^3*x^4 - 2*b^2*c^3*d^
3*x^2 + b^2*c*d^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(-c^2*d*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 8.31

$$\int \frac{1}{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \frac{\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)^2b^2c^4x^4-2\sqrt{-c^2x^2+1}\,acos(cx)^2b^2c^2x^2+\sqrt{-c^2x^2+1}\,acos(cx)^2b}}{}$$

input
```
int(1/(-c^2*d*x^2+d)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**4 - 2*sqrt( - c**2
*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*
*2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**4 - 4*sqrt( - c**
2*x**2 + 1)*acos(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a
*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a**2
*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)/(sqrt(d)*d**2)
```

3.80 $\int (d+ex^2)^4 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 317

$$\begin{aligned}
&\int (d+ex^2)^4 (a+b\arccos(cx))\,dx \\
&= -\frac{b(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4)\sqrt{1-c^2x^2}}{315c^9} \\
&\quad + \frac{4be(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)(1-c^2x^2)^{3/2}}{945c^9} \\
&\quad - \frac{2be^2(63c^4d^2+90c^2de+35e^2)(1-c^2x^2)^{5/2}}{525c^9} \\
&\quad + \frac{4be^3(9c^2d+7e)(1-c^2x^2)^{7/2}}{441c^9} - \frac{be^4(1-c^2x^2)^{9/2}}{81c^9} \\
&\quad + d^4x(a+b\arccos(cx)) + \frac{4}{3}d^3ex^3(a+b\arccos(cx)) + \frac{6}{5}d^2e^2x^5(a+b\arccos(cx)) + \frac{4}{7}de^3x^7(a+b\arccos(cx))
\end{aligned}$$

output
```
-1/315*b*(315*c^8*d^4+420*c^6*d^3*e+378*c^4*d^2*e^2+180*c^2*d*e^3+35*e^4)*
(-c^2*x^2+1)^(1/2)/c^9+4/945*b*e*(105*c^6*d^3+189*c^4*d^2*e+135*c^2*d*e^2+
35*e^3)*(-c^2*x^2+1)^(3/2)/c^9-2/525*b*e^2*(63*c^4*d^2+90*c^2*d*e+35*e^2)*
(-c^2*x^2+1)^(5/2)/c^9+4/441*b*e^3*(9*c^2*d+7*e)*(-c^2*x^2+1)^(7/2)/c^9-1/
81*b*e^4*(-c^2*x^2+1)^(9/2)/c^9+d^4*x*(a+b*arccos(c*x))+4/3*d^3*e*x^3*(a+b
*arccos(c*x))+6/5*d^2*e^2*x^5*(a+b*arccos(c*x))+4/7*d*e^3*x^7*(a+b*arccos(
c*x))+1/9*e^4*x^9*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.82

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx = \frac{315ax(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)-\frac{b\sqrt{1-c^2x^2}(4480e^4+320c^2e^3(81d+7ex^2)+48c^4e^2(1323\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x^2)^4*(a + b*ArcCos[c*x]),x]
```

output
```
(315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e
^4*x^8) - (b*Sqrt[1 - c^2*x^2]*(4480*e^4 + 320*c^2*e^3*(81*d + 7*e*x^2) +
48*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) + 8*c^6*e*(11025*d^3 + 39
69*d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*
e*x^2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x
*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*
ArcCos[c*x])/99225
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5171, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5171}$$

$$bc\int \frac{x(35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4)}{315\sqrt{1-c^2x^2}}dx+d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{315}bc\int \frac{x\left(35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4\right)}{\sqrt{1-c^2x^2}}dx+d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{630}bc\int \frac{35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4}{\sqrt{1-c^2x^2}}dx^2+d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{630}bc\int \left(\frac{35\left(1-c^2x^2\right)^{7/2}e^4}{c^8}-\frac{20\left(9dc^2+7e\right)\left(1-c^2x^2\right)^{5/2}e^3}{c^8}+\frac{6\left(63d^2c^4+90dec^2+35e^2\right)\left(1-c^2x^2\right)^{3/2}e^2}{c^8}-\right.$$
$$d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))+$$
$$\frac{1}{630}bc\left(\frac{40e^3\left(1-c^2x^2\right)^{7/2}\left(9c^2d+7e\right)}{7c^{10}}-\frac{70e^4\left(1-c^2x^2\right)^{9/2}}{9c^{10}}-\frac{12e^2\left(1-c^2x^2\right)^{5/2}\left(63c^4d^2+90c^2de+35e^2\right)}{5c^{10}}+\frac{8e\right.$$

input
```
Int[(d + e*x^2)^4*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(315*c^8*d^4 + 420*c^6*d^3*e + 378*c^4*d^2*e^2 + 180*c^2*d*e^3 +
 35*e^4)*Sqrt[1 - c^2*x^2])/c^10 + (8*e*(105*c^6*d^3 + 189*c^4*d^2*e + 135
*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2)^(3/2))/(3*c^10) - (12*e^2*(63*c^4*d^2 +
 90*c^2*d*e + 35*e^2)*(1 - c^2*x^2)^(5/2))/(5*c^10) + (40*e^3*(9*c^2*d + 7
*e)*(1 - c^2*x^2)^(7/2))/(7*c^10) - (70*e^4*(1 - c^2*x^2)^(9/2))/(9*c^10))
)/630 + d^4*x*(a + b*ArcCos[c*x]) + (4*d^3*e*x^3*(a + b*ArcCos[c*x]))/3 +
(6*d^2*e^2*x^5*(a + b*ArcCos[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcCos[c*x]))/
7 + (e^4*x^9*(a + b*ArcCos[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.39

method	result
parts	$a\left(\frac{1}{9}e^4x^9+\frac{4}{7}d\,e^3x^7+\frac{6}{5}d^2e^2x^5+\frac{4}{3}d^3e\,x^3+d^4x\right)+\frac{b\left(\frac{c\arccos(cx)e^4x^9}{9}+\frac{4c\arccos(cx)d\,e^3x^7}{7}+\frac{6c\arccos\cdots}{}\right.}{}$
derivativedivides	$\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\arccos(cx)d^4c^9x+\frac{4\arccos(cx)d^3c^9e\,x^3}{3}+\frac{6\arccos(cx)d^2c^9e^2x^5}{5}+\cdots\right.}{}$
default	$\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\arccos(cx)d^4c^9x+\frac{4\arccos(cx)d^3c^9e\,x^3}{3}+\frac{6\arccos(cx)d^2c^9e^2x^5}{5}+\cdots\right.}{}$
orering	$\frac{x\left(20825c^{10}e^5x^{10}+132525c^{10}d\,e^4x^8+366282c^{10}d^2e^3x^6+1400c^8e^5x^8+604170c^{10}d^3e^2x^4+12960c^8d\,e^4x^6+1025325c^{10}\cdots\right.}{}$

input
```
int((e*x^2+d)^4*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^4*x^9+4/7*d*e^3*x^7+6/5*d^2*e^2*x^5+4/3*d^3*e*x^3+d^4*x)+b/c*(1/9
*c*arccos(c*x)*e^4*x^9+4/7*c*arccos(c*x)*d*e^3*x^7+6/5*c*arccos(c*x)*d^2*e
^2*x^5+4/3*c*arccos(c*x)*d^3*e*x^3+arccos(c*x)*d^4*c*x+1/315/c^8*(35*e^4*(
-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c^4
*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*x^
2+1)^(1/2))-315*d^4*c^8*(-c^2*x^2+1)^(1/2)+180*d*c^2*e^3*(-1/7*c^6*x^6*(-c
^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^
(1/2)-16/35*(-c^2*x^2+1)^(1/2))+378*d^2*c^4*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)
^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+420*d^3*c^
6*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.02

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$=\frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\,(35\,bc^9e^4x\cdots}{}$$

input
```
integrate((e*x^2+d)^4*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^
2*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9
 + 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315
*b*c^9*d^4*x)*arccos(c*x) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8*d^4 + 88200*
b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 + 25920*b*c^2*d*e^3 + 100*(81*b*c^8*d*e^
3 + 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 + 1620*b*c^6*d*
e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e + 7938*b*c^6*d^2*e^2 + 324
0*b*c^4*d*e^3 + 560*b*c^2*e^4)*x^2)*sqrt(-c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.89

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$= \begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{acos}(cx) + \frac{4bd^3ex^3\operatorname{acos}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{acos}(cx)}{5} + \frac{4bde^3x^7\,\mathrm{a}}{7} \\ \left(a+\frac{\pi b}{2}\right)\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**4*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**
3*x**7/7 + a*e**4*x**9/9 + b*d**4*x*acos(c*x) + 4*b*d**3*e*x**3*acos(c*x)/
3 + 6*b*d**2*e**2*x**5*acos(c*x)/5 + 4*b*d*e**3*x**7*acos(c*x)/7 + b*e**4*
x**9*acos(c*x)/9 - b*d**4*sqrt(-c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqrt(-c
**2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 4*b
*d*e**3*x**6*sqrt(-c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(-c**2*x**2 + 1
)/(81*c) - 8*b*d**3*e*sqrt(-c**2*x**2 + 1)/(9*c**3) - 8*b*d**2*e**2*x**2*s
qrt(-c**2*x**2 + 1)/(25*c**3) - 24*b*d*e**3*x**4*sqrt(-c**2*x**2 + 1)/(245
*c**3) - 8*b*e**4*x**6*sqrt(-c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sq
rt(-c**2*x**2 + 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(-c**2*x**2 + 1)/(245*
c**5) - 16*b*e**4*x**4*sqrt(-c**2*x**2 + 1)/(945*c**5) - 64*b*d*e**3*sqrt(
-c**2*x**2 + 1)/(245*c**7) - 64*b*e**4*x**2*sqrt(-c**2*x**2 + 1)/(2835*c**
7) - 128*b*e**4*sqrt(-c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), ((a + pi*b/2)
*(d**4*x + 4*d**3*e*x**3/3 + 6*d**2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x
**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.36

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx = \frac{1}{9}\,ae^4x^9 + \frac{4}{7}\,ade^3x^7 + \frac{6}{5}\,ad^2e^2x^5 + \frac{4}{3}\,ad^3ex^3 + \frac{4}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^3e + \frac{2}{25}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd^2e^2 + \frac{4}{245}\left(35\,x^7\arccos\left(cx\right) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right. + \frac{1}{2835}\left(315\,x^9\arccos\left(cx\right) - \left(\frac{35\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\sqrt{-c^2x}}{c^8}\right.\right. + ad^4x + \frac{\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)bd^4}{c}$$

input
```
integrate((e*x^2+d)^4*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/
9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1
)/c^4))*b*d^3*e + 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2
 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 +
 4/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x
^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8
)*c)*b*d*e^3 + 1/2835*(315*x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^
2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqr
t(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*e^4 + a*d^4*x
+ (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d^4/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.50

$$\int (d+ex^2)^4 (a+b\arccos(cx))\,dx = \frac{1}{9}be^4x^9\arccos(cx) + \frac{1}{9}ae^4x^9 + \frac{4}{7}bde^3x^7\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^4x^8}{81c} + \frac{4}{7}ade^3x^7 + \frac{6}{5}bd^2e^2x^5\arccos(cx) - \frac{4\sqrt{-c^2x^2+1}bde^3x^6}{49c} + \frac{6}{5}ad^2e^2x^5 + \frac{4}{3}bd^3ex^3\arccos(cx) - \frac{6\sqrt{-c^2x^2+1}bd^2e^2x^4}{25c} - \frac{8\sqrt{-c^2x^2+1}be^4x^6}{567c^3} + \frac{4}{3}ad^3ex^3 + bd^4x\arccos(cx) - \frac{4\sqrt{-c^2x^2+1}bd^3ex^2}{9c} - \frac{24\sqrt{-c^2x^2+1}bde^3x^4}{245c^3} + ad^4x - \frac{\sqrt{-c^2x^2+1}bd^4}{c} - \frac{8\sqrt{-c^2x^2+1}bd^2e^2x^2}{25c^3} - \frac{16\sqrt{-c^2x^2+1}be^4x^4}{945c^5} - \frac{8\sqrt{-c^2x^2+1}bd^3e}{9c^3} - \frac{32\sqrt{-c^2x^2+1}bde^3x^2}{245c^5} - \frac{16\sqrt{-c^2x^2+1}bd^2e^2}{25c^5} - \frac{64\sqrt{-c^2x^2+1}be^4x^2}{2835c^7} - \frac{64\sqrt{-c^2x^2+1}bde^3}{245c^7} - \frac{128\sqrt{-c^2x^2+1}be^4}{2835c^9}$$

input

```
integrate((e*x^2+d)^4*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/9*b*e^4*x^9*arccos(c*x) + 1/9*a*e^4*x^9 + 4/7*b*d*e^3*x^7*arccos(c*x) -
1/81*sqrt(-c^2*x^2 + 1)*b*e^4*x^8/c + 4/7*a*d*e^3*x^7 + 6/5*b*d^2*e^2*x^5*
arccos(c*x) - 4/49*sqrt(-c^2*x^2 + 1)*b*d*e^3*x^6/c + 6/5*a*d^2*e^2*x^5 +
4/3*b*d^3*e*x^3*arccos(c*x) - 6/25*sqrt(-c^2*x^2 + 1)*b*d^2*e^2*x^4/c - 8/
567*sqrt(-c^2*x^2 + 1)*b*e^4*x^6/c^3 + 4/3*a*d^3*e*x^3 + b*d^4*x*arccos(c*
x) - 4/9*sqrt(-c^2*x^2 + 1)*b*d^3*e*x^2/c - 24/245*sqrt(-c^2*x^2 + 1)*b*d*
e^3*x^4/c^3 + a*d^4*x - sqrt(-c^2*x^2 + 1)*b*d^4/c - 8/25*sqrt(-c^2*x^2 +
1)*b*d^2*e^2*x^2/c^3 - 16/945*sqrt(-c^2*x^2 + 1)*b*e^4*x^4/c^5 - 8/9*sqrt(
-c^2*x^2 + 1)*b*d^3*e/c^3 - 32/245*sqrt(-c^2*x^2 + 1)*b*d*e^3*x^2/c^5 - 16
/25*sqrt(-c^2*x^2 + 1)*b*d^2*e^2/c^5 - 64/2835*sqrt(-c^2*x^2 + 1)*b*e^4*x^
2/c^7 - 64/245*sqrt(-c^2*x^2 + 1)*b*d*e^3/c^7 - 128/2835*sqrt(-c^2*x^2 + 1
)*b*e^4/c^9
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^4\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^4,x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.56

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$=\frac{-99225\sqrt{-c^2x^2+1}\,b\,c^8d^4+99225a\,c^9d^4x+11025a\,c^9e^4x^9+99225acos(cx)\,b\,c^9d^4x+132300acos(cx)\,b}{}$$

input
```
int((e*x^2+d)^4*(a+b*acos(c*x)),x)
```

output
```
(99225*acos(c*x)*b*c**9*d**4*x + 132300*acos(c*x)*b*c**9*d**3*e*x**3 + 119
070*acos(c*x)*b*c**9*d**2*e**2*x**5 + 56700*acos(c*x)*b*c**9*d*e**3*x**7 +
 11025*acos(c*x)*b*c**9*e**4*x**9 - 99225*sqrt( - c**2*x**2 + 1)*b*c**8*d*
*4 - 44100*sqrt( - c**2*x**2 + 1)*b*c**8*d**3*e*x**2 - 23814*sqrt( - c**2*
x**2 + 1)*b*c**8*d**2*e**2*x**4 - 8100*sqrt( - c**2*x**2 + 1)*b*c**8*d*e**
3*x**6 - 1225*sqrt( - c**2*x**2 + 1)*b*c**8*e**4*x**8 - 88200*sqrt( - c**2
*x**2 + 1)*b*c**6*d**3*e - 31752*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*e**2*x
**2 - 9720*sqrt( - c**2*x**2 + 1)*b*c**6*d*e**3*x**4 - 1400*sqrt( - c**2*x
**2 + 1)*b*c**6*e**4*x**6 - 63504*sqrt( - c**2*x**2 + 1)*b*c**4*d**2*e**2
- 12960*sqrt( - c**2*x**2 + 1)*b*c**4*d*e**3*x**2 - 1680*sqrt( - c**2*x**2
 + 1)*b*c**4*e**4*x**4 - 25920*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**3 - 2240
*sqrt( - c**2*x**2 + 1)*b*c**2*e**4*x**2 - 4480*sqrt( - c**2*x**2 + 1)*b*e
**4 + 99225*a*c**9*d**4*x + 132300*a*c**9*d**3*e*x**3 + 119070*a*c**9*d**2
*e**2*x**5 + 56700*a*c**9*d*e**3*x**7 + 11025*a*c**9*e**4*x**9)/(99225*c**
9)
```

3.81 $\int (d+ex^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 225

$$\int (d+ex^2)^3 (a+b\arccos(cx))\,dx = -\frac{b(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3)\sqrt{1-c^2x^2}}{35c^7} + \frac{be(35c^4d^2+42c^2de+15e^2)(1-c^2x^2)^{3/2}}{105c^7} - \frac{3be^2(7c^2d+5e)(1-c^2x^2)^{5/2}}{175c^7} + \frac{be^3(1-c^2x^2)^{7/2}}{49c^7} + d^3x(a+b\arccos(cx)) + d^2ex^3(a+b\arccos(cx)) + \frac{3}{5}de^2x^5(a+b\arccos(cx)) + \frac{1}{7}e^3x^7(a+b\arccos(cx))$$

output
```
-1/35*b*(35*c^6*d^3+35*c^4*d^2*e+21*c^2*d*e^2+5*e^3)*(-c^2*x^2+1)^(1/2)/c^
7+1/105*b*e*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^(3/2)/c^7-3/175*b*
e^2*(7*c^2*d+5*e)*(-c^2*x^2+1)^(5/2)/c^7+1/49*b*e^3*(-c^2*x^2+1)^(7/2)/c^7
+d^3*x*(a+b*arccos(c*x))+d^2*e*x^3*(a+b*arccos(c*x))+3/5*d*e^2*x^5*(a+b*ar
ccos(c*x))+1/7*e^3*x^7*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)$$
$$-\frac{b\sqrt{1-c^2x^2}(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex}{3675c^7}$$
$$+b\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)\arccos(cx)$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 - c^2*x^
2]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 294*d*e*x^
2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*
x^6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)*
ArcCos[c*x]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5171, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5171}$$

$$bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{35\sqrt{1-c^2x^2}}dx+d^3x(a+b\arccos(cx))+d^2ex^3(a+ b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{35}bc\int \frac{x\left(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3\right)}{\sqrt{1-c^2x^2}}dx+d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$\Big\downarrow$ 2331

$$\frac{1}{70}bc\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{\sqrt{1-c^2x^2}}dx^2+d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$\Big\downarrow$ 2389

$$\frac{1}{70}bc\int \left(-\frac{5\left(1-c^2x^2\right)^{5/2}e^3}{c^6}+\frac{3\left(7dc^2+5e\right)\left(1-c^2x^2\right)^{3/2}e^2}{c^6}-\frac{\left(35d^2c^4+42dec^2+15e^2\right)\sqrt{1-c^2x^2}e}{c^6}+\frac{35d^3c^{6}\cdots}{}\right.$$
$$d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$\Big\downarrow$ 2009

$$d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))+$$
$$\frac{1}{70}bc\left(-\frac{6e^2\left(1-c^2x^2\right)^{5/2}\left(7c^2d+5e\right)}{5c^8}+\frac{10e^3\left(1-c^2x^2\right)^{7/2}}{7c^8}+\frac{2e\left(1-c^2x^2\right)^{3/2}\left(35c^4d^2+42c^2de+15e^2\right)}{3c^8}-\frac{2\sqrt{1\cdots}}{}\right.$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*Sqrt[1 - c^2*
x^2])/c^8 + (2*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2)^(3/2))/(
3*c^8) - (6*e^2*(7*c^2*d + 5*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (10*e^3*(1
- c^2*x^2)^(7/2))/(7*c^8)))/70 + d^3*x*(a + b*ArcCos[c*x]) + d^2*e*x^3*(a
+ b*ArcCos[c*x]) + (3*d*e^2*x^5*(a + b*ArcCos[c*x]))/5 + (e^3*x^7*(a + b*A
rcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{1}{7}e^{3}x^{7}+\frac{3}{5}d\,e^{2}x^{5}+d^{2}e\,x^{3}+d^{3}x\right)+\frac{b\left(\frac{c\arccos(cx)e^{3}x^{7}}{7}+\frac{3c\arccos(cx)d\,e^{2}x^{5}}{5}+c\arccos(cx)d^{2}e\,x^{3}+\operatorname{arcco}\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^{3}c^{7}x+d^{2}c^{7}e\,x^{3}+\frac{3}{5}d\,c^{7}e^{2}x^{5}+\frac{1}{7}e^{3}c^{7}x^{7}\right)}{c^{6}}+\frac{b\left(\arccos(cx)d^{3}c^{7}x+\arccos(cx)d^{2}c^{7}e\,x^{3}+\frac{3\arccos(cx)d\,c^{7}e^{2}x^{5}}{5}+\frac{\arccos(cx)e^{3}c^{7}x^{7}}{7}\right.}{}}{}$
default	$\frac{\frac{a\left(d^{3}c^{7}x+d^{2}c^{7}e\,x^{3}+\frac{3}{5}d\,c^{7}e^{2}x^{5}+\frac{1}{7}e^{3}c^{7}x^{7}\right)}{c^{6}}+\frac{b\left(\arccos(cx)d^{3}c^{7}x+\arccos(cx)d^{2}c^{7}e\,x^{3}+\frac{3\arccos(cx)d\,c^{7}e^{2}x^{5}}{5}+\frac{\arccos(cx)e^{3}c^{7}x^{7}}{7}\right.}{}}{}$
orering	$\frac{x\left(325e^{4}x^{8}c^{8}+1792c^{8}d\,e^{3}x^{6}+4410c^{8}d^{2}e^{2}x^{4}+30c^{6}e^{4}x^{6}+9800c^{8}d^{3}e\,x^{2}+294c^{6}d\,e^{3}x^{4}+1225c^{8}d^{4}+2450c^{6}d^{2}e^{2}x^{2}+60c^{4}\right.}{1225(e\,x^{2}+d)c^{8}}$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+d^3*x)+b/c*(1/7*c*arccos(c*x)*e^3*x
^7+3/5*c*arccos(c*x)*d*e^2*x^5+c*arccos(c*x)*d^2*e*x^3+arccos(c*x)*d^3*c*x
+1/35/c^6*(5*e^3*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1
)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))-35*d^3*c
^6*(-c^2*x^2+1)^(1/2)+21*d*c^2*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c
^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+35*d^2*c^4*e*(-1/3*c^2*
x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02

$$\int \left(d+ex^{2}\right)^{3}(a+b\arccos(cx))\,dx$$
$$=\frac{525\,ac^{7}e^{3}x^{7}+2205\,ac^{7}de^{2}x^{5}+3675\,ac^{7}d^{2}ex^{3}+3675\,ac^{7}d^{3}x+105\left(5\,bc^{7}e^{3}x^{7}+21\,bc^{7}de^{2}x^{5}+35\,bc^{7}d^{2}ex\right.}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 +
3675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^
2*e*x^3 + 35*b*c^7*d^3*x)*arccos(c*x) - (75*b*c^6*e^3*x^6 + 3675*b*c^6*d^3
 + 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 + 10*b*c^4*e^3)
*x^4 + 240*b*e^3 + (1225*b*c^6*d^2*e + 588*b*c^4*d*e^2 + 120*b*c^2*e^3)*x^
2)*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.75

$$\int (d+ex^2)^3 (a+b\arccos(cx))\,dx$$
$$= \begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{acos}(cx) + bd^2ex^3\operatorname{acos}(cx) + \frac{3bde^2x^5\operatorname{acos}(cx)}{5} + \frac{be^3x^7\operatorname{acos}(cx)}{7} - b\ldots \\ \left(a+\frac{\pi b}{2}\right)\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*acos(c*x) + b*d**2*e*x**3*acos(c*x) + 3*b*d*e**2*x**5*acos(c*x)/5
 + b*e**3*x**7*acos(c*x)/7 - b*d**3*sqrt(-c**2*x**2 + 1)/c - b*d**2*e*x**2
*sqrt(-c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c)
- b*e**3*x**6*sqrt(-c**2*x**2 + 1)/(49*c) - 2*b*d**2*e*sqrt(-c**2*x**2 + 1
)/(3*c**3) - 4*b*d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(25*c**3) - 6*b*e**3*x**
4*sqrt(-c**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(-c**2*x**2 + 1)/(25*c*
*5) - 8*b*e**3*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b*e**3*sqrt(-c**2
*x**2 + 1)/(245*c**7), Ne(c, 0)), ((a + pi*b/2)*(d**3*x + d**2*e*x**3 + 3*
d*e**2*x**5/5 + e**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.32

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = \frac{1}{7}\,ae^3x^7+\frac{3}{5}\,ade^2x^5+ad^2ex^3$$
$$+\frac{1}{3}\left(3\,x^3\arccos\left(cx\right)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^2e$$
$$+\frac{1}{25}\left(15\,x^5\arccos\left(cx\right)-\left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bde^2$$
$$+\frac{1}{245}\left(35\,x^7\arccos\left(cx\right)-\left(\frac{5\,\sqrt{-c^2x^2+1}x^6}{c^2}+\frac{6\,\sqrt{-c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{-c^2x^2+1}x^2}{c^6}+\frac{16\,\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$+ad^3x+\frac{\left(cx\arccos\left(cx\right)-\sqrt{-c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arccos(c*x) - c
*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(
15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arccos(c*x)
 - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-
c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*
x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.41

$$\int (d+ex^2)^3 (a+b\arccos(cx))\,dx = \frac{1}{7}be^3x^7\arccos(cx) + \frac{1}{7}ae^3x^7 + \frac{3}{5}bde^2x^5\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^3x^6}{49c} + \frac{3}{5}ade^2x^5 + bd^2ex^3\arccos(cx) - \frac{3\sqrt{-c^2x^2+1}bde^2x^4}{25c} + ad^2ex^3 + bd^3x\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bd^2ex^2}{3c} - \frac{6\sqrt{-c^2x^2+1}be^3x^4}{245c^3} + ad^3x - \frac{\sqrt{-c^2x^2+1}bd^3}{c} - \frac{4\sqrt{-c^2x^2+1}bde^2x^2}{25c^3} - \frac{2\sqrt{-c^2x^2+1}bd^2e}{3c^3} - \frac{8\sqrt{-c^2x^2+1}be^3x^2}{245c^5} - \frac{8\sqrt{-c^2x^2+1}bde^2}{25c^5} - \frac{16\sqrt{-c^2x^2+1}be^3}{245c^7}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/7*b*e^3*x^7*arccos(c*x) + 1/7*a*e^3*x^7 + 3/5*b*d*e^2*x^5*arccos(c*x) -
1/49*sqrt(-c^2*x^2 + 1)*b*e^3*x^6/c + 3/5*a*d*e^2*x^5 + b*d^2*e*x^3*arccos
(c*x) - 3/25*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^4/c + a*d^2*e*x^3 + b*d^3*x*arcc
os(c*x) - 1/3*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^2/c - 6/245*sqrt(-c^2*x^2 + 1)*
b*e^3*x^4/c^3 + a*d^3*x - sqrt(-c^2*x^2 + 1)*b*d^3/c - 4/25*sqrt(-c^2*x^2
+ 1)*b*d*e^2*x^2/c^3 - 2/3*sqrt(-c^2*x^2 + 1)*b*d^2*e/c^3 - 8/245*sqrt(-c^
2*x^2 + 1)*b*e^3*x^2/c^5 - 8/25*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^5 - 16/245*sq
rt(-c^2*x^2 + 1)*b*e^3/c^7
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.50

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{3675acos(cx)\,b\,c^7d^3x + 3675acos(cx)\,b\,c^7d^2e\,x^3 + 2205acos(cx)\,b\,c^7d\,e^2x^5 + 525acos(cx)\,b\,c^7e^3x^7 - 3675}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(3675*acos(c*x)*b*c**7*d**3*x + 3675*acos(c*x)*b*c**7*d**2*e*x**3 + 2205*a
cos(c*x)*b*c**7*d*e**2*x**5 + 525*acos(c*x)*b*c**7*e**3*x**7 - 3675*sqrt(
- c**2*x**2 + 1)*b*c**6*d**3 - 1225*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*e*x
**2 - 441*sqrt( - c**2*x**2 + 1)*b*c**6*d*e**2*x**4 - 75*sqrt( - c**2*x**2
 + 1)*b*c**6*e**3*x**6 - 2450*sqrt( - c**2*x**2 + 1)*b*c**4*d**2*e - 588*s
qrt( - c**2*x**2 + 1)*b*c**4*d*e**2*x**2 - 90*sqrt( - c**2*x**2 + 1)*b*c**
4*e**3*x**4 - 1176*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**2 - 120*sqrt( - c**2
*x**2 + 1)*b*c**2*e**3*x**2 - 240*sqrt( - c**2*x**2 + 1)*b*e**3 + 3675*a*c
**7*d**3*x + 3675*a*c**7*d**2*e*x**3 + 2205*a*c**7*d*e**2*x**5 + 525*a*c**
7*e**3*x**7)/(3675*c**7)
```

3.82 $\int (d+ex^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int (d+ex^2)^2 (a+b\arccos(cx))\,dx = -\frac{b(15c^4d^2+10c^2de+3e^2)\sqrt{1-c^2x^2}}{15c^5} + \frac{2be(5c^2d+3e)(1-c^2x^2)^{3/2}}{45c^5} - \frac{be^2(1-c^2x^2)^{5/2}}{25c^5} + d^2x(a+b\arccos(cx)) + \frac{2}{3}dex^3(a+b\arccos(cx)) + \frac{1}{5}e^2x^5(a+b\arccos(cx))$$

output
```
-1/15*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*(-c^2*x^2+1)^(1/2)/c^5+2/45*b*e*(5*c
^2*d+3*e)*(-c^2*x^2+1)^(3/2)/c^5-1/25*b*e^2*(-c^2*x^2+1)^(5/2)/c^5+d^2*x*(
a+b*arccos(c*x))+2/3*d*e*x^3*(a+b*arccos(c*x))+1/5*e^2*x^5*(a+b*arccos(c*x
))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right) - \frac{b\sqrt{1-c^2x^2}(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5} + 15bx\left(15d^2+10dex^2+3e^2x^4\right)\arccos(cx)\right)$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 - c^2*x^2]*(24*e^2 +
 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 +
 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCos[c*x])/225
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5171, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5171}$$

$$bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{15\sqrt{1-c^2x^2}}dx + d^2x(a+b\arccos(cx)) + \frac{2}{3}dex^3(a+b\arccos(cx)) + \frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bc\int \frac{x(3e^2x^4+10dex^2+15d^2)}{\sqrt{1-c^2x^2}}dx+d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{30}bc\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{1-c^2x^2}}dx^2+d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{30}bc\int \left(\frac{3(1-c^2x^2)^{3/2}e^2}{c^4}-\frac{2(5dc^2+3e)\sqrt{1-c^2x^2}e}{c^4}+\frac{15d^2c^4+10dec^2+3e^2}{c^4\sqrt{1-c^2x^2}}\right)dx^2+d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))+\frac{1}{30}bc\left(\frac{4e(1-c^2x^2)^{3/2}(5c^2d+3e)}{3c^6}-\frac{6e^2(1-c^2x^2)^{5/2}}{5c^6}-\frac{2\sqrt{1-c^2x^2}(15c^4d^2+10c^2de+3e^2)}{c^6}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[1 - c^2*x^2])/c^6 + (4*e*
(5*c^2*d + 3*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) - (6*e^2*(1 - c^2*x^2)^(5/2))
/(5*c^6)))/30 + d^2*x*(a + b*ArcCos[c*x]) + (2*d*e*x^3*(a + b*ArcCos[c*x])
)/3 + (e^2*x^5*(a + b*ArcCos[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{c\arccos(cx)e^2x^5}{5}+\frac{2c\arccos(cx)de\,x^3}{3}+\arccos(cx)d^2cx+\frac{3e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arccos(cx)d^2c^5x+\frac{2\arccos(cx)d\,c^5e\,x^3}{3}+\frac{\arccos(cx)e^2c^5x^5}{5}+\frac{e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2}{}\right.}{5}\right.}{c}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arccos(cx)d^2c^5x+\frac{2\arccos(cx)d\,c^5e\,x^3}{3}+\frac{\arccos(cx)e^2c^5x^5}{5}+\frac{e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2}{}\right.}{5}\right.}{c}}{c}$
orering	$\frac{x\left(81e^3x^6c^6+395c^6d\,e^2x^4+1275c^6d^2e\,x^2+12c^4e^3x^4+225c^6d^3+200c^4d\,e^2x^2-900c^4d^2e+48c^2e^3x^2-400c^2d\,e^2-96e^3\right)(}{225(e\,x^2+d)c^6}$

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arccos(c*x)*e^2*x^5+2/3*c*arc
cos(c*x)*d*e*x^3+arccos(c*x)*d^2*c*x+1/15/c^4*(3*e^2*(-1/5*c^4*x^4*(-c^2*x
^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))-15*d^
2*c^4*(-c^2*x^2+1)^(1/2)+10*d*c^2*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(
-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\int \left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$
$$=\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\arccos\left(cx\right)-\left(9\,bc^4e^2x\right.}{225\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^
5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*arccos(c*x) - (9*b*c^4*e^2*
x^4 + 225*b*c^4*d^2 + 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4*d*e + 6*b*c^2
*e^2)*x^2)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.63

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\,\mathrm{acos}\,(cx)+\frac{2bdex^3\,\mathrm{acos}\,(cx)}{3}+\frac{be^2x^5\,\mathrm{acos}\,(cx)}{5}-\frac{bd^2\sqrt{-c^2x^2+1}}{c}-\frac{2bdex^2\sqrt{-c^2x^2+1}}{9c}- \\ \left(a+\frac{\pi b}{2}\right)\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acos(c*x)
+ 2*b*d*e*x**3*acos(c*x)/3 + b*e**2*x**5*acos(c*x)/5 - b*d**2*sqrt(-c**2*x
**2 + 1)/c - 2*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(-c
**2*x**2 + 1)/(25*c) - 4*b*d*e*sqrt(-c**2*x**2 + 1)/(9*c**3) - 4*b*e**2*x*
*2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(-c**2*x**2 + 1)/(75*c**5
), Ne(c, 0)), ((a + pi*b/2)*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{2}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bde$$
$$+\frac{1}{75}\left(15x^5\arccos(cx)-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)be^2$$
$$+ad^2x+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2
+ 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arccos(c*x)
 - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-
c^2*x^2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 +
1))*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.28

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \frac{1}{5}\,be^2x^5\arccos\left(cx\right) + \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,bdex^3\arccos\left(cx\right) - \frac{\sqrt{-c^2x^2+1}be^2x^4}{25\,c} + \frac{2}{3}\,adex^3 + bd^2x\arccos\left(cx\right) - \frac{2\sqrt{-c^2x^2+1}bdex^2}{9\,c} + ad^2x - \frac{\sqrt{-c^2x^2+1}bd^2}{c} - \frac{4\sqrt{-c^2x^2+1}be^2x^2}{75\,c^3} - \frac{4\sqrt{-c^2x^2+1}bde}{9\,c^3} - \frac{8\sqrt{-c^2x^2+1}be^2}{75\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/5*b*e^2*x^5*arccos(c*x) + 1/5*a*e^2*x^5 + 2/3*b*d*e*x^3*arccos(c*x) - 1/
25*sqrt(-c^2*x^2 + 1)*b*e^2*x^4/c + 2/3*a*d*e*x^3 + b*d^2*x*arccos(c*x) -
2/9*sqrt(-c^2*x^2 + 1)*b*d*e*x^2/c + a*d^2*x - sqrt(-c^2*x^2 + 1)*b*d^2/c
- 4/75*sqrt(-c^2*x^2 + 1)*b*e^2*x^2/c^3 - 4/9*sqrt(-c^2*x^2 + 1)*b*d*e/c^3
 - 8/75*sqrt(-c^2*x^2 + 1)*b*e^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.39

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \frac{225acos(cx)\,b\,c^5d^2x+150acos(cx)\,b\,c^5de\,x^3+45acos(cx)\,b\,c^5e^2x^5-225\sqrt{-c^2x^2+1}\,b\,c^4d^2-50\sqrt{-c^2x^2}}{}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(225*acos(c*x)*b*c**5*d**2*x + 150*acos(c*x)*b*c**5*d*e*x**3 + 45*acos(c*x
)*b*c**5*e**2*x**5 - 225*sqrt( - c**2*x**2 + 1)*b*c**4*d**2 - 50*sqrt( - c
**2*x**2 + 1)*b*c**4*d*e*x**2 - 9*sqrt( - c**2*x**2 + 1)*b*c**4*e**2*x**4
- 100*sqrt( - c**2*x**2 + 1)*b*c**2*d*e - 12*sqrt( - c**2*x**2 + 1)*b*c**2
*e**2*x**2 - 24*sqrt( - c**2*x**2 + 1)*b*e**2 + 225*a*c**5*d**2*x + 150*a*
c**5*d*e*x**3 + 45*a*c**5*e**2*x**5)/(225*c**5)
```

3.83 $\int (d+ex^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = -\frac{b(3c^2d+e)\sqrt{1-c^2x^2}}{3c^3}+\frac{be(1-c^2x^2)^{3/2}}{9c^3} + dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

output
```
-1/3*b*(3*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3+1/9*b*e*(-c^2*x^2+1)^(3/2)/c^3+d
*x*(a+b*arccos(c*x))+1/3*e*x^3*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = adx+\frac{1}{3}aex^3-\frac{bd\sqrt{1-c^2x^2}}{c} + be\left(-\frac{2}{9c^3}-\frac{x^2}{9c}\right)\sqrt{1-c^2x^2} + bdx\arccos(cx)+\frac{1}{3}bex^3\arccos(cx)$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 - (b*d*Sqrt[1 - c^2*x^2])/c + b*e*(-2/(9*c^3) - x^2/(9
*c))*Sqrt[1 - c^2*x^2] + b*d*x*ArcCos[c*x] + (b*e*x^3*ArcCos[c*x])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5171, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5171}$$

$$bc\int \frac{x\left(ex^2+3d\right)}{3\sqrt{1-c^2x^2}}dx+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{x\left(ex^2+3d\right)}{\sqrt{1-c^2x^2}}dx+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{6}bc\int \frac{ex^2+3d}{\sqrt{1-c^2x^2}}dx^2+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}bc\int \left(\frac{3dc^2+e}{c^2\sqrt{1-c^2x^2}}-\frac{e\sqrt{1-c^2x^2}}{c^2}\right)dx^2+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2e\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}\left(3c^2d+e\right)}{c^4}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(3*c^2*d + e)*Sqrt[1 - c^2*x^2])/c^4 + (2*e*(1 - c^2*x^2)^(3/2))
/(3*c^4)))/6 + d*x*(a + b*ArcCos[c*x]) + (e*x^3*(a + b*ArcCos[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\arccos(cx)x^3e}{3}+\arccos(cx)dcx+\frac{e\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-\frac{2\sqrt{-c^2x^2+1}}{3}\right)-3d\,c^2\sqrt{-c^2x^2+1}}{3c^2}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\arccos(cx)d\,c^3x+\frac{\arccos(cx)e\,c^3x^3}{3}+\frac{e\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-\frac{2\sqrt{-c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{-c^2x^2+1}\right)}{c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\arccos(cx)d\,c^3x+\frac{\arccos(cx)e\,c^3x^3}{3}+\frac{e\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-\frac{2\sqrt{-c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{-c^2x^2+1}\right)}{c^2}}{c}$
orering	$\frac{x(5e^2x^4c^4+30c^4de\,x^2+9c^4d^2+2c^2e^2x^2-18c^2de-4e^2)(a+b\arccos(cx))}{9(e\,x^2+d)c^4}-\frac{(c^2e\,x^2+9c^2d+2e)(cx-1)(cx+1)\left(2ex(a\right.}{9c^4(e\,x^2+d)}$

input
```
int((e*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arccos(c*x)*x^3*e+arccos(c*x)*d*c*x+1/3/c^2*(
e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))-3*d*c^2*(-c^2*x
^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int\left(d+ex^2\right)\left(a+b\arccos(cx)\right)dx$$
$$=\frac{3\,ac^3ex^3+9\,ac^3dx+3\left(bc^3ex^3+3\,bc^3dx\right)\arccos\left(cx\right)-\left(bc^2ex^2+9\,bc^2d+2\,be\right)\sqrt{-c^2x^2+1}}{9\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*arccos(c*
x) - (b*c^2*e*x^2 + 9*b*c^2*d + 2*b*e)*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \begin{cases} adx + \frac{aex^3}{3} + bdx\,\mathrm{acos}\,(cx) + \frac{bex^3\,\mathrm{acos}\,(cx)}{3} - \frac{bd\sqrt{-c^2x^2+1}}{c} - \frac{bex^2\sqrt{-c^2x^2+1}}{9c} - \frac{2be\sqrt{-c^2x^2+1}}{9c^3} & \text{for } c \neq 0 \\ \left(a+\frac{\pi b}{2}\right)\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**3/3 + b*d*x*acos(c*x) + b*e*x**3*acos(c*x)/3 - b
*d*sqrt(-c**2*x**2 + 1)/c - b*e*x**2*sqrt(-c**2*x**2 + 1)/(9*c) - 2*b*e*sq
rt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), ((a + pi*b/2)*(d*x + e*x**3/3), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{3}aex^3 + \frac{1}{9}\left(3x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)be + adx + \frac{\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*s
qrt(-c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1
))*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2) (a+b\arccos(cx))\,dx = \frac{1}{3}bex^3\arccos(cx) + \frac{1}{3}aex^3 + bdx\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bex^2}{9c} + adx - \frac{\sqrt{-c^2x^2+1}bd}{c} - \frac{2\sqrt{-c^2x^2+1}be}{9c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/3*b*e*x^3*arccos(c*x) + 1/3*a*e*x^3 + b*d*x*arccos(c*x) - 1/9*sqrt(-c^2*
x^2 + 1)*b*e*x^2/c + a*d*x - sqrt(-c^2*x^2 + 1)*b*d/c - 2/9*sqrt(-c^2*x^2
+ 1)*b*e/c^3
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \begin{cases} \frac{a\,x\,(e\,x^2+3\,d)}{3} - b\,e\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\,\left(\frac{2}{c^2}+x^2\right)}{9} - \frac{x^3\,\mathrm{acos}(c\,x)}{3}\right) - \frac{b\,d\,\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} & \text{if } 0<c \\ \int (a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)\,dx & \text{if } \neg 0<c \end{cases}$$

input
```
int((a + b*acos(c*x))*(d + e*x^2),x)
```

output
```
piecewise(0 < c, - b*e*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*acos(
c*x))/3) + (a*x*(3*d + e*x^2))/3 - (b*d*((- c^2*x^2 + 1)^(1/2) - c*x*acos(
c*x)))/c, ~0 < c, int((a + b*acos(c*x))*(d + e*x^2), x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int (d+ex^2)(a+b\arccos(cx))\,dx$$
$$= \frac{9acos(cx)\,b\,c^3dx + 3acos(cx)\,b\,c^3e\,x^3 - 9\sqrt{-c^2x^2+1}\,b\,c^2d - \sqrt{-c^2x^2+1}\,b\,c^2e\,x^2 - 2\sqrt{-c^2x^2+1}\,be + 9}{9c^3}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(9*acos(c*x)*b*c**3*d*x + 3*acos(c*x)*b*c**3*e*x**3 - 9*sqrt( - c**2*x**2
+ 1)*b*c**2*d - sqrt( - c**2*x**2 + 1)*b*c**2*e*x**2 - 2*sqrt( - c**2*x**2
 + 1)*b*e + 9*a*c**3*d*x + 3*a*c**3*e*x**3)/(9*c**3)
```

3.84 $\int (a+b \arccos (c x)) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a+b \arccos (c x)) \, dx = a x-\frac{b \sqrt{1-c^2 x^2}}{c}+b x \arccos (c x)$$

output
```
a*x-b*(-c^2*x^2+1)^(1/2)/c+b*x*arccos(c*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+b \arccos (c x)) \, dx = a x-\frac{b \sqrt{1-c^2 x^2}}{c}+b x \arccos (c x)$$

input
```
Integrate[a + b*ArcCos[c*x],x]
```

output
```
a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}$$

input
```
Int[a + b*ArcCos[c*x],x]
```

output
```
a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$ax+\frac{b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	32
parts	$ax+\frac{b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	32
derivativedivides	$\frac{cxa+b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	34
orering	$x(a+b\arccos(cx))+\frac{(cx-1)(cx+1)b}{c\sqrt{-c^2x^2+1}}$	39

input
```
int(a+b*arccos(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/c*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (a+b\arccos(cx))\,dx = \frac{bcx\arccos(cx)+acx-\sqrt{-c^2x^2+1}b}{c}$$

input
```
integrate(a+b*arccos(c*x),x, algorithm="fricas")
```

output
```
(b*c*x*arccos(c*x) + a*c*x - sqrt(-c^2*x^2 + 1)*b)/c
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (a+b\arccos(cx))\,dx = ax+b\left(\begin{cases} x\operatorname{acos}(cx)-\frac{\sqrt{-c^2x^2+1}}{c} & \text{for } c\neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*acos(c*x),x)
```

output
```
a*x + b*Piecewise((x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (pi*x/
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+b\arccos(cx))\,dx = ax + \frac{\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)b}{c}$$

input
```
integrate(a+b*arccos(c*x),x, algorithm="maxima")
```

output
```
a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+b\arccos(cx))\,dx = ax + \frac{\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)b}{c}$$

input
```
integrate(a+b*arccos(c*x),x, algorithm="giac")
```

output
```
a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (a+b\arccos(cx))\,dx = a\,x - \frac{b\sqrt{1-c^2\,x^2}}{c} + b\,x\,\mathrm{acos}(c\,x)$$

input
```
int(a + b*acos(c*x),x)
```

output
```
a*x - (b*(1 - c^2*x^2)^(1/2))/c + b*x*acos(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a + b\arccos(cx))\,dx = \frac{acos(cx)\,bcx - \sqrt{-c^2x^2+1}\,b + acx}{c}$$

input
```
int(a+b*acos(c*x),x)
```

output
```
(acos(c*x)*b*c*x - sqrt( - c**2*x**2 + 1)*b + a*c*x)/c
```

3.85 $\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 541

$$
\begin{aligned}
\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx = {} & \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\
& - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

output
```
1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2
)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x))*ln(1+e^(1/2
)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/
e^(1/2)+1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(
-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x))*ln(
1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d
)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(
-d)^(1/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,e^(1/2)
*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)
+I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(
-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.56

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2),x]
```

output
```
(2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 4*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[
e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*
d + e]] - 4*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*S
qrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + I*b*ArcCos[c*x]
*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] +
 (2*I)*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*S
qrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*ArcCos[c*x]*L
og[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (
2*I)*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqr
t[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*ArcCos[c*x]*Log
[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*
b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] +
Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + I*b*ArcCos[c*x]*Log[1 + (I*
(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (2*I)*b*ArcSin
[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2
*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - b*PolyLog[2, ((-I)*(-(c*Sqrt[d]) +
Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + b*PolyLog[2, (I*(-(c*Sqrt[d
]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + b*PolyLog[2, ((-I)*(c*
Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - b*PolyLog[2, (I*(
c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]])/(2*Sqrt[d]*Sq...
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 5173}$$

$$\int \left(\frac{\sqrt{-d}(a+b\arccos(cx))}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\arccos(cx))}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*S
qrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x])*Log[1 + (Sqr
t[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqr
t[e]) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] + I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x])*Log[
1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*Sqrt
[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[
-d] - I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, (Sqrt
[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e
]) + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqr
t[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*Ar
cCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.43

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{ibc\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\arccos(cx)\ln\left(\frac{\_R1-cx-i\sqrt{-c^2x^2+1}}{\_R1}\right)+\mathrm{dilo}\right.}{\_R1^2e+2c^2d+e}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\arccos(cx)\ln\left(\frac{\_R1-cx-i\sqrt{-c^2x^2+1}}{\_R1}\right)+\right.}{\_R1^2e+2c^2d+e}\right.}{2}\right.}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\arccos(cx)\ln\left(\frac{\_R1-cx-i\sqrt{-c^2x^2+1}}{\_R1}\right)+\right.}{\_R1^2e+2c^2d+e}\right.}{2}\right.}{}$

input
```
int((a+b*arccos(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*I*b*c*sum(_R1/(_R1^2*e+2*c^2*d+e
)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(
-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/2*I*b*
c*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^
(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(
4*c^2*d+2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{a + b \arccos(cx)}{d + ex^2} \, dx = \int \frac{b \arccos{(cx)} + a}{ex^2 + d} \, dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a + b \arccos(cx)}{d + ex^2} \, dx = \int \frac{a + b \operatorname{acos}{(cx)}}{d + ex^2} \, dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arccos(cx)}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a+\left(\int \frac{acos(cx)}{e\,x^2+d}dx\right)bde}{de}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(acos(c*x)/(d + e*x*
*2),x)*b*d*e)/(d*e)
```

3.86 $\int (d+ex^2)^3 (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 569

$$\begin{aligned}\int (d+ex^2)^3 (a+b\arccos(cx))^2\,dx = &-2b^2d^3x-\frac{4b^2d^2ex}{3c^2}-\frac{16b^2de^2x}{25c^4}-\frac{32b^2e^3x}{245c^6}\\
&-\frac{2}{9}b^2d^2ex^3-\frac{8b^2de^2x^3}{75c^2}-\frac{16b^2e^3x^3}{735c^4}\\
&-\frac{6}{125}b^2de^2x^5-\frac{12b^2e^3x^5}{1225c^2}-\frac{2}{343}b^2e^3x^7\\
&-\frac{2bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}\\
&-\frac{4bd^2e\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^3}\\
&-\frac{16bde^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^5}\\
&-\frac{32be^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^7}\\
&-\frac{2bd^2ex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c}\\
&-\frac{8bde^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^3}\\
&-\frac{16be^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^5}\\
&-\frac{6bde^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c}\\
&-\frac{12be^3x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^3}\\
&-\frac{2be^3x^6\sqrt{1-c^2x^2}(a+b\arccos(cx))}{49c}\\
&+d^3x(a+b\arccos(cx))^2+d^2ex^3(a+b\arccos(cx))^2\\
&+\frac{3}{5}de^2x^5(a+b\arccos(cx))^2\\
&+\frac{1}{7}e^3x^7(a+b\arccos(cx))^2\end{aligned}$$

output
```
-2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2-16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^
6-2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2-16/735*b^2*e^3*x^3/c^4-6/125*b^
2*d*e^2*x^5-12/1225*b^2*e^3*x^5/c^2-2/343*b^2*e^3*x^7-2*b*d^3*(-c^2*x^2+1)
^(1/2)*(a+b*arccos(c*x))/c-4/3*b*d^2*e*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x)
)/c^3-16/25*b*d*e^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5-32/245*b*e^3*
(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^7-2/3*b*d^2*e*x^2*(-c^2*x^2+1)^(1/2
)*(a+b*arccos(c*x))/c-8/25*b*d*e^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x)
)/c^3-16/245*b*e^3*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5-6/25*b*d*e
^2*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c-12/245*b*e^3*x^4*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))/c^3-2/49*b*e^3*x^6*(-c^2*x^2+1)^(1/2)*(a+b*arcco
s(c*x))/c+d^3*x*(a+b*arccos(c*x))^2+d^2*e*x^3*(a+b*arccos(c*x))^2+3/5*d*e^
2*x^5*(a+b*arccos(c*x))^2+1/7*e^3*x^7*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.78

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$= \frac{11025a^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-210ab\sqrt{1-c^2x^2}(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(}{}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*
a*b*Sqrt[1 - c^2*x^2]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(12
25*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*
d*e^2*x^4 + 75*e^3*x^6)) - 2*b^2*c*x*(25200*e^3 + 840*c^2*e^2*(147*d + 5*e
*x^2) + 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3 +
42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*(3
5*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 - c^2*x^2]*(24
0*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45
*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))
*ArcCos[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e
^3*x^6)*ArcCos[c*x]^2)/(385875*c^7)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (a+b\arccos(cx))^2 dx$$

$$\Big\downarrow \ 5173$$

$$\int \left(d^3(a+b\arccos(cx))^2+3d^2ex^2(a+b\arccos(cx))^2+3de^2x^4(a+b\arccos(cx))^2+e^3x^6(a+b\arccos(cx))^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{2bd^2ex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c}-\frac{6bde^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c}-\frac{2be^3x^6\sqrt{1-c^2x^2}(a+b\arccos(cx))}{49c}-\frac{32be^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^7}-\frac{16bde^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^5}-\frac{16be^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^5}-\frac{4bd^2e\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^3}-\frac{8bde^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^3}-\frac{12be^3x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^3}+d^3x(a+b\arccos(cx))^2+d^2ex^3(a+b\arccos(cx))^2+\frac{3}{5}de^2x^5(a+b\arccos(cx))^2+\frac{1}{7}e^3x^7(a+b\arccos(cx))^2-\frac{32b^2e^3x}{245c^6}-\frac{16b^2de^2x}{25c^4}-\frac{16b^2e^3x^3}{735c^4}-\frac{4b^2d^2ex}{3c^2}-\frac{8b^2de^2x^3}{75c^2}-\frac{12b^2e^3x^5}{1225c^2}-2b^2d^3x-\frac{2}{9}b^2d^2ex^3-\frac{6}{125}b^2de^2x^5-\frac{2}{343}b^2e^3x^7$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
-2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) - (16*b^2*d*e^2*x)/(25*c^4) - (32*b
^2*e^3*x)/(245*c^6) - (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) - (
16*b^2*e^3*x^3)/(735*c^4) - (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225
*c^2) - (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
]))/c - (4*b*d^2*e*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^3) - (16*b*
d*e^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(25*c^5) - (32*b*e^3*Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 - c^2*x^2
]*(a + b*ArcCos[c*x]))/(3*c) - (8*b*d*e^2*x^2*Sqrt[1 - c^2*x^2]*(a + b*Arc
Cos[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))
/(245*c^5) - (6*b*d*e^2*x^4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(25*c)
- (12*b*e^3*x^4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(245*c^3) - (2*b*e^
3*x^6*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(49*c) + d^3*x*(a + b*ArcCos[
c*x])^2 + d^2*e*x^3*(a + b*ArcCos[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcCos[c*x
])^2)/5 + (e^3*x^7*(a + b*ArcCos[c*x])^2)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{c^4d^2e\left(9\arccos(cx)^2c^3x^3\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{c^4d^2e\left(9\arccos(cx)^2c^3x^3\right.}{}\right.}{}}{}$
parts	$a^2\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b^2\left(55125\arccos(cx)^2c^7x^7e^3+231525\arccos(cx)^2c^7x^5d\,e^2+3858\right.}{}$
orering	$\frac{x\left(47625c^8e^5x^{10}+328917c^8d\,e^4x^8+1128666c^8d^2e^3x^6+10080c^6e^5x^8+5951050c^8d^3e^2x^4+146016c^6d\,e^4x^6-385875c^8d\right.}{}$

input

```
int((e*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/c*(a^2/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b
^2/c^6*(c^6*d^3*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2))
+1/9*c^4*d^2*e*(9*arccos(c*x)^2*c^3*x^3-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c
^2*x^2-2*c^3*x^3-12*arccos(c*x)*(-c^2*x^2+1)^(1/2)-12*c*x)+1/375*c^2*d*e^2
*(225*arccos(c*x)^2*c^5*x^5-90*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^4*x^4-18*c
^5*x^5-120*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-40*c^3*x^3-240*arccos(c*
x)*(-c^2*x^2+1)^(1/2)-240*c*x)+1/25725*e^3*(3675*arccos(c*x)^2*c^7*x^7-105
0*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^6*x^6-150*c^7*x^7-1260*(-c^2*x^2+1)^(1/
2)*arccos(c*x)*c^4*x^4-252*c^5*x^5-1680*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2
*x^2-560*c^3*x^3-3360*arccos(c*x)*(-c^2*x^2+1)^(1/2)-3360*c*x))+2*a*b/c^6*
(arccos(c*x)*d^3*c^7*x+arccos(c*x)*d^2*c^7*e*x^3+3/5*arccos(c*x)*d*c^7*e^2
*x^5+1/7*arccos(c*x)*e^3*c^7*x^7+1/7*e^3*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-
6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^
2*x^2+1)^(1/2))-d^3*c^6*(-c^2*x^2+1)^(1/2)+3/5*d*c^2*e^2*(-1/5*c^4*x^4*(-c
^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+d
^2*c^4*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.98

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$= \frac{1125\,(49\,a^2-2\,b^2)c^7e^3x^7+189\,(49\,(25\,a^2-2\,b^2)c^7de^2-20\,b^2c^5e^3)x^5+35\,(1225\,(9\,a^2-2\,b^2)c^7d^2e-117}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 - 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 - 2*b^2)*c^7
*d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 - 2*b^2)*c^7*d^2*e - 1176*b
^2*c^5*d*e^2 - 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^
7*d*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*arccos(c*x)^2 + 105
*(3675*(a^2 - 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e - 2352*b^2*c^3*d*e^2 - 4
80*b^2*c*e^3)*x + 22050*(5*a*b*c^7*e^3*x^7 + 21*a*b*c^7*d*e^2*x^5 + 35*a*b
*c^7*d^2*e*x^3 + 35*a*b*c^7*d^3*x)*arccos(c*x) - 210*(75*a*b*c^6*e^3*x^6 +
 3675*a*b*c^6*d^3 + 2450*a*b*c^4*d^2*e + 1176*a*b*c^2*d*e^2 + 240*a*b*e^3
+ 9*(49*a*b*c^6*d*e^2 + 10*a*b*c^4*e^3)*x^4 + (1225*a*b*c^6*d^2*e + 588*a*
b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2 + (75*b^2*c^6*e^3*x^6 + 3675*b^2*c^6*d^
3 + 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 + 240*b^2*e^3 + 9*(49*b^2*c^6*
d*e^2 + 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e + 588*b^2*c^4*d*e^2 + 12
0*b^2*c^2*e^3)*x^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.75

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e*
*3*x**7/7 + 2*a*b*d**3*x*acos(c*x) + 2*a*b*d**2*e*x**3*acos(c*x) + 6*a*b*d
*e**2*x**5*acos(c*x)/5 + 2*a*b*e**3*x**7*acos(c*x)/7 - 2*a*b*d**3*sqrt(-c*
*2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(-c**2*x**2 + 1)/(3*c) - 6*a*b*d*e*
*2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(-c**2*x**2 + 1)
/(49*c) - 4*a*b*d**2*e*sqrt(-c**2*x**2 + 1)/(3*c**3) - 8*a*b*d*e**2*x**2*s
qrt(-c**2*x**2 + 1)/(25*c**3) - 12*a*b*e**3*x**4*sqrt(-c**2*x**2 + 1)/(245
*c**3) - 16*a*b*d*e**2*sqrt(-c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*s
qrt(-c**2*x**2 + 1)/(245*c**5) - 32*a*b*e**3*sqrt(-c**2*x**2 + 1)/(245*c**
7) + b**2*d**3*x*acos(c*x)**2 - 2*b**2*d**3*x + b**2*d**2*e*x**3*acos(c*x)
**2 - 2*b**2*d**2*e*x**3/9 + 3*b**2*d*e**2*x**5*acos(c*x)**2/5 - 6*b**2*d*
e**2*x**5/125 + b**2*e**3*x**7*acos(c*x)**2/7 - 2*b**2*e**3*x**7/343 - 2*b
**2*d**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(-c**2*
x**2 + 1)*acos(c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c
*x)/(25*c) - 2*b**2*e**3*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x)/(49*c) - 4*b*
*2*d**2*e*x/(3*c**2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(1
225*c**2) - 4*b**2*d**2*e*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c**3) - 8*b**2
*d*e**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(25*c**3) - 12*b**2*e**3*x**4*
sqrt(-c**2*x**2 + 1)*acos(c*x)/(245*c**3) - 16*b**2*d*e**2*x/(25*c**4) - 1
6*b**2*e**3*x**3/(735*c**4) - 16*b**2*d*e**2*sqrt(-c**2*x**2 + 1)*acos(...
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.23

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*e^3*x^7*arccos(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arccos
(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arccos(c*x)^2 + a^2*d^2*e*x^3
+ b^2*d^3*x*arccos(c*x)^2 + 2/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)
*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(-c^2*x^2
+ 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2
)*b^2*d^2*e + 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4
*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/3
75*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sq
rt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4
)*b^2*d*e^2 + 2/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 +
6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2
*x^2 + 1)/c^8)*c)*a*b*e^3 - 2/25725*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6
*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*
x^2 + 1)/c^8)*c*arccos(c*x) + (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 16
80*x)/c^6)*b^2*e^3 - 2*b^2*d^3*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^
2*d^3*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.45

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/7*b^2*e^3*x^7*arccos(c*x)^2 + 2/7*a*b*e^3*x^7*arccos(c*x) + 1/7*a^2*e^3*
x^7 - 2/343*b^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arccos(c*x)^2 - 2/49*sqrt(-c^2
*x^2 + 1)*b^2*e^3*x^6*arccos(c*x)/c + 6/5*a*b*d*e^2*x^5*arccos(c*x) - 2/49
*sqrt(-c^2*x^2 + 1)*a*b*e^3*x^6/c + 3/5*a^2*d*e^2*x^5 - 6/125*b^2*d*e^2*x^
5 + b^2*d^2*e*x^3*arccos(c*x)^2 - 6/25*sqrt(-c^2*x^2 + 1)*b^2*d*e^2*x^4*ar
ccos(c*x)/c + 2*a*b*d^2*e*x^3*arccos(c*x) - 6/25*sqrt(-c^2*x^2 + 1)*a*b*d*
e^2*x^4/c + a^2*d^2*e*x^3 - 2/9*b^2*d^2*e*x^3 - 12/1225*b^2*e^3*x^5/c^2 +
b^2*d^3*x*arccos(c*x)^2 - 2/3*sqrt(-c^2*x^2 + 1)*b^2*d^2*e*x^2*arccos(c*x)
/c - 12/245*sqrt(-c^2*x^2 + 1)*b^2*e^3*x^4*arccos(c*x)/c^3 + 2*a*b*d^3*x*a
rccos(c*x) - 2/3*sqrt(-c^2*x^2 + 1)*a*b*d^2*e*x^2/c - 12/245*sqrt(-c^2*x^2
 + 1)*a*b*e^3*x^4/c^3 + a^2*d^3*x - 2*b^2*d^3*x - 8/75*b^2*d*e^2*x^3/c^2 -
 2*sqrt(-c^2*x^2 + 1)*b^2*d^3*arccos(c*x)/c - 8/25*sqrt(-c^2*x^2 + 1)*b^2*
d*e^2*x^2*arccos(c*x)/c^3 - 2*sqrt(-c^2*x^2 + 1)*a*b*d^3/c - 8/25*sqrt(-c^
2*x^2 + 1)*a*b*d*e^2*x^2/c^3 - 4/3*b^2*d^2*e*x/c^2 - 16/735*b^2*e^3*x^3/c^
4 - 4/3*sqrt(-c^2*x^2 + 1)*b^2*d^2*e*arccos(c*x)/c^3 - 16/245*sqrt(-c^2*x^
2 + 1)*b^2*e^3*x^2*arccos(c*x)/c^5 - 4/3*sqrt(-c^2*x^2 + 1)*a*b*d^2*e/c^3
- 16/245*sqrt(-c^2*x^2 + 1)*a*b*e^3*x^2/c^5 - 16/25*b^2*d*e^2*x/c^4 - 16/2
5*sqrt(-c^2*x^2 + 1)*b^2*d*e^2*arccos(c*x)/c^5 - 16/25*sqrt(-c^2*x^2 + 1)*
a*b*d*e^2/c^5 - 32/245*b^2*e^3*x/c^6 - 32/245*sqrt(-c^2*x^2 + 1)*b^2*e^3*a
rccos(c*x)/c^7 - 32/245*sqrt(-c^2*x^2 + 1)*a*b*e^3/c^7
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^3 (a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,(e\,x^2+d)^3\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$= \frac{3675acos(cx)^2 b^2c^7d^3x - 7350\sqrt{-c^2x^2+1}\,acos(cx)\,b^2c^6d^3 + 7350acos(cx)\,ab\,c^7d^3x + 7350acos(cx)\,ab\,c^7}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(3675*acos(c*x)**2*b**2*c**7*d**3*x - 7350*sqrt( - c**2*x**2 + 1)*acos(c*x
)*b**2*c**6*d**3 + 7350*acos(c*x)*a*b*c**7*d**3*x + 7350*acos(c*x)*a*b*c**
7*d**2*e*x**3 + 4410*acos(c*x)*a*b*c**7*d*e**2*x**5 + 1050*acos(c*x)*a*b*c
**7*e**3*x**7 - 7350*sqrt( - c**2*x**2 + 1)*a*b*c**6*d**3 - 2450*sqrt( - c
**2*x**2 + 1)*a*b*c**6*d**2*e*x**2 - 882*sqrt( - c**2*x**2 + 1)*a*b*c**6*d
*e**2*x**4 - 150*sqrt( - c**2*x**2 + 1)*a*b*c**6*e**3*x**6 - 4900*sqrt( -
c**2*x**2 + 1)*a*b*c**4*d**2*e - 1176*sqrt( - c**2*x**2 + 1)*a*b*c**4*d*e*
*2*x**2 - 180*sqrt( - c**2*x**2 + 1)*a*b*c**4*e**3*x**4 - 2352*sqrt( - c**
2*x**2 + 1)*a*b*c**2*d*e**2 - 240*sqrt( - c**2*x**2 + 1)*a*b*c**2*e**3*x**
2 - 480*sqrt( - c**2*x**2 + 1)*a*b*e**3 + 3675*int(acos(c*x)**2*x**6,x)*b*
*2*c**7*e**3 + 11025*int(acos(c*x)**2*x**4,x)*b**2*c**7*d*e**2 + 11025*int
(acos(c*x)**2*x**2,x)*b**2*c**7*d**2*e + 3675*a**2*c**7*d**3*x + 3675*a**2
*c**7*d**2*e*x**3 + 2205*a**2*c**7*d*e**2*x**5 + 525*a**2*c**7*e**3*x**7 -
 7350*b**2*c**7*d**3*x)/(3675*c**7)
```

3.87 $\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 335

$$\begin{aligned}\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx = & -2b^2d^2x-\frac{8b^2dex}{9c^2}-\frac{16b^2e^2x}{75c^4}-\frac{4}{27}b^2dex^3-\frac{8b^2e^2x^3}{225c^2} \\ & -\frac{2}{125}b^2e^2x^5-\frac{2bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c} \\ & -\frac{8bde\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3} \\ & -\frac{16be^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^5} \\ & -\frac{4bdex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c} \\ & -\frac{8be^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^3} \\ & -\frac{2be^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c} \\ & +d^2x(a+b\arccos(cx))^2 \\ & +\frac{2}{3}dex^3(a+b\arccos(cx))^2 \\ & +\frac{1}{5}e^2x^5(a+b\arccos(cx))^2\end{aligned}$$

output
```
-2*b^2*d^2*x-8/9*b^2*d*e*x/c^2-16/75*b^2*e^2*x/c^4-4/27*b^2*d*e*x^3-8/225*
b^2*e^2*x^3/c^2-2/125*b^2*e^2*x^5-2*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c
*x))/c-8/9*b*d*e*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3-16/75*b*e^2*(-c^
2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5-4/9*b*d*e*x^2*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))/c-8/75*b*e^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3-2/
25*b*e^2*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+d^2*x*(a+b*arccos(c*x)
)^2+2/3*d*e*x^3*(a+b*arccos(c*x))^2+1/5*e^2*x^5*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.87

$$\int (d+ex^2)^2 (a+b\arccos(cx))^2 \, dx$$
$$= \frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4)-30ab\sqrt{1-c^2x^2}(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e}{}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 - c^2*x^2
]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x
^4)) - 2*b^2*c*x*(360*e^2 + 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*
d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^
4) + b*Sqrt[1 - c^2*x^2]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2
 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcCos[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e*
x^2 + 3*e^2*x^4)*ArcCos[c*x]^2)/(3375*c^5)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(d^2(a+b\arccos(cx))^2+2dex^2(a+b\arccos(cx))^2+e^2x^4(a+b\arccos(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{4bdex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}-\frac{2be^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c}-\frac{16be^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^5}-\frac{8bde\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}-\frac{8be^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^3}+d^2x(a+b\arccos(cx))^2+\frac{2}{3}dex^3(a+b\arccos(cx))^2+\frac{1}{5}e^2x^5(a+b\arccos(cx))^2-\frac{16b^2e^2x}{75c^4}-\frac{8b^2dex}{9c^2}-\frac{8b^2e^2x^3}{225c^2}-2b^2d^2x-\frac{4}{27}b^2dex^3-\frac{2}{125}b^2e^2x^5$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
-2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) - (16*b^2*e^2*x)/(75*c^4) - (4*b^2*d*
e*x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) - (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqr
t[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c - (8*b*d*e*Sqrt[1 - c^2*x^2]*(a + b*
ArcCos[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(
75*c^5) - (4*b*d*e*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) - (8*b
*e^2*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sq
rt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(25*c) + d^2*x*(a + b*ArcCos[c*x])^2
+ (2*d*e*x^3*(a + b*ArcCos[c*x])^2)/3 + (e^2*x^5*(a + b*ArcCos[c*x])^2)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{a^2\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{2c^2de\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2}\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{2c^2de\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2}\right.}{}\right.}{}}{}$
parts	$a^2\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b^2\left(675\arccos(cx)^2c^5x^5e^2+2250\arccos(cx)^2c^5x^3de+3375\arccos(cx)^2c^5x\,d^2-\right.}{}$
orering	$\frac{x\left(1647c^6e^4x^8+10924c^6d\,e^3x^6+77050c^6d^2e^2x^4+600c^4e^4x^6-4500c^6d^3e\,x^2+21808c^4d\,e^3x^4+3375c^6d^4-89000c^4d^2e^2\right.}{3375\left(e\,x^2+d\right)^2c^6}$

input

```
int((e*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/c*(a^2/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b^2/c^4*(c^4*d^2*
(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2))+2/27*c^2*d*e*(9
*arccos(c*x)^2*c^3*x^3-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-2*c^3*x^3-
12*arccos(c*x)*(-c^2*x^2+1)^(1/2)-12*c*x)+1/1125*e^2*(225*arccos(c*x)^2*c^
5*x^5-90*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^4*x^4-18*c^5*x^5-120*(-c^2*x^2+1
)^(1/2)*arccos(c*x)*c^2*x^2-40*c^3*x^3-240*arccos(c*x)*(-c^2*x^2+1)^(1/2)-
240*c*x))+2*a*b/c^4*(arccos(c*x)*d^2*c^5*x+2/3*arccos(c*x)*d*c^5*e*x^3+1/5
*arccos(c*x)*e^2*c^5*x^5+1/5*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2
*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))-d^2*c^4*(-c^2*x^2+1)^(1/2
)+2/3*d*c^2*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.04

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2 \, dx$$
$$= \frac{27\,(25\,a^2-2\,b^2)c^5e^2x^5+10\,(25\,(9\,a^2-2\,b^2)c^5de-12\,b^2c^3e^2)x^3+225\,(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 - 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 - 2*b^2)*c^5*d*e -
 12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^
2*c^5*d^2*x)*arccos(c*x)^2 + 15*(225*(a^2 - 2*b^2)*c^5*d^2 - 200*b^2*c^3*d
*e - 48*b^2*c*e^2)*x + 450*(3*a*b*c^5*e^2*x^5 + 10*a*b*c^5*d*e*x^3 + 15*a*
b*c^5*d^2*x)*arccos(c*x) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*d^2 + 100*a
*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e + 6*a*b*c^2*e^2)*x^2 + (9*b^2*
c^4*e^2*x^4 + 225*b^2*c^4*d^2 + 100*b^2*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c
^4*d*e + 6*b^2*c^2*e^2)*x^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.79

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2 \, dx$$
$$= \begin{cases} a^2d^2x+\frac{2a^2dex^3}{3}+\frac{a^2e^2x^5}{5}+2abd^2x\operatorname{acos}(cx)+\frac{4abdex^3\operatorname{acos}(cx)}{3}+\frac{2abe^2x^5\operatorname{acos}(cx)}{5}-\frac{2abd^2\sqrt{-c^2x^2+1}}{c}-\frac{4abdex^2\sqrt{}}{9c} \\ \left(a+\frac{\pi b}{2}\right)^2\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2
*x*acos(c*x) + 4*a*b*d*e*x**3*acos(c*x)/3 + 2*a*b*e**2*x**5*acos(c*x)/5 -
2*a*b*d**2*sqrt(-c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(9
*c) - 2*a*b*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 8*a*b*d*e*sqrt(-c**2*x
**2 + 1)/(9*c**3) - 8*a*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 16*a*
b*e**2*sqrt(-c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*acos(c*x)**2 - 2*b**2*
d**2*x + 2*b**2*d*e*x**3*acos(c*x)**2/3 - 4*b**2*d*e*x**3/27 + b**2*e**2*x
**5*acos(c*x)**2/5 - 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(-c**2*x**2 +
1)*acos(c*x)/c - 4*b**2*d*e*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c) - 2*
b**2*e**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**
2) - 8*b**2*e**2*x**3/(225*c**2) - 8*b**2*d*e*sqrt(-c**2*x**2 + 1)*acos(c*
x)/(9*c**3) - 8*b**2*e**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(75*c**3) -
16*b**2*e**2*x/(75*c**4) - 16*b**2*e**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(75
*c**5), Ne(c, 0)), ((a + pi*b/2)**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.31

$$\begin{aligned}
&\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx \\
&= \frac{1}{5}\,b^2e^2x^5\arccos(cx)^2 + \frac{1}{5}\,a^2e^2x^5 + \frac{2}{3}\,b^2dex^3\arccos(cx)^2 + \frac{2}{3}\,a^2dex^3 + b^2d^2x\arccos(cx)^2 \\
&\quad + \frac{4}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abde \\
&\quad - \frac{4}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos(cx) + \frac{c^2x^3+6\,x}{c^2}\right)b^2de \\
&\quad + \frac{2}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abe^2 \\
&\quad - \frac{2}{1125}\left(15\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos(cx) + \frac{9\,c^4x^5+20\,c^2x^3+}{c^4}\right. \\
&\quad - 2\,b^2d^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) \\
&\quad + a^2d^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)abd^2}{c}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*e^2*x^5*arccos(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arccos(c
*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arccos(c*x)^2 + 4/9*(3*x^3*arccos(c*x)
 - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d*e - 4/
27*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x
) + (c^2*x^3 + 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2
*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^
6)*c)*a*b*e^2 - 2/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2
 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*
c^2*x^3 + 120*x)/c^4)*b^2*e^2 - 2*b^2*d^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c
*x)/c) + a^2*d^2*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.47

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx = \frac{1}{5}\,b^2e^2x^5\arccos\left(cx\right)^2 + \frac{2}{5}\,abe^2x^5\arccos\left(cx\right) + \frac{1}{5}\,a^2e^2x^5 - \frac{2}{125}\,b^2e^2x^5 + \frac{2}{3}\,b^2dex^3\arccos\left(cx\right)^2 - \frac{2\,\sqrt{-c^2x^2+1}b^2e^2x^4\arccos\left(cx\right)}{25\,c} + \frac{4}{3}\,abdex^3\arccos\left(cx\right) - \frac{2\,\sqrt{-c^2x^2+1}abe^2x^4}{25\,c} + \frac{2}{3}\,a^2dex^3 - \frac{4}{27}\,b^2dex^3 + b^2d^2x\arccos\left(cx\right)^2 - \frac{4\,\sqrt{-c^2x^2+1}b^2dex^2\arccos\left(cx\right)}{9\,c} + 2\,abd^2x\arccos\left(cx\right) - \frac{4\,\sqrt{-c^2x^2+1}abdex^2}{9\,c} + a^2d^2x - 2\,b^2d^2x - \frac{8\,b^2e^2x^3}{225\,c^2} - \frac{2\,\sqrt{-c^2x^2+1}b^2d^2\arccos\left(cx\right)}{c} - \frac{8\,\sqrt{-c^2x^2+1}b^2e^2x^2\arccos\left(cx\right)}{75\,c^3} - \frac{2\,\sqrt{-c^2x^2+1}abd^2}{c} - \frac{8\,\sqrt{-c^2x^2+1}abe^2x^2}{75\,c^3} - \frac{8\,b^2dex}{9\,c^2} - \frac{8\,\sqrt{-c^2x^2+1}b^2de\arccos\left(cx\right)}{9\,c^3} - \frac{8\,\sqrt{-c^2x^2+1}abde}{9\,c^3} - \frac{16\,b^2e^2x}{75\,c^4} - \frac{16\,\sqrt{-c^2x^2+1}b^2e^2\arccos\left(cx\right)}{75\,c^5} - \frac{16\,\sqrt{-c^2x^2+1}abe^2}{75\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/5*b^2*e^2*x^5*arccos(c*x)^2 + 2/5*a*b*e^2*x^5*arccos(c*x) + 1/5*a^2*e^2*
x^5 - 2/125*b^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arccos(c*x)^2 - 2/25*sqrt(-c^2*x
^2 + 1)*b^2*e^2*x^4*arccos(c*x)/c + 4/3*a*b*d*e*x^3*arccos(c*x) - 2/25*sqr
t(-c^2*x^2 + 1)*a*b*e^2*x^4/c + 2/3*a^2*d*e*x^3 - 4/27*b^2*d*e*x^3 + b^2*d
^2*x*arccos(c*x)^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*d*e*x^2*arccos(c*x)/c + 2*
a*b*d^2*x*arccos(c*x) - 4/9*sqrt(-c^2*x^2 + 1)*a*b*d*e*x^2/c + a^2*d^2*x -
 2*b^2*d^2*x - 8/225*b^2*e^2*x^3/c^2 - 2*sqrt(-c^2*x^2 + 1)*b^2*d^2*arccos
(c*x)/c - 8/75*sqrt(-c^2*x^2 + 1)*b^2*e^2*x^2*arccos(c*x)/c^3 - 2*sqrt(-c^
2*x^2 + 1)*a*b*d^2/c - 8/75*sqrt(-c^2*x^2 + 1)*a*b*e^2*x^2/c^3 - 8/9*b^2*d
*e*x/c^2 - 8/9*sqrt(-c^2*x^2 + 1)*b^2*d*e*arccos(c*x)/c^3 - 8/9*sqrt(-c^2*
x^2 + 1)*a*b*d*e/c^3 - 16/75*b^2*e^2*x/c^4 - 16/75*sqrt(-c^2*x^2 + 1)*b^2*
e^2*arccos(c*x)/c^5 - 16/75*sqrt(-c^2*x^2 + 1)*a*b*e^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,(e\,x^2+d)^2\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{225 acos(cx)^2 b^2 c^5 d^2 x - 450\sqrt{-c^2x^2+1}\, acos(cx)\, b^2 c^4 d^2 + 450 acos(cx)\, ab\, c^5 d^2 x + 300 acos(cx)\, ab\, c^5 de\, x^3}{}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(225*acos(c*x)**2*b**2*c**5*d**2*x - 450*sqrt( - c**2*x**2 + 1)*acos(c*x)*
b**2*c**4*d**2 + 450*acos(c*x)*a*b*c**5*d**2*x + 300*acos(c*x)*a*b*c**5*d*
e*x**3 + 90*acos(c*x)*a*b*c**5*e**2*x**5 - 450*sqrt( - c**2*x**2 + 1)*a*b*
c**4*d**2 - 100*sqrt( - c**2*x**2 + 1)*a*b*c**4*d*e*x**2 - 18*sqrt( - c**2
*x**2 + 1)*a*b*c**4*e**2*x**4 - 200*sqrt( - c**2*x**2 + 1)*a*b*c**2*d*e -
24*sqrt( - c**2*x**2 + 1)*a*b*c**2*e**2*x**2 - 48*sqrt( - c**2*x**2 + 1)*a
*b*e**2 + 225*int(acos(c*x)**2*x**4,x)*b**2*c**5*e**2 + 450*int(acos(c*x)*
*2*x**2,x)*b**2*c**5*d*e + 225*a**2*c**5*d**2*x + 150*a**2*c**5*d*e*x**3 +
 45*a**2*c**5*e**2*x**5 - 450*b**2*c**5*d**2*x)/(225*c**5)
```

3.88 $\int (d+ex^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 156

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx = -2b^2dx-\frac{4b^2ex}{9c^2}-\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{4be\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}-\frac{2bex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}+dx(a+b\arccos(cx))^2+\frac{1}{3}ex^3(a+b\arccos(cx))^2$$

output
```
-2*b^2*d*x-4/9*b^2*e*x/c^2-2/27*b^2*e*x^3-2*b*d*(-c^2*x^2+1)^(1/2)*(a+b*ar
ccos(c*x))/c-4/9*b*e*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3-2/9*b*e*x^2*
(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+d*x*(a+b*arccos(c*x))^2+1/3*e*x^3*(
a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx$$
$$=\frac{9a^2c^3x(3d+ex^2)-6ab\sqrt{1-c^2x^2}(2e+c^2(9d+ex^2))-2b^2cx(6e+c^2(27d+ex^2))-6b(-3ac^3x(3d+e}{27c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 - c^2*x^2]*(2*e + c^2*(9*d + e*x
^2)) - 2*b^2*c*x*(6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x^2
) + b*Sqrt[1 - c^2*x^2]*(2*e + c^2*(9*d + e*x^2)))*ArcCos[c*x] + 9*b^2*c^3
*x*(3*d + e*x^2)*ArcCos[c*x]^2)/(27*c^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx$$
$$\Big\downarrow \text{5173}$$
$$\int \left(d(a+b\arccos(cx))^2+ex^2(a+b\arccos(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{2bex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}-\frac{4be\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}+dx(a+b\arccos(cx))^2+\frac{1}{3}ex^3(a+b\arccos(cx))^2-\frac{4b^2ex}{9c^2}-2b^2dx-\frac{2}{27}b^2ex^3$$

input
```
Int[(d + e*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-2*b^2*d*x - (4*b^2*e*x)/(9*c^2) - (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 - c^2*
x^2]*(a + b*ArcCos[c*x]))/c - (4*b*e*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
)/(9*c^3) - (2*b*e*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) + d*x*
(a + b*ArcCos[c*x])^2 + (e*x^3*(a + b*ArcCos[c*x])^2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.42

method	result
parts	$a^2\left(\frac{1}{3}x^3e+dx\right)+\frac{b^2\left(\frac{e\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2+1}\,\arccos(cx)c^2x^2-2c^3x^3-12\arccos(cx)\sqrt{-c^2x^2+1}-12cx\right)}{27c^2}+d\left(a\cdots\right.\right.}{c}$
derivativedivides	$\frac{\frac{a^2\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{e\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2+1}\,\arccos(cx)c^2x^2\cdots\right.}{27}\right.}{c^2}}{\cdots}$
default	$\frac{\frac{a^2\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(d\,c^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{e\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2+1}\,\arccos(cx)c^2x^2\cdots\right.}{27}\right.}{c^2}}{\cdots}$
orering	$\frac{x\left(19c^4e^3x^6+209c^4d\,e^2x^4+9c^4d^2e\,x^2+24c^2e^3x^4+27c^4d^3-232c^2d\,e^2x^2-48e^3x^2\right)(a+b\arccos(cx))^2}{27(e\,x^2+d)^2c^4}-\frac{(6c^4e^2x^6+11\cdots}{\cdots}$

input
```
int((e*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*x^3*e+d*x)+b^2/c*(1/27*e*(9*arccos(c*x)^2*c^3*x^3-6*(-c^2*x^2+1)^
(1/2)*arccos(c*x)*c^2*x^2-2*c^3*x^3-12*arccos(c*x)*(-c^2*x^2+1)^(1/2)-12*c
*x)/c^2+d*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)))+2*a*
b/c*(1/3*c*arccos(c*x)*x^3*e+arccos(c*x)*d*c*x+1/3/c^2*(e*(-1/3*c^2*x^2*(-
c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))-3*d*c^2*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \left(d+ex^2\right)(a+b\arccos(cx))^2\,dx$$
$$=\frac{(9\,a^2-2\,b^2)c^3ex^3+9\,(b^2c^3ex^3+3\,b^2c^3dx)\arccos{(cx)}^2+3\,(9\,(a^2-2\,b^2)c^3d-4\,b^2ce)x+18\,(abc^3ex^3+3}{27}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 - 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*arccos
(c*x)^2 + 3*(9*(a^2 - 2*b^2)*c^3*d - 4*b^2*c*e)*x + 18*(a*b*c^3*e*x^3 + 3*
a*b*c^3*d*x)*arccos(c*x) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d + 2*a*b*e + (b^2
*c^2*e*x^2 + 9*b^2*c^2*d + 2*b^2*e)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.82

$$\int \left(d+ex^2\right)(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{acos}{(cx)}+\frac{2abex^3\operatorname{acos}{(cx)}}{3}-\frac{2abd\sqrt{-c^2x^2+1}}{c}-\frac{2abex^2\sqrt{-c^2x^2+1}}{9c}-\frac{4abe\sqrt{-c^2x^2+1}}{9c^3}+b^2dx\,\mathrm{a} \\ \left(a+\frac{\pi b}{2}\right)^2\left(dx+\frac{ex^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*acos(c*x) + 2*a*b*e*x**3*a
cos(c*x)/3 - 2*a*b*d*sqrt(-c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(-c**2*x**2
 + 1)/(9*c) - 4*a*b*e*sqrt(-c**2*x**2 + 1)/(9*c**3) + b**2*d*x*acos(c*x)**
2 - 2*b**2*d*x + b**2*e*x**3*acos(c*x)**2/3 - 2*b**2*e*x**3/27 - 2*b**2*d*
sqrt(-c**2*x**2 + 1)*acos(c*x)/c - 2*b**2*e*x**2*sqrt(-c**2*x**2 + 1)*acos
(c*x)/(9*c) - 4*b**2*e*x/(9*c**2) - 4*b**2*e*sqrt(-c**2*x**2 + 1)*acos(c*x
)/(9*c**3), Ne(c, 0)), ((a + pi*b/2)**2*(d*x + e*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.42

$$\begin{aligned}
&\int (d+ex^2)\,(a+b\arccos(cx))^2\,dx \\
&= \frac{1}{3}\,b^2ex^3\arccos{(cx)}^2 + \frac{1}{3}\,a^2ex^3 + b^2dx\arccos{(cx)}^2 \\
&\quad + \frac{2}{9}\left(3\,x^3\arccos{(cx)} - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)abe \\
&\quad - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\arccos{(cx)} + \frac{c^2x^3+6\,x}{c^2}\right)b^2e \\
&\quad - 2\,b^2d\left(x + \frac{\sqrt{-c^2x^2+1}\arccos{(cx)}}{c}\right) \\
&\quad + a^2dx + \frac{2\,\left(cx\arccos{(cx)} - \sqrt{-c^2x^2+1}\right)abd}{c}
\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*e*x^3*arccos(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arccos(c*x)^2 + 2/9*
(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/
c^4))*a*b*e - 2/27*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)
/c^4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*b^2*e - 2*b^2*d*(x + sqrt(-c^2*x^
2 + 1)*arccos(c*x)/c) + a^2*d*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))
*a*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.49

$$\int (d+ex^2) (a+b\arccos(cx))^2 dx = \frac{1}{3} b^2ex^3 \arccos(cx)^2 + \frac{2}{3} abex^3 \arccos(cx) + \frac{1}{3} a^2ex^3 - \frac{2}{27} b^2ex^3 + b^2dx \arccos(cx)^2 - \frac{2\sqrt{-c^2x^2+1}b^2ex^2 \arccos(cx)}{9c} + 2abdx \arccos(cx) - \frac{2\sqrt{-c^2x^2+1}abex^2}{9c} + a^2dx - 2b^2dx - \frac{2\sqrt{-c^2x^2+1}b^2d \arccos(cx)}{c} - \frac{2\sqrt{-c^2x^2+1}abd}{c} - \frac{4b^2ex}{9c^2} - \frac{4\sqrt{-c^2x^2+1}b^2e \arccos(cx)}{9c^3} - \frac{4\sqrt{-c^2x^2+1}abe}{9c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/3*b^2*e*x^3*arccos(c*x)^2 + 2/3*a*b*e*x^3*arccos(c*x) + 1/3*a^2*e*x^3 -
2/27*b^2*e*x^3 + b^2*d*x*arccos(c*x)^2 - 2/9*sqrt(-c^2*x^2 + 1)*b^2*e*x^2*
arccos(c*x)/c + 2*a*b*d*x*arccos(c*x) - 2/9*sqrt(-c^2*x^2 + 1)*a*b*e*x^2/c
 + a^2*d*x - 2*b^2*d*x - 2*sqrt(-c^2*x^2 + 1)*b^2*d*arccos(c*x)/c - 2*sqrt
(-c^2*x^2 + 1)*a*b*d/c - 4/9*b^2*e*x/c^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*e*ar
ccos(c*x)/c^3 - 4/9*sqrt(-c^2*x^2 + 1)*a*b*e/c^3
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) (a+b\arccos(cx))^2 dx = \int (a+b\,\mathrm{acos}(c\,x))^2 \left(e\,x^2+d\right) dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d + ex^2) (a + b \arccos(cx))^2 \, dx$$
$$= \frac{9acos(cx)^2 b^2 c^3 dx - 18\sqrt{-c^2x^2+1}\, acos(cx)\, b^2 c^2 d + 18acos(cx)\, ab\, c^3 dx + 6acos(cx)\, ab\, c^3 e\, x^3 - 18\sqrt{-c^2}}{}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(9*acos(c*x)**2*b**2*c**3*d*x - 18*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2*c
**2*d + 18*acos(c*x)*a*b*c**3*d*x + 6*acos(c*x)*a*b*c**3*e*x**3 - 18*sqrt(
 - c**2*x**2 + 1)*a*b*c**2*d - 2*sqrt( - c**2*x**2 + 1)*a*b*c**2*e*x**2 -
4*sqrt( - c**2*x**2 + 1)*a*b*e + 9*int(acos(c*x)**2*x**2,x)*b**2*c**3*e +
9*a**2*c**3*d*x + 3*a**2*c**3*e*x**3 - 18*b**2*c**3*d*x)/(9*c**3)
```

3.89 $\int (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int (a+b\arccos(cx))^2\,dx=-2b^2x-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+x(a+b\arccos(cx))^2$$

output
```
-2*b^2*x-2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+x*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int (a+b\arccos(cx))^2\,dx=(a^2-2b^2)\,x-\frac{2ab\sqrt{1-c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1-c^2x^2}\right)\arccos(cx)}{c}+b^2x\arccos(cx)^2$$

input
```
Integrate[(a + b*ArcCos[c*x])^2,x]
```

output
```
(a^2 - 2*b^2)*x - (2*a*b*Sqrt[1 - c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 - c
^2*x^2])*ArcCos[c*x])/c + b^2*x*ArcCos[c*x]^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5131}$$

$$2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{24}$$

$$2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2$$

input
```
Int[(a + b*ArcCos[c*x])^2,x]
```

output
```
x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*Ar
cCos[c*x]))/c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{cx\,a^2+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
default	$\frac{cx\,a^2+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
parts	$x\,a^2+\frac{b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)}{c}+\frac{2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	75
orering	$x(a+b\arccos(cx))^2-\frac{2(a+b\arccos(cx))b}{c\sqrt{-c^2x^2+1}}+\frac{x(cx-1)(cx+1)\left(\frac{2b^2c^2}{-c^2x^2+1}-\frac{2(a+b\arccos(cx))b\,c^3x}{(-c^2x^2+1)^{\frac{3}{2}}}\right)}{c^2}$	103

input
```
int((a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2+b^2*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)
)+2*a*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int (a+b\arccos(cx))^2\,dx = \frac{b^2cx\arccos(cx)^2+2\,abcx\arccos(cx)+(a^2-2\,b^2)cx-2\,\sqrt{-c^2x^2+1}(b^2\arccos(cx)+ab)}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*arccos(c*x)^2 + 2*a*b*c*x*arccos(c*x) + (a^2 - 2*b^2)*c*x - 2*sqr
t(-c^2*x^2 + 1)*(b^2*arccos(c*x) + a*b))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int (a + b\arccos(cx))^2\,dx$$
$$= \begin{cases} a^2x + 2abx\operatorname{acos}(cx) - \frac{2ab\sqrt{-c^2x^2+1}}{c} + b^2x\operatorname{acos}^2(cx) - 2b^2x - \frac{2b^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{c} & \text{for } c \neq 0 \\ x\left(a + \frac{\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*acos(c*x) - 2*a*b*sqrt(-c**2*x**2 + 1)/c + b**
2*x*acos(c*x)**2 - 2*b**2*x - 2*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/c, Ne(
c, 0)), (x*(a + pi*b/2)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int (a + b\arccos(cx))^2\,dx = b^2x\arccos(cx)^2 - 2\,b^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) + a^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)ab}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arccos(c*x)^2 - 2*b^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*x
 + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx = b^2x\arccos{(cx)}^2 + 2\,abx\arccos{(cx)} + a^2x - 2\,b^2x - \frac{2\,\sqrt{-c^2x^2+1}b^2\arccos{(cx)}}{c} - \frac{2\,\sqrt{-c^2x^2+1}ab}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x - 2*b^2*x - 2*sqrt(-c^2*
x^2 + 1)*b^2*arccos(c*x)/c - 2*sqrt(-c^2*x^2 + 1)*a*b/c
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int (a+b\arccos(cx))^2\,dx = \begin{cases} x\left(a^2+\pi\,a\,b+\frac{\pi^2\,b^2}{4}\right) & \text{if } c=0 \\ a^2\,x+b^2\,x\left(\mathrm{acos}(c\,x)^2-2\right)-\frac{2\,b^2\,\mathrm{acos}(c\,x)\,\sqrt{1-c^2\,x^2}}{c}-\frac{2\,a\,b\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} & \text{if } c\neq 0 \end{cases}$$

input
```
int((a + b*acos(c*x))^2,x)
```

output
```
piecewise(c == 0, x*(a^2 + (b^2*pi^2)/4 + a*b*pi), c ~= 0, a^2*x + b^2*x*(
acos(c*x)^2 - 2) - (2*b^2*acos(c*x)*(- c^2*x^2 + 1)^(1/2))/c - (2*a*b*((-
c^2*x^2 + 1)^(1/2) - c*x*acos(c*x)))/c)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx$$
$$= \frac{acos(cx)^2 b^2cx - 2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2 + 2acos(cx)\,abcx - 2\sqrt{-c^2x^2+1}\,ab + a^2cx - 2b^2cx}{c}$$

input
```
int((a+b*acos(c*x))^2,x)
```

output
```
(acos(c*x)**2*b**2*c*x - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2 + 2*acos(
c*x)*a*b*c*x - 2*sqrt( - c**2*x**2 + 1)*a*b + a**2*c*x - 2*b**2*c*x)/c
```

3.90 $\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 821

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx = \frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\arccos(cx))^2\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\arccos(cx))^2\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\,\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\,\text{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$

output
```
1/2*(a+b*arccos(c*x))^2*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1
/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x))^2*ln(1+e^
(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)+1/2*(a+b*arccos(c*x))^2*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2)
)/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x
))^2*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(1/2)+I*b*(a+b*arccos(c*x))*polylog(2,-e^(1/2)*(c*x+I*(-
c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-I*b
*(a+b*arccos(c*x))*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1
/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+I*b*(a+b*arccos(c*x))*polylog(2
,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d
)^(1/2)/e^(1/2)-I*b*(a+b*arccos(c*x))*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1
)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(
3,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)-I*(c^2*d+e)^(1/2)))/(-
d)^(1/2)/e^(1/2)+b^2*polylog(3,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^
(1/2)-I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,-e^(1/2)*(c*x+I
*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+
b^2*polylog(3,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(c*(-d)^(1/2)+I*(c^2*d+e)
^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2), x]
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{\sqrt{-d}(a+b\arccos(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\arccos(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}-\frac{\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}+\frac{i(a+b\arccos(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}-\frac{i(a+b\arccos(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}+\frac{i(a+b\arccos(cx))\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}-\frac{i(a+b\arccos(cx))\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}+\frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))^2\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{c\sqrt{-d}-i\sqrt{dc^2+e}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))^2\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{\sqrt{-d}c+i\sqrt{dc^2+e}}+1\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[c*x])^2*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I
*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x])^2*Log[1 +
(Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]
*Sqrt[e]) + ((a + b*ArcCos[c*x])^2*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*
Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x]
)^2*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])
/(2*Sqrt[-d]*Sqrt[e]) + (I*b*(a + b*ArcCos[c*x])*PolyLog[2, -((Sqrt[e]*E^(
I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - (
I*b*(a + b*ArcCos[c*x])*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
 - I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + (I*b*(a + b*ArcCos[c*x])*Poly
Log[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/(
Sqrt[-d]*Sqrt[e]) - (I*b*(a + b*ArcCos[c*x])*PolyLog[2, (Sqrt[e]*E^(I*ArcC
os[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - (b^2*Pol
yLog[3, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/
(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] - I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, -((Sqrt[e]*
E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e])
+ (b^2*PolyLog[3, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d +
 e])])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [F]

$$\int \frac{(a+b\arccos(cx))^2}{e\,x^2+d}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\int \frac{(b\arccos(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{d+ex^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))**2/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2+2\left(\int \frac{acos(cx)}{e\,x^2+d}dx\right)abde+\left(\int \frac{acos(cx)^2}{e\,x^2+d}dx\right)b^2de}{de}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2 + 2*int(acos(c*x)/(d +
 e*x**2),x)*a*b*d*e + int(acos(c*x)**2/(d + e*x**2),x)*b**2*d*e)/(d*e)
```

3.91 $\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 388

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \frac{d^2\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc} + \frac{de\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{2bc^3} + \frac{e^2\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8bc^5} + \frac{de\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{2bc^3} + \frac{3e^2\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16bc^5} + \frac{e^2\,\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16bc^5} - \frac{d^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} - \frac{de\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3} - \frac{e^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5} - \frac{de\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3} - \frac{3e^2\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5} - \frac{e^2\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5}$$

output
```
d^2*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c+1/2*d*e*Ci((a+b*arccos(c*x))/b)*s
in(a/b)/b/c^3+1/8*e^2*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^5+1/2*d*e*Ci(3*
(a+b*arccos(c*x))/b)*sin(3*a/b)/b/c^3+3/16*e^2*Ci(3*(a+b*arccos(c*x))/b)*s
in(3*a/b)/b/c^5+1/16*e^2*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c^5-d^2*co
s(a/b)*Si((a+b*arccos(c*x))/b)/b/c-1/2*d*e*cos(a/b)*Si((a+b*arccos(c*x))/b
)/b/c^3-1/8*e^2*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c^5-1/2*d*e*cos(3*a/b)*
Si(3*(a+b*arccos(c*x))/b)/b/c^3-3/16*e^2*cos(3*a/b)*Si(3*(a+b*arccos(c*x))
/b)/b/c^5-1/16*e^2*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = -\frac{-2(8c^4d^2+4c^2de+e^2)\,\mathrm{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-e(8c^2d+3e)\,\mathrm{CosIntegral}\left(3\left(\frac{a}{b}+\mathrm{arcco}\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcCos[c*x]),x]
```

output
```
-1/16*(-2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*CosIntegral[a/b + ArcCos[c*x]]*Sin
[a/b] - e*(8*c^2*d + 3*e)*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b]
- e^2*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*a)/b] + 16*c^4*d^2*Cos[a/b
]*SinIntegral[a/b + ArcCos[c*x]] + 8*c^2*d*e*Cos[a/b]*SinIntegral[a/b + Ar
cCos[c*x]] + 2*e^2*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 8*c^2*d*e*Cos
[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + 3*e^2*Cos[(3*a)/b]*SinInteg
ral[3*(a/b + ArcCos[c*x])] + e^2*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[
c*x])])/(b*c^5)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d^2}{a+b\arccos(cx)}+\frac{2dex^2}{a+b\arccos(cx)}+\frac{e^2x^4}{a+b\arccos(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5}+\frac{3e^2 \sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5}+\frac{e^2 \sin\left(\frac{5a}{b}\right) \text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5}-\frac{e^2 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5}-\frac{3e^2 \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5}-\frac{e^2 \cos\left(\frac{5a}{b}\right) \text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5}+\frac{de \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3}+\frac{de \sin\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3}-\frac{de \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3}-\frac{de \cos\left(\frac{3a}{b}\right) \text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3}+\frac{d^2 \sin\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}-\frac{d^2 \cos\left(\frac{a}{b}\right) \text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) + (d*e*CosIntegral
[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(2*b*c^3) + (e^2*CosIntegral[(a + b*ArcC
os[c*x])/b]*Sin[a/b])/(8*b*c^5) + (d*e*CosIntegral[(3*(a + b*ArcCos[c*x]))
/b]*Sin[(3*a)/b])/(2*b*c^3) + (3*e^2*CosIntegral[(3*(a + b*ArcCos[c*x]))/b
]*Sin[(3*a)/b])/(16*b*c^5) + (e^2*CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*S
in[(5*a)/b])/(16*b*c^5) - (d^2*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b]
)/(b*c) - (d*e*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(2*b*c^3) - (e
^2*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(8*b*c^5) - (d*e*Cos[(3*a)
/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(2*b*c^3) - (3*e^2*Cos[(3*a)/b
]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(16*b*c^5) - (e^2*Cos[(5*a)/b]*S
inIntegral[(5*(a + b*ArcCos[c*x]))/b])/(16*b*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{16\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^4d^2-16\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)c^4d^2+8\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^2de-8\,\mathrm{Ci}(\mathrm{arcc}}{}$
default	$-\frac{16\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^4d^2-16\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)c^4d^2+8\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^2de-8\,\mathrm{Ci}(\mathrm{arcc}}{}$

input
```
int((e*x^2+d)^2/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^5*(16*Si(arccos(c*x)+a/b)*cos(a/b)*c^4*d^2-16*Ci(arccos(c*x)+a/b)*
sin(a/b)*c^4*d^2+8*Si(arccos(c*x)+a/b)*cos(a/b)*c^2*d*e-8*Ci(arccos(c*x)+a
/b)*sin(a/b)*c^2*d*e+8*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*c^2*d*e-8*Ci(3*a
rccos(c*x)+3*a/b)*sin(3*a/b)*c^2*d*e+2*Si(arccos(c*x)+a/b)*cos(a/b)*e^2-2*
Ci(arccos(c*x)+a/b)*sin(a/b)*e^2+3*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*e^2-
3*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*e^2+Si(5*arccos(c*x)+5*a/b)*cos(5*a/b
)*e^2-Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*e^2)/b
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(d+ex^2)^2}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*acos(c*x)),x)
```

output
```
Integral((d + e*x**2)**2/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
e^2*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c^5) + 2*d*
e*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + d^2*co
s_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - e^2*cos(a/b)^5*sin_integral
(5*a/b + 5*arccos(c*x))/(b*c^5) - 2*d*e*cos(a/b)^3*sin_integral(3*a/b + 3*
arccos(c*x))/(b*c^3) - d^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c)
- 3/4*e^2*cos(a/b)^2*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c^5)
- 1/2*d*e*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + 3/4*e^2*c
os(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^5) + 1/2*d*e*c
os_integral(a/b + arccos(c*x))*sin(a/b)/(b*c^3) + 5/4*e^2*cos(a/b)^3*sin_i
ntegral(5*a/b + 5*arccos(c*x))/(b*c^5) + 3/2*d*e*cos(a/b)*sin_integral(3*a
/b + 3*arccos(c*x))/(b*c^3) - 3/4*e^2*cos(a/b)^3*sin_integral(3*a/b + 3*ar
ccos(c*x))/(b*c^5) - 1/2*d*e*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c
^3) + 1/16*e^2*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c^5) - 3/16
*e^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^5) + 1/8*e^2*cos_in
tegral(a/b + arccos(c*x))*sin(a/b)/(b*c^5) - 5/16*e^2*cos(a/b)*sin_integra
l(5*a/b + 5*arccos(c*x))/(b*c^5) + 9/16*e^2*cos(a/b)*sin_integral(3*a/b +
3*arccos(c*x))/(b*c^5) - 1/8*e^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/
(b*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*acos(c*x)),x)
```

output
```
int((d + e*x^2)^2/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \left(\int \frac{x^4}{acos\left(cx\right)b+a}dx\right)e^2 + 2\left(\int \frac{x^2}{acos\left(cx\right)b+a}dx\right)de + \left(\int \frac{1}{acos\left(cx\right)b+a}dx\right)d^2$$

input
```
int((e*x^2+d)^2/(a+b*acos(c*x)),x)
```

output
```
int(x**4/(acos(c*x)*b + a),x)*e**2 + 2*int(x**2/(acos(c*x)*b + a),x)*d*e +
 int(1/(acos(c*x)*b + a),x)*d**2
```

3.92 $\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 180

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \frac{d\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc} + \frac{e\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{e\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc^3} - \frac{d\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} - \frac{e\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{e\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}$$

output
```
d*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c+1/4*e*Ci((a+b*arccos(c*x))/b)*sin(a
/b)/b/c^3+1/4*e*Ci(3*(a+b*arccos(c*x))/b)*sin(3*a/b)/b/c^3-d*cos(a/b)*Si((
a+b*arccos(c*x))/b)/b/c-1/4*e*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c^3-1/4*e
*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \frac{(4c^2d+e)\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+e\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)-4c^2d\cos}{4bc^3}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcCos[c*x]),x]
```

output
```
((4*c^2*d + e)*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + e*CosIntegral[3*(
a/b + ArcCos[c*x])]*Sin[(3*a)/b] - 4*c^2*d*Cos[a/b]*SinIntegral[a/b + ArcC
os[c*x]] - e*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] - e*Cos[(3*a)/b]*SinI
ntegral[3*(a/b + ArcCos[c*x])])/(4*b*c^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d}{a+b\arccos(cx)}+\frac{ex^2}{a+b\arccos(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3}+\frac{e\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}-\frac{e\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3}-\frac{e\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}+\frac{d\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}-\frac{d\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)/(a + b*ArcCos[c*x]),x]
```

output
```
(d*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) + (e*CosIntegral[(a
+ b*ArcCos[c*x])/b]*Sin[a/b])/(4*b*c^3) + (e*CosIntegral[(3*(a + b*ArcCos[
c*x]))/b]*Sin[(3*a)/b])/(4*b*c^3) - (d*Cos[a/b]*SinIntegral[(a + b*ArcCos[
c*x])/b])/(b*c) - (e*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(4*b*c^3
) - (e*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(4*b*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{d\left(\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\right)}{b}-\frac{e\left(\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)\right)}{4c^2b}}{c}$
default	$\frac{-\frac{d\left(\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\right)}{b}-\frac{e\left(\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)\right)}{4c^2b}}{c}$

input
```
int((e*x^2+d)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*(Si(arccos(c*x)+a/b)*cos(a/b)-Ci(arccos(c*x)+a/b)*sin(a/b))/b-1/4*
e/c^2*(Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)-Ci(3*arccos(c*x)+3*a/b)*sin(3*a/
b))/b-1/4*e/c^2*(Si(arccos(c*x)+a/b)*cos(a/b)-Ci(arccos(c*x)+a/b)*sin(a/b)
)/b)
```

Fricas [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \int \frac{ex^2+d}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \int \frac{d+ex^2}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*acos(c*x)),x)
```

output
```
Integral((d + e*x**2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \int \frac{ex^2+d}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = {} & \frac{e\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc^3} \\ & + \frac{d\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc} \\ & - \frac{e\cos\left(\frac{a}{b}\right)^3\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc^3} \\ & - \frac{d\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc} \\ & - \frac{e\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ & + \frac{e\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ & + \frac{3e\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^3} \\ & - \frac{e\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc^3}\end{aligned}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
e*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + d*cos_
integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - e*cos(a/b)^3*sin_integral(3*a
/b + 3*arccos(c*x))/(b*c^3) - d*cos(a/b)*sin_integral(a/b + arccos(c*x))/(
b*c) - 1/4*e*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + 1/4*e*
cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c^3) + 3/4*e*cos(a/b)*sin_inte
gral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*e*cos(a/b)*sin_integral(a/b + ar
ccos(c*x))/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx=\int \frac{e\,x^2+d}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((d + e*x^2)/(a + b*acos(c*x)),x)
```

output
```
int((d + e*x^2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx=\left(\int \frac{x^2}{acos\,(cx)\,b+a}dx\right)e+\left(\int \frac{1}{acos\,(cx)\,b+a}dx\right)d$$

input
```
int((e*x^2+d)/(a+b*acos(c*x)),x)
```

output
```
int(x**2/(acos(c*x)*b + a),x)*e + int(1/(acos(c*x)*b + a),x)*d
```

3.93 $\int \frac{1}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

output
```
Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c-cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\arccos(cx)}\,dx = -\frac{-\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-1),x]
```

output
```
-((-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b +
 ArcCos[c*x]])/(b*c))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5135, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\arccos(cx)}dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcCos[c*x])^(-1),x]
```

output
```
-((-(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) + Cos[a/b]*SinIntegral[(
a + b*ArcCos[c*x])/b])/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49
default	$\frac{-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49

input
```
int(1/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-Si(arccos(c*x)+a/b)*cos(a/b)/b+Ci(arccos(c*x)+a/b)*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate(1/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{b\arccos(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\frac{\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - cos(a/b)*sin_integral(a/b
 + arccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\arccos(cx)}\,dx = \int \frac{1}{a + b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int(1/(a + b*acos(c*x)),x)
```

output
```
int(1/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{1}{a + b\arccos(cx)}\,dx = \int \frac{1}{acos\,(cx)\,b + a}dx$$

input
```
int(1/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b + a),x)
```

3.94 $\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\arccos\,(cx))}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx=\int \frac{1}{(ex^2+d)(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}(cx))(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx=\int \frac{1}{(ex^2+d)(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{acos\,(cx)\,bd + acos\,(cx)\,be\,x^2 + ad + ae\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*d + acos(c*x)*b*e*x**2 + a*d + a*e*x**2),x)
```

3.95 $\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx = \mathrm{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 4.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\arccos(cx))}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} \, dx = \int \frac{1}{(ex^2+d)^2 (b\arccos(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b
*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 96.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} \, dx = \int \frac{1}{(a+b\operatorname{acos}(cx))(d+ex^2)^2} \, dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} \, dx = \int \frac{1}{(ex^2+d)^2 (b\arccos(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 6.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^2(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx$$
$$= \int \frac{1}{acos(cx)\,b\,d^2 + 2acos(cx)\,bde\,x^2 + acos(cx)\,b\,e^2x^4 + a\,d^2 + 2ade\,x^2 + a\,e^2x^4}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*d**2 + 2*acos(c*x)*b*d*e*x**2 + acos(c*x)*b*e**2*x**4 +
 a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4),x)
```

3.96 $\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 497

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \frac{d^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} + \frac{2dex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} + \frac{e^2x^4\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{d^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{de\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3} - \frac{e^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5} - \frac{3de\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3} - \frac{9e^2\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5} - \frac{5e^2\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5} - \frac{d^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{de\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3} - \frac{e^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5} - \frac{3de\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3} - \frac{9e^2\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5} - \frac{5e^2\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5}$$

output
```
d^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))+2*d*e*x^2*(-c^2*x^2+1)^(1/2)/
b/c/(a+b*arccos(c*x))+e^2*x^4*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-d^2
*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2/c-1/2*d*e*cos(a/b)*Ci((a+b*arccos(c*
x))/b)/b^2/c^3-1/8*e^2*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2/c^5-3/2*d*e*co
s(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^3-9/16*e^2*cos(3*a/b)*Ci(3*(a+b*a
rccos(c*x))/b)/b^2/c^5-5/16*e^2*cos(5*a/b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c
^5-d^2*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c-1/2*d*e*sin(a/b)*Si((a+b*arc
cos(c*x))/b)/b^2/c^3-1/8*e^2*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c^5-3/2*
d*e*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^3-9/16*e^2*sin(3*a/b)*Si(3*
(a+b*arccos(c*x))/b)/b^2/c^5-5/16*e^2*sin(5*a/b)*Si(5*(a+b*arccos(c*x))/b)
/b^2/c^5
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx =$$

$$-\frac{-\frac{16bc^4d^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}-\frac{32bc^4dex^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}-\frac{16bc^4e^2x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}+2(8c^4d^2+4c^2de+e^2)\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/16*((-16*b*c^4*d^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (32*b*c^4*d
*e*x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (16*b*c^4*e^2*x^4*Sqrt[1 -
 c^2*x^2])/(a + b*ArcCos[c*x]) + 2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*Cos[a/b]*
CosIntegral[a/b + ArcCos[c*x]] + 3*e*(8*c^2*d + 3*e)*Cos[(3*a)/b]*CosInteg
ral[3*(a/b + ArcCos[c*x])] + 5*e^2*Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCo
s[c*x])] + 16*c^4*d^2*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 8*c^2*d*e*
Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 2*e^2*Sin[a/b]*SinIntegral[a/b +
 ArcCos[c*x]] + 24*c^2*d*e*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])]
 + 9*e^2*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + 5*e^2*Sin[(5*a)
/b]*SinIntegral[5*(a/b + ArcCos[c*x])])/(b^2*c^5)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d^2}{(a+b\arccos(cx))^2}+\frac{2dex^2}{(a+b\arccos(cx))^2}+\frac{e^2x^4}{(a+b\arccos(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{e^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{de\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3}-\frac{3de\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3}-\frac{de\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3}-\frac{d^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{d^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}+\frac{d^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{2dex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{e^2x^4\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (2*d*e*x^2*Sqrt[1 - c^
2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (e^2*x^4*Sqrt[1 - c^2*x^2])/(b*c*(a +
b*ArcCos[c*x])) - (d^2*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c
) - (d*e*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(2*b^2*c^3) - (e^2*C
os[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(8*b^2*c^5) - (3*d*e*Cos[(3*a)
/b]*CosIntegral[(3*(a + b*ArcCos[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cos[(3*a)
/b]*CosIntegral[(3*(a + b*ArcCos[c*x]))/b])/(16*b^2*c^5) - (5*e^2*Cos[(5*a
)/b]*CosIntegral[(5*(a + b*ArcCos[c*x]))/b])/(16*b^2*c^5) - (d^2*Sin[a/b]*
SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c) - (d*e*Sin[a/b]*SinIntegral[(a
 + b*ArcCos[c*x])/b])/(2*b^2*c^3) - (e^2*Sin[a/b]*SinIntegral[(a + b*ArcCo
s[c*x])/b])/(8*b^2*c^5) - (3*d*e*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos
[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos
[c*x]))/b])/(16*b^2*c^5) - (5*e^2*Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcCo
s[c*x]))/b])/(16*b^2*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.60

method	result	size
derivativedivides	Expression too large to display	796
default	Expression too large to display	796

input
```
int((e*x^2+d)^2/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^5*(-2*(-c^2*x^2+1)^(1/2)*b*e^2-8*sin(3*arccos(c*x))*b*c^2*d*e+9*ar
ccos(c*x)*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b*e^2+9*arccos(c*x)*Ci(3*arcc
os(c*x)+3*a/b)*cos(3*a/b)*b*e^2+2*arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)
*b*e^2+2*arccos(c*x)*Ci(arccos(c*x)+a/b)*cos(a/b)*b*e^2+5*arccos(c*x)*Si(5
*arccos(c*x)+5*a/b)*sin(5*a/b)*b*e^2+5*arccos(c*x)*Ci(5*arccos(c*x)+5*a/b)
*cos(5*a/b)*b*e^2+16*Si(arccos(c*x)+a/b)*sin(a/b)*a*c^4*d^2+16*Ci(arccos(c
*x)+a/b)*cos(a/b)*a*c^4*d^2-8*(-c^2*x^2+1)^(1/2)*b*c^2*d*e+16*arccos(c*x)*
Si(arccos(c*x)+a/b)*sin(a/b)*b*c^4*d^2+16*arccos(c*x)*Ci(arccos(c*x)+a/b)*
cos(a/b)*b*c^4*d^2+24*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a*c^2*d*e+24*Ci(3
*arccos(c*x)+3*a/b)*cos(3*a/b)*a*c^2*d*e+8*Si(arccos(c*x)+a/b)*sin(a/b)*a*
c^2*d*e+8*Ci(arccos(c*x)+a/b)*cos(a/b)*a*c^2*d*e+24*arccos(c*x)*Si(3*arcco
s(c*x)+3*a/b)*sin(3*a/b)*b*c^2*d*e+24*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*
cos(3*a/b)*b*c^2*d*e+8*arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b*c^2*d*e+
8*arccos(c*x)*Ci(arccos(c*x)+a/b)*cos(a/b)*b*c^2*d*e-sin(5*arccos(c*x))*b*
e^2-3*sin(3*arccos(c*x))*b*e^2-16*(-c^2*x^2+1)^(1/2)*b*c^4*d^2+9*Si(3*arcc
os(c*x)+3*a/b)*sin(3*a/b)*a*e^2+9*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a*e^2
+2*Si(arccos(c*x)+a/b)*sin(a/b)*a*e^2+2*Ci(arccos(c*x)+a/b)*cos(a/b)*a*e^2
+5*Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)*a*e^2+5*Ci(5*arccos(c*x)+5*a/b)*cos(
5*a/b)*a*e^2)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x
) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(d+ex^2)^2}{(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral((d + e*x**2)**2/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*c*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate((5*c^2*e^2*x^5 + 2*
(3*c^2*d*e - 2*e^2)*x^3 + (c^2*d^2 - 4*d*e)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1
)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2213 vs. $2(471) = 942$.

Time = 0.24 (sec) , antiderivative size = 2213, normalized size of antiderivative = 4.45

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
sqrt(-c^2*x^2 + 1)*b*c^4*e^2*x^4/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 5*b*e
^2*arccos(c*x)*cos(a/b)^5*cos_integral(5*a/b + 5*arccos(c*x))/(b^3*c^5*arc
cos(c*x) + a*b^2*c^5) - 6*b*c^2*d*e*arccos(c*x)*cos(a/b)^3*cos_integral(3*
a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - b*c^4*d^2*arccos(
c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2
*c^5) - 5*b*e^2*arccos(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arc
cos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 6*b*c^2*d*e*arccos(c*x)*cos(
a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) +
 a*b^2*c^5) - b*c^4*d^2*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x
))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 2*sqrt(-c^2*x^2 + 1)*b*c^4*d*e*x^2/
(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 5*a*e^2*cos(a/b)^5*cos_integral(5*a/b
+ 5*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 6*a*c^2*d*e*cos(a/b)^
3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) -
a*c^4*d^2*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c^5*arccos(c*x) +
a*b^2*c^5) - 5*a*e^2*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arccos(c*x
))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 6*a*c^2*d*e*cos(a/b)^2*sin(a/b)*sin
_integral(3*a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - a*c^4
*d^2*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2
*c^5) + 25/4*b*e^2*arccos(c*x)*cos(a/b)^3*cos_integral(5*a/b + 5*arccos(c*
x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 9/2*b*c^2*d*e*arccos(c*x)*cos(a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{(e\,x^2+d)^2}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*acos(c*x))^2,x)
```

output
```
int((d + e*x^2)^2/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{x^4}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)e^2 + 2\left(\int \frac{x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)de + \left(\int \frac{1}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)d^2$$

input
```
int((e*x^2+d)^2/(a+b*acos(c*x))^2,x)
```

output
```
int(x**4/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*e**2 + 2*int(x**2
/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*d*e + int(1/(acos(c*x)**2
*b**2 + 2*acos(c*x)*a*b + a**2),x)*d**2
```

3.97 $\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 248

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \frac{d\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{ex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{d\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{e\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3e\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3} - \frac{d\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{e\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3e\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}$$

output
```
d*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))+e*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a
+b*arccos(c*x))-d*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2/c-1/4*e*cos(a/b)*Ci
((a+b*arccos(c*x))/b)/b^2/c^3-3/4*e*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b
^2/c^3-d*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c-1/4*e*sin(a/b)*Si((a+b*arc
cos(c*x))/b)/b^2/c^3-3/4*e*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.77

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = -\frac{-\frac{4bc^2d\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \frac{4bc^2ex^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)} + (4c^2d+e)\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) + 3e\cos\left(\frac{3a}{b}\right)\text{CosInte}}{}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/4*((-4*b*c^2*d*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (4*b*c^2*e*x^2*
Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) + (4*c^2*d + e)*Cos[a/b]*CosIntegra
l[a/b + ArcCos[c*x]] + 3*e*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*x])]
 + 4*c^2*d*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + e*Sin[a/b]*SinIntegra
l[a/b + ArcCos[c*x]] + 3*e*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])]
)/(b^2*c^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d}{(a+b\arccos(cx))^2}+\frac{ex^2}{(a+b\arccos(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}-\frac{3e\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}-\frac{e\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}-\frac{d\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{d\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}+\frac{d\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{ex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

input
```
Int[(d + e*x^2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (e*x^2*Sqrt[1 - c^2*x^2]
)/(b*c*(a + b*ArcCos[c*x])) - (d*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/
b])/(b^2*c) - (e*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(4*b^2*c^3)
- (3*e*Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcCos[c*x]))/b])/(4*b^2*c^3) -
(d*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c) - (e*Sin[a/b]*SinI
ntegral[(a + b*ArcCos[c*x])/b])/(4*b^2*c^3) - (3*e*Sin[(3*a)/b]*SinIntegra
l[(3*(a + b*ArcCos[c*x]))/b])/(4*b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{\frac{d\left(\arccos(cx)\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\arccos(cx)\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b+\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)a+\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right.}{(a+b\arccos(cx))b^2}}{}$
default	$-\frac{\frac{d\left(\arccos(cx)\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\arccos(cx)\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b+\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)a+\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right.}{(a+b\arccos(cx))b^2}}{}$

input
```
int((e*x^2+d)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*(arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b+arccos(c*x)*Ci(arccos(
c*x)+a/b)*cos(a/b)*b+Si(arccos(c*x)+a/b)*sin(a/b)*a+Ci(arccos(c*x)+a/b)*co
s(a/b)*a-(-c^2*x^2+1)^(1/2)*b)/(a+b*arccos(c*x))/b^2-1/4*e/c^2*(3*arccos(c
*x)*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+3*arccos(c*x)*Ci(3*arccos(c*x)+3*
a/b)*cos(3*a/b)*b+3*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a+3*Ci(3*arccos(c*x
)+3*a/b)*cos(3*a/b)*a-sin(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^2-1/4*e/c^
2*(arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b+arccos(c*x)*Ci(arccos(c*x)+a
/b)*cos(a/b)*b+Si(arccos(c*x)+a/b)*sin(a/b)*a+Ci(arccos(c*x)+a/b)*cos(a/b)
*a-(-c^2*x^2+1)^(1/2)*b)/(a+b*arccos(c*x))/b^2)
```

Fricas [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{ex^2+d}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{d+ex^2}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((d + e*x**2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{ex^2+d}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*c*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x) + a*b*c)*integrate((3*c^2*e*x^3 + (c^2*d - 2*e)*x)*sqr
t(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(236) = 472$.

Time = 0.20 (sec) , antiderivative size = 860, normalized size of antiderivative = 3.47

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-3*b*e*arccos(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3
*arccos(c*x) + a*b^2*c^3) - b*c^2*d*arccos(c*x)*cos(a/b)*cos_integral(a/b
+ arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*b*e*arccos(c*x)*cos(a
/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) +
a*b^2*c^3) - b*c^2*d*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/
(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2 + 1)*b*c^2*e*x^2/(b^3*c^
3*arccos(c*x) + a*b^2*c^3) - 3*a*e*cos(a/b)^3*cos_integral(3*a/b + 3*arcco
s(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - a*c^2*d*cos(a/b)*cos_integral(
a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*e*cos(a/b)^2*si
n(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^
3) - a*c^2*d*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x)
 + a*b^2*c^3) + 9/4*b*e*arccos(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arccos
(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*b*e*arccos(c*x)*cos(a/b)*co
s_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*b*e*
arccos(c*x)*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c
*x) + a*b^2*c^3) - 1/4*b*e*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(
c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2 + 1)*b*c^2*d/(b^3*
c^3*arccos(c*x) + a*b^2*c^3) + 9/4*a*e*cos(a/b)*cos_integral(3*a/b + 3*arc
cos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*e*cos(a/b)*cos_integra
l(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*a*e*sin(a/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{e\,x^2+d}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((d + e*x^2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d + e*x^2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{x^2}{acos(cx)^2 b^2+2acos(cx)ab+a^2}dx\right)e + \left(\int \frac{1}{acos(cx)^2 b^2+2acos(cx)ab+a^2}dx\right)d$$

input
```
int((e*x^2+d)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**2/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*e + int(1/(acos(c
*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*d
```

3.98 $\int \frac{1}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

output
```
(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-cos(a/b)*Ci((a+b*arccos(c*x))/b)/
b^2/c-sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{b\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) - \sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-2),x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[a/b]*CosIntegral[a/b + Ar
cCos[c*x]] - Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5133, 5225, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx}{b}+\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{3784}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-$$
$$\frac{\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3780

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

$\Big\downarrow$ 3783

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\mathrm{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

input
```
Int[(a + b*ArcCos[c*x])^(-2),x]
```

output
```
Sqrt[1 - c^2*x^2]/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(a + b
*ArcCos[c*x])/b] + Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74
default	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74

input
```
int(1/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))/b-(Si(arccos(c*x)+a/b)*sin(a/b)+
Ci(arccos(c*x)+a/b)*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**2,x)
```

output
```
Integral((a + b*acos(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^2)*integrate
(sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x +
 1))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.24

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = -\frac{b\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} - \frac{b\arccos(cx)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} - \frac{a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} - \frac{a\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} + \frac{\sqrt{-c^2x^2+1}\,b}{b^3c\arccos(cx)+ab^2c}$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*arccos(c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*arccos(c*x)
 + a*b^2*c) - b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*
c*arccos(c*x) + a*b^2*c) - a*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3
*c*arccos(c*x) + a*b^2*c) - a*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^
3*c*arccos(c*x) + a*b^2*c) + sqrt(-c^2*x^2 + 1)*b/(b^3*c*arccos(c*x) + a*b
^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acos(c*x))^2,x)
```

output
```
int(1/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{acos\left(cx\right)^2 b^2 + 2acos\left(cx\right)ab + a^2}dx$$

input
```
int(1/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.99 $\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 34.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \ 5175$$

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\arccos\,(cx))^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccos(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 39.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx))^2\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 319, normalized size of antiderivative = 15.95

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e*x^2 + a*b*c*d + (b^2*c*e*x^2 + b^2*c*d)*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x))*integrate((c^2*e*x^3 - (c^2*d + 2*e)*x)*sqrt(c*x + 1)*
sqrt(-c*x + 1)/(a*b*c^3*e^2*x^6 - a*b*c*d^2 + (2*a*b*c^3*d*e - a*b*c*e^2)*
x^4 + (a*b*c^3*d^2 - 2*a*b*c*d*e)*x^2 + (b^2*c^3*e^2*x^6 - b^2*c*d^2 + (2*
b^2*c^3*d*e - b^2*c*e^2)*x^4 + (b^2*c^3*d^2 - 2*b^2*c*d*e)*x^2)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b
*c*e*x^2 + a*b*c*d + (b^2*c*e*x^2 + b^2*c*d)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}\,dx$$
$$=\int \frac{1}{acos\,(cx)^2\,b^2d+acos\,(cx)^2\,b^2e\,x^2+2acos\,(cx)\,abd+2acos\,(cx)\,abe\,x^2+a^2d+a^2e\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*d + acos(c*x)**2*b**2*e*x**2 + 2*acos(c*x)*a*b*d
+ 2*acos(c*x)*a*b*e*x**2 + a**2*d + a**2*e*x**2),x)
```

3.100 $\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 71.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5175}$$

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 \left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2} dx = \int \frac{1}{(ex^2+d)^2(b\arccos{(cx)}+a)^2} dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d
*e*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2
)*arccos(c*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2} dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 438, normalized size of antiderivative = 21.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2} dx = \int \frac{1}{(ex^2+d)^2(b\arccos{(cx)}+a)^2} dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e^2*x^4 + 2*a*b*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 + 2*b^2*c*d
*e*x^2 + b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(
(3*c^2*e*x^3 - (c^2*d + 4*e)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*e^3*
x^8 + (3*a*b*c^3*d*e^2 - a*b*c*e^3)*x^6 - a*b*c*d^3 + 3*(a*b*c^3*d^2*e - a
*b*c*d*e^2)*x^4 + (a*b*c^3*d^3 - 3*a*b*c*d^2*e)*x^2 + (b^2*c^3*e^3*x^8 + (
3*b^2*c^3*d*e^2 - b^2*c*e^3)*x^6 - b^2*c*d^3 + 3*(b^2*c^3*d^2*e - b^2*c*d*
e^2)*x^4 + (b^2*c^3*d^3 - 3*b^2*c*d^2*e)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-
c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b*c*e^2*x^4 + 2*a*b
*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 + 2*b^2*c*d*e*x^2 + b^2*c*d^2)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.75

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx$$
$$= \int \frac{1}{acos\,(cx)^2\,b^2d^2 + 2acos\,(cx)^2\,b^2de\,x^2 + acos\,(cx)^2\,b^2e^2x^4 + 2acos\,(cx)\,ab\,d^2 + 4acos\,(cx)\,abde\,x^2 + 2ac}$$

input
```
int(1/(e*x^2+d)^2/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*d**2 + 2*acos(c*x)**2*b**2*d*e*x**2 + acos(c*x)**
2*b**2*e**2*x**4 + 2*acos(c*x)*a*b*d**2 + 4*acos(c*x)*a*b*d*e*x**2 + 2*aco
s(c*x)*a*b*e**2*x**4 + a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4),x)
```

3.101 $\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \mathrm{Int}\left(\sqrt{d+ex^2}(a+b\arccos(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \sqrt{e\,x^2+d}\,(a+b\arccos\,(cx))\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \int \sqrt{ex^2+d}(b\arccos(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 9.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \int (a+b\operatorname{acos}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral((a + b*acos(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \int \sqrt{ex^2+d}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int\sqrt{e\,x^2+d}\,acos(cx)\,dx\right)be}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int(sqrt(d + e*x**2)*acos(c*x),x)*b*e)/(2*e)
```

3.102 $\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \operatorname{Int}\left(\frac{a+b\arccos(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arccos\left(cx\right)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\arccos\left(cx\right)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acos}\left(cx\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int \frac{acos(cx)}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a + int(acos(c*x)/sqr
t(d + e*x**2),x)*b*e)/e
```

3.103 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}} - \frac{b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

output
```
x*(a+b*arccos(c*x))/d/(e*x^2+d)^(1/2)-b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/
c/(e*x^2+d)^(1/2))/d/e^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x\left(bcx\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)+2(a+b\arccos(cx))\right)}{2d\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)] + 2*(a + b*ArcCos[c*x])))/(2*d*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5171, 27, 353, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 5171$$

$$bc\int \frac{x}{d\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 353$$

$$\frac{bc\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2d}+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 66$$

$$\frac{bc\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{d}+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 218$$

$$\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}-\frac{b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTan[(Sqrt[e]*Sqrt[1 -
c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [F]

$$\int \frac{a + b \arccos \left(c x \right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(61) = 122$.

Time = 0.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.14

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bex^2+bd)\sqrt{-e}\log\left(8c^4e^2x^4+c^4d^2-6c^2de+8(c^4de-c^2e^2)x^2-4(2c^3ex^2\right.}{4(de^2}\right.$$

$$\left.-\frac{(bex^2+bd)\sqrt{e}\arctan\left(\frac{(2c^2ex^2+c^2d-e)\sqrt{-c^2x^2+1}\sqrt{ex^2+d}\sqrt{e}}{2(c^3e^2x^4-cde+(c^3de-ce^2)x^2)}\right)-2(bex\arccos(cx)+aex)\sqrt{ex^2+d}}{2(de^2x^2+d^2e)}\right]$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*e*x^2 + b*d)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e +
8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(-c^2*x^2 +
1)*sqrt(e*x^2 + d)*sqrt(-e) + e^2) - 4*(b*e*x*arccos(c*x) + a*e*x)*sqrt(e*
x^2 + d))/(d*e^2*x^2 + d^2*e), -1/2*((b*e*x^2 + b*d)*sqrt(e)*arctan(1/2*(2
*c^2*e*x^2 + c^2*d - e)*sqrt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e)/(c^3*e^
2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*e*x*arccos(c*x) + a*e*x)*sq
rt(e*x^2 + d))/(d*e^2*x^2 + d^2*e)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c^2*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\,ad+\sqrt{e}\,ae\,x^2+\left(\int \frac{acos(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right) b\,d^2e+\left(\int \frac{}{\sqrt{e\,x^2+}}\right.}{de\left(e\,x^2+d\right)}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*a*d + sqrt(e)*a*e*x**2 + int(acos(c*x)/(
sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d**2*e + int(acos(c*x)/
(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2*x**2)/(d*e*(d +
 e*x**2))
```

3.104 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d\,(d+ex^2)^{3/2}}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}-\frac{2b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

output
```
-1/3*b*c*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)+1/3*x*(a+b*arccos(
c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccos(c*x))/d^2/(e*x^2+d)^(1/2)-2/3*b*
arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{3d\,(c^2d+e)\,\sqrt{d+ex^2}} + \sqrt{d+ex^2}\left(\frac{ax}{3d\,(d+ex^2)^2}+\frac{2ax}{3d^2\,(d+ex^2)}\right) + \frac{bcx^2\sqrt{\frac{d+ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{3d^2\sqrt{d+ex^2}} + \frac{bx(3d+2ex^2)\arccos(cx)}{3d^2\,(d+ex^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(b*c*Sqrt[1 - c^2*x^2])/(d*(c^2*d + e)*Sqrt[d + e*x^2]) + Sqrt[d + e*
x^2]*((a*x)/(3*d*(d + e*x^2)^2) + (2*a*x)/(3*d^2*(d + e*x^2))) + (b*c*x^2*
Sqrt[(d + e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(3*d^
2*Sqrt[d + e*x^2]) + (b*x*(3*d + 2*e*x^2)*ArcCos[c*x])/(3*d^2*(d + e*x^2)^
(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5171, 27, 435, 87, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 5171$$

$$bc\int \frac{x\left(2ex^2+3d\right)}{3d^2\sqrt{1-c^2x^2}\,(ex^2+d)^{3/2}}dx + \frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\arccos(cx))}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\int \frac{x(2ex^2+3d)}{\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bc\int \frac{2ex^2+3d}{\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx^2}{6d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{bc\left(2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2d\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{bc\left(4\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}-\frac{2d\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{bc\left(-\frac{4\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{2d\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6d^2}$$

input

```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^(5/2),x]
```

output

```
(x*(a + b*ArcCos[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCos[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) + (b*c*((-2*d*Sqrt[1 - c^2*x^2])/((c^2*d + e)*Sq
rt[d + e*x^2]) - (4*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])
])/(c*Sqrt[e])))/(6*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [F]

$$\int \frac{a+b\arccos\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(122) = 244.

Time = 0.19 (sec) , antiderivative size = 686, normalized size of antiderivative = 4.70

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-e}\log\left(8\,c^4e^2x^4+c\right.}{}\right.$$

$$-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{e}\arctan\left(\frac{(2\,c^2ex^2+c^2d-e)\sqrt{-c^2x^2+1}\sqrt{ex^2+d}\sqrt{e}}{2\,(c^3e^2x^4-cde+(c^3de-ce^2)x^2)}\right)-(2}{3\,(c^2d^5e+d^4}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d
*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(-c^2*x^2 + 1)*sqrt(
e*x^2 + d)*sqrt(-e) + e^2) - 2*(2*(a*c^2*d*e^2 + a*e^3)*x^3 + 3*(a*c^2*d^2
*e + a*d*e^2)*x + (2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^2*d^2*e + b*d*e^2)
*x)*arccos(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(-c^2*x^2 + 1))*sqrt(e*x
^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e
^2 + d^3*e^3)*x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d
- e)*sqrt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e)/(c^3*e^2*x^4 - c*d*e + (c^
3*d*e - c*e^2)*x^2)) - (2*(a*c^2*d*e^2 + a*e^3)*x^3 + 3*(a*c^2*d^2*e + a*d
*e^2)*x + (2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^2*d^2*e + b*d*e^2)*x)*arcc
os(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(-c^2*x^2 + 1))*sqrt(e*x^2 + d))
/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3
*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\,(cx)}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\arccos\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sq
rt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,adex + 2\sqrt{e\,x^2+d}\,a\,e^2x^3 - 2\sqrt{e}\,a\,d^2 - 4\sqrt{e}\,ade\,x^2 - 2\sqrt{e}\,a\,e^2x^4 + 3\Big(}{}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - 2*sqrt(e)*a
*d**2 - 4*sqrt(e)*a*d*e*x**2 - 2*sqrt(e)*a*e**2*x**4 + 3*int(acos(c*x)/(sq
rt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*
x**4),x)*b*d**4*e + 6*int(acos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*
x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2*x**2 + 3*int(a
cos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e
*x**2)*e**2*x**4),x)*b*d**2*e**3*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 + e**2
*x**4))
```

3.105 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 226

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{15d(c^2d+e)(d+ex^2)^{3/2}} - \frac{2bc(3c^2d+2e)\sqrt{1-c^2x^2}}{15d^2(c^2d+e)^2\sqrt{d+ex^2}} + \frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}} + \frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}} + \frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}} - \frac{8b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

output
```
-1/15*b*c*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(3/2)-2/15*b*c*(3*c^2*d
+2*e)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d+e)^2/(e*x^2+d)^(1/2)+1/5*x*(a+b*arccos
(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arccos(c*x))/d^2/(e*x^2+d)^(3/2)+8/15
*x*(a+b*arccos(c*x))/d^3/(e*x^2+d)^(1/2)-8/15*b*arctan(e^(1/2)*(-c^2*x^2+1
)^(1/2)/c/(e*x^2+d)^(1/2))/d^3/e^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{ax(15d^2+20dex^2+8e^2x^4) - \frac{bcd\sqrt{1-c^2x^2}(d+ex^2)(e(5d+4ex^2)+c^2d(7d+6ex^2))}{(c^2d+e)^2} + 4bcx^2(d+ \cdots}{15d \cdots}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 - c^2*x^2]*(d + e*x
^2)*(e*(5*d + 4*e*x^2) + c^2*d*(7*d + 6*e*x^2)))/(c^2*d + e)^2 + 4*b*c*x^2
*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*
x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcCos[c*x])/(15*d^3*(d +
 e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5171, 27, 7266, 1193, 27, 87, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{5171}$$

$$bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{15d^3\sqrt{1-c^2x^2}\,(ex^2+d)^{5/2}}dx + \frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}} + \frac{4x(a+b\arccos(cx))}{15d^2\,(d+ex^2)^{3/2}} + \frac{x(a+b\arccos(cx))}{5d\,(d+ex^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{\sqrt{1-c^2x^2}(ex^2+d)^{5/2}}dx}{15d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{7266}$$

$$\frac{bc\int \frac{8e^2x^4+20dex^2+15d^2}{\sqrt{1-c^2x^2}(ex^2+d)^{5/2}}dx^2}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{1193}$$

$$\frac{bc\left(-\frac{2\int -\frac{3\left(4e\left(dc^2+e\right)x^2+d\left(7dc^2+6e\right)\right)}{\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx^2}{3(c^2d+e)}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{bc\left(\frac{2\int \frac{4e\left(dc^2+e\right)x^2+d\left(7dc^2+6e\right)}{\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx^2}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{87}$$

$$\frac{bc\left(\frac{2\left(4(c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2d\sqrt{1-c^2x^2}\left(3c^2d+2e\right)}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{66}$$

$$\frac{bc\left(\frac{2\left(8(c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}-\frac{2d\sqrt{1-c^2x^2}\left(3c^2d+2e\right)}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}+\frac{bc\left(\frac{2\left(-\frac{8\left(c^2d+e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{2d\sqrt{1-c^2x^2}\left(3c^2d+2e\right)}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcCos[c*x])
)/(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcCos[c*x]))/(15*d^3*Sqrt[d +
e*x^2]) + (b*c*((-2*d^2*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(d + e*x^2)^(3/2))
 + (2*((-2*d*(3*c^2*d + 2*e)*Sqrt[1 - c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^
2]) - (8*(c^2*d + e)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2]
)])/(c*Sqrt[e])))/(c^2*d + e)))/(30*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int \frac{a + b \arccos(cx)}{(e\,x^2 + d)^{\frac{7}{2}}} dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(192) = 384$.

Time = 0.29 (sec) , antiderivative size = 1324, normalized size of antiderivative = 5.86

$$\int \frac{a + b \arccos(cx)}{(d + ex^2)^{7/2}} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output
```
[-1/15*(2*(b*c^4*d^5 + 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*
e^5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 +
 3*(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*log(8*c^4*e^2
*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 +
c^3*d - c*e)*sqrt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e) + e^2) - (8*(a*c^
4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 + 2*a*c^2*d^2*e
^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e + 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x + (8*
(b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 + 2*b*c^2*
d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)
*arccos(c*x) - (7*b*c^3*d^4*e + 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 + 2*b*c
*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 + 9*b*c*d^2*e^3)*x^2)*sqrt(-c^2*x^2 + 1))*
sqrt(e*x^2 + d))/(c^4*d^8*e + 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 + 2*c
^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 + 2*c^2*d^5*e^4 + d^4*e^5)*x^4
+ 3*(c^4*d^7*e^2 + 2*c^2*d^6*e^3 + d^5*e^4)*x^2), -1/15*(4*(b*c^4*d^5 + 2*
b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*d^3*e^2 + 3*
(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*e + 2*b*c^2
*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sq
rt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e -
 c*e^2)*x^2)) - (8*(a*c^4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4
*d^3*e^2 + 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e + 2*a*c^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \int \frac{b\arccos(cx)+a}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*
x^2 + d)^(5/2)*d)) + b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)/((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(7/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{15\sqrt{e\,x^2+d}\,a\,d^2ex+20\sqrt{e\,x^2+d}\,ad\,e^2x^3+8\sqrt{e\,x^2+d}\,a\,e^3x^5-8\sqrt{e}\,a\,d^3-24\sqrt{}}{}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(7/2),x)
```

output
```
(15*sqrt(d + e*x**2)*a*d**2*e*x + 20*sqrt(d + e*x**2)*a*d*e**2*x**3 + 8*sq
rt(d + e*x**2)*a*e**3*x**5 - 8*sqrt(e)*a*d**3 - 24*sqrt(e)*a*d**2*e*x**2 -
 24*sqrt(e)*a*d*e**2*x**4 - 8*sqrt(e)*a*e**3*x**6 + 15*int(acos(c*x)/(sqrt
(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d + e*x**2)*d*
e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**6*e + 45*int(acos(c*x)/(sq
rt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d + e*x**2)*
d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**5*e**2*x**2 + 45*int(aco
s(c*x)/(sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d
+ e*x**2)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**4*e**3*x**4 +
15*int(acos(c*x)/(sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 +
 3*sqrt(d + e*x**2)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**3*e*
*4*x**6)/(15*d**3*e*(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6))
```

3.106 $\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx=\operatorname{Int}\left(\sqrt{d+ex^2}(a+b\arccos(cx))^2,x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{e\,x^2+d}\,(a+b\arccos\,(cx))^2\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \int \sqrt{ex^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 9.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \int (a+b\operatorname{acos}{(cx)})^2\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral((a + b*acos(c*x))**2*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{ex^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,a^2ex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a^2d+4\left(\int\sqrt{e\,x^2+d}\,acos(cx)\,dx\right)abe+2\left(\int\sqrt{e\,x^2+d}\,acos(cx)^2\right.}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d + e*x**2)*a**2*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a**2*d + 4*int(sqrt(d + e*x**2)*acos(c*x),x)*a*b*e + 2*int(sqrt(d +
 e*x**2)*acos(c*x)**2,x)*b**2*e)/(2*e)
```

3.107 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 8.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a^2+2\left(\int\frac{acos(cx)}{\sqrt{e\,x^2+d}}dx\right)abe+\left(\int\frac{acos(cx)^2}{\sqrt{e\,x^2+d}}dx\right)b^2e}{e}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2 + 2*int(acos(c*x
)/sqrt(d + e*x**2),x)*a*b*e + int(acos(c*x)**2/sqrt(d + e*x**2),x)*b**2*e)
/e
```

3.108 $\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(e*x^2 + d)/(e^
2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c^2*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 222, normalized size of antiderivative = 10.09

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a^2ex+\sqrt{e}\,a^2d+\sqrt{e}\,a^2e\,x^2+2\left(\int \frac{acos(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)ab\,d^2e+2}{}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a**2*e*x + sqrt(e)*a**2*d + sqrt(e)*a**2*e*x**2 + 2*int(
acos(c*x)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a*b*d**2*e + 2
*int(acos(c*x)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a*b*d*e**
2*x**2 + int(acos(c*x)**2/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x
)*b**2*d**2*e + int(acos(c*x)**2/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*
x**2),x)*b**2*d*e**2*x**2)/(d*e*(d + e*x**2))
```

3.109 $\int \frac{(a+b \arccos(cx))^2}{(d+ex^2)^{5/2}}\, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\, dx = \text{Int}\left(\frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}, x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 7.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\, dx = \int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\, dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5175}$$

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(e*x^2 + d)/(e^
3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 58.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(d + e*x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.91

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(
(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x))*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2
*e*x^2 + d^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\arccos(cx))^2}{(d + ex^2)^{5/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\arccos(cx))^2}{(d + ex^2)^{5/2}}\, dx = \int \frac{(a + b\,\mathrm{acos}(c\,x))^2}{(e\,x^2 + d)^{5/2}}\, dx$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 22.18

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,a^2dex+2\sqrt{e\,x^2+d}\,a^2e^2x^3-2\sqrt{e}\,a^2d^2-4\sqrt{e}\,a^2de\,x^2-2\sqrt{e}\,a^2e^2x}{}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a**2*d*e*x + 2*sqrt(d + e*x**2)*a**2*e**2*x**3 - 2*sqr
t(e)*a**2*d**2 - 4*sqrt(e)*a**2*d*e*x**2 - 2*sqrt(e)*a**2*e**2*x**4 + 6*in
t(acos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d
+ e*x**2)*e**2*x**4),x)*a*b*d**4*e + 12*int(acos(c*x)/(sqrt(d + e*x**2)*d*
*2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*a*b*d**3
*e**2*x**2 + 6*int(acos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d
*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*a*b*d**2*e**3*x**4 + 3*int(acos(c
*x)**2/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x
**2)*e**2*x**4),x)*b**2*d**4*e + 6*int(acos(c*x)**2/(sqrt(d + e*x**2)*d**2
 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b**2*d**3*
e**2*x**2 + 3*int(acos(c*x)**2/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)
*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b**2*d**2*e**3*x**4)/(3*d**2*e*
(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.110 $\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\arccos(cx)},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\arccos\left(cx\right)}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x)),x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 200.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{acos\,(cx)\,b+a}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

3.111 $\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \ 5175$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{e\,x^2+d}\;(a+b\arccos(cx))}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\operatorname{acos}{(cx)})\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\ \sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,acos\,(cx)\,b+\sqrt{e\,x^2+d}\,a}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)*b + sqrt(d + e*x**2)*a),x)
```

3.112 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\arccos(cx))} \, dx$$

$$\Big\downarrow \; 5175$$

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}} (a+b\arccos(cx))}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2
*b*d*e*x^2 + b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,acos\,(cx)\,bd+\sqrt{e\,x^2+d}\,acos\,(cx)\,be\,x^2+\sqrt{e\,x^2+d}\,a}$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)*b*d + sqrt(d + e*x**2)*acos(c*x)*b*e*x**
2 + sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2),x)
```

3.113 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}(a+b\arccos(cx))}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.95

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^
3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 7.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,acos\,(cx)\,b\,d^2+2\sqrt{e\,x^2+d}\,acos\,(cx)\,bde\,x^2+\sqrt{e\,x^2+}}$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)*b*d**2 + 2*sqrt(d + e*x**2)*acos(c*x)*b*
d*e*x**2 + sqrt(d + e*x**2)*acos(c*x)*b*e**2*x**4 + sqrt(d + e*x**2)*a*d**
2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d + e*x**2)*a*e**2*x**4),x)
```

3.114 $\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 6.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2} \, dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{d+ex^2}}{(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 237, normalized size of antiderivative = 10.77

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate((2*
c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a
*b*c^3*e*x^4 - a*b*c*d + (a*b*c^3*d - a*b*c*e)*x^2 + (b^2*c^3*e*x^4 - b^2*
c*d + (b^2*c^3*d - b^2*c*e)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)), x) - sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1))/(b^2*c*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{e\,x^2+d}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{e\,x^2+d}}{\left(acos\left(cx\right)b+a\right)^2}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

3.115 $\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \; 5175$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{e\,x^2+d}\,\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccos(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx))^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 365, normalized size of antiderivative = 16.59

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((a*b*c^3*d^2 + a*b*c*d*e + (a*b*c^3*d*e + a*b*c*e^2)*x^2 + (b^2*c^3*d^2
+ b^2*c*d*e + (b^2*c^3*d*e + b^2*c*e^2)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x))*integrate(sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a
*b*c^3*e^2*x^6 - a*b*c*d^2 + (2*a*b*c^3*d*e - a*b*c*e^2)*x^4 + (a*b*c^3*d^
2 - 2*a*b*c*d*e)*x^2 + (b^2*c^3*e^2*x^6 - b^2*c*d^2 + (2*b^2*c^3*d*e - b^2
*c*e^2)*x^4 + (b^2*c^3*d^2 - 2*b^2*c*d*e)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)), x) - sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b*
c*e*x^2 + a*b*c*d + (b^2*c*e*x^2 + b^2*c*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\left(c\,x\right))^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,\left(a\cos\left(cx\right)b+a\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

3.116 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 26.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2
*x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e
*x^2 + a*b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^2(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 3.28 (sec) , antiderivative size = 456, normalized size of antiderivative = 20.73

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e^2*x^4 + 2*a*b*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 + 2*b^2*c*d
*e*x^2 + b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(
(2*c^2*e*x^3 - (c^2*d + 3*e)*x)*sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x +
1)/(a*b*c^3*e^3*x^8 + (3*a*b*c^3*d*e^2 - a*b*c*e^3)*x^6 - a*b*c*d^3 + 3*(a
*b*c^3*d^2*e - a*b*c*d*e^2)*x^4 + (a*b*c^3*d^3 - 3*a*b*c*d^2*e)*x^2 + (b^2
*c^3*e^3*x^8 + (3*b^2*c^3*d*e^2 - b^2*c*e^3)*x^6 - b^2*c*d^3 + 3*(b^2*c^3*
d^2*e - b^2*c*d*e^2)*x^4 + (b^2*c^3*d^3 - 3*b^2*c*d^2*e)*x^2)*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-
c*x + 1))/(a*b*c*e^2*x^4 + 2*a*b*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 +
2*b^2*c*d*e*x^2 + b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}}\,(acos\,(cx)\,b+a)^2}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*acos(c*x))^2,x)
```

3.117 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 53.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}(a+b\arccos(cx))^2}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 6.77

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2
+ a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*ar
ccos(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3
)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 26.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^2\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 4.44 (sec) , antiderivative size = 578, normalized size of antiderivative = 26.27

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e^3*x^6 + 3*a*b*c*d*e^2*x^4 + 3*a*b*c*d^2*e*x^2 + a*b*c*d^3 + (b^2
*c*e^3*x^6 + 3*b^2*c*d*e^2*x^4 + 3*b^2*c*d^2*e*x^2 + b^2*c*d^3)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate((4*c^2*e*x^3 - (c^2*d + 5*e)*x
)*sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*e^4*x^10 + (4*a*b*
c^3*d*e^3 - a*b*c*e^4)*x^8 - a*b*c*d^4 + 2*(3*a*b*c^3*d^2*e^2 - 2*a*b*c*d*
e^3)*x^6 + 2*(2*a*b*c^3*d^3*e - 3*a*b*c*d^2*e^2)*x^4 + (a*b*c^3*d^4 - 4*a*
b*c*d^3*e)*x^2 + (b^2*c^3*e^4*x^10 + (4*b^2*c^3*d*e^3 - b^2*c*e^4)*x^8 - b
^2*c*d^4 + 2*(3*b^2*c^3*d^2*e^2 - 2*b^2*c*d*e^3)*x^6 + 2*(2*b^2*c^3*d^3*e
- 3*b^2*c*d^2*e^2)*x^4 + (b^2*c^3*d^4 - 4*b^2*c*d^3*e)*x^2)*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*
x + 1))/(a*b*c*e^3*x^6 + 3*a*b*c*d*e^2*x^4 + 3*a*b*c*d^2*e*x^2 + a*b*c*d^3
 + (b^2*c*e^3*x^6 + 3*b^2*c*d*e^2*x^4 + 3*b^2*c*d^2*e*x^2 + b^2*c*d^3)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 8.50

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx=\int \frac{}{\sqrt{e\,x^2+d}\,acos\,(cx)^2\,b^2d^2+2\sqrt{e\,x^2+d}\,acos\,(cx)^2\,b^2de\,x^2+\sqrt{e}}$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)**2*b**2*d**2 + 2*sqrt(d + e*x**2)*acos(c
*x)**2*b**2*d*e*x**2 + sqrt(d + e*x**2)*acos(c*x)**2*b**2*e**2*x**4 + 2*sq
rt(d + e*x**2)*acos(c*x)*a*b*d**2 + 4*sqrt(d + e*x**2)*acos(c*x)*a*b*d*e*x
**2 + 2*sqrt(d + e*x**2)*acos(c*x)*a*b*e**2*x**4 + sqrt(d + e*x**2)*a**2*d
**2 + 2*sqrt(d + e*x**2)*a**2*d*e*x**2 + sqrt(d + e*x**2)*a**2*e**2*x**4),
x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file